Université de Montréal
& Université de Technologie de Troyes (co-tutelle)

Approches générales de résolution pour les problèmes multi-attributs de tournées de véhicules et confection d'horaires

par
Thibaut VIDAL

Département d'informatique et de recherche opérationnelle
Faculté des arts et des sciences

Thèse présentée
en vue de l'obtention du grade de
Philosophiæ Doctor (Ph.D.) en Informatique

Mars, 2013

Cette thèse a été évaluée par un jury composé des personnes suivantes:

Teodor Gabriel CRAINIC,	Directeur de recherche
Michel GENDREAU,	Co-directeur
Christian PRINS,	Co-directeur
Daniele VIGO,	Rapporteur
Richard EGLESE,	Rapporteur
Jean-Yves POTVIN,	Examinateur
Caroline PRODHON,	Examinateur
Marc SEVAUX,	Examinateur.

Université de Montréal
& Université de Technologie de Troyes (co-tutelle)

Cette thèse intitulée:

Approches générales de résolution pour les problèmes multi-attributs de tournées de véhicules et confection d'horaires

présentée par:

Thibaut VIDAL

a été évaluée par un jury composé des personnes suivantes:

Teodor Gabriel CRAINIC,	Directeur de recherche
Michel GENDREAU,	Co-directeur
Christian PRINS,	Co-directeur
Daniele VIGO,	Rapporteur
Richard EGLESE,	Rapporteur
Jean-Yves POTVIN,	Examinateur
Caroline PRODHON,	Examinateur
Marc SEVAUX,	Examinateur.

Thèse acceptée le: 10 Décembre 2012

RÉSUMÉ

Le problème de tournées de véhicules (VRP) implique de planifier les itinéraires d'une flotte de véhicules afin de desservir un ensemble de clients à moindre coût. Ce problème d'optimisation combinatoire NP-difficile apparait dans de nombreux domaines d'application, notamment en logistique, télécommunications, robotique ou gestion de crise dans des contextes militaires et humanitaires. Ces applications amènent différents contraintes, objectifs et décisions supplémentaires ; des "attributs" qui viennent compléter les formulations classiques du problème. Les nombreux VRP Multi-Attributs (MAVRP) qui s'ensuivent sont le support d'une littérature considérable, mais qui manque de méthodes généralistes capables de traiter efficacement un éventail significatif de variantes. Par ailleurs, la résolution de problèmes *riches*, combinant de nombreux attributs, pose d'importantes difficultés méthodologiques.

Cette thèse contribue à relever ces défis par le biais d'analyses structurelles des problèmes, de développements de stratégies métaheuristiques, et de méthodes unifiées. Nous présentons tout d'abord une étude transversale des concepts à succès de 64 méta-heuristiques pour 15 MAVRP afin d'en cerner les "stratégies gagnantes". Puis, nous analysons les problèmes et algorithmes d'ajustement d'horaires en présence d'une séquence de tâches fixée, appelés problèmes de *timing*. Ces méthodes, développées indépendamment dans différents domaines de recherche liés au transport, ordonnancement, allocation de ressource et même régression isotonique, sont unifiés dans une revue multidisciplinaire.

Un algorithme génétique hybride efficace est ensuite proposé, combinant l'exploration large des méthodes évolutionnaires, les capacités d'amélioration agressive des métaheuristiques à voisinage, et une évaluation bi-critère des solutions considérant coût et contribution à la diversité de la population. Les meilleures solutions connues de la littérature sont retrouvées ou améliorées pour le VRP classique ainsi que des variantes avec multiples dépôts et périodes. La méthode est étendue aux VRP avec contraintes de fenêtres de temps, durée de route, et horaires de conducteurs. Ces applications mettent en jeu de nouvelles méthodes d'évaluation efficaces de contraintes temporelles relaxées, des phases de décomposition, et des recherches arborescentes pour l'insertion des pauses des conducteurs. Un algorithme de gestion implicite du placement des dépôts au cours de recherches locales, par programmation dynamique, est aussi proposé. Des études expérimentales approfondies démontrent la contribution notable des nouvelles stratégies au sein de plusieurs cadres méta-heuristiques.

Afin de traiter la variété des attributs, un cadre de résolution heuristique modulaire est présenté ainsi qu'un algorithme génétique hybride unifié (UHGS). Les attributs sont gérés par des composants élémentaires adaptatifs. Des expérimentations sur 26 variantes du VRP et 39 groupes d'instances démontrent la performance remarquable de UHGS qui, avec une unique implémentation et paramétrage, égalise ou surpasse les nombreux algorithmes dédiés, issus de plus de 180 articles, révélant ainsi que la généralité ne s'obtient pas forcément aux dépends de l'efficacité pour cette classe de problèmes. Enfin, pour traiter les problèmes riches, UHGS est étendu au sein d'un cadre de résolution parallèle coopératif à base de décomposition, d'intégration de solutions partielles, et de recherche guidée.

L'ensemble de ces travaux permet de jeter un nouveau regard sur les MAVRP et les problèmes de timing, leur résolution par des méthodes méta-heuristiques, ainsi que les méthodes généralistes pour l'optimisation combinatoire.

Mots clefs : Recherche opérationnelle, Optimisation Combinatoire, Logistique, Transport, Problème de Tournées de Véhicules, Ordonnancement, Heuristique, Métaheuristique, Algorithmes Génétiques, Algorithmes Parallèles

ABSTRACT

The Vehicle Routing Problem (VRP) involves designing least cost delivery routes to service a geographically-dispersed set of customers while taking into account vehicle-capacity constraints. This NP-hard combinatorial optimization problem is linked with multiple applications in logistics, telecommunications, robotics, crisis management in military and humanitarian frameworks, among others. Practical routing applications are usually quite distinct from the academic cases, encompassing additional sets of specific constraints, objectives and decisions which breed further new problem variants. The resulting "Multi-Attribute" Vehicle Routing Problems (MAVRP) are the support of a vast literature which, however, lacks unified methods capable of addressing multiple MAVRP. In addition, some *rich* VRPs, i.e. those that involve several attributes, may be difficult to address because of the wide array of combined and possibly antagonistic decisions they require.

This thesis contributes to address these challenges by means of problem structure analysis, new metaheuristics and unified method developments. The "winning strategies" of 64 state-of-the-art algorithms for 15 different MAVRP are scrutinized in a unifying review. Another analysis is targeted on "timing" problems and algorithms for adjusting the execution dates of a given sequence of tasks. Such methods, independently studied in different research domains related to routing, scheduling, resource allocation, and even isotonic regression are here surveyed in a multidisciplinary review.

A Hybrid Genetic Search with Advanced Diversity Control (HGSADC) is then introduced, which combines the exploration breadth of population-based evolutionary search, the aggressive-improvement capabilities of neighborhood-based metaheuristics, and a bi-criteria evaluation of solutions based on cost and diversity measures. Results of remarkable quality are achieved on classic benchmark instances of the capacitated VRP, the multi-depot VRP, and the periodic VRP. Further extensions of the method to VRP variants with constraints on time windows, limited route duration, and truck drivers' statutory pauses are also proposed. New route and neighborhood evaluation procedures are introduced to manage penalized infeasible solutions w.r.t. to time-window and duration constraints. Tree-search procedures are used for drivers' rest scheduling, as well as advanced search limitation strategies, memories and decomposition phases. A dynamic programming-based neighborhood search is introduced to optimally select the depot, vehicle type, and first customer visited in the route during local searches. The notable contribution of these new methodological elements is assessed within two different metaheuristic frameworks.

To further advance general-purpose MAVRP methods, we introduce a new component-based heuristic resolution framework and a Unified Hybrid Genetic Search (UHGS), which relies on modular self-adaptive components for addressing problem specifics. Computational experiments demonstrate the groundbreaking performance of UHGS. With a single implementation, unique parameter setting and termination criterion, this algorithm matches or outperforms all current problem-tailored methods from more than 180 articles, on 26 vehicle routing variants and 39 benchmark sets. To address *rich* problems, UHGS was included in a new parallel cooperative solution framework called "Integrative Cooperative Search (ICS)", based on problem decompositions, partial solutions integration, and global search guidance.

This compendium of results provides a novel view on a wide range of MAVRP and timing problems, on efficient heuristic searches, and on general-purpose solution methods for combinatorial optimization problems.

Keywords : Operations Research, Combinatorial Optimization, Logistics, Transportation, Vehicle Routing Problem, Scheduling, Heuristic, Metaheuristic, Genetic Algorithms, Parallel Algorithms

TABLE DES MATIÈRES

LISTE DES TABLEAUX

LISTE DES FIGURES

LISTE DES ANNEXES

LISTE DES SIGLES

Table 1: Acronyms

AAR	ASSIGN Attribute Resource
ABHC	Attribute-Based Hill Climber
ACO	Ant Colony Optimization
AIX	Assignment and Insertion Crossover
ALNS	Adaptive Large Neighbourhood Search
AM	Adaptive Memory
ANOVA	ANalysis Of VAriance
ASSIGN	Assignment Attribute
AUS	Australia
Avg	Average
BFM	Basic Fatigue Management
BKS	Best Known Solution
CAN	Canada
CCVRP	Cumulative Vehicle Routing Problem
CMA-ES	Evolutionary Strategy with Covariance Matrix Adaptation
CNV	Cumulated Number of Vehicles
CPU	Central Processing Unit
CSG	Complete Solver Group
CTD	Cumulated Total Distance
CVRP	Capacitated Vehicle Routing Problem
DUR	Duration
EA	Evolutionary Algorithm
ELS	Evolutionary Local Search
EU	Europe
EVAL	Route-Evaluation Attribute
FIFO	First-In First-Out
FLEX	Flexible Travel Time
GA	Genetic Algorithm
GAP	Generalized Assignment Problem
GLS	Guided Local Search
GPU	Graphic Processing Unit
GRASP	Greedy Randomized Adaptive Search Procedure
GSC	Global Search Coordinator
HGA	Hybrid Genetic Algorithm
HGS	Hybrid Genetic Search
HGSADC	Hybrid Genetic Search with Advanced Diversity Control
ICS	Integrative Cooperative Search
ILP	Integer Linear Programming
ILS	Iterated Local Search
ILS-SP	Iterated Local Search and Set Partitioning
IRC	Isotonic Regression Problem
LDVRP	Vehicle Routing Problem with Load-Dependent Cost
LIFO	Last-In First-Out
LISP	Longest Increasing Subsequence Problem

Table 2: Acronyms (continued)

LNS	Large Neighbourhood Search
LS	Local Search
LSC	Local Search Coordinator
MAVRP	Multi-Attribute Vehicle Routing Problem
MDPVRP	Multi-Depot Periodic Vehicle Routing Problem
MDPVRPTW	Multi-Depot Periodic Vehicle Routing Problem with Time Windows
MDVFMP	Multi-Depot Vehicle Fleet Mix Problem
MDVRP	Multi-Depot Vehicle Routing Problem
MLS	Minimum Lower Set
MPM	Metra-Potential Method
MTVRP	Vehicle Routing Problem with Multiple Trips
MTW	Multiple Time Windows
NP	Non-Deterministic Polynomial time
OVRP	Open Vehicle Routing Problem
PATSO	Port Arrival Dates and Speed Optimization
PAV	Pool Adjacent Violators algorithm
PDP	Pickup and Delivery Problem
PI	Pattern Improvement
PIX	Periodic Crossover with Insertions
PR	Path Relinking
PSG	Partial Solver Group
PSO	Particle Swarm Optimization
PVRP	Periodic Vehicle Routing Problem
PVRPTW	Periodic Vehicle Routing Problem with Time Windows
RI	Route Improvement
R-to-R	Record-To-Record
SA	Simulated Annealing
SDVRP	Site-Dependent Vehicle Routing Problem
SDVRPTW	Site-Dependent Vehicle Routing Problem with Time Windows
SEQ	Sequence Attribute
SS	Scatter Search
Std.	Standard
TDVRPTW	Time-Dependent Vehicle Routing Problem with Time Windows
TDVRP	Time-Dependent Vehicle Routing Problem
TS	Tabu Search
TSP	Traveling Salesman Problem
TW	Time Window
UHGS	Unified Hybrid Genetic Search
ULS	Unified Local Search
US	United States of America
UTS	Unified Tabu Search
VFMP	Vehicle Fleet Mix Problem
VFMP-FV	Vehicle Fleet Mix Problem with Fixed and Variable costs
VFMPTW	Vehicle Fleet Mix Problem with Time Windows
VND	Variable Neighbourhood Descent
VNS	Variable Neighbourhood Search
VRP	Vehicle Routing Problem
VRPB	Vehicle Routing Problem with Backhauls
VRPPD	Vehicle Routing Problem with Pickups and Deliveries
VRPSD	Vehicle Routing Problem with Split Deliveries
VRPSTW	Vehicle Routing Problem with Soft Time Windows
VRPTW	Vehicle Routing Problem with Time Windows
VRPTWLB	Vehicle Routing Problem with Time Windows and Lunch Break
VRTDSP	Vehicle Routing and Truck Driver Scheduling Problem

REMERCIEMENTS

Cette thèse est le résultat d'une combinaison de travail, de confiance, de rencontres imprévues, d'enthousiasme, et de circonstances. Tout aurait été totalement différent sans l'aide considérable de nombreux proches, amis et collègues. Je tiens à remercier, en particulier,

- Mes directeurs, au nombre de trois : Teodor Gabriel Crainic, Michel Gendreau et Christian Prins. Je vous remercie pour m'avoir donné la chance de faire cette thèse, pour votre confiance sans faille, votre soutien, votre engagement, et tout l'enseignement que vous m'avez transmis.
- Le Conseil de Recherches en Sciences Naturelles et en Génie du Canada (CRSNG), le Fonds Québécois de la Recherche sur la Nature et les Technologies (FQRNT) et le conseil Régional de la région Champagne-Ardenne pour le soutien financier.
- Le CIRRELT et le LOSI, des environnements de recherche accueillants et dynamiques, ainsi que le RQCHP et Calcul Canada, qui m'ont mis à disposition des ressources informatiques providentielles.
- Richard Eglese, Jean-Yves Potvin, Caroline Prodhon, Marc Sevaux et Daniele Vigo, membres du jury. C'était un honneur de vous présenter cette thèse. Merci pour votre participation et votre temps, ainsi que les commentaires, suggestions et corrections détaillées.
- Julien Moncel et Denis Naddef : c'est grâce à une discussion providentielle, il y a quelques années à Grenoble, le jour de la clôture des inscriptions pour le master recherche, que j'ai finalement emprunté une carrière académique.
- Amis doctorants des deux côtés de l'océan et d'ailleurs : une dédicace spéciale à Elivelton, Karine, Sixtine, Phuong, Sanjay à Montréal, Atefeh, Julien et Slim à Troyes, Renaud de Nantes, Anand, Puca, Vinicius, Marcos de Niteroi, Diego à Gardanne, et bien d'autres encore.
- L'équipe administrative et technique : une dédicace spéciale à Lucie, Nath, Johanne, Josée, Véro et Marie-Jo pour avoir relevé les défis administratifs avec autant de bonne humeur. Daniel, Luc et Pierre, à qui le système informatique du CIRRELT obéit au doigt et à l'oeil, capables de réparer les cataclysmes que mes tâches en panne ont parfois causé à la grille de calcul.
- De nombreux amis et collègues, en particulier, mais sans se limiter à, Nadia Lahrichi, Walter Rei, Asvin Goel, Jean-François Cordeau, Gilbert Laporte, Jean-Yves Potvin, Patrice Marcotte, Michel Toulouse, Caroline Prodhon, Nacima Labadie, Christophe et Andrea Duhamel, Murat Afsar, Sandra Ngueveu, Nabil Absi, Dominique Feillet et Frédéric Semet. Votre enthousiasme m'a amené à avancer avec encore plus d'entrain.
- Ma famille pour ce constant soutien tout au long de ces trois années, ma mère qui a surement trouvé un moyen pour me transformer en futur amateur de science dès les premières années. Enfin Sophie, venue vivre à Montréal puis à Troyes, qui m'a soutenu jour après jour, malgré le froid, les mauvais jours et le travail considérable de ces trois années. Merci du fond du coeur.

AVANT-PROPOS

La recherche d'un plus court chemin, la confection de tournées et d'horaires, le placement d'objets 2D (puzzles) sont autant de problèmes que le cerveau humain résout jour après jour de manière suffisamment efficace pour ses besoins (Vickers et al. 2004, Dry et al. 2006). Ce sont cependant des problèmes d'optimisation combinatoire, pour lesquels le nombre de solutions croît de manière extrêmement rapide, exponentielle au moins, avec la taille des données : nombre de destinations, activités ou objets. Le nombre total de séquences possibles pour visiter 80 lieux est 80! ($\approx 7.10^{118}$), un nombre beaucoup plus grand que le nombre d'atomes ($\approx 10^{80}$) estimé dans l'univers. Ainsi, le dicton populaire "à chaque problème une solution", passe sous silence la difficulté à localiser une bonne solution (ainsi d'ailleurs que le caractère indécidable de certains problèmes).

Le développement rapide de la théorie de la complexité algorithmique a conduit à jeter depuis les années 1970 un nouveau regard sur un grand nombre de problèmes d'optimisation combinatoire qualifiés de "Nondeterministic Polynomial time" (NP) (Garey and Johnson 1979). De manière simpliste, la caractéristique de ces problèmes est qu'il est relativement aisé de vérifier l'optimalité d'une solution, mais excessivement difficile de trouver la solution optimale. L'existence ou la non existence d'algorithmes de résolution efficaces "polynomiaux" (pour lesquels le nombre d'opérations élémentaires de calcul est une fonction polynomiale de la taille des données d'entrée) pour les problèmes NP constitue une question célèbre, toujours non résolue à l'heure actuelle, mais pour laquelle une réponse positive est rarement envisagée. La conjecture précédente ne signifie cependant pas que tout effort de résolution est vain, mais plutôt que les méthodes exactes actuelles pour ces problèmes atteignent inévitablement leurs limites sur certains jeux de données de grande taille.

Toutefois, dans le monde industriel au sein de systèmes complexes et de grande taille, dans le domaine de la logistique, de la production manufacturière, ou des réseaux d'approvisionnement entre autres, la résolution efficace et précise de tels problèmes NP de grande taille est liée à des enjeux économiques majeurs. Le XXI$^{\text{ème}}$ siècle est ainsi un âge d'or pour la science informatique, où des enjeux considérables de recherche et d'application restent à relever afin de mieux guider la prise de décision. Cette thèse vient rajouter une pierre à l'édifice en proposant de nouvelles méthodologies de résolution approchée (heuristiques et méta-heuristiques) efficaces pour les problèmes de tournées de véhicules, une classe importante de problèmes d'optimisation combinatoire. Elle s'inscrit dans cette dynamique générale de recherche qui touche à la fois à des questions méthodologiques fondamentales et à des applications pratiques indispensables. Améliorer la gestion de ces problèmes permet certes d'augmenter la compétitivité économique, mais aussi de progresser vers des systèmes de transport plus intelligents et écologiques en adéquation avec les besoins de la société.

INTRODUCTION

0.1 Contexte et défis

Les problèmes d'optimisation combinatoire visent à trouver une meilleure solution au sein d'un ensemble fini et discret, mais généralement très large, d'alternatives. Les applications sont nombreuses en mathématiques appliquées, aide à la décision, informatique et bio-informatique, intelligence artificielle, sciences économiques, ainsi que de nombreuses autres disciplines. Ces problèmes sont aussi au coeur des systèmes industriels, qui dans un contexte d'intégration, de globalisation et de recherche de perfectionnement, cherchent à prendre les meilleures décisions malgré la présence de données approximatives, d'objectifs et contraintes antagonistes, et d'une multitude de solutions.

Le recherche d'une solution exacte "optimale" est souvent impossible pour des instances de problèmes de grande taille. Afin toutefois d'obtenir de bonnes solutions, de nombreuses approches heuristiques et métaheuristiques ont été proposées dans la littérature. Ces métaheuristiques sont des stratégies de recherche "globales", qui ne donnent pas de garantie de performance mais visent à guider l'exploration des solutions pour augmenter les chances de trouver une solution de qualité. Certaines exploitent des concepts de recherche simples : perturber légèrement pour trouver un meilleur arrangement, éviter de repasser sur les mêmes solutions. D'autres s'inspirent de phénomènes de notre environnement : sélection naturelle, insectes sociaux et intelligence collective, minimisation d'énergie libre et équilibre de cristallisation...

Les métaheuristiques ont permis d'aboutir à des algorithmes très efficaces pour de nombreux problèmes combinatoires classiques comme les problèmes de voyageur de commerce, de tournées de véhicules, d'affectation quadratique, de sac à dos, de couverture d'ensemble, entre autres. Malheureusement, les applications pratiques correspondent rarement à ces cas académiques, présentant généralement des contraintes ou objectifs particuliers, des *attributs* supplémentaires qui conduisent à de nouvelles variantes de problèmes. Cette multitude de nouvelles variantes amène un défi méthodologique de taille, car on ne connait pas de méthode systématique efficace pour ces problèmes, et des concepts qui apparaissaient comme prometteurs sur plusieurs attributs se révèlent parfois être inefficaces lorsque ces attributs sont combinés ensemble, ou lorsque des problèmes proches sont considérés, si bien que des développements spécialisés sont nécessaires dans la plupart des cas.

Cette thèse vise à contribuer à l'avancement des méthodes heuristiques et métaheuristiques pour les problèmes d'optimisation combinatoire multi-attributs, en portant une attention particulière sur une classe de problèmes très importante : le problème de tournées de véhicules (Vehicle Routing Problem – VRP) et ses variantes. Le VRP vise à planifier les meilleurs itinéraires d'une flotte de véhicules afin de desservir n clients dispersés géographiquement. Même en présence d'un seul véhicule, le nombre de solutions possibles, assimilable au nombre de permutations $n!$, est considérable. Une amélioration même mineure des solutions peut avoir des conséquences économiques et environnementales colossales, le secteur de la logistique et du transport représentant en France pour l'année 2007 un chiffre d'affaires de 150 milliards d'euros ainsi que 34% des émissions de dioxyde de carbone impliquées dans le phénomène de réchauffement climatique (Ministère de

l'écologie, du développement durable, des transports et du logement, France 2007, 2010). Par ailleurs, les modèles mathématiques de VRP apparaissent au-delà des domaines du transport et de la logistique, au sein de problèmes de production, de disposition d'atelier, d'organisation hospitalière, de télécommunications, de robotique, de maintenance, de réponse à des situations de crise dans des contextes militaires et humanitaires...

Une requête sur *Scopus* avec la clef "vehicle routing" sur une période de cinq ans, allant de 2007 à 2011, aboutit à 1258 publications référencées dont 648 articles de conférences et 566 articles de journaux. Cette impressionnante dynamique de recherche est en grande partie la conséquence du grand nombre de variantes émergentes. Les attributs du VRP visent à mieux gérer les spécificités des cas d'applications en considérant des objectifs variés (distance, coût, satisfaction client, impact environnemental, résilience, risque, équité...), en prenant en compte des caractéristiques fines du système (heures de livraison, contraintes de chargement, pauses et temps de travail, largeur des routes, embouteillages...), la nature des données (imprécision, agrégation, incertitude, dynamisme...) ou en intégrant plusieurs ensembles de décisions afin d'obtenir une meilleure solution globale (gestion de production, d'inventaire, localisation de noeuds de distribution...).

Les problèmes de tournées de véhicules multi-attributs (MAVRP) ainsi rencontrés amènent des défis méthodologiques majeurs, de par leur difficulté et leur très grande variété. Beaucoup de ces défis de recherche font écho à des questionnements fondamentaux en optimisation combinatoire. On vise notamment à développer de nouvelles méthodes métaheuristiques qui utilisent d'avantage la connaissance accumulée au cours de la recherche afin de mieux la guider ; exploitent la structure particulière des problèmes rencontrés (et notamment la quasi-convexité de l'espace des solutions, c.f. Reeves 1998) ; établissent un équilibre subtil entre recherche intensifiée autour de caractéristiques de solutions de haute qualité et l'exploration de caractéristiques inconnues ; utilisent des formulations de problèmes, au travers de relaxations de contraintes ou de différentes représentations de solutions, afin qu'il soit plus aisé de progresser vers de meilleures solutions ; tirent profit d'hybridations entre différentes méthodologies et de recherches parallèles simultanées, entre autres...

De plus, certains problèmes combinant plusieurs attributs et qualifiés de problèmes *riches* (Hartl et al. 2006), sont particulièrement difficiles à traiter du fait de la variété des décisions combinées, parfois antagonistes. La plupart des méthodes actuelles pour ces problèmes sont basées sur une résolution itérative de problèmes projetés considérant des sous-ensembles de décisions. Chaque décision n'est alors prise qu'en présence d'une vision très incomplète ou approximative du problème, ce qui conduit fréquemment à des solutions de mauvaise qualité. L'intégration des décisions est un enjeu de recherche majeur dans ce cadre.

Aussi, le nombre de combinaisons envisageables d'attributs, et donc de variantes de VRP, est en soi combinatoire. Étudier chaque nouveau problème de manière indépendante n'est pas une démarche scientifiquement acceptable dans ce cadre. Identifier le niveau de généralité des méthodes proposées est un challenge important, qui requiert des développements théoriques ou des expérimentations sur une vaste gamme de problèmes. Or, dans le cas des MAVRP, le nombre d'algorithmes académiques capables de résoudre plus de cinq variantes se comptent sur les doigts de la main, si bien qu'il est indispensable de mettre au point des méthodes unifiées pour traiter une gamme de problèmes et ainsi ouvrir la porte à des expérimentations à plus grande échelle.

Enfin, d'un point de vue applicatif, la nécessité de développer un algorithme spécialisé pour chaque nouvelle variante "exotique" engendre un délai de développement souvent beaucoup trop long pour les besoins industriels, limitant l'impact des nouvelles technologies en matière d'optimisation. Le développement de méthodes généralistes ou rapidement adaptables est capital pour la pratique. Cependant, le choix du niveau de généralité de la méthode est critique, car certains travaux théoriques (Wolpert 1997, Droste et al. 2002) viennent illustrer une idée générale selon laquelle aucun algorithme ne peut être meilleur sur tout problème, et que toute amélioration est le fruit de l'exploitation de connaissance particulière. A l'inverse, une méthode trop spécifique perdrait tout son intérêt applicatif. Déterminer le niveau de généralité et cibler la bonne classe de variantes est ainsi un challenge en soi, qui doit être relevé avant même de pouvoir développer une méthodologie unifiée performante.

0.2 Objectifs, démarche et contributions

L'objectif principal de cette thèse est de proposer de nouvelles métaheuristiques généralistes pour gérer efficacement la variété et les combinaisons d'attributs des problèmes de tournées de véhicules et d'optimisation combinatoire. Plusieurs développements intermédiaires sont nécessaires à cette fin, visant à mieux comprendre la structure des problèmes, introduire de nouvelles stratégies métaheuristiques efficaces, et enfin aboutir à des méthodologies généralistes. Le développement et les contributions de cette thèse sont ainsi organisés en trois axes complémentaires.

La premier axe de recherche vise ainsi à aboutir à une meilleure vision d'ensemble des attributs, problèmes, et méthodes. Deux contributions principales sont présentées dans ce cadre. Une première synthèse de la littérature répertorie les attributs des problèmes de tournées de véhicules et les méthodes de résolution heuristiques, analyse en détail les concepts des 64 meilleures méthodes pour 15 MAVRP différents, et mène à une meilleur compréhension des concepts à succès. Une seconde analyse, pluridisciplinaire, cible les sous-problèmes d'ajustement temporel des tâches (*timing*), qui apparaissent fréquemment au sein de problèmes riches de tournées de véhicules ou d'ordonnancement non régulier. Ce dernier état de l'art, unique en son genre pour cette famille de problèmes, vient mettre en relation des méthodes issues de nombreux domaines, ordonnancement de projet ou de production, optimisation de réseaux et de tournées, allocation de ressource, régression statistique, et identifie les meilleures approches pour une grande variété d'attributs.

Le deuxième axe de recherche vise à proposer de nouvelles méthodologies heuristiques efficaces sur un large éventail de problèmes de tournées de véhicules. Nous ciblons une classe particulière de métaheuristiques, les algorithmes génétiques hybrides (HGA), car ces méthodes à base de populations, croisements, et recherche locales, qui favorisent l'émergence et la diffusion de fragments de solutions élites, sont susceptibles d'être particulièrement efficaces sur des problèmes d'optimisation combinatoire "biens structurés" comme le VRP où les fragments de bonnes solutions ont une grande probabilité d'être présents dans les meilleures solutions. De ce fait, nous proposons un nouvel HGA avec contrôle adaptatif de diversité. La spécificité de cet algorithme réside dans une évaluation bi-critère des solutions, qui prend en compte à la fois leur qualité mais aussi leur contribution à la diversité. Les meilleurs résultats de la littérature sont obtenus pour des problèmes fondamentaux

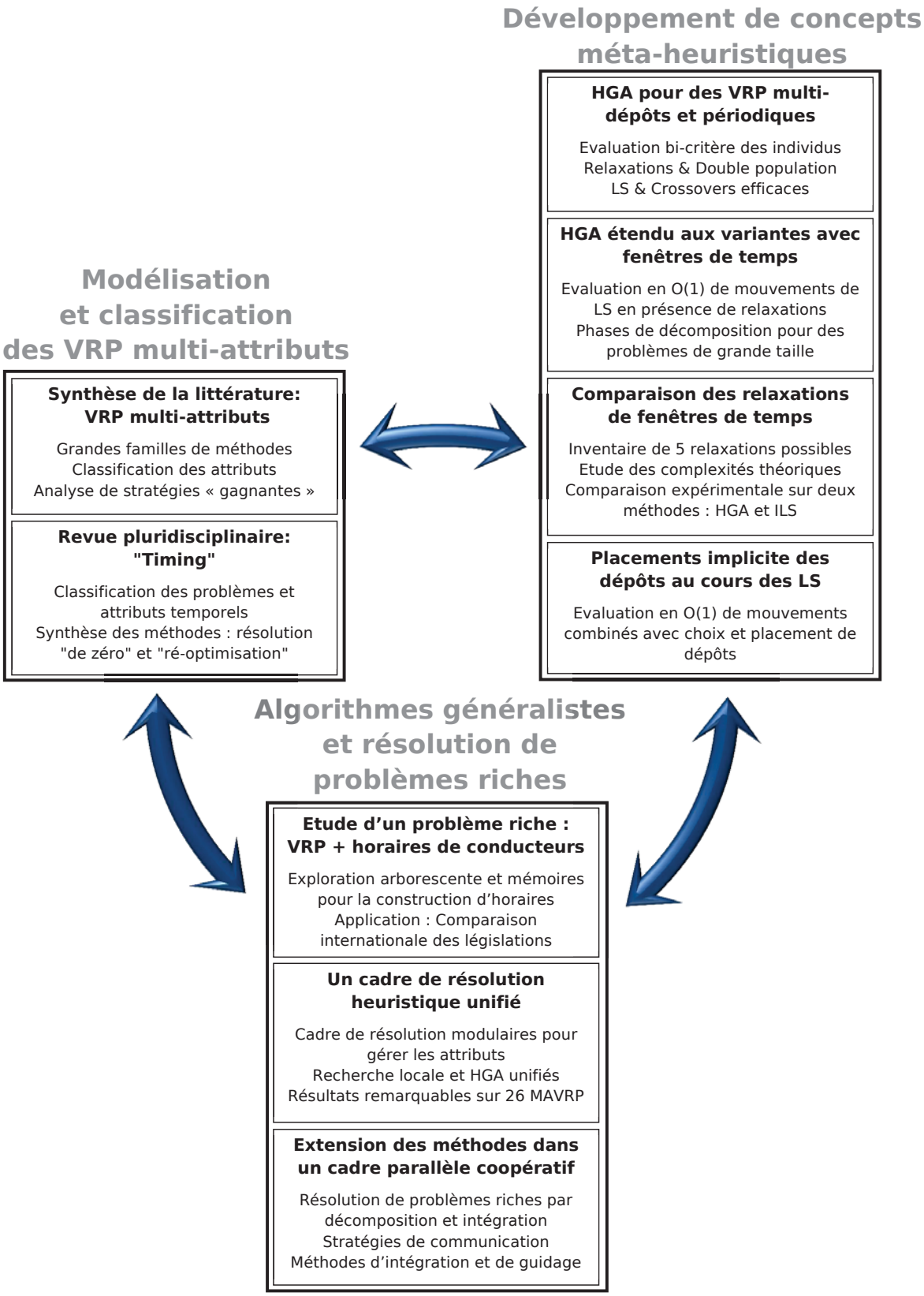


Figure 1 – Organisation des contributions. Trois axes de recherche complémentaires pour progresser vers des méthodes généralistes pour les problèmes de tournées multi-attributs

comme le VRP ainsi que ses variantes avec dépôts multiples, périodes de planification multiples, et fenêtre de temps. D'autres développements amènent à mieux tirer profit des relaxations de contraintes de fenêtres de temps, et à gérer implicitement certaines décisions, comme le placement et le choix des dépôts, par des méthodes programmation dynamique.

Le dernier axe de recherche s'appuie sur les contributions précédentes pour progresser vers des méthodes efficaces et généralistes pour les problèmes de tournées de véhicules multi-attributs. Nous résolvons tout d'abord un MAVRP particulièrement difficile, qui prend en compte les règles précises de la législation concernant les horaires et pauses des conducteurs. Cette méthodologie nous permet de qualifier l'impact des règles de la législation sur la performance économique du transport et la fatigue des conducteurs. Puis, un cadre de résolution heuristique modulaire est introduit, ainsi qu'un algorithme génétique hybride *unifié* pour les MAVRP. Avec une seule implémentation et jeu de paramètres, cet algorithme égalise ou surpasse toutes les meilleures méthodes dédiées de la littérature pour 26 variantes classiques du VRP. Enfin, un cadre de résolution parallèle coopératif à base de décomposition, d'intégration de solutions et de guidage est proposé pour résoudre efficacement des problèmes de tournées de véhicules et d'optimisation combinatoire riches.

La Figure 1 illustre de façon symbolique les liens étroits entre les axes de recherche considérés, et situe dans ce cadre les différentes contributions et chapitres de la thèse. Les développements méthodologiques de cette thèse sont systématiquement appuyés de développements algorithmiques et d'expérimentations sur des jeux de données issues de la littérature, qui permettent de comparer les méthodes proposées à de nombreux autres algorithmes de la littérature et d'examiner l'apport de différentes stratégies. Les différentes contributions, ainsi que les liens qui les unissent, sont décrites plus en détail dans la prochaine section.

0.3 Détail des contributions

Axe I : Analyse des attributs et méthodes. Une revue de littérature et analyse détaillée des heuristiques pour les VRP multi-attributs (MAVRP) est tout d'abord conduite dans le Chapitre 1. Cette revue répond au besoin de classifier et organiser les attributs relativement à leur impact sur les méthodologies de résolution. Trois grandes catégories d'attributs sont identifiées : les attributs Assign, qui impactent les choix d'affectation des routes et ressources aux clients, les attributs Seq, qui changent la structure et la nature des routes, et les attributs Eval, qui impactent l'évaluation du coût ou de la faisabilité des séquences, incluant éventuellement la résolution de sous-problèmes inhérents aux routes fixées, comme la recherche d'un placement géométrique des objets ou la détermination des horaires.

D'autre part, les concepts principaux de 64 métaheuristiques, sélectionnées pour leur remarquable performance sur 15 MAVRP classiques avec différents attributs, sont finement étudiés au cours d'une analyse transversale. Il apparait notamment dans cette revue que les meilleures méthodes heuristiques ne tiennent pas leur performance d'une unique idée "gagnante", mais sont plutôt le résultat d'un équilibre et d'une complémentarité de concepts, incluant par exemple plusieurs espaces de recherche, des procédures de modifications de solutions avec différents degrés d'impacts, différentes mémoires à plus ou moins long terme, etc... Aucune méthodologie métaheuristique ne

s'illustre comme étant une meilleure stratégie sur l'ensemble des problèmes. Toutefois, les méthodes à base de voisinage, comme les recherches tabou et variantes de recuit simulé, ont été généralement plus étudiées et exploitées dans le passé (Osman 1993, Gendreau et al. 1994, Cordeau et al. 1997), alors que le succès des méthodes évolutionnaires à base de populations est plus récent (Prins 2004, Reimann et al. 2004), conduisant à de nombreuses perspectives de recherche prometteuses. Certaines "stratégies gagnantes" identifiées seront par la suite exploitées et perfectionnées dans cette thèse lors du développement spécifique d'algorithmes spécialisés ou généralistes pour les VRPs.

Enfin, comme le souligne cette revue de littérature, de nombreux attributs de type Eval, impactant seulement l'évaluation des séquences, sont liés à des considérations temporelles sur les horaires de service. La résolution efficace des sous-problèmes de détermination d'horaires est alors la clef pour résoudre les MAVRP associées. Ces sous-problèmes, que nous appelons problèmes de *timing*, se retrouvent dans d'autres domaines de la littérature, dans des problèmes de transport, de plus courts chemins, d'ordonnancement de projet ou de production, d'allocation de ressources, ou de régression isotonique en statistique. Cependant, quasiment aucun lien n'avait été établi entre ces problèmes traités dans plusieurs domaines, si bien que des algorithmes efficaces similaires ont été redécouverts de nombreuses fois dans la littérature.

Afin d'établir l'état de l'art pour ces problèmes, décisif pour nos applications, une synthèse de littérature pluridisciplinaire sur les problèmes de timing est proposée dans le Chapitre 2. Les attributs temporels sont classifiés, et des algorithmes de résolution efficaces issus de nombreux domaines de recherche sont identifiés et analysés. Cette analyse considère non-seulement la résolution *séquentielle* de problèmes de timing en "partant de zéro", mais aussi la résolution de séries de sous-problèmes successifs au sein de recherches locales ou de méthodes de branchement par des méthodes de *ré-optimisation*. La bibliothèque de méthodes efficaces ainsi identifiée est un atout considérable pour la résolution de problèmes riches de tournées de véhicules multi-attributs ou d'ordonnancement non réguliers, car dans bien des cas cette richesse se manifeste exclusivement au travers de la résolution de ces sous-problèmes temporels. Cette analyse vient ainsi compléter et clore notre première partie de recherche, dédiée à l'analyse de la structure des attributs, des problèmes, et des méthodes de la littérature.

Axe II : Développements métaheuristiques. Nous introduisons de nouveaux développements méthodologiques afin d'étendre des concepts à succès issus des métaheuristiques évolutionnaires, introduire de nouveaux concepts prometteurs pour les variantes du VRP, et ainsi contribuer au deuxième défi énoncé dans ce plan de thèse. Dans ce cadre, le Chapitre 3 propose un nouvel algorithme génétique hybride *à contrôle adaptatif de diversité* (*Hybrid Genetic Search with Advanced Diversity Control – HGSADC*) très efficace pour les VRP multi-périodes (PVRP) et multi-dépôts (MDVRP) éventuellement combinés, ainsi que pour le VRP classique. Cette métaheuristique combine l'exploration large des méthodes évolutionnaires à base de populations, les capacités d'amélioration agressive des métaheuristiques à voisinage, les relaxations de contraintes pour cibler les frontières de non-faisabilité, et des méthodes avancées de gestion de la diversité dans la population. HGSADC réinterprète le concept de *survie du plus apte*, mettant en place une

évaluation bi-critère des individus qui considère à la fois la qualité des solutions, mais aussi leur contribution à la diversité. Cette stratégie permet de préserver la variété des éléments de solutions au sein de la population et de réduire les risques de convergence prématurée pour explorer des régions prometteuses de l'espace de recherche. De vastes expérimentations démontrent la grande performance de cette méthode, en terme d'efficacité de calcul et de qualité de solution, identifiant soit les meilleures solutions de la littérature ou les solutions optimales quand elles sont connues, soit de nouvelles meilleures solutions pour tous les jeux de tests connus et étudiés depuis des années sur les trois classes de problèmes. Finalement, une comparaison expérimentale des stratégies de gestion de population et d'évaluation des individus, sur plusieurs problèmes, révèle la contribution importante des méthodes proposées.

Les attributs étudiés dans le Chapitre 3 font partie de la catégorie Assign, impactant les choix d'affectation. Afin d'étendre la méthode HGSADC à d'autres types d'attributs, notamment ceux de type Eval, et d'étudier ses concepts principaux sur une plus grande variété de problèmes, le Chapitre 4 présente l'application de cette métaheuristique au VRP avec fenêtres de temps (VRP with time windows : VRPTW). Cette application donne lieu à de nouveaux développements méthodologiques, concernant notamment l'exploitation de solutions irréalisables ne respectant pas certaines contraintes sur les routes (fenêtres de temps, capacités et durées), et l'exploration efficace de voisinages avec des méthodes de pré-évaluation sur les segments de routes. Le concept de relaxation des contraintes temporelles de Nagata et al. (2010) est utilisé. Celui-ci permet de payer des pénalités pour des "retours dans le temps" en cas d'arrivée tardive à un client. Nous proposons une manière simple et efficace dans ce cadre pour évaluer en temps constant amorti $O(1)$ les solutions intermédiaires irréalisables produites par des mouvements de recherche locale à base d'un nombre borné d'échanges d'arcs. Les meilleurs résultats de la littérature sont obtenus sur les problèmes-tests classiques pour des variantes de VRP avec n'importe quelle combinaison d'attributs de fenêtres de temps, contraintes de durées, compatibilités véhicules-clients, dépôts multiples et périodes de planification multiples.

L'application de HGSADC aux problèmes avec fenêtres de temps, dans le Chapitre 4, a soulevé des questions de recherche liées à l'utilisation efficace de solutions irréalisables et aux choix de relaxation effectué dans les recherches locales et les heuristiques en général. Cette utilisation de solutions irréalisables avait déjà été identifiée comme un concept "gagnant" des métaheuristiques étudiées dans le Chapitre 1, cependant peu d'éléments théoriques ou empiriques de la littérature venaient jusqu'alors appuyer ces conjectures. De ce fait, le Chapitre 5 présente une analyse empirique des relaxations de contraintes de fenêtres de temps au sein de méthodes de recherche locale pour le VRPTW. Les résultats, produits pour deux types d'objectifs (minimisation de la distance totale et minimisation de la taille de flotte) confirment l'utilité de solutions irréalisables au cours de la recherche. Aussi, plusieurs alternatives de relaxation sont comparées : arrivées tardives pénalisées, arrivées en avance, retours dans le temps, ou accélérations des services et des déplacements. Des différences notables sont observées. Les résultats expérimentaux montrent que l'approche de "retours dans le temps" (Nagata et al. 2010), utilisée et généralisée lors de l'application d'HGSADC aux VRP avec fenêtres de temps, permet d'obtenir à la fois de meilleures solutions, et de les trouver plus rapidement.

Enfin, l'étude des problèmes avec dépôts multiples a soulevé des questions méthodologiques quand à l'exploration, combinée ou non, de différentes alternatives d'affectation aux dépôts et de choix de séquences. Dans le Chapitre 3 nous avions traité le MDVRP comme sous-problème du PVRP en séparant la représentation de solution et les voisinages en relation aux ressources d'affectation (décomposition sur les périodes de planification ou les dépôts) et en créant des opérateurs (croisement, recherche locale) spécifiques aux choix d'affectations. Le Chapitre 6 décrit une nouvelle méthode efficace, basée sur la programmation dynamique, pour évaluer de manière combinée et en temps amorti $O(1)$ le choix optimal du dépôt, du type de véhicule, et du premier client de la séquence à être visité après le dépôt (*rotation* optimale) lors de méthodes de recherche locale et de Split. Cette méthodologie permet non seulement d'évaluer efficacement un plus grand nombre d'alternatives combinées d'affectation et de séquence, mais aussi de reléguer les décisions d'affectation au sein de sous-problème d'évaluation de tournées, afin de ne plus avoir à considérer explicitement cet aspect combinatoire au sein des voisinages de recherche locale et des représentations de solution. Ces approches sont implantées et testées au sein de deux cadres métaheuristiques, une recherche locale itérée, et la méthode HGSADC, sur des VRP multi-dépôts avec flotte illimitée, homogène ou hétérogène. Les expériences mettent en valeur la contribution importante des méthodes proposées au sein des deux types de métaheuristiques.

Axe III : Résolution de problèmes riches et algorithmes généralistes. Les derniers chapitres de cette thèse se consacrent à la résolution de VRP *riches* combinant de nombreux attributs, comme le problème de tournées de véhicules et horaires de conducteurs combinés, et au développement de nouvelles méthodes heuristiques généralistes pour traiter une grande variété de problèmes.

Le Chapitre 7 présente tout d'abord une application à un problème riche de tournées de véhicules et horaires de conducteurs (VRP and truck driver scheduling : VRTDSP). En effet, de nombreux pays ont établi des législations complexes sur les séquences de temps de conduite et de pause (European Union 2002, 2006, National Transport Commission 2008c,b, Transport Canada 2005, Federal Motor Carrier Safety Administration 2011). Ne pas prendre en compte ces contraintes conduirait à des solutions totalement irréalistes dans le cadre du transport longue-distance. Toutefois, les VRP combinés avec des contraintes de législation sont des problèmes riches particulièrement difficile à résoudre. Ils amènent parfois plus de quinze règles supplémentaires sur les horaires, où plusieurs types de tâches sont discernées : conduite, travail hors conduite, repos hebdomadaire, repos journalier, pause... et peuvent être parfois étendues ou coupées dans des cas exceptionnels. Le sous-problème même de placement des pauses, pour une séquence de visite choisie, est en soi un problème très difficile, dont la polynomialité est à ce jour une question ouverte pour la plupart des législations.

Selon la classification du Chapitre 1, ces attributs sur les horaires impactent exclusivement l'évaluation des routes (type Eval). Une extension de HGSADC est proposée pour résoudre cette famille de problèmes pour les règles de l'Union Européenne, des États-Unis, du Canada, et de l'Australie. Des méthodes d'évaluation de routes efficaces, basées sur une exploration arborescente des combinaisons de tâches et pauses en présence de relaxations des contraintes de fenêtres de

temps, sont utilisées. Par ailleurs, dans notre cadre, le problème même de placement des pauses est un problème complexe. Des méthodes polynomiales quadratiques (en fonction du nombre de clients) permettent de le résoudre pour le cas des Etats-Unis, tandis que pour les autres pays la complexité (et polynomialité) du problèmes est toujours une question ouverte. Dans ce cadre où chaque évaluation de route est complexe et coûteuse en temps de calcul, nous introduisons des techniques d'accélération de recherche, basées sur des restrictions de voisinage ainsi que des mémoires sur les évaluations de routes et sur les sous-séquences de visites sont utilisées à la manière des méthodes de ré-optimisation pour les problèmes de timing L'algorithme ainsi obtenu améliore les solutions existantes de la littérature pour le VRTDSP avec règles européennes. Enfin, il permet de traiter les différentes législations en une seule implémentation, ouvrant ainsi la porte à une comparaison internationale de l'effet des règles sur les coûts, les horaires, et la fatigue estimée des conducteurs. En particulier, nos expérimentations indiquent que les règles européennes sont les plus sûres, tandis que les règles Canadiennes sont les plus profitables économiquement. Les règles Australiennes, par contre, semblent être dominées à la fois en termes de risque et d'efficacité économique. Ces observations démontrent l'intérêt pratique direct des méthodes développées, qui peuvent être utile aux acteurs de la chaine logistique pour leurs besoins d'optimisation, et aux organismes décisionnaires et syndicats pour évaluer l'impact de nouveaux choix de règles sur le transport longue-distance.

En outre, cette thèse contribue à la résolution généraliste de la grande variété de VRP multi-attributs, par le biais d'un cadre modulaire de résolution heuristique, et une implémentation métaheuristique généraliste, appelée *Unified Hybrid Genetic Search (UHGS)*. Ces contributions sont décrites dans le Chapitre 8. Proposer une telle méthode généraliste pour les MAVRP ne signifiait pas ignorer les caractéristiques spécifiques des problèmes, car il était nécessaire dans tous les cas de prendre en compte les contraintes, décisions supplémentaires et objectifs spécifiques, mais impliquait plutôt de restreindre les procédures dédiées à des éléments minimes et modulaires de la méthode. Ainsi, alors que les algorithmes modulaires et librairies de composants de la littérature pour l'optimisation combinatoire étaient auparavant principalement dédiés à la multiplicité des méthodes (différents modules pour différentes stratégies heuristiques), le cadre proposé gère la *multiplicité des variantes* de problèmes grâce à des composants de résolution élémentaires s'adaptant automatiquement aux attributs considérés. UHGS se base donc sur des procédures dont l'implantation est indépendante du problème : opérateurs de recherche locale unifiés, croisement, segmentation de routes (*Split* généralisé), gestion de diversité. Les traitements spécifiques au problème sont confinés dans des composants modulaires qui fournissent les fonctions nécessaires aux choix d'affectation, de séquence, et aux évaluations efficaces de routes, et sont automatiquement sélectionnés et adaptés relativement aux attributs du problème. Des études expérimentales de grande ampleur sur 26 variantes fondamentales de VRP et 39 ensembles de problèmes de test de la littérature, un total de 1008 instances, démontrent la performance remarquable de UHGS qui, avec une unique implémentation et paramétrage, égalise ou surpasse les meilleures méthodes dédiées de la littérature. De plus, toutes les solutions optimales connues pour ces problèmes ont été retrouvées. Ces développements démontrent que la généralité ne s'obtient pas forcément aux dépends de l'efficacité pour cette classe de problèmes.

Finalement, pour gérer efficacement non seulement la *variété* mais aussi la difficulté inhérente aux *combinaisons* des attributs, et afin de progresser plus en avant vers la résolution de VRP *riches* contenant plusieurs types d'attributs décisionnels, les métaheuristiques précédentes ont été étendues au sein d'un cadre de résolution parallèle coopérative à base de décompositions et recompositions successives de solutions, appelé *Integrative Cooperative Search (ICS)*. Cette contribution est présentée dans le Chapitre 9. L'idée fondamentale d'ICS est de combiner les idées de décomposition, de simplification intelligente et opportuniste, et de recherche coopérative. Les décompositions visent à générer des problèmes avec un nombre d'attributs réduits, pour lequel des méthodes de résolution efficaces, comme UHGS, peuvent être utilisées aux mieux de leurs capacités. Les solutions issues de ces sous-problèmes plus simples sont intégrées par des algorithmes dédiés afin de reconstituer des solutions complètes et ainsi traiter le problème initial. Les différents composants coopèrent ensemble par le biais d'une mémoire centrale enrichie de mécanismes de guidage. Ce développement méthodologique permet de produire des solutions de plus grande qualité sur les problèmes traités auparavant, combinant attributs périodiques et multi-dépôts. De plus, cette application a soulevé plusieurs considérations méthodologiques impliquant notamment la reconstitution de solutions complètes à partir de solutions partielles, et l'exploitation efficace de mémoires pour *guider* la recherche future. De nombreuses expérimentations ont été développées pour analyser l'impact de différentes stratégies alternatives à ce sujet. Ces contributions s'appliquent avec succès aux problèmes de tournées de véhicules multi-attributs, et sont applicables en toute généralité à une large gamme de problèmes d'optimisation combinatoire.

0.4 Structure du document

La structure de ce document concorde avec la démarche de recherche annoncée. La première partie, Chapitres 1 et 2, présente les contributions en terme d'analyse de problèmes et méthodes : les heuristiques pour les MAVRP, et la synthèse des problèmes de timing. La deuxième partie, au sein des Chapitres 3 à 6, détaille les différentes contributions méthodologiques pour les métaheuristiques, la méthode HGSADC, son application aux variantes du VRPTW, ainsi que l'étude des relaxations des contraintes de fenêtres de temps et des choix de rotation de séquences et d'affectation implicites. La dernière partie, Chapitres 7 à 9, décrit l'extension de HGSADC aux problèmes riches de VRTDSP, le nouveau cadre de résolution général pour les MAVRP et le traitement des problèmes riches avec la méthode ICS.

Première partie

ANALYSE STRUCTURELLE DES PROBLÈMES ET DES MÉTHODES

CHAPITRE 1

MÉTAHEURISTIQUES POUR LES PROBLÈMES DE TOURNÉES DE VÉHICULES MULTI-ATTRIBUTS, REVUE ET SYNTHÈSE DE LA LITTÉRATURE

1.1 Fil conducteur et contributions

Les *attributs* des problèmes de tournées de véhicules sont des caractéristiques et contraintes additionnelles qui visent à mieux prendre en compte les spécificités des applications réelles. Les variantes engendrées sont le support d'une littérature fournie, qui comporte une large variété d'heuristiques. Ce chapitre passe en revue les principaux types d'attributs, et décrit les heuristiques et méta-heuristiques classiques pour les problèmes de tournées de véhicules multi-attributs. Puis, il analyse de manière fine les concepts et "stratégies gagnantes" de 64 méta-heuristiques remarquablement efficaces sur plusieurs attributs. Cette analyse transversale est une étape importante pour progresser vers des méthodes efficaces pour une large gamme de problèmes.

1.2 Article I : Metaheuristics for multi-attribute vehicle routing problems, a survey and synthesis

Ce chapitre a été accepté pour publication sous forme d'article de journal : Vidal, T., Crainic, T.G., Gendreau, M., Prins, C. (2012). Metaheuristics for multi-attribute vehicle routing problems, a survey and synthesis. *European Journal of Operational Research*, Forthcoming.

Abstract: The attributes of Vehicle Routing Problems are additional characteristics or constraints that aim to better take into account the specificities of real applications. The variants thus formed are supported by a well-developed literature, including a large variety of heuristics. This article first reviews the main classes of attributes, providing a survey of heuristics and meta-heuristics for Multi-Attribute Vehicle Routing Problems (MAVRP). It then takes a closer look at the concepts of 64 remarkable meta-heuristics, selected objectively for their outstanding performance on 15 classic MAVRP with different attributes. This cross-analysis leads to the identification of "winning strategies" in designing effective heuristics for MAVRP. This is an important step in the development of general and efficient solution methods for dealing with the large range of vehicle routing variants.

Keywords: Vehicle routing, multi-attribute problems, heuristics, meta-heuristics, survey, analysis, algorithmic design principles.

1.3 Introduction

Vehicle routing problems have been the subject of intensive research for more than 50 years, due to their great scientific interest as difficult combinatorial optimization problems and their importance in many application fields, including transportation, logistics, communications, manufacturing, military and relief systems, and so on. The "traditional" Capacitated Vehicle Routing Problem (CVRP) involves designing least cost delivery routes to service a geographically-dispersed customer set, while respecting vehicle-capacity constraints. This NP-hard optimization problem combines the characteristics of a Multiple Knapsack Problem aiming to assign loads to capacitated vehicles, and a Travelling Salesman Problem (TSP) that aims to find the best route for each vehicle, i.e., the least costly sequence of visits for the customers assigned to it.

The extremely broad range of actual applications where routing issues are found leads to the definition of many VRP variants with additional characteristics and constraints, that we call *attributes*, aiming to capture a higher level of system detail or decision choices, including but not limited to richer system structures (e.g., several depots, vehicle fleets, and commodities), customer requirements (e.g., multi-period visits and within-period time windows), vehicle operation rules (e.g., load placement, route restrictions on total distance or time, and driver work rules), and decision context (e.g., traffic congestion and planning over extended time horizons). These attributes complement the traditional CVRP formulations and lead to a variety of *Multi-Attribute Vehicle Routing Problems* (MAVRPs), making up a vast research, development, and literature domain. The dimensions of most problem instances of interest hinders the applicability of exact methods, while the few software systems presented as general heuristic solvers are increasingly challenged by the growing variety of attributes. Finally, some MAVRPs combining multiple attributes together, the so-called *rich* VRPs, may be especially difficult to solve because of the compound, and possibly antagonist, decisions they involve.

Thousands of heuristics, meta-heuristics, and solution concepts tailored to some specific MAVRPs have been proposed in the literature. The vehicle routing domain, vast and difficult to classify, has been historically articulated around several streams of research dedicated to a number of major attributes. Such diverse research lines would be justified if the nature of the various problem settings would call for radically different solution approaches. Yet, MAVRPs naturally share many common features, and most heuristic strategies developed for specific problems can be applied to a broader range of VRP variants. The identification of such fundamental design elements for MAVRP metaheuristics is of primary interest to progress toward more generalist and efficient VRP algorithms, thus providing the means to quickly address various application cases and rich VRP settings without extensive problem-tailored algorithmic developments.

To respond to these challenges, we introduce a unifying synthesis and analysis of MAVRP solution methods, providing the means to identify the main concepts of successful heuristics and metaheuristics. The analysis is based on two main ideas. On the one hand, we analyse from a general perspective detached from the particular characteristics of the attributes. On the other hand, we adopt a synthetic approach to deal with the abundance of contributions. Thus, in

particular, the scope of the analysis has been limited to settings with complete and exact data, demands on nodes (no arc-routing settings), and a single objective.

We identified, classified, and analysed fifteen notable MAVRPs, which have been the object of a consistent body of well-acknowledged research resulting in a considerable number of heuristic methods and a number of common benchmark sets of test instances. The simple classification we use is method-oriented rather than application-oriented, i.e. attributes are discerned relatively to their impact on the resolution approaches rather than relatively to the real-life constraint or objective it originates from. We then selected three to five of the most efficient heuristics, in terms of average solution quality, for each of these MAVRP variants with different kinds of attributes. The resulting sixty-four methods were then analysed in detail, resulting in the identification of broad concepts and main algorithmic-design principles, an objective synthesis of "winning strategies", perspectives and major research challenges.

The article unfolds in three main parts. Section 1.4 recalls the "traditional" CVRP and reviews the fundamental elements of heuristics developed to address it. Most of these elements are also found in the next sections when analysing heuristics for multi-attribute problems. Section 1.5 introduces an attribute-classification system and presents the selected MAVRPs and the corresponding subset of selected high-performance heuristics, thus providing the necessary material for our unifying analysis of state-of-the-art MAVRP heuristics in Section 1.6. Section 1.7 concludes with a discussion of a number of challenges for the field and possible research perspectives.

1.4 Heuristics for the CVRP

The CVRP was introduced in the seminal article by Dantzig and Ramser (1959) under the name "Truck Dispatching Problem". It was only several years later, following the publication of the article by Christofides (1976), that the current name of the problem became widespread. Like numerous previous articles, we define the CVRP as follows.

Let $G = (\mathcal{V}, \mathcal{E})$ be a complete undirected graph with $|\mathcal{V}| = n + 1$ nodes. The node $v_0 \in \mathcal{V}$ represents a depot, where a fleet of m identical vehicles is based, and where the product to be distributed is stored. The other nodes $v_i \in \mathcal{V} \backslash \{v_0\}$, for $i \in \{1, \ldots, n\}$, represent the customers, characterized by demands for non-negative amounts of product q_i. Edges $(i, j) \in \mathcal{E}$ represent the possibility of travelling directly from a node (customer or depot) $v_i \in \mathcal{V}$ to a different node $v_j \in \mathcal{V}$ for a transportation cost of c_{ij}. The objective of the CVRP is to find a set of m or less vehicle routes, i.e. sequences of deliveries to customers, such that all customers are visited exactly one time, the total cumulated demand on any route does not exceed a capacity limit Q, the total length of each route is limited to a value T, and the total transportation cost is minimized. Applications considering an unlimited fleet can be modeled by setting $m = n$ as at most one vehicle per customer is needed.

The CVRP has been the subject of intensive research since the 1960s. Numerous exact methods, heuristics, and metaheuristics have been presented in the literature, as illustrated by various surveys (see Baldacci et al. 2007, Cordeau et al. 2007, Gendreau et al. 2008b, Eksioglu et al. 2009, Potvin

2009 and Laporte 2009, for the most recent) and books (Golden and Assad 1988, Toth and Vigo 2002a, Golden et al. 2008).

The CVRP is modeled in Equations (1.1-1.8) as an . Many different formulations of this problem can be found in the literature. Introduced by Fisher and Jaikumar (1981), the integer linear programming formulation (1.1) - (1.8) has the advantage of presenting explicitly the combined assignment and sequencing characteristics of the CVRP. It is based on two families of binary variables, y_{ik}, designating the assignment of customer i to vehicle k by the value 1 (and 0, otherwise; $y_{0k} = 1$ signals vehicle k operates), and x_{ijk}, taking the value 1 when vehicle k visits node v_j immediately after node v_i ($i \neq j$).

$$\text{Min.} \quad \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=1}^{m} c_{ij}x_{ijk} \tag{1.1}$$

$$\text{Subject to} \quad \sum_{k=1}^{m} y_{ik} = 1 \qquad i = 1,\ldots,n \tag{1.2}$$

$$\sum_{k=1}^{m} y_{0k} \leq m \tag{1.3}$$

$$\sum_{i=1}^{n} q_i y_{ik} \leq Q \qquad k = 1,\ldots,m \tag{1.4}$$

$$\sum_{j=0}^{n} x_{ijk} = \sum_{j=0}^{n} x_{jik} = y_{ik} \qquad i = 0,\ldots,n;\ k = 1,\ldots,m \tag{1.5}$$

$$\sum_{v_i \in S}\sum_{v_j \in S} x_{ijk} \leq |S| - 1 \qquad k = 1,\ldots,m; S \in V\backslash\{0\}; |S| \geq 2 \tag{1.6}$$

$$y_{ik} \in \{0,1\} \qquad i = 0,\ldots,n;\ k = 1,\ldots,m \tag{1.7}$$

$$x_{ijk} \in \{0,1\} \qquad i = 0,\ldots,n;\ j = 0,\ldots,n;\ k = 1,\ldots,m \tag{1.8}$$

Constraints (1.2) - (1.4) present the structure of a multiple knapsack problem problem with m resources (knapsacks). These constraints enforce, respectively, the assignment of each customer to a single vehicle, the maximum number of vehicles operating out of the depot, and the vehicle capacity. Constraints (1.5) - (1.6) are then related to the structure of the routes, guaranteeing the selection of an adequate number of arcs entering into and exiting from each node (depot and customers), and eliminating sub-tours (i.e., routes that don't pass through the depot). The number of constraints of this latter type grows exponentially with the number of customers. The CVRP includes the TSP as a special case when $m = 1$ and $Q = +\infty$, and is thus NP-hard.

An additional constraint on the maximum length of each route, Relation (1.9), is often found in the literature. A service duration τ_i is associated to each customer, the sum of the customer service times and the travel time of the route being then limited to T.

$$\sum_{i=0}^{n}\sum_{j=0}^{n}(c_{ij} + \tau_i)x_{ijk} \leq T \qquad k = 1,\ldots,m \tag{1.9}$$

Only relatively small CVRPs can currently be consistently solved to optimality. The largest symmetric instances solved by Fukasawa et al. (2006) and Baldacci et al. (2008b) have a maximum of 135 customers. Some exact methods for asymmetric problems have also been proposed in Fischetti et al. (1994) and Pessoa et al. (2008). Because of this, heuristics and metaheuristics constitute a very active research domain in the literature. These approaches for the classic CVRP are surveyed in the following, discerning some main categories of methods: *constructive heuristics*, *local improvement heuristics*, *metaheuristics*, *hybrid methods*, and *parallel and cooperative metaheuristics*.

1.4.1 Constructive heuristics

Mainly proposed between the 1960s and the 1980s, a large number of heuristics attempted to produce solutions constructively. One key characteristic of these heuristics is that they operate in a *greedy* manner, producing a set of definitive decisions (e.g., customer insertion or the merging of two routes) that cannot be reversed afterwards.

The *savings method* of Clarke and Wright (1964) is the best-known example of a constructive heuristic. Starting from an initial solution s_0 in which each customer is served by a different route, the heuristic searches for and merges two route extremities i and j, maximizing the distance saved $s_{ij} = c_{i0} + c_{0j} - c_{ij}$, under the condition that the merged route is feasible. The original method has been revised and improved several times, notably by Gaskell (1967) and Yellow (1970) who parametrized the original equation to give more, or less, importance to the distance to the depot ($s_{ij} = c_{i0} + c_{0j} - \lambda c_{ij}$ with $\lambda \geq 0$), thus correcting a flaw in the original method, which produced routes with a high "circular" tendency. Mole and Jameson (1976) and Solomon (1987) further generalize these constructive procedures, considering customer insertions into the routes and using additional parameters and simple local-improvement procedures.

Another constructive heuristic, called "sweep" (Gillett and Miller 1974), is remarkable in its simplicity. The approach explores the customers circularly, in increasing polar angle around the depot. Each customer is successively inserted in this order at the end of the current route. If this insertion is infeasible because of the route constraints, then a new route is initiated. At the end of this construction phase, Gillett and Miller (1974) proposed to apply a λ-*opt* improvement heuristic (see Section 1.4.2) to post-optimize each route separately.

Other heuristics perform the assignment and sequencing in two separate phases. The "route-first cluster-second" approach (Newton and Thomas 1974, Bodin and Berman 1979, Beasley 1983) first constructs a giant circuit that visits all customers, like a TSP solution. This giant tour is then cut into several routes from the depot. The segmentation problem can be solved exactly as a shortest path problem in an acyclic graph.

Proposed by Fisher and Jaikumar (1981), the "cluster-first route-second" approach, first creates customer clusters, and then optimizes the order of visits for each cluster as a TSP subproblem. The creation of the clusters is performed by solving a Generalized Assignment Problem (GAP) for the customers, around m locations chosen to represent zones with a high customer density. A linear estimate of the route costs is used as the objective function of the GAP. This approach is strongly linked to the visual solution approach of human planners. In addition, the priority given to the assignment allows capacity constraints to be better dealt with for highly constrained

problems presenting few feasible solutions. This specificity is significant in the CVRP literature where most constructive heuristics manage the capacity constraints as a by-product of a policy exclusively dedicated to the geometrical creation of routes.

The heuristics presented in this section are generally capable of producing solutions that are within 10% or 15% of the optimum in a very short time. A detailed review of these methods can be found in Laporte and Semet (2002). Today, constructive methods are still used to produce initial solutions for a wide range of heuristics, and have been adapted to many MAVRPs. Furthermore, certain metaheuristics (e.g., GRASP or Ant Colony Optimization) rely on iteratively calling on constructive heuristics, biased by information gathered during the global search, to create new solutions.

1.4.2 Local-improvement heuristics

Sequence-based combinatorial optimization problems lend themselves well to the application of local search (LS) improvement heuristics (see Aarts and Lenstra 2003 for a comprehensive introduction). Based on an initial solution s, a local search heuristic explores a *neighbourhood* $\mathcal{N}(s)$, generally defined by perturbations (*moves*) on s, in order to find an improving solution s' that replaces s for a new iteration of the heuristic. The local search stops at a solution $\bar{s}$ when no improving solution can be found in $\mathcal{N}(\bar{s})$. This solution is a *local optimum* of the problem and the neighbourhood used. The set of solutions – or states characterizing solutions – linked by neighbourhood relationships is usually called *search space*, while the succession of states reached in the course of the method constitutes a *search trajectory* in the graph thus formed. Many neighbourhoods have been defined in the VRP literature. For the sake of brevity, we will only describe those which are still frequently used and named as such in the current literature.

A first category, coming directly from the TSP literature, relies on arc exchanges to optimize separately the routes. In the terminology of Lin (1965), a neighbourhood of the type λ-*opt* contains the set of solutions obtained by deleting and reinserting λ arcs. The neighbourhood size is $|\mathcal{N}^{\lambda\text{-}opt}| = O(n^{\lambda})$. The most commonly used neighbourhoods in the literature include *2-opt* and *3-opt*, as well as *Or-exchange* (Or 1976). The latter neighbourhood involves relocating sequences of visits of bounded length, and constitutes a subset of 3-opt of size $O(n^2)$. Examples of the 2-opt and Or-exchange moves are illustrated in Figure 1.1. Noteworthy is also the *GENI* insertion operator (Gendreau et al. 1992), which effectively evaluates combined customer insertions in a route with restricted 3-opt or 4-opt optimization.

Other CVRP local-search neighbourhoods allow several routes to be improved simultaneously, generally by exchanging arcs or moving visits between the sequences. Among the most commonly used neighbourhoods of this type, the *insert* neighbourhood (also called *shift* neighbourhood in Osman 1993) consists of moving a visit from one route to another, while a *swap* (also called *1-interchange*) exchanges 2 visits between their respective routes. The *2-opt** neighbourhood (Potvin and Rousseau 1995) is based on the deletion and reinsertion of two arc pairs from two different routes. This neighbourhood, which can also be assimilated to an exchange of "route ends", is sometimes called *crossover* neighbourhood, and is illustrated in Figure 1.2. The three previously mentioned neighbourhoods contain $O(n^2)$ solutions.

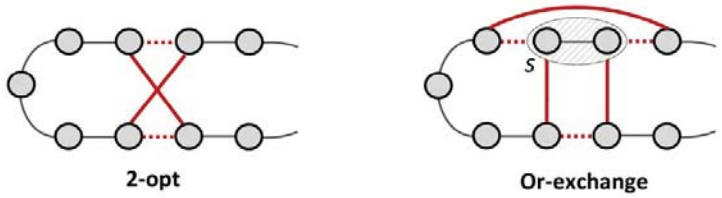


Figure 1.1: *2-opt* and *Or-exchange* illustration. The deleted/inserted arcs are indicated with dotted/bold lines.

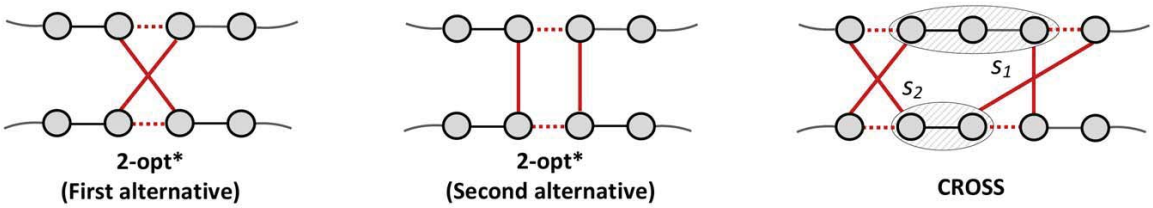


Figure 1.2: 2-opt* and CROSS-exchange illustration. The deleted/inserted arcs are indicated with dotted/bold lines.

Finally, the CROSS-exchange neighbourhood (Taillard et al. 1997) exchanges two (one being potentially empty) customer sequences s_1 and s_2, thus generalizing the three neighbourhoods introduced previously, *insert*, *swap*, and *2-opt**. The customer sequences s_1 and s_2 can eventually be reversed in this process to produce a slightly larger neighbourhood, called *I-CROSS* in Bräysy (2003). The CROSS and I-CROSS neighbourhoods are of size $O(n^4)$ and would be costly to evaluate exhaustively. In practice, the size of the exchanged sequences is often limited by a value L_{max}, so that the size of the neighbourhood becomes $O(L_{max}^2 n^2)$. CROSS and I-CROSS are themselves special cases of λ-interchanges moves (Osman 1993), which involve exchanging any (potentially non-consecutive) set of less than λ customers between two routes.

Even the evaluation of quadratically sized neighbourhoods can be impracticable for certain large problems. Thus, further *neighbourhood pruning* procedures are frequent in the literature. A common method, called *granular search*, requires computing for each node v_i a list $\Gamma(v_i)$ of spatially related neighbours, and only considering moves that involve v_i and $v_j \in \Gamma(v_i)$ (Gendreau et al. 1992, Johnson and McGeoch 1997, Toth and Vigo 2003). Another type of limitation, introduced early in the TSP literature by Christofides and Eilon (1972), Lin and Kernighan (1973), and generalized for the CVRP by Irnich et al. (2006) under the name of *sequential search*, is based on the observation that all profitable λ-opt can be broken down into a list of arc exchanges $(\phi_1, \ldots, \phi_\lambda)$ with gains $(g_1, \ldots, g_\lambda)$, such that all subsets of $k \leq \lambda$ first arc exchanges have a positive partial gain $\sum_{i=1}^{k} g_i$. This observation allows to rapidly eliminate a lot of unpromising neighbour solutions.

Of critical concern is the efficient serial evaluation of moves, route feasibility and costs, which represents the bottleneck of most local searches. Several techniques can lead to computational effort

reductions. The management of judicious variables on subsequences of visits (e.g. partial demands or distances), in particular, can lead to incremental move evaluations in amortized $O(1)$ time. Similar approaches are used with success on other MAVRPs (c.f. Section 1.6). Memory structures may also be used to store route evaluations or move evaluations within tables or hashtables.

Large neighbourhoods, with an exponential number of solutions, have also been widely studied and used in the literature. The procedure of Lin and Kernighan (1973) is a remarkably effective method for optimizing a TSP sequence. Like the *ejection chains* strategy developed by Glover (1992, 1996) and extensively applied to the CVRP by Rego (2001), this procedure attempts to find a cycle that alternates existing and non-existing arcs in the current solution, so that the solution obtained by replacing existing arcs in a cycle with the non-existing arcs is feasible and improving. Such a method can be viewed as an incomplete investigation of a λ-opt neighbourhood with large λ values. Closely related to the previous concepts, the cyclical transfers of Thompson and Psaraftis (1993) explore a large neighbourhood obtained by moving k customers within b routes. The search for an improving neighbour solution is formulated as a negative-cost cycle detection problem in an auxiliary graph. Although NP-hard, this latter subproblem can be solved effectively by means of heuristics.

Other ruin-and-recreate neighbourhoods (Shaw 1998) operate deletions and reinsertions of customer visits within customer sequences. Methods of this kind vary in the nature of their destruction and reconstruction operators, and may exploit heuristic methods, constraint programming, or integer programming for reconstruction. Finally, generalizing the work of Sarvanov and Doroshko (1981) for the TSP, De Franceschi et al. (2006) and Toth (2008) propose neighbourhoods based on fixing some customers and re-assigning unfixed customers between fixed ones, which are explored by solving an integer-programming model. Other large neighbourhoods and exploration techniques are reviewed in Ahuja et al. (2002) and Pisinger and Ropke (2010). Additional literature reviews on local-search methods for the VRP can be found in Van Breedam (1995), Thompson and Psaraftis (1993), Kindervater and Savelsbergh (1997), Laporte and Semet (2002), Bräysy and Gendreau (2005a) and Funke et al. (2005). Local search constitutes an essential building block of metaheuristics for the CVRP, described in the next section.

1.4.3 Metaheuristics

The term "metaheuristic" was first coined by Glover (1986) to designate a broad class of heuristic methods that continue the search beyond the first encountered local optimum. A somewhat crude but telling definition characterizes metaheuristics as *heuristics guiding other heuristics*.

Metaheuristics constitute a core research domain in combinatorial optimization as illustrated by many literature reviews (e.g., Osman and Laporte 1996, Blum and Roli 2003, Gendreau and Potvin 2005) and books (e.g., Corne et al. 1999, Glover and Kochenberger 2003, Dréo et al. 2003, Gendreau and Potvin 2010). The CVRP is a testing ground particularly appreciated for such methods, as illustrated by the reviews of Gendreau et al. (2002), Cordeau et al. (2005), Gendreau et al. (2008b), Laporte (2009) and Potvin (2009). We distinguish between so-called neighbourhood-centred methods, which generally proceed by iteratively exploring the neighbourhoods of a single incumbent solution, population-based strategies evolving a set of solutions by generating one of

several "new" solutions out of combinations of existing ones, and approaches that either combine elements of different metaheuristics, the so-called hybrids, or harness the exploration capabilities of several solution methods exploiting their interaction, the parallel and cooperative search methods.

1.4.3.1 Neighbourhood-centred search

Simulated Annealing (SA) (Kirkpatrick et al. 1983, Černý 1985) overcomes the limitation of local-improvement heuristics, the rapid attraction to a local optimum, by accepting solution-deterioration moves with a probability governed by a statistical process, the so-called *temperature* parameter. The higher the temperature, the more likely it is to accept a deteriorating move. Temperature evolves dynamically during the search relatively to a cooling scheme, first favouring a vast exploration and frequent degradations, then gradually accepting fewer and fewer degradations to intensify the search for good-quality solutions. For the CVRP, efficient deterministic "Record-to-Record" (R-to-R) variants (Dueck 1993, Li et al. 2005) accept any neighbor solution which is *not much worse* than the incumbent solution, and prevent degradations that are too significant relatively to the best-found solution s^*, subject to re-starting the search from s^*.

Tabu search (Glover 1986, 1989, 1990, Glover and Laguna 1998) associates a search trajectory centred on the choice the best neighbour of the incumbent solution, with learning capabilities, generally represented as short-, medium- and long-term memories on solution elements, which replace or significantly complement the randomization used in other metaheuristics. The method is thus escaping from local optima when the best neighbor solution is not improving. This decision process is enhanced by two mechanisms, the first aiming to avoid cycling and relying on short-term memories to reject solutions that contain recently examined *tabu* elements, the second accepting solutions that fulfil some *aspiration criteria* such as "the best solution in value or containing a given solution element". Of central importance are the medium- and long-term memories used to manage significant trajectory-inflecting procedures known as *intensification*, e.g., focusing the search around elite solutions while promoting high-quality elements, and *diversification*, e.g., moving the search to an under-explored area of the search space, promoting infrequent elements, and so on. The challenge of balancing diversification and intensification is still a key research question in the literature.

Tabu search led to very effective CVRP metaheuristics, including TABUROUTE (Gendreau et al. 1994), Adaptive Memory (AM) variants (Taillard 1993, Rochat and Taillard 1995, Tarantilis 2005), and the Unified Tabu Search (UTS) (Cordeau et al. 1997, 2001a). In TABUROUTE and UTS, diversification and intensification occur through penalties (incentives, respectively) on frequently (rarely) encountered solution elements, while AM approaches regularly redirect the search to a region around a new solution built out of promising fragments from a memory.

Concepts from tabu search have inspired other metaheuristics. Long-term memories for penalizing frequent solution elements can also be viewed as a basis of *Guided Local Search* (Voudouris and Tsang 1999), applied by Kilby et al. (1999), Tarantilis et al. (2007), Kytöjoki et al. (2007), and Zachariadis and Kiranoudis (2010a) to the CVRP. In this case, modifying the search space by means of penalties is a primary tool for escaping from local optima. Similarly, aspiration

criteria take a preponderant role in the *Attribute Based Hill Climber (ABHC)* method (Whittley and Smith 2004, Derigs and Kaiser 2007).

Variable neighbourhood Search (VNS) (Mladenović and Hansen 1997, Hansen et al. 2010) exploits the fact that a local optimum is defined for a given neighbourhood. Thus, changing the nature of the neighbourhood during the search, or at least some of its parameters, provide the means for further solution improvements. The order of neighbourhood evaluations and the solution acceptance criteria can be either deterministic or probabilistic. For the CVRP, additional solution perturbation mechanisms and long-term memories inspired from tabu search are sometimes employed (Kytöjoki et al. 2007, Fleszar et al. 2009, Chen et al. 2010). Metaheuristic hybrids (see Section 1.4.3.3) based on VNS are thus frequent.

In the same spirit, the *Adaptive Large neighbourhood Search (ALNS)* by Pisinger and Ropke (2007) exploits the benefits of varied neighbourhoods based on ruin-and-recreate moves (Shaw 1998). The frequency of use of these neighbourhoods is adapted throughout the search relatively to their past performance. Finally, the *Iterated Local Search (ILS)* (see Lourenço et al. 2010, for a recent review) applies successively a local-improvement phase, which ends up in a local optima, and a perturbation phase to escape from the local optima. Scaling appropriately the strength of the perturbation operator is a crucial point of the method. Prins (2009a) provides a simple and efficient application of ILS to the CVRP, where several solutions are iteratively produced from the same incumbent solution by means of improvement and perturbation mechanisms, the best one being selected for the next iteration.

1.4.3.2 Population-based methods

Population-based methods are often inspired from natural mechanisms. *Genetic Algorithms (GA)* and *Evolutionary Algorithms (EA)* were introduced during the late 1950s, and developed in their current form in Holland (1975). These algorithms interpret genetic laws and natural selection to evolve a population of individuals assimilated to solutions, through elitist selection, crossover, and mutation operators. With EA, it is also common to simultaneously make the search strategies (e.g., operator parameters) evolve with the solutions. Traditional GA and EA have a tendency to progress too slowly, however, and have thus been enhanced with various mechanisms, such as local search, which is also sometimes called an "education operator". The algorithms thus obtained are sometimes called "genetic local searches" (Mühlenbein et al. 1988) or "memetic algorithms" (Moscato 1989, Moscato and Cotta 2010).

Some of these enhanced genetic methods performed remarkably well on classical CVRP benchmark instances (Prins 2004, Alba and Dorronsoro 2006, Marinakis et al. 2006, Nagata et al. 2010, Vidal et al. 2012a). We refer to Potvin (2009) for a thorough coverage of the field. It is noteworthy that many successful genetic algorithms for the CVRP use a *giant-tour* solution representation *without trip delimiters* (Prins 2004), along with clustering procedures (Beasley 1983) to optimally *Split* a tour into routes. This strategy, based on route-first cluster-second constructive procedures, enables to reduce the number of alternative solutions (there are less giant tours than VRP solutions) and rely on very simple genetic crossover operators working on permutations. In addition, an

adequate management or promotion of population diversity appears to be of critical importance (Prins 2004, Sörensen and Sevaux 2006, Vidal et al. 2012a).

The *Path Relinking (PR)* and *Scatter Search (SS)* metaheuristics (Glover 1977, Resende et al. 2010) are other population methods based on solution recombinations. These methods promote strategic recombination over randomization, and differ essentially from the GA-type of methods in the manner in which solutions are crossed and in the size of the solution pool, which is generally smaller. Recombinations in PR involve an initial solution s^{DEP} and a guiding solution s^{GD}, both selected from an elite solution population. Characteristics of s^{GD} are progressively inserted in s^{DEP} in order to create a trajectory connecting these two solutions, potentially containing new improving solutions. On the other hand, the recombinations operators used in SS can involve more than two solutions. Path relinking was applied to the CVRP by Ho and Gendreau (2006).

Ant Colony Optimization (ACO) approaches (Dorigo and Stützle 2004) were inspired by the social behaviour of ants foraging for food and are for now the swarm-type of method most used in optimization. ACO was applied to the CVRP by Bullnheimer et al. (1999), Bell and Mc Mullen (2004), Doerner et al. (2004), Reimann et al. (2004) and Yu et al. (2009), among others. The individual behaviour of ants is embodied by constructive heuristics, exploiting information on the search history (i.e., pheromones). Other swarm-inspired methods were proposed for the CVRP by Marinakis and Marinaki (2011) (*bee colonies*) and Marinakis and Marinaki (2010) (*particle swarms*). All these methods are exploiting some form of learning, as are *neural networks* (Ghaziri 1996, Vakhutinsky and Golden 1994, Créput and Koukam 2008), and *artificial immune systems* (Masutti 2008), to name a few. These algorithms are often combined with local-improvement procedures, thus complicating the task to estimate the proper impact of collective intelligence paradigms on the search performance.

1.4.3.3 Hybrid metaheuristics

Hybrid metaheuristics blend concepts from various solution methodologies, metaheuristic classes most often, to take advantage of their respective strengths. The blending may take the form of a juxtaposition of methods (e.g., two algorithms called on consecutively) or an indissociable inclusion of elements from one method into a fully-functional different metaheuristic (e.g., tabu search-inspired memories in VNS). Hybrids may exclusively combine metaheuristic concepts, or also involve algorithmic ideas and modules from mathematical programming, constraint programming, tree-search procedures, and so on.

Although much effort has been recently put into properly defining the scope of hybrid metaheuristics (Raidl et al. 2010, Blum et al. 2011), the term remains very general and covers very different strategies. One can indeed argue that metaheuristics, described as *heuristics guiding other heuristics*, are hybrid in nature. This shows the shortcomings of a too-encompassing definition or, even, of trying to find a precise definition. Within the scope of this paper, we identify hybridisation as a strong concept in metaheuristic design, rather than a well-defined class of methods, aiming to take advantage of the synergy among different solution-method ideas to explore a broad variety of solution strategies, often yielding superior results.

A large variety of hybrid methods has thus been proposed for the CVRP. Several approaches involve combined neighbourhood-centred search concepts, such as SA+tabu (Osman 1993), GRASP+ILS (Prins 2009a), ILS+VND (Chen et al. 2010), tabu+ILS (Cordeau and Maischberger 2012), among others. Hybridization schemes of this kind are frequent in recent local search-based methods, which are frequently enriched with restart procedures (a main characteristic of GRASP), probabilistic acceptance of deteriorating moves (a main characteristic of SA), variable neighbourhoods (VNS), or long-term memories and penalties on solutions attributes (GLS).

Population- and neighbourhood-search hybrids are also widespread. The wide majority of population-based approaches for the CVRP actually integrates some kind of local-search components, and can be characterized as hybrid. Furthermore, two of the three most efficient current CVRP metaheuristics (Nagata and Bräysy 2009b, Vidal et al. 2012a) combine GA and LS. Other advanced hybridization schemes involve combined GA+tabu (Perboli et al. 2008), or combined population-based concepts such as GA+PSO (Marinakis and Marinaki 2010) and PR+PSO (Marinakis et al. 2010).

Finally, a number of metaheuristics for the CVRP integrate integer or constraint programming components to recombine promising elements of solutions into complete solutions (Rochat and Taillard 1995, Tarantilis 2005, Alvarenga et al. 2007, Groër et al. 2011), or to explore large neighbourhoods based on ruin-and-recreate (Shaw 1998, De Franceschi et al. 2006, Salari et al. 2010). One actually observes a trend towards proposing *matheuristics* for VRP, combining metaheuristic and mathematical programming components, and explicitly using the model formulation in defining elements of the method (Doerner and Schmid 2010).

1.4.3.4 Parallel and cooperative metaheuristics

Parallel metaheuristics (Toulouse et al. 1996, Alba 2005, Crainic and Toulouse 2010) are concerned with the efficient exploitation of simultaneous work (often on several processors) to solve a given problem instance, and have proved of great interest for routing problems (Crainic 2008).

Several types of parallelism may be distinguished according to how parallelism is obtained, how communications among the tasks are defined, as well as how the global search is conducted. In the most straightforward classification, *low-level* parallelism involves decomposing parts of the algorithm into independent tasks, thus providing the means to exploit parallel resources without changing the general behaviour of the method. To be efficient, such a strategy must target the computationally expensive "bottleneck" procedures, which most frequently are the evaluation of moves in neighbourhood-centred methods, and crossover, selection, and evaluation in population-based ones. To our knowledge, although many papers are concerned with the development of such strategies for metaheuristics in general, few studies on low-level procedures have been directly focused on the CVRP. A notable exception is the recent work of Schulz (2011), considering the efficient solving of CVRPs on Graphic Processing Units (GPU). In this case, the change in hardware has direct implications on the resolution methodology.

In contrast, metaheuristics based on *high-level* parallelism either partition the set of decisions, leading to problem decompositions, or conduct multiple concurrent searches on one or several search spaces. The simplest method of the latter kind, noted as *parallel independent multi-search*,

involves to gather the best final solution of a set of methods not linked by any communication or information exchange. This parallel implementation of the multi-start strategy can offer very interesting performances for the CVRP. Yet, to fully profit from parallelism, more advanced *cooperation schemes* integrate mechanisms to share information during the course of the methods and, in the most advanced settings, to create new information out of the exchanged data. Thus, the nature of the information shared, the frequency of the communications, and the scope (utilisation) of the received information are the main characteristics of cooperation strategies.

For the CVRP, as for most combinatorial optimization cases, the most efficient parallel metaheuristics are built on asynchronous communications, triggered individually by the cooperating algorithms, and often taking the form of exchanges of solutions or elements of solutions. Most multi-search strategies are based on either *adaptive* (Rochat and Taillard 1995, Badeau et al. 1997) or *central memory* (Rego 2001, Jin et al. 2012, Cordeau and Maischberger 2012, Groër et al. 2011) principles. The former gathers promising solution fragments and constructs new solutions out of such fragments. Tabu searches improve these new solutions, and return the best found solutions to the memory. In central memory-based cooperation, participating solution methods, which may be metaheuristics, exact algorithms, or any other method, exchange solutions and, possibly, various other data, through a common data repository (the "central memory"). Thus, all information is always available on request to any of the cooperating processes and, moreover, can be used to generate new relevant information, e.g., new solutions, performance measures on solution components, promising areas of the search space, and so on. Currently, tabu search threads (in a hybrid setting for Cordeau and Maischberger 2012) cooperate in most central-memory methods proposed for the CVRP, while Groër et al. (2011) also added integer programming solvers.

Other parallel strategies arose in the field of evolutionary computation. According to fine-grained parallel ideas, individuals are arrayed according to some geometrical form (2-dimensional toroidal grid in Alba and Dorronsoro 2006) and interact only with the (four, in this case) individuals directly connected to. This sets up a diffusion mechanism of good individual characteristics throughout the population. GA cooperation is generally built according to a coarse-grained strategy, where populations evolve separately and cooperate through migrations of elite solutions (e.g., Dorronsoro et al. 2007).

Doerner et al. (2006) performed extensive sensitivity analyses on several cooperative ACO metaheuristics that communicate synchronously through exchanges of solutions, ants, or pheromones. Experiments reproduce the results obtained for the parallel strategies for other metaheuristics, and show that parallel methods tend to outperform sequential ones, that sharing populations of elite solutions is more relevant than solely broadcasting the global best solution, and that episodic re-initializations of pheromone matrices contribute towards a better search. Furthermore, exploiting the spatial decomposition of Reimann et al. (2004) in a parallel context leads to increased speed-ups.

1.4.4 Relative performance of CVRP heuristics

Two main sets of instances have been widely used in the literature to compare the performance of heuristics in the last 30 years of research on the CVRP. The 14 benchmark instances of Christofides

et al. (1979) include between 50 and 199 customers, which are spatially randomly distributed for the first 10 instances, and otherwise clustered. The 20 large-scale instances (pr01-pr20) of Golden et al. (1998b), include between 200 and 483 customers and present geometric symmetries. Comparisons of VRP approaches are often based on the quality of the solutions in presence of *similar* computational effort. Nowadays, many metaheuristics reach systematically the best known solution (BKS) on almost all instances from Christofides et al. (1979). A comparison of state-of-the-art metaheuristics based on this benchmark tends to be less statistically significant, as only slight differences on 3 or 4 instances are now reported. To state the best performing methods, we therefore rely on the larger scale instances of Golden et al. (1998b), for which the results of well-performing approaches remain significantly different.

We start by emphasizing a number of good practices for reporting results and establishing a fair comparison between CVRP heuristics. We also refer to Barr et al. (1995) for further discussions and insights. A performance report must contain at least an estimate of the solution quality and CPU time of the proposed method, as well as the computing environment: programming language, processor, operating system, compiler and compiler options. Any performance analysis should be presented with a minimum of statistical information. As such, for non-deterministic algorithms, the average solution quality and CPU time on several runs for each problem instance should be reported, as well as the standard deviation to assess both quality and robustness. For deterministic algorithms, we suggest either to use a large-enough set of instances, or shuffle the customer indices to perform several different runs. A heuristic must also not require any other information than the problem instance. Using exterior knowledge, e.g, an optimal solution to trigger termination, or some best known characteristics as a starting point (fleet size for example in presence of a fleet minimization objective), is not a fair practice.

The most standard search-effort measure in the CVRP literature considers the total CPU time, and thus not only the time to reach the best solution. Other conventions should be clearly stated. The factors of Dongarra (2011) are commonly used to assess the relative speed of two processors by scaling the CPU time of two algorithms run on different machines. These factors must be used with caution however, mostly to compare order of magnitudes, since differences in operating systems, compilers, and memory organization have a huge impact on the measures. The best alternative remains to compare on the same computer whenever possible. Reporting the best solutions of the method on K runs may also provide valuable insights on the potential of solution improvement. In that case, K must be specified, and the computation effort of the assimilated multi-start method is the CPU time for all runs. Similarly, the effort of a parallel method is evaluated as the sum of the CPU times on all threads and processors. The communication overhead, approximated as the difference between the real wall-clock time and the CPU time should also be reported.

Table 1.1 provides a brief comparative analysis of the performance of the best CVRP heuristics on the instances of Golden et al. (1998b). The solution quality is measured as the average Gap (%) to the current Best Known Solutions (BKS) in the literature for each instance. The CPU time is indicated in column "T". The scaled computational effort $T^{\#} = n_{\text{RUNS}} \times n_{\text{CPU}} \times T \times f(\text{CPU})$ is also reported, where $f(\text{CPU})$ stands for the CPU speed factor relatively to a Pentium IV 3.0 GHz

(Dongarra 2011). We insist on the fact that these scaled times provide only rough estimates of the computation effort.

For the sake of brevity, we restricted the comparison to methods providing detailed results on all 20 instances, and with an average Gap smaller than 2%. Algorithm performances are also presented graphically in Figure 1.3 relatively to the dual objective of solution quality and scaled computational effort.

Table 1.1: Best performing metaheuristics for CVRP on Golden et al. (1998b) instances

Acronym	**Reference**	**Approach**	**Runs**	**Gap**	**T(min)**	**CPU**	**T$^{\#}$(min)**
VCGLR11s	Vidal et al. (2012a) slow	Hybrid GA	Avg 10	0.161%	113	Opt 2.4G	92.7
VCGLR11f	Vidal et al. (2012a) fast	Hybrid GA	Avg 10	0.267%	34.8	Opt 2.4G	28.5
NB09	Nagata and Bräysy (2009b)	Hybrid GA	Avg 10	0.273%	35.6	Opt 2.4G	29.2
GGW11	Groër et al. (2011)	Para. R-to-R	Best 5	0.296%	5.00	8×Xe 2.3G	129
MB07s	Mester and Bräysy (2007) slow	EA+ELS	Single	0.327%	24.4	P IV 2.8G	22.4
ZK10	Zachariadis and K. (2010a)	GLS+Tabu	Avg 10	0.430%	40.5	T5500 1.6G	26.7
JCL11	Jin et al. (2011)	Guided Tabu	Avg 10	0.448%	47.1	5×Xe 2.66G	180
MM11	Marinakis and Marinaki (2011)	Bees mating	Best 50	0.560%	3.96	P-M 1.86G	117
JCL12	Jin et al. (2012)	Coop Tabu	Avg 10	0.600%	41.9	8×Xe 3.0G	330
P09	Prins (2009a)	GRASP+ELS	Single	0.630%	7.27	P-IV 2.8G	6.09
RDH04	Reimann et al. (2004)	ACO	Avg 10	0.930%	49.3	P-III 900M	7.05
T05	Tarantilis (2005)	Ad.M.+Tabu	Single	0.931%	45.5	P-II 400M	2.02
CM11	Cordeau and M. (2012)	Iter. Tabu	Avg 10	0.939%	31.3	Xe 2.93G	30.8
MM10	Marinakis and Marinaki (2010)	GA+PSO	Avg 50	0.987%	4.20	P-M 1.86G	2.48
DK07	Derigs and Kaiser (2007)	ABHC	Single	1.017%	113	Cel 2.4G	106
GGW10	Groër et al. (2010)	R-to-R + EC	Single	1.186%	1.28	Xe 2.3G	0.82
MB07f	Mester and Bräysy (2007) fast	EA+ELS	Single	1.230%	0.22	P-IV 2.8G	0.20
PR07	Pisinger and Ropke (2007)	ALNS	Avg 10	1.347%	10.8	P-IV 3.0G	10.8
LGW05	Li et al. (2005)	R-to-R	Single	1.390%	1.13	Ath 1.0G	0.33
MMP06	Marinakis et al. (2006)	Hybrid GA	Single	1.559%	3.44	P-III 667G	0.23
P04	Prins (2004)	Hybrid GA	Single	1.662%	66.6	P-III 1.0G	10.6

As shown in Table 1.1, many current state-of-the-art methods exploit neighbourhood-centred searches such as local-improvement heuristics, record-to-record, or tabu search. The best performances are achieved, however, by hybrid methods combining neighbourhood-centred search with collective intelligence and population concepts (MB07, NB09, VCGLR11), and by parallel cooperative methods based on tabu search and solution-recombination procedures (T05, JCL11, JCL12, GGW11). Fourteen algorithms produce solutions that are very close to the BKS, with deviations of less than 1%. Seven methods (VCGLR11s, VCGLR11f, MB07s, P09, T05, GGW10, MB07f) constitute a dominating set with regards to the bi-objective of quality and computational efficiency. The metaheuristics of GGW11, ZK10, JCL10, P09, CM11, LGW05 and P04, in particular, stand out by their simplicity and still produce results of remarkable quality. Some other early neighborhood-centred metaheuristics, such as Golden et al. (1998b) and Toth and Vigo (2003), are also noteworthy for the same reasons, yielding average gaps of 3 to 4% with older and slower computing environments. Finally, even larger instances including thousands of customers have been tackled by LGW05, MB07, as well as Kytöjoki et al. (2007), thus filling the need for quick methods for large CVRPs.

In light of the methods presented in this section, the "traditional" CVRP is remarkably well addressed heuristically, at least for the classic benchmark instances, and several metaheuristic

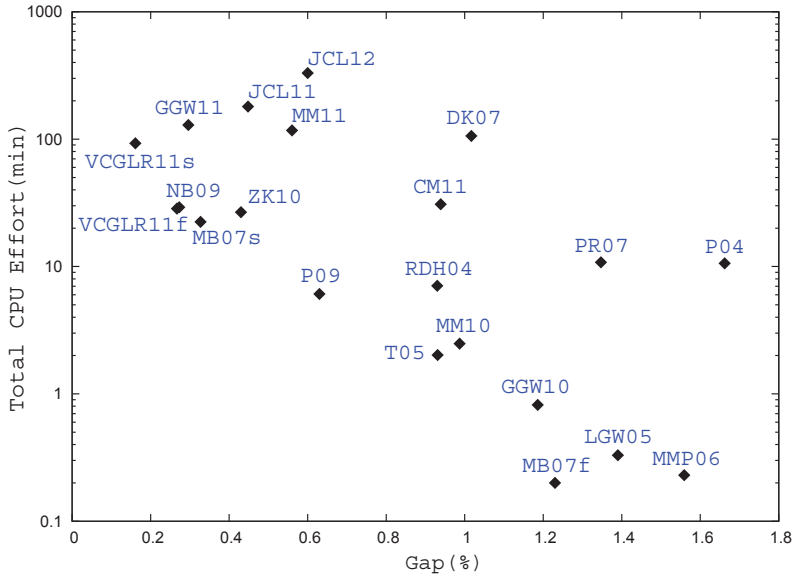


Figure 1.3: State-of-the-art CVRP methods: solution quality and scaled computation time

frameworks seem adequate to lead to high-quality solutions. The problem remains still an interesting testing ground for many studies aimed towards more efficient local search methods, new metaheuristic concepts, hybrid and cooperative methods.

In addition, the literature shows that more emphasis has been put in recent years on VRPs with additional attributes, for which applications are still very challenging. In fact, although many of the methods presented in this section can be rapidly adapted to these VRP extensions, very few general-purpose methods are able to handle the wide range of MAVRPs, and particular methods targeting individual cases were generally proposed. The objective of the following sections is to present and classify the main MAVRP attributes, and analyse the concepts of successful heuristics, as a first step on the path toward the development of agile methods able to handle a greater variety of VRP variants.

1.5 MAVRP: Classification and State-of-the-Art Heuristics

Most VRP attributes directly derive from the requirements of real applications. They are the subject of a vast amount of studies, grouping several thousands of articles. Several classification systems for VRP attributes have been proposed. Bodin (1975) and Bodin and Golden (1981) inventoried about a dozen attributes of VRP related to service times. With the same outlook, Ronen (1988) proposed a similar taxonomy centred on practical problem characteristics, and discussed the challenges of the solution methods. Desrochers et al. (1990) introduced a more complete classification system, as well as a four-field notation inspired by Graham et al. (1979). This notation served as a support for the creation of a management system for algorithms and models, based on inference mechanisms for selecting or producing appropriate solution methods (Desrochers et al. 1999). Eksioglu et al. (2009) provided the most complete taxonomy of the

MAVRP literature, integrating most common variants as well as several general observations on the nature of the articles. This taxonomy was accompanied by bibliometric data, illustrating the growth in the number of articles, the main authors, subjects, and journals. In addition to the previously listed taxonomies, other papers proposed thematic literature reviews of routing problems (Assad 1988, Desrosiers et al. 1995, Bräysy et al. 2008b,c, Andersson et al. 2010). The annotated bibliographies by Laporte and Osman (1995) and Gendreau et al. (2008b) are also noteworthy, providing pointers towards hundreds of articles dealing with MAVRPs and other related problems.

Unlike scheduling, however, where the classification system of Graham et al. (1979) is still used and updated, none of the previously-listed classification systems has been used on a large scale in the vehicle routing literature. This is probably connected to the large variety of attributes, which makes all exhaustive taxonomies extremely difficult. In addition, most of the previous classifications are application-oriented, based on the actual origin of the different types of constraints (related to, e.g., particular vehicle, network, driver or customers requirements). Moreover, even though these classification systems permitted the organization of the different attributes and contributions, few hints were given as to what heuristic concepts to privilege for the different types of attributes. Thus, the objective of the next section is to fill this gap with a new classification which, even though rather simple, emphasizes the relationships between problem attributes and recent heuristics.

1.5.1 Three main classes of attributes

To analyse the concepts of MAVRP heuristics, we distinguish three main classes of attributes, relative to their impact on three aspects of the problem that must imperatively be dealt with by solvers: the *Assignment of customers and routes to resources* (**ASSIGN**), the *Sequence choices* (**SEQ**), and the *Evaluation of fixed sequences* (**EVAL**). This simple classification is intimately connected with the resolution methodologies, as dealing with these three problem aspects leads to a complete solution method. Please note that an attribute may impact several aspects of the problem, and thus possibly appear in several categories.

ASSIGN attributes impact the assignment of limited resources, e.g., vehicles, vehicle types, depots, and service periods over a planning horizon, to customer services and routes. Most common ASSIGN attributes include *multiple depots*, *heterogeneous fleets*, *multiple periods*, *split deliveries*, *site dependencies*, *inventory*, *location* and *profits collection*. Furthermore, two main sub-families of ASSIGN attributes may be distinguished. Some variants of the VRP, e.g., multiple depot or heterogeneous VRP, are concerned with the assignment of *resources to routes*. In these settings, an entire route can potentially be re-assigned. In other cases, such as the VRP with multiple periods (PVRP) or inventory routing, the assignment issues are performed on *resources to customers*, and re-assigning a full route is likely to be infeasible due to the independent assignment constraints.

SEQ attributes directly impact the nature and structure of the routes. In a *backhaul* setting, for example, the route is a compound of two sequences of linehaul and backhaul services, respectively. In presence of *multiple trips* or *intermediate facilities*, the routes pass several times at depots, while in the *generalized* VRP, groups of customers are defined, and only one visit per group is performed. In *truck-and-trailer* problems, the routes involve sections with and without trailer. Finally, some

other SEQ attributes are related to specificities of the graph, e.g., routing on a *tree* or a *shoreline*, and dramatically impact the structure of the routes and the required sequencing methods.

Finally, EVAL attributes impact a large variety of evaluations and constraint checks that must be performed once the route contents and orders are chosen, including the optimization of the remaining variables, such as service times for problems with time characteristics, idle-time and break placement, speed choices, or the explicit consideration of product placement in trucks. The literature is extremely rich on attributes of this kind, some of the most common being *time windows*, *time-dependent route durations or costs*, *loading constraints*, *open routes*, and *working regulations*. The wide majority of EVAL attributes are inherent to *separate* routes, and thus the evaluations of routes can still be performed independently in the related VRP variants. However, there are also some *linking* EVAL attributes, such as *synchronization*, which link together the fixed-route evaluations, and result in very challenging problems.

Separating attributes among the three previously described categories allows to emphasize relationships between problems, and also estimate the solution method adjustments necessary to deal with them. For example, an EVAL attribute may be managed by an existing algorithm completed with appropriate sequence evaluation methods, while maintaining the resource assignment and sequence creation procedures. In a similar manner, an ASSIGN attribute may be tackled with new assignment procedures without impacting the route evaluations.

Table 1.2: Some frequently encountered attributes in the literature

ASSIGN		SEQ		EVAL	
Multi-Depots	Ombuki-B. and H. (2009)	**Backhauls**	Parragh et al. (2008a)	**Open**	Li et al. (2007b)
Heterogeneous	Baldacci et al. (2008a)	**1→1 Pick&D.**	Cordeau et al. (2008)	**Time windows**	Gendreau and T. (2010)
Multi-Periods	Francis et al. (2008)	**Multiple Trips**	Salhi and Petch (2007)	**Time dependent**	Ichoua et al. (2003)
Split Deliv.	Archetti and Speranza (2011)	Multi-Echelons	Hemmelmayr et al. (2012)	**HOS Regulations**	Rancourt et al. (2012)
Prize Collect.	Vansteenwegen et al. (2010)	Truck & Trailer	Villegas et al. (2011)	**2D-3D Loading**	Iori and Martello (2010)
Location	Nagy and Salhi (2007)	Generalized	Baldacci et al. (2009)	Soft & Multiple TW	Ibaraki et al. (2005, 2008)
Site Dependent	Cordeau and M. (2012)	Graph specifics: Tree, Shoreline...	Chandran and R. (2008)	Duration Constr.	Savelsbergh (1992)
Inventory	Coelho et al. (2012)			**Other time feat.**	Vidal et al. (2011)
Consistency	Groër et al. (2008)			Cumulative costs	Ribeiro and Laporte (2012)
				Simult Pick & Deliv	Subramanian et al. (2010)
				Pollution/Green	Bektas and Laporte (2011)
				Synchronization	Drexl (2012)

Table 1.2 gathers attributes frequently encountered in the literature, displaying for each either a recent survey, or a paper proposing a solid literature review. Fourteen of these attributes, marked in boldface, were selected to serve as support to our study on MAVRP heuristics, relatively to two main criteria: 1) the resulting VRP variant is the subject of a significant literature, including exact and heuristic methods, and is possibly mentioned in specialized literature reviews; and 2) benchmark instances are available for comparisons between methods. The first criterion illustrates the importance of the variant in the domain, while the second guarantees that some remarkably efficient algorithms can be objectively selected. The 14 resulting variants are now briefly reviewed in Sections (1.5.2 - 1.5.4). In each case, we describe the respective MAVRP, the size of instances currently solvable with exact methods, the current classic benchmark instances, and a selection of well performing heuristics in terms of average solution quality. Note that "pure" versions of MAVRP with one attribute are studied, since otherwise the number of combinations of attributes is exponential, and since pure variants are covered by a wider literature.

1.5.2 Heuristics for VRP variants with ASSIGN attributes

Multiple depots. The multi-depot VRP (MDVRP) deals with a number of depots $d > 1$. Each vehicle is assigned to a single depot, which is generally both the origin and the destination of the vehicle's route. Some variants, called "non-fixed" problems, relax this latter requirement. Furthermore, in the classical MDVRP, no limit on supply at depots is considered. Recent elements of literature review can be found in Ombuki-Berman and Hanshar (2009) and Vidal et al. (2012a). The best exact method (Baldacci and Mingozzi 2009) can solve problem instances up to 75 customers, as well as a few instances with up to 199 customers. When considering metaheuristics, the best solutions on the classic instances presented in Cordeau et al. (1997) were produced by the ALNS of Pisinger and Ropke (2007), the fuzzy logic-guided hybrid GA of Lau et al. (2010), the ILS and set covering approach of Subramanian (2012), the Hybrid Genetic Search with Advanced Diversity Control (HGSADC) of Vidal et al. (2012a), and the parallel UTS of Cordeau and Maischberger (2012).

Heterogeneous fleet. Customers are assigned to vehicle types with different characteristics: capacity, maximum route times, fixed costs, and variable costs in terms of the distance. When the number of vehicles is not constrained, the problem is usually referred to as the *Vehicle Fleet Mix Problem (VFMP)*, otherwise the more difficult version is called Heterogeneous VRP (HVRP) (see Baldacci et al. 2008a, for a review). The exact algorithm of Baldacci and Mingozzi (2009) solves most problem instances with 75 customers or less, as well as some instances with 100 customers. The state-of-the-art metaheuristics, evaluated on the HVRP instances of Taillard (1999) and Li et al. (2007a), are of various kinds: tabu search (Brandão 2011), hybrid GA (Prins 2009b), or ILS and VNS (Penna et al. 2011) with set covering phases (Subramanian et al. 2012).

Multiple periods. A time dimension is introduced in the Periodic VRP (PVRP) as route planning is to be performed over a horizon of several periods. Each customer requires a total number of services according to some acceptable combinations of visit periods called *patterns*. The assignment of customer visits is thus subject to compatibility constraints with the patterns. The PVRP is reviewed in Francis et al. (2008). Exact methods (Baldacci et al. 2011a) are able to solve some instances with up to 100 customers and 6 time periods. Benchmark instances for PVRP metaheuristics are gathered in Cordeau et al. (1997). Several efficient neighbourhood-centred searches have been designed, such as UTS (Cordeau et al. 1997, 2001a) and its parallel extension (Cordeau and Maischberger 2012), the VNS of Hemmelmayr et al. (2009), and the hybrid record-to-record and integer programming matheuristic of Gulczynski et al. (2011). The population-based approach of Alegre et al. (2007), dedicated to large temporal horizons, focuses on assignment optimization, while using constructive methods to create routes. Also, the HGSADC of Vidal et al. (2012a) produces the current best solutions by combining the GA search breadth with efficient LS, relaxations schemes, and diversity management procedures.

The PVRP has led to several other notable problem extensions (Groër et al. 2008, Gulczynski et al. 2011). The issue of service consistency, i.e., visiting regular customers on each period at similar time and with the same driver, has received a notable attention. Recent methods dealing with this attribute are based on record-to-record travel (Groër et al. 2008), tabu search (Tarantilis et al. 2012), and ALNS (Kovacs et al. 2012). All these methods optimize a template of visits to

frequent customers, which remains the same on every day, in which some additional non-frequent deliveries are inserted.

Split deliveries. Customer demands can be satisfied by several vehicles, each moving a partial load. This variant is called VRP with split deliveries (VRPSD), as reviewed in Gulczynski et al. (2008) and Archetti and Speranza (2011). Instances with up to 50 customers (Belenguer et al. 2000, Lee et al. 2006) can be exactly solved. Recent metaheuristics have been evaluated with two different fleet-size policies on the benchmark instances of Archetti et al. (2006), Belenguer et al. (2000) and Chen et al. (2007). In the first setting, the fleet size is unlimited, and state-of-the-art methods rely on hybrid GA with giant-tour representation (Boudia et al. 2007), *Attribute Based Hill Climber* (ABHC) (Derigs et al. 2009), and integer programming optimization with tabu search (Archetti et al. 2008). In the second setting, a solution with minimum number of vehicles is imposed, and the best performances are achieved by the scatter search of Mota et al. (2007) and the tabu search with vocabulary building of Aleman and Hill (2010).

Prize collection. For several customers, service is optional but rewarded with a prize. Hence, customers must be implicitly distributed among two subsets, following whether their service is omitted or performed. Several objectives were dealt with in the literature, notably the optimization of a weighted sum of route lengths and prizes (Dell'Amico et al. 1995), or the maximization of the prizes under a route length constraint, usually called the *team orienteering problem* (see the reviews of Feillet et al. 2005 and Vansteenwegen et al. 2010). Exact methods can solve instances with up to 100 customers (Boussier et al. 2006). Most efficient metaheuristics, evaluated on the instance set of Chao et al. (1996), rely on population concepts. Ke et al. (2008) proposed a hybrid ACO method with a local search. Souffriau et al. (2010) introduced a path relinking method, in which elements of the solution set undergo an *ageing* process. Bouly et al. (2009) introduced a hybrid GA based on giant-tour solution representation, which is hybridized later on with PSO in Dang et al. (2011). Finally, Archetti et al. (2006) proposed a hybrid tabu search and VNS.

1.5.3 Heuristics for VRP variants with SEQ attributes

Backhauls. Customers are separated into two groups: delivery customers (i.e., *linehaul customers*) and pickup customers (i.e., *backhaul customers*). All routes mixing both groups of customers must serve all linehaul customers before the first backhaul customer, thus leading to different route structures. We refer to Toth and Vigo (2002b) and Parragh et al. (2008a) for detailed reviews on the VRPB. The classic instances were first introduced in Goetschalckx and Jacobs-Blecha (1989) and Toth and Vigo (1997). Some of these, with a maximum 100 customers, were solved exactly in Toth and Vigo (1997) and Mingozzi et al. (1999). The best metaheuristics, on the instance sets of Goetschalckx and Jacobs-Blecha (1989), include the ALNS of Ropke and Pisinger (2006a); the tabu search of Brandão (2006), which, as Zachariadis and Kiranoudis (2012), uses long-term memories to direct the search toward inadequately exploited characteristics; and finally the ACO of Gajpal and Abad (2009), which concurrently evolves two ant families to work on assignment and sequences.

Pickups & Deliveries. Each service is characterized by a pair of locations designating the pickup and delivery spots. All pickups must be made before the deliveries. This type of problem is

dealt with in numerous literature reviews as a *one-to-one Pickup and Delivery Problem (PDP)* (Desaulniers et al. 2002, Berbeglia et al. 2007, Cordeau et al. 2008, Berbeglia et al. 2010) or simply as *VRP with Pickup and Deliveries* (VRPPD) (Parragh et al. 2008a,b). This problem is often coupled with time-window constraints. Ropke et al. (2007) solved exactly instances involving up to 96 requests. The classic benchmark instances have been introduced in Li and Lim (2001). Efficient neighbourhood-centred metaheuristics have been proposed, including the ALNS of Ropke and Pisinger (2006b) and the two-phase method of Bent and Van Hentenryck (2006), which combines simulated annealing (SA) to reduce the number of routes with large neighbourhood search (LNS) to optimize the distance. These methods were recently outperformed by the memetic algorithm of Nagata and Kobayashi (2011), which exploits a well-designed crossover focused on transmitting parent characteristics without introducing too many new arcs in the offspring. For problem variants arising from the domain of transportation on demand, the so-called *dial-a-ride problems*, UTS (Cordeau and Laporte 2003) and the VNS of Parragh et al. (2010) produce solutions of good quality.

Multiple trips. During its tour, a vehicle can reach several times the depot to load or unload. By doing so, the global constraints on the routes, such as the maximum duration, are still considered. The exact method of Mingozzi et al. (2012) can solve some instances of multi-trip VRP (MTVRP) with up to 120 customers. Three algorithms produce on average the largest number of feasible solutions, or the solutions with least amount of infeasibility on the classic instances of Taillard et al. (1996). The original tabu search and adaptive memory approach of Taillard (1993) remain still competitive. In addition, good results have been obtained with the adaptive memory-based search of Olivera and Viera (2007), and by Alonso et al. (2008), who generalized UTS to a PVRP with multiple trips and vehicle-customer compatibility constraints.

1.5.4 Heuristics for VRP variants with EVAL attributes

Time windows. The VRP with time windows (VRPTW) is certainty the most extensively studied VRP variant to date. Time windows are associated to customer visits and depot, each arc being characterized by a route duration. Waiting time is allowed upon an early arrival to a customer, while a late arrival is forbidden. Recent literature reviews can be found in Bräysy and Gendreau (2005b,a) and Gendreau and Tarantilis (2010). The classic VRPTW instances were introduced in Solomon (1987) and Gehring and Homberger (1999). Most efficient exact methods (Kallehauge et al. 2006, Jepsen et al. 2008, Baldacci et al. 2011c) can solve most instances with up to 100 customers, and a few instances with up to 1000 customers. However, exact resolution is highly dependent upon the characteristics of the instance and the width of time windows. Actual state-of-the-art VRPTW metaheuristics are of various kinds. The guided EA of Repoussis et al. (2009b) combines evolution, ruin-and-recreate mutations, and guided local search. Prescott-Gagnon et al. (2009) proposed a LNS combined with branch-and-price for solution reconstruction. The HGA proposed by Nagata et al. (2010) uses a particularly effective crossover operator. This latter method, as well as the path relinking of Hashimoto and Yagiura (2008) and HGSADC of Vidal et al. (2013), apply time-constraint relaxations during the search to benefit from infeasible solutions in the search space.

Time-dependent. In practical settings, when facing network congestion especially, travel times on an arc depend on the departure date, leading to a *Time-Dependent VRP* (TDVRP). This problem is frequently combined with time-window constraints, and a *First-In, First-Out (FIFO)* property for the travel times is frequently assumed, meaning that a vehicle starting earlier arrives at its destination earlier. Specialized literature reviews were conducted by Malandraki and Daskin (1992), Ichoua et al. (2003) and Fleischmann et al. (2004). Among the particularly efficient heuristics, the adaptive memory search of Ichoua et al. (2003) manages a population of good-quality routes, which are recombined and improved by tabu search. The ILS of Hashimoto et al. (2008) draws its strength from a temporary relaxation of the problem combined with efficient neighbourhood evaluation procedures. Balseiro et al. (2011) proposed a cooperative ACO, hybridized with local searches and ejection chains, which rely on two ant colonies to perform respectively fleet-size and distance minimization. The classic benchmark instances originate from Ichoua et al. (2003) and Balseiro et al. (2011).

Other time attributes. Several other time attributes on routes have been introduced in the literature, such as speed choices, waiting-time constraints, and multiple time windows, time-dependent service costs, or the minimization of the average time to reach customers, also called *cumulative VRP* (CCVRP). All these variants require determining the service times to customers for the routes produced during the search in order to evaluate their cost and feasibility. The resulting sub-problems, called *optimal start time problems* or *timing problems*, are reviewed in Hashimoto et al. (2010), and in Vidal et al. (2011) within a multidisciplinary unifying framework. Some ILS heuristics allowed to address effectively MAVRP with general or convex piecewise linear service costs as a function of service times (Ibaraki et al. 2005, 2008), and with flexible travel time (Hashimoto et al. 2006). These three heuristics are based on remarkably efficient move evaluations for the problems considered. For the CCVRP, Ngueveu et al. (2010) and Ribeiro and Laporte (2012) successfully extended the hybrid GA with giant tour representation of (Prins 2004), and the ALNS of Pisinger and Ropke (2007), respectively.

Hours of service regulations. Regulations related to long-distance transportation impose complex rules for driving time and driver breaks. Combining the VRP with break scheduling leads to difficult route feasibility checks. The recent literature on this subject is mainly oriented on the laws in the United States and the European Union. When considering a fixed sequence of visits, the break scheduling sub-problem can be solved exactly in $O(n^2)$ for the laws of the U.S. (Goel and Kok 2012). For the E.U. laws, the complexity of the resulting problem has not yet been determined (Goel 2010). Most metaheuristics have been compared on the benchmark instances of Goel (2009) with E.U. regulations. Since routes are costly to evaluate, neighbourhood-centred approaches are usually preferred. Both Goel and Kok (2012) and Prescott-Gagnon et al. (2010) rely on LNS, the latter method using integer programming for visit reinsertions. Rancourt et al. (2012) designed a tabu search to address the U.S. regulations with multiple time windows. Finally, Goel and Vidal (2012) proposed an efficient hybrid genetic algorithm to address a wide range of regulations.

2D and 3D loading constraints. Less-than-truckload routing activities are the source of a large range of constraints related to the 2D and 3D packing of objects (*2L-CVRP and 3L-CVRP*), and their effective loading and unloading. These attribute lead to intricate problems that mix both

multi-capacity bin-packing and VRP. Classic 2L-CVRP and 3L-CVRP benchmark instances have been proposed in Gendreau et al. (2008a) and Gendreau et al. (2006). The most effective heuristics for the 2L-CVRP include the ACO of Fuellerer et al. (2009), and the GRASPxELS of Duhamel et al. (2011) which solves a problem relaxation as a project scheduling problem with resource constraints, and yields the current best solution quality. For the 3L-CVRP, the best current methods are based on tabu search combined with advanced packing heuristics (Bortfeldt 2012, Zhu et al. 2012). Other lines of research consider the explicit placement of different products in different compartments, and the transportation of hazardous material, with additional constraints related to product incompatibility and spacing (Iori and Martello 2010).

Open. Related to the invoicing practices of road transportation suppliers, the last return to the depot is not counted towards the transportation costs in the *Open VRP (OVRP)*. This variant has been reviewed by Li et al. (2007b). Currently, the exact method of Letchford et al. (2006) can solve problems with up to 100 customers. The OVRP is very similar to the "traditional" CVRP from the point of view of a heuristic approach, and a lot of effective methods are adaptations of metaheuristics originally intended for the CVRP. These approaches have been tested on the classic CVRP instances of Christofides et al. (1979) and Golden et al. (1998b), in which the tour-length limits, when applicable, have been reduced by 10%. Among the best methods, we find the tabu search with route-evaluations memories of Zachariadis and Kiranoudis (2010b), and the VNS of Fleszar et al. (2009). High performance was also achieved by the hybrid EA of Repoussis et al. (2010) and the ILS-VNS and integer programming hybrid of Subramanian (2012).

1.6 A Synthesis of "Winning" MAVRP Strategies

In the previous sections, champion methods were identified for 14 different MAVRPs and for the classic CVRP. All together, these approaches constitute a set of 64 successful algorithms for 15 different MAVRP, which are "anatomized" in the following. The analysis we develop is backed by quantitative observations on the frequency of appearance of elements of methodology in the successful approaches. One drawback of quantitative evaluations is that they favor seminal widespread approaches over single path breaking papers. Thus, detailed discussions on alternative strategies, even when represented a single time among the 64 methods, are presented to complete the analysis.

Table 1.3: Main metaheuristic concepts used in the 64 winning methods

Neighbourhood-centred	**Freq.**	**Population-based**	**Freq.**
Tabu Search	17	Genetic or Evolutionary Algorithm	16
Iterated Local Search	7	Ant Colony Optimization	4
Variable Neighbourhood Search	5	Scatter Search	2
Adaptive Large Neighbourhood Search	4	Path Relinking	2
Simulated Annealing & Record-to-record	3	Particle Swarm Optimization	1

Table 1.3 first surveys the main metaheuristic frameworks used in the 64 algorithms. These methods are visibly of various natures, neighbourhood- and population-based methods tending to

be equally represented, contrasting with claims (frequent in the literature) for a best metaheuristic type. As such, different metaheuristic frameworks may lead to state-of-the-art algorithms for some variants when cleverly designed and complemented by adequate diversification and intensification strategies.

To better understand which elements of methodology make these particular applications a success, we examine in detail 19 selected characteristics, presented in Table 1.4. Tables 1.5 and 1.6 then provide a summary of our analysis, each line being associated to a method, and each of the 19 columns (3 - 21) corresponding to a feature that is potentially present. An X sign where line i meets column j indicates that method i relies on concept j. The rest of this section details how these features are used in the 64 state-of-the-art metaheuristics under consideration.

Table 1.4: Fundamental metaheuristic features

Search space	1) presence of infeasible solutions
	2) use of indirect representations of solutions
Neighbourhoods	3) presence of multiple neighbourhoods
	4) use of polynomially enumerable neighbourhoods
	5) use of pruning procedures
	6) use of large neighbourhoods
	7) use of solution recombinations
Trajectory	8) presence of random components
	9) continuous aspect of trajectories
	10) discontinuous aspect
	11) mixed aspect
Control and memories	12) use of populations
	13) diversity management
	14) parameter adaptation
	15) advanced guidance mechanisms
Hybrid strategies	16) use of hybridization
	17) matheuristics with integer programming
Parallelism	18) use of parallelism or cooperation concepts
Problem decompositions	19) use of problem decompositions

Search Space. Metaheuristics are generally described relatively to the concept of search space, that is, a set of solutions, or more generally a set of states describing solutions, in which the method evolves. Basing the search-space definition on solutions is appropriate for the CVRP. For many MAVRPs, however, defining a complete solution goes beyond route description, as additional decisions related to attributes must be specified. Many metaheuristics are then designed to explore a search space made of indirect representations of solutions, containing, for example, only the route information, on which an efficient *decoder* algorithm can be applied to extract one or several complete solutions. This widely applied methodology is in itself a structural problem decomposition.

In the heuristics surveyed, 12/64 methods rely explicitly on indirect solution representations and decoders. The resulting search spaces may then be smaller and more prone to lead to high quality solutions. A well-known example is the representation of Prins (2004) as a *giant tour without trip delimiters*, used in many of the selected GAs (Boudia et al. 2007, Prins 2009b, Ngueveu

Table 1.5: Successful metaheuristics for CVRP and MAVRPs with ASSIGN attributes

		SP.		NEIGHBOUR.					TRAJEC.				CONTROL							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		RELAXATION	SOL. REPRESENT.	MULT NEIGHB.	ENUMERATIVE	PRUNING	LARGE	RECOMBIN.	RANDOMNESS	CONTINUOUS	DISCONTINOUS	MIXED	POPULATIONS	DIV. MANAG.	PARAM. ADAPT.	GUIDANCE	HYBRIDIZATION	MATHEUR.	PARALLELISM	DECOMPOSITION
CAPACITATED VRP																				
Mester and Bräysy (2007)	Guided EA + ELS			X	X	X	X		X	X					X	X	X			
Nagata and Bräysy (2009b)	HGA	X		X	X	X		X	X		X		X				X			
Zachariadis and K. (2010a)	Tabu			X	X	X			X	X						X				
Groër et al. (2011)	Parallel R-to-R			X	X	X			X	X	X	X	X		X		X	X	X	X
Vidal et al. (2012a)	HGA + Div.Man.	X	X	X	X	X		X	X		X		X	X	X		X			
MULTIPLE DEPOTS																				
Pisinger and Ropke (2007)	ALNS	X		X			X		X	X					X	X	X			
Lau et al. (2010)	Genetic							X	X		X		X	X	X					
Subramanian (2012)	ILS + SP			X	X			X	X	X	X	X	X		X		X	X		
Vidal et al. (2012a)	HGA + Div.Man.	X	X	X	X	X		X	X		X		X	X	X		X			
Cordeau and M. (2011)	Parallel Tabu	X		X	X		X	X	X	X					X	X			X	
HETEROGENEOUS FLEET																				
Prins (2009b)	HGA		X	X	X			X	X		X		X	X			X			
Brandão (2011)	Tabu	X		X	X	X			X	X					X		X			
Penna et al. (2011)	ILS + VNS			X	X				X	X							X			
Subramanian et al. (2012)	ILS + SP			X	X			X	X	X	X	X	X		X		X	X		
MULTIPLE PERIODS																				
Alegre et al. (2007)	Scatter Search		X	X	X			X	X		X		X	X						X
Hemmelmayr et al. (2009)	VNS	X		X	X				X	X					X					
Gulczynski et al. (2011)	Rec-to-Rec + IP			X	X		X		X	X							X	X		
Vidal et al. (2012a)	HGA + Div.Man.	X	X	X	X	X		X	X		X		X	X	X		X			
Cordeau and M. (2011)	Parallel Tabu	X		X	X		X	X	X	X					X	X			X	
SPLIT DELIVERIES																				
Boudia et al. (2007)	HGA		X	X	X			X	X		X		X	X			X			
Derigs et al. (2009)	ABHC			X	X					X						X				
Archetti et al. (2008)	Tabu + IP			X	X				X	X						X	X	X		
Mota et al. (2007)	Scatter Search			X	X			X			X		X							
Aleman and Hill (2010)	Tabu + Voc Build			X	X	X		X		X	X	X	X			X	X			
PRIZE COLLECTING																				
Archetti et al. (2006)	Tabu + VNS	X		X	X				X	X	X	X			X		X			
Ke et al. (2008)	ACO				X				X		X					X	X			
Souffriau et al. (2010)	Path Relinking			X	X			X	X		X		X			X				
Bouly et al. (2009)	HGA + LNS		X	X	X		X	X	X		X		X	X			X			
Dang et al. (2011)	PSO + HGA		X	X	X		X	X	X		X		X	X		X	X			

et al. 2010), and in the GRASP+ELS of Duhamel et al. (2011). In this case, the optimal insertion of depot visits in the tour can be solved in a quadratic number of route evaluations with a shortest path-based *Split* procedure. In Alegre et al. (2007), solutions are characterized exclusively by decisions on assignments to time periods. The role of the decoder is assumed by a VRP algorithm (a quick constructive method in this case) that creates the routes for each period separately. Other decoding methods can be found in the literature. For example, Salhi and Petch (2007) rely on an abstract solution representation in the form of circular sectors is used. Decoding is performed by means of a cluster-first route-second heuristic that relies on the sectors for the clustering phase.

Table 1.6: Successful metaheuristics for MAVRPs with SEQ and EVAL attributes

		SP.		NEIGHBOUR.					TRAJEC.				CONTROL							
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
		RELAXATION	SOL. REPRESENT.	MULT NEIGHB.	ENUMERATIVE	PRUNING	LARGE	RECOMBIN.	RANDOMNESS	CONTINUOUS	DISCONTINOUS	MIXED	POPULATIONS	DIV. MANAG.	PARAM. ADAPT.	GUIDANCE	HYBRIDIZATION	MATHEUR.	PARALLELISM	DECOMPOSITION
BACKHAULS																				
Brandão (2006)	Tabu	X		X	X				X	X					X	X				
Ropke and Pisinger (2006a)	ALNS	X		X			X		X	X					X	X	X			
Gajpal and Abad (2009)	ACO			X	X	X			X		X					X	X			
Zachariadis and K. (2012)	Attrib. driven LS			X	X	X				X						X				
PICK-UP AND DELIVERIES																				
Bent and V.H. (2006)	SA + LNS			X	X	X	X		X	X							X	X		
Ropke and Pisinger (2006b)	ALNS	X		X			X		X	X					X	X	X			
Cordeau and Laporte (2003)	Tabu	X			X	X				X					X	X				
Parragh et al. (2010)	VNS	X		X	X		X		X	X					X					
Nagata and Kobayashi (2011)	HGA			X	X	X	X	X	X		X		X			X	X			
MULTIPLE TRIPS																				
Taillard et al. (1996)	Adapt. M. + Tabu	X		X	X			X	X	X	X	X	X			X	X		X	X
Olivera and Viera (2007)	Adapt. M. + Tabu	X		X	X	X		X	X	X	X	X	X			X	X			
Alonso et al. (2008)	Tabu	X			X					X					X	X				
TIME WINDOWS																				
Hashimoto et Y. (2008)	Path Relinking	X		X	X	X	X	X	X		X		X		X	X				
Repoussis et al. (2009b)	Guided EA			X	X	X	X	X	X	X	X	X	X	X	X	X	X			
P.-Gagnon et al. (2009)	LNS + Col. Gen.			X			X		X	X							X	X		
Nagata et al. (2010)	HGA	X		X	X	X		X	X		X		X				X			
Vidal et al. (2013)	HGA + Div.Man.	X	X	X	X	X		X	X		X		X	X	X		X			X
TIME DEPENDENT																				
Ichoua et al. (2003)	Adapt. M. + Tabu			X	X			X	X	X	X	X	X			X	X		X	X
Hashimoto et al. (2008)	ILS	X		X	X	X			X	X										
Balseiro et al. (2011)	ACO	X		X	X	X			X		X					X	X		X	
OTHER TIME FEATURES																				
Ibaraki et al. (2005, 2008)	ILS	X		X	X	X	X		X	X										
Ngueveu et al. (2010)	HGA		X	X	X			X	X		X		X	X			X			
Ribeiro and Laporte (2012)	ALNS	X		X			X		X	X					X	X	X			
HOURS OF SERVICE REGULATIONS																				
Goel and Kok (2012)	LNS			X	X		X		X	X										
P.-Gagnon et al. (2010)	LNS + Col. Gen.			X			X		X	X							X	X		
Rancourt et al. (2012)	Tabu	X		X	X					X					X					
Goel and Vidal (2012)	HGA + Div.Man.	X	X	X	X	X		X	X		X		X	X	X		X			
2D & 3D LOADING CONSTRAINTS																				
Fuellerer et al. (2009)	ACO	X		X	X				X		X				X	X				X
Duhamel et al. (2011)	GRASP + ELS	X	X	X	X				X	X	X	X								
Zhu et al. (2012)	Tabu	X		X	X				X	X					X					
Bortfeldt (2012)	Tabu			X	X	X				X	X									
OPEN VRP																				
Fleszar et al. (2009)	VNS	X		X	X				X	X										
Repoussis et al. (2010)	Guided EA			X	X	X	X	X	X	X	X	X	X	X	X	X	X			
Zachariadis and K. (2010b)	Tabu			X	X	X			X	X						X				
Subramanian (2012)	ILS + SP			X	X			X	X	X	X	X	X		X		X	X		
		31	12	60	57	26	20	29	56	42	35	12	28	14	30	29	39	9	6	6

Finally, multiple structurally different search spaces, relying on different (representations, decoder) pairs, may efficiently reduce the risks of getting trapped in a local optimum.

Another main characteristic of the search space comes from the potential use of infeasible solutions. Since the early literature on tabu search with the *strategic oscillation* concept (Glover 1986, Glover and Hao 2011), studies report that a controlled exploitation of infeasible solutions may enhance the search, by allowing it to transition more easily between structurally different feasible solutions. Furthermore, the use of infeasible solutions may contribute toward improving the *robustness* of the method, which is less dependent upon the availability of a feasible initial solution (finding a feasible solution is often in itself a NP-hard problem).

About half of the selected MAVRP heuristics (31/64) rely on penalized infeasible solutions in the search space, which violate either the route constraints (load, duration, or time windows), the fleet size limit, or do not service all customers. Moreover, iteratively decrementing the fleet size limit while relaxing route constraints provides the means to address the "fleet size minimization" objective without relying on complex route elimination procedures. Elements of sensitivity analyses on the role of infeasible solutions in the context of PVRP and MDVRP can be found in Vidal et al. (2012a). In the methods surveyed, relaxations of route constraints are usually privileged over fleet-size relaxations, as it can be difficult to progress from a solution with too many routes to a feasible solution.

Neighbourhoods. With the exception of some methods that use exclusively large neighbourhoods, and the GA of Lau et al. (2010) which appears to rely exclusively on crossover and random mutation, all mentioned MAVRP heuristics are based on at least one type of *enumerable* neighbourhood using the arc exchanges described in Section 1.4.2. The size of these enumerable neighbourhoods is usually $O(n^2)$ in practice. Exponentially large neighbourhoods are also frequently used (20/64). Besides ruin-and-recreate neighbourhoods or perturbation mechanisms that are well represented in recent methods, cyclic transfers or ejection chains are also used (Ibaraki et al. 2005), as well as variants of the Sarvanov-Doroshko IP refinement heuristic (Gulczynski et al. 2011). Finally, 29/64 methods combine solutions, or fragments of solutions, into new solutions, thus transmitting good sequence elements as the evolutionary, genetic, scatter search and path relinking algorithms do. Not only GA and EA use these mechanisms. Consider for example the adaptive memory approaches of Taillard et al. (1996), Ichoua et al. (2003) and Olivera and Viera (2007), which operate recombinations of solution fragments, and the set covering based approach of Groër et al. (2011) involving recombinations of routes issued from multiple solutions.

Almost all the methods surveyed (60/64) rely on multiple neighbourhoods, either successively, or in a compound way. The successive exploration of multiple neighbourhoods makes the basis of the VNS methodology, and is recognized as an important success factor for metaheuristics in general, especially on complex problems with multiple constraints and characteristics such as MAVRPs. The methods of Archetti et al. (2006) and Parragh et al. (2010) push very far the concept of neighbourhood variation by exploiting structurally-different, enumerative and large, neighbourhoods. Other VNS for MAVRPs may gradually increase the size of the enumerative neighbourhoods by varying the number of arcs to be exchanged (Hemmelmayr et al. 2009), but, strictly speaking, do not involve structural neighbourhood differences.

Searching efficiently these neighbourhoods is critical for performance, as it generally makes for the biggest part of the computation effort. Therefore, many techniques aim at pruning the neighbourhoods (26/64 algorithms), or at enumerating them more efficiently. Move restrictions based on customer neighbourhood lists (granular search of Toth and Vigo (2003)) are frequently used (Ibaraki et al. 2005, Mester and Bräysy 2007, Olivera and Viera 2007, Hashimoto and Yagiura 2008, Vidal et al. 2012a), as well as neighbourhood limitation strategies based on recently modified solution features (Nagata and Bräysy 2008, 2009b, Nagata et al. 2010, Nagata and Kobayashi 2011). In the presence of EVAL attributes, re-optimization information developed on subsequences of successive customers can increase the efficiency of neighbour evaluations (Kindervater and Savelsbergh 1997, Cordeau and Laporte 2003, Nagata et al. 2010, Vidal et al. 2013). Lower bounds, multi-phase (feasibility-first or cost-first for example) or approximate evaluations of neighbours can be used to reduce complexity (Ichoua et al. 2003, Bortfeldt 2012).

Memories of previous computations, aimed at reducing computational redundancy without changing the method behaviour, are also frequently used. Although such procedures may be viewed as a matter of algorithmic engineering, and thus not necessarily mentioned, they are critical to reach a good performance, especially on problems for which route evaluations are costly such as the 2L- or 3L-CVRP, or the VRP with break scheduling. Most common memories of this kind are dedicated to manage move informations (Cordeau and Laporte 2003, Alegre et al. 2007, Zachariadis and Kiranoudis 2010a, Vidal et al. 2012a) and route evaluations (Duhamel et al. 2011). Addressing all the attributes of the problem with well-designed neighbourhood-centred searches is, and should remain, a primary concern when addressing complex MAVRPs.

Search trajectories. The inclusion of random components in the various algorithm choices, mentioned explicitly in 56/64 methods, is a dominant characteristic of search trajectories. Randomisation is a prerequisite of asymptotic convergence properties of metaheuristics such as SA or GA. In practice, however, it is mostly used as a simple and efficient way to avoid cyclic behaviour and increase the diversity of solutions. Only a few current methods for MAVRPs are deterministic. For example, although tabu search has been first built on deterministic arguments (Glover 1986), recent applications involve random diversification operations, or tabu lists whose sizes vary probabilistically. Adding random noise to the objective function, as in Pisinger and Ropke (2007), is another way to exploit randomization to diversify the search.

The amount of change from one solution to the next is also characteristic of the methods. In neighbourhood-centred methods, successive solutions tend to be in close proximity, sharing many common elements. This kind of trajectory can be qualified as continuous, unlike the trajectories of most population-based metaheuristics with crossovers, which are discontinuous, and display a "jumpy" behaviour between successive solutions. Finally, mixed trajectories, combining continuous search and jumps, aim to profit from both kinds of exploration.

We identified 42/64 methods that use mostly a continuous trajectory, and 35/64 methods that often use discontinuous trajectories. Twelve algorithms use mixed trajectories, with large continuous search phases as well as regular jumps. These are neighbourhood-centred metaheuristics that include mechanisms to change abruptly the search region by "jumping" to an elite solution (e.g., Archetti et al. 2006, Groër et al. 2011), during GRASP restarts (Duhamel et al. 2011), or

when complete solutions are reconstituted from fragments or separate routes (Taillard et al. 1996, Ichoua et al. 2003, Olivera and Viera 2007). Note that, ruin-and-recreate LNS and perturbation moves were included among the *continuous* class. Our main motivation is that the effective amount of arcs that are actually changed from one LNS iteration to the next can remain rather small, and such moves are generally operated in a spirit of single solution improvement.

Memories and control. The judicious acquisition, management, and exploitation of knowledge on the problem and on the past search history is a complex task that belongs to the core of metaheuristics. Glover (1986) described three types of memories in the case of tabu search: short-term memories (e.g., tabu lists), which allow the search to be influenced locally in order to evade local optima, and medium- and long-term memories (e.g., memories on solutions elements), which are used to direct the overall exploration of the search space. These kind of memories have since been developed into various forms, and exploited for many means in other metaheuristics, including those surveyed for MAVRPs.

In particular, 28/64 of the selected metaheuristics bring into play populations as memories to manage promising or good-quality solutions, solution representations, routes, or solutions fragments. This is naturally the case for GA-based methods, path relinking, and scatter search, as well as metaheuristics relying on adaptive (Taillard et al. 1996, Ichoua et al. 2003, Olivera and Viera 2007) or central memory cooperation (Cordeau and Maischberger 2012, Groër et al. 2011). The populations of solution elements are used as the support for recombination procedures, including through set covering formulations, yielding new incumbent solutions.

Usually, a mix of diverse and high-quality elements is stored, thus aiming to find a balance between exploring new solution elements and focusing on champion features. Maintaining both diversity and elitism simultaneously in a population is a difficult task, as the aggressive local-improvement procedures, used in most efficient metaheuristics, tend to strongly drive the population towards a few local optima, resulting in premature convergence. Population-diversity management has thus been shown to be a key success factor in achieving good performance for MAVRPs (Prins 2004, Goel and Vidal 2012, Vidal et al. 2012a). It is especially critical in addressing rich VRPs combining several attributes, as finding new high-quality solutions on such intricate problems seems to require a good diversity of solution elements.

Half of the above-mentioned methods operate diversity management procedures, relying usually on a distance metric between individuals for both measuring diversity and driving the population management. For MAVRPs, this metric is usually based on solution differences in the objective space (Prins 2004, Ngueveu et al. 2010) or similarities in the route sequences (Prins 2009b, Vidal et al. 2012a), or are designed specifically for the attributes considered (those of the ASSIGN category especially, e.g., Alegre et al. 2007, Vidal et al. 2012a). Diversity can then be controlled by different means. Lau et al. (2010) rely on fuzzy logic to adapt search parameters relatively to population diversity and quality measures. Prins (2004), as well as several other recent genetic algorithms with population management (Sörensen and Sevaux 2006), impose distance constraints for acceptance in the population. Souffriau et al. (2010) implement ageing concepts to discard too "old" solutions from the pool. Finally, HGSADC (Vidal et al. 2012a) does not consider diversity as a constraint, but as an integral part of the objective that competes with solution quality during

individual evaluations. Empirical studies show that the latter mechanism leads to a higher solution diversity and quality.

Population management parameters are not the only ones to be adapted throughout the search. Parameter adaptation tends to be widespread in the methods analysed (30/64) to drive the infeasibility penalties (Cordeau et al. 1997, Vidal et al. 2012a), mutation or crossover rates (Repoussis et al. 2009b, Lau et al. 2010), or other algorithm strategies such as the frequency of use of operators and neighbourhoods (Ropke and Pisinger 2006a, Pisinger and Ropke 2007). Evolving search parameters directly within the genetic material of individuals is a common practice in EAs, while general metaheuristics adaptation is a main focus of hyper-heuristics (Burke et al. 2010).

More advanced forms of *guidance*, aiming to explicitly collect, analyse, and exploit knowledge on the past search to orient the future trajectories, are used in 29/64 methods. In MAVRP metaheuristics, the information is usually built as statistics on solution features, arcs, sets of arcs, routes, or problem specific attributes. The search context, e.g., the value of the incumbent solution and, eventually, the evolution of the value of the best solution (overall or for the current phase of the search), the value of particular counters resulting from the search history, and so on, is also part of the knowledge which is built.

This body of information, once collected and analysed, serves as support for *guidance actions*. The purpose of such actions is generally to either intensify the search, by focusing on promising solution features, or diversify it towards under-explored areas of the search space. Various methods are used in the methods surveyed to undertake such intensification and diversification actions, such as, penalties or incentives on solution attributes (see Cordeau and Laporte 2003, Derigs et al. 2009, Repoussis et al. 2009b, 2010, among others), "jumps" toward elite solutions or new solutions recombined from elite elements (Taillard et al. 1996, Ichoua et al. 2003, Brandão 2006, Olivera and Viera 2007), target solutions in path relinking (Hashimoto et al. 2008, Souffriau et al. 2010), neighbourhood choices governed by pheromone matrices (Ke et al. 2008, Fuellerer et al. 2009, Balseiro et al. 2011), or history-based ruin-and-recreate operators (Ropke and Pisinger 2006a, Pisinger and Ropke 2007, Ribeiro and Laporte 2012). Guidance actions may be undertaken continuously, as part of the fundamental search pattern of the metaheuristic (e.g., path relinking or TABUROUTE and UTS incorporating dynamically adjusted penalties on solution stagnation or infeasibility elements), or discreetly through a purposeful move.

Balancing intensification and diversification is particularly important for MAVRPs, where many problem features may be exploited in order to drive the search more efficiently. It is thus well-known that *statistically frequent* features of high-quality solutions are more likely to appear in the global optimum, thus explaining partly the recent success for MAVRP of population-based metaheuristics (Jones 1995), which favour the apparition and transmission of good solution elements, called *building blocks* in Holland (1975). Similarly, concepts of identification and combination of statistically promising solution attributes appeared with tabu search under the name of *vocabulary building* (Glover and Laguna 1998, Aleman and Hill 2010). Problem knowledge can thus be exploited in many ways in MAVRPs to intensify the search around relevant solution elements. Much of this same information can also be used for diversification, as it does, and should not play second fiddle. Indeed, as MAVRP search spaces, although sometimes metaphorically described as globally

convex "big valleys" (Boese 1995, Kubiak 2007), remain nonetheless rugged and some near-optimal solutions may be substantially different from the global optimum, diversification procedures play a critical role in search efficiency.

Finally, among the papers surveyed, many sensitivity analyses on parameters seek a good balance between intensification and diversification, through modifications of diversity management, tabu lists, temperature controls in SA, pheromone matrices, thresholds choices in R-to-R, neighbourhood choices in LNS or VNS, and so on. However, due to the balance that must be established, such parameters are subject to correlations, and advanced calibration methods, meta-calibration (De Landgraaf et al. 2007) or other statistical methods (Nannen and Eiben 2007) that address all parameters together may be necessary.

Hybridization. The metaheuristics surveyed rely to a large extent (39/64) on hybridization. By decreasing order of appearance, we report genetic algorithms and ACO methods combined with local search, sometimes using large neighbourhoods; tabu search methods combined with diversification operators based on solution recombinations (Ichoua et al. 2003, Olivera and Viera 2007); and hybrid neighbourhood-centred methods combining SA and LNS (Gajpal and Abad 2009), tabu search and VNS (Archetti et al. 2006), or ILS with VNS (Penna et al. 2011). Nine hybrid matheuristics involve mathematical programming components. These components are used to handle attributes of the problem, such as loading constraints (Fuellerer et al. 2009) or split deliveries (Archetti et al. 2008). In other cases, exact methods are used to search large neighbourhoods (Bent and Van Hentenryck 2006, Prescott-Gagnon et al. 2010, Gulczynski et al. 2011), or recombine solution elements (Groër et al. 2011).

Parallelism and cooperation. With the exception of multi-start methods which can be considered as a straightforward form of parallelism, 6/64 efficient methods relying on advanced parallelism and cooperation mechanisms were identified. Most such methods involve neighbourhood-centred heuristics, tabu search in particular, that communicate through an *adaptive memory* of elements of solutions (Ichoua et al. 2003) or through a *central memory* of complete solutions (Cordeau and Maischberger 2012, Groër et al. 2011). Integer programming solvers are used in Groër et al. (2011) to recreate solutions from the routes present in memory. In Balseiro et al. (2011), cooperation is based on pheromone exchanges between two ant colonies which simultaneously optimize travel times and fleet size.

It should finally be mentioned that more advanced cooperative metaheuristics are emerging for rich MAVRPs. In particular, Le Bouthillier and Crainic (2005a) introduced an advanced cooperative method for the VRPTW based on central memory. The method was complemented in Le Bouthillier and Crainic (2005b) with advanced guidance features. It served then as a building block of the *Integrative Cooperative Search (ICS)* framework (Crainic et al. 2009, Lahrichi et al. 2012), which relies on a structural problem decomposition among several such central memories. Each central memory involves several *partial solvers* that cooperate to produce *partial solutions* of the sub-problems, while integrators take on the role of reconstituting complete solutions from partial solutions picked in the partial memories. A global search coordinator is in charge of guiding the overall search as well as modifying the parameters and procedures.

Problem Decompositions. MAVRPs lend themselves well to various decomposition approaches, centred on assignments or geometry (Ostertag 2008, Bent and Van Hentenryck 2010), temporal aspects (Bent and Van Hentenryck 2010), or on solution attribute subsets (Crainic et al. 2009, Lahrichi et al. 2012). Among the methods analysed, Ichoua et al. (2003), Fuellerer et al. (2009) and Vidal et al. (2013) separate temporarily the routes of an elite solution using geometrical arguments, the different customer sets corresponding to sub-problems that are solved separately. Such decompositions thus makes it possible to improve the assignments of an elite solution in a view of intensification. Structural problem decompositions, involving successive or simultaneous solutions of sub-problems presenting less attributes, are also used. Alegre et al. (2007) apply to the PVRP a scatter search to optimize the assignment to periods, while a simple CVRP heuristic is repeatedly used for route creation. Decompositions become essential to handle rich MAVRPs but, in this context, the sequential approaches that independently solve problem characteristics consecutively are not sufficient to attain high-quality solutions. A clever management of the successive decompositions, sub-problem resolutions, and full solution reconstructions becomes thus essential.

1.7 Conclusions and Perspectives

This unifying survey and synthesis responds to the considerable challenge related to the abundance of VRP variants and to the relatively few general classifications and analyses of these problems and solution methods. The survey underlines that, while few general and efficient metaheuristics were proposed in the literature for this important class of problems, MAVRPs naturally share many common features, and most heuristic strategies developed for specific problems can be applied to a broader range of VRP variants. Hence, we conducted this analysis from a general perspective detached from the particular characteristics of the VRP attributes, and adopted a synthetic approach providing the means to cope with the abundance of contributions. We analysed in detail sixty-four successful metaheuristics for fifteen well-studied MAVRPs, identifying the main concepts and algorithmic-design principles, and highlighting the winning strategies of many efficient metaheuristics for a wide variety of variants.

When considering state-of-the-art methods, we observed recurrent notions such as *mix, variability, hybridisation, cooperation, diversity, multiplicity*, as well as *balance, equilibrium, trade-off*. It appears that most successful metaheuristics are not determined by a single factor but are the result of a good balance between several elements of methodology: the use of different search spaces, variable neighbourhoods, mixed continuous and discontinuous search, short-, medium- and long-term memories, trade-off between diversification and intensification, cooperation and collective intelligence, hybridisation, and so on. In brief, *in cooperation and diversity lies strength*. The performance of those methods indicates that each element plays an important role. On the one hand, long-term memories, jumps, recombinations and, generally, advanced guidance mechanisms providing diversification and, when relevant, population-diversity management methods have the potential to make the search progress in the general "big rugged valley" of MAVRPs. On the other

hand, short and medium-term memories and well-designed solution-improvement methods provide the aggressive search capabilities to complete the refinement of solutions.

We also observed that a clever implementation of algorithms is a necessary condition to yield competitive and scalable methods. Neighbourhood-pruning procedures (granularity, sequential searches) or memories on already evaluated routes, route segments, and moves, are necessary in many cases. Furthermore, one may notice that many algorithms rely on randomization and dedicate most of their computing time to evaluating various potential choices, without taking much advantage of history and already performed computations that may in many cases be profitably used. More intelligent guidance schemes have thus the potential to lead to performance improvements.

The research avenues for developing efficient MAVRP heuristics are numerous. We conclude the paper by summing up some open research questions.

In the previous sections, we identified a number of search-space, neighbourhoods, and trajectory choices leading to successful MAVRP metaheuristics. One may then ask to what extent these choices should depend upon the variant of the problem, and whether it is possible to identify desirable search spaces and neighbourhoods for some broad MAVRP classes. Of a similar nature are studies related to the definition of population-diversity metrics (e.g., what type of distance for MAVRPs) and management methods, and whether it should it be dependent upon the particular problem setting. Designing adequate and general neighbourhood pruning procedures for MAVRPs is another important issue of similar nature, which may also be stated in terms of making current mechanisms, e.g., granular and sequential search, efficiently applicable to a large variety of attributes and problem settings. Such algorithmic developments and proof-of-concept studies make up a very challenging research area.

The integration of diversification and the appropriate balance between intensification and diversification are critical factors for efficient MAVRP metaheuristics. This area is closely related to the development of advanced mechanisms to extract knowledge out of the explored search-space areas and to globally guide the metaheuristics. Links to the fields of hyper-heuristics and landscape analysis should also be more thoroughly explored.

As this survey illustrates, a number of metaheuristic families, tabu search, adaptive large neighbourhood search, and hybrid genetic algorithms, in particular, are widely acknowledged for their performance on a variety of MAVRPs. Given how differently these metaheuristics define and explore the search space, they are very likely to lead to extremely effective hybrid algorithms and parallel cooperative methods. This is an extremely rich and promising research field, particularly given the trend toward problem settings including a continuously increasing number of attributes and solution methods capable of addressing these attributes simultaneously.

To conclude, more general-purpose solvers, capable of handling a wide range of MAVRPs, are necessary to efficiently address practical routing applications in a timely manner. Many research questions have been answered by personalizing algorithms for each particular variant and by case-by-case improvements. However, solving generically (e.g., using a single solver and parameter set) a wide range of MAVRPs requires a better understanding of the problem foundations and the methods. This unifying survey and synthesis is a step toward reaching these goals.

CHAPITRE 2

UNE SYNTHÈSE UNIFICATRICE DES PROBLÈMES ET ALGORITHMES DE "TIMING"

2.1 Fil conducteur et contributions

Ce chapitre présente une revue de littérature et synthèse pluridisciplinaire visant à étudier les problèmes de détermination d'horaires d'activités sous un ordre d'exécution fixé, que nous appelons problèmes de *timing*, ainsi que les méthodes de résolution efficaces développées à cet effet dans la littérature. Les modèles de *timing* apparaissent fréquemment comme sous-problèmes lors de la résolution de problèmes de tournées de véhicules multi-attributs ou d'ordonnancement non-réguliers, ainsi que dans d'autres problèmes de réseaux, d'allocation de ressources, ou de statistiques. Cette analyse considère non-seulement la résolution *séquentielle* de problèmes de timing en "partant de zéro", mais aussi la résolution de séries de sous-problèmes successifs au sein de recherches locales ou de méthodes de branchement par des méthodes de *ré-optimisation*. La bibliothèque de méthodes efficaces ainsi identifiée est un atout pour la résolution d'une vaste gamme de problèmes d'optimisation combinatoire riches faisant intervenir des attributs temporels, car très souvent la difficulté de ces problèmes est inhérente la gestion de ces dernières caractéristiques.

2.2 Article II : A unifying view on timing problems and algorithms

Un article basé sur ce chapitre a été soumis pour publication : Vidal, T., Crainic, T.G., Gendreau, M., Prins, C. (2012) A Unifying View on Timing Problems and Algorithms. *Computers & Operations Research*, submitted for publication.

Abstract: Timing problems involve the choice of task execution dates within a predetermined processing sequence, and under various additional constraints or objectives such as time windows, time-dependent costs and so on. Their efficient solving is critical in branch and bound and neighborhood search methods for vehicle routing, scheduling with idle times, as well as in various applications in network optimization and statistical inference. Timing related problems have been studied for years, yet research on this subject suffers from a lack of consensus, and most knowledge is scattered among operations research and applied mathematics domains. This article introduces a classification for timing problems and features, as well as a unifying multidisciplinary analysis of timing algorithms. In relation to frequent application cases within branching schemes or neighborhood searches, the efficient resolution of series of similar timing subproblems is also analyzed. A dedicated formalism of re-optimization "by concatenation" is introduced to that extent. The knowledge developed through this analysis is valuable for modeling work and algorithmic design, for a wide range of combinatorial optimization problems with time characteristics, including rich vehicle routing settings and emerging non-regular scheduling applications, among others.

Keywords: Timing problems; scheduling; routing; statistical inference; branch and bound; neighborhood search.

2.3 Introduction

Time-related constraints and objectives appear in a variety of flavors within scheduling, project management, data transmission, routing, network optimization and numerous other fields. Several problem settings in these domains involve the allocation and arrangement of elementary *activities* under time requirements, such as tasks, visits to customers, production of objects, and so on. They may thus be seen as a combination of three issues (Desrosiers et al. 1995): the repartition of activities among resources, called *resource allocation*, the choice of an execution order for activities on each resource, called *sequencing*, and finally the adjustment of activity execution dates with respect to this order, called *scheduling* or *timing*. We will favor the name *timing* in this paper, as the word *scheduling* is already employed for various other settings in the literature.

In most situations, the combinatorial aspect of resource allocation and sequencing issues leads to NP-hard problem settings. Most heuristic and exact solution approaches perform a search through a large number of sequence and resource allocation alternatives, using repeatedly a timing algorithm to produce adequate execution dates, filter feasible solutions and evaluate costs. In all these cases, the timing algorithm is called extensively, and thus its complexity impacts dramatically the performance of the solution method, making the difference between successful algorithmic approaches and failure.

Timing problems therefore are the cornerstone of many algorithms. Yet, the literature dedicated to this subject remains scarce and scattered. Most developments on timing are made in relation to particular fields such as project planning, shortest path, routing, scheduling, and statistical inference, which bring into play, quite unexpectedly, the same formulations. Few relationships between domains have been actually exploited and, thus, close concepts and solution methods are independently developed within different formalisms, being rarely assessed in a more general context. The large number of problem variants arising from real-life settings bring forth additional challenges related to the variety of timing problems that must be dealt with. Among typical features we find target execution dates, (possibly multiple) time windows on activities, penalized lateness and earliness, speed decisions, time-dependent activity durations and costs, congestion, learning issues, and so on and so forth. Although efficient timing algorithms have been designed for some of these characteristics taken separately, problems involving combinations of characteristics become much more complex, and there is generally no systematic way to evolve concepts developed for the separate problems into a methodology for the new ones.

This paper contributes to the *timing field*, by means of a multidisciplinary review and analysis of timing features, problems, and algorithmic approaches. A large assortment of problems, often treated independently in the literature under various names, are identified and classified in relation to their structure. Successful solution methods and solving concepts are inventoried and analyzed. In the most noticeable cases, this analysis led to identify more than 26 algorithms from different

research fields as similar implementations of three main general approaches (Sections 2.7.3-2.7.4). Not only does this review gather the keys for a stand-alone resolution of a large variety of timing problems, but it also analyzes the efficient resolution of timing problems within the context of global search methods, e.g. neighborhood-based heuristics and branch-and-bound-based approaches. For these applications, managing global information through the successive resolution of similar timing instances can lead to dramatic reductions of the overall computational effort. To this extent, a *re-optimization framework* is introduced in the second part of this paper. The body of knowledge developed in this paper is critical for both modeling work and algorithmic design, enabling to point out relationships between problems and their respective complexities. A portfolio of state-of-the-art timing algorithms is identified, which will prove useful to build more generalist solvers for many variants of difficult combinatorial optimization problems. To our knowledge, no such unifying review and methodological analysis of this rich body of issues has been performed in the past.

The remainder of this paper is organized as follows: Section 2.4 formally defines timing problems, while Section 2.5 presents examples of applications. Section 2.6 provides a detailed classification of the main timing features encountered in the literature as well as notations. Our methodological analysis of timing problems and their *independent resolution* is then organized in Sections 2.7, 2.8, and 2.9 relatively to the previous classification. Section 2.10 finally introduces a *re-optimization framework* that encompasses state-of-the-art approaches for solving *series* of related timing instances. Section 2.11 highlights a number of challenging avenues of research in the timing field, and concludes.

2.4 Problem statement

In this paper, the term *activities* is used to represent, independently of the field of application, elementary operations that must be managed. The term *date* always stands for a point in time, whereas the words *duration* or *time* are employed for relative values (e.g., processing time). Without loss of generality, objective minimization is considered.

Definition 2.4.1 (General timing problem) *Let $A = (a_1, \ldots, a_n)$ be a sequence of n activities with processing times $p_1, \ldots, p_n$. The execution dates of these activities $\mathbf{t} = (t_1, \ldots, t_n)$ constitute the decision variables of the timing problem, and are required to follow a total order with respect to the subscripts, such that $t_i \leq t_{i+1}$ for $i \in \{1, \ldots, n\text{-}1\}$.Additional so-called* features *complete the description of the problem, providing the means to address particular settings. Let m be the number of such features. A feature F^x for $x \in \{1, \ldots, m\}$ corresponds to the specification of a set of m_x characteristic functions $f^x_y(\mathbf{t})$ for each $y \in \{1, \ldots, m_x\}$. A role is associated to each feature F^x, either as "objective" ($F^x \in \mathcal{F}^{\text{OBJ}}$) or as "constraint" ($F^x \in \mathcal{F}^{\text{CONS}}$). The general timing problem aims to find a feasible timing solution $\mathbf{t} = (t_1, \ldots, t_n)$, respecting order and features constraints (Equations 2.2-2.3), and minimizing the weighted sum of contributions from all feature objectives*

(Equation 2.1).

$$\min_{\mathbf{t}=(t_1,\ldots,t_n)\in\Re^{n+}} \quad \sum_{F^x\in\mathcal{F}^{\text{OBJ}}} \alpha_x \sum_{1\leq y\leq m_x} f_y^x(\mathbf{t}) \tag{2.1}$$

$$s.t. \quad t_i + p_i \leq t_{i+1} \qquad 1 \leq i < n \tag{2.2}$$

$$f_y^x(\mathbf{t}) \leq 0 \qquad F^x \in \mathcal{F}^{\text{CONS}}\ ,\ 1 \leq y \leq m_x \tag{2.3}$$

Two illustrative examples of features follow. The feature *deadlines* D, for example, completes the model with $m_D = n$ additional parameters, representing latest execution dates d_i for activities, and involve the characteristic functions $f_i^D(\mathbf{t}) = (t_i - d_i)^+$ $(i = 1, \ldots, n)$, where a^+ denotes $\max(a, 0)$. When D takes the role of a *constraint*, $f_i^D(\mathbf{t}) = (t_i - d_i)^+ \leq 0 \Leftrightarrow t_i \leq d_i$ yields the standard formulation of deadlines, while a role as objective leads to standard tardiness optimization criteria. The feature *time-lags* TL involves up to $m_{\text{TL}} = n^2$ additional parameters δ_{ij} on minimum elapsed time between the execution dates of activity pairs. The characteristic functions are $f_{ij}^{\text{TL}}(\mathbf{t}) = (t_j - \delta_{ij} - t_i)^+$.

Timing problems can be viewed as shifting activity execution dates on a single resource, without never changing the processing order. This issue is equivalent to idle-time insertion when processing times are fixed. Most basic versions of timing are straightforward to solve, while various features arising from application cases can lead to dramatic increases in problem difficulty. Traditionally, in the scheduling domain, some constraints and objectives, such as due dates, are based on activity completion dates $C_i = t_i + p_i$. Without loss of generality, these problems are reformulated to involve only execution dates. It must also be noted that features have been defined independently from their role as constraint or objective for two main reasons. First, many algorithms are concerned with the effective calculation of some quantities, like total duration for example, that enable to tackle related constraints or objectives in the same way. Secondly, since constraints can be transformed into objectives by Lagrangian relaxation, it is sometimes artificial to discriminate problems involving a given feature either as constraint or objective. Our study will thus be targeted on features, independently of their role, the latter being specified only when relevant to the method.

To emphasize the relations with practical problem settings, Section 2.5 now details some major problems in the fields of operations research and applied mathematics leading to underlying timing structures.

2.5 Timing issues and major application fields

Production and project scheduling. The development of just-in-time policies leads to challenging non-regular scheduling settings for which earliness or idle times are a major concern. Time-dependent processing times and costs (Alidaee and Womer 1999, Cheng 2004, Gawiejnowicz 2008), especially, represent very active areas of research (Kanet and Sridharan 2000). In the earliness and tardiness (E/T) scheduling problem, for example, a sequence of activities $(a_1, \ldots, a_n)$ is given with target execution dates d_i and processing times p_i, as well as penalty factors for earliness ϵ_i and tardiness τ_i. The goal is to determine the sequence of activities and their execution dates on a single machine, such that linear penalties incurred for early or late processing are

minimized. This scheduling problem presents a non-regular objective, and is NP-hard in the strong sense (Garey et al. 1988). Most recent approaches for (E/T) scheduling consider branch and bound, neighborhood search or other metaheuristic frameworks working on the activity sequence (Baker and Scudder 1990). For every sequence explored during the search, a timing algorithm is applied to compute the activity execution dates and thus the sequence cost. This timing problem is formulated in Equations (2.4-2.5).

$$\text{(E/T) timing:} \quad \min_{(t_1,\ldots,t_n)\in\Re^{n+}} \quad \sum_{i=1}^{n}\{\epsilon_i(d_i - t_i)^+ + \tau_i(t_i - d_i)^+\} \tag{2.4}$$

$$s.t. \quad t_i + p_i \leq t_{i+1} \qquad 1 \leq i < n \tag{2.5}$$

This problem involves two main features: release dates and deadlines. The characteristic functions of these features, involved in the objective, are $f_i(\mathbf{t}) = (r_i - t_i)^+$ and $f_i(\mathbf{t}) = (t_i - d_i)^+$, respectively. This timing problem is known to be solvable in $O(n \log n)$ (Sections 2.7.3 and 2.7.4). Yet, as the timing resolution is the main bottleneck for most (E/T) scheduling approaches, extensive research has been conducted to solve series of timing instances more efficiently within neighborhood searches. The use of global information through the search leads to timing "re-optimization" procedures working in amortized $O(\log n)$ complexity, and even $O(1)$ for some particular cases, as described in Section 2.10.

Network optimization. Timing subproblems are also frequently encountered in network optimization settings, e.g., resource-constrained shortest paths, delivery-man and minimum latency, vehicle routing and scheduling, among others (Solomon and Desrosiers 1988, Desrochers et al. 1990, Desrosiers et al. 1995, Vidal et al. 2012c). Thus, for example, the vehicle routing problem with time windows (VRPTW) consists in designing vehicle itineraries to service a set of geographically scattered customers within allowed time intervals. The VRPTW makes for one of the most intensively studied combinatorial optimization problem, as underlined by dozens of literature reviews on the subject (see Kallehauge et al. 2005, Bräysy and Gendreau 2005b,a, Gendreau and Tarantilis 2010, for the most recent). Timing subproblems arise when checking the feasibility, or estimating the minimal amount of violation with respect to time-windows, for the sequences (routes) produced in the course of the search. Time windows are a combination of both deadline and release date features, and the resulting timing formulations are closely related to those present in (E/T) scheduling.

One also observes a recent important focus on "richer" VRPs (Hartl et al. 2006, Vidal et al. 2012c), which explicitly take into account various combined constraints and objectives issued from application cases. These complex combinatorial optimization problems frequently involve temporal considerations, time-dependent travel speed, crew costs, customer requirements in terms of visit times, employee breaks and duty times, learning or fatigue effects, fair repartition of working time among employees, and so on. Such characteristics must be directly managed within route evaluations in heuristics and exact methods, thus leading to a large variety of timing subproblems. Having at hand efficient and generic timing solvers is thus an important step towards designing

robust solvers for rich VRP settings.

Energy optimization. Norstad et al. (2010) introduce a ship routing problem with convex speed optimization, which presents two interlaced issues: the design of a ship itinerary, and the optimization of arrival dates and speed (PATSO) to reduce fuel consumption. For a fixed sequence of visits, the latter subproblem is formulated in Equations (2.6-2.9).

$$\text{(PATSO):} \quad \min_{\mathbf{t},\mathbf{v}} \quad \sum_{i=1}^{n-1} d_{i,i+1}\ c(v_{i,i+1}) \tag{2.6}$$

$$s.t. \quad t_i + p_i + d_{i,i+1}/v_{i,i+1} \leq t_{i+1} \qquad 1 \leq i \leq n-1 \tag{2.7}$$

$$r_i \leq t_i \leq d_i \qquad 1 \leq i \leq n \tag{2.8}$$

$$v_{min} \leq v_{i,i+1} \leq v_{max} \qquad 1 \leq i \leq n-1 \tag{2.9}$$

In the previous formulation, the decision variables are the travel speeds $v_{i,i+1}$ for $i \in \{1,\ldots,n-1\}$ for each port-to-port leg, and the arrival dates at ports t_i for $i \in \{1,\ldots,n\}$. The objective is to minimize the fuel consumption on all trips, $c(v)$ representing the energy consumption per mile as a convex and increasing function of speed. Let v_{opt} be the minimum of this function. $d_{i,i+1}$ represents the leg distances, and p_i stands for processing times at ports. Equations (2.7-2.9) ensure that port arrival and departure dates are consistent with leg speeds, that time windows at port arrivals are respected, and finally that speeds are within a feasible range.

This problem can be reformulated to rely exclusively on arrival dates by defining an extended cost/speed function $\hat{c}(v)$, which accounts for the fact that waiting times can be used in case of sub-optimal low speed values (Equations 2.10-2.13).

$$\text{(PATSO-2):} \quad \min_{\mathbf{t}} \quad \sum_{i=1}^{n} d_{i,i+1}\hat{c}\left(\frac{d_{i,i+1}}{t_{i+1}-t_i}\right) \tag{2.10}$$

$$\text{s.t.} \quad t_i + p_i + d_{i,i+1}/v_{max} \leq t_{i+1} \qquad 1 \leq i \leq n-1 \tag{2.11}$$

$$r_i \leq t_i \leq d_i \qquad 1 \leq i \leq n \tag{2.12}$$

$$\text{with} \quad \hat{c}(v) = \begin{cases} c(v_{opt}) & \text{if } v \leq v_{opt} \\ c(v) & \text{otherwise} \end{cases} \tag{2.13}$$

The latter model falls into the category of timing problems. It involves time-window features characterized by functions $f_i(\mathbf{t}) = (t_i - d_i)^+ + (r_i - t_i)^+$ with a role as constraints, as well as flexible processing times characterized by convex functions $f_i(\mathbf{t}) = c_i(t_{i+1} - t_i)$ in the objective, such that $c_i(\Delta t_i) = d_{i,i+1}\hat{c}(d_{i,i+1}/\Delta t_i)$. Norstad et al. (2010) introduced a Recursive Smoothing Algorithm (RSA) to solve the previous timing problem with a worst case complexity of $O(n^2)$. A similar algorithm was then used in the context of a vehicle routing problem with CO_2 minimization (Demir et al. 2012). Several other timing algorithms and re-optimization procedures are known for these settings (Sections 2.8.3 and 2.10.5.6).

Statistical Inference. The isotonic regression problem under a total order (IRC) constitutes an intensively studied particular case of our models. Given a vector $\mathbf{N} = (N_1, \ldots, N_n)$ of n real numbers, IRC seeks a vector of non-decreasing values $\mathbf{t} = (t_1, \ldots, t_n)$ *as close as possible* to $\mathbf{N}$ according to a distance metric $\| \ \|$ (generally the Euclidean distance), as in Equations (2.14-2.15):

$$\text{(IRC):} \quad \min_{\mathbf{t}=(t_1,\ldots,t_n)} \quad \|\mathbf{t} - \mathbf{N}\| \tag{2.14}$$

$$\text{s.t.} \quad t_i \leq t_{i+1} \qquad 1 \leq i < n \tag{2.15}$$

As underlined by the seminal books of Barlow et al. (1972) and Robertson et al. (1988), IRC is the key to performing many restricted maximum likelihood estimates in statistics, and is linked with various applications such as image processing and data analysis. It appears here as a timing problem with separable convex costs, similar to those encountered when solving vehicle routing problems with time windows or (E/T) scheduling settings.

Other applications. Timing formulations also appear under more general mathematical formalisms, under the name of *projection onto order simplexes* in Grotzinger and Witzgall (1984), or as particular cases of several convex optimization problems with underlying network structures (Hochbaum 2002, Ahuja et al. 2003). We now develop a new classification, as well as notations, for the main classes of timing features and problems.

2.6 Features : classification and reductions

This section introduces a classification of the main timing features in the literature, and levers notations for the related problems. Reduction relationships between features are then investigated.

2.6.1 Classification and notations

The features are here classified relatively to the structure of their characteristic functions. We rely to that extent on a *feature dimension* measure ξ, which illustrates the links that a feature creates between decision variables.

Definition 2.6.1 (Feature dimension) *The dimension $\xi(F^x)$ of a feature F^x is defined as the maximum number of variables involved together in any characteristic function $f^x_y(\mathbf{t})$ for $y \in \{1, \ldots, m_x\}$.*

Table 2.1 presents the most common features in the literature relatively to their dimension ξ. The first column provides an abbreviation for each feature. The next columns describe the parameters, characteristic functions and dimensions of these features. Finally, we report the most frequent roles of each feature in the literature.

Most of the features presented in Table 2.1 are well-known in the scheduling or vehicle routing literature. Features D, C, and W, can be qualified as *regular*, as they involve non-decreasing characteristic functions $f^x_y(\mathbf{t})$. The set of *active schedules "such that no operation can be made to start sooner by permissible left shifting"* (Giffler and Thompson 1960) is dominating for regular

Table 2.1: Classification of timing features and notations

Symbol	Parameters	Char. functions	ξ	Most frequent roles
C	Deadline t_{max} on last activity	$f(\mathbf{t}) = (t_n - t_{max})^+$	1	Deadline on last activity, lateness of last activity, makespan
W	Weights w_i	$f_i(\mathbf{t}) = w_i t_i$	1	Weighted execution dates
D	Deadlines d_i	$f_i(\mathbf{t}) = (t_i - d_i)^+$	1	Deadline constraints, tardiness
R	Release dates r_i	$f_i(\mathbf{t}) = (r_i - t_i)^+$	1	Release-date constraints, earliness.
TW	Time windows $TW_i = [r_i, d_i]$	$f_i(\mathbf{t}) = (t_i - d_i)^+ + (r_i - t_i)^+$	1	Time-window constraints, soft time windows.
MTW	Multiple TW $MTW_i = \cup[r_{ik}, d_{ik}]$	$f_i(\mathbf{t}) = \min_k [(t_i - d_{ik})^+ + (r_{ik} - t_i)^+]$	1	Multiple time-window constraints
$\Sigma c_i^{\text{CVX}}(t_i)$	Convex $c_i^{\text{CVX}}(t_i)$	$f_i(\mathbf{t}) = c_i^{\text{CVX}}(t_i)$	1	Separable convex objectives
$\Sigma c_i(t_i)$	General $c_i(t)$	$f_i(\mathbf{t}) = c_i(t_i)$	1	Separable objectives, time-dependent activity costs
DUR	Total dur. δ_{max}	$f(\mathbf{t}) = (t_n - \delta_{max} - t_1)^+$	2	Duration or overall idle time
NWT	No wait	$f_i(\mathbf{t}) = (t_{i+1} - p_i - t_i)^+$	2	No wait constraints, min idle time
IDL	Idle time ι_i	$f_i(\mathbf{t}) = (t_{i+1} - p_i - \iota_i - t_i)^+$	2	Limited idle time by activity, min idle time excess
$P(t)$	Time-dependent proc. times $p_i(t_i)$	$f_i(\mathbf{t}) = (t_i + p_i(t_i) - t_{i+1})^+$	2	Processing-time constraints, min activities overlap
TL	Time-lags δ_{ij}	$f_i(\mathbf{t}) = (t_j - \delta_{ij} - t_i)^+$	2	Min excess with respect to time-lags
$\Sigma c_i(\Delta t_i)$	General $c_i(t)$	$f_i(\mathbf{t}) = c_i(t_{i+1} - t_i)$	2	Separable functions of durations between successive activities, flex. processing times
$\Sigma c_i(t_i, t_{i+1})$	General $c_i(t, t')$	$f_i(\mathbf{t}) = c_i(t_i, t_{i+1})$	2	Separable objectives or constraints by successive pairs of variables
$\Sigma c_{ij}(t_i, t_j)$	General $c_{ij}(t, t')$	$f_{ij}(\mathbf{t}) = c_i(t_i, t_j)$	2	Separable objectives or constraints by any pairs of variables
$c(\mathbf{t})$	General $c(t)$	$f(\mathbf{t}) = c(\mathbf{t})$	–	Any feature

features. Solving the timing problem is then straightforward by means of a minimum idle time policy (Section 2.7.1). However, these regular features present notably different behaviors with regards to re-optimization, thus motivating a detailed study. Other features from Table 2.1 are *non-regular*. They lead to more complex timing problems for which the insertion of idle time can improve the objective or the satisfaction of constraints.

Single- and two-dimensional features are directly linked to physical quantities, execution dates and durations, respectively. As seen in Table 2.1, such features are frequently encountered in timing formulations. Higher-dimension features are more unusual in the literature, and can, by definition, lead to a wide range of difficult problems. Indeed, any mathematical program can be viewed as a combination of unary and binary mathematical operators of the form $x = f(y, z)$, and thus can be reformulated with constraints and objectives separable in groups of three variables. Hence, any problem on continuous totally ordered variables can be viewed as a timing problem with three-dimensional features. A reasonable limit to define *"what a timing problem is"* is then to consider, as in the present paper, only applications and problems presenting explicitly the aspect of an activity sequence.

We introduce a notation specifying for each problem the features considered, as well as information regarding their role. Each problem is tagged as a two-component string $\{O|C\}$, where O is a list of features involving the objective and C lists features that participate to constraints. Separating features in the field O with a comma indicates a weighted sum of objectives. The sign $\cup$ is used for multi-objective problems and the sign $>$ indicates an order of priority. Particular parameter characteristics are reported in parentheses after the feature symbol. For example, problems with common deadlines can be marked with $(d_i = d)$, null processing times as $(p_i = 0)$, and so on.

To illustrate, consider the problem of speed optimization of Section 2.5. This problem presents a separable and convex objective as a function of durations between successive activities, along with time-window constraints. It can thus be categorized as $\{\Sigma c^{\text{CVX}}(\Delta t)|TW\}$. The (E/T) timing problem presents linear penalties around a target execution date. These penalties can be assimilated to relaxed simultaneous release dates and deadlines, leading to the notation $\{R, D(r_i = d_i)|\}$. Finally, the vehicle routing literature includes problem settings with a hierarchical objective aiming first to minimize the amount of time-window violations, then duration excess, and finally time-lag violations (Cordeau and Laporte 2003, Berbeglia et al. 2011). Such a problem setting can be characterized as $\{TW > D > TL|\}$.

2.6.2 Feature reductions

We use reduction relationships to illustrate the level of generality and complexity of timing features. A rich body of polynomial reductions has been developed in the scheduling literature. Most timing problems, however, are polynomially solvable, and the use of polynomial reduction relationships leads to consider most problems in the same class of equivalence. We thus seek stronger reduction properties to distinguish them. We also aim to build relationships between features instead of complete problems, leading to the following definition of feature reductions:

Definition 2.6.2 (Reducibility among timing features) *A feature F is said to be reducible to feature F' if any timing problem $\mathcal{T}$ involving F and other features $\{F^1, \ldots, F^k\}$ admits a linear many-one reduction to a timing problem $\mathcal{T}'$ involving $F' \cup \{F^1, \ldots, F^k\}$.*

An overview of feature reductions is given in Figure 2.1, where an arrow from feature F^i to F^j indicates that feature F^i can be reduced to F^j. Four different categories of features are identified by different shades of gray. On the left, we present features involving at most one decision variable (the first part of Table 2.1) and separable costs. Progressing to the right, the next gray shade represents two-dimensional features that involve only pairs of consecutive activities, then features involving any pair of activities, and finally other features. We also demarcate the area of "NP-hard" features, which alone are sufficient to lead to NP-hard timing problems.

The hierarchy of reductions presented in Figure 2.1 gives an indication on the level of generality of the features. Some features, such as $\Sigma c_i^{\text{CVX}}(t_i)$, $\Sigma c_i(t_i)$, $\Sigma c_i(\Delta t_i)$, and TL, generalize many other features while remaining polynomially solvable. An algorithm addressing such general features can tackle many problems, while specialized algorithms for simpler combinations of features may be more efficient. Both specialized and general algorithms are critical for practical applications and

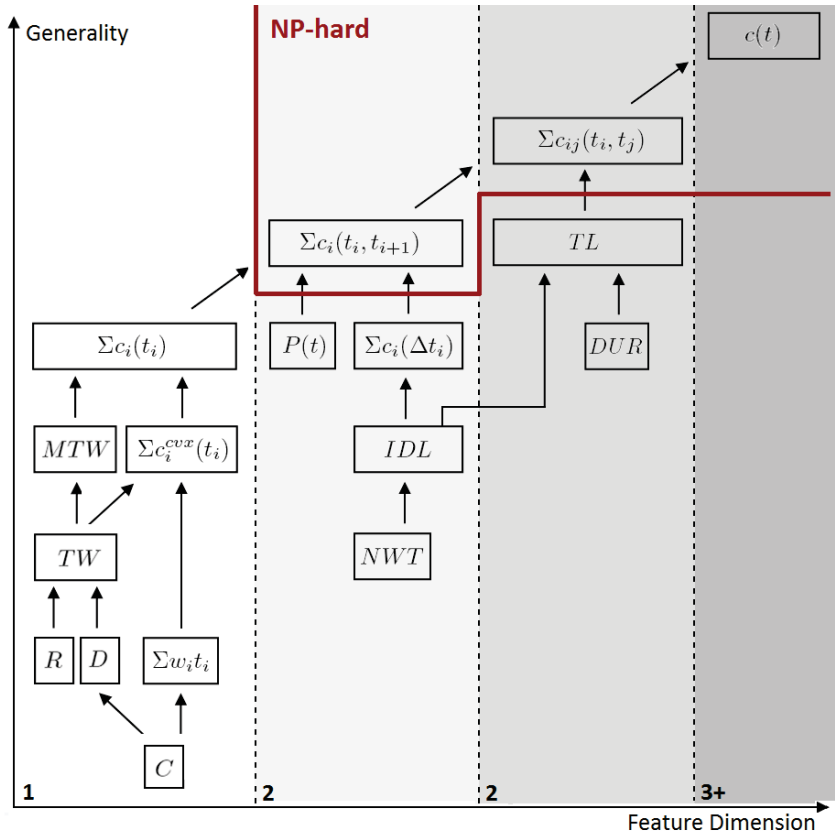


Figure 2.1: Hierarchy of timing features

deserve a detailed study. We thus provide a methodological review ordered by increasing generality of the main features, timing problems and solution methods in the literature. We start with the most simple cases of single-dimensional features in Section 2.7, and follow with two-dimensional features in Section 2.8, to conclude our analysis of stand-alone methods for timing in Section 2.9.

2.7 Single-dimensional features

Problems with single-dimensional features are analyzed according to their difficulty and generality, starting with simple regular features, following with time-window TW features, separable convex costs $\Sigma c_i^{\text{CVX}}(t_i)$ and, finally, general separable costs $\Sigma c_i(t_i)$. The latter feature encompasses multiple time windows MTW and generalizes all problems in this category.

2.7.1 Makespan, deadlines and weighted execution dates

Maximum execution dates C, deadlines D, and weighted execution dates W features lead to several well-documented objectives in the scheduling literature, aiming to minimize makespan, tardiness, lateness or total weighted starting or completion times among others (Graham et al. 1979, Pinedo 2008). W as an objective also arises in various routing settings, such as the delivery-man problem (Fischetti et al. 1993), the minimum latency problem (Blum et al. 1994), and the cumulative TSP or VRP (Bianco et al. 1993, Ngueveu et al. 2010), where the goal is to service a

set of customer as early as possible. These features are *regular*, as any backward shift of execution date is beneficial for both feasibility and objective value. A very simple algorithm follows, which will be referred to as the "minimum idle time policy": *For each activity a_i of A in the sequence order, schedule a_i at its earliest possible execution date. If a_i cannot be scheduled, declare problem infeasibility and stop. If all activities have been successfully scheduled, declare problem feasibility.* An optimal solution is thus retrieved in n searches of the earliest feasible execution date, leading to a $O(n)$ complexity.

2.7.2 Release dates and time windows

Release dates and time-window features appear frequently in vehicle routing and scheduling applications. Time-window features generalize release dates R and deadlines D, as any release date r_i or deadline d_i can be transformed into a time window with an infinite value on the right $[r_i, +\infty]$ or the left $[-\infty, d_i]$. Two main issues are often considered regarding these features. The first involves stating the feasibility of a sequence of activities under time-window constraints, whereas the second problem involves the minimization of infeasibility with respect to the time windows, and thus involves characteristic functions $f_i(\mathbf{t}) = (t_i - d_i)^+ + (r_i - t_i)^+$ in the objective.

Feasibility problem. Solving the feasibility problem $\{|TW\}$ is straightforward, as the minimum idle time policy, presented in Section 2.7.1, is dominating in this respect. For a sequence of n activities $(a_1, \ldots, a_n)$, the algorithm starts with $t_1 = r_1$, then chooses each subsequent activity execution date to minimize idle time: $t_{i+1} = \max(t_i + p_i, r_{i+1})$. Hence, feasibility can be checked in $O(n)$ *from scratch*. Yet, more efficient feasibility checking procedures are available to solve *series* of timing instances in local-search context. These procedures are presented in Section 2.10.

Infeasibility minimization. Many real-case applications allow lateness or earliness, the so-called *soft* time-window settings, as a way to gain flexibility. Relaxations are also very frequently used in heuristics, as managing intermediate infeasible solutions contributes to improve the capabilities of exploration. Different conventions for soft time windows have been reported in the literature, relative to earliness allowance and the way infeasibility is penalized. Several contributions, such as Taillard et al. (1997) and Cordeau et al. (2001a), focus on the problem $\{D|R\}$, where late activities are allowed with penalties, but not early activities. This case falls within the scope of regular features (Section 2.7.1), and choosing for each activity the earliest execution date is optimal. The problem can thus be solved with linear complexity $O(n)$.

However, when early activities are allowed, as in $\{TW|\}$ (Garey et al. 1988, Baker and Scudder 1990, Koskosidis et al. 1992, Balakrishnan 1993, Ibaraki et al. 2005), the objective function is no longer non-decreasing. Supposing that activity a_i is finished earlier than the beginning of the time-window of a_{i+1}, a choice must be made whether to insert idle time to reach a_{i+1}, or pay a penalty to better satisfy the time windows of remaining activities. The resulting timing setting can thus become more complex. As an example, we show in Appendix I.1 that the problem of minimizing the number of time-window infeasibilities, $\{TW(unit)|\}$, generalizes the Longest Increasing Subsequence Problem (LISP). LISP has been the subject of extensive research, and

admits a computational lower bound of $\Omega(n \log n)$ in the comparison tree model (Fredman 1975). It should be noted that$\{TW|\}$ is also special case of separable piecewise linear convex cost functions. Sections 2.7.3 and 2.7.4 provide general efficient algorithms to address this wide range of problems, leading to an $O(n \log n)$ algorithm for soft time-window relaxations.

2.7.3 Separable convex costs

Separable convex costs Σc_i^{CVX} include a wide range of problem settings as particular cases. The feature TW, and thus R, D, and C, can be reduced to $\Sigma c_i^{\text{CVX}}(t_i)$, as any time-window constraint can be formulated as a piecewise convex cost by associating arbitrary large costs to both sides of the feasibility interval. This feature also encompasses various other settings from the literature such as earliness-tardiness scheduling (Baker and Scudder 1990), isotonic regression problems with respect to a total order (Barlow et al. 1972, Robertson et al. 1988), extensions of team orienteering problems (Feillet et al. 2005) in which the profit value can decrease with time (Erkut and Zhang 1996), various convex penalty functions for time-window infeasibility (Sexton and Bodin 1985a,b, Ioachim et al. 1998, Ibaraki et al. 2005) and time-dependent convex processing costs (Tagmouti et al. 2007), among others. The timing problem $\{\Sigma c_i^{\text{CVX}}(t_i)|\}$ is formulated in Equations (2.16-2.17). Functions $c_i^{\text{CVX}}(t_i)$ are supposed to take infinite value for $t_i < 0$.

$$\min_{(t_1,\ldots,t_n)\in\Re^n} \quad \sum_{i=1}^{n} c_i^{\text{CVX}}(t_i) \tag{2.16}$$

$$s.t. \quad t_i + p_i \leq t_{i+1} \qquad 1 \leq i < n \tag{2.17}$$

Many methods have been proposed for this setting. We study, in this subsection, approaches specially designed for separable convex cost functions. Dynamic programming based algorithms, relying on fundamentally different concepts, are grouped in Section 2.7.4.

We base our analysis of algorithms for $\{\Sigma c_i^{\text{CVX}}(t_i)|\}$ on a set of optimality conditions using the active set formalism (Chakravarti 1989, Best and Chakravarti 1990, Best et al. 2000). The necessary and sufficient conditions we propose are more general than those developed previously in the literature, being applicable to any set of proper convex cost functions, including non-smooth cases.

Definition 2.7.1 (Activity blocks and prefix blocks) *A block B is defined as a sequence of activities $(a_{B(1)}, \ldots, a_{B(|B|)})$ processed consecutively such that $t_i + p_i = t_{i+1}$ for all $i \in \{B(1), \ldots, B(|B|) - 1\}$. For $k \in \{B(1), \ldots, B(|B|) - 1\}$, we also define the prefix block $B^k = (a_{B(1)}, \ldots, a_k)$. Let p_{ij} for $1 \leq i \leq j \leq n$ be the cumulative processing duration of activities $(a_i, \ldots, a_j)$. The execution cost C_B of a block B as a function of its first activity execution date $t_{B(1)}$ is given in Equation (2.18).*

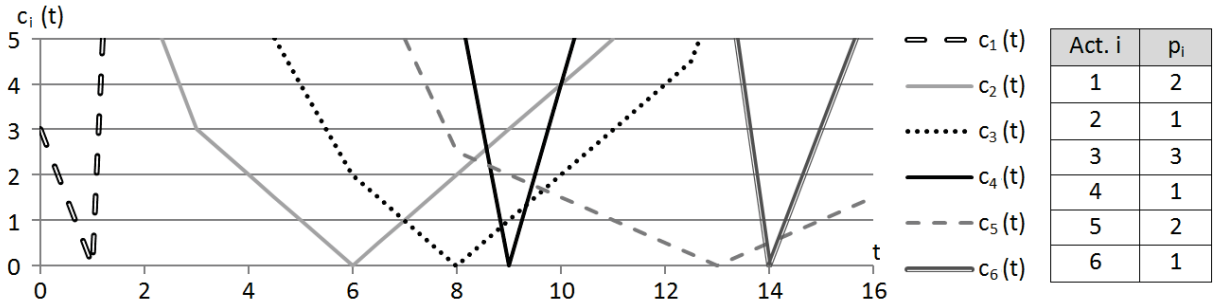
$$C_B(t_{B(1)}) \quad = \quad c_{B(1)}(t_{B(1)}) + \sum_{i=B(1)+1}^{B(|B|)} c_i(t_{B(1)} + p_{B(1)i-1}) \tag{2.18}$$

When the costs are proper convex functions (such that $\exists x | f(x) < +\infty$ and $\forall x, f(x) > -\infty$), the set of execution dates for the first activity minimizing this block execution cost is an interval $[T_B^{-*}, T_B^{+*}]$. These assumptions enable to state the following necessary and sufficient optimality conditions:

Theorem 2.7.1 *Let costs $c_i(t_i)$ for $i \in \{1, \ldots, n\}$ be proper convex, possibly non-smooth, functions. A solution $\mathbf{t}^* = (t_1^*, \ldots, t_n^*)$ of $\{\Sigma c_i^{\text{CVX}}(t_i)|\}$ with activity blocks $(B_1, \ldots, B_m)$, is optimal if and only if the three following conditions are satisfied:*

1. *Blocks are optimally placed, $t^*_{B_i(1)} \in [T^{-*}_{B_i}, T^{+*}_{B_i}]$ for each block B_i;*
2. *Blocks are strictly spaced, $t^*_{B_i(1)} + p_{B_i(1)B_i(|B_i|)} < t^*_{B_{i+1}(1)}$ for each pair of blocks (B_i, B_{i+1});*
3. *Blocks are consistent, $T^{+*}_{B_i^k} \geq t^*_{B_i(1)}$ for each block B_i and prefix block B_i^k.*

Condition 2 and 3 are direct consequences of the primal and the dual feasibility, respectively. Proof is given in Appendix I.2. Surveying the literature, we distinguish two main categories of methods: those which maintain primal feasibility, and those which maintain dual feasibility. These algorithms are issued from various domains. In the case of isotonic regression in particular, only precedence constraints among decision variables are considered (and thus $p_i = 0$ for all i), yet these methods can generally be extended to solve problems with processing times with only minor modifications. Hence, we illustrate all algorithms on a simple problem, for which the cost functions and the processing times are given in Figure 2.2.



Act. i	p_i
1	2
2	1
3	3
4	1
5	2
6	1

Figure 2.2: Illustrative example with six activities: cost functions and durations

Primal methods. A first category of methods is based on respecting the primal feasibility conditions and iteratively restoring the dual conditions. The first method of this kind, called *Minimum Lower Set* (*MLS*) algorithm, has been proposed by Brunk (1955) for isotonic regression problems. The MLS algorithm starts with a single big block, then iteratively finds for each block B the biggest prefix block B^k violating dual conditions. If no such violation is found, this block is optimal, otherwise the current block is split in two and the procedure is recursively called on each sub-block until no dual conditions violation may be found. The algorithm can be implemented in $O(n^2)$ unimodal function minimizations.

Later on, Best and Chakravarti (1990) introduced a primal feasible algorithm for IRC in $O(n)$ unimodal function minimizations. Again, activities are sequentially examined in each block to find

the first violation of dual conditions (and not the most important violation). If such a violation exists, the block under consideration is split at this place. The leftmost block has an earlier optimal starting date, and thus can possibly be merged with one or several previously scheduled blocks to reach an optimal execution date. In the presence of quadratic costs, a closed form exists for the function minimums, and the complexity of this algorithm becomes $O(n)$ elementary operations.

The method of Garey et al. (1988), originally designed for (E/T) scheduling, iterates on activities in the sequence order and yields at any step i an optimal solution to the subproblem containing only the first i activities. Each new activity is inserted at the end of the schedule, to be left-shifted and possibly merged with previous activity blocks until no improvement may be achieved. The algorithm runs in $O(n \log n)$ when relying on heap data structures.

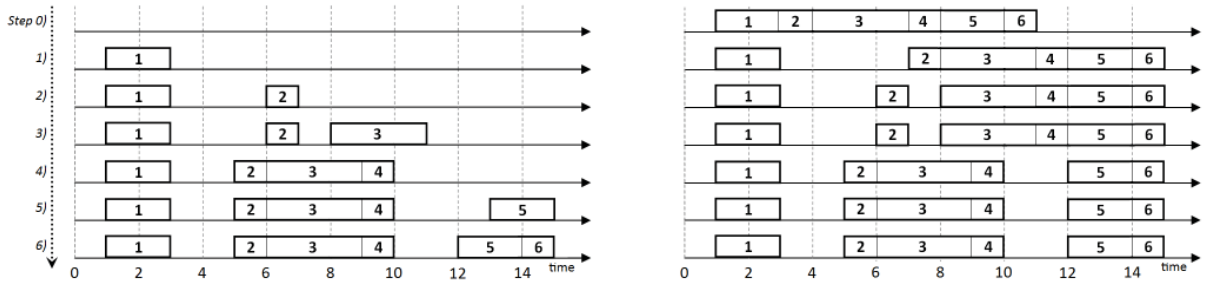


Figure 2.3: Comparison between Garey et al. (1988) algorithm (left part of the figure), and Best and Chakravarti (1990) algorithm (right part of the figure)

Figure 2.3 illustrates the algorithms of Best and Chakravarti (1990) and Garey et al. (1988) on the example of Figure 2.2. The problem is solved in six steps, illustrated from top to bottom along with the incumbent solutions, representing activities as rectangles with a length proportional to the processing time. The activity blocks in presence are very similar. Indeed, these two algorithms can be viewed as two variations of the same underlying primal feasible method, with the exception that Garey et al. (1988) considers non-inserted activities as non-existing in the current solution, whereas Best and Chakravarti (1990) maintains these non-scheduled activities in one final block which does not respect Condition 3.

The method of Garey et al. (1988) was extended by Lee and Choi (1995) and Pan and Shi (2005) to address (E/T) scheduling problems with distinct penalty weights for earliness and tardiness, that is $\{D, R(d_i = r_i)|\}$, in $O(n \log n)$ elementary operations. Szwarc and Mukhopadhyay (1995) and Feng and Lau (2007) also proposed to identify the tasks that are necessarily processed without idle time (in the same block) before solving. Chrétienne and Sourd (2003) applied the algorithm to project scheduling with piecewise convex cost functions, and Hendel and Sourd (2007) to timing problems with convex piecewise linear or quadratic costs. These algorithms work in $O(n)$ unimodal function minimizations, but differ in terms of the data structures used to represent the functions and thus on the complexity of the function minimizations. When the cost functions are Piecewise Linear (PiL), the method of Hendel and Sourd (2007) attains a complexity of $O(\varphi_c \log n)$, where φ_c is the total number of pieces in the activity cost functions of the sequence. Finally, Davis and Kanet (1993) proposed another primal method for (E/T) scheduling similar to Garey et al. (1988),

and generalized to PiL convex costs by Wan and Yen (2002). Activities are iteratively added to the solution in reverse sequence order. Each activity is scheduled at date 0, and then shifted onwards (while possibly merging blocks), until no improvement can be achieved.

Dual feasible methods. Simultaneously with the MLS algorithm, another seminal method for IRC was proposed by Ayer et al. (1955) under the name of *Pool Adjacent Violators* (*PAV*). Starting with an initial solution consisting of n separate blocks, one for each activity, successive pairs of blocks (B_i, B_{i+1}) not satisfying primal conditions are iteratively identified. Such blocks are merged, and the next iteration is started. The order in which these block couples are identified does not affect the final result of the algorithm. An illustration of the method on the previous example is given in Figure 2.4. The algorithm iteratively merges the first pair of blocks that does not verify primal conditions. We notice that the optimal solution is reached after three merges (at Step 3).

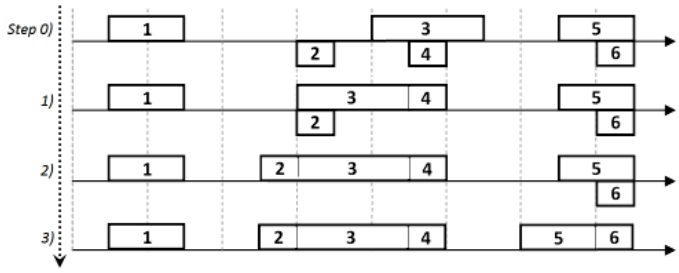


Figure 2.4: The PAV algorithm illustrated on timing problems

Chakravarti (1989) proved that PAV is a dual feasible method for the linear formulation of the problem when the distance considered is $\| \ \|_1$ ($c(\mathbf{t}) = \Sigma|t_i - N_i|$), while Grotzinger and Witzgall (1984) and Best and Chakravarti (1990) showed that PAV is a dual algorithm for IRC when quadratic costs are considered (Euclidean distance).

The PAV algorithm was also generalized to convex functions by Best et al. (2000) and Ahuja and Orlin (2001), achieving a complexity of $O(n)$ unimodal minimizations. It is noteworthy that, under a totally different formalism, an equivalent algorithm was discovered by Dumas et al. (1990) for a general vehicle routing setting with convex time-dependent service costs. For IRC with the $\| \ \|_1$ distance, the PAV algorithm can be implemented in $O(n \log^2 n)$ elementary operations using balanced search trees (Pardalos 1995), or $O(n \log n)$ complexity using scaling techniques (Ahuja and Orlin 2001). Finally, $O(n)$ algorithms are known in the case of quadratic costs (Grotzinger and Witzgall 1984, Dumas et al. 1990, Pardalos and Xue 1999).

2.7.4 Separable costs and multiple time windows

Without the previous convexity assumption, the timing problems $\{\Sigma c_i(t_i)|\}$ become more complex, and many authors have focused on separable PiL costs. The feature MTW especially (Christiansen and Fagerholt 2002, Tricoire et al. 2010) can also be linearly reduced to a $\{\Sigma c_i(t_i)|\}$ problem when a closed form representation of the multiple time windows, such as a list of feasible intervals, is available.

In presence of non-negative and Lower Semi-Continuous (LSC) costs ($c_i(t_i) \leq lim_{\epsilon \to 0} \min\{c_i(t_i + \epsilon), c(t_i - \epsilon)\}$ at every discontinuity point), the timing problem $\{\Sigma c_i(t_i)|\}$ can be efficiently solved by dynamic programming. A large range of backward and forward approaches has thus been proposed in the routing and scheduling literature (Yano and Kim 1991, Sourd 2005, Ibaraki et al. 2005, Hendel and Sourd 2006, Ibaraki et al. 2008).

Solving $\{\Sigma c_i(t_i)|\}$ by forward dynamic programming involves the *forward minimum cost* function $F_i(t)$, which evaluates the minimum cost to execute the sequence of activities $(a_1, \ldots, a_i)$ while starting the last activity before t ($t_i \leq t$). $F_i(t)$ functions can be computed by means of Equation (2.19), starting from the case $i = 1$ with a single activity where $F_1(t) = \min_{x \leq t} c_1(x)$. The optimal solution value of the timing problem is $z^* = F_n(+\infty)$, and the optimal activity execution dates can be retrieved from the $F_i(t)$ functions.

$$F_i(t) = \min_{0 \leq x \leq t} \{c_i(x) + F_{i-1}(x - p_{i-1})\} \qquad 1 < i \leq n \tag{2.19}$$

The symmetric way to solve this problem by backward programming involves the *backward minimum cost* function $B_i(t)$, which evaluates the minimum cost to execute the sequence of activities $(a_i, \ldots, a_n)$, while executing the first activity a_i after t ($t_i \geq t$). $B_i(t)$ functions are computed by backward recursion, starting with $B_n(t) = \min_{x \geq t} c_n(x)$ and using Equation (2.20). The optimal solution value of the timing problem is $z^* = B_1(-\infty)$.

$$B_i(t) = \min_{x \geq t} \{c_i(x) + B_{i+1}(x + p_i)\} \qquad 1 \leq i < n \tag{2.20}$$

These methods can be implemented in $O(n\varphi_c)$, where φ_c stands for the total number of pieces in the activity cost functions c_i. When the costs are also convex, the use of efficient tree data structures leads to a complexity of $O(\varphi_c \log \varphi_c)$ (Ibaraki et al. 2008), matching the best available approaches in $O(n \log n)$ for the particular cases related to IRC, (E/T) scheduling, and soft time windows. Finally, besides their good complexity for solving independent timing problems, these dynamic programming open the way to efficient re-optimization procedures for solving series of similar timing instances. These latter methods are surveyed in Section 2.10.

2.7.5 State-of-the-art: single-dimensional features

We introduced and analyzed in this section the main single-dimensional features from the literature, independently of the field of application. Single-dimensional features are related to many prominent problems such as LISP and IRC, which have been the subject of extensive research. Various algorithms were examined and state-of-the-art methods for each particular feature and problem were identified. In the particular case of $\{\Sigma c_i^{\text{CVX}}(t_i)|\}$, 26 methods from various fields, such as routing, scheduling and isotonic regression, were classified into three main families: primal, dual, and dynamic programming methods.

Efficient linear methods are known for the most general problem of this category, $\{\Sigma c_i^{\text{CVX}}(t_i)|\}$, in presence of either convex or LSC and piecewise linear cost functions. Linear complexity is the best possible complexity when addressing the problem *from scratch*. As illustrated in Section 2.10,

the resolution of series of similar timing instances in a local search context can be performed more efficiently by means of re-optimization procedures, and research challenges remain open in this area, even for single-dimensional features.

2.8 Two-dimensional features

We now focus on the problems with two-dimensional features. These features are also often considered in presence of time-window TW constraints. The presentation is structured in relation to the level of problem complexity and generality. Starting with the duration feature DUR, which involves exclusively the first and last activities together, we then examine two-dimensional features involving successive activities: no-wait NWT, idle time IDL, and flexible $c_i(\Delta t_i)$ or time-dependent $P(t)$ processing times. Finally, features involving any pair of activities, such as time-lags TL, and cost functions separable by pairs of variables $\Sigma c_{ij}(t_i, t_j)$ are analyzed.

2.8.1 Total duration and total idle time

Accounting for total duration or idle-time is meaningful when one has the possibility to delay the beginning of operations. Otherwise, considering the maximum execution date feature C is sufficient. Whereas delaying the start of production is generally not an option in scheduling problems, it becomes particularly relevant in routing, as real-life objectives and driver's wages are frequently based on duration. We mention Savelsbergh (1992) for duration minimization under time-window and duration constraints in VRPs, Cordeau et al. (2004) that generalizes the previous approach for soft time-windows and duration constraints, Desaulniers and Villeneuve (2000) for shortest path settings with linear idle-time costs and time windows, and Desaulniers et al. (1998) and Irnich (2008b) for a general framework that enables to tackle duration and idle time features, among others, in various time-constrained routing and crew scheduling problems. It should be noted that computing the *total duration* or the *total idle time* is equivalent in the presence of fixed processing times. Therefore, w.l.o.g. we will focus on duration in this section.

To manage duration features in $\{DUR|TW\}$ and $\{|DUR, TW\}$, Savelsbergh (1992) proposed to first rely on a minimum idle time policy, and then shift activity execution dates forward to reduce the total duration. The related amount of shift was introduced many years ago in the project scheduling literature (Malcolm et al. 1959), as the latest processing date for an activity that does not cause a delay in the calendar. It is also known in the VRP literature under the name of *forward time slack* (Savelsbergh 1985, 1992). The following quantities are computed for each activity a_i: the earliest feasible execution date T_i, the cumulative idle time W_i on the subsequence $(a_1, \ldots, a_i)$ according to these execution dates, and the partial forward time slack F_i on the subsequence $(a_1, \ldots, a_i)$. These values are computed recursively by means of Equations (2.21-2.23), starting with the case of a single activity where $T_1 = r_1$, $W_1 = 0$ and $F_1 = d_1 - r_1$.

$$T_i = \max(T_{i-1} + p_{i-1}, r_i) \qquad 1 < i \leq n \tag{2.21}$$

$$W_i = W_{i-1} + T_i - T_{i-1} - p_{i-1} \qquad 1 < i \leq n \tag{2.22}$$

$$F_i = \min(F_{i-1}, d_i - T_i + W_i) \qquad 1 < i \leq n \tag{2.23}$$

The problem admits a feasible solution if and only if $T_i \leq d_i$ for all i. The execution date of the first activity in an optimal solution is given by $t_1^* = r_1 + \min\{F_n, W_n\}$. The other dates are computed using the minimum idle time policy. Both feasibility checking and duration minimization problems are thus solved in $O(n)$. Kindervater and Savelsbergh (1997), Desaulniers and Villeneuve (2000) and Irnich (2008b) proposed different calculations of this optimal schedule. As pointed out by Parragh et al. (2012), all these approaches are equivalent.

Tricoire et al. (2010) recently considered a more complex timing problem aiming to minimize duration under MTW constraints $\{DUR|MTW\}$. Each activity a_i is associated with a set of k_i time windows, $MTW_i = \{[r_{i1}, d_{i1}], \ldots, [r_{ik_i}, d_{ik_i}]\}$. The authors proposed a procedure that first removes some unnecessary time-window segments, not suitable for any feasible solution, while detecting infeasible timing problems. In a second step, the procedure examines a subset of *dominant* schedules, such that "no better solution exists with the same last activity execution date". For a given execution date t_n of the last activity, a *dominant* schedule with minimum duration can easily be found using the backward recursion of Equation (2.24).

$$t_{i-1} = \max\{t \mid t \leq t_i - p_{i-1} \ \wedge \ t \in MTW_i\} \tag{2.24}$$

Starting from the dominant schedule $\bar{\mathbf{t}}$ with earliest completion time, the method iteratively identifies the last activity a_i followed by idle time: $\bar{t}_i + p_i < \bar{t}_{i+1}$. If activity a_i does not admit a later time window, the algorithm terminates. Otherwise, the execution date of activity a_i is set to the beginning of the next time window, and the execution dates of activities situated afterwards in the sequence are re-computed with a minimum idle time policy. This leads to a *dominant* schedule which becomes $\bar{\mathbf{t}}$ in the next iteration. Tricoire et al. (2010) proved that at least one dominant schedule explored in the course of the algorithm is optimal. If each customer is associated to at least one time window, the overall method can be implemented in $O(n\varphi_{\text{MTW}})$, φ_{MTW} representing the number of time windows in the problem.

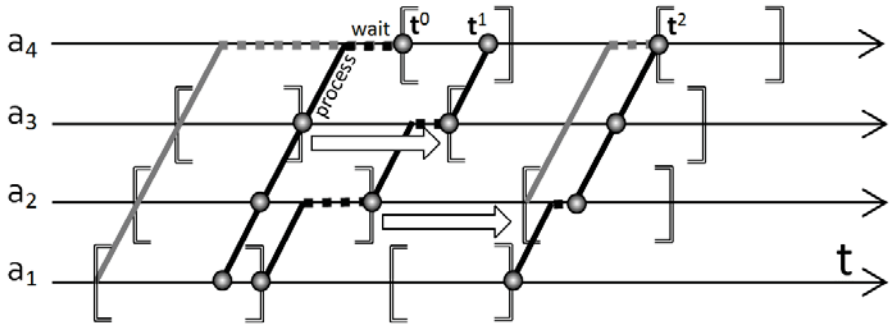


Figure 2.5: Duration minimization under multiple time-window constraints

This algorithm is illustrated in Figure 2.5 on a small example with four activities. Activities are represented from bottom to top with their time windows. The earliest completion date is computed with a minimum idle time policy, illustrated in gray lines. The initial dominant schedule $\bar{\mathbf{t}}^0$, in black, is then determined by backward recursion using Equation (2.24). This schedule presents

waiting time after activity a_3, and thus the execution date of this activity is delayed to the next time window, leading to a dominant schedule $\bar{\mathbf{t}}^1$. Now the latest activity followed by waiting time is a_2. Its execution date is delayed, and leads to the dominant schedule $\bar{\mathbf{t}}^2$. The latest activity followed by waiting time is a_1. As there is no later time window for this activity, the algorithm terminates. Among the three dominant schedules explored by the method, the best solution with minimum duration has been reached by $\bar{\mathbf{t}}^2$, and is optimal.

2.8.2 No wait and idle time

No-wait NWT and idle-time IDL features appear in various settings involving, among others, deterioration of products, maximum waiting times in passenger transportation, fermentation processes in the food industry, and cooling in metal-casting processes. IDL reduces to NWT when $\iota_i = 0$. No-wait constraints $t_i = t_{i+1}$ can also be addressed by problem reformulation, merging unnecessary variables. When no waiting time is allowed on the entire activity sequence, the timing problem becomes a minimization problem of a sum of single-variable functions. Two main categories of problems have been considered in the literature for NWT and IDL: the feasibility problem under idle-time and time-window constraints, and the optimization problem when some of these features appear in the objective function, treated in Section 2.8.3 in a more general context.

Feasibility checking under maximum idle time and time-window constraints has been frequently studied in the routing literature. Hunsaker and Savelsbergh (2002) designed an algorithm to check the feasibility of itineraries in dial-a-ride settings. This algorithm contains a $O(n)$ checking method for the special case of $\{|IDL, TW\}$. The solution to $\{|IDL, TW\}$ is found in two scans of the activity sequence. The first pass considers the relaxed subproblem $\{|TW\}$, determining for each activity i the earliest feasible execution dates T_i complying with time-windows and processing times. This calculation is performed by means of Equation (2.21). In a second pass, starting with $T'_n = T_n$, the algorithm proceeds backwards in the sequence to determine the earliest feasible execution dates $T'_i = \max(T'_{i+1} - p_i - \iota_i, T_i)$ and check idle-time constraints. The problem is declared infeasible if for any $i \in \{1, \ldots, n\}$, $T_i > d_i$ or $T'_i > d_i$.

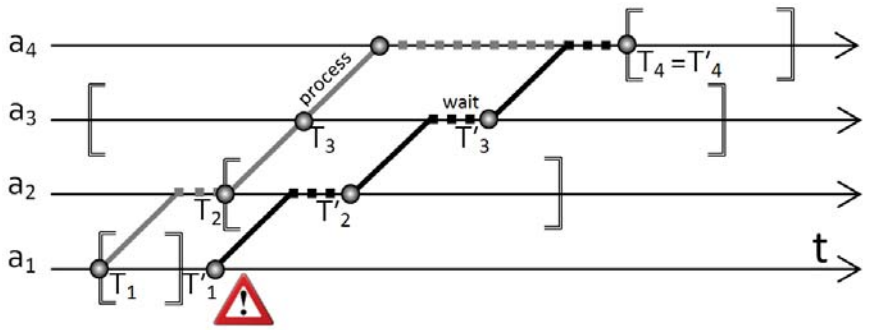


Figure 2.6: Feasibility checking under time-windows and idle-time constraints

The two passes are illustrated in Figure 2.6 for a small problem with four activities, represented from bottom to top with their time windows. The first pass (T_i values) has been represented in gray, while the backward scan, in black, provides the earliest feasible execution date for each

activity T'_i. As shown in the figure, this value exceeds the time window of activity a_1 ($T'_1 > d_1$), and thus the timing problem illustrated is infeasible.

2.8.3 Flexible processing times

Resource allocation, time-resource trade-offs, and project crashing problems have been thoroughly studied in the operations research literature, since they are critical in many applications, including energy optimization (see Section 2.5), project scheduling, portfolio selection, production economics, optimization of search effort, advertising, and so on and so forth (Kelley and Walker 1959, Talbot 1982, Ibaraki and Katoh 1988, Patriksson 2008). The processing time or resource allocation in all these problems can be increased or reduced at a cost. We refer to this feature as "flexible processing times" $\Sigma c_i(\Delta t_i)$. The characteristic function of this feature is a general separable function of successive execution time differences $t_{i+1} - t_i$. As such, it generalizes both *NWT* and *IDL*.

When only *NWT*, *IDL*, and $\Sigma c_i(\Delta t_i)$ features are encountered, the timing problem $\{\Sigma c_i(\Delta t_i)|\}$ (Equations 2.25-2.27) can be decomposed along $t_{i+1} - t_i$ values, and the independent minimization of every $c_i(\Delta t_i)$ leads to the optimal solution where $t_{i+1} - t_i = \text{argmin}_{\Delta t_i} \Sigma c_i(\Delta t_i)$.

When the c_i functions are convex and in presence of an additional constraint on the total duration, the problem $\{\Sigma c_i(\Delta t_i)|DUR\}$ can be reformulated (Equations 2.28-2.30) as a Simple Resource Allocation Problem (SRA) by setting $t_0 = 0$ w.l.o.g. and using the change of variables $x_i = t_{i+1} - t_i$.

$$\min_{(t_1,\ldots,t_n)\in\Re^{n+}} \quad \sum_{i=1}^{n-1} c_i(t_{i+1} - t_i) \tag{2.25}$$

$$s.t. \quad t_n - t_1 \leq d_{max} \tag{2.26}$$

$$t_i + p_i \leq t_{i+1} \qquad 1 \leq i < n \tag{2.27}$$

$$\min_{(t_1,\ldots,t_n)\in\Re^{n}} \quad \sum_{i=1}^{n-1} c_i(x_i) \tag{2.28}$$

$$s.t. \quad \sum_{i=1}^{n-1} x_i \leq d_{max} \tag{2.29}$$

$$x_i \in [p_i, +\infty] \qquad 1 \leq i < n \tag{2.30}$$

Non-linear resource allocation problems have been the subject of hundreds of papers. Two main families of methods for this problem are generally discerned. *Lagrangian methods* (Charnes and Cooper 1958, Everett 1963, Srikantan 1963) seek to iteratively progress towards the optimal value of the single dual variable associated to Equation (2.29) while respecting primal feasibility. *Pegging* algorithms (Sanathanan 1971, Luss and Gupta 1975, Kodialam and Luss 1998) relax the constraints of Equations (2.29-2.30), iteratively solving the relaxed problems and fixing the values of variables that do not satisfy primal constraints. A third approach originates from concepts

originally developed for the discrete problem version. Hochbaum (1994) proposes a greedy algorithm with arbitrary increments coupled with scaling techniques, yielding the best known computational complexity for this problem, $O(n \log n \log(d_{max}/\epsilon n))$, where ϵ is the target precision. The method increments progressively the x_i variable yielding the least reduced cost, leading to the optimum of this convex problem. Finally, a dynamic programming approach has been introduced in Karush (1962), which iteratively characterizes the best cost as a function of the resource consumption (total time in our case).

In particular, relationships can be established between these approaches and the three classes of methods surveyed in Section 2.7.3 for timing problems with separable convex costs. *Pegging* algorithms work similarly to the dual feasible methods of Section 2.7.3, such as PAV or the algorithm of Dumas et al. (1990), by iteratively solving relaxed problems and re-introducing the constraints. These constraints are here re-introduced by setting variables to constraint bounds, rather than joining pairs of activities (merging blocks) as in the PAV algorithm. The equivalent to the primal methods of Section 2.7.3 is less immediate. The greedy method of Hochbaum (1994) is one of this kind, but requires advanced scaling techniques to solve the continuous case. Also, even though most authors opted to optimize the single Lagrangian variable μ associated to Equation (2.29) rather than the primal variables, a consequence of the KKT conditions is that optimizing μ is equivalent to choosing iteratively the slope for each $c_i(x_i)$ and, in fact, iteratively fixing primal variables. Thus, Lagrangian approaches for this problem were eventually qualified by Patriksson (2008) as *explicitly dual* but also *implicitely primal*.

Adding time-window constraints to the model of Equations (2.25-2.27) leads to other timing settings, which can no longer be assimilated to resource allocation problems. In the special case of Norstad et al. (2010), a timing problem $\{\Sigma c_i^{\text{CVX}}(\Delta t_i)|TW\}$ is addressed with the objective $z(t) = \sum_{i=1}^{n} d_{i,i+1} \bar{c}\left(\frac{t_{i+1}-t_i}{d_{i,i+1}}\right)$ and non-increasing and convex $\bar{c}(\Delta t)$ functions. In presence of such functions, removing time-window constraints leads to optimal solutions presenting a constant ratio $\frac{t_{i+1}-t_i}{d_{i,i+1}}$ for all i, and thus constant speed on all legs. The *recursive smoothing algorithm* (RSA) exploits this property by maintaining this ratio constant on subsequences and progressively re-introducing violated time-window constraints. The overall method works in $O(n^2)$ elementary operations once the minimum of each function $\bar{c}(\Delta t)$ is given. It is noteworthy that this method is another example of a *dual feasible* algorithm based on the relaxation and re-introduction of constraints, conceptually similar to PAV and *pegging* methods.

In more general settings, when flexible processing times are combined with single-dimensional features such as time windows or time-dependent activity costs, the problem can become more difficult. Sourd (2005) and Hashimoto et al. (2006) have independently studied $\{\Sigma c_i(\Delta t_i), \Sigma c_i(t_i)|\}$ (Equation 2.31) with piecewise linear functions in the context of (E/T) scheduling and vehicle routing, and report its NP-hardness.

$$\min_{(t_1,\ldots,t_n)\in\Re^{n+}} \quad \sum_{i=1}^{n} c_i(t_i) + \sum_{i=1}^{n-1} c'_i(t_{i+1} - t_i) \tag{2.31}$$

When the functions c_i and c'_i are PiL with integer breakpoints, a dynamic programming algorithm is proposed to solve the problem in $O(T^2)$, where T represents an upper bound on the

schedule durations. Note that this dynamic programming algorithm can be viewed as an extension of the resource allocation algorithm of Karush (1962). The method can be implemented with a forward dynamic programming function $F_i(t)$ (Equations 2.32-2.33), which evaluates the minimum cost to process the subsequence of activities $(a_1, \ldots, a_i)$, starting the last activity exactly at time t $(t_i = t)$. The resulting optimal cost is given by $z^* = \min_t F_n(t)$.

$$F_1(t) = c_1(t) \tag{2.32}$$

$$F_i(t) = c_i(t) + \min_{0 \leq x \leq t} \{F_{i-1}(x) + c'_{i-1}(t-x)\} \qquad 1 < i \leq n \tag{2.33}$$

A polynomial dynamic programming algorithm working in $O(n(\varphi_c + \widehat{\varphi_c} \times \varphi'_c))$ exists for the case where the functions $c'_i(\Delta t)$ are PiL and convex (Sourd 2005, Hashimoto et al. 2006). φ_c and φ'_c represent the total number of pieces in the cost functions c_i and c'_i, and $\widehat{\varphi_c}$ stands for the number of convex pieces in the cost functions c_i. Efficient re-optimization procedures have also been proposed (Section 2.10). Finally, DUR involved in the objective can be seen as a special case of $\Sigma c_i(\Delta t_i)$ where $c_i^{\text{DUR}}(\Delta t_i) = \Delta t_i$. Since MTW is also reducible to $\Sigma c_i(t_i)$, the previous algorithm thus provides an alternative way to solve $\{DUR|MTW\}$ or $\{|DUR, MTW\}$ in $O(n+n\varphi_{\text{MTW}})$, where φ_{MTW} represents the total number of time windows.

2.8.4 Time-dependent processing times

In several application settings, activity processing times may vary as a function of the execution dates. In machine and project scheduling for example, learning, deterioration effects and other time-dependencies can have a large impact (see Alidaee and Womer 1999, Cheng 2004). Network congestion is a major concern for vehicle routing and data transmission (Van Woensel et al. 2008, Kok et al. 2009) and, thus, the time-dependent processing-time feature $P(t)$ appears in various network optimization problems: shortest path (Cooke and Halsey 1966, Dreyfus 1969, Halpern 1977), traveling salesman (Malandraki and Dial 1996), vehicle routing (Beasley 1981, Malandraki and Daskin 1992, Ichoua et al. 2003, Donati et al. 2008, Hashimoto et al. 2008), and so on.

The literature on the subject can generally be separated between discrete and continuous settings. Discrete optimization models generally involve time-space networks which are less likely to present the timing issues studied in this article, whereas several continuous models have led to explicit timing problems with $P(t)$ features, as in Ichoua et al. (2003), Fleischmann et al. (2004), Donati et al. (2008), and Hashimoto et al. (2008). These models involve constraints of the type $t_i + p_i(t_i) \leq t_{i+1}$ within a timing formulation with other additional features. A *FIFO* assumption on functions p_i can be made:

$$\text{FIFO assumption: } \forall i \; x \geq y \Longrightarrow x + p_i(x) \geq y + p_i(y) \tag{2.34}$$

FIFO implies that any delay in an activity execution date results in a delay in its completion date. The assumption is meaningful in several settings, e.g. vehicle routing, as two vehicles that behave similarly on the same route are supposed to remain in the same arrival order, whatever congestion happens (Ichoua et al. 2003).

Time-dependent processing-time features are generally assumed to result in more complex timing problems. However, one should clearly identify the source of the difficulty, which is frequently imputable to the calculation and access to $p_{ij}(t)$ throughout the search, and not necessarily to the timing problem solving method. Assuming that $p_{ij}(t)$ can be evaluated in constant time and under FIFO, $\{D|R, P(t)\}$ is still solvable in $O(n)$ by means of a *minimum idle time policy* (Fleischmann et al. 2004). In the same spirit, Donati et al. (2008) apply the time-slack approach of Savelsbergh (1992) to $\{|TW, P(t)\}$. Still, dedicated methodologies are necessary for other settings such as $\{DUR|TW, P(t)\}$ and for re-optimization procedures (Section 2.10).

A general time-dependent timing problem $\{\Sigma c_i(t_i)|P(t)\}$, Equations (2.35-2.36), is addressed in Hashimoto et al. (2008). All functions considered are PiL, non-negative and lower semicontinuous.

$$\min_{(t_1,\ldots,t_n)\in\mathbb{R}^{n+}} \quad \sum_{i=1}^{n} c_i(t_i) \tag{2.35}$$

$$s.t. \quad t_i + p_i(t_i) \leq t_{i+1} \qquad 1 \leq i < n \tag{2.36}$$

The authors propose a dynamic programming approach, which extends the method of Section 2.7.4. It involves the functions $F_i(t)$, which represent the minimum cost to process the subsequence of activities $(a_1, \ldots, a_i)$ while starting the last activity before t ($t_i \leq t$). These functions can be computed by means of Equations (2.37-2.38), and the optimal solution cost is given by $F_n(+\infty)$.

$$F_1(t) = \min_{0\leq x\leq t} c_1(x) \tag{2.37}$$

$$F_i(t) = \min_{0\leq x\leq t} \{c_i(x) + \min_{x'+p_i(x')\leq x} F_{i-1}(x')\} \qquad 1 < i \leq n \tag{2.38}$$

Under the assumption of Equation (2.39), which is slightly weaker than FIFO, the method can be implemented in $O(n(\varphi_c + \varphi_p))$, where φ_c and φ_p are the total number of pieces in cost and processing-time functions. This method for $\{\Sigma c_i(t_i)|P(t)\}$ thus presents the same quadratic complexity as in the case without time dependency (Section 2.7.4). When the previous assumption does not hold, the dynamic programming method of Hashimoto et al. (2008) is not polynomial, and the question remains open whether $\{\Sigma c_i(t_i)|P(t)\}$ is polynomially solvable.

$$\text{(HYI) assumption: } \forall i \; x + p_i(x) = y + p_i(y) \Longrightarrow x + p_i(x) = z + p_i(z) \; \forall z \in [x, y] \tag{2.39}$$

2.8.5 Time lags

The two-dimensional features surveyed in the previous sections involved linking constraints and objectives between the first and last tasks, in the case of DUR, or between pairs of successive variables in the case of NWT and IDL. We now review the time-lag TL feature, which brings into play a time difference $t_j - t_i$ between any two activity execution dates t_i and t_j. This feature is thus a generalization of NWT, IDL and DUR.

To the best of our knowledge, early research on time lags has been conducted by Mitten (1959) for flowshop scheduling problems. This feature has been used since to model many problem characteristics in various domains, such as the deterioration of food or chemical products, glue

drying, customer requirements in some dial-a-ride problems, elevator dispatching, quarantine durations, and so on. Time-lag scheduling problems on a single machine have also been shown by Brucker et al. (1999) to generalize all shop, multi-purpose machines, and multi-processor scheduling problems. Hence, timing problems with TL are likely to be difficult.

The most basic problem with TL feature relates to feasibility checking under time-lag constraints of the form $t_i + \delta_{ij} \leq t_j$. When $\delta_{ij} \geq 0$, the constraint is called positive time lag, and corresponds to a minimum delay between activities a_i and a_j, whereas $\delta_{ij} \leq 0$ corresponds to a negative time lag, and involves a maximum delay of $-\delta_{ij}$ between the activities a_j and a_i. Equality constraints $t_i + \delta_{ij} = t_j$ involve both positive and negative time lags. The resulting timing problem $\{|TL\}$ can be seen as a special case of project scheduling on a chain of activities, and the METRA potential method (MPM) of Roy (1959, 1962) can be applied. In MPM, the time-lag constraints are represented on a graph $G = (V, A)$, where each activity a_i is associated with a node $v_i \in V$, and each arc (v_i, v_j), associated with a weight w_{ij}, represents a temporal constraint of the form $t_j - t_i \geq w_{ij}$. The feasibility of $\{|TL\}$ is equivalent to the non-existence of a positive length cycle in this graph (Bartusch et al. 1988, Dechter et al. 1991). The algorithm of Floyd-Warshall can be employed to solve this problem in $O(n^3)$, but the longest-path procedure of Hurink and Keuchel (2001), also in $O(n^3)$, is shown to provide faster results in practice. Potts and Whitehead (2007) also considered a coupled-operation scheduling problem with only $n/2$ time-lag constraints, and timing feasibility is checked in $O(n^2)$. The authors underlined the computational burden of such timing algorithms, which strongly degrades the performance of neighborhood searches or branch and bound procedures.

Hunsaker and Savelsbergh (2002) studied a case of $\{|TL, TW\}$ timing in the context of dial-a-ride problems. Activities represent customer requests on pick-up and deliveries services, which occur by pairs, such that any pick-up always precedes its corresponding delivery in the sequence. Each such pair of activities is linked by a single positive time-lag constraint. The total number of time-lag constraints is thus $n/2$. The problem also involves time windows and maximum idle times for each activity. The authors claim that the resulting feasibility problem can be solved in three passes on the sequence of activities with linear complexity. Yet, the algorithm presents a small flaw, which is straightforward to correct (Tang et al. 2010), but leads to a $O(n \log n)$ complexity (Haugland and Ho 2010). A $O(n)$ complexity is finally achieved in Firat and Woeginger (2011) by means of a reduction to a shortest path problem on a weighted interval graph.

The same setting is also addressed in Gschwind and Irnich (2012). The authors describe a labeling procedure based on $|M_i|$ resources for each pickups and each delivery activity a_i. M_i stands for the current number of open pickups at a_i. This labeling procedure provides another way to check the feasibility of a fixed activity sequence $\{|TL, TW\}$ in $O(n^2)$. Also, it enables to evaluate the feasibility of an extended sequence with one additional activity a_{n+1} in $O(M_{n+1})$ given the information on the original sequence. Such forward extension function is critical when solving shortest path problems with underling timing features.

Cordeau and Laporte (2003) and Berbeglia et al. (2011) consider a dial-a-ride setting with an additional duration constraint on the entire trip duration. The authors solve heuristically a Lagrangian relaxation of the problem with a hierarchical objective. Total trip duration infeasibility

is minimized, then time-window infeasibility and, finally, time-lag infeasibility, that is the timing problem $\{DUR > D > TL|R\}$. The algorithm first minimizes duration and time-window infeasibility as in Section 2.8.1, then iteratively delays some pick-up services to reduce time-lag infeasibility without increasing any other violation. A computational complexity of $O(n^2)$ is achieved. It was observed (Private Communication 2010), however, that the previous approach only guarantees optimality under an additional assumption that we call *LIFO*, which requires that for any $1 \leq i < j < k < l \leq n$, no activities a_i, a_j, a_k, a_l present "entangled" time-lag constraints of the form $t_k - t_i \leq \delta_{ik}$ and $t_l - t_j \leq \delta_{jl}$. The LIFO assumption is frequently enforced in the vehicle routing literature, especially when transporting passengers, or in presence of complex loading constraints. In this case, the last object or customer received in the vehicle is the first one to leave. Without this assumption, the difficulty of many problems with time lags strongly increases, and no specialized efficient algorithm is actually known for $\{DUR > D > TL|R\}$ and similar problems.

2.8.6 Separable costs by pairs of variables

Separable costs by pairs of variables $\Sigma c_{ij}(t_i, t_j)$ generalize all problems combining single or two-dimensional features. The timing problem with this feature alone is NP-hard, as it includes $\{\Sigma c_i(t_i), \Sigma c_i(\Delta t_i)|\}$ as special case (see Section 2.8.2). When the objective function is convex, the problem $\{\Sigma c_{ij}^{\text{CVX}}(t_j - t_i), \Sigma c_i^{\text{CVX}}(t_i)|\}$ given in Equations (2.40-2.41), is equivalent to the *convex cost dual network flow problem* for which weakly polynomial algorithms are available (Ahuja et al. 2003).

$$\min_{\mathbf{t}=(t_1,\ldots,t_n)\in\Re^{n+}} \quad \sum_{(i,j)\in Q} c_{ij}^{\text{CVX}}(t_j - t_i) + \sum_{1\leq i\leq n} c_i^{\text{CVX}}(t_i) \tag{2.40}$$

$$s.t. \quad t_i + p_i \leq t_{i+1} \qquad 1 \leq i < n \tag{2.41}$$

2.9 Conclusion on "stand-alone" timing methods

In contrast to single-dimensional features, which appeared as fairly well addressed in Section 2.7 by means of a few algorithms and concepts, two-dimensional features lead to more diverse problem structures and algorithms. Several simple cases with duration minimization or time-dependent processing times can be solved in linear time, but other problems with time-lag features actually require $O(n^3)$ algorithms to be solved exactly. Although polynomial, the latter methods can be impracticable in the context of local searches or branch-and-bound approaches.

Many practical timing settings result in models with linear constraints and linear or separable convex objectives. For these problems, the linear and convex programming theory ensures weakly polynomial solvability, and provides general solution methods (Khachiyan 1979, Karmarkar 1984, Hochbaum and Shanthikumar 1990). Some general problems, however, such as $\{\Sigma c_i(t_i), \Sigma c_i(\Delta t_i)|\}$ with PiL functions, are NP-hard, while for other problems, such as $\{\Sigma c_i(t_i)|P(t)\}$ with general piecewise linear functions $P(t)$, the existence or non-existence of polynomial algorithm is still open. Timing settings thus lead to a rich variety of problem structures and complexities.

In all these cases, whether polynomial algorithms are available or not, research is still open to provide more efficient algorithms exploiting the particular structure of the features and problems at hand. The present paper contributed by building a formalism, a classification of features, timing problems and methods. We gathered the most efficient stand-alone timing approaches from various fields of research to tackle both specialized timing settings, and more general features. The focus can now be turned on filling the gaps that have been highlighted in this review, and which continue to appear, following the rich variety of application cases with time constraints emerging nowadays. Finally, important avenues of research target the efficient solving of *series* of timing problems, in the particular context of neighborhood searches and branch-and-bound. Such approaches are presented in the next part of this paper.

2.10 Timing re-optimization

In the first part of this article, we examined how to address timing problems as a stand-alone issue. Yet, most neighborhood-search-based heuristics, metaheuristics, and some exact methods require to solve iteratively a large number of closely related timing instances. In this case, solving each timing problem "from scratch", without exploiting any knowledge on previous resolutions, can result in losses of information and redundant computations.

Most local searches for routing and scheduling problems (see Hoos and Stützle 2005 for a thorough presentation of LS) rely on a neighborhood based on a limited number of sequence changes. One or several timing subproblems are solved for each neighbor to estimate its feasibility and cost. Two classical neighborhoods, dedicated respectively to exchanging the tails of two vehicle routes in VRP context (2-opt* of Potvin and Rousseau 1995), and relocating a sequence of activities (Or-exchange of Or 1976), are illustrated in Figure 2.7. It is noticeable that large subsequences of activities (e.g. Seq.A. . . Seq.D on the figure) are shared by successive timing subproblems.

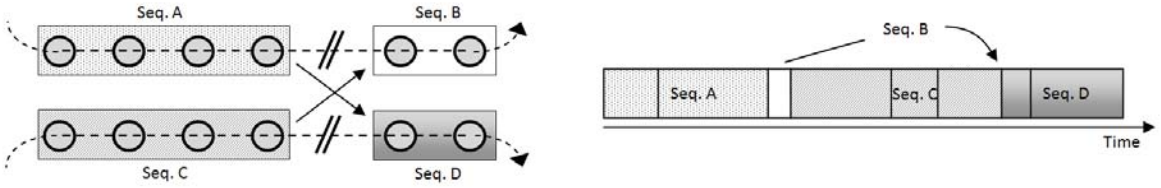


Figure 2.7: Sequence invariants in 2-opt* and relocate moves

Branch-and-bound procedures for problems with sequencing decisions can similarly involve timing subproblems at nodes of the search tree when evaluating sequences of activities, during lower bound computation and branch pruning (Hoogeveen and Van De Velde 1996, Sourd and Kedad-Sidhoum 2003). The search for improving columns, in columns generation approaches, also frequently involves elementary shortest paths with combined timing decisions and resource constraints (Desrochers et al. 1992, Prescott-Gagnon et al. 2009, Baldacci et al. 2011c).

It is noticeable that, in all these cases, numerous closely related timing problems must be solved, where long subsequences of consecutive activities remain unchanged, and only a minor proportion

of problem parameters (reduced cost values for column generation) is impacted. Several authors thus propose to keep *meaningful data* on the global search process to save computations and solve more efficiently these series of similar timing problems. Neighborhood searches have actually largely benefited from these techniques, as move evaluations (and thus the resolution of timing problems) take the largest part of the computational effort. These *re-optimization* methodologies therefore can lead to dramatic reductions in the computational burden of algorithms.

We now formally define *serial* timing problems in Section 2.10.1, and present a general framework for re-optimization methods based on sequence concatenations in Sections 2.10.2-2.10.3. Links with related re-optimization methodologies are analyzed in Section 2.10.4, before reviewing or introducing efficient concatenation-based re-optimization methods for each major timing feature and related problems in Section 2.10.5.

2.10.1 Problem statement: Serial timing

This section formally defines serial timing problems. Sequence-dependent processing times are also considered here, in relation to a large range of vehicle routing applications, which heavily rely on re-optimization methods.

Definition 2.10.1 (Serial timing) *Let $\mathcal{T}$ be an* incumbent timing problem *with n activities $(a_1, \ldots, a_n)$, sequence-dependent processing-times p_{ij}, and additional features with their data and characteristic functions $f_i^x(\mathbf{t})$. Features are separated into two sets $\mathcal{F}^{\text{OBJ}}$ and $\mathcal{F}^{\text{CONS}}$ following their role as objective or constraint (see Section 2.4). N permutation functions $\sigma^k : \{1, \ldots, n\} \to \{1, \ldots, n\}$ for $k \in \{1, \ldots, N\}$, are also given. The* serial timing *problem involves to solve the timing subproblems $\mathcal{T}^k$ of Equations (2.42-2.44), for $k \in \{1, \ldots, N\}$.*

$$(\mathcal{T}^{\|}): \quad \min_{\mathbf{t}=(\sqcup_\infty,\ldots,\sqcup_\setminus)\in\Re^{\setminus}+} \quad \sum_{F^x\in\mathcal{F}^{\text{OBJ}}} \alpha_x \sum_{1\le y\le m_x} f_y^x(\mathbf{t}) \tag{2.42}$$

$$s.t. \quad t_{\sigma^k(i)} + p_{\sigma^k(i)\sigma^k(i+1)} \le t_{\sigma^k(i+1)} \qquad 1 \le i < n \tag{2.43}$$

$$f_y^x(\mathbf{t}) \le 0 \qquad F^x \in \mathcal{F}^{\text{CONS}}\ ,\ 1 \le y \le m_x \tag{2.44}$$

To efficiently solve the previous problem, several types of re-optimization approaches have been developed in the literature to take advantage of the information developed during the successive subproblem solving. One such approach involves re-arranging previously developed schedules in relation to the new settings. For network-flow or shortest-path formulations especially, re-optimization methods related to a change of arcs or costs in the network have been thoroughly studied (Goto and Sangiovanni-Vincentelli 1978, Pallottino 2003, Frangioni and Manca 2006, Miller-Hooks and Yang 2005). Most timing problems can also be formulated as linear programs, on which sensitivity analysis and warm start following a problem modification have been studied for long, and can be tackled by means of a primal-dual simplex algorithm. Finally, a last methodology, on which we focus in the following, is based on the simple observation that a permutation of activities can be assimilated to a concatenation of some subsequences of consecutive activities. Hence, keeping information on subsequences (e.g., dynamic programming data) can lead to significant speed up in

solving (Kindervater and Savelsbergh 1997, Cordone 2000). We introduce in the next sections a framework for these concatenation properties and the re-optimization approaches that follow.

2.10.2 Breakpoints and concatenations

We first introduce some vocabulary, and then emphasize the links between operations on sequences of activities, such as activity relocations or changes of order relations, and properties of the resulting permutation functions. These observations lead to efficient re-optimization approaches "by concatenation".

Definition 2.10.2 (Permutation breakpoints) *Let $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ be a permutation. Any integer b such that $\sigma(b) + 1 \neq \sigma(b + 1)$ and $1 \leq i < n$ is called a breakpoint of σ, and corresponds to non consecutive values in the permutation representation.*

Let $b(\sigma)$ denote the number of breakpoints of σ, and $b_1^\sigma, \ldots, b_{b(\sigma)}^\sigma$ denote these breakpoints in increasing order. For instance, the permutation $\sigma_0 : \{1, 2, 3, 4, 5, 6\} \to \{4, \mathbf{5}, \mathbf{3}, 1, \mathbf{2}, 6\}$ has three breakpoints (indicated in boldface): $b_1^{\sigma_0} = 2$, $b_2^{\sigma_0} = 3$, and $b_3^{\sigma_0} = 5$. We now show the links between classical operations on activity sequences and the resulting permutation function properties in terms of breakpoints:

Lemma 2.10.1 (Order changes) *Let A' be an activity sequence obtained from A by changing l order relations and $\sigma_{A \to A'}$ the associated permutation function, then $b(\sigma_{A \to A'}) = l$.*

Lemma 2.10.2 (Activity relocations) *Let A' be an activity sequence obtained from A by relocating l activities and $\sigma_{A \to A'}$ the associated permutation function, then $b(\sigma_{A \to A'}) \leq 3l$.*

Any change of the order relationship results in exactly one breakpoint, while any relocation of activity can be assimilated to at most three changes of order relations, and thus yields three breakpoints. Situations where k order relations or activities are changed from one timing problem T to another problem T' occur frequently in the context of neighborhood searches working on sequences of activities and solving timing subproblems to evaluate cost or feasibility of each new sequence. The interest of breakpoints is highlighted in the following proposition. Although straightforward, it provides the basis of re-optimization methods working by concatenation.

Proposition 2.10.1 *Let $\sigma : \{1, \ldots, n\} \to \{1, \ldots, n\}$ be a permutation with breakpoints $b_1^\sigma, \ldots, b_{b(\sigma)}^\sigma$. Let A be a sequence of n activities. Then, $A' = \sigma(A)$ corresponds to the concatenation of exactly $b(\sigma) + 1$ subsequences of consecutive activities in A, as presented in Equation 2.45. A dummy breakpoint $b_0^\sigma = 1$ stands for the beginning of the sequence.*

$$A' = \bigoplus_{l=0,\ldots,b(\sigma)-1} (a_{\sigma(b_l^\sigma+1)}, \ldots, a_{\sigma(b_{l+1}^\sigma)}) \tag{2.45}$$

Any bounded number of operations transforming an activity sequence A into A' (relocation of activities, or changes of order relations) thus involves a permutation function with a bounded number of breakpoints, such that A' can be seen as a concatenation of a bounded number of

subsequences of A. Pre-processed informations from a bounded number of subsequences can then be exploited to produce information on their concatenation, to solve the timing subproblems without browsing all activities of the sequence at play. The next section formalizes the re-optimization operations and algorithms that can be developed to this extent.

2.10.3 Re-optimization "by concatenation"

Re-optimization by concatenation can be formalized through a set of four basic re-optimization operators, given in Table 2.2, which are used for data acquisition and exploitation on subsequences of consecutive activities. As the goal of this methodology is to avoid redundant computations, it is critical for all these operators to maintain the integrity of the input data.

Table 2.2: Re-optimization operators

Initialization:	Initialize the data $\mathcal{D}(A)$ of a sequence containing a single activity.
Forward extension:	Given an activity a_k and a sequence $A = (a_i, \ldots, a_j)$ with its data, determine the data $\mathcal{D}(A')$ for the sequence $A' = (a_k, a_i, \ldots, a_j)$.
Backward extension:	Given an activity a_k and a sequence $A = (a_i, \ldots, a_j)$ with its data, determine the data $\mathcal{D}(A')$ for the sequence $A' = (a_i, \ldots, a_j, a_k)$.
Evaluate concatenation:	Given L sequences of activities $\mathcal{D}(A_l), l = 1 \ldots l$ and their data, evaluate the feasibility and the optimal solution cost of the timing problem involving the concatenation of these sequences.

It should first be noted that the *forward extension* (respectively *backward extension*) operation of Table 2.2 directly derives from forward or backward dynamic programming concepts. Bi-directional dynamic programming approaches can also be assimilated to both forward and backward extension operations, as well as a single concatenation evaluation to provide the optimal solution (Righini and Salani 2006). The re-optimization approach "by concatenation", illustrated in Algorithm 2.1, is also based on these operators. Data is built on subsequences of the incumbent timing problem $\mathcal{T}$, by means of the forward and backward extension operators. These data, developed during a preprocessing phase or during the search, are then used to solve repetitively all the derived subproblems using the *evaluate concatenation* operator.

Algorithm 2.1 Re-optimization

1: Build re-optimization data on subsequences of the *incumbent timing problem* $\mathcal{T}$, using *initialize*, and *forward extension* or *backward extension*.
2: **for each** timing subproblem $\mathcal{T}^k$, $k \in \{1, \ldots, N\}$;
3: Determine the breakpoints involved in the permutation function σ^k;
4: Evaluate the optimal cost of $\mathcal{T}^k$, as the concatenation of $b(\sigma) + 1$ activity subsequences from $\mathcal{T}$ (Equation 2.45).

The efficiency of Algorithm 2.1 directly relies on the potential to develop concatenation operations that are less computationally complex than stand-alone methods. One should also pay attention to the price to pay in terms of data computing on subsequences, and whether the resulting computational effort is dominated by the quantity of derived timing subproblems to solve. There is no unique way to apply this method; problem specific design choices arise, for example,

when determining the nature of the data, the subsequences on which it is computed, as well as the phases dedicated to data computation.

We therefore illustrate an application in neighborhood search for routing problems with time constraints on routes. In this context, a local search improvement procedure based on sequence changes leads to a number of timing subproblems proportional to the number of neighborhood solutions to explore. The number of derived timing subproblems is often $N = \Omega(n^2)$ in the VRP or scheduling literature, and the derived activity sequences do not involve the concatenation of more than $k = 2, 3$ or 4 subsequences of the original problem.

The overall complexity of a neighborhood exploration, when solving each timing subproblem independently, is $Nc(T)$, $c(T)$ being the computational complexity of one stand-alone timing solution procedure. A straightforward re-optimization approach consists in exhaustively computing the data for each of the $n(n-1)$ subsequences of consecutive activities from $\mathcal{T}$, and then use it to evaluate all moves. Data computation is straightforward to perform in $O(n^2c(I) + n^2c(F/B))$. $c(I)$ and $c(F/B)$ stand for the computational complexity of initialization, and forward or backward extension, respectively. The overall complexity of the new neighborhood exploration procedure is thus $O(n^2c(I) + n^2c(F/B) + Nc(EC))$, $c(EC)$ being the complexity for evaluating the concatenation of less than 4 subsequences. Assuming that $N = \Omega(n^2)$, the computational complexity of neighborhood evaluation becomes $O(N[c(I) + c(F/B) + c(EC)])$ for re-optimization methods instead of $Nc(T)$ for independent solving. Re-optimization operators being less computationally complex than stand-alone methods, the resulting approach is likely to lead to reduced computational effort.

Concatenation operators involving more than two sequences are not actually available or not computationally suitable for efficient re-optimization for some settings such as $\{|MTW\}$ and most problems with two-dimensional features. In this case, forward and backward propagation may be used along with concatenations of two subsequences only to perform the timing subproblem evaluations. Such an example is given by the lexicographic search of Savelsbergh (1985), which enables to evaluate timing subproblems associated to well-known neighborhoods for vehicle routing problems exclusively by means of concatenation of two subsequences. Designing efficient evaluation orders in such contexts becomes a problem dependent issue, clearly impacted by the complexity of the operators at hand, and the nature of the permutations involved.

Finally, if the concatenation of many subsequences can be operated efficiently, but data creation constitutes the bottleneck in terms of computational effort, the subset of subsequences involved can be restricted to $O(n^{4/3})$ or $O(n^{8/7})$, using the hierarchical approach of Irnich (2008a).

The relevant data to compute can also be tailored relatively to the neighborhoods at play. For example, the 2-opt* move presented in Figure 2.7 involves only the concatenation of subsequences containing the first or last activity, which we call *prefix* or *suffix subsequences*. The number of such subsequences requiring data computation is thus reduced to $O(n)$.

2.10.4 General literature on the topic

A large range of vehicle routing and scheduling problems are addressed using local search on sequences. Serial timing issues thus frequently arise in these fields.

Following the seminal work of Savelsbergh (1985, 1992), Kindervater and Savelsbergh (1997) proposed a framework to manage several constraints on vehicle routes, such as precedence constraints, time windows, collection and deliveries under capacity constraints. Several of these constraints are explicitly, or can be assimilated to, timing features. To perform efficient feasibility checking, the authors develop *global variables* on partial routes, which are used in concatenation operations to evaluate moves consisting of a constant number of edge exchanges. Move evaluations are performed in lexicographic order to allow calculation of the global variables through the search.

Cordone (2000) and Duhamel (2001) report similar concepts of global data management on subsequences and concatenation operators. Although these methodologies are essentially dedicated to the VRPTW, they explore different possibilities related to the concept of macro-nodes. Indeed, when the information developed on subsequences has the same structure as the problem data on activities, subsequences of activities can be replaced by equivalent single activities during timing resolution. Concatenation concepts in this case enable to temporarily reduce the problem size by collapsing nodes into "macro-nodes", opening the way to algorithms based on aggregation of activities and multi-level approaches (Bean et al. 1987, Walshaw 2002, Elhallaoui et al. 2005).

Campbell and Savelsbergh (2004) presented a compilation of efficient insertion heuristics for many vehicle routing problems with additional characteristics such as shift time limits, variable delivery quantities, fixed and variable delivery times, and multiple routes per vehicle. These methods iteratively create solutions by adding customers to the routes. The authors show that by managing global data on the routes, the cost of feasibility of customer insertions can be evaluated in amortized $O(1)$ for many of these settings.

A rich body of dynamic programming-based timing algorithms is also presented in Ibaraki et al. (2005), Hashimoto et al. (2006, 2008), Hashimoto (2008), Ibaraki et al. (2008), and Hashimoto et al. (2010). Forward and backward propagation is used, with an additional "connect" operator to manage concatenation of two subsequences, thus leading to efficient re-optimization approaches by concatenation for several timing problems involving Piecewise Linear (PiL) functions.

The framework of Desaulniers et al. (1998) models many constraints and objectives on sequences of activities as resources that are subject to window constraints, and are extended from one activity to the next by means of resource extension functions (REFs). This framework proved extremely efficient to model many crew scheduling and routing problems and solve them by column generation (Desaulniers et al. 2005). It has also been recently extended by Irnich (2008a,b) to perform efficient neighborhood search under various constraints on routes, such as load dependent costs, simultaneous pickup and deliveries, maximum waiting and duty times. To that extent, REFs "generalized to segments" are built on subsequences to characterize their resource consumption. Inverse REFs are defined to give an upper bound on the resource consumptions that allow the sequence to be processed. Developing this data on subsequences enables then to evaluate efficiently the cost or feasibility of local search moves. This framework, however, requires rather restrictive conditions: the existence of REFs that can be generalized to subsequences and inversed. These conditions are satisfied only by a limited subset of the timing problems and features introduced previously, such as $\{|TW\}$, $\{|MTW\}$ or $\{DUR|TW\}$.

Several general methodologies thus exist in the literature to tackle timing problems within a neighborhood search context. However, these approaches are restricted by the types of concatenations allowed, the assumptions made on features, or the applicability of the models to a wide range of constraints. The timing formalism we propose and its generalization to re-optimization procedures by concatenation, following the concepts of Kindervater and Savelsbergh (1997), is less specialized and thus can encompass a wider range of timing settings and methods. As shown in the next sections, this framework unifies previous successful concepts, and provides a line of thought for the development of efficient algorithms for various timing problems. It can also be viewed as a generalization of Irnich (2008a,b) concepts applied to timing problems. Indeed, generalized REFs and their inverse provide, when they exist, the suitable data for re-optimization.

2.10.5 Re-optimization algorithms

We now review and analyze re-optimization approaches for main timing features and problems. As in the first part of this paper, the analysis is organized by increasing order of complexity and feature dimensions, starting with single-dimensional features in Sections 2.10.5.1 to 2.10.5.5, and finishing with two-dimensional features in Sections 2.10.5.6 to 2.10.5.8. For each timing setting we describe the re-optimization data, as well as the operators that can be used for forward and backward data computation and for evaluating the concatenation of several subsequences. These operators can be used within Algorithm 2.1 to develop efficient re-optimization approaches by concatenation.

2.10.5.1 Constant activity costs & cumulative resources.

In presence of constant activity costs or, more generally, in presence of a cumulated resource such as distance, load or time with a global constraint, evaluating the total cost of a solution from scratch would involve to browse each activity and cumulate the costs, or the resource consumption, resulting in an $O(n)$ complexity. To perform more efficient evaluations, a simple re-optimization approach involves applying Algorithm 2.1 with the following data and operators.

Data. Partial cost $C(A_i)$ (or resource consumed) by each subsequence A_i.

Data computation. Cumulating the cost (or resource consumption) on the subsequence.

Evaluate concatenation. Cumulating the partial costs (or resource consumptions) on subsequences: $C(A_1 \oplus \cdots \oplus A_k) = \sum C(A_i)$.

The evaluation of the concatenation of a bounded number of subsequences can thus be performed in a bounded number of operations, leading to $O(1)$ complexity for move evaluation when the data is available. Data can be processed in amortized constant time for each move during a local search procedure for many classic neighborhoods, using a *lexicographic order* (Kindervater and Savelsbergh 1997) for move evaluation, or developed in a preprocessing phase for a complexity of $O(n^2)$, which is often dominated by the neighborhood size.

2.10.5.2 Weighted execution dates & non-decreasing linear costs.

The feature W and, in general, non-decreasing linear time-dependent costs of the form $c_i(t_i) = w_i t_i + c_i$ with $w_i \geq 0$ for $i \in \{1, \ldots, n\}$ can be addressed with the following re-optimization data and operators.

Data. Total processing time $T(A_i)$ for all the activities of the sequence A_i but the last one. Waiting cost $W(A_i)$ related to a delay of one time unit in the sequence processing, and sequence cost $C(A_i)$ when started at time 0.

Data computation and evaluate concatenation. For a sequence A with a single activity, $T(A) = 0$, $W(A) = w_{A(1)}$ and $C(A) = c_{A(1)}$. Equations (2.46-2.48) enable both to evaluate the cost of concatenation and compute by induction the data for sequences with more activities.

$$W(A_1 \oplus A_2) = W(A_1) + W(A_2) \tag{2.46}$$
$$C(A_1 \oplus A_2) = C(A_1) + W(A_2)(T(A_1) + p_{A_1(|A_1|)A_2(1)}) + C(A_2) \tag{2.47}$$
$$T(A_1 \oplus A_2) = T(A_1) + p_{A_1(|A_1|)A_2(1)} + T(A_2) \tag{2.48}$$

The previous equations have been formulated to also manage sequence-dependent processing times. It is remarkable that, in this case, the re-optimization data is a simple generalization of single-activity characteristics to sequences of activities, similar to the *generalization to segments* concepts of Irnich (2008b).

2.10.5.3 Time-windows feasibility check.

Savelsbergh (1985) opened the way to efficient feasibility checking with regards to time windows in the context of local search. In the subsequent work of Kindervater and Savelsbergh (1997), the following re-optimization data and operators are introduced.

Data. Total processing time $T(A_i)$ for all the activities of the sequence A_i but the last one. Earliest execution date $E(A_i)$ of the last activity in any feasible schedule for A_i. Latest execution date $L(A_i)$ of the first activity in any feasible schedule for A_i. A record $isFeas(A_i)$ valuated to true if and only if a feasible schedule for A_i exists.

Data computation and evaluate concatenation. For a sequence A with a single activity, $T(A) = 0$, $E(A) = r_{A(1)}$, $L(A) = d_{A(1)}$ and $isFeas(A) = true$. Equations (2.49-2.52) enable to evaluate the cost of concatenation and compute the data for sequences with more activities in $O(1)$.

$$T(A_1 \oplus A_2) = T(A_1) + p_{A_1(|A_1|)A_2(1)} + T(A_2) \tag{2.49}$$
$$E(A_1 \oplus A_2) = \max\{E(A_1) + p_{A_1(|A_1|)A_2(1)} + T(A_2), E(A_2)\} \tag{2.50}$$
$$L(A_1 \oplus A_2) = \min\{L(A_1), L(A_2) - p_{A_1(|A_1|)A_2(1)} - T(A_1)\} \tag{2.51}$$
$$isFeas(A_1 \oplus A_2) \equiv isFeas(A_1) \ \wedge \ isFeas(A_2) \ \wedge \ (E(A_1) + p_{A_1(|A_1|)A_2(1)} \leq L(A_2)) \tag{2.52}$$

It should also be noted that the total duration to process the sequence is given by $DUR(A_i) = \max\{E(A_i) - L(A_i), T(A_i)$, and thus this approach also enables to solve $\{|DUR, TW\}$ and $\{DUR|TW\}$ serial timing problems in $O(1)$.

2.10.5.4 Earliness, tardiness, soft time-windows, and separable convex costs.

Timing problems with tardiness $\{D|\}$, earliness and tardiness $\{R, D(r_i = d_i)|\}$, or with soft time windows $\{TW|\}$ can be solved in a stand-alone way by means of a minimum idle time policy in $O(n)$, or variants of the PAV algorithm (Section 2.7.3) in $O(n \log n)$, respectively. Nevertheless, a better complexity can be achieved by means of re-optimization approaches.

Ibaraki et al. (2008) considers the efficient resolution of the serial timing problem $\{\Sigma c^{\text{CVX}}(t)|\}$, and attains an amortized logarithmic complexity per sub-problem when the activity costs are PiL non-negative, lower semicontinuous and convex. The re-optimization data corresponds to the dynamic programming functions described in Section 2.7.4. These functions are represented as segment pieces within a tree data structure, and computed only on *prefix* and *suffix* subsequences, containing the first or the last activity of the incumbent timing problem.

Data. Total duty time $T(A_i)$ on a partial sequence A_i. Optimal cost $\bar{F}(A_i)(t)$ of a schedule for A_i, when the first activity is executed before $t(t_{A_i(1)} \leq t)$. Optimal cost $\bar{B}(A_i)(t)$ of a schedule for A_i, when the last activity is executed after t.

Data computation. $\bar{F}(A_i)(t)$ and $\bar{B}(A_i)(t)$ are computed by means of forward and backward dynamic programming (Equations 2.19-2.20), respectively. The use of tree structures for function representations allows for data computations in $O(\varphi_c \log \varphi_c)$ for each subsequence, where φ_c is the total number of pieces in the cost functions of the timing subproblem.

Evaluate concatenation. Equation (2.53) returns the optimal cost $Z^*(A_1 \oplus A_2)$ of the timing problem related to the concatenation of two sequences. This value can be computed in $O(\log \varphi_c)$. When the number of pieces of cost functions is linear in the number of activities, as in $\{D|\}$, $\{R, D(r_i = d_i)|\}$ or $\{TW|\}$ settings, a $O(\log n)$ complexity is attained. The concatenation of a bounded number of subsequences can be also efficiently evaluated (see Ibaraki et al. 2008).

$$Z^*(A_1 \oplus A_2) = \min_{t \geq 0}\{F(A_1)(t) + B(A_2)(t + p_{A_1(|A_1|)A_2(1)})\} \tag{2.53}$$

Only Ergun and Orlin (2006) and Kedad-Sidhoum and Sourd (2010) have actually reported methods with an *evaluate concatenation* operator working in amortized $O(1)$ for $\{D|\}$ and $\{D, R(d_i = r_i)|NWT\}$ serial timing, respectively, in presence of particular types of permutation functions (neighborhood search based on swap, insert as well as compound moves), and for sequence-independent processing times. The cornerstone of these two approaches is that they call the functions $B(A_2)(t)$ associated to subsequences by series of $O(n)$ increasing values of t. The methodology requires some orderings, which are performed in a pre-processing phase in $O(n \log n)$. This complexity is generally dominated by the number N of timing subproblems. It is actually an open question whereas this type of approach can be extended to more general problems. This approach is likely to become far more complex when tackling sequence-dependent processing times,

in particular. Approximate serial timing procedures in $O(1)$ may also be used when computational time is critical (Taillard et al. 1997).

2.10.5.5 Separable cost functions and multiple time-windows.

Ibaraki et al. (2005) and Hendel and Sourd (2006) simultaneously introduced a re-optimization approach in $O(\varphi_c)$ for timing problems with non-convex PiL cost functions. This approach brings into play the same quantities and functions $T(A_i)$, $\bar{F}(A_i)(t)$ and $\bar{B}(A_i)(t)$ as in Section 2.10.5.4. In this case, simple linked lists are sufficient for function representation. Evaluating the concatenations is also performed by means of Equation (2.53), resulting in $O(\varphi_c)$ complexity.

2.10.5.6 Flexible processing times: a special case.

The flexible processing time feature involves separable functions of successive activity execution dates. Complex problems are raised when this feature is coupled with time-window constraints as in $\{\Sigma c_i(\Delta t_i)|TW\}$ (Equations 2.54-2.55), or with separable activity execution costs. The total order constraints can be directly taken into account in the objective, with cost functions c_{ij} such that for $\Delta t < 0$, $c_{ij}(\Delta t) = +\infty$.

$$\min_{(t_1,\ldots,t_n)\in\Re^{n+}} \quad \sum_{i=1}^{n} c'_{\sigma_i\sigma_{i+1}}(t_{\sigma_{i+1}} - t_{\sigma_i}) \tag{2.54}$$

$$s.t. \quad r_i \leq t_i \leq d_i \qquad\qquad 1 \leq i \leq n \tag{2.55}$$

We first present a re-optimization approach for a particular shape of cost functions c_{ij}, introduced by Nagata (2007) in the context of VRP with time-windows as an alternative time relaxation. Instead of allowing earliness or lateness with respect to time-window constraints, penalized processing time reductions are allowed. The resulting cost functions are given in Equation (2.56). In this case, no limit is fixed on the amount of processing time reduction, thus allowing negative processing times and non infinite c_{ij} values on $\mathbb{R}^{*-}$.

$$c'_{ij}(\Delta t) = \begin{cases} 0 & \text{if } \Delta t \geq p_{ij} \\ \alpha(p_{ij} - \Delta t) & \text{otherwise} \end{cases} \tag{2.56}$$

Nagata et al. (2010) and Hashimoto et al. (2008) introduced forward and backward dynamic programming functions to efficiently evaluate the merge of two sequences as a result of some particular VRP neighborhoods. We describe here the re-optimization approach of Vidal et al. (2013), which enables to work with any number of concatenations, and accounts for duration features. This approach is closely related to the method of Kindervater and Savelsbergh (1997) for $\{|DUR\}$ and $\{|DUR, TW\}$.

Data. Minimum duration $D(A_i)$ to perform all the activities of A_i but the last one. Earliest execution date $E(A_i)$ of the first activity in any feasible schedule with minimum idle time for A_i. Latest possible execution date $L(A_i)$ of the first activity in any feasible schedule with minimum

processing time reduction for A_i. Minimum processing time reduction $TW(A_i)$ in any feasible schedule for A_i.

Data computation and evaluate concatenation. For a sequence A with a single activity, $D(A) = TW(A) = 0$, $E(A) = r_{A(1)}$ and $L(A) = d_{A(1)}$. Equations (2.57-2.62) enable then to compute the re-optimization data and cost for a concatenation of two sequences.

$$E(A_1 \oplus A_2) = \max\{E(A_2) - D(A_1) + TW(A_1) - p_{A_1(|A_1|)A_2(1)}, E(A_1)\} - \delta_{WT} \tag{2.57}$$

$$L(A_1 \oplus A_2) = \min\{L(A_2) - D(A_1) + TW(A_1) - p_{A_1(|A_1|)A_2(1)}, L(A_1)\} + \delta_{TW} \tag{2.58}$$

$$D(A_1 \oplus A_2) = D(A_1) + D(A_2) + p_{A_1(|A_1|)A_2(1)} + \delta_{WT} \tag{2.59}$$

$$TW(A_1 \oplus A_2) = TW(A_1) + TW(A_2) + \delta_{TW} \tag{2.60}$$

$$\text{with } \delta_{WT} = \max\{E(A_2) - D(A_1) + TW(A_1) - p_{A_1(|A_1|)A_2(1)} - L(A_1), 0\} \tag{2.61}$$

$$\text{and } \delta_{TW} = \max\{E(A_1) + D(A_1) - TW(A_1) + p_{A_1(|A_1|)A_2(1)} - L(A_2), 0\} \tag{2.62}$$

This approach enables to evaluate in $O(1)$ the minimum amount of processing time reduction for any constant number of subsequence concatenations. In terms of computational complexity, the flexible processing time relaxation is a good option for local search methods. In contrast, the best re-optimization methods for soft time-window relaxations $\{TW|P\}$ or $\{D|R,P\}$ attain a complexity of $O(\log n)$ per sub-problem (Section 2.10.5.4).

2.10.5.7 General flexible processing-times.

Sourd (2005) and Hashimoto et al. (2006) addressed a general serial timing problem $\{\Sigma c_i(\Delta t_i), \Sigma c_i(t_i)|\}$ with flexible and sequence-dependent processing times and costs on activity execution dates. This problem is formulated in Equation (2.63). Functions $c_i(t)$ and $c'_{ij}(t)$ are assumed to be PiL, lower semicontinuous, non-negative and take infinite value for $t < 0$. Functions $c'_{\sigma_i\sigma_{i+1}}(\Delta t)$ are convex.

$$\min_{(t_1,\ldots,t_n)\in\Re^n} \quad \sum_{i=1}^{n} c_i(t_i) + \sum_{i=1}^{n-1} c'_{\sigma_i\sigma_{i+1}}(t_{\sigma_{i+1}} - t_{\sigma_i}) \tag{2.63}$$

Data. Optimal schedule cost $F(A_i)(t)$ when the last activity is executed exactly at time t. Optimal cost $B(A_i)(t)$ when the first activity is executed exactly at time t.

Data computation. For a sequence A with a single activity, $F(A)(t) = B(A)(t) = c_{A(1)}(t)$. Equations (2.64-2.65) can then be used to compute the $F(A_i)(t)$ and $B(A_i)(t)$ functions, respectively, on prefix and suffix subsequences of the incumbent timing problem.

$$F(A_i \oplus A)(t) = \min_{0\le x\le t}\{F(A_i)(x) + c'_{A_i(|A_i|)A(1)}(t-x)\} + c_{A(1)}(t) \tag{2.64}$$

$$B(A \oplus A_i)(t) = c_{A(1)}(t) + \min_{x\ge t}\{c'_{A(1)A_i(|A_i|)}(x-t) + B(A_i)(x)\} \tag{2.65}$$

Evaluate concatenation. Equation (2.66) returns the optimal cost $Z^*(A_1 \oplus A_2)$ of the timing problem related to the concatenation of a prefix and a suffix subsequence in $O(\varphi_c + \widehat{\varphi_c} \times \varphi'_c)$ (Hashimoto et al. 2006). φ_c and φ'_c are respectively the total number of pieces in cost functions c_i

and $c'_{\sigma_i\sigma_{i+1}}$. $\widehat{\varphi_c}$ represents the number of convex pieces in $c'_{\sigma_i\sigma_{i+1}}$.

$$Z^*(A_1 \oplus A_2) = \min_{t\geq 0}\{F(A_1)(t) + \min_{t'\geq t}(c'_{A_1(|A_1|)A_2(1)}(t'-t) + B(A_2)(t'))\} \tag{2.66}$$

Finally, it was noted in Section 2.8.3 that $\{DUR|MTW\}$ and $\{|DUR, MTW\}$ constitute special cases of $\{\Sigma c_i(\Delta t_i), \Sigma c_i(t_i)|\}$. Hence, the previous re-optimization approach applies, leading to an amortized complexity of $O(1+\varphi_{\text{MTW}})$ for solving successive $\{DUR|MTW\}$ or $\{|DUR, MTW\}$ subproblems, where φ_{MTW} denotes the total number of time windows in the problem. This is an improvement over the previous procedures in $O(n+n\varphi_{\text{MTW}})$ for these settings.

2.10.5.8 Time-dependent processing times.

Donati et al. (2008) solve the feasibility problem $\{|P(t), TW\}$ using an extension of the method of Savelsbergh (1985). Let $g_{ij}(t) = t + p_{ij}(t)$ be the completion date of an activity i started at t, and followed by activity j. Under the assumption that all $g_{ij}(t)$ are continuous and strictly increasing (any activity started strictly later will finish strictly later), the inverse function $g_{ij}^{-1}(t)$ can be defined and the following re-optimization data and operators enable to perform efficient feasibility checks.

Data. Earliest possible execution date $E(A_i)$ of the last activity in any feasible schedule for A_i. Latest possible execution date $L(A_i)$ of the first activity in any feasible schedule for A_i.

Data computation. For a sequence A with a single activity, $E(A) = r_{A(1)}$, and $L(A) = d_{A(1)}$. Equations (2.67-2.70) then enables to determine the re-optimization data on prefix and suffix subsequences by forward and backward dynamic programming.

$$E(A_1 \oplus A) = \max\{r_{A(1)}, g_{A_1(|A_1|)A(1)}(E(A_1))\} \tag{2.67}$$

$$isFeas(A_1 \oplus A) \equiv isFeas(A_1) \ \wedge \ \{E(A_1 \oplus A) \leq d_{A(1)}\} \tag{2.68}$$

$$L(A \oplus A_2) = \min\{d_{A(1)}, g^{-1}_{A(1)A_2(1)}(L(A_2))\} \tag{2.69}$$

$$isFeas(A \oplus A_2) \equiv isFeas(A_2) \ \wedge \ \{L(A \oplus A_2) \geq r_{A(1)}\} \tag{2.70}$$

Evaluate concatenation. Equation (2.71) enables to state on the feasibility of any concatenation of a pair of prefix and suffix subsequences.

$$isFeas(A_1 \oplus A_2) \equiv isFeas(A_1) \wedge isFeas(A_2) \wedge \{E(A_1) + p_{A_1(|A_1|)A_2(1)}(E(A_1)) \leq L(A_2)\} \tag{2.71}$$

Under the assumption that function $g^{-1}(t)$ is evaluated in $O(1)$, the previous re-optimization framework enables to evaluate the feasibility of a concatenation of two subsequences in $O(1)$. However, it does not allow to concatenate more than two subsequences, as opposed to the fixed processing time setting treated in Section 2.10.5.3, thus limiting the range of local search moves that can be evaluated efficiently without relying on a lexicographic search order.

Finally, Hashimoto et al. (2008) proposed a dynamic programming approach similar to the one of Sections 2.10.5.4 and 2.10.5.6 to manage the general case with time-dependent (and sequence-dependent) processing times with separable execution costs $\{\Sigma c_i(t_i)|P(t)\}$. Evaluations

of concatenations of pairs of subsequences A_1 and A_2 are performed in $O(\varphi_c + \varphi_p)$, where φ_c and φ_p denote respectively the total number of pieces in the cost and processing-time functions.

2.10.6 Summary and perspectives on re-optimization

Table 2.3 summarizes the complexity of stand-alone resolution and re-optimization operations for the main timing settings in the literature. The leftmost column lists the problems, while the next block of columns present stand-alone approaches and their complexity. Following to the right, the next columns are dedicated to summarize re-optimization approaches, the complexity of forward and backward data construction, "F/B", the complexity of concatenation of two, "C2", and more than two subsequences, "C3+", if available. Column "Sd" finally indicates whether the re-optimization approach enables to tackle problems with sequence-dependent parameters. Additional assumptions on the problems are listed in the last column.

Re-optimization has clearly proven useful to address more efficiently several timing settings, reducing in many case the computational complexity by a factor of n or φ_c. Still, the complexity gain remains smaller in some particular cases. Solving the timing problem $\{D|R\}$ from scratch, for example, can be done in $O(n)$ but the best re-optimization approach attains only a complexity of $O(\log n)$. This latter timing problem takes a prominent role in many VRP heuristics that allow time-constraints relaxations. Whether a $O(1)$ re-optimization algorithm exists for $\{D|R\}$ remains an open question. In addition, no re-optimization algorithm is available, to our knowledge, for several problems such as $\{DUR|MTW\}$, $\{|DUR, MTW\}$, $\{DUR|TW, P(t)\}$, $\{|TL, TW\}$ and $\{DUR > D > TL|R\}$. Finally, another promising research path concerns the study of re-optimization data and concatenation operators involving more than two subsequences and their application to sequence-dependent problems. Such operators are still missing in several cases such as $\{\Sigma c_i(t_i)|P(t)\}$ or $\{\Sigma c_i^{\text{CVX}}(\Delta t_i), \Sigma c_i(t_i)|\}$, but are necessary to achieve high-performance local searches in many combinatorial optimization problems with time features.

2.11 General conclusion

In this chapter, a rich body of problems with time characteristics and totally ordered variables has been identified and classified. Many algorithms from different fields of research were analyzed to identify key problem features and main solution concepts. As timing subproblems frequently arise in the context of local search, both the stand-alone resolution of problems, and the efficient resolution of series of problems by means of re-optimization approaches were analyzed. A general re-optimization framework based on decomposition and recombination of sequences was introduced, and links to other re-optimization approaches were highlighted.

A subset of timing methods, originating from various research fields, and constituting the actual state-of-the-art for timing problems with different features, has been identified. These algorithms are a keystone for addressing many rich combinatorial optimization problems with time characteristics, for which the timing sub-problem represents the core of the originality and difficulty. Having a library of timing algorithms at hand opens also the door to further developments on more generic solvers that relegate the problem difficulties to known subproblems and methods.

Table 2.3: Complexity of algorithms and re-optimization operators for common timing problems

Problem	From Scratch		Re-opt. by concat.	F/B	C2	C3+	Sd	Assumptions
Const. act. costs	—	$O(n)$	—	$O(1)$	$O(1)$	$O(1)$	✓	
$\{W\|\}$	Min idle time	$O(n)$	—	$O(1)$	$O(1)$	$O(1)$	✓	
$\{\|TW\}$	Min idle time	$O(n)$	Savelsbergh (1985) & Kind. and Sav. (1997)	$O(1)$	$O(1)$	$O(1)$	✓	
$\{D\|\}$	Min idle time	$O(n)$	Ergun and Orlin (2006)	$O(\log n)$ $O(1)$*	—			penalty coefficient depending upon act.
$\{D, R(d_i = r_i)\|NWT\}$	Min idle time	$O(n)$	Kedad-Sidhoum and Sourd (2010)	$O(\log n)$ $O(1)$*	—			penalty coefficient depending upon act.
$\{D, R(d_i = r_i)\|\}$	Garey et al. (1988) & Ahuja and Orlin (2001)	$O(n \log n)$	Ibaraki et al. (2008)	$O(\log n)$	$O(\log n)$	✓		
$\{D\|R\}$	Min idle time	$O(n)$	Ibaraki et al. (2008)	$O(\log n)$	$O(\log n)$	✓		
$\{\sum c_i^{\text{cvx}}(t_i)\|\}$	Ibaraki et al. (2008)	$O(n \log \varphi_c)$	Ibaraki et al. (2008)	$O(\log \varphi_c)$	$O(\log \varphi_c)$	✓		cost f. ≥ 0, PiL & LSC
$\{\sum c_i(t_i)\|\}$	Ibaraki et al. (2005)	$O(n\varphi_c)$	Ibaraki et al. (2005)	$O(\varphi_c)$	$O(\varphi_c)$	✓		cost f. ≥ 0, PiL & LSC
$\{\|MTW\}$	Min idle time	$O(n + \varphi_{\text{MTW}})$	Ibaraki et al. (2005)	$O(\log \varphi_{\text{MTW}})$	—	✓		
$\{DUR\|TW\}$, $\{\|DUR, TW\}$	Malcolm et al. (1959)	$O(n)$	Savelsbergh (1992) & Kind. and Sav. (1997)	$O(1)$	$O(1)$	✓	✓	
$\{DUR\|MTW\}$, $\{\|DUR, MTW\}$	Tricoire et al. (2010)	$O(n\varphi_{MTW})$	Hashimoto et al. (2006)	$O(\varphi_{\text{MTW}})$	—	✓		
$\{\|IDL, TW\}$	Hunsaker and S. (2002)	$O(n)$	—	—	—			
$\{\sum c_i^{\text{cvx}}(\Delta t_i)\|DUR\}$	Hochbaum (1994)	$O(n \log n \log(d_{max}/\epsilon n))$	—	—	—			with ϵ precision
$\{\sum c_i^{\text{cvx}}(\Delta t_i), \Sigma c_i(t_i)\|\}$	Sourd (2005) & Hashimoto et al. (2006)	$O(n(\varphi_c + \widehat{\varphi_c} \times \varphi_c'))$	Sourd (2005) & Hashimoto et al. (2006)	$O(\varphi_c + \widehat{\varphi_c} \times \varphi_c')$	—	✓	✓	cost f. ≥ 0, PiL & LSC
$\{D\|R, P(t)\}$	Min idle time	$O(n)$	—	—	—			FIFO assumption
$\{\|TW, P(t)\}$	Donati et al. (2008)	$O(n)$	Donati et al. (2008)	$O(1)$	—	✓	✓	FIFO assumption
$\{\sum c_i(t_i)\|P(t)\}$	Hashimoto et al. (2008)	$O(n(\varphi_c + \varphi_p))$	Hashimoto et al. (2008)	$O(\varphi_c + \varphi_p)$	—	✓	✓	cost f. ≥ 0, PiL & LSC & HYI assumption
$\{\|TL, TW\}$	Hurink and Keuchel (2001)	$O(n^3)$	—	—	—			
$\{\|TL, TW\}$	Firat and Woeginger (2011)	$O(n)$	Gschwind and Irnich (2012)	O(M)	—	✓	✓	$O(n)$ TL constraints, M $\Leftrightarrow$ nb. open deliv.
$\{DUR > D > TL\|R\}$	Cordeau and Laporte (2003)	$O(n^2)$	—	—	—			$O(n)$ TL constraints & LIFO assumption
$\{\sum c_{ij}^{\text{cvx}}(t_j - t_i), \Sigma c_i^{\text{cvx}}(t_i)\|\}$	Ahuja et al. (2003)	$O(n^3 \log n \log(nU))$	—	—	—			U is an upper bound of execution dates

Many interesting avenues of research arise as a result of this work. First of all, for several timing features studied in this article, more efficient stand-alone, re-optimization methods and complexity lower bounds should be investigated (Section 2.10.6). The impact of sequence-dependency on re-optimization is also another interesting concern, as sequence-dependency constitutes a fundamental delimitation between routing-related problems and scheduling settings. Identifying precisely its impact on re-optimization procedures would lead to better insights on local search-based methods for these two important classes of problems. Finally, even if the focus of this paper was on time characteristics, other cumulative resources such as loads, energy or workforce, lead to similar features and models. The work performed on timing can thus prove useful for an even broader range of applications.

Deuxième partie

MÉTAHEURISTIQUES POUR LES PROBLÈMES DE TOURNÉES DE VÉHICULES

CHAPITRE 3

UN ALGORITHME GÉNÉTIQUE HYBRIDE POUR LES PROBLÈMES DE TOURNÉES DE VÉHICULES MULTI-DÉPÔTS ET PÉRIODIQUES

3.1 Fil conducteur et contributions

En relation aux concepts étudiés dans les deux revues de littérature, des études expérimentales poussées sont développées afin de proposer des concepts méta-heuristiques performants pour les variantes du VRP, et ainsi contribuer au deuxième défi énoncé dans le plan de thèse. Ces travaux ont abouti à la création d'un algorithme génétique hybride *à contrôle adaptatif de diversité* (HGSADC) extrêmement efficace pour les VRP périodiques, multi-dépôts éventuellement combinées, ainsi que le VRP classique. Cet algorithme hybride combine l'exploration large des méthodes évolutionnaires à base de populations, les capacités d'amélioration agressive des méta-heuristiques à voisinage, les relaxations de contraintes pour cibler les frontières de non-faisabilité, et des méthodes avancées de gestion de la diversité dans la population. HGSADC réinterprète le concept de *survie du plus apte*, en évaluant les individus non pas seulement en relation à la qualité de la solution associée, mais aussi en relation à leur contribution dans la diversité de la population. Cette stratégie permet de préserver la variété de l'information présente dans la population sous forme de matériel génétique, et de réduire les risques de convergence prématurée, pour explorer de vastes régions prometteuses de l'espace de recherche. Des études expérimentales approfondies sur des variantes de VRP avec multiple dépôts et multiple périodes montrent la grande performance de la méthode, en termes de qualité de solution et efficacité de calcul. Pour tous les jeux de tests de la littérature sur ces problèmes, HGSADC retrouve soit les meilleures solutions de la littérature, y compris celles qui sont prouvées optimales, ou de meilleures solutions. Les meilleurs résultats de la littérature sont aussi obtenus pour le VRP classique.

3.2 Article III : A hybrid genetic algorithm for multi-depot and periodic vehicle routing problems

Ce chapitre a fait l'objet d'une publication sous forme d'article de journal : Vidal, T., Crainic, T. G., Gendreau, M., Lahrichi, N., & Rei, W. (2012). A Hybrid Genetic Algorithm for Multi-Depot and Periodic Vehicle Routing Problems. *Operations Research*, 60(3), 611–624.

Abstract: We propose an algorithmic framework that successfully addresses three vehicle routing problems: the multi-depot VRP, the periodic VRP, and the multi-depot periodic VRP with capacitated vehicles and constrained route duration. The meta-heuristic combines the exploration breadth of population-based evolutionary search, the aggressive-improvement capabilities of neighborhood-based meta-heuristics, and advanced population-diversity management schemes. Extensive computational experiments show that the method performs impressively, in terms of

computational efficiency and solution quality, identifying either the best known solutions, including the optimal ones, or new best solutions for all currently available benchmark instances for the three problem classes. The proposed method also proves extremely competitive for the capacitated VRP.

Keywords: Multi-depot multi-period vehicle routing problems, hybrid population-based meta-heuristics, adaptive population diversity management

3.3 Introduction

Vehicle Routing Problem (VRP) formulations are used to model an extremely broad range of issues in many application fields, transportation, supply chain management, production planning, and telecommunications, to name but a few (Toth and Vigo 2002a, Hoff et al. 2010). Not surprisingly, starting with the seminal work of Dantzig and Ramser (1959), routing problems make up an extensively and continuously studied field, as illustrated by numerous conferences, survey articles (e.g., Cordeau et al. 2007, Laporte 2009), and books (Toth and Vigo 2002a, Golden et al. 2008).

Surveying the literature one notices, however, that not all problem classes have received an equal nor adequate degree of attention. This is the case for the problems with multiple depots and periods. A second general observation is that most methodological developments target a particular problem variant, the capacitated VRP (*CVRP*) or the VRP with time windows (*VRPTW*), for example, very few contributions aiming to address a broader set of problem settings. This also applies to the problem classes targeted in this paper.

Our objective is to contribute toward addressing these two challenges. We propose an algorithmic framework that successfully addresses three VRP variants: the multi-depot VRP, *MDVRP*, the periodic VRP, *PVRP*, and the multi-depot periodic VRP, *MDPVRP*, with capacitated vehicles and constrained route duration. The literature on these problems is relatively scarce (Francis et al. 2008) despite their relevance to many applications, e.g., raw material supply (Alegre et al. 2007), refuse collection (Beltrami and Bodin 1974, Russell and Igo 1979, Teixeira et al. 2004, Coene et al. 2010), food collection or distribution (Golden and Wasil 1987, Parthanadee and Logendran 2006), and maintenance operations (Hadjiconstantinou and Baldacci 1998, Blakeley et al. 2003).

We propose a meta-heuristic that combines the exploration breadth of population-based evolutionary search, the aggressive-improvement capabilities of neighborhood-based meta-heuristics, and advanced population-diversity management schemes. The method, that we name *Hybrid Genetic Search with Adaptive Diversity Control* (*HGSADC*), performs impressively in terms of both solution quality and computational efficiency. HGSADC outperforms the state-of-the-art methods for the three problem classes on all currently available benchmark instances, identifying either the best known solutions, including the optimal ones, or new best solutions. Moreover, with very limited adaptation, HGSADC proves to be extremely competitive for the CVRP.

The two main contributions of this article are: 1) A new meta-heuristic, which is highly effective for four important vehicle routing problem classes, the MDVRP, the PVRP, the MDPVRP, and the CVRP. The method is built on general components that are applicable, provided minor adjustments,

to an even wider range of VRP variants. 2) New population-diversity management mechanisms to allow a broader access to reproduction, while preserving the characteristics of elite solutions. In this respect, we revisit the traditional *survival-of-the-fittest* paradigm to enhance the evaluation of individuals by making it rely on both solution cost and diversity (distance-to-the-others) measures. Thus, diversity appears as an integral part of the objective, which contrasts with classical diversity-management procedures that have traditionally imposed diversity constraints for the acceptance of solutions in the population. Empirical studies show that these new mechanisms do not only avoid premature population convergence efficiently, but also lead to higher quality solutions in reduced computation time when compared to traditional approaches. It must be emphasized that the proposed population-diversity management mechanisms could be applied to almost any problem that one can solve using population-based evolutionary search methods.

The paper is organized as follows. Section 3.4 states the notation and formal definition of the three classes of VRPs we address, while the relevant literature is surveyed in Section 3.5. The proposed meta-heuristic is detailed in Section 3.6, its performances are analyzed in Section 3.7, and we conclude in Section 3.8.

3.4 Problem Statement

We formally state the MDVRP, PVRP, and MDPVRP, introducing the notation used in this paper and the transformation of the MDPVRP into a PVRP, which supports the algorithmic developments.

The CVRP can be defined as follows. Let $G = (\mathcal{V}, \mathcal{A})$ be a complete graph with $|\mathcal{V}| = n + 1$ vertices, divided in two sets $\mathcal{V} = \mathcal{V}^{\text{DEP}} \cup \mathcal{V}^{\text{CST}}$. The unique vertex $v_0 \in \mathcal{V}^{\text{DEP}}$ represents the depot where the product to be distributed is kept and a fleet of m identical vehicles with capacity Q is based. Vertices $v_i \in \mathcal{V}^{\text{CST}}$ stand for customers $i,\ i = 1, \ldots, n$, requiring service and characterized by a non-negative demand q_i and a service duration τ_i. Arcs $a_{ij} \in \mathcal{A},\ i, j \in \mathcal{V}$ represent the direct-travel possibility from v_i to v_j with travel time equal to c_{ij}. The duration of a vehicle route is computed as the total travel and service time required to serve the customers, and is limited to T. The goal is to design a set of vehicle routes servicing all customers, such that vehicle-capacity and route-duration constraints are respected, and the total travel time is minimized.

Several depots, d, are available to service customers in the Multi-Depot VRP, m representing the number of vehicles available at each depot. In this case, vertices $v_0, \ldots, v_d$ make up the set $\mathcal{V}^{\text{DEP}}$, while the remaining vertices $\mathcal{V}^{\text{CST}}$ stand for customers. A time dimension is introduced in the Periodic VRP as route planning is to be performed over a horizon of t periods. Each customer i is characterized by a service frequency f_i, representing the number of visits to be performed during the t periods, and a list L_i of possible visit-period combinations, called *patterns*. The PVRP aims to select a pattern for each customer and construct the associated routes to minimize the total cost over all periods. Finally, the Multi-Depot Periodic VRP extends the two previous problem settings, asking for the selection of a depot and a visit pattern for each customer, with services in different periods to the same customer being required to originate at the same depot. The CVRP is NP-Hard and so are the three problem classes that generalize it and are addressed in this paper.

The MDPVRP reduces to a PVRP when $d = 1$ and to a MDVRP when $t = 1$. Furthermore, the three problem settings share similar mathematical structures. We take advantage of this property and, in the spirit of the problem transformation from MDVRP to PVRP of Cordeau et al. (1997), we transform a MDPVRP with d depots and t periods into an equivalent PVRP with $d \times t$ periods corresponding to (depot, period) couples. (Appendix II.1 details these mathematical structures and problem transformations). This transformation provides the means to address the three problem classes with the same solution method and reduces the number of problem characteristics. Of course, the method must be computationally efficient to deal with the increased number of periods and the corresponding increase in problem dimension. As the computational results displayed in Section 3.7 show, we achieve this goal.

3.5 Literature Review

This section provides a brief literature review of contributions for the PVRP, the MDVRP, and the MDPVRP. The purpose of this review is twofold. First, to present the most recently proposed meta-heuristic algorithms, particularly population-based ones, for the considered problems. Second, to distinguish the leading solution approaches for the three problem settings.

Some population and neighborhood-based meta-heuristics already exist in the PVRP literature. Drummond et al. (2001) proposed an island-based parallel evolutionary method, which evolves individuals representing schedules (patterns), the fitness of each individual being obtained by constructing routes for each period with a savings heuristic. Alegre et al. (2007) proposed a scatter search procedure designed especially for PVRPs with a large number of periods. As in Drummond et al. (2001), the core of the method is dedicated to the improvement of visit schedules, while a neighborhood-based improvement procedure is used to design routes for each period. Contrasting with the two previous methods, Matos and Oliveira (2004) proposed an ant colony optimization (ACO) approach that first optimizes routes, then schedules. This ACO method addresses the PVRP as a VRP, where each customer is duplicated as many times as indicated by its frequency, the resulting routes being then distributed to periods by solving a graph coloring problem.

Until recently, however, the most successful contributions to this problem were based on the serial exploration of neighborhoods. The local search approach of Chao et al. (1995) was the first to use deteriorating moves to escape from poor local optima, and also allow relaxation of vehicle-capacity limits to enhance the exploration of the solution space. The tabu search proposed by Cordeau et al. (1997) introduced an innovative guidance scheme, which collects statistics on customer assignments to periods and vehicle routes in order to penalize recurring assignments within the solutions obtained and, thus, gradually diversify the search. For a long period of time, this method stood as the state-of-the-art solution approach for both the PVRP and the MDVRP, as well as, in its Unified Tabu Search (*UTS*) version (Cordeau et al. 2001a), for a number of other VRP variants. To date, however, the best PVRP results have been produced by the variable neighborhood search (VNS) of Hemmelmayr et al. (2009) and the record-to-record travel approach of Gulczynski et al. (2011), the latter combining local search and an integer-programming-based

large neighborhood search. Finally, one should notice the VNS algorithm with multilevel refinement strategy of Pirkwieser and Raidl (2010a), specifically tailored for large-size instances.

Population-based meta-heuristics proposed for the MDVRP often took advantage of geometric aspects of the problem. Thus, Thangiah and Salhi (2001) represented solutions as circles in the 2D space, whereas Ombuki-Berman and Hanshar (2009) introduced a mutation operator that targeted the depot assignment to "borderline" customers (i.e., which are close to several depots). Lau et al. (2010) explored a different genetic-algorithm idea by using diversity measures and fuzzy logic to adapt the mutation and crossover rates during the search. Finally, a parallel ACO approach was proposed by Yu et al. (2011).

As underlined in Ombuki-Berman and Hanshar (2009), various neighborhood-based single-trajectory searches have been proposed in the MDVRP literature. The most successful approach remains the adaptive large neighborhood search (ALNS) method of Pisinger and Ropke (2007), which implements the ruin-and-recreate paradigm with an adaptive selection of operators.

In the case of the MDPVRP, most proposed algorithms do not consider all characteristics simultaneously, but rather apply a successive-optimization approach. Thus, the method developed by Hadjiconstantinou and Baldacci (1998) starts by first assigning all customers to a particular depot. Given these a priori assignments, customer visits are then successively inserted among available periods to obtain feasible visit combinations. The depot-period VRP subproblems obtained are then separately solved using a tabu search algorithm. Finally, a last phase attempts to improve the solution by modifying some period or depot assignments. The overall solution strategy then repeats this sequence of heuristics for a fixed number of iterations. Other such approaches were proposed by Kang et al. (2005) and Yang and Chu (2000), where schedules for each depot and period are first determined, followed by the design of the corresponding routes.

We are aware of only two methods that aim to address problems similar to the MDPVRP as a whole. Parthanadee and Logendran (2006) implemented a tabu search method for a complex variant of the MDPVRP with backorders. The authors also study the impact of interdependent operations between depots, where the depot assignment of a customer may vary according to the periods considered. Significant gains are reported on small test instances when such operations are applied. Crainic et al. (2009) introduced the Integrative Cooperative Search (ICS) framework, which relies on problem decomposition by attributes, concurrent resolution of subproblems, integration of the elite partial solutions yielded by the subproblems, and adaptive search-guidance mechanisms. The authors used the MDPVRP with time windows to illustrate the methodology with very promising results, but did not report results for the problems addressed in this paper. Moreover, ICS targets complex problem settings and we provide a simpler way to treat the MDPVRP.

A number of exact methods were also proposed for one or another of the problems we address. Noteworthy are the recent contributions of Baldacci and Mingozzi (2009) and Baldacci et al. (2011a) addressing the MDVRP and the PVRP. Exact methods are limited in the size of instances they may handle, but these particular approaches have proven quite successful in solving to optimality several instances that are used as a test bed for the algorithm we propose.

This brief review supports the general statement made previously that no satisfactory method has yet been proposed for the three problem settings. Furthermore, the contributions to the

MDPVRP literature are very scarce, those addressing all the problem characteristics simultaneously being scarcer still. Most solution methods proposed address the periodic and multi-depot VRP settings, with neighborhood-based methods yielding, until now, the best results on standard benchmark instances. However, evolutionary methods have proven recently to be efficient on the standard VRP (Prins 2004, Nagata and Bräysy 2009b) and on a number of other variants, e.g., the VRPTW (Bräysy et al. 2004a). Noteworthy is the contribution of Prins (2004), who introduced an important methodological element, namely the solution representation for the VRP as a TSP tour without delimiters along with a polynomial time algorithm to partition the sequence of customers into separate routes. This approach was later applied by Lacomme et al. (2005) and Chu et al. (2006) to the periodic capacitated arc routing problem, which shares a number of common characteristics with the PVRP. We adopt this solution representation for the population-based method we propose to efficiently address the periodic and multi-depot problems, as well as the MDPVRP as a whole. This methodology is described in the next section.

3.6 The Hybrid Genetic Search with Adaptive Diversity Control Meta-heuristic

The *Hybrid Genetic Search with Adaptive Diversity Control* (*HGSADC*) meta-heuristic we propose is based on the Genetic Algorithm (GA) paradigm introduced by Holland (1975) but includes a number of advanced features, in terms of solution evaluation, offspring generation and improvement, and population management, which contribute to its originality and high performance level.

The general scheme of the meta-heuristic we propose is displayed in Algorithm 1. The method evolves a population of individuals, managing feasible and infeasible solutions, which are kept in two separate groups (*subpopulations*). It applies successively a number of operators to select two parent individuals and combine them, yielding a new individual (*offspring*), which is first enhanced using local search procedures (*education* and *repair*), and then included in the appropriate subpopulation in relation to its feasibility. Of particular interest is the evaluation mechanism we propose, which is used to select both parents for mating (Line 3 of Algorithm 1) and individuals to survive to the next generation (Line 8). The mechanism takes into account not only the solution cost (Section 3.6.1), which is often the norm, but also the contribution the individual makes to the diversity of the gene pool. It thus contributes to maintain a high level of diversity among individuals, and plays an important role in the overall performance of the proposed methodology.

We initiate the description of HGADSC with the definition of the search space, Section 3.6.1, followed by the representation and evaluation of individuals, Sections 3.6.2 and 3.6.3, respectively. We then proceed with detailed discussions of parent selection and crossover, Section 3.6.4, education and repair, Section 3.6.5, and population management, Section 3.6.6.

3.6.1 Search space

The meta-heuristic literature indicates that allowing a controlled exploration of infeasible solutions may enhance the performance of the search, which may more easily transition between structurally different feasible solutions (Glover and Hao 2011). We thus define the search space $\mathcal{S}$

Algorithm 3.1 HGSADC

1: Initialize population
2: **while** *number of iterations without improvement* $< It_{NI}$, and *time* $< T_{max}$
3: Select parent solutions P_1 and P_2
4: Generate offspring C from P_1 and P_2 (crossover)
5: Educate offspring C (local search procedure)
6: **if** C is infeasible **then** insert C into infeasible subpopulation; repair with probability P_{rep}
7: **if** C is feasible **then** insert C into feasible subpopulation
8: **if** *maximum subpopulation size reached* **then** select survivors
9: Adjust penalty parameters for violating feasibility conditions
10: **if** *best solution not improved for* It_{div} *iterations* **then** diversify population
11: Return best feasible solution

as a set of feasible and infeasible solutions $s \in \mathcal{S}$, the latter being obtained by relaxing the limits on vehicle capacities and maximum route travel time (as in Gendreau et al. 1994, Cordeau et al. 1997).

Let $\mathcal{R}(s)$ represent the set of routes making up solution s. Each route $r \in \mathcal{R}(s)$ starts from a depot $\sigma_0^r \in \mathcal{V}^{\text{DEP}}$, visits a sequence of n_r customers $\sigma_1^r, \ldots, \sigma_{n_r}^r \in \mathcal{V}^{\text{CST}}$, and returns to the same depot $\sigma_{n_r+1}^r = \sigma_0^r$. It is characterized by load $q(r) = \sum_{i=1}^{n_r} q_{\sigma_i^r}$, driving time $c(r) = \sum_{i=0}^{n_r} c_{\sigma_i^r \sigma_{i+1}^r}$, and total duration $\tau(r) = c(r) + \sum_{i=1}^{n_r} \tau_{\sigma_i^r}$.

Let ω^{Q} and ω^{D} represent the penalties for exceeding the vehicle capacity and the route maximum duration, respectively. The *penalized cost* of a route r is then defined in Equation 3.1 as its driving time plus, when the route is infeasible, the weighted sum of its excess duration and/or load.

$$\phi(r) = c(r) + \omega^{\text{D}} \max\{0, \tau(r) - T)\} + \omega^{\text{Q}} \max\{0, q(r) - Q\} \tag{3.1}$$

The penalized cost $\phi(s)$ of a solution s is then computed as the sum of the penalized costs of all its routes $\phi(s) = \sum_{r \in \mathcal{R}(s)} \phi(r)$, and is used to compute the fitness of the individuals.

3.6.2 Solution representation

Solutions $s \in \mathcal{S}$ are characterized by their customer schedules, depot assignments, and routes. The individuals representing them in the HGSADC population are thus represented as a set of three chromosomes: 1) the *pattern chromosome*, which registers for each customer i its pattern $\pi_i(P)$; 2) the *depot chromosome*, containing the depot assignment $\delta_i(P)$ of each customer i; and 3) the *giant tour chromosome*, containing for each combination (depot o, period l), a sequence $\mathcal{V}_{ol}(P)$ of customers **without trip delimiters**, obtained by concatenating all routes from depot o during period l, in an arbitrary order, and removing visits to depots. Figure 3.1 illustrates this representation scheme for a small MDPVRP problem with two periods, two depots, and eight customers.

The representation of the routes out of the same period and depot as a giant tour provides the means to use simple and efficient crossover procedures working on permutations, but requires an algorithm to find the optimal segmentation of the tour into routes and, thus, retrieve both the

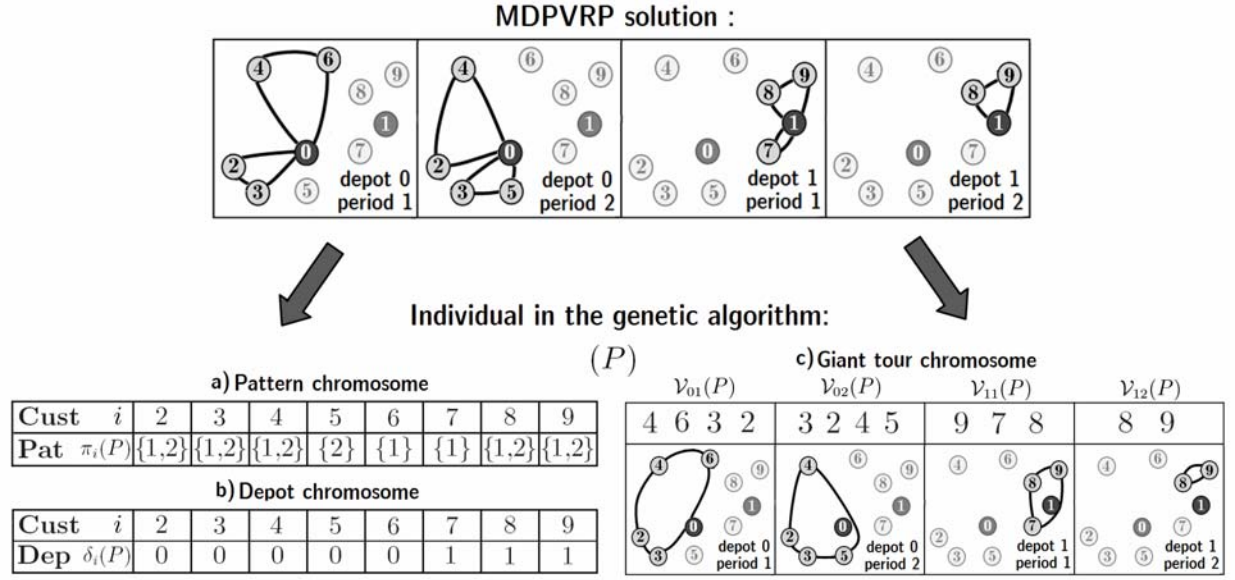


Figure 3.1: From a MDPVRP solution to the individual chromosome representation

solution and its cost. The first successful utilization of a giant-tour representation within a genetic algorithm was reported by Prins (2004), who also introduced an efficient algorithm to optimally extract the routes from the tour. This algorithm, named *Split*, reduces the problem of finding the route delimiters to a shortest path problem on an auxiliary acyclic graph. It is straightforward to adapt to the setting with penalized costs and limited fleet size, and can be implemented in polynomial time $O(mn^2)$, as explained in Appendix II.2.

3.6.3 Evaluation of individuals

The individual-evaluation function in population-based meta-heuristics aims to determine for each individual a relative value with respect to the entire population. Often based on the value of the objective function of the problem at hand (e.g., the value of the individual compared to the average value of the population), this so-called *fitness* measure is then used to perform various selections, e.g., parents for mating or individuals to advance to the next generation, the latter being named *survivors* in the following. Such an approach is, however, generally myopic with respect to the possible impact of the evaluation and selection processes on the diversity of the population, a critical performance factor for this class of meta-heuristics. We therefore propose a mechanism that addresses both objectives, the *evaluation function* accounting for the cost of an individual and its contribution to the population diversity.

We define the *diversity contribution* $\Delta(P)$ of an individual P as the average distance to its n_{close} closest neighbors, grouped in set $\mathcal{N}_{\text{CLOSE}}$, computed according to Equation (3.2). Several distance measures were tested in the experiments leading to the final algorithm. A normalized Hamming distance $\delta^{\text{H}}(P_1, P_2)$, based on the differences between the service patterns and depot assignments of two individuals P_1 and P_2, appeared the most adequate for the multi-depot, multi-period routing

problems we address. This distance is computed according to Equation (3.3), where $\mathbb{1}(cond)$ is a valuation function that returns 1 if the condition *cond* is true, 0, otherwise.

$$\Delta(P) = \frac{1}{n_{close}} \sum_{P_2 \in \mathcal{N}_{\text{CLOSE}}} \delta^{\text{H}}(P, P_2) \tag{3.2}$$

$$\delta^{\text{H}}(P_1, P_2) = \frac{1}{2n} \sum_{i=1,\ldots,n} (\mathbb{1}(\pi_i(P_1) \neq \pi_i(P_2)) + \mathbb{1}(\delta_i(P_1) \neq \delta_i(P_2)) \tag{3.3}$$

Let $fit(P)$ and $dc(P)$ in $\{1, \ldots, nbIndiv\}$ stand for the *rank* of an individual P in a subpopulation of size $nbIndiv$, with respect to its penalized cost $\phi(P)$ and diversity contribution $\Delta(P)$, respectively. The *biased fitness* function $BF(P)$ we propose combines the cost and diversity ranks, and is given by Equation (3.4), where $nbElit$ is the number of elite individuals one desires to survive to the next generation.

$$BF(P) = fit(P) + \left(1 - \frac{nbElit}{nbIndiv}\right) dc(P), \tag{3.4}$$

The ranks and biased-fitness measures are continuously updated for the two subpopulations and are used to evaluate the quality of an individual during parent (Section 3.6.4) and survivor (Section 3.6.6) selections. The biased-fitness is thus an adaptive mechanism aiming to balance the drive for the best individual (elitism) and the possible loss of information usually associated with this drive. This concern for continuous and "early" (parent selection) population-diversity control complements the periodic population-management mechanism introduced in Section 3.6.6.

3.6.4 Parent Selection and Crossover

The offspring generation scheme of HGADSC selects two parents, P_1 and P_2, and yields a single individual C. Parent selection is performed through a binary tournament, which twice randomly (with uniform probability) picks two individuals from the complete population, grouping the feasible and infeasible subpopulations, and keeps the one with the best biased fitness. Feasible and infeasible individuals may thus be selected to undergo crossover in order to lead the search close to the borders of feasibility, where we expect to find high quality solutions.

We propose a new *periodic crossover with insertions* (*PIX*) dedicated to periodic routing problems and designed to transmit good sequences of visits, while enabling pattern, depot, and route recombinations. We aimed for a versatile crossover, which would allow for both a wide exploration of the search space and small refinements of "good" solutions. The possibility for the offspring to inherit genetic material from its parents in nearly equal proportions is required to provide the crossover with the former capability, while copying most of one parent along with small parts of the other provides the latter. To ensure PIX has both capabilities meant avoiding *a priori* determined rules on how much genetic material the offspring inherits from each parent, as well as rules based on simple random selection of individual characteristics.

Algorithm 2 displays the detailed pseudo-code for the PIX crossover procedure, which is illustrated in Figure 3.2 on a problem with two periods and two depots, and described in the rest of this section. The crossover begins in Step 0 by determining the period and depot inheritance rule.

Algorithm 3.2 PIX

1: Step 0: Inheritance rule.
2: Pick two random numbers between 0 and td according to a uniform distribution. Let n_1 and n_2 be respectively the smallest and the largest of these numbers;
3: Randomly select n_1 (depot, period) couples to form the set Λ_1;
4: Randomly select $n_2 - n_1$ remaining couples to form the set Λ_2;
5: The remaining $td - n_2$ couples make up the set Λ_{mix}.

6: Step 1: Inherit data from P_1.
7: **for** each (depot, period) (o, l), belonging to set
8: Λ_1: Copy the sequence of customer visits from $\mathcal{V}_{o,l}(P_1)$ to $\mathcal{V}_{o,l}(C)$;
9: Λ_{mix}: Randomly (uniform distribution) select two chromosome-cutting points α_{kl} and β_{kl}; copy the α_{kl} to β_{kl} substring of $\mathcal{V}_{o,l}(P_1)$ to $\mathcal{V}_{o,l}(C)$.

10: Step 2: Inherit data from P_2.
11: **for** each (depot, period) $(o, l) \in \Lambda_2 \bigcup \Lambda_{mix}$ selected in random order
12: Consider each customer visit i in $\mathcal{V}_{o,l}(P_2)$ and copy it at the end of $\mathcal{V}_{o,l}(C)$ when
1) The depot choice $\delta_i(C)$ is equal to o or undefined (no visit to i has been copied to C yet);
2) One visit pattern of customer i, at least, contains the set $\pi_i(C) \cup l$ of visit periods.

13: Step 3. Complete customer services.
14: Perform the *Split* algorithm and extract the routes for each (depot, period) pair;
15: **if** the service-frequency requirements are satisfied for all customers **then stop**; Otherwise,
16: **while** customers with unsatisfied service-frequency requirements exist, **repeat**:
17: Randomly select a customer i for which service-frequency requirements are not satisfied;
18: Let $\mathcal{F}$ be the set of admissible (depot, period) combinations (o, l) with respect to its pattern list L_i and the visits already included in C. Let $\psi(i, o, l)$ be the minimum penalized cost (Section 4.1) for the insertion of customer i into a route from depot o in period l. Insert i into $(o^*, l^*) = \text{argmin}_{(o,l)\in\mathcal{F}}\, \psi(i, o, l)$.

Let Λ_1, Λ_2, and Λ_{mix} be the sets of (depot, period) couples corresponding to inheriting material from the first parent, P_1, the second parent, P_2, or both parents, respectively. The procedure then first determines the cardinality of each set and, then, fills them up sequentially by randomly selecting the appropriate number of (depot, period) couples. We assume for the example of Figure 3.2 that $\Lambda_1 = \{(\text{d1,p2})\}$, $\Lambda_2 = \{(\text{d1,p1})\}$, and $\Lambda_{mix} = \{(\text{d0,p1}), (\text{d0,p2})\}$.

Steps 1 and 2 are dedicated to taking genetic material from the two parents and combining their giant-tour chromosomes. Step 1 targets the first parent and copies for each selected (depot, period) couple either the complete material, if it belongs to Λ_1, or a random subsequence, if it belongs to Λ_{mix}. Thus, in Figure 3.2, all customers serviced from depot 1 in period 2 ($\Lambda_1 = \{(d1, p2)\}$) are inherited from P_1, while only subsets are inherited for depot 0 at periods 1 and 2 ($\Lambda_{mix} = \{(\text{d0,p1}), (\text{d0,p2})\}$). Step 2 targets the second parent and, thus, the material from (depot, period) couples in $\Lambda_2 \bigcup \Lambda_{mix}$. The inheritance is restricted by the selections performed in Step 1, however, and, thus, customers are inserted at the end of the corresponding sequence when the depot and pattern compatibility conditions specified at line 12 of Algorithm 2 are satisfied. Given the random order (d0,p2), (d1,p1), (d0,p1) of $\Lambda_2 \bigcup \Lambda_{mix}$ in the illustration, the visits from P_2 that conform to this

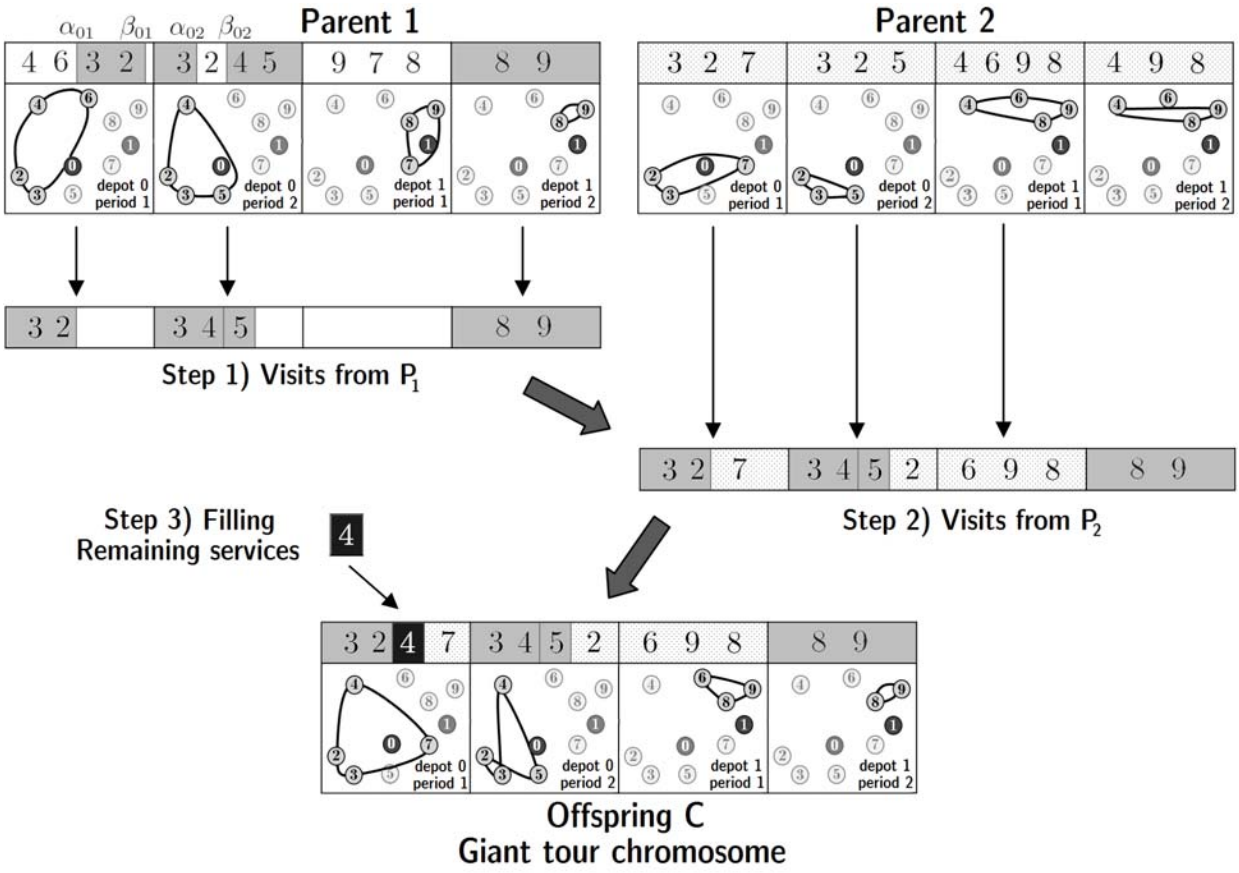


Figure 3.2: The PIX crossover

test and are transmitted to C are marked through dots on white background in Figure 3.2. Thus, for example, the (d1,p1) pair of P_2 yields only the subsequence [6,9,8] out of [4,6,9,8], because a visit to customer 4 was copied during Step 1 from (d0,p2) of P_1.

The offspring built at the end of the Step 2 might not be feasible, however, because of customers with unsatisfied service-frequency requirements. The goal of Step 3 is then to perform the necessary insertions of additional visits. Most insertion mechanisms used in vehicle routing and traveling salesman problems could be used. Yet, to enhance the precision of the insertion, and because the routes must be extracted in all cases, before undertaking the next phase of the meta-heuristic (Section 3.6.5), we anticipate this extraction, using the Split algorithm, and perform best-insertion directly into the actual routes based on the corresponding biased-fitness measure. In Figure 3.2, the necessary services to customer 4 are not fulfilled in C following the first crossover steps, and a possible result of least-cost insertion is illustrated.

3.6.5 Education

An *Education* operator is applied with probability P_m to improve the quality of the offspring solution (the routes were extracted in Step 3 of the PIX procedure). Education goes beyond the

classical genetic-algorithm concepts of random mutation and enhancement through hill-climbing techniques, as it includes several local-search procedures based on neighborhoods for the VRP. A *Repair* phase eventually completes the Education operator when the educated offspring is infeasible.

Two sets of local-search procedures are defined. The nine *route improvement* (RI) procedures are dedicated to optimize each VRP subproblem separately, whereas the *pattern improvement* (PI) procedure relies on a quick and simple move to improve the visit assignments of customers by changing their patterns and depots. These local searches are called in the RI, PI, RI sequence.

Route Improvement. Let $r(u)$ stand for the route containing vertex u in the given (depot, period) routing subproblem, and (u_1, u_2) identify the partial route from u_1 to u_2. Define the neighborhood of vertex u, customer or depot, as the hn closest vertices, where $h \in [0, 1]$ is a *granularity threshold* restricting the search to nearby vertices (Toth and Vigo 2003). Let v be a neighbor of u, and x and y the successors of u in $r(u)$ and v in $r(v)$, respectively. The Route Improvement phase iterates, in random order, over each vertex u and each of its neighbors v, and evaluate the following moves:

- (M1) If u is a customer visit, remove u and place it after v;
- (M2) If u and x are customer visits, remove them, then place u and x after v;
- (M3) If u and x are customer visits, remove them, then place x and u after v;
- (M4) If u and v are customer visits, swap u and v;
- (M5) If u, x, and v are customer visits, swap u and x with v;
- (M6) If u, x, v, and y are customer visits, swap u and x with v and y;
- (M7) If $r(u) = r(v)$, replace (u, x) and (v, y) by (u, v) and (x, y);
- (M8) If $r(u) \neq r(v)$, replace (u, x) and (v, y) by (u, v) and (x, y);
- (M9) If $r(u) \neq r(v)$, replace (u, x) and (v, y) by (u, y) and (x, v).

The first three moves correspond to *insertions*, while moves M4 to M6 are generally called *swaps*. These moves can be applied indifferently on the same or different routes. Move M7 is a 2-opt intra-route move, while moves M8 and M9 are 2-opt* inter-route moves. Moves are examined in random order, the first yielding an improvement being implemented. The Route-Improvement phase stops when all possible moves have been successively tried without success.

Pattern Improvement. Let $\bar{o}$ and $\bar{p}$ be the depot and pattern, respectively, of customer i in the current solution. The Pattern-Improvement procedure iterates on customers in random order and computes, for each customer i, depot o, and pattern $p \in L_i$, $\Psi(i, o, p) = \sum_{l \in p} \psi(i, o, l)$, the minimum cost to satisfy the visit requirements of i from the depot o according to the visit pattern p. If a (i, o, p) combination exists such that $\Psi(i, o, p) < \Psi(i, \bar{o}, \bar{p})$, then all visits to customer i are removed, and a new visit is inserted in the best location in each sequence corresponding to depot o and period $l \in p$. The procedure stops when all customers have been successively considered without a modification.

The Pattern-Improvement procedure is significantly faster when the optimal position and insertion cost of each customer is stored for each route. It is also worth noting that, sometimes, the current pattern and depot choices are kept, but a better insertion of customers is found. The

resulting move is then, in fact, a combination of intra-period M1 insertions. This may prove particularly interesting for the exceptional case when the move was not attempted in RI because of proximity conditions. The Pattern-Improvement phase thus fulfills the double role of changing the patterns and attempting moves between distant vertices.

The individual yielded by the RI, PI, RI education sequence may be feasible, in which case, we call it *naturally feasible*, or infeasible, and it is inserted into the appropriate subpopulation. Infeasible individuals are subject to the Repair procedure with probability P_{rep}. When Repair is successful, the resulting individual is added to the feasible subpopulation (the infeasible one is not deleted from the infeasible subpopulation). Repair consists in temporarily multiplying the penalty parameters by 10 and re-starting the RI, PI, RI sequence. When the resulting individual is still infeasible, penalty parameters are temporarily multiplied by 100 and the sequence is started again. This significant increase of penalties aims at redirecting the search toward feasible solutions.

3.6.6 Population management and search guidance

The population management mechanism complements the selection, crossover, and education operators in identifying and propagating the characteristics of good solutions, enhancing the population diversity, and providing the means for a thorough and efficient search. The two subpopulations dedicated to feasible and infeasible individuals are independently managed to contain between μ and $\mu + \lambda$ individuals, the former representing the minimum subpopulation size, and the latter the generation size. Any incoming individual is directly included in the appropriate subpopulation with respect to its feasibility, and thus acceptance in the population is systematically granted. Any subpopulation reaching its maximum size will undergo a survivors selection phase to discard λ individuals and thus return to its minimum size. Four main components thus constitute the general behavior of the population: initialization, adjustment of the penalties for infeasible individuals, diversification, and selection of survivors.

Initialization. To initialize the subpopulations, 4μ individuals are created by randomly choosing a pattern and a depot for each customer and producing for each period the associated service sequence in random order. These initial individuals undergo education, repair with probability 0.5, and are inserted into the appropriate subpopulation in relation to their feasibility. Survivor selection is activated, as described later on, when a subpopulation reaches the maximum size. At the end of initialization, one of the two subpopulations may be incomplete, with less than μ individuals.

Penalty parameter adjustment. The penalty parameters are initially set to $\omega^{\mathrm{D}} = 1$ and $\omega^{\mathrm{Q}} = \bar{c}/\bar{q}$, where $\bar{c}$ represents the average distance between two customers and $\bar{q}$ is the average demand. The parameters are then dynamically adjusted during the execution of the algorithm, to favor the generation of naturally-feasible individuals as defined in Section 3.6.5. Let ξ^{REF} be a target proportion of naturally-feasible individuals, and ξ^{Q} and ξ^{D} the proportion in the last 100 generated individuals of naturally-feasible one with respect to vehicle capacity and route duration, respectively. The following adjustment is then performed every 100 iterations, where PAR = Q, D:

- if $\xi^{\text{PAR}} \leq \xi^{\text{REF}} - 0.05$, then $\omega^{\text{PAR}} = \omega^{\text{PAR}} \times 1.2$;
- if $\xi^{\text{PAR}} \geq \xi^{\text{REF}} + 0.05$, then $\omega^{\text{PAR}} = \omega^{\text{PAR}} \times 0.85$.

Diversification is called when It_{div} iterations occurs without improving the best solution. It is performed by eliminating all but the best $\mu/3$ individuals of each subpopulation, and creating 4μ new individuals as in the initialization phase. This process introduces a significant amount of new genetic material, which revives the search further, even when the population has lost most of its diversity.

Diversity and selection of survivors. A major challenge in population-based algorithms is avoiding premature convergence of the population. The issue is even more challenging when, as in our case, education compounds the parent selection tendency to favor individuals with good characteristics, thus reducing the genetic material diversity in the population. The mechanism we propose aims to address this challenge by simultaneously identifying and preserving the most promising solution characteristics, and ensuring the diversity of both subpopulations.

The first component of this mechanism is made up of the definition of the biased-fitness function and the explicit consideration of diversity during parents selection (Section 3.6.3). The second takes place whenever one of the two subpopulations reaches the maximum size $\mu + \lambda$. Named *Survivor selection*, the procedure determines the μ individuals that will go on to the next generation, such that the population diversity, in terms of visit patterns, is preserved and elite individuals in terms of cost are protected. The λ discarded individuals are thus either clones (Prins 2004) or bad with respect to cost and contribution to diversity as measured by their biased fitness.

Let a *clone* be an individual P_2 with either the same pattern and depot assignments as another individual P_1, i.e., $\delta^{\text{H}}(P_1, P_2) = 0$, or the same solution cost. The procedure successively eliminates, first, clones, and then, bad individuals, as described in Algorithm 3. Proposition 3.6.1 formalizes the elitism property of the Survivor-selection procedure.

Algorithm 3.3 Survivor selection

1: **for** $i = 1 \ldots \lambda$
2: $X \leftarrow$ all individuals having a clone
3: **if** $X \neq \emptyset$ **then** remove $P \in X$ with maximum Biased Fitness
4: **else** remove P in the subpopulation with maximum Biased Fitness
5: Update distance and Biased Fitness measures

Proposition 3.6.1 *An individual $P \notin X$, among the nbElit best individuals of the subpopulation in terms of cost, will not be removed from the subpopulation by the Survivor-selection procedure.*

Proof Let J be the individual with the worst cost in the subpopulation, i.e., $fit(J) = nbIndiv$, and thus $BF(J) \geq nbIndiv + 1$. P belongs to the best $nbElit$ solutions in terms of cost, thus $BF(P) \leq nbElit + (1 - \frac{nbElit}{nbIndiv})(nbIndiv) \leq nbIndiv$. Individual P will not be removed as J has a worst biased fitness. □

3.7 Computational Experiments

We conducted several sets of experiments to evaluate the performance of HGSADC and to assess the impact on this performance of a number of algorithmic components. The former is performed through comparisons to results of state-of-the-art methods and to Best Known Solutions (BKS) for the three multi-period, multi-depot settings (Section 3.7.2), as well as for the capacitated VRP (Section 3.7.3). The latter is discussed in Section 3.7.4, while the calibration of the meta-heuristic is discussed in Section 3.7.1.

HGSADC was implemented in C++. Experiments were run on a AMD Opteron 250 computer with 2.4 Ghz clock. To facilitate comparisons with previous work, all CPU times reported in this section and Appendix II were converted into their equivalent Pentium IV 3.0 Ghz run times using Dongarra (2011) factors (see Appendix II.3).

3.7.1 Calibration of the HGSADC algorithm

As for most meta-heuristics, evolutionary ones in particular, HGSADC relies on a set of correlated parameters and configuration choices for its key operators. In order to identify good parameter values, we adopted the *meta-calibration* approach (Mercer and Sampson 1978), which was shown to perform particularly well for genetic-algorithm calibration (Smit and Eiben 2009).

Meta-calibration involves solving the problem of parameter optimization by means of meta-heuristics. In this scope, any evaluation of a set of parameters implies launching automatically the algorithm to be calibrated (HGSADC here) on a restricted set of *training instances* and measuring its effectiveness. We used a meta-evolutionary method, the Evolutionary Strategy with Covariance Matrix Adaptation (CMA-ES) of Hansen and Ostermeier (2001) to perform this optimization, as it necessitates few parameter evaluations to converge towards good solutions.

The calibration was run independently for each problem class, with the dual objective of measuring the dependency of the best parameter set upon the problem class, and identifying an eventual set of parameters suitable for all problem classes considered. Table 3.1 provides a summary of HGSADC parameters, together with the range of values we estimate to be appropriate due to either the parameter definition (e.g., probabilities and proportions), conceptual requirements (a local distance measure is assumed to implicate not more than 25% of the population), or values found in the literature (e.g., subpopulations sizes). The calibration results for each class, along with the final choice of parameter values for HGSADC, are also presented.

Except for the generation size λ, the optimum set of parameters appears independent of the problem type. We therefore averaged these results to get the final parameter values of Table 3.1, with the exception of the probability to educate a new individual (the education rate P_m). Calibrated education rates are generally very high, with an average value of 0.8. Additional tests indicated similarly good performance as long as $P_m \geq 0.7$. Hence we selected the value $P_m = 1$, which corresponds to a systematic education of all individuals, and reduces the number of parameters in use. The only parameter that is problem dependent is λ, which is set to 40 for the PVRP, 70 for MDVRP, and 100 for the MDPVRP.

Table 3.1: Calibration Results

Parameter		Range	PVRP	MDVRP	MDPVRP	**Final Params**
μ	Population size	[5,200]	18	24	30	**25**
λ	Number of offspring in a generation	[1,200]	33	87	146	**40 / 70 / 100**
el	Proportion of elite individuals, such that $nbElit = el \times \mu$	[0,1]	0.38	0.45	0.36	**0.4**
nc	Proportion of close individuals considered for distance evaluation, such that $n_{close} = nc \times \mu$	[0,0.25]	0.24	0.18	0.15	**0.2**
P_m	Education rate	[0,1]	0.86	0.86	0.70	**1.0**
P_{rep}	Repair rate	[0,1]	0.57	0.61	0.33	**0.5**
h	Granularity threshold in RI	[0,1]	0.53	0.36	0.35	**0.4**
ξ^{REF}	Reference proportion of feasible individuals	[0,1]	0.10	0.30	0.20	**0.2**

3.7.2 Results on periodic and multi-depot VRPs

HGSADC was tested on the MDVRP and PVRP benchmark instances of Cordeau et al. (1997), grouped into two sets, S_1 and S_2, containing respectively 33 and 42 instances of various sizes, from 50 to 417 customers. It was compared to state-of-the-art methods for these problems: the tabu search of Cordeau et al. (1997) (CGL), the scatter search of Alegre et al. (2007) (ALP), the VNS of Hemmelmayr et al. (2009) (HDH), and the record-to-record-ILP approach of Gulczynski et al. (2011) (GGW) for the PVRP; CGL, the fuzzy-logic guided-GA of Lau et al. (2010) (LCTP), and the adaptive large neighborhood search of Pisinger and Ropke (2007) (PR) for the MDVRP.

To study the behavior of HGSADC with respect to the number of iterations, three different stopping conditions were tested for (It_{NI}, T_{max}), $(10^4, 10min)$, $(2.10^4, 30min)$, and $(5.10^4, 1h)$. In all cases, the diversification parameter was set to $It_{div} = 0.4It_{NI}$. The instance set contains a few very large problems with more than 450 visits to customers over the different periods, for which the population size was reduced by two, and the computation time limit was increased.

Tables 3.2 and 3.3 sum up the comparison of average results from 10 independent runs of HGSADC, with various stopping conditions, to results reported for state-of-the-art algorithms. We first report the averages, over all instances, of the instance computation times (line *Time*) and the percentages of deviation from the BKS (*Gap overall*). We then present average deviations to BKS for the two instance sets, as well as for large problems with more than 150 customers. More detailed results are provided in Appendix II.4.

Table 3.2: HGSADC performance on PVRP instances

	CGL	**ALP**	**HDH**			**GGW**	**HGSADC**		
	(1 run)	—	(Avg. 10 runs)			(1 run)	(Avg. 10 runs)		
	15.10^3 it	—	10^7 it	10^8 it	10^9 it	—	10^4 it	2.10^4 it	5.10^4 it
Time	4.28 min	3.64 min	3.34 min	33.4 min	334 min	10.36 min	5.56 min	13.74 min	28.21 min
Gap overall	+1.82%	—	+1.45%	+0.76%	+0.39%	—	+0.20%	+0.12%	+0.07%
Gap S_1	+1.62%	+1.40%	+1.43%	+0.73%	+0.37%	+0.94%	+0.14%	+0.09%	+0.04%
Gap S_2	+2.48%	—	+1.53%	+0.83%	+0.44%	—	+0.38%	+0.23%	+0.17%
Gap $n \geq 150$	+3.23%	—	+2.16%	+1.18%	+0.62%	—	+0.35%	+0.20%	+0.14%

Table 3.3: HGSADC performance on MDVRP instances

	CGL	**PR**		**LCTP**	**HGSADC**		
	(1 run)	(Avg. 10 runs)		(Avg. 50 runs)	(Avg. 10 runs)		
	15.10^3 it	25.10^3 it	50.10^3 it	—	10^4 it	2.10^4 it	5.10^4 it
Time	small	1.97 min	3.54 min	2.06 min	4.24 min	8.99 min	19.11 min
Gap overall	+0.96%	+0.52%	+0.34%	+0.49%	-0.01%	-0.04%	-0.06%
Gap S_1	+0.58%	+0.54%	+0.35%	+0.39%	+0.00%	-0.02%	-0.03%
Gap S_2	+1.85%	+0.47%	+0.34%	+0.71%	-0.04%	-0.10%	-0.12%
Gap $n \geq 150$	+1.40%	+0.68%	+0.45%	+0.70%	-0.03%	-0.08%	-0.10%

With respect to these experiments, HGSADC seems to perform remarkably well in comparison to other algorithms. During short runs of 10^4 iterations, an average overall gap of +0.20% relative to the previous BKS is achieved for the PVRP, compared to more than +1.40% for the other approaches. Similar performance is observed for MDVRP, with an average gap of −0.01% indicating that the new method is on average better than the previous BKS on all instances. Actually, during these short runs, HGSADC produced new best average results for 41 out of 42 PVRP instances and for all 33 MDVRP instances. It is noteworthy that the average standard deviation per instance obtained by HGSADC is 0.15% for PVRP and 0.05% for MDVRP, meaning that the algorithm is very reliable. New BKS were obtained for 20 instances out of 42 for the PVRP, and 9 instances out of 33 for the MDVRP.

The average computation time is short, barely higher than for other methods, and suitable for many operational decisions. For MDVRP problems especially, only 2.15 min are required, on average, to find the final solution for short MDVRP runs, the rest of the time being spent to reach the time-limit termination criteria. Using the termination criteria $(5.10^4, 1h)$, previous BKS are retrieved on all runs for 21 instances out of 33, while known optimal solutions from Baldacci and Mingozzi (2009) and Baldacci et al. (2011a) are retrieved on every run. It is noticeable that HGSADC obtains in a few minutes better PVRP results than HDH, the previous state-of-the-art method for PVRP, even when HDH runs for 10^9 iterations (100 times the number of iterations for standard HDH runs), corresponding to some 300 minutes of run time.

No benchmark instance set was available for the MDPVRP. We therefore built a set of 10 MDPVRP instances by merging the PVRP and MDVRP instances of the second set provided by Cordeau et al. (1997). Each of the 10 MDVRP instances was combined with the PVRP instance with the same number of customers. The number of periods and the patterns were taken from the PVRP instance, the depots from the MDVRP one, and the number of vehicles was fixed to the smallest number such that a feasible solution could be found by HGSADC. The full data sets can be obtained from the authors.

The maximum run time was increased to 30 minutes for these experiments (It_{NI} remains set to 10^4) to account for the higher difficulty of the MDPVRP. The average results of 10 runs of HGSADC on these new instances are reported in Appendix II.4, and compared to the best solutions ever found during all our experiments. An average error gap of +0.42% was observed, which is reasonable given the increased problem difficulty. The average standard deviation per instance is

now 0.26%, illustrating the increased irregularity of the search space. Keeping the best solution of the 10 runs leads to significantly better solutions, with an average error gap of +0.13%, but requires more computational resources. This approach corresponds to the well-known independent-search strategy for parallel meta-heuristics (Crainic and Toulouse 2010). More sophisticated parallel-search strategies, based on cooperation, in particular, could be used to improve the exploration of the search space and reach better results.

3.7.3 Results on the capacitated VRP

The CVRP is a special case of multi-depot periodic problems, when $d = 1$ and $t = 1$. HGSADC can thus be used to address the CVRP with very minor changes in the distance measure and the parameters, even though its operators were designed for multi-period settings.

Detailed results of experiments on 34 well-known CVRP instances from the literature are reported in Appendix II.5. Very competitive results to state-of-the-art methods were obtained in similar computation times. An overall gap to the BKS of 0.10% was thus observed, which is equal to the performance of the best, highly specialized, algorithm in the literature (Nagata and Bräysy 2009b). We also retrieved 12 new best known solutions for Golden et al. (1998a) instances. The diversity management method we propose seems to compensate for the lack of problem-tailored operators, and opens several promising avenues of research.

3.7.4 Sensitivity analysis of algorithmic components

A second set of experiments targeted the analysis of the impact on the performance of the proposed meta-heuristic of various algorithmic components. Sensitivity analysis was thus performed on "traditional" hybrid genetic components by "removing" each of them in turn.

The "No-Education" version was obtained by setting the probability of offspring education P_m to 0, which also meant that no repair was performed. Local-search improvement methods are thus exclusively used to produce the initial population. In the "No-Population" version, $\lambda = 1,\ \mu = 0$, and $nbElite = 1$. Thus, only one individual appears in each subpopulation, the population management mechanism then behaving as a steady-state population management where the offspring replaces the parent only if it improves. The crossover either combines an individual with itself, or both feasible and infeasible individuals. Only one parent was selected by binary tournament for the "No-Crossover" version, underwent education and was inserted into the population. The parent selection was performed only in the feasible subpopulation for the "No-Infeasible" algorithm, a constant high penalty value being enforced through the search. Finally, setting the repair probability P_{rep} to 0 yielded the "No-Repair" version. The results are reported in Table 3.4, each column corresponding to the average time and gap to BKS of HGSADC without the respective component.

It is noticeable that all these algorithmic components play an important role in the good performance of the proposed meta-heuristic, the most crucial being education followed by population, crossover, infeasible solutions, and repair to a lesser extent.

Table 3.4: Sensitivity analysis on main HGSADC components

Benchmark		No-Edu	No-Pop	No-Cross	No-Inf	No-Rep	HGSADC
PVRP	T	0.89 min	4.21 min	4.42 min	5.39 min	5.20 min	5.56 min
	%	+4.24%	+2.19%	+1.94%	+0.80%	+0.19%	+0.20%
MDVRP	T	0.83 min	3.49 min	4.21 min	4.45 min	3.58 min	4.24 min
	%	+7.10%	+9.54%	+7.04%	+0.45%	+0.07%	-0.01%
MDPVRP	T	0.89 min	9.29 min	11.21 min	15.47 min	13.22 min	15.96 min
	%	+25.22%	+16.90%	+8.39%	+1.40%	+0.54%	+0.42 %

The second part of the sensitivity analysis was dedicated to the adaptive population diversity control mechanism, which is a cornerstone of the proposed methodology. We therefore compared its performance to those of two mechanisms from the literature, mechanisms that proved their worth in their respective contexts. Two new algorithms were thus derived from HGSADC to conform to each of these two rules, as well as a variant without diversity control (identified as *HGS0*).

The *HGS1* variant involves a dispersal rule in the objective space as in Prins (2004). Let F be the fitness function, defined as the cost, and Δ_F a fitness spacing parameter. Acceptance of an individual I in the population is granted only if $|F(I) - F(C)| \geq \Delta_F$ for all individuals C already in the population. The second variant, named *HGS2*, relies on the population management framework of Sörensen and Sevaux (2006). Let Δ_D be a spacing parameter and δ_H the distance measure presented in Section 3.6.3. To be added to the population, an individual I must obey a dispersal rule, i.e., it must verify $\delta_H(I, C) \geq \Delta_D$ for all C already in the population. In our implementation, the value of Δ_D changes during run time: strong distance constraints are imposed at the beginning of the search to encourage exploration, whereas the value of Δ_D decreases progressively toward zero as the method approaches the termination criteria, to encourage the exploitation of good solutions. For both methods, we use an incremental population management and only individuals with a fitness below the median of the population can be discarded.

Table 3.5: Comparison of population-diversity management mechanisms

Benchmark		HGS0	HGS1	HGS2	HGSADC
PVRP	T	4.68 min	5.15 min	5.37 min	5.56 min
	%	+0.70%	+0.62%	+0.39%	+0.20%
MDVRP	T	3.37 min	3.55 min	4.49 min	4.24 min
	%	+0.80%	+0.61%	+0.10%	-0.01%
MDPVRP	T	13.16 min	14.00 min	15.94 min	15.96 min
	%	+2.95%	+2.95%	+2.37%	+0.42%

Table 3.5 reports the average gaps to BKS and average run times for each method on the instances presented in Section 3.7.2. One observes that the results verify that applying the dispersal rule with respect to the solution space (HGS2) is more effective than using the dispersal rule with

respect to the objective space (HGS1), which is an indication of the interest of the hybrid evolution strategy of HGSADC. One also observes that proceeding without diversity management yields rather poor results compared to all other strategies. The best results are definitely obtained with the proposed adaptive diversity management method, which yields the best average gap for an equivalent computational effort.

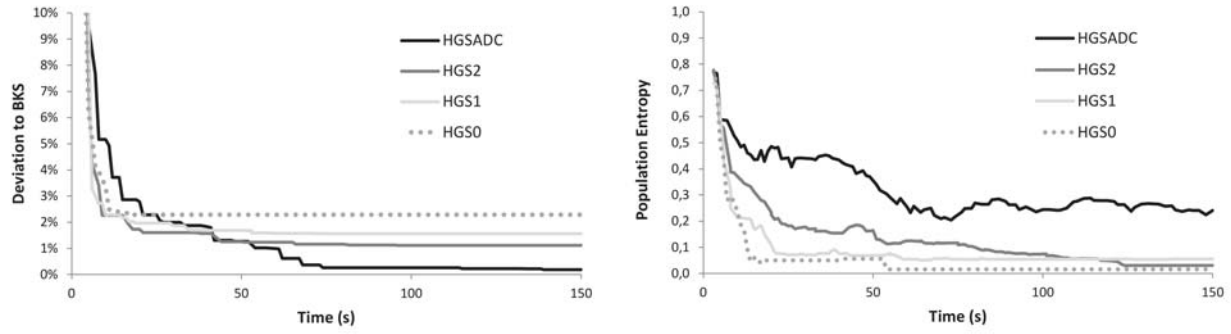


Figure 3.3: Population entropy and error gap to the BKS for the diversity management strategies on MDPVRP instance pr03

Figure 3.3 illustrates the behavior of the four population-diversity management strategies during one of the runs (150 seconds) on MDPVRP instance pr03, as measured by the population entropy and the gap to the BKS. The population entropy is computed as the average distance from one individual to another. All algorithms close the gap to less than 2.50% within a few seconds. The methods that use diversity management are able, however, to efficiently continue searching and, thus, to reach better solutions. The proposed HGSADC meta-heuristic is still regularly improving its best found solution as the time limit approaches, despite being already very close to the best-known solution (a gap of 0.19% only). The no-diversity management strategy, HGS0, provides a perfect example of premature convergence. In less than one minute, one observes no additional improvement of the best solution, very low entropy, and quite likely very little evolution in the population. HGSADC, on the other hand, maintains a healthy diversity in the population, as illustrated by a rather high level of entropy at 0.3. In comparison, the two alternate strategies, HGS1 and HGS2, display lower entropy levels, around 0.1.

We conclude that the proposed diversity management mechanism is particularly effective for the problem classes considered in this paper. In the experiments we conducted, it allowed to avoid premature convergence and to reach high quality solutions.

3.8 Conclusions and Research Perspectives

We proposed a new hybrid genetic search meta-heuristic to efficiently address several classes of multi-depot and periodic vehicle routing problems, for which few efficient algorithms are currently available. Given the great practical interest of the problem considered, the proposed methodology opens the way to significant progress in the optimization of distribution networks.

The paper introduces several methodological contributions, in particular, in the crossover and education operators, the management of infeasible solutions, the individual evaluation procedure driven both by solution cost and the contribution to population diversity and, more generally, the adaptive population management mechanism that enhances diversity, allows a broader access to reproduction, and preserves the memory of what characterizes good solutions represented by the elite individuals. The combination of these concepts provides the capability of the proposed *Hybrid Genetic Search with Adaptive Diversity Control* meta-heuristic to reach high quality solutions on the literature benchmarks. The method actually identifies either the best known solutions, including the optimal ones, or new best solutions for all benchmark instances, thus outperforming the current state-of-the-art meta-heuristics for each particular problem class. Moreover, with minimal adjustments, it obtains comparable results to the best methods for the CVRP.

Among the many interesting avenues of research, we mention the interest to explore the impact of the adaptive diversity control mechanism for other classes of problems, and to validate its good performance using theoretical models. We also plan to generalize the methodology to problems with additional attributes, and thus progress toward addressing *rich VRP* problem settings, as well as real world applications.

CHAPITRE 4

UN ALGORITHME GÉNÉTIQUE HYBRIDE POUR UNE GRANDE FAMILLE DE PROBLÈMES DE TOURNÉES DE VÉHICULES AVEC FENÊTRES DE TEMPS

4.1 Fil conducteur et contributions

Les attributs étudiés dans le précédent chapitre faisaient partie de la catégorie des attributs de type Assign, impactant les choix d'affectation. Ce chapitre étudie l'extension de la méthodologie HGSADC à un autre type d'attribut : les fenêtres de temps pour les services aux clients. Cet attribut est de type Eval selon la classification du Chapitre 1. De nouvelles méthodes d'évaluation de voisinages de recherche locale sont introduites pour évaluer efficacement des solutions irréalisables ne respectant pas certaines contraintes sur les routes (fenêtres de temps, capacité et durée). Aussi, pour traiter des problèmes de grande taille comprenant jusque 4500 services sur 10 jours, des phases de décomposition sont utilisées. La méthode ainsi obtenue produit des résultats remarquables sur une vaste gamme de problèmes de tournées de véhicules avec fenêtres de temps, contraintes de durée de routes, multiple dépôts et multiple périodes, surpassant toutes les méthodes précédentes de la la littérature pour ces problèmes.

4.2 Article IV : A hybrid genetic algorithm for a large class of vehicle routing problems with time windows

Ce chapitre a fait l'objet d'une publication sous forme d'article de journal : Vidal, T., Crainic, T.G., Gendreau, M., Prins, C. (2012). A hybrid genetic algorithm for a large class of vehicle routing problems with time windows. *Computers & Operations Research*, 40(1), 475–489.

Abstract: The paper presents an efficient Hybrid Genetic Search with Advanced Diversity Control for a large class of time-constrained vehicle routing problems, introducing several new features to manage the temporal dimension. New move evaluation techniques are proposed, accounting for penalized infeasible solutions with respect to time-window and duration constraints, and allowing to evaluate moves from any classical neighbourhood based on arc or node exchanges in amortized constant time. Furthermore, geometric and structural problem decompositions are developed to address efficiently large problems. The proposed algorithm outperforms all current state-of-the-art approaches on classical literature benchmark instances for any combination of periodic, multi-depot, site-dependent, and duration-constrained vehicle routing problem with time windows.

Keywords: Vehicle routing problems, time windows, hybrid genetic algorithm, diversity management, neighbourhood search, decomposition

4.3 Introduction

Vehicle routing problems (VRP) with time constraints and requirements relative to customer assignments to vehicle types, depots, or planning periods constitute a class of difficult optimization problems. These settings are linked with numerous practical applications including logistics, goods transportation, refuse collection, maintenance operations, and relief supply (see Golden et al. 2008, for a large variety of application cases). Much has already been dedicated to specific VRP with additional "attributes" such as time windows, multiple depots, or delivery-period choices. As illustrated by numerous reviews (Bräysy and Gendreau 2005b,a, Francis et al. 2008, Gendreau et al. 2008b, Gendreau and Tarantilis 2010, for the most recent), almost every prominent metaheuristic paradigm, including evolutionary methods, ant colony optimization, tabu search, simulated annealing, other improved local search approaches or ruin-and-recreate, has been applied to at least one of the previous settings. Yet, besides highly problem-tailored methods, the literature critically lacks good generalist approaches able to efficiently address a wide range of problem variants, and VRP combining several problem attributes (also called multi-attribute VRP in Crainic et al. 2009 and rich VRP in Hartl et al. 2006) still constitute major challenges for both research and applications. Within these settings, it is well known that imposing time-window constraints on customer services (and depot availability) raises significant challenges related to the smaller proportion of feasible solutions, the increased computation burden required to evaluate moves in neighbourhood search, and the antagonist influence of temporal and spatial characteristics.

This paper introduces a new *Hybrid Genetic Search with Advanced Diversity Control* (*HGSADC*), that addresses some of these challenges. In particular, it addresses efficiently a wide range of large-scale *vehicle routing problems with time windows* (*VRPTW*), route-duration constraints, and additional attributes involving requirements for customer assignments to particular vehicles types, depots or planning periods. The main characteristic of HGSADC stands in a different approach to population diversity management, the contribution of a particular individual to the diversity of the population appearing as a proper objective to be optimized (Vidal et al. 2012a). We also introduce a number of new algorithmic features targeting specifically the temporal characteristics of the problems. We thus propose simple move evaluation procedures that accommodate penalized infeasibility with regard to duration and time-window constraints, and work in amortized $O(1)$ for any neighbourhood based on a bounded number of arc exchanges or node relocations. We also develop neighbourhood pruning procedures based on the temporal dimension, and decomposition principles to address efficiently large problem instances involving up to 1000 customers and 4500 services. The resulting algorithm is simple and efficient, outperforming all existing approaches on classical benchmarks for the VRPTW, its variants with multiple depots (*MDVRPTW*), multiple periods (*PVRPTW*), and vehicle-site dependencies (*SDVRPTW*).

The main contributions of this paper are thus 1) a generalization of the concept of HGSADC to a large class of VRPTW variants presenting mixed temporal and geometrical characteristics; 2) new procedures to efficiently search neighbourhoods when considering infeasible solutions with regards to duration and time-window constraints; 3) decomposition principles within the genetic

framework allowing to address efficiently large instances; and 4) a state-of-the-art meta-heuristic for four classes of vehicle routing problems with time windows.

The remainder of this paper is organized as follows: Section 4.4 states the notation and formally defines the problems. Summarized elements of the literature are presented in Section 4.5. The proposed meta-heuristic is described in Section 4.6, while extensive computational experiments are reported in Section 4.7. Section 4.8 concludes.

4.4 Problem Statement

We first formally state the VRPTW. A general PVRPTW is then defined, including other notable variants, such as the MDVRPTW and the SDVRPTW, as special cases. Route-duration constraints are included in all cases but do not appear in the acronyms.

Let $G = (\mathcal{V}, \mathcal{A})$ be a complete directed graph. Vertex $v_0 \in \mathcal{V}$ represents a single depot, where a fleet of m identical vehicles with capacity Q is located, and a product to be delivered is kept. Each other vertex $v_i \in \mathcal{V}^{\text{CST}}$, with $\mathcal{V}^{\text{CST}} = \mathcal{V} \backslash \{v_0\}$ and $i \in \{1, \ldots, n\}$, stands for a customer to be serviced, characterized by a non-negative demand q_i, a service duration τ_i, as well as an interval of allowable visit times $[e_i, l_i]$, called time window. By definition, $q_0 = \tau_0 = 0$. Arcs $(i, j) \in \mathcal{A}$ represent the possibility to travel from v_i to v_j with a distance c_{ij} and a duration δ_{ij}. A feasible route r is defined as a circuit in G that starts and ends at v_0, such that the total demand of customers in r is smaller than or equal to Q. While performing its route, a vehicle may stop and wait in order to reach the next customer within its time window, but the route duration, computed as the difference between the start time and the return time at v_0, is limited to D. The VRPTW aims to construct up to m vehicle routes, to visit each customer vertex once within its time window, while minimizing the total distance.

In the generalized PVRPTW, route planning is performed for a horizon of t periods. Distances c_{ijl} and durations δ_{ijl} can be dependent upon the period. Each customer v_i is characterized by a frequency f_i, representing the total number of services requested on the planning horizon, and a list L_i of allowable visit combinations, called patterns. The objective is to select a pattern for each customer, and construct the associated routes to minimize the total distance over all periods. A mathematical integer programming formulation of this problem is given in Appendix III.1.

We also consider two related problem classes. The MDVRPTW involves $d > 1$ depots. It is also generally assumed that any route originates and returns to the same depot. This problem was shown to constitute a special case of the generalized PVRPTW (Cordeau et al. 1997), where depots are assimilated to periods ($t = d$), any customer $v_i \in \mathcal{V}^{\text{CST}}$ has a frequency $f_i = 1$ and can be serviced in any period $L_i = \{\{1\}, \ldots, \{t\}\}$, and δ_{0il} and c_{0il} values are set in each period to correctly account for the distance to the assimilated depots. The SDVRPTW involves w vehicle types, with compatibility constraints between customers and vehicles. Each customer $v_i \in \mathcal{V}^{\text{CST}}$ can be serviced only by a subset $R_i \in \{1 \ldots w\}$ of vehicle types. As shown in Cordeau and Laporte (2001), this problem constitutes another particular case of PVRPTW, where each vehicle type is assimilated to a different period ($t = w$) and $L_i = \{\{k\} : k \in R_i\}$.

It was shown in Vidal et al. (2012a), that any VRP with multiple depots and periods (MDPVRP) can be transformed into an equivalent PVRP, by associating a different period to each (period, depot) pair from the former problem. In the same spirit, we can transform a problem combining multi-depot, site-dependent, multi-period, and time-window attributes into a PVRPTW, by associating a period for each (depot, period, vehicle type) in the original problem. This transformation thus enables to address all the previous VRPTW variants and their combinations by means of a single algorithm for the PVRPTW. The inherent difficulty related to combined (depot, period, vehicle type) choices leads to a large number of periods in the new problem. The proposed algorithm has thus been designed to successfully tackle large PVRPTW instances.

4.5 Literature Review

We initiate this review surveying proposed meta-heuristics for the VRPTW, which is one of the most intensively studied NP-hard combinatorial optimization problems in the last thirty years. Exact methods are still not able to address most large-size applications, and their performance strongly varies with the time-window characteristics. Heuristic and meta-heuristic approaches have thus been the methodology of choice (see Bräysy and Gendreau 2005b,a, Gendreau and Tarantilis 2010, for extensive reviews), and have been mostly evaluated and compared on standard benchmark instances introduced by Solomon (1987) and Gehring and Homberger (1999) relative to their computational efficiently and the quality of the solutions obtained. Most authors have focused on primarily minimizing fleet size, and then distance, but a few exceptions exist (Alvarenga et al. 2007, Labadi et al. 2008). As a consequence, state-of-the-art VRPTW heuristics are generally based on two stages, dedicated first to minimizing fleet size and then distance.

Most successful approaches involve local search improvement procedures based on arc- and node-exchange neighbourhoods, and are coupled with various other concepts listed in the following:

- *Evolution strategies* (Alvarenga et al. 2007, Mester and Bräysy 2005, Hashimoto et al. 2008, Labadi et al. 2008, Repoussis et al. 2009b, Nagata et al. 2010);
- *Solutions recombinations* (Alvarenga et al. 2007, Hashimoto et al. 2008, Labadi et al. 2008, Repoussis et al. 2009b, Nagata et al. 2010);
- *Ruin and recreate* (Pisinger and Ropke 2007, Prescott-Gagnon et al. 2009, Repoussis et al. 2009b);
- *Ejection chains* (Lim and Zhang 2007, Nagata and Bräysy 2009a, Nagata et al. 2010);
- *Guidance and memories* (Mester and Bräysy 2005, Le Bouthillier and Crainic 2005b, Repoussis et al. 2009b);
- *Parallel and cooperative search* (Le Bouthillier and Crainic 2005a,b);
- *Mathematical programming hybrids* (Prescott-Gagnon et al. 2009).

The most competitive result are currently offered by the hybrid genetic algorithm of Nagata et al. (2010). The method combines powerful route minimization procedures, with a very effective *edge assembly crossover*, and extremely efficient local search procedures. The Iterated Local Search

(ILS) of Ibaraki et al. (2008), the Adaptive Large Neighbourhood Search (ALNS) of Pisinger and Ropke (2007), and the Unified Tabu Search (UTS) of Cordeau et al. (1997, 2001a, 2004) are also worth mentioning. These methods stand out in terms of simplicity and wider applicability, as both have been extended to address various VRP variants.

Variants of the VRPTW, combining time windows with multi-period, multi-depot, or site dependency, arise in many practical applications such as maintenance operations (Weigel and Cao 1999, Blakeley et al. 2003), refuse collection (Teixeira et al. 2004, Sahoo et al. 2005), or product distribution (Golden and Wasil 1987, Privé et al. 2005, Jang et al. 2006, Chiu et al. 2006, Parthanadee and Logendran 2006). Although the interest in these problem settings is growing, a somewhat restricted number of contributions have been proposed in the literature addressing them. Most of these implemented some form of neighbourhood-based meta-heuristic search. UTS (Cordeau et al. 2001a, Cordeau and Laporte 2001, Cordeau et al. 2004) is currently the only method addressing all variants considered in this paper. UTS exploits long-term memories to penalize frequently-encountered solution features. A parallel variant of this approach has been recently proposed by Cordeau and Maischberger (2012).

Specific to the PVRPTW, Pirkwieser and Raidl (2008) propose a Variable Neighborhood Search (VNS), further enhanced by means of multi-start strategies, column generation hybridizations, or multi-level strategies in (Pirkwieser and Raidl 2009a,b, 2010b). Yu and Yang (2011) propose a coarse-grained parallel Ant Colony Optimization (ACO) algorithm, and Nguyen et al. (2011) develop a hybrid genetic approach. The latter method combines the strength of population-based search, and Tabu and VNS improvement methods applied to the offspring. For the MDVRPTW, we report the hybrid Tabu search and savings approach of Tamashiro et al. (2010), the parallel VNS approaches of Polacek et al. (2004, 2008), and a genetic algorithm and ACO hybrid by Ostertag (2008). Noteworthy is also the approach of Chiu et al. (2006), which considers total duty time minimization (including waiting times). Other than UTS, a single hybrid Tabu search and VNS approach by Belhaiza (2010) may be reported for the SDVRPTW.

The literature is extremely scarce on VRPTW variants combining multiple features. We mention the Tabu search of Parthanadee and Logendran (2006) able to address multi-depot periodic VRPTW (MDPVRPTW). Crainic et al. (2009) introduced the Integrative Cooperative Search (ICS) framework to target highly complex combinatorial optimization settings. ICS is a central-memory cooperative multi-search involving problem decompositions by decision sets, integration of elite partial solutions yielded by the subproblems, and adaptive guidance mechanisms. A MDPVRPTW application was presented, but no definitive results have been published yet.

We conclude this section focusing on the design of efficient local-search methods, because most methods presented in this section dedicate the greatest part of their computational effort to the serial exploration of neighbourhoods, basically edge exchanges. Efficient move evaluation procedures are thus determining for both algorithmic speed and scalability.

Many methods rely on search spaces that include infeasible solutions with respect to time-window constraints, aiming to explore a wider diversity of structurally different feasible solutions. Several relaxation alternatives have been proposed in the literature. Cordeau et al. (2001a) and Repoussis et al. (2009b), among others, allow penalized late services to customers, while Ibaraki

et al. (2008) also allow penalized early services. Using such relaxations however leads to less efficient move-evaluation procedures, working in $O(n)$ or $O(\log n)$ (Ibaraki et al. 2008). A different relaxation was recently used by Hashimoto et al. (2008) and Nagata et al. (2010). An assumption is made that upon a late arrival, a penalized return in time can be employed to reach the time window. The authors demonstrated that several classical neighbourhood moves can be evaluated in amortized $O(1)$ in this relaxation scheme. However, neither intra-route moves, nor duration constraints are actually managed in $O(1)$.

This review clearly underlines several gaps in the actual state of the art. Thus, for example, while population-based methods have shown their worth on the classic VRPTW, there is a lack of really efficient methods of this type for more complex variants such as PVRPTW, MDVRPTW, SDVRPTW and their combinations. Also, most current efficient methods for VRPTW are intricate, hard to reproduce, and largely rely on specific problem-tailored procedures. Hence, there is a need for more general and simple methods, broadly applicable to a large variety of practical settings with combined features. Finally, the temporal aspects lead to important challenges regarding infeasible-solution management and neighbourhood-evaluation procedures, which have a strong impact on the efficiency and scalability of VRPTW meta-heuristics. The concepts developed in this paper contribute towards addressing these issues.

4.6 The HGSADC Methodology

This section describes the proposed Hybrid Genetic Search with Adaptive Diversity Control algorithm for time window-constrained VRP variants. For matters of presentation clarity, we describe the approach for the VRPTW and PVRPTW, the latter encompassing the MDVRPTW, SDVRPTW and other problems as special cases.

HGSADC (Vidal et al. 2012a) is a hybrid meta-heuristic combining the exploration capabilities of genetic algorithms with efficient local search-based improvement procedures and diversity management mechanisms. In HGSADC, population diversity is considered as an objective to be optimized along with solution quality through individual evaluations and selections. The general behavior of HGSADC is sketched in Algorithm 4.1.

The method evolves feasible and infeasible solutions in two separate subpopulations. Genetic operators are iteratively applied to select two parents from the subpopulations (Line 3 of Algorithm 4.1), combine them into an offspring (Line 4), which undergoes a local search-based *Education*, is *Repaired* if infeasible, and is finally inserted into the suitable subpopulation (Lines 5-7). Each subpopulation is managed separately to trigger a *Survivor Selection* phase when a maximum size is reached, adapt infeasibility penalties, and call a *Diversification* mechanism (after each It_{div} successive iterations without improvement, Lines 8-10) whenever the search stagnates. In this application, structural and geometrical decompositions phases are also performed (after each It_{dec} iterations, Line 11) to tackle large problems, the subproblems being addressed by means of recursive calls to HGSADC. The method terminates when It_{NI} successive iterations have been performed without improvement.

Algorithm 4.1 HGSADC

```
1: Initialize population
2: while number of iterations without improvement < It_NI, and
   time < T_max
3:    Select parent solutions P_1 and P_2
4:    Create offspring C from P_1 and P_2 (crossover)
5:    Educate C (local search procedure)
6:    if C infeasible then
          Insert C into infeasible subpopulation,
          Repair with probability P_rep
7:    if C feasible then
          Insert C into feasible subpopulation
8:    if maximum subpopulation size reached then
          Select survivors
9:    if best solution not improved for It_div iterations, then
          Diversify population
10:   Adjust penalty parameters for infeasibility
11:   if number of iterations = k × It_dec where k ∈ ℕ*, then
          Decompose the master problem
          Use HGSADC on each subproblem
          Reconstitute three solutions, and insert them in the population
12: Return best feasible solution
```

The main components of the method are described in the following subsections. The search space is presented in Section 4.6.1. Sections 4.6.2-4.6.4 briefly recall the solution representation, individual evaluation, selection, and crossover operators which, as the population management of Section 4.6.6, remain unchanged from Vidal et al. (2012a). We then detail, in Section 4.6.5, the new neighbourhood-based evaluation procedures within *Education* and *Repair*, specifically developed for the temporal characteristics of the problems. Finally, Section 4.6.7 presents structural and geometrical problem decompositions that enable to address efficiently large instances. All these components together lead to a highly efficient algorithm for VRPTW variants.

4.6.1 Search space

The efficient exploitation of penalized infeasible solutions is known to contribute significantly to the performance of heuristics (Glover and Hao 2011, Vidal et al. 2012a). The search space of HGSADC involves infeasible solutions with respect to route constraints: load, duration, and time windows. The fleet-size limit is always respected, as a solution with too many vehicles may require sophisticated and computationally costly route-reduction methods to be repaired. Time windows are relaxed following the lines of Nagata (2007). Upon a late arrival to a customer, one pays for a "time warp" to reach the end of the time window. This choice of relaxation is motivated by the availability of efficient penalty evaluation procedures within neighbourhood searches (Section 4.6.5).

Figure 4.1 (inspired by Nagata et al. 2010) illustrates the previous assumptions on a route with five stops, which are represented from bottom to top with their time windows. The time dimension

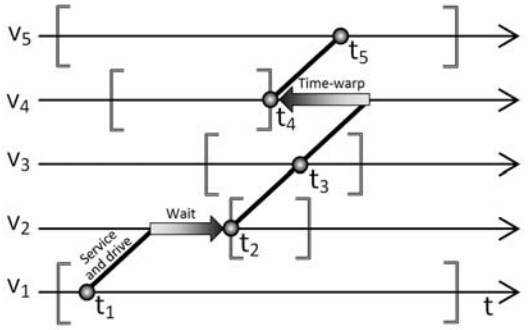


Figure 4.1: Illustration of waiting times and time warps

corresponds to the horizontal axis, while the vertical axis represents the progression on the route. A possible schedule is represented in bold line. This schedule presents some waiting time before service to v_2, and a time warp, triggered by a late arrival to v_4. Time warps are in some sense symmetric to waiting times, although waiting times are not penalized. Let r be a route, which starts from depot v_0 ($\sigma_0^r = 0$), visits n_r customers $(\sigma_1^r, \ldots, \sigma_{n_r}^r) \in \mathcal{V}^{\text{CST}}$, and returns to the depot $\sigma_{n_r+1}^r = 0$. Let $\mathbf{t}^r = (t_0^r, \ldots, t_{n_r+1}^r)$ be the visit times associated to each stop. On the way from a vertex σ_i^r to σ_{i+1}^r, the incurred time warp is given by $tw_{i,i+1} = \max\{t_i^r + \tau_{\sigma_i^r} + \delta_{\sigma_i^r \sigma_{i+1}^r} - t_{i+1}^r, 0\}$. The following quantities characterize route r:

- Load $q(r) = \sum_{i=1,\ldots,n_r} q_{\sigma_i^r}$;
- Distance $c(r) = \sum_{i=0,\ldots,n_r-1} c_{\sigma_i^r \sigma_{i+1}^r}$;
- Time-warp use $tw(r) = \sum_{i=0,\ldots,n_r-1} tw_{i,i+1}$;
- Duration $\tau(r) = t_{n_r+1}^r - t_0^r + tw(r)$

The *penalized cost* $\phi(r)$ of route r with schedule $\mathbf{t}^r$, presented in Equation (4.1), is defined as its total distance plus the weighted sum of its excess duration, load, and time-warp use.

$$\begin{aligned}\phi(r) = c(r) &+ \omega^{\text{D}} \max\{0, \tau(r) - D)\} \\ &+ \omega^{\text{Q}} \max\{0, q(r) - Q\} + \omega^{\text{TW}} \times tw(r)\end{aligned} \tag{4.1}$$

Finally the penalized cost $\phi(s)$ of solution s, involving a set of routes $\mathcal{R}(s)$, is given by the sum of the penalized costs of all its routes.

4.6.2 Solution representation

The solution representation defined previously in HGSADC (Vidal et al. 2012a) is fairly general, and can be applied without change in the present context. A solution is represented as a two-chromosome *individual without trip delimiters*. The chromosomes account for the visit-period choices for each customer and the sequences of services for each period, respectively. The representation without trip delimiters, introduced in Prins (2004), allows for simple recombination operators working on sequences, without the need to explicitly account for the individual routes. Then, to obtain a full solution from an individual representation, a polynomial *Split* algorithm,

based on a shortest path procedure, is applied for each period to optimally partition the sequence of customers into routes. In our context, we use a *Split* algorithm that respects the maximum number of routes (Chu et al. 2006), and includes in the auxiliary graph penalized infeasible routes r regarding duration, load, and time-window constraints, and such that $q(r) \leq 2Q$. Reversely, any PVRPTW solution, represented by its routes for each period, can be transformed into an individual by removing visits to the depot.

4.6.3 The diversity and cost objective for evaluating individuals

Any individual P in the population is characterized by its solution cost $\phi(P)$ (Section 4.6.1), and its *diversity contribution* $\Delta(P)$ defined as the average distance from P to its closest neighbours $\mathcal{N}_{close}$ in the subpopulation (Equation 4.2).

$$\Delta(P) = \frac{1}{|\mathcal{N}_{close}|} \sum_{P_2 \in \mathcal{N}_{\text{CLOSE}}} \delta(P, P_2) \tag{4.2}$$

For the PVRPTW and SDVRPTW, we rely on a Hamming distance δ measuring the proportion of customers with identical patterns or vehicle type assignments, as in Vidal et al. (2012a). For the VRPTW and MDVRPTW, experiments led to choose the *broken pairs* distance (see Prins 2009b), which evaluates the amount of common arcs.

The evaluation, *biased fitness*, $BF(P)$ of an individual P (Equation 4.3) is then a "diversity and cost objective" that involves both the rank $fit(P)$ of P in the subpopulation with regards to solution cost $\phi(P)$, and its rank $dc(P)$ in terms of diversity contribution $\Delta(P)$ (Equation 4.2). $BF(P)$ depends upon the actual number of individuals in the subpopulation N_{indiv}, and a parameter N_{elite} ensuring elitism properties during survivor selection. This trade-off between diversity and elitism is critical for a thorough and efficient search.

$$BF(P) = fit(P) + \left(1 - \frac{N_{elite}}{N_{indiv}}\right) dc(P) \tag{4.3}$$

4.6.4 Parent selection and crossover

An iteration of HGSADC corresponds to the generation of a new individual by a succession of genetic operations. Two parents are first selected by binary tournament in the union of both feasible and infeasible populations, and used as input to the crossover operator. The PIX crossover (Vidal et al. 2012a) is used for the PVRPTW. PIX enables to inherit good sequences of visits from both parents, and also to recombine visit patterns. For the VRPTW, we rely on the simple Ordered Crossover (OX) (see Prins 2004, for instance). These crossovers allow both small solution refinement and more important structural changes.

4.6.5 Neighbourhood search for VRPTW education and repair

An offspring resulting from the crossover operator undergoes the *Split* procedure to extract its routes. A neighbourhood search-based improvement operator, called *education*, is then systematically applied, followed by a *repair* phase, called with probability P_{rep}, when the resulting

solution is infeasible. *Repair* increases the penalty values by a factor of 10 and calls *education*, aiming to restore the solution feasibility. This process is repeated with a penalty increase of 100 if the offspring remains infeasible.

Education and repair are essential for a fast progression toward high-quality solutions. Yet, these procedures tend inevitably to make for the largest part (90-95%) of the overall computational effort, such that high computational efficiency is required. Three basic aspects are decisive for performance: 1) a suitable choice of neighbourhood, restricted to relevant moves while being large enough to allow some structural solution changes; 2) memory structures to evade redundant move computations; and 3) highly efficient neighbour cost and feasibility evaluations. We introduce new methodologies to address these aspects relatively to the specificities of time-constrained VRPs.

4.6.5.1 Neighbourhood choices and restrictions

Following Vidal et al. (2012a), *education* is performed by means of two local search-based procedures. The *route improvement* procedure (RI) is dedicated to optimize services from each period separately, while the *pattern improvement* procedure (PI) relies on a quick and simple move to improve assignment choices. These local searches, called in the sequence RI,PI,RI, provide the means to address efficiently both service sequencing and assignment characteristics.

The PI procedure evaluates for each customer in random order the best combination of re-insertions within periods. Any improving re-insertion is directly performed until no more improvement can be found. For a customer v_i, the number of possible places of insertion is $O(N + m \times t)$, N representing the total number of customer services in all periods, and $m \times t$ representing the total number of routes, to account for insertions after the depot. Once insertion costs at all different places are known (and thus also the best cost insertion for each period), the best visiting period combination is computed for each customer v_i in $O(f_i|L_i|)$.

The RI procedure explores for each period a neighbourhood based on relocations and exchanges of customer visit sequences, with eventual inversions. A broader range of moves than in Vidal et al. (2012a) is exploited to cope with the increased variety of VRPTW solution structures, along with more advanced neighbourhood pruning procedures. The following neighbourhoods are evaluated:

- $\mathcal{N}_1$ (Swap and relocate) : Swap two disjoint visit sequences $(\sigma_i^r, \ldots, \sigma_j^r)$ and $(\sigma_{i'}^{r'}, \ldots, \sigma_{j'}^{r'})$, containing between 0 and 2 visits. Combine this with the reversal of one or both sequences.
- $\mathcal{N}_2$ (2-opt*) : Swap two visit sequences $(\sigma_i^r, \ldots, \sigma_{n_r}^r)$ and $(\sigma_{i'}^{r'}, \ldots, \sigma_{n_{r'}}^{r'})$, involving the extremities of two distinct routes.
- $\mathcal{N}_3$ (2-opt) : Reverse a visit sequence $(\sigma_i^r, \ldots, \sigma_j^r)$.

Neighbourhoods $\mathcal{N}_1$ and $\mathcal{N}_2$ can involve one empty sequence. $\mathcal{N}_1$ involves eventually the same route or different routes. The size of these neighbourhoods is $O(n^2)$. One example of "swap" move from $\mathcal{N}_1$ is illustrated in Figure 4.2. This move exchanges one visit from a route r with two visits from a distinct route r'.

In the context of time-window constraints, moves in RI are explored in two phases: first the moves between "new" routes not existing in the parents, and then, the other moves. Each neighbourhood subset is searched in random order. The best improving move, when existing, is

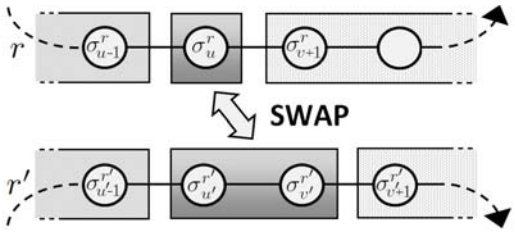


Figure 4.2: Example of "swap" move in $\mathcal{N}_1$

applied as soon as 5% of the neighbourhood has been explored since last move acceptance. This strategy is motivated by the need to perform quick improvements, focus on the new solution elements, and remain close to the characteristics of the original individuals, but aim for a better performance than that usually offered by first-improvement heuristics.

To further increase the computational efficiency of education, neighbourhoods are also pruned by means of *customer correlation* measures. The set of correlated neighbours is usually defined in the context of traveling salesman and classical vehicle routing problems in relation to a spatial proximity measure (Johnson and McGeoch 1997, Toth and Vigo 2003). Time-constrained VRP involve another dimension, however, related to time proximity, as well as additional asymmetry issues. Correlation relationships are thus harder to define (Ibaraki et al. 2005). We define for customer v_i a set $\Gamma(v_i)$ of correlated customers as the $|\Gamma|$ closest customers v_j in relation to the correlation measure $\gamma(v_i, v_j)$ of Equation (4.4). These customers can be viewed as the most relevant options for a direct visit from v_i, and thus arcs (v_i, v_j) for $v_j \in \Gamma(v_i)$ can be seen as a subset of "promising" arcs.

$$\begin{aligned}\gamma(v_i, v_j) = c_{ij} &+ \gamma^{\mathrm{WT}} \max\{e_j - \tau_i - \delta_{ij} - l_i, 0\} \\ &+ \gamma^{\mathrm{TW}} \max\{e_i + \tau_i + \delta_{ij} - l_j, 0\}\end{aligned} \tag{4.4}$$

This correlation measure corresponds to a weighted sum of the distance, the minimum waiting time, and the minimum penalty on a direct service from v_i to v_j. Values for the γ^{TW} and γ^{WT} coefficients, which balance the role of these spatial and temporal components, are discussed in Section 4.7.1. Neighbourhoods $\mathcal{N}_1$ and $\mathcal{N}_2$ are then restricted to sequences $(\sigma_i^r, \ldots, \sigma_j^r)$ and $(\sigma_{i'}^{r'}, \ldots, \sigma_{j'}^{r'})$ such that $\sigma_i^r \in \Gamma(\sigma_{i'-1}^{r'})$ or $\sigma_{i'}^r \in \Gamma(\sigma_{i-1}^{r'})$, while $\mathcal{N}_3$ is restricted to sequences $(\sigma_i^r, \ldots, \sigma_j^r)$ such that $\sigma_j^r \in \Gamma(\sigma_{i-1}^r)$ or $\sigma_{j+1}^r \in \Gamma(\sigma_i^r)$. This restriction ensures that at least one "promising" arc is introduced within each move. The resulting neighbourhood size becomes $O(|\Gamma|n)$.

4.6.5.2 Memories

Memories are used in PI to store for each customer v_i the minimum cost insertion $\psi(i, r, l)$ in each route r and each period l. For RI, the cost of the best move is stored for each pair of customers. These values are valid until the routes under consideration are modified. These techniques lead to notable reductions in the overall computational effort.

4.6.5.3 Move evaluations

When infeasible solutions are used, evaluating moves implies to compute the change in total arc costs, as well as the variation of duration, load, and time-window infeasibility of the routes. Calculation of cost and load variation is straightforward to perform in amortized $O(1)$ for moves based on a constant number of arc exchanges (Kindervater and Savelsbergh 1997). Nagata et al. (2010) also provided the means to compute infeasibility in $O(1)$ for some neighbourhoods, including 2-opt*, inter-route swaps, and inter-route inserts. This method can not address as efficiently intra-route moves or more complex neighborhoods, however, and does not actually manage duration features. We thus introduce new procedures to evaluate combined duration and time-window infeasibility in amortized $O(1)$. The proposed approach is widely applicable to any classical neighbourhood based on a constant number of arc exchanges or sequence relocations.

We first observe that any such move can be viewed as a separation of routes into subsequences, which are then concatenated into new routes. This simple property is formalized in Kindervater and Savelsbergh (1997), Irnich (2008a), and Vidal et al. (2011). In the example of Figure 4.2, the move produces indeed two new routes, $(\sigma_0^r, \ldots, \sigma_{i-1}^r) \oplus (\sigma_{i'}^{r'}, \sigma_{j'}^{r'}) \oplus (\sigma_{j+1}^r, \ldots, \sigma_{n_r}^r)$ and $(\sigma_0^{r'}, \ldots, \sigma_{i'-1}^{r'}) \oplus (\sigma_i^r) \oplus (\sigma_{j'+1}^{r'}, \ldots, \sigma_{n_{r'}}^{r'})$, where $\oplus$ represents the concatenation operator. Our move evaluation approach follows from this observation, and uses induction on the concatenation operation to develop suitable *re-optimization data* on subsequences of consecutive visits in the incumbent solution.

For each such subsequence σ, containing visits to depots or customers, we compute the minimum duration $D(\sigma)$, minimum time-warp use $TW(\sigma)$, earliest $E(\sigma)$ and latest visit $L(\sigma)$ to the first vertex allowing a schedule with minimum duration and minimum time-warp use, as well as the cumulated distance $C(\sigma)$ and load $Q(\sigma)$. This data is straightforward to compute for a sequence σ^0 involving a single vertex v_i, as $D(\sigma^0) = \tau_i$, $TW(\sigma^0) = 0$, $E(\sigma^0) = e_i$, $L(\sigma^0) = l_i$, $C(\sigma^0) = 0$ and $Q(\sigma^0) = q_i$. Proposition 4.6.1 then enables to compute the same data on concatenations of sequences. Equations (4.5 - 4.10) are frequently used in the VRP literature to calculate loads and costs. The other statements, which target the temporal aspects of the problem, are proven in III.2.

Proposition 4.6.1 (Concatenation of two sequences) *Let $\sigma = (\sigma_i, \ldots, \sigma_j)$ and $\sigma' = (\sigma'_{i'}, \ldots, \sigma'_{j'})$ be two subsequences of visits. The concatenated subsequence $\sigma \oplus \sigma'$ is characterized by the following data:*

$$D(\sigma \oplus \sigma') = D(\sigma) + D(\sigma') + \delta_{\sigma_j \sigma'_{i'}} + \Delta_{WT} \tag{4.5}$$

$$TW(\sigma \oplus \sigma') = TW(\sigma) + TW(\sigma') + \Delta_{TW} \tag{4.6}$$

$$E(\sigma \oplus \sigma') = \max\{E(\sigma') - \Delta, E(\sigma)\} - \Delta_{WT} \tag{4.7}$$

$$L(\sigma \oplus \sigma') = \min\{L(\sigma') - \Delta, L(\sigma)\} + \Delta_{TW} \tag{4.8}$$

$$C(\sigma \oplus \sigma') = C(\sigma) + C(\sigma') + c_{\sigma_j \sigma'_{i'}} \tag{4.9}$$

$$Q(\sigma \oplus \sigma') = Q(\sigma) + Q(\sigma') \tag{4.10}$$

where $\Delta = D(\sigma) - TW(\sigma) + \delta_{\sigma_j \sigma'_{i'}}$, $\Delta_{WT} = \max\{E(\sigma') - \Delta - L(\sigma), 0\}$ *and* $\Delta_{TW} = \max\{E(\sigma) + \Delta - L(\sigma'), 0\}$.

The neighbourhood evaluation procedure we propose relies on Proposition 4.6.1 to first develop data on relevant consecutive visit subsequences (and their reversal) in a preprocessing phase, and then to evaluate the penalties and costs of routes issued from the moves. Classical neighbourhoods in the literature correspond to a concatenation of less than five subsequences. Hence, given the data on subsequences, any move evaluation is performed in constant time. As shown in Vidal et al. (2011), this property stands for any move issued from a constant number of arc exchanges or customer visit relocations.

Experiments showed that, for some problem instances with long routes with more than 50 customers, data preprocessing on all $O(n^2)$ subsequences can play a non negligible role in the overall computation effort. The 1-level or 2-level strategy of Irnich (2008a) can be employed to limit this preprocessing to $O(n^{4/3})$ or $O(n^{8/7})$ subsequences, while maintaining the constant time evaluation of moves. To make it even simpler, we limited the preprocessing to "prefix" (and "suffix") subsequences containing the first (the last) customer of a route, and subsequences of size smaller than 20. This data enables to evaluate inter-route moves in constant time, and allows an evaluation of intra-route moves as a concatenation of less than 7 subsequences for the instances considered.

4.6.6 Population management and search guidance

The main components of HGSADC regarding population management remain unchanged from Vidal et al. (2012a). The two subpopulations are set up to contain between μ and $\mu + \lambda$ individuals. To initialize the populations, 4μ individuals are created by randomly choosing the patterns and routes, using *Education* and, when infeasible, *Repair*. These individuals are then included in the appropriate subpopulations, which then evolve through iterative generation, education, and selections of individuals. Any solution produced by the education and repair operators is transformed into an individual by removing visits to depots (Section 4.6.2), and is included in the appropriate subpopulation with respect to its feasibility. It can thus be selected for mating immediately after education. Any subpopulation reaching the size $\mu + \lambda$ undergoes a *survivor-selection* phase, where λ individuals are discarded. Let a "clone" be an individual with the same solution cost, or null distance to another with respect to the metric defined in Section 4.6.3. Survivor selection removes iteratively λ times the worst clone in terms of biased fitness (Equation 4.3), or the worst individual when no clone exists. The use of the biased fitness for survivor selection promotes both elitism and innovation during the search (Vidal et al. 2012a).

The proportion of feasible solutions following education with regards to duration, capacity, and time-window constraints is monitored during the search on the last 100 generated individuals. Penalty coefficients $\omega^{\mathrm{D}}, \omega^{\mathrm{Q}}$, and ω^{TW} (Section 4.6.1) are also adjusted each 100 iterations. Let parameter ξ^{REF} stand for the target proportion of feasible individuals. If the proportion of feasible individuals relatively to one type of constraint (duration, load or time-window) falls below $\xi^{REF} - 5\%$ (or, rises to more than $\xi^{REF} + 5\%$), then the corresponding penalty is increased (or, decreased).

A *diversification* phase finally occurs whenever $It_{div} = 0.4It_{NI}$ iterations are performed without improving the best solution. Diversification retains the best $\mu/3$ individuals of each subpopulation, to be completed with 4μ new individuals as in the Initialization phase, thus introducing new genetic material.

All these components contribute towards a more thorough search, and complement advantageously the aggressive local improvement abilities of education and repair operators.

4.6.7 Decomposition phases

Variants of vehicle routing problems with time windows lend themselves well to various decomposition approaches, mostly based on geometry (Taillard 1993, Reimann et al. 2004, Ostertag 2008, Bent and Van Hentenryck 2010), temporal aspects (Bent and Van Hentenryck 2010), or problem structure (Crainic et al. 2009), which enable to address large instances more efficiently.

We introduce a simple decomposition framework for population-based methods, which takes full advantage of their associated pool of solutions. The approach proceeds in four steps: 1) features from one elite solution are exploited to define subproblems; 2) initial individuals for the subproblems are created from the genetic material of the complete problem population; 3) the algorithm, here HGSADC, is called to address these subproblems; 4) a set of complete solutions is finally reconstructed.

We rely on simple route-based geometrical decompositions in the VRPTW case. For the PVRPTW, fixing assignments to periods naturally decomposes the problem. It is worth noticing that the "integration" of partial solutions into solutions of the complete problem is here straightforward, as it simply involves gathering the routes. Also, any improvement in any subproblem leads to an improvement of the elite complete solution, as subproblems are built from the features of this solution.

In the current implementation, decomposition phases occur every It_{dec} iterations, and are only used on problems with more than 120 customers. The elite solution is randomly selected from the 25% best feasible individuals (or infeasible if no feasible solution has been found). In the VRPTW case, routes from this elite solution are swept circularly around the depot by polar angle of barycentre, and included in a set $\mathcal{R}_{dec}$. Each time the number of customers in routes from $\mathcal{R}_{dec}$ becomes larger than 120, or when all the routes have been swept, a VRPTW subproblem is created with the customers from $\mathcal{R}_{dec}$ and the set is emptied. Subproblems of this size can be very efficiently handled by HGSADC. In the case of the PVRPTW, we fix the periods of service, leading to a subproblem for each period.

Initial populations of subproblem solutions are created from the complete solutions population, by retaining from the route chromosomes of complete individuals only the services that occur in the subproblem at the right period. For the PVRTPW, visits from solutions with patterns different from those of the elite solution can be missing. These visits are completed by means of a least cost insertion heuristic. HGSADC is then run on each of these subproblems until $It_{dec}/2$ iterations without improvement are performed. Combining the best solution of each subproblem yields an elite solution, to be added to the population of the complete problem. The same process is repeated with the second and the third best solutions to produce two additional elite individuals.

4.7 Computational Experiments

Extensive computational experiments were performed to analyze the impact of the parameter settings (Section 4.7.1), assess HGSADC performance when compared to state-of-the-art methods for each problem (Section 4.7.2), and evaluate the role of the new decomposition phases (Section 4.7.3). The algorithm is coded in C++, compiled with "g++ -O3", and run on a Intel Xeon 2.93 Ghz processor.

We rely on a variety of classical benchmark instance sets: Solomon and Desrosiers (1988) and Gehring and Homberger (1999) VRPTW instances, Cordeau et al. (2001a) and Cordeau and Laporte (2001) instances for PVRPTW, MDVRPTW, SDVRPTW with duration constraints and the PVRPTW instances of Pirkwieser and Raidl (2009b) without duration constraints. These instances involve from 48 to 1008 customers, up to 9 depots, 10 periods, and 6 vehicle types. It should be noted that the distances of Pirkwieser and Raidl (2009b) instances have been truncated to the first digit by previous authors. We used this convention exclusively in this case to perform a fair comparison. We also introduce new larger-dimension instances for PVRPTW, MDVRPTW, and SDVRPTW, involving 360 to 960 customers, and up to 4608 total services, following the generation procedure of Cordeau et al. (1997, 2001a). These instances are available upon request.

Finally, the traditional objective for VRPTW is fleet minimization in priority, and then route length minimization. To apply HGSADC with this objective, we first constrain the fleet size to a large value of 100, and iteratively interrupt the run and reduce the fleet size whenever a feasible solution is found. As soon as HGSADC fails to find a feasible solution, we return to the last feasible fleet value and perform a final run. It is noteworthy that both objectives are here addressed in single-stage-single-algorithm mode contrasting with current state-of-the-art VRPTW methods, which generally rely on two distinct procedures and concepts.

4.7.1 Parameter calibration

The parameter values from Vidal et al. (2012a) were shown to perform well on a large range of VRP variants. In order to study the applicability and generality of the HGSADC framework with limited changes, we voluntarily limited the role of parameter tuning, to focus exclusively on the new ones, related to the neighbourhood evaluation procedures (γ^{WT}, γ^{TW}), and the decomposition phases. The remaining parameter values are imported from Vidal et al. (2012a), where an extensive meta-calibration approach had been used to produce good parameter values on closely related PVRP and MDVRP problems. Thus, $N_{elite} = 8$ and $|\mathcal{N}_{close}| = 3$; $P_{rep} = 0.5$ and $|\Gamma| = 40$; $(\mu, \lambda) = (25, 40)$ and $\xi^{REF} = 0.2$ (Section 4.6.6). Finally, the termination criteria $It_{NI} = 5000$ is used to compare with other authors in similar run times, while $It_{dec} = 2000$ to balance the computation time dedicated to decomposition phases and the regular run.

The nature of VRPTW solutions tends to strongly vary in relation to the distribution and tightness of time windows. For some problems, high-quality solutions involve some long arcs and relatively small waiting times, while for less tightly constrained problems, closer to the classical VRP, long arcs become very unlikely. The parameters γ^{TW} and γ^{WT}, balancing the role of geometrical and temporal aspects during neighbourhood pruning, are thus critical, and need to

be calibrated relatively to the instances at hand. To that extent, we selected 10 problems with various structures (R1-2, R1-4, R2-1, R2-3, RC1-1, RC1-2, RC2-3, RC2-4, C1-2, and C2-4) from the 200-customer instances of Gehring and Homberger (1999). 20 runs were performed for each instance and each of the 36 combinations of parameters $(\gamma^{TW}, \gamma^{WT}) \in \{0; 0.2; 0.5; 1; 2; 5\}^2$. To reduce computation time and amplify the impact of good move choices, we also reduced $|\Gamma|$ to 20, $It_{NI} = 2000$, and turned off the decomposition phases. The minimum number of vehicles has been reached on all runs but five out of $20 \times 36 = 720$ total experiments. These five marginal results have been discarded in order to simply compare on the basis of distance. Table 4.1 presents the gap of HGSADC to the best known solution (BKS) in the literature for each parameter setting, averaged on the 20 (19 in some cases) experiments and 10 instances. The line and column indicated in boldface corresponds to the best mean for each parameter taken independently.

Table 4.1: Performance of HGSADC on a selection of 200-customer VRPTW instances for various γ^{TW} and γ^{WT} settings

γ^{TW} \ γ^{WT}	0.0	0.2	0.5	1.0	2.0	5.0
0.0	0.70	**0.64**	0.69	0.72	0.71	0.82
0.2	0.59	**0.59**	0.65	0.57	0.71	0.72
0.5	0.63	**0.53**	0.56	0.63	0.65	0.67
1.0	**0.63**	**0.59**	**0.51**	**0.59**	**0.59**	**0.63**
2.0	0.60	**0.51**	0.61	0.63	0.62	0.61
5.0	0.56	**0.56**	0.57	0.56	0.70	0.61

HGSADC performance appears to increase with parameter values close to $(\gamma^{TW}, \gamma^{WT}) = (1.0, 0.2)$. To state the statistical significance of these observations, we analyzed the distribution of the average deviation to BKS among the 20×36 experiments. The Shapiro-Wilk test rejects the distribution normality assumption with high confidence ($p = 0.000$). Noteworthy is the fact that Levene's test also rejects ($p = 0.011$) the equality of variances among the 36 groups of experiments, hence these parameters affect both the algorithm performance and stability.

Following these observations, a classic ANOVA is not relevant. We thus compared the settings $(\gamma^{TW}, \gamma^{WT}) = (0, 0)$ and $(\gamma^{TW}, \gamma^{WT}) = (1.0, 0.2)$ on 50 new runs, using new random number generator seeds. Average deviations of +0.68% and +0.53% were retrieved. A Wilcoxon test on these paired samples yields $p = 0.000$, thus rejecting with very high confidence the null hypothesis that "both parameter settings lead to the same solution quality". The new setting $(1.0, 0.2)$, which accounts for the temporal aspect in neighbourhood pruning, leads to a significant increase in quality when compared to $(0, 0)$, which corresponds to the classic distance-driven granular search policy.

4.7.2 Comparison of performances

We compare HGSADC with state-of-the art methods for the PVRPTW in Tables 4.2 and 4.3, for the MDVRPTW in Table 4.4, for the SDVRPTW in Table 4.5, and for the VRPTW in Tables 4.6-4.7. We also report the performance of HGSADC on the new large scale instances in Table 4.8. The first group of columns displays the instance identifier, number of customers n, maximum fleet

size m, number of periods t, depots d, and vehicle types w when applicable. The next group of columns compares the average and best results, as well as the average run time of HGSADC with state-of-the-art methods for each problem:

- PisR: ALNS of Pisinger and Ropke (2007)
- LZ: Two-phase ejections chains and iterated local search of Lim and Zhang (2007)
- PDR: Branch-and-price based LNS of Prescott-Gagnon et al. (2009)
- RTI: Arc-guided evolutionary algorithm of Repoussis et al. (2009b)
- NB-100 and NB-f(n): Hybrid GA based on EAX crossover of Nagata et al. (2010) with a population size of 100 and $f(n) = n/20000$.
- CLM: UTS of Cordeau et al. (2004)
- PR08: VNS of Pirkwieser and Raidl (2008)
- CM-8P and CM-64: Parallel iterative UTS of Cordeau and Maischberger (2012) on 8 and 64 processors
- NCT: GA+VNS+Tabu of Nguyen et al. (2011)
- PR09: Multiple VNS+ILP (15,10) of Pirkwieser and Raidl (2009b)
- PR10: VNS+ILP of Pirkwieser and Raidl (2010b)
- PBDH: Cooperative VNS of Polacek et al. (2008)
- B: Hybrid VNS and Tabu of Belhaiza (2010)

Table 4.2: Results on Cordeau et al. (2001a) PVRPTW instances

Inst	**n**	**m**	**t**	**CLM**	**PR08**	**CM-8P**	**CM-64P**		**HGSADC**		**prev BKS**	**HGSADC**
				1 run	Best X	Avg 10	Avg 5	Avg 10	Best 10	T(min)	—	All exp.
p01a	48	3	4	2915.58	**2909.02**	**2909.02**	**2909.02**	2909.05	**2909.02**	1.13	2909.02	2909.02
p02a	96	6	4	5094.39	5036.27	5046.78	5037.60	5031.50	**5026.57**	3.28	5026.57	5026.57
p03a	144	9	4	7284.32	7138.70	7134.11	7097.55	7091.51	**7050.72**	8.11	7062.00	7023.90
p04a	192	12	4	8087.06	7882.06	7923.48	7857.08	7818.75	**7791.93**	17.93	7807.32	7755.77
p05a	240	15	4	8752.72	8492.45	8518.20	8434.02	8368.98	**8341.93**	31.03	8358.96	8311.17
p06a	288	18	4	10961.78	10713.75	10756.53	10664.56	10595.85	**10477.01**	65.36	10542.10	10473.24
p07a	72	5	6	6891.76	6787.72	6799.12	6799.73	6788.67	**6783.23**	3.76	6782.68	6783.23
p08a	144	10	6	9990.46	9721.25	9729.13	9684.86	9623.72	**9593.43**	17.00	9603.92	9574.80
p09a	216	15	6	13796.75	13463.96	13459.28	13371.16	13285.89	**13247.38**	45.94	13299.80	13201.06
p10a	288	20	6	18135.6	17650.89	17503.69	17365.50	17058.89	**16999.88**	95.96	17261.30	16920.96
p01b	48	3	4	2297.21	**2277.44**	2278.41	**2277.44**	**2277.44**	**2277.44**	0.83	2277.44	2277.44
p02b	96	6	4	4335.11	4137.45	4200.75	4139.10	4130.64	**4122.03**	4.88	4124.76	4121.50
p03b	144	9	4	5699.78	5575.27	5601.34	5571.88	5555.77	**5521.71**	8.44	5489.84	5489.33
p04b	192	12	4	6619.56	6476.67	6482.60	6433.16	6400.55	**6352.28**	27.80	6383.28	6347.77
p05b	240	15	4	7138.28	6970.33	6902.39	6846.96	6838.54	**6790.44**	47.47	6800.45	6777.54
p06b	288	18	4	9039.29	8819.32	8760.22	8695.84	8647.15	**8595.10**	77.48	8659.44	8582.72
p07b	72	5	6	5580.22	5504.67	5514.35	5494.67	5491.08	**5481.61**	3.64	5481.61	5481.61
p08b	144	10	6	7914.39	7729.32	7772.38	7726.38	7665.10	**7619.95**	16.75	7656.13	7599.01
p09b	216	15	6	11269.13	10885.93	10871.81	10750.42	10653.60	**10589.68**	68.10	10579.50	10532.51
p10b	288	20	6	14145.37	13943.61	13778.33	13576.78	13502.65	**13442.57**	109.98	13490.80	13406.89
Avg Gap to BKS				+3.54%	+1.28%	+1.31%	+0.64%	+0.17%	-0.24%	—		
Avg Time (min)				16.0	NC	8 × 7.6	64 × 11.32	32.74	10 × 32.74	—		
Processor				P4-2G	Opt-2.2G	Xe-2.93G	Xe-2.93G		Xe-2.93G			

Table 4.3: Results on Pirkwieser and Raidl (2009b) PVRPTW instances without duration constraints, distances truncated to the first digit

Inst	**n**	**t**	**PR09**	**PR10**	**NCT**	**HGSADC**		
			Avg 30	Avg 30	Avg 10	Avg 10	Best 10	T(min)
R4	100	4	3454.50	3467.08	3441.86	3441.34	**3434.18**	3.03
C4	100	4	2787.14	2828.83	2778.19	2768.76	**2766.22**	2.68
RC4	100	4	3641.61	3659.66	3628.41	3630.81	**3620.70**	3.92
R6	100	6	4475.85	4474.04	4445.81	4443.48	**4428.88**	4.52
C6	100	6	3777.97	3807.68	3742.74	3728.94	**3723.20**	3.96
RC6	100	6	5030.33	5021.79	4967.34	4971.14	**4952.94**	4.85
R8	100	8	—	5526.47	5443.08	5456.67	**5428.80**	5.08
C8	100	8	—	4971.18	4860.52	4827.88	**4809.46**	4.23
RC8	100	8	—	5994.37	5902.67	5876.73	**5840.84**	5.88
Avg Gap to BKS			NC	+1.75%	+0.38%	+0.19%	-0.09%	
Avg Time (min)			0.86	0.61	97.51	4.24	10 × 4.24	
Processor			Qd-2.83G	Qd-2.83G	C2-2.4G		Xe-2.93G	

Table 4.4: Results on Cordeau et al. (2001a) MDVRPTW instances

Inst	**n**	**m**	**d**	**CLM**	**PBDH**	**CM-8P**	**CM-64P**	**HGSADC**			**prev BKS**	**HGSADC**
				1 run	Best X	Avg 10	Avg 5	Avg 10	Best 10	T(min)	—	All exp.
p01a	48	2	4	**1074.12**	**1074.12**	**1074.12**	**1074.12**	**1074.12**	**1074.12**	0.31	1074.12	1074.12
p02a	96	3	4	1766.94	1763.66	1762.80	**1762.21**	1762.61	**1762.21**	1.15	1762.21	1762.61
p03a	144	4	4	2420.89	2388.73	2394.77	2380.24	2374.27	**2373.65**	1.75	2373.65	2373.65
p04a	192	5	4	2868.64	2847.56	2841.51	2822.80	2817.39	**2815.75**	5.89	2815.48	2815.11
p05a	240	6	4	3059.40	3015.27	3007.80	2987.01	2968.71	**2964.65**	8.68	2965.18	2962.25
p06a	288	7	4	3701.08	3674.60	3638.40	3616.69	3598.77	**3588.78**	13.43	3590.58	3588.78
p07a	72	2	6	1425.87	**1418.22**	**1418.22**	**1418.22**	**1418.22**	**1418.22**	0.51	1418.22	1418.22
p08a	144	3	6	2118.50	2103.21	2111.16	2101.50	2097.35	**2096.73**	2.39	2096.73	2096.73
p09a	216	4	6	2777.91	2753.61	2739.40	2723.81	2716.15	**2712.56**	5.20	2717.69	2712.56
p10a	288	5	6	3546.24	3541.01	3505.30	3481.58	3477.56	**3465.92**	15.22	3469.29	3464.65
p01b	48	2	4	1025.14	1011.65	**1005.73**	**1005.73**	**1005.73**	**1005.73**	0.51	1005.73	1005.73
p02b	96	3	4	1486.26	1488.32	1473.65	1468.30	1466.49	**1464.50**	1.68	1464.50	1464.50
p03b	144	4	4	2033.75	2012.37	2004.69	2001.83	2001.82	**2001.81**	2.94	2001.81	2001.81
p04b	192	5	4	2228.64	2239.02	2212.38	2199.70	2197.41	**2195.33**	6.55	2195.33	2195.33
p05b	240	6	4	2555.95	2498.85	2476.87	2443.94	2454.28	**2433.15**	12.56	2434.94	2433.15
p06b	288	7	4	2978.60	2909.45	2875.70	2862.38	2844.06	**2836.67**	15.97	2852.25	2836.67
p07b	72	2	6	1250.18	1247.51	1238.51	1237.00	1237.78	**1236.24**	1.05	1236.24	1236.24
p08b	144	3	6	1870.34	1809.25	1793.90	1790.44	1789.76	**1788.18**	3.30	1788.18	1788.18
p09b	216	4	6	2338.74	2294.19	2283.35	2276.32	2264.56	**2261.08**	8.59	2263.74	2257.13
p10b	288	5	6	3147.79	3093.51	3060.46	3016.73	3006.18	**2993.31**	22.18	2995.08	2984.01
Avg Gap to BKS				+2.32%	+1.28%	+0.74%	+0.27%	+0.10%	-0.06%	—		
Avg Time (min)				14.9	146.94	8 × 4.15	64 × 6.57	6.49	10 × 6.49	—		
Processor				P4-2G	P4-3.6G	Xe-2.93G	Xe-2.93G		Xe-2.93G			

We indicate in boldface for each problem instance the best performing method. In the last two columns are given for each instance the previous best-known solution (BKS) ever reported in the literature, and the best solution obtained by HGSADC during all our experiments. New BKS produced by HGSADC are underlined. Finally, the last three lines provide average measures over all instances: the percentage of error relative to the previous BKS, the computation time, and the type of processor used by each author. For concision matters, we report only average results by group of instances for the PVRPTW instances of Pirkwieser and Raidl (2009b), the VRPTW

Table 4.5: Results on Cordeau and Laporte (2001) SDVRPTW instances.

Inst	**n**	**m**	**w**	**CLM**	**B**	**CM-8P**	**CM-64P**	**HGSADC**			**prev BKS**	**HGSADC**
				1 run	Best X	Avg 10	Avg 5	Avg 10	Best 10	T(min)	—	All exp.
p01a	48	2	4	1666.47	**1655.42**	**1655.42**	**1655.42**	**1655.42**	**1655.42**	0.23	1655.42	1655.42
p02a	96	3	4	2915.55	2938.37	**2904.13**	**2904.13**	**2904.13**	**2904.13**	0.70	2904.13	2904.13
p03a	144	4	4	3406.21	3338.58	3322.38	3317.25	3320.44	**3304.13**	1.60	3304.13	3304.13
p04a	192	5	4	4532.58	4560.96	4486.13	4448.48	4437.19	**4427.25**	5.85	4438.97	4427.25
p05a	240	6	4	5859.03	5908.41	5722.89	5666.23	5681.48	**5647.76**	11.64	5620.56	5626.42
p06a	288	7	4	5773.24	5966.33	5767.97	5710.96	5666.20	**5637.48**	12.68	5670.66	5627.82
p07a	72	2	6	2181.77	2169.06	**2168.88**	**2166.88**	**2166.88**	**2166.88**	0.42	2166.88	2166.88
p08a	144	3	6	3983.08	3944.33	3908.93	3885.31	3880.58	**3873.40**	2.35	3874.32	3873.40
p09a	216	4	6	5008.54	4985.19	4854.70	4838.82	4797.72	**4777.61**	5.60	4801.47	4772.55
p10a	288	5	6	6171.16	6118.20	5935.56	5881.95	5876.38	**5858.82**	11.58	5868.03	5817.28
p11a	1008	27	4	—	16784.67	—	—	15198.10	**15080.68**	120.46	16418.21	14982.35
p12a	720	14	6	—	12485.43	—	—	11475.15	**11402.01**	112.06	12106.20	11330.23
p01b	48	2	4	1433.24	1429.37	**1429.35**	**1429.35**	**1429.35**	**1429.35**	0.22	1429.35	1429.35
p02b	96	3	4	2516.83	2494.34	2489.83	2482.48	**2479.56**	**2479.56**	0.99	2479.56	2479.56
p03b	144	4	4	2814.61	2801.51	2785.84	2780.40	2779.09	**2775.61**	2.28	2775.61	2774.30
p04b	192	5	4	3762.38	3746.99	3695.27	3673.01	3660.66	**3649.72**	6.57	3655.48	3649.72
p05b	240	6	4	4955.04	4730.63	4678.58	4654.10	4625.79	**4611.16**	8.06	4613.09	4609.20
p06b	288	7	4	5008.27	5019.64	4823.92	4777.44	4755.59	**4729.96**	15.29	4752.04	4716.36
p07b	72	2	6	1864.11	**1837.94**	1839.08	**1837.94**	**1837.94**	**1837.94**	0.51	1837.94	1837.94
p08b	144	3	6	3215.06	3163.99	3161.03	3149.49	3152.69	**3149.77**	2.15	3144.91	3144.91
p09b	216	4	6	4033.63	4033.21	3959.74	3940.15	3894.67	**3883.94**	8.90	3894.64	3883.94
p10b	288	5	6	5158.89	5114.38	5014.48	5009.00	4962.62	**4932.40**	12.03	4967.59	4927.95
p11b	1008	27	4	—	14655.00	—	—	13226.60	**13067.52**	120.32	14015.50	12998.63
p12b	720	14	6	—	10864.70	—	—	9857.89	**9777.44**	120.17	10267.50	9708.45
Gap p01-10				+2.76%	+2.23%	+0.82%	+0.39%	+0.12%	-0.13%			
Gap p11-12				—	+3.94%	—	—	-5.57%	-6.38%			
T. (min) p01-10				13.3	2.94	8 × 4.53	64 × 5.60	5.48	10 × 5.48			
T. (min) p11-12				—	45.73	—	—	118.25	10 × 118.25			
Processor				P4-2G	Qd-2.66G	Xe-2.93G	Xe-2.93G		Xe-2.93G			

A limit of 2 hours has been set for HGSADC runs. The average gaps and time are reported separately for instances sets p01-10 and p11-12, which are of very different sizes.

Table 4.6: Results on Solomon and Desrosiers (1988) VRPTW instances

Inst	**n**	**PisR**	**PDR**	**RTI**	**NB-100**	**NB-f(n)**	**HGSADC**	
		Best 10	Best 5	Best 3	1 run	Best 5	Avg 5	Best 5
R1	100	11.92 | 1212.39	**11.92** | **1210.34**	11.92 | 1210.82	**11.92** | **1210.34**	**11.92** | **1210.34**	11.92 | 1211.49	11.92 | 1210.69
R2	100	2.73 | 957.72	2.73 | 955.74	2.73 | 952.67	2.73 | 952.08	**2.73** | **951.03**	2.73 | 952.05	2.73 | 951.51
C1	100	**10.0** | **828.38**	**10.0** | **828.38**	**10.0** | **828.38**	**10.0** | **828.38**	**10.0** | **828.38**	**10.0** | **828.38**	**10.0** | **828.38**
C2	100	**3.0** | **589.86**	**3.0** | **589.86**	**3.0** | **589.86**	**3.0** | **589.86**	**3.0** | **589.86**	**3.0** | **589.86**	**3.0** | **589.86**
RC1	100	11.5 | 1385.78	**11.5** | **1384.16**	11.50 | 1384.30	11.50 | 1384.72	**11.5** | **1384.16**	11.5 | 1384.81	11.5 | 1384.17
RC2	100	3.25 | 1123.49	3.25 | 1119.44	3.25 | 1119.72	3.25 | 1119.45	**3.25** | **1119.24**	3.25 | 1119.40	**3.25** | **1119.24**
CNV		405	405	405	405	**405**	405	405
CTD		57 332	57 240	57 216	57 205	**57 187**	57 218	57 196
T(min)		10 × 2.5 min	5 × 30 min	3 × 17.9 min	3.2 min	5 × 5.0 min	2.68 min	5 × 2.68 min
Processor		P4-3G	Opt-2.3G	P4-3G	Opt-2.4G	Opt-2.4G	Xe-2.93G	

instances of Solomon and Desrosiers (1988) and Gehring and Homberger (1999). Calculation of the average deviation to BKS is also based on groups. Detailed results are available upon request. Finally, notice that results are presented in the format "Fleet Size|Distance" for VRPTW benchmarks.

Table 4.7: Results on Gehring and Homberger (1999) large-scale VRPTW instances

Inst	n	LZ	PDR	RTI	NB-100	NB-f(n)	HGSADC	
		1 run	Best 5	Best 3	1 run	Best 5	Avg 5	Best 5
R1	200	18.2 | 3639.60	18.2 | 3615.69	18.2 | 3640.11	18.2 | 3615.15	**18.2 | 3612.36**	18.20 | 3621.72	18.2 | 3613.16
R2	200	4.0 | 2950.09	4.0 | 2937.67	4.0 | 2941.99	4.0 | 2930.04	**4.0 | 2929.41**	4.00 | 2933.19	**4.0 | 2929.41**
C1	200	18.9 | 2726.11	18.9 | 2718.77	18.9 | 2721.90	18.9 | 2718.44	**18.9 | 2718.41**	18.90 | 2720.13	**18.9 | 2718.41**
C2	200	6.0 | 1834.24	6.0 | 1831.59	6.0 | 1833.36	6.0 | 1831.64	6.0 | 1831.64	6.00 | 1832.08	**6.0 | 1831.59**
RC1	200	18.0 | 3205.51	18.0 | 3192.56	18.0 | 3224.63	18.0 | 3182.48	**18.0 | 3178.68**	18.00 | 3195.26	18.0 | 3180.48
RC2	200	4.3 | 2574.10	4.3 | 2559.32	4.3 | 2554.33	4.3 | 2536.54	4.3 | 2536.22	4.30 | 2538.29	**4.3 | 2536.20**
CNV		694	694	694	694	**694**	694	694
CTD		169 296	168 556	169 163	168 143	**168 067**	168 407	168 092
Time		93.2 min	5 × 53 min	90 min	4.7 min	5 × 4.1 min	8.40 min	5 × 8.40 min
R1	400	36.4 | 8489.53	36.4 | 8420.52	36.4 | 8514.11	36.4 | 8413.23	36.4 | 8403.24	36.40 | 8423.07	**36.4 | 8402.57**
R2	400	8.0 | 6271.57	8.0 | 6213.48	8.0 | 6258.82	8.0 | 6149.49	**8.0 | 6148.57**	8.00 | 6168.98	8.0 | 6152.92
C1	400	37.6 | 7229.04	37.6 | 7182.75	37.6 | 7273.90	37.6 | 7179.71	37.6 | 7175.72	37.60 | 7184.65	**37.6 | 7170.47**
C2	400	11.7 | 3942.93	11.9 | 3874.58	11.7 | 3941.70	11.7 | 3898.02	11.7 | 3899.00	11.68 | 3916.83	**11.6 | 3952.95**
RC1	400	36.0 | 8005.25	36.0 | 7940.65	36.0 | 8088.46	36.0 | 7931.66	36.0 | 7922.23	36.00 | 7942.81	**36.0 | 7907.14**
RC2	400	8.5 | 5431.15	8.6 | 5269.09	8.4 | 5516.59	**8.4 | 5293.74**	8.4 | 5297.86	8.52 | 5233.33	8.5 | 5215.21
CNV		1382	1385	1381	1381	1381	1382	**1381**
CTD		393 695	389 011	395 936	388 548	388 466	388 697	**388 013**
Time		295.9 min	5 × 89 min	180 min	34.0 min	5 × 16.2 min	34.1 min	5 × 34.1 min
R1	600	54.5 | 18381.28	54.5 | 18252.13	54.5 | 18781.79	54.5 | 18194.38	54.5 | 18186.24	54.50 | 18111.58	**54.5 | 18023.18**
R2	600	11.0 | 12847.31	11.0 | 12808.59	11.0 | 12804.60	**11.0 | 12319.75**	11.0 | 12330.49	11.00 | 12385.20	11.0 | 12352.38
C1	600	57.4 | 14103.61	57.4 | 14106.03	**57.3 | 14236.86**	57.4 | 14054.70	57.4 | 14067.34	57.40 | 14078.12	57.4 | 14058.46
C2	600	17.4 | 7725.86	17.5 | 7632.37	17.4 | 7729.80	17.4 | 7601.94	17.4 | 7605.07	17.40 | 7635.68	**17.4 | 7594.41**
RC1	600	55.0 | 16274.17	55.0 | 16266.14	55.0 | 16767.72	55.0 | 16179.39	55.0 | 16183.95	55.00 | 16156.47	**55.0 | 16097.05**
RC2	600	11.5 | 10935.91	11.7 | 10990.85	11.4 | 11311.81	11.4 | 10591.87	**11.4 | 10586.14**	11.50 | 10568.26	11.5 | 10511.86
CNV		2068	2071	**2066**	2067	2067	2068	2068
CTD		802 681	800 797	**816 326**	789 420	789 592	789 353	786 373
Time		646.9 min	5 × 105 min	270 min	80.4 min	5 × 25.3 min	99.4 min	5 × 99.4 min
R1	800	72.8 | 31755.57	72.8 | 31797.42	72.8 | 32734.57	72.8 | 31486.74	72.8 | 31492.81	72.80 | 31385.55	**72.8 | 31311.38**
R2	800	15.0 | 20601.22	15.0 | 20651.81	15.0 | 20618.21	**15.0 | 19873.04**	15.0 | 19914.97	15.00 | 19995.82	15.0 | 19933.39
C1	800	75.4 | 25026.42	75.4 | 25093.38	75.2 | 25911.44	75.3 | 24990.42	**75.2 | 25151.83**	75.50 | 24898.96	75.4 | 24876.38
C2	800	23.4 | 11598.81	23.5 | 11569.39	23.4 | 11835.72	23.4 | 11438.52	23.4 | 11447.27	23.38 | 11474.16	**23.3 | 11475.05**
RC1	800	72.0 | 31267.84	72.0 | 33170.01	72.0 | 33795.61	72.0 | 31020.22	72.0 | 31278.28	72.0 | 29655.52	**72.0 | 29404.32**
RC2	800	15.6 | 16992.79	15.8 | 16852.38	15.5 | 17536.54	**15.4 | 16438.90**	15.4 | 16484.31	15.50 | 16513.49	15.4 | 16495.82
CNV		2742	2745	2739	2739	**2738**	2741.8	2739
CTD		1 372 427	1 391 344	1 424 321	1 352 478	**1 357 695**	1 339 235	1 334 963
Time		1269.4 min	5 × 129 min	360 min	126.8 min	5 × 27.6 min	215 min	5 × 215 min
R1	1000	91.9 | 48827.23	91.9 | 49702.32	91.9 | 51414.26	91.9 | 48287.98	91.9 | 48369.71	91.90 | 47928.13	**91.9 | 47759.66**
R2	1000	19.0 | 30164.60	19.0 | 30495.26	19.0 | 30804.79	**19.0 | 28913.40**	19.0 | 29003.42	19.00 | 29159.07	19.0 | 29076.45
C1	1000	94.4 | 41699.32	94.3 | 41783.27	94.2 | 43111.60	94.1 | 41683.29	94.1 | 41748.60	94.42 | 41550.55	**94.1 | 41572.86**
C2	1000	29.3 | 16589.74	29.5 | 16657.06	29.3 | 16810.22	29.1 | 16498.61	29.1 | 16534.36	28.90 | 16723.59	**28.8 | 16796.45**
RC1	1000	90.0 | 44818.54	90.0 | 45574.11	90.0 | 46753.61	90.0 | 44743.18	90.0 | 44860.60	90.00 | 44448.97	**90.0 | 44333.40**
RC2	1000	18.3 | 25064.88	18.5 | 25470.33	18.4 | 25588.52	18.3 | 23939.62	18.3 | 24055.31	18.24 | 24209.03	**18.2 | 24131.13**
CNV		3429	3432	3428	3424	3424	3424.6	**3420**
CTD		2 071 643	2 096 823	2 144 830	2 040 661	2 045 720	2 040 193	**2 036 700**
Time		1865.4 min	5 × 162 min	450 min	186.4 min	5 × 35.3 min	349 min	5 × 349 min
Processor		P4-3G	P4-2.8G	Opt-2.3G	P4-3G	Opt-2.4G	Xe-2.93G	

HGSADC appears to be highly competitive in terms of solution quality for all the problem settings considered. The average computation time remains short, similar to other methods, and suitable for many operational decisions. The proposed approach outperforms all other algorithms

Table 4.8: Results on new large-scale PVRPTW, MDVRPTW, and SDVRPTW instances.

Inst	n	m			t	**PVRPTW**			**MDVRPTW**			**SDVRPTW**		
		P	MD	SD		Avg 5	T(min)	Best 5	Avg 5	T(min)	Best 5	Avg 5	T(min)	Best 5
pr11a	360	24	10	11	4	21120,94	61,74	20937,29	6772,00	16,81	6720,71	9958,05	13,91	9924,11
pr12a	480	30	13	14	4	26677,56	192,16	26483,68	8259,57	30,00	8179,80	12371,95	30,44	12251,66
pr13a	600	38	16	17	4	31909,12	297,03	31808,00	9751,22	54,85	9667,20	14562,53	44,93	14491,25
pr14a	720	44	19	21	4	37066,65	302,25	36954,39	11235,13	65,65	11124,01	16620,70	70,12	16547,86
pr15a	840	50	22	25	4	41847,30	301,05	41699,07	13078,26	132,44	13013,97	19283,90	111,09	19090,19
pr16a	960	58	26	29	4	48855,14	307,29	48375,16	14415,89	133,63	14299,87	21803,57	176,25	21413,65
pr17a	360	22	7	8	6	28889,82	65,28	28818,04	6340,66	17,23	6304,30	10581,02	12,20	10547,07
pr18a	520	30	10	12	6	37491,40	263,63	37385,82	8381,71	44,25	8308,32	14009,53	21,56	13963,49
pr19a	700	38	13	16	6	49103,78	300,39	48993,72	10734,60	74,42	10677,61	18998,48	95,29	18855,51
pr20a	880	48	16	20	6	60474,34	302,59	60144,66	12142,60	107,37	11963,91	22655,72	150,56	22513,44
pr21a	420	22	4	6	12	54562,68	213,11	54257,26	6321,20	28,00	6260,53	13775,90	16,22	13758,32
pr22a	600	30	6	8	12	73226,99	297,44	72978,33	8047,87	76,05	7985,37	17661,05	45,00	17572,09
pr23a	780	38	8	10	12	91424,98	300,02	90951,34	9984,75	137,72	9937,43	21974,90	97,06	21793,32
pr24a	960	48	10	12	12	114892,01	308,38	114712,30	11971,74	197,17	11923,72	26875,82	148,26	26775,76
pr11b	360	18	8	9	4	16102,27	86,18	15992,20	4852,67	18,04	4839,44	8011,50	15,89	7962,22
pr12b	480	24	11	11	4	20822,71	177,36	20753,17	6084,33	29,09	6063,26	9566,13	33,45	9508,68
pr13b	600	30	14	14	4	25050,30	291,83	24972,94	7282,25	70,99	7254,17	11609,39	74,81	11562,67
pr14b	720	36	17	17	4	29976,52	301,40	29790,14	8796,77	98,92	8732,29	13693,79	157,09	13623,28
pr15b	840	48	20	20	4	41715,58	300,02	41609,04	10496,39	129,48	10439,72	15589,83	191,21	15437,52
pr16b	960	56	23	23	4	49558,36	306,31	49470,50	11565,39	170,31	11483,22	17920,79	252,35	17834,61
pr17b	360	18	6	6	6	23138,63	94,09	22989,05	4847,58	15,78	4806,01	8629,90	13,20	8562,99
pr18b	520	24	9	9	6	32201,55	274,33	32093,04	6555,95	39,45	6526,72	11525,05	53,61	11477,72
pr19b	700	32	12	12	6	42467,74	300,53	42332,28	8295,30	80,55	8227,25	14918,54	92,15	14894,65
pr20b	880	42	15	15	6	53119,63	302,15	52863,23	10378,54	150,74	10325,80	18666,10	228,78	18566,66
pr21b	420	18	4	4	12	43195,88	261,51	43098,26	4887,57	36,75	4866,57	11309,41	27,17	11246,57
pr22b	600	24	6	6	12	58942,49	303,13	58814,76	6537,15	73,28	6488,50	14354,29	57,21	14288,26
pr23b	780	30	7	8	12	74755,07	302,99	74357,84	8603,85	163,99	8523,41	17635,56	125,08	17576,39
pr24b	960	40	8	10	12	94551,24	303,18	94395,56	10997,66	298,51	10890,08	23420,33	227,26	23176,90
Avg. Gap & T(min)						+0.42%	254.19	+0.00%	+0.71%	88.98	+0.00%	+0.60 %	92.22	+0.00%
Processor							Xe-2.93G			Xe-2.93G			Xe-2.93G	

To save computation time, the termination criterion was reduced to $It_{NI} = 2500$ iterations without improvement, and a time limit of 5h is imposed. On the PVRPTW instances, which are very large, the population size was divided by two to speed up the convergence.

in the literature for the PVRPTW, MDVRPTW, and SDVRPTW, including the parallel iterative UTS of Cordeau and Maischberger (2012), which requires a large overall computational effort distributed on 64 processors. For the VRPTW, HGSADC is comparable to the hybrid genetic algorithm of Nagata et al. (2010) in terms of solution quality, is less computationally efficient, but relies on less problem-tailored components, and does not necessitate dedicated route minimization procedures.

The average standard deviation, measured on the instances of Cordeau et al. (2001a), ranges between 0.17% for the MDVRPTW and 0.27% for the PVRPTW, thus illustrating the good stability of the algorithm. HGSADC performance appears to be higher on problems with tight time windows and short routes. This observation goes in accordance with Bent and Van Hentenryck (2010), which showed that geometrical decomposition tends to perform better for this kind of instances.

These experiments retrieved or improved 105/109 BKS for the PVRPTW, MDVRPTW, and SDVRPTW, and 75/109 of those have been strictly improved. For the VRPTW, HGSADC

retrieved or improved 292/356 BKS, and strictly improved 158/356. Five new BKS with one less vehicle have been produced on the large scale VRPTW instances. In particular, on the VRPTW instances of Solomon and Desrosiers (1988), HGSADC found the same best solutions as Nagata et al. (2010). The new BKS on the instances of Pirkwieser and Raidl (2009b) and Gehring and Homberger (1999) are presented in III.3.

4.7.3 Sensitivity analysis on method components

Addressing large-scale time-constrained VRP with the HGSADC methodology led to several challenges, which were answered in this paper by means of new procedures for neighbourhood evaluation and pruning, and problem decompositions. This section analyses the role of several of these components. We measure the impact of the decomposition phases, the contribution of infeasible solutions to the search, which required new move evaluation procedures, and the diversity and cost objective.

Three versions of the algorithm were thus derived by removing in turn a different element from the method. The first does not rely on infeasible solutions with respect to time windows, setting high penalties to time-window violations, and relies on the "feasible" subpopulation only. The second does not apply decomposition phases. The last one uses a "traditional" evaluation of individuals driven exclusively by solution cost. Table 4.9 compares the average results on 5 runs, as an average deviation to the BKS, of these derived methods on various benchmark instances studied in this paper. The objective of fleet minimization being not straightforward to tackle without relying on TW-infeasible solutions, some instances were not considered in this case.

Table 4.9: Sensitivity analysis on the role of diversity and cost objective, time window-infeasible solutions, and decomposition phases.

Benchmark	**No Diversity objective**			**No TW infeasibility**			**No Decomposition**			**HGSADC**		
	Fleet	Dist	T(min)	Fleet	Dist	T(min)	Fleet	Dist	T(min)	Fleet	Dist	T(min)
PVRPTW	—	+1.23%	17.6	—	+0.59%	26.0	—	+0.21%	22.56	—	+0.17%	33.2
MDVRPTW	—	+1.24%	4.73	—	+0.80%	5.79	—	+0.11%	5.29	—	+0.10%	6.49
SDVRPTW	—	+1.33%	3.59	—	+0.67%	4.84	—	+0.10%	4.06	—	+0.12%	5.48
PVRPTW new	—	+1.68%	232	—	+0.97%	252	—	+1.45%	238	—	+0.42%	254
MDVRPTW new	—	+3.91%	62.9	—	+1.83%	85.9	—	+0.52%	78.3	—	+0.60%	92.2
SDVRPTW new	—	+2.91%	44.3	—	+1.56%	74.2	—	+0.65%	99.6	—	+0.71%	89.9
VRPTW $n = 200$	+0.00%	+0.55%	5.93	—	—	—	+0.00%	+0.21%	6.21	+0.00%	+0.18%	8.40
VRPTW $n = 400$	+0.48%	+0.53%	27.9	—	—	—	+0.32%	+0.22%	29.1	+0.22%	+0.15%	34.1

These experiments confirm the pertinence of the HGSADC framework for this class of problems, as the diversity and cost objective contributes largely to the performance of the proposed method. They also underline the major role of the time window-infeasible solutions, thus giving full meaning to the new move evaluation procedures.

Decomposition phases contribute significantly to the search performance on large PVRPTW instances, for which subproblems can involve up to 560 customers. This impact is less substantial on MDVRPTW and SDVRPTW instances, which present few customer visits (between 90 and 240) per resource (depot or vehicle type), and generate only small- or medium-size VRPTW sub-problems.

In this latter case, VRPTW routes were already efficiently optimized by the algorithm, without the need for decomposition, solution improvements being more likely to come from combined route and assignment optimizations.

4.8 Conclusions

We presented a new Hybrid Genetic Search with Advanced Diversity Control to efficiently address a large class of VRPTW variants, including MDVRPTW, PVRPTW, and SDVRPTW. Several new features were introduced to efficiently evaluate and prune neighbourhoods, and decompose large instances. Their important contribution to the performance of the algorithm in terms of solution quality and computing efficiency, as well as that of the HGSADC methodology combining cost and contribution-to-diversity factors in evaluating and selecting individuals, has been demonstrated by extensive sensitivity analysis.

Comprehensive computational experiments and comparisons to state-of-the-art methods showed that the proposed algorithm performs impressively, in terms of both solution quality and computational efficiency, outperforming all current state-of-the-art methods for the considered problems, and producing numerous new best known solutions on classical literature benchmark instances.

Among the future developments we intend to undertake, there is the development of more advanced mixed structural and spatial decompositions, in order to decompose a PVRPTW into smaller PVRPTW subproblems thus providing the means to keep on working on service assignments during the decomposition phases. We also plan to keep on generalizing the method, progressing towards VRP variants with even more attributes and “rich” settings.

CHAPITRE 5

UNE ÉTUDE EMPIRIQUE DES RELAXATIONS DE FENÊTRES DE TEMPS

5.1 Fil conducteur et contributions

L'application de HGSADC aux problèmes avec fenêtres de temps, dans le chapitre précédent, a soulevé des interrogations méthodologiques sur l'utilisation efficace de solutions irréalisables et aux choix de relaxation effectué dans les recherches locales. Cette utilisation de solutions irréalisables avait déjà été identifiée comme un concept récurrent des heuristiques étudiées au cours du Chapitre 1, cependant peu d'éléments théoriques ou empiriques de la littérature venaient jusqu'alors appuyer ces conjectures. De ce fait, ce chapitre présente une analyse empirique des relaxations de contraintes de fenêtres de temps au sein de méthodes de recherche locale pour le VRPTW. Les résultats expérimentaux montrent que l'approche de "retours dans le temps" (Nagata et al. 2010), utilisée précédemment et généralisée pour l'application de HGSADC aux variantes avec fenêtres de temps, permet d'obtenir de meilleures solutions en un temps de calcul moindre.

5.2 Article V : Empirical studies on time-window relaxations in vehicle routing heuristics

Ce chapitre a fait l'objet d'une publication sous forme d'article de conférence internationale : Vidal, T., Crainic, T.G., Gendreau, M., Prins, C. (2012). Empirical studies on time-window relaxations in vehicle routing heuristics. *Proceedings of the XVIth CLAIO / XLIVth SBPO conference*, Rio de Janeiro, Brazil.

Abstract: The contribution of infeasible solutions in heuristic searches for vehicle routing problems does not make, up to date, a consensus in the metaheuristics community. On one hand, infeasible solutions may allow to transition between structurally different feasible solutions, thus enhancing the search. On the other hand they also lead to more complex move evaluation procedures and to wider search spaces. Choices of constraint relaxations may thus dramatically impact the method performance, both in terms of speed and solution quality. This paper analyzes the impact of infeasible solutions on heuristic searches through various empirical studies on local search improvement procedures for the vehicle routing problem with time windows (VRPTW). As a result of these analyses, relaxations appear clearly to have a positive impact on the solution quality. Especially, the time-window relaxation scheme of Nagata et al. (2010) clearly contributes to increase the performance of neighborhood searches for this problem.

Keywords: Constraint relaxations, neighborhood search, vehicle routing, time windows

5.3 Introduction

The vehicle routing problem with time-windows (VRPTW) is one of the most intensively studied NP-hard combinatorial optimization problems in transportation logistics, due to its major practical applications and its remarkable difficulty, most exact methods being still rarely able to solve instances of more than 100 customers. As a result, a wide range of heuristics and metaheuristics (see the surveys of Bräysy and Gendreau 2005b,a, Gendreau and Tarantilis 2010) have been proposed to address real-life settings.

Most efficient metaheuristics rely on some sort of local search-based improvement procedure, dedicating a large part of the computation effort to the serial exploration of neighborhoods. Efficient move evaluations are thus critical for algorithmic performance and scalability. Furthermore, even finding a feasible solution to the VRPTW is a NP-hard problem (Savelsbergh 1985). Hence, any method built on the postulate that an initial feasible solution can be rapidly found, e.g. by a constructive procedure, may fail on tightly-constrained problem instances.

The use of intermediate infeasible solutions with relaxed time-window constraints appears in this context as an alternative to guarantee the availability of some initial solutions. Several relaxation schemes have been proposed through the years, such as penalized late arrival to customers (Taillard et al. 1997), early and late arrival (Ibaraki et al. 2005), or penalized returns in time (Nagata et al. 2010). It is frequently conjectured that infeasible solutions enable to better transition during the search between structurally different feasible solutions (Cordeau et al. 2001a, Vidal et al. 2013). In particular, a clever management of penalties may enable to focus the search towards borders of feasibility, a place where high quality solutions are more susceptible to appear (Glover and Hao 2011). However, relaxations can also lead to more complex move evaluations (Ibaraki et al. 2005, Vidal et al. 2011) and larger search spaces. Hence, it becomes necessary to determine whether 1) the use of infeasible solutions contributes significantly to the search 2) if one relaxation scheme is more suitable to progress towards good feasible solutions, and 3) to quantify the additional computational burden related to infeasibility evaluations on practical problems.

This paper contributes towards answering these questions by means of dedicated experimental analyses. Four local-search improvement procedures, differing only by their relaxation scheme, are compared on the well-known benchmark instances of Solomon (1987). Sensitivity analyses are conducted to assess on the computational effort related to relaxations, on the contributions of relaxations in the search performance, relatively to different objectives and initial solution procedures. As a result of these analyses, the “return in time” relaxation of Nagata et al. (2010) appears to contribute to reach higher quality solutions, mitigates the dependency upon a good initial solution, with only a minor impact on the algorithm speed.

The paper is organized as follows. A review of relaxation schemes for the VRPTW is conducted in Section 5.5, followed by a presentation of state-of-the-art neighborhood evaluation procedures for each relaxation in Section 5.6. Empirical analysis on the impact of relaxations are performed in Section 5.7, and finally Section 5.8 concludes.

5.4 Problem statement and notations

The vehicle routing problem with time windows (VRPTW) can be defined on a complete undirected graph $G = (\mathcal{V}, \mathcal{E})$, where $v_0 \in \mathcal{V}$ stands for a central depot, and vertices $\mathcal{V}^{\text{CST}} = \mathcal{V} \backslash \{v_0\}$ represent n geographically dispersed customers requiring service. Each edge $(i, j) \in \mathcal{E}$ represents a traveling possibility from vertex v_i to vertex v_j with distance d_{ij} (equal to the travel time w.l.o.g.). Each customer $v_i \in \mathcal{V}^{\text{CST}}$ is characterized by a non-negative demand q_i, a service time τ_i, as well as an interval of allowed service times $[e_i, l_i]$, called time window. A fleet of m identical vehicles is located at the depot, where the product to be delivered to customers is kept. The capacity of each vehicle is limited to Q units of product. The VRPTW aims to design at most m sequences of visits σ^k for $k \in \{1 \dots m\}$, starting from the depot $\sigma^k(1) = 0$, visiting customers $\sigma^k(2), \dots, \sigma^k(|\sigma^k| - 1)$ during their time-windows, and returning to the depot $\sigma^k(|\sigma^k|) = 0$. The traditional objective involves minimizing the number of routes in priority, and then distance.

$$\text{Minimize} \quad \sum_{v_i \in \mathcal{V}} \sum_{v_j \in \mathcal{V}} \sum_{k=1}^{m} d_{ij} x_{ijk} \tag{5.1}$$

$$\text{Subject to:} \quad \sum_{v_j \in \mathcal{V}} \sum_{k=1}^{m} x_{ijk} = 1 \qquad v_i \in \mathcal{V}^{\text{CST}} \tag{5.2}$$

$$\sum_{v_j \in \mathcal{V} \backslash \{v_{n+1}\}} x_{jik} - \sum_{v_j \in \mathcal{V} \backslash \{v_0\}} x_{ijk} = 0 \qquad v_i \in \mathcal{V}^{\text{CST}} \ ; \ k \in \{1, \dots, m\} \tag{5.3}$$

$$\sum_{v_j \in \mathcal{V} \backslash \{v_0\}} x_{0jk} = 1 \qquad k \in \{1, \dots, m\} \tag{5.4}$$

$$\sum_{v_j \in \mathcal{V} \backslash \{v_{n+1}\}} x_{j,n+1,k} = 1 \qquad k \in \{1, \dots, m\} \tag{5.5}$$

$$\sum_{v_i \in \mathcal{V}} \sum_{v_j \in \mathcal{V}} q_i x_{ijk} \leq Q \qquad k \in \{1, \dots, m\} \tag{5.6}$$

$$x_{ijk}(t_{ik} + d_{ij} + \tau_i - t_{jk}) \leq 0 \qquad v_i \in \mathcal{V} \ ; \ v_j \in \mathcal{V} \ ; \ k \in \{1, \dots, m\} \tag{5.7}$$

$$e_i \leq t_{ik} \leq l_i \qquad v_i \in \mathcal{V} \ ; \ k \in \{1, \dots, m\} \tag{5.8}$$

$$x_{ijk} \in \{0, 1\} \qquad v_i \in \mathcal{V} \ ; \ v_j \in \mathcal{V} \ ; \ k \in \{1, \dots, m\} \tag{5.9}$$

$$t_{ik} \in \Re^+ \qquad v_i \in \mathcal{V} \ ; \ k \in \{1, \dots, m\} \tag{5.10}$$

Equations (5.1-5.10) recall the mathematical formulation of the VRPTW as a multi-commodity network flow. For convenience, v_0 has been separated into two vertices v_0 and v_{n+1}, standing respectively for the depot at the origin and destination. The binary variables x_{ijk} are set to 1 if and only if vehicle k visits v_j immediately after v_i, and the linear variables t_{ik} provide the service date to customer v_i, when serviced by vehicle k. Equations (5.2-5.6) set up the basis VRP network structure, and Equations (5.7-5.8) formulate the choices of service times t_{ik}. Equation (5.7) also eliminates sub-tours and can be linearized by means of big M values.

The next section reviews the use of these relaxations in the literature, and analyzes efficient methods to evaluate penalties in each case.

5.5 Time-window relaxations in VRPTW heuristics

Considerable effort has been dedicated during the last decades on solving the VRPTW by means of metaheuristics, leading to a plethora of approaches, reviewed in Bräysy and Gendreau (2005a) and Gendreau and Tarantilis (2010) among others.

Table 5.1: Infeasible solutions in state of the art VRPTW heuristics

Authors	**Approach**	**TW Relax.**
Taillard et al. (1997)	Tabu Search	Late service
Gambardella et al. (1999)	Ant Colony Optimization & Local Search	NO
Homberger and Gehring (1999)	Evolution Strategies & Local Search	NO
Liu and Shen (1999)	Customers relocations with deteriorating moves	NO
Cordeau et al. (2001a)	Unified Tabu search	Late service
Gehring and Homberger (2002)	Evolution Strategies & Tabu Search	NO
Bräysy (2003)	Node ejection chains & Variable neighborhood descent	NO
Berger et al. (2003)	GA & Local & Large Neighborhood Search	Late service
Bent and Van Hentenryck (2004)	Simulated Annealing & Large Neighborhood Search	Late service
Bräysy et al. (2004b)	Injection Tree & Iterative Improvement	NO
Homberger and Gehring (2005)	Evolution Strategies & Tabu Search	NO
Ibaraki et al. (2005)	Iterated local search	Early/Late
Le Bouthillier and Crainic (2005a)	Cooperative GA and Tabu Searches	Late service
Le Bouthillier and Crainic (2005b)	Guided Cooperative GA and Tabu Searches	Late service
Mester and Bräysy (2005)	Active Guided Evolution Strategies	NO
Lim and Zhang (2007)	Generalized Ejection Chains	NO
Pisinger and Ropke (2007)	Adaptive Large Neighborhood Search	NO
Hashimoto and Yagiura (2008)	Path Relinking	Return in time
Hashimoto et al. (2008)	Iterated Local Search	Early/Late
Ibaraki et al. (2008)	Iterated Local Search	Early/Late
Prescott-Gagnon et al. (2009)	Branch-and-price based Large Neighborhood Search	NO
Repoussis et al. (2009b)	Evolutionary Algorithm & Local Search	Late service
Nagata et al. (2010)	Hybrid GA with edge assembly crossover	Return in time
Vidal et al. (2013)	Hybrid GA with cost/diversity objective	Return in time

Table 5.1 provides a review of recent state-of-the-art methods and their time-window relaxations. 13/24 of these state-of-the-art methods rely on time-window infeasible solutions. Three main relaxations are used. The *Late service* relaxation allows linearly penalized late services but not early service to customers, while the *Early/Late* relaxation allows both early and late services. These two relaxations are often called "soft time windows" settings in the literature, and correspond to a Lagrangean relaxation of Equation (5.8). Finally, the relaxation of Nagata et al. (2010) allows the use of penalized *Returns in time* to reach customers in their time-windows, and can be viewed as a Lagrangean relaxation of Equation (5.7). Up to this date, the hybrid genetic algorithms of Nagata et al. (2010) and Vidal et al. (2013), which rely on this latter uncommon relaxation, have produced the best overall solutions. The contribution of this particular time-window relaxation in resolution approaches is an open question which is investigated in this paper.

5.6 Move evaluation methods and their computational complexity

Most metaheuristics for the VRPTW rely extensively on Local Search (LS) improvement procedures, exploring iteratively from an incumbent solution s a neighborhood $\mathcal{N}(s)$ of solutions

defined relatively to a limited number of movements on the sequences of visits, called *moves*. In practice, move evaluations make for the largest part of the overall computation effort in recent heuristics, and therefore must be very efficient.

It is well-known in the literature (Kindervater and Savelsbergh 1997, Irnich 2008a, Vidal et al. 2011, 2013) that any classical VRP move based on a bounded number of edge exchanges or vertices relocations, such as RELOCATE, SWAP, 2-OPT, 2-OPT*, or CROSS exchanges, can be assimilated to a recombination of a bounded number of subsequences of consecutive visits (and reverse subsequences for 2-opt) from the incumbent solution. Since local-search neighborhoods generally involve recurrent subsequences of visits, the management of meaningful information on subsequences can save redundant computations and increase the local search performance.

Keeping track of partial demands and distances, for example, on each $O(n^2)$ subsequence from the incumbent solution, provides the means to evaluate the demand and distance of any recombined route with a bounded number of sums. This opens the way to $O(1)$ time load-feasibility checks and distance computations during the local search, compared to $O(n)$ for a straightforward method that browses the routes and cumulates loads and distances.

In the literature, the sequence information is either pre-processed before move exploration and updated whenever a route change is performed, or computed on the fly if a *lexicographic order* is used for move evaluations (Savelsbergh 1985, 1992). In addition, pre-processing can be limited to the $O(L_{max}n)$ subsequences that contain either the first node of the route, the last node, or of less than L_{max} nodes, while still allowing for efficient inter-route RELOCATE, SWAP, 2-OPT*, and CROSS exchanges. We opted in this paper to rely on pre-processing, since the computational effort required to manage the information on subsequences is generally negligible when compared to the effort required by move evaluations, and because it allows for a random exploration of local search moves which increases the solution variety.

In a similar manner, preprocessing meaningful information on subsequences can contribute to reduce the computational complexity of time-window feasibility checks or, when applicable, time-penalties evaluations on the routes issued from the local search moves. Still, as reviewed in the following, not all relaxation schemes allow for efficient move evaluation techniques, and some relaxations result in non-trivial sub-problems for determining service times (called timing problems in Vidal et al. 2011) on each new route.

5.6.1 No infeasible solution

When no infeasible solution is used, checking route feasibility within a local search can be efficiently done with the approach of Savelsbergh (1985, 1992) and Kindervater and Savelsbergh (1997). For any subsequence σ, the sum of travel and service times $T(\sigma)$, the earliest possible sequence completion time $E(\sigma)$, and the latest feasible starting date $L(\sigma)$ are pre-processed. These values can be obtained by induction on the concatenation operation, starting with the base case of a sequence $\sigma_0 = (v_i)$ containing a single visit where $T(\sigma_0) = \tau_i$, $E(\sigma_0) = e_i + \tau_i$ and $L(\sigma_0) = l_i$,

and using Equations (5.11-5.14) to derive the same information on larger subsequences.

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2) \tag{5.11}$$

$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\} \tag{5.12}$$

$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\} \tag{5.13}$$

$$isFeas(\sigma_1 \oplus \sigma_2) \equiv isFeas(\sigma_1) \ \wedge \ isFeas(\sigma_2) \ \wedge \ (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{5.14}$$

These equations, furthermore, enable to check in $O(1)$ time the feasibility of routes issued from moves, assimilated to a recombination of a bounded number of sequences.

5.6.2 Early/Late and Late service

Penalized late or early services have been mentioned in early works by Sexton and Choi (1986) in the context of a Pickup and Delivery Problem with Time-Windows. Such a relaxation represents a trade-off between service quality and routing costs which is frequently encountered in practice. It is also eventually used within heuristics to achieve better performance.

However, allowing both early and late deliveries leads to an additional decision problem, since upon an early arrival to a customer, choice must be made on either waiting or paying a penalty for early service. For a fixed route σ, these decisions can be modeled as a $\{R, D|\}$ timing problem (Vidal et al. 2011) formulated in Equations (5.15-5.16). Coefficients α and β represent penalties for early and late service.

$$\min_{t_1,\ldots,t_{|\sigma|}} \quad \sum_{i=1}^{n_k} \alpha(e_{\sigma(i)} - t_i)^+ + \sum_{i=1}^{n_k} \beta(t_i - l_{\sigma(i)})^+ \tag{5.15}$$

$$\text{s.t.} \quad t_i + \tau_{\sigma(i)} + d_{\sigma(i)\sigma(i+1)} \leq t_{i+1} \qquad 1 \leq i < |\sigma| \tag{5.16}$$

The previous model is encountered in various fields of operations research, in transportation logistics, project and machine scheduling literature, as well as in statistics as a generalization of the isotonic regression problem (Robertson et al. 1988). Therefore, various solution algorithms are available, some of which provide a solution in $O(n \log n)$ time (see Garey et al. 1988 and Dumas et al. 1990, among others).

The special case where only late services are allowed ($\alpha = +\infty$) is straightforward to solve in $O(n)$ with a minimum idle time policy by servicing each customer as early as possible. The related route evaluation procedure will be denoted here as "straightforward evaluation".

In addition, Hendel and Sourd (2006) and Ibaraki et al. (2005, 2008) proposed a more efficient move evaluation procedure, generally applicable to any vehicle routing problem with separable piecewise linear service costs $c_i(t_i)$ as a function of time. This "advanced evaluation" requires managing two types of information on subsequences during the search: a function $F(\sigma)(t)$ representing the minimum cost to service the sequence σ while arriving at the last customer before time t, and a function $B(\sigma)(t)$ stating the minimum cost of servicing σ after time t. These functions are represented explicitly in the method using appropriate data structures.

For a sequence $\sigma_0 = (v_i)$ with a single vertex, $F_{\sigma_0}(t) = \min_{x \leq t} c_i(x)$ and $B_{\sigma_0}(t) = \min_{x \geq t} c_i(x)$. These values can then be computed by forward dynamic programming, or backward dynamic programming, respectively, on longer sequences using Equations (5.17-5.18).

$$F(\sigma \oplus v_i)(t) = \min_{0 \leq x \leq t} \{c_i(x) + F(\sigma)(x - \tau_{\sigma(|\sigma|)} - d_{\sigma(|\sigma|),i})\} \tag{5.17}$$

$$B(v_i \oplus \sigma)(t) = \min_{x \geq t} \{c_i(x) + B(\sigma)(x + \tau_i + t_{i,\sigma(1)})\} \tag{5.18}$$

Equation (5.19) then provides the optimal service cost $Z^*(\sigma_1 \oplus \sigma_2)$ for a route issued of the concatenation of two subsequences σ_1 and σ_2.

$$Z^*(\sigma_1 \oplus \sigma_2) = \min_{x \geq 0} \{F(\sigma_1)(x) + B(\sigma_2)(x + \tau_{\sigma_1(|\sigma_1|)} + t_{\sigma_1(|\sigma_1|)\sigma_2(1)})\} \tag{5.19}$$

This equation allows, among other, for efficient evaluations of 2-OPT* neighborhoods. Moreover, any route resulting from the concatenation of three subsequences $(\sigma_1 \oplus \sigma_L \oplus \sigma_2)$, where σ_L is a sequence of bounded size, can be evaluated by relying on Equation (5.17) $|\sigma_L|$ successive times to yield the information on $\sigma' = \sigma_1 \oplus \sigma_L$, and computing $Z^*(\sigma' \oplus \sigma_2)$ with Equation (5.19). This latter strategy can be used to account for inter-route moves such as RELOCATE, SWAP or CROSS.

Route evaluations based on this methodology achieve a time complexity of $O(\Sigma_i \xi(c_i))$, where $\xi(c_i)$ represents the number of pieces in each function $c_i(t_i)$. The methodology applies for the wide majority of intra-route VRP neighborhoods. In the particular case where all functions $c_i(t_i)$ are convex, which is the case in soft time windows settings, more advanced implementations based either on heap data structures (Hendel and Sourd 2006), or on search trees (Ibaraki et al. 2008), achieve a route evaluation complexity of $O(\log \Sigma_i \xi(c_i))$. In soft time-windows settings, $\xi(c_i) = 3$ for any customer v_i, and thus moves can be evaluated in amortized $O(\log n)$. Hence, when considering such advanced neighborhood evaluation procedures, the two relaxations schemes *Early/Late* and *Late Service* lead to the same move evaluation complexity despite the additional decisions related to allowable earliness.

5.6.3 Returns in time

The relaxation proposed by Nagata et al. (2010) is based on linearly penalized "time warps" to reach time windows upon a late arrival. As shown in the following, despite its very limited practical significance, the relaxation proves to be particularly useful to allow intermediate infeasible solution in heuristics while still allowing for amortized constant time move evaluations.

A possible way to efficiently perform move evaluations (Vidal et al. 2013) requires computing on any subsequence σ the minimum duration $D(\sigma)$ to perform the services, the minimum time warp usage $TW(\sigma)$, and the earliest $E(\sigma)$ and latest visit $L(\sigma)$ to the first vertex allowing a schedule with minimum duration and minimum time-warp use. For a sequence $\sigma_0 = (v_i)$ containing a single vertex $D(\sigma_0) = \tau_i$, $TW(\sigma_0) = 0$, $E(\sigma_0) = e_i$ and $L(\sigma_0) = l_i$. The same information can then be computed on larger subsequences by induction on the concatenation operator using Equations (5.20-5.26).

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + D(\sigma_2) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + \Delta_{WT} \tag{5.20}$$

$$TW(\sigma_1 \oplus \sigma_2) = TW(\sigma_1) + TW(\sigma_2) + \Delta_{TW} \tag{5.21}$$

$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_2) - \Delta, E(\sigma_1)\} - \Delta_{WT} \tag{5.22}$$

$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_2) - \Delta, L(\sigma_1)\} + \Delta_{TW} \tag{5.23}$$

$$\textit{where } \Delta = D(\sigma_1) - TW(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \tag{5.24}$$

$$\Delta_{WT} = \max\{E(\sigma_2) - \Delta - L(\sigma_1), 0\} \tag{5.25}$$

$$\Delta_{TW} = \max\{E(\sigma_1) + \Delta - L(\sigma_2), 0\} \tag{5.26}$$

The previous equations lead to amortized $O(1)$ time move evaluations. This complexity is identical to the case where no infeasible solutions are used (Section 5.6.1).

5.6.4 Flexible service and travel times

A drawback of the relaxation of Section 5.6.3 lies in the fact that the amount of time warp is not limited, opening the way to routes that can potentially serve some customers with early time windows, move on towards other customers later in the day, and pay for a large time warp to start again deliveries to early customers. To avoid this issue, we investigate another relaxation alternative based on flexible travel and service times, but which forbids too small and negative travel durations.

Flexible travel times are usually computationally expensive to deal with (Hashimoto et al. 2006). Yet, the relaxation we propose is a very simple case, since the penalty $p_{ij}(\delta t)$ as a function of the service and travel duration is a simple piecewise linear function given in Equation (5.27), which is equivalent to pay linearly for a duration gain in presence of a minimum duration constraint. In our experiments, the minimum duration allowed for a service to a customer v_i is $\tau_i^{min} = \tau_i/2$, and the minimum duration for driving from any vertex v_i to any vertex v_j is set to $d_{ij}^{min} = d_{ij}/2$.

$$p_{ij}(\delta t) = \begin{cases} +\infty & \text{if } \delta t < d_{ij}^{min} + \tau_i^{min} \\ \alpha \times (d_{ij} - \delta t) & \text{if } d_{ij}^{min} + \tau_i^{min} \leq \delta t < d_{ij} + \tau_i \\ 0 & \text{if } d_{ij} + \tau_i \leq \delta t \end{cases} \tag{5.27}$$

Route evaluations in presence of this relaxation can be managed by means of a combination of the previous methodologies. First, Equations (5.11-5.14) are used to check whether the path is time-window feasible when maximum speed is used. Infeasibility at this step means that a forbidden high speed or negative travel time is used, and thus the route is declared infeasible. In the other case, some acceptable infeasability can exist and Equations (5.20-5.26) are used to measure the necessary amount of *return in time* along the path, which also corresponds to the necessary speedups. These two steps are perated in $O(1)$ such that the relaxation cost related to flexible travel times can be measured in amortized constant time.

5.7 Empirical comparison of relaxations

In this Section, experimental investigations are conducted on several important questions related to relaxations schemes for the VRPTW, and especially:

1. What is the impact of relaxation schemes on heuristics performance, does one relaxation lead to solutions of higher quality ?
2. Do relaxation schemes mitigate the need for good starting solutions in heuristics ?
3. What is the practical computational burden related to different relaxations for problems of practical sizes ?

To answer to these questions, four variants of a simple local-search improvement procedure, differing only by their relaxation scheme, have been compared on the well-known benchmark instances of Solomon (1987) with 100 customers. These 56 instances are grouped in 6 categories which differ by the characteristics of the geographical distribution of customers. Customers are uniformly distributed in the R1 and R2 problem classes, clustered in the C1 and C2 classes, whereas RC1 and RC2 mix both uniform and clustered customer distributions. Class C1, R1 and RC1 contain problems with short time horizon and small vehicle capacities, while C2, R2 and RC2 have longer time horizon, larger vehicle capacities, and lead to longer routes.

Experiments have been conducted on two different problem objectives: a hierarchical objective involving first the minimization of fleet size and then distance, and the distance-minimization objective. Experiments have been conducted with two different types of initial solutions to analyze their impact, either a random solution obtained by randomly assigning and positioning customers into routes, or a solution produced by the I1 insertion heuristic of Solomon (1987).

Three alternative relaxations, with either late services "Late", returns in time "Return", and flexible travel times "Flex" have been tested and compared with a strategy where infeasible solutions are forbidden "No Inf". The efficient move evaluation strategies presented in Section 5.6 are implemented for each relaxation scheme. The *straightforward evaluation* strategy in $O(n)$ time is used for the Late relaxation. Faster $O(\log n)$ evaluations are known to be achievable (Section 5.6.2), but the price to pay in terms of algorithm development is very high.

5.7.1 Performance of relaxations with regards to distance minimization

The local search procedure used in our experiments is based on classic vehicle routing neighborhoods: 2-opt, 2-opt* as well as CROSS and I-CROSS exchanges restricted to sequences of size smaller than $L_{max} = 2$ (see Vidal et al. 2012c for a detailed presentation of these neighborhoods). Moves are explored in random order, any improving move being directly applied, until no improvement can be found in the whole neighborhood.

This procedure is applied several times from different initial solutions, the best overall final solution being returned. Each run involves two phases as described in Algorithm 5.1. First, an initial solution is created with either the random construction procedure, or the I1 insertion heuristic of Solomon (1987), and the local search is applied with moderately small penalty coefficients ($\alpha = \beta = 1$). This process can lead to an infeasible solution, such that a second local search run with higher penalty coefficients is performed to restore the solution feasibility ($\alpha = \beta = 100$).

Algorithm 5.1 Duration Minimization

```
1: for t = 1,...,nbTry
2:     penalties = 1
3:     sol = LocalSearch(newInitialSol())
4:     if not isFeasible(sol) then
5:         penalties = 100
6:         sol = LocalSearch(sol)
7: returnBestFeasibleSolution()
```

Table 5.2 reports the solutions retrieved by each method during a sample run, using either the random initialization procedure or the I1 heuristic of Solomon (1987). The initial solution provided by the I1 heuristic is also displayed in Column 2. Distances are aggregated by problem classes. The last two lines indicate the Cumulated Total Distance (CTD) for all the 56 instances and the average computation time per instance.

Table 5.2: Distance minimization on the VRPTW benchmark instances of Solomon (1987)

Inst.	**SolI1**	**No Inf**	**Late**	**Return**	**Flex**	**No Inf**	**Late**	**Return**	**Flex**
		SolomonI1 Initial Solution				**Random Initial Solution**			
R1	1431.97	1225.25	1219.11	1220.13	**1219.41**	1268.97	1231.70	1226.08	1229.73
R2	1326.64	963.53	957.11	947.77	942.87	982.44	**940.38**	947.41	940.79
C1	936.48	844.00	835.17	835.67	**834.26**	860.82	835.14	840.73	835.32
C2	696.57	605.62	603.62	603.08	**600.59**	709.37	650.52	645.45	649.61
RC1	1578.28	1401.49	1399.11	**1389.83**	1396.54	1482.79	1406.57	1401.53	1404.57
RC2	1653.61	1139.12	1077.93	1093.02	1079.40	1130.21	1072.63	1075.60	**1070.59**
CTD	71633	58067	57319	57275	**57125**	60361	57679	57678	57621
T(sec)	0.03	3.41	20.06	6.59	7.90	6.25	17.55	8.03	10.00

Table 5.2 demonstrates the significant contribution of time-window relaxations to the solution quality. Relaxations lead to an improvement in distance of -1.30% to -1.65% when the constructive initialization procedure is used, otherwise it reaches -4.65% to -4.76% when random initial solutions are used. Furthermore, relaxations tend also to mitigate the impact of low quality initial solutions. When passing from a random initialization to a constructive procedure, the distance deteriorates by $+0.63\%$ to $+0.87\%$ in presence of relaxations, whereas the distance strongly deteriorates by $+3.95\%$ if only feasible solutions are used.

The different relaxations appear to perform equally well with regards to distance minimization, however the Return relaxation is less time-consuming. This observation goes in accordance with the theoretical results of Section 5.6, and would motivate the choice of this particular relaxation for distance minimization.

5.7.2 Performance of relaxations with regards to fleet minimization

When relaxations are used, addressing the hierarchical objective of fleet size minimization and distance can be simply done by iteratively decrementing the fleet size limit and running the local search algorithm of the previous section, as described in Algorithm 5.2.

However, when only feasible solutions are explored, minimizing the number of vehicles leads to new theoretical issues. Indeed, the method must start from a feasible solution, which has inevitably

Algorithm 5.2 Fleet Size Minimization

```
1: t = 0 ;
2: while t < nbTry
3:     sol = LocalSearch(newInitialSol())
4:     if isFeasible(sol) then fleetSize = fleetSize - 1; t = 0 ; else t = t+1 ;
5: fleetSize = fleetSize + 1;
6: for t = 1,...,nbTry
7:     LocalSearch(newInitialSol())
8: returnBestFeasibleSolution()
```

a too large number of routes, and reduce the number of routes during the search. Several tailored procedures have been proposed to that extent, involving eventually the use of auxiliary objectives to progress towards empty routes as in Gendreau et al. (1996) and Bent and Van Hentenryck (2004), or route-removal operations (Nagata and Bräysy 2009b). However, these procedures are too different in their behavior to conduct a fair comparison with relaxation-based heuristics. For this reason, only the results for heuristics relying on infeasible solutions are reported in the following experiments.

Table 5.3 reports computational results on one run of the fleet minimization algorithm for each relaxation scheme, using either the constructive heuristic I1 or the random procedure for solution initialization. For each problem class, the average number of vehicles and distance is indicated. The last two lines indicate the Cumulated Total Distance (CTD) for all the 56 instances and the average computation time per instance.

Table 5.3: Fleet minimization on the VRPTW benchmark instances of Solomon (1987)

Inst.	**SolI1**	**Late**	**Return**	**Flex**	**Late**	**Return**	**Flex**
		SolomonI1	**Initial**	**Solution**	**Random**	**Initial**	**Solution**
R1	13.42	12.67	12.50	**12.42**	13.17	12.50	12.58
	1431.97	1231.65	1241.02	1246.45	1257.77	1258.19	1242.94
R2	3.18	3.09	**2.91**	**2.91**	4.27	**2.91**	3.00
	1326.64	1006.06	1019.56	1017.65	998.95	1026.30	1011.49
C1	10.00	**10.00**	**10.00**	**10.00**	**10.00**	**10.00**	**10.00**
	936.48	840.41	838.63	838.90	859.98	858.21	861.25
C2	3.13	**3.00**	**3.00**	**3.00**	3.75	**3.00**	**3.00**
	696.57	604.90	603.34	603.11	674.27	646.22	629.17
RC1	13.50	12.25	**12.13**	**12.13**	12.25	12.25	**12.13**
	1578.28	1415.26	1414.58	1435.54	1417.72	1420.09	1431.37
RC2	3.75	3.50	**3.25**	**3.25**	3.38	**3.25**	**3.25**
	1653.61	1246.05	1228.97	1241.17	1218.00	1238.56	1254.65
CNV	459	426	419	**418**	450	420	421
CTD	71633	59539	59630	59940	60301	60550	60314
T(sec)	0.03	42.61	14.02	17.10	59.97	19.94	24.89

Relaxations appear to have a large impact on the method performance when dealing with the hierarchical objective of fleet and distance minimization. In particular, the Return and Flex relaxations produce solutions of better quality, with a 418 and 419 cumulated vehicles respectively, than the Late relaxation, which leads to 426 vehicles.

Return and Flex relaxations are also less dependent upon the availability of a good initial solution. Indeed, the cumulated fleet size reaches 420 and 421 vehicles when a random initial solution is used, whereas with a Late relaxation the fleet size increases to 450 vehicles. A potential explanation is that infeasible insertions at the beginning of routes are likely to result in massive penalties in the Late relaxation scheme, leading to imbalanced insertion capacities within the routes, thus reducing the ability to progress towards feasible solutions in tightly constrained settings with few routes.

5.8 Conclusions

In this paper, four simple neighborhood search procedures, differing only by their relaxation scheme, have been tested on the vehicle routing problem with time windows. Experimental results demonstrate the positive contribution of time-window relaxations in the method performance for two different objectives, as well as their ability to mitigate the impact of low quality starting solutions. The relaxation of Nagata et al. (2010), especially, based on the concept of returns in time, yields the best results in terms of both solution quality and computation efficiency, and thus appear as a promising option for VRPTW metaheuristics.

Finally, as a perspective of research, this study will be completed by comparison of relaxations within an iterated local search and the hybrid genetic algorithm of Vidal et al. (2013), thus providing the means to challenge the previous observations on more advanced metaheuristic procedures.

CHAPITRE 6

GESTION IMPLICITE DES PLACEMENTS DE DÉPÔT DANS LES HEURISTIQUES ET MÉTA-HEURISTIQUES À BASE DE RECHERCHE LOCALE

6.1 Fil conducteur et contributions

L'application de HGSADC aux problèmes avec dépôts multiples, en Section 3 a permis de soulever des questions méthodologiques quand à l'exploration, combinée ou non, de différentes alternatives d'affectation aux dépôts et de choix de séquences. Ce chapitre présente une méthode efficace de programmation dynamique pour évaluer en temps $O(1)$ amorti le placement et le choix optimal des dépôts dans les routes, lors de méthodes de recherche locale et de Split. Cette approche permet d'une part d'explorer des alternatives combinées d'affectation et de séquence et donc d'augmenter la gamme des mouvements considérés. D'autre part les décisions d'affectation peuvent être ainsi reléguées au sein de sous-problèmes d'évaluation de routes, afin de ne plus avoir à considérer explicitement cet aspect combinatoire au sein des voisinages de recherche locale et des représentations de solution. Ces approches sont implantées et testées au sein de deux cadres méta-heuristiques, une recherche locale itérée, et la méthode HGSADC, sur des VRP multi-dépôts avec flotte illimitée, homogène ou hétérogène. Les expériences mettent en valeur la contribution importante des méthodes proposées au sein des deux types de méta-heuristiques.

6.2 Article VI : Implicit depot choice and positioning in vehicle routing heuristics

Un article basé sur ce chapitre a été soumis pour publication : Vidal, T., Crainic, T.G., Gendreau, M., Prins, C. (2012) Implicit depot assignments and rotations in vehicle routing heuristics. *European Journal of Operational Research*, submitted for publication.

Abstract: Vehicle routing variants with multiple depots and mixed fleet present intricate combinatorial aspects related to sequencing choices, vehicle type choices, depot choices, and depot positioning. This paper introduces a dynamic programming methodology for efficiently evaluating compound neighborhoods combining sequence-based moves with an optimal choice of vehicle and depot, and an optimal determination of the first customer to be visited in the route, called *rotation*. The assignment choices, making the richness of the problem, are thus no more addressed in the solution structure, but implicitly determined during each move evaluation. Two meta-heuristics relying on these concepts, an iterated local search and a hybrid genetic algorithm, are presented. Extensive computational experiments demonstrate the remarkable performance of these methods on classic benchmark instances for multi-depot vehicle routing problems with and without fleet mix, as well as the notable contribution of the implicit depot choice and positioning methods to the search performance. The proposed concepts are fairly general, and widely applicable to many

other vehicle routing variants.

Keywords: Vehicle routing, multi-depot, fleet mix, dynamic programming, local search, genetic algorithms

6.3 Introduction

Vehicle Routing Problems (VRP) with combined assignment choices, such as multi-depot and mixed-fleet settings, appear prominently in many applications related to transportation, production planning, robotics, maintenance, health care or emergency relief. These combinatorial optimization problems require two levels of decisions, related respectively to the sequencing of visits to customers into routes, and the assignment of customers to some global resources, such as depots or vehicle types. Heuristics and meta-heuristics that rely on separate optimization procedures for addressing each aspect, e.g. separate families of local searches, large neighborhoods or crossovers to work on the order of visits, the depot choices or the vehicle types, may overlook a wide range of potential solution refinements involving joint changes in the sequencing and assignment decisions, e.g. swapping two customers and in the meantime changing the vehicle or the depot assigned to the routes. Thus, most advanced meta-heuristics combine these decisions within purposeful optimization procedures to achieve notable performance gains (see Prins 2009b, for example), though the number of combined solution changes tends to become computationally expensive to investigate.

To contribute towards addressing this challenge, this paper proposes a new bidirectional dynamic programming approach to optimally manage the choices of vehicle, depot, and first customer visited in a route, the so-called optimal *rotation*, directly at the level of route evaluations in vehicle routing heuristics. We thus introduce a new Local Search (LS) in which the neighborhoods are solely based on customer-visits relocations and arc exchanges, while dynamic programming-based route evaluation functions produce optimal depot placements, choices and rotations for each alternative route. Since several advanced meta-heuristics for vehicle routing problems requires a Split algorithm to optimally segment a solution represented as a giant tour into several routes, we also derive an advanced *Split* algorithm with compound vehicle assignments, depot choices and rotations. The proposed enhanced procedures work with the same computational complexity as the classic ones from the literature. Thus, the additional capability we introduce does not lead to any additional computational overhead.

As a proof of concept, these methodologies are integrated and tested within two meta-heuristics: a simple multi-start Iterated Local Search (ILS) similar to the one of Prins (2009a) for the capacitated VRP, and a more elaborate Hybrid Genetic Search with Advanced Diversity Control (HGSADC) similar to the one of Vidal et al. (2012a,b, 2013). Two specific problems are investigated, the multi-depot fleet mix problem requiring combined decisions on assignments to vehicles types and depot along with sequencing choices, and the classic multi-depot VRP. Extensive computational experiments, on small- and large-scale benchmark instances with up to 960 customers demonstrate

the remarkable performance of the proposed meta-heuristics, as well as the notable contribution of the combined neighborhoods to the search performance.

To facilitate the presentation, we first introduce in Section 6.4 the problems, notations and variants considered in our experiments. Sections 6.5 and 6.6 describe the proposed methodology for optimally managing depot, vehicle choices and rotations within route evaluations, and presents the advanced Split method. The integration of these procedures into a neighborhood-based and a population-based meta-heuristic is discussed in Section 6.7. The computational experiments are reported in Section 6.8, and Section 6.9 concludes.

6.4 Vehicle routing problems and variants

Vehicle Routing Problems (VRP) aim to design least cost vehicle routes to service geographically dispersed customers (Toth and Vigo 2002a, Cordeau et al. 2007, Golden et al. 2008, Laporte 2009, Vidal et al. 2012c). Emphasis is still growing on this family of problems after 50 years of research, mostly because of their major economic impact in many application fields, but also because of the considerable amount of problem variants that must be dealt with to adequately address practical settings. Practical applications, indeed, lead to a variety of problem *attributes* that complement the classic VRP model, and seek to account for customer requirements (e.g., schedules, consistency), network and vehicle characteristics (mixed fleet, multiple depots), and driver's needs (working hour regulations, lunch breaks) among others.

Two attributes especially, *mixed-fleet* and *multi-depot*, are recurrent in many large-scale logistics applications. The problem combining these attributes, known as the Multi-Depot Vehicle Fleet Mix Problem (MDVFMP), can be defined as follows. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with $\mathcal{V} = \mathcal{V}^{\text{DEP}} \cup \mathcal{V}^{\text{CST}}$ be a complete undirected graph, in which the d nodes $v_o \in \mathcal{V}^{\text{DEP}}$ represent depots with infinite capacity, and the n nodes $v_i \in \mathcal{V}^{\text{CST}}$ stand for customers. Each customer v_i is characterized by a demand for a non-negative amount of product q_i. The edges $(i, j) \in \mathcal{E}$ represent the possibility of traveling between customers v_i and v_j for a total travel distance of c_{ij}. Finally, w types of vehicles are available in unlimited quantity, any vehicle k being characterized by a base acquisition/depreciation cost e_k, a per-distance-unit cost u_k, and a capacity Q_k. As such, the minimum cost $\Phi(x, q)$ to perform a route with distance x and total demand q is given in Equation (6.1).

$$\Phi(x, q) = \min_{k \in \{1,\dots,w\}/q \leq Q_k} \{e_k + u_k x\} \tag{6.1}$$

The MDVFMP aims to find a set of routes, as well as their assignment to vehicles and depots, to service each customer once and minimize the total cost. Each route assigned to any vehicle k shall start and end at the same depot location and carry less than q_k of units of products. A mathematical formulation is given in Equations (6.2-6.10). In this model, $\mathcal{F}$ represents a fleet containing a large number of vehicles of each type. The binary variable y_{io} represents the depot assignment decision, taking value 1 if and only if customer v_i is assigned to depot v_o. The binary variable x_{ijko} takes value 1 if and only if customer v_j is serviced immediately after v_i by vehicle k from depot o. The objective, presented in Equation (6.2), includes the base cost of e_k of any used vehicle k, and the distance-based cost $u_k c_{ij}$. Equations (6.3-6.5) ensure that exactly one depot

is chosen for each customer. Equation (6.6) ensures the conservation of the flow. Equation (6.7) enforces the capacity limits for the vehicles, and sub-tours are eliminated by Equation (6.8).

$$\text{Minimize} \sum_{k\in\mathcal{F}}\sum_{v_o\in\mathcal{V}^{\text{DEP}}}(\sum_{v_i\in\mathcal{V}} e_k x_{oiko} + \sum_{(v_i,v_j)\in\mathcal{E}} u_k c_{ij} x_{ijko}) \tag{6.2}$$

$$\text{Subject to:} \sum_{v_o\in\mathcal{V}^{\text{DEP}}} y_{io} = 1 \qquad v_i \in \mathcal{V}^{\text{CST}} \tag{6.3}$$

$$\sum_{v_i\in\mathcal{V}}\sum_{k\in\mathcal{F}} x_{ijko} - y_{io} = 0 \qquad v_j \in \mathcal{V}^{\text{CST}}\ ;\ v_o \in \mathcal{V}^{\text{DEP}} \tag{6.4}$$

$$\sum_{v_j\in\mathcal{V}} x_{ojko'} = 0 \qquad v_o \in \mathcal{V}^{\text{DEP}}\ ;\ v_{o'} \in \mathcal{V}^{\text{DEP}}\ ;\ v_o \neq v_i\ ;\ k \in \mathcal{F} \tag{6.5}$$

$$\sum_{v_j\in\mathcal{V}} x_{jiko} - \sum_{v_j\in\mathcal{V}} x_{ijko} = 0 \qquad v_i \in \mathcal{V}\ ;\ v_o \in \mathcal{V}^{\text{DEP}}\ ;\ k \in \mathcal{F} \tag{6.6}$$

$$\sum_{v_i\in\mathcal{V}}\sum_{v_j\in\mathcal{V}} q_i x_{ijko} \leq Q_k \qquad v_o \in \mathcal{V}^{\text{DEP}}\ ;\ k \in \mathcal{F} \tag{6.7}$$

$$\sum_{v_i\in S}\sum_{v_j\in S} x_{ijko} \leq |S| - 1 \qquad S \in \mathcal{V}^{\text{CST}}\ ;\ |S| \geq 2\ ;\ v_o \in \mathcal{V}^{\text{DEP}}\ ;\ k \in \mathcal{F} \tag{6.8}$$

$$x_{ijko} \in \{0,1\} \qquad v_i \in \mathcal{V};\ v_j \in \mathcal{V}\ ;\ v_o \in \mathcal{V}^{\text{DEP}}\ ;\ k \in \mathcal{F} \tag{6.9}$$

$$y_{iko} \in \{0,1\} \qquad v_i \in \mathcal{V}\ ;\ v_o \in \mathcal{V}^{\text{DEP}}\ ;\ k \in \mathcal{F} \tag{6.10}$$

The MDVFMP includes several prominent problems as special cases, such as the Vehicle Fleet Mix Problem (VFMP) when $d = 1$, the Multi-Depot VRP (MDVRP) when $w = 1$, and the Capacitated VRP (CVRP) when $(w, d) = (1, 1)$. The MDVFMP is also NP-hard as a generalization of the CVRP.

Contributions explicitly targeted on the MDVFMP are not frequent in the literature, and we are currently aware of a single meta-heuristic from Salhi and Sari (1997). This neighborhood-based method relies on the same concepts as the Variable Neighborhood Search, and proceeds by changing the neighborhoods by increasing size whenever a local optimum is encountered. Advanced moves that change both the assignment and the sequencing are used once several simpler neighborhoods have been exhausted. The MDVFMP is also a sub-problem of the settings encountered by Irnich (2000), Dondo and Cerdá (2007) and Goel and Gruhn (2008), although a limited fleet has been considered in most cases along with some other problem attributes.

In contrast, the literature dedicated to its two immediate sub-problems, the MDVRP and the VFMP, is much more furnished. An extensive survey of all methods for these two sub-problems is outside the scope of this section, and we refer to Ombuki-Berman and Hanshar (2009), Vidal et al. (2012c) and Subramanian et al. (2012) to that extent. Several meta-heuristics produce solutions of remarkable quality. For the MDVRP, the current state of the art results are produced by the Hybrid Genetic Search with Advanced Diversity Control (HGSADC) of Vidal et al. (2012a), which relies on efficient crossover and LS-improvement procedures to create new individuals. Of particular interest is the individual evaluation used in HGSADC, which relies on both solution quality and

contribution to the population diversity. High-quality solutions have also been generated by the Adaptive Large Neighborhood Search (ALNS) of Pisinger and Ropke (2007), the parallel iterated tabu search heuristic of Cordeau and Maischberger (2012) and the hybrid iterated local search and integer programming approach of Subramanian (2012).

For the VFMP, state-of-the-art heuristics are based either on ILS and integer programming (Subramanian et al. 2012, Subramanian 2012), tabu search (Brandão 2009), variable neighborhood search (Imran et al. 2009), or hybrid genetic algorithms (Liu et al. 2009, Prins 2009b). The previously-mentioned meta-heuristics enable to adequately address large scale problem instances, while smaller-size instances may be manageable with exact methods. The integer programming approach of Baldacci and Mingozzi (2009) especially, based on a set partitioning formulation, has demonstrated its ability to solve most MDVRP and VFMP instances with up to 100 customers.

It should finally be noted that most successful hybrid genetic algorithms for the previous problems (Liu et al. 2009, Prins 2009b, Vidal et al. 2012a,b, 2013) rely on a solution representation as a giant tour without trip delimiters that does not consider the route segmentation. Such a representation reduces the search space and allows for the use of simple crossovers based on permutations, but requires a *Split* algorithm to optimally segment the tour into routes whenever a full solution is needed (Prins 2004). This latter splitting problem is generally solved as a shortest path problem on an acyclic auxiliary directed graph (Beasley 1983). For the CVRP, the VFMP and the MDVRP, advanced Split algorithms with supplementary capabilities have been proposed, in order to combine the segmentation choices with decisions on vehicle type-to-customer assignments (Prins 2009b), depot selections (Kansou and Yassine 2010), or potential route inversions or rotations (Prins et al. 2009). These enhanced procedures were shown to contribute significantly to solution quality, however, although implicit depot choice and positioning has been studied in the context of Split algorithms, no such methodology has been to this date proposed in the context of LS. The next section contributes to fill this gap in the state of the art, by introducing a new dynamic programming-based approach to efficiently explore compound LS neighborhoods with sequence changes, optimal vehicle, depot choices and route rotations.

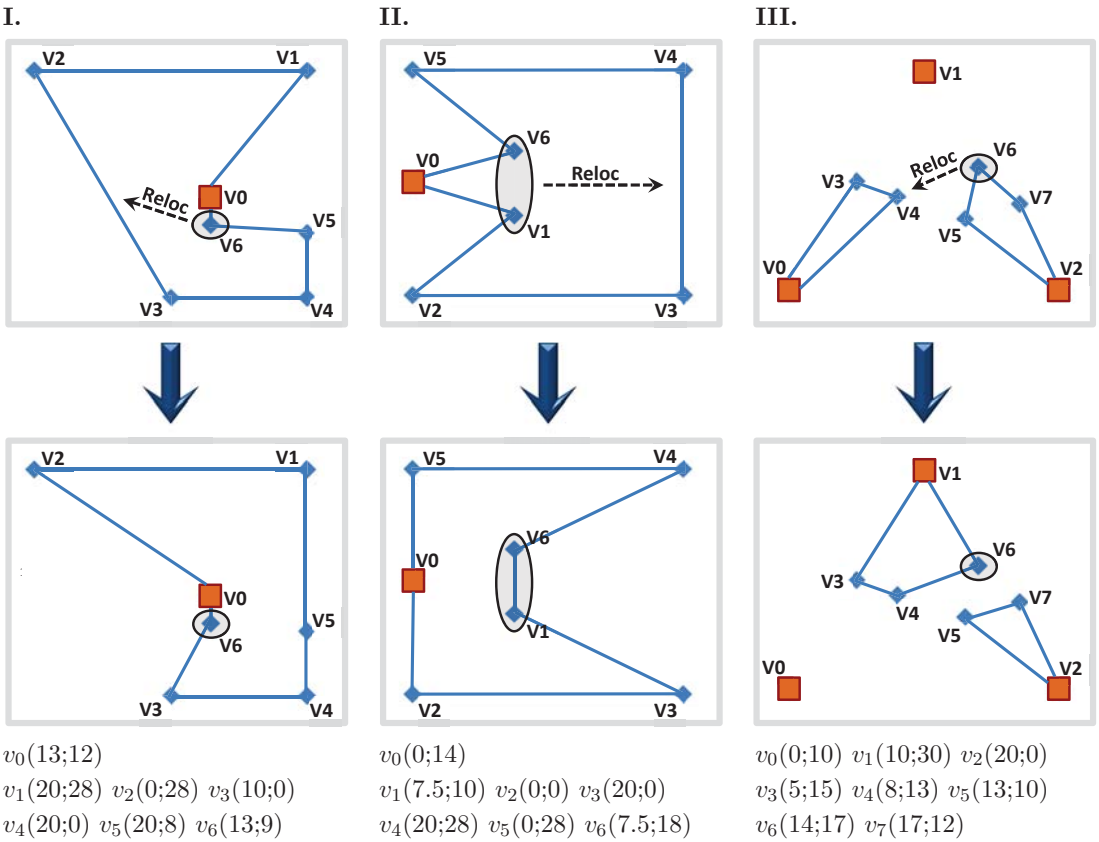
6.5 Compound neighborhoods with implicit depot positioning and vehicle choices

When dealing with vehicle routing variants with combined assignment choices – for example to depots, days, drivers or vehicle types –, the success of heuristics is often linked to their capacity to tightly integrate decisions together. Any change in the routes, especially, may influence the best way to assign these routes to vehicles, depots, or intervals in drivers schedules, or to rotate the route, such that the exploration of compound neighborhoods is often a key to find new combined choices that could otherwise look unfavorable when examined separately.

Figure 6.5 displays three euclidean problem instances where these situations arise. Customers are represented with diamonds, while depots are represented by larger squares. For the VRP instance illustrated on the right, the vehicle capacity is $Q = 3$ and any customer v_i for $i \in \{3, \ldots, 7\}$ requests one unit of product. All solutions illustrated on the top of this figure are not the global

optimum of the problem, but in fact precisely the second best solution (we listed all the solutions when generating the instances). The global optimum is presented below.

It is noteworthy that, even for these small problems with seven nodes, for situations I and II the classical TSP neighborhoods such as 2-opt or Or-opt are insufficient to lead to an optimal solution. The same happens in situation III with the 2-opt*, Relocate, Swap, Cross and I-Cross VRP neighborhoods (see Vidal et al. 2012c for a review of these neighborhoods). Nevertheless, by looking further at the solution changes required to attain the unique global optimum of our instances, we observe that for cases I and II a simple Or-Opt move of customer v_1 or customers (v_1, v_6), combined with an optimal placement of the depot v_0, i.e. rotation of the route, resolves the situation. For the case III, the key is to relocate customer v_6 with (v_3, v_4) and change the depot assigned to the route in a compound way. There is no other manner to attain this solution by local search without accepting deteriorating transitional moves.



$v_0(13;12)$
$v_1(20;28)$ $v_2(0;28)$ $v_3(10;0)$
$v_4(20;0)$ $v_5(20;8)$ $v_6(13;9)$

$v_0(0;14)$
$v_1(7.5;10)$ $v_2(0;0)$ $v_3(20;0)$
$v_4(20;28)$ $v_5(0;28)$ $v_6(7.5;18)$

$v_0(0;10)$ $v_1(10;30)$ $v_2(20;0)$
$v_3(5;15)$ $v_4(8;13)$ $v_5(13;10)$
$v_6(14;17)$ $v_7(17;12)$

Figure 6.1: Solution improvements using compound moves

Such multi-attribute compound neighborhoods open the way to critical solution refinements, but may be computationally expensive to explore. To give an example, searching any classical 2-opt, 2-opt*, Relocate or Swap neighborhood with combined depot choices, vehicle assignments and rotations would take $O(dwn^3)$ elementary operations when using a straightforward approach that tests all combinations. This complexity is too high to address large problems with meta-heuristics, which usually involve many LS runs.

To address this challenge, we introduce a new efficient search procedure to explore composite VRP neighborhoods based on a bounded number of edge exchanges and node relocations with an optimal choice of vehicle type, depot, and rotation. The proposed approach examines classic VRP neighborhoods of size $O(n^2)$ to produce new alternative sequences of visits, and relies on dynamic programming and incremental route evaluations to optimally determine the other attribute decisions. It exploits the fact that any sequence-based neighborhood involving a bounded number of edge exchanges and vertices relocations, can be assimilated to a recombination of a bounded number of sequences of consecutive visits (Kindervater and Savelsbergh 1997, Vidal et al. 2011). Figure 6.2 (inspired from Vidal et al. 2011) illustrates this property on a 2-OPT* move, involving two edge exchanges, and on a RELOCATE move involving a vertex relocation.

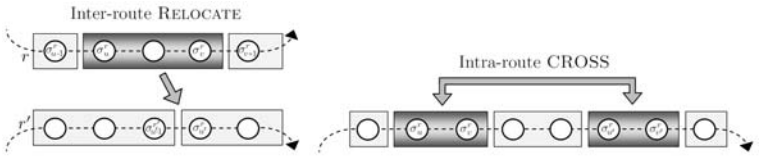


Figure 6.2: Moves assimilated to recombinations of sequences

The proposed method requires pre-processing dynamic-programming information on the $O(n^2)$ sub-sequences of each incumbent solution, in order to speed up the computations of optimal depot positions for each route produced by the LS. The following values are processed on each subsequence σ of visits to customers : the distance $C(\sigma)$ of the sequence (no visit to the depot is considered), the minimum distance supplement $\hat{C}(\sigma)$ to also visit one depot between the first and the last delivery, and the sum of customers' demands $Q(\sigma)$. By convention, $\hat{C}(\sigma_0)$ is set to $+\infty$ if the sequence σ_0 is empty or restricted to a single customer. Propositions 1 and 2 enable to exploit this informations to perform efficient move evaluations.

Proposition 6.5.1 *Let the* distance supplement $\hat{c}_{ij} = \min_{v_o \in \mathcal{V}^{\text{DEP}}}\{c_{io} + c_{oj} - c_{ij}\}$ *be defined as the additional distance required to visit the closest depot between customers v_i and v_j rather than driving directly. The minimum distance to perform the sequence of visits with the best depot choice and route rotation is then given by* $Z(\sigma) = \min\{\hat{C}(\sigma) + c_{\sigma_2(|\sigma_2|)\sigma_1(1)}, C(\sigma) + \hat{c}_{\sigma_1(|\sigma_1|)\sigma_2(1)}\}$.

Proposition 6.5.2 *For a sequence $\sigma_0 = (v_i)$ containing a single vertex $\sigma_0 = (v_i)$, $C(\sigma_0) = 0$, $\hat{C}(\sigma_0) = +\infty$, and $Q(\sigma) = q_i$. Furthermore, these values can be derived on larger sequences by induction on the concatenation operation $\oplus$ using Equations (6.11-6.13):*

$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + c_{\sigma_1(|\sigma_1|)\sigma_2(1)} + C(\sigma_2) \tag{6.11}$$

$$\hat{C}(\sigma_1 \oplus \sigma_2) = \min\{\hat{C}(\sigma_1), \hat{c}_{\sigma_1(|\sigma_1|)\sigma_2(1)}, \hat{C}(\sigma_2)\} \tag{6.12}$$

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \tag{6.13}$$

Equations (6.11-6.13) can indeed be used in a pre-processing phase to derive characteristic information on the subsequences of each new incumbent solution, by iteratively appending single customers at the extremities. This pre-processing phase requires $O(n^2)$ calls to the previous

operations. Neighbor solutions, assimilated to recombinations of subsequences, are then efficiently evaluated using these equations and the values developed on sequences. Because any local-search move can be assimilated to a bounded number $O(1)$ of concatenations, $O(1)$ calls to these equations lead to the cost for the new routes.

Any use of Equations (6.11-6.13) requires $O(d)$ elementary operations in a case with multiple depots, otherwise $O(1)$. The factor d appears within the computation of $\hat{c}_{ij}$ values. Yet, the $\hat{c}_{ij}$ values can be preprocessed at the beginning of the algorithm in $O(dn^2)$, and then be used for all LS runs. Since for the wide majority of neighborhood-based heuristics the number of successive LS is, by far, greater than the number of depots d, the amortized compound move evaluation complexity also drops down from $O(d)$ to $O(1)$ in multi-depot settings.

This methodology thus enables to compute the route distances resulting from any compound sequence-based move with a best choice of rotation and depot in $O(1)$ amortized time. The implicit rotation optimization is applicable to many VRP variants, and the additional implicit choice of depot has to potential to strongly enhance the capacities of meta-heuristics for multi-depot settings. Finally, in presence of a mixed fleet, the optimal vehicle type can also be chosen in a compound way in $O(w)$ time, and even $O(1)$ once some cost tables of pseudo-polynomial size are preprocessed (Prins 2009b).

6.6 Split algorithm with vehicle choices, depot assignments and rotations

Before going further with meta-heuristics implementations relying on these concepts, we detail another methodological result which follows from the previous property: an advanced Split algorithm which has the ability to optimally segment the giant tour, choose the vehicles and the depots, and rotate the routes.

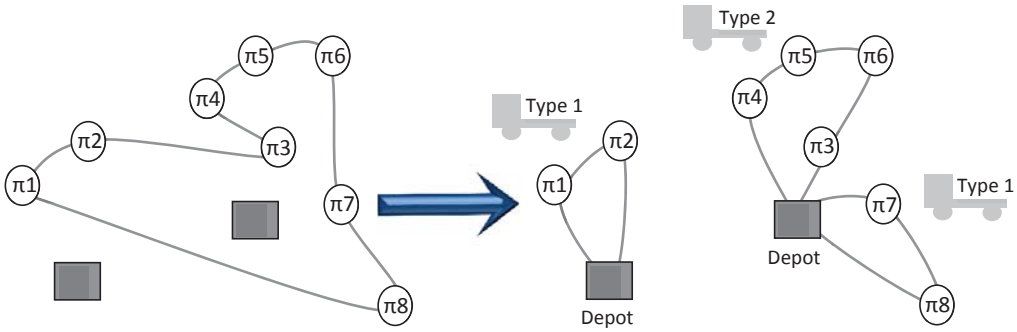


Figure 6.3: Advanced Split requirements

Consider the example illustrated in Figure 6.3. Let π_i be the i^{th} customer in the giant tour. The proposed algorithm computes an optimal compound segmentation in three routes corresponding to three sequences of customers (π_1,π_2), (π_3,...,π_6) and (π_7,π_8), with the best depot and vehicle choices. Also, notice that for the sequence (π_3,...,π_6) it is advantageous not to visit the depot between the sequence extremities π_3 and π_6, as it would be in a classic Split procedure, but rather between π_3 and π_4. The compound detection of these rotations is also managed by the method.

As in Beasley (1983) and subsequent works, our advanced Split assimilates the problem of optimally segmenting a giant tour to a shortest path problem on a directed acyclic auxiliary graph $\mathcal{H} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ contains $n+1$ nodes indexed from 0 to n. Each arc $(i,j) \in \mathcal{A}$ with $i < j$ represents a potential trip visiting the subsequence of customers π_{i+1} to π_j. The cost h_{ij} of arc (i,j) corresponds to the cost to serve the associated subsequence of customers. The proposed approach accounts for rotations and assignments within this cost definition, and thus h_{ij} stands for the best cost to service the sequence of customers π_{i+1} to π_j, considered as cyclic (π_j is followed by π_{i+1}) while optimally choosing the depot, vehicle type, and rotation. Section 6.5 gives the methods to compute all these values on subsequences in $O(n^2)$ time.

Algorithm 6.1 Split with vehicle choice, depot choice and positioning (rotations)

//Compute the costs h_{ij} for $0 \leq i < j \leq n$ in the auxiliary graph $\mathcal{H}$
for $i = 1$ to n
  //Single node case
  $\sigma = \{\pi_i\}$, $C(\sigma) = 0$, $\hat{C}(\sigma) = +\infty$, $Q(\sigma) = q_i$ and $h_{i-1,i} = \Phi(\hat{c}_{\pi_i\pi_i}, q_i)$
  //Append customers to compute costs on larger routes
  for each $j = i+1$ to n
    if $Q(\sigma) + q_{\pi_j} \leq Q_{max}$ **then**
      $C(\sigma) = C(\sigma) + c_{\pi_{j-1}\pi_j}$
      $\hat{C}(\sigma) = \min\{\hat{C}(\sigma), \hat{c}_{\pi_{j-1}\pi_j}\}$
      $Q(\sigma) = Q(\sigma) + q_{\pi_j}$
      $h_{i-1,j} = \Phi(\min\{\hat{C}(\sigma) + c_{\pi_j\pi_i}, C(\sigma) + \hat{c}_{\pi_j\pi_i}\}, Q(\sigma))$
Solve the shortest path problem on $\mathcal{H}$ with costs h_{ij}, using Bellman algorithm
Return the set of routes associated to the set of arcs of the shortest path

Once all the costs are determined, applying the Bellman algorithm for undirected acyclic graphs (see Cormen et al. 2001) on $\mathcal{H}$ leads to the best compound segmentation with vehicle types, depot choices and positioning within the routes. The overall splitting algorithm, given in Algorithm 6.1, works in $O(n^2)$ time. This complexity of $O(n^2)$ is the same as for the basic version of Split (Beasley 1983, Prins 2004) without depot choices and rotations. In this algorithm, Q_{max} denotes the capacity of the largest vehicle type. For the sake of simplicity, we separated the algorithm into a cost computation phase (equivalent to generating the graph $\mathcal{H}$), and a shortest path resolution. These two phases could be done simultaneously to avoid storing the costs, but without impact on the theoretical complexity.

This advanced Split procedure can thus be viewed as a byproduct of advanced subsequence evaluation procedures. The same observation arose in Vidal et al. (2012b), where different operators for sequence evaluations were shown to provide the keys to design splitting algorithms for a wide family of VRP variants. We believe that Split algorithms and local searches with pre-computations on subsequences take their roots on the same dynamic programming concepts, and that as we progress in our understanding of these procedures, these common fundamentals become more apparent.

6.7 Two meta-heuristic applications

The previous move evaluations and Split methodologies have been tested within two types of methods, a multi-start ILS following the guidelines of Prins (2009b), representing perhaps one of the simplest alternative for a neighborhood-based meta-heuristic, and an elaborate population-based meta-heuristic such as HGSADC of Vidal et al. (2012a,b, 2013).

6.7.1 An iterated local search application

The ILS framework is based on the iterative application of shaking operators and LS-improvement procedures on an incumbent solution to obtain new –hopefully better– solutions. This process is repeated until a maximum number of iterations without improvement $n_{\text{IT-ILS}}$ is attained. In addition, as recommended by Prins (2009a), three enhancements have been added to the method. Firstly, n_{C} solutions are generated at each iteration instead of one, the best solution being kept for the next iteration. This variant of ILS is sometimes called *evolutionary local search* (Wolf and Merz 2007). Secondly, the overall method is started n_{R} times from different initial solutions. Finally, two alternative search spaces are considered, a giant-tour solution representation being used during the shaking phases, while a complete solution representation is used during LS-improvement procedures. We thus rely on the advanced Split procedure of Section 6.6 to pass from a giant-tour solution representation to a complete solution while efficiently managing assignment choices and rotations. The general structure of the meta-heuristic is presented in Algorithm 6.2. The algorithm starts from a random solution (random permutation of the giant-tour), and terminates when all n_{R} restarts have been exhausted, or when a maximum time limit T_{MAX} is attained.

Algorithm 6.2 Iterated local search framework

$s_{\text{BEST-EVER}} \leftarrow$
for $i_{\text{R}} = 1$ to n_{R}
  $s_{\text{BEST}} \leftarrow$; $i_{\text{ILS}} = 0$
  $(s_{\text{FEAS}}, s_{\text{INFEAS}}) \leftarrow$InitializeRandom()
  while $i_{\text{ILS}} < n_{\text{IT-ILS}}$ and Time() $< T_{\text{MAX}}$
    $S_{\text{CHILDREN}} \leftarrow$
    for $s = s_{\text{FEAS}}$ and s_{INFEAS}
      for $i_{\text{C}} = 1$ to n_{C}
        if $i_{\text{ILS}} = k \times n_{\text{R}}$ with $k \in \mathcal{N}^{*+}$ **then** $s_{\text{CURR}} \leftarrow s_{\text{BEST}}$ **else** $s_{\text{CURR}} \leftarrow s$
        $s_{\text{CURR}} \leftarrow$Shaking($s$)
        $s_{\text{CURR}} \leftarrow$AdvancedSplit($s_{\text{CURR}}$)
        $s_{\text{CURR}} \leftarrow$LocalSearch($s_{\text{CURR}}$)
        $S_{\text{CHILDREN}} \leftarrow s_{\text{CURR}}$
        if Infeasible(s_{CURR}) **then** $S_{\text{CHILDREN}} \leftarrow$Repair($s_{\text{CURR}}$)
    $s_{\text{FEAS}} \leftarrow$BestFeasible($S_{\text{CHILDREN}}$)
    $s_{\text{INFEAS}} \leftarrow$BestInFeasible($S_{\text{CHILDREN}}$)
    if Cost(s_{FEAS}) $<$ Cost(s_{BEST}) **then** $s_{\text{BEST}} \leftarrow s_{\text{FEAS}}$; $i_{\text{ILS}} = 0$
    else $i_{\text{ILS}} = i_{\text{ILS}} + 1$
    AdaptPenaltyCoefficients()
if Cost(s_{BEST}) $<$ Cost($s_{\text{BEST-EVER}}$) **then** $s_{\text{BEST-EVER}} \leftarrow s_{\text{BEST}}$
return $s_{\text{BEST-EVER}}$

In addition, several studies highlighted the benefits of using intermediate penalized infeasible solutions with respect to route constraints (Cordeau et al. 1997, Glover and Hao 2011, Vidal et al. 2012a,b, 2013). Hence, both load and distance constraints are relaxed in our ILS, the cost $\Phi(x, q)$ of a route with distance x and total demand q begin given in Equation (6.14), where ω^{D} and ω^{Q} stand for the unit penalties for distance and load excess, respectively. These penalty coefficients are adapted as in Vidal et al. (2012a,b, 2013) relatively to the proportion of feasible solutions generated by the LS.

$$\Phi(x, q) = \min_{k \in \{1,\dots,w\}} \{e_k + u_k x + \omega^{\mathrm{D}} \max\{0, x - D)\} + \omega^{\mathrm{Q}} \max\{0, q - Q_k\}\} \tag{6.14}$$

The structure of the method has been slightly changed to account for infeasible solutions. First, it is straightforward to extend the Split algorithm to include penalized infeasibility by modifying the subsequence evaluations, and examining subsequences σ with a total demand smaller than two times the maximum capacity ($Q(\sigma) + q_{\pi_j} \leq 2 \times Q_{max}$ – Line 7 of Algorithm 6.1). In addition, two incumbent solutions are used in the ILS, one feasible, and one infeasible. At each iteration of the ILS, each solution is used to produce n_{C} child solutions. In the event that no feasible solution is currently known, then only the infeasible incumbent solution is used. Furthermore, any infeasible solution undergoes a Repair procedure, where the penalty coefficients are multiplied by a factor of 10, and the LS-improvement procedure is applied to focus the search towards feasible solutions. A last attempt with a factor of 100 is performed if the infeasibility remains. The resulting repaired solution is included in the set of offspring.

The LS-improvement procedure is based on 2-OPT, 2-OPT*, CROSS and I-CROSS neighborhoods restricted of sequences of less than two vertices. The optimal depot choice and position, and vehicle type choice is determined in a compound way using the approach of Section 6.5. A granularity threshold (Toth and Vigo 2003) is set to limit the neighborhood size, the moves being tested only between a vertex v_i and one of its Γ closest neighbors. Moves are explored in a random order, any improving move being directly applied.

Shaking is done by swapping two couples of random customers within the giant tour. Finally, experimental analyses that adding a probability of *Recall*, that is, jumping back to the current best solution, enhances the search performance by letting the method focus further on elite solution characteristics. We thus included this procedure, and after every n_R successive solution generations without improvement, consider the best solution as starting point for the next shaking and LS.

6.7.2 A hybrid genetic search application

To further investigate the contribution of the proposed strategies in the context of a population-based method, we derived an extension of the HGSADC framework of Vidal et al. (2012a,b, 2013) using the implicit depot management during LS and Split. The method relies on the following main elements:

- An hybridization between genetic algorithms and efficient local-improvement procedures. The same LS as in Section 6.7.1 is used.

- A solution representation as a giant tour without trip delimiters. It should be noted that since the depot choice and management is done implicitly by the LS and Split algorithm of Section 6.6, there is no need to separate the solution representation among different depots.
- A two sub-population scheme to manage feasible and penalized infeasible solutions w.r.t. route constraints.
- A bi-criteria evaluation of individuals, driven by both solution quality and contribution to the population diversity. This evaluation is used for parent selections and survivors selections when the population size becomes too large.

Algorithm 6.3 HGSADC framework

$S_{\text{POP}} = (S_{\text{FEAS}}, S_{\text{INF}}) \leftarrow$ InitializePopulation()
$s_{\text{BEST}} \leftarrow$; $i_{\text{HGS}} = 0$; $i_{\text{HGS-TOT}} = 0$
while $i_{\text{HGS}} < n_{\text{IT-HGS}}$ and Time() $< T_{\text{MAX}}$
  $s_{\text{P1}} \leftarrow$ BinTournamentSelection(S_{POP})
  $s_{\text{P2}} \leftarrow$ BinTournamentSelection(S_{POP})
  $s_{\text{OFF}} \leftarrow$ Crossover $(s_{\text{P1}}, s_{\text{P2}})$
  $s_{\text{OFF}} \leftarrow$ AdvancedSplit(s_{OFF})
  $s_{\text{OFF}} \leftarrow$ LocalSearch(s_{OFF})
  $S_{\text{POP}} \leftarrow s_{\text{OFF}}$
  if Infeasible(s_{OFF}) and Alea() $< P_{\text{REP}}$ **then** $s_{\text{OFF}} \leftarrow$ Repair(s_{OFF}) ; $S_{\text{POP}} \leftarrow s_{\text{OFF}}$
  $i_{\text{HGS-TOT}} = i_{\text{HGS-TOT}} + 1$
  if Cost(s_{OFF}) $<$ Cost(s_{BEST}) **then** $s_{\text{BEST}} \leftarrow s_{\text{OFF}}$; $i_{\text{HGS}} = 0$
  else $i_{\text{HGS}} = i_{\text{HGS}} + 1$
  if TooLarge(S_{FEAS}) **then** SelectSurvivors(S_{FEAS})
  if TooLarge(S_{INF}) **then** SelectSurvivors(S_{INF})
  if $i_{\text{HGS}} = k \times It_{div}$ with $k \in \mathcal{N}^*$ **then** Diversification(S_{POP})
  if $i_{\text{HGS-TOT}} = k \times It_{dec}$ with $k \in \mathcal{N}^*$ **then** DecompositionPhase()
  AdaptPenaltyCoefficients()
return s_{BEST}

The structure of the method is described in Algorithm 6.3. Initial individuals are first randomly generated and inserted in the appropriate sub-population relatively to their feasibility. HGSADC then iteratively selects two individuals by binary tournament in the merged sub-populations to serve as input the ordered crossover (OX) operator (see Prins 2004), yielding a single offspring s_{OFF}. This offspring undergoes the advanced Split procedure to optimally delimit and rotate its routes, assign vehicles and depots. Then, s_{OFF} undergoes the LS-improvement procedure of Section 6.7.1 and is inserted in the population. In case of infeasibility, it is *repaired* with probability P_{REP} and added again to the population.

Each sub-population is monitored separately, a *survivor selection* phase being triggered whenever a maximum size is attained. In addition, *diversification* and *decomposition* procedures (Vidal et al. 2013) are regularly used to enhance exploration and intensify the search around elite solution characteristics. The best found solution is returned once $n_{\text{IT-HGS}}$ successive iterations without improvement have been performed or when a time limit T_{MAX} is attained.

6.8 Computational experiments

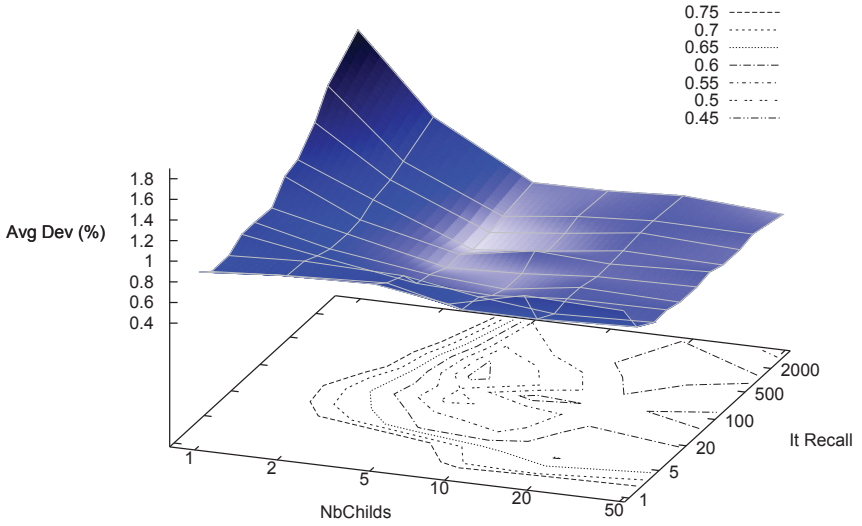
Extensive computational experiments are conducted to analyze the contribution of the compound neighborhoods within the ILS and HGSADC metaheuristics, and compare these methods with state-of-the-art algorithms from the literature. Three main VRP variants are considered: the CVRP and the MDVRP, two seminal problems covered by a huge literature, and the multi-depot vehicle fleet mix problem (MDVFMP), a good example of *rich* problem with compound attributes. The goal, with the ILS experiments, is not to put forward a new champion method, but rather to test the impact of the new compound neighborhoods on a simple LS-based metaheuristic.

Five sets of benchmark instances are used for the tests. For the CVRP, we rely on the classic benchmark instances of Christofides et al. (1979) (Set A) and Golden et al. (1998b) (Set B). Set A includes ten instances with uniformly geographically distributed customers, and four instances with clustered customers. Set B ranges from 200 to 483 customers and displays geometric symmetries.

Three sets of instances (Sets C-E) are used for multi-depot settings. These instances present a mix of uniformly distributed customers and clustered customers. Sets C and D of Cordeau et al. (1997) include 33 MDVRP instances with 50 to 360 customers and two to nine depots. The last Set E, of fourteen large-scale instances, has been introduced in Vidal et al. (2013) for the MDVRP with time windows, and is used for the MDVRP by removing time-window constraints. This set involves 360 to 960 customers and four to twelve depots. MDVFMP instances were also derived, as in Salhi and Sari (1997), by keeping the same customer locations and demands, and generating five types of vehicles v_k such that $Q_k = (0.4 + 0.2k) * \hat{Q}$, $e_k = 70 + 10k$ and $u_k = 0.7 + 0.1k$ for $k \in \{1, \ldots, 5\}$, $\hat{Q}$ standing for the vehicle capacity in the original instance. An unlimited fleet is considered in all experiments.

The run time limit for the methods has been set to $T_{max} = 20\ min$ on the medium-scale instance sets (Sets A to D) and $T_{max} = 5\ h$ for the larger instances (Set E). The parameter setting of HGSADC is kept the same as in Vidal et al. (2012a,b, 2013), since an extensive parameter calibration had already been conducted for multi-depot settings. The termination criteria of ILS has been set to $n_{\text{It-ILS}} = 25000/n_{\text{C}}$, so as to generate 25000 non-improving children before stopping, and the number of restarts is set to $n_{\text{R}} = 5$ to compare with HGSADC and other authors using similar computational effort. The two remaining free parameters, n_{C} and n_{R} were simultaneously calibrated. Several possible values of each parameter were selected, and the method was run 30 times for each configuration on a subset of instances, using different random seeds. For each parameter configuration, the gap to the Best Known Solutions (BKS) in the literature, averaged on all test instances, is reported in Figure 6.4.

As observed in Figure 6.4, the combination of parameters $(n_{\text{C}}, n_{\text{R}}) = (10, 200)$ yields the solutions of highest quality. These values were kept for all computational experiments. The algorithm has been coded in C++, compiled with "g++ -O3", and run on an Opteron 250 2.4 GHz CPU for the medium-scale instances (Sets A to D), and an Opteron 275 2.2 GHz CPU for large-scale instances (Set E).



n_C \ n_R	1	2	5	10	20	50	100	200	500	1000
2	0.887	0.778	0.781	0.872	0.874	0.861	1.052	**1.100**	1.309	1.551
4	0.949	0.826	0.815	0.752	0.705	0.698	0.720	**0.801**	0.842	0.908
10	**0.976**	**0.872**	**0.791**	**0.602**	**0.572**	**0.489**	**0.501**	**0.433**	**0.449**	**0.492**
20	0.870	0.702	0.673	0.573	0.521	0.519	0.624	**0.508**	0.528	0.577
40	1.042	0.699	0.598	0.629	0.624	0.578	0.585	**0.599**	0.622	0.624
100	1.011	0.740	0.631	0.639	0.598	0.594	0.626	**0.579**	0.575	0.612
200	0.896	0.632	0.710	0.678	0.568	0.665	0.523	**0.519**	0.747	0.700

Figure 6.4: ILS performance – Avg Gap to BKS (%) – with different parameter settings

6.8.1 Impact of compound neighborhoods

The impact of the new compound neighborhoods, within LS and Split, is assessed by means of comparative analyses of several variants of the proposed meta-heuristics, in which the compound optimization of some aspects – rotations, depot choices – has been activated or not. The methods with compound neighborhoods are notated ILS+ and HGSADC+ in the following.

The ILS variant without compound rotation optimization is referred to as *ILS-noR*. Not using both implicit rotations and assignments leads a variant called *ILS-noRD*. In the latter case, customer-to-depots assignments are explicitly managed within the solution representation, using one giant-tour per depot, and applying independently the classic Split procedure on the different giant-tours. The local search of Section 6.7.1 is used on each separate subset of customers associated to a different depot to perform route improvements (RI), and an additional LS-neighborhood based on customer-to-depot re-assignments is considered during an assignment improvement (AI) phase. These phases are called in the sequence AI-RI-AI-RI.

In the case of HGSADC, deactivating the compound rotations leads to a variant called *HGSADC-noR*. Coming back to an explicit management of the depots means using the original algorithm of Vidal et al. (2012a) on the MDVRP instances with unlimited fleet, notated *HGSADC-noRD*. In this case, the solutions are represented as one giant tour per depot, the PIX crossover of Vidal et al.

(2012a) is applied to impact both assignment and sequencing decisions, and the LS-improvement procedure is again separated between route- and assignment-improvement moves.

Tables 6.1 and 6.2 report the results of the two meta-heuristics with and without implicit rotations on CVRP instances. The quality of the solutions is compared to those of the best previous methods in the literature: the original HGSADC of Vidal et al. (2012a) (VCGLR12), the HGA with edge-assembly crossover of Nagata and Bräysy (2009b) (NB09), and the parallel record-to-record and set covering algorithm of Groër et al. (2011) (GGW11). In these tables, the first three columns display the instances names and characteristics, then the average solution values for the different methods are reported, and finally, the Best Known Solution (BKS) ever found, in previous literature and in our experiments. Best solution values are indicated in boldface for each problem, and new BKS are underlined. The last lines of the tables display the average gap of each method w.r.t. the BKS on each instance set, the average time per run for the methods, and the type of processor used.

In a similar fashion, Tables 6.3 and 6.4 report the results of the two meta-heuristics with and without implicit rotations and assignment choices on the MDVRP instances. A comparison of solution quality is done with the best current MDVRP algorithms: the ALNS of Pisinger and Ropke (2007) (PR07), the ILS of Subramanian (2012) (S12) and the original HGSADC of Vidal et al. (2012a) (VCGLR12). Some BKS, indicated with a "$*$" are known to be optimal (Baldacci and Mingozzi 2009). Results are reported for these three methods in presence of a fleet size limit. Still, as indicated by Cordeau et al. (1997), in many cases the maximum fleet size is large and does not impact the optimal solution value. Problems for which the fleet size limit appears to have an incidence are indicated in italics.

These results first highlight the notable contribution of implicit depot assignment, which lead to average MDVRP solutions of better quality for both ILS (0.781% for ILS-noR compared to 2.585% for ILS-noRD) and HGSADC (0.100% compared to 0.252%) with similar run time. This contribution is higher on large-scale problems, for which larger solution improvements were still achievable. The impact of the implicit depot management is very large for the ILS approach, which seems to have difficulties to achieve high-quality solutions on large-scale benchmarks without compound sequencing and assignment moves. For these problems, a more thorough exploration of assignment alternatives appear to be necessary, and the compound moves contribute to fulfill this goal.

The use of implicit rotations enhances the solution quality for both ILS and HGSADC, on both CVRP and MDVRP experiments. The largest improvement is again observed on the MDVRP with ILS, with an average gap decrease from 0.781% to 0.515%. No impact on the computational time is observed on MDVRP experiments for both ILS and HGSADC, and thus the effort spent in evaluating compound moves is paid off in terms of convergence speed. For the CVRP, the computational time increases by a factor of two. However, the current HGSADC+ implementation was derived from an existing code, and "skips" depots when evaluating routes using IF instructions. A complete new implementation without any mention of depots may run much faster. Implicit rotation management also lead to LS simplifications, by enabling to define moves only between sequences of visits to customers, thus avoiding to deal with special cases related to depots.

Table 6.1: Impact of implicit rotations within ILS – CVRP instances

Inst	n	**VCGLR12**	**NB09**	**GGW11**	**ILS-noR**			**ILS+**			**BKS**
		Avg 10	Avg 10	Best 5	Avg 10	Best 10	T(min)	Avg 10	Best 10	T(min)	
A-p01	50	**524.61**	**524.61**	**524.61**	**524.61**	**524.61**	0.89	**524.61**	**524.61**	1.90	524.61
A-p02	75	**835.26**	835.61	**835.26**	836.16	835.32	2.25	835.67	**835.26**	4.56	835.26
A-p03	100	**826.14**	**826.14**	**826.14**	826.78	**826.14**	4.27	826.26	**826.14**	9.30	826.14
A-p04	150	**1028.42**	**1028.42**	**1028.42**	1034.18	1031.96	8.11	1033.39	1031.29	16.87	1028.42
A-p05	199	1294.06	1291.84	1291.50	1315.50	1306.95	11.55	1313.98	1303.45	24.89	1291.29
A-p06	50	**555.43**	**555.43**	**555.43**	**555.43**	**555.43**	1.16	**555.43**	**555.43**	1.91	555.43
A-p07	75	**909.68**	910.41	**909.68**	**909.68**	**909.68**	3.18	**909.68**	**909.68**	3.47	909.68
A-p08	100	**865.94**	**865.94**	**865.94**	**865.94**	**865.94**	3.76	**865.94**	**865.94**	6.29	865.94
A-p09	150	**1162.55**	1162.56	**1162.55**	1166.28	1165.44	9.82	1166.70	1164.11	15.60	1162.55
A-p10	199	1400.23	1398.30	1399.91	1418.05	1416.58	14.60	1416.41	1412.91	24.79	1395.85
A-p11	120	**1042.11**	**1042.11**	**1042.11**	**1042.11**	**1042.11**	5.15	**1042.11**	**1042.11**	11.62	1042.11
A-p12	100	**819.56**	**819.56**	**819.56**	**819.56**	**819.56**	2.19	**819.56**	**819.56**	4.52	819.56
A-p13	120	1543.07	1542.99	1542.36	1549.56	1545.96	7.24	1549.90	1547.81	12.19	1541.14
A-p14	100	**866.37**	**866.37**	**866.37**	**866.37**	**866.37**	2.86	**866.37**	**866.37**	4.84	866.37
B-pr01	240	5627.00	5632.05	5636.96	5647.13	5644.44	22.89	5648.59	5644.42	46.95	5623.47
B-pr02	320	8446.65	8440.25	8447.92	8458.11	8452.72	39.27	8460.78	8451.05	85.09	8404.61
B-pr03	400	**11036.22**	**11036.22**	**11036.22**	11056.98	11041.02	59.85	11056.85	11043.41	120.78	11036.22
B-pr04	480	13624.53	13618.55	13624.52	13646.49	13632.39	85.03	13660.90	13642.33	165.73	13592.88
B-pr05	200	**6460.98**	**6460.98**	**6460.98**	**6460.98**	**6460.98**	13.19	**6460.98**	**6460.98**	28.19	6460.98
B-pr06	280	8412.90	8413.41	8412.90	8413.46	8413.36	24.33	8413.78	8413.36	58.46	8400.33
B-pr07	360	10157.63	10186.93	10195.59	10201.16	10195.59	44.44	10204.74	10195.59	103.81	10102.68
B-pr08	440	11646.58	11691.54	11691.76	11814.54	11734.96	66.48	11847.72	11743.37	142.68	11635.34
B-pr09	255	581.79	581.46	581.92	594.23	593.01	19.61	594.81	592.63	36.12	579.71
B-pr10	323	739.86	739.56	739.82	758.47	757.27	25.85	757.93	756.25	77.97	736.26
B-pr11	399	916.44	916.27	916.14	940.97	936.87	39.64	941.03	936.98	105.76	912.84
B-pr12	483	1106.73	1108.21	1112.73	1141.83	1139.49	62.55	1140.38	1136.28	144.35	1102.69
B-pr13	252	859.64	858.42	858.45	880.25	878.14	16.77	877.55	875.82	32.21	857.19
B-pr14	320	1082.41	1080.84	**1080.55**	1112.22	1110.56	23.42	1106.18	1102.59	55.75	1080.55
B-pr15	396	1343.52	1344.32	1341.41	1380.38	1377.45	40.30	1375.84	1374.52	78.75	1337.92
B-pr16	480	1621.02	1622.26	1619.45	1672.93	1668.66	58.34	1666.15	1663.46	114.03	1612.50
B-pr17	240	708.09	707.78	707.79	713.98	712.70	16.49	713.45	712.24	32.10	707.76
B-pr18	300	998.44	995.91	997.25	1019.51	1016.16	24.17	1017.44	1012.10	46.92	995.13
B-pr19	360	1367.83	1366.70	1366.26	1395.49	1392.94	36.62	1394.29	1390.17	81.92	1365.60
B-pr20	420	1822.02	1821.65	1820.88	1871.64	1863.90	56.16	1872.68	1869.40	106.13	1818.32
Gap Set-A		0.047%	0.033%	0.028%	0.363%	0.258%		0.336%	0.215%		
Gap Set-B		0.267%	0.273%	0.296%	1.873%	1.665%		1.795%	1.552%		
Gap All		0.176%	0.174%	0.186%	1.251%	1.086%		1.194%	1.002%		
T(min)		21.57	21.51	8×3.92			25.07			53.13	
CPU		Opt 2.4G	Opt 2.4G	Xeon 2.3G		Opt 2.4G			Opt 2.4G		

Table 6.2: Impact of implicit rotations within HGSADC – CVRP instances

Inst	n	**VCGLR12**	**NB09**	**GGW11**	**HGSADC-noR**			**HGSADC+**			**BKS**
		Avg 10	Avg 10	Best 5	Avg 10	Best 10	T(min)	Avg 10	Best 10	T(min)	
A-p01	50	**524.61**	**524.61**	**524.61**	**524.61**	**524.61**	1.13	**524.61**	**524.61**	2.33	524.61
A-p02	75	**835.26**	835.61	**835.26**	**835.26**	**835.26**	1.83	**835.26**	**835.26**	3.32	835.26
A-p03	100	**826.14**	**826.14**	**826.14**	**826.14**	**826.14**	3.23	**826.14**	**826.14**	7.02	826.14
A-p04	150	**1028.42**	**1028.42**	**1028.42**	1028.56	**1028.42**	5.91	**1028.42**	**1028.42**	12.40	1028.42
A-p05	199	1294.06	1291.84	1291.50	1293.88	1291.45	10.38	1292.17	1291.45	22.11	1291.29
A-p06	50	**555.43**	**555.43**	**555.43**	**555.43**	**555.43**	1.29	**555.43**	**555.43**	2.26	555.43
A-p07	75	**909.68**	910.41	**909.68**	**909.68**	**909.68**	2.29	**909.68**	**909.68**	3.41	909.68
A-p08	100	**865.94**	**865.94**	**865.94**	**865.94**	**865.94**	3.75	**865.94**	**865.94**	6.67	865.94
A-p09	150	**1162.55**	1162.56	**1162.55**	**1162.55**	**1162.55**	6.60	**1162.55**	**1162.55**	11.46	1162.55
A-p10	199	1400.23	1398.30	1399.91	1399.62	1396.54	17.17	1398.12	**1395.85**	28.33	1395.85
A-p11	120	**1042.11**	**1042.11**	**1042.11**	**1042.11**	**1042.11**	4.29	**1042.11**	**1042.11**	10.34	1042.11
A-p12	100	**819.56**	**819.56**	**819.56**	**819.56**	**819.56**	2.41	**819.56**	**819.56**	5.20	819.56
A-p13	120	1543.07	1542.99	1542.36	1542.86	1542.86	5.79	1542.52	**1541.14**	10.17	1541.14
A-p14	100	**866.37**	**866.37**	**866.37**	**866.37**	**866.37**	3.41	**866.37**	**866.37**	5.83	866.37
B-pr01	240	5627.00	5632.05	5636.96	5625.75	**5623.47**	25.65	5625.22	**5623.47**	62.00	5623.47
B-pr02	320	8446.65	8440.25	8447.92	8447.92	8447.92	39.04	8444.29	8413.82	103.07	8404.61
B-pr03	400	**11036.22**	**11036.22**	**11036.22**	11051.23	**11036.22**	64.80	**11036.22**	**11036.22**	151.02	11036.22
B-pr04	480	13624.53	13618.55	13624.52	13645.38	13624.53	93.06	13645.38	13624.53	174.19	13592.88
B-pr05	200	**6460.98**	**6460.98**	**6460.98**	**6460.98**	**6460.98**	16.39	**6460.98**	**6460.98**	34.00	6460.98
B-pr06	280	8412.90	8413.41	8412.90	8412.90	8412.90	29.32	8412.90	8412.90	62.69	8400.33
B-pr07	360	10157.63	10186.93	10195.59	10182.45	10115.58	54.07	10168.95	10141.06	130.77	10102.68
B-pr08	440	11646.58	11691.54	11691.76	11649.45	**11635.34**	78.31	11640.99	**11635.34**	161.10	11635.34
B-pr09	255	581.79	581.46	581.92	581.86	**579.71**	28.70	581.93	580.50	91.74	579.71
B-pr10	323	739.86	739.56	739.82	739.01	737.43	81.40	739.78	737.65	151.96	736.26
B-pr11	399	916.44	916.27	916.14	915.41	913.69	111.23	915.87	913.15	230.69	912.84
B-pr12	483	1106.73	1108.21	1112.73	1107.36	1104.96	177.02	1106.99	1105.24	267.33	1102.69
B-pr13	252	859.64	858.42	858.45	859.74	**857.19**	20.11	859.99	858.39	52.81	857.19
B-pr14	320	1082.41	1080.84	**1080.55**	1082.33	**1080.55**	29.25	1081.68	**1080.55**	55.16	1080.55
B-pr15	396	1343.52	1344.32	1341.41	1343.87	1340.62	60.73	1341.68	1339.75	159.65	1337.92
B-pr16	480	1621.02	1622.26	1619.45	1619.53	1616.09	98.69	1618.65	1616.43	226.49	1612.50
B-pr17	240	708.09	707.78	707.79	707.95	707.79	15.98	707.93	707.79	35.04	707.76
B-pr18	300	998.44	995.91	997.25	997.26	**995.13**	35.10	998.32	997.25	66.46	995.13
B-pr19	360	1367.83	1366.70	1366.26	1367.20	1366.48	52.51	1366.95	1366.40	100.58	1365.60
B-pr20	420	1822.02	1821.65	1820.88	1821.09	1819.59	87.84	1821.88	1820.45	173.46	1818.32
Gap Set-A		0.047%	0.033%	0.028%	0.043%	0.012%		0.023%	0.001%		
Gap Set-B		0.267%	0.273%	0.296%	0.272%	0.102%		0.253%	0.119%		
Gap All		0.176%	0.174%	0.186%	0.178%	0.065%		0.158%	0.070%		
T(min)		21.57	21.51	8×3.92			37.31			77.09	
CPU		Opt 2.4G	Opt 2.4G	Xeon 2.3G		Opt 2.4G			Opt 2.4G		

Table 6.3: Impact of implicit rotations and depot choices within ILS – MDVRP instances.

Inst	n	d	**PR07**	**VCGLR12**	**S12**	**ILS-noRD**			**ILS-noR**			**ILS+**			**BKS**
			Avg 10	Avg 10	Avg 10	Avg 10	Best 10	T(min)	Avg 10	Best 10	T(min)	Avg 10	Best 10	T(min)	
C-p01	50	4	**576.87**	**576.87**	**576.87**	**576.87**	**576.87**	0.53	**576.87**	**576.87**	0.50	**576.87**	**576.87**	0.49	576.87*
C-p02	50	4	**473.53**	**473.53**	**473.53**	**473.53**	**473.53**	0.81	473.67	**473.53**	1.21	473.63	**473.53**	1.41	473.53*
C-p03	75	2	*641.19*	*641.19*	*641.19*	641.19	641.19	0.96	641.08	**640.65**	0.99	641.08	**640.65**	1.00	640.65*
C-p04	100	2	*1006.09*	*1001.24*	*1001.04*	1001.84	**999.21**	2.57	1002.12	**999.21**	1.88	1000.01	**999.21**	1.81	999.21*
C-p05	100	2	752.34	**750.03**	750.21	**750.03**	**750.03**	2.09	750.92	**750.03**	2.84	750.72	**750.03**	2.81	750.03
C-p06	100	3	883.01	**876.50**	**876.50**	878.65	**876.50**	2.06	879.82	**876.50**	1.63	877.43	**876.50**	1.60	876.50*
C-p07	100	4	889.36	882.24	**881.97**	886.85	884.66	2.29	886.72	**881.97**	1.72	885.71	**881.97**	1.71	881.97*
C-p08	249	2	4421.03	4397.71	4393.70	4426.63	4401.79	19.65	4424.72	4401.17	15.54	4392.96	4382.91	14.85	4372.78
C-p09	249	3	3892.50	3863.45	3864.22	3903.12	3891.91	19.30	3899.90	3882.66	14.69	3889.82	3878.25	12.75	3858.66
C-p10	249	4	3666.85	3634.03	3634.72	3679.20	3660.30	19.92	3657.40	3635.52	14.43	3651.73	3635.71	12.45	3631.11
C-p11	249	5	3573.23	3546.95	3546.15	3588.35	3560.27	19.52	3586.44	3565.95	14.73	3574.43	3557.57	12.94	3546.06
C-p12	80	2	1319.13	**1318.95**	**1318.95**	**1318.95**	**1318.95**	1.09	**1318.95**	**1318.95**	1.35	**1318.95**	**1318.95**	1.51	1318.95*
C-p13	80	2	**1318.95**	**1318.95**	**1318.95**	**1318.95**	**1318.95**	0.98	**1318.95**	**1318.95**	1.24	**1318.95**	**1318.95**	1.23	1318.95
C-p14	80	2	**1360.12**	**1360.12**	**1360.12**	**1360.12**	**1360.12**	0.98	**1360.12**	**1360.12**	1.34	**1360.12**	**1360.12**	1.17	1360.12
C-p15	160	4	2519.64	**2505.42**	**2505.42**	**2505.42**	**2505.42**	4.18	2507.46	**2505.42**	3.82	**2505.42**	**2505.42**	3.25	2505.42
C-p16	160	4	2573.95	**2572.23**	**2572.23**	**2572.23**	**2572.23**	3.32	**2572.23**	**2572.23**	3.15	**2572.23**	**2572.23**	3.17	2572.23
C-p17	160	4	**2709.09**	**2709.09**	2710.21	2716.89	**2709.09**	3.43	2711.32	**2709.09**	3.48	2710.21	**2709.09**	3.03	2709.09
C-p18	240	6	3736.53	**3702.85**	**3702.85**	3713.02	**3702.85**	13.98	3731.55	3714.56	9.82	3711.32	**3702.85**	10.32	3702.85
C-p19	240	6	3838.76	**3827.06**	3827.55	3828.29	**3827.06**	7.95	3834.44	**3827.06**	7.06	3831.98	**3827.06**	7.32	3827.06
C-p20	240	6	4064.76	**4058.07**	**4058.07**	4064.76	**4058.07**	9.26	4088.92	**4058.07**	8.70	4069.80	**4058.07**	8.02	4058.07
C-p21	360	9	5501.58	**5474.84**	**5474.84**	5525.93	5490.11	20.00	5540.82	5506.26	32.59	5530.58	5496.40	20.00	5474.84
C-p22	360	9	5722.19	**5702.16**	5705.84	5719.73	**5702.16**	19.66	5724.26	5714.46	19.99	5717.64	**5702.16**	18.63	5702.16
C-p23	360	9	6092.66	**6078.75**	**6078.75**	6118.29	**6078.75**	20.00	6129.08	6112.46	30.04	6124.99	6112.46	20.00	6078.75
D-pr01	48	4	**861.32**	**861.32**	**861.32**	**861.32**	**861.32**	1.02	**861.32**	**861.32**	1.18	**861.32**	**861.32**	1.12	861.32
D-pr02	96	4	*1308.17*	*1307.34*	*1308.53*	1299.08	1297.44	3.96	**1296.25**	**1296.25**	3.03	**1296.25**	**1296.25**	2.82	1296.25
D-pr03	144	4	1810.66	1803.80	1804.09	1804.55	**1803.80**	6.61	1803.81	**1803.80**	5.79	1803.81	**1803.80**	5.66	1803.80
D-pr04	192	4	*2073.16*	*2058.31*	*2060.93*	2054.15	**2042.45**	11.41	2047.99	**2042.45**	8.13	2044.26	**2042.45**	8.09	2042.45
D-pr05	240	4	*2350.31*	*2335.81*	*2338.12*	2377.66	2342.77	20.00	2333.43	2326.50	12.77	2330.19	2326.35	11.41	2324.12
D-pr06	288	4	*2695.74*	*2680.95*	*2685.23*	2684.11	2675.71	20.00	2679.96	2673.00	18.72	2670.77	2668.76	18.06	2663.56
D-pr07	72	6	*1089.56*	*1089.56*	*1089.56*	1075.53	**1075.12**	1.85	**1075.12**	**1075.12**	1.64	**1075.12**	**1075.12**	1.55	1075.12
D-pr08	144	6	*1675.74*	*1664.99*	*1665.08*	1667.48	1660.16	6.44	1660.21	1658.71	4.32	1658.68	**1658.23**	4.30	1658.23
D-pr09	216	6	*2144.84*	*2133.52*	*2135.37*	2159.27	2147.68	18.88	2144.57	2136.67	8.88	2139.08	**2131.70**	8.85	2131.70
D-pr10	288	6	*2905.43*	*2885.39*	*2882.41*	2861.08	2826.91	20.00	2819.33	2812.39	20.38	2815.00	2810.25	17.96	2805.53
E-pr11	360	4	–	–	–	5144.80	5096.71	164.31	5071.03	5029.61	70.36	5026.89	4999.04	72.89	4994.67
E-pr12	480	4	–	–	–	6593.86	6522.94	281.29	6446.22	6424.10	149.53	6418.54	6391.48	168.58	6367.67
E-pr13	600	4	–	–	–	8100.34	8054.60	300.02	7791.39	7746.68	261.83	7743.51	7689.19	293.95	7645.29
E-pr14	720	4	–	–	–	9681.79	9588.94	300.03	9279.51	9211.41	300.02	9239.96	9184.68	300.06	9101.67
E-pr15	840	4	–	–	–	11390.61	11235.84	300.03	10817.77	10782.85	300.02	10762.94	10726.17	300.03	10598.70
E-pr16	960	4	–	–	–	13025.41	12720.01	300.04	12196.66	12143.63	300.03	12128.21	12062.69	300.02	11919.71
E-pr17	360	6	–	–	–	4901.09	4880.87	172.32	4817.03	4797.91	68.73	4792.53	4772.62	80.47	4761.70
E-pr18	520	6	–	–	–	6869.04	6770.66	300.02	6585.97	6536.84	186.31	6562.69	6531.79	176.90	6504.36
E-pr19	700	6	–	–	–	9362.54	9170.60	300.03	8804.79	8770.25	300.02	8775.67	8736.22	300.01	8639.44
E-pr20	880	6	–	–	–	10730.24	10553.01	300.03	10040.07	10000.46	300.02	9980.76	9946.18	300.03	9825.50
E-pr21	420	12	–	–	–	4839.75	4712.22	295.36	4632.10	4613.14	99.47	4605.51	4596.09	109.73	4582.62
E-pr22	600	12	–	–	–	6651.39	6466.61	300.03	6230.97	6193.52	283.34	6207.59	6181.67	264.14	6141.63
E-pr23	780	12	–	–	–	9041.86	8778.31	300.04	8176.64	8119.01	300.01	8128.24	8078.99	300.02	8014.10
E-pr24	960	12	–	–	–	11547.32	11020.10	300.06	10134.47	10067.97	300.03	10054.52	10040.63	300.04	9909.49
Gap/T Set-C			0.413%	0.049%	0.049%	0.378%	0.148%	8.46	0.429%	0.157%	8.38	0.268%	0.093%	7.02	
Gap/T Set-D			1.171%	0.747%	0.795%	0.778%	0.298%	11.02	0.250%	0.096%	8.48	0.133%	0.046%	7.98	
Gap/T Set-E			–	–	–	7.501%	5.495%	279.54	1.740%	1.193%	229.98	1.194%	0.745%	233.35	
Gap/T All			–	–	–	2.585%	1.772%	89.75	0.781%	0.453%	74.41	0.515%	0.277%	74.64	
T(min) Set-CD			3.95	5.18	10.45	–	–	9.23	–	–	8.41	–	–	7.31	
CPU			P-IV 3G	Opt 2.4G	I7 2.9G	Opt 2.2 & Opt 2.4G			Opt 2.2 & Opt 2.4G			Opt 2.2 & Opt 2.4G			

Table 6.4: Impact of implicit rotations and depot choices within HGSADC – MDVRP instances.

Inst	n	d	**PR07**	**VCGLR12**	**S12**	**HGSADC-noRD**			**HGSADC-noR**			**HGSADC+**			**BKS**
			Avg 10	Avg 10	Avg 10	Avg 10	Best 10	T(min)	Avg 10	Best 10	T(min)	Avg 10	Best 10	T(min)	
C-p01	50	4	**576.87**	**576.87**	**576.87**	**576.87**	**576.87**	0.65	**576.87**	**576.87**	1.10	**576.87**	**576.87**	1.09	576.87*
C-p02	50	4	**473.53**	**473.53**	**473.53**	**473.53**	**473.53**	0.90	**473.53**	**473.53**	1.77	**473.53**	**473.53**	1.69	473.53*
C-p03	75	2	*641.19*	*641.19*	*641.19*	640.92	**640.65**	1.47	**640.65**	**640.65**	2.56	**640.65**	**640.65**	2.53	640.65*
C-p04	100	2	*1006.09*	*1001.24*	*1001.04*	1000.18	**999.21**	3.48	1001.04	1001.04	3.59	1000.66	**999.21**	3.43	999.21*
C-p05	100	2	752.34	**750.03**	750.21	**750.03**	**750.03**	2.84	**750.03**	**750.03**	5.12	**750.03**	**750.03**	4.89	750.03
C-p06	100	3	883.01	**876.50**	**876.50**	**876.50**	**876.50**	1.99	**876.50**	**876.50**	3.31	**876.50**	**876.50**	3.02	876.50*
C-p07	100	4	889.36	882.24	**881.97**	883.83	**881.97**	3.02	**881.97**	**881.97**	3.75	**881.97**	**881.97**	3.75	881.97*
C-p08	249	2	4421.03	4397.71	4393.70	4386.59	4384.15	17.83	4382.51	4375.49	26.36	4383.63	4375.49	19.93	4372.78
C-p09	249	3	3892.50	3863.45	3864.22	3864.85	3859.17	16.48	3865.11	3859.76	27.73	3860.77	3859.17	19.51	3858.66
C-p10	249	4	3666.85	3634.03	3634.72	3639.61	**3631.11**	18.43	3631.71	**3631.11**	23.54	3631.71	**3631.11**	17.71	3631.11
C-p11	249	5	3573.23	3546.95	3546.15	3553.28	**3546.06**	13.13	3547.47	**3546.06**	17.31	3547.37	**3546.06**	17.14	3546.06
C-p12	80	2	1319.13	**1318.95**	**1318.95**	**1318.95**	**1318.95**	1.57	**1318.95**	**1318.95**	2.77	**1318.95**	**1318.95**	2.85	1318.95*
C-p13	80	2	**1318.95**	**1318.95**	**1318.95**	**1318.95**	**1318.95**	1.91	**1318.95**	**1318.95**	2.94	**1318.95**	**1318.95**	2.85	1318.95
C-p14	80	2	**1360.12**	**1360.12**	**1360.12**	**1360.12**	**1360.12**	1.88	**1360.12**	**1360.12**	2.38	**1360.12**	**1360.12**	2.53	1360.12
C-p15	160	4	2519.64	**2505.42**	**2505.42**	**2505.42**	**2505.42**	4.42	**2505.42**	**2505.42**	7.66	**2505.42**	**2505.42**	7.79	2505.42
C-p16	160	4	2573.95	**2572.23**	**2572.23**	**2572.23**	**2572.23**	5.28	**2572.23**	**2572.23**	7.71	**2572.23**	**2572.23**	7.75	2572.23
C-p17	160	4	**2709.09**	**2709.09**	2710.21	**2709.09**	**2709.09**	5.27	**2709.09**	**2709.09**	8.29	**2709.09**	**2709.09**	8.31	2709.09
C-p18	240	6	3736.53	**3702.85**	**3702.85**	**3702.85**	**3702.85**	8.81	**3702.85**	**3702.85**	13.48	**3702.85**	**3702.85**	14.01	3702.85
C-p19	240	6	3838.76	**3827.06**	3827.55	**3827.06**	**3827.06**	9.90	**3827.06**	**3827.06**	15.34	**3827.06**	**3827.06**	15.11	3827.06
C-p20	240	6	4064.76	**4058.07**	**4058.07**	**4058.07**	**4058.07**	9.55	**4058.07**	**4058.07**	16.55	**4058.07**	**4058.07**	16.20	4058.07
C-p21	360	9	5501.58	**5474.84**	**5474.84**	5476.36	**5474.84**	19.83	**5474.84**	**5474.84**	29.18	**5474.84**	**5474.84**	20.07	5474.84
C-p22	360	9	5722.19	**5702.16**	5705.84	**5702.16**	**5702.16**	19.94	**5702.16**	**5702.16**	33.19	**5702.16**	**5702.16**	20.00	5702.16
C-p23	360	9	6092.66	**6078.75**	**6078.75**	**6078.75**	**6078.75**	19.97	**6078.75**	**6078.75**	38.69	6080.43	**6078.75**	20.00	6078.75
D-pr01	48	4	**861.32**	**861.32**	**861.32**	**861.32**	**861.32**	1.12	**861.32**	**861.32**	2.10	**861.32**	**861.32**	2.05	861.32
D-pr02	96	4	*1308.17*	*1307.34*	*1308.53*	**1296.25**	**1296.25**	2.70	**1296.25**	**1296.25**	4.74	**1296.25**	**1296.25**	4.67	1296.25
D-pr03	144	4	1810.66	1803.80	1804.09	**1803.80**	**1803.80**	4.93	**1803.80**	**1803.80**	8.54	**1803.80**	**1803.80**	8.84	1803.80
D-pr04	192	4	*2073.16*	*2058.31*	*2060.93*	2043.09	**2042.45**	10.99	**2042.45**	**2042.45**	13.24	**2042.45**	**2042.45**	12.84	2042.45
D-pr05	240	4	*2350.31*	*2335.81*	*2338.12*	2325.91	**2324.12**	16.40	2326.30	**2324.12**	25.88	2325.09	**2324.12**	19.51	2324.12
D-pr06	288	4	*2695.74*	*2680.95*	*2685.23*	2664.85	**2663.56**	19.26	2663.88	**2663.56**	34.66	2664.57	**2663.56**	20.00	2663.56
D-pr07	72	6	*1089.56*	*1089.56*	*1089.56*	**1075.12**	**1075.12**	1.78	**1075.12**	**1075.12**	3.01	**1075.12**	**1075.12**	3.04	1075.12
D-pr08	144	6	*1675.74*	*1664.99*	*1665.08*	1658.52	**1658.23**	6.36	**1658.23**	**1658.23**	7.73	**1658.23**	**1658.23**	7.85	1658.23
D-pr09	216	6	*2144.84*	*2133.52*	*2135.37*	2132.70	**2131.70**	10.39	2132.96	**2131.70**	17.23	2132.95	**2131.70**	15.65	2131.70
D-pr10	288	6	*2905.43*	*2885.39*	*2882.41*	2809.72	2807.17	18.60	2808.19	**2805.53**	38.64	2808.63	2807.11	20.00	2805.53
E-pr11	360	4	–	–	–	5018.62	**4994.67**	73.02	5001.94	**4994.67**	88.13	5004.67	**4994.67**	81.90	4994.67
E-pr12	480	4	–	–	–	6396.26	6371.96	127.92	6392.27	**6367.67**	109.81	6389.08	6375.87	139.18	6367.67
E-pr13	600	4	–	–	–	7683.73	7651.43	250.91	7668.48	**7645.29**	215.08	7668.70	7648.71	175.03	7645.29
E-pr14	720	4	–	–	–	9185.56	9163.63	293.97	9136.79	9106.37	294.81	9129.58	**9101.67**	296.51	9101.67
E-pr15	840	4	–	–	–	10682.96	10614.31	300.44	10627.80	10607.65	300.11	10638.93	**10598.70**	296.53	10598.70
E-pr16	960	4	–	–	–	12071.44	12016.08	301.22	11948.52	**11919.71**	300.38	11948.39	11921.00	300.32	11919.71
E-pr17	360	6	–	–	–	4772.85	**4761.70**	77.00	4765.72	4762.19	73.01	4766.29	**4761.70**	73.10	4761.70
E-pr18	520	6	–	–	–	6539.55	6504.81	158.53	6518.19	6504.72	192.22	6517.60	6504.62	162.11	6504.36
E-pr19	700	6	–	–	–	8707.25	8667.11	290.42	8669.38	8641.05	296.36	8669.61	**8639.44**	296.34	8639.44
E-pr20	880	6	–	–	–	9883.17	9834.39	300.74	9845.57	9826.77	300.28	9845.52	**9825.50**	300.26	9825.50
E-pr21	420	12	–	–	–	4603.04	4583.67	75.69	4595.10	**4582.62**	88.00	4596.54	4586.41	88.90	4582.62
E-pr22	600	12	–	–	–	6183.22	6154.31	198.82	6162.68	6147.01	207.01	6157.81	**6141.63**	233.83	6141.63
E-pr23	780	12	–	–	–	8101.79	8050.99	289.35	8037.81	8021.97	283.35	8032.95	**8014.10**	294.29	8014.10
E-pr24	960	12	–	–	–	10049.78	10004.94	300.01	9938.18	**9909.49**	300.48	9926.27	9910.02	300.57	9909.49
Gap/T Set-C			0.413%	0.049%	0.049%	0.056%	0.012%	8.20	0.027%	0.012%	12.80	0.023%	0.003%	10.09	
Gap/T Set-D			1.171%	0.747%	0.795%	0.037%	0.006%	9.25	0.026%	0.000%	15.58	0.025%	0.006%	11.45	
Gap/T Set-E			–	–	–	0.729%	0.275%	217.00	0.270%	0.026%	217.79	0.257%	0.020%	217.06	
Gap/T All			–	–	–	0.252%	0.089%	70.62	0.100%	0.014%	74.45	0.093%	0.009%	72.03	
T(min) Set-CD			3.95	5.18	10.45	–	–	8.52	–	–	13.64	–	–	10.50	
CPU			P-IV 3G	Opt 2.4G	I7 2.9G	Opt 2.2 & Opt 2.4G			Opt 2.2 & Opt 2.4G			Opt 2.2 & Opt 2.4G			

Table 6.5: Performance of HGSADC – MDVFMP instances

Inst	n	d	**SS97**	**HGSADC+**			**BKS**
			–	Avg 10	Best 10	T(min)	
C-p01	50	4	1526.7	**1477.73**	**1477.73**	1.87	1477.73
C-p02	50	4	992.8	**957.73**	**957.73**	2.44	957.73
C-p03	75	2	1611.1	**1569.67**	**1569.67**	3.37	1569.67
C-p04	100	2	2361.9	**2292.64**	**2292.64**	4.64	2292.64
C-p05	100	2	1498.4	**1453.64**	**1453.64**	7.42	1453.64
C-p06	100	3	2277.5	**2208.66**	**2208.66**	5.34	2208.66
C-p07	100	4	2297.1	**2198.91**	**2198.91**	5.19	2198.91
C-p08	249	2	6718.6	6448.26	**6441.36**	20.00	6441.36
C-p09	249	3	6211.4	6021.41	**5998.70**	20.00	5998.70
C-p10	249	4	6018.7	5817.81	**5807.53**	20.00	5807.53
C-p11	249	5	6030.8	5773.28	**5770.42**	19.74	5770.42
C-p12	80	2	2108.2	**2072.18**	**2072.18**	3.60	2072.18
C-p13	80	2	2126.8	**2096.39**	**2096.39**	3.56	2096.39
C-p14	80	2	**2160.1**	**2160.12**	**2160.12**	4.18	2160.12
C-p15	160	4	4116.2	**3973.47**	**3973.47**	9.61	3973.47
C-p16	160	4	4178.9	**4119.76**	**4119.76**	9.93	4119.76
C-p17	160	4	4344.1	4323.09	**4309.09**	13.95	4309.09
C-p18	240	6	6217.0	**5887.43**	**5887.43**	19.80	5887.43
C-p19	240	6	6233.6	**6130.36**	**6130.36**	19.47	6130.36
C-p20	240	6	6493.1	6481.23	**6469.21**	20.00	6469.21
C-p21	360	9	9184.6	8710.75	**8709.26**	20.00	8709.26
C-p22	360	9	9332.0	9164.65	**9151.64**	20.00	9151.64
C-p23	360	9	9706.6	9728.87	**9714.41**	20.00	9714.41
D-pr01	48	4	–	**1181.47**	**1181.47**	2.09	1181.47
D-pr02	96	4	–	**1901.39**	**1901.39**	6.81	1901.39
D-pr03	144	4	–	**2712.71**	**2712.71**	9.97	2712.71
D-pr04	192	4	–	3371.35	**3370.85**	24.15	3370.85
D-pr05	240	4	–	4068.98	**4066.52**	28.20	4066.52
D-pr06	288	4	–	4677.35	**4669.16**	62.08	4669.16
D-pr07	72	6	–	**1550.87**	**1550.87**	3.56	1550.87
D-pr08	144	6	–	**2705.46**	**2705.46**	10.73	2705.46
D-pr09	216	6	–	3642.57	**3637.39**	28.84	3637.39
D-pr10	288	6	–	4980.33	**4973.74**	48.63	4973.74
E-pr11	360	4	–	7342.40	**7323.10**	150.65	7323.10
E-pr12	480	4	–	9472.47	**9436.13**	251.09	9436.13
E-pr13	600	4	–	11567.96	**11526.27**	291.36	11526.27
E-pr14	720	4	–	13824.41	**13778.18**	300.01	13778.18
E-pr15	840	4	–	16393.71	**16352.74**	301.06	16352.74
E-pr16	960	4	–	18509.70	**18471.52**	300.46	18471.52
E-pr17	360	6	–	7048.26	**7025.01**	168.08	7025.01
E-pr18	520	6	–	9810.34	**9775.42**	245.35	9775.42
E-pr19	700	6	–	13312.76	**13280.04**	301.52	13280.04
E-pr20	880	6	–	15683.36	**15631.64**	301.52	15631.64
E-pr21	420	12	–	7087.02	**7072.79**	170.80	7072.79
E-pr22	600	12	–	9804.69	**9776.65**	288.99	9776.65
E-pr23	780	12	–	12878.26	**12840.99**	301.05	12840.99
E-pr24	960	12	–	16360.55	**16343.33**	300.02	16343.33
Gap/T Set-C			2.769%	0.067%	0.000%	11.92	
Gap/T Set-D			–	0.053%	0.000%	22.51	
Gap/T Set-E			–	0.282%	0.000%	262.28	
Gap/T All			–	0.128%	0.000%	88.75	
T(min) Set-C			4.10	–	–	11.92	
CPU			VAX 4000	Opt 2.2 & Opt 2.4G			

These experiments finally illustrate the good performance of the proposed HGSADC+ meta-heuristic, which matches or outperforms other current state-of-the-art CVRP and MDVRP with an average gap to BKS of 0.158% for the CVRP, and 0.093% for the MDVRP. In all cases, HGSADC variants produce solutions of better quality than ILS. As a counterpart, ILS is simpler and in general faster. All known optimal solutions have been retrieved. Run times remains moderate on medium-scale instances, of a magnitude comparable to those of other methods in the literature. The standard deviation of solution costs from HGSADC+ is small (0.069%, 0.071% for the CVRP and MDVRP), thus showing that high-quality solutions are produced in a consistent manner.

6.8.2 Addressing a rich problem – the MDVFMP

A second set of experiments has been conducted to investigate the performance of the proposed HGSADC+ on a rich vehicle routing variant, the MDVFMP. The literature is scarce on this particular problem. We compare the proposed method to the variable neighborhood heuristic of Salhi and Sari (1997) (SS97) and to the best known solution founds during multiple runs. Table 6.5 reports the results of the methods on the modified MDVRP instances, using the same format as previously.

As highlighted in Table 6.5, major solution improvements were still achievable on these instances. The average gap obtained with HGSADC+ is 0.067%, compared to 2.769% for SS97. Still, it should be noted that SS97 was executed on an old system and processor, and that extended runs on modern computers may lead to decreased gaps. Nevertheless, HGSADC+ demonstrate its ability to find the best known solutions in a consistent manner on this difficult problem: for 14/23 instances of set C with 50 to 240 customers the best known solution has been reached on all 10 runs, and the overall standard deviation of solution costs remains very small (0.089%).

6.9 Conclusions

In this paper, an efficient dynamic programming methodology was introduced for managing compound customer-to-depots assignments, rotations and vehicles choices within neighborhood searches for vehicle routing. Two meta-heuristics based on these concepts, an ILS and a HGSADC, have been proposed. These approaches produce solutions of remarkable quality on classic CVRP, MDVRP and MDVFMP benchmark instances with unlimited fleet. Extensive experiments demonstrate the notable contribution of the proposed implicit depot management to the search performance. The implicit rotations have a smaller but noticeable impact, and may simplify several aspects of LS implementations. The proposed methodology is general, and broadly applicable to many VRP variants.

Promising avenues of research involves generalizing the approach to other multi-attribute VRPs, possibly with limited fleet. Finally, this research is part of a general effort aiming to identify efficiently manageable subproblems to reduce the size of solution spaces, and similar subproblems and management methods may be investigated on other VRP variants and combinatorial optimization settings.

Troisième partie

RÉSOLUTION DE PROBLÈMES RICHES ET APPROCHES GÉNÉRALISTES

CHAPITRE 7

PROBLÈMES DE TOURNÉES DE VÉHICULES ET D'HORAIRES DE CONDUCTEURS

7.1 Fil conducteur et contributions

Les derniers chapitres de cette thèse se consacrent à la résolution de VRP *riches* combinant de nombreux attributs et au développement de nouvelles méthodes heuristiques généralistes pour traiter une grande *variété* de problèmes.

Ce chapitre présente tout d'abord une application de HGSADC au VRTDSP, un problème riche particulièrement difficile, qui requiert la détermination simultanée de routes et de combinaisons de pauses légalement acceptables pour les conducteurs. Les règles de la législation sont très complexes, et ne pas les prendre en compte dans les méthodes de résolution conduirait inévitablement à des solutions irréalistes pour le transport de longue distance. Par ailleurs, le méthodologie proposée dans ce chapitre permet de résoudre le VRTDSP pour une grande variété de législations, permettant ainsi de conduire une comparaison internationale expérimentale de l'impact des règlements gouvernementaux sur la compétitivité économique, les horaires, et la fatigue des conducteurs. Nous montrons en particulier que les règles de l'Union Européenne sont les plus sûres, tandis que les règles du Canada sont les plus avantageuses économiquement. Les règles de l'Australie sont dominées à la fois en termes de coût et sûreté par les autres règles. Finalement, les États-Unis fournissent une alternative intermédiaire entre coût et sureté, les récents changements de règles à venir en 2013 tendant vers plus de sécurité.

7.2 Article VII : Hours of service regulations in road freight transport – an optimization-based international assessment

Un article basé sur ce chapitre a été soumis pour publication : Goel, A., Vidal., T. Hours of service regulations in road freight transport – an optimization-based international assessment. *Transportation Science*, submitted for publication.

Abstract: Driver fatigue is internationally recognized as a significant factor in approximately 15 to 20% of commercial road transport crashes. In their efforts to increase road safety and improve working conditions of truck drivers, governments world wide are enforcing stricter limits on the amount of working and driving time without rest. This paper describes an effective optimization algorithm for minimizing transportation costs for a fleet of vehicles considering business hours of customers and complex hours of service regulations. The algorithm combines the exploration capacities of population-based metaheuristics, the quick improvement abilities of local search, with efficient tree search procedures for checking compliance with hours of service regulations. The proposed approach can be used to assess the impact of different hours of service regulations from a carrier-centric point of view. Extensive computational experiments conducted for various sets of

regulations in the United States, Canada, the European Union, and Australia are conducted to provide an international assessment of the impact of different rules on transportation costs and accident risks. Our experiments demonstrate that European Union rules lead to the highest safety, while Canadian regulations are the most competitive in terms of economic efficiency. Australian regulations appear to have unnecessarily high risk rates with respect to operating costs. The recent rule change in the United States reduces accident risk rates with a moderate increase in operating costs.

Keywords: Hours of Service Regulations; Fatigue; Road Safety; Truck Driver Scheduling; Vehicle Routing and Scheduling.

7.3 Introduction

Driver fatigue is a significant factor in approximately fifteen to twenty percent of commercial road transport crashes (Williamson et al. 2001, European Transport Safety Council 2001, Federal Motor Carrier Safety Administration 2008). In Europe it is estimated that one out of two long haul drivers has fallen asleep while driving (European Transport Safety Council 2001). One out of five long distance road transport drivers in Australia reported at least one fatigue related incident on their last trip and one out of three drivers reported breaking road rules on at least half of their trips (Williamson et al. 2001). A survey among truck drivers in the United States revealed that one out of six truck drivers has dozed at wheel in the month prior to the survey, and less than one out of two truck drivers reported that delivery schedules are always realistic (McCartt et al. 2008). Undoubtedly, fatigue is a threat to road safety and companies must give drivers enough time for breaks and rest periods during their trips.

In their efforts to increase road safety and improve working conditions, governments world wide are adopting stricter regulations concerning driving and working hours of truck drivers. These regulations impose maximum limits on the amount of driving and working within certain time periods and minimum requirements on the number and duration of break and rest periods which must be taken by drivers. Compulsory break and rest periods have a significant impact on total travel durations, which are typically more than twice as long as the pure driving time required in long distance haulage. Consequently, motor carriers must take applicable hours of service regulations into account when generating routes and truck driver schedules. Not doing so would inevitably result in unrealistic schedules, large delays, violations of regulations and reduced road safety.

In this paper, a hybrid genetic algorithm is introduced for the problem of determining a set of routes for a fleet of vehicles, such that each customer is visited within given time windows, that each driver can comply with applicable hours of service regulations, and that transportation costs are minimized. The proposed optimization method is specifically designed to efficiently handle explicit schedule generation during route evaluations and can be applied for various hours of service regulations world wide. Extensive computational experiments on benchmark instances developed for vehicle routing and truck driver scheduling in the European Union demonstrate the remarkable

performance of the method in comparison to previous approaches. In particular, 103/112 best known solutions for these instances were either obtained or improved, and 72/112 were strictly improved. This shows that our approach can provide a valuable tool for transport operators to minimize costs and likewise give drivers enough time for recuperation.

Furthermore, policy makers, unions, and transport companies can use our approach to assess the impact of regulations or agreements to find the best trade-off between road safety, working conditions of truck drivers as well as speed and costs of transportation. In this paper we assess and compare of hours of service regulations in the United States, Canada, the European Union, and Australia with regards to operating costs and accident risks. Specific subsets of rules such as split breaks and rests or extended driving times and reduced rests in the European Union as well as the impact of the recent rule change in the United States are analyzed.

7.4 Hours of service regulations

This section presents the hours of service regulations in the United States, Canada, the European Union, and Australia. For the sake of conciseness only the most important rules for a planning horizon of six days (i.e. Monday to Saturday) are described. For more details about these regulations the reader is referred to Federal Motor Carrier Safety Administration (2011), Transport Canada (2005), European Union (2006, 2002) and National Transport Commission (2008a,b,c).

7.4.1 United States

In December 2011, the Federal Motor Carrier Safety Agency published new hours of service regulations in the United States. These regulations distinguish between *on-duty time* and *off-duty time*. On-duty time refers to all time a driver is working and includes driving activities as well as other work such as loading and unloading. Off-duty time refers to any time during which a driver is not performing any work.

The regulations limit the maximum amount of accumulated driving time between two rest periods to 11 hours. After accumulating 11 hours of driving, the driver must be off duty for 10 consecutive hours before driving again. The regulations prohibit a driver from driving after 14 hours have elapsed since the end of the last rest period. However, a driver may conduct other work after 14 hours have elapsed since the end of the last rest period. Furthermore, a driver must not drive after accumulating 60 hours of on-duty time within a period of 7 days. Alternatively, a driver must not drive after accumulating 70 hours of on-duty time within a period of 8 days. For the sake of conciseness, however, this second option is not considered in the remainder of this paper.

Above rules are the same as in the previous regulations. The new regulations, furthermore, include new rules which will become effective in July 2013. According to these new rules a truck driver must not drive if 8 hours or more have elapsed since the end of the last off-duty period of at least 30 minutes.

7.4.2 Canada

Canadian hours of service regulations are described in Transport Canada (2005) and interpreted in Canadian Council of Motor Transport Administrators (2007). Two sets of regulations exist, one of which applies to driving conducted south of latitude 60° N and one to driving north of latitude 60° N. In this paper we focus on the subset of regulations applicable for driving south of latitude 60° N, because this is the area of major economic concern. On- and off-duty times are defined as described above for U.S. hours of service regulations. The regulations demand that a driver must not drive after accumulating 13 hours of driving time, after accumulating 14 hours of on-duty time, or after 16 hours of time have elapsed since the end of the last period of at least 8 consecutive hours of off-duty time. In any of these cases the driver may only commence driving again after taking another period of at least 8 consecutive hours of off-duty time.

Furthermore, the regulations impose restrictions on the maximum amount of on-duty time and the minimum amount of off-duty time during a *day*. According to the regulations a *day* means a 24-hour period that begins at some time designated by the motor carrier. For simplicity and w.l.o.g. let us assume in the remainder that this time is midnight. The regulations demand that a driver does not drive for more than 13 hours in a day and that a driver accumulates at least 10 hours of off-duty time in a day. At least 2 of these hours must not be part of a period of 8 consecutive hours of off-duty time as required by the provisions described in the previous paragraph. However, if a period of more than 8 consecutive hours of off-duty time is scheduled, the amount exceeding the 8th hour may contribute to these 2 hours. Off-duty periods of less than 30 minutes do not count toward the minimum off-duty time requirements. Eventually, the regulations demand that a driver does not drive after accumulating 70 hours of on-duty time within a period of 7 days.

7.4.3 European Union

In the European Union, truck drivers must comply with regulation (EC) No 561/2006 and the national implementations of Directive 2002/15/EC.

Regulation (EC) No 561/2006 distinguishes between four driver activities: rest periods, breaks, driving time, and other work. Rest periods are periods during which a driver may freely dispose of her or his time and have the purpose of giving drivers enough time to sleep. Breaks are short periods exclusively used for recuperation during which a driver must not carry out any work. Driving time refers to the time during which a driver is operating a vehicle and includes any time during which the vehicle is temporarily stationary due to reasons related to driving, e.g. traffic jams. Other work refers to any work except for driving and includes time spent for loading or unloading, cleaning and technical maintenance, customs, and so on.

Regulation (EC) No 561/2006 demands that a driver takes a break of at least 45 minutes after accumulating $4\frac{1}{2}$ hours of driving. A daily rest period of at least 11 hours must be completed within 24 hours after the end of the previous rest period, and the accumulated driving time between two rest periods shall not exceed 9 hours. Furthermore, the driving time in a week must not exceed 56 hours, and the accumulated driving and working time in a week must not exceed 60 hours.

The basic set of rules described above are sufficient to comply with regulation (EC) No 561/2006. The regulation, furthermore, allows a driver to take break and rest periods in two parts. A break period may be taken in two parts if the first part is a period of at least 15 minutes and the second part is a period of at least 30 minutes. A rest period may be taken in two parts if the first part is a period of at least 3 hours and the second part is a period of at least 9 hours. If a rest period is taken in two parts, the second part must be completed within 24 hours after the end of the previous rest period. Within a planning horizon of one week a driver is allowed to reduce the duration of at most three rest periods to 9 hours. Furthermore, the amount of driving between two rest periods may be extended twice a week to at most 10 hours.

According to Directive 2002/15/EC, a truck driver must not work for more than 6 hours without taking at least 30 minutes of break time. If a truck driver works for more than 9 hours at least 45 minutes of break time must be taken. The break time may be taken in several periods of at least 15 minutes each. The directive, furthermore, applies additional rules for night work. However, these rules not are not considered in the scope of this paper because they differ throughout the member states of the European Union.

7.4.4 Australia

In Australia, motor carriers accredited in the National Heavy Vehicle Accreditation Scheme may operate according to the Basic Fatigue Management Standard (National Transport Commission 2008b). Motor carriers without accreditation must comply with the *standard hours* option of the Australian Heavy Vehicle Driver Fatigue described in National Transport Commission (2008c).

Standard Hours

Australian motor carriers without accreditation must comply with the following constraints on driver schedules:

1. In any period of $5\frac{1}{2}$ hours a driver must not work for more than $5\frac{1}{4}$ hours and must have at least 15 continuous minutes of rest time.

2. In any period of 8 hours a driver must not work for more than $7\frac{1}{2}$ hours and must have at least 30 minutes rest time in blocks of not less than 15 continuous minutes.

3. In any period of 11 hours a driver must not work for more than 10 hours and must have at least 60 minutes rest time in blocks of not less than 15 continuous minutes.

4. In any period of 24 hours a driver must not work for more than 12 hours and must have at least 7 continuous hours of stationary rest time.

5. In any period of 7 days a driver must not work for more than 72 hours and must have at least 24 continuous hours of stationary rest time.

When evaluating whether a truck driver schedule complies with these provisions, the duration of each work period is rounded up to the nearest multiple of 15 minutes and the duration of each rest period is rounded down to the nearest multiple of 15 minutes.

Basic Fatigue Management

Australian motor carriers accredited in the National Heavy Vehicle Accreditation Scheme (NHVAS) may operate according to the Basic Fatigue Management (BFM) option which imposes the following constraints:

1. In any period of $6\frac{1}{4}$ hours a driver must not work for more than 6 hours and must have at least 15 continuous minutes of rest time.

2. In any period of 9 hours a driver must not work for more than $8\frac{1}{2}$ hours and must have at least 30 minutes rest time in blocks of not less than 15 continuous minutes.

3. In any period of 12 hours a driver must not work for more than 11 hours and must have at least 60 minutes rest time in blocks of not less than 15 continuous minutes.

4. In any period of 24 hours a driver must not work for more than 14 hours and must have at least 7 continuous hours of stationary rest time.

5. In any period of 7 days a driver must not accumulate more than 36 hours of long/night work time; the term *long/night work time* refers to any work time in excess of 12 hours in a 24 hour period plus any work time between midnight and 6.00 AM.

The BFM option limits the amount of driving and working to at most 144 hours of work within 14 days. As the accumulated amount of driving and working within a period of 7 days is not explicitly constrained, we will assume a limit of 72 hours in the remainder. The duration of work and rest periods is rounded in the same way as in the standard hours options.

7.4.5 Discussion

It is interesting to see that all regulations have some specific characteristics which make it difficult to analytically compare their impact on road freight transport. Table 7.1 illustrates some of the main characteristics of the different regulations.

	US	**CAN**	**EU (Basic)**	**EU (All)**	**AUS (Std.)**	**AUS (BFM)**
Duration of a long rest period	10	8	11	9	7	7
Driving time between two long rest periods	11	13	9	10	12	14
On-duty time between two long rest periods	14^+	14^+	$12\frac{1}{4}$	$14\frac{1}{4}$	12	14
Time elapsed between two long rest periods	14^+	16^+	13	15	17	17
Driving time within six days	60	70	56	56	72	72
On-duty time within six days	60^+	70^+	60	60	72	72

Table 7.1: Comparison of the regulations

All regulations require long rest periods to be regularly taken. Requirements on when to take these rest periods as well as their minimum duration differ between the regulations. With 11 hours, the longest continuous rest period is required by the basic regulations in the European Union in which rest periods may neither be split nor reduced and driving time may not be extended. When

exploiting all of the rules of the regulations this minimum duration can be reduced to 9 hours. The accumulated amount of driving between two long rest periods differs significantly and ranges from 9 or 10 hours in the European Union to 13 hours in Canada and 14 hours in Australia if the BFM option is used. It is worth noting that, according to the current rules in the United States and in Canada, a driver may drive for the full amount of driving that is allowed between two long rest periods without taking a break. According to the new rules in the United States, as well as in the European Union and Australia, drivers must take short breaks after accumulating a certain amount of driving and/or work time.

Australian regulations do not differentiate between on-duty periods in which the driver is driving or working. Hours of service regulations in the United States and in Canada, on the other hand, do not explicitly limit the amount of on-duty time between rest periods and allow drivers to keep on working when the respective driving time limits are reached. In the table these limits are indicated with a "+". The maximum amounts of driving and working within a period of six days differ significantly between the regulations and, again, European Union regulations have the most restrictive limits.

7.5 Problem statement and related work

As hours of service regulation have a significant impact on travel times, transport companies must consider respective regulations when generating vehicle routes. The resulting decision problem is a variant of vehicle routing problem with time windows (VRPTW). The *vehicle routing and truck driver scheduling problem (VRTDSP)* aims to find a set of routes for a fleet of vehicles, such that each customer requesting service is visited within given time windows, that the accumulated load to be delivered to (or collected from) the customers of a route does not exceed the capacity of the vehicle, that each truck driver can comply with applicable hours of service regulations, and that transportation costs, considered proportional to the travel distance, are minimized.

The VRPTW has attracted a lot of attention in the operations research literature. The most efficient exact methods (Kallehauge et al. 2006, Jepsen et al. 2008, Baldacci et al. 2011c) can solve most instances with up to 100 customers, and a few instances with up to 1000 customers. However, their performance heavily depends upon the specificities of instances and the width of time windows. Hence, metaheuristics are currently the method of choice to address practical settings. In the VRPTW literature, almost every prominent metaheuristic paradigm has been applied, including tabu search (Gendreau et al. 1994, Cordeau et al. 2001a), adaptive large neighborhood search (Pisinger and Ropke 2007), iterated local search (Ibaraki et al. 2005, 2008), genetic algorithms and evolution strategies (Mester and Bräysy 2005, Labadi et al. 2008, Repoussis et al. 2009b, Nagata et al. 2010, Vidal et al. 2013), path relinking (Hashimoto et al. 2008), other metaheuristic hybrids (Prescott-Gagnon et al. 2009), and cooperative and parallel methods (Le Bouthillier and Crainic 2005a,b). A comprehensive review of recent VRPTW heuristics is conducted in Gendreau and Tarantilis (2010). Overall, hybrid methods combining genetic algorithms with local search are well represented in the current state-of-the-art methods (Nagata et al. 2010, Vidal et al. 2013).

The problem of determining whether time window constraints of all customers in a route can be complied with has been studied for long (Savelsbergh 1985, 1992). When using efficient data structures this problem can be solved in $O(1)$ operations for each route determined within the course of a local search approach. A comprehensive overview of vehicle routing variants with time features, including multiple time windows, time-dependent costs and travel times, flexible travel times, etc. is given by Vidal et al. (2011). It is worth noting that for most of these variants the problem of determining adequate service date to customers for a fixed sequence of visits can be modeled as a linear or convex mathematical program on continuous variables. As this is not the case when hours of service regulations must be complied with, determining whether all locations in a route can be visited within given time windows can become a particularly difficult task.

Regulations concerning working hours of mobile staff in the transportation sector have been studied since the 1960s. An early survey on airline crew scheduling is presented by Arabeyre et al. (1969). Kohl and Karisch (2004) describe typical rules and regulations arising in airline crew rostering. Various approaches have been developed for combined aircraft routing and crew scheduling (Cordeau et al. 2001b, Mercier et al. 2005, Sandhu and Klabjan 2007), for simultaneous vehicle and driver scheduling for mass transit systems (Haase et al. 2001, Valouxis and Housos 2002, Freling et al. 2003, Huisman et al. 2005) and for limousine rental (Laurent and Hao 2007). Ernst et al. (2004) provide a comprehensive annotated bibliography on personnel scheduling which covers crew and driver scheduling problems for airlines, railways, and mass transit systems.

Until very recently, hours of service regulations in road freight transport have received little attention in the literature. Scheduling in road freight transportation differs significantly from scheduling in airlines, railways, and mass transit systems which typically operate on time tables. In road freight transport arrival times are usually given by time windows. As travel times between customer locations depend on previous driving and rest patterns and as many different driving and rest patterns are possible, efficient solution procedures are required to determine whether all customer locations in a route can be visited within given time windows. Comprehensive models of different hours of service regulations world wide are provided by Archetti and Savelsbergh (2009), Goel and Kok (2012), Goel (2010), Goel and Rousseau (2011), Goel et al. (2012), and Goel (2012b). These works present exact methods for the problem of determining whether a truck driver schedule complying with specific hours of service regulations exists for a fixed sequence of visits to customers with respective time windows. For current U.S. hours of service regulations, this problem is known to be solvable in polynomial time (Archetti and Savelsbergh 2009, Goel and Kok 2012). For the other regulations and the new rules in the United States, the existence of a polynomial algorithm for this scheduling problem is still an open research question.

Heuristic approaches for the VRTDSP have been introduced by Goel (2009), Kok et al. (2010), and Prescott-Gagnon et al. (2010) for EU regulations, and by Rancourt et al. (2012) for U.S. regulations. Other specific variants have also been addressed by Xu et al. (2003) and Zapfel and Bogl (2008). So far, no approach for the VRTDSP in Canada, Australia, or the new rules in the United States has been presented, no approach can handle more than one set of rules, and no international comparison on the impact of different hours of service regulations on motor carrier profitability has been made.

7.6 An optimization method for combined vehicle routing and truck driver scheduling

We introduce a new metaheuristic for the VRTDSP for different hours of service regulations around the world. This approach relies on two main building blocks, namely the hybrid genetic search with advanced diversity control (HGSADC) for route optimization (Vidal et al. 2012a), and the truck driver scheduling procedures of Goel and Kok (2012), Goel and Rousseau (2011), Goel (2010), Goel et al. (2012) and Goel (2012b).

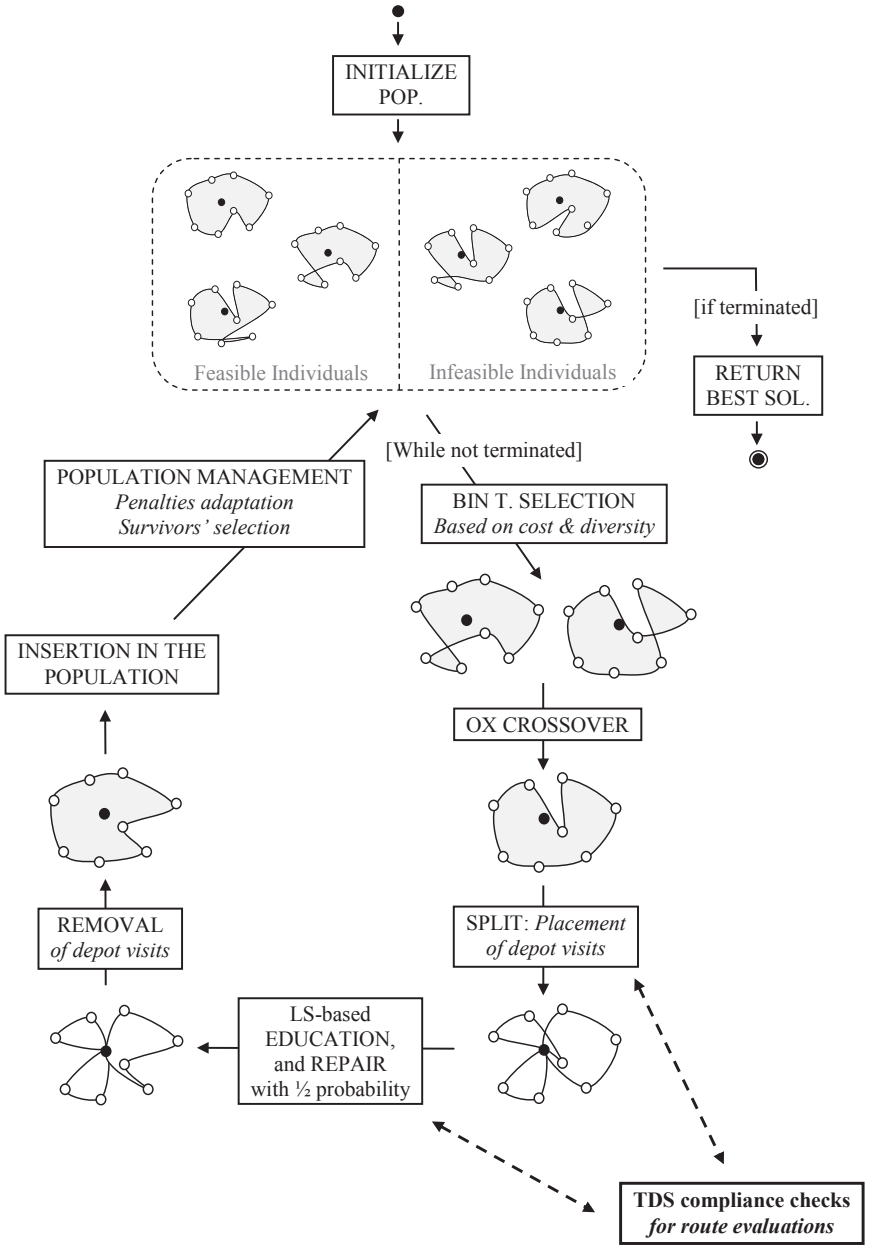


Figure 7.1: General behavior of the hybrid genetic algorithm with adaptive diversity control for the VRTDSP

The general behavior of the proposed HGSADC for the VRTDSP is represented in Figure 7.1. As a member of the family of genetic algorithms (GA), the HGSADC evolves a population of individuals representing different solutions, by means of elitist selection, mutation and recombination operations. Furthermore, unlike classical GA, the proposed approach relies on an incomplete solution representation *without trip delimiters* with dedicated *Split* and *Removal* procedures to pass from individual representations to full solutions (see Section 7.6.1). Both feasible and individual solutions are produced and evaluated relatively to their cost, feasibility, and contribution to diversity (see Section 7.6.2). To generate new individuals, a crossover operator is used as well as local search-based *Education* and *Repair* procedures (see Section 7.6.3). The feasible and infeasible individuals produced by the previous operations are managed in two separate *sub-populations* (see Section 7.6.4).

Any route created in the course of the search, especially during *Education*, *Repair* and *Split*, must be evaluated with respect to capacity and time window constraints. In order to evaluate compliance with time windows, truck driver schedules complying with applicable hours of service regulations must be generated (see Section 7.6.5). The computational challenges that must be tackled to achieve an efficient method are discussed in Section 7.6.6.

7.6.1 Solution representation

Each individual in HGSADC is represented as a *giant tour* without *trip delimiters* (Prins 2004). This representation allows the use of simple permutation-based crossover operators, and has been used successfully for many vehicle routing variants. A *Split* procedure fulfills the role of partitioning a given giant tour into several vehicle routes to obtain the associated VRTDSP solution, thus providing the means to evaluate individuals and apply local search-based improvement procedures. In reverse, generating a giant tour from a solution is done by ordering the routes by increasing barycenter's polar angle around the depot, and then removing depot occurrences. Figure 7.2 illustrates the relationship between giant tour and solution representation.

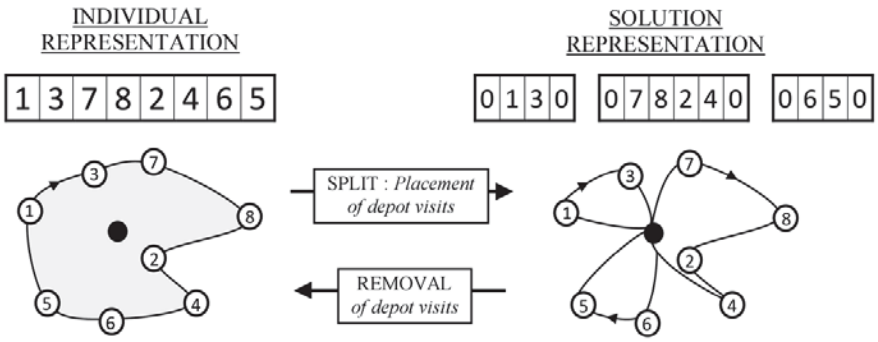


Figure 7.2: From individual to solution representation

The problem of optimally segmenting a giant tour by inserting visits to the depot is modeled as a shortest path problem on a directed acyclic auxiliary graph (Beasley 1983). In this graph, each arc is associated to a potential route servicing a subsequence of consecutive visits from the

giant tour, which must be evaluated with respect to cost and route constraints (including hours of service regulations). There are $\mathcal{O}(nb)$ such arcs to evaluate, where n denotes the number of customers and $b \leq n$ represents a bound on the number of customers per route. Once arc costs in this graph are determined, the splitting problem is solved in $\mathcal{O}(nb)$ using the Bellman algorithm. If the fleet is limited to m vehicles, a path with less than m edges can be found in $\mathcal{O}(mnb)$.

7.6.2 Evaluation of individuals

The VRTDSP can be qualified as a "tightly constrained" problem in the sense that only a relatively small proportion of all possible sequences of customer locations represent feasible solutions. To better transition between structurally different solutions in the course of the search, penalized infeasibility with respect to capacity and time window constraints is allowed, and the evaluation of individuals is based on both *penalized costs* and *contribution to diversity* metrics.

The *penalized cost* $\phi_{\mathcal{P}}^{\text{COST}}(p)$ of an individual p is defined as the sum of the penalized costs its routes, determined relatively to load, distance, and lateness measures. Computation of distance and load on a route is straightforward, whereas evaluating lateness in presence of hours of service regulations requires to explicitly build truck driver schedules. This difficult and computationally intensive task is discussed in Section 7.6.5. For a route r with distance $\varphi^{\text{D}}(r)$, load $\varphi^{\text{Q}}(r)$, and lateness $\varphi^{\text{L}}(r)$, the penalized cost $\phi(r)$ is then given by

$$\phi(r) = \varphi^{\text{D}}(r) + \omega^{\text{Q}} \max\{0, \varphi^{\text{Q}}(r) - Q\} + \omega^{\text{L}}\varphi^{\text{L}}(r), \tag{7.1}$$

where ω^{Q} and ω^{L} are penalty coefficients for capacity violation and lateness. Like in Vidal et al. (2012a), these coefficients are adapted during the search relatively to the proportion of feasible individuals.

The *diversity contribution* $\phi_{\mathcal{P}}^{\text{DIV}}(p)$ of an individual p to its sub-population $\mathcal{P}$ is defined as the average proportion of arcs in common with each of the μ^{CLOSE} most similar individuals in the sub-population (Vidal et al. 2012a).

The *biased fitness* $f_{\mathcal{P}}(p)$ of an individual p is defined in Equation (7.2) as the weighted sum of the rank $f_{\mathcal{P}}^{\text{COST}}(p)$ of p in its sub-population $\mathcal{P}$ in terms of penalized cost and of its rank $f_{\mathcal{P}}^{\text{DIV}}(p)$ in $\mathcal{P}$ in terms of diversity contribution. The parameter μ^{ELITE} balances the role of both components.

$$f_{\mathcal{P}}(p) = f_{\mathcal{P}}^{\text{COST}}(p) + \left(1 - \frac{\mu^{\text{ELITE}}}{|\mathcal{P}|}\right) f_{\mathcal{P}}^{\text{DIV}}(p) \tag{7.2}$$

The biased fitness thus reflects the amount of innovation, the cost, and the feasibility of solutions.

7.6.3 Generation of new individuals

Sub-populations are initially filled with randomly generated individuals, which are *Educated*, and *Repaired* as described in the next paragraphs. The method proceeds by iteratively selecting two "parents" in the combined population of feasible and infeasible individuals by a *binary tournament* (Goldberg and Deb 1991) based on the biased fitness measure. These parents serve as input of the *ordered crossover* (OX) (see Prins 2004) to produce a new individual called *offspring*. This

offspring is converted into a full solution by means of the *Split* procedure, before being *Educated*, and *Repaired* with probability $\pi^{\text{REP}} = 0.5$ if infeasible.

Education is a local search procedure based on well-known VRP neighborhoods such as 2-opt, 2-opt*, and CROSS-exchanges. As in Vidal et al. (2013), neighboring solutions are explored in a random order and any improving move is directly applied. To reduce the computational effort, only moves between related customers with regards to distance and time characteristics are attempted.

The *Repair* operator temporarily increases the penalty coefficients by a factor of 10 and calls *Education* to redirect the search towards feasible solutions.

7.6.4 Population management

All individuals produced by means of the previous operations are included in the appropriate sub-population. Each individual can start to "reproduce" immediately after being created. Sub-populations are independently managed to contain between μ^{MIN} and $\mu^{\text{MIN}} + \mu^{\text{GEN}}$ individuals. Whenever a sub-population reaches a maximum size $\mu^{\text{MIN}} + \mu^{\text{GEN}}$, a *survivor selection* phase is triggered. This phase involves to remove μ^{GEN} times the worst individual with regards to the biased fitness function $f_{\mathcal{P}}$ previously defined, privileging the removal of individuals that appear identically several times in the sub-population. The previous cycle of operations is repeated until a maximum number of individual creations without improvement λ^{IT} is reached. The best found solution is finally returned.

7.6.5 Truck driver scheduling for route evaluations

The routes produced in the course of the search must be evaluated with respect to time window constraints. For this, a schedule complying with hours of service regulations must be generated which minimizes lateness in customer service times. In this process, any voluntary increase in service lateness to a customer with an eye to reduced lateness at subsequent customers is forbidden.

For a route $r = (r_1, r_2, \ldots, r_{n_r})$ with n_r locations, a forward labeling algorithm is used which iteratively generates a set of schedules S_i for each partial route $(r_1, r_2, \ldots, r_i)$, $i \in \{1, \ldots, n_r\}$. The algorithm begins with a set of truck driver schedules $\mathcal{S}_1$ for the partial route consisting solely of node r_1. In each subsequent iteration, for $2 \leq i \leq n_r$, each schedule from $\mathcal{S}_{i-1}$ is extended into new schedules for the partial route $(r_1, r_2, \ldots, r_i)$ by subsequently appending driving, working and off-duty periods to the end of the schedule and by extending the duration of off-duty periods already scheduled. Different types of off-duty periods must be scheduled depending on the regulations. As any voluntary increase in service lateness is forbidden, only schedules with a minimal lateness value are included in the set $\mathcal{S}_i$. A *dominance* relationship is then used to prune schedules from $\mathcal{S}_i$.

Figure 7.3 illustrates the search tree of the truck driver scheduling procedure for a route $r = (r_1, r_2, r_3, r_4)$ and current U.S. hours of service regulations. The scheduling method for current U.S. hours of service regulations extends each non-dominated schedule into two child schedules, one of them comprising an additional rest period immediately before service. Schedules s_{32}, s_{33}, and s_{42} are pruned using a dominance relationship based on the completion time, the accumulated driving time since the last rest period, and the time elapsed since the last rest period.

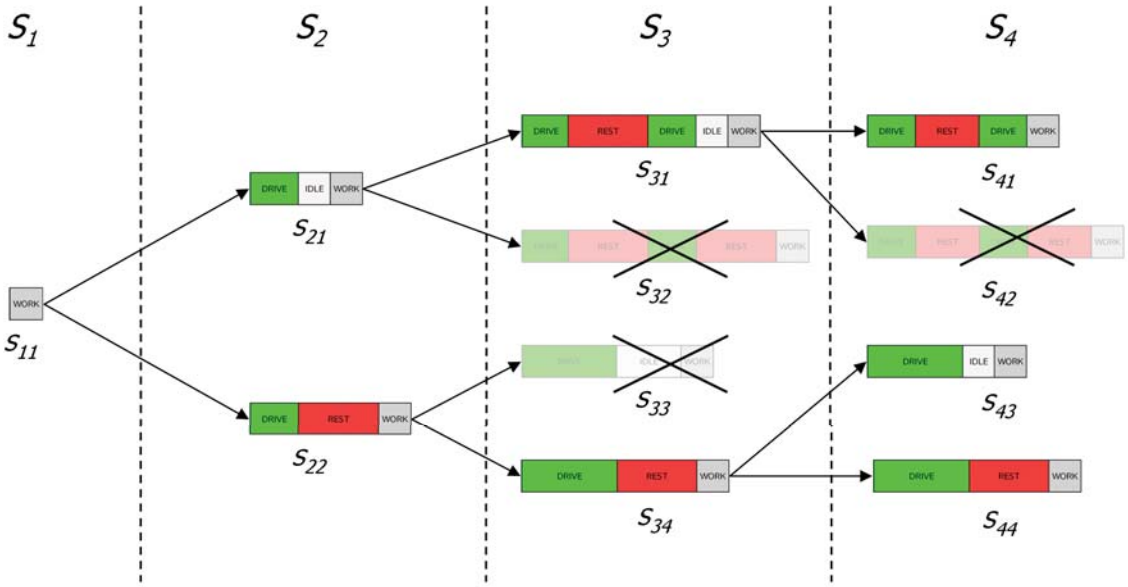


Figure 7.3: Truck driver scheduling procedure for a route with four locations

The details on how schedules are extended, how many alternative schedules need to be generated, and the dominance relationship depend on the specific rules of the regulations. Different forward labeling algorithm for hours of service regulations in the United States, Canada, the European Union, and Australia can be found in Goel and Kok (2012), Goel (2012b), Goel and Rousseau (2011), Goel (2010), and Goel et al. (2012). In this paper, we use adaptations of these algorithms that allow penalized lateness with respect to time window constraints, and also account for multiple time windows as done in Goel and Kok (2012). For European Union regulations, we extended the approach of Goel (2010) in order to consider the possibility of reducing the duration of rest periods to 9 hours and extending the amount of driving between two rest periods to 10 hours. The method was thus modified in such a way that additional schedules exploiting these possibilities are generated whenever this could be beneficial. Further modifications were also made to include the same set of rules from Directive 2002/EC/15 as in Prescott-Gagnon et al. (2010). The approaches for Canadian and Australian regulations presented by Goel and Rousseau (2011) and Goel et al. (2012) were based on the assumption that all time values are a multiple of 15 minutes. We modified these approaches in such a way that arbitrary time values can be used. This is achieved by increasing the completion time of any partial schedule to a multiple of 15 minutes whenever a driver is released from duty. By this, all off-duty periods start and end at a multiple of 15 minutes, and the modified approaches can be used without further changes.

7.6.6 Addressing the challenge of computational efficiency

Hybrid genetic algorithms are known to rely on a large number of route evaluations, especially due to the local search-based *Education* and *Repair* procedures. One major algorithmic result is to show that, even in presence of computationally expensive route evaluations and scheduling

procedures, an efficient overall hybrid genetic method can be developed. Essential components for this are adequate memory structures, neighborhood pruning, and schedule pruning procedures.

Memories. Since early research on VRP variants, it has been observed that the same customer sequences appear in many solutions generated throughout the solution process. Adequate data structures on partial routes can thus lead to notable computational savings (Savelsbergh 1985, 1992). To illustrate this, consider the evaluation of a 2-opt* neighborhood, which involves to replace two arcs (r_i, r_j) and $(r'_{i'}, r'_{j'})$ from two different routes r and r', by arcs $(r_i, r'_{j'})$ and $(r_{i'}, r'_j)$. As illustrated in Figure 7.4, the partial route $(r_1, \ldots, r_i)$ appears several times in the neighboring solutions. Hence, a large number of redundant computations are avoided by storing partial truck driver schedules associated to such subsequences.

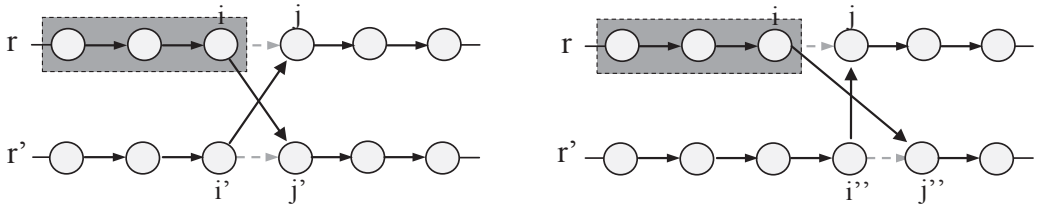


Figure 7.4: Common subsequences through 2opt move evaluations

Furthermore, memories for move and route evaluations are used to avoid redundant computations. During the local search, moves are sorted relatively to the nodes and the routes they impact. The evaluation $f(x, r, r')$ of any move x between routes r and r' is stored, along with a *chronological* information indicating *when*, for example at which iteration of the local search, this value has been calculated. Similarly, chronological information indicates for each route *when* this route has been last modified. A move is not evaluated if none of the routes it impacts has been modified since its last evaluation.

We observed that the *Education*, *Repair* and *Split* procedures, when applied to different individuals, are naturally bound to evaluate some identical routes. High-quality routes are particularly likely to appear in many individuals. To avoid redundant computations, we added a *long-term global memory* to store the results of the route evaluations. This memory is implemented as a hash-table. To limit memory usage, each route evaluation is stored along with a counter for frequency of appearance. Whenever 5 million route evaluations are stored, the half least frequently encountered route evaluations are discarded. This long-term memory led to an algorithm speed-up ranging from 2 to 10 relatively to the instances used.

Local search restrictions and search tree reductions. Local search moves have been restricted to pairs of related customers, which are spatially close, or require service in close periods of time (Vidal et al. 2013). The resulting neighborhood size, once pruned, is $O(\Gamma n)$, where Γ is a method parameter representing the number of close customers to consider. Thanks to memory structures, each of the $O(\Gamma n)$ moves is evaluated at most once, and upon the application of a move, $O(2\Gamma\bar{n})$ moves must be recomputed, $\bar{n}$ representing the average number of customers in a route.

The total number of route evaluations during a local search is thus $O(\Gamma n + \alpha^{\text{IMP}}(n)\Gamma\bar{n})$, where $\alpha^{\text{IMP}}(n)$ is the number of moves before reaching a local optimum.

Fast route evaluations are crucial for the overall running time of the HGSADC. As the search tree generated during route evaluations may grow very large for some of the regulations, various techniques for limiting its size have been applied. For Canadian and Australian regulations, Goel and Rousseau (2011) and Goel et al. (2012) presented heuristic forward labeling methods which only generate a small subset of all possible partial schedules, thus reducing the size of the search tree significantly. In the European Union, the possibility of reducing the duration of rest periods and extending the amount of driving time between rest periods results in a dramatic increase in the size of the search tree. To speed up route evaluations in this case, the size of the search tree has been reduced using a combination of two techniques. First, we use a heuristic dominance relationship which does not take into account the number of reduced rests and extended driving periods. Second, the number of different schedules in $\mathcal{S}_i$ at each iteration i of the truck driver scheduling method is limited to at most $\Gamma' = 5$. To do so, the set of non-dominated partial schedules $\mathcal{S}_i$ is ordered by completion time, and only the schedules at positions $1 + \lfloor (j-1)(|\mathcal{S}_i|-1)/(\Gamma'-1) \rfloor$ for all $1 \leq j \leq \Gamma'$ are kept.

All these elements lead to rapid *Split* and local search-based *Education* procedures for the VRTDSP, and thus enable to efficiently apply the HGSADC framework to this difficult problem.

7.7 Computational experiments

Extensive computational experiments have been conducted to evaluate the performance of the proposed algorithm, and to assess the impact of different hours of service regulations world wide. These experiments are based on the 56 benchmark instances for the VRTDSP proposed by Goel (2009), which are derived from the VRPTW benchmarks of Solomon (1987). The instances are grouped into six classes. In classes R1 and R2 customers are randomly distributed in a square region. In classes C1 and C2 customers are clustered, and in classes RC1 and RC2 the customer distribution is mixed. In all instances, 100 customers with a demand of at most 50 units must be served. In the R1, C1, and RC1 classes the capacity of each vehicle is 200 units, in the R2 and RC2 classes the capacity of each vehicle is 1000 units, and in the C2 class the capacity of each vehicle is 700 units. The average size of time windows per instance ranges from less than 7 hours to more than 107 hours. The service time at every customer is set to one hour. The planning horizon is 144 hours and the maximum required driving time (without compulsory breaks and rests) to go from one point in the square region to another is approximately one day.

To demonstrate the performance of the proposed method, the solutions obtained by HGSADC on these original instances were compared with the solutions of the best current methods. To assess the impact of different hours of service regulations world wide, we also derived a modified instance set to improve realism. As time window requirements of customers are usually tied to business hours and most customers cannot be visited in the night, we removed the time between 8.00 PM and 8.00 AM from time windows in the original VRTDSP instances. However, to maintain feasibility of the instances, the night time of time windows with a duration of 24 hours or less

was not removed. Thus some customers have a single time window and others have multiple time windows tied to business hours. Any feasible solution of a modified instance is obviously feasible for its original counterpart.

For all experiments, the HGSADC parameters proposed by Vidal et al. (2013) are used, with the exception of the population size parameters $\mu^{\text{MIN}} = 10$, $\mu^{\text{GEN}} = 5$, and the neighborhood pruning parameter $\Gamma = 10$, which are set to small values to quickly converge towards high quality solutions. The termination criterion is set to $\lambda^{\text{IT}} = 500$. The algorithm has been implemented in C++, and run on an Intel Xeon X7350 2.93 Ghz processor.

7.7.1 Comparison with best known solutions

The most advanced method for solving the VRTDSP known to the authors is the approach presented by Prescott-Gagnon et al. (2010) which combines column generation techniques with large neighbourhood search. This approach was tested on the instances presented by Goel (2009) for different subsets of rules applicable in the European Union. The authors used a hierarchical objective with the primary goal of minimizing the size of the vehicle fleet and the secondary goal of minimizing the total travel distance. We addressed this hierarchical objective by setting a constraint on the fleet size of 20 vehicles, and then iteratively decrementing the fleet size constraint whenever a feasible solution is found with HGSADC.

Tables 7.2 and 7.3 show the results for two subsets of rules in the European Union. The subset labeled *EU (All)* contains all the rules described in Section 7.4.3. The subset labeled *EU (No split)* contains all the rules except for those allowing to split breaks and rest periods and those allowing to reduce the duration of daily rest periods or to extend the accumulated amount of driving time between two rest periods. As in Prescott-Gagnon et al. (2010), five runs of HGSADC have been performed for each instance and each set of rules. The tables report for each method and each problem class the average and best solution values with respect to the hierarchical objective, i.e. the accumulated fleet size and the accumulated distance. The best solutions are indicated in boldface. The last lines report the accumulated fleet size and distance on all instances, the average computation time per instance, and the processor used. Detailed results per instance are reported in Appendix IV.

Table 7.2: Method performance on Goel (2009) instances - EU (No split)

	Prescott-Gagnon et al. (2010)				HGSADC			
	Avg. Fleet	Avg. Dist.	Best Fleet	Best Dist.	Avg. Fleet	Avg. Dist.	Best Fleet	Best Dist.
R1	98.40	11855.28	98.00	11855.34	98.80	11769.13	**98.00**	**11835.89**
R2	64.40	10341.83	63.00	10262.50	62.60	10294.36	**62.00**	**10279.25**
C1	90.00	7628.71	**90.00**	**7628.47**	90.40	7630.25	90.00	7628.73
C2	39.40	5847.00	40.00	5792.67	40.00	5754.04	**40.00**	**5753.30**
RC1	72.00	8945.84	72.00	8903.44	72.00	8915.07	**72.00**	**8892.74**
RC2	52.50	8938.95	50.00	8976.28	50.00	8960.99	**50.00**	**8917.25**
All	416.70	53557.61	413.00	53418.70	413.80	53323.84	**412.00**	**53307.16**
	Avg. CPU: 11 min (OPT 2.3 Ghz)				Avg. CPU: 54 min (XE 2.83 Ghz)			

Table 7.3: Method performance on Goel (2009) instances - EU (All)

	Prescott-Gagnon et al. (2010)				HGSADC			
	Avg. Fleet	Avg. Dist.	Best Fleet	Best Dist.	Avg. Fleet	Avg. Dist.	Best Fleet	Best Dist.
R1	97.00	11710.92	97.00	11659.63	96.20	11800.47	**96.00**	**11806.72**
R2	62.40	10208.45	60.00	10273.19	59.80	10177.15	**59.00**	**10153.30**
C1	90.00	7628.56	90.00	7628.47	90.00	7444.86	**90.00**	**7444.86**
C2	37.00	5559.58	37.00	5519.58	36.00	5505.79	**36.00**	**5501.50**
RC1	72.00	8890.88	72.00	8858.12	72.00	8834.31	**72.00**	**8806.01**
RC2	49.20	8772.75	49.00	8726.37	49.00	8654.63	**49.00**	**8604.17**
All	407.60	52771.14	405.00	52665.36	403.00	52417.21	**402.00**	**52316.56**
	Avg. CPU: 88 min (OPT 2.3 Ghz)				Avg. CPU: 228 min (XE 2.83 Ghz)			

For both sets of rules, the proposed method produces solutions of higher quality than the approach of Prescott-Gagnon et al. (2010), which was designed specifically for European Union regulations. For the *EU (No split)* set of rules, HGSADC produces new best known solutions for 29 of the 56 instances and obtains equally good solutions for 22 of the instances. For the *EU (All)* set of rules, HGSADC produces new best known solutions for 43 of the 56 instances and obtains equally good solutions for nine of the instances. For these rules, the average solution quality is better than the best solution quality found by Prescott-Gagnon et al. (2010).

Computation times are higher than those of Prescott-Gagnon et al. (2010), but still of the same order of magnitude. As our main goal is to assess the impact of hours of service regulations world wide, a special emphasis has been put on the quality of the scheduling methods. Smaller CPU times could thus be achieved by using faster heuristic scheduling procedures within route evaluations.

A Wilcoxon test on the 112 average solution pairs from HGSADC and Prescott-Gagnon et al. (2010) confirms with high confidence ($p < 0.0001$) the statistical significance of the solution quality improvements. In average, on the subset of 106/112 instances for which the minimum fleet size was obtained on all five runs, the standard deviation on distance measures is +0.21%, thus illustrating the good reliability of the method.

7.7.2 An international comparison of hours of service regulations

To assess the impact of different hours of service regulations world wide, we conducted experiments for the different regulations described in Section 7.4 on the modified Goel (2009) instances obtained by removing the night time from long time windows. As most fleet operators have a fixed fleet size which cannot be increased or reduced on a weekly basis, the minimization of distance has been selected as the primary objective in the experiments described in this section. For each instance, we associated a fleet size limit which is a few vehicles larger than the minimum feasible value obtained in preliminary experiments. The fleet size limit for each of the instances is reported in Tables IV.2 to IV.4 in Appendix IV.

Table 7.4 reports for each class of instances and each type of regulation the best solution found in five runs of our algorithm. The last lines indicate respectively the cumulated distance (CTD) on

all instances, the percentage of increase in total distance in comparison to the case in which no hours of service regulations are considered (Inc %), the cumulated number of vehicles (CNV), and the computation time (CPU) averaged on all instances and runs.

The column titled *US (current)* reports the results obtained using the exact truck driver scheduling method presented by Goel and Kok (2012) for current hours of service regulations in the United States with a limit of 60 hours of on-duty time within 7 days. The column titled *US (2013)* reports the results obtained using the exact truck driver scheduling presented by Goel (2012b) for the new regulations in the United States, becoming effective in July 2013. Due to the complexity of Canadian regulations, using an exact approach for truck driver scheduling results in prohibitively slow running times. Therefore, the heuristic truck driver scheduling procedure *CAN2* introduced in Goel and Rousseau (2011) was used. The columns titled *EU (No split)*, *EU (Split)*, and *EU (All)* report the results obtained for European Union regulations. For *EU (No split)* and *EU (All)* the same scheduling methods as in Section 7.7.1 are used. For *EU (Split)* the exact truck driver scheduling method presented by Goel (2010) is used which considers all rules except for those allowing to reduce the duration of daily rest periods and to extend the amount of driving time between rest periods. The columns titled *AUS (Std.)* and *AUS (BFM)* report the results obtained for the standard option and the BFM option of Australian regulations. For both options, the heuristic truck driver scheduling procedure *AUS1* introduced in Goel et al. (2012) is used. Finally, the column titled with *None* reports the results obtained by our approach without considering hours of service regulations. All algorithms were modified as described in Section 7.6.5.

It must be noted that the approach for Australian regulations does not consider the 36 hour limit on long/night work of the BFM option. Although the limit could theoretically have an impact for the benchmark instances considered in this paper, we observed that for all solutions obtained in our experiment a feasible schedule with respect to all rules is found.

Table 7.4: Best solutions found for modified Goel (2009) instances

	US		**CAN**	**EU**			**AUS**		**None**
	(current)	**(2013)**		**(No split)**	**(Split)**	**(All)**	**(Std.)**	**(BFM)**	
R1	11666.19	11690.82	11688.77	11817.42	11764.29	11748.14	11819.65	11752.88	11620.10
R2	10078.91	10123.65	10074.01	10276.13	10232.13	10181.37	10261.73	10180.27	10002.36
C1	7447.15	7447.15	7447.14	7637.43	7636.20	7451.15	7625.02	7447.15	7447.15
C2	5427.60	5655.66	5124.82	5857.09	5677.43	5533.44	5466.39	5153.82	4730.51
RC1	8856.83	8863.28	8868.42	8945.68	8922.60	8892.30	8921.56	8890.82	8821.35
RC2	8540.56	8653.45	8552.40	8916.51	8827.14	8710.67	8878.58	8634.84	8325.21
CTD	52017.23	52434.00	51755.55	53450.26	53059.78	52517.07	52972.93	52059.78	50946.68
Inc %	+2.1%	+2.9%	+1.6%	+4.9%	+4.2%	+3.1%	+4.0%	+2.2%	+0.0%
CNV	432	437	430	452	447	440	444	432	411
CPU	11 min	21 min	64 min	23 min	180 min	228 min	26 min	19 min	7 min

With a value of 1.6%, the smallest increase in total distance compared to the case without hours of service regulations is obtained for Canadian hours of service regulations. Current U.S. hours of service regulations result in an increase of 2.1%, which becomes 2.9% when the 2013 rule

change is enforced. In the European Union a similar increase of 3.1% is obtained when exploiting all rules of the regulations. The regulations give a strong incentive of exploiting the possibilities of reducing the duration of rest periods to 9 hours and extending the driving time between rest periods to 10 hours. Without these optional rules, the total distance increases by 4.2% if the possibility of taking break and rest periods in two parts is exploited and by 4.9% otherwise. One might think that the intention of European lawmakers was to give motor carriers the possibility of reacting on unforeseeable traffic conditions by allowing to reduce the duration of rest period and to extend the driving time between rest periods on some days of the week. However, if this was the case, these optional rules are unlikely to fulfill this purpose as economic pressure can force motor carriers to exploit these options on a regular basis and not only in the case of unexpected delays. Australian motor carriers without accreditation have with 4.0% the highest increase in total distance. As travel distances only increase by 2.2% when using the BFM option, there is a strong incentive for Australian motor carriers to be accredited for the BFM option.

To further analyze the impact of hours of service regulations, we determined for each of the routes of the best solutions a schedule with minimal duration using the iterative dynamic programming approach of Goel (2012a). Table 7.5 reports for each set of solutions the cumulated schedule duration (CSD), the average percentage of on-duty time with respect to the schedule duration (OD), and the average amount of on-duty time between two long rest periods (OBR). Time values are reported in hours and minutes (hh:mm).

As illustrated in Table 7.5, schedule characteristics appear to be consistent with the properties of the regulations. The impact of hours of service regulations on the total schedule duration, average on-duty ratio, and duty time between rests periods is evidenced. Exploiting the optional rules in the European Union leads to reduced schedule durations and to increased on-duty ratios and on-duty time between rests. Similar observations can be made when comparing AUS (Std.) with AUS (BFM) regulations, and US (2013) with US (current). Overall, the highest on-duty ratio (45.98%) is achieved for Canadian regulations followed by the current regulations in the United States (45.01%). On-duty ratios for the new regulations in the United States (43.41%), European Union regulations (43.19%) and the rules of the BFM option in Australia (43.02%) are comparable.

It is also worth noting that hours of service regulations do not have a high impact on distances for the C1 class. Analyzing the schedules, we observe that only one third of the time spent by the drivers in the solutions for the C1 class is on-duty time. Consequently, there is plenty of time available that potentially can be used for taking breaks and rest periods. For the C2 class, on the other hand, we obtain a high on-duty ratio of 58.98% and an average amount of on-duty time per vehicle above 72 hours when not considering hours of service constraints. As the average amount of on-duty time exceeds the weekly limits of the regulations and as less time can be used for taking breaks and rest periods, the impact of hours of service regulations is the highest for the C2 class.

Although the fleet size is not considered in the objective function, fleet sizes differ in the solutions as a result of minimizing the total distance. The largest increase in fleet size compared to the case without regulations is obtained for the instances in the C2 class. For most other instances and most of the regulations, at most one additional vehicle is required. Again, CAN, US (current), and AUS (BFM) regulations lead to the smallest fleet sizes.

Table 7.5: Schedule characteristics

		US		CAN	EU			AUS		None
		(current)	(2013)		(No split)	(Split)	(All)	(Std.)	(BFM)	
R1	CSD	7701:17	7781:22	7614:30	8493:44	8226:12	7897:22	8791:03	8203:23	6976:20
	OD	45.72%	45.31%	46.30%	41.81%	43.04%	44.79%	40.40%	43.13%	50.34%
	OBR	9:01	8:56	9:04	7:56	8:20	8:46	7:47	8:19	
R2	CSD	6676:13	6866:41	6654:21	7487:26	7296:31	7051:50	7652:05	7482:32	6423:20
	OD	46.50%	45.34%	46.64%	41.99%	42.97%	44.31%	41.05%	41.76%	48.10%
	OBR	9:23	8:59	9:27	8:29	8:33	9:05	8:01	8:09	
C1	CSD	7110:20	7152:27	6973:48	7572:27	7476:26	7308:34	7404:59	7237:57	7054:48
	OD	33.55%	33.35%	34.21%	32.01%	32.41%	32.65%	32.70%	32.96%	33.82%
	OBR	6:42	6:33	6:57	6:11	6:14	6:26	5:17	5:22	
C2	CSD	4025:31	4645:37	3564:01	4945:59	4875:29	4508:54	4631:22	3814:39	2951:55
	OD	46.71%	41.45%	51.06%	39.75%	39.59%	42.17%	40.77%	47.86%	58.98%
	OBR	10:37	9:31	11:10	8:32	9:03	9:32	9:05	10:31	
RC1	CSD	5022:03	5201:13	5052:00	5784:08	5673:59	5204:16	5816:18	5391:15	4607:28
	OD	51.09%	49.36%	50.84%	44.67%	45.45%	49.44%	44.34%	47.72%	55.54%
	OBR	9:42	9:27	9:25	8:17	8:21	9:08	8:03	8:48	
RC2	CSD	5386:09	5535:45	5347:53	6075:56	5952:10	5541:58	6058:10	5639:57	4753:37
	OD	46.47%	45.62%	46.85%	42.43%	43.01%	45.77%	42.43%	44.71%	51.74%
	OBR	9:36	9:26	9:28	8:34	8:41	9:19	8:34	8:56	
All	CSD	5986:56	6197:11	5867:46	6726:37	6583:28	6252:09	6725:39	6294:57	5461:15
	OD	45.01%	43.41%	45.98%	40.44%	41.08%	43.19%	40.28%	43.02%	49.75%
	OBR	9:10	8:48	9:15	7:60	8:12	8:43	7:48	8:21	

To evaluate the impact of hours of service regulations on road safety we used the fatigue and risk index calculator available from Health and Safety Executive (2006). This calculator can be used to estimate the average risk of the occurrence of an accident given a specific work schedule and is described in Spencer et al. (2006). The risk indices are calculated from separate components considering the amount of sleep loss that is likely to accumulate throughout the course of a work schedule, the effect of start time and length of the individual daily shifts, and the break patterns within these shifts. When using the calculator to assess the risk associated to the solutions obtained by our method, we interpreted any off-duty period of at least 7 hours duration as the end of a daily shift and specified the required input accordingly. Table 7.6 shows the average risk indices obtained for the different hours of service regulations. The indices represent the estimated relative accident risk and an index of two represents a twice as high average accident risk as an index of one. We normalized the risk indices with respect to the average risk associated to the *EU (No split)* rule set.

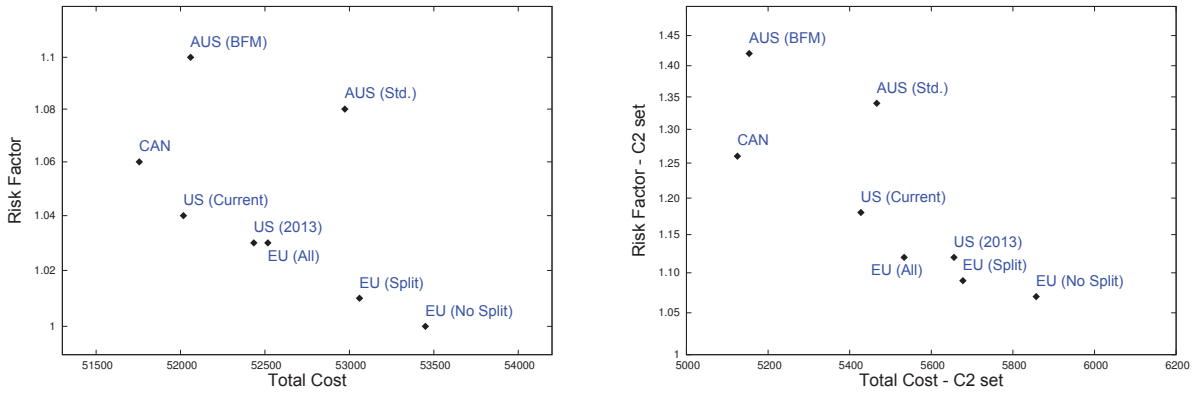
For European Union rules we observe that the possibility of taking breaks and rest periods in two parts and the respective reduction in the minimum duration of rest periods has only little impact on the average risk. When exploiting all optional rules in the European Union, the risk is 3% higher compared to the basic set of rules. Looking at the individual risk components it

Table 7.6: Average risk indices

	US		CAN	EU			AUS	
	(current)	**(2013)**		**(No split)**	**(Split)**	**(All)**	**(Std.)**	**(BFM)**
R1	1.03	1.04	1.07	0.99	0.99	1.03	1.11	1.14
R2	1.08	1.05	1.11	1.02	1.02	1.04	1.11	1.13
C1	0.93	0.92	0.88	0.92	0.95	0.94	0.85	0.86
C2	1.18	1.12	1.26	1.07	1.09	1.12	1.34	1.42
RC1	1.06	1.05	1.08	1.00	1.00	1.04	1.10	1.15
RC2	1.08	1.07	1.11	1.03	1.04	1.08	1.16	1.21
All	1.04	1.03	1.06	1.00	1.01	1.03	1.08	1.10

appears that this increase in risk is mainly related to an increased duration of daily shifts. In the United States, the additional break requirement that will be enforced in 2013 will reduce the risk by approximately 1%, reaching a value comparable to the risk in the European Union when all optional rules are exploited. The risk associated to Canadian regulations is notably higher compared to regulations in the United States and the European Union. The standard and the BFM rules of Australian regulations have the largest risk indices. It appears that due to the short minimum rest duration of seven hours in Australia and eight hours in Canada, the risk resulting from likely accumulated sleep loss throughout the course of a work schedule is the largest contributor to this increase in risk.

Figure 7.5: Costs vs. risks



As for the economic impact analyzed earlier, the largest variation in risk indices is observed for the C2 set. For this set the BFM rules in Australia result in a risk index of 1.42 which is 33% higher than the minimum average risk index for this set. Furthermore, the 2013 rule change in the United States leads to a risk reduction of 5% for this set. Figure 7.5 illustrates the tradeoff between total costs and average risks associated to the different rules. The graph on the left illustrates the respective values for all of the instances whereas the graph on the right illustrates the values for the instances of set C2. We can see that except for Australian regulations, there is no clear dominance of one set of rules over another. The rules resulting in small operating

costs are associated with a higher risk index and rules associated with a small risk index have higher operating costs. Australian rules, however, appear to result in unnecessarily high risk levels in relation to the economic impact of these regulations. Apparently, the break requirements of Australian regulations are not sufficient to compensate the negative impact of short rest periods on associated risk values.

7.8 Conclusions

In this paper we proposed a hybrid genetic search with advanced diversity control (HGSADC) for solving the combined vehicle routing and truck driver scheduling problem. By combining the exploration capacities of population-based approaches, the quick improvement abilities of local search, along with efficient procedures for checking compliance with hours of service regulations, the proposed approach outperforms all current methods designed for this difficult family of problems. Our approach is the first which is not specifically designed for a particular set of rules and can be a valuable tool for transport operators world wide.

We conducted extensive experiments to assess the impact of hours of service regulations in the United States, Canada, the European Union, and Australia. The results indicate that Australian regulations have unnecessarily high risk levels with respect to the resulting operating costs. For the other regulations, average accident risk rates appear to be negatively correlated to operating costs. European Union rules lead to the highest safety, while in terms of economic efficiency Canadian regulations are the most competitive. The recent rule change in the United States will bring a reduction in accident risks. The largest decrease in the associated accident risk of the new rules can be observed for the set of instances in which the previous rules had the largest associated risk indices.

Our optimization-based approach can be used to realistically assess the impact of hours of service regulations from a carrier-centric point of view. The decision whether the economic impact of hours of service regulations is justified by improved road safety is a question that has to be discussed and answered by society, policy makers, transport operators, and truck drivers. Our optimization-based approach to analyze the impact of hours of service regulations can be an important building block in such a discussion.

Where transport operators can choose among alternative rule sets our approach can bring important insight concerning the best choice of rules to operate under. Our experiments indicate that accreditation for the BFM option can bring significant advantages for transport operators in Australia. Furthermore, we observed that there are strong economic incentives for European operators to exploit all optional rules of the regulations, in particular, reducing the duration of rest periods and extending driving times.

CHAPITRE 8

RÉSOLUTION GÉNÉRALISTE DE PROBLÈMES DE TOURNÉES DE VÉHICULES MULTI-ATTRIBUTS

8.1 Fil conducteur et contributions

Ce chapitre propose de nouvelles méthodes pour la résolution généraliste d'une grande variété de problèmes de tournées de véhicules multi-attributs. Un cadre modulaire de résolution heuristique est décrit, ainsi qu'une implémentation de méta-heuristique généraliste efficace, appelée *Unified Hybrid Genetic Search* (UHGS). UHGS se base sur des procédures dont l'implantation est indépendante du problème : opérateurs de recherche locale unifiés, croisement, *Split* généralisé, gestion de diversité. Les traitements spécifiques au problème sont confinés dans des composants modulaires qui fournissent les fonctions nécessaires aux choix d'affectation, de séquence, et aux évaluations efficaces de routes, et sont automatiquement sélectionnés et adaptés relativement aux attributs du problème. Des études expérimentales de grande ampleur sur 26 variantes fondamentales de VRP, 39 jeux de test et au total 1008 instances, illustrent la performance remarquable de la méthode qui, avec une unique implémentation et paramétrage, égalise ou surpasse les meilleures méthodes dédiées de la littérature issues de plus de 180 articles. La généralité n'altère ainsi pas forcément la performance pour cette classes de problèmes.

8.2 Article VIII : A unified solution framework for multi-attribute vehicle routing problems

Un article basé sur ce chapitre a été soumis pour publication : Vidal, T., Crainic, T.G., Gendreau, M., Prins, C. A Unified Solution Framework for Multi-Attribute Vehicle Routing Problems. *Operations Research*, submitted for publication.

Abstract: Vehicle routing problem attributes are extra characteristics and decisions that complement the academic problem formulations and aim to properly account for real-life application needs. Hundreds of methods have been introduced in recent years for specific attributes, but the development of a single, general-purpose algorithm, which is both efficient and applicable to a wide family of variants remains a considerable challenge. Yet, such a development is critical for understanding the proper impact of attributes on resolution approaches, and to answer the needs of actual applications. This paper contributes towards addressing these challenges with a component-based design for heuristics, targeting multi-attribute vehicle routing problems, and an efficient general-purpose solver. The proposed Unified Hybrid Genetic Search metaheuristic relies on problem-independent unified local search, genetic operators, and advanced diversity management methods. Problem specifics are confined to a limited part of the method and are addressed by means of assignment, sequencing, and route-evaluation components, which are automatically selected and adapted and provide the fundamental opertors to manage attribute specificities. Extensive computational experiments on

26 prominent vehicle routing variants and 39 benchmark instance sets demonstrate the remarkable performance of the method, which matches or outperforms the current state-of-the-art problem-tailored algorithms, and reveals that generality does not necessarily alter efficiency for these settings.

Keywords: Vehicle routing; multiple attributes; general-purpose solver; component-based design.

8.3 Introduction

General-purpose solvers for combinatorial optimization are algorithms that can be used to address large classes of problem settings without requiring extensive adaptations, user involvement and expertise. The development of such solvers is critical to the understanding of the impact of problem characteristics on the performance of solution methods, as well as to the capability to efficiently address new problem settings and applications displaying particular sets of characteristic combinations. One thus aims for high-performance general-purpose solvers, achieving a subtle balance between generality of scope and specificity in exploiting particular problem characteristics, to identify high-quality solutions for the broadest set of problem settings possible within limited computation time. Such developments are very challenging, however. Indeed, a number of theoretical results indicate that high generality is paid for in terms of performance (Wolpert 1997), while dedicated algorithms cannot address problem variants without extensive adaptation.

We focus on vehicle routing problems (VRPs), one of the major classes of combinatorial optimization problems with an extremely broad range of applications yielding a very large number of variants born of the requirement to manage a wide variety of characteristics and decisions, called *attributes* in Vidal et al. (2012c), to account for the particular customer, vehicle, driver, and network settings and to combine routing considerations with other tactical or strategic choices. The number of VRP attributes that need to be jointly considered is continuously increasing, yielding a considerable variety of *Multi-Attribute Vehicle Routing Problems* (*MAVRP*s).

The current state-of-the-art and knowledge does not offer the means to use exact solution methods for combinatorial optimization as general-purpose solvers for MAVRPs. Actually, such solvers cannot even efficiently address realistic-scale instances of most MAVRPs of interest. Consequently, literally hundreds of papers were published recently, proposing supposedly different solution methods for VRP variants with diverse combinations of sets of attributes. As for the most general vehicle routing metaheuristics proposed in the literature (Cordeau et al. 1997, 2001a, Ropke and Pisinger 2006a,b, Subramanian 2012), they usually address a single difficult compound problem formulation including several variants as special cases, but still require extensive adaptation when the main problem settings is modified. The field thus lacks an efficient general-purpose MAVRP solver and building one represents a considerable research challenge.

Our objective is to address this challenge and propose a unique general-purpose solver providing high performance in terms of solution quality and computational efficiency for a very broad and diverse set of multi-attribute vehicle routing problem settings. Furthermore, the methodology

we use in setting up this solver may points to promising developments in related fields such as scheduling.

We thus introduce a component-based heuristic solution framework, most of which is general in purpose, the specifics of each problem setting being confined to a very limited part of the method. The framework relies on adaptive *assignment*, *sequencing*, and *route-evaluation* components to interface with problem-specific knowledge. These components are automatically selected and adapted to address the problem attributes. We then use this framework to derive efficient general-purpose *Unified Local Search (ULS)* and *Unified Hybrid Genetic Search (UHGS)* methods for MAVRPs. ULS is built out of the route-evaluation components. It is designed for efficiency through information storage and addresses a very large set of MAVRPs with state-of-the-art move evaluations. The proposed UHGS exploits a giant-tour solution representation with a unified *Split* procedure, relies on route constraint relaxations to orient the search towards feasibility frontiers, harnesses the improvement capabilities of ULS to *educate* new individuals, and considers the diversity contribution of an individual as a proper component of its fitness to enhance exploration.

Extensive computational experiments demonstrate the remarkable performance of UHGS on the classical VRP as well as on MAVRP with multiple periods, multiple depots, vehicle-site dependencies, soft, multiple, and general time windows, backhauls, cumulative or load-dependent costs, simultaneous pickup and delivery, fleet mix, time dependency, service site choice, driving and working hour regulations, and many of their combinations. With a single implementation and parameter setting for all the 26 different VRP variants and the 39 sets of benchmark instances considered, UHGS matches or outperforms all current state-of-the-art problem-tailored algorithms in the literature, thus showing that generality does not alter efficiency for this class of problems.

The contributions of this work are the following: 1) A component-based heuristic design is proposed for multi-attribute vehicle routing problems, which efficiently isolates problem-specific adaptations from the generic framework; 2) A general-purpose local search is built on these principles, its computational complexity matching the state-of-the-art approaches on many MAVRPs despite its high generality; 3) A unified solution representation, *Split* algorithm, and genetic operators are introduced; 4) A Unified Hybrid Genetic Search is built from these components, addressing a large set of variants with a single implementation and set of parameters, and yielding solutions of remarkable quality on prominent VRP variants and benchmark instance sets.

This paper is structured as follows. Section 8.4 states the problem, reviews the main classes of general-purpose MAVRP solvers, and introduces the proposed component-based heuristic design. Section 8.5 details the ULS and its route-evaluation operators. Section 8.6 describes the UHGS. Computational experiments on a wide range of problems are reported in Section 8.7. Section 8.8 concludes.

8.4 Problem Statement and General Methodology

Vehicle routing problems have been studied for more than 50 years, serving as support for a vast literature, including numerous surveys (see Gendreau et al. 2008b, Andersson et al. 2010, Vidal et al. 2012c, among others), books (Toth and Vigo 2002a, Golden et al. 2008), and overall

more than a thousand dedicated journal articles (Eksioglu et al. 2009). The research effort on the topic is still growing today, because of its major economic impact, the large difficulty of many settings, and the considerable variety of attributes combinations encountered in practice.

8.4.1 Vehicle routing problems, notations and attributes

The classical Capacitated Vehicle Routing Problem (CVRP) can be stated as follows. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a complete undirected graph with $|\mathcal{V}| = n + 1$ vertices, vertex $v_0 \in \mathcal{V}$ representing a depot, where a fleet of m identical vehicles with capacity Q is based, the other vertices $v_i \in \mathcal{V} \backslash \{v_0\}$ for $i \in \{1, \ldots, n\}$ representing customers characterized by a demand for q_i units of product. Edges $(i, j) \in \mathcal{E}$ illustrate the possibility to travel from a customer v_i to a customer v_j for a cost d_{ij} (assimilated to the distance). The CVRP requires designing up to m cycles (vehicle routes) starting and ending at a depot v_0 in order to service each customer once.

Many VRP variants with *attributes* have emerged due to the requirements of practical applications. These particular versions aim at better accounting for customer requirements (e.g., time-dependent service costs, time windows, multiple planning periods), network and vehicle characteristics (multiple depots, congestion, heterogeneous fleet, vehicle-site dependencies), driver needs (working hour regulations, lunch breaks), or at better integrating the decisions in a tactical or strategic planning (inventory or location routing). The large variety of actual settings, characteristics and VRP attributes is addressed by a vast literature, including hundreds of methods targeting various attribute combinations. For the purpose of conciseness, we do not include a detailed literature review on all VRP variants addressed in this paper. Comprehensive surveys can be found in Gendreau et al. (2008b), Golden et al. (2008), Andersson et al. (2010), and Vidal et al. (2012c).

Following Vidal et al. (2012c), three main categories of attributes are discerned in this paper. ASSIGN attributes are problem particularities requiring decisions on the assignment of customers to some globally constrained ASSIGN Attribute Resources (AARs), for example, depots, days or vehicle types. SEQ attributes are problem characteristics that explicitly impact the structure and geometry of the routes such as, backhaul trips, multiple trips, or multi-echelon attributes. Finally, EVAL attributes affect the way routes are evaluated. This latter class of attributes encompasses advanced route costs or feasibility evaluations, as well as the eventual optimization of additional decisions on routes (e.g., service dates, waiting times, packing of objects in the vehicle) when the sequence of visits if fixed. Each family of attributes thus impacts the resolution methodologies in a very different way.

8.4.2 General-purpose solution approaches for MAVRPs.

Three main approaches for achieving generality may be identified when analyzing the literature on general-purpose MAVRP solvers that we identify as *rich solvers* and *modeling and solution frameworks*, examined in this subsection, and *component-based frameworks*, which are the topic of the next one.

Rich solvers are designed to address a multi-attribute VRP formulation generalizing several variants associated to subsets of its attributes. Several well-known VRP heuristics are included in this category and are displayed in Table 8.1: the Unified Tabu Search (UTS; Cordeau et al. 1997, 2001a, Cordeau and Laporte 2001, 2003, Cordeau et al. 2004), the Adaptive Large Neighborhood Search algorithm (ALNS; Ropke and Pisinger 2006a,b, Pisinger and Ropke 2007), the Iterated Local Searches of Ibaraki et al. (2005, 2008) and Hashimoto et al. (2006, 2008) (ILS), and Subramanian (2012) (ILS-SP), the latter being hybridized with integer programming components, and the exact integer programming approach of Baldacci and Mingozzi (2009), Baldacci et al. (2011a,b) (IPSP), based on a set partitioning formulation. Keeping in line with the focus of the paper on general-purpose algorithms, the table indicates for each method the largest subset of MAVRPs that was addressed in a **single implementation**, generally the one from the original paper. Most successful methodologies were extended later on to other variants, but separate developments were generally required. The subset of variants addressed by the general-purpose UHGS methodology we propose is also displayed for comparison purposes.

Table 8.1: Attributes addressed by some well-known rich VRP solvers

Type	Attribute	Acronym	UTS	ALNS	ILS	ILS-SP	IPSP	UHGS
ASSIGN	Multiple depots	MDVRP	X	X		X	X	X
	Multiple periods	PVRP	X				X	X
	Heterogeneous fleet	HVRP				X	X	X
	Site-dependent	SDVRP	X	X			X	X
	Split deliveries	VRPSD						
SEQ	Multiple trips	MTVRP						
	Pickup & deliveries	VRPPD	X	X			X	
	Backhauls	VRPB		X				X
EVAL	Open	OVRP		X		X		X
	Cumulative	CCVRP						X
	Load-dependent costs	LDVRP						X
	Simultaneous P.&D.	VRPSDP		X		X		X
	Vehicle Fleet Mix	VFMP				X	X	X
	Duration constraints	DurVRP	X		X			X
	Hard TW	VRPTW	X	X	X		X	X
	Soft TW	VRPSTW			X			X
	Multiple TW	VRPMTW			X			X
	General TW	VRPGTW			X			X
	Time-dep. travel time	TDVRP			X			X
	Flexible travel time	VRPFTT			X			
	Lunch breaks	VRPLB						X
	Work hours reg	VRTDSP						X
	Service choice (Generalized VRP)	GVRP[2]						X

Hybrid Genetic Algorithms (HGA), with *giant-tour* solution representations and local search solution enhancements (Prins 2004), have proven their ability in addressing many MAVRPs (Labadi et al. 2008, Prins 2009b, Ngueveu et al. 2010, Vidal et al. 2012a), as well as a large class of mixed node and arc routing problem variants (Prins and Bouchenoua 2005). We did not include them in this classification, however, because no unifying implementation of this class of methods has been proposed up to date, particular hard-coded implementations of solution representation, crossover, *Split*, and local search procedures being proposed for different MAVRPs. Generalizing these procedures to a wider range of variants is an important challenge that we address in this paper.

Each rich solver included in Table 8.1 relies on a "rich" multi-attribute VRP formulation, a periodic VRP with time windows (UTS), a pick-up and delivery problem with time windows (ALNS), a VRP with general time windows, time-dependent, and flexible travel-times (ILS), or a heterogeneous pickup-and-delivery problem with time windows (ILS-SP). Yet, relying on such formulations to achieve generality presents two main limitations. First, problems become more intricate and difficult to address as the number of attributes one must consider simultaneously grows. Second, all the features of the general model are still present when particular variants, with less attributes, are considered, resulting in loss of efficiency through wasted computations induced by deactivated attributes and, sometimes, higher complexity for some algorithm components. The methodology we propose avoids these pitfalls.

Modeling and solution frameworks seek to capture the general properties of the attributes to transform them into machine-readable components. Thus, the framework of Desaulniers et al. (1998) formulates a number of classes of attributes as resources (e.g., load, distance, time), which are extended to successive customer visits through *resource extension functions (REFs)* subject to interval constraints. This framework was applied to various crew scheduling and routing problem variants, the resulting formulations being then solved efficiently by column generation (Desaulniers et al. 2005).

It is well known that the performance of many heuristics for MAVRPs is directly linked to the capability of efficiently evaluating new routes produced during the search. Hence, a large body of literature focuses on reducing the complexity of route evaluation in presence of difficult EVAL attributes (Savelsbergh 1985, 1992, Garcia 1996, Kindervater and Savelsbergh 1997, Campbell and Savelsbergh 2004). These approaches share the common characteristic that they develop meaningful information on subsequences of successive visits (partial routes) to speed up evaluations of new routes. Using this methodology, time windows, simultaneous pickups and deliveries, and load-dependent costs attributes can be efficiently managed in the course of local searches, leading to notable gains in computational complexity.

Merging these two avenues of research, Irnich (2008b) considered forward and backward extension of resources, as well as the management of generalized resources extension functions on subsequences of visits to perform efficient route evaluations. This extended REF methodology was combined with *sequential search* concepts, leading to a unified solution approach (Irnich 2008a). Yet, strong properties on REFs inversion and generalization to segments are required for the framework to apply.

Finally, Puranen (2011) introduced a domain model able to express VRP variants and transform them into a routing metamodel workable by optimization methods. The routing metamodel is based on the concepts of *actors*, *activities*, *resources*, and *capabilities*. It exploits both the concept of resource extension functions, and a generalization called mapping-ordering constraints. The methodology covers the complete resolution process flow, from the domain model, to the routing metamodel and its resolution. However, few computational experiments were presented to demonstrate the capabilities of the approach.

8.4.3 Proposed component-based framework

As underlined in this review, a few unifying methodologies have been proposed for multi-attribute VRPs. However, these approaches are limited in the classes, properties and number of attributes they manage. Modeling and solution frameworks (Desaulniers et al. 1998, Irnich 2008a,b, Puranen 2011) do provide remarkable formalisms for many attributes, but in counterpart require strong properties to be efficiently applied, such as the existence of REFs which are invertible and generalizable to segments.

In this paper, we investigate a different approach towards general-purpose MAVRP resolution inspired from component-based design. Indeed, even a general-purpose solver must ultimately account for the specific attributes, objectives, and constraints of the particular problem setting at hand. Yet, to achieve a high level of generality, these problem attributes must be confined to restricted components of the algorithm, defined relatively to a subset of functionalities. These components should be *polymorphic* (Meyer 1997), meaning that they can be implemented differently as required by problem specifics, but their functionalities can always be used in the same manner, independently of the problem, and without requiring the knowledge of what is inside the component. The implementation of components requires a library of basic attribute-dependent operators, out of which the algorithm can automatically select the necessary operators relatively to the problem specification. Furthermore, components can be designed to offer the possibility to integrate attribute-specific strategies, opening the way, for example, to efficient route-evaluation procedures managing meaningful data on sequences.

It is worth noticing that related designs have been used in the combinatorial optimization literature to build general-purpose heuristic solvers or software libraries (e.g., Fink and Voss 2003, Cahon et al. 2004), hyper-heuristics (Burke et al. 2010), and cooperative methods (Crainic and Toulouse 2010). Component-based heuristic approaches are rare in the VRP literature (Du and Wu 2001, Groër et al. 2010). While polymorphism has been efficiently used to generate adaptable resolution strategies, i.e., configurable metaheuristics or local-search strategies, it has not yet provided the means to address the challenge of the broad variability in problem settings. Moreover, although hyper-heuristics and cooperative methods achieve more robust solving by making several basic methods adapt or cooperate, they are still dependent upon the availability of these basic problem-tailored methods.

We restrict in this paper the scope of the proposed approach to the VRP class in order to keep the length of the paper within acceptable limits and discus in detail its main components and logic. Indeed, similarly to several other combinatorial optimization problems, MAVRPs present a particular structure combining decisions on assignment (and partitioning), sequencing, and fixed-sequence optimization and evaluation. Consequently, as indicated in Section 8.4.1, we identify three categories of attributes, defined relatively to their impact on the heuristic resolution: ASSIGN attributes requiring the assignment of routes and customers to global resources (depots, days, vehicle types), SEQ attributes determining the structure of the network and the sequences of visits, and EVAL attributes modifying the solution evaluations. We introduce three adaptive components, which account for these attributes, providing the following functionalities:

- **Assignment.** Select and check the feasibility of customer and route re-assignments to different ASSIGN attribute resources (day, depot, vehicle type...);
- **Sequence choice.** Generate neighbor solutions with different sequence alternatives with regards to SEQ attributes;
- **Route evaluations.** Evaluate a fixed route and optimize side decisions related to EVAL attributes (timing or loading sub-problems).

We show in the next sections how these components can serve as building blocks for a wide range of general-purpose neighborhood- or population-based metaheuristics for MAVRPs. Section 8.5 first describes a unified local search with efficient route-evaluation operators designed to address MAVRPs with EVAL attributes. Section 8.6 follows with a description of the proposed Unified Hybrid Genetic Search for MAVRPs.

8.5 Unified Local Search for Vehicle Routing Problems

Designing a general-purpose high-performance local search for MAVRPs is an important research challenge in itself. We therefore introduce first the methodology we propose to address this challenge, before proceeding to the complete UHGS framework. The emphasis is on *EVAL* attributes, which impact the heuristic resolution during route evaluations, such as loading constraints or timing aspects. In the proposed approach, these problem specifics are confined to *route-evaluation* components, which are adaptive problem-dependent elements of the methodology providing the basic functionalities for route, move evaluation, and feasibility statements. Since high performance is sought, these components were designed to store information on sub-sequences and avoid redundant computations. We first define these components, proceeding then to the corresponding route-evaluation operators and, finally, to the ULS method.

8.5.1 Route-evaluation components

The route-evaluation components exploit the fact that any local-search move issued from a bounded number of edge exchanges and node relocations can be assimilated to a recombination of a bounded number of sequence of visits from an incumbent solution (Kindervater and Savelsbergh 1997, Vidal et al. 2011). As illustrated in Figure 8.1, an inter-route RELOCATE move of a sequence of visits $[\sigma_r(u), \ldots, \sigma_r(v)]$ next to a visit $\sigma_{r'}(w)$ yields two recombined routes $\rho = [\sigma_r(1), \ldots, \sigma_r(u-1)] \oplus [\sigma_r(v+1), \ldots, \sigma_r(|r|)]$ and $\rho' = [\sigma_{r'}(1), \ldots, \sigma_{r'}(w)] \oplus [\sigma_r(u), \ldots, \sigma_r(v)] \oplus [\sigma_{r'}(w+1), \ldots, \sigma_{r'}(|r'|)]$, $\oplus$ denoting the concatenation operator.

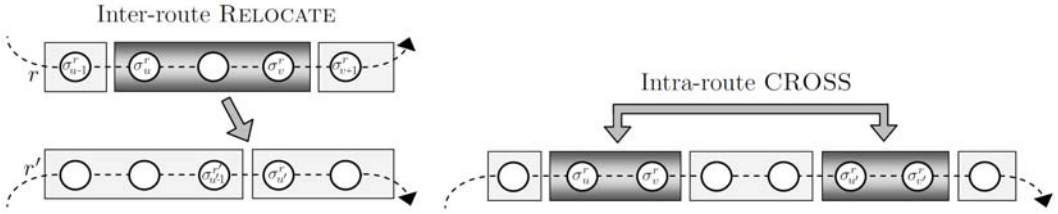


Figure 8.1: Moves assimilated to recombinations of sequences

We thus introduce in Table 8.2 five functionalities of route-evaluation components. The first three functionalities, called INIT(σ), FORW(σ), and BACK(σ) provide the means to initialize and build the re-optimization information on sequences by forward and backward concatenation of single visits, respectively. Within a local search, they can be used during a pre-processing phase to build the information on sub-sequences. The evaluation of new sequences made of a concatenation of several sub-sequences is then performed by using an evaluator, which takes advantage from the previously developed information on sub-sequences. Two evaluators, EVAL2(σ_1, σ_2) or EVALN($\sigma_1, \ldots, \sigma_n$), are presented. The former considers the concatenation of two segments, while the latter allows for any number of segments. The reasons for designing two different functionalities relate to the fact that all attributes do not allow for an efficient implementation of EVALN and thus, in some well-defined settings, the algorithm must rely on EVAL2 and other construction functionalities to perform route evaluations (Section 8.5.3).

Table 8.2: Route-evaluation component functionalities

Functionalities for data construction:	
INIT(σ)	Initialize the data $\mathcal{D}(v_0)$ for a sub-sequence containing a single visit.
FORW(σ)	Compute the data of $\mathcal{D}(\sigma \oplus v_i)$ from the data of sub-sequence σ and vertex v_i.
BACK(σ)	Compute the data of $\mathcal{D}(v_i \oplus \sigma)$ from the data of vertex v_i and sub-sequence σ.
Functionalities for route evaluations:	
EVAL2(σ_1, σ_2)	Evaluate the cost and feasibility of the concatenated sequence $\sigma_1 \oplus \sigma_2$.
EVALN($\sigma_1, \ldots, \sigma_n$)	Evaluate the cost and feasibility of the concatenated sequence $\sigma_1 \oplus \cdots \oplus \sigma_n$.

The route-evaluation component provides the basic structure to obtain state-of-the-art local search procedures for all *EVAL* attributes. It relies on a library of route-evaluation operators, specific to each attribute, which are selected automatically by the method relatively to the problem specification. Route-evaluation operators for different attributes are presented in Section 8.5.2. A unified local search based on these operators is presented in Section 8.5.3.

8.5.2 Route-evaluation operators for several attributes

Route-evaluation operators are specific to each attribute, but always respect the five functionality scheme described in Section 8.5.1. Three cases of attributes arise.

For the first case, some type of information on subsequences, including cost characterization, is efficiently computable by induction on the concatenation operation, such that a single equation can serve as the basis for all functionalities. Such a situation corresponds in the framework of Irnich (2008b) to the case of REFs that are invertible and generalizable to segments. Among the MAVRPs that can be managed in this way, we find the VRP with capacity, distance constraints, backhauls, cumulative costs, hard (eventually multiple) time windows, simultaneous deliveries and pickups, or lunch breaks.

In the second case, which includes soft time windows and time-dependent travel times, among others, the structure of the re-optimization information is more complex and FORW(σ) or BACK(σ) functionalities may become more computationally expensive than quick concatenation evaluations. In addition, EVALN may not be available in all cases.

Finally, a more advanced role may be given to the route-evaluation operator for some MAVRPs. These operators can indeed assume the optimization of additional decisions on visit locations within groups of customers (case of the generalized VRP), explicitly determine the break times placement for drivers (VRP with truck driver schedule regulations), or position the objects in the vehicle (VRP with loading constraints). Bi-directional shortest path procedures, tree search methods, or integer programming components are then potentially employed in the operators. We now describe route-evaluation operators for several important attributes.

Capacity and distance. The classical CVRP is perhaps the simplest setting for which information preprocessing is frequently used. Indeed, it is natural to manage for each sub-sequence σ its partial load $Q(\sigma)$ and partial distance $D(\sigma)$ to speed-up the load constraint checks and distance computations. Equations (8.1) and (8.2) enable to compute these quantities by induction on the concatenation operation, and provide the means to perform both Forw, Back and EvalN functionalities in $O(1)$ time. It is also worth noting that other globally constrained resources accumulated on arcs or vertices on the routes can be managed in the same way (see Irnich 2008b).

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \tag{8.1}$$

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{8.2}$$

Cumulative costs. The Cumulative VRP (CCVRP) is based on a different objective seeking the minimization of the sum of arrival times to customers. Evaluating the cost of a route subject to some modifications requires more advanced methods than for the classical CVRP, since arrival times to many customers in the route are impacted. Still, evaluations remain manageable in amortized $O(1)$ operations for several families of classical local search neighborhoods (Ngueveu et al. 2010). Vidal et al. (2011) and Silva et al. (2012) show that three types of information on subsequences are sufficient to efficiently evaluate route costs: the duration $D(\sigma)$ to perform the sequence of visits σ, the cumulative cost $C(\sigma)$ when starting at time 0, thus representing the cost of the sequence, and the delay cost $W(\sigma)$ for each unit of time delay in the starting date. For a sequence σ_0 containing a single vertex, the information can be initialized by setting $D(\sigma_0) = 0$ as no travel time is performed, $C(\sigma_0) = 0$, and $W(\sigma_0) = 1$ when the vertex is a customer, otherwise $W(\sigma_0) = 0$. Equations (8.3-8.5) then enable to compute this information by induction on the concatenation operation, thus allowing to efficiently implement all route-evaluation functions.

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{8.3}$$

$$C(\sigma_1 \oplus \sigma_2) = C(\sigma_1) + W(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + C(\sigma_2) \tag{8.4}$$

$$W(\sigma_1 \oplus \sigma_2) = W(\sigma_1) + W(\sigma_2) \tag{8.5}$$

Load-dependent costs. The fuel consumption f_{ij} of a vehicle is estimated in Xiao et al. (2012) to grow linearly with the load q_{ij} on a segment, and thus $f_{ij} = (f_1 q_{ij} + f_2) d_{ij}$, where f_1 represents the fuel cost per mile and unit of load, and f_2 stands for the base cost per mile. We propose an efficient evaluation of fuel consumption on a route which involves the computation of cumulated

demand $Q(\sigma)$, distance $D(\sigma)$, and the load-factor $F(\sigma)$ (load-times-distance) on sequences. The fuel consumption $C(\sigma)$ can be derived from this information since $C(\sigma) = f_1F(\sigma) + f_2D(\sigma)$. For a sequence σ_0 containing a single vertex v_i, $Q(\sigma_0) = q_i$, $D(\sigma_0) = 0$, and $F(\sigma_0) = 0$. Furthermore, Equations (8.6-8.8) enable to compute these values by induction on larger subsequences, leading to route evaluations in $O(1)$ time.

$$D(\sigma_1 \oplus \sigma_2) = D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + D(\sigma_2) \tag{8.6}$$

$$Q(\sigma_1 \oplus \sigma_2) = Q(\sigma_1) + Q(\sigma_2) \tag{8.7}$$

$$F(\sigma_1 \oplus \sigma_2) = F(\sigma_1) + Q(\sigma_2)(D(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)}) + F(\sigma_2) \tag{8.8}$$

Backhauls. In the VRP with Backhauls (VRPB), to each customer is either associated a delivery quantity $q_i \neq 0$ of a product, or a pickup quantity $p_i \neq 0$ of a different product. The capacity of the vehicle is limited to Q product units. Furthermore, a structural route constraint is imposed, pick-up customers being necessarily serviced at the end of the route, after at least one delivery customer. This structural constraint can be modeled directly in the distance matrix by setting $c_{ij} = +\infty$ if vertex v_i corresponds to a pickup customer and v_j is a delivery customer, and by setting the distance from the depot $c_{0j} = +\infty$ for any pickup customer v_j. Evaluating the routes then requires checking the load constraints and summing up the distances. Three types of information are developed on sequences σ to that extent: the partial distance $D(\sigma)$, the total delivery quantity $Q^{\mathrm{D}}(\sigma)$, and the total pickup quantity $Q^{\mathrm{P}}(\sigma)$. Since the two types of products are never jointly in the vehicle because of structural route constraints, checking load feasibility on a sequence involves simply to check whether $Q^{\mathrm{D}}(\sigma) \leq Q$ and $Q^{\mathrm{P}}(\sigma) \leq Q$. Hence, both $Q^{\mathrm{D}}(\sigma)$ and $Q^{\mathrm{P}}(\sigma)$ can be independently evaluated as previously described in Equation (8.1) to perform route evaluations.

Simultaneous deliveries and pickups. The VRP with simultaneous deliveries and pickups (VRPSDP) also involves two different products to be respectively delivered and picked-up. In contrast with the VRPB, no structural constraint is imposed on the routes, and a vertex can require both a delivery and a pick-up. As the vehicle can now contain both types of products simultaneously, load feasibility must be ensured at each vertex of the trip. To that extent, three kinds of data are managed on subsequences: $Q^{\mathrm{D}}(\sigma)$ and $Q^{\mathrm{P}}(\sigma)$, the sum of deliveries and pick-ups on the sequence σ, respectively, and $Q^{\mathrm{MAX}}(\sigma)$, the maximum load in the vehicle while processing the sequence σ when starting with an initial load of $Q^{\mathrm{D}}(\sigma)$. These values can be computed by induction on the concatenation operation using Equations (8.9-8.11), leading to efficient constant time route-evaluation functionalities.

$$Q^{\mathrm{P}}(\sigma_1 \oplus \sigma_2) = Q^{\mathrm{P}}(\sigma_1) + Q^{\mathrm{P}}(\sigma_2) \tag{8.9}$$

$$Q^{\mathrm{D}}(\sigma_1 \oplus \sigma_2) = Q^{\mathrm{D}}(\sigma_1) + Q^{\mathrm{D}}(\sigma_2) \tag{8.10}$$

$$Q^{\mathrm{MAX}}(\sigma_1 \oplus \sigma_2) = \max\{Q^{\mathrm{MAX}}(\sigma_1) + Q^{\mathrm{D}}(\sigma_2), Q^{\mathrm{MAX}}(\sigma_2) + Q^{\mathrm{P}}(\sigma_1)\} \tag{8.11}$$

Another variant of VRPSDP has been addressed in Kindervater and Savelsbergh (1997), where a single commodity was considered and products picked-up at a location could be used to service

further customers in the route, leading to different equations.

Time windows and duration constraints. The VRP with hard time windows (VRPTW) imposes interval constraints $[e_i, l_i]$ on arrival dates to each customer v_i, as well as service durations s_i (by default $s_0 = 0$). Waiting time is allowed on the route. The VRPTW is the first variant on which information on sub-sequences was managed and exploited (Savelsbergh 1985, 1992, Garcia 1996, Kindervater and Savelsbergh 1997). These authors proposed to characterize any sub-sequence with four types of information: a feasibility statement $F(\sigma)$, the sum of travel and service times $T(\sigma)$, the earliest possible completion time for the sequence of visits $E(\sigma)$, and the latest feasible starting date $L(\sigma)$. For a sequence $\sigma_0 = (v_i)$ containing a single vertex, $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$, and $F(\sigma_0) = true$. Equations (8.12-8.15) enable then to compute by induction the information for a concatenation of sequences.

$$T(\sigma_1 \oplus \sigma_2) = T(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2) \tag{8.12}$$

$$E(\sigma_1 \oplus \sigma_2) = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\} \tag{8.13}$$

$$L(\sigma_1 \oplus \sigma_2) = \min\{L(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\} \tag{8.14}$$

$$F(\sigma_1 \oplus \sigma_2) \equiv F(\sigma_1) \ \wedge \ F(\sigma_2) \ \wedge \ (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{8.15}$$

When the departure date of the vehicle is not fixed, starting dates have an influence on the total waiting time on the route. The minimum duration for the route can still be obtained from the previous information as $DUR(\sigma) = \max\{E(\sigma_i) - L(\sigma_i), T(\sigma_i)\}$. Route evaluations are thus manageable in $O(1)$ time.

Lunch breaks and depot choices. Lunch breaks appear in several real-life applications (Sahoo et al. 2005, Bostel et al. 2008), but have been the focus of only moderate attention in the literature. Let the VRPTW with lunch breaks (VRPTWLB) be defined as a VRPTW variant such that for any non-empty route a single break of duration s_{LB} must be taken between $[e_{\text{LB}}, l_{\text{LB}}]$ at one dedicated location v^{LB} chosen in a set of potential locations $\mathcal{V}^{\text{LB}}$. Let also the variant with flexible breaks (VRPTWFB) represent the case where the location of the break is unconstrained. As shown in the following, lunch placement choices can be addressed by adequately designing the route-evaluation operators, having thus a minor impact only on the general algorithm structure.

Consider the case of the VRPTWFB. Any sub-sequence σ can be characterized by two sets of information: a data set $T(\sigma)$, $E(\sigma)$, $L(\sigma)$, $F(\sigma)$, characterizing the time windows as in Equations (8.12-8.15) when no break has been taken in the sub-sequence, and another data set $E'(\sigma)$, $L'(\sigma)$, $F'(\sigma)$, characterizing the case where a break is taken *somewhere* between the first and the last visit of σ. By definition, $T'(\sigma) = T(\sigma) + s_{\text{LB}}$ for any σ. Initially, for a sequence $\sigma_0 = (v_i)$ containing a single vertex, $T(\sigma_0) = s_i$, $E(\sigma_0) = e_i + s_i$, $L(\sigma_0) = l_i$ and $F(\sigma_0) = true$. Furthermore, breaks are exclusively taken inside the sequence and thus, a sequence made of a single visit should not include a break, such that $E'(\sigma_0) = +\infty$, $L'(\sigma_0) = 0$ and $F'(\sigma_0) = false$. Computing $T(\sigma_1 \oplus \sigma_2)$, $E(\sigma_1 \oplus \sigma_2)$, $L(\sigma_1 \oplus \sigma_2)$, $F(\sigma_1 \oplus \sigma_2)$ can be done as previously with Equations (8.12-8.15). Computing their counterparts with breaks by induction comes to select a best case out of three: the break is either

taken during σ_1 (Case 1), between σ_1 and σ_2 (Case 2), or during σ_2 (Case 3). These computations are displayed in Equations (8.16-8.27).

$$E'(\sigma_1 \oplus \sigma_2) = \min(\{E'_{\text{case i}} | F'_{\text{case i}} = true\} \cup +\infty) \tag{8.16}$$
$$L'(\sigma_1 \oplus \sigma_2) = \max(\{L'_{\text{case i}} | F'_{\text{case i}} = true\} \cup -\infty) \tag{8.17}$$
$$F'(\sigma_1 \oplus \sigma_2) = F'_{\text{case 1}} \vee F'_{\text{case 2}} \vee F'_{\text{case 3}} \tag{8.18}$$
$$E'_{\text{case 1}} = \max\{E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T(\sigma_2), E(\sigma_2)\} \tag{8.19}$$
$$E'_{\text{case 2}} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + s_{\text{LB}} + T(\sigma_2), e_{\text{LB}} + s_{\text{LB}} + T(\sigma_2), E(\sigma_2)\} \tag{8.20}$$
$$E'_{\text{case 3}} = \max\{E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} + T'(\sigma_2), E'(\sigma_2)\} \tag{8.21}$$
$$L'_{\text{case 1}} = \min\{L'(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T'(\sigma_1)\} \tag{8.22}$$
$$L'_{\text{case 2}} = \min\{L(\sigma_1), l_{\text{LB}} - T(\sigma_1), L(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - s_{\text{LB}} - T(\sigma_1)\} \tag{8.23}$$
$$L'_{\text{case 3}} = \min\{L(\sigma_1), L'(\sigma_2) - d_{\sigma_1(|\sigma_1|)\sigma_2(1)} - T(\sigma_1)\} \tag{8.24}$$
$$F'_{\text{case 1}} = F'(\sigma_1) \wedge F(\sigma_2) \wedge (E'(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{8.25}$$
$$F'_{\text{case 2}} = F(\sigma_1) \wedge F(\sigma_2) \wedge (E(\sigma_1) \leq l_{\text{LB}}) \wedge (E(\sigma_1) + s_{\text{LB}} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L(\sigma_2)) \tag{8.26}$$
$$F'_{\text{case 3}} = F(\sigma_1) \wedge F'(\sigma_2) \wedge (E(\sigma_1) + d_{\sigma_1(|\sigma_1|)\sigma_2(1)} \leq L'(\sigma_2)) \tag{8.27}$$

It is worth mentioning that a similar methodology can be used to adjust dynamically, within the evaluation of the routes, the break location choices in the VRPTWLB case, as well as the choice and placement of depot visits in a multi-depot setting. Integrating these decisions in the evaluation operators enables to combine the placement or assignment features within local search moves, and considerably reduce the combinations of choices to be worked out in the remaining parts of the method.

Soft and general time windows. For all previously-mentioned attributes, constant-size characteristic data was available for the segments, as well as general concatenation equations (segment REFs in the terminology of Irnich 2008b). However, several MAVRPs fall outside of this class. This is the case for the VRP with soft time windows (VRPSTW), which allows penalized late arrivals to customers, and, more generally, for the generalization of the VRPTW where the service cost $c_i(t_i)$ of each customer v_i is a piecewise linear function of the service time. For this latter variant, the placement of departure times and waiting times, and thus the determination of a good schedule for a fixed route, makes for a non-trivial *timing problem with separable time-dependent processing costs* (Vidal et al. 2011) and no judicious $O(1)$ size data structure is known to characterize the sub-sequences and their exact cost when concatenated together.

In this case, the route-evaluation information can be developed as a set of piecewise functions (Hendel and Sourd 2006, Ibaraki et al. 2005, 2008). Each sub-sequence is characterized by a function $F(\sigma)(t)$ representing the minimum cost to service the sequence σ while arriving at the last customer before time t, and $B(\sigma)(t)$ stating the minimum cost of servicing σ after time t.

For a sequence $\sigma_0 = (v_i)$ with a single vertex, $F_{\sigma_0}(t) = \min_{x \le t} c_i(x)$ and $B_{\sigma_0}(t) = \min_{x \ge t} c_i(x)$. The construction operator FORW relies on forward dynamic programming (Equation 8.28) to build explicitly the information for the concatenation of a sequence σ with a vertex v_i. In reverse, the functionality BACK is based on backward dynamic programming (Equation 8.29). Equation (8.30) provides the cost $Z^*(\sigma_1 \oplus \sigma_2)$ of the concatenated sequence $\sigma_1 \oplus \sigma_2$ when $F(\sigma_1)(t)$ and $B(\sigma_2)(t)$ are available, thus leading to an efficient EVAL2 functionality.

$$F(\sigma \oplus v_i)(t) = \min_{0 \le x \le t} \{c_i(x) + F(\sigma)(x - s_{\sigma(|\sigma|)} - d_{\sigma(|\sigma|),i})\} \tag{8.28}$$

$$B(v_i \oplus \sigma)(t) = \min_{x \ge t} \{c_i(x) + B(\sigma)(x + s_i + d_{i,\sigma(1)})\} \tag{8.29}$$

$$Z^*(\sigma_1 \oplus \sigma_2) = \min_{x \ge 0} \{F(\sigma_1)(x) + B(\sigma_2)(x + s_{\sigma_1(|\sigma_1|)} + d_{\sigma_1(|\sigma_1|)\sigma_2(1)})\} \tag{8.30}$$

In our implementations, the data structures $F(\sigma)(t)$ and $B(\sigma)(t)$ are managed as linked lists of function pieces characterized by interval, origin value and slope. The data construction functionalities FORW(σ), BACK(σ) and EVAL2 work in $O(\Sigma_i \xi(c_i))$ time, $\xi(c_i)$ representing the number of pieces of a piecewise cost function c_i. However the EVALN functionality is not efficiently manageable. In the particular case where all functions $c_i(t)$ are convex, more advanced implementations based either on heaps (Hendel and Sourd 2006) or on search trees (Ibaraki et al. 2008) achieve a complexity of $O(\log \Sigma_i \xi(c_i))$ for both EVAL2 and EVALN.

Other time features. The literature contains various other EVAL attributes related to time, such as duration constraints, multiple time windows, time-dependent trip durations, flexible travel times, and minimum and maximum intervals of time between pairs of services. We refer to Vidal et al. (2011) for a comprehensive review and analysis of state-of-the-art algorithms for the underlying *timing* sub-problems for route evaluations, and their incremental resolution during local searches. These approaches were used to generate UHGS route-evaluation operators for the related problems with time characteristics.

Service site choices. In the Generalized Vehicle Routing Problem (GVRP), each request v_i is associated to a set of λ_i alternative locations $L_i = \{l_{i1}, \ldots, l_{i|\lambda_i|}\}$. Exactly one location of each set must be serviced. As illustrated in Baldacci et al. (2009), the GVRP is relevant for several practical applications and directly generalizes other variants of vehicle routing.

Recent metaheuristics for this problem (Moccia et al. 2012) include local-search procedures on the order of services that do not consider explicitly in the neighborhoods the locations to be serviced. Evaluating a sequence of services then becomes finding the associated shortest sequence of visits to locations, leading to a shortest path problem.

Again, efficient route evaluations require to store for each sequence of services appropriate data to speed-up the shortest path computation. In this case, the information to be stored for a sequence σ is the shortest path $S(\sigma)[i,j]$ between the i^{th} location of $\sigma(1)$ and the j^{th} location of $\sigma(|\sigma|)$, where $i \in \{1, \ldots, \lambda_{\sigma(1)}\}$ and $j \in \{1, \ldots, \lambda_{\sigma(|\sigma|)}\}$. For a sequence $\sigma_0 = (v_i)$ containing a single service, $S(\sigma_0)[x,x] = 0$ for any $x \in \{1, \ldots, \lambda_i\}$ and $S(\sigma_0)[x,y] = +\infty$ if $x \neq y$. Equation (8.31)

enables then to develop this information on larger subsequences by induction on the concatenation operation.

$$S(\sigma_1 \oplus \sigma_2)[i,j] = \min_{1 \leq x \leq \lambda_{\sigma_1(|\sigma_1|)}, 1 \leq y \leq \lambda_{\sigma_2(1)}} S(\sigma_1)[i,x] + d_{xy} + S(\sigma_2)[y,j] \quad \forall i \in \{1,\ldots,\lambda_{\sigma_1(1)}\}, \forall j \in \{1,\ldots,\lambda_{\sigma_2(|\sigma_2|)}\} \tag{8.31}$$

Equation (8.31) provides the means to perform efficiently in $O(\lambda^2)$ operations all route-evaluation functionalities, λ standing for the maximum number of locations associated to a service. This complexity is notably better than the complexity of computing each shortest path from scratch, which would be $O(n_r\lambda^2)$ operations for a route containing n_r services.

Hours of service regulations. Governments worldwide impose complex regulations on truck-driver schedules to limit the amount of work and driving within intervals of time and impose a minimum frequency and duration for break and rest periods. Because of their large impact on driving times, these regulations should be accounted for when optimizing the routes, leading to combined vehicle routing and truck-driver scheduling problems (VRTDSP). However, even checking the existence of a feasible placement of breaks for a fixed sequence of visits makes for a highly complex problem which is known to be solvable in a quadratic time for United States hours of service regulations (Goel and Kok 2012), while no polynomial algorithm is known for many other cases, with European Union, Canadian, and Australian rules.

Despite this high complexity, most efficient methods for the VRTDSP integrate break scheduling feasibility checks directly in the local search (Prescott-Gagnon et al. 2010, Goel and Vidal 2012), and thus during each route evaluation. In the proposed methodology, these break-scheduling procedures are used inside the route-evaluation operators. A set of schedule alternatives is maintained for each subsequence of consecutive visits. The schedule information is extended to larger subsequences by appending new driving and break activities at the end of the schedules, and selecting only a relevant subset by means of dominance relationships. Our current implementation is exclusively based on forward operators, and thus EVAL2(σ_1,σ_2) is performed by iteratively completing the schedule of σ_1 with services of σ_2.

Summary. As reviewed in this section, efficient route-evaluation operators relative to different VRP attributes may require to develop radically different information on sequences, and use more or less complex evaluation procedures. Still, all previously-mentioned approaches respect the same five functionalities scheme, based on the forward or backward propagation of labels (or, generally, of any information to characterize the sequences), and the evaluation of the concatenation of two or more sequences using the information developed on sequences. As shown in the following, this library of route-evaluation operators provides the means to create a general-purpose state-of-the-art local search for many MAVRPs.

8.5.3 Unified local search procedure

The five functionalities of the route-evaluation component can be used to build any local search method with moves involving a bounded number of edge exchanges and node relocations, since all these moves can be evaluated as a recombination of partial subsequences from the incumbent solution. The general ULS process is illustrated in Algorithm 8.1. To efficiently evaluate moves, ULS manages information on subsequences of consecutive visits (and reverse subsequences in presence of moves that impact the route orientation), using the INIT, FORW, and BACK route construction functionalities. This information is built during a pre-processing phase at the beginning of the local search, and is then updated whenever any route is modified. Moves are then evaluated by means of EVAL2 and EVALN.

Algorithm 8.1 Unified local search based on route-evaluation operators

```
Detect the good combination of evaluation operators relatively to the problem attributes
Build re-optimization data on subsequences using the INIT, FORW and BACK operators.
while some improving moves exist in the neighborhood 𝒩
   for each move μ_i in 𝒩
      for each route r_j^μ produced by the move
         Determine the k sub-sequences [σ_1, ..., σ_k] that are concatenated to produce r_j^μ
         if k = 2, then NEWCOST(r) = EVAL2(σ_1,σ_2)
         else if k > 2, then NEWCOST(r) = EVALN(σ_1,...,σ_k)
      if ACCEPTCRITERIA(μ_i) then perform the move μ and update the re-optimization data
      on for each route r_j^μ using the INIT, FORW and BACK operators.
```

In the specific implementation of this paper, the neighbor solutions issued from moves are explored in random order, using the acceptance criterion of Vidal et al. (2013) and terminating whenever no improving move can be found in the whole neighborhood. As in Vidal et al. (2013), the classical 2-OPT*, and 2-OPT neighborhoods are used, as well as the inter-route and intra-route CROSS and I-CROSS neighborhoods, restricted to subsequences of length smaller than $L_{max} = 2$ and including relocate moves as special cases. Only moves involving *neighbor vertices* in terms of distance and time characteristics (Toth and Vigo 2003, Vidal et al. 2013) are attempted, leading to a neighborhood size of $O(L_{max}^2 \Gamma n)$ instead of $O(L_{max}^2 n^2)$, where Γ stands for the number of neighbor vertices per vertex.

It should be noted that all *inter-route* moves such as CROSS, I-CROSS and 2-OPT*, require either EVAL2(σ_1, σ_2) or EVALN$(\sigma_1, \sigma_L, \sigma_2)$, where σ_L is a sequence of size bounded by L_{max}. When no efficient EVALN is available, in presence of attributes such as soft and general time windows for example, this first family of *inter-route* moves can still be evaluated efficiently as EVALN$(\sigma_1, \sigma_L, \sigma_2)$ can be replaced by less than L_{max} successive calls to FORW to yield the information on $\sigma' = \sigma_1 \oplus \sigma_L$, with a final call to EVAL2(σ', σ_2). *Intra-route* CROSS and I-CROSS and 2-OPT moves require calling EVALN on a set of 3 to 5 subsequences. If no efficient EVALN is available, the same reasoning for replacement can still be used, but in this case the number of necessary calls to FORW becomes linear in the route size since the size of intermediate subsequences is not bounded. However, since intra-route moves are usually in minority, this increased number of operations did not impact the method speed.

The good combination of route-evaluation operators is automatically determined relatively to the problem attributes according to the component-based framework of Section 8.4.3, and thus the route-evaluation operators allow to use advanced move evaluation techniques which were until now considered as problem-specific in a unified framework for MAVRPs. The resulting unified local search is efficient and applicable to many VRP variants. It can be extended into any generic neighborhood-based metaheuristic such as tabu search, iterated local search, or variable neighborhood search. Relatively to the recent advances in genetic algorithms and diversity management for vehicle routing, we opted to combine this procedure with the approach of Vidal et al. (2012a) to obtain a Unified Hybrid Genetic Search (UHGS). Such integration requires addressing several additional challenges, related to the design of a generic solution representation, genetic operators, and population management methods. The next section explains how to address them.

8.6 Unified Hybrid Genetic Search

The proposed UHGS is an extension of the Hybrid Genetic Search with Advanced Diversity Control of Vidal et al. (2012a), and aims to address MAVRPs in a unified manner by means of the proposed component-based design. The method stands out from previous works since all its elements (solution representation, genetic operators, local searches) are fully generic and detached from the attributes of the problem, relying on the subset of adaptive *assignment* and *route-evaluation* components to make the interface with problem-specific knowledge (Section 8.4.3). Note that in this work, only single-echelon problems with a route structure, e.g., a single sequence, are addressed, thus allowing to rely on a unique *sequencing component* based on standard VRP neighborhoods (Section 8.5.3). This Section briefly recalls the general structure of UHGS, then details in turn each element of the unified method.

8.6.1 General algorithmic structure

UHGS combines four main optimization methodologies: 1) hybridization of genetic algorithms with local search procedures; 2) the use of penalized infeasible solutions, managed through two distinct sub-populations during the search; 3) a solution representation *without trips delimiters* (Prins 2004) with an optimal *Split* procedure for delimiter computation; 4) an advanced population management method with *diversity-and-cost objective* for solution evaluation.

The structure of UHGS is illustrated in Figure 8.2. UHGS iteratively selects two individuals in the merged sub-populations to serve as input of a crossover operator, yielding a single offspring. After going through the *Split* procedure to compute trip delimiters, the offspring is *Educated* by means of a local search, *Repaired* with probability P_{rep} when infeasible, and transfered to the suitable sub-population. Each sub-population is managed separately to trigger a *Survivor Selection* procedure when reaching a maximum size. *Diversification procedures* and *decomposition phases* are regularly used to further enhance the diversity and intensify the search around elite solution characteristics. The algorithm terminates when It_{max} successive iterations (individual generations) have been performed without improving the best solution, or when a time limit T_{max} is reached.

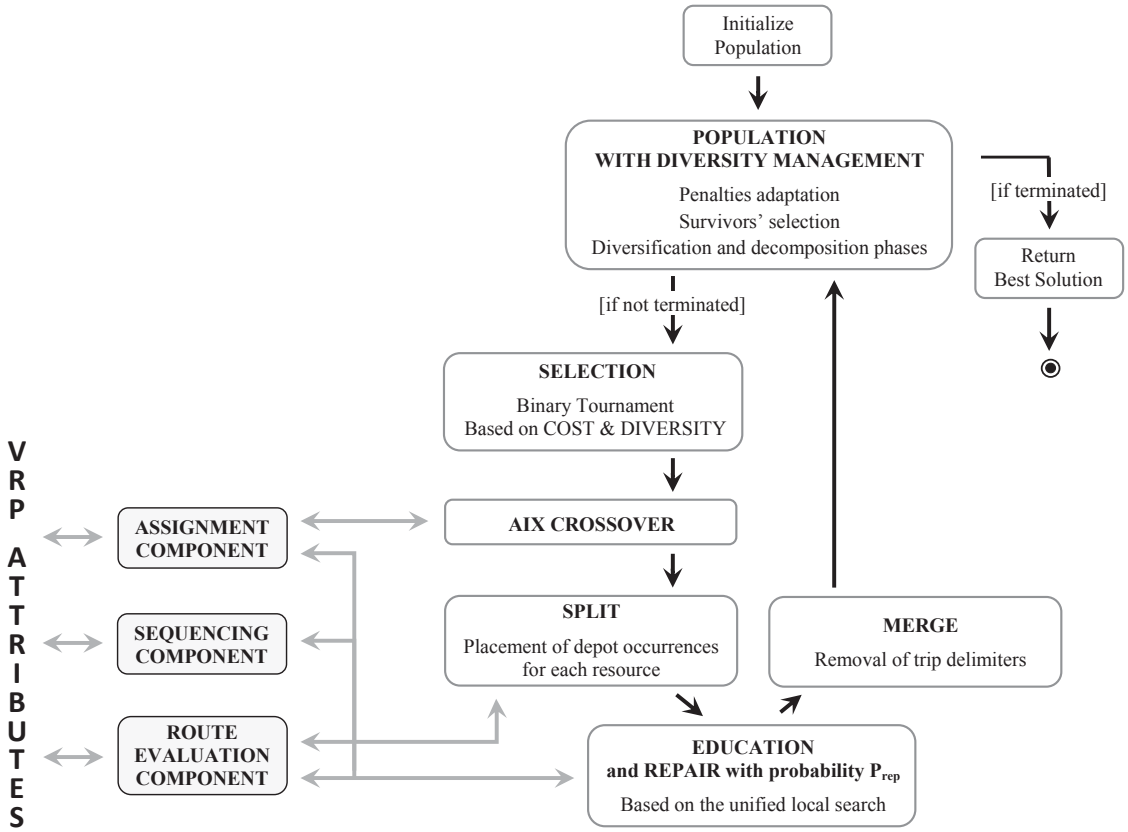


Figure 8.2: UHGS structure and relationships with problem attributes

8.6.2 Unified solution representation and Split

MAVRPs generally involve two levels of decisions relative to the assignment of customer services to some ASSIGN Attribute Resources (AARs), and the optimization of routes for each AAR. In accordance with this problem structure, solutions are represented in the course of UHGS as a collection of giant tours without explicit mention of visits to the depot (Prins 2004). As illustrated in Figure 8.3, each giant tour corresponds to a different combination of AAR, for example, a (vehicle type/day) couple in a heterogeneous periodic VRP. Problems without ASSIGN attributes lead to only one AAR, and thus to a solution representation as a single giant tour.

Not considering trip delimiters in the solution representation allows for simpler crossover procedures such as the ones introduced in Lacomme et al. (2005), Bostel et al. (2008), and Vidal et al. (2012a). On the other hand, a *Split* algorithm must be applied on each giant tour to insert depot visits, to support for solution evaluations and local-search procedures.

A fully generic *Split* procedure for MAVRPs based on route-evaluation components is introduced in Algorithm 8.2. For any giant tour $\tau = (\tau_1, \ldots, \tau_\nu)$ containing ν customers, the splitting problem is assimilated to a shortest path problem on a directed acyclic auxiliary graph $\mathcal{G}' = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ includes $\nu + 1$ nodes notated 0 to ν. Any arc $a_{ij} \in \mathcal{A}$ with $i < j$ represents the route originating from the depot, visiting customers σ_{i+1} to σ_j, and returning. The cost of each arc is set to the cost of the associated route.

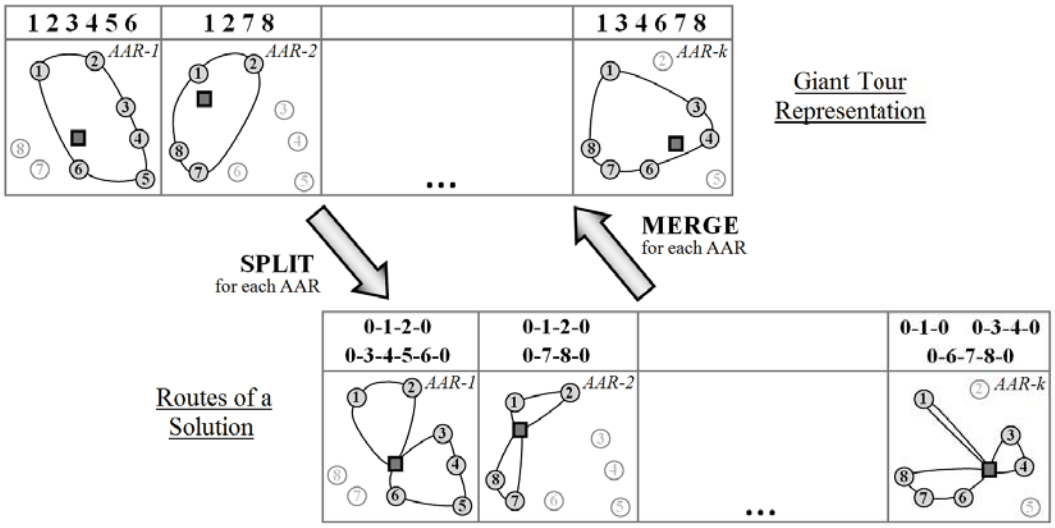


Figure 8.3: Solution representation as a giant tour per AAR, illustration of the Split procedure and its reverse Merge operation

Algorithm 8.2 Generic Split

1: **for** each node $i \in \{1, \ldots, \nu\}$
2: $SeqData(\sigma) = \textsc{Init}(\{v_0\})$ *//Initialize with depot vertex*
3: **for** each node $j \in \{i+1, \ldots, \min(i+\bar{r}, \nu)\}$
4: $SeqData(\sigma) = \textsc{Forw}(\sigma, \{\tau_j\})$ *//Append a new customer to the route end*
5: $\phi(a_{ij}) = \textsc{Eval2}(\sigma, \{v_0\})$ *//Evaluate the route*
6: Solve the shortest path problem on $\mathcal{G}' = (\mathcal{V}, \mathcal{A})$ with cost $\phi(a_{ij})$ for each arc a_{ij}
7: Return the set of routes associated to the set of arcs of the shortest path

All arc costs can be computed by calling $\mathcal{O}(\nu^2)$ times the Forw and Eval2 functions (Lines 1-5 of Algorithm 8.2). Setting a maximum value $\bar{r}$ on the number of customers in a route enables to reduce this number of calls to $\mathcal{O}(\nu\bar{r})$. Once this pre-processing is achieved, the shortest path is solved by means of m iterations of the Bellman-Ford algorithm (see Cormen et al. 2001) in presence of a fleet size limit to m. If no limit on the fleet size is imposed, a shortest path based on the topological order of indexes is used. The final complexity of the proposed unified *Split* algorithm is $\mathcal{O}([m + \xi(\textsc{Forw}) + \xi(\textsc{Eval2})]\nu\bar{r})$, where $\xi(\textsc{Forw})$ and $\xi(\textsc{Eval2})$ represent respectively the complexity of the Forw and Eval2 functions. This algorithm is applicable to all VRP variants mentioned in Section 8.5.2.

8.6.3 Evaluation of individuals

An individual p in UHGS is evaluated relatively to its *feasibility*, *cost*, and *contribution to the population diversity*. Define the *penalized cost* $\phi_{\mathcal{P}}^{\text{COST}}(p)$ of p as the sum on all routes of total distance and penalized excesses relatively to load and other constraint violations of N_{ATT} EVAL attributes. For any route σ with distance $\varphi^{\text{D}}(\sigma)$, load excess $\varphi^{\text{Q}}(\sigma)$, and excesses $\varphi^{\text{E}_i}(\sigma)$ for $i \in \{1, \ldots, N_{\text{ATT}}\}$ relatively to EVAL attributes, the penalized cost $\phi(r)$ is given by Equation (8.32),

where ω^{Q} and ω^{E_i} for $i \in \{1, \ldots, N_{\mathrm{ATT}}\}$ represent the associated penalty coefficients. The set of excesses $\varphi^{\mathrm{E}_i}(\sigma)$ depends upon the EVAL attributes of the problem, and can include the excess of pickup load (variants of VRPB), excess in duration (variants of DurVRP), time-window relaxations in the sense of Nagata et al. (2010) or service lateness (VRTDSP, TDVRP). Penalty coefficients are adapted during the search relatively to the proportion of feasible individuals as in Vidal et al. (2012a, 2013).

$$\phi(r) = \varphi^{\mathrm{D}}(r) + \omega^{\mathrm{Q}}\varphi^{\mathrm{Q}}(r) + \sum_{i=1}^{N_{\mathrm{ATT}}} \omega^{\mathrm{E}_i}\varphi^{\mathrm{E}_i}(\sigma) \tag{8.32}$$

Define the *diversity contribution* $\phi_{\mathcal{P}}^{\mathrm{DIV}}(p)$ of an individual p as its average distance with the μ^{CLOSE} most similar individuals in the sub-population. The Hamming distance on assignment decisions is used in presence of ASSIGN attributes, while in the other case, the broken pairs distance (Prins 2009b) is automatically used to measure the proportion of common edges.

Equation (8.33) finally states the *biased fitness* $f_{\mathcal{P}}(p)$ of an individual p in the sub-population $\mathcal{P}$ as a weighted sum of its penalized cost rank $f_{\mathcal{P}}^{\mathrm{COST}}(p)$ and its rank $f_{\mathcal{P}}^{\mathrm{DIV}}(p)$ relative to its diversity contribution. This trade-off between diversity and cost is balanced by parameter μ^{ELITE} and was shown to play an essential role in the performance of the method.

$$f_{\mathcal{P}}(p) = f_{\mathcal{P}}^{\mathrm{COST}}(p) + \left(1 - \frac{\mu^{\mathrm{ELITE}}}{|\mathcal{P}|}\right) f_{\mathcal{P}}^{\mathrm{DIV}}(p) \tag{8.33}$$

8.6.4 Selection and Crossover

Two parent individuals are iteratively selected during the course of UHGS by binary tournament within the merged feasible and infeasible sub-populations to serve as input to the crossover and produce a single offspring. The Assignment and Insertion Crossover (AIX) is then used for problems involving at least one ASSIGN attribute, otherwise the simple Ordered Crossover (OX) is applied (see Prins 2004).

AIX is a direct generalization of the PIX crossover of Vidal et al. (2012a). This crossover first decides for each of the n_{AAR} ASSIGN Attribute Resources whether the genetic material of p_1, p_2 or both parents is transmitted. To that extent, two random numbers are first picked between 0 and n_{AAR} according to a uniform distribution. Let n_1 and n_2 be the smallest and the largest of these numbers, respectively. For n_1 random AARs, the genetic material will be inherited from p_1 exclusively. For $(n_2 - n_1)$ other random AARs, the material will be inherited from p_2 exclusively. Finally, for the remaining $(n_{\mathrm{AAR}} - n_2)$ AARs, the material will be jointly inherited from p_1 and p_2. Such a method yields the possibility of unequal inheritance of genetic material from parents, even in presence of a large number of AARs, and thus provides the means to perform both small solution refinements and complex structural recombinations.

The selected material from p_1 is then fully transmitted, resulting in a partial assignment of customers to AARs, which is by definition a subset of a feasible assignment. The method then proceeds by inheriting in turn each selected delivery from p_2 and appending it to the end of the giant tour corresponding to its AAR. At each insertion, the *assignment* component (Section 8.4.3)

is called to check whether this inheritance of a customer still allows for completion into a feasible assignment relatively to ASSIGN attributes. If this latter property in not fulfilled, then the delivery is not transmitted to the offspring.

Finally, as the previous constraints can lead to an incomplete offspring, missing visits to customers are inserted in turn in a random order to the best location relatively to the penalized route cost. Good insertion procedures require the knowledge of the depot occurrences, and thus this final step of AIX is completed after using the unified *Split* algorithm (Section 8.6.2).

8.6.5 Education and Repair

An offspring issued from the crossover undergoes the *Split* procedure, and is then improved by means of an *Education* operator based on two local searches. First, the local search procedure of Section 8.5.3 is applied independently for each AAR to perform Route Improvements (RI). Second, an Assignment Improvement (AI) procedure is designed to optimize on assignment decisions. Finally, RI is applied a last time.

The AI procedure is the generalization of the *pattern improvement* procedure of Vidal et al. (2012a). AI tentatively removes all services to a customer, and chooses the best combination of insertion locations in the AARs for reinsertion. AI relies on the *assignment* component to list the tentative combinations of resource assignments, and evaluates each insertion position by means of the *route-evaluation* component. Any insertion is thus assimilated to a call to $\textsc{EvalN}(\sigma_1, \sigma_0, \sigma_2)$ where σ_0 contains a single vertex. Customer re-assignments are exhaustively tried in random order, the best re-assignment position being systematically chosen. AI stops when no improving re-assignment can be found.

The solutions issued from *Education* are directly accepted in the appropriate sub-population relatively to their feasibility. Furthermore, any infeasible solution with penalty (see Section 8.6.3) is *Repaired* with probability P_{rep}. The *Repair* operator temporarily increases the penalty coefficients by a factor of 10 and calls *Education* to redirect the search towards feasible solutions.

8.6.6 Population management

Sub-populations are independently managed to always contain between μ^{MIN} and $\mu^{\text{MIN}} + \mu^{\text{GEN}}$ individuals, by triggering a *survivor selection* phase each time a sub-population reaches a maximum size of $\mu^{\text{MIN}} + \mu^{\text{GEN}}$. *Survivor selection* consists in iteratively removing μ^{GEN} times the worst individual with regards to the biased fitness of Section 8.6.3, privileging first the removal of *clone* individuals with null distance to at least another individual. To regularly introduce new genetic material, these population management mechanisms are completed by diversification phases (Vidal et al. 2012a, 2013) which take place after each It_{div} successive iterations without improvement of the best solution, and consist in retaining the best $\mu/3$ individuals and replacing the others by new individuals. This advanced population management procedure coupled with the diversity-based evaluation of individuals plays a main role in the success of the overall algorithm.

8.6.7 Decomposition Phases

Finally, a decomposition phase is triggered after each It_{dec} iterations. In a decomposition phase, an elite solution selected within the 25% best feasible individuals is used to define subproblems, by fixing the assignments to AAR resources and by separating routes in several subsets as in Vidal et al. (2013). UHGS is then independently applied on each subproblem, including the partial solution issued from the elite individual in the initial population, and three different best solutions are reconstituted out of the best solutions of the subproblems. This decomposition phase introduces a strong intensification around elite characteristics, and contributes to finding high-quality solutions for problems of large size. For this reason, it is only activated for problem instances involving more than 150 customers.

8.7 Computational Experiments

Extensive computational experiments were conducted on a wide range of MAVRPs to assess the performance of the general-purpose UHGS relatively to the best problem-tailored algorithms for each setting. Both "academic" problems and rich multi-attribute VRPs have been addressed in these studies. A single parameter setting, the same as in Vidal et al. (2012a, 2013), was used for all the experiments in order to examine the applicability of the method without extensive problem-tailored parameter customization. The termination criteria was set to ($It_{max} = 5000$; $T_{max} = 30min$) to compare with other authors in similar time. Problems requiring fleet minimization were solved by iteratively decrementing the fleet size limit and running UHGS until no feasible solution can be found. The algorithm was implemented in C++ and run on Opteron 250 2.4GHz and Opteron 275 2.2GHz processors.

Table 8.3: List of acronyms for benchmark instances and methods

Benchmark instances:					
B11	Bektas et al. (2011)	G84	Golden (1984)	LS99	Liu and Shen (1999)
CGL97	Cordeau et al. (1997)	G09	Goel (2009)	MG06	Montané and Galvão (2006)
CL01	Cordeau and Laporte (2001)	GH99	Gehring and Homberger (1999)	SD88	Solomon and Desrosiers (1988)
CMT79	Christofides et al. (1979)	GJ89	Goetschalckx and J.-B. (1989)	SN99	Salhi and Nagy (1999)
F94	Fisher (1994)	GWKC98	Golden et al. (1998b)		
State-of-the-art algorithms:					
B10	Belhaiza (2010)	KTDHS12	Kritzinger et al. (2012)	RT10	Repoussis and Tarantilis (2010)
BDHMG08	Bräysy et al. (2008a)	MB07	Mester and Bräysy (2007)	RTBI10	Repoussis et al. (2010)
BER11	Bektas et al. (2011)	MCR12	Moccia et al. (2012)	RTI09a	Repoussis et al. (2009a)
BLR11	Balseiro et al. (2011)	NB09	Nagata and Bräysy (2009b)	RTI09b	Repoussis et al. (2009b)
BPDRT09	Bräysy et al. (2009)	NBD10	Nagata et al. (2010)	S12	Subramanian (2012)
CM12	Cordeau and M. (2012)	NPW10	Ngueveu et al. (2010)	SDBOF10	Subramanian et al. (2010)
F10	Figliozzi (2010)	P09	Prins (2009b)	SPUO12	Subramanian et al. (2012)
FEL07	Fu et al. (2007)	PBDH08	Polacek et al. (2008)	XZKX12	Xiao et al. (2012)
GA09	Gajpal and Abad (2009)	PDDR10	Prescott-Gagnon et al. (2010)	ZTK10	Zachariadis et al. (2010)
GG11	Groër et al. (2011)	PR07	Pisinger and Ropke (2007)	ZK10	Zachariadis and Kiranoudis (2010b)
HDH09	Hemmelmayr et al. (2009)	PR08	Pirkwieser and Raidl (2008)	ZK11	Zachariadis and Kiranoudis (2011)
ISW09	Imran et al. (2009)	RL12	Ribeiro and Laporte (2012)	ZK12	Zachariadis and Kiranoudis (2012)

Table 8.3 displays the list of acronyms for the benchmark instances and methods used in the comparative analysis. Tables 8.4 and 8.5 compare the results of UHGS with the current best methods in the literature for each problem class taken separately. Columns (1-4) indicate the

Table 8.4: Performance analysis on several VRP variants with various objectives

Variant	**Bench.**	***n***	**Obj.**	**State-of-the-art methods**				
				Author	Avg.%	Best%	T(min)	CPU
CVRP	CMT79	[50,199]	C	GG11	—	+0.03%	*2.38*	8×Xe 2.3G
				MB07	+0.03%	—	2.80	P-IV 2.8G
				UHGS*	**+0.02%**	**+0.00%**	11.90	Opt 2.4G
CVRP	GWKC 98	[200,483]	C	GG11	—	+0.29%	*5.00*	8×Xe 2.3G
				NB09	+0.27%	+0.16%	21.51	Opt 2.4G
				UHGS*	**+0.15%**	**+0.02%**	71.41	Opt 2.4G
VRPB	GJ89	[25,200]	C	ZK12	+0.38%	+0.00%	1.09	T5500 1.67G
				GA09	+0.09%	+0.00%	1.13	Xe 2.4G
				UHGS	**+0.01%**	**+0.00%**	0.99	Opt 2.4G
CCVRP	CMT79	[50,199]	C	NPW10	+0.74%	+0.28%	5.20	Core2 2G
				RL12	+0.37%	+0.07%	2.69	Core2 2G
				UHGS	**+0.01%**	**-0.01%**	1.42	Opt 2.2G
CCVRP	GWKC 98	[200,483]	C	NPW10	+2.03%	+1.38%	94.13	Core2 2G
				RL12	+0.34%	+0.07%	21.11	Core2 2G
				UHGS	**-0.14%**	**-0.23%**	17.16	Opt 2.2G
VRPSDP	SN99	[50,199]	C	SDBOF10	+0.16%	+0.00%	*0.37*	256×Xe 2.67G
				ZTK10	—	+0.11%	—	T5500 1.66G
				UHGS	**+0.01%**	**+0.00%**	2.79	Opt 2.4G
VRPSDP	MG06	[100,400]	C	SDBOF10	+0.30%	+0.17%	*3.11*	256×Xe 2.67G
				UHGS	+0.20%	+0.07%	12.00	Opt 2.4G
				S12	**+0.08%**	**+0.00%**	7.23	I7 2.93G
VFMP-F	G84	[20,100]	C	ISW09	—	+0.07%	8.34	P-M 1.7G
				SPUO12	+0.12%	+0.01%	0.15	I7 2.93G
				UHGS	**+0.04%**	**+0.01%**	1.13	Opt 2.4G
VFMP-V	G84	[20,100]	C	ISW09	—	+0.02%	8.85	P-M 1.7G
				SPUO12	+0.17%	+0.00%	0.06	I7 2.93G
				UHGS	**+0.03%**	**+0.00%**	0.85	Opt 2.4G
VFMP-FV	G84	[20,100]	C	P09	—	+0.02%	0.39	P4M 1.8G
				UHGS	+0.01%	+0.00%	0.99	Opt 2.4G
				SPUO12	**+0.01%**	**+0.00%**	0.13	I7 2.93G
LDVRP	CMT79	[50,199]	C	XZKX12	+0.48%	+0.00%	1.3	NC 1.6G
				UHGS	**-0.28%**	**-0.33%**	2.34	Opt 2.2G
LDVRP	GWKC98	[200,483]	C	XZKX12	+0.66%	+0.00%	3.3	NC 1.6G
				UHGS	**-1.38%**	**-1.52%**	23.81	Opt 2.2G
PVRP	CGL97	[50,417]	C	HDH09	+1.69%	+0.28%	3.09	P-IV 3.2G
				UHGS*	+0.43%	+0.02%	6.78	Opt 2.4G
				CM12	**+0.24%**	**+0.06%**	*3.55*	64×Xe 3G
MDVRP	CGL97	[50,288]	C	CM12	+0.09%	+0.03%	*3.28*	64×Xe 3G
				S12	+0.07%	+0.02%	11.81	I7 2.93G
				UHGS*	**+0.08%**	**+0.00%**	5.17	Opt 2.4G
GVRP	B11	[16,262]	C	BER11	+0.06%	—	0.01	Opt 2.4G
				MCR12	+0.11%	—	0.34	Duo 1.83G
				UHGS	**+0.00%**	**-0.01%**	1.53	Opt 2.4G

Table 8.5: Performance analysis on several VRP variants with various objectives (continued)

Variant	**Bench.**	***n***	**Obj.**	**State-of-the-art methods**				
				Author	Avg.%	Best%	T(min)	CPU
OVRP	CMT79 &F94	[50,199]	F/C	RTBI10	0%/+0.32%	—	9.54	P-IV 2.8G
				S12	—/+0.16%	0%/+0.00%	2.39	I7 2.93G
				UHGS	**0%/+0.11%**	**0%/+0.00%**	1.97	Opt 2.4G
OVRP	GWKC 98	[200,480]	F/C	ZK10	0%/+0.39%	0%/+0.21%	14.79	T5500 1.66G
				S12	0%/+0.13%	0%/+0.00%	64.07	I7 2.93G
				UHGS	**0%/-0.11%**	**0%/-0.19%**	16.82	Opt 2.4G
VRPTW	SD88	100	F/C[4]	RTI09	0%/+0.11%	0%/+0.04%	17.9	Opt 2.3G
				UHGS*	0%/+0.04%	0%/+0.01%	2.68	Xe 2.93G
				NBD10	**0%/+0.02%**	**0%/+0.00%**	5.0	Opt 2.4G
VRPTW	HG99	[200,1000]	F/C[4]	RTI09b	—	+0.16%/+3.36%	270	Opt 2.3G
				NBD10	+0.20%/+0.42%	+0.10%/+0.27%	21.7	Opt 2.4G
				UHGS*	**+0.18%/+0.11%**	**+0.08%/-0.10%**	141	Xe 2.93G
OVRPTW	SD88	100	F/C[4]	RTI09a	+0.89%/+0.42%	0%/+0.24%	10.0	P-IV 3.0G
				KTDHS12	0%/+0.79%	0%/+0.18%	10.0	Xe 2.67G
				UHGS	**+0.09%/-0.10%**	**0%/-0.10%**	5.27	Opt 2.2G
TDVRPTW	SD88	100	C	KTDHS12	+1.03%	0%	10.0	Xe 2.67G
				UHGS	**-0.93%**	**-1.03%**	11.59	Opt 2.2G
VFMPTW	LS99	100	D	BDHMG08	—	+0.59%	10.15	Ath 2.6G
				RT10	+0.22%	—	16.67	P-IV 3.4G
				UHGS	**-0.15%**	**-0.24%**	4.58	Opt 2.2G
VFMPTW	LS99	100	C	BDHMG08	—	+0.25%	3.55	Ath 2.6G
				BPDRT09	—	+0.17%	0.06	Duo 2.4G
				UHGS	-0.38%	-0.49%	4.82	Opt 2.2G
PVRPTW	CL01	[48,288]	C	PR08	—	+1.75%	—	Opt 2.2G
				CM12	+1.10%	+0.76%	*11.3*	64×Xe 3G
				UHGS*	**+0.63%**	**+0.22%**	32.7	Xe 2.93G
MDVRPTW	CL01	[48,288]	C	PBDH08	—	+1.37%	147	P-IV 3.6G
				CM12	+0.36%	+0.15%	*6.57*	64×Xe 3G
				UHGS*	**+0.19%**	**+0.03%**	6.49	Xe 2.93G
SDVRPTW	CL01	[48,288]	C	B10	+2.23%	—	2.94	Qd 2.67G
				CM12	+0.62%	+0.36%	*5.60*	64×Xe 3G
				UHGS*	**+0.36%**	**+0.10%**	5.48	Xe 2.93G
VRPSTW (type 1, α=100)	SD88	100	F/TW/C[5]	F10	0%	—	9.69	P-M 1.6G
				UHGS	**-3.05%**	**-4.42%**	18.62	Opt 2.2G
VRPSTW (type 1, α=1)	SD88	100	C+TW	KTDHS12	+0.62%	+0.00%	10.0	Xe 2.67G
				UHGS	**-0.13%**	**-0.18%**	5.82	Opt 2.2G
VRPSTW (type 2, α=100)	SD88	100	F/TW/C[5]	FEL07	0%	—	5.98	P-II 600M
				UHGS	**-13.91%**	**-13.91%**	41.16	Opt 2.2G
VRPSTW (type 2, α=1)	SD88	100	C+TW	**UHGS**	**+0.26%**	**0%**	29.96	Opt 2.2G
MDPVRPTW	New	[48,288]	C	**UHGS**	**+0.77%**	**0%**	16.89	Opt 2.2G
VRTDSP (E.U. rules)	G09	100	F/C	PDDR10	0%/0%	0%/0%	88	Opt 2.3G
				UHGS*	**-0.56%/-0.54%**	**-0.85%/-0.70%**	228	Xe 2.93G

$*$ These results have been originally presented in Vidal et al. (2013, 2012a) and Goel and Vidal (2012).

[4] The gaps to BKS in terms of fleet size and distance are averaged on groups of instances C1-100,R1-100,...,RC2-1000.

[5] For the sake of brevity, only the fleet size is reported in this Table.

variant considered, the origin of the benchmark instances, the number n of customers in these instances, and the objective function ("C" standing for distance, "D" for duration, i.e., the time elapsed between departure and return, "T" for travel time, "F" for fleet size, "TW" for time-window violations). Hierarchical objectives are presented by decreasing order of priority, separated with the sign "/". The last column reports, for each state-of-the-art method, the gap of an average or single run with respect to the current Best Known Solutions (BKS), the gap of the best solution produced by the method, the average run time to achieve these results (for parallel methods, the computation time on a single CPU is reported in italics as well as the number of CPUs), and the type of processor used. The algorithm yielding the best result quality, for each benchmark instances set and problem class, is indicated in boldface. Table 8.4 also includes the results from previous HGSADC applications on the PVRP, MDVRP, CVRP, and VRTDSP with European Union regulations (Vidal et al. 2013, 2012a, Goel and Vidal 2012) since UHGS works identically when instantiated on these problems. Detailed results are reported in Appendix V.

As reported in Tables 8.4 and 8.5, UHGS produces high-quality solutions for all problems and benchmark sets, from pure academic problems such as the CVRP to a large variety of VRP variants and rich settings. In addition to its high generality and its potential to address many MAVRPs, UHGS matches or outperforms the wide majority of problem-tailored approaches on each separate benchmark and problem class. The average standard deviation of solutions, measured separately for each single objective problem, ranges between 0.002% (GVRP) and 0.66% (MDPVRPTW), thus showing that the metaheuristic produces high-quality solutions in a consistent manner. The average run time remains in most cases smaller than 10 minutes for average-sized problems (100 to 200 customers), being thus adequate for daily or weekly planning. Overall, 954 BKS out of 1008 have been either retrieved or improved during these experiments, and 550 BKS out of 1008 have been strictly improved.

8.8 Conclusions and Perspectives

A new unified local search and a Unified Hybrid Genetic Search (UHGS) relying on a component-based design have been introduced to address a large variety of difficult VRP variants. The methods rely on adaptive assignment, sequencing, and route-evaluation components to address problem specifics. These components serve as the basis to generate generic local-search improvement, *Split*, and genetic operators. Furthermore, the use of individual fitness measures based on both diversity and quality, and advanced population management schemes provides the means for a thorough and efficient search. The remarkable performance of UHGS has been demonstrated on a wide range of problems. On 26 VRP variants and 39 sets of benchmark instances, UHGS matches or outperforms the current state-of-the-art problem-tailored algorithms. Overall, 954 of the 1008 best known solutions have been either retrieved or improved. Hence, it appears that the proposed heuristic design is particularly efficient for dealing with MAVRPs and, furthermore, that generality does not necessarily play against performance for the considered classes of VRP variants.

This general-purpose solver opens the way to experimentations and sensitivity analyses of local search and metaheuristic components on a wide range of structurally different problems.

Perspectives of research involve the generalization of the method towards a wider variety of ASSIGN and SEQ attributes, multi-objective and stochastic settings. Further methodological analyses should also be conducted to fully identify the compatibility relationships and complexity implications of attribute combinations.

CHAPITRE 9

UNE APPROCHE PARALLÈLE COOPÉRATIVE À BASE DE DÉCOMPOSITION ET INTÉGRATION POUR LES PROBLÈMES DE TOURNÉES DE VÉHICULES MULTI-ATTRIBUTS

9.1 Fil conducteur et contributions

Finalement, pour gérer efficacement non seulement la *variété* mais aussi la difficulté inhérente aux *combinaisons* des attributs, et afin de progresser plus en avant vers la résolution de VRP *riches* contenant plusieurs types d'attributs décisionnels, les méta-heuristiques précédentes ont été étendues au sein d'un cadre de résolution parallèle coopérative à base de décomposition et recompositions successives de solutions, appelé *Integrative Cooperative Search (ICS)*. L'idée fondamentale de ICS est de combiner les idées de décomposition, simplification intelligente et opportuniste, et de recherche coopérative. Le mécanisme de décomposition vise à générer des problèmes avec un nombre d'attributs réduits, pour lequel des méthodes de résolution efficaces, comme UHGS, peuvent être utilisées aux mieux de leurs capacités. Les solutions issues de ces problèmes plus simples sont intégrées au sein d'une cadre de recherche coopératif pour traiter le problème initial. Cette application permet de produire des solutions de plus grande qualité sur les problèmes traités auparavant, et soulève plusieurs considérations méthodologiques impliquant notamment la reconstitution de solutions complètes à partir de solutions partielles, et l'exploitation efficace de mémoires pour *guider* la recherche future.

9.2 Article IX : Integrative Cooperative Search for Multi-Attribute Vehicle Routing Problems

La méthodologie ICS a fait l'objet d'un article commun soumis pour publication : Lahrichi, N., Crainic, T.G., Gendreau, M., Rei, W., Crisan, G.C., Vidal, T. (2012). An Integrative Cooperative Search Framework for Multi-Decision-Attribute Combinatorial Optimization. *INFORMS Journal on Computing*, submitted for publication. Plusieurs co-auteurs ont contribué dans ce travail. Mon rôle a été d'appliquer le concept ICS aux VRP riches, en proposant et expérimentant des choix de solvers partiels, de modes de communication, de méthodes d'intégration et de guidage efficaces pour ces problèmes.

Abstract: We introduce the *Integrative Cooperative Search* (*ICS*), a multi-thread cooperative search method for multi-attribute combinatorial optimization problems. ICS musters the combined capabilities of a number of independent exact or meta-heuristic solution methods. A number of these methods work on sub-problems defined by suitably selected subsets of decision-set attributes of the problem, while others combine the resulting partial solutions into complete ones and, eventually, improve them. All these methods cooperate through an adaptive search-guidance mechanism, using

the central-memory cooperative search paradigm. Extensive numerical experiments explore the behavior of ICS and how the interest of the method through an application to the multi-depot, periodic vehicle routing problem, for which ICS improves the results of the current state-of-the-art methods.

Keywords: Multi-attribute combinatorial optimization, integrative cooperative search, meta-heuristics, decision-set decomposition, multi-depot, periodic vehicle routing

9.3 Introduction

Combinatorial optimization problems prominently appear in many theoretical and real-life settings. A large number of methodological developments targeted these problems proposing exact, heuristic, and meta-heuristic solution methods. Parallel computing enhanced these optimization methods providing the means to accelerate the resolution process and, for meta-heuristics, to obtain higher-quality solutions for a broad range of problems (Crainic and Nourredine 2005, Crainic and Toulouse 2010).

Yet, although solution methods become more powerful, the combinatorial problems one faces grow continuously in size and difficulty, as defined by the number of interacting characteristics defining their feasibility structures and optimality criteria. Such increasingly larger sets of characteristics severely challenge our methodological capability to efficiently address the corresponding problem settings. Thus, the general approach when addressing such *multi-attribute*, also informally known as *rich*, problem settings is to either simplify them, or to sequentially solve a series of restricted ones where part of the overall problem might be fixed, ignored, or both (Section 9.4 further discusses the issue).

It is well-known, however, that such approaches lead to sub-optimal solutions. Moreover, one observes in many application settings, including vehicle routing, network design, and carrier service network design, the need to comprehensively address the associated combinatorial optimization formulations to simultaneously account for "all" relevant attributes. The current literature does not offer a satisfactory answer to this challenge in terms of methods able to efficiently address multi-attribute problem settings and provide good solutions. The goal of this paper is to contribute toward addressing this challenge.

We focus on decision-based characteristics, or *decision-set attributes*, i.e., the sets of decisions defining the particular problem setting, and introduce the *Integrative Cooperative Search* (*ICS*), a multi-thread cooperative search method for multi-attribute combinatorial optimization problems. ICS musters the combined capabilities of a number of independent solution methods, exact or meta-heuristic. A number of these methods work on sub-problems defined by suitably selected subsets of decision-set attributes of the problem, while others combine the resulting partial solutions into complete ones and, eventually, improve them. These methods cooperate through an adaptive search-guidance mechanism, using the central-memory cooperative search paradigm. Our goal is to present and discuss the ICS concept, its structure, main building blocks, and operating principles, as well as to present a proof-of-concept of its efficiency.

The main contributions of this paper are to: 1) Introduce and formally describe a new metaheuristic solution framework for multi-attribute combinatorial optimization problems; ICS is general, flexible, and scalable in the number of attributes defining the problem at hand; 2) Define and exploit a functional decomposition of such problems along decisional attributes; 3) Show the interest of ICS through an application to a well-known multi-attribute case, the *multi-depot, periodic vehicle routing problem* (*MDPVRP*), for which ICS improves the results of the current state-of-the-art methods, and discuss how to apply the methodology to other combinatorial optimization problem classes; 4) Experimentally examine the role and impact of various ICS components.

This paper is organized as follows. Section 9.4 discusses the motivation for our work, and identifies the sources of inspiration for the methodology we propose. Section 9.5 introduces the fundamental concepts underlining the Integrative Cooperative Search methodology, particularly the decision-set attribute-based decomposition and the ICS algorithmic structure. We illustrate these concepts and methods through an application to the MDPVRP in Section 9.6. Extensive experiments are presented and discussed in Section 9.7. We finally conclude.

9.4 Motivation and Inspiration

We aim to address extended versions of classical combinatorial-optimization problems, which are NP-hard in their basic forms. When real-world cases are considered, "new" characteristics have to be considered while searching for feasible solutions of high quality. These characteristics take usually the form of a broad set of conditions defining the solution feasibility or optimality, or both, and may generally be represented through sets of decision variables. The resulting set of decisions is then much broader than for the classical problem settings usually found in the literature, each new group of decisions compounding the difficulty of the problem and complicating the solution process, particularly when one aims to address them simultaneously.

Two examples to illustrate, selected from two major problem classes of significant methodological interest and widely encountered in actual applications. Network design aims to select among a set of possible facilities on arcs, nodes, or both, such that the demand for utilization of the resulting network is satisfied at minimum total cost accounting for the cost of both selecting the facilities and using the network. Many particular design and location-decision problem settings have been defined during the years, the vast majority focusing on the facility-selection and demand-flow decisions (Magnanti and Wong 1984, Grotschel et al. 1995, Drezner and Hamacher 2004, Crainic 2000). Crainic et al. (2006) describe a realistic setting of a wireless network design problem, where the decision set includes not only decisions of the selection of base stations to cover a given territory, but also on selecting the number of antennae for each of these, as well as their height, power, tilt, and orientation.

A similar richness of attributes may be increasingly observed in the vehicle routing field (where the "rich" qualification was first linked to a VRP setting; Hartl et al. 2006). There exists an extensive literature on the Capacitated Vehicle Routing Problem (CVRP), which aims to construct cost-efficient routes to deliver the demand of a given set of customers with a fleet of vehicles of limited capacity operating out of a unique depot (Toth and Vigo 2002a, Golden et al. 2008). An

even greater volume of contributions address extensions of the CVRP reflecting the extreme variety of actual applications, including but not limited to multiple depots, vehicle fleets, or commodities, customer requirements for multi-period visits or within-period time windows, route restrictions on total distance or time, and so on and so forth (Vidal et al. 2012c). Several of these generalizations yielded problem classes of their own, the Vehicle Routing Problem with Time Windows (VRPTW) being probably the most well known of these.

Most generalizations found in the literature either add decisional features, e.g., selecting the number of antennae in addition of the location and number of stations in the wireless network design problem, or are concerned with adding characteristics to particular problem elements, e.g., the timing concerns in the VRPTW sequence of customer visits. While not making problems easier, the latter type of generalization may be addressed within particular algorithmic components, e.g., route generation and sequencing. Adding an extra characteristic to the definition of this group of problem elements might require a more involved algorithmic component (Vidal et al. 2011), but does not change fundamentally the nature of the problem.

Adding more decision sets to the problem, on the other hand, makes it significantly harder to address and, thus, the general approach when addressing such multi-attribute problems is to either simplify them, or to sequentially solve a series of particular cases, where part of the overall problem is fixed or ignored, or both (e.g. Golden et al. 2002, Hadjiconstantinou and Baldacci 1998, Hartl et al. 2006, Homberger and Gehring 1999, 2005). It is well-known that this leads to sub-optimal solutions. Moreover, one observes in many application settings, including vehicle routing, network design, and carrier service network design, the need to comprehensively address the corresponding combinatorial formulation accounting for "all" relevant decision-set attributes simultaneously.

Among the few exceptions to the state of the literature described above, we single out the *Multi-Depot Periodic VRP* (*MDPVRP*) problem class, with or without time windows, encompassing decisions on selecting patterns of multi-period visits for customers, as well as on assigning each customer to a depot for each of its selected visits. This multi-decision-set problem was considered until quite recently as difficult to address as a whole. New exact (Baldacci and Mingozzi 2009, Baldacci et al. 2011a) and meta-heuristic (Vidal et al. 2012a) contributions have re-defined the state-of-the-art for this class of problems. We have selected the MDPVRP to illustrate the performance of the methodology we propose to take advantage of this new set of best-known solutions (BKS).

So, how to proceed to build a method, which is general, scalable, and efficient, to address multi-attribute combinatorial optimization problems in a comprehensive manner? We were inspired by three major methodological concepts, decomposition, simplification, and cooperative search.

Decomposition is a fundamental technique in mathematical programming and parallel/distributed computing. Proceeding through various strategies, e.g., projection and relaxation in mathematical programming, its main objective is to transform a difficult-to-address formulation into a number of much simpler ones. All such methods encompass three main mechanisms. The first defines how the problem is transformed and how derived sub-problems are specified. The second specifies the information exchanged among the sub-problems. The third brings together the results obtained working on the sub-problems to create complete solutions to the original problem and, eventually, continues the decomposition-based algorithm. A so-called master problem (actually,

one of the sub-problems resulting from the decomposition) is performing this task in the context of mathematical-programming decomposition, as well as for low-level and most domain-decomposition parallel strategies (Crainic and Toulouse 2010).

Simplification is and has always been widely used to transform difficult problems into settings easier to address. As part of the modeling component of operations research, and of all disciplines based on formal representations of the problems contemplated, it is indeed an indispensable instrument for problem solving. The challenge in using simplification is reaching the right equilibrium between an easy-to-address problem setting and a high-usefulness of results for decision making for the original problem.

Cooperative search (Crainic 2005, Crainic and Toulouse 2008, 2010) has emerged as one of the most successful meta-heuristic methodologies to address hard optimization problems (e.g., Crainic and Nourredine 2005, Crainic 2008). Described generally as a parallel strategy for meta-heuristics, cooperative multi-search is based on harnessing the "solving" capabilities of several solution methods through mechanisms to asynchronously share information during the course of addressing a given problem instance and, in the most advanced settings, to create new information out of the exchanged data. The nature of the information shared, how the sharing proceeds, as well as the global and local (i.e., at the level of each collaborating solution method) utilization of the exchanged and received information, respectively, are the main characteristics of cooperative-search strategies. We focus on a widely-used class of cooperative strategies where the asynchronous communications are generally triggered individually by the cooperating algorithms, taking the form of exchanges of solutions or elements of solutions, and proceeding through a common data repository, often referred to as *adaptive* or *central memory* (Rego 2001, Le Bouthillier and Crainic 2005a,b, Jin et al. 2012, Cordeau and Maischberger 2012, Groër et al. 2011). Notice that, while "central"-memory mechanisms are clearly adaptive, "adaptive memory" is still sometimes used in the original sense of gathering fragments of good solutions, which are then used to construct new search starting points (Rochat and Taillard 1995, Badeau et al. 1997). We therefore use "central memory" in this paper.

Figure 9.1 illustrates the structure of a central-memory-based cooperative search algorithm. Several solution methods (four in the figure) participate to the collaborative search. As illustrated by the double-ended arrows, the flow of information exchange proceeds through the central-memory structure, a management-and-guidance module monitoring the traffic, managing the information deposited in the central memory and, eventually, using it to generate new relevant information, e.g., new solutions, performance measures on solution components, promising areas of the search space, and so on. Each method, which may be a meta-heuristic, an exact algorithm, or any other method and is simply identified as *solver*, thus makes available relevant information to the other participating methods by sending it to (depositing it into) the central memory. This information generally includes improved solutions or solution components, as well as so-called contextual information, e.g., performance measures and memories, to contribute building an image of the status of the search. The sending of information is triggered by the internal logic of the method, e.g., on identifying a new best solution or before a diversification phase, as is the request for new information sent to the central memory (the two events are usually coupled). The required

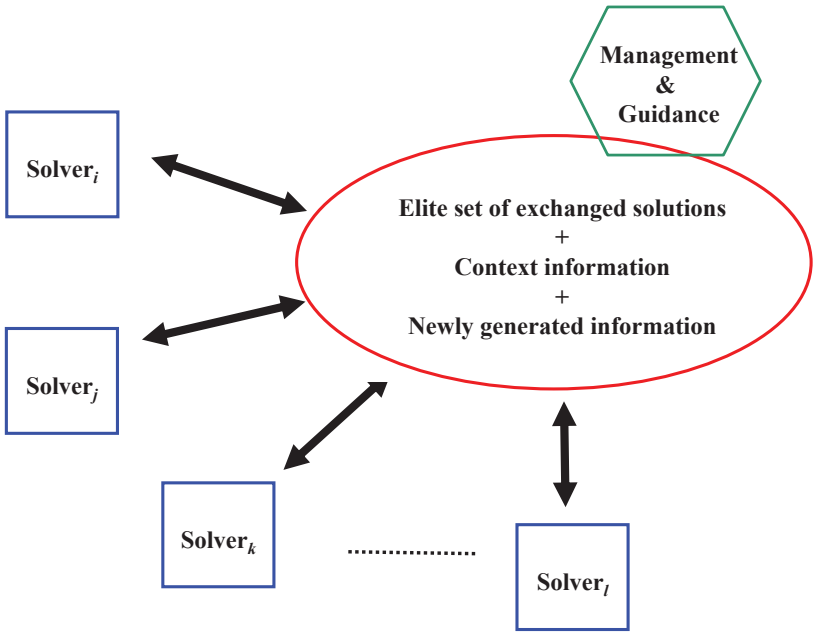


Figure 9.1: The Central-Memory Multi-Search Cooperative Scheme

information is used to inflect the search trajectory and may take the form of complete or partial solutions, or indications on regions of the search space to explore or to avoid.

Illustrations of successful application of these principles to problems with several attributes may be found, e.g., in Berger and Barkaoui (2004) and Crainic et al. (2006). The former addressed a vehicle routing problem with time windows by evolving two populations, one focusing on minimizing the total traveled distance, the other on minimizing the violation of temporal requirements to generate feasible solutions. The latter addressed the wireless network design problem mentioned previously through a parallel cooperative meta-heuristic that uses Tabu Search (TS) solvers on limited subsets of attributes only, while a Genetic Algorithm (GA) amalgamates the partial solutions attained by the TS procedures into complete solutions to the initial problem.

9.5 ICS Fundamental Concepts and Structure

We now describe the fundamental concepts and structure of the proposed ICS methodology, which builds on the ideas and methodological developments reviewed in the previous section.

9.5.1 The ICS idea

The fundamental ICS concept combines the ideas of decomposition, intelligent simplification and opportunism, and cooperative search. We aim for a decomposition mechanism that yields simpler but meaningful problem settings for which one can either opportunistically use existing high-performing methodology or more easily develop new solution approaches. The resulting simpler problem settings together with the required mechanisms to reconstruct and enhance full solutions are then brought together into a cooperative-search framework aimed to address the initial

formulation. Figure 9.2 illustrates the main elements of the methodology and their interconnection links.

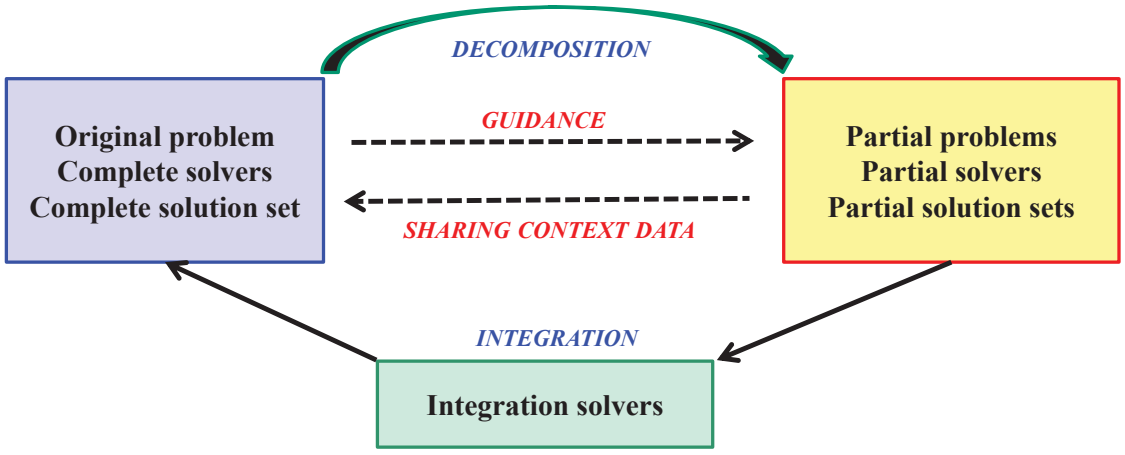


Figure 9.2: The ICS Methodology Idea

Decomposition is the first item that needs to be addressed, in particular, the problem dimensions along which it is to be applied and the mechanisms to reconstruct complete solutions to the initial problem. Given our objective of opportunist simplification for an intelligent use within a cooperative-search framework, we propose a structural problem decomposition along sets of decisions variables.

Let $x \in \mathcal{X}$ be the set of decision variables of the problem at hand, and let δ be a set of indices identifying a particular subset of x. The *decision-set attribute decomposition* then identifies $|\Delta|$ sets of decision variables, and defines a *partial-problem* formulation $\Pi_\delta(\tilde{x_\delta})$ for each decision set x_δ, $\delta \in \Delta$, by fixing the corresponding variables to suitably selected values $\tilde{x_\delta}$ (and appropriately adjusted constraints, if needed). Notice that this definition of decomposition does not change the dimensionality of the partial problems with respect to that of the original setting. Actually, any solution to any partial problem constructed according to this definition may be considered for the complete formulation.

The selection of the decision sets is specific to each application case. Thus, in particular, defining Δ according to spatio-temporal characteristics of the problem elements (e.g., node coordinates in a space-time network representation) yields the well known data-decomposition procedure used in a number of parallel and sequential solution methods. The choice for ICS is governed by the objective of opportunistic simplification indicated above and, thus, decision variables are clustered to yield known or identifiable optimization problem settings. Thus, for example, fixing the customer-to-depot assignments in the MDPVRP illustration of Section 9.6 yields a PVRP, while fixing the patterns for all customers yields a MDVRP.

Notice that the proposed decomposition does not require the sets $\delta \in \Delta$ to induce a partition of the feasible domain. Furthermore, with respect to the mathematical programming literature, the decision-set decomposition may be viewed as a multiple simultaneous projection (Geoffrion 1970a,b) performed through variable fixing. Further notice that there is no a priori limit on the number of sets. Actually, the decomposition process should aim for a "best" compromise between

the ease of addressing each partial problem and the difficulty in building complete solutions to the original problem from the solutions to the partial problems.

Indeed, decomposition very often needs to be coupled with *integration* providing an efficient way to use the solutions obtained by addressing the partial problems to build a complete solution to the initial formulation. This functionality makes up the second main element of the ICS methodology. Integration is generally not an easy task, however. Integration solvers need to address three, possibly contradictory, challenges: 1) solution quality, 2) transmission of critical features (which may be incompatible) from the partial solutions, and 3) computational efficiency. Thus, the simple integrator consisting in transferring directly to the complete-solution set a solution (feasible or not) to a partial problem achieves the latter but fails in most cases to achieve the first two goals. Even when this simple integrator achieves solution quality, it is generally due to a good partial solution and not through combining, and respecting critical features of, several partial solutions, as often required by decomposition schemes. More advanced methods are thus required and evolutionary methods, genetic algorithms and path relinking, in particular, have proved their flexibility and stability in combining solution characteristics to yield high-quality solutions, often at the price of higher computational efforts. A formal definition of integration problems for meta-heuristics is provided by Crainic et al. (2012).

We complete the ICS concept with a third main element, a purposeful-evolution mechanism geared to produce high-quality complete solutions while avoiding a heavy-handed control of the process (which has been shown to be unproductive for most meta-heuristics). We build upon the central-memory cooperative search meta-heuristic paradigm presented above, integrating a dynamically adaptive guidance mechanism based on monitoring the activities of the partial solvers, the partial and complete solutions produced, and the context information shared by the solvers. The ICS methodology, its structure, elements, and operational mechanisms are detailed next.

9.5.2 The ICS Method

ICS implements the decision-set attribute decomposition described in the previous section, and takes the form of a self-adaptive cooperative meta-heuristic concurrently evolving and combining several populations, one corresponding to solutions to the original problem, each of the others addressing specific dimensions of the problem resulting from the decision-set attributes used to decompose the problem.

Figure 9.3 illustrates the structure and components of ICS as inspired by the central-memory cooperative search meta-heuristic paradigm. Following the initial decomposition of the original problem into partial problems through decision-variable fixing as defined previously, each partial problem is addressed by one or several solution methods within a *Partial Solver Group* (*PSG*), two of which are illustrated in Figure 9.3. Concurrently with PSG activities, *integrators* select partial solutions from PSGs (represented by full arrows in the figure), combine them to create complete ones, which are then sent (short slashed arrow) to the *Complete Solver Group* (*CSG*). (All solutions are "complete" in our setting; we use the terms "partial" and "complete" solution, however, to indicate the solver group of origin.) The latter could enhance these solutions, when appropriate solvers are available, but its main task is to extract out of the complete-solution set

the information required by the smooth but purposeful guidance of the partial and global searches. Dotted-line arrows represent the exchange of information supporting the cooperation under the supervision and lightly-handed guidance of the *Global Search Coordinator* (*GSC* - the Global M&G hexagonal box in the figure).

According to this decomposition-cooperation strategy, the problem is concurrently tackled by several solvers, which indirectly and asynchronously interact through a two-layer central-memory and guidance mechanism. It is noteworthy that the representation of solutions for all solvers and in all central memories is identical no matter the particular partial or complete problem addressed or the specific solution method used. This characteristic, derived from the choice of fixing rather than eliminating variables when defining partial problems, facilitates communications, information extraction and knowledge creation from exchanged solutions, as well as the development of generic operators and solvers. The cooperation is built on these indirect exchanges through collections of elite solutions, communications being triggered either by the internal logic of each (partial) solver deciding when to send its "good" (e.g., just improved current best) solutions to the central memory or to request new solutions from the same population, or by the reaction of a local or global search coordinator monitoring the status of the memories and the search trajectory.

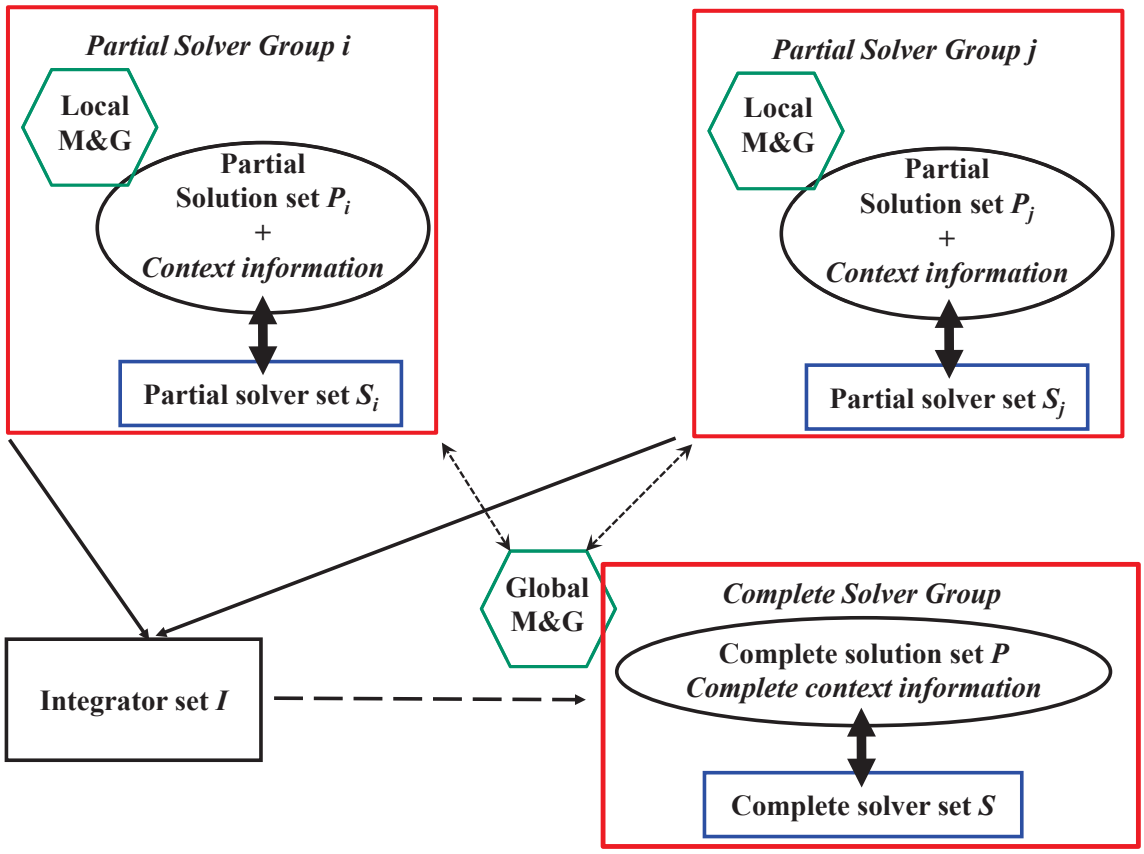


Figure 9.3: The Integrative Cooperative Search Structure

This framework ensured a high search efficiency, as solvers never interact directly, and has been shown to display very good performance in terms of solution quality, speed, and robustness in its "classical" central-memory incarnation (see the surveys of, e.g., Crainic and Nourredine

2005, Crainic 2008). ICS expands this concept to include Partial Solvers and Integrators into the cooperation, as well as a richer set of tasks for the Search Coordinators. We now examine each component in more details.

Partial Solver Group. Each PSG investigates a partial problem obtained through the decision-set decomposition procedure. It thus focuses on a particular subset of the original decision variables, the values of the other ones being fixed, directly or indirectly, through communications with the Global Search Coordinator. The PSG i is composed of a set of *Partial Solvers* $\mathcal{S}_i$ that work with complete solutions, but may modify only the values for the decision subset they are assigned to. Partial Solvers construct $\mathcal{P}_i$, the corresponding set (population) of elite *partial solutions*.

Each PSG operates according to the central-memory cooperative search paradigm. Thus, Partial Solvers may be simple constructive methods (providing initial solutions), metaheuristics or exact methods tailored to the subset of attributes they are dedicated to, or post-optimization methods. The type of solvers depends only on the available methods to efficiently address the particular problem. The number of solvers depends on the availability of appropriate solution methods and the choice of the analyst. ICS does not impose any actual limit, but the usual concerns related to an efficient implementation of parallel methods (e.g., computer architecture, communication protocol and latency, programming language, etc.) also apply in this context.

Communications and exchanges among Partial Solvers are performed through the central-memory mechanism made up of the population of elite solutions, context information, and the *Local Search Coordinator* (*LSC*) providing supervision, management, communications, and guidance functionalities. The actual Partial Solvers involved in cooperation and the corresponding cooperation mechanism are application specific and, thus, so is the local context information. The latter may consist in simple pointers to the best solutions and a relative order of the elements in the population, or may be enriched with memories related to, e.g., solution elements, solutions, performance measures for the Partial Solvers involved, and partial solution structures (see, e.g., Le Bouthillier and Crainic 2005b, Jin et al. 2012), to be used in guiding the Partial Solvers and the search trajectory of the group.

Communications between the central memory and the Partial Solvers is generally initiated by the latter to deposit or request, or both, solutions and context information. Guidance instructions issued by the local or global search coordinators may reverse this general policy. Indeed, the LSC may include functionalities to monitor the performance of the Partial Solvers and act when this performance becomes locally unsatisfactory (e.g., a given Partial Solver did not send any solution for a given time or all its latest solutions received by the LSC are weak compared to the best solutions in memory). The LSC could then, for example, force a Partial Solver to restart from a suitable solution in memory, modify the search parameters, or even change the solution method.

Similar behavior could also be triggered by messages from the GSC that monitors the progress of the global search. We discuss in more depth the role of the GSC later in this section, but want to emphasize that a thorough exploration of the original problem solution space would often require the modification of the focus of particular Partial Solver Groups. The modification will generally proceed through changes to the $\tilde{x_\delta}$ values defining the search space of given $\Pi_\delta(\tilde{x_\delta})$ PSGs, but could also involve the definition of the Δ partition. This modification is communicated to the

corresponding LSC that, in turn, will instruct Partial Solvers, as well as work on the solutions and context information (initialize or modify values) in memory.

Integrators select solutions in the populations of one or more PSGs, combine them to yield complete solutions, and transmit some or all of these new solutions to the Complete Solver Group. Integrators are defined by the particular rules and procedures used to perform these tasks. Similarly to the Partial Solvers, Integrators can be very simple procedures selecting partial solutions to pass directly to the CSG, or comprehensive exact or heuristic solution methods.

Integrators play an essential role in the ICS methodology. While Partial Solvers address a single aspect of the original problem, with a number of attributes fixed, Integrators build complete solutions by mixing partial solutions with promising features from these various populations. The selection operators thus need to take into account this objective by picking up not only the currently best solution in a given population, but rather a small number of diverse elite solutions. Similarly, in order to contribute to the progress of the global search, the usual quality and diversity criteria should guide the choice of complete solutions to be sent to the CSG (when the Integrator yields more than a single solution). More than one Integrator can be involved in an ICS implementation. Using different solution methodologies would then contribute toward the diversity objective.

Complete Solver Group and **Global Search Coordinator**. The CSG is the component of ICS where complete solutions are kept, and sometimes enhanced (e.g., Crainic et al. 2006), and from where the final solution to the original problem is obtained once the stopping conditions are verified.

The CSG is organized similarly to the PSGs, but the complete-solver set is optional or could hold a few methods only. Indeed, in most applications of interest for ICS, methods targeting the respective problem, if they exist, would be inadequate in computing efficiency or solution quality, or both. ICS is intended to replace them and, in such cases, the inclusion of solvers would not be warranted. On the other hand, post-optimization techniques may profitably be used to locally enhance solutions (e.g., customer sequencing for VRP or flow distribution for network design). Post-optimization methods could therefore belong to the complete-solver set.

Most importantly, it is the CSG that produces the *complete context information* required by the GSC for the global guidance of the search. Indeed, the purposeful-evolution objective of ICS requires the monitoring and guidance of the progress of the search. This is the main role of the Global Search Coordinator. More specifically, the GSC monitors the evolution of the complete solution set, and those of the partial populations, as well as the behavior of the solver groups and integrators. This activity enables it to build and maintain the context information of the global search. This later may include various performance measures (e.g., the cost or elaborate functions that reflect multiple characteristics) and indicators (e.g., membership to a specific class of solutions, or information on the solver that yielded it) for each solution in the complete solution set. It may also include an image of the global search through statistical information (memories) on the evolution of solutions in the complete and partial populations, the contribution of solutions and their components (e.g., routes or arcs in VRP) to the evolution of the search, the relative performances of Partial Solvers and Integrators, etc.

The complete context information provides the means to detect undesired situations, e.g., loss of diversity in the partial or complete elite population, stagnation in the improvement of the best solution quality, awareness that some zones of the solution space - defined by particular values for particular decision sets - have been scarcely explored, if at all, and the search should be diversified in that direction, and so on. Thresholds on such global performance measures trigger guidance operations for ICS performed by sending "instructions" to Partial Solvers and Integrators. The particular type of guidance is application specific, but instructions may modify the values $\tilde{x_\delta}$ of the fixed attributes for a specific Partial Solver to orient the search toward a different area, change the attribute subset under investigation (i.e., change the decomposition of the decision-set attributes), or modify/replace the solution method in a Partial Solver or Integrator. The last two types of instructions significantly modify the structure of the global search and should reflect major observations in the behavior of the method, e.g., a solver is constantly under-performing compared to the others. Their usage should therefore be rather infrequent.

The first type of instructions, on the contrary, is the core guidance mechanism of the method and is implemented in the application discussed later on in this paper. In their simplest form, instructions take the form of a particular solution or solution subset being sent to re-initialize a particular partial population (re-initialization may be complete or partial, retaining in the latter case a few very good and diverse solutions) and thus re-start the Partial Solver searches. Additional context information (e.g., the promising arc patterns of Le Bouthillier and Crainic 2005b) may complete the guiding instructions, to further inflect the search trajectory toward regions that appear promising from the point of view of the global search.

This general ICS framework can be applied to any multi-attribute combinatorial problem class for which a decision set decomposition may be defined. The next section illustrates the application of the ICS methodology to such a problem class.

9.6 Application to the MDPVRP

We illustrate the ICS methodology with an application to the multi-depot periodic vehicle routing problem (MDPVRP). Our aim is double. First, to document the instantiation of ICS components and general method on a well-known multi-attribute problem and, second, to evaluate the performance of the method. We first briefly recall the MDPVRP setting and then detail the ICS application.

9.6.1 The multi-depot periodic vehicle routing problem

Briefly, see Vidal et al. (2012a) for a full description, the MDPVRP (Mingozzi 2005) is defined on a multi-period planning horizon, each customer requiring service several times during this planning horizon according to one of a particular set of pre-defined visit patterns (i.e., lists of periods when visits may occur). Service is provided out of a set of depot, operating in all periods, by a homogeneous fleet of limited-capacity vehicles. The distribution of the fleet among depots is known and the same for all periods. The MDPVRP aims to select a depot and a visit pattern for

each customer, with services in different periods to the same customer being required to originate at the same depot, such that the total cost of distribution is minimized.

The literature, we refer the reader to Vidal et al. (2012a) for a detailed review, shows a richer set of contributions for two restrictions of the MDPVRP, the periodic VRP (PVRP) where service proceeds out of a single depot, and the multi-depot VRP (MDVRP) where the planning horizon has a single period only. Most contributions to the MDPVRP did not consider all attributes simultaneously, but rather applied a successive-optimization approach (e.g., Hadjiconstantinou and Baldacci 1998, Kang et al. 2005, Yang and Chu 2000). We are aware of only two methods that address problems similar to the MDPVRP with a comprehensive approach. Parthanadee and Logendran (2006) implemented a tabu search method for a complex variant of the MDPVRP with backorders. Vidal et al. (2012a) proposed a hybrid genetic methodology, named *Hybrid Genetic Search with Adaptive Diversity Control* (*HGSADC*), combining the exploration capability of population-based evolutionary search, the aggressive-improvement capabilities of neighborhood-based meta-heuristics to enhance (educate) solution newly created by genetic operators, and advanced population-diversity management mechanisms to evaluate and select solutions. HGSADC is the current state-of-the-art method for the MDPVRP, as well as for the PVRP and the MDVRP. We therefore use HGSADC as solver in our ICS implementation for the MDPVRP and give more details in the next subsection.

9.6.2 ICS for the MDPVRP

To apply ICS to a given problem, we have to specify the decision-set attributes for the decomposition, the state of the art algorithms making up the solvers to address the resulting partial problems, the organization of each Partial Solver Group, the integrators to reconstruct solutions, and finally the Complete Solver Group and the coordination mechanisms.

We are opportunistic in this implementation and decompose the MDPVRP along the depot and period decision sets to create two partial problems, managed through two PSGs called *PSG-fixDep* and *PSG-fixPat*, respectively, which complement the CSG dedicated to the complete MDPVRP. The partial problem addressed in the *PSG-fixDep* group is a set of independent PVRPs optimizing pattern assignment and routing decisions, given customer-to-depot assignments. (The depot-specific PVRPs can be optimized independently when depot assignments are fixed.) Symmetrically, the second partial problem, addressed in *PSG-fixPat*, is a multiple MDVRP optimizing depot assignment and routing decisions for each period given pattern-to-customer selections. Notice that, fixing the pattern-to-customer selections simplifies the problem, but does not lead to a separable formulation as previously, since the depot assignment for each customer must be consistent among different periods.

Solver Groups. Two algorithms are used in this implementation, namely, GUTS, a generalized version of the Unified Tabu Search (UTS) of Cordeau et al. (2001a) and the Hybrid Genetic Search with Adaptive Diversity Control (HGSADC) of Vidal et al. (2012a). UTS is a tabu search-based meta-heuristic implementing advanced insertion neighborhoods and allowing the exploration of infeasible solutions by dynamically adjusted penalties on violation of vehicle capacity and route duration constraints. We implemented a generalized version, GUTS, which can be used either as

complete or partial solver by fixing the appropriate attributes. HGSADC harnesses the strength of local-search improvement procedures and the exploration capacities of population-based methods, including advanced crossover operators and diversity management mechanisms. A new evaluation of individuals driven by both solution quality and contribution to the population diversity is proposed to further promote innovation within selection and population management procedures. This strategy was shown to be highly successful on periodic and multi-depot problems. We therefore rely on it to manage individuals in the elite sets of partial and complete solver groups with the same parameters as in Vidal et al. (2012a).

As illustrated in Algorithm 9.1, both GUTS and HGSADC can be used to work on a solution S_{INI} received from an elite set with fixed depot or pattern assignments. These solvers are designed to use pattern-change, depot-change, and inter-route movements, as well as intra-route optimization, but only on the solution components they are allowed to work on. In the case of GUTS, S_{INI} is simply considered an initial solution to be enhanced by the algorithm. In the case of HGSADC, initial individuals are generated as in Vidal et al. (2012a), considering the allowed pattern and depot assignment alternatives, to create an initial population to which S_{INI} is added. These two algorithms are then run until $It_{\text{END}} = 5000$ successive iterations (number of local-search moves in the case of GUTS or number of generated individuals for HGSADC) have been performed without improvement of the best solution. When this threshold is reached, the solver restarts from a new solution from the elite set. Any improving solution is sent to the elite set if it remains the best for at least $It_{\text{COMM}} = 200$ successive iterations, and thus the elite set only receives solutions known to require some effort for further improvement. Furthermore, the solver receives after each It_{COMM} iterations a new solution selected from the elite set. This solution becomes the new starting solution of GUTS and it is included in the population of HGSADC, if it improves upon the current best solution of the solver. The solutions sent to the solvers are selected from the elite sets by binary tournament, using the individual evaluation function of Vidal et al. (2012a). To avoid sending the same solution multiple times, any solution which a solver has worked on until the termination criteria is marked with a flag, and cannot be received again by the same solver. A solution, which has been considered by all types of solvers without success, remains in the elite set for the purpose of integration, migration, and bookkeeping only.

Various combinations of GUTS and HGSADC, or parts thereof, yield different Partial Solver Groups. Preliminary experiments showed that"pure" cooperation, involving only GUTS or HGSADC solvers, outperforms "mixed" ones. The speed and quality of the evolution performed by HGSADC is the main reason behind this observation, as including one more HGSADC was experimentally shown to be in all cases more profitable than one additional GUTS. We thus investigate in Section 9.7 different versions of ICS, one with only neighborhood-based solvers (UTS) in the solver groups, and the other with HGSADC only. For the latter implementation, two different strategies have been evaluated. The first is the previously described *encapsulated* setting, where HGSADC is used with a dedicated population to improve single solutions, exchanging solutions ("migrating" individuals) with the elite set. The second is the *sharing* setting, where the elite set directly serves as population for all HGSADC solvers involved in the cooperation. In this latter case, flags are not used to mark solutions, and the role of the partial solver simply comes to iteratively receiving a

Algorithm 9.1 Partial solvers

UTS CASE:
Repeat
RECEIVE an initial solution S_{INI}
$S_{\text{CUR}} \leftarrow S_{\text{INI}}$ and $S_{\text{BEST}} \leftarrow S_{\text{INI}}$

while Number of Local Search moves since last best solution $\leq It_{\text{END}}$

//One GUTS iteration
For each customer, search for a better insertion position given the fixed attributes (pattern or depot assignments);
Apply best non-tabu move to S_{CUR};
Adjust diversification parameters;
Adjust infeasibility penalties;
Update tabu list and status

//Update best and eventual communication
if $cost(S_{\text{CUR}}) < cost(S_{\text{BEST}})$ **then**
$S_{\text{BEST}} \leftarrow S_{\text{CUR}}$
if WhenFound(S_{BEST}) $= It_{\text{COMM}}$ **then**
SEND S_{BEST}
if Nb LS moves $= k \times It_{\text{COMM}}$ **then**
RECEIVE a solution S_{INI}
if $cost(S_{\text{INI}}) < cost(S_{\text{BEST}})$ **then**
SEND S_{CUR} and $S_{\text{CUR}} \leftarrow S_{\text{INI}}$

HGSADC CASE:
Repeat
RECEIVE an initial solution S_{INI}
Create an initial local population $\mathcal{P}$
Add S_{INI} to $\mathcal{P}$ and $S_{\text{BEST}} \leftarrow S_{\text{INI}}$
while Number of generated individuals since last best solution $\leq It_{\text{END}}$

//One HGSADC iteration
$(S_1, S_2) \leftarrow$ Select two parents in $\mathcal{P}$ by binary tournament;
$S_{\text{CUR}} \leftarrow$ Crossover(S_1, S_2);
$S_{\text{CUR}} \leftarrow$ LocalSearch(S_{CUR})
Add S_{CUR} to $\mathcal{P}$
if $|\mathcal{P}| >$ MaxPopSize then manage the population by removing some individuals;
Adjust infeasibility penalties;

//Update best and eventual communication
if $cost(S_{\text{CUR}}) < cost(S_{\text{BEST}})$ **then**
$S_{\text{BEST}} \leftarrow S_{\text{CUR}}$
if WhenFound(S_{BEST}) $= It_{\text{COMM}}$ **then**
SEND S_{BEST}
if Nb generated indivs $= k \times It_{\text{COMM}}$ **then**
RECEIVE a solution S_{INI}
if $cost(S_{\text{INI}}) < cost(S_{\text{BEST}})$ **then**
SEND S_{CUR} and add S_{INI} to $\mathcal{P}$

pair of individuals, performing a crossover and a local search, and sending the resulting solution to the elite set (Line 2-4 in the HGSADC iteration of Algorithm 9.1). Notice that, in the sharing versions, the two individuals selected in the respective elite set for the crossover operation do not necessarily have the same depot or patterns associated to customers, and that the crossover operator will further modify these assignments.

Integration. Four integrators operate in parallel in the proposed MDPVRP application. The first, named I_{BEST}, selects the best solution from the population of each PSG and transfers it directly to the complete solution set of the GSG. The best solutions of each PSG are thus rapidly made available for sharing with the other PSGs and for extraction of relevant information for global guidance. The other three integrators aim to create new complete solutions out of pairs of partial solutions S_{FIXDEP} and S_{FIXPAT} randomly selected among the best 25% of the *PSG-fixDep* and *PSG-fixPat* populations, respectively. The I_{CROSS} integrator relies on the crossover operator of HGSADC, which extracts from each parent a subset of promising solution elements and is particularly efficient for combining two solutions. I_{CROSS} consists in applying 50 different times the crossover operator on S_{FIXDEP} and S_{FIXPAT}, and improving the respective resulting solution with local search. The best obtained solution is sent to the global solver group. The last two integrators, detailed in Crainic et al. (2012), aim to promote attributes obtained by solvers working on different facets of the problem while modifying part of these solutions to move to a different search subspace. The I_{AND} integrator fixes the attributes shared by the selected partial solutions and, then, addresses

the resulting restricted formulation by means of HGSADC, to return the best solution. The I_{PEN} integrator modifies the original complete formulation by adding cost incentives to direct HGSADC toward the depot-to-customer assignments of S_{FIXPAT} and the pattern-to-customer assignments of S_{FIXDEP}.

Global Solver Group and Search Coordination. The Global Search Coordinator assumes the fourfold role of collecting statistical information on the past search, monitoring the status of the solver groups, triggering migration of individuals when necessary, and building new guiding solutions to orient the search towards promising features. The statistical information is developed on (client, depot, pattern) triplets entering the complete solution set, by monitoring the number of occurrences (frequency) of each triplet in the population as well as in the best, average and worst sub-populations (Le Bouthillier and Crainic 2005b), and finally the best cost of a solution containing any given triplet.

The GSC monitors the elite sets to check whether more than five non-flagged individuals are available (UTS and encapsulated-HGSADC implementations), and whether improving solutions have been received during the $It_{\text{CONTROL}} = 2000$ last solutions. If one of these two criteria is not fulfilled, the GSC sends three solutions to the corresponding elite set. The three solutions are either randomly selected (equiprobably) from the complete solution set, or are three *guiding* solutions, as defined in the next paragraph. In the shared-HGSADC implementations, this migration is preceded by a reset of the population, where all the solutions are replaced by new initial random solutions. It should be noted that the solutions to be migrated do not necessarily have the same values for the fixed attributes, and thus the solver groups process individuals with identical sets of fixed attributes, but potentially different attribute values.

The GSC generates guiding solutions by selecting *promising* triplets to create feasible pattern and depot customer assignments. "Promising" is based on the complete search history and is meant as appearing in at least one solution with a cost difference of less than 3% with respect to the best found solution. Once these patterns and depots are selected, complete solutions with routes are generated by local search, to serve as initial population (HGSADC implementations) or solution (UTS implementations) and be optimized with the solver of choice. The termination criterion for GUTS or HGSADC is here set to $It'_{\text{END}} = 2000$ iterations without improvement. Since creating such an individual is time consuming, guiding individuals are generated by the GSC in background, and stored in a temporary pool of guiding individuals. These solutions significantly contribute to inflect the search trajectory towards different alternatives of assignment combinations.

We experimented with two settings of the CSG, with and without a complete-solver set. The same two algorithms, GUTS and HGSADC, are used in the former case, targeting the full MDPVRP. The results are presented next.

9.7 Experimental Results and Analyses

Extensive experimental analyses were conducted to 1) assess the performance of ICS when compared to state-of-the-art sequential methods and, 2) investigate several implementation alternatives, including the choice of solver type (GUTS vs. HGSADC) and of cooperation strategy in

the case of HGSADC (encapsulated vs. shared), as well as the potential inclusion of global solvers in the complete solver group.

The ICS method was implemented in C++. Experiments were run on an Altix 4700 server, using double Core processors Itanium 2 with 1.5GHz cadence and a NUMAlink communication network. 14 processors were used for the MDPVRP experiments:

- 3 CPUs managing each one of the elite sets;
- 6 CPUs for solvers: three for each of the two PSGs when no global solver was used, two for each PSG and two for the CSG, otherwise;
- 3 CPUs for the integrators, I_{BEST} and I_{CROSS} being run on the same CPU;
- 2 CPUs for the GSC, one for creating the guiding individuals, and the other fulfilling the remaining tasks.

This choice of task-to-CPU assignment is mostly due to implementation simplicity. Computational experiments show that, in practice, only 10 CPU out of 14 perform computationally-intensive tasks. The other four CPUs, dedicated to the elite set management and GSC monitoring, could be implemented on a single core in a more advanced implementation. We therefore consider in analyzing the results that the effective workload of ICS is 10 times the load of a sequential method.

Experiments were performed on six versions of ICS for the MDPVRP. *HGSADC-ICS1*, *HGSADC-ICS2*, and *GUTS-ICS* represent the population-based HGSADC sharing and encapsulated versions and the GUTS implementation, respectively, without general solvers. Their counterparts, *HGSADC-ICS1+*, *HGSADC-ICS2+*, and *GUTS-ICS+*, include the MDPVRP solvers. Each method was run ten times on each instance, recording the best solutions after 5, 10, 15 and 30 minutes minutes of computation time.

The ten MDPVRP instances pr01 - pr10 of Vidal et al. (2012a), derived from the Cordeau et al. (2001a) data sets, were used for these tests. These instances present a uniform geographical distribution of customers coupled with a few customer clusters. The largest instances include up to 288 customers, 6 depots, 6 days, and a total of 864 deliveries. Detailed descriptions are provided in the Annex.

Table 9.1 sums up the performance comparison of the ICS versions and the two sequential solvers at different instances of computing time. Solution quality is reported as the average percentage of cost deviation (on all instances and all runs) with respect to the best known solutions in the MDPVRP literature, reported in Vidal et al. (2012a). In addition, Tables 9.2 and 9.3 provide detailed comparisons of the quality of solutions between the GUTS-ICS+ version and the sequential GUTS, and between the "best" population-based ICS version, HGSADC-ICS2+, and the sequential HGSADC, respectively. The first column in these tables specifies the instance solved, while the next columns report the average solution quality, the run time, and the best solution found by the sequential method. The two next groups of columns report the average and best results of the ICS versions at different points in time, and finally the last column reports the previous BKS from the literature. For each line, the best solution is reported in boldface. New best known solutions are underlined. Detailed results for the other ICS implementations are provided in Appendix VI.

The reported results underline the large contribution of the ICS framework in enhancing neighborhood-based searches such as GUTS. Thus, the solution quality improves from a 2.77%

Version	Gap 5 min	Gap 10 min	Gap 15 min	Gap 30 min
HGSADC	NC	NC	NC	+0.42%
HGSADC-ICS1	+0.78%	+0.51%	+0.40%	+0.29%
HGSADC-ICS2	+1.12%	+0.65%	+0.49%	+0.32%
HGSADC-ICS1+	+0.59%	+0.37%	+0.29%	+0.21%
HGSADC-ICS2+	+0.79%	+0.34%	+0.23%	+0.17%
GUTS	NC	NC	NC	+2.77%
GUTS-ICS	+1.62%	+1.04%	+0.78%	+0.58%
GUTS-ICS+	+1.29%	+0.94%	+0.71%	+0.52%

Table 9.1: Average performance of ICS versions and sequential algorithms

Inst	GUTS			GUTS-ICS+								BKS
	Avg	T(min)	Best	Avg				Best				
				5 min	10 min	15 min	30 min	5 min	10 min	15 min	30 min	
pr01	**2019.07**	0.14	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	2019.07
pr02	3547.72	2.39	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	3547.45
pr03	4639.50	18.87	4578.14	4495.73	4488.85	4482.96	4482.21	4483.24	**4480.87**	**4480.87**	**4480.87**	4480.87
pr04	5246.39	20.03	5225.67	5187.92	5167.56	5161.60	5153.83	5154.93	5153.73	5150.40	**5149.09**	5134.17
pr05	5738.40	21.10	5666.35	5726.11	5693.80	5652.20	5630.05	5686.88	5624.05	5618.88	**5599.41**	5570.45
pr06	6759.41	20.06	6723.52	6640.94	6621.20	6604.15	6586.48	6566.44	6566.09	6566.09	**6565.69**	6524.92
pr07	4644.93	0.71	4644.93	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	4502.02
pr08	6217.23	20.82	6174.15	6052.42	6024.29	6024.20	6024.15	6024.41	**6023.98**	**6023.98**	**6023.98**	6023.98
pr09	8644.82	23.85	8548.63	8408.71	8356.79	8338.22	8302.88	8324.61	8294.79	8278.58	**8277.63**	8257.80
pr10	10241.67	26.45	10120.30	10270.76	10178.57	10102.29	10040.94	9990.04	9946.60	9935.10	**9930.32**	9818.42
Gap	+2.77%		+2.10%	+1.29%	+0.94%	+0.71%	+0.52%	+0.57%	+0.37%	+0.33%	+0.28%	

Table 9.2: Performance of GUTS-ICS

Inst	HGSADC			HGSADC-ICS2+								BKS
	Avg	T(min)	Best	Avg				Best				
				5 min	10 min	15 min	30 min	5 min	10 min	15 min	30 min	
pr01	**2019.07**	0.35	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	2019.07
pr02	**3547.45**	1.49	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	3547.45
pr03	4491.08	7.72	**4480.87**	4480.94	**4480.87**	**4480.87**	**4480.87**	**4480.87**	**4480.87**	**4480.87**	**4480.87**	4480.87
pr04	5151.73	22.10	**5144.41**	5152.10	5145.63	5145.15	5144.42	**5144.41**	**5144.41**	**5144.41**	**5144.41**	5134.17
pr05	5605.60	10.00	5581.10	5649.40	5597.52	5588.82	5582.86	5610.52	5573.79	5572.44	**5558.92**	5570.45
pr06	6570.28	10.00	6549.57	6615.02	6554.82	6542.43	6538.54	6582.20	6534.18	6533.67	**6529.67**	6524.92
pr07	4502.06	2.18	4502.06	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	4502.02
pr08	6029.58	7.96	**6023.98**	6024.03	**6023.98**	**6023.98**	**6023.98**	**6023.98**	**6023.98**	**6023.98**	**6023.98**	6023.98
pr09	8310.19	27.79	8271.66	8312.31	8287.48	8283.34	8274.42	8283.95	8256.99	**8256.47**	**8256.47**	8257.80
pr10	9972.35	30.00	**9852.87**	10215.12	10003.86	9938.42	9907.24	10099.90	9962.31	9887.07	9868.98	9818.42
Gap	+0.42%		+0.13%	+0.79%	+0.34%	+0.23%	+0.17%	+0.50%	+0.19%	+0.11%	+0.06%	

Table 9.3: Performance of HGSADC-ICS2+ with encapsulated population-based solvers

gap to a 0.52% gap when relying on the cooperative framework. The improvement in the case of population-based searches appears smaller but still significant, reducing the average gap from 0.42% to 0.17%. This is impressive, given the smaller potential for improvement due to the high-quality solutions provided by HGSADC. It should be noted that ICS also enables to obtain high-quality solutions in reduced time, the average solution quality being higher after 10 minutes of ICS than after 30 minutes of HGSADC. Finally, during the overall experimentation process, HGSADC-ICS2+ qallowed us to find three new best known solutions for the considered problems: 5558.02 for pr05, 8254.73 for pr09, and 9776.28 for pr10 (these solutions may be obtained from the authors).

We complete this analysis by examining the performance distribution of all these methods with respect to their respective trade-offs between solution quality and overall CPU effort. Figure 9.4 illustrates this analysis, by displaying average and best-solution results for GUTS and HGSADC-based methods. The computation effort is computed multiplying the run time and the number of effectively working CPUs. Because the total effort to achieve best solutions is that of all the corresponding runs, the computation effort for these results is scaled accordingly.

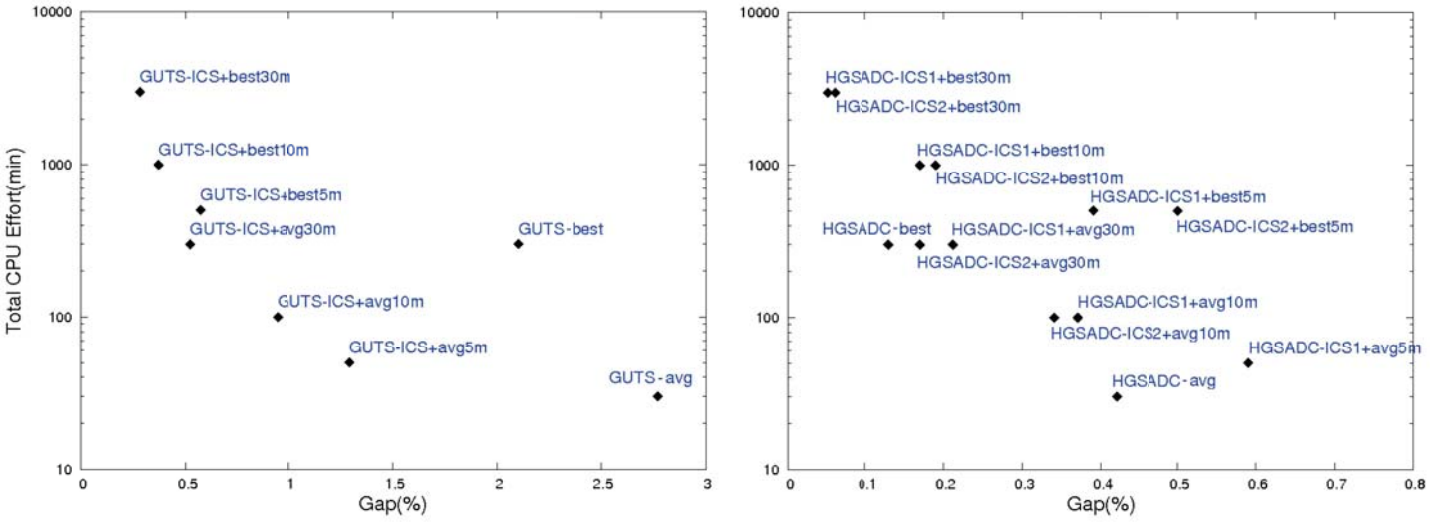


Figure 9.4: Solution Quality versus Computational Effort

Figure 9.4 shows that the exploration capabilities of the GUTS solvers are greatly enhanced by the cooperative framework, GUTS-ICS implementations leading to a variety of non-dominated (time/CPU consumption) trade-offs. Four variants of HGSADC constitute a Pareto set with respect to the twofold objective of solution quality and CPU effort: HGSADC-ICS1+best30m, HGSADC-best, HGSADC-ICS2+avg10m, and HGSADC-avg. For an equal CPU effort, a 10-processor parallel multi-search with HGSADC (HGSADC-best), seems to produce solutions of moderately higher quality than HGSADC-ICS2+ (deviation of 0.13% versus 0.17%). However, we must remark that HGSADC-ICS2+ with general solvers only rely on 2 solvers working on the general problem, while the best run of HGSADC out of 10 is comparable to the joint effort of 10 parallel general solvers. This emphasizes the excellent performance of ICS.

We now turn to discussing the results of a series of experiments targeting the impact of two particular ICS design features, the encapsulating or sharing of populations within PSGs, and the inclusion of general solvers in the CSG.

The results show that for short computation times (5 or 10 minutes), genetic algorithms that work directly on the elite sets (sharing ICS) seem to conduct to higher-quality solutions. This observation relates to the fact that such a tight cooperation enables a fast exchange of solutions for an aggressive intensification. On longer runs, the encapsulated version of ICS, where each genetic solver operates its own population, seems to lead to better best solutions, thanks to an increased diversity of the search trajectories.

Finally, using general solvers within the CSG yields moderate increases in solution quality in all experiments. This is not surprising as it corroborates the general observation that addressing the complete problem formulation within the resolution approach is highly profitable, if possible. Indeed, of particular importance is the fact that, even when general solvers are not used, the ICS solution quality does only slightly decrease (a 0.08% difference in the worst case). This illustrates the remarkable ability of ICS to produce results of similar quality to the ones of the best methods addressing the complete problem, with only solvers that work on partial aspects of the problem. This observation is critical when dealing with multi-attribute, rich problems for which integrated solvers are either not available or not sufficiently efficient. ICS becomes the method of choice in these cases.

9.8 Conclusions

We introduced the Integrative Cooperative Search framework to efficiently address the challenges of rich, multi-attribute combinatorial optimization problems.

ICS decomposes such complex problems along decision-set attributes and musters, within a multi-thread cooperative search framework, the combined capabilities of a number of independent exact or meta-heuristic solution methods. A number of these methods work on sub-problems defined by suitably selected subsets of decision-set attributes of the problem, while others combine the resulting partial solutions into complete ones and, eventually, improve them. These methods cooperate through an adaptive search-guidance mechanism, using the central-memory cooperative search paradigm.

We illustrated the interest of the Integrative Cooperative Search methodology through an application to the multi-depot, periodic vehicle routing problem, for which ICS enhances the results of the current state-of-the-art methods.

To apply ICS to other combinatorial optimization problems is sufficient to identify decision-set attributes yielding suitable partial problems. Thus, for example, the recently proposed Unified Hybrid Genetic Search methodology (Vidal et al. 2012b) offers the means to rapidly devise high-performing meta-heuristics for a wide range of routing problem settings. Similarly, multi-attribute design problems may be addressed through ICS integrating efficient tabu search and path relinking meta-heuristics (Crainic et al. 2006). This emphasizes the generality and broad applicability of the proposed ICS methodology.

CONCLUSION

Les contributions de cette thèse concernent à la fois l'analyse et la synthèse des problèmes de tournées de véhicules multi-attributs et de timing, ainsi que l'amélioration et le développement de nouvelles méthodes dédiées et généralistes, qui ont le potentiel d'être étendues à de nombreux autres problèmes d'optimisation combinatoire.

Le développement de synthèses et d'analyses de concepts au sein de revues de littérature unificatrices, dans les chapitres 1 et 2, a permis de poser un nouveau regard sur les problèmes de tournées de véhicules multi-attributs, ainsi que sur un vaste domaine de recherche lié aux problèmes de *timing*, riche en problématiques et perspectives. Des liens ont été établis entre la structure des problèmes et les méthodes de résolution. Ainsi, pour le VRP, trois types d'attributs (Assign, Seq et Eval) ont été différenciés en considérant leur impact sur les méthodes de résolution heuristique. Par ailleurs, les stratégies gagnantes de 64 métaheuristiques ont été analysées en détail. Pour les problèmes de timing, l'étude de la structure des problèmes, éventuellement par le biais de leurs conditions d'optimalité (cas des *"coûts convexes séparables"*), a conduit à classifier de nombreux algorithmes de différents domaines comme autant de variations de trois principes fondamentaux, liés au respect des contraintes primales, duales, ou à des concepts de programmation dynamique. Cette vaste analyse pluridisciplinaire de la littérature a permis d'identifier une librairie d'algorithmes efficaces pour résoudre les problèmes de timing pour des attributs variés de manière indépendante, et de manière groupée par des méthodes de *ré-optimisation* dans des contextes de recherche locale. Ces méthodes donnent la clef pour gérer les caractéristiques temporelles difficiles de nombreux problèmes de tournées de véhicules riches et d'ordonnancement non-régulier.

Les Chapitres 3 à 6 ont décrit de nouvelles méthodologies heuristiques efficaces pour plusieurs problèmes de tournées de véhicules fondamentaux. Un algorithme génétique hybride (HGSADC) efficace a été proposé, combinant l'exploration large des méthodes évolutionnaires à base de populations, les capacités d'amélioration agressive des méta-heuristiques à voisinage, les relaxations de contraintes pour cibler les frontières de non-faisabilité, et des méthodes avancées de gestion de population et d'évaluation des individus considérant à la fois des critères de qualité et de contribution à la diversité. HGSADC s'est avéré être particulièrement efficace sur les jeux de tests classiques de la littérature, sur des problèmes fondamentaux comme le VRP classique, et d'autres variantes avec dépôts multiples et contraintes de périodes de visites, obtenant toutes les solutions optimales connues et retrouvant ou améliorant toutes les meilleures solutions de la littérature pour ces problèmes. Par ailleurs, une extension de cet algorithme aux problèmes de tournées de véhicules avec fenêtres de temps et contraintes de durée de route a donné lieu à d'autres contributions méthodologiques, concernant l'exploitation efficaces de relaxation de contraintes temporelles et l'utilisation d'approches de décomposition. De nouvelles méthodes ont été proposées pour évaluer en temps constant amorti $O(1)$ les solutions intermédiaires irréalisables vis-à-vis des contraintes temporelles, produites par des mouvements de recherche locale. Des études expérimentales approfondies ont mis en valeur la contribution majeure des nouvelles stratégies d'évaluation des individus et de gestion de population, l'apport des méthodes de décomposition, et l'impact positif des solutions intermédiaires irréalisables et dans la méthodologie HGSADC, ainsi

que dans des approches simples de descente par recherche locale et de recherche locale itérée. De plus, une méthode simple de programmation dynamique a été proposée pour choisir en temps $O(1)$ le meilleur dépôt et son placement dans les routes. Cette approche ouvre la porte à des évaluations efficaces de familles de mouvements combinés, et permet de reléguer les décisions d'affectation au sein de sous-problème d'évaluation de routes afin de ne plus avoir à considérer explicitement cet aspect combinatoire au sein des voisinages de recherche locale et des représentations de solution.

Les Chapitres 7 à 9 ont exploité les analyses et développements méthodologiques précédents pour progresser vers des méthodes efficaces et généralistes pour les problèmes de tournées de véhicules multi-attributs. Dans ce cadre, une famille de problèmes riches particulièrement difficiles, les problèmes combinés de tournées de véhicules et optimisation des horaires de conducteurs, a été considérée. Ces problèmes combinés ont été traités efficacement en couplant l'algorithme génétique hybride avec des stratégies avancées d'évaluation des routes basées sur des recherches arborescentes et des mémoires. Un cadre de résolution modulaire a par ailleurs été proposé, et appliqué pour produire un algorithme génétique hybride unifié pour les MAVRP. Tandis que les algorithmes modulaires et librairies de composants de la littérature pour l'optimisation combinatoire étaient auparavant principalement dédiés à la multiplicité des méthodes (différents modules pour différentes stratégies heuristiques), le cadre proposé a permis de gérer la *multiplicité des variantes de problèmes* grâce à des composants de résolution élémentaires s'adaptant automatiquement aux attributs considérés. L'algorithme UHGS bâti sur ces principes, testé sur 26 variantes fondamentales de VRP avec une unique implémentation et paramétrage, égalise ou surpasse toutes les meilleures méthodes dédiées de la littérature issues de plus de 180 articles. Cet algorithme est une contribution fondamentale à la fois d'un point de vue pratique et méthodologique, de tels algorithmes généralistes étant nécessaires pour mettre à l'épreuve des concepts de résolution sur une vaste gamme de problèmes, mais aussi pour répondre rapidement aux besoins des cas application pratiques. De plus, afin de traiter plus efficacement le challenge des *combinaisons d'attributs* et des problèmes riches, les méta-heuristiques précédentes ont été étendues au sein d'un cadre de résolution parallèle coopérative à base de décomposition et intégration successives de solutions (ICS-UHGS). Cette application a donné lieu à des développements concernant la gestion efficace des problèmes partiels obtenus par projection, de l'intégration des solutions, et du guidage de la recherche. Les expérimentations ont démontré la contribution importante du cadre parallèle coopératif, des méthodes d'intégration et de guidage, et ont illustré la capacité d'ICS-UHGS à traiter efficacement un problème *riche* en utilisant exclusivement des algorithmes de résolution sur des projections, ouvrant ainsi la porte à de nouvelles perspectives pour la résolution efficace de problèmes riches d'optimisation combinatoire.

Enfin, le fort potentiel d'impact pratique des méthodologies développées dans cette thèse a été démontré à plusieurs reprises. Le Chapitre 7, plus particulièrement, a utilisé ces méthodes d'optimisation pour quantifier l'impact des législations sur les temps de conduite, et ainsi comparer et évaluer l'impact de différentes législations autour du monde sur la compétitivité économique, les horaires, et la fatigue des conducteurs. Il apparait notamment que les règles de l'Union Européenne sont les plus sûres, tandis que les règles du Canada sont les plus avantageuses économiquement. Les règles de l'Australie sont dominées en termes de coût et sûreté par les autres règles. Finalement, les règles des États-Unis sont une alternative intermédiaire entre coût et sureté, les récents changements

de règles à venir en 2013 tendant vers plus de sécurité. Ces observations démontrent l'intérêt pratique direct des méthodes proposées, qui peuvent apporter une aide précieuse non seulement aux acteurs des chaines logistiques à des fins d'optimisation, mais aussi aux organismes décisionnaires afin de mesurer l'impact réel de choix de règles sur le transport longue-distance. Enfin, il faut noter que les méthodes heuristiques proposées ont été testées sur de nombreux ensembles de problèmes, dont un bon nombre sont issus de cas d'applications pratiques et de jeux de données réels. La grande généralité et performance de UHGS en fait un outil unique, dans la littérature scientifique sur les tournées de véhicules, pour répondre rapidement et efficacement aux besoins d'applications industrielles. Cette thèse a ainsi conduit à de nouvelles contributions cruciales en termes d'analyse, de synthèse, et de nouvelles méthodologies de résolution pour gérer à la fois la difficulté, la variété et la combinaison des problèmes. Les implications académiques et pratiques de ces travaux sont manifestes au vu de l'ampleur des expérimentations et variétés de problèmes traités. Les questions et les pistes de recherches ouvertes sont, en conséquence, nombreuses.

En particulier, l'étude unificatrice de la classe de problèmes de timing a permis d'identifier des brèches dans l'état de l'art. Des algorithmes plus performants pourraient être recherchés pour des problèmes de timing avec contraintes de laps de temps minimum ou maximum, durée d'exécution dépendant du temps et contrainte de durée, entre autres. Les bornes inférieures de complexité sont rares et particulièrement utiles, ainsi que le serait une extension de la méthodologie à des problèmes multi-objectifs ou stochastiques.

Concernant les MAVRP, certains problèmes riches apparaissent comme particulièrement difficiles, notamment les variantes avec synchronisation de flux (rencontres, échanges de marchandises), les variantes stochastiques avec des recours complexes faisant éventuellement changer l'itinéraire de plusieurs véhicules, ou les problèmes impliquant plusieurs modes de transport.

Plusieurs méthodologies développées dans cette thèse, et en particulier l'évaluation multicritère des individus impliquant qualité et contribution à la diversité ainsi que le cadre de résolution heuristique modulaire généraliste, ont vocation à être étendues et testées sur d'autres problèmes d'optimisation combinatoire. Des études supplémentaires pourraient être conduites sur les différentes mesures de distance et de diversité utilisées dans les méthodes de population, et il serait pertinent d'étudier si les attributs ont un impact significatif sur le meilleur choix de mesure.

La prochaine génération de métaheuristiques pourrait tirer parti de voisinages plus larges et structurellement différents, permettant d'intégrer au mieux au niveau de la recherche locale des décisions combinées sur différents attributs des problèmes. Un progrès pourrait être effectué vers des métaheuristiques plus "intelligentes" et moins randomisées, qui profitent au mieux de l'historique de la recherche et d'informations locales, pour aiguiller au mieux la recherche future et trouver un équilibre subtil entre diversification et intensification correctement ciblée. Aussi, nous avons observé dans cette thèse que le succès sur les MAVRP n'est pas réellement lié à un cadre métaheuristique particulier, mais à une combinaison de concepts à succès. Il existe des méthodes performantes de plusieurs types, incluant la recherche tabou, la recherche voisinage large, recherche locale itérée, méthode à base de population et croisements... Étant donné la manière très différente avec laquelle ces méthodes explorent l'espace des solutions, il est très probable que des heuristiques hybrides très

efficaces puissent être générées, le défi majeur à ce niveau restant cependant de trouver le meilleur compromis entre simplicité des concepts et performance.

Finalement, beaucoup de progrès restent à venir vis-à-vis de la résolution généraliste de problèmes de tournées de véhicules. Entre autres, l'utilisation de langages formels de modélisation peut être considérée, ainsi que des approches plus systématiques pour gérer la combinaison de modules spécifiques aux attributs et la génération des voisinages de recherche locale relativement à la structure du graphe et la nature des solutions. Ayant montré qu'une unique méthode peut être performante sur une vaste gamme de problèmes, nous espérons que plus d'articles futurs visent non seulement à introduire de nouveaux éléments méthodologiques, mais aussi à mieux examiner leur potentiel d'application à des plusieurs variantes de VRP. Les algorithmes unifiés produits dans cette thèse – ILS, UHGS, ICS – ont le potentiel d'être une aide considérable pour mener à bien ces pistes de recherche en permettant d'introduire de nouveaux éléments méthodologiques et d'examiner leur impact sur une vaste gamme de problèmes.

BIBLIOGRAPHIE

Aarts, E.H.L., J.K. Lenstra, eds. 2003. *Local search in combinatorial optimization*. Princeton Univ Pr.

Ahuja, R, D Hochbaum, J Orlin. 2003. Solving the convex cost integer dual network flow problem. *Management Science* **49**(7) 950–964.

Ahuja, Ravindra K., James B. Orlin. 2001. A Fast Scaling Algorithm for Minimizing Separable Convex Functions Subject to Chain Constraints. *Operations Research* **49**(5).

Ahuja, R.K., Ö. Ergun, J.B. Orlin, A.P. Punnen. 2002. A survey of very large-scale neighborhood search techniques. *Discrete Applied Mathematics* **123**(1-3) 75–102.

Alba, E. 2005. *Parallel metaheuristics : a new class of algorithms*. Wiley-Interscience, Hoboken, NJ.

Alba, E., B. Dorronsoro. 2006. Computing nine new best-so-far solutions for Capacitated VRP with a cellular Genetic Algorithm. *Information Processing Letters* **98**(6) 225–230.

Alegre, J., M. Laguna, J. Pacheco. 2007. Optimizing the periodic pick-up of raw materials for a manufacturer of auto parts. *European Journal of Operational Research* **179**(3) 736–746.

Aleman, R.E., R.R. Hill. 2010. A tabu search with vocabulary building approach for the vehicle routing problem with split demands. *International Journal of Metaheuristics* **1**(1) 55–80.

Alidaee, B., N.K. Womer. 1999. Scheduling with time dependent processing times : review and extensions. *Journal of the Operational Research Society* **50**(7) 711–720.

Alonso, F., M. J. Alvarez, J. E. Beasley. 2008. A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility restrictions. *Journal of the Operational Research Society* **59**(7) 963–976.

Alvarenga, G.B., G.R. Mateus, G. De Tomi. 2007. A genetic and set partitioning two-phase approach for the vehicle routing problem with time windows. *Computers & Operations Research* **34**(6) 1561–1584.

Andersson, H., A. Hoff, M. Christiansen, G. Hasle, A. Lokketangen. 2010. Industrial aspects and literature survey : Combined inventory management and routing. *Computers & Operations Research* **37**(9) 1515–1536.

Arabeyre, J.P., J. Fearnley, F.C. Steiger, W. Teather. 1969. The Airline Crew Scheduling Problem : A Survey. *Transportation Science* **3**(2) 140–163.

Archetti, C., A. Hertz, M. G. Speranza. 2006. Metaheuristics for the team orienteering problem. *Journal of Heuristics* **13**(1) 49–76.

Archetti, C., M. Savelsbergh. 2009. The trip scheduling problem. *Transportation Science* **43**(4) 417–431.

Archetti, C., M.G. Speranza. 2011. Vehicle routing problems with split deliveries. *International Transactions in Operational Research, Forthcoming* .

Archetti, C., M.G. Speranza, M.W.P. Savelsbergh. 2008. An optimization-based heuristic for the split delivery vehicle routing problem. *Transportation Science* **42**(1) 22–31.

Assad, A.A. 1988. Modeling and Implementation Issues in Vehicle Routing. B.L. Golden, A.A. Assad, eds., *Vehicle Routing : Methods and Studies*. North-Holland Amsterdam, 7–45.

Ayer, M., H.D. Brunk, G.M. Ewing, W.T. Reid, E. Silverman. 1955. An empirical distribution function for sampling with incomplete information. *The Annals of Mathematical Statistics* **26**(4) 641–647.

Badeau, P., F. Guertin, M. Gendreau, J.-Y. Potvin, E.D. Taillard. 1997. A parallel tabu search heuristic for the vehicle routing problem with time windows. *Transportation Research Part C : Emerging Technologies* **5**(2) 109–122.

Baker, K.R., G.D. Scudder. 1990. Sequencing with earliness and tardiness penalties : a review. *Operations Research* **38**(1) 22–36.

Balakrishnan, N. 1993. Simple heuristics for the vehicle routeing problem with soft time windows. *Journal of the Operational Research Society* **44**(3) 279–287.

Baldacci, R., E. Bartolini, G. Laporte. 2009. Some applications of the generalized vehicle routing problem. *Journal of the Operational Research Society* **61**(7) 1072–1077.

Baldacci, R., E. Bartolini, A. Mingozzi, A. Valletta. 2011a. An Exact Algorithm for the Period Routing Problem. *Operations Research* **59**(1) 228–241.

Baldacci, R., M. Battarra, D. Vigo. 2008a. Routing a heterogeneous fleet of vehicles. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, New York, 3–27.

Baldacci, R., N. Christofides, A. Mingozzi. 2008b. An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts. *Mathematical Programming* **115**(2) 351–385.

Baldacci, R., A. Mingozzi. 2009. A unified exact method for solving different classes of vehicle routing problems. *Mathematical Programming* **120**(2) 347–380.

Baldacci, R., A. Mingozzi, R.W. Calvo. 2011b. An exact method for the capacitated location-routing problem. *Operations Research* **59**(5) 1284–1296.

Baldacci, R., A. Mingozzi, R. Roberti. 2011c. New route relaxation and pricing strategies for the vehicle routing problem. *Operations Research* **59**(5) 1269–1283.

Baldacci, R., P. Toth, D. Vigo. 2007. Recent advances in vehicle routing exact algorithms. *4OR* **5**(4) 269–298.

Balseiro, S.R., I. Loiseau, J. Ramonet. 2011. An Ant Colony algorithm hybridized with insertion heuristics for the Time Dependent Vehicle Routing Problem with Time Windows. *Computers & Operations Research* **38**(6) 954–966.

Barlow, R.E., D.J. Bartholomew, J.M. Bremner, H.D. Brunk. 1972. *Statistical inference under order restrictions : The theory and application of isotonic regression*. Wiley New York.

Barr, R.S., B.L. Golden, J.P. Kelly, M.G.C. Resende, W.R. Stewart JR. 1995. Designing and reporting on computational experiments with heuristic methods. *Journal of Heuristics* **1**(1) 9–32.

Bartusch, M., R.H. Möhring, F.J. Radermacher. 1988. Scheduling project networks with resource constraints and time windows. *Annals of Operations Research* **16**(1) 199–240.

Bean, J.C., J.R. Birge, R.L. Smith. 1987. Aggregation in dynamic programming. *Operations Research* **35**(2) 215–220.

Beasley, J.E. 1981. Adapting the savings algorithm for varying inter-customer travel times. *Omega* **9**(6) 658–659.

Beasley, J.E. 1983. Route first–cluster second methods for vehicle routing. *Omega* **11**(4) 403–408.

Bektas, T., G. Erdogan, S. Ropke. 2011. Formulations and branch-and-cut algorithms for the generalized vehicle routing problem. *Transportation Science* **45**(3) 299–316.

Bektas, T., G. Laporte. 2011. The pollution-routing problem. *Transportation Research Part B : Methodological* **45**(8) 1232–1250.

Belenguer, J. M., M. C. Martinez, E. Mota. 2000. A Lower Bound for the Split Delivery Vehicle Routing Problem. *Operations Research* **48**(5) 801–810.

Belhaiza, S. 2010. Hybrid Variable Neighborhood - Tabu Search Algorithm for the Site Dependent Vehicle Routing Problem with Time Windows. Tech. rep., GERAD, Montreal, Canada.

Bell, J., P. Mc Mullen. 2004. Ant colony optimization techniques for the vehicle routing problem. *Advanced Engineering Informatics* **18**(1) 41–48.

Beltrami, E.J., L.D. Bodin. 1974. Networks and vehicle routing for municipal waste collection. *Networks* **4**(1) 65–94.

Bent, R., P. Van Hentenryck. 2006. A two-stage hybrid algorithm for pickup and delivery vehicle routing problems with time windows. *Computers & Operations Research* **33**(4) 875–893.

Bent, R., P. Van Hentenryck. 2010. Spatial, temporal, and hybrid decompositions for large-scale vehicle routing with time windows. D. Cohen, ed., *Proceedings of CP'10*, *LNCS*, vol. 6308. Springer Berlin Heidelberg, 99–113.

Bent, Russell, Pascal Van Hentenryck. 2004. A Two-Stage Hybrid Local Search for the Vehicle Routing Problem with Time Windows. *Transportation Science* **38**(4) 515–530.

Berbeglia, G., J.-F. Cordeau, I. Gribkovskaia, G. Laporte. 2007. Static pickup and delivery problems : a classification scheme and survey. *Top* **15**(1) 1–31.

Berbeglia, G., J.-F. Cordeau, G. Laporte. 2010. Dynamic pickup and delivery problems. *European Journal of Operational Research* **202**(1) 8–15.

Berbeglia, G., J.-F. Cordeau, G. Laporte. 2011. A Hybrid Tabu Search and Constraint Programming Algorithm for the Dynamic Dial-a-Ride Problem. *INFORMS Journal on Computing* .

Berger, J, M Barkaoui. 2004. A parallel hybrid genetic algorithm for the vehicle routing problem with time windows. *Computers & Operations Research* **31**(12) 2037–2053.

Berger, J., M. Barkaoui, O. Bräysy. 2003. A route-directed hybrid genetic approach for the vehicle routing problem with time windows. *INFOR* **41**(2) 179–194.

Bertsekas, D.P., A. Nedi, A.E. Ozdaglar. 2003. *Nonlinear Programming*. Athena Scientific, Nashua, NH.

Best, M.J., N. Chakravarti. 1990. Active set algorithms for isotonic regression, a unifying framework. *Mathematical Programming* **47**(1-3) 425–439.

Best, M.J, N. Chakravarti, V.A. Ubhaya. 2000. Minimizing separable convex functions subject to simple chain constraints. *SIAM Journal on Optimization* **10**(3) 658–672.

Bianco, L., A. Mingozzi, S. Ricciardelli. 1993. The traveling salesman problem with cumulative costs. *Networks* **23**(2) 81–91.

Blakeley, F., B. Bozkaya, B. Cao, W. Hall, J. Knolmajer. 2003. Optimizing periodic maintenance operations for Schindler Elevator Corporation. *Interfaces* **33**(1) 67–79.

Blum, A., P. Chalasani, D. Coppersmith, B. Pulleyblank, P. Raghavan, M. Sudan. 1994. The minimum latency problem. *Proceedings of the twenty-sixth annual ACM symposium on Theory of computing*. ACM, New York, USA, 163–171.

Blum, C., J. Puchinger, G. Raidl, A. Roli. 2011. Hybrid metaheuristics in combinatorial optimization : A survey. *Applied Soft Computing* **11**(6) 4135–4151.

Blum, C., A. Roli. 2003. Metaheuristics in Combinatorial Optimization : Overview and Conceptual Comparison. *ACM Computing Surveys (CSUR)* **35**(3) 268–308.

Bodin, L., B. Golden. 1981. Classification in vehicle routing and scheduling. *Networks* **11**(2) 97–108.

Bodin, L. D. 1975. A taxonomic structure for vehicle routing and scheduling problems. *Computers & Urban Society* **1**(1) 11–29.

Bodin, L.D., L. Berman. 1979. Routing and scheduling of school buses by computer. *Transportation Science* **13**(2) 113.

Boese, K.D. 1995. Cost versus distance in the traveling salesman problem. Tech. rep.

Bortfeldt, A. 2012. A hybrid algorithm for the capacitated vehicle routing problem with three-dimensional loading constraints. *Computers & Operations Research* **39**(9) 2248–2257.

Bostel, N., P. Dejax, P. Guez, F. Tricoire. 2008. Multiperiod planning and routing on a rolling horizon for field force optimization logistics. Bruce Golden, S. Raghavan, Edward Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*, vol. 43. Springer, New York, 503–525.

Boudia, M., C. Prins, M. Reghioui. 2007. An Effective Memetic Algorithm with Population Management for the Split Delivery Vehicle Routing Problem. *Hybrid Metaheuristics*, *LNCS*, vol. 4771. Springer Berlin Heidelberg, 16–30.

Bouly, H., D.-C. Dang, A. Moukrim. 2009. A memetic algorithm for the team orienteering problem. *4OR* **8**(1) 49–70.

Boussier, S., D. Feillet, M. Gendreau. 2006. An exact algorithm for team orienteering problems. *4OR* **5**(3) 211–230.

Brandão, J. 2006. A new tabu search algorithm for the vehicle routing problem with backhauls. *European Journal of Operational Research* **173**(2) 540–555.

Brandão, J. 2009. A deterministic tabu search algorithm for the fleet size and mix vehicle routing problem. *European Journal of Operational Research* **195**(3) 716–728.

Brandão, J. 2011. A tabu search algorithm for the heterogeneous fixed fleet vehicle routing problem. *Computers & Operations Research* **38**(1) 140–151.

Bräysy, O. 2003. A Reactive Variable Neighborhood Search for the Vehicle-Routing Problem with Time Windows. *INFORMS Journal on Computing* **15**(4) 347–368.

Bräysy, O., W. Dullaert, M. Gendreau. 2004a. Evolutionary algorithms for the vehicle routing problem with time windows. *Journal of Heuristics* **10**(6) 587–611.

Bräysy, O., W. Dullaert, G. Hasle, D. Mester, M. Gendreau. 2008a. An Effective Multirestart Deterministic Annealing Metaheuristic for the Fleet Size and Mix Vehicle-Routing Problem with Time Windows. *Transportation Science* **42**(3) 371–386.

Bräysy, O., M. Gendreau. 2005a. Vehicle routing problem with time windows, Part I : Route construction and local search algorithms. *Transportation Science* **39**(1) 104–118.

Bräysy, O., M. Gendreau. 2005b. Vehicle routing problem with time windows, Part II : Metaheuristics. *Transportation Science* **39**(1) 119–139.

Bräysy, O., M. Gendreau, G. Hasle, A. Lokketangen. 2008b. A Survey of Heuristics for the Vehicle Routing Problem Part I : Basic Problems and Supply Side Extensions. Tech. rep., SINTEF, Norway.

Bräysy, O., M. Gendreau, G. Hasle, A. Lokketangen. 2008c. A Survey of Heuristics for the Vehicle Routing Problem Part II : Demand Side Extensions. Tech. rep., SINTEF, Norway.

Bräysy, O., G. Hasle, W. Dullaert. 2004b. A multi-start local search algorithm for the vehicle routing problem with time windows. *European Journal of Operational Research* **159**(3) 586–605.

Bräysy, O., P.P. Porkka, W. Dullaert, P.P. Repoussis, C.D. Tarantilis. 2009. A well-scalable metaheuristic for the fleet size and mix vehicle routing problem with time windows. *Expert Systems with Applications* **36**(4) 8460–8475.

Brucker, P., T. Hilbig, J. Hurink. 1999. A branch and bound algorithm for a single-machine scheduling problem with positive and negative time-lags. *Discrete Applied Mathematics* **94**(1-3) 77–99.

Brunk, H. D. 1955. Maximum Likelihood Estimates of Monotone Parameters. *The Annals of Mathematical Statistics* **26**(4) 607–616.

Bullnheimer, B., R.F. Hartl, C. Strauss. 1999. An improved ant system algorithm for the vehicle routing problem. *Annals of Operations Research* **89** 319–328.

Burke, E.K., M. Hyde, G. Kendall, G. Ochoa, E. Özcan, J.R. Woodward. 2010. A classification of hyper-heuristic approaches. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer US, 449–468.

Cahon, S., N. Melab, E.-G. Talbi. 2004. ParadisEO : A framework for the reusable design of parallel and distributed metaheuristics. *Journal of Heuristics* **10**(3) 357–380.

Campbell, A.M., M. Savelsbergh. 2004. Efficient insertion heuristics for vehicle routing and scheduling problems. *Transportation science* **38**(3) 369–378.

Canadian Council of Motor Transport Administrators. 2007. Commercial Vehicle Drivers Hours of Service Regulations – Application Guide.

Chakravarti, N. 1989. Isotonic Median Regression : A Linear Programming Approach. *Mathematics of Operations Research* **14**(2) 303–308.

Chandran, B., S. Raghavan. 2008. Modeling and Solving the Capacitated Vehicle Routing Problem on Trees. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, New York, 616–622.

Chao, I., B. Golden, E.A. Wasil. 1996. The team orienteering problem. *European Journal of Operational Research* **88**(3) 464–474.

Chao, I.M., B.L. Golden, E. Wasil. 1995. An improved heuristic for the period vehicle routing problem. *Networks* **26**(1) 25–44.

Charnes, A., W.W. Cooper. 1958. The theory of search : optimum distribution of search effort. *Management Science* **5**(1) 44–50.

Chen, P., H.-K. Huang, X.-Y. Dong. 2010. Iterated variable neighborhood descent algorithm for the capacitated vehicle routing problem. *Expert Systems with Applications* **37**(2) 1620–1627.

Chen, S., B. Golden, E. Wasil. 2007. The split delivery vehicle routing problem : Applications, algorithms, test problems, and computational results. *Networks* **49**(4) 318–329.

Cheng, T. 2004. A concise survey of scheduling with time-dependent processing times. *European Journal of Operational Research* **152**(1) 1–13.

Chiu, H.N., Y.S. Lee, J.H. Chang. 2006. Two approaches to solving the multi-depot vehicle routing problem with time windows in a time-based logistics environment. *Production Planning & Control* **17**(5) 480–493.

Chrétienne, P., F. Sourd. 2003. PERT scheduling with convex cost functions. *Theoretical Computer Science* **292**(1) 145–164.

Christiansen, M., K. Fagerholt. 2002. Robust ship scheduling with multiple time windows. *Naval Research Logistics* **49**(6) 611–625.

Christofides, N. 1976. The Vehicle Routing Problem. *RAIRO Operations Research* **10**(2) 55–70.

Christofides, N., S. Eilon. 1972. Algorithms for Large-Scale Travelling Salesman Problems. *Operational Research Quarterly* **23**(4) 511.

Christofides, N., A. Mingozzi, P. Toth. 1979. The vehicle routing problem. N. Christofides, A. Mingozzi, P. Toth, C. Sandi, eds., *Combined Optimization*. Wiley, Chichester, UK, 315–338.

Chu, F., N. Labadi, C. Prins. 2006. A Scatter Search for the periodic capacitated arc routing problem. *European Journal of Operational Research* **169**(2) 586–605.

Clarke, G., J. W. Wright. 1964. Scheduling of Vehicles from a Central Depot to a Number of Delivery Points. *Operations Research* **12**(4) 568–581.

Coelho, L.C., J.-F. Cordeau, G. Laporte. 2012. Thirty Years of Inventory-Routing. Tech. rep., CIRRELT, Montreal, Canada.

Coene, S., A. Arnout, F.C.R Spieksma. 2010. On a periodic vehicle routing problem. *Journal of the Operational Research Society* **61**(12) 1719–1728.

Cooke, K.L., E. Halsey. 1966. The shortest route through a network with time-dependent internodal transit times. *Journal of Mathematical Analysis and Applications* **14** 493–498.

Cordeau, J.-F., M. Gendreau, A. Hertz, G. Laporte, J.S. Sormany. 2005. New heuristics for the vehicle routing problem. A. Langevin, D. Riopel, eds., *Logistics systems : design and optimization*. Springer, 279–297.

Cordeau, J.-F., M. Gendreau, G. Laporte. 1997. A tabu search heuristic for periodic and multi-depot vehicle routing problems. *Networks* **30**(2) 105–119.

Cordeau, J.-F., G. Laporte. 2001. A tabu search algorithm for the site dependent vehicle routing problem with time windows. *INFOR* **39**(3) 292–298.

Cordeau, J.-F., G. Laporte. 2003. A tabu search heuristic for the static multi-vehicle dial-a-ride problem. *Transportation Research Part B : Methodological* **37**(6) 579–594.

Cordeau, J.-F., G. Laporte, A. Mercier. 2001a. A unified tabu search heuristic for vehicle routing problems with time windows. *Journal of the Operational Research Society* **52**(8) 928–936.

Cordeau, J.-F., G. Laporte, A. Mercier. 2004. Improved tabu search algorithm for the handling of route duration constraints in vehicle routing problems with time windows. *Journal of the Operational Research Society* **55**(5) 542– 546.

Cordeau, J.-F., G. Laporte, S. Ropke. 2008. Recent models and algorithms for one-to-one pickup and delivery problems. Bruce Golden, S. Raghavan, Edward Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, New York, 327–357.

Cordeau, J.-F., G. Laporte, M.W.P. Savelsbergh, D. Vigo. 2007. Vehicle Routing. C. Barnhart, G. Laporte, eds., *Transportation*. Elsevier, North-Holland, Amsterdam, 367–428.

Cordeau, J.-F., M. Maischberger. 2012. A Parallel Iterated Tabu Search Heuristic for Vehicle Routing Problems. *Computers & Operations Research* **39**(9) 2033–2050.

Cordeau, J.-F., G. Stojkovic, F. Soumis, J. Desrosiers. 2001b. Benders Decomposition for Simultaneous Aircraft Routing and Crew Scheduling. *Transportation Science* **35**(4) 375–388.

Cordone, R. 2000. A note on time windows constraints in routing problems. Tech. rep., Department of Electronics and Information, Polytechnic of Milan.

Cormen, T.H., C. Stein, R.L. Rivest, C.E. Leiserson. 2001. *Introduction to Algorithms*. McGraw-Hill Higher Education.

Corne, D., M. Dorigo, F. Glover. 1999. *New ideas in optimisation*. McGraw-Hill, Maidenhead, UK, England.

Crainic, Teodor Gabriel. 2000. Service network design in freight transportation. *European Journal of Operational Research* **122**(2) 272–288.

Crainic, T.G. 2005. Parallel computation, co-operation, tabu search. C. Rego, B. Alidaee, eds., *Metaheuristic Optimization Via Memory and Evolution : Tabu Search and Scatter Search*. Kluwer Academic Publishers, Norwell, MA, 283–302.

Crainic, T.G. 2008. Parallel solution methods for vehicle routing problems. B.L. Golden, S. Raghavan, E.A. Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, New York, 171–198.

Crainic, T.G., G.C. Crisan, M. Gendreau, N. Lahrichi, W. Rei. 2009. Multi-thread integrative cooperative optimization for rich combinatorial problems. *Proceedings of IPDPS '09*.

Crainic, T.G., B. Di Chiara, M. Nonato, L. Tarricone. 2006. Tackling electrosmog in completely configured 3G networks by parallel cooperative meta-heuristics. *IEEE Wireless Communications* **13**(6) 34–41.

Crainic, T.G., N. El Hachemi, M. Gendreau, N. Lahrichi, W. Rei, T. Vidal. 2012. Solution Integration in Combinatorial Optimization : Applications to Cooperative Search and Multi-Attribute Vehicle Routing. Tech. rep., CIRRELT, Montréal, QC, Canada.

Crainic, T.G., H. Nourredine. 2005. Parallel Meta-Heuristics Applications. E. Alba, ed., *Parallel Metaheuristics*. John Wiley & Sons, Hoboken, NJ, 1–49.

Crainic, T.G., M. Toulouse. 2008. Explicit and Emergent Cooperation Schemes for Search Algorithms. V. Maniezzo, R. Battiti, J.-P. Watson, eds., *Learning and Intelligent Optimization*, *LNCS*, vol. 5315. Springer-Verlag, Berlin, Heidelberg, 95–109.

Crainic, T.G., M. Toulouse. 2010. Parallel Meta-heuristics. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer US, Boston, MA, 497–541.

Créput, J.-C., A. Koukam. 2008. The memetic self-organizing map approach to the vehicle routing problem. *Soft Computing* **12**(11) 1125–1141.

Dang, D.-C., R. Guibadj, A. Moukrim. 2011. A PSO-Based Memetic Algorithm for the Team Orienteering Problem. *Applications of Evolutionary Computation*, *LNCS*, vol. 6625. Springer Berlin Heidelberg, 471–480.

Dantzig, G.B., J.H. Ramser. 1959. The truck dispatching problem. *Management Science* **6**(1) 80–91.

Davis, J.S., J.J. Kanet. 1993. Single-machine scheduling with early and tardy completion costs. *Naval Research Logistics* **40** 85–101.

De Franceschi, R., M. Fischetti, P. Toth. 2006. A new ILP-based refinement heuristic for vehicle routing problems. *Mathematical Programming* **105**(2) 471–499.

De Landgraaf, W.A., AE Eiben, V Nannen. 2007. Parameter calibration using meta-algorithms. *Proceedings of CEC'07*. IEEE, 71–78.

Dechter, R, I. Meiri, J. Pearl. 1991. Temporal constraint networks. *Artificial Intelligence* **49** 61–95.

Dell'Amico, M., F. Maffioli, P. Värbrand. 1995. On prize-collecting tours and the asymmetric travelling salesman problem. *International Transactions in Operational Research* **2**(3) 297–308.

Demir, E., T. Bektas, G. Laporte. 2012. An adaptive large neighborhood search heuristic for the Pollution-Routing Problem. *European Journal of Operational Research* **223**(2) 346–359.

Derigs, U., R. Kaiser. 2007. Applying the attribute based hill climber heuristic to the vehicle routing problem. *European Journal of Operational Research* **177**(2) 719–732.

Derigs, U., B. Li, U. Vogel. 2009. Local search-based metaheuristics for the split delivery vehicle routing problem. *Journal of the Operational Research Society* 1356–1364.

Desaulniers, G., J. Desrosiers, A. Erdmann, M.M. Solomon, F. Soumis. 2002. VRP with pickup and delivery. P. Toth, D. Vigo, eds., *The Vehicle Routing Problem*. SIAM, Philadelphia, PA, USA, 225–242.

Desaulniers, G., J. Desrosiers, I. Ioachim, M.M. Solomon, F. Soumis, D. Villeneuve. 1998. A unified framework for deterministic time constrained vehicle routing and crew scheduling problems. T.G. Crainic, G. Laporte, eds., *Fleet Management and Logistics*. Kluwer Academic Publishers, Boston, MA, 129–154.

Desaulniers, G., J. Desrosiers, M.M. Solomon, eds. 2005. *Column generation*. Springer.

Desaulniers, G., D. Villeneuve. 2000. The Shortest Path Problem with Time Windows and Linear Waiting Costs. *Transportation Science* **34**(3) 312–319.

Desrochers, M., J. Desrosiers, M. Solomon. 1992. A new Optimization Algorithm for the Vehicle Routing Problem with Time Windows. *Operations Research* **40**(2) 342–354.

Desrochers, M, C.V. Jones, J.K. Lenstra, M.W.P. Savelsbergh, L. Stougie. 1999. Towards a model and algorithm management system for vehicle routing and scheduling problems. *Decision support systems* **25**(2) 109–133.

Desrochers, M., J.K. Lenstra, M.W.P. Savelsbergh. 1990. A classification scheme for vehicle routing and scheduling problems. *European Journal of Operational Research* **46**(3) 322–332.

Desrosiers, J., Y. Dumas, M.M. Solomon, F. Soumis. 1995. Time Constrained Routing and Scheduling. M. Ball, T. L. Magnanti, C.L. Monma, G.L. Nemhauser, eds., *Network Routing*. North-Holland Amsterdam, 35–139.

Doerner, K.F., R.F. Hartl, S. Benkner, M. Lucka. 2006. Parallel cooperative savings based ant colony optimization : multiple search and decomposition approaches. *Parallel processing letters* **16**(3) 351.

Doerner, K.F., R.F. Hartl, G. Kiechle, M. Lucka, M. Reimann. 2004. Parallel ant systems for the capacitated vehicle routing problem. J. Gottlieb, G. Raidl, eds., *Evolutionary Computation in Combinatorial Optimization*, *LNCS*, vol. 3004. Springer Berlin Heidelberg, 72–83.

Doerner, K.F., V. Schmid. 2010. Survey : Matheuristics for Rich Vehicle Routing Problems. *Hybrid Metaheuristics*, *LNCS*, vol. 6373. Springer Berlin Heidelberg, 206–221.

Donati, A., R. Montemanni, N. Casagrande, A.E. Rizzoli, L.M. Gambardella. 2008. Time dependent vehicle routing problem with a multi ant colony system. *European Journal of Operational Research* **185**(3) 1174–1191.

Dondo, R., J. Cerdá. 2007. A cluster-based optimization approach for the multi-depot heterogeneous fleet vehicle routing problem with time windows. *European Journal of Operational Research* **176**(3) 1478–1507.

Dongarra, J. 2011. Performance of various computers using standard linear equations software. Tech. rep., University of Tennessee.

Dorigo, M., T. Stützle. 2004. *Ant colony optimization*. Cambridge, MIT Press.

Dorronsoro, B., D. Arias, F. Luna, A.J. Nebro, E. Alba. 2007. A grid-based hybrid cellular genetic algorithm for very large scale instances of the CVRP. W. Smari, ed., *Proceedings of HPCS'07*. Prague, Czech Republic, 759–765.

Dréo, J., A. Pétrowski, P. Siarry, É. Taillard. 2003. *Métaheuristiques pour l'optimisation difficile*. Eyrolles.

Drexl, M. 2012. Synchronization in vehicle routing – a survey of VRPs with multiple synchronization constraints. *Transportation Science* **46**(3) 297–316.

Dreyfus, SE. 1969. An appraisal of some shortest-path algorithms. *Operations Research* **17**(3) 395–412.

Drezner, Z, HW Hamacher. 2004. *Facility location : applications and theory*. Springer Verlag, Berlin.

Droste, S., T. Jansen, I. Wegener. 2002. Optimization with randomized search heuristics – the (A)NFL theorem, realistic scenarios, and difficult functions. *Theoretical Computer Science* **287**(1) 131–144.

Drummond, L.M.A., L.S. Ochi, D.S. Vianna. 2001. An asynchronous parallel metaheuristic for the period vehicle routing problem. *Future Generation Computer Systems* **17**(4) 379–386.

Dry, M., M.D. Lee, D. Vickers, P. Hughes. 2006. Human performance on visually presented traveling salesperson problems with varying numbers of nodes. *Journal of Problem Solving* **1**(1) 20–32.

Du, T.C., J.-L. Wu. 2001. Using object-oriented paradigm to develop an evolutional vehicle routing system. *Computers in Industry* **44**(3) 229–249.

Dueck, G. 1993. New optimization heuristics : The great deluge algorithm and the record-to-record travel. *Journal of Computational Physics* **104**(1) 86–92.

Duhamel, C. 2001. Un Cadre Formel pour les Méthodes par Amélioration Itérative - Application à deux problèmes d'Optimisation dans les réseaux. Ph.D. thesis, Université Blaise Pascal, Clermont-Ferrand.

Duhamel, C., P. Lacomme, A. Quilliot, H. Toussaint. 2011. A multi-start evolutionary local search for the two-dimensional loading capacitated vehicle routing problem. *Computers & Operations Research* **38**(3) 617–640.

Dumas, Y., F. Soumis, J. Desrosiers. 1990. Optimizing the Schedule for a Fixed Vehicle Path with Convex Inconvenience Costs. *Transportation Science* **24**(2) 145–152.

Eksioglu, B., A.V. Vural, A. Reisman. 2009. The vehicle routing problem : A taxonomic review. *Computers & Industrial Engineering* **57**(4) 1472–1483.

Elhallaoui, I., D. Villeneuve, F. Soumis, G. Desaulniers. 2005. Dynamic Aggregation of Set-Partitioning Constraints in Column Generation. *Operations Research* **53**(4) 632–645.

Ergun, O., J. Orlin. 2006. Fast neighborhood search for the single machine total weighted tardiness problem. *Operations Research Letters* **34**(1) 41–45.

Erkut, E., J. Zhang. 1996. The Maximum Collection Problem with Time-Dependent Rewards. *Naval Research Logistics* **43**(5) 749–763.

Ernst, A.T., H. Jiang, M. Krishnamoorthy, B. Owens, D. Sier. 2004. An Annotated Bibliography of Personnel Scheduling and Rostering. *Annals of Operations Research* **127**(1) 21–144.

European Transport Safety Council. 2001. The Role of Driver Fatigue in Commercial Road Transport Crashes. ETSC Review.

European Union. 2002. Directive 2002/15/EC of the European Parliament and of the Council of 11 March 2002 on the organisation of the working time of persons performing mobile road transport activities. *Official Journal of the European Communities* **L 80** 35–39.

European Union. 2006. Regulation (EC) No 561/2006 of the European Parliament and of the Council of 15 March 2006 on the harmonisation of certain social legislation relating to road transport and amending Council Regulations (EEC) No 3821/85 and (EC) No 2135/98 and repealing Co. Official Journal of the European Union L 102, 11.04.2006.

Everett, H. 1963. Generalized Lagrange multiplier method for solving problems of optimum allocation of resources. *Operations Research* **11**(3) 399–417.

Federal Motor Carrier Safety Administration. 2008. Hours of Service of Drivers. Federal Register Vol. 73, No. 224, p. 69569 - 69586.

Federal Motor Carrier Safety Administration. 2011. Hours of Service of Drivers. Federal Register Vol. 76, No. 248, p. 81134 - 81188.

Feillet, D., P. Dejax, M. Gendreau. 2005. Traveling Salesman Problems with Profits. *Transportation Science* **39**(2) 188–205.

Feng, G., H.C. Lau. 2007. Efficient algorithms for machine scheduling problems with earliness and tardiness penalties. *Annals of Operations Research* **159**(1) 83–95.

Figliozzi, M.A. 2010. An iterative route construction and improvement algorithm for the vehicle routing problem with soft time windows. *Transportation Research Part C : Emerging Technologies* **18**(5) 668–679.

Fink, A., S. Voss. 2003. HOTFRAME : A Heuristic Optimization Framework. S. Voss, D.L. Woodruff, eds., *Optimization Software Class Libraries*. Operations Research/Computer Science Interfaces Series, Vol. 18, Springer, 81–154.

Firat, M., G.J. Woeginger. 2011. Analysis of the dial-a-ride problem of Hunsaker and Savelsbergh. *Operations Research Letters* **39**(1) 32–35.

Fischetti, M., G. Laporte, S. Martello. 1993. The Delivery Man Problem and Cumulative Matroids. *Operations Research* **41**(6) 1055–1064.

Fischetti, M., P. Toth, D. Vigo. 1994. A branch-and-bound algorithm for the capacitated vehicle routing problem on directed graphs. *Operations Research* **42**(5) 846–859.

Fisher, M.L. 1994. Optimal solution of vehicle routing problems using minimum k-trees. *Operations Research* **42**(4) 626–642.

Fisher, M.L., R. Jaikumar. 1981. A generalized assignment heuristic for vehicle routing. *Networks* **11**(2) 109–124.

Fleischmann, B., M. Gietz, S. Gnutzmann. 2004. Time-Varying Travel Times in Vehicle Routing. *Transportation Science* **38**(2) 160–173.

Fleszar, K., I. Osman, K. Hindi. 2009. A variable neighbourhood search algorithm for the open vehicle routing problem. *European Journal of Operational Research* **195**(3) 803–809.

Francis, P.M., K.R. Smilowitz, M. Tzur. 2008. The period vehicle routing problem and its extensions. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, New York, 73–102.

Frangioni, A., A. Manca. 2006. A Computational Study of Cost Reoptimization for Min-Cost Flow Problems. *INFORMS Journal on Computing* **18**(1) 61–70.

Fredman, M.L. 1975. On computing the length of longest increasing subsequences. *Discrete Mathematics* **11**(1) 29–35.

Freling, R., D. Huisman, A.P.M. Wagelmans. 2003. Models and algorithms for integration of vehicle and crew scheduling. *Journal of Scheduling* **6**(1) 63–85.

Fu, Z., R. Eglese, L.Y.O. Li. 2007. A unified tabu search algorithm for vehicle routing problems with soft time windows. *Journal of the Operational Research Society* **59**(5) 663–673.

Fuellerer, G., K.F. Doerner, R.F. Hartl, M. Iori. 2009. Ant colony optimization for the two-dimensional loading vehicle routing problem. *Computers & Operations Research* **36**(3) 655–673.

Fukasawa, R., H. Longo, J. Lysgaard, M.P. Aragão, M. Reis, E. Uchoa, R.F. Werneck. 2006. Robust branch-and-cut-and-price for the capacitated vehicle routing problem. *Mathematical Programming* **106**(3) 491–511.

Funke, B., T. Grünert, S. Irnich. 2005. Local search for vehicle routing and scheduling problems : Review and conceptual integration. *Journal of Heuristics* **11**(4) 267–306.

Gajpal, Y., P. Abad. 2009. Multi-ant colony system (MACS) for a vehicle routing problem with backhauls. *European Journal of Operational Research* **196**(1) 102–117.

Gambardella, L.M., E. Taillard, G. Agazzi. 1999. MACS-VRPTW : A multiple ant colony system for vehicle routing problems with time windows. F. Glover D. Corne, M. Dorigo, ed., *New Ideas in Optimization*. McGraw-Hill, 63–76.

Garcia, B.-L. 1996. Une approche générique des méthodes par amélioration itérative. Application à la résolution de problèmes d'optimisation dans les réseaux. Ph.D. thesis, Laboratoire d'Informatique de l'Université Blaise Pascal, Clermont-Ferrand, France.

Garey, M R, D S Johnson. 1979. *Computers and lntractability : A Guide to the Theory of NP–Completeness*.

Garey, M.R., R.E. Tarjan, G.T. Wilfong. 1988. One-processor scheduling with symmetric earliness and tardiness penalties. *Mathematics of Operations Research* **13**(2) 330–348.

Gaskell, T.J. 1967. Bases for vehicle fleet scheduling. *Operational Research Quarterly* **18**(3) 281–295.

Gawiejnowicz, S. 2008. *Time-dependent scheduling*. Springer, New York.

Gehring, H., J. Homberger. 1999. A parallel hybrid evolutionary metaheuristic for the vehicle routing problem with time windows. *Proceedings of EURO-GEN'99*. 57–64.

Gehring, H., J. Homberger. 2002. Parallelization of a two-phase metaheuristic for routing problems with time windows. *Journal of heuristics* **8**(2) 251–276.

Gendreau, M., A. Hertz, G. Laporte. 1992. New insertion and postoptimization procedures for the traveling salesman problem. *Operations Research* **40**(6) 1086–1094.

Gendreau, M., A. Hertz, G. Laporte. 1994. A tabu search heuristic for the vehicle routing problem. *Management Science* **40**(10) 1276–1290.

Gendreau, M., M. Iori, G. Laporte, S. Martello. 2006. A Tabu Search Algorithm for a Routing and Container Loading Problem. *Transportation Science* **40**(3) 342–350.

Gendreau, M., M. Iori, G. Laporte, S. Martello. 2008a. A Tabu search heuristic for the vehicle routing problem with two-dimensional loading constraints. *Networks* **51**(1) 4–18.

Gendreau, M., G. Laporte, J.-Y. Potvin. 2002. *Metaheuristics for the capacitated VRP*. SIAM, Philadelphia, USA, 129–154.

Gendreau, M., G. Laporte, R. Seguin. 1996. A Tabu Search Heuristic for the Vehicle Routing Problem with Stochastic Demands and Customers. *Operations Research* **44**(3) 469–477.

Gendreau, M., J.-Y. Potvin. 2005. Metaheuristics in Combinatorial Optimization. *Annals of Operations Research* **140**(1) 189–213.

Gendreau, M., J.-Y. Potvin, eds. 2010. *Handbook of Metaheuristics*. International Series in Operations Research & Management Science, Springer.

Gendreau, M., J.Y. Potvin, O. Bräysy, G. Hasle, A. Lokketangen. 2008b. Metaheuristics for the vehicle routing problem and its extensions : A categorized bibliography. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, New York, 143–169.

Gendreau, M., C.D. Tarantilis. 2010. Solving large-scale vehicle routing problems with time windows : The state-of-the-art. Tech. rep., CIRRELT.

Geoffrion, A.M. 1970a. Elements of Large-Scale Mathematical Programming : Part I : Concepts. *Management Science* **16**(11) 652–675.

Geoffrion, AM. 1970b. Elements of large-scale mathematical programming. Part II : Synthesis of algorithms and bibliography. *Management Science* **16**(11) 676–691.

Ghaziri, H. 1996. Supervision in the self-organizing feature map : Application to the vehicle routing problem. I.H. Osman, J.P. Kelly, eds., *Meta-heuristics : Theory & applications*. Kluwer, Boston, 651–660.

Giffler, B., G.L. Thompson. 1960. Algorithms for Solving Production-Scheduling Problems. *Operations Research* **8**(4) 487–503.

Gillett, B.E., L.R. Miller. 1974. A heuristic algorithm for the vehicle-dispatch problem. *Operations Research* **22**(2) 340–349.

Glover, F. 1977. Heuristics for integer programming using surrogate constraints. *Decision Sciences* **8** 156–166.

Glover, F. 1986. Future paths for integer programming and links to artificial intelligence. *Computers & Operations Research* **13**(5) 533–549.

Glover, F. 1989. Tabu Search – Part I. *ORSA Journal on Computing* **1**(3) 190–206.

Glover, F. 1990. Tabu Search – Part II. *ORSA Journal on Computing* **2**(1) 4–32.

Glover, F. 1992. New ejection chain and alternating path methods for traveling salesman problems. O. Balci, R. Sharda, S. Zenios, eds., *Computer Science and Operations Research : New Developments in Their Interfaces*. Pergamon Press, 449–509.

Glover, F. 1996. Ejection chains, reference structures and alternating path methods for traveling salesman problems. *Discrete Applied Mathematics* **65**(1-3) 223–253.

Glover, F., J.-K. Hao. 2011. The case for strategic oscillation. *Annals of Operations Research* **183**(1) 163–173.

Glover, F., M. Laguna. 1998. *Tabu search*. Kluwer Academic Publishers, Boston, MA.

Glover, F.W., G.A. Kochenberger, eds. 2003. *Handbook of Metaheuristics*. Springer.

Goel, A. 2009. Vehicle Scheduling and Routing with Drivers' Working Hours. *Transportation Science* **43**(1) 17–26.

Goel, A. 2010. Truck driver scheduling in the European Union. *Transportation Science* **44**(4) 429–441.

Goel, A. 2012a. The Canadian minimum duration truck driver scheduling problem. *Computers & Operations Research* **39**(10) 2359–2367.

Goel, A. 2012b. Truck driver scheduling in the United States and the 2013 rule change. Tech. rep., Zaragoza Logistics Center.

Goel, A., C. Archetti, M. Savelsbergh. 2012. Truck driver scheduling in Australia. *Computers & Operations Research* **39**(5) 1122–1132.

Goel, A., V. Gruhn. 2008. A general vehicle routing problem. *European Journal of Operational Research* **191**(3) 650–660.

Goel, A., L. Kok. 2012. Truck driver scheduling in the United States. *Transportation Science* **46**(3) 317–326.

Goel, A., T. Vidal. 2012. Hours of service regulations in road freight transport : an optimization-based international assessment. Tech. rep., CIRRELT.

Goel, Asvin, L.M. Rousseau. 2011. Truck Driver Scheduling in Canada. *Journal of Scheduling, Articles in Advance* .

Goetschalckx, M., C. Jacobs-Blecha. 1989. The vehicle routing problem with backhauls. *European Journal of Operational Research* **42**(1) 39–51.

Goldberg, D.E., K. Deb. 1991. A comparative analysis of selection schemes used in genetic algorithms. *Foundations of Genetic Algorithms* 69–93.

Golden, B. 1984. The fleet size and mix vehicle routing problem. *Computers & Operations Research* **11**(1) 49–66.

Golden, B., S. Raghavan, E. Wasil, eds. 2008. *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, New York.

Golden, B.L., A.A. Assad. 1988. *Vehicle routing : methods and studies*. North-Holland Amsterdam.

Golden, B.L., A.A. Assad, E.A. Wasil. 2002. *Routing vehicles in the real world : applications in the solid waste, beverage, food, dairy, and newspaper industries*. SIAM, Philadelphia, PA, USA, 245–286.

Golden, B.L., E.A. Wasil. 1987. Computerized Vehicle Routing in the Soft Drink Industry. *Operations Research* **35**(1) 6–17.

Golden, B.L., E.A. Wasil, J.P. Kelly, I.M. Chao. 1998a. Metaheuristics in vehicle routing. T.G. Crainic, G. Laporte, eds., *Fleet Management and Logistics*. Kluwer Academic Publishers, Boston, MA, 33–56.

Golden, B.L., E.A. Wasil, J.P. Kelly, I.M. Chao. 1998b. The impact of metaheuristics on solving the vehicle routing problem : algorithms, problem sets, and computational results. T.G. Crainic, G. Laporte, eds., *Fleet management and logistics*. Kluwer Academic Publishers, Boston, MA, 33–56.

Goto, S., A. Sangiovanni-Vincentelli. 1978. A new shortest path updating algorithm. *Networks* **8**(4) 341–372.

Graham, R.L., E.L. Lawler, J.K. Lenstra, A.H.G Rinnooy Kan. 1979. Optimization and approximation in deterministic sequencing and scheduling : a survey. *Annals of Discrete Mathematics* **5** 287–326.

Groër, C., B. Golden, E. Wasil. 2008. The Consistent Vehicle Routing Problem. *Manufacturing & Service Operations Management* **11**(4) 630–643.

Groër, C., B. Golden, E. Wasil. 2010. A Library of Local Search Heuristics for the Vehicle Routing Problem. *Mathematical Programming Computation* **2**(2) 79–101.

Groër, C., B. Golden, E. Wasil. 2011. A Parallel Algorithm for the Vehicle Routing Problem. *INFORMS Journal on Computing* **23**(2) 315–330.

Grotschel, M., C.L. Monma, M. Stoer. 1995. Design of survivable networks. M. Ball, T. Magnanti, C. Monma, G. Nemhauser, eds., *Network Models*, *Handbooks in Operations Research and Management Science*, vol. 7. North-Holland Amsterdam, 617–672.

Grotzinger, S.J., C. Witzgall. 1984. Projections onto Order Simplexes. *Applied Mathematics and Optimization* **270**(12) 247–270.

Gschwind, T., S. Irnich. 2012. Effective Handling of Dynamic Time Windows and Synchronization with Precedences for Exact Vehicle Routing. Tech. rep., Johannes Gutenberg University, Mainz, Germany.

Gulczynski, D., B. Golden, E. Wasil. 2011. The period vehicle routing problem : New heuristics and real-world variants. *Transportation Research Part E : Logistics and Transportation Review* **47**(5) 648–668.

Gulczynski, D.J., B. Golden, E. Wasil. 2008. Recent Developments in Modeling and Solving the Split Delivery Vehicle Routing Problem. Z. Chen, S. Raghavan, eds., *Tutorials in Operations Research*. INFORMS, Hanover, MD, 170–180.

Haase, K., G. Desaulniers, J. Desrosiers. 2001. Simultaneous Vehicle and Crew Scheduling in Urban Mass Transit Systems. *Transportation Science* **35**(3) 286–303.

Hadjiconstantinou, E., R. Baldacci. 1998. A multi-depot period vehicle routing problem arising in the utilities sector. *Journal of the Operational Research Society* **49**(12) 1239–1248.

Halpern, J. 1977. Shortest route with time dependent length of edges and limited delay possibilities in nodes. *Zeitschrift für Operations Research* **21**(3) 117–124.

Hansen, N., A. Ostermeier. 2001. Completely Derandomized Self-Adaptation in Evolution Strategies. *Evolutionary Computation* **9**(2) 159–195.

Hansen, P., N. Mladenović, J.A. Moreno Pérez. 2010. Variable neighbourhood search : methods and applications. *Annals of Operations Research* **175**(1) 367–407.

Hartl, R.F., G. Hasle, G.K. Janssens. 2006. Special issue on Rich Vehicle Routing Problems. *Central European Journal of Operations Research* **14**(2) 103–104.

Hashimoto, H. 2008. Studies on Local Search-Based Approaches for Vehicle Routing and Scheduling Problems. Ph.D. thesis, Kyoto University.

Hashimoto, H., T. Ibaraki, S. Imahori, M. Yagiura. 2006. The vehicle routing problem with flexible time windows and traveling times. *Discrete Applied Mathematics* **154**(16) 2271–2290.

Hashimoto, H., M. Yagiura. 2008. A Path Relinking Approach with an Adaptive Mechanism to Control Parameters for the Vehicle Routing Problem with Time Windows. J. Hemert, C. Cotta, eds., *Evolutionary Computation in Combinatorial Optimization*, *LNCS*, vol. 4972. Springer Berlin Heidelberg, 254–265.

Hashimoto, H., M. Yagiura, T. Ibaraki. 2008. An iterated local search algorithm for the time-dependent vehicle routing problem with time windows. *Discrete Optimization* **5**(2) 434–456.

Hashimoto, H., M. Yagiura, S. Imahori, T. Ibaraki. 2010. Recent progress of local search in handling the time window constraints of the vehicle routing problem. *4OR* **8**(3) 221–238.

Haugland, D., S. Ho. 2010. Feasibility Testing for Dial-a-Ride Problems. B. Chen, ed., *Algorithmic Aspects in Information and Management*, *LNCS*, vol. 6124. Springer Berlin / Heidelberg, 170–179.

Health and Safety Executive. 2006. http ://www.hse.gov.uk/research/rrhtm/rr446.htm.

Hemmelmayr, V.C., J.-F. Cordeau, T.G. Crainic. 2012. An adaptive large neighborhood search heuristic for Two-Echelon Vehicle Routing Problems arising in city logistics. *Computers & Operations Research* **39**(12) 3215–3228.

Hemmelmayr, V.C., K.F. Doerner, R.F. Hartl. 2009. A variable neighborhood search heuristic for periodic routing problems. *European Journal of Operational Research* **195**(3) 791–802.

Hendel, Y., F. Sourd. 2006. Efficient neighborhood search for the one-machine earliness-tardiness scheduling problem. *European Journal of Operational Research* **173**(1) 108–119.

Hendel, Y., F. Sourd. 2007. An improved earliness-tardiness timing algorithm. *Computers & Operations Research* **34**(10) 2931–2938.

Ho, S.C., M. Gendreau. 2006. Path relinking for the vehicle routing problem. *Journal of Heuristics* **12**(1-2) 55–72.

Hochbaum, D. 2002. Solving integer programs over monotone inequalities in three variables : A framework for half integrality and good approximations. *European Journal of Operational Research* **140**(2) 291–321.

Hochbaum, D.S. 1994. Lower and upper bounds for the allocation problem and other nonlinear optimization problems. *Mathematics of Operations Research* **19**(2) 390–409.

Hochbaum, D.S., J.G. Shanthikumar. 1990. Convex separable optimization is not much harder than linear optimization. *Journal of the ACM (JACM)* **37**(4) 843–862.

Hoff, A., H. Andersson, M. Christiansen, G. Hasle, A. Lø kketangen. 2010. Industrial aspects and literature survey : Fleet composition and routing. *Computers & Operations Research* **37**(12) 2041–2061.

Holland, J.H. 1975. *Adaptation in natural and artificial systems. an introductory analysis with applications to biology, control and artificial intelligence*. Ann Arbor : The University of Michigan Press.

Homberger, J., H. Gehring. 1999. Two evolutionary metaheuristics for the vehicle routing problem with time windows. *INFOR* **37**(3) 297–318.

Homberger, J., H. Gehring. 2005. A two-phase hybrid metaheuristic for the vehicle routing problem with time windows. *European Journal Of Operational Research* **162**(1) 220–238.

Hoogeveen, J.A., S.L. Van De Velde. 1996. A branch-and-bound algorithm for single-machine earliness-tardiness scheduling with idle time. *INFORMS Journal on Computing* **8**(4) 402–412.

Hoos, H.H., T. Stützle. 2005. *Stochastic local search : Foundations and applications*. Morgan Kaufmann.

Huisman, Dennis, Richard Freling, Albert P. M. Wagelmans. 2005. Multiple-Depot Integrated Vehicle and Crew Scheduling. *Transportation Science* **39**(4) 491–502.

Hunsaker, B., M.W.P. Savelsbergh. 2002. Efficient feasibility testing for dial-a-ride problems. *Operations Research Letters* **30**(3) 169–173.

Hurink, J., J. Keuchel. 2001. Local search algorithms for a single-machine scheduling problem with positive and negative time-lags. *Discrete Applied Mathematics* **112**(1-3) 179–197.

Ibaraki, T., S. Imahori, M. Kubo, T. Masuda, T. Uno, M. Yagiura. 2005. Effective Local Search Algorithms for Routing and Scheduling Problems with General Time-Window Constraints. *Transportation Science* **39**(2) 206–232.

Ibaraki, T., S. Imahori, K. Nonobe, K. Sobue, T. Uno, M. Yagiura. 2008. An iterated local search algorithm for the vehicle routing problem with convex time penalty functions. *Discrete Applied Mathematics* **156**(11) 2050–2069.

Ibaraki, T., N. Katoh. 1988. *Resource allocation problems : algorithmic approaches*. MIT Press, Cambridge, MA, USA.

Ichoua, S., M. Gendreau, J.Y. Potvin. 2003. Vehicle dispatching with time-dependent travel times. *European Journal of Operational Research* **144**(2) 379–396.

Imran, A., S. Salhi, N.A. Wassan. 2009. A variable neighborhood-based heuristic for the heterogeneous fleet vehicle routing problem. *European Journal of Operational Research* **197**(2) 509–518.

Ioachim, I., S. Gélinas, F. Soumis, J. Desrosiers. 1998. A dynamic programming algorithm for the shortest path problem with time windows and linear node costs. *Networks* **31**(3) 193–204.

Iori, M., S. Martello. 2010. Routing problems with loading constraints. *Top* **18**(1) 4–27.

Irnich, S. 2000. A multi-depot pickup and delivery problem with a single hub and heterogeneous vehicles. *European Journal of Operational Research* **122**(2) 310–328.

Irnich, S. 2008a. A Unified Modeling and Solution Framework for Vehicle Routing and Local Search-Based Metaheuristics. *INFORMS Journal on Computing* **20**(2) 270–287.

Irnich, S. 2008b. Resource extension functions : properties, inversion, and generalization to segments. *OR Spectrum* **30** 113–148.

Irnich, S., B. Funke, T. Grünert. 2006. Sequential search and its application to vehicle-routing problems. *Computers & Operations Research* **33**(8) 2405–2429.

Jang, W., H.H. Lim, T.J. Crowe, G. Raskin, T.E. Perkins. 2006. The Missouri Lottery Optimizes Its Scheduling and Routing to Improve Efficiency and Balance. *Interfaces* **36**(4) 302–313.

Jepsen, M., B. Petersen, S. Spoorendonk, D. Pisinger. 2008. Subset-Row Inequalities Applied to the Vehicle-Routing Problem with Time Windows. *Operations Research* **56**(2) 497–511.

Jin, J., T.G. Crainic, A. Lokketangen. 2011. A Guided Cooperative Parallel Tabu Search for the Capacitated Vehicle Routing Problem. *Proceedings of NIK'11*. 49–60.

Jin, J., T.G. Crainic, A. Lokketangen. 2012. A parallel multi-neighborhood cooperative Tabu search for Capacitated Vehicle Routing Problem. *European Journal of Operational Research* **222**(3) 441–451.

Johnson, D.S., L.A. McGeoch. 1997. The traveling salesman problem : A case study in local optimization. E.H.L. Aarts, J.K. Lenstra, eds., *Local search in Combinatorial Optimization*. Princeton Univ Pr, 215–310.

Jones, T. 1995. Fitness distance correlation as a measure of problem difficulty for genetic algorithms. *Proceedings ICGA'95*. Morgan Kaufmann, San Francisco, CA, USA, 184–192.

Kallehauge, B., J. Larsen, O. Madsen. 2006. Lagrangian duality applied to the vehicle routing problem with time windows. *Computers & Operations Research* **33**(5) 1464–1487.

Kallehauge, B., J. Larsen, O.B.G. Madsen, M.M. Solomon. 2005. Vehicle routing problem with time windows. G. Desaulniers, J. Desrosiers, M.M. Solomon, eds., *Column Generation*. Springer, New York, 67–98.

Kanet, J.J., V Sridharan. 2000. Scheduling with inserted idle time : problem taxonomy and literature review. *Operations Research* **48**(1) 99–110.

Kang, K.H., Y.H. Lee, B.K. Lee. 2005. An Exact Algorithm for Multi Depot and Multi Period Vehicle Scheduling Problem. *Computational Science and Its Applications – ICCSA 2005* **3483** 350–359.

Kansou, A., A. Yassine. 2010. New upper bounds for the multi-depot capacitated arc routing problem. *International Journal of Metaheuristics* **1**(1) 81–95.

Karmarkar, N. 1984. A New Polynomial-Time algorithm for Linear Programming. *Combinatorica* **4**(4) 373–395.

Karush, W. 1962. A general algorithm for the optimal distribution of effort. *Management Science* **9**(1) 50–72.

Ke, L., C. Archetti, Z. Feng. 2008. Ants can solve the team orienteering problem. *Computers & Industrial Engineering* **54**(3) 648–665.

Kedad-Sidhoum, S., F. Sourd. 2010. Fast neighborhood search for the single machine earliness-tardiness scheduling problem. *Computers & Operations Research* **37**(8) 1464–1471.

Kelley, J.E., M.R. Walker. 1959. Critical-path planning and scheduling. *Proceedings of Eastern joint Computer conference*. ACM Press, New York, New York, USA, 160–173.

Khachiyan, L.G. 1979. A polynomial algorithm in linear programming. *Soviet Mathematics Doklady* **20** 191–194.

Kilby, P., P. Prosser, P. Shaw. 1999. Guided local search for the vehicle routing problem with time windows. *Meta-Heuristics : Advances and Trends in Local Search Paradigms for Optimization*. Kluwer Academic Publishers, 321–361.

Kindervater, G.A.P., M.W.P. Savelsbergh. 1997. Vehicle routing : Handling edge exchanges. E.H.L. Aarts, J.K. Lenstra, eds., *Local Search in Combinatorial Optimization*. Princeton Univ Pr, 337–360.

Kirkpatrick, S., C.D. Gelatt, M.P. Vecchi. 1983. Optimization by simulated annealing. *Science* **220**(4598) 671–680.

Kodialam, M.S., H. Luss. 1998. Algorithms for separable nonlinear resource allocation problems. *Operations Research* **46**(2) 272–284.

Kohl, N., S. Karisch. 2004. Airline Crew Rostering : Problem Types , Modeling , and Optimization. *Annals of Operations Research* **127**(1) 223–257.

Kok, A.L., E.W. Hans, J.M.J. Schutten. 2009. Vehicle routing under time-dependent travel times : the impact of congestion avoidance. Tech. rep., University of Twente.

Kok, A.L., C.M. Meyer, H. Kopfer, J.M.J. Schutten. 2010. A dynamic programming heuristic for the vehicle routing problem with time windows and European Community social legislation. *Transportation Science* **44**(4) 442–454.

Koskosidis, Y A, W B Powell, M M Solomon. 1992. An Optimization-Based Heuristic for Vehicle Routing and Scheduling with Soft Time Window Constraints. *Transportation Science* **26**(2) 69–85.

Kovacs, A.A., S.N. Parragh, R.F. Hartl. 2012. A template based adaptive large neighborhood search for the consistent vehicle routing problem.

Kritzinger, S., F. Tricoire, K.F. Doerner, R.F. Hartl, T. Stützle. 2012. A Unified Framework for Routing Problems with a Fixed Fleet Size. Tech. rep., Johannes Kepler University, Linz, Austria.

Kubiak, M. 2007. Distance measures and fitness-distance analysis for the capacitated vehicle routing problem. *Metaheuristics : Progress in Complex Systems Optimization*, vol. 39. Springer, New York, 345–364.

Kytöjoki, J., T. Nuortio, O. Bräysy, M. Gendreau. 2007. An efficient variable neighborhood search heuristic for very large scale vehicle routing problems. *Computers & Operations Research* **34**(9) 2743–2757.

Labadi, N., C. Prins, M. Reghioui. 2008. A memetic algorithm for the vehicle routing problem with time windows. *RAIRO Operations Research* **42**(3) 415–431.

Lacomme, P., C. Prins, W. Ramdane-Cherif. 2005. Evolutionary algorithms for periodic arc routing problems. *European Journal of Operational Research* **165**(2) 535–553.

Lahrichi, N., T.G. Crainic, M. Gendreau, W. Rei, G.C. Crisan, T. Vidal. 2012. An Integrative Cooperative Search Framework for Multi-Decision-Attribute Combinatorial Optimization. Tech. rep., CIRRELT.

Laporte, G. 2009. Fifty Years of Vehicle Routing. *Transportation Science* **43**(4) 408–416.

Laporte, G., I.H. Osman. 1995. Routing problems : A bibliography. *Annals of Operations Research* **61**(1) 227–262.

Laporte, G., F. Semet. 2002. *Classical heuristics for the Capacitated VRP*. SIAM, Philadelphia, PA, USA, 109–128.

Lau, H.C.W., T.M. Chan, W.T. Tsui, W.K. Pang. 2010. Application of Genetic Algorithms to Solve the Multidepot Vehicle Routing Problem. *IEEE Transactions on Automation Science and Engineering* **7**(2) 383–392.

Laurent, B., J.-K. Hao. 2007. Simultaneous vehicle and driver scheduling : A case study in a limousine rental company. *Computers & Industrial Engineering* **53**(3) 542–558.

Le Bouthillier, A., T.G. Crainic. 2005a. A cooperative parallel meta-heuristic for the vehicle routing problem with time windows. *Computers & Operations Research* **32**(7) 1685–1708.

Le Bouthillier, A., T.G. Crainic. 2005b. A guided cooperative search for the vehicle routing problem with time windows. *Intelligent Systems, IEEE* **20**(4) 36–42.

Lee, C., M. Epelman, C. White III, Y.A. Bozer. 2006. A shortest path approach to the multiple-vehicle routing problem with split pick-ups. *Transportation Research Part B : Methodological* **40**(4) 265–284.

Lee, C.Y., J.Y. Choi. 1995. A genetic algorithm for job sequencing problems with distinct due dates and general early-tardy penalty weights. *Computers & Operations Research* **22**(8) 857–869.

Letchford, A.N., J. Lysgaard, R.W. Eglese. 2006. A branch-and-cut algorithm for the capacitated open vehicle routing problem. *Journal of the Operational Research Society* **58**(12) 1642–1651.

Li, F., B. Golden, E. Wasil. 2005. Very large-scale vehicle routing : new test problems, algorithms, and results. *Computers & Operations Research* **32**(5) 1165–1179.

Li, F., B. Golden, E. Wasil. 2007a. A record-to-record travel algorithm for solving the heterogeneous fleet vehicle routing problem. *Computers & Operations Research* **34**(9) 2734–2742.

Li, F., B. Golden, E. Wasil. 2007b. The open vehicle routing problem : Algorithms, large-scale test problems, and computational results. *Computers & Operations Research* **34**(10) 2918–2930.

Li, H., A. Lim. 2001. A metaheuristic for the pickup and delivery problem with time windows. *Proceedings of ICTAI'01*. IEEE Comput. Society, 160–167.

Lim, A., X. Zhang. 2007. A two-stage heuristic with ejection pools and generalized ejection chains for the vehicle routing problem with time windows. *INFORMS Journal on Computing* **19**(3) 443–57.

Lin, S. 1965. Computer solutions of the traveling salesman problem. *Bell System Technical Journal* **44**(10) 2245–2269.

Lin, S., B.W. Kernighan. 1973. An effective heuristic algorithm for the traveling-salesman problem. *Operations Research* **21**(2) 498–516.

Liu, F.-H., S.-Y. Shen. 1999. The fleet size and mix vehicle routing problem with time windows. *Journal of the Operational Research Society* **50**(7) 721–732.

Liu, S., W. Huang, H. Ma. 2009. An effective genetic algorithm for the fleet size and mix vehicle routing problems. *Transportation Research Part E : Logistics and Transportation Review* **45**(3) 434–445.

Lourenço, H.R., O.C. Martin, T. Stützle. 2010. Iterated Local Search : Framework and Applications. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer, Boston, MA, 363–397.

Luss, H., S.K. Gupta. 1975. Allocation of effort resources among competing activities. *Operations Research* **23**(2) 360–366.

Magnanti, T.L., R.T. Wong. 1984. Network design and transportation planning : Models and algorithms. *Transportation Science* **18**(1) 1–55.

Malandraki, C., M.S. Daskin. 1992. Time dependent vehicle routing problems : Formulations, properties and heuristic algorithms. *Transportation Science* **26**(3) 185–200.

Malandraki, C., R.B. Dial. 1996. A restricted dynamic programming heuristic algorithm for the time dependent traveling salesman problem. *European Journal of Operational Research* **90**(1) 45–55.

Malcolm, D.G., J.H. Roseboom, C.E. Clark, W. Fazar. 1959. Application of a technique for research and development program evaluation. *Operations Research* **7**(5) 646–669.

Marinakis, Y., M. Marinaki. 2010. A Hybrid Genetic - Particle Swarm Optimization Algorithm for the Vehicle Routing Problem. *Expert Systems with Applications* **37**(2) 1446–1455.

Marinakis, Y., M. Marinaki. 2011. Bumble Bees Mating Optimization Algorithm for the Vehicle Routing Problem. *Handbook of Swarm Intelligence*. Springer Berlin Heidelberg, 347–369.

Marinakis, Y., M. Marinaki, G. Dounias. 2010. A hybrid particle swarm optimization algorithm for the vehicle routing problem. *Engineering Applications of Artificial Intelligence* **23**(4) 463–472.

Marinakis, Y., A. Migdalas, P.M. Pardalos. 2006. A new bilevel formulation for the vehicle routing problem and a solution method using a genetic algorithm. *Journal of Global Optimization* **38**(4) 555–580.

Masutti, T. 2008. A neuro-immune algorithm to solve the capacitated vehicle routing problem. *Artificial Immune Systems*, *LNCS*, vol. 5132. Springer Berlin Heidelberg, 210–219.

Matos, A.C., R.C. Oliveira. 2004. *An Experimental Study of the Ant Colony System for the Period Vehicle Routing Problem*, *LNCS*, vol. 3172. Springer Berlin Heidelberg, 286–293.

McCartt, A.T., L.A. Hellinga, M.G. Solomon. 2008. Work schedules of long-distance truck drivers before and after 2004 hours-of-service rule change. *Traffic injury prevention* **9**(3) 201–10.

Mercer, R.E., J.R. Sampson. 1978. Adaptive search using a reproductive metaplan. *Kybernetes* **7**(3) 215–228.

Mercier, A., J.-F. Cordeau, F. Soumis. 2005. A computational study of Benders decomposition for the integrated aircraft routing and crew scheduling problem. *Computers & Operations Research* **32**(6) 1451–1476.

Mester, D., O. Bräysy. 2005. Active guided evolution strategies for large-scale vehicle routing problems with time windows. *Computers & Operations Research* **32**(6) 1593–1614.

Mester, D., O. Bräysy. 2007. Active-guided evolution strategies for large-scale capacitated vehicle routing problems. *Computers & Operations Research* **34**(10) 2964–2975.

Meyer, B. 1997. *Object-oriented software construction*. Prentice Hall, Englewood Cliffs, USA.

Miller-Hooks, E., B. Yang. 2005. Updating Paths in Time-Varying Networks Given Arc Weight Changes. *Transportation Science* **39**(4) 451–464.

Mingozzi, A. 2005. The Multi-depot Periodic Vehicle Routing Problem. *Lectures Notes in Computer Science* 347–350.

Mingozzi, A., S. Giorgi, R. Baldacci. 1999. An exact method for the vehicle routing problem with backhauls. *Transportation Science* **33**(3) 315–329.

Mingozzi, A, R Roberti, P Toth. 2012. An exact algorithm for the multi-trip vehicle routing problem. *INFORMS Journal on Computing, Articles in Advance* .

Ministère de l'écologie – du développement durable – des transports et du logement – France. 2007. Le Compte des Transports. Tech. Rep. tome 1, Ministère De L'Ecologie, Du Developpement Durable, Des Transports Et Du Logement.

Ministère de l'écologie – du développement durable – des transports et du logement – France. 2010. L'Environnement en France. Tech. rep.

Mitten, L.G. 1959. Sequencing n Jobs on Two Machines with Arbitrary Time Lags. *Management Science* **5**(3) 293–298.

Mladenović, N., P. Hansen. 1997. Variable neighborhood search. *Computers & Operations Research* **24**(11) 1097–1100.

Moccia, L., J.-F. Cordeau, G. Laporte. 2012. An incremental tabu search heuristic for the generalized vehicle routing problem with time windows. *Journal of the Operational Research Society* **63**(2) 232–244.

Mole, R.H., S.R. Jameson. 1976. A Sequential Route-Building Algorithm Employing a Generalised Savings Criterion. *Operational Research Quarterly* **27**(2) 503–511.

Montané, F.A.T., R.D. Galvão. 2006. A tabu search algorithm for the vehicle routing problem with simultaneous pick-up and delivery service. *Computers & Operations Research* **33**(3) 595–619.

Moscato, P. 1989. On Evolution, Search, Optimization, Genetic Algorithms and Martial Arts : Towards Memetic Algorithms. Tech. rep., California Institute of Technology, Pasadena, California USA.

Moscato, P., C. Cotta. 2010. A Modern Introduction to Memetic Algorithms. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*, vol. 146. Springer, Boston, MA, 141–183.

Mota, E., V. Campos, Á. Corberán. 2007. A new metaheuristic for the vehicle routing problem with split demands. *Evolutionary Computation in Combinatorial Optimization*, *LNCS*, vol. 4446. Springer Berlin Heidelberg, 121–129.

Mühlenbein, H., M. Gorges-Schleuter, O. Krämer. 1988. Evolution algorithms in combinatorial optimization. *Parallel Computing* **7**(1) 65–85.

Nagata, Y. 2007. Effective memetic algorithm for the vehicle routing problem with time windows : Edge assembly crossover for the VRPTW. *Proceedings of the Seventh Metaheuristics International Conference, Montreal, Canada (on CD-ROM)*.

Nagata, Y., O. Bräysy. 2008. Efficient local search limitation strategies for vehicle routing problems. J. Hemert, C. Cotta, eds., *Evolutionary Computation in Combinatorial Optimization*, *LNCS*, vol. 4972. Springer Berlin Heidelberg, 48–60.

Nagata, Y., O. Bräysy. 2009a. A powerful route minimization heuristic for the vehicle routing problem with time windows. *Operations Research Letters* **37**(5) 333–338.

Nagata, Y., O. Bräysy. 2009b. Edge Assembly-Based Memetic Algorithm for the Capacitated Vehicle Routing Problem. *Networks* **54**(4) 205–215.

Nagata, Y., O. Bräysy, W. Dullaert. 2010. A penalty-based edge assembly memetic algorithm for the vehicle routing problem with time windows. *Computers & Operations Research* **37**(4) 724–737.

Nagata, Y., S. Kobayashi. 2011. A memetic algorithm for the pickup and delivery problem with time windows using selective route exchange crossover. *Proceedings of PPSN'11*, *LNCS*, vol. 6238. Springer Berlin Heidelberg, 536–545.

Nagy, G., S. Salhi. 2007. Location-routing : Issues, models and methods. *European Journal of Operational Research* **177**(2) 649–672.

Nannen, V., A.E. Eiben. 2007. Efficient relevance estimation and value calibration of evolutionary algorithm parameters. *Proceedings of CEC'07*. IEEE, 103–110.

National Transport Commission. 2008a. Heavy Vehicle Driver Fatique Reform – Advanced Fatigue Management explained. Information Bulletin.

National Transport Commission. 2008b. Heavy Vehicle Driver Fatique Reform – Basic Fatigue Management explained. Information Bulletin.

National Transport Commission. 2008c. Heavy Vehicle Driver Fatique Reform – Standard Hours explained. Information Bulletin.

Newton, R.M., W.H. Thomas. 1974. Bus routing in a multi-school system. *Computers & Operations Research* **1**(2) 213–222.

Ngueveu, S.U., C. Prins, R. Wolfler Calvo. 2010. An effective memetic algorithm for the cumulative capacitated vehicle routing problem. *Computers & Operations Research* **37**(11) 1877–1885.

Nguyen, P.K., T.G. Crainic, M. Toulouse. 2011. A Hybrid Genetic Algorithm for the Periodic Vehicle Routing Problem with Time Windows. Tech. rep., CIRRELT.

Norstad, I., K. Fagerholt, G. Laporte. 2010. Tramp ship routing and scheduling with speed optimization. *Transportation Research Part C : Emerging Technologies* .

Olivera, A., O. Viera. 2007. Adaptive memory programming for the vehicle routing problem with multiple trips. *Computers & Operations Research* **34**(1) 28–47.

Ombuki-Berman, B., F. Hanshar. 2009. Using genetic algorithms for multi-depot vehicle routing. F.B. Pereira, J. Tavares, eds., *Bio-inspired Algorithms for the Vehicle Routing Problem*. Springer, 77–99.

Or, I. 1976. Traveling salesman-type combinatorial problems and their relation to the logistics of regional blood banking. Ph.D. thesis, Northwestern University, Evanston, IL.

Osman, I.H. 1993. Metastrategy simulated annealing and tabu search algorithms for the vehicle routing problem. *Annals of Operations Research* **41**(1-4) 421–451.

Osman, I.H., G. Laporte. 1996. Metaheuristics : A bibliography. *Annals of Operations Research* **63**(5) 511–623.

Ostertag, A. 2008. Decomposition strategies for large scale multi depot vehicle routing problems. Ph.D. thesis, Universität Wien.

Pallottino, S. 2003. A new algorithm for reoptimizing shortest paths when the arc costs change. *Operations Research Letters* **31**(2) 149–160.

Pan, Y., L. Shi. 2005. Dual Constrained Single Machine Sequencing to Minimize Total Weighted Completion Time. *IEEE Transactions on Automation Science and Engineering* **2**(4) 344–357.

Pardalos, P. 1995. Efficient computation of an isotonic median regression. *Applied Mathematics Letters* **8**(2) 67–70.

Pardalos, P.M., G Xue. 1999. Algorithms for a class of isotonic regression problems. *Algorithmica* **23**(3) 211–222.

Parragh, S.N., J.-F. Cordeau, K.F. Doerner, R.F. Hartl. 2012. Models and algorithms for the heterogeneous dial-a-ride problem with driver-related constraints. *OR Spectrum* **34**(3) 593–633.

Parragh, S.N., K.F. Doerner, R.F. Hartl. 2008a. A survey on pickup and delivery problems Part I : Transportation between customers and depot. *Journal für Betriebswirtschaft* **58**(1) 21–51.

Parragh, S.N., K.F. Doerner, R.F. Hartl. 2008b. A survey on pickup and delivery problems Part II : Transportation between pickup and delivery locations. *Journal für Betriebswirtschaft* **58**(2) 81–117.

Parragh, S.N., K.F. Doerner, R.F. Hartl. 2010. Variable neighborhood search for the dial-a-ride problem. *Computers & Operations Research* **37**(6) 1129–1138.

Parthanadee, P., R. Logendran. 2006. Periodic product distribution from multi-depots under limited supplies. *IIE Transactions* **38**(11) 1009–1026.

Patriksson, M. 2008. A survey on the continuous nonlinear resource allocation problem. *European Journal of Operational Research* **185**(1) 1–46.

Penna, P.H.V., A. Subramanian, L.S. Ochi. 2011. An Iterated Local Search heuristic for the Heterogeneous Fleet Vehicle Routing Problem. *Journal of Heuristics, forthcoming* .

Perboli, G., F. Pezzella, R. Tadei. 2008. EVE-OPT : a hybrid algorithm for the capacitated vehicle routing problem. *Mathematical Methods of Operations Research* **68**(2) 361–382.

Pessoa, A., M.P. de Aragão, E. Uchoa. 2008. Robust branch-cut-and-price algorithms for vehicle routing problems. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, Boston, MA, 297–325.

Pinedo, M. 2008. *Scheduling : theory, algorithms, and systems*. Prentice Hall.

Pirkwieser, S., G.R. Raidl. 2008. A variable neighborhood search for the periodic vehicle routing problem with time windows. *Proceedings of the 9th EU Meeting on Metaheuristics for Logistics and Vehicle Routing, Troyes, France* .

Pirkwieser, S., G.R. Raidl. 2009a. A Column Generation Approach for the Periodic Vehicle Routing Problem with Time Windows. *Proceedings of the International Network Optimization Conference 2009*. Pisa, Italy.

Pirkwieser, S., G.R. Raidl. 2009b. Multiple Variable Neighborhood Search Enriched with ILP Techniques for the Periodic Vehicle Routing Problem with Time Windows. M. Blesa, C. Blum, L. Di Gaspero, A. Roli, M. Sampels, A. Schaerf, eds., *Hybrid Metaheuristics*, *LNCS*, vol. 5818. Springer Berlin / Heidelberg, 45–59.

Pirkwieser, S., G.R. Raidl. 2010a. Multilevel variable neighborhood search for periodic routing problems. P. Cowling, P. Merz, eds., *Evolutionary Computation in Combinatorial Optimisation*, *LNCS*, vol. 6022. Springer Berlin / Heidelberg, 226–238.

Pirkwieser, S., R. Raidl. 2010b. Matheuristics for the Periodic Vehicle Routing Problem with Time Windows. *Proceedings of the 3rd International Workshop on model-based metaheuristics (Metaheuristics 2010)*. Vienna, Austria.

Pisinger, D., S. Ropke. 2007. A general heuristic for vehicle routing problems. *Computers & Operations Research* **34**(8) 2403–2435.

Pisinger, D., S. Ropke. 2010. Large neighborhood search. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer, Boston, MA, 399–419.

Polacek, M., S. Benkner, K.F. Doerner, R.F. Hartl. 2008. A cooperative and adaptive variable neighborhood search for the multi depot vehicle routing problem with time windows. *BuR-Business Research.* **1**(2) 207–218.

Polacek, M., R.F. Hartl, K. Doerner, M. Reimann. 2004. A Variable Neighborhood Search for the Multi Depot Vehicle Routing Problem with Time Windows. *Journal of Heuristics* **10**(6) 613–627.

Potts, C.N., J.D. Whitehead. 2007. Heuristics for a coupled-operation scheduling problem. *Journal of the Operational Research Society* **58**(10) 1375–1388.

Potvin, J.-Y. 2009. State-of-the Art Review : Evolutionary Algorithms for Vehicle Routing. *INFORMS Journal on Computing* **21**(4) 518–548.

Potvin, J.-Y., J.-M. Rousseau. 1995. An Exchange Heuristic for Routeing Problems with Time Windows. *Journal of the Operational Research Society* **46**(12) 1433–1446.

Prescott-Gagnon, E., G. Desaulniers, M. Drexl, L.-M. Rousseau. 2010. European Driver Rules in Vehicle Routing with Time Windows. *Transportation Science* **44**(4) 455–473.

Prescott-Gagnon, E., G. Desaulniers, L.M. Rousseau. 2009. A branch-and-price-based large neighborhood search algorithm for the vehicle routing problem with time windows. *Networks* **54**(4) 190–204.

Prins, C. 2004. A Simple and Effective Evolutionary Algorithm for the Vehicle Routing Problem. *Computers & Operations Research* **31**(12) 1985–2002.

Prins, C. 2009a. A GRASP - Evolutionary Local Search Hybrid for the Vehicle Routing Problem. F.B. Pereira, J. Tavares, eds., *Bio-Inspired Algorithms for the Vehicle Routing Problem*. Springer Berlin Heidelberg, 35–53.

Prins, C. 2009b. Two memetic algorithms for heterogeneous fleet vehicle routing problems. *Engineering Applications of Artificial Intelligence* **22**(6) 916–928.

Prins, C., S. Bouchenoua. 2005. A memetic algorithm solving the VRP, the CARP, and more general routing problems with nodes, edges and arcs. W. Hart, N. Krasnogor, J. Smith, eds., *Recent advances in memetic algorithms*. Studies in Fuzziness and Soft Computing, Springer, 65–85.

Prins, C., N. Labadi, M. Reghioui. 2009. Tour splitting algorithms for vehicle routing problems. *International Journal of Production Research* **47**(2) 507–535.

Privé, J., J. Renaud, F. Boctor, G. Laporte. 2005. Solving a vehicle-routing problem arising in soft-drink distribution. *Journal of the Operational Research Society* **57**(9) 1045–1052.

Puranen, T. 2011. Metaheuristics Meet Metamodels - A Modeling Language and a Product Line Architecture for Route Optimization Systems. Ph.D. thesis, University of Jyväskylä, Finland.

Raidl, R., J. Puchinger, C. Blum. 2010. Metaheuristic Hybrids. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer US, Boston, MA, 469–496.

Rancourt, M.È., J.-F. Cordeau, G. Laporte. 2012. Long-Haul Vehicle Routing and Scheduling with Working Hour Rules. *Transportation Science, Articles in Advance* .

Reeves, Colin R. 1998. Fitness Landscapes. E.K. Burke, G. Kendall, eds., *Search methodologies : introductory tutorials in optimization and decision support techniques*. Springer, Berlin, 587–610.

Rego, C. 2001. Node-ejection chains for the vehicle routing problem : Sequential and parallel algorithms. *Parallel Computing* **27**(3) 201–222.

Reimann, M., K.F. Doerner, R.F. Hartl. 2004. D-Ants : Savings Based Ants divide and conquer the vehicle routing problem. *Computers & Operations Research* **31**(4) 563–591.

Repoussis, P.P., C.D. Tarantilis. 2010. Solving the Fleet Size and Mix Vehicle Routing Problem with Time Windows via Adaptive Memory Programming. *Transportation Research Part C : Emerging Technologies* **18**(5) 695–712.

Repoussis, P.P., C.D. Tarantilis, O. Bräysy, G. Ioannou. 2010. A hybrid evolution strategy for the open vehicle routing problem. *Computers & Operations Research* **37**(3) 443–455.

Repoussis, P.P., C.D. Tarantilis, G. Ioannou. 2009a. An Evolutionary Algorithm for the Open Vehicle Routing Problem with Time Windows. F.B. Pereira, J. Tavares, eds., *Bio-inspired Algorithms for the Vehicle Routing Problem*. Studies in Computational Intelligence, Springer, 55–75.

Repoussis, P.P., C.D. Tarantilis, G. Ioannou. 2009b. Arc-Guided Evolutionary Algorithm for the Vehicle Routing Problem With Time Windows. *IEEE Transactions on Evolutionary Computation* **13**(3) 624–647.

Resende, M.G.C., C.C. Ribeiro, F. Glover, R. Marti. 2010. Scatter search and path-relinking : Fundamentals, advances, and applications. M. Gendreau, J.-Y. Potvin, eds., *Handbook of Metaheuristics*. Springer US, Boston, MA, 87–107.

Ribeiro, G.M., G. Laporte. 2012. An Adaptive Large Neighborhood Search Heuristic for the Cumulative Capacitated Vehicle Routing Problem. *Computers & Operations Research* **39**(3) 728–735.

Righini, G, M Salani. 2006. Symmetry helps : Bounded bi-directional dynamic programming for the elementary shortest path problem with resource constraints. *Discrete Optimization* **3**(3) 255–273.

Robertson, T., F.T. Wright, R.L. Dykstra. 1988. *Order Restricted Statistical Inference*. Wiley Series in Probability and Statistics, John Wiley & Sons, New York.

Rochat, Y., E.D. Taillard. 1995. Probabilistic Diversification and Intensification in Local Search for Vehicle Routing. *Journal of Heuristics* **1**(1) 147–167.

Rockafellar, R.T. 1970. *Convex analysis*. Princeton Univ Press.

Ronen, D. 1988. Perspectives on practical aspects of truck routing and scheduling. *European Journal of Operational Research* **35**(2) 137–145.

Ropke, S., J.F. Cordeau, G. Laporte. 2007. Models and branch-and-cut algorithms for pickup and delivery problems with time windows. *Networks* **49**(4) 258–272.

Ropke, S., D. Pisinger. 2006a. A unified heuristic for a large class of Vehicle Routing Problems with Backhauls. *European Journal of Operational Research* **171**(3) 750–775.

Ropke, S., D. Pisinger. 2006b. An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows. *Transportation Science* **40**(4) 455–472.

Roy, B. 1959. Contribution de la théorie des graphes à l'étude de certains problèmes linéaires. *Comptes rendus de l'académie des sciences de Paris* **248** 2437–2439.

Roy, B. 1962. Graphes et ordonnancement. *Revue Française de Recherche Opérationnelle* **25** 323–333.

Russell, R., W. Igo. 1979. An assignment routing problem. *Networks* **9**(1) 1–17.

Sahoo, S., S. Kim, B.-I. Kim, B. Kraas, A. Popov Jr. 2005. Routing Optimization for Waste Management. *Interfaces* **35**(1) 24–36.

Salari, M., P. Toth, A. Tramontani. 2010. An ILP improvement procedure for the Open Vehicle Routing Problem. *Computers & Operations Research* **37**(12) 2106–2120.

Salhi, S., G. Nagy. 1999. A cluster insertion heuristic for single and multiple depot vehicle routing problems with backhauling. *Journal of the Operational Research Society* **50**(10) 1034–1042.

Salhi, S., R. J. Petch. 2007. A GA Based Heuristic for the Vehicle Routing Problem with Multiple Trips. *Journal of Mathematical Modelling and Algorithms* **6**(4) 591–613.

Salhi, S., M. Sari. 1997. A multi-level composite heuristic for the multi-depot vehicle fleet mix problem. *European Journal of Operational Research* **103**(1) 95–112.

Sanathanan, L. 1971. On an allocation problem with multistage constraints. *Operations Research* **19**(7) 1647–1663.

Sandhu, R., D. Klabjan. 2007. Integrated Airline Fleeting and Crew-Pairing Decisions. *Operations Research* **55**(3) 439–456.

Sarvanov, V.I., N.N. Doroshko. 1981. The approximate solution of the travelling salesman problem by a local algorithm with scanning neighborhoods of factorial cardinality in cubic time (in Russian). *Software : Algorithms and Programs 31, Mathematical Institute of the Belorussian Academy of Sciences, Minsk* 11–13.

Savelsbergh, M.W.P. 1985. Local Search in Routing Problems with Time Windows. *Annals of Operations Research* **4**(1) 285–305.

Savelsbergh, M.W.P. 1992. The Vehicle Routing Problem with Time Windows : Minimizing Route Duration. *ORSA Journal on Computing* **4**(2) 146–154.

Schulz, C. 2011. Efficient Local Search on the GPU Investigations on the Vehicle Routing Problem. *SINTEF Report 2011-05-24* .

Sexton, T.R., L.D. Bodin. 1985a. Optimizing single vehicle many-to-many operations with desired delivery times : I. Scheduling. *Transportation Science* **19**(4) 378–410.

Sexton, T.R., L.D. Bodin. 1985b. Optimizing single vehicle many-to-many operations with desired delivery times : II. Routing. *Transportation Science* **19**(4) 411–435.

Sexton, T.R., Y.-M. Choi. 1986. Pickup and delivery of partial loads with "soft" time windows. *American Journal of Mathematical and Management Sciences* **6**(3-4) 369–398.

Shaw, P. 1998. *Using constraint programming and local search methods to solve vehicle routing problems*, *LNCS*, vol. 1520. Springer Berlin Heidelberg, 417–431.

Silva, M.M., A. Subramanian, T. Vidal, L.S. Ochi. 2012. A simple and effective metaheuristic for the Minimum Latency Problem. *European Journal of Operational Research* **221**(3) 513–520.

Smit, S.K., A.E. Eiben. 2009. Comparing Parameter Tuning Methods for Evolutionary Algorithms. *Proceedings of the 2009 IEEE Congress on Evolutionary Computation*. IEEE Press, 399–406.

Solomon, M.M. 1987. Algorithms for the vehicle routing and scheduling problems with time window constraints. *Operations Research* **35**(2) 254–265.

Solomon, M.M., J. Desrosiers. 1988. Time Window Constrained Routing and Scheduling Problems. *Transportation Science* **22**(1) 1–13.

Sörensen, K., M. Sevaux. 2006. MAPM : memetic algorithms with population management. *Computers & Operations Research* **33**(5) 1214–1225.

Souffriau, W., P. Vansteenwegen, G. Vanden Berghe, D. Van Oudheusden. 2010. A Path Relinking approach for the Team Orienteering Problem. *Computers & Operations Research* **37**(11) 1853–1859.

Sourd, F. 2005. Optimal timing of a sequence of tasks with general completion costs. *European Journal of Operational Research* **165**(1) 82–96.

Sourd, F., S. Kedad-Sidhoum. 2003. The one-machine problem with earliness and tardiness penalties. *Journal of Scheduling* **6**(6) 533–549.

Spencer, M.B., K.A. Robertson, S. Folkard. 2006. The development of a fatigue / risk index for shiftworkers. Tech. rep., Health and Safety Executive, UK.

Srikantan, K.S. 1963. A problem in optimum allocation. *Operations Research* **11**(2) 265–273.

Subramanian, A. 2012. Heuristic, Exact and Hybrid Approaches for Vehicle Routing Problems. Ph.D. thesis, Universidade Federal Fluminense, Niterói, Brazil.

Subramanian, A., L.M.A. Drummond, C. Bentes, L.S. Ochi, R. Farias. 2010. A parallel heuristic for the Vehicle Routing Problem with Simultaneous Pickup and Delivery. *Computers & Operations Research* **37**(11) 1899–1911.

Subramanian, A., P.H.V. Penna, E. Uchoa, L.S. Ochi. 2012. A Hybrid Algorithm for the Heterogeneous Fleet Vehicle Routing Problem. *European Journal of Operational Research* **221**(2) 285–295.

Szwarc, W., S.K. Mukhopadhyay. 1995. Optimal timing schedules in earliness-tardiness single machine sequencing. *Naval Research Logistics* **42**(7) 1109–1114.

Tagmouti, M., M. Gendreau, J.-Y. Potvin. 2007. Arc routing problems with time-dependent service costs. *European Journal of Operational Research* **181**(1) 30–39.

Taillard, E. 1993. Parallel iterative search methods for vehicle routing problems. *Networks* **23**(8) 661–673.

Taillard, E. 1999. A heuristic column generation method for the heterogeneous fleet VRP. *RAIRO Operations Research* **33**(1) 1–14.

Taillard, E., P. Badeau, M. Gendreau, F. Guertin, J.-Y. Potvin. 1997. A tabu search heuristic for the vehicle routing problem with soft time windows. *Transportation Science* **31**(2) 170–186.

Taillard, E., G. Laporte, M. Gendreau. 1996. Vehicle Routeing with Multiple Use of Vehicles. *Journal of the Operational Research Society* **47**(8) 1065.

Talbot, F.B. 1982. Resource-Constrained Project Scheduling with Time-Resource Tradeoffs : The Non-preemptive Case. *Management Science* **28**(10) 1197–1210.

Tamashiro, H., M. Nakamura, T. Okazaki, D. Kang. 2010. A Tabu Search Approach combined with An Extended Saving Method for Multi-depot Vehicle Routing Problems with Time Windows. *Soft Computing* **15**(1) 31–39.

Tang, J., Y. Kong, H. Lau, A.W.H. Ip. 2010. A note on "Efficient feasibility testing for dial-a-ride problems". *Operations Research Letters* **38**(5) 405–407.

Tarantilis, C.D. 2005. Solving the vehicle routing problem with adaptive memory programming methodology. *Computers & Operations Research* **32**(9) 2309–2327.

Tarantilis, C.D., F. Stavropoulou, P.P. Repoussis. 2012. A template-based Tabu Search algorithm for the Consistent Vehicle Routing Problem. *Expert Systems with Applications* **39**(4) 4233–4239.

Tarantilis, C.D., E.E. Zachariadis, C.T. Kiranoudis. 2007. A guided tabu search for the heterogeneous vehicle routeing problem. *Journal of the Operational Research Society* **59**(12) 1659–1673.

Teixeira, J., A.P. Antunes, J.P. De Sousa. 2004. Recyclable waste collection planning - a case study. *European Journal of Operational Research* **158**(3) 543–554.

Thangiah, S.R., S. Salhi. 2001. Genetic clustering : an adaptive heuristic for the multidepot vehicle routing problem. *Applied Artificial Intelligence* **15**(4) 361–383.

Thompson, P.M., H.N. Psaraftis. 1993. Cyclic transfer algorithms for multivehicle routing and scheduling problems. *Operations Research* **41**(5) 935–946.

Toth, P. 2008. An integer linear programming local search for capacitated vehicle routing problems. B. Golden, S. Raghavan, E. Wasil, eds., *The Vehicle Routing Problem : Latest Advances and New Challenges*. Springer, New York, 275–295.

Toth, P., D. Vigo. 1997. An exact algorithm for the vehicle routing problem with backhauls. *Transportation Science* **31**(4) 372–385.

Toth, P., D. Vigo, eds. 2002a. *The vehicle routing problem*. Society for Industrial Mathematics, Philadelphia, PA, USA.

Toth, P., D. Vigo. 2002b. VRP with Backhauls. P. Toth, D. Vigo, eds., *The Vehicle Routing Problem*. SIAM, Philadelphia, PA, USA, 195–224.

Toth, P., D. Vigo. 2003. The Granular Tabu Search and Its Application to the Vehicle-Routing Problem. *INFORMS Journal on Computing* **15**(4) 333–346.

Toulouse, M., T.G. Crainic, M. Gendreau. 1996. Communication Issues in Designing Cooperative Multi-Thread Parallel Searches. I.H. Osman, J.P. Kelly, eds., *Meta-heuristics : Theory & Applications*. Kluwer Academic Publishers, Norwell, MA, 501–522.

Transport Canada. 2005. Commercial Vehicle Drivers Hours of Service Regulations. SOR/2005-313.

Tricoire, F., M. Romauch, K.F. Doerner, R.F. Hartl. 2010. Heuristics for the multi-period orienteering problem with multiple time windows. *Computers & Operations Research* **37**(2) 351–367.

Vakhutinsky, A.I., B.L. Golden. 1994. Solving vehicle routing problems using elastic nets. *Proceedings of ICNN'94* **7**(2) 4535–4540.

Valouxis, C., E. Housos. 2002. Combined bus and driver scheduling. *Computers & Operations Research* **29**(3) 243–259.

Van Breedam, A. 1995. Improvement heuristics for the vehicle routing problem based on simulated annealing. *European Journal of Operational Research* **86**(3) 480–490.

Van Woensel, T., L. Kerbache, H. Peremans, N. Vandaele. 2008. Vehicle routing with dynamic travel times : A queueing approach. *European Journal of Operational Research* **186**(3) 990–1007.

Vansteenwegen, P., W. Souffriau, D.V. Oudheusden. 2010. The orienteering problem : A survey. *European Journal of Operational Research* **209**(1) 1–10.

Černý, V. 1985. Thermodynamical approach to the traveling salesman problem : An efficient simulation algorithm. *Journal of optimization theory and applications* **45**(1) 41–51.

Vickers, D, T Mayo, M Heitmann, M Lee, P Hughes. 2004. Intelligence and individual differences in performance on three types of visually presented optimisation problems. *Personality and Individual Differences* **36**(5) 1059–1071.

Vidal, T., T.G. Crainic, M. Gendreau, N. Lahrichi, W. Rei. 2012a. A Hybrid Genetic Algorithm for Multi-Depot and Periodic Vehicle Routing Problems. *Operations Research* **60**(3) 611–624.

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2011. A Unifying View on Timing Problems and Algorithms. Tech. rep., CIRRELT.

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2012b. A Unified Solution Framework for Multi-Attribute Vehicle Routing Problems. Tech. rep., CIRRELT.

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2012c. Heuristics for Multi-Attribute Vehicle Routing Problems : A Survey and Synthesis. *European Journal of Operational Research, Forthcoming* .

Vidal, T., T.G. Crainic, M. Gendreau, C. Prins. 2013. A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time-windows. *Computers & Operations Research* **40**(1) 475–489.

Villegas, J.G., C. Prins, C. Prodhon, A.L. Medaglia, N. Velasco. 2011. A GRASP with evolutionary path relinking for the truck and trailer routing problem. *Computers & Operations Research* **38**(9) 1319–1334.

Voudouris, C., E. Tsang. 1999. Guided local search and its application to the traveling salesman problem. *European Journal of Operational Research* **113**(2) 469–499.

Walshaw, C. 2002. A multilevel approach to the travelling salesman problem. *Operations Research* **50**(5) 862–877.

Wan, G., B.P.-C. Yen. 2002. Tabu search for single machine scheduling with distinct due windows and weighted earliness/tardiness penalties. *European Journal of Operational Research* **142**(2) 271–281.

Weigel, D., B. Cao. 1999. Applying GIS and OR techniques to solve Sears technician-dispatching and home-delivery problems. *Interfaces* **29**(1) 112–130.

Whittley, I.M., G.D. Smith. 2004. The attribute based hill climber. *Journal of Mathematical Modelling and Algorithms* **3**(2) 167–178.

Williamson, A., S. Sadural, A. Feyer, R. Friswell. 2001. Driver Fatigue : A Survey of Long Distance Heavy Vehicle Drivers in Australia. Tech. rep., National Road Transport Commission.

Wolf, S., P. Merz. 2007. Evolutionary local search for the super-peer selection problem and the p-hub median problem. T. Bartz-Beielstein, M.J. Belsa Aguilera, C. Blum, B. Naujoks, A. Roli, G. Rudolph, M. Sampels, eds., *Hybrid Metaheuristics*, *LNCS*, vol. 4771. Springer Berlin Heidelberg, 1–15.

Wolpert, D.H. 1997. No free lunch theorems for optimization. *IEEE Transactions on Evolutionary Computation* **1**(1) 67–82.

Xiao, Y., Q. Zhao, I. Kaku, Y. Xu. 2012. Development of a fuel consumption optimization model for the capacitated vehicle routing problem. *Computers & Operations Research* **39**(7) 1419–1431.

Xu, Hang, Zhi-Long Chen, Srinivas Rajagopal, Sundar Arunapuram. 2003. Solving a Practical Pickup and Delivery Problem. *Transportation Science* **37**(3) 347–364.

Yang, W.T., L.C. Chu. 2000. A heuristic algorithm for the multi-depot periodic Vehicle Routing Problem. *Journal of Information & Optimization Sciences* **22** 359–367.

Yano, C.A., Y.-D. Kim. 1991. Algorithms for a class of single-machine weighted tardiness and earliness problems. *European Journal of Operational Research* **52**(2) 167–178.

Yellow, P.C. 1970. A Computational Modification to the Savings Method of Vehicle Scheduling. *Operational Research Quarterly* **21**(2) 281–283.

Yu, B., Z. Yang, B. Yao. 2009. An improved ant colony optimization for vehicle routing problem. *European Journal of Operational Research* **196**(1) 171–176.

Yu, B., Z.-Z. Yang, J.-X. Xie. 2011. A parallel improved ant colony optimization for multi-depot vehicle routing problem. *Journal of the Operational Research Society* **62**(1) 183–188.

Yu, B., Z.Z. Yang. 2011. An ant colony optimization model : The period vehicle routing problem with time windows. *Transportation Research Part E : Logistics and Transportation Review* **47**(2) 166–181.

Zachariadis, E.E., C.T. Kiranoudis. 2010a. A strategy for reducing the computational complexity of local search-based methods for the vehicle routing problem. *Computers & Operations Research* **37**(12) 2089–2105.

Zachariadis, E.E., C.T. Kiranoudis. 2010b. An open vehicle routing problem metaheuristic for examining wide solution neighborhoods. *Computers & Operations Research* **37**(4) 712–723.

Zachariadis, E.E., C.T. Kiranoudis. 2011. A local search metaheuristic algorithm for the vehicle routing problem with simultaneous pick-ups and deliveries. *Expert Systems with Applications* **38**(3) 2717–2726.

Zachariadis, E.E., C.T. Kiranoudis. 2012. An effective local search approach for the Vehicle Routing Problem with Backhauls. *Expert Systems with Applications* **39**(3) 3174–3184.

Zachariadis, E.E., C.D. Tarantilis, C.T. Kiranoudis. 2010. An adaptive memory methodology for the vehicle routing problem with simultaneous pick-ups and deliveries. *European Journal of Operational Research* **202**(2) 401–411.

Zapfel, G, M Bogl. 2008. Multi-period vehicle routing and crew scheduling with outsourcing options. *International Journal of Production Economics* **113**(2) 980–996.

Zhu, W., H. Qin, A. Lim, L. Wang. 2012. A two-stage tabu search algorithm with enhanced packing heuristics for the 3L-CVRP and M3L-CVRP. *Computers & Operations Research* **39**(9) 2178–2195.

Annexe I

SUPPLEMENT TO "TIMING PROBLEMS AND ALGORITHMS"

I.1 Reduction of LISP to $\{TW(unit)|P\}$

Given a vector $\mathbf{N} = (N_1, \ldots, N_n)$ of n real numbers, LISP aims to find the maximum length L of a non-decreasing subsequence of numbers: $L = max\{k|1 \leq i_1 < \cdots < i_k \leq n \text{ and } N_{i_1} \leq ... \leq N_{i_k}\}$.

From a LISP instance, we construct the following instance $\mathcal{T}$ of $\{TW(unit)|\}$, with n activities such that for $i \in \{1, \ldots, n\}$, $e_i = l_i = N_i$ and $p_i = 0$. This instance is created in n elementary algorithmic operations.

Let $z^*(\mathcal{T})$ be the optimal solution cost of $\mathcal{T}$. This solution naturally initiates as many activities as possible without penalties, within a non-decreasing order of execution dates. Hence, the activities achieved without penalty correspond to the LISP subsequence sought in the original problem, whose length is $L^* = n - z^*(\mathcal{T})$. Hence, LISP admits a many-one reduction to $\{TW(unit)|\}$. Conversely, $\{TW(unit)|\}$ generalizes LISP.

I.2 Proof of Theorem 2.7.1: Block optimality conditions

We first recall the corresponding timing problem, stated in Equations (I.1-I.2).

$$\min_{(t_1,\ldots,t_n)\in\Re^n} \quad \sum_{i=1}^{n} c_i(t_i) \tag{I.1}$$

$$s.t. \quad t_i + p_i \leq t_{i+1} \qquad\qquad 1 \leq i < n \tag{I.2}$$

The functions $c_i(t) : \Re \to \Re$ are assumed to be convex, but not necessarily smooth. We note $\delta c_i(t)$ the subdifferential of c_i at t, which is necessarily a non-empty interval, as a byproduct of the convexity assumption and the space of definition. We first recall two useful properties on subdifferentials from Rockafellar (1970), to follow with the proof of Theorem (2.7.1).

Proposition I.2.1 *Let $f_1, \ldots, f_m$ be subdifferentiable functions on $\Re^n$, then:*

$$\delta(f_1(x) + \cdots + f_m(x)) \supset \delta f_1(x) + \cdots + \delta f_m(x) \ \forall x \tag{I.3}$$

Theorem I.2.1 *Let $f_1, \ldots, f_m$ be a set of proper convex functions on $\Re^n$ having at least one common point in the relative interior of their domains $ri(dom(f_i))$, then:*

$$\delta(f_1(x) + \cdots + f_m(x)) = \delta f_1(x) + \cdots + \delta f_m(x) \ \forall x \in \bigcap ri(dom(f_i)) \tag{I.4}$$

Constraint qualifications hold as the constraints are linear, and any solution with idle time between each activity is feasible, thus taking place in the relative interior of the polytope. Hence, strong duality applies, leading to the following necessary and sufficient optimality conditions. A solution $\mathbf{t}^* = (t_1^*, \ldots, t_n^*)$ of Problem (I.1-I.2) is optimal if and only if a set of Lagrangian multipliers

$(\lambda_1^*, \ldots, \lambda_{n-1}^*)$ exists, such that conditions (I.5) are satisfied (Bertsekas et al. 2003, Propositions 5.7.1 and 6.4.2).

$$\begin{cases} t_{i-1}^* + p_{i-1} \leq t_i^* & i = 2, \ldots, n \\ 0 \in \delta c_1(t_1^*) + \lambda_1^* & \\ 0 \in \delta c_i(t_i^*) + \lambda_i^* - \lambda_{i-1}^* & i = 2, \ldots, n-1 \\ 0 \in \delta c_n(t_n^*) - \lambda_{n-1}^* & \\ \lambda_{i-1}^*(t_{i-1}^* + p_{i-1} - t_i^*) = 0 & i = 2, \ldots, n \\ \lambda_i^* \geq 0 & i = 1, \ldots, n \end{cases} \tag{I.5}$$

Solution $\mathbf{t}^*$ can be represented as a succession of blocks of activities $(B_1, \ldots, B_m)$, such that activities within each block are processed without idle time, and the last activities of blocks are followed by non-zero idle time. The previous definition, combined with primal feasibility, yields $t_{B_j(|B_j|)}^* + p_{B_j(|B_j|)} < t_{B_j(|B_j|)+1}^*$, and thus $\lambda_{B_j(|B_j|)}^* = 0$ for $j \in \{1, \ldots, m-1\}$. Conditions (I.5) are thus equivalent to primal feasibility (equivalent to the first condition of (2.7.1) combined with the definition of blocks) and the following independent systems of equations for each block:

$$\forall j \in \{1, \ldots, m\} \begin{cases} \textbf{i)} & 0 \in \delta c_{B_j(1)}(t_{B_j(1)}^*) + \lambda_{B_j(1)}^* & \\ \textbf{ii)} & 0 \in \delta c_i(t_{B_j(1)}^* + p_{B_j(1)i-1}) + \lambda_i^* - \lambda_{i-1}^* & i = B_j(1)+1, \ldots, B_j(|B_j|)-1 \\ \textbf{iii)} & 0 \in \delta c_{B_j(|B_j|)}(t_{B_j(1)}^* + p_{B_j(1)B_j(|B_j|-1)})) - \lambda_{B_j(|B_j|)-1}^* & \\ \textbf{iv)} & \lambda_i^* \geq 0 \qquad i = B_j(1), \ldots, B_j(|B_j|) & \end{cases} \tag{I.6}$$

Necessary condition proof. For $1 \leq i \leq j \leq n$, let p_{ij} be the cumulative processing duration of activities a_i to a_j. Relying on Proposition (I.2.1), one can sum i) , ii) and iii) of (I.6), leading to:

$$0 \in \delta c_{B_j(1)}(t_{B_j(1)}^*) + \sum_{i=B_j(1)+1}^{B_j(|B_j|)} \delta c_i(t_{B_j(1)}^* + p_{B_j(1)i-1}) \Rightarrow 0 \in \delta C_{B_j}(t_{B_j(1)}^*) \ , \tag{I.7}$$

and thus, following the definition of the optimal block execution cost of Equation (2.18), $t_{B_i(1)}^* \in [T_{B_i}^{-*}, T_{B_i}^{+*}]$. Any optimal solution thus verifies the first statement of Theorem (2.7.1). Finally, for any block B_j and prefix block B_j^k, summing i) , ii) and iii) for $j \in \{B_j(1)+1, \ldots, k\}$ leads to:

$$-\lambda_k^* \in \delta c_{B_j(1)}(t_{B_j(1)}^*) + \sum_{i=B_j(1)+1}^{k} \delta c_i(t_{B_j(1)}^* + p_{B_j(1)i-1}) \Rightarrow -\lambda_k^* \in \delta C_{B_j}(t_{B_j^k(1)}^*) \ , \tag{I.8}$$

which can be reformulated as $T_{B_j^k}^{+*} \geq t_{B_j(1)}^*$ and implies the last statement of Theorem (2.7.1).

Sufficient condition proof. Consider a solution $\mathbf{t} = (t_1, \ldots, t_n)$ with its blocks $(B_1, \ldots, B_m)$, respecting conditions of Theorem (2.7.1). Following block definitions and primal feasibility, it only remains to prove that Conditions (I.6) are respected for each block. We choose the following

Lagrangian multipliers, which are non-negative as $T^{+*}_{B^k_i} \geq t_{B_i(1)} \Rightarrow \exists x \ \leq 0 \in \delta C_{B^i_j}(t_{B_j(1)}))$:

$$\forall j \in \{1,\ldots,m\} \begin{cases} \lambda^*_i = -\min(x \in \delta C_{B^i_j}(t_{B_j(1)})) & i = B_j(1)+1,\ldots,B_j(|B_j|)-1 \\ \lambda^*_{B_j(|B_j|)} = 0 \end{cases} \tag{I.9}$$

Proposition (I.2.1) then involves that for $i \in \{1,\ldots,m\}$ and $j \in \{B_j(1)+1,\ldots,B_j(|B_j|)-1\}$;

$$-\lambda^*_i \in \delta C_{B^i_j}(t_{B_j(1)}) = \delta c_{B_j(1)}(t_{B_j(1)}) + \sum_{k=B_j(1)+1}^{i} \delta c_k(t_{B_j(1)} + p_{B_j(1)k-1}) \tag{I.10}$$

In the case where $i = B_j(1)$, Equation (I.10) proves statement i) of (I.6). Also, $\lambda^*_{B_j(1)} \in \delta(-c_{B_j(1)})(t_{B_j(1)})$, and we can combine this statement with Equation (I.10) for $i = B_j(1)+1$, using Proposition I.2.1), leading to:

$$\begin{aligned} \lambda^*_{B_j(1)+1} - \lambda^*_{B_j(1)} &\in \delta(-c_{B_j(1)})(t_{B_j(1)}) + \delta c_{B_j(1)}(t_{B_j(1)}) + \delta c_{B_j(1)+1}(t_{B_j(1)} + p_{B_j(1)}) \\ &= \delta c_{B_j(1)+1}(t_{B_j(1)} + p_{B_j(1)}) \end{aligned} \tag{I.11}$$

The remaining statements of Equation ii) and Equation iii) in (I.6) are proven by recurrence. Assuming that for a given $i \in \{B_j(1)+1,\ldots,B_j(|B_j|)-1\}$, $\lambda^*_{i-1} - \lambda^*_i \in \delta c_i(t_{B_j(1)} + p_{B_j(1)i})$, then

$$\begin{aligned} \lambda^*_i - \lambda^*_{i+1} &= -\lambda^*_{i+1} + \lambda^*_{i-1} - (\lambda^*_{i-1} - \lambda^*_i) \\ &\in \delta c_{B_j(1)}(t_{B_j(1)}) + \sum_{k=B_j(1)}^{i+1} \delta c_k(t_{B_j(1)} + p_{B_j(1)k-1}) + \delta(-c_{B_j(1)})(t_{B_j(1)}) \\ &+ \sum_{k=B_j(1)}^{i-1} \delta(-c_k)(t_{B_j(1)} + p_{B_j(1)k-1}) + \delta(-c_i)(t_{B_j(1)} + p_{B_j(1)i-1}) \\ &\subset \delta c_{i+1}(t_{B_j(1)} + p_{B_j(1)i}) \end{aligned} \tag{I.12}$$

All the sufficient optimality conditions are thus satisfied by solution $\mathbf{t} = (t_1,\ldots,t_n)$. $\square$

Annexe II

SUPPLEMENT TO "A HYBRID GA FOR MULTI-DEPOT AND PERIODIC VRP"

Section II.1 details the multi-depot periodic VRP formulation and the transformation proposition of the MDPVRP into the PVRP. Additional precisions and examples for the Split procedures are given in Section II.2. The remaining three sections are focused on the experimental studies, and provide respectively details on time comparisons between different CPUs (Section II.3), detailed results on multi-depot and periodic VRP benchmarks (Section II.4), and the results for the CVRP benchmarks (Section II.5).

II.1 Problem formulation and transformations

A PVRP formulation was introduced in Cordeau et al. (1997). We now introduce a MDPVRP formulation, as a five-index vehicle flow formulation, and show how it reduces to the PVRP.

Let c_{ij} be the routing cost from vertex v_i to vertex $v_j \in \mathcal{V}$. Let the binary constants a_{pl} be equal to 1 if and only if day l belongs to visit combination (pattern) p and 0 otherwise. Two sets of binary variables are defined. For every $v_i \in \mathcal{V}^{\text{CST}}$, $p \in L_i$, and $v_o \in \mathcal{V}^{\text{DEP}}$, y_{ipo} equals 1 if and only if customer i is assigned to visit combination p and depot o. For any $(v_i, v_j) \in \mathcal{V}^2$, $k = 1 \dots m$, $l = 1 \dots t$, and $v_o \in \mathcal{V}^{\text{DEP}}$, x_{ijklo} takes value 1 if and only if vehicle k coming from depot o on day l visits v_j immediately after v_i. Using by convention $\tau_o = 0, \forall v_o \in \mathcal{V}^{\text{DEP}}$, the MDPVRP can be stated as follows:

$$\text{Minimize} \quad \sum_{v_i \in \mathcal{V}} \sum_{v_j \in \mathcal{V}} \sum_{k=1}^{m} \sum_{l=1}^{t} \sum_{v_o \in \mathcal{V}^{\text{DEP}}} c_{ij} x_{ijklo} \tag{II.1}$$

$$\text{Subject to:} \sum_{p \in L_i} \sum_{v_o \in \mathcal{V}^{\text{DEP}}} y_{ipo} = 1 \qquad v_i \in \mathcal{V}^{\text{CST}} \tag{II.2}$$

$$\sum_{v_j \in \mathcal{V}} \sum_{k=1}^{m} x_{ijklo} - \sum_{p \in L_i} a_{pl} y_{ipo} = 0 \qquad v_i \in \mathcal{V}^{\text{CST}} \ ; \ v_o \in \mathcal{V}^{\text{DEP}} \ ; \ l = 1 \dots t \tag{II.3}$$

$$\sum_{v_j \in \mathcal{V}} x_{ojklo} \leq 1 \qquad v_o \in \mathcal{V}^{\text{DEP}} \ ; \ k = 1 \dots m \ ; \ l = 1 \dots t \tag{II.4}$$

$$\sum_{v_j \in \mathcal{V}} x_{ijklo} = 0 \qquad v_i \in \mathcal{V}^{\text{DEP}} \ ; \ v_o \in \mathcal{V}^{\text{DEP}} \ ; \ v_o \neq v_i \ ; \ k = 1 \dots m \ ; \ l = 1 \dots t \tag{II.5}$$

$$\sum_{v_j \in \mathcal{V}} x_{jiklo} - \sum_{v_j \in \mathcal{V}} x_{ijklo} = 0 \qquad v_i \in \mathcal{V} \ ; \ v_o \in \mathcal{V}^{\text{DEP}} \ ; \ k = 1 \dots m \ ; \ l = 1 \dots t \tag{II.6}$$

$$\sum_{v_i \in \mathcal{V}} \sum_{v_j \in \mathcal{V}} q_i x_{ijklo} \leq Q \qquad v_o \in \mathcal{V}^{\text{DEP}} \ ; \ k = 1 \dots m \ ; \ l = 1 \dots t \tag{II.7}$$

$$\sum_{v_i \in \mathcal{V}} \sum_{v_j \in \mathcal{V}} (c_{ij} + \tau_i) x_{ijklo} \leq T \qquad v_o \in \mathcal{V}^{\text{DEP}} \; ; \; k = 1 \ldots m \; ; \; l = 1 \ldots t \qquad \text{(II.8)}$$

$$\sum_{v_i \in S} \sum_{v_j \in S} x_{ijklo} \leq |S| - 1 \qquad S \in \mathcal{V}^{\text{CST}} \; ; \; |S| \geq 2 \; ; \; v_o \in \mathcal{V}^{\text{DEP}} \; ; \; k = 1 \ldots m \; ; \; l = 1 \ldots t \qquad \text{(II.9)}$$

$$x_{ijklo} \in \{0, 1\} \qquad v_i \in \mathcal{V}; \; v_j \in \mathcal{V} \; ; \; v_o \in \mathcal{V}^{\text{DEP}} \; ; \; k = 1 \ldots m \; ; \; l = 1 \ldots t \qquad \text{(II.10)}$$

$$y_{ipo} \in \{0, 1\} \qquad v_i \in \mathcal{V} \; ; p \in L_i \; ; \; v_o \in \mathcal{V}^{\text{DEP}} \qquad \text{(II.11)}$$

Constraints II.2 ensure that exactly one depot and one visit combination are assigned to each customer. Constraints II.3 guarantee that customer visits occur only on the periods related to the chosen visit combination on a vehicle coming from the assigned depot, while constraints II.4 and II.5 enforce respectively the single use of vehicles, and compatibility issues between depot assignments and route starting and ending points. Equations II.6 are flow conservation constraints and II.7 and II.8 enforce limits on the capacity of vehicles and the duration of routes. Subtours are eliminated through II.9. This MDPVRP model includes both the MDVRP and the PVRP as a special case when $t = 1$, and $d = 1$ respectively.

Cordeau et al. (1997) showed that an MDVRP could be reduced into a PVRP, by associating a different period to each depot, such that each customer i has a frequency $f_i = 1$ and may be served during any period. A particularity of the resulting PVRP is that routing costs c_{ijl} depend upon the period l considered.

In the same spirit, the MDPVRP also happens to have the same general structure as the PVRP. Indeed, a simple change of indexes associating a period to each (depot, period) pair, transforms the MDPVRP formulation into a PVRP. This transformation is summarized in Proposition II.1.1.

Proposition II.1.1 *The MDPVRP reduces to a PVRP with period-dependent routing costs.*

Proof: Let $\mathcal{I}$ be an MDPVRP instance with t periods, d depot vertices in $\mathcal{V}^{\text{DEP}}$, and m vehicles per depot. Each customer $i \in \mathcal{V}^{\text{CST}}$ has frequency f_i and pattern list $L_i = \{\{p^i_{11}, \ldots, p^i_{1f_i}\}, \ldots, \{p^i_{|L_i|1}, \ldots, p^i_{|L_i|f_i}\}\}$. We now define an equivalent PVRP instance $\mathcal{J}$, which has $t' = td$ periods, a single depot vertex $v'_0 \in \mathcal{V}'_{dep}$, the same set of customers $\mathcal{V}'_{cus} = \mathcal{V}^{\text{CST}}$, and m vehicles available at each period. Each customer i in the new problem still must be served with frequency $f'_i = f_i$, with a pattern list L'_i containing $d \times |L_i|$ patterns defined by Equation II.12. Also, travel costs (durations) c'_{ijl} in the new PVRP are period-dependent to take into account that vehicles operating in period l in the new PVRP were based at depot $v_{\lfloor l/d \rfloor}$ in the MDPVRP. The new distance between v'_0 and any customer v_i on period l is thus equal to $c_{v_{\lfloor l/d \rfloor} i}$ in the old problem, while all other distances between customer pairs remain constant among periods.

$$L'_i = \bigcup_{\substack{o \in \{1, \ldots, d\} \\ r \in \{1, \ldots, |L_i|\}}} \{p^i_{r1} + ot, \ldots, p^i_{rf_i} + ot\} \qquad \text{(II.12)}$$

Solving the PVRP instance $\mathcal{J}$ leads to a solution of the MDPVRP instance $\mathcal{I}$ in a straightforward manner, as the routes for period l and depot v_o in $\mathcal{I}$ correspond to the routes of period $l + ot$ in $\mathcal{J}$. □

II.2 The Split algorithm

The *Split* algorithm (Prins 2004) addresses the problem of finding the route delimiters as a shortest path problem.

For a given visiting sequence, let c_i be the customer in position i. Define an auxiliary graph $\mathcal{H} = (\mathcal{V}, \mathcal{A})$, where $\mathcal{V}$ contains $n + 1$ nodes indexed from 0 to n. For each pair of nodes $i < j$, arc (i, j) represents the trip $r_{i+1,j}$ starting from the depot, visiting customers c_{i+1} to c_j, and coming back to the depot. For each trip, travel time and load are given by Equations (II.13) and (II.14). If the total load of a trip including arc (i, j) exceeds $Q_{max} = 2Q$, arc (i, j) is excluded from $\mathcal{A}$. This avoids solutions that are too far from feasibility and reduces the number of arcs. The cost of arcs is noted $\phi(r_{i+1,j})$ and accounts for penalized infeasible solutions:

$$q(r_{i+1,j}) = \sum_{l=i+1,j} q_{S_l} \tag{II.13}$$

$$t(r_{i+1,j}) = c_{0,S_{i+1}} + \sum_{l=i+1,j-1} (s_{S_l} + c_{S_l,S_{l+1}}) + s_{S_j} + c_{S_j,0} \tag{II.14}$$

An optimal segmentation of the giant tour into routes consists in identifying a minimum-cost path from 0 to n in $\mathcal{H}$ containing less than m edges, where m is the number of vehicles available per period. This minimum-cost path can be computed in m iterations of the Bellman-Ford algorithm (see Cormen et al. 2001, for an implementation), each iteration executing in $O(n^2)$. When the demand or the distance between customers is "large", it is possible to impose a bound b on the number of valid trips ending at a given customer i. Thus the complexity of an iteration becomes $O(n \times b)$, and the Split algorithm works in $O(m \times n \times b)$.

Figure II.1 illustrates the Split algorithm on a sequence of 5 customers c_1 to c_5. The first graph shows the cost of each arc and the demands at nodes (in bold). In this example, the vehicle capacity is set to $Q = 6$, thus $Q_{max} = 12$, the maximum route duration to $D = 150$, all customers i have an identical service time $d_i = 10$, and the penalty parameters are $\omega^{\mathrm{Q}} = 10$ and $\omega^{\mathrm{D}} = 1$. The corresponding graph $\mathcal{H}$ displays on each arc the route cost including penalties. For example, the route servicing customers c_3, c_4, and c_5 has a cost of $165 + 20 + 15$, the penalties of 20 and 15 corresponding to the load excess of two units and duration, respectively. The optimal solution of the minimum-cost path problem is of cost 260, and made up of the three following routes: route 1 visits c_1, route 2 visits c_2, c_3, and c_4, and route 3 visits c_5.

Note that in the actual implementation of the algorithm, building the graph $\mathcal{H}$ explicitly is not mandatory. A detailed example of pseudo-code for this procedure can be found in (Chu et al. 2006).

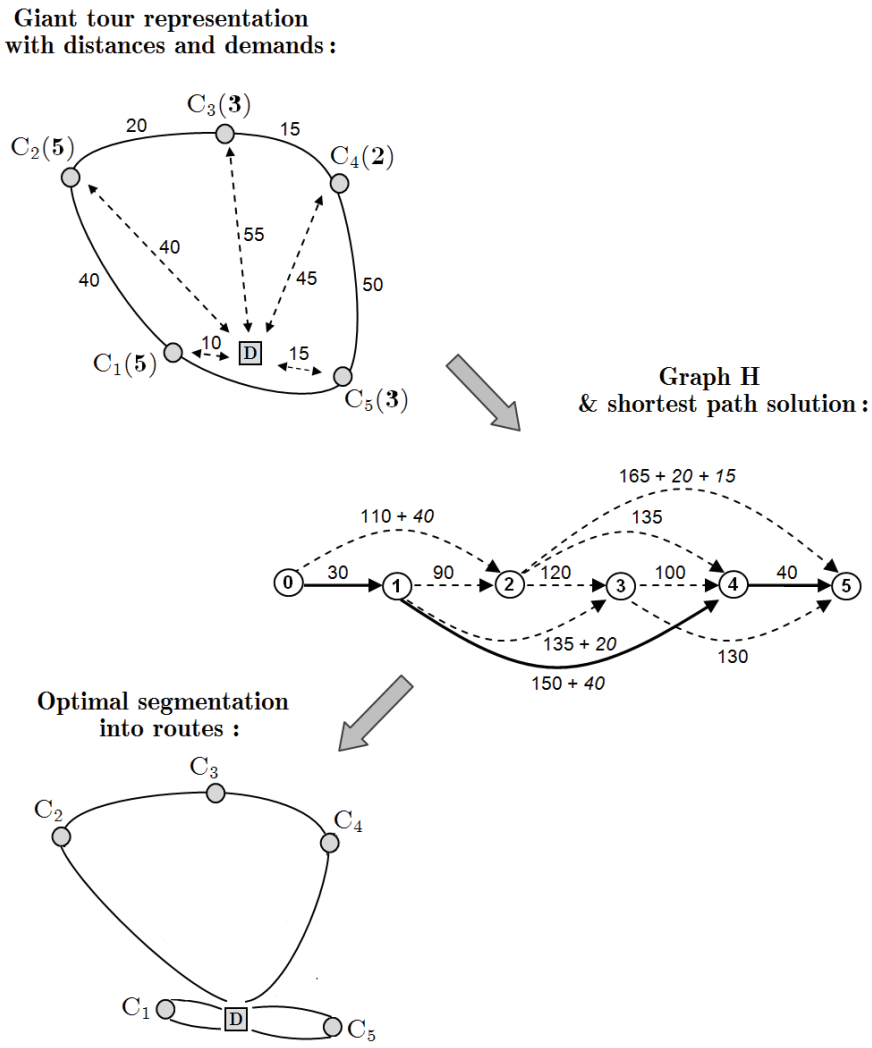


Figure II.1: Illustration of a Split graph and shortest-path solution

II.3 Comparison of run times

All CPU times in this paper were scaled into their equivalent Pentium IV 3.0 Ghz run times. This conversion is based on the assumption that CPU time is approximately linearly proportional to the amount of floating point operations per second (flop/s) performed by the processor. Various types of processors have been tested by solving dense systems of linear equations, to report their (flop/s) measures in Dongarra (2011). We provide in Table II.1 these values for each of the algorithms we compared with, along with the resulting scaling factors used for time conversions. No (flop/s) measure could be found for Pentium IV 2.8 Ghz, Pentium IV 3.2 Ghz, and we made the assumption that the processor speed should be approximately linear with frequency among the processors from the same family.

Table II.1: Scaling factors for computation times

Authors	**Processor**	**MFlop/s**	**Factor**
Prins (2004)	Pentium III 1.0 Ghz	250	0.16
This paper	AMD Opteron 250 2.4 Ghz	1291	0.82
Nagata and Bräysy (2009b)	AMD Opteron 250 2.4 Ghz	1291	0.82
Mester and Bräysy (2007)	Pentium IV 2.8 Ghz	—	0.92
Pisinger and Ropke (2007)	Pentium IV 3.0 Ghz	1573	1.00
Gulczynski et al. (2011)	Pentium IV 3.0 Ghz	1573	1.00
Hemmelmayr et al. (2009)	Pentium IV 3.2 Ghz	—	1.08

II.4 Detailed results on PVRP, MDVRP and MDPVRP

Tables II.2, II.3, and II.4 present respectively PVRP, MDVRP and MDPVRP results. The first group of columns (1-4) display the instance identifier, number of customers, vehicles, and periods. The next group of columns lists the state-of-the-art methods: the tabu search of Cordeau et al. (1997) (CGL), the scatter search of Alegre et al. (2007) (ALP), the VNS of Hemmelmayr et al. (2009) (HDH), and the record-to-record-ILP approach of Gulczynski et al. (2011) (GGW) for the PVRP; CGL, the fuzzy-logic guided-GA of Lau et al. (2010) (LCTP), and the adaptive large neighborhood search of Pisinger and Ropke (2007) (PR) for the MDVRP (none are available for MDPVRP). This list is followed by the results of the method we propose. We indicate in boldface the best average result among algorithms for each instance, as well as, in the last two columns, the previous best-known solution (BKS), and the best solution obtained by HGSADC during all our experiments. Optimality has been proved for several solutions, marked with *, by Baldacci and Mingozzi (2009), Baldacci et al. (2011a). When upper bounds are improved, the new state-of-the-art solutions are underlined. Finally, the last lines provide average measures over sets of instances: the computation time for each method, and the average percentage of error (Gap) relative to the previous BKS.

II.5 Experiments on capacitated VRP instances

The VRP appears naturally as a special case of the multi-depot periodic VRPs. The HGSADC method is flexible enough to be used to address this fundamental problem with the following minor changes:

- The distance measure of Section 4.3, originally designed for periodic problems, is replaced by the *broken pairs* distance (Prins 2009b) measuring the proportion of arc similarities between two solutions;
- The PIX crossover is replaced by the simple OX crossover (Prins 2004) for permutation based solutions. Routes are also concatenated in a cyclic way around the depot to produce the giant tour representation. This should better match related customer visits in the crossover;

- Finally, as the route improvement local search procedure tackles now a large number of customers visits in the same day (up to 483 visits in Golden et al. (1998a) instances), the number of neighbor nodes taken into account in moves has been reduced to a small number (20).

All other operators and parameters, including our adaptive diversity management scheme, remain identical.

The algorithm is tested on the traditional benchmarks from the VRP literature. The 14 instances (S1-p01...p14) of Christofides et al. (1979), ranging from 50 to 199 customers, are geographically randomly distributed for the 10 first instances, and otherwise clustered. The 20 large-scale instances (S2-p01...p20) of Golden et al. (1998a), range from 200 to 483 customers and present geometric symmetries.

Table II.5 compares the average results of HGADSC to the actual state-of-the-art algorithms on these benchmarks. The termination criteria $(It_{NI}, T_{max}) = (10^4, \infty)$ is used to perform experiments in a number of iterations and computation time similar to the literature. The first two columns display the instance identifier and the number of customers, while the next group of columns compare the average performance of HGSADC to the performance of the hybrid genetic algorithm of Prins (2004) (P), the guided evolution strategies of Mester and Bräysy (2007) (MB), and the edge-assembly crossover based memetic algorithm of Nagata and Bräysy (2009b) (NB). On the CVRP, our algorithm differs from (P) only by its population management, infeasible solution use, and the way neighborhoods are restricted in the education operator. These experiments enable thus to state on the benefits of the concepts we introduce. Also, (MB) and (NB) constitute the actual state-of-the-art algorithms for the CVRP. We indicate in boldface the best average result among algorithms for each instance, as well as, in the last two columns, the previous best-known solution (BKS), (found in (MB), (NB) and Zachariadis and Kiranoudis 2010a), and the best solution obtained by HGSADC during all our experiments. When upper bounds are improved, the new state-of-the-art solutions are underlined. The last two lines provide average measures over sets of instances: the average percentage of error relative to the previous BKS, and computation time for each method.

Table II.6 finally reports the results of HGADSC with different termination criteria: $It_{NI} = \{10^4, 2.10^4, 5.10^4\}$. The table format remains the same as previously.

These experiments reveal that HGSADC is competitive with state-of-the-art methods in terms of solution quality and computation time, even though it has not been designed for the CVRP. The results of Table II.6 with increased computation times also underline the capability of HGSADC to maintain a very efficient search throughout the run. With a reasonably small increase in the run time (26.37 minutes for 2.10^4 iterations without without improvement compared to 17.69 minutes for Nagata and Bräysy (2009b) state-of-the-art algorithm), HGSADC solution quality already improves upon previously published algorithms.

In addition, 12 new best known solutions are reported on Golden et al. (1998a) instances. All these solutions present equal numbers of vehicles and shorter distances than previous BKS from the literature. In the special case of problem pr07, our algorithm also succeeds in finding a

solution with one less route (8 routes instead of 9 for previous BKS), and also shorter distance. This solution is reported in details.

pr07 – Total Distance: 10102.68 – 8 vehicles

Route	Distance	Load	Customer Visits
1	1298.05	900	0 22 58 94 130 166 167 168 204 240 276 312 348 347 311 275 239 203 202 238 274 310 346 345 309 273 237 236 272 308 344 343 307 271 235 199 163 127 91 55 20 0
2	1263.58	900	0 23 59 95 131 132 133 169 205 241 277 313 349 350 314 278 242 206 207 243 279 315 351 352 316 280 244 208 172 171 170 134 135 136 137 138 102 101 100 99 98 97 96 60 61 25 24 0
3	1179.91	900	0 26 62 63 64 65 66 67 103 139 175 174 173 209 245 281 317 353 354 318 282 246 210 211 247 283 319 355 356 320 284 248 212 176 177 141 140 104 105 69 68 32 31 30 29 28 27 0
4	1298.33	900	0 1 36 35 72 71 107 143 179 215 216 252 217 218 254 290 326 325 289 253 288 324 360 359 323 287 251 250 286 322 358 357 321 285 249 213 214 178 142 106 70 34 33 0
5	1267.10	900	0 6 42 41 40 39 38 74 73 109 110 146 147 148 149 150 186 222 258 294 330 329 293 257 221 185 184 220 256 292 328 327 291 255 219 183 182 181 145 180 144 108 37 2 3 4 5 0
6	1263.58	900	0 8 7 43 44 80 79 78 77 76 75 111 112 113 114 115 151 187 223 259 295 331 332 296 260 224 188 189 225 261 297 333 334 298 262 226 190 154 153 152 116 117 118 82 81 45 9 0
7	1242.66	900	0 11 10 46 47 83 119 120 156 155 191 227 263 299 335 336 300 264 228 192 193 229 265 301 337 338 302 266 230 194 158 157 121 122 123 124 125 89 88 87 86 85 84 48 49 13 12 0
8	1289.48	900	0 21 57 56 92 93 129 128 164 165 201 200 198 234 270 306 342 341 305 269 233 197 196 232 268 304 340 339 303 267 231 195 159 160 161 162 126 90 54 53 52 51 50 14 15 16 17 18 19 0

Table II.2: Results on Cordeau et al. (1997) PVRP instances

				Average						BKS	
Inst	**n**	**m**	**t**	**CGL** (1 run)	**ALP** —	**HDH** (10 runs)	**GGW**[1] (1 run)	**HGSADC** (10 runs)	**T(min)**	**prev BKS** —	**HGSADC** (all exp.)
S1-p01	50	3	2	**524.61**	531.02	**524.61**	**524.61**	**524.61**	0.22	524.61*	524.61*
S1-p02	50	3	5	1330.09	1324.74	1332.01	1335.08	**1322.87**	0.44	1322.87	1322.87
S1-p03	50	1	5	**524.61**	537.37	528.97	**524.61**	**524.61**	0.18	524.61*	524.61*
S1-p04	75	6	5	837.94	845.97	847.48	838.47	**836.59**	1.05	835.26*	835.26*
S1-p05	75	1	10	2061.36	2043.74	2059.74	2052.81	**2033.72**	2.27	2027.99	2024.96
S1-p06	75	1	10	840.30	840.10	884.69	**838.47**	842.48	0.89	835.26*	835.26*
S1-p07	100	4	2	829.37	829.65	829.92	827.39	**827.02**	0.88	826.14	826.14
S1-p08	100	5	5	2054.90	2052.51	2058.36	2054.25	**2022.85**	2.54	2034.15	2022.47
S1-p09	100	1	8	829.45	829.65	834.92	827.39	**826.94**	1.01	826.14	826.14
S1-p10	100	4	5	1629.96	1621.21	1629.76	1645.42	**1605.22**	1.80	1593.45	1593.43
S1-p11	126	4	5	817.56	782.17	791.18	781.68	**775.84**	4.60	779.06	770.89
S1-p12	163	3	5	1239.58	1230.95	1258.46	1225.78	**1195.29**	5.34	1195.88	1186.47
S1-p13	417	9	7	3602.76	—	3835.90	3624.45	**3599.86**	40.00	3511.62	3492.89
S1-p14	20	2	4	**954.81**	**954.81**	**954.81**	**954.81**	**954.81**	0.08	954.81*	954.81*
S1-p15	38	2	4	**1862.63**	**1862.63**	**1862.63**	**1862.63**	**1862.63**	0.17	1862.63*	1862.63*
S1-p16	56	2	4	**2875.24**	**2875.24**	**2875.24**	**2875.24**	**2875.24**	0.32	2875.24*	2875.24*
S1-p17	40	4	4	**1597.75**	**1597.75**	1601.75	**1597.75**	**1597.75**	0.27	1597.75*	1597.75*
S1-p18	76	4	4	3159.22	3157.00	3147.91	3215.44	**3131.09**	0.89	3136.69	3131.09
S1-p19	112	4	4	4902.64	4846.49	4851.41	4846.58	**4834.50**	2.26	4834.34	4834.34
S1-p20	184	4	4	**8367.40**	8412.02	**8367.40**	8369.70	**8367.40**	4.01	8367.40	8367.40
S1-p21	60	6	4	2184.04	2173.58	2180.33	2188.92	**2170.61**	0.90	2170.61*	2170.61*
S1-p22	114	6	4	4307.19	4330.59	4218.46	4317.18	**4194.23**	4.27	4193.95	4193.95
S1-p23	168	6	4	6620.50	6813.45	6644.93	6685.08	**6434.10**	4.29	6420.71*	6420.71*
S1-p24	51	3	6	3704.11	3702.02	3704.60	3741.78	**3687.46**	0.32	3687.46*	3687.46*
S1-p25	51	3	6	3781.38	3781.38	3781.38	3816.94	**3777.15**	0.59	3777.15*	3777.15*
S1-p26	51	3	6	**3795.32**	3795.33	**3795.32**	**3795.32**	**3795.32**	0.33	3795.32*	3795.32*
S1-p27	102	6	6	23017.45	22561.33	22153.31	21946.93	**21885.70**	3.52	21912.85	21833.87
S1-p28	102	6	6	22569.40	22562.44	22418.52	22383.90	**22272.60**	4.67	22246.69*	22242.51*[2]
S1-p29	102	6	6	24012.92	23752.15	22864.23	22628.81	**22564.05**	3.86	22543.75*	22543.76*
S1-p30	153	9	6	77179.33	76793.99	75579.23	75003.94	**74534.38**	9.99	74464.26	73875.19
S1-p31	153	9	6	79382.35	77944.79	77459.14	76842.07	**76686.65**	10.00	76322.04	76001.57
S1-p32	153	9	6	80908.95	81055.52	79487.97	78650.75	**78168.82**	10.00	78072.88	77598.00
S2-p01	48	2	4	2234.23	—	2209.11	—	**2209.02**	0.29	2209.02	2209.02
S2-p02	96	4	4	3836.49	—	3787.51	—	**3768.86**	2.49	3774.09	3767.50
S2-p03	144	6	4	5277.62	—	5243.09	—	**5174.80**	7.32	5175.15	5153.54
S2-p04	192	8	4	6072.67	—	6011.39	—	**5936.16**	10.00	5914.93	5877.37
S2-p05	240	10	4	6769.80	—	6778.00	—	**6651.76**	20.00	6618.95	6581.86
S2-p06	288	12	4	8462.37	—	8461.45	—	**8284.94**	20.00	8258.08	8207.21
S2-p07	72	3	6	5000.90	—	5007.01	—	**4996.14**	1.49	4996.14	4996.14
S2-p08	144	6	6	7183.39	—	7119.61	—	**7035.52**	10.00	6989.81	6970.68
S2-p09	216	9	6	10507.34	—	10259.09	—	**10162.22**	20.00	10075.40	10038.43
S2-p10	288	12	6	13629.25	—	13342.41	—	**13091.00**	20.00	12924.66	12897.01
Avg. Time				4.28 min	3.64 min	3.34 min	10.36 min	5.56 min			
Gap overall				+1.82%	—	+1.45%	—	+0.20%			
Gap S1				+1.62%	+1.40%	+1.43%	+0.94%	+0.14%			
Gap S2				+2.48%	—	+1.53%	—	+0.38%			
Gap $n \geq 150$				+3.23%	—	+2.16%	—	+0.35%			

[1] The instances used in GGW differ slightly in terms of customer locations. The reported results follow from a private communication with the authors, and were recalculated relatively to the exact CGL instances.

[2] Optimality was proven within 0.02% precision, and thus some "optimal" solutions can be slightly improved.

Table II.3: Results on Cordeau et al. (1997) MDVRP instances

				Average					BKS	
Inst	**n**	**m**	**d**	**CGL**	**PR**	**LCTP**	**HGSADC**	**T(min)**	**prev BKS**	**HGSADC**
				(1 run)	(5-10 runs)	(50 runs)	(10 runs)		—	(all exp.)
S1-p01	50	4	4	**576.87**	**576.87**	**576.87**	**576.87**	0.23	576.87*	576.87*
S1-p02	50	2	4	**473.87**	**473.53**	**473.53**	**473.53**	0.21	473.53*	473.53*
S1-p03	75	3	2	645.15	**641.19**	**641.19**	**641.19**	0.43	641.19	641.19
S1-p04	100	8	2	1006.66	1006.09	1001.59	**1001.23**	1.94	1001.04	1001.04
S1-p05	100	5	2	753.34	752.34	752.08	**750.03**	1.06	750.03	750.03
S1-p06	100	6	3	877.84	883.01	882.73	**876.50**	1.14	876.50*	876.50*
S1-p07	100	4	4	891.95	889.36	887.94	**884.43**	1.55	881.97*	881.97*
S1-p08	249	14	2	4482.44	4421.03	4438.15	**4397.42**	10.00	4387.38	4372.78
S1-p09	249	12	3	3920.85	3892.50	3916.32	**3868.59**	9.50	3873.64	3858.66
S1-p10	249	8	4	3714.65	3666.85	3669.76	**3636.08**	9.82	3650.04	3631.11
S1-p11	249	6	5	3580.84	3573.23	3581.88	**3548.25**	7.14	3546.06	3546.06
S1-p12	80	5	2	**1318.95**	1319.13	**1318.95**	**1318.95**	0.52	1318.95*	1318.95*
S1-p13	80	5	2	**1318.95**	**1318.95**	**1318.95**	**1318.95**	0.57	1318.95	1318.95
S1-p14	80	5	2	**1360.12**	**1360.12**	**1360.12**	**1360.12**	0.55	1360.12	1360.12
S1-p15	160	5	4	2534.13	2519.64	2514.06	**2505.42**	1.92	2505.42	2505.42
S1-p16	160	5	4	**2572.23**	2573.95	2578.37	**2572.23**	1.97	2572.23	2572.23
S1-p17	160	5	4	2720.23	**2709.09**	**2709.09**	**2709.09**	2.14	2709.09	2709.09
S1-p18	240	5	6	3710.49	3736.53	3728.44	**3702.85**	4.52	3702.85	3702.85
S1-p19	240	5	6	**3827.06**	3838.76	3840.53	**3827.06**	4.20	3827.06	3827.06
S1-p20	240	5	6	**4058.07**	4064.76	4063.26	**4058.07**	4.37	4058.07	4058.07
S1-p21	360	5	9	5535.99	5501.58	5525.68	**5476.41**	10.00	5474.84	5474.84
S1-p22	360	5	9	5716.01	5722.19	5733.13	**5702.16**	10.00	5702.16	5702.16
S1-p23	360	5	9	6139.73	6092.66	6098.75	**6078.75**	10.00	6078.75	6078.75
S2-p01	48	2	4	**861.32**	**861.32**	**861.32**	**861.32**	0.17	861.32	861.32
S2-p02	96	4	4	1314.99	1308.17	1312.85	**1307.34**	0.76	1307.34	1307.34
S2-p03	144	6	4	1815.62	1810.66	1809.69	**1803.80**	1.91	1806.60	1803.80
S2-p04	192	8	4	2094.24	2073.16	2076.27	**2059.36**	5.22	2060.93	2058.31
S2-p05	240	10	4	2408.10	2350.31	2391.42	**2340.29**	9.56	2337.84	2331.20
S2-p06	288	12	4	2768.13	2695.74	2728.94	**2681.93**	10.00	2685.35	2676.30
S2-p07	72	3	6	1092.12	**1089.56**	**1089.56**	**1089.56**	0.34	1089.56	1089.56
S2-p08	144	6	6	1676.26	1675.74	1666.60	**1665.05**	2.05	1664.85	1664.85
S2-p09	216	9	6	2176.79	2144.84	2152.05	**2134.17**	6.10	2136.42	2133.20
S2-p10	288	12	6	3089.62	2905.43	2919.59	**2886.59**	10.00	2889.49	2868.26
Avg. Time				small	3.54 min	2.06 min	4.24 min			
Gap overall				+0.96%	+0.34%	+0.49%	-0.01%			
Gap S1				+0.58%	+0.35%	+0.39%	+0.00%			
Gap S2				+1.85%	+0.34%	+0.71%	-0.04%			
Gap $n \geq 150$				+1.40%	+0.45%	+0.70%	-0.03%			

Table II.4: Results on new MDPVRP instances

Inst	**n**	**m**	**d**	**t**	**Average**	**T(min)** (10 runs)	**Best**	**BKS** (all exp.)
p01	48	1	4	4	**2019.07**	0.35	**2019.07**	2019.07
p02	96	1	4	4	**3547.45**	1.49	**3547.45**	3547.45
p03	144	2	4	4	4491.08	7.72	**4480.87**	4480.87
p04	192	2	4	4	5151.73	22.10	5144.41	5134.17
p05	240	3	4	4	5605.60	30.00	5581.10	5570.45
p06	288	3	4	4	6570.28	30.00	6549.57	6524.92
p07	72	1	6	6	4502.06	2.18	**4502.02**	4502.02
p08	144	1	6	6	6029.58	7.96	**6023.98**	6023.98
p09	216	2	6	6	8310.19	27.79	8271.66	8257.80
p10	288	3	6	6	9972.35	30.00	9852.87	9818.42
Avg Time					15.96 min		159.6 min	
Gap overall					+0.42%		+0.13%	
Gap $n \geq 150$					+0.77%		+0.26%	

Table II.5: Results for Christofides et al. (1979) and Golden et al. (1998a) CVRP instances

		Average					BKS	
Inst	**n**	**P**	**MB**	**NB**	**HGSADC**	**T(min)**	**prev BKS**	**HGSADC**
		(1 run)	(1 run)	(10 runs)	(10 runs)		—	(all exp.)
S1-p01	50	**524.61**	**524.61**	**524.61**	**524.61**	0.43	524.61	524.61
S1-p02	75	**835.26**	**835.26**	835.61	**835.26**	0.96	835.26	835.26
S1-p03	100	**826.14**	**826.14**	**826.14**	**826.14**	1.27	826.14	826.14
S1-p04	150	1031.63	**1028.42**	**1028.42**	**1028.42**	2.87	1028.42	1028.42
S1-p05	199	1300.23	**1291.29**	1291.84	1294.06	5.94	1291.29	1291.45
S1-p06	50	**555.43**	**555.43**	**555.43**	**555.43**	0.48	555.43	555.43
S1-p07	75	912.3	**909.68**	910.41	**909.68**	1.09	909.68	909.68
S1-p08	100	**865.94**	**865.94**	**865.94**	**865.94**	1.14	865.94	865.94
S1-p09	150	1164.25	**1162.55**	1162.56	**1162.55**	2.53	1162.55	1162.55
S1-p10	199	1420.2	1401.12	**1398.3**	1400.23	8.22	1395.85	1395.85
S1-p11	120	**1042.11**	**1042.11**	**1042.11**	**1042.11**	1.15	1042.11	1042.11
S1-p12	100	**819.56**	**819.56**	**819.56**	**819.56**	0.84	819.56	819.56
S1-p13	120	1542.97	**1541.14**	1542.99	1543.07	2.83	1541.14	1541.14
S1-p14	100	**866.37**	**866.37**	**866.37**	**866.37**	1.19	866.37	866.37
S2-p01	240	5648.04	5627.54	5632.05	**5627.00**	11.68	5626.81	5623.47
S2-p02	320	8459.73	8447.92	**8440.25**	8446.65	20.75	8431.66	8404.61
S2-p03	400	**11036.22**	**11036.22**	**11036.22**	**11036.22**	27.99	11036.22	11036.22
S2-p04	480	13728.80	13624.52	**13618.55**	13624.53	43.67	13592.88	13624.53
S2-p05	200	**6460.98**	**6460.98**	**6460.98**	**6460.98**	2.56	6460.98	6460.98
S2-p06	280	8412.90	**8412.88**	8413.41	8412.90	8.38	8404.26	8412.90
S2-p07	360	10267.50	10195.56	10186.93	**10157.63**	22.94	10156.58	10102.68
S2-p08	440	11865.40	11663.55	11691.54	**11646.58**	40.67	11663.55	11635.34
S2-p09	255	596.89	583.39	**581.46**	581.79	16.22	580.02	579.71
S2-p10	323	751.41	741.56	**739.56**	739.86	25.86	738.44	736.26
S2-p11	399	939.74	918.45	**916.27**	916.44	45.61	914.03	912.84
S2-p12	483	1152.88	1107.19	1108.21	**1106.73**	95.67	1104.84	1102.69
S2-p13	252	877.71	859.11	**858.42**	859.64	9.36	857.19	857.19
S2-p14	320	1089.93	1081.31	**1080.84**	1082.41	14.12	1080.55	1080.55
S2-p15	396	1371.61	1345.23	1344.32	**1343.52**	39.15	1340.24	1337.92
S2-p16	480	1650.94	1622.69	1622.26	**1621.02**	58.27	1616.33	1612.50
S2-p17	240	717.09	707.79	**707.78**	708.09	7.06	707.76	707.76
S2-p18	300	1018.74	998.73	**995.91**	998.44	14.40	995.13	995.13
S2-p19	360	1385.60	1366.86	**1366.70**	1367.83	27.91	1365.97	1365.60
S2-p20	420	1846.55	**1820.09**	1821.65	1822.02	38.23	1819.99	1818.32
Avg Time		—	14.20 min	17.64 min	17.69 min			
Gap overall		+1.00%	+0.13%	+0.10%	+0.10%			
Gap S1		+0.24%	+0.03%	+0.03%	+0.05%			
Gap S2		+1.54%	+0.20%	+0.15%	+0.14%			
Gap $n \geq 150$		+1.40%	+0.18%	+0.13%	+0.14%			

Table II.6: Behaviour of HGSADC on CVRP instances, when run times increase

Inst	**n**	**10^4it**		**2.10^4it**		**5.10^4it**			*HGSADC*
		Avg sol.	*T(min)*	*Avg sol.*	*T(min)*	*Avg sol.*	*T(min)*	*prev BKS*	*new BKS*
S1-p01	50	**524.61**	0.43	**524.61**	0.96	**524.61**	2.96	524.61	524.61
S1-p02	75	**835.26**	0.96	**835.26**	1.84	**835.26**	4.98	835.26	835.26
S1-p03	100	**826.14**	1.27	**826.14**	2.30	**826.14**	6.33	826.14	826.14
S1-p04	150	**1028.42**	2.87	1028.56	4.46	**1028.42**	10.65	1028.42	1028.42
S1-p05	199	1294.06	5.94	1294.41	7.83	**1291.74**	19.52	1291.29	1291.45
S1-p06	50	**555.43**	0.48	**555.43**	1.07	**555.43**	3.28	555.43	555.43
S1-p07	75	**909.68**	1.09	**909.68**	2.10	**909.68**	5.89	909.68	909.68
S1-p08	100	**865.94**	1.14	**865.94**	2.38	**865.94**	6.68	865.94	865.94
S1-p09	150	**1162.55**	2.53	**1162.55**	4.78	**1162.55**	12.00	1162.55	1162.55
S1-p10	199	1400.23	8.22	1398.58	18.37	**1397.70**	33.09	1395.85	1395.85
S1-p11	120	**1042.11**	1.15	**1042.11**	2.35	**1042.11**	6.65	1042.11	1042.11
S1-p12	100	**819.56**	0.84	**819.56**	1.73	**819.56**	5.06	819.56	819.56
S1-p13	120	1543.07	2.83	1543.04	5.37	**1542.86**	12.65	1541.14	1541.14
S1-p14	100	**866.37**	1.19	**866.37**	2.35	**866.37**	6.83	866.37	866.37
S2-p01	240	5627.00	11.68	5625.44	24.18	**5625.10**	66.90	5626.81	5623.47
S2-p02	320	8446.65	20.75	8438.79	39.52	**8419.25**	103.91	8431.66	8404.61
S2-p03	400	**11036.22**	27.99	**11036.22**	53.52	**11036.22**	116.43	11036.22	11036.22
S2-p04	480	**13624.53**	43.67	**13624.53**	50.67	**13624.53**	175.22	13592.88	13624.53
S2-p05	200	**6460.98**	2.56	**6460.98**	4.96	**6460.98**	13.25	6460.98	6460.98
S2-p06	280	**8412.90**	8.38	**8412.90**	15.94	**8412.90**	41.95	8404.26	8412.90
S2-p07	360	10157.63	22.94	**10132.61**	43.56	10134.90	108.49	10156.58	10102.68
S2-p08	440	11646.58	40.67	**11635.34**	63.65	**11635.34**	152.42	11663.55	11635.34
S2-p09	255	581.79	16.22	581.65	30.44	**581.08**	51.69	580.02	579.71
S2-p10	323	739.86	25.86	739.94	48.89	**738.92**	104.02	738.44	736.26
S2-p11	399	916.44	45.61	915.54	78.51	**914.37**	131.23	914.03	912.84
S2-p12	483	1106.73	95.67	1106.93	112.03	**1105.97**	196.52	1104.84	1102.69
S2-p13	252	859.64	9.36	859.24	19.24	**859.08**	25.56	857.19	857.19
S2-p14	320	1082.41	14.12	1082.21	18.13	**1081.99**	43.35	1080.55	1080.55
S2-p15	396	1343.52	39.15	1343.29	43.97	**1341.95**	109.14	1340.24	1337.92
S2-p16	480	1621.02	58.27	1619.49	79.24	**1616.92**	174.70	1616.33	1612.50
S2-p17	240	708.09	7.06	707.85	10.27	**707.84**	28.98	707.76	707.76
S2-p18	300	998.44	14.40	998.18	20.91	**996.95**	40.83	995.13	995.13
S2-p19	360	1367.83	27.91	1366.92	35.94	**1366.39**	81.49	1365.97	1365.60
S2-p20	420	1822.02	38.23	1822.09	45.03	**1819.75**	88.31	1819.99	1818.32
6.61 min		17.69 min		26.37 min		58.56 min			
Gap overall		+0.10%		+0.08%		+0.03%			
Gap S1		+0.05%		+0.04%		+0.02%			
Gap S2		+0.14%		+0.10%		+0.04%			
Gap $n \geq 150$		+0.14%		+0.10%		+0.04%			

Annexe III

SUPPLEMENT TO "A HYBRID GA FOR A LARGE CLASS OF VRPTW"

III.1 Mathematical model for the generalized PVRPTW

Equations (III.1-III.13) introduce a mathematical formulation of the generalized periodic vehicle routing problem with time windows. For convenience, the depot node v_0 has been modeled by two nodes v_0 and v_{n+1} representing the origin and destination nodes, respectively. We also identify the set of customer vertices as $\mathcal{V}^{\text{CST}} = \mathcal{V}\backslash\{v_0, v_{n+1}\}$. The model relies on binary constants a_{pl}, equal to 1 if and only if period l belongs to pattern p. Binary decision variables x_{ijkl} take value 1 if and only if vehicle k in period l visits v_j immediately after v_i. Binary variables y_{ip} take value 1 if and only if customer i is assigned to pattern p. Finally, the continuous variables t_{ikl} stand for the start of service of customer v_i, when serviced by vehicle k during period l.

$$\text{Minimize} \quad \sum_{v_i \in \mathcal{V}} \sum_{v_j \in \mathcal{V}} \sum_{k=1}^{m} \sum_{l=1}^{t} c_{ijl} x_{ijkl} \tag{III.1}$$

$$\text{Subject to:} \quad \sum_{p \in L_i} y_{ip} = 1 \qquad v_i \in \mathcal{V}^{\text{CST}} \tag{III.2}$$

$$\sum_{v_j \in \mathcal{V}} \sum_{k=1}^{m} x_{ijkl} - \sum_{p \in L_i} a_{pl} y_{ip} = 0 \qquad v_i \in \mathcal{V}^{\text{CST}}; l = 1 \ldots t \tag{III.3}$$

$$\sum_{v_j \in \mathcal{V}\backslash\{v_{n+1}\}} x_{jikl} - \sum_{v_j \in \mathcal{V}\backslash\{v_0\}} x_{ijkl} = 0 \qquad v_i \in \mathcal{V}^{\text{CST}}; k = 1 \ldots m; l = 1 \ldots t \tag{III.4}$$

$$\sum_{v_j \in \mathcal{V}\backslash\{v_0\}} x_{0jkl} = 1 \qquad k = 1 \ldots m; l = 1 \ldots t \tag{III.5}$$

$$\sum_{v_j \in \mathcal{V}\backslash\{v_{n+1}\}} x_{j,n+1,kl} = 1 \qquad k = 1 \ldots m; l = 1 \ldots t \tag{III.6}$$

$$\sum_{v_i \in \mathcal{V}} \sum_{v_j \in \mathcal{V}} q_i x_{ijkl} \leq Q \qquad k = 1 \ldots m; l = 1 \ldots t \tag{III.7}$$

$$x_{ijkl}(t_{ikl} + \delta_{ijl} + \tau_i - t_{jkl}) \leq 0 \qquad (v_i, v_j) \in \mathcal{V}^2; k = 1 \ldots m; l = 1 \ldots t \tag{III.8}$$

$$e_i \leq t_{ikl} \leq l_i \qquad v_i \in \mathcal{V}; k = 1 \ldots m; l = 1 \ldots t \tag{III.9}$$

$$t_{n+1,kl} - t_{0kl} \leq D \qquad k = 1 \ldots m; l = 1 \ldots t \tag{III.10}$$

$$x_{ijkl} \in \{0, 1\} \qquad (v_i, v_j) \in \mathcal{V}^2; k = 1 \ldots m; l = 1 \ldots t \tag{III.11}$$

$$y_{ip} \in \{0, 1\} \qquad v_i \in \mathcal{V}; p \in L_i \tag{III.12}$$

$$t_{ikl} \in \Re^+ \qquad v_i \in \mathcal{V}; k = 1 \ldots m; l = 1 \ldots t \tag{III.13}$$

Three main groups of constraints constitute the building blocks of the model. The first group, Constraints (III.2-III.3), corresponds to the assignment of customers to patterns, and its impact on routes. The next group of constraints (III.4-III.7) presents an underlying network flow structure to model the route choices, with flow conservation constraints (III.4), and capacity restrictions (III.7). Finally, the last group, Constraints (III.8-III.10), models the service time to customers, and enforces the temporal constraints (duration and time windows). Under the assumption that the graph does not admit a cycle with null total travel time, sub-tours are implicitly eliminated by Equations (III.8). This constraint can also be linearized using big M values.

III.2 Proof of Proposition 4.6.1: Minimum duration, time-warp use, and concatenation of sequences

When the starting date of a visit sequence is set, an optimal policy for minimizing duration and time-window infeasibility on the remaining vertices involves to service each customer as early as possible, and use time warp only upon a late arrival to reach the end of a customer's time window. To demonstrate Proposition 4.6.1, we prove by induction on the concatenation operator the stronger Proposition III.2.1, which characterizes the minimum duration and time-warp use to service a sequence σ of visits, as a function of the service date t of the first sequence's vertex (i.e., the departure date when the first vertex is a depot). This proposition also provides the means to calculate the characteristic functions of the concatenation of two sequences from the functions of the separate sequences.

Proposition III.2.1 *There exists a set of starting dates t that minimize both the time-warp use and the duration to service a sequence σ. This set is a segment, notated $\mathcal{T}^{\text{MIN}}(\sigma) = [E(\sigma), L(\sigma)]$. The minimum duration $D(\sigma)(t)$ and time-warp use $TW(\sigma)(t)$ as a function of t can be expressed as follows, where $D(\sigma)$ and $TW(\sigma)$ represent the minimum duration and time-warp use:*

$$D(\sigma)(t) = D(\sigma) + (E(\sigma) - t)^+ \tag{III.14}$$

$$TW(\sigma)(t) = TW(\sigma) + (t - L(\sigma))^+ \tag{III.15}$$

Furthermore, let $\sigma = (\sigma_i, \ldots, \sigma_j)$ and $\sigma' = (\sigma'_{i'}, \ldots, \sigma'_{j'})$ be two visit sequences, then the sequence $\sigma \oplus \sigma'$ is characterized by the following data:

$$D(\sigma \oplus \sigma') = D(\sigma) + D(\sigma') + \delta_{\sigma_j \sigma'_{i'}} + \Delta_{WT} \tag{III.16}$$

$$TW(\sigma \oplus \sigma') = TW(\sigma) + TW(\sigma') + \Delta_{TW} \tag{III.17}$$

$$E(\sigma \oplus \sigma') = \max\{E(\sigma') - \Delta, E(\sigma)\} - \Delta_{WT} \tag{III.18}$$

$$L(\sigma \oplus \sigma') = \min\{L(\sigma') - \Delta, L(\sigma)\} + \Delta_{TW} \tag{III.19}$$

where $\Delta = D(\sigma) - TW(\sigma) + \delta_{\sigma_j \sigma'_{i'}}$, $\Delta_{WT} = \max\{E(\sigma') - \Delta - L(\sigma), 0\}$ and $\Delta_{TW} = \max\{E(\sigma) + \Delta - L(\sigma'), 0\}$.

Proof For a sequence σ^0 with a single service to v_i, $D(\sigma^0) = \tau_i$, $TW(\sigma^0) = 0$, $E(\sigma^0) = e_i$, $L(\sigma^0) = l_i$. A schedule starting at t means to wait if $t \leq e_i$ or use a time warp if $t \geq l_i$, and then perform the service. Starting at t, the minimum duration is $D(\sigma)(t) = \tau_i + (e_i - t)^+$, while the minimum time-warp use is $TW(\sigma)(t) = (t - l_i)^+$, thus satisfying the statements of Proposition III.2.1.

Let σ and σ' be two visit sequences with their characteristic functions $D(\sigma)(t)$, $TW(\sigma)(t)$, $D(\sigma')(t)$ and $TW(\sigma')(t)$. The minimum duration of a schedule starting at t for $\sigma \oplus \sigma'$ is the sum of the duration to process the sequence σ at t, reach the first customer of σ' at time $t + D(\sigma)(t) - TW(\sigma)(t) + \delta_{\sigma_j \sigma'_{i'}}$, and perform the service of σ':

$$\begin{aligned}
&D(\sigma \oplus \sigma')(t) = D(\sigma)(t) + \delta_{\sigma_j \sigma'_{i'}} + D(\sigma')(t + D(\sigma)(t) - TW(\sigma)(t) + \delta_{\sigma_j \sigma'_{i'}}) \\
&= D(\sigma) + \max\{E(\sigma) - t, 0\} + \delta_{\sigma_j \sigma'_{i'}} + D(\sigma') \\
&+ \max\{E(\sigma') - t - \Delta + \max\{t - L(\sigma), 0\} - \max\{E(\sigma) - t, 0\}, 0\} \\
&= D(\sigma) + \delta_{\sigma_j \sigma'_{i'}} + D(\sigma') + \max\{E(\sigma') - \Delta - L(\sigma), 0, E(\sigma') - \Delta - t, E(\sigma) - t\} \\
&= D(\sigma) + \delta_{\sigma_j \sigma'_{i'}} + D(\sigma') + \max\{\delta_{WT}, \max\{E(\sigma') - \Delta, E(\sigma)\} - t\} \\
&= D(\sigma) + \delta_{\sigma_j \sigma'_{i'}} + D(\sigma') + \delta_{WT} + \max\{0, \max\{E(\sigma') - \Delta, E(\sigma)\} - \delta_{WT} - t\} \\
&= D(\sigma \oplus \sigma') + \max\{0, E(\sigma \oplus \sigma') - t\}
\end{aligned} \tag{III.20}$$

The function profile of Proposition (III.2.1) and the values of $D(\sigma \oplus \sigma')$ and $E(\sigma \oplus \sigma')$ are thus respected. The minimum time-warp use of a schedule starting at t can be also calculated in a similar manner as follows:

$$\begin{aligned}
&TW(\sigma \oplus \sigma')(t) = TW(\sigma)(t) + TW(\sigma')(t + D(\sigma)(t) - TW(\sigma)(t) + \delta_{\sigma_j \sigma'_{i'}}) \\
&= TW(\sigma) + \max\{t - L(\sigma), 0\} + TW(\sigma') \\
&+ \max\{t + \Delta - L(\sigma') + \max\{E(\sigma) - t, 0\} - \max\{t - L(\sigma), 0\}, 0\} \\
&= TW(\sigma) + TW(\sigma') + \max\{E(\sigma) + \Delta - L(\sigma'), 0, t + \Delta - L(\sigma'), t - L(\sigma)\} \\
&= TW(\sigma) + TW(\sigma') + \max\{\delta_{TW}, t + \max\{\Delta - L(\sigma'), -L(\sigma)\}\} \\
&= TW(\sigma) + TW(\sigma') + \delta_{TW} + \max\{0, t - (\min\{L(\sigma') - \Delta, L(\sigma)\} + \delta_{TW})\} \\
&= TW(\sigma \oplus \sigma') + \max\{0, t - L(\sigma \oplus \sigma')\}
\end{aligned} \tag{III.21}$$

Again, the profile of $TW(\sigma \oplus \sigma')(t)$, and the values of $TW(\sigma \oplus \sigma')$ and $L(\sigma \oplus \sigma')$ of Proposition (III.2.1) are correct. It only remains to show that $E(\sigma \oplus \sigma') \leq L(\sigma \oplus \sigma')$, which is equivalent to Equation III.22:

$$\begin{aligned}
&L(\sigma \oplus \sigma') - E(\sigma \oplus \sigma') = \min\{L(\sigma') - \Delta, L(\sigma)\} - \max\{E(\sigma') - \Delta, E(\sigma)\} \\
&+ \max\{E(\sigma') - \Delta - L(\sigma), 0\} + \max\{E(\sigma) + \Delta - L(\sigma'), 0\} \geq 0
\end{aligned} \tag{III.22}$$

If $\delta_{WT} = 0$, then $L(\sigma \oplus \sigma') - E(\sigma \oplus \sigma') \geq \min\{L(\sigma') - \Delta, L(\sigma)\} - \min\{L(\sigma') - \Delta, L(\sigma)\} \geq 0$.

Otherwise, if $\delta_{TW} = 0$, then $L(\sigma \oplus \sigma') - E(\sigma \oplus \sigma') \geq \min\{L(\sigma') - \Delta, L(\sigma)\} - \min\{E(\sigma') - \Delta, L(\sigma') - \Delta\} \geq 0$.

Finally, if $\delta_{WT} = \delta_{TW} = 0$, then $\min\{L(\sigma') - \Delta, L(\sigma)\} - \min\{E(\sigma') - \Delta, E(\sigma)\} \geq 0$, as $L(\sigma') - \Delta \geq E(\sigma') - \Delta$; $L(\sigma) \geq E(\sigma)$; $L(\sigma') - \Delta \geq E(\sigma)$ because of $\delta_{TW} = 0$; and $L(\sigma) \geq E(\sigma') - \Delta$ following from $\delta_{WT} = 0$.

All the statements of Proposition 2 are thus proven by induction on the concatenation operation, and Proposition 4.6.1 follows. □

III.3 Best solutions found on VRPTW and PVRPTW instances

Tables III.1 and III.2 present the best solutions found by HGSADC during all the experiments on the PVRPTW instances of Pirkwieser and Raidl (2009b), and the VRPTW instances of Gehring and Homberger (1999). New best known solutions are indicated in boldface.

Table III.1: HGSADC best solutions on Pirkwieser and Raidl (2009b) PVRPTW instances. Distances truncated to the first digit.

#	T4-R1	T4-C1	T4-RC1	T6-R1	T6-C1	T6-RC1	T8-R1	T8-C1	T8-RC1
1	**4082.0**	2907.4	3956.4	**5376.1**	**3981.2**	**5781.5**	**6471.3**	**4679.1**	**6847.2**
2	**3724.3**	**2882.9**	**3755.7**	**5201.6**	3841.7	**5333.3**	**6097.9**	4933.3	**5763.3**
3	3153.1	**2734.5**	**3449.9**	**3940.5**	**3523.6**	**4273.1**	**4687.0**	**4664.0**	**5424.9**
4	**2566.0**	**2419.0**	**2991.5**	**3335.8**	**3206.3**	**4062.0**	**4355.8**	**4591.6**	**4929.5**
5	3638.9	2884.1	**3932.6**	**4272.9**	4052.1	**5227.1**	5476.5	**5134.2**	**6203.4**

Table III.2: HGSADC best solutions on Gehring and Homberger (1999) large-scale VRPTW instances.

n	#	R1		R2		C1		C2		RC1		RC2	
200	1	20	4784.11	4	4483.16	20	2704.57	6	1931.44	18	**3602.80**	6	3099.53
	2	18	**4040.60**	4	3621.20	18	2917.89	6	1863.16	18	**3249.05**	5	2825.24
	3	18	3381.96	4	2880.62	18	2707.35	6	1775.08	18	3008.33	4	2601.88
	4	18	3057.81	4	1981.30	18	2643.31	6	1703.43	18	2851.68	4	2038.56
	5	18	4107.86	4	3366.79	20	2702.05	6	**1878.85**	18	**3371.00**	4	2911.46
	6	18	**3583.14**	4	2913.03	20	2701.04	6	1857.35	18	**3324.80**	4	2873.12
	7	18	3150.11	4	2451.14	20	2701.04	6	1849.46	18	3189.32	4	2525.83
	8	18	2951.99	4	1849.87	19	2775.48	6	1820.53	18	3083.93	4	**2292.53**
	9	18	**3760.58**	4	3092.04	18	2687.83	6	1830.05	18	3081.13	4	2175.04
	10	18	3301.18	4	2654.97	18	2643.55	6	1806.58	18	**3000.30**	4	2015.61
400	1	40	**10372.31**	8	9210.15	40	7152.06	12	4116.14	36	**8576.97**	11	6682.37
	2	36	**8926.70**	8	7606.75	36	**7686.38**	12	3930.05	36	**7905.66**	9	**6191.24**
	3	36	**7821.95**	8	**5911.07**	36	**7060.67**	11	**4018.02**	36	**7540.59**	8	4930.84
	4	36	**7282.78**	8	4241.47	36	**6803.24**	11	**3702.49**	36	**7310.35**	8	3631.01
	5	36	**9246.63**	8	7132.14	40	7152.06	12	3938.69	36	**8185.21**	9	5893.84
	6	36	**8387.62**	8	**6125.82**	40	7153.45	12	3875.94	36	**8177.80**	8	5766.61
	7	36	**7641.22**	8	**5018.53**	39	**7417.92**	12	3894.16	36	**7957.64**	8	5360.34
	8	36	**7275.13**	8	4018.01	37	**7349.01**	**11**	**4303.69**	36	**7797.02**	8	4799.02
	9	36	**8719.19**	8	**6400.10**	36	**7043.74**	12	3865.65	36	**7752.77**	8	4551.81
	10	36	**8113.93**	8	5805.87	36	6860.63	11	**3827.15**	36	**7609.21**	8	**4278.61**
600	1	59	**21408.13**	11	**18206.80**	60	**14095.64**	18	7774.16	55	**17118.70**	14	13368.77
	2	54	**18863.43**	11	14807.67	56	14163.31	17	**8273.78**	55	**16044.93**	12	**11555.51**
	3	54	**17040.40**	11	11200.10	56	13778.75	17	**7523.12**	55	**15273.98**	11	**9444.99**
	4	54	**15819.62**	11	**8029.37**	56	13563.17	17	6911.35	55	**14839.61**	11	**7076.49**
	5	54	**19771.90**	11	**15098.49**	60	**14085.72**	18	7575.20	55	**16693.26**	11	13138.99
	6	54	**18041.87**	11	12506.57	60	14089.66	18	**7471.17**	55	**16632.03**	11	11977.46
	7	54	**16615.13**	11	**10066.34**	58	14848.38	18	**7512.07**	55	**16145.64**	11	10779.24
	8	54	**15696.58**	11	7609.96	56	**14429.48**	17	**7547.67**	55	**15978.70**	11	10009.46
	9	54	**18708.67**	11	13483.92	56	**13693.42**	17	**8015.73**	55	**15922.60**	11	9583.65
	10	54	**17801.43**	11	12279.01	56	13637.34	17	**7255.69**	55	**15740.26**	11	9078.64
800	1	80	36860.69	15	**28117.94**	80	**25184.39**	24	11662.08	72	**31710.68**	18	**21014.85**
	2	72	**32598.51**	15	22811.82	74	25430.07	23	12378.53	72	**29034.99**	16	18197.14
	3	72	**29506.45**	15	17741.68	72	**24278.18**	23	**11410.69**	72	**27905.64**	15	14467.00
	4	72	**27931.57**	15	13219.06	72	**23841.11**	**22**	**11154.40**	72	**26875.90**	15	**11006.56**
	5	72	**33861.43**	15	24350.52	80	**25166.28**	24	11425.23	72	**30277.12**	15	19147.21
	6	72	**31154.87**	15	20480.79	80	**25160.85**	24	**11347.35**	72	**30262.33**	15	18160.91
	7	72	**29010.78**	15	16697.82	78	**25845.05**	24	**11370.84**	72	**29862.44**	15	16852.17
	8	72	**27766.11**	15	12748.16	74	**25293.09**	23	**11292.10**	72	**29194.16**	15	15808.99
	9	72	**32629.99**	15	22423.76	72	**24389.50**	23	**11645.22**	72	**28978.35**	15	15390.38
	10	72	**31187.35**	15	20459.29	72	**24090.10**	23	**10981.00**	72	**28797.79**	15	14454.62
1000	1	100	53657.99	19	42317.54	100	**42478.95**	30	16879.24	90	**46272.07**	20	30343.11
	2	91	**49105.21**	19	33567.91	90	**42300.76**	29	17130.76	90	**44129.42**	**18**	**26327.92**
	3	91	**45237.29**	19	25053.80	90	**40239.23**	**28**	**16886.80**	90	**42487.54**	18	20053.78
	4	91	**42845.99**	19	18039.77	90	**39501.23**	28	**15656.75**	90	**41613.58**	18	15747.13
	5	91	**51869.67**	19	36446.65	100	**42469.18**	30	**16561.29**	90	**45564.81**	18	27237.68
	6	91	**47849.05**	19	30223.14	100	**42471.28**	29	**16951.39**	90	**45303.67**	18	26986.30
	7	91	**44525.53**	19	23452.85	98	42873.78	**29**	**18162.46**	90	**44903.80**	18	25295.67
	8	91	**42597.89**	19	17602.31	93	**42220.24**	**28**	**16577.32**	90	**44366.01**	18	23787.26
	9	91	**50490.49**	19	33231.28	90	**40570.60**	29	16432.53	90	**44280.84**	18	23116.15
	10	91	**48578.49**	19	30598.69	90	**39933.06**	28	**15944.72**	90	**43896.78**	18	22076.90

Annexe IV

SUPPLEMENT TO "HOS REGULATIONS IN ROAD FREIGHT TRANSPORT"

Table IV.1 presents detailed results of our experiments conducted on the Goel (2009) instances. The table shows the average fleet size and distance as well as the best fleet size and distance obtained from the five HGSADC runs. Tables IV.2, IV.3, and IV.4 present detailed results of our experiments on the modified instances with multiple time windows. The tables report the average distance and fleet size as well as the best distance and fleet size obtained from the five HGSADC runs, for regulations in North America, Europe, and Australia, as well as the results obtained without hours of service constraints.

Table IV.1: Detailed results for Goel (2009) instances and European Union regulations

	EU (No split)				**EU (All)**			
Instance	Avg Fleet	Avg Dist.	Best Fleet	Best Dist.	Avg Fleet	Avg Dist.	Best Fleet	Best Dist.
R101	10.00	1332.93	10.00	1326.78	8.20	1452.47	8.00	1482.44
R102	8.80	1196.98	8.00	1283.86	8.00	1187.97	8.00	1176.58
R103	8.00	979.00	8.00	977.25	8.00	966.62	8.00	965.92
R104	8.00	859.96	8.00	859.27	8.00	853.26	8.00	852.99
R105	8.00	1114.32	8.00	1109.96	8.00	1095.53	8.00	1093.63
R106	8.00	1018.79	8.00	1017.71	8.00	999.53	8.00	997.83
R107	8.00	900.93	8.00	900.93	8.00	900.99	8.00	898.05
R108	8.00	839.30	8.00	838.54	8.00	839.63	8.00	837.99
R109	8.00	930.03	8.00	928.43	8.00	923.12	8.00	922.40
R110	8.00	885.03	8.00	881.30	8.00	876.75	8.00	875.80
R111	8.00	880.54	8.00	880.54	8.00	874.84	8.00	873.93
R112	8.00	831.31	8.00	831.31	8.00	829.75	8.00	829.14
R201	7.00	1261.50	7.00	1256.06	7.00	1210.10	7.00	1205.42
R202	6.00	1120.13	6.00	1116.22	6.00	1091.54	6.00	1087.36
R203	6.00	921.35	6.00	918.82	5.00	926.25	5.00	921.72
R204	5.00	776.27	5.00	774.96	5.00	771.22	5.00	770.21
R205	6.00	1018.80	6.00	1011.29	6.00	1000.76	6.00	997.44
R206	5.60	945.67	5.00	958.59	5.20	942.83	5.00	943.83
R207	5.00	857.69	5.00	854.01	5.00	835.75	5.00	830.96
R208	5.00	745.52	5.00	745.15	5.00	742.61	5.00	742.61
R209	6.00	905.89	6.00	904.21	5.00	926.08	5.00	910.70
R210	6.00	943.64	6.00	943.64	5.60	946.54	5.00	959.58
R211	5.00	797.92	5.00	796.30	5.00	783.46	5.00	783.46
C101	10.40	928.70	10.00	931.37	10.00	828.94	10.00	828.94
C102	10.00	908.70	10.00	904.52	10.00	828.94	10.00	828.94
C103	10.00	833.19	10.00	833.19	10.00	827.34	10.00	827.34
C104	10.00	819.81	10.00	819.81	10.00	819.81	10.00	819.81
C105	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94
C106	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94
C107	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94
C108	10.00	827.38	10.00	827.38	10.00	827.38	10.00	827.38
C109	10.00	825.65	10.00	825.65	10.00	825.65	10.00	825.65
C201	6.00	853.12	6.00	852.94	5.00	802.65	5.00	800.18
C202	5.00	811.71	5.00	811.15	5.00	692.66	5.00	692.66
C203	5.00	695.54	5.00	695.54	4.00	660.07	4.00	660.07
C204	4.00	661.57	4.00	661.57	4.00	651.91	4.00	650.28
C205	5.00	683.75	5.00	683.75	5.00	678.33	5.00	678.33
C206	5.00	680.78	5.00	680.78	4.00	675.27	4.00	675.27
C207	5.00	693.97	5.00	693.97	5.00	672.42	5.00	672.42
C208	5.00	673.61	5.00	673.61	4.00	672.49	4.00	672.30
RC101	9.00	1314.22	9.00	1305.09	9.00	1296.37	9.00	1286.03
RC102	9.00	1185.39	9.00	1180.34	9.00	1172.32	9.00	1159.33
RC103	9.00	1081.36	9.00	1080.40	9.00	1076.59	9.00	1075.81
RC104	9.00	993.19	9.00	993.19	9.00	993.13	9.00	993.13
RC105	9.00	1228.41	9.00	1227.14	9.00	1205.77	9.00	1203.02
RC106	9.00	1097.79	9.00	1093.62	9.00	1092.98	9.00	1092.80
RC107	9.00	1027.98	9.00	1027.89	9.00	1017.46	9.00	1016.96
RC108	9.00	986.73	9.00	985.05	9.00	979.69	9.00	978.93
RC201	8.00	1385.00	8.00	1384.01	7.00	1375.75	7.00	1344.99
RC202	7.00	1193.72	7.00	1193.12	7.00	1162.65	7.00	1162.28
RC203	6.00	1040.33	6.00	1036.96	6.00	1015.51	6.00	1012.13
RC204	5.00	878.88	5.00	877.17	5.00	860.81	5.00	860.17
RC205	7.00	1328.51	7.00	1313.71	7.00	1230.56	7.00	1228.09
RC206	6.00	1168.93	6.00	1160.40	6.00	1128.02	6.00	1124.17
RC207	6.00	1087.30	6.00	1079.01	6.00	1046.74	6.00	1038.04
RC208	5.00	878.32	5.00	872.87	5.00	834.60	5.00	834.30
All	413.80	53323.83	412.00	53307.16	403.00	52417.21	402.00	52316.57

Table IV.2: Detailed results for modified Goel (2009) instances and U.S. and Canadian regulations

Instance &		**US (current)**				**US (2013)**				**CAN**			
Fleet size		Avg Dist.	Avg Fleet	Best Dist.	Best Fleet	Avg Dist.	Avg Fleet	Best Dist.	Best Fleet	Avg Dist.	Avg Fleet	Best Dist.	Best Fleet
R101	12	1286.39	11.00	1285.50	11.00	1288.27	11.00	1285.50	11.00	1288.38	10.80	1287.09	11.00
R102	10	1163.08	9.20	1160.90	9.00	1168.72	9.20	1166.59	9.00	1171.53	10.00	1169.13	10.00
R103	10	985.53	9.00	985.34	9.00	985.34	9.00	985.34	9.00	986.16	9.00	984.43	9.00
R104	10	855.38	8.00	855.13	8.00	855.74	8.00	855.13	8.00	855.72	8.00	855.13	8.00
R105	10	1069.05	9.00	1068.77	9.00	1073.37	9.00	1071.85	9.00	1075.34	9.00	1073.83	9.00
R106	10	998.42	8.00	994.03	8.00	998.00	8.20	994.48	8.00	1006.43	8.20	1000.98	8.00
R107	10	904.27	8.00	900.90	8.00	905.73	8.00	901.72	8.00	905.94	8.00	902.03	8.00
R108	10	838.30	8.00	837.99	8.00	838.14	8.00	837.99	8.00	841.65	8.00	839.87	8.00
R109	10	973.74	9.00	967.95	9.00	975.91	9.00	969.65	9.00	970.86	9.00	962.40	9.00
R110	10	887.15	8.00	886.63	8.00	899.52	8.00	896.72	8.00	890.15	8.00	890.05	8.00
R111	10	895.70	8.00	893.07	8.00	896.17	8.00	893.68	8.00	894.73	8.40	893.86	8.00
R112	10	831.65	8.00	829.98	8.00	833.39	8.00	832.15	8.00	830.08	8.00	829.98	8.00
R201	9	1171.68	9.00	1171.68	9.00	1171.68	9.00	1171.68	9.00	1170.93	9.00	1168.57	9.00
R202	8	1063.43	8.00	1063.34	8.00	1065.31	8.00	1064.50	8.00	1053.40	8.00	1048.86	8.00
R203	7	898.56	6.40	895.14	6.00	915.60	7.00	910.07	7.00	908.88	6.60	899.85	6.00
R204	7	763.59	6.00	762.73	6.00	769.69	6.20	766.78	6.00	762.97	6.00	762.73	6.00
R205	7	1020.70	7.00	1018.51	7.00	1027.42	7.00	1021.96	7.00	1020.57	7.00	1017.57	7.00
R206	7	937.24	6.00	935.21	6.00	944.06	6.80	942.37	6.00	937.28	6.00	935.84	6.00
R207	7	840.54	6.00	834.44	6.00	841.99	6.00	840.37	6.00	842.50	6.00	840.37	6.00
R208	7	747.44	5.20	742.64	5.00	746.48	5.00	746.24	5.00	744.38	5.00	742.43	5.00
R209	7	918.23	6.20	916.33	6.00	920.37	6.40	918.15	6.00	918.06	6.00	917.88	6.00
R210	7	937.93	7.00	937.63	7.00	941.07	7.00	940.99	7.00	940.19	7.00	938.65	7.00
R211	7	803.77	6.00	801.26	6.00	801.65	5.80	800.54	5.00	802.30	6.00	801.26	6.00
C101	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C102	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C103	12	828.06	10.00	828.06	10.00	828.06	10.00	828.06	10.00	828.07	10.00	828.07	10.00
C104	12	819.81	10.00	819.81	10.00	819.81	10.00	819.81	10.00	819.81	10.00	819.81	10.00
C105	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C106	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C107	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C108	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C109	12	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00
C201	8	826.42	6.00	826.42	6.00	828.93	6.00	828.93	6.00	651.83	4.00	651.83	4.00
C202	7	755.92	5.00	753.59	5.00	769.14	5.60	761.88	5.00	647.41	4.00	647.41	4.00
C203	6	658.00	4.00	658.00	4.00	688.91	5.00	688.74	5.00	636.92	4.00	636.92	4.00
C204	6	638.00	4.00	638.00	4.00	680.35	4.80	666.74	4.00	634.17	4.00	634.17	4.00
C205	6	638.57	4.00	638.57	4.00	677.44	5.00	677.44	5.00	638.57	4.00	638.57	4.00
C206	6	637.33	4.00	637.33	4.00	676.72	5.00	676.25	5.00	638.57	4.00	638.57	4.00
C207	6	638.36	4.00	638.36	4.00	679.94	5.00	679.82	5.00	638.79	4.00	638.79	4.00
C208	6	637.70	4.00	637.33	4.00	676.35	5.00	675.85	5.00	638.57	4.00	638.57	4.00
RC101	11	1264.32	10.00	1260.57	10.00	1263.74	10.00	1263.21	10.00	1260.10	10.00	1259.30	10.00
RC102	11	1163.56	10.20	1157.66	10.00	1160.93	10.40	1157.66	10.00	1160.54	10.20	1157.67	10.00
RC103	11	1085.87	9.20	1080.70	9.00	1082.65	9.00	1080.70	9.00	1086.07	9.00	1082.67	9.00
RC104	11	993.13	9.00	993.13	9.00	993.13	9.00	993.13	9.00	993.13	9.00	993.13	9.00
RC105	11	1199.93	10.00	1197.25	10.00	1200.60	10.00	1200.60	10.00	1204.41	10.20	1201.23	10.00
RC106	11	1137.82	9.60	1132.84	9.00	1138.56	9.40	1132.84	9.00	1136.29	9.00	1134.90	9.00
RC107	11	1055.24	9.40	1045.69	9.00	1049.56	9.00	1045.69	9.00	1051.91	9.00	1049.76	9.00
RC108	11	989.86	9.00	988.98	9.00	990.41	9.00	989.44	9.00	989.77	9.00	989.77	9.00
RC201	9	1297.39	9.00	1296.88	9.00	1308.87	9.00	1308.37	9.00	1300.02	9.00	1299.09	9.00
RC202	8	1127.71	8.00	1127.68	8.00	1142.08	8.00	1142.08	8.00	1142.08	8.00	1142.08	8.00
RC203	7	999.28	7.00	998.46	7.00	1018.33	7.00	1017.34	7.00	995.68	7.00	995.68	7.00
RC204	7	846.89	6.00	846.09	6.00	857.26	6.00	856.46	6.00	848.74	6.00	847.73	6.00
RC205	7	1246.51	7.00	1239.91	7.00	1268.97	7.00	1268.16	7.00	1233.31	7.00	1231.23	7.00
RC206	7	1137.40	7.00	1133.52	7.00	1145.78	7.00	1144.94	7.00	1126.45	7.00	1126.45	7.00
RC207	7	1054.76	7.00	1048.07	7.00	1062.02	7.00	1054.39	7.00	1064.60	7.00	1062.27	7.00
RC208	7	849.94	6.00	849.94	6.00	862.39	6.00	861.70	6.00	849.11	6.00	847.87	6.00
All	508	52118.84	434.40	52017.23	432.00	52533.82	441.80	52434.00	437.20	51832.56	431.40	51755.55	430.00

Table IV.3: Detailed results for modified Goel (2009) instances and European Union regulations

Instance & Fleet size		**EU (No split)**				**EU (Split)**				**EU (All)**			
		Avg Dist.	Avg Fleet	Best Dist.	Best Fleet	Avg Dist.	Avg Fleet	Best Dist.	Best Fleet	Avg Dist.	Avg Fleet	Best Dist.	Best Fleet
R101	12	1300.78	12.00	1299.62	12.00	1294.23	11.00	1293.92	11.00	1290.68	11.00	1290.06	11.00
R102	10	1181.60	10.00	1173.01	10.00	1175.05	9.40	1172.56	9.00	1174.94	9.80	1173.56	9.00
R103	10	1006.61	10.00	1006.61	10.00	998.69	10.00	998.69	10.00	998.54	9.00	997.34	9.00
R104	10	864.61	9.00	864.61	9.00	863.23	8.00	862.92	8.00	860.87	8.00	860.54	8.00
R105	10	1093.31	9.00	1092.52	9.00	1084.13	10.00	1082.55	10.00	1084.64	9.20	1081.70	9.00
R106	10	1025.89	9.00	1021.10	9.00	1006.51	8.20	1001.86	8.00	1003.50	8.00	1001.57	8.00
R107	10	910.36	8.00	906.60	8.00	908.16	8.00	903.18	8.00	909.28	8.00	902.32	8.00
R108	10	843.51	8.00	838.54	8.00	838.35	8.00	837.99	8.00	837.99	8.00	837.99	8.00
R109	10	985.75	9.00	983.05	9.00	981.81	9.00	981.81	9.00	979.87	9.00	977.56	9.00
R110	10	908.49	8.00	901.45	8.00	908.28	8.00	901.45	8.00	906.75	8.00	901.09	8.00
R111	10	900.68	8.20	898.17	8.00	897.32	8.00	895.20	8.00	894.80	8.00	892.25	8.00
R112	10	833.98	8.00	832.16	8.00	833.44	8.00	832.16	8.00	834.54	8.00	832.16	8.00
R201	9	1189.68	9.00	1189.07	9.00	1188.18	9.00	1185.36	9.00	1181.30	9.00	1181.30	9.00
R202	8	1083.18	8.00	1082.86	8.00	1080.53	8.00	1080.42	8.00	1075.02	8.00	1069.76	8.00
R203	7	942.22	7.00	940.03	7.00	938.28	7.00	936.64	7.00	922.40	7.00	918.59	7.00
R204	7	777.94	6.20	775.16	6.00	775.27	6.20	770.59	6.00	770.24	6.00	768.57	6.00
R205	7	1054.68	7.00	1048.32	7.00	1037.79	7.00	1036.49	7.00	1036.90	7.00	1036.32	7.00
R206	7	951.25	7.00	948.79	7.00	946.68	7.00	945.08	7.00	944.67	6.80	942.81	7.00
R207	7	854.76	7.00	854.65	7.00	854.27	6.80	853.26	6.00	850.32	6.60	846.06	6.00
R208	7	753.56	6.00	750.96	6.00	754.67	6.00	750.96	6.00	753.26	5.00	750.33	5.00
R209	7	924.54	7.00	924.49	7.00	923.16	7.00	922.83	7.00	922.96	7.00	922.81	7.00
R210	7	950.49	7.00	947.88	7.00	947.79	7.00	946.13	7.00	943.60	7.00	943.05	7.00
R211	7	817.28	6.00	813.93	6.00	805.89	6.00	804.37	6.00	801.78	6.00	801.78	6.00
C101	12	920.37	11.00	920.37	11.00	920.41	11.00	920.37	11.00	828.94	10.00	828.94	10.00
C102	12	909.81	10.00	905.48	10.00	906.02	10.00	904.25	10.00	828.94	10.00	828.94	10.00
C103	12	834.75	10.00	834.75	10.00	834.75	10.00	834.75	10.00	828.06	10.00	828.06	10.00
C104	12	829.58	10.00	829.58	10.00	829.58	10.00	829.58	10.00	823.81	10.00	823.81	10.00
C105	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C106	12	834.79	10.00	834.79	10.00	834.79	10.00	834.79	10.00	828.94	10.00	828.94	10.00
C107	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C108	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C109	12	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00
C201	8	852.94	6.00	852.94	6.00	756.52	6.00	756.52	6.00	745.37	6.00	745.37	6.00
C202	7	798.37	6.00	798.37	6.00	758.38	6.00	758.38	6.00	734.37	5.60	731.09	5.00
C203	6	741.78	5.00	738.79	5.00	717.07	5.00	717.07	5.00	672.85	4.00	672.31	4.00
C204	6	716.48	5.00	711.43	5.00	702.01	5.00	701.08	5.00	680.38	4.60	672.66	4.00
C205	6	683.70	5.00	683.70	5.00	683.70	5.00	683.70	5.00	675.85	5.00	675.85	5.00
C206	6	694.90	5.00	694.10	5.00	691.43	5.00	691.43	5.00	680.42	5.00	680.03	5.00
C207	6	694.42	5.00	694.42	5.00	687.36	5.00	685.90	5.00	680.28	5.00	680.28	5.00
C208	6	683.65	5.00	683.34	5.00	683.55	5.00	683.34	5.00	675.85	5.00	675.85	5.00
RC101	11	1272.29	10.60	1268.97	11.00	1266.67	10.20	1266.32	10.00	1267.32	10.40	1264.96	10.00
RC102	11	1177.54	10.80	1175.75	10.00	1176.32	10.20	1175.37	10.00	1165.47	10.20	1162.65	10.00
RC103	11	1090.28	9.00	1088.59	9.00	1088.83	9.20	1085.73	9.00	1086.83	9.00	1083.62	9.00
RC104	11	993.30	9.00	993.30	9.00	993.30	9.00	993.30	9.00	994.08	9.00	994.08	9.00
RC105	11	1223.34	11.00	1223.34	11.00	1217.54	11.00	1213.76	11.00	1210.48	11.00	1208.42	11.00
RC106	11	1158.17	9.80	1147.20	9.00	1150.45	9.60	1139.92	9.00	1145.33	9.60	1137.08	9.00
RC107	11	1060.94	9.60	1058.07	9.00	1060.87	9.40	1057.72	9.00	1052.05	9.00	1052.05	9.00
RC108	11	990.94	9.00	990.47	9.00	991.09	9.00	990.47	9.00	990.16	9.00	989.44	9.00
RC201	9	1341.41	9.00	1340.63	9.00	1336.93	9.00	1336.93	9.00	1321.58	9.00	1320.71	9.00
RC202	8	1182.32	8.00	1179.05	8.00	1165.93	8.00	1164.41	8.00	1154.67	8.00	1152.92	8.00
RC203	7	1042.01	7.00	1041.49	7.00	1027.60	7.00	1027.13	7.00	1022.19	7.00	1017.34	7.00
RC204	7	869.17	6.00	869.17	6.00	867.32	6.00	867.23	6.00	857.45	6.00	857.45	6.00
RC205	7	1371.62	7.00	1371.62	7.00	1332.10	7.00	1328.00	7.00	1274.00	7.00	1273.66	7.00
RC206	7	1163.19	7.00	1162.27	7.00	1159.25	7.00	1154.98	7.00	1149.34	7.00	1148.95	7.00
RC207	7	1090.68	7.00	1074.65	7.00	1082.71	7.00	1072.58	7.00	1074.38	7.00	1068.64	7.00
RC208	7	882.27	6.40	877.63	6.00	876.95	6.20	875.89	6.00	873.36	6.00	870.99	6.00
All	508	53572.62	454.60	53450.26	452.00	53153.59	450.40	53059.78	447.00	52614.07	443.80	52517.07	440.00

Table IV.4: Detailed results for modified Goel (2009) instances and Australian regulations and without regulations

Instance &		**AUS (Std.)**				**AUS (BFM)**				**None**			
Fleet size		Avg Dist.	Avg Fleet	Best Dist.	Best Fleet	Avg Dist.	Avg Fleet	Best Dist.	Best Fleet	Avg Dist.	Avg Fleet	Best Dist.	Best Fleet
R101	12	1297.58	12.00	1297.56	12.00	1288.55	11.00	1288.55	11.00	1282.59	10.40	1278.95	10.00
R102	10	1178.81	10.00	1176.51	10.00	1175.12	10.00	1174.74	10.00	1160.75	9.40	1156.73	9.00
R103	10	1002.15	9.60	1000.08	9.00	1000.56	9.00	1000.43	9.00	981.99	8.20	981.51	9.00
R104	10	865.46	8.40	864.44	8.00	861.23	8.20	859.19	8.00	855.15	8.00	855.13	8.00
R105	10	1097.66	10.00	1097.12	10.00	1082.89	9.00	1078.67	9.00	1059.69	8.20	1056.52	8.00
R106	10	1023.13	8.80	1018.23	9.00	1007.05	8.20	1003.94	8.00	988.09	8.00	983.06	8.00
R107	10	911.82	8.00	908.46	8.00	905.50	8.00	901.29	8.00	902.87	8.00	900.03	8.00
R108	10	842.69	8.00	838.54	8.00	840.62	8.00	838.54	8.00	838.96	8.00	837.99	8.00
R109	10	985.61	9.00	983.94	9.00	981.34	9.00	976.66	9.00	968.59	9.00	962.40	9.00
R110	10	911.32	8.20	904.75	8.00	911.69	8.00	904.75	8.00	885.87	8.00	885.54	8.00
R111	10	899.12	8.00	897.61	8.00	898.18	8.00	893.96	8.00	893.70	8.00	892.26	8.00
R112	10	834.56	8.00	832.41	8.00	832.16	8.00	832.16	8.00	831.00	8.00	829.98	8.00
R201	9	1188.74	9.00	1188.27	9.00	1181.50	9.00	1179.79	9.00	1161.66	8.00	1159.14	8.00
R202	8	1085.24	8.00	1081.06	8.00	1068.09	8.00	1062.97	8.00	1044.94	7.60	1042.41	7.00
R203	7	937.05	7.00	935.60	7.00	926.13	7.00	922.60	7.00	892.50	6.20	890.85	6.00
R204	7	773.79	6.00	769.50	6.00	764.33	6.00	761.57	6.00	761.77	6.00	758.45	6.00
R205	7	1039.16	7.00	1038.09	7.00	1031.57	7.00	1028.97	7.00	1016.61	7.00	1013.37	7.00
R206	7	948.23	7.00	946.67	7.00	945.10	7.00	944.03	7.00	935.86	6.00	934.38	6.00
R207	7	855.64	7.00	854.40	7.00	854.03	7.00	852.28	7.00	827.18	6.00	827.18	6.00
R208	7	758.39	5.40	752.71	5.00	755.75	5.20	753.77	5.00	733.72	5.00	733.51	5.00
R209	7	924.24	7.00	924.18	7.00	922.83	7.00	922.83	7.00	913.79	6.00	911.96	6.00
R210	7	951.07	7.00	948.92	7.00	943.44	7.00	942.97	7.00	932.08	7.00	930.48	7.00
R211	7	823.50	6.20	822.33	6.00	811.91	6.20	808.51	6.00	802.52	5.80	800.63	5.00
C101	12	920.37	11.00	920.37	11.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C102	12	904.72	10.00	904.69	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C103	12	834.75	10.00	834.75	10.00	828.06	10.00	828.06	10.00	828.06	10.00	828.06	10.00
C104	12	823.81	10.00	823.81	10.00	819.81	10.00	819.81	10.00	819.81	10.00	819.81	10.00
C105	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C106	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C107	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C108	12	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00	828.94	10.00
C109	12	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00	825.65	10.00
C201	8	706.26	5.00	706.26	5.00	639.40	4.00	639.40	4.00	588.88	3.00	588.88	3.00
C202	7	730.26	5.00	728.82	5.00	664.66	4.00	664.66	4.00	588.88	3.00	588.88	3.00
C203	6	703.14	5.00	702.99	5.00	652.60	4.00	652.20	4.00	593.41	3.00	593.41	3.00
C204	6	669.32	4.00	669.32	4.00	646.87	4.00	646.70	4.00	593.03	3.00	593.03	3.00
C205	6	664.34	5.00	664.08	5.00	626.63	4.00	626.63	4.00	588.49	3.00	588.49	3.00
C206	6	669.16	4.20	667.32	4.00	641.26	4.00	640.89	4.00	588.49	3.00	588.49	3.00
C207	6	662.62	5.00	662.62	5.00	628.72	4.00	628.72	4.00	588.29	3.00	588.29	3.00
C208	6	665.87	4.60	664.98	4.00	654.62	4.00	654.62	4.00	601.05	3.00	601.05	3.00
RC101	11	1270.83	11.00	1270.15	11.00	1265.29	10.00	1263.82	10.00	1246.06	9.80	1243.20	9.00
RC102	11	1175.26	10.20	1173.04	10.00	1167.75	9.40	1164.55	9.00	1162.36	9.80	1156.30	9.00
RC103	11	1091.86	9.20	1088.18	9.00	1085.24	9.00	1084.06	9.00	1072.22	9.00	1072.22	9.00
RC104	11	993.30	9.00	993.30	9.00	993.14	9.00	993.13	9.00	993.13	9.00	993.13	9.00
RC105	11	1211.72	10.00	1208.22	10.00	1203.59	10.00	1203.38	10.00	1188.34	10.00	1186.15	10.00
RC106	11	1146.96	9.60	1139.01	9.00	1142.78	9.60	1134.71	9.00	1135.07	9.00	1132.52	9.00
RC107	11	1062.89	9.40	1060.22	9.00	1060.66	9.20	1057.72	9.00	1053.60	9.40	1049.76	9.00
RC108	11	990.22	9.00	989.44	9.00	989.75	9.00	989.44	9.00	989.44	9.00	988.09	9.00
RC201	9	1344.55	9.00	1342.10	9.00	1310.98	9.00	1307.20	9.00	1270.16	8.60	1266.50	9.00
RC202	8	1172.76	8.00	1171.30	8.00	1142.78	8.00	1142.78	8.00	1104.42	8.00	1103.98	8.00
RC203	7	1042.57	7.00	1042.57	7.00	1019.07	7.00	1017.77	7.00	958.40	5.20	954.86	5.00
RC204	7	870.87	6.00	870.67	6.00	860.73	5.80	859.57	5.00	812.18	5.00	812.18	5.00
RC205	7	1360.45	7.00	1339.21	7.00	1254.43	7.00	1243.12	7.00	1170.36	7.00	1170.36	7.00
RC206	7	1165.65	7.00	1161.43	7.00	1140.71	7.00	1138.82	7.00	1119.73	7.00	1118.13	7.00
RC207	7	1086.28	7.00	1078.14	7.00	1071.50	7.00	1062.89	7.00	1056.45	7.00	1051.32	7.00
RC208	7	876.44	6.00	873.15	6.00	868.36	6.00	862.70	6.00	849.52	6.00	847.86	6.00
All	508	53093.57	447.80	52972.93	444.00	52168.24	434.80	52059.78	432.00	51030.98	414.60	50946.68	411.00

Annexe V

SUPPLEMENT TO "A UNIFIED HYBRID GENETIC SEARCH"

Tables V.1 to V.28 present the detailed results of UHGS and other state-of-the-art methods for a variety of vehicle routing variants and benchmarks. The first group of columns displays the instance identifier, number of customers n, vehicle fleet limit m when applicable, and also the number of customer clusters c in the GVRP case, the number of vehicle types w in fleet size and mix settings, and the number of periods t and depots d in the MDPVRP case. The next group of columns presents the results of actual state-of-the-art methods for each problem, as well as the results of UHGS. When available, both average and best results on several runs (number of runs specified in the headings of the Table) are provided.

We indicate in boldface the best average result among algorithms for each instance as well as the previous Best Known Solution (BKS) in the last column. New best known solutions are underlined. Finally, average measures over sets of instances are presented in the last lines: the computation time for each method, the average percentage of error (Gap) relative to the previous BKS, and the processor used. Some specific details for each problem and benchmark are listed in the following.

VRP with Backhauls (VRPB). "Double" precision values have been used for distance computations. Comparison is made with methods that rely on the same assumption. A fleet size value m is specified in the instances. As in the previous works, we consider a fixed fleet size without allowing less or more than m vehicles. To that extent, the distance matrix has been modified by setting $d_{00} = +\infty$. Other variants of the VRPB, such as the vehicle routing with mixed backhauls, the VRPB with time windows or with multiple depots, can be addressed by UHGS. For the sake of conciseness, results on these variants are not reported in this paper.

Cumulative VRP (CCVRP). Following the guidelines of Ngueveu et al. (2010), the duration constraint is not considered and the fleet size limit is fixed to the minimum feasible value. In the original paper of NPC10, only the best solution on the benchmark instances of Christofides et al. (1979) were reported. This algorithm was then provided to Ribeiro and Laporte (2012) who ran more extensive experiments on all instances. We rely on these latter values for our experimental comparison.

VRP with Simultaneous Deliveries and Pickups (VRPSDP). "Double" precision values have been used for distances and demands. We only compare to recent methods that rely on the same convention.

Load Dependent VRP (LDVRP). As in previous work, maximum trip duration constraints are taken into account (sum of driving length plus service time) and the fleet size is unconstrained.

Generalized VRP (GVRP). Distances were rounded to the closest integer to compare with the recent works of Bektas et al. (2011) and Moccia et al. (2012) using the same assumptions.

Open VRP (OVRP). The hierarchical objective of fleet minimization and then distance is used. As almost all recent methods succeeded in reaching the same best known fleet size for each problem, this fleet size "m_{BKS}" is presented in a single column. The same convention as Repoussis et al. (2010) is used: the route duration from the benchmark instances of Christofides et al. (1979) is multiplied by a factor of 0.9, whereas for the benchmark instances of Golden et al. (1998b) the duration constraint is not considered.

Vehicle Fleet Mix Problem with Time Windows (VFMPTW). The benchmark instances of Liu and Shen (1999) are considered with three different fleet cost settings (type A,B,C instances). As in the former paper and several following works, we addressed the minimization of fixed fleet cost plus the *trip duration*, i.e., the time elapsed between departure from the depot and return minus the service time (Tables V.18-V.20). Is should be noted that the departure time is not constrained to be time 0, and several best known solutions require a delayed departure from the depot. Additional experiments have also been conducted to address the more standard objective of fixed fleet cost plus total distance (Tables V.21-V.23).

VRP with Soft Time Windows (VRPSTW). A classification and notation for soft time windows settings is introduced in Fu et al. (2007). Type 1 and type 2 soft time windows have been addressed in this paper. Several criteria have been considered by previous authors to assess on the quality of solutions, these criteria include the fleet size, the number of customers serviced outside of their time windows, the amount of lateness and earliness, and the route distance. To optimize on these objectives, we implemented a general formulation of service costs $c_i(t_i)$ to customer as a function of the service date t_i, given in Equation (V.1). In these equations, γ represents a fixed penalty for servicing a customer outside of its time window, and α and β are respectively the penalties for one unit of tardiness or earliness.

$$c_i(t_i) = \begin{cases} \gamma + \beta(e_i - t_i) & \text{if } t_i < e_i \\ 0 & \text{if } e_i \leq t_i \leq l_i \\ \gamma + \alpha(t_i - l_i) & \text{if } l_i < t_i \end{cases} \tag{V.1}$$

Two settings of type 1 soft time windows have been addressed. To address the hierarchical objective of minimizing first the fleet size, then the number of customers serviced outside of their time windows, then lateness, and finally distance, the parameters values have been to $\beta = +\infty$, $\gamma = 100000$ and $\alpha = 100$ and the fleet size has been minimized by iteratively reducing the fleet limit (Table V.24). Another objective, involving the minimization of distance plus lateness with $\alpha = 1$ has been also addressed (Table V.25). In this case, the other parameters have been set to $\beta = +\infty$ and $\gamma = 0$.

Furthermore, two settings of type 2 soft time windows have been addressed. The hierarchical objective has been addressed by setting $\beta = 100$, $\gamma = 100000$ and $\alpha = 100$ (Table V.26). We also provide results for the objective seeking to minimize the sum of distance, earliness and lateness by setting $\beta = 1$, $\gamma = 0$ and $\alpha = 1$ (Table V.27).

Time-Dependent VRP with Time Windows (TDVRPTW). The same setting as Kritzinger et al. (2012) is considered, involving the minimization of the total time dependent travel time. The same convention as the authors is used, and thus all routes are constrained to start at time 0, waiting time being allowed only at a customer location upon an early arrival. We consider the same fleet size limits as in Kritzinger et al. (2012). The benchmark instances of Solomon and Desrosiers (1988) are used. In the scope of this thesis, only the results for the first scenario travel time scenarios of Ichoua et al. (2003) are reported in Table V.17. The complete results are available in the published article journal associated to this paper. The planning horizon is divided into three parts of 20%, 60% and 20%, respectively, as in Ichoua et al. (2003). The arc category matrix of Balseiro et al. (2011) is used.

Multi-Depot Periodic VRP with Time Windows (MDPVRPTW). The set of MDPVRP instances from Vidal et al. (2012a), originally issued from the combination of multi-depot and periodic instances of Cordeau et al. (1997, 2001a), has been completed with the time windows values from Cordeau et al. (2001a). Experiments have been conducted to set the fleet size value close to the minimum fleet size. The corresponding values of m are reported, along with the results, in Table V.28.

Table V.1: Results on the VRPB, instances of Goetschalckx and Jacobs-Blecha (1989)

Inst	**n**	**m**	**GA09**		**ZK11**		**UHGS**			**BKS**
			Avg 8	Best X	Avg 10	Best 10	Avg 10	Best 10	T(min)	
A1	25	8	**229.89**	**229.89**	**229.89**	**229.89**	**229.89**	**229.89**	0.11	229.89
A2	25	5	**180.12**	**180.12**	**180.12**	**180.12**	**180.12**	**180.12**	0.12	180.12
A3	25	4	**163.41**	**163.41**	**163.41**	**163.41**	**163.41**	**163.41**	0.13	163.41
A4	25	3	**155.80**	**155.80**	**155.80**	**155.80**	**155.80**	**155.80**	0.16	155.80
B1	30	7	**239.08**	**239.08**	**239.08**	**239.08**	**239.08**	**239.08**	0.14	239.08
B2	30	5	**198.05**	**198.05**	**198.05**	**198.05**	**198.05**	**198.05**	0.15	198.05
B3	30	3	**169.37**	**169.37**	**169.37**	**169.37**	**169.37**	**169.37**	0.18	169.37
C1	40	7	**250.56**	**250.56**	**250.56**	**250.56**	**250.56**	**250.56**	0.22	250.56
C2	40	5	**215.02**	**215.02**	**215.02**	**215.02**	**215.02**	**215.02**	0.24	215.02
C3	40	5	**199.35**	**199.35**	**199.35**	**199.35**	**199.35**	**199.35**	0.23	199.35
C4	40	4	**195.37**	**195.37**	**195.37**	**195.37**	**195.37**	**195.37**	0.24	195.37
D1	38	12	**322.53**	**322.53**	**322.53**	**322.53**	**322.53**	**322.53**	0.18	322.53
D2	38	11	**316.71**	**316.71**	**316.71**	**316.71**	**316.71**	**316.71**	0.17	316.71
D3	38	7	**239.48**	**239.48**	**239.48**	**239.48**	**239.48**	**239.48**	0.19	239.48
D4	38	5	**205.83**	**205.83**	**205.83**	**205.83**	**205.83**	**205.83**	0.24	205.83
E1	45	7	**238.88**	**238.88**	**238.88**	**238.88**	**238.88**	**238.88**	0.27	238.88
E2	45	4	**212.26**	**212.26**	**212.26**	**212.26**	**212.26**	**212.26**	0.32	212.26
E3	45	4	**206.66**	**206.66**	**206.66**	**206.66**	**206.66**	**206.66**	0.36	206.66
F1	60	6	**263.17**	**263.17**	263.27	**263.17**	**263.17**	**263.17**	0.38	263.17
F2	60	7	**265.21**	**265.21**	265.66	**265.21**	**265.21**	**265.21**	0.39	265.21
F3	60	5	241.48	**241.12**	**241.12**	**241.12**	**241.12**	**241.12**	0.49	241.12
F4	60	4	**233.86**	**233.86**	234.60	**233.86**	**233.86**	**233.86**	0.54	233.86
G2	57	6	**245.44**	**245.44**	**245.44**	**245.44**	**245.44**	**245.44**	0.38	245.44
G3	57	5	**229.51**	**229.51**	229.97	**229.51**	**229.51**	**229.51**	0.43	229.51
G4	57	6	**232.52**	**232.52**	**232.52**	**232.52**	**232.52**	**232.52**	0.45	232.52
G5	57	5	**221.73**	**221.73**	**222.87**	**221.73**	**221.73**	**221.73**	0.46	221.73
G6	57	4	**213.46**	**213.46**	214.38	**213.46**	**213.46**	**213.46**	0.54	213.46
H1	68	6	269.00	**268.93**	270.06	**268.93**	**268.93**	**268.93**	0.62	268.93
H2	68	5	**253.37**	**253.37**	253.91	**253.37**	**253.37**	**253.37**	0.57	253.37
H3	68	4	**247.45**	**247.45**	**247.45**	**247.45**	**247.45**	**247.45**	0.64	247.45
H4	68	5	**250.22**	**250.22**	251.09	**250.22**	**250.22**	**250.22**	0.59	250.22
H5	68	4	**246.12**	**246.12**	**246.12**	**246.12**	**246.12**	**246.12**	0.62	246.12
H6	68	5	**249.14**	**249.14**	250.06	**249.14**	**249.14**	**249.14**	0.59	249.14
I1	90	10	350.40	**350.25**	351.08	**350.25**	350.37	**350.25**	0.89	350.25
I2	90	7	310.32	**309.94**	309.98	**309.94**	**309.94**	**309.94**	0.85	309.94
I3	90	5	294.84	**294.51**	294.79	**294.51**	**294.51**	**294.51**	0.99	294.51
I4	90	6	296.13	**295.99**	297.91	**295.99**	**295.99**	**295.99**	0.92	295.99
I5	90	7	301.83	**301.24**	303.49	**301.24**	**301.24**	**301.24**	0.82	301.24
J1	94	10	335.12	**335.01**	335.78	**335.01**	**335.01**	**335.01**	0.83	335.01
J2	94	8	**310.42**	**310.42**	312.51	**310.42**	**310.42**	**310.42**	0.84	310.42
J3	94	6	279.34	**279.22**	280.43	**279.22**	**279.22**	**279.22**	0.93	279.22
J4	94	7	296.58	**296.53**	298.32	**296.53**	**296.53**	**296.53**	1.12	296.53
K1	113	10	396.14	**394.07**	397.38	**394.07**	394.35	**394.07**	1.33	394.07
K2	113	8	362.56	**362.13**	365.46	**362.13**	**362.13**	**362.13**	1.40	362.13
K3	113	9	366.71	**365.69**	369.44	**365.69**	**365.69**	**365.69**	1.30	365.69
K4	113	7	350.32	**348.95**	349.72	**348.95**	**348.95**	**348.95**	1.29	348.95
L1	150	10	420.06	**417.90**	421.68	**417.90**	418.16	**417.90**	3.94	417.90
L2	150	8	401.36	**401.23**	405.20	**401.23**	**401.23**	**401.23**	2.96	401.23
L3	150	9	404.32	**402.68**	405.76	**402.68**	**402.68**	**402.68**	2.73	402.68
L4	150	7	384.83	**384.64**	388.14	**384.64**	**384.64**	**384.64**	2.35	384.64
L5	150	8	390.33	**387.56**	390.46	387.57	**387.56**	**387.56**	2.72	387.56
M1	125	11	399.12	**398.59**	400.50	**398.59**	398.66	**398.59**	1.58	398.59
M2	125	10	398.16	**396.92**	401.91	**396.92**	396.93	**396.92**	2.39	396.92
M3	125	9	377.81	**375.70**	378.07	**375.70**	375.93	**375.70**	2.70	375.70
M4	125	7	348.46	**348.14**	352.03	**348.14**	348.20	**348.14**	1.72	348.14
N1	150	11	408.17	**408.10**	411.72	**408.10**	**408.10**	**408.10**	2.72	408.10
N2	150	10	408.25	**408.07**	412.31	**408.07**	408.13	**408.07**	2.55	408.07
N3	150	9	394.70	**394.34**	398.76	**394.34**	394.94	**394.34**	2.46	394.34
N4	150	10	394.87	**394.79**	396.16	**394.79**	395.13	**394.79**	2.37	394.79
N5	150	7	374.12	**373.48**	376.90	**373.48**	373.55	**373.48**	3.12	373.48
N6	150	8	374.79	**373.76**	379.78	**373.76**	373.76	**373.76**	3.42	373.76
Time			1.13 min		1.09 min		0.99 min			
Gap			+0.09%	+0.00%	+0.38%	+0.00%	+0.01%	+0.00%		
CPU			T5500 1.67G		Xe 2.4G		Opt 2.4G			

Table V.2: Results on the CCVRP, instances of Christofides et al. (1979)

Inst	**n**	**m**	**NPC10**		**RL12**		**UHGS**			**BKS**
			Avg 5	Best 5	Avg 5	Best 5	Avg 10	Best 10	T(min)	
p01	50	5	**2230.35**	**2230.35**	2235.27	**2230.35**	**2230.35**	**2230.35**	0.44	2230.35
p02	75	10	2443.07	2421.90	2401.72	**2391.63**	2394.00	**2391.63**	0.70	2391.63
p03	100	8	4073.12	4073.12	4063.98	**4045.42**	**4045.42**	**4045.42**	0.93	4045.42
p04	150	12	5020.75	**4987.52**	4994.93	**4987.52**	**4987.52**	**4987.52**	1.84	4987.52
p05	199	17	5842.00	5810.20	5857.76	5838.32	5809.94	**5806.02**	4.02	5810.12
p11	120	7	7395.83	7317.98	7341.28	7315.87	**7314.55**	**7314.55**	1.35	7315.87
p12	100	10	3559.23	**3558.92**	3566.06	**3558.92**	**3558.93**	**3558.93**	0.64	3558.93
Time			5.20 min		2.69 min		1.42 min			
Gap			+0.74%	+0.28%	+0.37%	+0.07%	+0.01%	-0.01%		
CPU			Core2 2G		Core2 2G		Opt 2.2G			

Table V.3: Results on the CCVRP, instances of Golden et al. (1998b)

Inst	**n**	**m**	**NPC10**[1]		**RL12**		**UHGS**			**BKS**
			Avg 5	Best 5	Avg 5	Best 5	Avg 10	Best 10	T(min)	
pr01	240	9	54878.25	54815.17	54853.76	54786.92	54742.20	**54739.85**	7.40	54786.92
pr02	320	10	100918.54	100836.90	100934.34	100662.53	100562.52	**100560.16**	11.00	100662.53
pr03	400	10	171400.35	171277.26	172231.14	171613.59	170964.42	**170923.53**	23.95	171277.26
pr04	480	10	262830.96	262584.23	265207.46	263433.03	262044.19	**261993.33**	26.14	262584.23
pr05	200	5	114237.00	114163.64	114846.27	114494.66	**114163.63**	**114163.63**	6.95	114163.64
pr06	280	7	140456.96	140430.09	140929.71	140804.64	**140430.08**	**140430.08**	12.65	140430.09
pr07	360	8	186702.15	183282.64	181610.82	180481.56	178976.20	**178880.44**	26.66	180481.56
pr08	440	10	194510.99	194312.60	195174.85	194988.74	193683.21	**193659.14**	26.07	194273.58
pr09	255	14	4740.42	4730.70	4728.05	4725.58	4724.01	**4722.06**	6.97	4725.58
pr10	323	16	6747.10	6732.36	6717.76	6713.92	6720.04	**6713.26**	10.30	6713.92
pr11	399	18	9259.66	9243.05	9216.60	**9214.07**	9222.92	9219.42	14.61	9214.07
pr12	483	19	12649.21	12629.37	12543.04	12526.17	12516.98	**12500.52**	28.66	12526.17
pr13	252	26	3660.93	3653.07	3638.50	3628.30	3632.63	**3627.45**	7.16	3628.30
pr14	320	29	6045.20	5770.02	5257.95	5216.80	5206.53	**5187.56**	16.25	5216.80
pr15	396	33	7140.11	7077.48	7023.12	7010.41	7015.51	**7005.47**	18.70	7010.41
pr16	480	37	9339.45	9300.74	9268.30	9250.98	9247.68	**9239.10**	27.28	9250.98
pr17	240	22	3103.99	3089.99	3068.29	3065.46	3061.28	**3060.14**	6.06	3065.46
pr18	300	27	4582.44	4528.16	4244.60	4221.14	4211.80	**4199.43**	11.64	4221.14
pr19	360	33	5589.12	5570.35	5531.78	5523.38	5502.59	**5496.39**	17.06	5523.38
pr20	420	38	7473.69	7413.58	7240.86	7223.08	7188.59	**7184.19**	21.82	7223.08
Time			94.13 min		21.11 min		17.16 min			
Gap			+2.03%	+1.38%	+0.34%	+0.07%	-0.14%	-0.23%		
CPU			Core2 2G		Core2 2G		Opt 2.2G			

Table V.4: Results on the VRPSDP, instances of Salhi and Nagy (1999)

Inst	n	ZTK10	SDBOF10		S12		UHGS			BKS
		—	Avg 50	Best 50	Avg 10	Best 10	Avg 10	Best 10	T(min)	
CMT1X	50	469.80	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	0.72	466.77
CMT1Y	50	469.80	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	**466.77**	0.71	466.77
CMT2X	75	**684.21**	684.49	**684.21**	684.78	**684.21**	684.43	**684.21**	1.32	684.21
CMT2Y	75	**684.21**	684.43	**684.21**	684.59	**684.21**	684.36	**684.21**	1.35	684.21
CMT3X	100	**721.27**	**721.27**	**721.27**	721.46	**721.27**	**721.27**	**721.27**	1.69	721.27
CMT3Y	100	**721.27**	**721.27**	**721.27**	721.50	**721.27**	**721.27**	**721.27**	1.79	721.27
CMT12X	100	**662.22**	**662.22**	**662.22**	663.44	**662.22**	**662.22**	**662.22**	1.74	662.22
CMT12Y	100	**662.22**	662.25	**662.22**	663.12	**662.22**	**662.22**	**662.22**	1.69	662.22
CMT11X	120	**833.92**	842.78	**833.92**	848.65	846.23	**833.92**	**833.92**	2.75	833.92
CMT11Y	120	**833.92**	842.78	**833.92**	848.74	846.23	**833.92**	**833.92**	2.66	833.92
CMT4X	150	**852.46**	**852.46**	**852.46**	853.02	**852.46**	852.65	**852.46**	4.16	852.46
CMT4Y	150	**852.46**	**852.46**	**852.46**	852.73	**852.46**	852.72	**852.46**	3.95	852.46
CMT5X	199	1030.55	1029.66	**1029.25**	1029.52	**1029.25**	1029.60	**1029.25**	7.99	1029.25
CMT5Y	199	1030.55	1029.71	**1029.25**	**1029.25**	**1029.25**	1029.79	**1029.25**	6.60	1029.25
Time		—	256×0.36 min		—		2.79 min			
Gap		+0.11%	+0.16%	+0.00%	+0.30%	+0.21%	+0.01%	+0.00%		
CPU		T5500 1.66G	Xe 2.67G		I7 2.93G		Opt 2.4G			

Table V.5: Results on the VRPSDP, instances of Montané and Galvão (2006)

Inst	n	ZTK10	SDBOF10		S12		UHGS			BKS
		—	Avg 50	Best 50	Avg 10	Best 10	Avg 10	Best 10	T(min)	
r101	100	**1009.95**	1010.54	**1009.95**	1010.08	**1009.95**	1011.60	**1009.95**	1.23	1009.95
r201	100	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	**666.20**	2.39	666.20
c101	100	1220.99	1220.64	**1220.18**	1220.43	**1220.18**	1220.99	1220.99	1.10	1220.18
c201	100	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	**662.07**	1.63	662.07
rc101	100	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	**1059.32**	1.21	1059.32
rc201	100	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	**672.92**	1.99	672.92
R1_2_1	200	3376.30	3369.93	3360.02	3355.04	**3353.80**	3364.40	3355.37	6.77	3353.80
R2_2_1	200	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	**1665.58**	8.95	1665.58
C1_2_1	200	3643.82	3635.87	3629.89	3636.53	**3628.51**	3639.00	3637.42	7.25	3628.51
C2_2_1	200	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	**1726.59**	5.52	1726.59
RC1_2_1	200	3323.56	3317.51	3306.00	3306.73	**3303.70**	3315.35	3304.39	8.10	3303.70
RC2_2_1	200	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	**1560.00**	7.35	1560.00
R1_4_1	400	9691.60	9647.24	9618.97	9539.56	**9519.45**	9594.08	9547.85	24.53	9519.45
R2_4_1	400	3572.38	3557.43	3551.38	3549.49	**3546.49**	3555.10	**3546.49**	24.57	3546.49
C1_4_1	400	11179.36	11118.98	11099.54	11075.60	**11047.19**	11113.63	11077.26	29.67	11047.19
C2_4_1	400	3549.27	3558.92	3546.10	3543.65	**3539.50**	3541.44	**3539.50**	29.76	3539.50
RC1_4_1	400	9645.27	9564.86	9536.77	9478.12	**9447.53**	9509.39	9469.44	29.84	9447.53
RC2_4_1	400	3423.62	3404.62	**3403.70**	**3403.70**	**3403.70**	3404.08	**3403.70**	24.10	3403.70
Time		—	256×3.11 min		7.23 min		12.00 min			
Gap		+0.47%	+0.30%	+0.17%	+0.08%	+0.00%	+0.20%	+0.07%		
CPU		T5500 1.66G	Xe 2.67G		I7 2.93G		Opt 2.4G			

Table V.6: Results on the VFMP-F, only fixed vehicle costs, instances of Golden (1984)

Inst	**n**	**w**	**ISW09**	**P09**	**SPUO12**		**UHGS**			**BKS**
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
F3	20	5	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**	**961.03**	0.20	961.03
F4	20	3	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	0.23	6437.33
F5	20	5	**1007.05**	**1007.05**	1008.76	**1007.05**	**1007.05**	**1007.05**	0.23	1007.05
F6	20	3	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	0.23	6516.47
F13	50	6	**2406.36**	**2406.36**	2411.31	**2406.36**	2406.57	**2406.36**	1.02	2406.36
F14	50	3	**9119.03**	**9119.03**	**9119.03**	**9119.03**	**9119.03**	**9119.03**	0.88	9119.03
F15	50	3	**2586.37**	**2586.37**	**2586.37**	**2586.37**	**2586.37**	**2586.37**	0.73	2586.37
F16	50	3	**2720.43**	2729.08	2724.55	**2720.43**	**2720.43**	**2720.43**	0.66	2720.43
F17	75	4	1741.95	1746.09	1744.23	**1734.53**	1735.37	**1734.53**	1.75	1734.53
F18	75	6	**2369.65**	**2369.65**	2373.79	**2369.65**	2374.16	**2369.65**	1.73	2369.65
F19	100	3	8665.05	8665.12	8662.54	**8661.81**	8663.97	8662.86	3.70	8661.81
F20	100	3	4044.68	4044.78	4038.63	4032.81	4037.77	4034.42	2.26	4029.74
Time			8.34 min	0.71 min	0.15 min		1.13 min			
Gap			+0.07%	+0.12%	+0.13%	+0.01%	+0.04%	+0.01%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table V.7: Results on the VFMP-V, only variable vehicle costs, instances of Golden (1984)

Inst	**n**	**w**	**ISW09**	**P09**	**SPUO12**		**UHGS**			**BKS**
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
V3	20	5	NC	NC	**623.22**	**623.22**	**623.22**	**623.22**	0.17	623.22
V4	20	3	NC	NC	387.34	**387.18**	**387.18**	**387.18**	0.19	387.18
V5	20	5	NC	NC	**742.87**	**742.87**	**742.87**	**742.87**	0.20	742.87
V6	20	3	NC	NC	**415.03**	**415.03**	**415.03**	**415.03**	0.22	415.03
V13	50	6	**1491.86**	**1491.86**	1492.01	**1491.86**	**1491.86**	**1491.86**	0.72	1491.86
V14	50	3	**603.21**	**603.21**	605.00	**603.21**	**603.21**	**603.21**	0.56	603.21
V15	50	3	**999.82**	**999.82**	1001.03	**999.82**	**999.82**	**999.82**	0.61	999.82
V16	50	3	**1131.00**	**1131.00**	1131.85	**1131.00**	**1131.00**	**1131.00**	0.57	1131.00
V17	75	4	**1038.60**	**1038.60**	1042.48	**1038.60**	**1038.60**	**1038.60**	1.14	1038.60
V18	75	6	**1800.80**	**1800.80**	1802.89	**1800.80**	1801.40	1801.40	1.34	1800.80
V19	100	3	**1105.44**	**1105.44**	1106.71	**1105.44**	1106.93	**1105.44**	1.71	1105.44
V20	100	3	1533.24	1535.12	1534.23	**1530.43**	1531.82	**1530.43**	2.80	1530.43
Time			8.85 min	0.41 min	0.06 min		0.85 min			
Gap			+0.02%	+0.03%	+0.17%	+0.00%	+0.03%	+0.00%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table V.8: Results on the VFMP-FV, fixed and variable vehicle costs, instances of Golden (1984)

Inst	**n**	**w**	**ISW09**	**P09**	**SPUO12**		**UHGS**			**BKS**
			Best 5-7	Best 5	Avg 10	Best 10	Avg 10	Best 10	T(min)	
FV3	20	5	**1144.22**	**1144.22**	**1144.22**	**1144.22**	**1144.22**	**1144.22**	0.17	1144.22
FV4	20	3	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	**6437.33**	0.23	6437.33
FV5	20	5	**1322.26**	**1322.26**	**1322.26**	**1322.26**	**1322.26**	**1322.26**	0.17	1322.26
FV6	20	3	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	**6516.47**	0.23	6516.47
FV13	50	6	**2964.65**	**2964.65**	**2964.65**	**2964.65**	**2964.65**	**2964.65**	0.51	2964.65
FV14	50	3	**9126.90**	**9126.90**	**9126.90**	**9126.90**	**9126.90**	**9126.90**	0.79	9126.90
FV15	50	3	**2634.96**	2635.21	**2634.96**	**2634.96**	2635.06	**2634.96**	0.71	2634.96
FV16	50	3	3169.10	3169.14	**3168.92**	**3168.92**	**3168.92**	**3168.92**	0.80	3168.92
FV17	75	4	2008.14	**2004.48**	2007.12	**2004.48**	2007.04	**2004.48**	1.33	2004.48
FV18	75	6	3157.20	3153.16	3148.91	**3147.99**	3148.99	3148.99	1.28	3147.99
FV19	100	3	8665.88	8664.67	8662.89	**8661.81**	8663.04	**8661.81**	3.91	8661.81
FV20	100	3	4154.87	4154.49	4153.12	**4153.02**	**4153.02**	**4153.02**	1.73	4153.02
Time			8.42 min	0.39 min	0.13 min		0.99 min			
Gap			+0.05%	+0.02%	+0.01%	+0.00%	+0.01%	+0.00%		
CPU			PM 1.7G	PM 1.8G	I7 2.93G		Opt 2.4G			

Table V.9: Results on the LDVRP, instances of Christofides et al. (1979)

Inst	**n**	**XZKX12**		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	T(min)	
p01	50	751.43	751.11	**746.39**	**746.39**	0.50	751.11
p02	75	1188.62	1179.53	**1172.62**	**1172.62**	0.99	1179.53
p03	100	1153.56	**1147.83**	**1147.83**	**1147.83**	1.49	1147.83
p04	150	1461.69	1452.88	**1446.64**	**1446.64**	4.49	1452.88
p05	199	1865.30	1844.87	1840.54	**1834.31**	6.26	1844.87
p11	120	1516.42	1513.48	**1511.99**	**1511.99**	1.74	1513.48
p12	100	1175.59	**1174.02**	**1174.02**	**1174.02**	0.92	1174.02
Time		1.3 min		2.34 min			
Gap		+0.48%	+0.00%	-0.28%	-0.33%		
CPU		—		Opt 2.2G			

Table V.10: Results for the LDVRP, instances of Golden et al. (1998b)

Inst	**n**	**XZKX12**		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	T(min)	
pr01	240	7714.29	7683.952	7661.10	**7660.64**	12.24	7683.95
pr02	320	11195.02	11172.71	11178.93	**11148.74**	25.71	11172.71
pr03	400	14566.73	14497.64	14525.36	**14480.67**	25.90	14497.64
pr04	480	18605.37	18327.03	18225.56	**18206.84**	30.43	18327.03
pr05	200	8576.91	8561.53	8457.61	**8457.60**	5.71	8561.53
pr06	280	11121.04	11102.22	11056.72	**11056.47**	12.51	11102.22
pr07	360	13477.07	13422.16	13408.06	**13392.93**	27.56	13422.16
pr08	440	16098.60	15928.26	15538.15	**15491.34**	29.76	15928.26
pr09	255	858.34	850.80	835.55	**834.73**	23.17	850.80
pr10	323	1090.85	1083.00	1062.66	**1061.36**	27.86	1083.00
pr11	399	1360.20	1352.32	1319.47	**1316.59**	30.34	1352.32
pr12	483	1661.07	1630.81	1599.59	**1596.68**	30.02	1630.81
pr13	252	1269.37	1261.93	1235.32	**1232.99**	13.84	1261.93
pr14	320	1604.83	1595.48	1564.18	**1562.73**	22.87	1595.48
pr15	396	1987.76	1970.43	1934.13	**1930.84**	28.80	1970.43
pr16	480	2408.72	2391.12	2340.21	**2337.60**	30.00	2391.12
pr17	240	1033.88	1027.21	1018.17	**1018.02**	11.22	1027.21
pr18	300	1469.97	1462.31	1440.00	**1435.34**	20.15	1462.31
pr19	360	2014.26	2007.62	1967.85	**1966.77**	28.32	2007.62
pr20	420	2699.29	2687.85	2626.61	**2621.48**	30.18	2687.85
Time		3.3 min		23.81 min			
Gap		+0.66%	+0.00%	-1.38%	-1.52%		
CPU		—		Opt 2.2G			

Table V.11: Results on the GVRP, instances of Bektas et al. (2011)

Inst	n	c	m	BER11	MCL11	UHGS			BKS
				Single	Single	Avg 10	Best 10	T(min)	
A-n32-k5-C16-V2	32	16	2	**519.00**	**519.00**	**519.00**	**519.00**	0.64	519.00
A-n33-k5-C17-V3	33	17	3	**451.00**	**451.00**	**451.00**	**451.00**	0.69	451.00
A-n33-k6-C17-V3	33	17	3	**465.00**	**465.00**	**465.00**	**465.00**	0.69	465.00
A-n34-k5-C17-V3	34	17	3	**489.00**	**489.00**	**489.00**	**489.00**	0.73	489.00
A-n36-k5-C18-V2	36	18	2	**505.00**	**505.00**	**505.00**	**505.00**	0.83	505.00
A-n37-k5-C19-V3	37	19	3	**432.00**	**432.00**	**432.00**	**432.00**	0.76	432.00
A-n37-k6-C19-V3	37	19	3	**584.00**	**584.00**	**584.00**	**584.00**	0.83	584.00
A-n38-k5-C19-V3	38	19	3	**476.00**	**476.00**	**476.00**	**476.00**	0.89	476.00
A-n39-k5-C20-V3	39	20	3	**557.00**	**557.00**	**557.00**	**557.00**	0.99	557.00
A-n39-k6-C20-V3	39	20	3	**544.00**	**544.00**	**544.00**	**544.00**	1.05	544.00
A-n44-k6-C22-V3	44	22	3	**608.00**	**608.00**	**608.00**	**608.00**	1.36	608.00
A-n45-k6-C23-V4	45	23	4	**613.00**	**613.00**	**613.00**	**613.00**	1.10	613.00
A-n45-k7-C23-V4	45	23	4	**674.00**	**674.00**	**674.00**	**674.00**	1.21	674.00
A-n46-k7-C23-V4	46	23	4	**593.00**	**593.00**	**593.00**	**593.00**	1.00	593.00
A-n48-k7-C24-V4	48	24	4	**667.00**	**667.00**	**667.00**	**667.00**	1.26	667.00
A-n53-k7-C27-V4	53	27	4	**603.00**	**603.00**	**603.00**	**603.00**	1.33	603.00
A-n54-k7-C27-V4	54	27	4	**690.00**	**690.00**	**690.00**	**690.00**	1.39	690.00
A-n55-k9-C28-V5	55	28	5	**699.00**	**699.00**	**699.00**	**699.00**	1.32	699.00
A-n60-k9-C30-V5	60	30	5	**769.00**	**769.00**	**769.00**	**769.00**	1.46	769.00
A-n61-k9-C31-V5	61	31	5	**638.00**	**638.00**	**638.00**	**638.00**	1.59	638.00
A-n62-k8-C31-V4	62	31	4	**740.00**	**740.00**	**740.00**	**740.00**	1.89	740.00
A-n63-k10-C32-V5	63	32	5	**801.00**	**801.00**	**801.00**	**801.00**	1.63	801.00
A-n63-k9-C32-V5	63	32	5	**912.00**	**912.00**	**912.00**	**912.00**	1.71	912.00
A-n64-k9-C32-V5	64	32	5	**763.00**	**763.00**	**763.00**	**763.00**	1.84	763.00
A-n65-k9-C33-V5	65	33	5	**682.00**	**682.00**	**682.00**	**682.00**	1.62	682.00
A-n69-k9-C35-V5	69	35	5	**680.00**	**680.00**	**680.00**	**680.00**	1.83	680.00
A-n80-k10-C40-V5	80	40	5	**997.00**	**997.00**	**997.00**	**997.00**	2.65	997.00
B-n31-k5-C16-V3	31	16	3	**441.00**	**441.00**	**441.00**	**441.00**	0.63	441.00
B-n34-k5-C17-V3	34	17	3	**472.00**	**472.00**	**472.00**	**472.00**	0.70	472.00
B-n35-k5-C18-V3	35	18	3	**626.00**	**626.00**	**626.00**	**626.00**	0.65	626.00
B-n38-k6-C19-V3	38	19	3	**451.00**	**451.00**	**451.00**	**451.00**	0.86	451.00
B-n39-k5-C20-V3	39	20	3	**357.00**	**357.00**	**357.00**	**357.00**	0.79	357.00
B-n41-k6-C21-V3	41	21	3	**481.00**	**481.00**	**481.00**	**481.00**	1.08	481.00
B-n43-k6-C22-V3	43	22	3	**483.00**	**483.00**	**483.00**	**483.00**	1.15	483.00
B-n44-k7-C22-V4	44	22	4	**540.00**	**540.00**	**540.00**	**540.00**	1.13	540.00
B-n45-k5-C23-V3	45	23	3	**497.00**	**497.00**	**497.00**	**497.00**	1.28	497.00
B-n45-k6-C23-V4	45	23	4	**478.00**	**478.00**	**478.00**	**478.00**	1.23	478.00
B-n50-k7-C25-V4	50	25	4	**449.00**	**449.00**	**449.00**	**449.00**	1.08	449.00
B-n50-k8-C25-V5	50	25	5	**916.00**	**916.00**	**916.00**	**916.00**	1.36	916.00
B-n51-k7-C26-V4	51	26	4	**651.00**	**651.00**	**651.00**	**651.00**	1.44	651.00
B-n52-k7-C26-V4	52	26	4	**450.00**	**450.00**	**450.00**	**450.00**	1.18	450.00
B-n56-k7-C28-V4	56	28	4	**486.00**	492.00	**486.00**	**486.00**	1.35	486.00
B-n57-k7-C29-V4	57	29	4	**751.00**	**751.00**	**751.00**	**751.00**	1.43	751.00
B-n57-k9-C29-V5	57	29	5	**942.00**	**942.00**	**942.00**	**942.00**	1.60	942.00
B-n63-k10-C32-V5	63	32	5	**816.00**	**816.00**	**816.00**	**816.00**	1.74	816.00
B-n64-k9-C32-V5	64	32	5	**509.00**	**509.00**	**509.00**	**509.00**	1.37	509.00
B-n66-k9-C33-V5	66	33	5	**808.00**	**808.00**	**808.00**	**808.00**	1.88	808.00
B-n67-k10-C34-V5	67	34	5	**673.00**	**673.00**	**673.00**	**673.00**	1.91	673.00
B-n68-k9-C34-V5	68	34	5	**704.00**	**704.00**	**704.00**	**704.00**	1.87	704.00
B-n78-k10-C39-V5	78	39	5	**803.00**	804.00	**803.00**	**803.00**	2.38	803.00
G-n262-k25-C131-V12	262	131	12	3249.00	3319.00	3241.80	**3229.00**	22.98	3249.00
M-n101-k10-C51-V5	101	51	5	**542.00**	**542.00**	**542.00**	**542.00**	2.85	542.00
M-n121-k7-C61-V4	121	61	4	**719.00**	720.00	**719.00**	**719.00**	5.45	719.00

Table V.12: Results on the GVRP, instances of Bektas et al. (2011) (continued)

Inst	**n**	**c**	**m**	**BER11**	**MCL11**	**UHGS**			**BKS**
				Single	Single	Avg 10	Best 10	T(min)	
M-n151-k12-C76-V6	151	76	6	**659.00**	**659.00**	**659.00**	**659.00**	4.94	659.00
M-n200-k16-C100-V8	200	100	8	**791.00**	805.00	**786.00**	**786.00**	11.47	791.00
P-n101-k4-C51-V2	101	51	2	**455.00**	**455.00**	**455.00**	**455.00**	4.83	455.00
P-n16-k8-C8-V5	16	8	5	**239.00**	**239.00**	**239.00**	**239.00**	0.09	239.00
P-n19-k2-C10-V2	19	10	2	**147.00**	**147.00**	**147.00**	**147.00**	0.15	147.00
P-n20-k2-C10-V2	20	10	2	**154.00**	**154.00**	**154.00**	**154.00**	0.15	154.00
P-n21-k2-C11-V2	21	11	2	**160.00**	162.00	**160.00**	**160.00**	0.19	160.00
P-n22-k2-C11-V2	22	11	2	**162.00**	163.00	**162.00**	**162.00**	0.24	162.00
P-n22-k8-C11-V5	22	11	5	**314.00**	**314.00**	**314.00**	**314.00**	0.18	314.00
P-n23-k8-C12-V5	23	12	5	**312.00**	**312.00**	**312.00**	**312.00**	0.23	312.00
P-n40-k5-C20-V3	40	20	3	**294.00**	**294.00**	**294.00**	**294.00**	1.00	294.00
P-n45-k5-C23-V3	45	23	3	**337.00**	**337.00**	**337.00**	**337.00**	1.13	337.00
P-n50-k10-C25-V5	50	25	5	**410.00**	**410.00**	**410.00**	**410.00**	1.08	410.00
P-n50-k7-C25-V4	50	25	4	**353.00**	**353.00**	**353.00**	**353.00**	1.23	353.00
P-n50-k8-C25-V4	50	25	4	**392.00**	421.00	**392.00**	**392.00**	1.39	392.00
P-n51-k10-C26-V6	51	26	6	**427.00**	**427.00**	**427.00**	**427.00**	1.04	427.00
P-n55-k10-C28-V5	55	28	5	**415.00**	**415.00**	**415.00**	**415.00**	1.22	415.00
P-n55-k15-C28-V8	55	28	8	**555.00**	565.00	**555.00**	**555.00**	1.07	555.00
P-n55-k7-C28-V4	55	28	4	**361.00**	**361.00**	**361.00**	**361.00**	1.41	361.00
P-n55-k8-C28-V4	55	28	4	**361.00**	**361.00**	**361.00**	**361.00**	1.36	361.00
P-n60-k10-C30-V5	60	30	5	**443.00**	**443.00**	**443.00**	**443.00**	1.52	443.00
P-n60-k15-C30-V8	60	30	8	**565.00**	**565.00**	**565.00**	**565.00**	1.24	565.00
P-n65-k10-C33-V5	65	33	5	**487.00**	**487.00**	**487.00**	**487.00**	1.55	487.00
P-n70-k10-C35-V5	70	35	5	**485.00**	**485.00**	**485.00**	**485.00**	1.74	485.00
P-n76-k4-C38-V2	76	38	2	**383.00**	**383.00**	**383.00**	**383.00**	2.80	383.00
P-n76-k5-C38-V3	76	38	3	**405.00**	**405.00**	**405.00**	**405.00**	2.44	405.00
A-n32-k5-C11-V2	32	11	2	**386.00**	**386.00**	**386.00**	**386.00**	0.33	386.00
A-n33-k5-C11-V2	33	11	2	318.00	**315.00**	**315.00**	**315.00**	0.30	315.00
A-n33-k6-C11-V2	33	11	2	**370.00**	**370.00**	**370.00**	**370.00**	0.28	370.00
A-n34-k5-C12-V2	34	12	2	**419.00**	**419.00**	**419.00**	**419.00**	0.39	419.00
A-n36-k5-C12-V2	36	12	2	**396.00**	**396.00**	**396.00**	**396.00**	0.39	396.00
A-n37-k5-C13-V2	37	13	2	**347.00**	**347.00**	**347.00**	**347.00**	0.42	347.00
A-n37-k6-C13-V2	37	13	2	**431.00**	**431.00**	**431.00**	**431.00**	0.47	431.00
A-n38-k5-C13-V2	38	13	2	**367.00**	**367.00**	**367.00**	**367.00**	0.46	367.00
A-n39-k5-C13-V2	39	13	2	**364.00**	**364.00**	**364.00**	**364.00**	0.43	364.00
A-n39-k6-C13-V2	39	13	2	**403.00**	**403.00**	**403.00**	**403.00**	0.53	403.00
A-n44-k6-C15-V2	44	15	2	**503.00**	**503.00**	**503.00**	**503.00**	0.74	503.00
A-n45-k6-C15-V3	45	15	3	**474.00**	**474.00**	**474.00**	**474.00**	0.65	474.00
A-n45-k7-C15-V3	45	15	3	**475.00**	**475.00**	**475.00**	**475.00**	0.62	475.00
A-n46-k7-C16-V3	46	16	3	**462.00**	**462.00**	**462.00**	**462.00**	0.87	462.00
A-n48-k7-C16-V3	48	16	3	**451.00**	**451.00**	**451.00**	**451.00**	0.90	451.00
A-n53-k7-C18-V3	53	18	3	**440.00**	**440.00**	**440.00**	**440.00**	1.10	440.00
A-n54-k7-C18-V3	54	18	3	**482.00**	**482.00**	**482.00**	**482.00**	1.04	482.00
A-n55-k9-C19-V3	55	19	3	**473.00**	**473.00**	**473.00**	**473.00**	1.10	473.00
A-n60-k9-C20-V3	60	20	3	**595.00**	**595.00**	**595.00**	**595.00**	1.33	595.00
A-n61-k9-C21-V4	61	21	4	**473.00**	**473.00**	**473.00**	**473.00**	1.18	473.00
A-n62-k8-C21-V3	62	21	3	**596.00**	**596.00**	**596.00**	**596.00**	1.61	596.00
A-n63-k10-C21-V4	63	21	4	**593.00**	**593.00**	**593.00**	**593.00**	1.33	593.00
A-n63-k9-C21-V3	63	21	3	**642.00**	643.00	**642.00**	**642.00**	1.51	642.00
A-n64-k9-C22-V3	64	22	3	**536.00**	**536.00**	**536.00**	**536.00**	1.64	536.00
A-n65-k9-C22-V3	65	22	3	**500.00**	**500.00**	**500.00**	**500.00**	1.53	500.00
A-n69-k9-C23-V3	69	23	3	**520.00**	**520.00**	**520.00**	**520.00**	1.60	520.00
A-n80-k10-C27-V4	80	27	4	**710.00**	**710.00**	**710.00**	**710.00**	2.14	710.00
B-n31-k5-C11-V2	31	11	2	**356.00**	**356.00**	**356.00**	**356.00**	0.35	356.00

Table V.13: Results on the GVRP, instances of Bektas et al. (2011) (continued)

Inst	**n**	**c**	**m**	**BER11**	**MCL11**	**UHGS**			**BKS**
				Single	Single	Avg 10	Best 10	T(min)	
B-n34-k5-C12-V2	34	12	2	**369.00**	**369.00**	**369.00**	**369.00**	0.35	369.00
B-n35-k5-C12-V2	35	12	2	**501.00**	**501.00**	**501.00**	**501.00**	0.36	501.00
B-n38-k6-C13-V2	38	13	2	**370.00**	**370.00**	**370.00**	**370.00**	0.53	370.00
B-n39-k5-C13-V2	39	13	2	**280.00**	**280.00**	**280.00**	**280.00**	0.42	280.00
B-n41-k6-C14-V2	41	14	2	**407.00**	**407.00**	**407.00**	**407.00**	0.57	407.00
B-n43-k6-C15-V2	43	15	2	**343.00**	**343.00**	**343.00**	**343.00**	0.73	343.00
B-n44-k7-C15-V3	44	15	3	**395.00**	**395.00**	**395.00**	**395.00**	0.54	395.00
B-n45-k5-C15-V2	45	15	2	422.00	**410.00**	**410.00**	**410.00**	0.73	410.00
B-n45-k6-C15-V2	45	15	2	**336.00**	**336.00**	**336.00**	**336.00**	0.76	336.00
B-n50-k7-C17-V3	50	17	3	**393.00**	**393.00**	**393.00**	**393.00**	1.00	393.00
B-n50-k8-C17-V3	50	17	3	**598.00**	**598.00**	**598.00**	**598.00**	0.98	598.00
B-n51-k7-C17-V3	51	17	3	**511.00**	**511.00**	**511.00**	**511.00**	0.72	511.00
B-n52-k7-C18-V3	52	18	3	**359.00**	**359.00**	**359.00**	**359.00**	0.99	359.00
B-n56-k7-C19-V3	56	19	3	**356.00**	**356.00**	**356.00**	**356.00**	1.10	356.00
B-n57-k7-C19-V3	57	19	3	**558.00**	**558.00**	**558.00**	**558.00**	1.26	558.00
B-n57-k9-C19-V3	57	19	3	**681.00**	**681.00**	**681.00**	**681.00**	1.18	681.00
B-n63-k10-C21-V3	63	21	3	**599.00**	**599.00**	**599.00**	**599.00**	1.61	599.00
B-n64-k9-C22-V4	64	22	4	**452.00**	**452.00**	**452.00**	**452.00**	1.34	452.00
B-n66-k9-C22-V3	66	22	3	**609.00**	**609.00**	**609.00**	**609.00**	1.43	609.00
B-n67-k10-C23-V4	67	23	4	**558.00**	**558.00**	**558.00**	**558.00**	1.37	558.00
B-n68-k9-C23-V3	68	23	3	**523.00**	**523.00**	**523.00**	**523.00**	1.71	523.00
B-n78-k10-C26-V4	78	26	4	**606.00**	**606.00**	**606.00**	**606.00**	1.83	606.00
G-n262-k25-C88-V9	262	88	9	2476.00	2463.00	2469.00	**2460.00**	13.80	2463.00
M-n101-k10-C34-V4	101	34	4	**458.00**	**458.00**	**458.00**	**458.00**	2.62	458.00
M-n121-k7-C41-V3	121	41	3	**527.00**	**527.00**	**527.00**	**527.00**	4.18	527.00
M-n151-k12-C51-V4	151	51	4	**483.00**	**483.00**	**483.00**	**483.00**	5.19	483.00
M-n200-k16-C67-V6	200	67	6	**605.00**	**605.00**	**605.00**	**605.00**	6.06	605.00
P-n101-k4-C34-V2	101	34	2	374.00	**370.00**	**370.00**	**370.00**	3.26	370.00
P-n16-k8-C6-V4	16	6	4	**170.00**	**170.00**	**170.00**	**170.00**	0.05	170.00
P-n19-k2-C7-V1	19	7	1	**111.00**	**111.00**	**111.00**	**111.00**	0.07	111.00
P-n20-k2-C7-V1	20	7	1	**117.00**	**117.00**	**117.00**	**117.00**	0.07	117.00
P-n21-k2-C7-V1	21	7	1	**117.00**	**117.00**	**117.00**	**117.00**	0.07	117.00
P-n22-k2-C8-V1	22	8	1	**111.00**	**111.00**	**111.00**	**111.00**	0.10	111.00
P-n22-k8-C8-V4	22	8	4	**249.00**	**249.00**	**249.00**	**249.00**	0.12	249.00
P-n23-k8-C8-V3	23	8	3	**174.00**	**174.00**	**174.00**	**174.00**	0.13	174.00
P-n40-k5-C14-V2	40	14	2	**213.00**	**213.00**	**213.00**	**213.00**	0.55	213.00
P-n45-k5-C15-V2	45	15	2	**238.00**	**238.00**	**238.00**	**238.00**	0.71	238.00
P-n50-k10-C17-V4	50	17	4	**292.00**	**292.00**	**292.00**	**292.00**	0.69	292.00
P-n50-k7-C17-V3	50	17	3	**261.00**	**261.00**	**261.00**	**261.00**	0.84	261.00
P-n50-k8-C17-V3	50	17	3	**262.00**	**262.00**	**262.00**	**262.00**	0.84	262.00
P-n51-k10-C17-V4	51	17	4	**309.00**	**309.00**	**309.00**	**309.00**	0.73	309.00
P-n55-k10-C19-V4	55	19	4	**301.00**	**301.00**	**301.00**	**301.00**	1.01	301.00
P-n55-k15-C19-V6	55	19	6	**378.00**	**378.00**	**378.00**	**378.00**	0.76	378.00
P-n55-k7-C19-V3	55	19	3	**271.00**	**271.00**	**271.00**	**271.00**	1.15	271.00
P-n55-k8-C19-V3	55	19	3	**274.00**	**274.00**	**274.00**	**274.00**	1.15	274.00
P-n60-k10-C20-V4	60	20	4	**325.00**	**325.00**	**325.00**	**325.00**	1.31	325.00
P-n60-k15-C20-V5	60	20	5	**382.00**	**382.00**	**382.00**	**382.00**	1.10	382.00
P-n65-k10-C22-V4	65	22	4	**372.00**	**372.00**	**372.00**	**372.00**	1.25	372.00
P-n70-k10-C24-V4	70	24	4	**385.00**	**385.00**	**385.00**	**385.00**	1.55	385.00
P-n76-k4-C26-V2	76	26	2	320.00	**309.00**	**309.00**	**309.00**	2.02	309.00
P-n76-k5-C26-V2	76	26	2	**309.00**	**309.00**	**309.00**	**309.00**	2.31	309.00
Time				0.01 min	0.34 min		1.53 min		
Gap				+0.06%	+0.11%	+0.00%	-0.01%		
CPU				Opt 2.4G	Duo 1.83G		Opt 2.4G		

Table V.14: Results on the OVRP, instances of Christofides et al. (1979) and Fisher (1994)

Inst	**n**	**m_{BKS}**	**ZK10**		**RTBI10**	**S12**		**UHGS**			**BKS**
			Avg 10	Best 10	Single	Avg 10	Best 10	Avg 10	Best 10	T(min)	
p01	50	5	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	**416.06**	0.41	416.06
p02	75	10	568.38	**567.14**	**567.14**	**567.14**	**567.14**	568.15	**567.14**	0.51	567.14
p03	100	8	639.98	**639.74**	**639.74**	639.81	**639.74**	**639.74**	**639.74**	0.85	639.74
p04	150	12	733.93	**733.13**	**733.13**	**733.13**	**733.13**	**733.13**	**733.13**	1.73	733.13
p05	199	16	895.62	893.39	894.11	895.55	**883.50**	890.15	884.08	4.13	883.50
p06	50	6	—	—	**412.96**	**412.96**	**412.96**	**412.96**	**412.96**	0.55	412.96
p07	75	10	—	—	584.15	*582.07**	**583.19**	584.59	**583.19**	0.77	583.19
p08	100	9	—	—	**644.63**	644.95	**644.63**	644.79	**644.63**	1.79	644.63
p09	150	13	—	—	764.56	759.38	757.91	760.75	**757.07**	5.18	757.84
p10	199	17	—	—	888.46	877.68	**874.71**	875.49	**874.71**	6.10	874.71
p11	120	7	682.34	**682.12**	**682.12**	**682.12**	**682.12**	**682.12**	**682.12**	1.51	682.12
p12	100	10	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	**534.24**	0.61	534.24
p13	120	11	—	—	910.26	904.02	**899.16**	900.22	**899.16**	3.39	899.16
p14	100	11	—	—	**591.87**	**591.87**	**591.87**	**591.87**	**591.87**	1.70	591.87
f11	71	4	**177.00**	**177.00**	**177.00**	177.21	**177.00**	**177.00**	**177.00**	0.65	177.00
f12	134	7	770.57	**769.55**	**769.55**	770.00	**769.55**	769.68	**769.55**	1.71	769.55
Time			—		9.54 min	2.39 min		1.97 min			
Gap			—	—	+0.32%	*+0.16%**	+0.00%	+0.11%	+0.00%		
CPU			T5500 1.66G		P-IV 2.8G	I7 2.93G		Opt 2.4G			

∗ The minimum fleet size was not attained by S12 on all runs.

Table V.15: Results on the OVRP, instances of Golden et al. (1998b)

Inst	**n**	**m^***	**ZK10**		**RTBI10**	**S12**		**UHGS**			**BKS**
			Avg 10	Best 10	Single	Avg 10	Best 10	Avg 10	Best 10	T(min)	
pr01	240	9	4562.88	4557.38	4583.70	4551.74	4544.46	4546.35	**4543.00**	8.20	4544.46
pr02	320	10	7264.32	7253.20	7271.24	7229.56	7215.48	7218.41	**7213.56**	14.94	7215.48
pr03	400	9	9824.44	9793.72	9821.09	9784.52	9773.83	9763.39	**9750.63**	22.66	9773.83
pr04	480	10	12430.06	12415.36	12428.20	12393.40	12389.43	12387.82	**12380.66**	26.41	12389.43
pr05	200	5	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	**6018.52**	5.71	6018.52
pr06	280	7	7735.10	7731.00	7733.77	7728.77	7721.16	7704.91	**7704.59**	11.23	7721.16
pr07	360	8	9243.69	9193.15	9254.15	9205.01	9180.93	9132.27	**9127.70**	19.01	9180.93
pr08	440	10	10363.28	10347.70	10363.40	10342.10	10326.57	10316.60	**10289.70**	26.43	10326.57
Time			14.79 min		17.53 min	64.07 min		16.82 min			
Gap			+0.39%	+0.21%	+0.47%	+0.13%	+0.00%	-0.11%	-0.19%		
CPU			T5500 1.66G		P-IV 2.8G	I7 2.93G		Opt 2.4G			

Table V.16: Results on the OVRPTW, instances of Solomon and Desrosiers (1988)

Inst	**n**	**RTI09**		KTDHS12		**UHGS**			**BKS**
		Avg 10	Best 10	Avg 10	Best 10	Avg 10	Best 10	T(min)	
R101	100	**19.00/1192.85**	**19/1192.85**	**19.00/1192.95**	**19/1192.85**	**19.00/1192.85**	**19/1192.85**	2.86	19/1192.85
R102	100	17.00/1081.65	**17/1079.39**	**17.00/1079.39**	**17/1079.39**	**17.00/1079.39**	**17/1079.39**	2.81	17/1079.39
R103	100	13.00/1017.28	**13/1016.78**	13.00/1016.83	**13/1016.78**	**13.00/1016.78**	**13/1016.78**	3.57	13/1016.78
R104	100	9.53/844.32	9/869.63	9.00/837.28	9/834.44	9.00/833.52	**9/832.50**	4.86	9/834.44
R105	100	14.00/1055.98	**14/1055.04**	14.00/1055.34	**14/1055.04**	**14.00/1055.04**	**14/1055.04**	3.92	14/1055.04
R106	100	12.00/1001.04	12/1000.95	12.00/1003.15	12/1001.41	12.00/1000.93	**12/1000.36**	4.88	12/1000.95
R107	100	10.00/915.82	10/912.99	10.00/918.47	**10/910.75**	10.00/914.75	10/912.08	6.14	10/910.75
R108	100	9.00/760.30	9/760.30	9.00/765.63	9/760.30	9.00/760.32	**9/759.86**	4.80	9/760.30
R109	100	11.00/934.77	11/934.53	11.00/937.86	**11/934.15**	11.00/934.52	**11/934.15**	4.36	11/934.15
R110	100	10.00/851.01	**10/846.49**	10.00/881.91	10/874.64	10.00/885.18	**10/873.75**	5.08	10/846.49
R111	100	10.00/902.45	**10/895.21**	10.00/904.25	10/895.56	**10.00/895.21**	**10/895.21**	5.34	10/895.21
R112	100	9.47/814.33	9/811.73	9.00/815.43	9/805.17	9.00/811.76	**9/801.43**	5.81	9/805.17
C101	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.67	10/556.18
C102	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	1.00	10/556.18
C103	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	1.10	10/556.18
C104	100	**10.00/555.41**	**10/555.41**	**10.00/555.41**	**10/555.41**	**10.00/555.41**	**10/555.41**	1.06	10/555.41
C105	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.83	10/556.18
C106	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.85	10/556.18
C107	100	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	**10.00/556.18**	**10/556.18**	0.84	10/556.18
C108	100	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	0.95	10/555.80
C109	100	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	**10.00/555.80**	**10/555.80**	0.97	10/555.80
RC101	100	14.00/1228.76	**14/1227.37**	**14.00/1227.37**	**14/1227.37**	**14.00/1227.37**	**14/1227.37**	3.37	14/1227.37
RC102	100	12.00/1205.65	12/1203.05	12.00/1197.16	**12/1185.43**	12.50/1125.04	12/1195.20	4.12	12/1185.43
RC103	100	11.00/927.62	11/923.15	11.00/919.32	**11/918.65**	**11.00/918.65**	**11/918.65**	3.92	11/918.65
RC104	100	10.00/789.21	**10/787.02**	10.00/790.58	**10/787.02**	**10.00/787.02**	**10/787.02**	4.51	10/787.02
RC105	100	13.00/1220.96	13/1195.20	13.00/1201.73	13/1195.20	13.10/1186.34	**13/1185.43**	4.45	13/1195.20
RC106	100	11.00/1113.20	11/1095.65	11.00/1073.78	**11/1071.83**	11.00/1074.52	**11/1071.83**	4.24	11/1071.83
RC107	100	11.00/862.44	11/861.28	11.00/862.88	11/861.28	11.00/860.62	**11/860.62**	4.03	11/861.28
RC108	100	10.00/832.05	**10/831.09**	10.00/837.38	10/833.03	10.00/832.27	**10/831.09**	4.31	10/831.09
R201	100	**4.00/1182.43**	**4/1182.43**	4.00/1187.99	**4/1182.43**	**4.00/1182.43**	**4/1182.43**	5.80	4/1182.43
R202	100	3.00/1150.07	**3/1149.59**	3.00/1152.70	3/1151.14	**3.00/1149.59**	**3/1149.59**	9.73	3/1149.59
R203	100	3.00/894.34	**3/889.12**	3.00/900.51	3/894.40	3.00/890.02	**3/889.12**	11.72	3/889.12
R204	100	2.00/803.54	2/801.46	2.00/821.67	2/803.50	2.00/798.13	**2/797.83**	7.67	2/801.46
R205	100	3.00/950.21	**3/943.33**	3.00/966.18	3/952.83	**3.00/943.33**	**3/943.33**	11.21	3/943.33
R206	100	3.00/870.96	3/869.27	3.00/883.18	3/874.78	**3.00/865.32**	**3/865.32**	13.01	3/869.27
R207	100	2.77/865.93	2/857.08	2.00/880.56	2/857.08	**2.00/854.40**	**2/854.40**	6.65	2/857.08
R208	100	2.00/700.67	2/700.53	2.00/710.74	2/700.63	2.00/694.67	**2/694.24**	8.00	2/700.53
R209	100	3.00/853.86	**3/851.69**	3.00/864.91	**3/851.69**	3.00/852.42	**3/851.69**	11.57	3/851.69
R210	100	3.00/894.38	3/892.45	3.00/908.77	3/901.87	3.00/891.23	**3/890.02**	11.00	3/892.45
R211	100	2.00/887.41	2/886.90	2.00/894.55	2/874.49	2.00/852.15	**2/846.92**	8.14	2/874.49
C201	100	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	1.33	3/548.51
C202	100	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	**3.00/548.51**	**3/548.51**	2.09	3/548.51
C203	100	**3.00/548.13**	**3/548.13**	**3.00/548.13**	**3/548.13**	**3.00/548.13**	**3/548.13**	2.73	3/548.13
C204	100	**3.00/547.55**	**3/547.55**	3.00/549.02	**3/547.55**	**3.00/547.55**	**3/547.55**	3.21	3/547.55
C205	100	**3.00/545.83**	**3/545.83**	**3.00/545.83**	**3/545.83**	**3.00/545.83**	**3/545.83**	1.76	3/545.83
C206	100	**3.00/545.45**	**3/545.45**	**3.00/545.45**	**3/545.45**	**3.00/545.45**	**3/545.45**	1.87	3/545.45
C207	100	**3.00/545.24**	**3/545.24**	**3.00/545.24**	**3/545.24**	**3.00/545.24**	**3/545.24**	1.92	3/545.24
C208	100	**3.00/545.28**	**3/545.28**	**3.00/545.28**	**3/545.28**	**3.00/545.28**	**3/545.28**	2.16	3/545.28
RC201	100	4.00/1309.06	**4/1303.73**	4.00/1321.87	4/1304.50	**4.00/1303.73**	**4/1303.73**	6.50	4/1303.73
RC202	100	3.00/1329.52	3/1321.43	3.00/1335.13	3/1292.35	3.00/1289.86	**3/1289.04**	10.92	3/1292.35
RC203	100	3.00/995.02	3/993.29	3.00/1004.88	3/993.22	3.00/987.28	**3/977.56**	10.81	3/993.22
RC204	100	3.00/719.92	**3/718.97**	3.00/736.97	3/722.20	**3.00/718.97**	**3/718.97**	9.52	3/718.97
RC205	100	4.00/1190.67	**4/1189.84**	4.00/1193.05	**4/1189.84**	**4.00/1189.84**	**4/1189.84**	7.92	4/1189.84
RC206	100	3.00/1092.09	3/1091.79	3.00/1102.53	3/1092.66	**3.00/1087.97**	**3/1087.97**	10.72	3/1091.79
RC207	100	3.00/1005.32	**3/998.70**	3.00/1015.46	3/1006.06	3.00/999.29	**3/998.70**	9.43	3/998.70
RC208	100	3.00/772.76	3/769.40	3.00/786.41	3/778.32	3.00/769.12	**3/768.75**	11.94	3/769.40
Time		10.0 min		10.0 min		5.27 min			
Gap		+0.89%/+0.42%	0%/+0.24%	0%/+0.79%	0%/+0.18%	+0.09%/-0.10%	0%/-0.10%		
CPU		P-IV 3G		Xe 2.67G		Opt 2.2G			

Table V.17: Results on the TDVRPTW. Scenario 1 for time-dependent travel times. Instances of Solomon and Desrosiers (1988).

Inst	**n**	**m**	**KTDHS12**		**UHGS**			**BKS**
			Avg 10	Best 10	Avg 10	Best 10	T(min)	
C101	100	10	**755.86**	**755.86**	**755.86**	**755.86**	2.87	755.86
C102	100	10	**738.87**	**738.87**	**738.87**	**738.87**	3.33	738.87
C103	100	10	713.82	**713.50**	**713.50**	**713.50**	3.32	713.50
C104	100	10	683.96	**683.27**	**683.27**	**683.27**	3.22	683.27
C105	100	10	**747.41**	**747.41**	**747.41**	**747.41**	3.35	747.41
C106	100	10	**741.08**	**741.08**	**741.08**	**741.08**	3.27	741.08
C107	100	10	**735.84**	**735.84**	**735.84**	**735.84**	3.16	735.84
C108	100	10	**705.61**	**705.61**	**705.61**	**705.61**	2.99	705.61
C109	100	10	691.57	691.26	**691.16**	**691.16**	3.61	691.26
R101	100	17	1320.22	1311.57	**1310.20**	**1310.20**	3.68	1311.57
R102	100	16	1132.68	1130.36	**1128.23**	**1128.23**	3.16	1130.36
R103	100	13	926.72	918.03	912.23	**911.74**	4.85	918.03
R104	100	10	761.08	754.67	753.04	**752.73**	6.25	754.67
R105	100	14	1033.27	1020.20	1020.43	**1020.16**	4.90	1020.20
R106	100	12	949.94	940.86	939.38	**939.19**	5.16	940.86
R107	100	10	843.05	837.10	831.69	**831.50**	6.86	837.10
R108	100	9	741.89	734.76	726.33	**725.08**	8.71	734.76
R109	100	11	842.03	833.93	**829.39**	**829.39**	5.14	833.93
R110	100	10	817.95	806.34	796.76	**795.00**	7.21	806.34
R111	100	10	808.21	798.35	797.70	**797.50**	7.80	798.35
R112	100	9	739.17	723.25	717.74	**714.75**	7.28	723.25
RC101	100	15	1241.72	**1236.04**	**1236.04**	**1236.04**	4.14	1236.04
RC102	100	13	1085.29	**1072.60**	1073.20	**1072.60**	4.79	1072.60
RC103	100	11	937.77	931.49	**928.22**	**928.22**	4.23	931.49
RC104	100	10	864.99	858.35	844.62	**844.57**	5.42	858.35
RC105	100	14	1161.43	1151.13	1148.75	**1147.51**	5.03	1151.13
RC106	100	12	997.20	994.23	989.83	**988.73**	6.14	994.23
RC107	100	11	915.44	907.27	899.97	**898.27**	6.12	907.27
RC108	100	10	853.37	843.04	**838.10**	**838.10**	5.52	843.04
C201	100	3	**620.77**	**620.77**	**620.77**	**620.77**	9.19	620.77
C202	100	3	601.93	**601.19**	**601.19**	**601.19**	10.32	601.19
C203	100	3	589.01	585.76	**585.75**	**585.75**	10.70	585.76
C204	100	3	574.05	568.48	**566.07**	**566.07**	14.23	568.48
C205	100	3	**595.79**	**595.79**	**595.79**	**595.79**	10.27	595.79
C206	100	3	**571.07**	**571.07**	**571.07**	**571.07**	8.21	571.07
C207	100	3	577.03	**577.02**	**577.02**	**577.02**	8.47	577.02
C208	100	3	565.75	**565.74**	**565.74**	**565.74**	8.14	565.74
R201	100	4	996.46	984.55	981.00	**980.95**	15.01	984.55
R202	100	3	947.05	933.24	920.36	**910.46**	28.28	933.24
R203	100	3	762.06	752.14	739.98	**738.55**	27.16	752.14
R204	100	2	644.02	636.21	616.33	**615.83**	29.64	636.21
R205	100	3	789.42	767.56	**758.00**	**758.00**	18.40	767.56
R206	100	3	735.59	719.59	703.65	**703.14**	24.68	719.59
R207	100	2	719.74	691.36	669.18	**669.10**	30.00	691.36
R208	100	2	580.47	569.78	557.38	**556.37**	27.14	569.78
R209	100	3	684.13	666.75	657.91	**656.62**	20.26	666.75
R210	100	3	748.70	735.30	718.03	**711.94**	22.17	735.30
R211	100	2	706.45	691.10	651.23	**649.69**	29.96	691.10
RC201	100	4	1171.56	1164.73	**1146.39**	**1146.39**	11.96	1164.73
RC202	100	3	1056.94	1037.16	1025.99	**1025.07**	22.89	1037.16
RC203	100	3	826.91	814.80	803.39	**803.18**	21.21	814.80
RC204	100	3	649.31	642.18	**623.96**	**623.96**	19.83	642.18
RC205	100	4	1034.77	1024.01	991.45	**989.01**	21.33	1024.01
RC206	100	3	922.09	910.09	894.04	**893.92**	21.02	910.09
RC207	100	3	809.39	777.23	764.24	**762.47**	20.89	777.23
RC208	100	3	643.09	628.81	603.76	**600.85**	16.24	628.81
Time			10.00 min		11.59 min			
Gap			+1.03%	+0.00%	-0.93%	-1.03%		
CPU			Xe 2.67G		Opt 2.2G			

Table V.18: Results on the VFMPTW, minimization of duration, type A fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**RT10**	**UHGS**			**BKS**
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	4631.31	**4536.40**	4617.95	4608.62	6.03	4536.40
R102	100	5	4401.30	**4348.92**	4376.11	4369.74	6.38	4348.92
R103	100	5	4182.16	**4119.04**	4149.67	4145.68	4.65	4119.04
R104	100	5	3981.28	3986.35	3965.21	**3961.39**	5.23	3981.28
R105	100	5	4236.84	4229.67	4215.84	**4209.84**	5.11	4229.67
R106	100	5	4118.48	4130.82	4112.20	**4109.08**	6.32	4118.48
R107	100	5	4035.96	4031.16	4012.58	**4007.87**	5.45	4031.16
R108	100	5	3970.26	3962.20	3936.47	**3934.48**	5.12	3962.20
R109	100	5	4060.17	4052.21	4037.40	**4020.75**	5.06	4052.21
R110	100	5	3995.18	3999.09	3971.53	**3965.88**	5.30	3995.18
R111	100	5	4017.81	4016.19	3992.07	**3985.68**	6.05	4016.19
R112	100	5	3947.30	3954.65	3923.21	**3918.88**	6.77	3947.30
C101	100	3	**7226.51**	**7226.51**	**7226.51**	**7226.51**	3.19	7226.51
C102	100	3	**7119.35**	7137.79	**7119.35**	**7119.35**	2.82	7119.35
C103	100	3	7107.01	7141.03	7104.46	**7102.86**	2.46	7107.01
C104	100	3	**7081.50**	7086.70	7081.51	7081.51	2.22	7081.50
C105	100	3	7199.36	**7169.08**	7196.06	7196.06	3.40	7169.08
C106	100	3	7180.03	**7157.13**	7177.41	7176.68	3.67	7157.13
C107	100	3	7149.17	**7135.38**	7144.73	7144.49	3.19	7135.38
C108	100	3	7115.81	7113.57	**7111.23**	**7111.23**	2.78	7113.57
C109	100	3	7094.65	7092.49	**7091.66**	**7091.66**	2.39	7092.49
RC101	100	4	5253.97	5237.19	5225.17	**5217.90**	5.03	5237.19
RC102	100	4	5059.58	5053.62	5044.63	**5018.47**	5.71	5053.48
RC103	100	4	4868.94	4885.58	4830.08	**4822.21**	6.03	4868.94
RC104	100	4	4762.85	4761.28	4741.69	**4737.00**	4.10	4761.28
RC105	100	4	5119.80	5110.86	5110.51	**5097.35**	5.61	5110.86
RC106	100	4	4960.78	4966.27	4947.46	**4935.91**	6.60	4960.78
RC107	100	4	4828.17	4819.91	4791.19	**4783.08**	5.32	4819.91
RC108	100	4	4734.15	4749.44	4710.71	**4708.85**	5.17	4734.15
R201	100	4	3922.00	**3753.42**	3791.54	3782.88	7.66	3753.42
R202	100	4	3610.38	3551.12	3540.39	**3540.03**	13.37	3551.12
R203	100	4	3350.18	3336.60	3314.09	**3311.35**	9.07	3334.08
R204	100	4	3390.14	3103.84	3076.13	**3075.95**	8.87	3103.84
R205	100	4	3465.81	3367.90	3334.35	**3334.27**	9.25	3367.90
R206	100	4	3268.36	3264.70	3246.09	**3242.40**	9.01	3264.70
R207	100	4	3231.26	3158.69	3145.79	**3145.08**	9.40	3158.69
R208	100	4	3063.10	3056.45	3020.52	**3017.12**	8.07	3056.45
R209	100	4	3192.95	3194.74	3186.18	**3183.36**	9.49	3191.63
R210	100	4	3375.38	3325.28	3288.82	**3287.66**	10.21	3325.28
R211	100	4	3042.48	3053.08	3021.67	**3019.93**	9.08	3042.48
C201	100	4	5891.45	**5820.78**	5878.54	5878.54	5.17	5820.78
C202	100	4	5850.26	5783.76	**5776.88**	**5776.88**	5.15	5779.59
C203	100	4	5741.90	**5736.94**	5741.82	5741.12	5.72	5736.94
C204	100	4	5691.51	5718.49	**5680.46**	**5680.46**	4.31	5691.51
C205	100	4	5786.71	**5747.67**	5782.53	5781.15	6.56	5747.67
C206	100	4	5795.15	**5738.09**	5767.70	5767.70	4.74	5738.09
C207	100	4	5743.52	**5721.16**	5731.54	5731.44	5.14	5721.16
C208	100	4	5884.20	5732.95	**5725.03**	**5725.03**	4.52	5732.95
RC201	100	6	4740.21	**4701.88**	4740.49	4737.59	5.28	4701.88
RC202	100	6	4522.36	4509.11	**4487.48**	**4487.48**	4.48	4509.11
RC203	100	6	4312.52	4313.42	4305.63	**4305.49**	5.88	4312.52
RC204	100	6	4141.04	4157.32	4140.16	**4137.93**	6.68	4141.04
RC205	100	6	4652.57	**4585.20**	4625.21	4615.04	6.40	4585.20
RC206	100	6	4431.64	4427.73	4408.63	**4405.16**	5.14	4416.95
RC207	100	6	4310.11	4313.07	4295.07	**4290.14**	6.52	4310.11
RC208	100	6	4091.92	4103.31	4076.12	**4075.04**	5.74	4091.92
Time			13.14 min	16.67 min		5.86 min		
Gap			+0.72%	+0.08%	-0.13%	-0.21%		
CPU			Ath 2.6G	P-IV 3.4G		Opt 2.2G		

Table V.19: Results on the VFMPTW, minimization of duration, type B fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**RT10**	**UHGS**			**BKS**
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	2486.76	**2421.19**	2487.11	2486.77	3.89	2421.19
R102	100	5	2227.48	**2209.50**	2223.80	2222.15	4.31	2209.50
R103	100	5	1938.93	1953.50	1931.17	**1930.21**	4.18	1938.93
R104	100	5	1714.73	1713.36	1694.06	**1688.12**	4.34	1713.36
R105	100	5	2027.98	2030.83	**2017.56**	**2017.56**	3.83	2027.98
R106	100	5	1919.03	1919.02	1916.36	**1913.84**	5.10	1919.02
R107	100	5	1789.58	1780.52	1775.34	**1774.50**	4.27	1780.52
R108	100	5	**1649.24**	1665.78	1657.01	1654.68	5.83	1649.24
R109	100	5	1828.63	1840.54	1818.15	**1818.15**	5.09	1828.63
R110	100	5	1774.46	1788.18	1765.50	**1761.53**	5.77	1774.46
R111	100	5	1769.71	1772.51	1757.34	**1751.10**	5.57	1769.71
R112	100	5	1669.78	1667.00	1664.36	**1663.09**	6.33	1667.00
C101	100	3	**2417.52**	**2417.52**	**2417.52**	**2417.52**	2.06	2417.52
C102	100	3	2350.55	**2350.54**	2350.55	2350.55	2.98	2350.54
C103	100	3	2353.64	2347.99	2345.31	**2345.31**	4.02	2347.99
C104	100	3	2328.62	**2325.78**	2327.84	2327.84	2.43	2325.78
C105	100	3	**2373.53**	2375.04	**2373.53**	**2373.53**	3.35	2373.53
C106	100	3	2404.56	**2381.14**	2386.03	2386.03	3.17	2381.14
C107	100	3	2370.01	2357.67	2364.21	2364.21	3.10	2357.52
C108	100	3	**2346.38**	**2346.38**	**2346.38**	**2346.38**	3.28	2346.38
C109	100	3	2339.89	**2336.29**	**2336.29**	**2336.29**	2.60	2336.29
RC101	100	4	2462.60	2464.66	2461.29	**2456.10**	4.69	2462.60
RC102	100	4	2263.45	2272.68	2261.83	**2259.25**	4.24	2263.45
RC103	100	4	2035.62	2041.24	2028.38	**2025.30**	4.74	2035.62
RC104	100	4	1905.06	1916.85	1901.04	**1901.04**	4.37	1905.06
RC105	100	4	**2308.59**	2325.99	2329.30	2329.14	4.76	2308.59
RC106	100	4	2149.56	2160.45	2152.58	**2146.00**	3.68	2149.56
RC107	100	4	2000.77	2003.26	1990.20	**1989.34**	4.09	2000.77
RC108	100	4	1910.83	1908.72	1900.80	**1898.96**	3.26	1906.69
R201	100	4	2002.53	**1953.42**	1975.28	1973.43	6.39	1953.42
R202	100	4	1790.38	1751.12	1747.39	**1740.03**	8.05	1751.12
R203	100	4	1541.19	1536.60	1513.38	**1511.35**	6.44	1535.08
R204	100	4	1284.33	1303.84	1276.31	**1275.95**	7.58	1284.33
R205	100	4	1563.62	1560.07	**1534.27**	**1534.27**	6.45	1560.07
R206	100	4	1464.53	1464.70	1443.43	**1441.35**	5.92	1464.53
R207	100	4	1380.41	1358.69	1345.42	**1345.08**	6.97	1358.69
R208	100	4	1244.74	1256.45	1219.25	**1217.12**	6.00	1244.74
R209	100	4	1431.37	1394.74	1382.44	**1380.79**	7.75	1394.74
R210	100	4	1516.66	1525.28	1486.85	**1485.65**	7.72	1516.66
R211	100	4	1255.06	1253.08	1220.46	**1219.93**	7.36	1253.08
C201	100	4	1820.64	**1816.14**	1820.64	1820.64	2.90	1816.14
C202	100	4	1795.40	**1768.51**	1775.21	**1768.51**	5.22	1768.51
C203	100	4	**1733.63**	1734.82	**1733.63**	**1733.63**	3.29	1733.63
C204	100	4	1708.69	1716.18	**1680.46**	**1680.46**	3.30	1708.69
C205	100	4	1782.74	**1747.68**	1778.30	1778.30	5.48	1747.68
C206	100	4	1772.87	**1756.01**	1767.70	1767.70	3.84	1756.01
C207	100	4	1729.49	**1729.39**	1729.49	1729.49	3.48	1729.39
C208	100	4	1724.20	**1723.20**	1724.20	1724.20	3.40	1723.20
RC201	100	6	2343.79	**2230.54**	2331.33	2329.59	4.34	2230.54
RC202	100	6	2091.53	2022.15	2059.81	2057.66	6.69	2002.62
RC203	100	6	1852.74	1841.26	1825.14	**1824.54**	5.33	1841.26
RC204	100	6	1565.31	1575.18	1557.77	**1555.75**	5.50	1565.31
RC205	100	6	2195.75	**2166.62**	2179.31	2174.74	5.43	2166.62
RC206	100	6	1923.56	1893.13	**1883.08**	**1883.08**	4.33	1887.23
RC207	100	6	1745.85	1743.23	1719.07	**1714.14**	5.65	1743.23
RC208	100	6	1488.19	1526.78	**1483.20**	**1483.20**	4.80	1488.19
Time			9.12 min	16.67 min		4.80 min		
Gap			+0.59%	+0.23%	-0.16%	-0.25%		
CPU			Ath 2.6G	P-IV 3.4G		Opt 2.2G		

Table V.20: Results on the VFMPTW, minimization of duration, type C fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**RT10**	**UHGS**			**BKS**
			Best 3	Single	Avg 10	Best 10	T(min)	
R101	100	5	2199.78	**2134.90**	2199.79	2199.79	3.38	2134.90
R102	100	5	1925.55	**1913.37**	1926.50	1925.56	5.06	1913.37
R103	100	5	**1609.94**	1631.47	1616.42	1615.38	3.62	1609.94
R104	100	5	1370.84	1377.81	1365.32	**1363.26**	4.58	1370.84
R105	100	5	**1722.05**	1729.57	**1722.05**	**1722.05**	3.60	1722.05
R106	100	5	1602.87	1607.96	1603.06	**1599.04**	4.77	1602.87
R107	100	5	1456.02	1452.52	1447.86	**1442.97**	3.72	1452.52
R108	100	5	1336.28	1330.28	1321.96	**1321.68**	5.44	1330.28
R109	100	5	1507.77	1519.37	1508.36	**1506.59**	5.02	1507.77
R110	100	5	1446.41	1457.43	1446.96	**1443.92**	5.73	1446.41
R111	100	5	1447.88	1443.34	1427.82	**1423.47**	6.99	1443.34
R112	100	5	1335.41	1339.44	1329.24	**1329.07**	4.77	1335.41
C101	100	3	**1628.31**	1628.94	1628.94	1628.94	1.83	1628.31
C102	100	3	**1610.96**	**1610.96**	**1610.96**	**1610.96**	2.43	1610.96
C103	100	3	1619.68	**1607.14**	**1607.14**	**1607.14**	2.77	1607.14
C104	100	3	1613.96	**1598.50**	1599.90	1599.90	2.70	1598.50
C105	100	3	**1628.38**	1628.94	1628.94	1628.94	1.88	1628.38
C106	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	1.88	1628.94
C107	100	3	**1628.38**	1628.94	1628.94	1628.94	1.96	1628.38
C108	100	3	**1622.89**	**1622.89**	**1622.89**	**1622.89**	3.00	1622.89
C109	100	3	**1614.99**	**1614.99**	1615.93	1615.93	3.62	1614.99
RC101	100	4	2084.48	2089.37	2084.16	**2082.95**	4.91	2084.48
RC102	100	4	1895.92	1906.68	1898.52	**1895.05**	4.28	1895.92
RC103	100	4	1660.62	1666.24	1661.76	**1650.30**	3.98	1660.62
RC104	100	4	1537.09	1540.13	**1526.04**	**1526.04**	3.57	1537.09
RC105	100	4	1957.52	**1953.99**	1962.82	1957.14	4.71	1953.99
RC106	100	4	1776.08	1787.69	1775.84	**1774.94**	3.80	1776.08
RC107	100	4	1614.04	1622.90	1611.28	**1607.11**	3.83	1614.04
RC108	100	4	1535.14	1531.69	1524.10	**1523.96**	3.38	1531.69
R201	100	4	1729.92	1728.42	**1716.02**	**1716.02**	4.54	1728.42
R202	100	4	1537.35	1527.92	1524.96	**1515.03**	8.84	1527.92
R203	100	4	1308.70	1311.60	1287.36	**1286.35**	6.24	1308.70
R204	100	4	1062.46	1085.71	1051.19	**1050.95**	7.62	1062.46
R205	100	4	1311.84	1335.07	1309.29	**1309.27**	6.44	1311.84
R206	100	4	1251.51	1239.70	1216.87	**1216.35**	5.34	1239.70
R207	100	4	1149.23	1139.61	**1120.08**	**1120.08**	7.23	1139.61
R208	100	4	1009.26	1022.11	992.66	**992.12**	6.01	1009.26
R209	100	4	1178.45	1171.41	1156.97	**1155.79**	7.50	1171.41
R210	100	4	1289.35	1281.08	1259.42	**1257.89**	6.54	1281.08
R211	100	4	1013.84	1028.08	995.54	**994.93**	6.59	1013.84
C201	100	4	**1269.41**	**1269.41**	**1269.41**	**1269.41**	2.86	1269.41
C202	100	4	1242.66	1244.54	**1239.54**	**1239.54**	3.85	1242.66
C203	100	4	**1193.63**	1203.42	**1193.63**	**1193.63**	3.03	1193.63
C204	100	4	**1176.52**	1188.18	**1176.52**	**1176.52**	3.90	1176.52
C205	100	4	1245.62	1239.60	**1238.30**	**1238.30**	4.36	1239.60
C206	100	4	1245.05	**1229.23**	1238.30	1238.30	4.87	1229.23
C207	100	4	1215.42	1213.07	**1209.49**	**1209.49**	3.00	1213.07
C208	100	4	**1204.20**	1205.18	**1204.20**	**1204.20**	3.03	1204.20
RC201	100	6	2004.53	**1915.42**	1996.79	1996.79	3.67	1915.42
RC202	100	6	1766.52	**1677.62**	1733.23	1732.66	6.53	1677.62
RC203	100	6	1517.98	1504.35	1496.48	**1496.11**	6.15	1504.35
RC204	100	6	1238.66	1241.45	**1220.75**	**1220.75**	5.45	1238.66
RC205	100	6	1854.22	**1822.07**	1844.74	1844.74	5.07	1822.07
RC206	100	6	1590.22	1586.61	1557.19	**1553.65**	4.45	1586.61
RC207	100	6	1396.16	1406.26	1382.17	**1377.52**	6.06	1396.16
RC208	100	6	1145.84	1175.23	1141.47	**1140.10**	6.32	1145.84
Time			8.18 min	16.67 min		4.58 min		
Gap			+0.45%	+0.35%	-0.17%	-0.25%		
CPU			Ath 2.6G	P-IV 3.4G		Opt 2.2G		

Table V.21: Results on the VFMPTW, minimization of distance, type A fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**BPDRT09**	**UHGS**			**BKS**
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	4349.80	4342.72	4322.04	**4314.36**	4.61	4342.72
R102	100	5	4196.46	4189.21	4175.05	**4166.28**	6.03	4182.47
R103	100	5	4052.85	4051.62	4034.88	**4027.36**	5.35	4051.62
R104	100	5	3973.48	3972.65	3938.92	**3936.40**	4.81	3972.65
R105	100	5	4161.72	4152.50	4130.71	**4122.50**	6.49	4152.50
R106	100	5	4095.20	4085.30	4058.95	**4048.59**	5.57	4085.07
R107	100	5	4006.61	3996.74	3979.18	**3970.51**	5.56	3996.74
R108	100	5	3961.38	3949.50	3932.46	**3928.12**	4.68	3949.50
R109	100	5	4048.29	4035.89	4020.93	**4015.71**	4.80	4035.89
R110	100	5	3997.88	3991.63	3966.47	**3961.68**	6.49	3991.63
R111	100	5	4011.63	4009.61	3973.49	**3964.99**	5.28	4008.88
R112	100	5	3962.73	3954.19	3926.32	**3918.88**	4.92	3954.19
C101	100	3	7098.04	7097.93	**7093.45**	**7093.45**	2.96	7097.13
C102	100	3	7086.11	7085.47	**7080.17**	**7080.17**	2.14	7085.47
C103	100	3	7080.35	7080.41	**7079.21**	**7079.21**	2.09	7080.35
C104	100	3	7076.90	**7075.06**	**7075.06**	**7075.06**	2.19	7075.06
C105	100	3	7096.19	7096.22	**7093.45**	**7093.45**	3.33	7095.13
C106	100	3	7086.91	7088.35	**7083.87**	**7083.87**	2.28	7086.91
C107	100	3	7084.92	7090.91	**7084.61**	**7084.61**	2.23	7084.92
C108	100	3	7082.49	7081.18	**7079.66**	**7079.66**	2.19	7081.18
C109	100	3	7078.13	7077.68	**7077.30**	**7077.30**	2.04	7077.68
RC101	100	4	5180.74	5168.23	5154.95	**5150.86**	5.21	5168.23
RC102	100	4	5029.59	5025.22	5000.28	**4987.24**	4.81	5025.22
RC103	100	4	4895.57	4888.53	4821.61	**4804.61**	7.08	4888.53
RC104	100	4	4760.56	4747.38	4724.10	**4717.63**	5.30	4747.38
RC105	100	4	5060.37	5068.54	5035.76	**5035.35**	5.57	5060.37
RC106	100	4	4997.86	4972.11	4944.74	**4936.74**	5.63	4972.11
RC107	100	4	4865.76	4861.04	4795.35	**4788.69**	5.08	4861.04
RC108	100	4	4765.37	4753.12	4709.09	**4708.85**	4.78	4753.12
R201	100	4	3484.95	3530.24	**3446.78**	**3446.78**	6.51	3484.95
R202	100	4	3335.74	3335.61	**3308.16**	**3308.16**	7.68	3335.61
R203	100	4	3173.95	3164.03	**3141.09**	**3141.09**	5.65	3162.84
R204	100	4	3065.15	3029.83	3018.83	**3018.14**	6.96	3029.83
R205	100	4	3277.69	3261.19	3220.56	**3218.97**	6.40	3252.43
R206	100	4	3173.30	3165.85	3150.61	**3146.34**	10.30	3165.85
R207	100	4	3136.47	3102.79	3080.64	**3077.58**	8.70	3100.64
R208	100	4	3050.00	3009.13	2999.35	**2997.24**	5.37	3009.13
R209	100	4	3155.73	3155.60	3123.30	**3122.42**	6.37	3141.17
R210	100	4	3219.23	3206.09	3178.57	**3174.85**	6.93	3206.09
R211	100	4	3055.04	3026.02	3021.67	**3019.93**	9.10	3026.02
C201	100	4	5701.45	5700.87	**5695.02**	**5695.02**	3.71	5695.02
C202	100	4	5689.70	5689.70	**5685.24**	**5685.24**	3.78	5687.07
C203	100	4	5685.82	**5681.55**	**5681.55**	**5681.55**	4.21	5681.55
C204	100	4	5690.30	5677.69	**5677.66**	**5677.66**	4.27	5677.66
C205	100	4	5691.70	5691.70	**5691.36**	**5691.36**	3.98	5691.70
C206	100	4	5691.70	5691.70	**5689.32**	**5689.32**	3.82	5691.70
C207	100	4	5689.82	5692.36	**5687.35**	**5687.35**	4.24	5689.82
C208	100	4	5686.50	5689.59	**5686.50**	**5686.50**	3.86	5686.50
RC201	100	6	4407.68	4404.07	4378.21	**4374.09**	5.92	4398.21
RC202	100	6	4277.67	4266.96	4244.65	**4244.63**	4.63	4266.96
RC203	100	6	4204.85	4189.94	4171.47	**4170.17**	7.73	4185.70
RC204	100	6	4109.86	4098.34	**4087.11**	**4087.11**	5.79	4098.34
RC205	100	6	4329.96	4304.52	4295.41	**4291.93**	5.46	4304.52
RC206	100	6	4272.08	4272.82	4253.57	**4251.88**	5.12	4272.08
RC207	100	6	4232.81	4219.52	4186.43	**4185.98**	4.82	4213.66
RC208	100	6	4095.71	4093.83	4076.27	**4075.04**	4.08	4082.58
Time			4.13 min	—		5.09 min		
Gap			+0.25%	+0.06%	-0.41%	-0.48%		
CPU			Ath 2.6G	Duo 2.4G		Opt 2.2G		

Table V.22: Results on the VFMPTW, minimization of distance, type B fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**BPDRT09**	**UHGS**			**BKS**
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	**2226.94**	—	2229.24	2228.67	5.05	2226.94
R102	100	5	**2071.90**	—	2073.91	2073.63	3.59	2071.90
R103	100	5	1857.22	—	1855.83	**1853.66**	4.57	1857.22
R104	100	5	1707.31	—	1686.09	**1683.33**	5.37	1707.31
R105	100	5	1995.07	—	**1988.86**	**1988.86**	3.30	1995.07
R106	100	5	1903.95	—	1889.58	**1888.31**	4.64	1903.95
R107	100	5	1766.18	—	1754.75	**1753.35**	4.15	1766.18
R108	100	5	1666.89	—	1651.75	**1647.88**	4.54	1666.89
R109	100	5	1833.54	—	1819.14	**1818.15**	3.66	1833.54
R110	100	5	1781.15	—	1765.34	**1758.64**	5.11	1781.15
R111	100	5	1768.74	—	1747.08	**1740.86**	5.32	1768.74
R112	100	5	1675.76	—	1664.41	**1661.85**	5.36	1675.76
C101	100	3	2340.98	—	**2340.15**	**2340.15**	3.12	2340.98
C102	100	3	2326.53	—	**2325.70**	**2325.70**	2.61	2326.53
C103	100	3	2325.61	—	**2324.60**	**2324.60**	3.03	2325.61
C104	100	3	**2318.04**	—	**2318.04**	**2318.04**	2.44	2318.04
C105	100	3	2344.64	—	**2340.15**	**2340.15**	3.00	2344.64
C106	100	3	2345.85	—	**2340.15**	**2340.15**	3.46	2345.85
C107	100	3	2345.60	—	**2340.15**	**2340.15**	3.20	2345.60
C108	100	3	2340.17	—	**2338.58**	**2338.58**	3.18	2340.17
C109	100	3	**2328.55**	—	**2328.55**	**2328.55**	2.72	2328.55
RC101	100	4	2417.16	—	2416.23	**2412.71**	4.16	2417.16
RC102	100	4	2234.47	—	2213.93	**2213.92**	5.86	2234.47
RC103	100	4	2025.74	—	2016.28	**2016.28**	3.50	2025.74
RC104	100	4	1912.65	—	1908.66	**1897.04**	4.07	1912.65
RC105	100	4	2296.16	—	2293.21	**2287.51**	4.54	2296.16
RC106	100	4	2157.84	—	2141.32	**2140.86**	3.95	2157.84
RC107	100	4	2008.02	—	1990.13	**1989.34**	2.99	2008.02
RC108	100	4	1920.91	—	1900.72	**1898.96**	3.91	1920.91
R201	100	4	1687.44	—	1721.54	**1646.78**	5.96	1687.44
R202	100	4	1527.74	—	1509.06	**1508.16**	9.15	1527.74
R203	100	4	1379.15	—	**1341.09**	**1341.09**	3.75	1379.15
R204	100	4	1243.56	—	1218.47	**1218.14**	5.11	1243.56
R205	100	4	1471.97	—	1419.52	**1418.97**	7.28	1471.97
R206	100	4	1400.84	—	1350.32	**1346.34**	7.23	1400.84
R207	100	4	1333.53	—	1279.63	**1277.58**	6.28	1333.53
R208	100	4	1225.37	—	1198.41	**1197.24**	4.33	1225.37
R209	100	4	1370.30	—	1323.60	**1322.42**	6.15	1370.30
R210	100	4	1418.54	—	1376.24	**1374.31**	6.97	1418.54
R211	100	4	1263.72	—	1219.99	**1219.93**	6.79	1263.72
C201	100	4	1700.87	—	**1695.02**	**1695.02**	2.97	1700.87
C202	100	4	1687.84	—	**1685.24**	**1685.24**	2.83	1687.84
C203	100	4	1696.25	—	**1681.55**	**1681.55**	3.36	1696.25
C204	100	4	1705.94	—	**1677.66**	**1677.66**	3.47	1705.94
C205	100	4	1711.00	—	**1691.36**	**1691.36**	3.21	1711.00
C206	100	4	1691.70	—	**1689.32**	**1689.32**	2.99	1691.70
C207	100	4	1704.88	—	**1687.35**	**1687.35**	3.44	1704.88
C208	100	4	1689.59	—	**1686.50**	**1686.50**	3.24	1689.59
RC201	100	6	1965.31	—	1942.19	**1938.36**	5.94	1965.31
RC202	100	6	**1771.87**	—	1773.04	1772.81	5.92	1771.87
RC203	100	6	1619.55	—	1606.56	**1604.04**	5.99	1619.55
RC204	100	6	1501.10	—	**1490.25**	**1490.25**	4.28	1501.10
RC205	100	6	1853.58	—	1835.74	**1832.53**	5.11	1853.58
RC206	100	6	1761.49	—	**1725.44**	**1725.44**	5.79	1761.49
RC207	100	6	1666.03	—	1651.09	**1646.37**	5.65	1666.03
RC208	100	6	1494.11	—	**1483.20**	**1483.20**	4.39	1494.11
Time			3.45 min	—		4.50 min		
Gap			+0.00%	—	-0.94%	-1.10%		
CPU			Ath 2.6G	Duo 2.4G		Opt 2.2G		

Table V.23: Results on the VFMPTW, minimization of distance, type C fleet, instances of Liu and Shen (1999)

Inst	**n**	**w**	**BDHMG08**	**BPDRT09**	**UHGS**			**BKS**
			Best 3	Best 5	Avg 10	Best 10	T(min)	
R101	100	5	**1951.20**	1951.89	**1951.20**	**1951.20**	4.60	1951.20
R102	100	5	**1770.40**	1778.29	1785.35	1785.35	2.92	1770.40
R103	100	5	1558.17	1555.26	1552.64	**1552.34**	3.55	1555.26
R104	100	5	1367.82	1372.08	1356.70	**1355.15**	5.37	1361.46
R105	100	5	1696.67	1698.26	**1694.56**	**1694.56**	3.25	1696.67
R106	100	5	1589.25	1590.11	1590.25	**1583.17**	4.12	1589.25
R107	100	5	1435.21	1439.81	1433.44	**1428.08**	5.36	1435.21
R108	100	5	1334.75	1334.68	1315.47	**1314.88**	5.32	1334.68
R109	100	5	1515.22	1514.13	1507.97	**1506.59**	4.68	1507.10
R110	100	5	1457.42	1461.85	1448.83	**1443.92**	5.06	1457.42
R111	100	5	1439.43	1439.14	1423.43	**1420.15**	5.58	1435.93
R112	100	5	1358.17	1343.26	1329.70	**1327.58**	4.97	1337.68
C101	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.12	1628.94
C102	100	3	**1597.66**	**1597.66**	**1597.66**	**1597.66**	2.35	1597.66
C103	100	3	**1596.56**	**1596.56**	**1596.56**	**1596.56**	2.88	1596.56
C104	100	3	1594.06	1590.86	**1590.76**	**1590.76**	2.32	1590.86
C105	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	1.96	1628.94
C106	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.05	1628.94
C107	100	3	**1628.94**	**1628.94**	**1628.94**	**1628.94**	2.17	1628.94
C108	100	3	**1622.75**	**1622.75**	**1622.75**	**1622.75**	3.20	1622.75
C109	100	3	**1614.99**	**1614.99**	1615.93	1615.93	3.88	1614.99
RC101	100	4	2048.44	2053.55	2047.33	**2043.48**	4.39	2048.44
RC102	100	4	1860.48	1872.49	1849.38	**1847.92**	4.10	1860.48
RC103	100	4	1660.81	1663.08	1654.30	**1646.35**	4.19	1660.81
RC104	100	4	1536.24	1540.61	1523.42	**1522.04**	5.64	1536.24
RC105	100	4	1913.09	1929.89	1925.66	**1913.06**	4.01	1913.09
RC106	100	4	1772.05	1776.52	1770.95	1770.95	3.77	1761.63
RC107	100	4	1615.74	1633.29	1609.88	**1607.11**	4.08	1615.74
RC108	100	4	1527.35	1527.87	1524.10	**1523.96**	3.35	1527.35
R201	100	4	1441.46	1466.13	1462.03	1443.41	4.90	1439.76
R202	100	4	1298.10	1296.78	1297.43	**1283.16**	7.91	1288.70
R203	100	4	1145.38	1127.28	**1116.09**	**1116.09**	3.95	1127.28
R204	100	4	1019.77	1000.89	993.16	**993.14**	6.63	1000.89
R205	100	4	1222.03	1240.74	1196.73	**1193.97**	7.33	1222.03
R206	100	4	1138.26	1141.13	1123.21	**1121.34**	6.00	1138.26
R207	100	4	1086.42	1067.97	1055.39	**1052.58**	6.97	1067.97
R208	100	4	976.11	979.50	971.36	**969.90**	5.78	976.11
R209	100	4	1140.96	1140.38	1098.89	**1097.42**	5.94	1123.19
R210	100	4	1161.87	1170.29	1153.34	**1149.85**	6.80	1161.87
R211	100	4	1015.84	1008.54	**994.93**	**994.93**	6.51	1008.54
C201	100	4	**1194.33**	**1194.33**	**1194.33**	**1194.33**	4.82	1194.33
C202	100	4	1189.35	**1185.24**	**1185.24**	**1185.24**	2.57	1185.24
C203	100	4	**1176.25**	**1176.25**	**1176.25**	**1176.25**	3.60	1176.25
C204	100	4	1176.55	1176.55	**1175.37**	**1175.37**	4.32	1176.55
C205	100	4	**1190.36**	**1190.36**	**1190.36**	**1190.36**	4.46	1190.36
C206	100	4	**1188.62**	**1188.62**	**1188.62**	**1188.62**	4.01	1188.62
C207	100	4	**1184.88**	1187.71	**1184.88**	**1184.88**	3.66	1184.88
C208	100	4	1187.86	**1186.50**	**1186.50**	**1186.50**	2.99	1186.50
RC201	100	6	1632.41	1630.53	1623.78	**1623.36**	6.81	1630.53
RC202	100	6	1459.84	1461.44	1450.70	**1447.27**	5.29	1459.84
RC203	100	6	1295.07	1292.92	**1274.04**	**1274.04**	4.49	1292.92
RC204	100	6	1171.26	1162.91	1159.37	**1159.00**	6.01	1162.91
RC205	100	6	1525.28	1532.67	1517.09	**1512.53**	5.24	1525.28
RC206	100	6	1425.15	1420.89	1400.62	**1395.18**	3.92	1420.89
RC207	100	6	1332.40	1328.29	1319.56	**1314.44**	6.24	1328.29
RC208	100	6	1155.02	1152.92	**1140.10**	**1140.10**	6.81	1152.92
Time			3.08 min	—		4.56 min		
Gap			+0.26%	+0.27%	-0.36%	-0.51%		
CPU			Ath 2.6G	Duo 2.4G		Opt 2.2G		

Table V.24: Results on the type 1 VRPSTW (only lateness) with $\alpha = 100$, hierarchical objective involving first the minimization of the Fleet Size "Fleet", then the number of customers serviced outside of their time windows "TW", then the overall lateness "L", and finally distance "Dist". Instances of Solomon and Desrosiers (1988)

Inst	**n**	**F10**				**UHGS**								
		Single				Avg 10				Best 10				
		Fleet	TW	L	Dist	Fleet	TW	L	Dist	Fleet	TW	L	Dist	T(min)
R101	100	12	56	—	1128.70	11.00	27.70	1644.70	1329.70	11	27	1720.97	1331.85	11.78
R102	100	11	46	—	1058.70	10.00	22.20	1069.17	1226.84	10	21	1206.72	1252.78	18.64
R103	100	10	34	—	1027.40	9.80	7.50	258.62	1171.84	9	5	90.68	1208.63	11.05
R104	100	9	18	—	947.30	9.00	0.50	17.15	1009.30	9	0	0.00	1007.31	7.79
R105	100	11	42	—	1073.50	10.20	19.50	1106.53	1239.15	10	9	680.04	1381.88	13.74
R106	100	10	33	—	1047.40	9.90	8.10	433.32	1236.89	9	6	358.86	1259.11	12.59
R107	100	10	24	—	987.60	9.00	7.00	354.81	1029.30	9	7	343.45	1042.96	11.24
R108	100	9	14	—	947.20	9.00	0.00	0.00	963.50	9	0	0.00	960.88	8.10
R109	100	10	28	—	1001.40	10.00	5.00	208.62	1194.64	10	5	200.72	1183.42	8.80
R110	100	9	29	—	1013.40	9.20	9.50	461.42	1043.08	9	0	0.00	1118.84	12.57
R111	100	10	26	—	983.30	9.00	7.80	385.46	1038.01	9	6	443.34	1047.09	12.72
R112	100	9	17	—	940.90	9.00	0.00	0.00	988.59	9	0	0.00	982.14	9.96
R201	100	3	56	—	984.00	3.00	2.00	197.62	1502.80	3	2	174.47	1497.06	27.05
R202	100	3	40	—	943.50	2.00	21.30	4703.07	991.60	2	20	4757.71	991.27	30.61
R203	100	2	30	—	901.80	2.00	6.10	1124.60	991.09	2	6	1076.65	995.76	30.04
R204	100	2	19	—	836.30	2.00	0.00	0.00	830.79	2	0	0.00	825.52	26.74
R205	100	3	36	—	911.90	2.00	14.40	2998.08	983.30	2	14	2767.13	973.84	26.49
R206	100	2	25	—	956.90	2.00	3.50	652.23	994.39	2	3	465.02	993.13	29.92
R207	100	2	18	—	876.60	2.00	0.00	0.00	893.85	2	0	0.00	893.33	29.54
R208	100	2	11	—	833.40	2.00	0.00	0.00	727.17	2	0	0.00	726.82	14.16
R209	100	2	26	—	950.50	2.00	7.30	1814.72	990.71	2	7	1386.37	994.41	29.28
R210	100	2	29	—	963.80	2.00	6.00	1339.34	995.19	2	6	1264.94	997.75	29.75
R211	100	2	14	—	906.80	2.00	0.00	0.00	900.56	2	0	0.00	892.72	27.23
C101	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.32
C102	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.28
C103	100	—	—	—	—	10.00	0.00	0.00	828.06	10	0	0.00	828.06	2.12
C104	100	—	—	—	—	10.00	0.00	0.00	824.78	10	0	0.00	824.78	1.96
C105	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.73
C106	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.56
C107	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.48
C108	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.19
C109	100	—	—	—	—	10.00	0.00	0.00	828.94	10	0	0.00	828.94	2.05
C201	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	5.82
C202	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	8.35
C203	100	—	—	—	—	3.00	0.00	0.00	591.17	3	0	0.00	591.17	9.79
C204	100	—	—	—	—	3.00	0.00	0.00	590.60	3	0	0.00	590.60	8.08
C205	100	—	—	—	—	3.00	0.00	0.00	588.88	3	0	0.00	588.88	6.40
C206	100	—	—	—	—	3.00	0.00	0.00	588.49	3	0	0.00	588.49	6.53
C207	100	—	—	—	—	3.00	0.00	0.00	588.29	3	0	0.00	588.29	6.54
C208	100	—	—	—	—	3.00	0.00	0.00	588.32	3	0	0.00	588.32	6.51
RC101	100	11	44	—	1255.30	11.00	12.10	789.34	1486.11	11	12	808.44	1486.99	7.01
RC102	100	10	32	—	1230.10	10.30	10.80	476.57	1399.34	10	4	41.41	1539.29	10.51
RC103	100	10	25	—	1154.60	10.00	1.20	18.41	1320.22	10	1	4.18	1333.71	8.03
RC104	100	10	12	—	1083.90	10.00	0.00	0.00	1135.48	10	0	0.00	1135.48	5.57
RC105	100	11	38	—	1219.70	10.90	7.20	345.49	1516.88	10	6	278.42	1538.72	6.89
RC106	100	10	27	—	1150.30	10.00	7.60	308.59	1310.32	10	7	344.38	1307.76	8.85
RC107	100	10	28	—	1123.00	10.00	1.50	65.46	1300.57	10	1	64.05	1325.19	9.70
RC108	100	10	10	—	1071.60	10.00	0.00	0.00	1139.82	10	0	0.00	1139.82	6.97
RC201	100	3	48	—	1147.40	3.00	3.00	482.47	1663.69	3	3	482.47	1663.69	32.65
RC202	100	3	35	—	1073.50	3.00	0.00	0.00	1377.25	3	0	0.00	1365.65	26.22
RC203	100	3	29	—	906.30	2.10	13.80	3013.28	929.95	2	0	0.00	1064.14	30.49
RC204	100	2	14	—	850.70	2.00	2.00	261.17	915.94	2	2	257.81	916.79	29.27
RC205	100	3	40	—	1158.40	3.00	1.00	87.83	1623.09	3	1	9.38	1605.20	32.78
RC206	100	3	40	—	978.40	3.00	0.00	0.00	1149.37	3	0	0.00	1146.32	24.48
RC207	100	3	33	—	986.40	3.00	0.00	0.00	1061.49	3	0	0.00	1061.14	24.51
RC208	100	2	21	—	885.50	2.00	6.40	946.83	910.34	2	6	949.50	917.64	22.51
Time		9.69 min				18.62 min								
CPU		P-M 1.6G				Opt 2.2G								

Table V.25: Results on the type 1 VRPSTW (only lateness) with $\alpha = 1$. Minimization of the sum of distance and lateness under a fleet size limit. Instances of Solomon and Desrosiers (1988).

Inst	**n**	**m**	**KTDHS12**		**UHGS**			**BKS**
			Avg 10	Best 10	Avg 10	Best 10	T(min)	
R101	100	19	1562.98	**1562.58**	1562.89	**1562.58**	1.93	1562.58
R102	100	17	1379.62	**1379.11**	1379.21	**1379.11**	1.90	1379.11
R103	100	13	1160.64	1159.54	1159.51	**1159.28**	3.34	1159.54
R104	100	9	1009.02	1003.73	**999.77**	**999.77**	3.93	1003.73
R105	100	14	1348.89	**1347.75**	**1347.75**	**1347.75**	2.17	1347.75
R106	100	12	1237.29	**1236.58**	**1236.58**	**1236.58**	2.91	1236.58
R107	100	10	1089.84	1084.96	**1083.62**	**1083.62**	4.55	1084.96
R108	100	9	951.24	949.94	947.04	**946.60**	4.20	949.94
R109	100	11	1176.40	**1173.21**	**1173.21**	**1173.21**	3.54	1173.21
R110	100	10	1114.66	**1106.66**	1111.57	1107.26	3.55	1106.66
R111	100	10	1086.36	1080.25	1076.41	**1074.84**	4.97	1080.25
R112	100	9	981.82	972.11	975.78	**971.31**	5.13	972.11
C101	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.10	828.94
C102	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.15	828.94
C103	100	10	828.07	828.07	**828.06**	**828.06**	2.02	828.07
C104	100	10	**824.78**	**824.78**	**824.78**	**824.78**	1.96	824.78
C105	100	10	**828.94**	**828.94**	**828.94**	**828.94**	2.12	828.94
C106	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.91	828.94
C107	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.89	828.94
C108	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.94	828.94
C109	100	10	**828.94**	**828.94**	**828.94**	**828.94**	1.90	828.94
RC101	100	14	1591.59	**1590.22**	**1590.22**	**1590.22**	2.57	1590.22
RC102	100	12	1429.90	**1428.21**	**1428.21**	**1428.21**	2.83	1428.21
RC103	100	11	1242.33	**1239.54**	1239.73	**1239.54**	3.09	1239.54
RC104	100	10	1128.74	**1126.31**	**1126.31**	**1126.31**	3.25	1126.31
RC105	100	13	1451.38	**1450.84**	**1450.84**	**1450.84**	2.59	1450.84
RC106	100	11	1350.17	**1349.30**	1349.72	**1349.30**	3.55	1349.30
RC107	100	11	1208.96	**1208.81**	1208.98	**1208.81**	3.37	1208.81
RC108	100	10	1119.61	**1118.00**	1118.31	**1118.00**	3.72	1118.00
R201	100	4	1237.17	**1237.11**	**1237.11**	**1237.11**	4.13	1237.11
R202	100	3	1169.23	**1165.32**	**1165.32**	**1165.32**	6.19	1165.32
R203	100	3	942.96	937.35	934.01	**933.52**	10.55	937.35
R204	100	2	840.79	832.38	824.73	**824.02**	24.72	832.38
R205	100	3	1006.79	**994.43**	**994.43**	**994.43**	6.53	994.43
R206	100	3	920.13	912.81	**906.14**	**906.14**	7.39	912.81
R207	100	2	1044.87	908.70	888.44	**887.28**	24.63	908.70
R208	100	2	735.26	728.92	727.08	**726.82**	13.72	728.92
R209	100	3	917.21	909.30	**909.16**	**909.16**	6.98	909.30
R210	100	3	958.58	948.80	941.95	**938.34**	8.57	948.80
R211	100	2	923.85	901.18	892.50	**885.71**	19.66	901.18
C201	100	3	**591.56**	**591.56**	**591.56**	**591.56**	4.62	591.56
C202	100	3	**591.56**	**591.56**	**591.56**	**591.56**	6.90	591.56
C203	100	3	**591.17**	**591.17**	**591.17**	**591.17**	7.67	591.17
C204	100	3	**590.60**	**590.60**	**590.60**	**590.60**	6.84	590.60
C205	100	3	**588.88**	**588.88**	**588.88**	**588.88**	5.24	588.88
C206	100	3	**588.49**	**588.49**	**588.49**	**588.49**	5.84	588.49
C207	100	3	**588.29**	**588.29**	**588.29**	**588.29**	5.25	588.29
C208	100	3	**588.32**	**588.32**	**588.32**	**588.32**	5.89	588.32
RC201	100	4	1380.47	**1380.33**	**1380.33**	**1380.33**	4.34	1380.33
RC202	100	3	1322.17	**1317.28**	**1317.28**	**1317.28**	9.59	1317.28
RC203	100	3	1057.10	1046.05	1045.00	**1040.77**	10.73	1046.05
RC204	100	3	809.09	797.41	**797.04**	**797.04**	6.71	797.41
RC205	100	4	1305.97	1299.61	**1298.00**	**1297.65**	6.34	1299.61
RC206	100	3	1135.90	**1135.26**	**1135.26**	**1135.26**	6.56	1135.26
RC207	100	3	1073.58	1061.14	1058.16	**1056.88**	7.43	1061.14
RC208	100	3	834.82	829.00	827.90	**827.67**	7.98	829.00
Time			10.00 min		5.82 min			
Gap			+0.62%	+0.00%	-0.13%	-0.18%		
CPU			Xe 2.67G		Opt 2.2G			

Table V.26: Results on the type 2 VRPSTW (earliness and lateness) with $\alpha = 100$, hierarchical objective involving first the minimization of the Fleet Size "Fleet", then the number of customers serviced outside of their time windows "TW", then the overall earliness plus lateness "E+L", and finally distance "Dist". Instances of Solomon and Desrosiers (1988)

Inst	**n**	**FEL07** Best X				**UHGS** Avg 10				**UHGS** Best 10				
		Fleet	TW	E+L	Dist	Fleet	TW	E+L	Dist	Fleet	TW	E+L	Dist	T(min)
R101	100	14	44	—	1872.94	9.00	57.30	2998.08	1025.65	9	56	2742.45	1018.58	61.19
R102	100	13	29	—	1732.54	9.00	38.70	1825.32	1018.77	9	37	1934.47	1012.28	60.94
R103	100	12	9	—	1542.79	9.00	17.30	676.55	1020.72	9	16	621.25	1022.96	60.65
R104	100	10	0	—	1107.18	9.00	1.60	44.39	1014.39	9	1	19.05	1013.65	60.44
R105	100	—	—	—	—	9.00	39.10	2008.68	1030.78	9	37	2050.22	1037.70	61.04
R106	100	—	—	—	—	9.00	24.30	1107.09	1035.12	9	24	972.07	1021.48	60.73
R107	100	—	—	—	—	9.00	7.90	341.56	1031.17	9	7	294.55	1033.97	60.58
R108	100	10	0	—	968.34	9.00	0.00	0.00	980.60	9	0	0.00	970.15	60.37
R109	100	11	4	—	1379.87	9.00	22.40	961.34	1023.20	9	21	833.92	1028.18	60.66
R110	100	—	—	—	—	9.00	12.60	572.97	1025.82	9	11	552.12	1034.03	60.49
R111	100	—	—	—	—	9.00	8.10	334.60	1015.26	9	7	248.93	1013.65	60.65
R112	100	—	—	—	—	9.00	1.10	30.57	1018.88	9	1	1.25	1025.86	60.30
C101	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.55
C102	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.25
C103	100	10	0	—	918.08	10.00	0.00	0.00	828.06	10	0	0.00	828.06	30.15
C104	100	10	0	—	899.00	10.00	0.00	0.00	824.78	10	0	0.00	824.78	30.32
C105	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.43
C106	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.78
C107	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.55
C108	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.18
C109	100	10	0	—	828.94	10.00	0.00	0.00	828.94	10	0	0.00	828.94	30.15
RC101	100	13	26	—	1851.22	9.00	46.70	2631.68	1120.65	9	44	2787.38	1122.73	39.23
RC102	100	13	1	—	1772.42	9.00	31.30	1573.76	1123.74	9	29	1508.22	1119.92	38.13
RC103	100	11	0	—	1416.81	9.00	17.30	698.94	1125.40	9	16	640.61	1131.62	37.56
RC104	100	10	0	—	1262.55	9.00	5.80	155.00	1117.95	9	5	161.57	1126.69	33.99
RC105	100	12	1	—	1531.57	9.00	34.60	1648.93	1130.18	9	33	1580.23	1127.29	39.71
RC106	100	11	0	—	1224.72	9.00	28.63	1161.00	1123.24	9	28	1207.60	1111.51	38.08
RC107	100	—	—	—	—	9.00	18.67	720.80	1112.72	9	18	601.38	1106.99	35.73
RC108	100	—	—	—	—	9.00	11.20	352.37	1107.38	9	10	284.23	1123.11	35.30
R201	100	—	—	—	—	2.00	41.20	9159.21	985.32	2	40	8583.79	988.84	31.01
R202	100	—	—	—	—	2.00	25.90	4631.88	986.18	2	24	5062.88	986.21	31.00
R203	100	—	—	—	—	2.00	10.80	1924.46	979.73	2	10	1514.13	988.64	30.89
R204	100	—	—	—	—	2.00	0.00	0.00	873.07	2	0	0.00	851.66	30.96
R205	100	—	—	—	—	2.00	20.00	3979.80	985.48	2	19	3289.15	979.97	30.89
R206	100	—	—	—	—	2.00	9.00	1699.39	983.37	2	7	1624.34	988.52	30.89
R207	100	—	—	—	—	2.00	1.50	135.63	973.57	2	1	26.44	933.74	30.91
R208	100	—	—	—	—	2.00	0.00	0.00	741.26	2	0	0.00	730.54	30.69
R209	100	—	—	—	—	2.00	12.70	2383.53	980.17	2	10	1910.46	963.47	30.85
R210	100	—	—	—	—	2.00	12.30	2352.23	981.78	2	11	2015.68	983.07	30.88
R211	100	—	—	—	—	2.00	0.70	40.99	968.68	2	0	0.00	931.99	30.90
C201	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	30.19
C202	100	—	—	—	—	3.00	0.00	0.00	591.56	3	0	0.00	591.56	30.33
C203	100	—	—	—	—	3.00	0.00	0.00	591.17	3	0	0.00	591.17	30.38
C204	100	—	—	—	—	3.00	0.00	0.00	590.93	3	0	0.00	590.60	30.37
C205	100	—	—	—	—	3.00	0.00	0.00	588.88	3	0	0.00	588.88	30.17
C206	100	—	—	—	—	3.00	0.00	0.00	588.49	3	0	0.00	588.49	30.22
C207	100	—	—	—	—	3.00	0.00	0.00	588.29	3	0	0.00	588.29	30.18
C208	100	—	—	—	—	3.00	0.00	0.00	588.32	3	0	0.00	588.32	30.24
RC201	100	—	—	—	—	2.00	52.70	12622.22	908.21	2	50	12420.98	912.51	31.01
RC202	100	—	—	—	—	2.00	33.80	7893.30	911.08	2	33	7335.92	907.59	30.98
RC203	100	—	—	—	—	2.00	16.30	3181.25	907.44	2	15	2418.65	910.63	30.89
RC204	100	—	—	—	—	2.00	3.40	601.79	902.05	2	2	460.90	894.01	30.72
RC205	100	—	—	—	—	2.00	39.30	8993.74	909.28	2	37	8706.52	911.66	30.96
RC206	100	—	—	—	—	2.00	35.30	8265.45	906.10	2	33	7424.35	913.25	30.82
RC207	100	—	—	—	—	2.00	25.00	5291.51	908.94	2	24	4805.29	908.80	30.85
RC208	100	—	—	—	—	2.00	12.70	1828.13	911.30	2	11	1694.30	912.64	30.74
Time		5.98 min				41.16 min								
CPU		P-II 0.6G				Opt 2.2G								

Table V.27: Results on the type 2 VRPSTW (earliness and lateness) with $\alpha = 1$. Minimization of the sum of distance, earliness and lateness under a fleet size limit. Instances of Solomon and Desrosiers (1988)

Inst	**n**	**m**	**UHGS**		
			Avg 10	Best 10	T(min)
R101	19	100	1546.91	1546.91	24.13
R102	17	100	1377.38	1377.38	26.93
R103	13	100	1158.83	1158.31	30.14
R104	9	100	1004.57	1000.33	30.10
R105	14	100	1342.57	1342.57	30.03
R106	12	100	1223.09	1223.09	30.12
R107	10	100	1080.90	1079.12	30.43
R108	9	100	948.23	945.64	30.08
R109	11	100	1164.68	1164.68	30.11
R110	10	100	1108.30	1104.59	30.16
R111	10	100	1065.76	1065.76	30.08
R112	9	100	991.50	969.91	30.10
C101	10	100	828.94	828.94	30.10
C102	10	100	828.94	828.94	30.08
C103	10	100	828.06	828.06	30.20
C104	10	100	824.78	824.78	29.34
C105	10	100	828.94	828.94	30.12
C106	10	100	828.94	828.94	30.07
C107	10	100	828.94	828.94	30.07
C108	10	100	828.94	828.94	30.33
C109	10	100	828.94	828.94	30.09
RC101	14	100	1584.20	1584.20	29.67
RC102	12	100	1409.36	1409.36	30.07
RC103	11	100	1231.67	1231.67	30.16
RC104	10	100	1123.25	1121.84	30.12
RC105	13	100	1433.37	1433.37	30.18
RC106	11	100	1334.89	1334.89	30.39
RC107	11	100	1203.06	1203.06	30.45
RC108	10	100	1115.44	1115.44	30.12
R201	4	100	1235.14	1235.14	30.28
R202	3	100	1159.76	1159.76	30.13
R203	3	100	937.04	934.10	30.15
R204	2	100	837.21	820.90	30.18
R205	3	100	996.24	994.43	30.12
R206	3	100	910.99	906.54	30.11
R207	2	100	937.79	906.81	30.19
R208	2	100	735.31	730.52	30.13
R209	3	100	911.61	909.16	30.11
R210	3	100	948.91	938.77	30.14
R211	2	100	921.81	912.39	30.17
C201	3	100	591.56	591.56	30.09
C202	3	100	591.56	591.56	30.11
C203	3	100	591.17	591.17	30.11
C204	3	100	590.60	590.60	30.09
C205	3	100	588.88	588.88	30.12
C206	3	100	588.49	588.49	30.13
C207	3	100	588.29	588.29	30.11
C208	3	100	588.32	588.32	30.11
RC201	4	100	1380.33	1380.33	30.12
RC202	3	100	1312.05	1312.05	30.10
RC203	3	100	1047.43	1044.74	30.13
RC204	3	100	796.91	796.68	30.13
RC205	4	100	1300.98	1297.86	30.11
RC206	3	100	1135.44	1135.26	30.12
RC207	3	100	1061.92	1056.88	30.12
RC208	3	100	832.30	827.67	30.13
Time			29.96 min		
Gap			+0.26%	+0.00%	
CPU			Opt 2.2G		

Table V.28: Results on the new MDPVRPTW instances.

Inst	**n**	**m**	**t**	**d**	**UHGS**		
					Avg 10	Best 10	T(min)
pr01	48	1	4	4	2483.81	2482.78	0.87
pr02	96	2	4	4	4474.72	4468.60	2.93
pr03	144	3	4	4	5758.43	5735.59	6.93
pr04	192	4	4	4	6708.98	6680.76	18.96
pr05	240	4	4	4	7275.26	7202.79	29.02
pr06	288	5	4	4	8263.29	8207.18	29.68
pr07	72	1	6	6	5497.20	5496.76	2.06
pr08	144	2	6	6	7791.98	7716.08	11.21
pr09	216	3	6	6	10579.81	10504.77	29.49
pr10	288	4	6	6	13612.91	13343.55	30.00
pr11	48	1	4	4	2043.74	2043.74	0.85
pr12	96	2	4	4	3851.07	3825.34	3.47
pr13	144	3	4	4	4781.02	4755.10	9.48
pr14	192	4	4	4	5535.82	5471.17	21.36
pr15	240	4	4	4	5871.13	5830.71	30.00
pr16	288	5	4	4	6913.56	6832.53	30.01
pr17	72	1	6	6	4787.41	4782.74	2.47
pr18	144	2	6	6	6465.43	6402.79	21.78
pr19	216	3	6	6	8940.97	8785.80	30.01
pr20	288	3	6	6	10930.86	10662.62	30.00
pr01b	48	1	4	4	2423.29	2423.29	0.64
pr02b	96	2	4	4	4521.26	4486.88	2.84
pr03b	144	3	4	4	5664.54	5649.74	7.20
pr04b	192	4	4	4	6724.52	6694.32	16.69
pr05b	240	4	4	4	7345.82	7284.81	28.18
pr06b	288	5	4	4	8591.64	8551.01	30.00
pr07b	72	1	6	6	5255.06	5255.06	1.67
pr08b	144	2	6	6	7468.25	7444.70	10.10
pr09b	216	3	6	6	10930.40	10797.34	29.67
pr10b	288	4	6	6	12673.68	12494.65	30.00
pr11b	48	1	4	4	2100.23	2100.23	0.80
pr12b	96	2	4	4	3782.07	3748.45	2.85
pr13b	144	3	4	4	4891.31	4883.31	9.40
pr14b	192	4	4	4	5474.16	5442.94	21.13
pr15b	240	4	4	4	5902.56	5809.17	29.71
pr16b	288	5	4	4	6992.78	6941.25	29.90
pr17b	72	1	6	6	4794.89	4794.89	1.86
pr18b	144	2	6	6	6332.54	6288.46	22.25
pr19b	216	3	6	6	9003.87	8825.36	30.00
pr20b	288	3	6	6	10998.39	10774.34	30.00
Time						16.09 min	
Gap					+0.77%	+0.00%	
CPU						Opt 2.2G	

Annexe VI

SUPPLEMENT TO "INTEGRATIVE COOPERATIVE SEARCH"

We first display the characteristics of the test instances and then present the detailed result tables for the four ICS versions not included in the body of the text. Each entry is the average of ten repetitions.

The MDPVRP benchmark instances introduced in Vidal et al. (2012a) were produced by merging the instances for the MDVRP and the PVRP described in Cordeau et al. (2001a). The characteristics of the instances are summarized in Table VI.1. The number of customers in each instance is represented by n, while t is the number of periods in the PVRP and the number of depots in the MDVRP. m is the number of vehicles for the MDPVRP. This number of vehicles available each day at each depot is set to the minimum number of vehicles needed. D and Q are the capacity and the duration constraints on the vehicles, respectively.

Inst	**n**	**t**	**m**	**D**	**Q**
pr01	48	4	1	500	200
pr02	96	4	1	480	195
pr03	144	4	2	460	190
pr04	192	4	2	440	185
pr05	240	4	3	420	180
pr06	288	4	3	400	175
pr07	72	6	1	500	200
pr08	144	6	1	475	190
pr09	216	6	2	450	180
pr10	288	6	3	425	170

Table VI.1: MDPVRP instances

Inst	GUTS			GUTS-ICS								BKS
	Avg	T(min)	Best	Avg				Best				
				5 min	10 min	15 min	30 min	5 min	10 min	15 min	30 min	
pr01	**2019.07**	0.14	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	2019.07
pr02	3547.72	2.39	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	3547.45
pr03	4639.50	18.87	4578.14	4492.07	4483.49	4482.74	4481.51	4483.29	4480.90	**4480.87**	**4480.87**	4480.87
pr04	5246.39	20.03	5225.67	5188.12	5171.35	5168.81	5163.29	5170.63	5156.16	5156.16	**5148.06**	5134.17
pr05	5738.40	21.10	5666.35	5762.78	5674.16	5644.62	5633.24	5663.68	5635.88	5626.66	**5616.02**	5570.45
pr06	6759.41	20.06	6723.52	6716.27	6615.74	6603.22	6585.77	6575.16	6559.11	6555.71	**6548.86**	6524.92
pr07	4644.93	0.71	4644.93	4502.06	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	4502.02
pr08	6217.23	20.82	6174.15	6038.41	6024.40	6024.19	6024.08	6024.00	**6023.98**	**6023.98**	**6023.98**	6023.98
pr09	8644.82	23.85	8548.63	8430.12	8356.94	8322.31	8306.93	8323.97	8305.61	8286.66	**8277.93**	8257.80
pr10	10241.67	26.45	10120.30	10421.26	10324.62	10192.87	10071.34	10106.10	10092.50	10018.80	**10006.50**	9818.42
Gap	+2.77%		+2.10%	+1.62%	+1.04%	+0.78%	+0.58%	+0.69%	+0.55%	+0.43%	+0.36%	

Table VI.2: Performance of GUTS-ICS

Inst	HGSADC			HGSADC-ICS1								BKS
	Avg	T(min)	Best	Avg				Best				
				5 min	10 min	15 min	30 min	5 min	10 min	15 min	30 min	
pr01	**2019.07**	0.35	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	2019.07
pr02	**3547.45**	1.49	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	3547.45
pr03	4491.08	7.72	**4480.87**	4485.72	4483.03	4482.07	4480.88	**4480.87**	**4480.87**	**4480.87**	**4480.87**	4480.87
pr04	5151.73	22.10	5144.41	5162.75	5156.36	5153.18	5149.17	5154.83	5146.29	5146.14	**5144.41**	5134.17
pr05	5605.60	10.00	5581.10	5674.21	5622.02	5608.19	5598.38	5606.55	5580.53	5577.83	**5574.61**	5570.45
pr06	6570.28	10.00	6549.57	6602.40	6579.26	6568.33	6555.72	6574.52	6548.81	6543.54	**6542.32**	6524.92
pr07	4502.06	2.18	4502.06	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	4502.02
pr08	6029.58	7.96	**6023.98**	6024.29	6024.03	6024.03	6023.99	**6023.98**	**6023.98**	**6023.98**	**6023.98**	6023.98
pr09	8310.19	27.79	8271.66	8325.98	8297.34	8290.60	8281.60	8265.31	8262.99	**8261.74**	**8261.74**	8257.80
pr10	9972.35	30.00	9852.87	10138.28	10052.64	10003.22	9949.62	9970.82	9907.04	9904.36	**9830.38**	9818.42
Gap	+0.42%		+0.13%	+0.78%	+0.51%	+0.40%	+0.29%	+0.35%	+0.17%	+0.16%	+0.07%	

Table VI.3: Performance of HGSADC-ICS1 with shared populations

Inst	HGSADC			HGSADC-ICS1+								BKS
	Avg	T(min)	Best	Avg				Best				
				5 min	10 min	15 min	30 min	5 min	10 min	15 min	30 min	
pr01	**2019.07**	0.35	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	2019.07
pr02	**3547.45**	1.49	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	3547.45
pr03	4491.08	7.72	**4480.87**	4482.33	4481.35	4480.99	**4480.87**	**4480.87**	**4480.87**	**4480.87**	**4480.87**	4480.87
pr04	5151.73	22.10	5144.41	5158.64	5152.78	5150.57	5148.79	5146.63	5145.62	5145.62	**5144.45**	5134.17
pr05	5605.60	10.00	5581.10	5631.65	5601.88	5593.40	5587.97	5613.77	5590.25	5576.38	**5573.91**	5570.45
pr06	6570.28	10.00	6549.57	6599.25	6567.80	6560.05	6551.72	6573.99	6550.10	6543.19	**6536.97**	6524.92
pr07	4502.06	2.18	4502.06	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	4502.02
pr08	6029.58	7.96	**6023.98**	6024.09	6024.03	6024.01	6024.01	**6023.98**	**6023.98**	**6023.98**	**6023.98**	6023.98
pr09	8310.19	27.79	8271.66	8306.52	8287.13	8278.86	8273.77	8288.08	8263.17	8261.70	**8256.67**	8257.80
pr10	9972.35	30.00	9852.87	10065.30	9993.48	9950.46	9902.44	9995.04	9879.76	9859.08	**9826.14**	9818.42
Gap	+0.42%		+0.13%	+0.59%	+0.37%	+0.29%	+0.21%	+0.39%	+0.17%	+0.11%	+0.05%	

Table VI.4: Performance of HGSADC-ICS1+ with shared populations

Inst	HGSADC			HGSADC-ICS2								BKS
	Avg	T(min)	Best	Avg				Best				
				5 min	10 min	15 min	30 min	5 min	10 min	15 min	30 min	
pr01	**2019.07**	0.35	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	**2019.07**	2019.07
pr02	**3547.45**	1.49	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	**3547.45**	3547.45
pr03	4491.08	7.72	**4480.87**	4495.57	4482.45	4480.91	4480.91	**4480.87**	**4480.87**	**4480.87**	**4480.87**	4480.87
pr04	5151.73	22.10	5144.41	5168.41	5155.54	5150.79	5148.27	5154.51	**5144.41**	**5144.41**	**5144.41**	5134.17
pr05	5605.60	10.00	5581.10	5682.73	5639.67	5617.91	5604.86	5621.61	5570.54	5570.54	**5568.28**	5570.45
pr06	6570.28	10.00	6549.57	6638.89	6585.02	6569.59	6551.77	6605.94	6560.20	6541.38	**6528.58**	6524.92
pr07	4502.06	2.18	4502.06	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	**4502.02**	4502.02
pr08	6029.58	7.96	**6023.98**	6029.07	6024.01	**6023.98**	**6023.98**	**6023.98**	**6023.98**	**6023.98**	**6023.98**	6023.98
pr09	8310.19	27.79	8271.66	8341.98	8308.42	8292.10	8277.61	8291.09	8263.99	8261.47	**8261.10**	8257.80
pr10	9972.35	30.00	9852.87	10339.20	10139.79	10075.60	9984.54	10147.90	9986.01	9959.13	**9896.27**	9818.42
Gap	+0.42%		+0.13%	+1.12%	+0.65%	+0.49%	+0.32%	+0.63%	+0.25%	+0.19%	+0.10%	

Table VI.5: Performance of HGSADC-ICS2 with encapsulated population-based solvers