

# **ITEM: an Integrated Transport Economy Model**

## **Price Elasticities of Freight Transport Demand in Canada:**

### **Three Canadian Markets and Four Commodities**

by

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## ABSTRACT

This analysis will shed light on the effects of a price variation on freight transport demand for two modes: rail and truck. Laferrière (1992) has developed a method for comparing the price elasticities of different econometric models. This method consists of obtaining elasticity for a particular market without the sample that was used for calibration by approximating the price elasticity following an enumeration methodology. This will be done for seven econometric models for three Canadian markets and four commodities. Overall, this paper develops a methodology that is exportable for freight transportation around the world. Further investigation by exploring preliminary results would allow seeing if price elasticities of freight transport demand are homogenous and transferable over time and space.

## SOMMAIRE

Cette analyse s'attardera à l'étude de l'effet d'une variation de prix sur la demande de transport de marchandises pour deux modes; le train et le camion. Il développera particulièrement une méthodologie adaptée au transport de marchandises. Laferrière (1992) a développé une méthode pour comparer les élasticités-prix de différents modèles économétriques. Cette méthode consiste à obtenir des élasticités-prix, sans l'échantillon initial de calibration en approximant les élasticités par énumération. Cette méthode sera appliquée à sept modèles économétriques, à trois marchés Canadiens et quatre commodités. L'exploration de résultats préliminaires pourrait suggérer que les élasticité-prix soient homogènes et transférables dans l'espace et le temps. Un approfondissement de la méthode ajoutant plusieurs commodités et marchés contribuerait à supporter ou rejeter cette conclusion.

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## 1 INTRODUCTION

We know that the demand for transport between two destinations depends on the price of such transportation via a particular mode and depends equally on the price of transportation via other modes. It is also clear that other factors can affect travel demand. A Montreal/Toronto trip for an agent surely depends on the price of the flight tickets, but also depends on his income, on the number of people in his social unit, on the time it takes to get to destination. It is easy to take these factors individually and see their specific effect on transport demand. According to the economic theory, when the price of a transport mode goes up, the demand for that same mode goes down. Inversely, when the price goes down, the demand tends to grow.

This analysis will shed light on the effects of a price variation on freight transport demand for two modes: rail and truck. The task is done while considering the effects of other factors on transport demand. Thus, as mentioned above, econometrics tools and models are at the centre of the analysis as they allow for a multidimensional approach. This approach gives way to the question of interest: *Etceterus paribus*, what is the effect of a price increase on freight transport demand? Multivariate analysis will lead to clear answers allowing for a better understanding of the transport industry and enabling government agency in their pursuit of better management.

It is also important to define a concept that will be at the heart of the study: the concept of price-elasticity. Elasticity is simply defined as a change in % in the dependent variable induced by a 1 % change in the independent variable. It follows that transport price-

elasticity is the change in % of transport demand for mode  $i$  induced by a 1% change in the price of mode  $i$ ,  $i$  being a particular mode. Transport economics is much like any economics in that price-elasticity of demand is generally thought to be negative. When prices are up, demand is down. Note that the interpretations of price-elasticities in this study are in absolute value. For instance, an elasticity of  $-2$  will be interpreted as greater than an elasticity of  $-1$ .

There is a vast literature on factors affecting transport demand. It is worth mentioning that the literature on this subject focuses on transport demand for a mode in particular, which is called modal demand, in relation to its price and to the price of its alternative. Very few studies however treat the comparison between different models. Oum (1989), Goodwin (1989) and Fang (1996) all made review of models but never tried comparing price elasticities of each model on a common basis, read sample. A lot less articles and study exist on the question of price elasticity of freight transport. Oum (1992), Gaudry and Picard (1998) discuss different models, Translog, and the Box-Cox respectively, but they do not allow for generalization as Oum's articles relies on one market and as Gaudry's articles relies on old Canadian data. This paper will fill this gap.

Moreover, the question is of great economic interest. Theory tends to assert that the price elasticity of freight transport demand is null due to the small part that tariffs represent in the price of the commodity transported. However, many empirical studies show that price elasticity does vary enormously mainly due to inter-modal competition, road

network complexity and the number of firm involved in the market. This study will clear this paradox for Canadian markets.

This study takes root in the work of Dr. Richard Laferrière. Laferrière (1992) has developed a method for comparing the price elasticities of different econometric models. He compared for the first time different estimates from econometric demand models. Within these models he specifically studied the effect of a price change on transport demand for persons. He more precisely observed the price elasticity of passenger travel for four Canadian markets in the following modes: car, train, bus and plane. As these elasticities came from different models, first off comparison was impossible requiring a calibration of the different models. The method was developed to deal with this problem.

It is therefore interesting to ask the same questions that motivated him and apply his method to rail and truck freight transport demand. Do models calibrated more then 20 years ago still permit for the evaluation of actual demand sensibility? Which models are applicable to any given market type? Are the price elasticities derived from econometric models sufficiently homogenous to suggest a consensus? Which modes have elastic demand? Which transportation modes are sensitive to the price of other modes?

These elasticities are to be calculated with seven econometric models of inter-city freight transport demand. The method for comparing these demand models is set out in section 2. Section 3 will essentially be a literature review. The details for the seven models can be found in section 4. Section 5 will then set out some price-elasticity properties for each



of the models demonstrated, answering some of the questions raised earlier. As this research primarily deals with the development of a methodology adapted to freight transportation, subsequent research will lead to empirical results

## **2 METHODOLOGY FOR MODEL COMPARISON**

This section will highlight the methodology that will give way to elasticity comparison. First, the Canadian markets and market shares of interest will be defined. Second, the models that will be used in this study will be explained. Finally, the enumeration method developed by Laferrière (1992) will be made explicit.

### **2.1 Market Definition, Market Shares and Prices**

As mentioned earlier, price elasticity of transport demand differs from one market to the other. The transport demand literature often presents price elasticities that have been calculated using the same data as those to calibrate the econometric model. When calculated this way, price elasticities cannot be compared with the price elasticities of other empirical studies when they are based on different data.

In order to by pass these difficulties, this analysis will present on a common basis the price elasticities of freight transport demand obtained from different econometric models. This comparison will be done for three Canadian markets for four commodities. Table 2.1.1 reports the market share of each mode in each market by commodity.

Table 2-1 Modal Rail and Truck Share by Commodity and Region for Canadian Market 1997

Commodities	Wood	Paper and	Stone, Clay and	Iron, Steel
Markets	Products	Plastic	Glass	Products
<b>Rail</b>				
Alberta	0,0551	0,057486	0,3450124	0,0125943
Ontario	0,0340	0,170043	0,1286857	0,1092208
Quebec	0,0973	0,208823	0,0105574	0,0440182
Saskatchewan	0,0079	0,014044	0	0,0245079
<b>Truck</b>				
Alberta	0,0407	0,094313	0,0115488	0,0004768
Ontario	0,0856	0,124651	0,1774252	0,0236236
Quebec	0,1200	0,038447	0,130151	0,1565715
Saskatchewan	0,0067	0,039195	1,091E-05	0,0060036

Source: Statistic Canada, Matrix 4915, 1997.

The commodity prices for each of the market will be obtained shortly. Subsequently certain simplifying assumptions have to be made. The prices that will be used are the average rail and truck rate in cents per ton-mile use by Oum (1989). Table 2.2 reports those prices.

Table 2-2 Average Rate in cents per ton-mile for Freight Transportation in Canada(1979)

Mode	Rate
Rail	6.29
Truck	14.32

Source: Oum Tae, "Alternative Demand Models and their Elasticity Estimate", Journal of Transport economic and Policy, May 1989, pp.172.

The market shares of each mode for disaggregate data have been calculated using the same method as used in the econometric models that are examine in this study. Indeed, there exist many ways to calculate a share and one has to be careful with the manipulation of the database in order to remain at a level allowing for comparison. Thus, the market share for a mode in a particular market is:

$S_{im}/(S_{iM}+S_{jM})$   $i,j = \text{rail, truck,}$   $m = \text{particular market,}$   $M = \text{all markets}$

## 2.2 Model Highlights and Definitions

The freight transport demand models used in this paper can be grouped under two broad categories: probability models (Probit-Logit) and market share models (Linear, Log-Linear, Translog). The first category deals with the probability that an economic agent will choose a certain mode for an inter-city shipment. The second category gives the proportion of trips per mode.

### 2.2.1 Logit and Probit Models

The development of a probability model takes many forms. Two of those are of interest: the Logit and the Probit. Both models have behavioural interpretation. Consider the Probit, where some  $y$  is observed taking the value 1 or 0. Define a latent  $y^*$  such that;

$$Y_i^* = X_i \beta + \epsilon_i \quad (2.1)$$

$y^*$  is not observe and  $y$  take the value 1 or 0 following this rule;

$$y_i = 1 \text{ if } y_i^* > 0 \quad (2.2)$$

$$= 0 \text{ otherwise}$$

Assume that  $\epsilon_i \sim N(0, \sigma^2)$  and the rule of equation 2.2 yield a Probit;

$$\begin{aligned} \text{Prob}(y_i = 1) &= \text{prob}(y_i^* > 0) \\ &= \text{prob}(X_i \beta + \epsilon_i > 0) \\ &= \text{prob}(\epsilon_i > -X_i \beta) \end{aligned}$$

$$\begin{aligned}
&= \text{prob} (\varepsilon_i / \sigma > -\mathbf{X}_i \boldsymbol{\beta} / \sigma) \\
&= \Phi (\mathbf{X}_i \boldsymbol{\beta} / \sigma)
\end{aligned} \tag{2.3}$$

The distribution of the Probit is symmetric. It then follows that:

$$\text{Prob} (y_i = 0) = 1 - \Phi (\mathbf{X}_i \boldsymbol{\beta} / \sigma) \tag{2.4}$$

With the first derivative;

$$\partial E(y) / \partial \mathbf{X}_k = \phi (\mathbf{X} \boldsymbol{\beta}) \beta_k \tag{2.5}$$

The derivation of the Logit is identical to the Probit.

$$\text{Prob} (y_i = 1) = F (\mathbf{X}_i \boldsymbol{\beta}) \tag{2.6}$$

Choosing  $F$  to be a logistic distribution we get:

$$\begin{aligned}
\text{Prob} (y_i = 1) &= \Lambda(\mathbf{X}_i \boldsymbol{\beta}) \\
&= \exp (\mathbf{X}_i \boldsymbol{\beta}) / (1 + \exp (\mathbf{X}_i \boldsymbol{\beta}))
\end{aligned} \tag{2.7}$$

with the first derivative;

$$\partial E(y) / \partial \mathbf{X}_k = \exp (\mathbf{X}_i \boldsymbol{\beta}) / (1 + \exp(\mathbf{X}_i \boldsymbol{\beta}))^2 \beta_k \tag{2.8}$$

## 2.2.2 Modal Share Models

They are four market share models in this study. They are enumerated below.

### 2.2.2.1 The Linear Model.

The Linear model is derived the following way:

$$Y = \alpha + \beta X + \mu \tag{2.9}$$

where  $Y$  is a function of  $X$ ,  $\beta$  is a coefficient and  $\mu$  is an error term. The first derivative is:

$$F_{yk} = \partial Y / \partial X_k * X_k / Y \tag{2.10}$$

#### 2.2.2.2 The Log-Linear Model

The equation of the models is;

$$\ln Y = \alpha + \beta \ln X + \mu \quad (2.11)$$

where Y is a function of the log of X,  $\beta$  is a coefficient and  $\mu$  is an error term. The first derivative is simply:

$$F_{yk} = b_k \quad (2.12)$$

#### 2.2.2.3 The Box-Cox Model

The Box-Cox model is obtained the following way:

$$Y_i(\lambda) = \sum_j \beta_j x_{ij}(\lambda) + \mu \quad (2.13)$$

where Y is a function of x,  $\beta$  a coefficient,  $\lambda$  is the Box-Cox term allowing for free variation of the functional form and  $\mu$  is an error term. The first derivative is:

$$F_{yk} = b_k (Y/X_k)^\lambda \quad (2.14)$$

#### 2.2.2.4 The Translog Model

The final model in this category is the Translog model. It is often obtained in the following way:

let

$$C_s = C_s (Y, K, M, P_l, P_t, P_r) \quad (2.15)$$

$$T_s = dC_s (Y, K, M, P_l, P_t, P_r) / dP_t \quad (2.16)$$

$$R_s = dC_s (Y, K, M, P_l, P_t, P_r) / dP_r \quad (2.17)$$

where  $C_s$  refers to short run variable cost,  $T_s$  and  $R_s$  refer to the short-run demand functions for truck and rail respectively;  $Y$  is output;  $K$  and  $M$  are fixed inputs, capital and materials respectively;  $P_l$ ,  $P_t$  and  $P_r$ , refer to the price of labor, the price of truck and rail transportation respectively.

Denoting the fixed inputs, capital and materials, and output by the vector  $X(X = K, M, Y)$  and the price of labor, rail and truck services by the vector  $P(P = P_l, P_r, P_t)$ , the Translog cost function is written :

$$\begin{aligned} \ln C_s = & \alpha_0 + \sum \alpha_i \ln P_i + \sum_h \beta_h \ln X_h + 1/2 \sum_i \sum_j A_{ij} \ln P_i \ln P_j \\ & + \sum_i \sum_h \beta_{ih} \ln P_i \ln X_h + 1/2 \sum_h \sum_s C_{hs} \ln X_h \ln X_s \end{aligned} \quad (2.18)$$

The derivation of this cost function with respect to the price of mode  $i$  yields the input share equation of that mode:

$$S_i = \partial \ln C_s / \partial \ln P_i = \alpha_i + \sum A_{ij} \ln P_j + \sum_h \beta_{ih} \ln X_h \quad (2.19)$$

where  $S_i$  is the firm market share in terms of expenditure on mode  $i$ . The first derivative with respect to price yields;

$$\partial \ln S_i / \partial \ln P_i = A_i / S_i \quad (2.20)$$

### **2.3 Enumeration Method for Elasticity Comparison (Laferrière, 1992)**

In the case of probability models, a sample containing observation on modal choices for a group of agent is necessary to calibrate the model. This is where market shares and the prices intervene. When the model is estimated, it is then possible to calculate the

elasticity (E) for a mode of transport  $i$  with respect to its price  $P_i$  for each agent  $k$  in the sample. This yields:  $E_{pi}^i(k)$ . Once this is done, the price elasticity of a particular market is easily obtainable. Using the elasticity of the agent and its weight in the sample ( $f_k$ ), we get:

$$E_{pi}^i(\text{market}) = \sum_k E_{pi}^i(k) * f_k \quad (2.1)$$

The calculation of price elasticity for an agent  $k$  is done using three pieces of information: a parameter ( $\beta$ ), the cost of freight shipment by mode  $i$  ( $P_i$ ) for the  $k$  agent and the probability that  $k$  will not choose mode  $i$  ( $1 - \text{prob}_i(k)$ ). This yields:

$$E_{pi}^i(k) = \beta * P_i(k) * (1 - \text{prob}_i(k)). \quad (2.10)$$

In order to conduct this enumeration method, the sample that was used to calibrate the model is of capital importance. As said above, a comparison of price elasticities obtained from different sample is impossible and renders this type analysis inadequate. Therefore, one must consider another way.

One way to obtain aggregate elasticity for a particular market ( $m$ ) without the sample that was used for calibration is to approximate the price elasticity.  $E_{pi}^i(k)$  becomes  $E_{pi}^i(\text{approx})$  where  $P_i$  is the price of the representative market ( $m$ ). Substituting the probability of choosing mode  $i$  by the market share of the other modes, equation 2.10 becomes:

$$E_{pi}^i(\text{approx}) = \beta * P_i(k) * (1 - S_i(k)) \quad (2.11)$$

The equation simplifies the task at hand for two reasons. First, a single value for the price of each mode is necessary for calculating price elasticity using (2.11). Second,  $E_{pi}^i(\text{approx})$  is based on  $S_i$ , which is observed, rather than  $\text{prob}_i(k)$ . The approximation involves directly calculating the aggregate elasticity with the market price of market  $m$  and the market share of agent  $k$ . Laferrière (1992) has shown that the difference between  $E_{pi}^i(\text{approx})$  and  $E_{pi}^i(\text{market})$  is small and negligible.

Consider the following three equations representing the aggregate price elasticity of the share of mode  $i$  associated with a probability model:

$$E_{pi}^i(\text{market}) = \sum_k E_{pi}^i(k) * f_k \quad (2.12)$$

$$E_{pi}^i(k) = \beta * P_i(k) * (1 - \text{prob}_i(k)). \quad (2.13)$$

$$\text{prob}_i(k) = \exp [\beta * P_i(k) + A_i(k)] / \sum_j \exp [\beta * P_j(k) + A_j(k)] \quad (2.14)$$

where:

$\text{prob}_i(k)$  = the probability that agent  $k$  will choose mode  $i$ ;

$E_{pi}^i(k)$  = the elasticity of the probability that  $k$  will choose mode  $i$  in respect to the price of mode  $i$ ;

$E_{pi}^i(\text{market})$  = the aggregate own price elasticity with the enumeration method;

$f_k$  = the weight of agent  $k$  in the population;



$P_{hji}(k)$  =the price of mode  $i$  for the agent  $k$ ,

$A_{hji}(k)$  =factors associated with mode  $i$  other than price.

Aggregate elasticities are obtained as a result of a weighted sum of the elasticities of individual. However, since sample data are not available, this method is useless. In this study, equation (2.11) is used. The similarity between the two methods is shown in Table 2.1. Table 2.1 illustrates the result from a Logit model applied to urban San Diego. Laferrière (1992) argues that although this is not a scientific justification, the comparison confirms that  $E_{pi}^i(\text{approx})$  yields reasonable elasticity estimates. This method will be applied to freight transport demand data for ten Canadian markets.

Table 2-3 Direct Price Elasticity with Two Different Methods, Santiago, Chile\*

	$E_{pi}^i(\text{market})$	$E_{pi}^i(\text{approx})$
Alameda	-0.251	-0.370
Alameda	-0.064	-0.075
Alameda	-0.213	-0.240
Alameda	-0.015	-0.015
Alameda	-0.049	-0.095
Alameda	-0.154	-0.193
Alameda	-0.070	-0.077
Alameda	-0.169	-0.189
Alameda	-0.141	-0.160

Laferrière, Richard, "Price Elasticities of Inter-city Travel Demand", Final Report of the Royal Commission on National Passenger Transportation, vol. 4, 1992, pp.1625.

### 3 STUDIES WITH PRICE ELASTICITY COMPARISONS

The nature of this section is to underline the contribution of other authors to the subject at hands. It will consist on surveying articles that did compare elasticity estimates both within the same sample. Again, the subject of this analysis will be to compare elasticity estimates that come from different models and sample under a common basis using the method explained previously.

The theory and the empirical estimation of transport demand have both been subject to many refinements over the past 30 years. Such refinements occurred in the advancement of discrete choice modeling, with the development of computing techniques, the use of more flexible functional forms and the linkage between consumer theory empirical demand models. This part of the study focuses on the major empirical studies of own and cross price elasticity of freight transport demand.

Recent surveys of the demand for freight demand transport include Oum (1989, 1992) and Gaudry and Picard (1998). Oum's 1989 paper is as much of a survey as it is a systematic comparison of four freight demand models done with 1979 Canadian data for 14 commodity groups. His paper concentrates on aggregate freight demand models. He compares the Linear demand model (eq. 2.9); the Log-Linear demand model (eq.2.11); the Logit model applied to aggregate market data share (eq. 2.6) and the Translog demand system model based on the neo-classical demand theory (2.19). In his comparison, Oum explicitly underlines the advantage and the disadvantage of using each model.

The Linear demand model is simple and easily interpreted but the linear effect may not be realistic. The Log Linear demand model is widely used and deals with the non-robust assumption of the previous model. Moreover, in the Log-Linear model, the coefficient themselves are the elasticities. The model is linked to a Cobb-Douglas production function that can be easily interpreted. The main drawback is that each elasticity is invariant across all data points. The Logit model is derived from the utility maximizing choice process of rational actors and can easily be estimated but the IIA (Independence of Irrelevant Assumption) may not always be respected. The Translog demand model is a generalization of the Cobb-Douglas providing a quadratic approximation of the unknown true function. It is consistent with neo-classical theory of consumption and allows for free variation of the elasticities of substitution between modes and of the own and cross elasticities. The main drawback is that it requires increased computational techniques.

Oum maintains that there is no general method for comparing and selecting the best model from a set alternative model when these models do not have nesting relationship. The author evaluates the model by comparing the reasonableness of the elasticities, the magnitudes of the parameters and of various elasticity estimates. He uses two approaches for comparison. First, he compares the empirical results of the models allowing isolation of the pure effects of the model. He then compares the empirical results of the best models. This permits the examination of the extent to which the choice of the functional form affects the results. Table 3.1 recaptures equation 2.6 to 2.19 in a more concisely.

Table 3-1 Price-Elasticities of Freight Transport Demand Models: Five cases.

	Models	Equation	Elasticity
1.	Linear	$Y = \alpha + \beta X + \mu$	$F_{yk} = \partial Y / \partial X_k * X_k / Y$
2.	Log-Linear	$\ln Y = \alpha + \beta \ln X + \mu$	$F_{yk} = b_k$
3.	Box-Cox	$Y_i(\lambda) = \sum_j \beta_j x_{ij}(\lambda) + \mu$	$F_{yk} = b_k (Y/X_k)^\lambda$
4.	Logit	$S_{ij} = \exp(-\beta' x_{ij}) /$ $1 + \exp(-\beta' x_{ij})$	$F_{ij} = \beta x_{ij} * (1 - S_{ij})$
5.	Translog	$S_{ijt} = a_{11} \ln (P_{it}/P_{jt}) + a_{11}(\beta_i$ $\ln Z_{iit} - \beta_2 \ln Z_{jit}) + a_{11}(\gamma_i \ln Z_{ijt}$ $- \gamma_2 \ln Z_{j1t}) + a_{11}(\delta_i + \delta_j) \ln D_t$	$\sigma_{ij} = a_{ij} / (S_i * S_j) + 1$ $E_{ij} = [(a_{ij} - S_i S_j) / S_t] = \sigma_{ij} S_j$ $F_{ij} = \sigma_{ij} + \eta * E(P_y, P_f) S_j$

The reader can refer to Oum's article for a comprehensive and complete layout of model formulation.

In order to compare the reasonableness of demand elasticities for the various models, Oum presents them evaluated at the mean data point for all commodities in table form. Table 3.2 shows these results.  $F_r$  refers to the ordinary elasticity of demand for rail mode  $F_t$  refers to the ordinary elasticity of demand for truck;  $F_{rt}$  refers to the cross price elasticity of the rail mode with respect to freight rate of the truck mode and  $F_{tr}$  inversely.

Table 3-2 1979 Elasticities for All Commodities.

	Translog	Log-Linear	Linear	Box-Cox	Logit
$F_r$	-0.598	-1.517	-0.638	-1.384	-0.830
$F_{rt}$	-0.692	-1.341	-0.048	-1.140	-0.928
$F_t$	0.498		0.059		-0.175
$F_{tr}$	0.592	0.453	0.838	0.403	-0.616

Source: Oum Tae, "Alternative Demand Models and their Elasticity Estimate", Journal of Transport economic and Policy, May 1989, pp.181.

The ordinary cross-price elasticities of the Logit demand model are negative. This is counter intuitive. As the price of the alternative  $j$  goes up, the demand for mode  $i$  should increase or at the least be stable. This is due the effects of aggregation on the variables. The own-price elasticities for ordinary demand derived from the Box-Cox and the Log-Linear model tend to be a little high as they are greater than one. The Translog and the Linear demand model appear to perform better. Both signs of own and cross elasticities are respected as well as the magnitude. In short, the Translog model performs better suggesting that a theory-based approach is robust.

Oum's 1992 article is a real review of elasticity estimates of transport demand model embracing all passenger travel and freight shipment. He considers four functional forms: Log-Linear, aggregate Logit, Translog and discrete choice models or mode choice elasticities for two modes, truck and rail, with 11 commodities. His tables are reproduced in this paper. Table 3.3 refers to price elasticities for rail mode and table 3.4 refers to price elasticities of truck mode. The elasticity estimates presented cover a wide range across commodity group and across functional form for the same commodity group.

Table 3-3 Demand Elasticities of Rail Freight: Selected Commodities and Functional Forms.\*

Models	Log-Linear	Aggregate Logit	Translog	Discrete Choice Model
Commodities				
Aggregate commodities	1.52	0.25,- 0.35,0.83,0.34,- 1.06	0.09,-0.29,0.60	n.a.
Chemicals	n.a.	0.66	0.69	2.25
Fabricated metal products	n.a.	1.57	2.16	n.a.
Food Products	0.02, 1.18	1.36	2.58,1.04	n.a.
Iron and steel products	n.a.	n.a.	2.54,1.20	0.02
Machinery	n.a.	0.16,-1.73	2.27,-3.50	0.61
Paper, plastic & rubber products	0.67	0.87	1.85	0.17,-1.09
Petroleum products	n.a.	n.a.	0.99	0.53
Stone, clay & glass products	n.a.	0.69	1.68	0.82
Textile	n.a.	2.03	n.a.	0.56
Transport equipment	n.a.	n.a.	0.92,-1.08	2.68
Wood & wood products	0.05	0.76	1.97,-0.58	0.08

\*All elasticities are in negative value.

Source: Oum Tae, "Concept of Price Elasticities of Transport Demand and Recent Empirical estimates", *Journal of Transport Economic and Policy*, May 1992, pp.139-169.

Many factors contribute to this diversity. He mentions them in his conclusion but does not address them.

During the past years, modal choice analysis has made extensive use of Logit model. Recently, the use of Box-Cox transformations on explanatory variables of passenger studies has proved to be more efficient than Linear Logit modeling. Little application existed on freight transport demand model. Friedstom and Madslie (1994) found that the transformation renders the model more efficient. Chow and Waters (1994) stated that

despite the knowledge that functional is important, the subject has not received much attention.

Table 3-4 Demand Elasticities of Truck Freight: Selected Commodities and Functional Forms. \*

Models	Log-Linear	Aggregate Logit	Translog	Discrete Choice Model
Commodities				
Aggregate commodities	1.34	0.93	0.69	n.a.
Chemicals	n.a.	n.a.	0.98	2.31
Fabricated metal products	n.a.	n.a.	1.36	0.18
Food Products	1.18,1.54	0.97	0.52,0.65,1	0.99
Machinery	n.a.	n.a.	1.08,-1.23	0.78
Paper, plastic & rubber products	n.a.	n.a.	1.05	0.29
Petroleum products	n.a.	n.a.	0.52	0.66
Stone, clay & glass products	n.a.	n.a.	1.03	2.04
Transport equipment	n.a.	n.a.	0.52,-0.67	2.96
Wood & wood products	n.a.	n.a.	0.56,1.55	0.14

\* All elasticity estimates are in negative value.

Source: Oum Tae, "Concept of Price Elasticities of Transport Demand and Recent Empirical estimates", *Journal of Transport Economic and Policy*, May 1992, pp.139-169.

The study of Gaudry and Picard (1998) attempts to fill this gap by testing different specifications of the Box-Cox Logit over the normal Linear Logit. They do so by using 1979 data from Canada for 64 commodity groups among 67 geographical regions, by three transportation modes: truck (private and for hire), rail and ship. Their study is much like Oum's 1988 article except that they concentrate on Box-Cox specifications.

Their model of reference is the following. The Box-Cox explaining  $S_r$ , the market share of the rail mode, is:

$$S_r = e^{U_r} / e^{U_r} + e^{U_t} \tag{3.1}$$

where,  $U_r$  and  $U_t$  are representative utility function of rail and truck:

$$U_r = \beta_0 + \beta_1 P_r^{(\lambda_1)} + \beta_2 t_r^{(\lambda_2)} \quad (3.2)$$

$$U_t = \beta_3 P_t^{(\lambda_1)} + \beta_4 t_t^{(\lambda_2)} \quad (3.3)$$

where  $P$ , the shipper's cost per ton and  $t$ , the transit time, are subjected to the following Box-Cox transformation:

$$X^{(\lambda)} = X^\lambda - 1 \text{ if } \lambda \neq 0 \quad (3.4A)$$

$$= \text{Ln}(X) \text{ if } \lambda = 0 \quad (3.4B)$$

They are interested in comparing this general model against five nested sub models to compare the impact of the Box-Cox transformation. Those sub models are in table 3.5. More precisely, they want to compare case 4, the Linear Logit, with less restrictive specifications. Because a difference in cost per ton between two modes is assumed not to have the same effect in market share when this difference is small or great, the linear property of the Linear Logit might not be credible.

After estimating the six models, they decided that the best model to be compared with model 4 was the general model. They found that the general model was significantly superior to model 4 when comparing the log likelihood values. Gaudry and Picard



Table 3-5 General model and five nested cases.

Functions	Representative utility functions
Model number	
General model	$\beta_0 + \beta_1 P_r^{(\lambda_1)} + \beta_2 t_r^{(\lambda_2)}$ $\beta_3 P_1^{(\lambda_1)} + \beta_4 t_r^{(\lambda_2)}$
Model I	General model + ( $\beta_1 = \beta_2$ )
Model II	General model + ( $\lambda_1 = \lambda_3$ )
Model III	General model + ( $\beta_1 = \beta_3, \lambda_1 = \lambda_2$ )
Model IV	General model + ( $\beta_1 = \beta_3, \lambda_1 = \lambda_2 = 1$ )
Model V	General model + ( $\beta_1 = \beta_3, \lambda_1 = \lambda_2 = 0$ )

calculated, for each commodity, an average elasticity weighted by the relative magnitude of each mode and by the size of the O-D pair flow. Equation 3.5 shows the price elasticity of the rail share:

$$\eta(S_r, P_r) = \sum_{l=1}^L \eta(S_r, P_r)^l (e^{U_{l,r}} / e^{U_{l,r}} + e^{U_{l,r}}) F^l / \sum_{l=1}^L (e^{U_{l,r}} / e^{U_{l,r}} + e^{U_{l,r}}) \quad (3.5)$$

where;

$\eta(S_r, P_r)^l = (P_r)^\lambda \beta_{P_r} (e^{U_{l,r}} / e^{U_{l,r}} + e^{U_{l,r}})$  is the rail share price elasticity evaluated for O-D pair l;

$P_r$  is the rail fare;

$\beta_{P_r}$  is the rail coefficient;

$(e^{U_{l,r}} / e^{U_{l,r}} + e^{U_{l,r}})$  is the estimated rail share,  $S_r$ , for O-D pair l;

$U_m^l$  is the representative utility of mode m for O-D pair l;

$F^l$  is the total tonnage flow on O-D pair l;

L is the number of O-D pairs.

Since the interest of this study is the comparison of price elasticity, table 3.6 presents the price elasticity of each model. The reader can see that even if both sets of elasticity measures are comparable, there is a significant difference between them especially in the direct price elasticity estimate where the fare rail elasticity is augmented nearly 50% and where the truck fare elasticity is diminished also nearly 50%. Gaudry and Picard argue that this result shows the need for a more careful specification of the relationship between mode choice probability and the values of the variables affecting mode choice.

Table 3-6 Share elasticity estimates for the general model and model 4

Fare	Fare rail	Fare truck
General Model		
$\eta(S_r)$	-0.764	1.303
$\eta(S_t)$	0.194	-0.344
Model IV		
$\eta(S_r)$	-0.415	0.694
$\eta(S_t)$	0.152	-0.237
Difference		
$\eta(S_r)$	46%	-47%
$\eta(S_t)$	-21%	31%

Source: Gaudry, Marc, Picard, Guy, "Exploration of a Box-Cox Logit Model of Inter-city Freight Mode Choice", *Transportation Research*, Vol. 34, No. 1, 1998, pp. 9.

The next section will turn to model formulation and description. It will discuss and present the models developed by different authors that will be used to bring about new and up to date price elasticity estimates under a common basis for Canadian markets.

## 4 MODEL OVERVIEW

The following presents the different models that will be compared using the enumeration method described in section 2. Each model presentation will consist of a two-step description. First, the mathematic structure and derivation of the model will be set forth. This will be done for two reasons; making explicit the functional form of the model and highlighting the method for calculating price-elasticities. Second, brief results of the models will be presented. The idea will be to give a sense of what the coefficient and elasticity estimates of each models look like and to underline (coefficient, price, commodity, shares) what will be necessary for this study to conduct its own calculation. The reader will not find a discussion of those results, as the lack of space cannot render justice to the articles. The reader is then invited to read upon them in the appropriate journals.

### 4.1 The Friedlander & Spady model (1979)

#### 4.1.1 The Model

To derive transportation demand functions, the authors use a short-run variable cost function and its associated input demand equation:

$$C_s = C_s (Y, K, M, P_t, P_t, P_r) \quad (4.1)$$

$$T_s = dC_s (Y, K, M, P_t, P_t, P_r) / dP_t \quad (4.2)$$

$$R_s = dC_s (Y, K, M, P_t, P_t, P_r) / dP_r \quad (4.3)$$

where  $C_s$  refers to short run variable cost,  $T_s$  and  $R_s$  refer to the short-run demand functions for truck and rail respectively;  $Y$  is output;  $K$  and  $M$  are fixed inputs, capital

and materials respectively;  $P_l$ ,  $P_t$  and  $P_r$ , refer to the price of labour, the price of truck and rail transportation respectively.

Because freight transportation is a productive input, the cost of transportation must include the rate of inventory cost associated with shipping. Thus,  $P_t$  and  $P_r$  must reflect inventory cost as well as rates. However, inventory costs are related to shipment attributes. The authors constructed this price function:

$$P_i = \psi^i (r_i(q^i), q^i) \quad (4.4)$$

where  $r_i$  is the rate of mode  $i$  and  $q_i$  is a vector of shipment characteristics associated with mode  $i$ . It is worth noting that the authors estimated the coefficient of average length of haul and average load by Instrumental Variable Technique since they are jointly determined. They assume that the firm's cost function can be expressed by a Translog approximation (Christensen, Jorgenson and Lau, 1973). The demand function is of the same form.

Denoting the fixed inputs, capital and materials, and output by the vector  $X(X = K, M, Y)$  and the price of labour, rail and truck services by the vector  $P(P = P_l, P_r, P_t)$ , the Translog cost function is written :

$$\begin{aligned} \ln C_s = & \alpha_0 + \sum \alpha_i \ln P_i + \sum_h \beta_h \ln X_h + 1/2 \sum_i \sum_j A_{ij} \ln P_i \ln P_j \\ & + \sum_i \sum_h \beta_{ih} \ln P_i \ln X_h + 1/2 \sum_h \sum_s C_{hs} \ln X_h \ln X_s \end{aligned} \quad (4.5)$$

The derivation of this cost function with respect to the price of mode  $i$  yields the input share equation of that mode:

$$S_i = \partial \ln C_s / \partial \ln P_i = \alpha_i + \sum A_{ij} \ln P_j + \sum_h \beta_{ih} \ln X_h \quad (4.6)$$

where  $S_i$  is the firm market share in terms of expenditure on mode  $i$ . The authors then impose the usual conditions for a firm's cost minimizing behaviour, i.e. homogeneity of degree 1.

Since the input share equation (5) is not a direct demand equation, elasticity estimate of demand for rail and truck cannot be obtained directly from (5). Based on Berndt and Wood (1975), they can however derive the elasticities in the following way.

Consider the elasticity of substitution between two inputs Uzawa Hirofimi (1961):

$$\sigma_{ij} = CC_{ij} / C_i C_j \quad (4.7)$$

where  $C_i$  and  $C_j$  are the first derivative of the cost function with respect to the price of input  $i$  and  $j$ .  $C_{ij}$  is the second derivative with respect to the relevant input price. When using the Translog cost function, we get:

$$\sigma_{ij} = (A_{ij} + S_i S_j) / S_i S_j \quad (4.8)$$

$$\sigma_{ii} = (A_{ii} + S_i^2 - S_i) / S_i^2 \quad (4.9)$$

Friedlander and Spady then follow Allen (1956) to show that conventional demand elasticities are related to the elasticities of substitution in the following way:

$$E_{ii} = S_i \sigma_{ii} \quad (4.10)$$

$$E_{ij} = S_j \sigma_{ij} \quad (4.11)$$

Performing substitution :

$$E_{ii} = A_{ii} / S_i + S_i - 1, \quad i = 1, t, r \quad (4.12)$$

$$E_{ij} = A_{ij} / S_i + S_j, \quad i, j = 1, t, r \quad (4.13)$$

where  $E_{ii}$  and  $E_{ij}$  represent the partial own price elasticity and partial cross price elasticity of demand.

#### 4.1.2 The Results

To estimate the input share equation, a cross-section of 96 three-digit manufacturing industries in 1972 producing in each of five broad regions was chosen. Since all data are from the Census of Transportation, all fore hire trucking is lumped together. The market shares are in ton-mile. The parameter estimates are in table 4.1 and the elasticity estimates in table 4.2.

Table 4-1 Estimated Input Share Equations, Rail and Truck

Modes	Rail (I = r)	Truck (i = t)
Variables	Coefficient	Coefficient
Constant	0.051	0.033959
ln $P_{it}$	-0.00248	-0.00739
ln $P_{ir}$	-0.0338	-0.00248
ln $P_{il}$	0.03629	0.01087
ln K	-0.02393	-0.0346
ln M	0.00241	0.04334
ln Y	0.01927	-0.01912

Source: Friedlaender F. Ann, Spady H. Richard, "A Derived Demand Function for Freight Transportation", Journal of Economics and Statistics, 1979, pp.

Table 4-2 Own Price elasticity of Demand and Cross Price Elasticity of Demand by Mode, Commodity and Region.

Commodities	Food	Wood	Paper	Stone, Clay	Iron, Steel	Fab. Metal	Non-elect	Electrical
Regions	Products	Products	Plastic	Glass	Products	Products	Machinery	Machinery
<b>Own Price Elasticity</b>								
Rail								
All Region	-2.583	-1.971	-1.847	-1.681	-2.542	-2.164	-2.271	-3.547
Official	-2.68	-2.106	-1.897	-1.757	-2.784	-8.656	-1.988	-3.816
Southern	-4.009	-1.899	-1.857	-1.811	-1.816	-2.966	-2.59	-5.062
Western	-1.569	-1.448	-1.682	na	na	-2.175	-2.106	-2.438
South-western	-1.841	-1.898	-1.555	-1.607	na	-2.84	-2.602	Na
Mountain-Pacific	-1.777	-2.179	-2.063	-1.613	na	irregular	-2.766	-1.661
Truck								
All Region	-1.001	-1.547	-1.054	-1.031	-1.083	-1.0364	-1.085	-1.23
Official	-1.01	-1.719	-1.083	-1.061	-1.091	-1.581	-1.066	-1.312
Southern	-1.037	-0.1455	-1.066	-1	-1.059	-1.194	-1.017	-1.177
Western	-0.963	-1.546	-1.047	na	na	-1.342	-1.14	-1.196
South-western	-0.987	-1.277	-1.016	-1.026	na	-1.282	-1.175	Na
Mountain-Pacific	-0.956	-1.323	-0.97	-1.022	na	-1.168	-1.097	-1.073
<b>Cross Price Elasticity</b>								
Truck-Rail								
All Region	0,004	-0,129	0,003	0,16	-0,013	-0,099	-0,01	-0,061
Officials	-0,002	-0,186	-0,004	0,005	-0,018	-0,164	0,0002	-0,089
Southern	-0,015	-0,098	0	0,013	0,002	-0,051	0,007	-0,056
Western	0,0324	-0,101	0,12	na	na	-0,08	-0,25	-0,044
Southwestern	0,13	-0,5	0,26	0,023	na	-0,074	-0,043	na
Mountain-Pacific	0,19	-0,75	0,01	0,02	na	-0,053	-0,026	0,008
Rail-Truck								
All Region	-0,23	-0,05	0,007	0,25	-0,053	-0,059	-0,032	-0,151
Officials	-0,33	-0,672	-0,009	0,008	-0,071	-0,545	-0,006	-0,177
Southern	-0,145	-0,035	0,002	0,029	0,004	-0,108	-0,033	-0,26
Western	0,066	-0,023	0,018	na	na	-0,064	-0,038	-0,071
Southwestern	0,035	-0,041	0,039	0,033	na	-0,107	-0,08	na
Mountain-Pacific	0,056	-0,065	0,031	0,033	na	1,555	-0,176	0,01

Source: Friedlaender F. Ann, Spady H. Richard, "A Derived Demand Function for Freight Transportation", Journal of economics and Statistics, 1979, pp.435

Many considerations have to be addressed in applying the enumeration. In this case the Translog model does not require coefficient manipulation as the price does intervene in the derivative. Elasticities for Canadian markets can then be obtained just by replacing

the market shares of the regions by the market share of the 1997 Canadian markets within the same commodity group. The wood, paper, iron and stone commodity seem to have more stable and reasonable ( $<2$ ) elasticities. In large, this model offers all the information necessary for the enumeration method.

## 4.2 The Oum model (1979)

### 4.2.1 The Model

The model constructed by Oum in 1979 is much like that of Friedlander and Spady in that it is a Translog model. The input demand function is obtained by applying the Hotteling-Sheppard lemma to the cost minimizing input demand function. The Translog cost function is:

$$\ln C^T (P_{1t}, P_{2t}, P_{3t}) = \ln a_0 + \sum_{i=1}^3 a_i \ln P_{it} + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 b_{ij} \ln P_{it} \ln P_{jt} \quad (4.14)$$

where;

$P_{it}$  is the freight rate of mode  $i$  in year  $t$ ,  $i = 1(\text{rail}), 2(\text{truck}), 3(\text{ship})$

$a_0$  is a scale factor of cost and

$a_i$ 's and  $b_{ij}$  are the parameter of the cost functions.

The author applies the usual homogeneity condition. He then logarithmically differentiate equation 3.14:

$$\partial \ln C^T / \partial \ln P_{it} = a_i + \frac{1}{2} \sum_{j=1}^3 (b_{ij} + b_{ji}) \ln P_{jt} \quad (4.15)$$

The input demand function is:



$$X_{it} = [ a_i + \frac{1}{2} \sum_{j=1}^3 (b_{ij} + b_{ji}) \ln P_{jt} + \frac{1}{2} \sum_{j=1}^3 (b_{ij} + b_{ji}) t_j T_t ] * C_t^T / P_{it} \quad (4.17)$$

where;

T is a trend variable reflecting the state of the technology and t is a coefficient.

Taking into account on the price factors, the cost minimizing expenditure share function is:

$$X_{it} = a_i + \frac{1}{2} \sum_{j=1}^3 (b_{ij} + b_{ji}) \ln P_{jt} + A_i \quad (4.18)$$

where  $A_i$  reflects all other non-price factor.

The elasticities are obtained in a similar fashion as in p.21. Consider the Allen Partial elasticity of substitution between two inputs:

$$\sigma_{ij} = CC_{ij} / C_i C_j \quad (4.19)$$

where  $C_i$  and  $C_j$  are the first derivative of the cost function with respect to the price of input  $i$  and  $j$ , and  $C_{ij}$  is the second derivative with respect to the relevant input price.

When using the Translog cost function, we get:

$$\sigma_{ij} = (A_{ij} + S_i S_j) / S_i S_j \quad (4.20)$$

$$\sigma_{ii} = (A_{ii} + S_i^2 - S_i) / S_i^2 \quad (4.21)$$

Oum then follows Allen (1956) to show that conventional demand elasticities are related to the elasticities of substitution in the following way:

$$E_{ii} = S_i \sigma_{ii} \quad (4.22)$$

$$E_{ij} = S_j \sigma_{ij} \quad (4.23)$$

Performing substitution :

$$E_{ii} = A_{ii} / S_i + S_i - 1, \quad i = 1, t, r \quad (4.24)$$

$$E_{ij} = A_{ij} / S_i + S_j, \quad i, j = 1, t, r \quad (4.25)$$

where  $E_{ii}$  and  $E_{ij}$  represent the partial own price elasticity and partial cross price elasticity of demand.

#### 4.2.2 The Results

The data used for the estimation of the Translog cost function was the price index and revenue shares of three modes of transport, rail, truck and ship for the Canadian grain market of Thunder-Bay/St-Lawrence from 1945 to 1974. Oum presents results over six time frames but this study will focus on the last time frame of his estimation. The market shares are in tone-mile. The results are presented in table 4.1 to 4.3.

Table 4-3 Estimated Coefficient for the Rail and Truck Mode, Grain Transport Demand

Modes	Rail ( $i = r$ )	Truck ( $i = t$ )
Variables	Coefficient	Coefficient
Constant	0.1128	0.1625
$\ln P_{ir}$	-0.1405	-0.0193
$\ln P_{it}$	0.1754	-0.035
$\ln P_{is}$	-0.035	0.0157

Source: Oum Tae, "Derived Demand for Freight Transport and inter-Modal Competition in Canada", Journal of Transport Economics and Policy, May 1979, pp.161

Table 4-4 Allen Partial Elasticity of Substitution for Grain for the Thunder Bay/St-Lawrence Seaway Region 1997.

Elasticity	Value
Rail	-0.693
Rail/Truck	0.302
Rail/Ship	1.455
Truck	-0.323
Truck/Rail	
Truck/Ship	0.277
Ship	-7.036

Sources: Oum Tae, "Derived Demand for Freight Transport and inter-Modal Competition in Canada", *Journal of Transport Economics and Policy*, May 1979, pp.163.

The own and cross elasticities all have the correct signs. They are less than unity. This suggests a robust estimation method for the database. In this case, the grain commodity could be used in the enumeration method. Again, several considerations have to be made. One of them is that in the available database, the grain commodity is decomposed in several types of grain. Aggregation will be necessary. This may have an effect on the results. In large, this model offers all the information necessary for the enumeration method.

Table 4-5 Own Price elasticity of Demand and Cross Price Elasticity of Demand by mode for grain for the Thunder Bay/St-Lawrence Seaway Region, 1974

Elasticity	Value
Rail	-0.291
Rail/Truck	0.144
Rail/Ship	0.147
Truck	-0.155
Truck/Rail	0.127
Truck/Ship	0.028
Ship	-0.744
Ship/Rail	0.611
Ship/Truck	0.133

Sources: Oum Tae, "Derived Demand for Freight Transport and Inter-Modal Competition in Canada", *Journal of Transport Economics and Policy*, May 1979, pp.163.

### 4.3 The Oum Model 1989

The models presented here were obtained from Oum's 1989 paper. This paper was also used in the literature review of this study. It offers four models that can be used for our purposes. Each model was discussed earlier so that only a brief description will follow.

#### 4.3.1 The Linear Model

The equation of the model is:

$$Y_i = \alpha + \beta X_i + \mu \quad (4.29)$$

Where

Y is the demand for transport mode *i*;

X is a vector of modal attributes:

$\beta$  is a parameter, and,

$\mu$  is an error term.

The elasticity is simply:

$$F_{yk} = \partial Y / \partial X_k * X_k / Y \quad (4.30)$$

#### 4.3.2 The Log Linear Model

The equation of the Log-Linear model is well known and takes the form:

$$\ln Y = \alpha + \beta \ln X + \mu \quad (4.31)$$

where;

$\ln Y$  is the demand for transport mode  $i$ ;

$\ln X$  is a vector of modal attributes;

$\beta$  is a parameter, and,

$\mu$  is an error term.

The direct price elasticity of a logarithmic form is simply:

$$F_{yk} = b_k \quad (4.32)$$

#### 4.3.3 The Translog Model

This functional form has taken enough space. The reader is referred to equation 4.14 to 4.25 for a description of the model.

#### 4.3.4 The Box-Cox

Oum argues that it is possible and suggested to apply to the Linear and the Log-Linear the following Box-Cox transformation:

$$Y_i(\lambda) = \sum_j \beta_j x_{ij}(\lambda) + \sum_j \gamma_j Z_{ij} + \mu \quad (4.33)$$

where  $X_{ij}$  is the  $i$ th observation on regressor  $j$ , which can be lagged, and  $Z_{ji}$  is the  $i$ th observation on the regressor  $j$  that cannot.  $\lambda$  is the Box-Cox parameter. The elasticity is then:

$$F_{yk} = b_k (Y/X_k)^\lambda \quad (4.34)$$

#### 4.3.5 The Aggregate Logit Model

The equation of the Logit is:

$$\text{Prob}_{ij} = \exp(-\beta' x_{ij}) / 1 + \exp(-\beta' x_{ij}) \quad (4.33)$$

where;

$Prob_{ij}$  is the probability share of mode  $i$  or  $j$ ;

$X_{ij}$  is a vector of modal attribute, and;

$\beta$  is a coefficient.

The direct and cross are given by:

$$F_{ii} = \beta x_{ii} * (1 - Prob_{ii}) \quad (4.34)$$

$$F_{ij} = -\beta x_{ij} * (Prob_{ij}) \quad (4.35)$$

It is worth mentioning again that only three pieces of information are necessary to evaluate this elasticity:  $\beta$ ,  $x$  and  $Prob$ .

#### 4.3.6 The Results.

Data for Canadian inter-regional aggregate freight flows in 1979 were used to estimate these models. The market shares are in ton-mile. Table 4.1 reports the result of the Linear model. The coefficients have the expected signs. The big character indicates non-significance. Table 4.2 indicates the results for the Log-Linear model. Table 4.3 indicates the result of the Box-Cox estimate. The result for the Logit can be found in table 4.4. Finally, the result of the Translog model can be found table 4.5. Table 4.6 reports the elasticity estimates of each model.

Table 4-6 Linear Model Parameter Estimates for All Commodities

Modes		
Variables	Rail	Truck
Constant	47508	-8636.2
$P_r$	-8405	8786
$P_t$	<b>221</b>	<b>-143</b>

Sources: Oum Tae, "Alternative Demand Models and their Elasticity Estimates", Journal of Transport Economics and Policy, May 1989, pp.177.

Table 4-7 Log-Linear Model Parameter Estimates for All Commodities

Modes		
Variables	Rail	Truck
Constant	-1.28	3.269
$P_r$	-1.517	0.453
$P_t$	<b>-0.142</b>	-1.341

Sources: Oum Tae, "Alternative Demand Models and their Elasticity Estimates", Journal of Transport Economics and Policy, May1989, pp.178.

Table 4-8 Box-Cox Model Parameter Estimates for All Commodities

Modes		
Variables	Rail	Truck
Constant	-1.41	6.7
$P_r$	-2.023	-1.57
$P_t$	<b>-0.158</b>	0.584

Sources: Oum Tae, "Alternative Demand Models and their Elasticity Estimates", Journal of Transport Economics and Policy, May1989, pp.179.

Table 4-9 Logit Model Parameter Estimates for All Commodities

Dependent Var.	
Independent Var.	Log Truck/Rail Volume
Constant	-8E-07
Freight rate	-0.0428

Sources: Oum Tae, "Alternative Demand Models and their Elasticity Estimates", Journal of Transport Economics and Policy, May1989, pp.180.

Table 4-10 Translog Model Parameter Estimates for All Commodities

Dependent Var.	Log of the Weighted
Independent Var.	Average Freight Rate
Constant	1.863
$P_r$	0.543
$P_t$	0.457

Sources: Oum Tae, "Alternative Demand Models and their Elasticity Estimates", Journal of Transport Economics and Policy, May1989, pp.180.

Table 4-11 Elasticities for All 1979 commodities

Models	Translog	Log-Linear	Linear	Box-Cox	Logit
$F_r$	-0.598	-1.517	-0.638	-1.384	-0.83
$F_t$	-0.692	-1.341	-0.048	-1.14	-0.928
$F_{rt}$	0.498		0.059		-0.175
$F_{tr}$	0.592	0.453	0.838	0.403	-0.616

Sources: Oum Tae, "Alternative Demand Models and their Elasticity Estimates", Journal of Transport Economics and Policy, May1989, pp.181.

This model presentation appeals directly to the enumeration method in that it presents a probability modal that is the Logit. It will then be necessary to replace the probabilities of choosing a mode obtained by Oum by the markets share of that same mode for the 1997 Canadian Markets. Also, it will be necessary to perform a coefficient transformation accounting for the fact that the price, as seen in equation 2.11 and 2.13, intervenes in the derivative and thus in the elasticity. The transformation to beta will account for inflation. Beta will simply be multiplied by the  $IPC_{97}/IPC_{79}$ . In some cases, it will be necessary to account for a change of monetary units.

#### 4.4 The Abdelwahab Model (1998)

##### 4.4.1 The Model

His paper presents empirical estimates of market elasticities of demand and elasticities of mode choice probabilities in the inter-city freight transport market. The results are derived from a mixed discrete/continuous choice of mode and shipment size. The basic structure of the model consists of a system of three switching simultaneous equations:

$$I_i = X_i\gamma + Y_{1i}\eta_1 + Y_{2i}\eta_2 - \varepsilon_i \quad (4.36)$$

$$\text{Truck: } Y_{1i} = X_{1i}\beta_1 + \varepsilon_{1i} \quad \text{iff } I_i^* > 0 \quad (4.37)$$



$$\text{Rail: } Y_{2i} = X_{2i}\beta_2 + \varepsilon_{2i} \quad \text{iff } I_i^* \leq 0 \quad (4.38)$$

where;

- $I_i^*$  = unobserved index which determines the mode choice,  
 $Y_{1i}, Y_{2i}$  = endogenous dependent variables (e.g. shipment size by truck and rail respectively),  
 $X_i, X_{1i}, X_{2i}$  = vectors of exogenous independent variables (e.g. commodity, market, and modal attribute; a list of 18 variables),  
 $\beta_1, \beta_2, \gamma, \eta_1, \eta_2$  = vectors of estimated parameter, and,  
 $\varepsilon_1, \varepsilon_{1i}, \varepsilon_{2i}$  = residual terms

The first equation represents the discrete mode choice, the second and third equation are the choice of shipment size by truck and rail respectively. They are two types of elasticities in discrete choice modeling, such as the Probit: aggregate and disaggregate elasticities. Abdelwahab calculates both of them.

A disaggregate elasticities represents the sensibility of a shipper's probability to a change in attribute value. In the case of the Probit, the direct elasticity is:

$$E^{P_{ni}}_{X_{ik}} = \partial P_{ni} / \partial X_{ik} * X_{ik} / P_{ni} \quad (4.39)$$

Given the authors specification of the Probit model, the elasticity of  $P_{ni}$  with respect to  $X_{ik}$  can also be written as:

$$E^{P_{ni}}_{X_{ik}} = \partial / \partial X_{ik} (\Phi(V_i - V_j)) * X_{ik} / \Phi(V_i - V_j) = \pi_k X_{ik} \phi(V_i - V_j) / \Phi(V_i - V_j) \quad (4.40)$$

where  $\pi_k$  is the parameter estimate of  $X_{ik}$ . The cross elasticity is also derived for the case of the binary Probit:

$$E^{Pni}_{Xjk} = -\pi_k X_{jk} \phi(V_i - V_j) / \Phi(V_i - V_j) \quad (4.41)$$

Direct aggregate elasticities measure the sensibility of a group of shippers to a change in modal attributes. It is given by:

$$E^{Pni}_{Xik} = \sum_{n=1}^N P_{ni} E^{PniXik} / \sum_{n=1}^N P_{ni} \quad (4.42)$$

In a similar fashion, cross aggregate elasticity is given by:

$$E^{Pni}_{Xjk} = -\sum_{n=1}^N P_{ni} E^{PniXjk} / \sum_{n=1}^N P_{ni} \quad (4.44)$$

The parameter estimates of the joint mode choice/shipment size model are used to obtain the market elasticity of demand for the different model attributes. Let the aggregate demand for mode  $i$  be:

$$D_i(x_i(t)) = \sum_n w_n P_{ni}(x_{ni}(t)) \quad (4.36)$$

where  $D_i$  is the aggregate demand for mode  $i$ ,  $n$  is an index referring to observation on mode  $i$ ,  $x_{ni}(t)$  is a set of explanatory variables,  $w_n$  is a weighting factor defined as the product of sampling weight and the quantity shipped the  $n$ th shipper. In the article by Abdelwahab,  $w_n$  is equal to 1 and is replaced by the shipment size of the  $n$ th shipper,  $S_n$ .

Then the direct aggregate market elasticity of demand with respect to  $x_{ni}^k$  is:

$$E^k_i = \sum_n x_{ni}^k [\pi_k S_n \phi(x_{ni}) + \alpha_k \Phi(x_{ni})] / \sum_n S_n \Phi(x_{ni}) \quad (4.37)$$

Again, in a similar fashion, the cross-elasticity of demand,  $E^k_{ij}$ , is obtain:

$$E^k_{ij} = \sum_n x_{nj}^k [-\pi_k S_n \phi(x_{nj}) + \alpha_k \Phi(x_{nj})] / \sum_n S_n \Phi(x_{nj}) \quad (4.38)$$

#### 4.4.2 The Results

The coefficient estimates are reported in table 4.1. Disaggregate elasticity of mode choice probabilities are expressed in table 4.2. All estimation and calculation are done with data drawn from the US commodity Transportation Survey over five regions for 8

commodity groups in 1977. They are calculated at the mean value of each modal attribute. The market shares are in ton-miles.

Table 4-12 Coefficient estimates by Commodities

Modes	Rail (i = r)	Truck (i = t)
Variables		
In <i>Pir</i>	-0,0136	0,10336
In <i>Pit</i>	0,05916	-0,00272

Source: Abdelwahab Walid, "Elasticities of Mode Choice Probabilities and Market Elasticities of Demand", *Transportation Research*, vol. 34, No. 4, pp.260.

The observation of the elasticities demonstrates that the elasticity of choosing rail is always larger than the elasticity of choosing truck.

Table 4-13 Disaggregate Price Elasticity of Mode Choice Probabilities

Modes	Rail	Truck
Modes		
Rail	-1.8772	1.5428
Truck	1.7525	-1.4403

Source: Abdelwahab Walid, "Elasticities of Mode Choice Probabilities and Market Elasticities of Demand", *Transportation Research*, vol. 34, No. 4, pp.261.

The market elasticities of demand are defined in table 4.3. Again the reader finds the common four values when discussing elasticities: own-price elasticity of demand for truck and rail; cross-price elasticity of demand of rail-truck and truck-rail. It is known from equation 2.5 that price also intervenes in the derivative of the Probit model. This indicates that coefficient manipulation is again necessary. The transformation to betas will have to account for the US-Canada dollar exchange rate and for the Canadian inflation. The beta will simply be multiplied by the 97 exchange rate and by the  $IPC_{97}/IPC_{77}$ . Another consideration concerning the market shares has to be addressed. The wood commodity in its data set contains the paper commodity. In the Canadian database, and in

the estimate by Friedlander, this is not case. Aggregation of the two commodities is possible. But as the elasticity estimates of Friedlander for paper and wood resemble the elasticity estimates obtained for both commodities in aggregate form from the Abdewahab model, the same coefficient will be used for both commodities in the enumeration method.

Table 4-14 Own Price Elasticity of Demand and Cross Price Elasticity of Demand by Mode, Commodity and Region.

Commodities	Food	Chemical	Plastic, Rubber	Non-Fab. Metal	Electrical	Stone, Clay	Wood
Region	Products	Petroleum	& Leather	& Fab. Metal	Machinery	Glass	Products
<b>Own Price Elasticity</b>							
<b>Rail</b>							
All Region	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Official	-1.499	-1.0534	-1.2348	-0.9084	-1.1644	-0.9558	-1.2816
Southern	-1.6463	-1.5471	n.a.	-1.3317	-2.159	n.a.	-1.701
Western	-1.7232	-1.9696	n.a.	-1.3664	-1.9821	n.a.	-1.5596
South-western	-2.3426	-1.3902	n.a.	-2.4892	n.a.	n.a.	n.a.
Mountain-Pacific	-1.322	-1.3817	n.a.	-1.9179	n.a.	n.a.	-2.1058
<b>Truck</b>							
All Region	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Official	-1.1963	-0.927	-1.1358	-0.7972	-1.1938	-0.7494	-1.0591
Southern	-1.6313	-1.608	n.a.	-1.3256	-2.5254	n.a.	-1.612
Western	-1.3363	1.7147	n.a.	-1.1194	-2.0473	n.a.	-1.2785
South-western	-2.1899	-1.4802	n.a.	-2.1784	n.a.	n.a.	n.a.
Mountain-Pacific	-1.32	-1.008	n.a.	-1.6217	n.a.	n.a.	-1.8012
<b>Cross Price Elasticity</b>							
<b>Truck-Rail</b>							
All Region	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Officials	1.2612	1.0786	1.2812	0.9326	1.1991	0.9818	1.1125
Southern	1.7256	1.5711	n.a.	1.3593	2.1964	n.a.	1.7065
Western	1.4186	2.0074	n.a.	1.4034	2.0253	n.a.	1.341
South-western	2.3327	1.4055	n.a.	2.5324	n.a.	n.a.	n.a.
Mountain-Pacific	1.201	1.4291	n.a.	1.9551	n.a.	n.a.	2.1428
<b>Rail-Truck</b>							
All Region	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Officials	1.488	1.0421	1.2592	0.9042	1.1672	0.9525	1.293
Southern	1.614	1.5011	n.a.	1.3078	2.1037	n.a.	1.6793
Western	1.6646	1.9273	n.a.	1.3614	1.913	n.a.	1.5826
south-western	2.2256	1.335	n.a.	2.4256	n.a.	n.a.	n.a.
Mountain-Pacific	1.2781	1.399	n.a.	1.8785	n.a.	n.a.	2.0529

Source: Abdelwahab Walid, "Elasticities of Mode Choice Probabilities and Market Elasticities of Demand", Transportation Research, vol. 34, No. 4, pp.263.

## 4.5 The Winston model (1981)

### 4.5.1 The Model

Let a receiver maximize  $EU_i(Z)$ , where  $Z$  is vector of modal attributes. Winston maintains that the empirical estimation of such a model requires a random utility model of the form:

$$U_i^*(Z_i, S) = V(Z_i, S) + \varepsilon_i(Z_i, S) \quad (4.39)$$

Where the expected value of the  $\varepsilon_i(Z_i, S)$  becomes  $e_i$  and is interpreted as the unobserved modal attributes, and commodity and firms. Expressing the expectation of  $V(Z_i, S)$  as  $V(Z_i, S) + \delta_i$  yields the random expected utility model:

$$EU_i^k(Z_i, S) = V(Z_i, S) + e_i + \delta_i \quad (4.40)$$

As it is well known, the two common statistical formations for this kind of problem are the Logit and the Probit. The author chose the Probit since the model reveals that the errors are not IID. The Probit was developed in equation 2.1 to 2.5.

It is worth however to consider the way by which the author develops market elasticities. To begin, the aggregate demand for mode  $j$  in his analysis is expressed in the following way:

$$D_j(Z(t)) = \sum_i w_i P_{ij}(Z_{ij}(t)) \quad (4.41)$$

Where  $i$  is an index referring to those observations where mode  $j$  is available and  $w_i$  is a weight defined as the product of the choice-based sample weight and the quantity shipped for the  $i$ th observation. He assumes that a uniform percentage change for each variable of interest will take place. Hence, the elasticity of market demand with respect to such a change is the elasticity with respect to  $t$  evaluated at  $t = 1$ . Thus:

$$E_j = \partial D_j(Z_j(t)) / \partial t * t / D_j(Z_j(t)) \quad (4.42)$$

Substituting for aggregate demand;

$$\begin{aligned} \partial D_j / \partial t &= w_i \partial P_{ij} / \partial Z_{ij}^r(t) * \partial Z_{ij}^r(t) / \partial t \\ &= \sum_i w_i Z_{ij}^r \partial P_{ij} / \partial Z_{ij}^r(t) \Big|_{t=1} \end{aligned} \quad (4.43)$$

where  $Z_{ij}^r$  is the specific variable for which the market elasticity is calculated. For this study is the price.

#### 4.5.2 The Results

The data set that was uses for the estimation is the same that was used by Friedlander and Abdelwahab (ICC). The year is 1976-1977. The coefficient result and the elasticity estimates can be found in table 4.1 and 4.2 respectively.

Table 4-15 Coefficient estimates by commodities

Commodities	Unregulate d Agr.	Machinery	Textile	Chemical	Plastics	Stone	Primary Metal	Paper	Wood
Modes									
Rail- Common- Private	-2.026	-6.242	-0.69	-13.8	-3.29	-4.10	-6.99	-14.08	-24.14

Source: Winston Clifford, "A Disaggregate Model of the Demand for Inter-city Freight Transportation", *Econometrica*, Vol. 49, No. 4 July 1981, pp. 992.

In general, the result exhibit great differences across commodities but they are all, except for paper and wood, not significant. This problem will have to be address when the enumeration method is applied.

Table 4-16 Market elasticities by commodities and by modes

Commodities	Unregulate d Agr.	Machinery	Textile	Chemical	Plastics	Stone	Primary Metal	Paper	Wood
Rail-	-1.11	-.61	-.56	-2.25	-1.03	-.82	-.019	-.17	-.08
Private	-.99	-.78	-.43	-2.31	-2.01	-2.04	-.18	-.29	-.14
Common		-0.4	-.77	-1.87	-2.97	-2.17	-.28		

Source: Winston Clifford, "A Disaggregate Model of the Demand for Inter-city Freight Transportation", *Econometrica*, Vol. 49, No. 4 July 1981, pp. 997.

In order to apply the enumeration method to this model, it is again necessary to transform the coefficient obtained by Winston, thus adjusting for the exchange rate and for inflation via  $IPC_{81}/IPC_{97}$ . Further attention will be come surrounding the type of modal share used in his paper. Winston makes a difference between private and common transport. In this paper, all private transport<sup>1</sup> are not includes in the modal shares of the database.

#### 4.6 The Wilson, Wilson and Koo Model (1988)

##### 4.6.1 The Model

A theoretical framework is develop for two related markets, rail and truck transport. His modal is base on the theory of the firm where the demand side is characterize by a large number of price-taking firm. The supply side is divided in two: one side consist of a large number of price-taking firms and the other consists of one profit-maximizing firm.



The theory is known as the dominant firm price leadership in the case of differentiated products. Let the market supply function be:

$$T^s = T^s(t, ) \quad (4.44)$$

Where  $t$  and  $T^s$  are the transport price and the quantity of service provided by the firm respectively. It is then necessary to formulate two demand equations, one for rail and one for truck.

$$R^D = R^D(r, t, ) \quad (4.45)$$

$$T^D = T^D(t, r, ) \quad (4.46)$$

where  $R^D$  and  $T^D$  are the quantities demanded and  $r$  and  $t$  are the price of each modes.

The specified modal is a bloc recursive equation system. Notice that the rail function is behavioural one and cannot be treated and residual as the substitution effect is imperfect.

Thus, the demand curve relevant to the dominant mode's decision process is given by:

$$R^D = R^D [r, t(r)] \quad (4.47)$$

Incorporating their conjectural variation can derive the elasticity of the total effect of a change in rail rates. From equation 4.47;

$$dR = R_r dr + R_t \partial t / \partial r dr \quad (4.48)$$

and

$$\partial r / \partial R = 1 / (R_r + R_t) * \partial t / \partial r \quad (4.49)$$

where  $R_r$  and  $R_t$  are partial derivatives of  $R^D$  with respect to rail and truck rates.

The inverse elasticity can be derived from equation 4.49 to derive the following:

$$\xi^* = \xi^R + \xi_{cp}^R \partial t / \partial r * r / t \quad (4.50)$$

---

<sup>1</sup> Private transport is transportation that takes place within a company domain of practice. An example of private transport includes transport of merchandise by supermarkets.

where  $\xi^*$  is the total elasticity of change in the rail rates,  $\xi^R$  is the own price elasticity of demand,  $\xi_{cp}^R$  is the rail cross-price elasticity and  $\partial t / \partial r$  is the conjectural variation.

The specified model can be understood as a bloc recursive equation system;

$$QTS_t = f_1(QTS_{t-1}, PT_t, F_t, I_t, T, e_{1t}) \quad \text{Truck supply} \quad (4.51)$$

$$QTD_t = f_2(PR_t, PT_t, C_t, U_t, e_{2t}) \quad \text{Truck Demand} \quad (4.52)$$

$$QRD_t = f_3(PR_t, PT_t, C_t, U_t, e_{3t}) \quad \text{Rail Demand} \quad (4.53)$$

$$Q_t = QTD_t + QRD_t \quad \text{Identity} \quad (4.54)$$

where

Q = aggregate shipments;

QTD = QTS = truck shipments;

QRD = rail shipments;

PT = truck rates;

PR = rail rates

F = fuel rates

I = input cost index;

T = time trend;

C = index of rail car availability;

U = dummy variable equal to one after multicar shipment were introduced and

E = disturbance term

From the structure of the model, one can see that changes in rail price in the modal market cause simultaneous exchanges in truck demand, equilibrium prices and supply in that market, and because the change in truck rate, the rail demand function shifts.<sup>2</sup>

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<sup>2</sup>  $\uparrow P_r \rightarrow \downarrow Q_r \rightarrow \uparrow P_t \rightarrow \uparrow Q_t$

#### 4.6.2 The Results

The estimation procedure required for this model as to take into account that many equation are over specified. Thus OLS was used for the estimation of the rail pricing equation, an autoregressive three-stage least square was used for equation 4.51 to 4.53. The equation system was estimated for wheat from North Dakota to the main terminal Markets of Minneapolis and Duluth. The time period of the study was from July 1973 to June 1983. Shipment data are in tone miles and were aggregate across states. The estimates result can be found in table 4.1, and the elasticity result are in table 4.2.

Table 4-17 System Estimates for Structural Equations

Modes	Tuck Demand	Rail Demand
Variable		
PT	824.740	201.704
PR	-476.319	-11.540
U	-1.660	27.582
C	60.8	
PR*C	51	
PT*C	-86.5	

Source: Wilson & al, "modal Competition and Pricing in Grain Transport", Journal of Transport and Economic Policy", September 198, pp, 223.

Table 4-18 Elasticities of Intermodal Competition

Variable	Rail Rate	Truck Rate
Equation		
Truck Demand	0.70	-0.73
Rail Demand	-1.18	2.30

Source: Wilson & al, "modal Competition and Pricing in Grain Transport", Journal of Transport and Economic Policy", September 198, pp, 224.

The complexity of the modal at hand does not change the fact that it is a linear modal. To conduct the enumeration method only two elements are need the price coefficient and C. As the price does not intervене in the derivative, no coefficient transformation is necessary.

## **4.7 The Lewis Widup Model (1982)**

### **4.7.1 The model**

Lewis and Widup present a Translog model with the prices expressed in ration term. Except from that fact, the model is developed exactly like equation 4.5 to 4.13. The reader he is referred.

### **4.7.2 The Results**

## **5 PROPERTIES OF THE MODELS**

Before proceeding to the analysis of demand elasticities of the four Canadian markets, it is necessary to underline some properties of the econometric model previously defined.

### **5.1 The Influence of Price and Market Shares on Price Elasticities<sup>3</sup>.**

Trips per mode of transport are a function of the total number of trips by all modes and the share of each modal trip. Evidently, the number of trips per mode is equal to the product of the total number of trip by all modes and the share of each modal trip. Under this definition, the price elasticity of the demand for trips by mode  $m$  ( $\eta^m(\text{mode})$ ) is equal to the sum of the price elasticity of total demand ( $\eta(\text{total})$ ) and the price elasticity of the share of mode  $i$  in total trips ( $\eta(\text{share})$ ). This is expressed by:

$$(\eta^m(\text{mode})) = (\eta(\text{total})) + (\eta(\text{share})) \quad (5.1)$$

As seen earlier, the price elasticities of modal share ( $\eta(\text{share})$ ) are calculated using different econometric model, probability model, and modal split model. However, the price elasticities of total demand ( $\eta(\text{total})$ ) are obtainable only from aggregate freight demand models. In short, equation 4.34 can be decomposed into equation 5.1.

It has been shown that ( $\eta(\text{share})$ ) in probability model is calculated using a coefficient, a market share and a price. The same goes for ( $\eta(\text{total})$ ) at the aggregate level. It is then clear that ( $\eta^m(\text{mode})$ ) will vary from market to market because prices and market share differ. It will also depend on the level of aggregation. All the results that were previously mentioned in section 4 support this claim. Indeed, the great variation in the magnitudes of elasticities estimate in the different studies are not surprising when considering the wide variation possible in:

- 1.the type of model;
- 2.the type of data;
- 3.the grouping of data;
- 4.the type and extension of the market; and
- 5.the definition of demand.

Knowing this, it becomes interesting to discuss the separate effect of market share (share effect) and of the mode's price (price effect) on price elasticities.

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<sup>3</sup> These properties were previously discuss for similar models of passenger Travel in Laferrière (1992).

### 5.1.1 Share effect on Price Elasticities of Modal Share

Own Elasticities: The direct price elasticity ( $\eta^m_{C_m}(\text{share})$ ) of modal share decreases as the share of mode  $m$  ( $S_m$ ) increases. In economic term, the greater the modal share, the less sensitive is that share to the mode's price ( $C_m$ ). All the models mentioned in section 4, except for the Linear and the Log-Linear, imply this property.

$$\Delta^+ S_m \rightarrow \Delta^- \eta^m_{C_m}(\text{share}) \quad (\text{P.1})$$

Cross-price elasticity:  $\eta^m_{C_i}(\text{share})$  is proportional to the share of the mode ( $S_i$ ) whose price ( $C_i$ ) has varied. In economics term, the price elasticity of mode  $m$  with respect to the price of mode  $i$  increase as the share of mode  $i$  ( $S_i$ ) increase. Then if  $S_i$  is large, the adjustment following a change in the price of mode  $i$  will be small forcing the other mode to adjust more. Again all the models that will be used in this analysis imply this property.

$$\Delta^+ S_i \rightarrow \Delta^+ \eta^m_{C_i}(\text{share}) \quad (\text{P.2})$$

### 5.1.2 Share Effect on Price Elasticity of the Total Demand.

Total freight demand elasticity ( $\eta(\text{total})$ ), with respect to the price of the modes of transport are proportional to modal shares. This only respects equation 5.1 with property (P.1)<sup>4</sup>. This property always holds in theory but it is not verified in the models explained in section four because many studies do not attend the question. The property is nevertheless stated because it will be of interest for extracting information from market share and modal share elasticities in the Canadian market context.

$$\Delta^+ S_m \rightarrow \Delta^+ \eta^m_{C_m}(\text{total}) \quad (\text{P.3})$$

### 5.1.3 Price Effect on Price Elasticities of the Modal Share

Here a change in a mode's price does not have the same effect on modal share for all the models previously defined. The next few paragraphs will show that as the price of a mode increase, the price elasticity of that mode may go up, down or remain unchanged.

One can expect the price elasticity of the modal share to increase as the price increases. The Logit and the Probit model (Oum 1989 E. and Abdalwahab 1998) conform to this property.<sup>5</sup>

$$\Delta^+ C_m \rightarrow \Delta^+ \eta^m_{C_m}(\text{share}), \Delta^+ \eta^i_{C_m}(\text{share}), \quad (\text{P.4})$$

Price elasticity of the modal share for the Translog and Log-Linear model (Friedlander 1979, Oum 1979, Oum 1989 B, C) are invariant to a mode price change. This is simply because the price ( $C_m$ ) does not intervene in the elasticity derivation.<sup>6</sup>

$$\Delta^+ C_m \rightarrow \eta^m_{C_m}(\text{share}) \text{ constant}, \eta^i_{C_m}(\text{share}) \text{ constant}, \quad (\text{P.5})$$

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<sup>4</sup>  $(\eta^m(\text{mode})) = (\eta(\text{total})) + (\eta^m(\text{share})) \rightarrow (\eta(\text{total})) = (\eta^m(\text{mode})) - (\eta^m(\text{share})), S_m \uparrow \rightarrow (\eta^m(\text{share})) \downarrow \rightarrow (\eta(\text{total})) \uparrow$ .

<sup>5</sup> See equation 4.36, 4.40 and 4.42.

<sup>6</sup> See equation 4.12, 4.24 and 4.32.

Price elasticity of the Box-Cox model (Oum 1989 D) decreases as the price increases because the price variable intervenes in the Box-Cox transformation at the denominator.<sup>7</sup>

$$\Delta^+ C_m \rightarrow \Delta^- \eta^m_{C_m}(\text{share}), \Delta^- \eta^l_{C_m}(\text{share}), \quad (\text{P.6})$$

#### 5.1.4 Price Effect on Price Elasticity of Total Demand

Properties P.4 to P.6 can directly be applied to the elasticity of total demand ( $\eta(\text{total})$ ). This conclusion can be explained because aggregate elasticities calculate using Logit or Probit models let the prices intervene in the same manner. Similarly, modal split analyses offered by the Translog, Log-Linear models are not affected by price change. Finally, the Box-Cox aggregate model lets the prices intervene at the denominator, so that again the property is respected.

Therefore, it is possible to substitute share by total in properties P.4, P.5 and P.6 and call the properties P.4\*, P.5\* and P.6\*.

## 6 Conclusion

This analysis has shed light on the effects of a price variation on freight transport demand for two modes: rail and truck. Following Laferrière (1992), a methodology was developed for comparing the price elasticities of different econometric models for freight transportation. This has been done for seven econometric models for three Canadian markets and four commodities. General properties were drawn from those models offering theoretical answers to many questions such as: Are the price elasticities derived

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<sup>7</sup> See equation 4.34.



from econometric models sufficiently homogenous to suggest a consensus? Which modes have elastic demand? Which transportation modes are sensitive to the price of other modes? Freight elasticity properties suggest that price elasticities derived from the chosen econometric models are homogenous over time and space but that mode elasticity depends on the model used as counter intuitive results were sometimes seen. A methodology as now has been set to improve on these answers. Further empirical investigation would allow for acceptance or refutation of this claim. The author of the present paper has already undertaken this part of the research.

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