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Choice of Contract Maturity with Applications to International and Mortgage Lending

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Résumé

Cette thèse contribue à la littérature sur la durée des contrats en identifiant les bénéfices et les coûts de l'engagement de long terme dans le cas des contrats de dette internationale, et dans le cas des contrats d'emprunt hypothécaire ; notamment du point de vue de l'agent. Plus précisément, je caractérise et compare deux structures alternatives de contrats de dette qui diffèrent uniquement par l'ajustement qu'elles prévoient face à un changement exogène du taux d'intérêt futur. D'une part, un contrat de long terme qui ne prévoit aucun ajustement des remboursements au taux d'intérêt réalisé, et de ce fait constitue un instrument sans risque pour l'emprunteur. D'autre part, un contrat de court terme qui prévoit un ajustement complet des remboursements, et de ce fait constitue un instrument risqué pour l'emprunteur. Intuitivement, le choix de s'engager à long terme est semblable au choix entre un actif à rendement sans risque et un actif à rendement risqué. Les facteurs importants qui déterminent la durée du contrat sont le degré d'aversion pour le risque et le profil de revenu de l'emprunteur.

Dans le chapitre 1, mon modèle de choix de la durée des contrats permet d'expliquer le recours aux contrats d'hypothèque à taux flexible par des agents averses au risque. Ma contribution, à la littérature sur le choix de contrats d'hypothèque, est de montrer que les contrats d'hypothèque à taux variable permettent aux agents de s'auto assurer contre le risque de taux, en ajustant leurs paiements en conséquence. Sous certaines conditions, renoncer à l'assurance d'un contrat d'hypothèque à taux fixe est optimal. Dans le chapitre 2, j'estime une forme réduite du modèle de choix de contrats d'hypothèque, basée sur les résultats du chapitre 1. Je compare mes estimations aux résultats présents dans la littérature empirique. La contribution de ce chapitre est de mettre à profit une nouvelle base de données pour étudier le choix de contrats d'hypothèque. Dans le chapitre 3, mon modèle de choix de la durée des contrats permet d'expliquer le recours excessif, dans les années 1990, à la dette extérieure de court terme par les pays émergents. Ma contribution, à la littérature sur les mouvements de capitaux internationaux, est de montrer que la dette de long terme et la dette de court terme sont des substituts imparfaits. Toute politique visant la réduction de l'endettement à court terme devrait proposer des instruments alternatifs.

Mots-clés : Hypothèque à taux fixe, Hypothèque à taux variable, Crise asiatique, Dette extérieure à court terme, Epargne de précaution, Flexibilité, Auto assurance, Durée du contrat.

Abstract

This dissertation contributes to the literature of contract length by identifying benefits and costs of long-term commitment in the case of both foreign debt contracts, and mortgage loan contracts; in particular from the borrower point of view. More precisely, I characterize and compare two alternatives structures of debt contracts which differ only by the extent to which they accommodate for an exogenous shock on the future interest-rate. On one hand, long-term contracts do not adjust to the realized interest-rate, so that they are risk-free for the borrower. Short-term contracts on the other hand, adjust the repayments to the realized rate of interest. Therefore, they are risky for the borrower. Intuitively, the choice of long-term commitment is similar to the choice between a risky asset and a risk-free asset. The key factors that govern this choice are both the degree of risk aversion, and the income profile of the borrower.

In the first chapter, my model of maturity choice helps explain why risk-averse borrowers choose adjustable-rate mortgage contracts. My contribution, specifically to the literature on mortgage choice, is to prove that adjustable-rate mortgage loans allow for self-insurance against the interest-rate risk, by adjusting repayments consequently. Under some conditions, it becomes optimal to renounce to the insurance attached to fixed-rate mortgage loans. In the second chapter, I run an estimation of a reduce-form equation, based on Chapter 1's findings. Then, I compare the estimates with the corresponding empirical literature. The novelty in this chapter comes from using a new database on mortgage indebtedness. In the third chapter, my model of maturity choice helps explain the surge in short-term foreign borrowing by Emerging countries prior to the Asian crisis. My contribution, specifically to the literature on international capital flows, is to prove that long-term and short-term debt instruments are imperfect substitutes. Hence, any policy aiming at discouraging short-term indebtedness should propose alternatives instruments.

Key words : Fixed-rate mortgage, Adjustable-rate mortgage, Asian crisis, Maturity mismatch, Precautionary saving, Flexibility, Self-insurance, Contract length.

Table des matières

Table des figures

Liste des tableaux

à mes parents

à feue ma tutrice, Nicole

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Introduction

Dans plusieurs situations réelles pour lesquelles une analyse de type principal-agent semble adaptée, la *durée* est un paramètre clé d'un contrat entre parties. L'intérêt pour la durée des contrats découle du fait qu'elle puisse être différente de la durée de la relation entre les parties. Sur le marché du travail, par exemple, un employeur et un employé d'une industrie saisonnière signent généralement un contrat à durée déterminée mais qu'ils renouvellent chaque saison. Il en va de même pour certains contrats de dette sur les marchés des capitaux. Afin de rembourser un emprunt lié à l'achat d'une maison, une banque et un ménage peuvent signer plusieurs contrats d'emprunt, successifs, de durée unitaire inférieure à la durée de l'amortissement. D'un point de vue aussi bien normatif que positif, il est important de comprendre dans quelle mesure la durée d'un contrat devrait dépendre des facteurs environnants.

En l'absence de toutes frictions dans l'économie, des parties entrant dans une relation de plusieurs périodes devraient signer un contrat de long terme couvrant toute la durée de leur relation, et prenant en compte toutes les contingences possibles liées à la relation. Il en découle que, expliquer la durée des contrats reviendrait à expliquer les bénéfices et les coûts associés à l'engagement de long terme. En pratique, par exemple, l'absence de contrats complets serait due au coût prohibitif de la spécification et de l'application de toutes les contingences possibles (Williamson (1985)). Les études sur les contrats *complets* ont essayé de répondre principalement aux deux questions suivantes : quelle est la durée optimale d'un contrat ? Et, sous quelles conditions une succession de contrats de court terme est aussi efficiente qu'un contrat de long terme ?

Premièrement, les recherches sur la durée optimale d'un contrat concluent qu'elle dépend du degré d'incertitude qui affecte les parties, indépendamment du fait que cette incertitude soit exogène ou non, idiosyncrasique ou non. Parmi les pionniers de cette littérature, Gray (1978) a montré que la durée optimale d'un contrat est négativement reliée au niveau d'incertitude. Dans le modèle de Gray, la durée optimale est telle qu'elle minimise le coût total moyen de contracter (coût de transaction) sur la période du contrat. Un contrat de long terme possède l'avantage d'amortir sur une période plus longue les coûts fixes liés à la négociation du contrat. Tandis qu'un contrat de court terme, réduit les coûts liés à l'incertitude. En effet, Gray suppose que le degré d'incertitude augmente dans le temps. D'autres études ont nuancé la conclusion de Gray (1978). Harris et Holmström (1987) ont montré que si les coûts de transactions augmentent avec le niveau d'incertitude, alors il est possible que la durée optimale du contrat augmente aussi avec le niveau d'incertitude. Dans le modèle de Harris et Holmström, le coût est lié à l'acquisition de l'information sur l'unique variable d'état du système (et du contrat). Le choix d'acquérir de l'information s'apparente à réduire la durée du contrat. Aussi, lorsque le niveau d'incertitude augmente l'information se détériore, mais plus rapidement que l'augmentation du niveau d'incertitude. Au delà d'un certain seuil les pertes liées à la non-acquisition de l'information deviennent supérieures au coût de son acquisition. Enfin, Dye (1985) a montré que la durée optimale d'un contrat varie dans le temps, et sous certaines conditions est indépendante du niveau d'incertitude.

Deuxièmement, une succession de contrats de court terme est aussi efficiente qu'un contrat de long terme lorsque trois conditions sont réunies. Selon la première condition, l'agent n'a pas accès aux marchés des capitaux tandis que le principal y a un accès illimité. Selon la deuxième condition, aux dates de renégociation, l'information nécessaire pour identifier toutes contingences pertinentes est connaissance commune. Et enfin selon la troisième condition, le principal et l'agent maximisent des objectifs conflictuels, de sorte que la frontière de Pareto soit décroissante. Sous ces conditions, Malcomson et Spinnewyn (1988) et Rey et Salanié (1990) ont montré que le lissage optimal de la consommation peut être atteint indépendamment du type de contrats. Fudenberg, Holmström et Milgrom (1990) arrivent au même résultat en supposant que l'agent possède un accès illimité aux capitaux au même taux d'intérêt que le principal, pourvu que les parties possèdent la même information aux dates de renégociation. Pour éliminer l'information asymétrique découlant de l'accès de l'agent aux marchés des capitaux, il suffit par exemple que la fonction d'utilité de ce dernier exclue les effets de richesse.

Cette thèse contribue à la littérature sur la durée des contrats en identifiant les bénéfices et les coûts de l'engagement de long terme dans le cas des contrats de dette internationale, et dans le cas des contrats d'emprunt hypothécaire ; notamment du point de vue de l'agent. Plus précisément, je caractérise et compare deux structures alternatives de contrats de dette qui diffèrent uniquement par l'ajustement qu'elles prévoient face à un changement exogène du taux d'intérêt futur. D'une part, un contrat de long terme qui ne prévoit aucun ajustement des remboursements au taux d'intérêt réalisé, et de ce fait constitue un instrument sans risque pour l'agent.[1] D'autre part, un contrat de court terme qui prévoit un ajustement complet des remboursements, et de ce fait constitue un instrument risqué pour l'agent. Implicitement, la durée des contrats est supposée exogène, et négativement corrélée au niveau d'incertitude compris dans le contrat. En l'absence d'incertitude dans l'économie, il est équivalent de signer un contrat de long terme ou une succession de contrats de court terme.[2]

[1] Je suppose que le principal absorbe tout le risque.

[2] Lorsque la variance du taux d'intérêt futur est nulle.

Dans le cas plus général où l'incertitude est présente, le choix de s'engager ou non à long terme est semblable au choix entre un actif à rendement sans risque et un actif à rendement risqué. Ainsi, les facteurs importants qui déterminent le choix d'un contrat sont le degré d'aversion pour le risque et le profil de revenu de l'agent. Je montre, par exemple, que les bénéfices de l'engagement à long terme augmentent avec le degré d'aversion au risque. Intuitivement, plus un agent est averse au risque plus il bénéficie de l'assurance du contrat de long terme, toutes choses égales par ailleurs. Tandis que les coûts de l'engagement à long terme augmentent avec le niveau de revenus accumulés avant l'occurrence du choc sur le taux d'intérêt. Intuitivement, plus un agent est riche plus il est capable de s'auto assurer, toutes choses égales par ailleurs.

Dans le chapitre 1, mon modèle de choix de la durée des contrats permet d'expliquer le recours aux contrats d'hypothèque à taux flexible par des agents averses au risque. Ma contribution, à la littérature sur le choix de contrats d'hypothèque, est de montrer que les contrats d'hypothèque à taux variable permettent aux agents de s'auto assurer contre le risque de taux, en ajustant leurs paiements en conséquence. Sous certaines conditions, renoncer à l'assurance d'un contrat d'hypothèque à taux fixe est optimal. Dans le chapitre 2, j'estime une forme réduite du modèle de choix de contrats d'hypothèque, basée sur les résultats du chapitre 1. Je compare mes estimations aux résultats présents dans la littérature empirique. La contribution de ce chapitre est de mettre à profit une nouvelle base de données pour étudier le choix de contrats d'hypothèque. Mes estimations permettent de conclure que le degré d'aversion au risque et le niveau de richesse du ménage sont des facteurs importants du choix de contrats d'hypothèque. Dans le chapitre 3, mon modèle de choix de la durée des contrats permet d'expliquer le recours excessif, dans les années 1990, à la dette extérieure de court terme par les pays émergents. Ma contribution, à la littérature sur les mouvements de capitaux internationaux, est de montrer que la dette de long terme et la dette de court terme sont des substituts imparfaits. Toute politique visant la réduction de l'endettement à court terme devrait proposer des instruments alternatifs.

Chapitre 1

Households' choice between fixed- and adjustable-rate mortgages : On the value of flexibility in contracting

1.1 Introduction

Mortgage choice is an important decision that many households make at least once in their lifetime. Because of the many dimensions in which mortgage loans can differ (caps, margins, prepayment, points, refinancing charges, ...), such a choice is very complex one. At the very least however, practitioners and economists classify mortgages between fixed-rate (FRM) and adjustable-rate (ARM), that is according to their adjustability to interest-rate changes. Under an FRM contract, the coupon rate is fixed once-and-for-all at the time the contract is signed. Under an ARM on the other hand, the coupon rate is adjusted to a given index at pre-specified intervals. As a result, FRM contracts insulate borrowers against interest-rate risk, so that uncertainty is borne entirely by the lender. On the contrary, ARM contracts reflect the risk of rising interest-rates to borrowers. We are concerned with how households' idiosyncrasies influence their mortgage choice along this classification. Beyond the intuitive role of risk aversion as favoring FRM contracts (see for instance Capozza and Gau (1983)), the theoretical literature has yet to explain how a risk-averse borrower can benefit from interest-rate risk in the first place.

In this paper, we investigate the idea that the flexibility that comes with an ARM contract should promote such risk taking. Indeed, peculiar to the mortgage choice is the fact that interest-rate uncertainty gets resolved before the mortgage contract comes to an end. Thus, the borrower is able to self-insure by spreading this risk over the subsequent periods that follow resolution. It can be accomplished by setting the remaining payments so as to reduce the overall exposure to risk; something possible under an ARM contract as payments are contingent to interest-rate realizations. This in turn should incite risk-averse individuals to bear risk, provided that they are better-off than under an FRM contract.[1]

[1] In this respect, the mortgage choice is an illustration that the optimal attitude toward risk and the way this risk will be allocated cannot be dissociated (see Gollier (2002)).

We opt for a three-period model, where a risk averse household must finance the purchase of a house from a deterministic income stream. The future interest rate is uncertain and the only means to transfer resources over time are : an adjustable-rate mortgage or a fixed-rate mortgage. The model predicts, among other things, that : more risk averse households are more likely to select FRM loans over ARM loans because they tend to shy away from fluctuating consumption paths as those implied by the repayment schedules of ARM loans. Moreover, for households with similar wealth, those with a faster income growth are more likely to choose FRM loans because a higher share of their discounted lifetime income is subject to uncertainty. Finally, households which are sufficiently rich prior to the interest-rate adjustment, are more likely to select adjustable-rate loans over fixed-rate loans. In fact, they enjoy similar advantages as investing in a risky asset while being able to self-insure against interest-rates increases.

The rest of the paper is organized as follows. Section 2 presents the environment. Section 3 displays the mechanisms between the mortgage choice. Section 4 presents the self-insurance effect. In section 5, we consider the case of CRRA utility function. In section 6, we conduct an empirical investigation of the model implications. Conclusions are presented in section 7.

1.2 Summary of previous research

Few papers theoretically discuss the choice between adjustable-rate and fixed-rate mortgages, let alone provide a rationale for choosing ARM loans. Capone and Cunningham (1992) sketch the importance of risk aversion and holding periods. However, their analysis ignores self-insurance as they find no rationale for risk-averse borrowers to hold ARMs, when FRMs with the same holding period are available. Baesel and Biger (1980) and Statman (1982) focus on inflation and labor income. All these papers approach the demand for mortgage in the specific case where a borrower maximizes the expected utility of its lifetime wealth at retirement, with *no intermediary consumption* on which to spread interest-rate risk.[2] However, it should only be under restrictive circumstances that the repayment schedule (and de facto mortgage demand) is unrelated to a borrower's lifetime consumption profile.[3] The reason is that the typical household portfolio is quite undiversified. A mortgage loan is usually its major liability (see Guiso *et al.* (2001)) which, in addition, is carried over a long portion of its lifetime.[4] A notable exception to this special case, however, has been Campbell and Cocco (2003).

Campbell and Cocco numerically solve a life-cycle model and assume a constant relative risk aversion utility (CRRA). They find that households with smaller houses relative to income, more stable income and lower risk aversion should find ARMs most attractive. All this is consistent with our theoretical analysis based on a CRRA utility function (section 6). But Campbell and Cocco (2003) make no mention of the underlying assumptions, as we do in this paper. We are not aware of any previous work that treats early resolution of uncertainty in the mortgage choice.

Empirical studies, on the other hand, have highlighted the central role of borrowers' risk aversion in the mortgage choice decision. Capone and Cunningham (1992) find that, for any given holding period, borrowers who choose ARM contracts have lower degrees of risk aversion. In Sa-Aadu and Sirmans (1995), this role shows up through a life-cycle effect, as younger households appear more sensitive to interest-rate changes than older borrowers.[5] Using Canadian data, Breslaw, Irvine and

[2]Gollier (2002) provided the theoretical foundation of allocating risk on consumption over several periods after uncertainty is resolved.

[3]Evidence of correlations between mortgage-related features and lifetime consumption profiles can be found in Englehardt (1996). Using data drawn from the PSID, he find that down payment constraints affect consumption profiles by forcing first-time home buyers to reduce consumption when young in order to access to homeownership later on.

[4]Canada is no exception as mortgages are the single most important debt; accounting for 75% of the overall value (see Statistics Canada (2005)). Guiso *et al.* (2001) also found that a house is usually the major asset in the typical household portfolio. Ironically, and contrary to the literature on the demand for mortgage, the literature on the demand for housing has long operated within the life-cycle framework (see for example Dusansky and Wilson (1993)).

[5]Liquidity constraints is a factor as well. Since old households are less likely to face binding constraints, they can afford to take more risk than young households.

Rahman (1996) found that borrowers with family responsibilities also display relatively more aversion to interest-rate risk. Campbell and Cocco (2003) examine the mortgage choice in a dynamic stochastic equilibrium model. They demonstrate that risk aversion is important in generating patterns that are consistent with U.S. data.

1.3 The model

There are two individuals, a borrower and a lender. They live for three periods. The borrower's consumption in period t is denoted by $C_t > 0$, for all t. Preferences are time-separable, and momentary utility is given by the function u which is continuous, thrice differentiable with $u' > 0$ and $u'' < 0$. We impose the Inada condition $\lim_{C\to 0} u'(C) = +\infty$. For each period, the borrower earns an income $Y_t > 0$ units of the consumption good. This income stream is nonstochastic. The borrower enters the first period having signed a mortgage contract for the purchase of a house worth H units of consumption.[6] The house is to be paid in the remainder of the agent's life. Let D_t denote period t's repayment.

The lender is risk neutral, it only cares about the total discounted mortgage payments and not about their timing. It can borrow or save unlimited amounts at the market (gross) rate of interest R_t, between t and $t+1$; R_1 is fixed, but we assume that R_2 is stochastic, with mean equals to $\bar{R}_2$. Denote by R the long-term interest-rate (between period 1 and period 3), we assume that *the term structure of interest rates is such that* $R = E(R_1 R_2) = R_1 \bar{R}_2$. This assumption will guarantee that the lender is indifferent in offering the two types of contract that we will consider, fixed-rate and adjustable-rate. Competition in the mortgage industry forces the lender to offer contracts that maximize the expected utility of the borrower, while making zero-profit. Thus, its behavior can be represented by an optimal borrowing/lending problem. The lender will not loan beyond what the borrower can repay throughout its lifetime, at the more burdensome interest-rate that is,

$$Y_1 + \frac{Y_2}{R_1} + \frac{Y_3}{R_1 R_2^{max}} \geq H$$

where R_2^{max} is the upper bound of the admissible interest-rate in period 2.

[6] It is equivalent to assuming that : in period 0, the house is bought, there is no consumption, the borrower's period-income is zero and the interest-rate $R_0 = 1$.

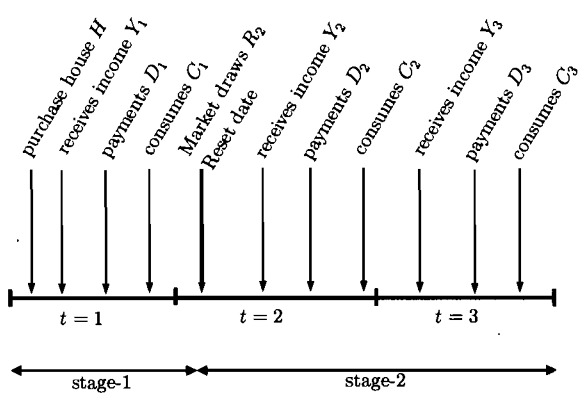


FIG. 1.1 – Timing of the events

We assume that *the borrower has no access to credit markets.* This assumption is intended to tie up the repayment schedule to the borrower's optimal saving path. The exclusive relationship with the lender may be due to the fact that once the borrower takes on a mortgage loan, subsequent borrowing with another lender is compromised. Indeed, given the size of a mortgage loan relative to income, acquiring a house moves the borrower significantly closer to a binding borrowing constraint. Moreover, there is no room for refinancing because the housing price is fixed so that home equity is always zero at time 1, by assumption. Consequently, a mortgage contract is the only way available to smooth consumption over time.

1.4 Mortgage contracts

In this section, we describe and give a preliminary analysis of fixed-rate and adjustable-rate mortgage contract. In doing so we introduce the basic elements of the borrower-lender relationship.

1.4.1 Fixed-rate mortgage contracts

A complete mortgage contract must specify the repayment schedule. Each repayment consists in two components : an interest-repayment on the outstanding loan balance, and an "ad-hoc" repayment of the principal (see Appendix 1.1 for details).[7] The amortization period and the term are identical and last three periods. Under the condition $R_1 Y_1 + Y_2 \leq R_1 H$, full payment of the mortgage loan prior to the maturity is prohibited We also rule out default on repayments. Mortgage contracts differ on whether periodic mortgage interest (plus principal) payments are fixed or adjustable with respect to interest rate changes. Under FRM contracts they are fixed. The key feature is that the interest-repayment between period 2 and period 3 is calculated with the *expected* interest-rate $\bar{R}_2$.

An optimal FRM contract is the solution to the following maximization problem :

$$W^F \equiv \max_{(C_t, D_t)_{t=1}^3} u(C_1) + \beta u(C_2) + \beta^2 u(C_3)$$

$$\text{subject to :} \qquad C_t = Y_t - D_t, \qquad D_t > 0, \text{ for } t = 1, 2, 3,$$

$$D_1 + \frac{D_2}{R_1} + \frac{D_3}{R_1 \bar{R}_2} = H.$$

where β is the discount factor, $0 < \beta < 1$. The first constraint is the period-t budget constraint. It states that the borrower's consumption in any period is determined by its income net of the mortgage

[7] Notice that, since payments of the principal are endogenous the optimal mortgage payments D_t need not be equal for all t.

repayment. The second and third constraints are imposed by the mortgage contract. The second-constraint states that, in each period, the borrower makes a non-negative mortgage-repayment. Consequently, borrowing to finance current consumption is prohibited. The third constraint represents the lender's zero-profit condition. It states that the present discounted value of payments must equal the mortgage loan value.[8]

Solving the FRM problem is pretty straightforward. We use the first constraint to substitute consumption in the objective function. The problem is now to find the optimal repayment schedule that maximizes the expected utility under the zero-profit constraint. At the optimum, the last-repayment trivially equals the outstanding loan balance, so that we are left with an unconstrained maximization problem, that is :

$$W^F \equiv \max_{D_1,D_2} u(Y_1 - D_1) + \beta u(Y_2 - D_2) + \beta^2 u\left[Y_3 + \bar{R}_2\left(R_1 D_1 + D_2 - R_1 H\right)\right]$$

The first-order conditions for an interior solution are :

$$u'(Y_1 - D_1^F) = \beta R_1 u'(Y_2 - D_2^F) \tag{1.1}$$

$$u'(Y_2 - D_2^F) = \beta \bar{R}_2 u'\left(Y_3 + \bar{R}_2(R_1 D_1^F + D_2^F - R_1 H)\right) \tag{1.2}$$

Under our assumptions, these conditions are sufficient as well as necessary for an optimum. Equations (1) and (2) characterize efficient intertemporal smoothing.

For the purpose of the exposition, we will consider the following equivalent problem in the rest of the paper :

$$W^F \equiv \max_{D_1} u(Y_1 - D_1) + \beta \left\{ \max_{D_2} u(Y_2 - D_2) + \beta u\left[Y_3 + \bar{R}_2\left(R_1 D_1 + D_2 - R_1 H\right)\right] \right\} \tag{P^F}$$

[8] Payment D_1 should not be confused with a downpayment. In fact, we assume that there is no downpayment requirement in the mortgage.

since it is deterministic, the FRM problem is trivially time-consistent, and therefore it is equivalent to choose D_2 in period 1 or in period 2.

1.4.2 Adjustable-rate mortgage contracts

An adjustable-rate mortgage contract has the same components as an FRM contract, namely the repayment made at the start of each period to the lender. The key feature of an ARM contract is that the interest-payment between period 2 and period 3 is decided once uncertainty about R_2 is resolved. Consequently, under an ARM contract a full adjustment of payments D_2 and D_3 is made at date 2.

An optimal ARM contract is the solution to the following maximization problem :

$$W^A \equiv \max_{(C_1,D_1)} u(C_1) + \beta \mathbf{E} \left\{ \max_{(C_t,D_t)_{t=2}^3} u(C_2) + \beta u(C_3) \right\}$$

$$\text{subject to : } \quad C_1 = Y_1 - D_1, \qquad D_1 > 0,$$

for every realized value of R_2

$$C_t = Y_t - D_t > 0, \qquad D_t > 0,\, t = 2, 3,$$

$$D_1 + \frac{D_2}{R_1} + \frac{D_3}{R_1 R_2} = H.$$

where $\mathbf{E}$ is the expectation operator with respect to the distribution of R_2. The important difference with the FRM problem is that all constraints must hold for every realization of R_2.

To solve this problem, we follow similar steps as with the FRM problem, namely we replace all the constraints into the objective function so as to obtain an unconstrained maximization problem where the last-period consumption equals final wealth, that is :

$$W^A \equiv \max_{D_1} u(Y_1 - D_1) + \beta \mathbf{E} \left\{ \max_{D_2} u(Y_2 - D_2) + \beta u \left[Y_3 + R_2 \left(R_1 D_1 + D_2 - R_1 H \right) \right] \right\} \qquad (P^A)$$

Now, the second maximization operator is important as the subsequent mortgage payments depend on the ex-post interest-rate. Obviously, early resolution of uncertainty is valuable because payments can adjust to the new information (see Appendix A.2). We solve this problem by backward induction. The first-order conditions, for an interior solution, are : given a first-period repayment z, we obtain optimal values of D_2^A by solving the following equation :

$$u'(Y_2 - D_2^A(z)) \;\; = \;\; \beta R_2 u'\Big(Y_3 + R_2(R_1 z + D_2^A(z) - R_1 H)\Big) \tag{1.3}$$

for each realization of R_2. The optimal first-period repayment then follows from

$$u'(Y_1 - D_1^A) \;\; = \;\; \beta^2 R_1 \mathbf{E}\Big\{R_2 u'\Big(Y_3 + R_2(R_1 D_1^A + D_2^A(D_1^A) - R_1 H)\Big)\Big\} \tag{1.4}$$

Moreover, for any two realized interest-rates R_2^i and R_2^j with $i \neq j$, the following equation must hold

$$\frac{u'(C_2^i)}{R_2^i u'(C_3^i)} = \frac{u'(C_2^j)}{R_2^j u'(C_3^j)} \tag{1.5}$$

Equations (3) and (4) characterize efficient intertemporal smoothing just like the FRM contract. In addition, *Eq.*(4) shows that under the ARM contract the marginal benefit of future wealth is uncertain, hence, D_1 depends on the borrower's optimal *exposure to risk* as determined by $R_2 D_2'$ for all R_2, where $D_2' = (R_1 D_1 + D_2 - R_1 H) \leqslant 0$. Part of this exposure is implicit and positively related to the housing value; mechanically, $R_1 H$ is a scale parameter which controls the variance of the random variable R_2. The other part is explicit (endogenous) and controlled by the amount of money devoted to repayments in the first-two periods, i.e., $(R_1 D_1 + D_2)$. More specifically, once uncertainty is resolved, the contingent repayment D_2 varies according to *Eq.*(5) so as to keep the ratio of marginal utilities unchanged across states of the nature (i.e., *ex-post* interest-rates).

We can interpret *Eq.*(5) as a Borch rule for co-insurance between ("individuals" named) period 2 and period 3. Thus, sharing risk in this case essentially means making a contingent "transfer" D_2 from period 2 to period 3. If, for instance, the size of the payment D_2 is bigger for high R_2s, then $R_2 D_2'$ unambiguously decreases with respect to R_2 (in absolute value). In this case, bigger transfers from period 2 to period 3 at states where the risk is high, reduce the overall exposure to risk. Intuitively, the borrower displays some *self-insurance* behavior in this case. However, if the size of the payment D_2 is lower for high R_2s, then the overall exposure to risk is increased eventually. In this particular case, the borrower takes on added risk in order to increase its final wealth. Intuitively, it displays some *gambling* behavior in that he bets that high interest-rates will occur.

In this paper, our goal is to compare W^F and W^A. In the absence of risk, the borrower is indifferent between an ARM contract and an FRM contract, i.e., $W^A = W^F$. Away from this trivial case, however, an ARM contract entails some risk which is absent from an FRM contract. However, it comes with a larger set of strategies as the borrower can now adjust its payments to the *ex-post* interest-rate. So, it is unclear which contract is optimal *ex-ante*.

1.5 Determinants of the mortgage choice

To start, consider the second-stage problem the household faces after the renewal date, that is,

$$v(z, R_2) \equiv \max_{D_2'} u(Y_2' - D_2') + \beta u \left(Y_3 + R_2 D_2'\right)$$

where $Y_2' = Y_2 + R_1(z - H)$ and v is the indirect utility function given the first-period repayment z, and the rate of interest R_2 (possibly equals to $\bar{R}_2$). Notice that P^A features the expected indirect utility, while P^F features the indirect utility evaluated at the expected interest rate $\bar{R}_2$. Basically, the mortgage choice is governed by the borrower's attitude towards interest-rate risk as measured by v's curvature with respect to R_2, i.e., v_{22}. When $v_{22} > 0$ the household is thought as being prone to interest-rate risk. When $v_{22} < 0$, on the other hand, the household is thought as being averse to interest-rate risk.

To see it, assume that v_{22} is uniformly negative. From Jensen's inequality we have that :

$$\mathbf{E}v\left(z, R_2\right) < v\left(z, \bar{R}_2\right)$$

Multiplying both sides of the inequality by β and adding up $u(Y_1 - z)$, one obtains :

$$u\left(Y_1 - z\right) + \beta\mathbf{E}v\left(z, R_2\right) < u\left(Y_1 - z\right) + \beta v\left(z, \bar{R}_2\right).$$

This last inequality holds for any z, therefore it must hold when taking the maximum operator on both sides :

$$\max_z u\left(Y_1 - z\right) + \beta\mathbf{E}v\left(z, R_2\right) < \max_z u\left(Y_1 - z\right) + \beta v\left(z, \bar{R}_2\right).$$

From P^A and P^F, this is equivalent to

$$W^A < W^F.$$

We conclude that :

$$\text{If} \qquad v_{22} < 0 \qquad \text{then} \qquad W^A < W^F.$$

Consequently, the household will choose the FRM contract. The opposite can be shown when $v_{22} > 0$, i.e., $W^F < W^A$. However, the indirect utility function v is endogenous so that a simple comparison between the two contracts is not always possible.

1.5.1 Borrower's attitude towards interest-rate risk

In this section, we show that the household's attitude towards interest-rate risk is determined by the way this risk is allocated in the second-stage problem. Basically, the second-stage problem is a standard consumption/saving problem under certainty, in that given the "income" flow Y_2' and Y_3, today and tomorrow, respectively, the individual pursues intertemporal smoothing by borrowing D_2', at the (*risk-free*) interest-rate R_2.

First, we present a geometric treatment. In Figures 2 and 3, we represent the budget constraint when the interest-rate is R_2 (line Y_3KB) ; and the indifference curve (from the second-stage utility index) which is tangent to the segment KB. Values of v are drawn from the utility at tangency points (point E), for different combinations of z and R_2. We are concerned with the welfare effects of the variability of R_2. With an increase in R_2, for instance, the budget constraint pivots clockwise around K so that today's consumption C_2 falls in Figure 2. This happens for two reasons : first, in order to compensate for a relatively cheaper consumption in period 3, so as to keep the same welfare level (substitution effect) ; and second, because the indebted-household grows poorer (income effect). As for C_3, it decreases as well. Overall, a lower indifference level is attained. However, since a falling interest-rate brings a welfare gain, the net effect on welfare of the interest-rate risk is ambiguous.

It depends on both the level of the outstanding debt at the beginning of the stage-2 problem and the curvature of the indifference curve. If the outstanding debt at the beginning of the second-stage is low enough, then a declining interest-rate increases final wealth, i.e., C_3 ; while a rising rate has no effect on welfare. It follows that dispersion of the interest-rate is welfare-improving provided the indifference curve is curvy enough around E. If such is the case at every level of R_2, then the household unambiguously chooses the ARM contract.[9]

[9]It should be pointed out that Y_2' cannot be too high, in order to insure that $D_3 > 0$, i.e., $Y_2' < C_2$. For clarity sake, we have just considered the case $Y_2' > 0$, but everything carries on with Y_2' negative.

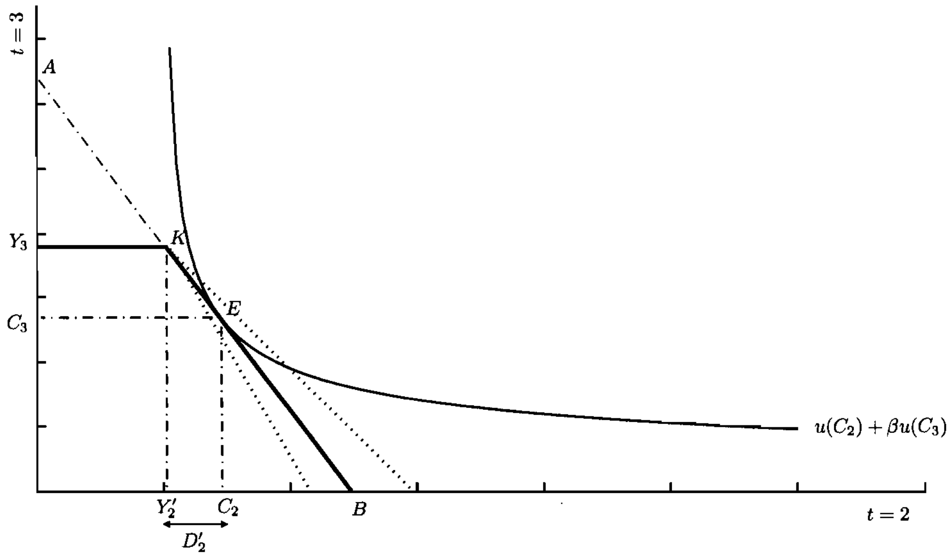


FIG. 1.2 – Income profile is away from the autarky point

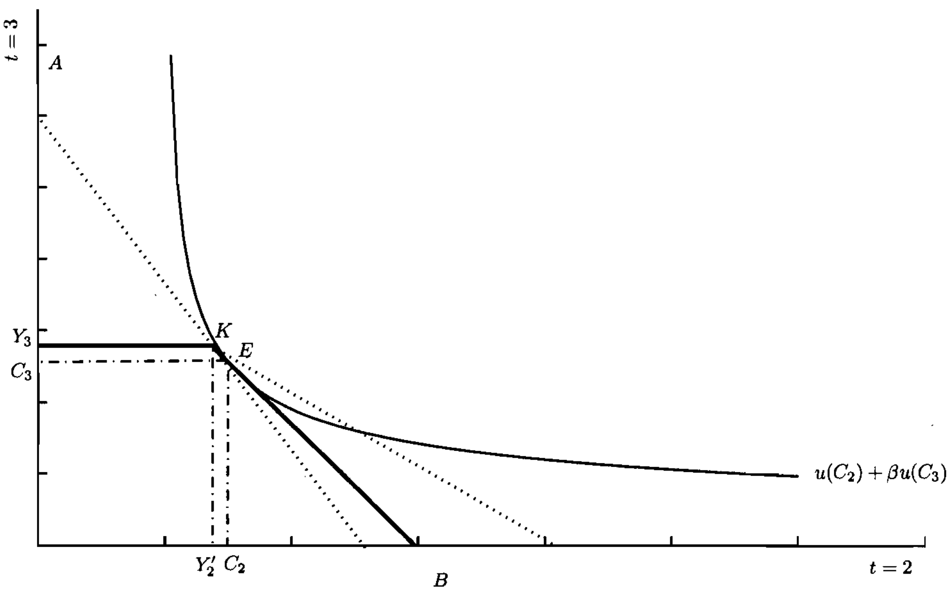


FIG. 1.3 – Income profile is around the autarky point

We can show, mathematically, that the household's attitude towards risk depends on the allocation problem in stage-2. From the envelope theorem

$$v_2(z, R_2) = \beta u'(C_3)\left(-\frac{D_3}{R_2}\right) < 0$$

then, taking the derivative of v_2 with respect to R_2 yield

$$v_{22}(z, R_2) = \beta u''(C_3)\left(-\frac{D_3}{R_2}\right)\frac{\partial C_3}{\partial R_2} - \beta u'(C_3)\frac{\partial C_2}{\partial R_2} \tag{1.6}$$

In words, v_{22}'s sign depends on the total effect of interest-rate changes on consumption levels C_2 and C_3. The Slutsky equation describes this total effect. Denote by $\gamma_t = -u''(C_t)/u'(C_t)$ the degree of absolute risk aversion. Total differentiation of the first-order condition $Eq.(3)$ and some calculations (see the Appendix A.4) yields

$$\frac{\partial C_2}{\partial R_2} = \frac{-1}{R_2(\gamma_2 + R_2\gamma_3)} - D_3\frac{\gamma_3}{R_2(\gamma_2 + R_2\gamma_3)} \tag{1.7}$$

which is negative because both the substitution effect – i.e., first-term in the LHS of $Eq.(7)$– and the income effect – i.e., first-term in the RHS of $Eq.(7)$– are negative. The total effect on C_3, however, is ambiguous. To see it, take the first-derivative of the budget constraint with respect to R_2 and notice that $\partial D_t/\partial R_2 = -\partial C_t/\partial R_2$, it yields

$$\frac{\partial C_3}{\partial R_2} = \frac{R_2}{R_2(\gamma_2 + R_2\gamma_3)} - D_3\frac{\gamma_2}{R_2(\gamma_2 + R_2\gamma_3)} \tag{1.8}$$

The sign of the substitution effect on C_3 is the opposite of $\partial C_2/\partial R_2$, while the sign of the income effect is the same, that is, C_2 and C_3 are normal goods. Overall, C_3 increases when the substitution effect dominates the income effect, which in turn implies that the indirect utility function v is convex at this specific R_2. If this is true for all realized values of R_2, then the borrower always chooses the ARM contract. For the indirect utility function to be concave, however, it is necessary that $\partial C_3/\partial R_2$ be negative; that is, the income effect dominates the substitution effect on C_3.

One can show that : if the borrower does not earn enough income in the first-two periods to pay off its entire loan, and if its elasticity of substitution is uniformly lower than one; then $\partial C_3/\partial R_2 \leqslant 0$ everywhere (see Lemma 1, in the Appendix 1.5).

Condition 1. *$R_1 Y_1 + Y_2 \leqslant R_1 H$ and $\nu_t \geqslant 1$ for all t.*

Under Condition 1, v_{22}'s sign is ambiguous. Hence, it guarantees that the borrower won't choose either ARM or FRM for all parameter values (as it would be the case if $v_{22} < 0$ or $v_{22} > 0$). It will depend on the elasticity of substitution and the outstanding debt in the second-stage.[10] To see it, rewrite $Eq.(8)$ as follows

$$\frac{\partial C_3}{\partial R_2} = \frac{1 - \dfrac{D_3}{R_2 C_2}\nu_2}{\gamma_2 + R_2 \gamma_3} \tag{8'}$$

where $\nu_t = -C_t u''(C_t)/u'(C_t)$ represents the inverse of the elasticity of substitution (and the degree of relative risk aversion). The sign of the numerator depends on both the elasticity of substitution and the size of last-period repayment relative to consumption in period 2. The lower the elasticity of substitution, or the higher the ratio of last-period repayment to second-period consumption, the more likely $\partial C_3/\partial R_2$ is negative. Obviously, it happens away from the autarky point (see Figure 3).

So far, we have just considered the stage-2 problem for isolated value of R_2. The design of payments D_2 and D_3 contingent on the realized interest-rate has important implications for the exposure to risk under the ARM contract. The borrower displays some self-insurance behavior when the income effect dominates the substitution effect everywhere. Although, the optimal exposure to

[10] Notice that Y_2' is negative under Condition 1.

risk does not have to be zero, because renouncing to consumption in period 2 (and period 1) is costly in terms of marginal utility, doing so brings positive return in expectation. Under Condition 1, the borrower can find optimal to choose the ARM contract while at the same time insuring himself. However, in the limiting case where it is too costly to renounce to consumption (i.e., too costly to insure himself), the borrower will find optimal to choose (the full insurance of) an FRM contract.[11]

On the other hand, when the substitution effect dominates the income effect everywhere, the borrower displays some gambling behavior. Choosing the ARM contract is thus unambiguously optimal.[12]

1.5.2 Borrower's allocation of risk over time

In this section, we study the role of the first-period repayment in the mortgage choice. With a bigger first-period repayment, for instance, a lower debt burden is carried into the second-stage. This affects the autarky point, and thus the mortgage choice.

In the stage-1 problem, the borrower decides consumption, and savings, which proceeds become available for consumption in period 3.[13] However, unlike the FRM contract, first-period saving under the ARM contract is subject to uncertainty. So, comparing first-period repayment under the two contracts is similar to asking : how, starting from a zero-risk situation, an increasing risk affects saving under the ARM contract. We distinguish two channels : first, increasing risk translates into a higher return on savings in the stage-2 problem, for each state ; second, increasing risk exogenously alters the exposure to risk from the standpoint of the stage-1 problem.

Let us parameterize the risk as $R_2 = \bar{R}_2 + \eta\varepsilon$, where ε is a zero-mean random variable, η is

[11] If ν_t is high enough.

[12] The amount invested in R_2 is similar to a short position in a risky asset.

[13] This feature is specific to long-term saving instruments in general, and mortgages in particular.

a positive scale parameter, and the support of ε is such that $\bar{R}_2 + \eta\varepsilon$ is no less than one.[14] For instance, $\bar{R}_2 + \eta\varepsilon$ could describe a family of uniformly distributed random variables, all having a mean equals to $\bar{R}_2$, with variance parameterized by η. The FRM problem is obtained by $\eta = 0$. A small increase of η from zero, constitutes a MPS (mean preserving spread) shift of the distribution of R_2. Combining *Eq.*(3) and *Eq.*(4), and differentiating it with respect to η, we obtain :

$$\frac{\partial C_1}{\partial \eta} = \beta^2 R_1 \left\{ \frac{\mathbf{E}\left[\varepsilon u'(C_3)\right]}{D} + \frac{\mathbf{E}\left[\varepsilon R_2 \frac{\partial C_3}{\partial \eta} u''(C_3)\right]}{D} \right\} \tag{1.9}$$

where D is the second-order condition of the ARM problem

$$D \equiv \mathbf{E}\left\{ \frac{u''(C_1)\, u''(C_2) + \beta R_2^2 u''(C_1)\, u''(C_3) + \beta^2 R_1^2 R_2^2 u''(C_2)\, u''(C_3)}{u''(C_2) + \beta R_2^2 u''(C_3)} \right\} < 0$$

with $\partial C_3/\partial \eta$ given by *Eq.*(8).[15]

An increased riskiness has two effects on the first-period consumption. A direct effect – i.e., first-term in the brackets of *Eq.*(9)– which follows from changing the dispersion of the return from saving around the mean. Since $u'(C_3)$ is uniformly increasing with respect to ε, the sign of the direct effect depends solely on the sign of $\partial C_3/\partial \varepsilon$; which is the same as $\partial C_3/\partial R_2$'s sign. When the income effect dominates the substitution effect everywhere, i.e. $\partial C_3/\partial \varepsilon \leqslant 0$ for all R_2, the direct effect is negative. In turn, first-period consumption tends to fall as the borrower needs additional resources in the second-stage problem for the Euler equation *Eq.*(4) to hold. Graphically, if all tangency points lie around the autarky point, as in Figure 2, a higher dispersion moves them away from it by rotating the budget constraint clockwise for each initial R_2. Hence, by increasing first-period repayment, Y_2' is reduced so that tangency points still lie around the autarky point. On the other hand, when the substitution effect dominates the income effect everywhere, i.e. $\partial C_3/\partial \varepsilon > 0$ for all

[14]Davis (1989) considered a similar parameterization.

[15]The effect of risk on first-period payments follows from $\partial D_1/\partial \eta = -\partial C_1/\partial \eta$.

R_2, the direct effect is positive. Notice that, risk aversion alone suffices to sign this effect.

Furthermore, an indirect effect – i.e., second-term in the brackets of $Eq.(9)$– follows from carrying some risk. Under prudence, for instance, if it is optimal to hold a non-zero risk then individuals tend to increase saving in the face of uncertainty. While risk aversion defines an individual's attitude toward uncertainty, prudence defines how an individual reacts to it. Leland (1968) and Sandmo (1970) first formalized this concept and proved that a convex marginal utility, i.e. $u''' > 0$, is attached to such a *prudent* behavior.[16] Later on, Kimball (1990) introduced the degree of absolute prudence, defined as $\phi_t = -u'''(C_t)/u''(C_t)$. In our setup, the analysis of the precautionary effect is complicated by the fact that payments adjust to the realized interest-rate. To see it, notice that the indirect effect is negative if the derivative of $R_2(\partial C_3/\partial\eta)u''(C_3)$ with respect to ε is uniformly positive. The computation of that derivative yields the following expression :

$$\frac{\partial}{\partial\varepsilon}\left[R_2\frac{\partial C_3}{\partial\eta}u''(C_3)\right] = \left[\eta\frac{\partial C_3}{\partial\eta} + R_2\frac{\partial^2 C_3}{\partial\eta\partial\varepsilon}\right]u''(C_3) + R_2\frac{\partial C_3}{\partial\eta}\frac{\partial C_3}{\partial\varepsilon}u'''(C_3)$$

which is a weighted sum of second- and third-derivatives of the utility function. The weight on $u'''(C_3)$ is positive since it has the same sign as $(\partial C_3/\partial R_2)^2$. It follows that if $\partial C_3/\partial\eta$ is independent of ε (i.e., $\partial^2 C_3/\partial\eta\partial\varepsilon = 0$), and if the risk is small (i.e., $\eta \approx 0$), then the indirect effect is negative provided that prudence (i.e., ϕ_3) is sufficiently high. In this case, the indirect effect can unambiguously be termed as the precautionary effect.

When payments adjust to the realized interest-rate, however, $\partial C_3/\partial\eta$ is no longer independent of ε (i.e., $\partial^2 C_3/\partial\eta\partial\varepsilon \neq 0$). Consequently, the precautionary effect should now reflect the endogenous change in the exposure to risk. More specifically, a positive weight on $u''(C_3)$ reduces the

[16] Drèze and Modigliani (1972) generalized Leland and Sandmo's results to non-additively-separable utility functions.

precautionary effect, while a negative weight reinforces it. The quantity $\partial^2 C_3/\partial\eta\partial\varepsilon$ measures the marginal change in the exposure to risk, as a result of allocating the risk between period 2 and period 3. Obviously, it depends on the dynamic of both substitution and income effects, but we are not aware of any theoretical result on this dynamic. Under Condition 1 and for a iso-elastic utility specification, $\partial^2 C_3/\partial\eta\partial\varepsilon$ is positive (see Appendix A.4). Intuitively, self-insurance reduces the carried risk so that the precautionary effect is weakened. In general, the total effect is ambiguous. In the Appendix A.5, we show that for the total effect to be negative, it suffices that v_{122} be uniformly positive (see Lemma 2).

1.6 Mortgage choice with constant relative risk aversion

This section aims at clarifying all ambiguities in the model by drawing qualitative results based on CRRA preferences. This preference class is standard in finance and macroeconomics, and most importantly is consistent with realistic consumption patterns under uncertainty as opposed to say CARA or quadratic preferences.[17]

Consider the constant relative risk aversion (CRRA) utility function of the form :

$$u(C) = (C^{1-\nu} - 1)/(1-\nu), \qquad \text{for } \nu > 0,$$

where ν is the degree of relative risk aversion, and $\nu + 1$ is the degree of relative prudence. With this specification, it is readily verified that the optimal consumption plan under the FRM contract can be described as follows :

$$C_t^F = \theta_t^F \, (\bar{\omega} - H) \qquad t = 1, 2, 3,$$

[17] In a similar three-period framework, Blundell and Stocker (1999) examine the impact of *income risk* on consumption patterns, when the timing of the resolution of uncertainty varies. They conclude that "the preferences of the CRRA class [are] perhaps the most realistic for modelling actual saving behavior in empirical work, because they can capture the most plausible precautionary behavior for rich and poor households."

where $\bar{\omega} = Y_1 + \frac{Y_2}{R_1} + \frac{Y_3}{R_1\bar{R}_2}$, $\theta_1^F = (R_1\bar{R}_2)/\left[R_1\bar{R}_2 + (\beta R_1)^{1/\nu}(\bar{R}_2 + (\beta\bar{R}_2)^{1/\nu})\right]$, $\theta_2^F = \theta_1^F(\beta R_1)^{1/\nu}$ and $\theta_3^F = \theta_2^F(\beta\bar{R}_2)^{1/\nu}$. Typically, the borrower uses the FRM contract to allocate fixed shares of its lifetime's net wealth to consumption in each period. In fact, any change in the timing of the income profile that leaves the wealth level unchanged, will also leave the consumption profile unaffected. Under the ARM contract on the other hand, no closed-form solution is available for the entire consumption profile. But, we can still obtain the consumption plan of the second-stage problem, that is :

$$C_t^A = \theta_t^A\left(Y_2 + \frac{Y_3}{R_2} - R_1(H - D_1^A)\right) \qquad t = 2, 3$$

$\theta_2^A = R_2/(R_2+(\beta R_2)^{1/\nu})$ and $\theta_3^A = (\beta R_2)^{1/\nu}\theta_2^A$. Again, at each R_2, a fixed share of the second-stage net wealth is allocated to C_2 and C_3, respectively. However, the timing of the income profile becomes relevant for the first-period consumption because it is affected by uncertainty. We will compute C_1^A numerically.

1.6.1 The effect of uncertainty

To single out the borrower's reactions toward risk, we use as benchmark the fixed-rate mortgage problem when the borrower does not use the mortgage contract to smooth consumption but simply to access homeownership.

In terms of parameters, it means setting $\beta R_1 = \beta\bar{R}_2 = 1$ and $Y_t = Y$ for all t, so that the agent has no incentives to borrow or save, and the consumption profile is flat and independent of ν. For the remainder of the section, we will use $Y = 2$ and $\beta = 0.8$ unless stated otherwise. The future interest-rate is assumed to be $R_2 = \bar{R}_2 + \eta\varepsilon$, where ε is uniformly distributed over the interval $[-\epsilon, \epsilon]$ and discrete on a grid of 11 points, with $\epsilon = \bar{R}_2 - 1$ (so that the lower bound is no less than one). The comparative statics experiment consists in scaling up the variance of the distribution of

R_2 by increasing η. We execute this experiment, numerically, for several degrees of risk aversion and housing values.

From Figure 4, it appears that, in absence of risk (i.e., $\eta = 0$) the first-period shares are equal under the two contracts. Unlike first-period shares under the FRM contract, θ_1^A is affected by both the increasing risk and the changing parameter ν.[18] For low values of risk aversion, the borrower gambles on interest-rate, that is, the direct effect is positive. Moreover, it dominates the precautionary effect so that the borrower enjoys additional risk to the extent that wealth is sent to the risk-free period 1. In this case, first-period share increases with respect to risk, and is greater than its counterpart in the FRM contract everywhere. At higher degrees of risk aversion, the borrower eventually stops gambling and the direct effect becomes negative. In addition, since the degree of prudence is increased as well, the precautionary effect reinforces the direct effect so that the borrower sends much wealth into risky periods in order to insure himself.

From Figure 5, it appears that raising the housing value H generates the same reactions towards risk as raising ν. In fact, by increasing the overall indebtedness, a higher housing value makes gambling on high interest-rates less attractive, consequently the direct effect goes from being positive to being negative. In the meantime, a higher H increases the implicit risk exposure so as to raise the optimal exposure to risk and thus the precautionary effect (everything else being equal, specially ν, η and ε's variance). Overall, θ_1^A is increases at first with respect to risk for low values of H, and then decreases for higher values.

Based on Figures 5 and 6, we conclude that the substitution effect dominates the income effect for low values of H, or low values of ν, while the income effect dominates the substitution effect for high values of H, or high values of ν.[19] In Figure 6, we provide additional support about

[18]The share is computed as $\theta_1^j = C_1^j/(\bar{\omega} - H)$, we use it in order to eliminate level effects.

[19]It has to do with the fact that the two-period utility index attached to the second-stage problem is ordinarily

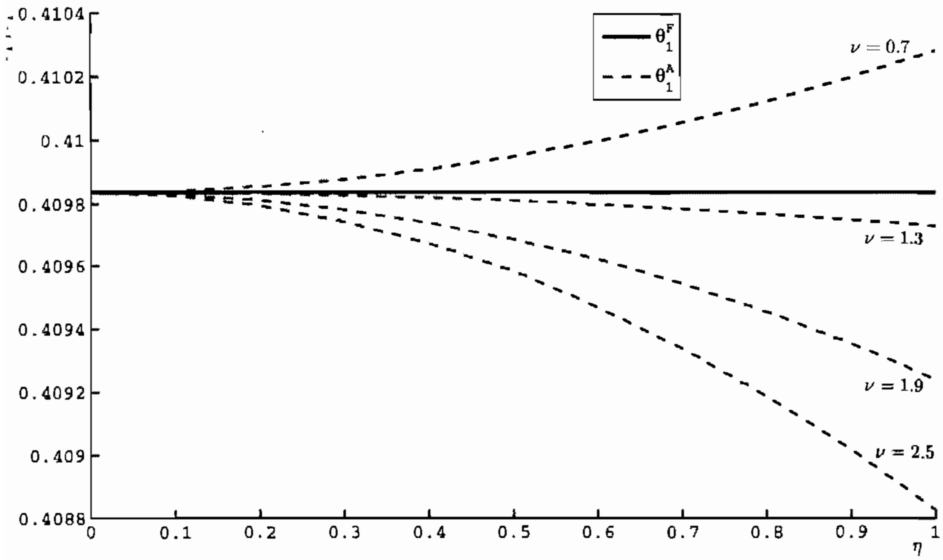


FIG. 1.4 – Effect of uncertainty wrt ν

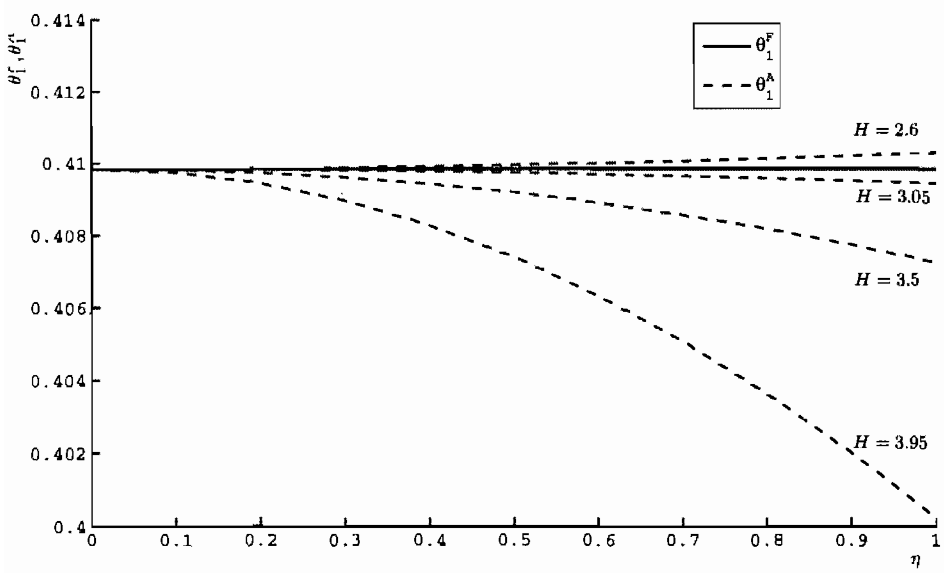


FIG. 1.5 – Effect of uncertainty wrt H

the dynamics of both effects with respect to the housing value and the elasticity of substitution, respectively. Let us first set $\eta = 1$. We then give an illustration of each possible configuration of C_3 over the interval of R_2. Panels 1 and 2 depict the cases where $\partial C_3/\partial R_2$ keeps the same sign over the interval of R_2, these cases are well understood (Panels 1 and 2 are obtained for the combination $H = 2.9$, $\nu = 0.7$, and $H = 2.75$, $\nu = 1.1$, respectively). Computations suggest that the total effect of risk on first-period consumption is dominated by the direct effect. Panels 3 and 4 depict hybrid cases where $\partial C_3/\partial R_2$ changes sign over the interval of R_2 (Panels 3 and 4 are obtained for the combination $H = 3.2$, $\nu = 0.7$ and $H = 2.6; \nu = 1.1$, respectively).[20] In Panel 3, C_3 first increases, and then decreases. In Panel 4, C_3 first decreases, and then increases. We summarizes these finding as follows :

Consider a borrower with a constant degree of relative risk aversion. Then, he finds optimal to choose the ARM contract : (i) if he is not too averse to risk; or (ii) if the housing value is not too high.

Starting from a situation where the borrower is indifferent between an ARM contract and a FRM contract, a borrower with a lower degree of risk aversion requires less insurance than what the FRM contract offers, so that he would rather self-insure and bear some risk in exchange for additional wealth. From the same initial situation, a borrower with a lower housing value faces less implicit risk while has more revenues early on to speculate on future interest-rates. He thus needs less insurance than what a FRM contract offers so that he finds optimal to choose the ARM contract. We illustrate Proposition 1 in Figure 7 (in Appendix A.5, we report some numerical result

equivalent to a CES utility function with elasticity of substitution equal to $1/\nu$ (this intuition is due to Larry Epstein (1980)). As such, we recall that, C_2 and C_3 are complements when $\nu > 1$, and perfect complements (Leontief) in the limiting case $\nu \to \infty$. They are substitutes, on the other hand, when $\nu < 1$, and perfect substitutes in the limiting case $\nu \to 0$. In our setup, the budget constraint (income effect) makes this classification depend on other parameters as well. Geometrically, it has to do with the fact that preferences of the CRRA-class are (quasi)-homothetic and that the curvature of the contour of a typical indifference curve is controlled by a single parameter, i.e., ν.

[20]These cases are rather scarce, for 100 combinations of H and ν we found just the two cases that appear in Panels 3 and 4, respectively.

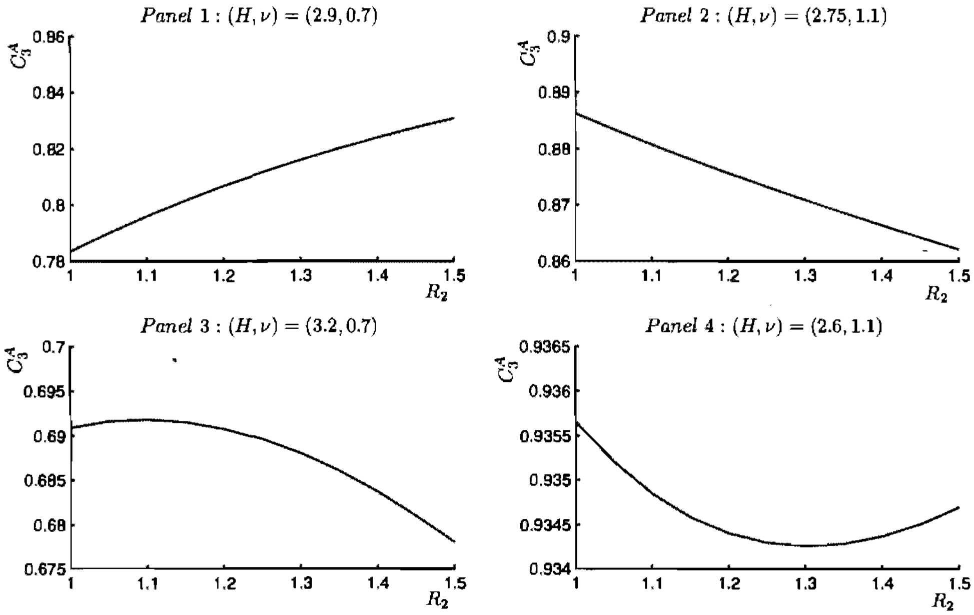


FIG. 1.6 – Last-period consumption wrt to interest-rate

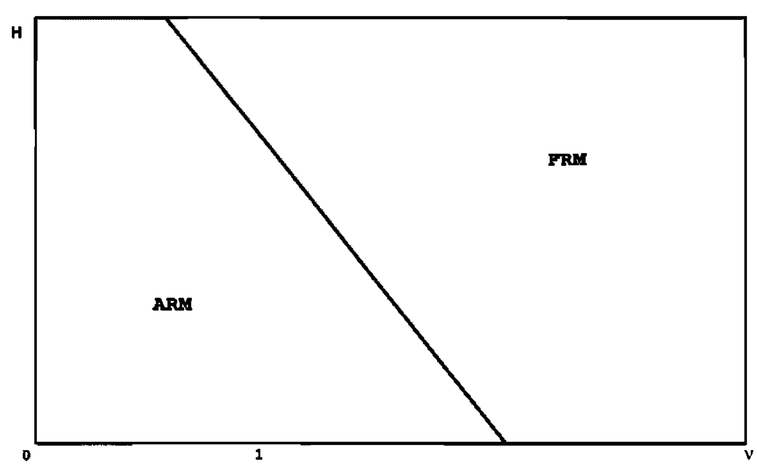


FIG. 1.7 – Mortgage choice, housing value and relative risk aversion

for different housing values and degrees of relative risk aversion).

1.6.2 The effect of consumption smoothing

We now proceed to reintroduce consumption smoothing by departing from the flat income profile assumption. How it affects the trade-off between mortgage contracts is summarized in the following proposition :

Consider a borrower with a constant degree of relative risk aversion. Then, he finds optimal to choose the ARM contract : (i) if its income is relatively more concentrated early on in life, i.e., Y_1 *or* Y_2 *is high enough ; or (ii) if income growth, i.e,* Y_{23} *is not too high.*

What matters for the trade-off between mortgage contracts is how rich the borrower is early in its life, i.e. how big is Y_2' in Figures 2 and 3. From Figure 3, for instance, the higher Y_2' is, the lower will be the outstanding debt when entering period 3, i.e., D_3. This in turn brings closer the tangency points and the autarky point, and reduces welfare losses that follow from rising interest-rates. So, if we start from a situation where the borrower is indifferent between an ARM contract and a FRM contract, higher income in the first-two periods will lead the borrower to choose the ARM contract.

However, increasing future revenue, i.e., Y_3, has the opposite effect on the mortgage trade-off. In fact, increasing Y_3 reduces the incurred gains of a borrower who is close to the autarky point, or increases the incurred losses for a borrower who is already away from the autarky point. This in turn, favors the FRM contract.

1.7 Conclusion and Empirical implications

By choosing an adjustable-rate mortgage (ARM), households speculate on the future interest-rate, and take a risk that is absent from a fixed-rate mortgage (FRM). In this framework, the following predictions can be tested empirically : first, the higher the interest-rate under an FRM loan, the more likely households choose ARM loans because it becomes increasingly attractive to bet on a lower interest-rate in the future. Second, more risk averse households are more likely to select FRM loans over ARM loans because they tend to shy away from fluctuating consumption paths as those implied by repayments schedule under ARM loans. Third, for households with similar wealth, those with a faster income growth are more likely to choose FRM loans because a higher share of their discounted lifetime income is then subject to uncertainty. Fourth, households with enough wealth are more likely to select adjustable-rate loans over fixed-rate loans because they enjoy similar advantages as investing in a risky asset while being able to self-insure against interest-rates increases.

Contrary to the standard practice of banks to ignore the future stream of income of the borrower when lending, this paper suggests that it actually matters for the mortgage choice. In practice, borrowers have a private information over their future incomes, hence taking them into account will create a moral hazard problem. An avenue for future research will be to characterize the incentive-compatible mortgage contract that accounts when the bank account for future incomes.

Chapitre 2

Households' choice between fixed- and adjustable-rate mortgages : An Empirical Analysis

2.1 Introduction

In this paper, we follow the findings in Chapter 1 in order to identify the determinants of the mortgage choice. By choosing an adjustable-rate mortgage (ARM), households speculate on the future interest-rate, and take a risk that is absent from a fixed-rate mortgage (FRM). Heuristically, this binary choice parallels a discrete portfolio problem of choosing between a risk-free and a risky asset with the important feature that the borrower can self-insure against the interest-rate risk. We showed that : first, the higher the interest-rate under an FRM loan, the more likely households choose ARM loans because it becomes increasingly attractive to bet on a lower interest-rate in the future. Second, more risk averse households are more likely to select FRM loans over ARM loans because they tend to shy away from fluctuating consumption paths as those implied by repayments schedule under ARM loans. Third, for households with similar wealth, those with a faster income growth a more likely to choose FRM loans because a higher share of their discounted lifetime income is then subject to uncertainty. Fourth, households with enough wealth are more likely to select adjustable-rate loans over fixed-rate loans because they enjoy similar advantages as investing in a risky asset while being able to self-insure against interest-rates increases.

The role of borrower characteristics in predicting mortgage instruments selection has generated numerous interests within the empirical literature. At the heart of the debate, two views have been considered : whether the selection is primarily motivated by aggregate factors, such as the affordability of the loan (Dhillon, Shilling and Sirmans (1987), Sa-Aadu and Sirmans (1995), and Breslaw, Irvine and Rahman (1996)), or by borrower idiosyncracies, such as the degree of risk aversion or the income stream (Basel and Biger (1980), and Statman (1982)). Something that has been lacking is an integrated theory of the mortgage choice to rely upon for the empirical work. This paper attempts to fill this void.

We take advantage of unique supplements in the 1996 survey of the Panel Study of Income Dynamics (PSID) to address these effects.[1] Most of these variables appear in the existing literature. But, the difficulty in pinning down their effects on the mortgage decision is due, in part, to the lack of publicly available data set on mortgage loans; or alternatively to the lack of appropriate proxies for variables such as risk aversion.[2] One convenience with using households survey data is that self-reported answers are available for many aspects of the household behavior.[3] For instance, in order to palliate the absence of information on households balance sheets, we rely on an indirect measure to classify households according to their wealth level. Homeowners are said to have low level of wealth if they belong to the group of liquidity-constrained households, based on self-reported aspects of their credit status. Households in this group are younger, belong to larger families, earn lower annual incomes and own housing that worth less than their unconstrained counterpart.

We find strong evidence that the spread between FRM and interest-rates, as well as borrowers' level of wealth, influence the mortgage decision. However, the motivation for more affordable loan seems to govern the decision because the probability to choose an adjustable-rate loan is higher for less liquid borrowers. We also find that risk considerations matter to the mortgage choice decision. In fact, households with a higher risk aversion choose the fixed-rate mortgage with a higher probability. Surprisingly, households that choose government insurance programs are more likely to choose FRM loans. Once we account for those variables, life-cycle variables have little explanatory power.

The rest of the paper is organized as follows : we summarize the relevant empirical literature in Section 2. We conduct a general description of the 1996 PSID, as well as the sample selec-

[1] Hurst and Stafford (2004) exploit this data set as well, but their focus is not on the choice between adjustable-rate and fixed-rate mortgage loans.

[2] For competitive reasons banks and other financial institutions refuse to disclosed data drawn from their pool of mortgage-applicants.

[3] The caveat, compared to data sets from financial institutions, is that reported amounts are often subject to measurement errors, specially for the reported mortgage transactions.

tion in Section 3. Section 4 provides further description of the working sample along the lines of theoretically-motivated variables. In Sections 5 and 6, we present the empirical methodology and the results, respectively.

2.2 Literature review

Brueckner and Follain (1988) pioneered a methodology to estimate the role of borrowers' characteristics in the mortgage choice decision. Using a mortgage-pool data drawn from the Residential Mortgage Finance Database, they estimate a binomial probit to determine the role of borrower risk aversion, income-path, mobility as well as the level of interest rate and the mortgage affordability. The key in Brueckner and Follain's methodology is to predict the interest rate of rejected mortgage-instruments, say a FRM loan for a borrower who chooses an ARM loan, by using a subsample of a FRM loan after correcting for selectivity bias. They found no significant evidence of the role of borrowers' characteristics. According to their research, only the interest-rate differential between FRM and ARM matters. However, except for rates of interest, Brueckner and Follain have no choice but to rely on rather imperfect proxies to capture borrower characteristics available in their data set. For instance, the "presence of children in the household" serves as an operational measure for the effect of risk aversion, discount rate and strength of the demand for housing. Clearly, it may capture effects that have nothing to do with the mortgage demand as well.

Capone and Cunningham (1992) recognize that the selection of a mortgage-instrument cannot be separated from the likelihood to default on the loan, the idea being that borrowers make their mortgage decision using information about their expected mortgage tenure and their aversion to risk. Basically, they extend Brueckner and Follain (1988) by including indices that account for the mortgage termination, and they consider several proxies for risk aversion. Data for this study are drawn from origination files of the North American Mortgages Corporation, and comprise only single-family property in Texas. They find that : for any given tenure, borrowers who choose ARM contracts have lower degrees of risk aversion. The sample studied, however, belongs to a period of high and volatile interest rates, something they could not control for because ARM loans were not widely available prior to 1981.

Sa-Aadu and Sirmans (1995) treat mortgage instruments as differentiated products along the degree of exposure to risk that they offer. Using the pool of borrowers from a federally chartered savings association, they estimate a multinomial model along the five alternative contracts. Since all mortgages are originated by the same institution, Sa-Aadu and Sirmans find rates of rejected contracts by picking rates in the sample of contracts originated at a similar date. They conclude that borrower characteristics influence the demand for mortgage. More specifically, they find that borrowers who are more mobile, young or expecting their income to rise are more likely to select ARM loans. Given that samples of applicants drawn from financial institutions are usually subject to systematic truncation, Sa-Aadu and Sirmans's matching technic leaves their estimates vulnerable to important selectivity bias.

Dhillon, Sa-Aadu and Shilling (1996) extend previous studies by relying solely on commercial mortgages instead of residential mortgages, in order to check whether borrower characteristics matter. They reproduce Brueckner and Follain's study on a pool of commercial mortgages originated by a life insurance company. They found that, contrary to individual borrowers, income stream generated by project of commercial borrowers have little effect on their mortgage selection. However, Dhillon, *et al.* fail to address the fact that a potential effect is likely to show up between the length period of the mortgage and companies projects duration. In fact, as it is the case for the maturity choice of debt in a corporate environment, firms likely match long-term projects with long-term debt, and vice versa for short-term projects.

Few papers address this question on non-U.S. data. Breslaw, Irvine and Rahman (1996) use a large database of new loans compiled by the Canada Mortgage and Housing Corporation (CMHC), a mortgage insurance company. To some extent the set of mortgage instruments in Canada is less complex than in the U.S., that is, being closer to a binary decision between ARM and FRM. Breslaw, Irvine and Rahman find no role for borrower idiosyncracies. In particular they conclude that income has more influence on the purchase decision than on the type of mortgage instrument selected. But it seems that policy-related incentives from a clear housing policy in Canada, that include almost systematic resort to insurance from the CMHC, make the mortgage demand fairly homogenous. One example is a very low dispersion of the gross debt service to income ratio among sampled households. Leece (2000) uses the British Household Panel Survey (BHPS) to study the mortgage choice decision. Unlike the version of the PSID that we use in this paper, the BHPS do not asked surveyed households the type of mortgage instrument they carry. Leece relies on reported mortgage payments and interest rates to deduct whether the loan is an ARM or a FRM. Surprisingly, he finds no evidence of a systematic response to rates differential between fixed-rate and adjustable-rate mortgages. But, his study is vulnerable to important classification errors.

2.3 Data

2.3.1 Background on U.S. mortgage market

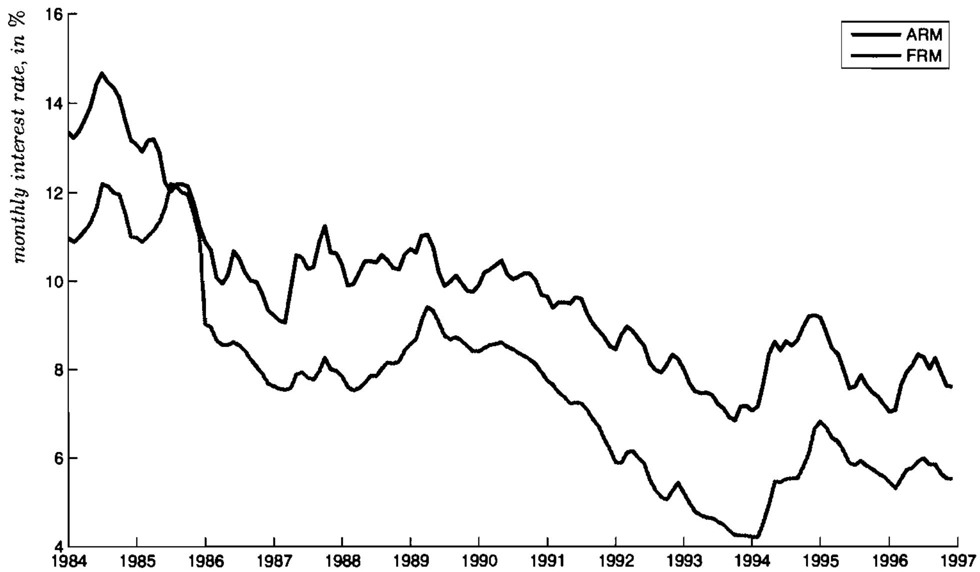
The typical mortgage instrument in the U.S. is the fixed-rate mortgage (FRM) with an amortization period of thirty years. However, a fifteen years FRM, as well as several alternative adjustable-rate mortgages (ARMs) also exist. ARM loans started to be supplied widely after an important deregulation for federally chartered institutions in 1981. ARMs are differentiated by the frequency of interest-rate adjustment to a pre-specified index, ranging from six months to 5 years. Four indices dominate the market : the one year constant maturity Treasury yield, the one year LIBOR, the Eleventh District Cost-of-Funds Index (EDCOFI), and the Federal Housing Finance Board (FHFB) national average contract interest rate (Stanton and Wallace (1999)). Depending on the period, the most popular oscillates between the 1-year, the 3-year and the 5-year ARM ; in 1996, for instance, 1-year ARMs were the most prevalent (Freddie Mac (1996)).

Since lending institutions rely on bond market to finance their mortgage loans, the spread between FRM interest-rates and ARM interest-rates (hereafter *FRM-ARM rate differential*) is usually positive.[4] In Figure 1, we represent the monthly rates between 1984 and 1996 for the 1-year ARM indexed to the Treasury yield and the 30-year FRM, respectively. Understandably, rates increase with the fixity of term. For instance, in 1996 the FRM-ARM rate differential were 1.97%, 0.91% and 0.54%, for 1-year, 3-year and 5-year ARMs, respectively, based on the 1-year or 3-year constant maturity Treasury yield (Freddie Mac (1996)).[5]

Another important aspect of the U.S. mortgage market is the mortgage-insurance system. Households mostly have access to three means to insure their mortgages against default risk, which is defined as the risk of loss that could occur if the borrower fails to make a payment on the loan (see Brueggeman and Fisher (2005)). The first means is the conventional insurance, it is offered by private insurers and which covers the lender for only a portion of the loan. Regulations require that all conventional mortgages with an originating down payment lower than 20% must secure private insurance. Hence, the interest-rate charge on this type of loan might be higher than the rate on uninsured conventional mortgage. The second means works through Federal Housing Administration (FHA) insurance programs, which unlike conventional insurance fully insure the lender.

[4] Except if the yield curve is inverted.

[5] While the FRM rate corresponds to the average 30-year FRM commitment rate of 7.67 percent from Freddie Macs Primary Mortgage Market Survey.



Source: Primary Mortgage Market Survey, Freddie Mac

FIG. 2.1 – 30-year FRM vs. 1-year ARM

This program was initially created to facilitates access to homeownership. However, there is loan maximums on FHA insured mortgages, so that households must turn to conventional insurance to obtained a higher loan or to avoid making a big payment up front for down payment. The third mortgage-insurance is the Veteran Affairs (VA) guarantee programs which target veterans or their spouses. Unlike FHA programs, VA programs provide loan guarantees instead of default insurance.

2.3.2 Description and selection

The data for our study are based on a sample of American households extracted from the Panel of Study of Income Dynamics (PSID). Since 1968, this database has been gathered each year by the Survey of Research Center of the University of Michigan. The survey contains detailed information on earnings, income, demographic and mortgage-related characteristics. In particular, homeowners were asked to indicate the year they obtain the loan, whether it is original or refinanced, the current interest-rate (r), the outstanding loan balance (B), the number of years remaining on the mortgage (n) and the annual mortgage payment, i.e. payment of interest and principal (A). For one year, and for only one year, questions about homeowners' choice of mortgage instrument were included in the annual survey. For 1996, the survey asked mortgage borrowers to indicate whether the interest rates on the loan were fixed or adjustable. From the initial 8,511 households in the survey, 2,837 held a mortgage loan. Our working sample consists on 1,792 households. Among them, a total of 1,427 households answered "Fixed" to the question : "Is the interest rate on that loan fixed or variable (adjustable) ?" ; and 365 answered "variable". A proportion of 79.63% held a FRM contract.

A variety of different criteria led to the exclusion of many observations in the database. First, we exclude data for which the mortgage instrument selected is neither an ARM nor a FRM (59 homeowners). Second, we drop all observations for which the mortgage were obtained prior to 1981, because ARM contracts were not widely offered (278 homeowners). Third, we exclude observations for which the current interest-rate (296 homeowners) or the year the mortgage loan were obtained (24 homeowners) are not reported. Fourth, we also exclude from the working sample, observations for which the consistency of the mortgage transaction (as defined below) could not be performed; mostly due to missing values for property taxes, homeowner insurance premium, outstanding loan balance, mortgage payment or number of years remaining on the mortgage. At this stage, we are left with 2074 observations. Finally, we exclude households from the analysis if the mortgage interest rate (r_m) implied from the recorded values of A, B and n lays outside the range 2% to 27% (as in Quigley (1987)), or is inconsistent with the recorded interest-rate r. The implied mortgage interest-

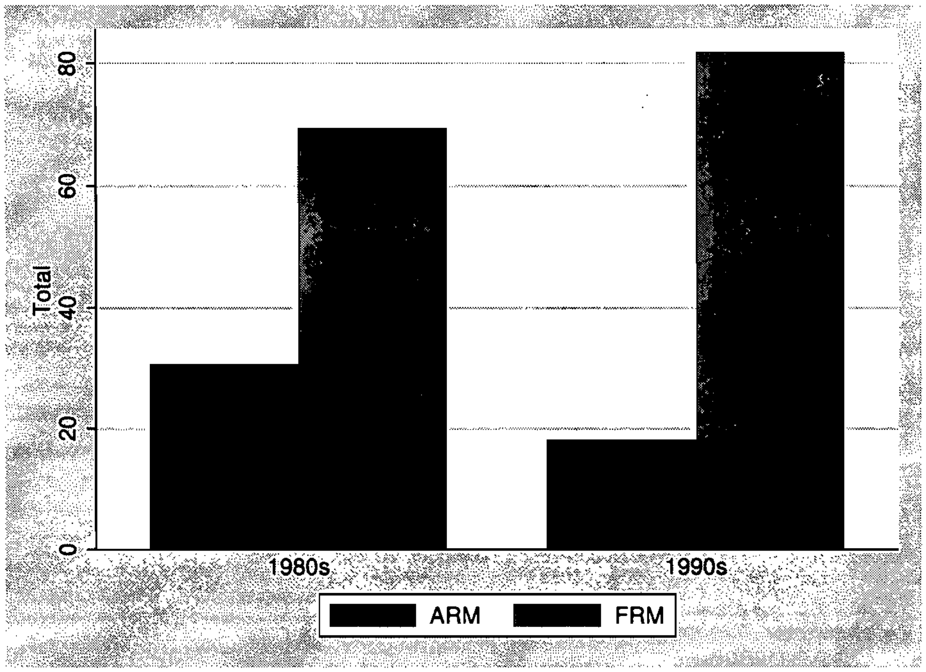


FIG. 2.2 – Mortgage loans originating in the 1980s and 1990s, by type (percentage)

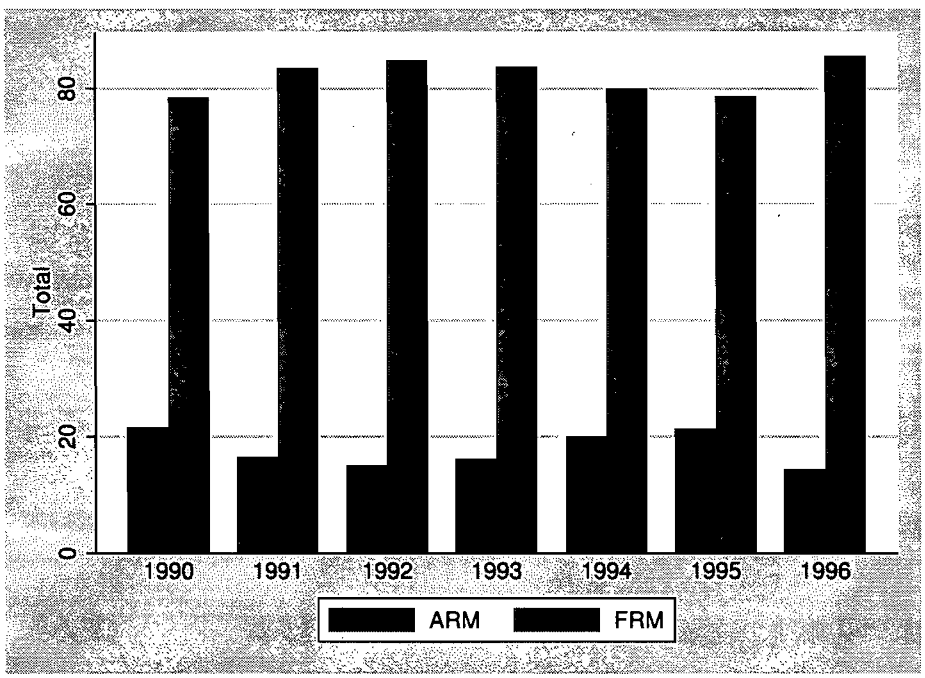


FIG. 2.3 – Mortgage loans originating in the 1990s, by type (percentage)

TAB. 2.1 – Mortgage selection for various sub-samples (total)

Variables	ARM (1)	FRM (2)
All	365	1,427
Original	262	825
Originated after 1995	76	308
Original and originated after 1995	60	184

NOTES : "Original" stands for original loan and terms (not refinanced); while "Originated after 1995" corresponds to mortgage loan originated in 1995 or in 1996. Data definitions and sources for all variables are reported in the Appendix.

rate is defined as the value of r_m that satisfies :

$$B - \sum_{i=1}^{n} \frac{A}{(1+r_m)^i} = 0.$$[6]

We are left with 1,792 homeowners. The vast majority of these mortgage loans are meant to finance one-family houses (around 91%), and are worth on average $137,114. From Figures 2 and 3, a bulk of these mortgage loans were obtained in the 1990s (1,481 loans), with the FRMs being consistently the dominant instruments. In columns 1 and 2 of Table 1, we also report the total number of observations for original mortgage loans and recent loans origination. More than half of the sample is composed of original loans (1,104 observations), among which 384 were obtained after 1995.

We report additional description of the working sample. In Table 2, we break up the mortgage choice along various sociodemographic characteristics of the head of the borrowing household. The

[6]The difference between the contract rate and the implied rate may come from fees and charges. We use the command "**annurate**" in Matlab to get the implied mortgage interest-rate.

TAB. 2.2 – Mortgage selection and sociodemographic characteristics (proportion)

Variables	ARM (1)	FRM (2)	ARM+FRM (3)
Sex			
Male	20.23	79.77	87.44
Female	21.33	78.67	12.56
Race			
Black	17.52	82.48	17.52
White	21.05	78.95	78.74
Other	19.4	80.6	3.74
Marital status			
Married	20.06	79.94	78.74
Divorced	22.17	77.83	11.33
Age (years old)			
less than 35	21.13	78.87	21.65
35 to 45	20.52	79.48	42.97
over 45	19.71	80.29	35.38
Education (years)			
less than 13	19.13	80.87	46.37
13 to 16	19.42	80.68	38.78
over 16	26.69	73.31	14.84

NOTE : Data definitions and sources for all variables are reported in the Appendix.

majority of them are male (87.44%) under 45 years old, white, married and with less than 13 years of schooling completed. The mortgage selection among sex groups is similar to the sample proportion; roughly, 20% choose ARM loans. However, a slightly bigger proportion of divorced heads chose adjustable-rate loans compared to both married heads and the sample average. In terms of age and education, it stands out that older people (over 45 years old) hold a relatively bigger proportion of FRMs; while the highest fraction of ARM contracts is found among the more educated heads (26.69%). At the other extreme, households with a black head hold the lowest fraction of adjustable-rate loans (17.52%).

In Table 3, we compute mean sample averages along family size, age and education of the head, gross income of the household, house value, outstanding debt and time at the current job. In terms of age, family size house value and time at the current job, the difference between mortgage types is not flagrant. The difference in average outstanding debt probably follows from the fact FRM loans have a longer term in general. However, people who seek ARM loans tend to be more educated (around 16 years of schooling completed) and have slightly higher earnings, a higher fraction of this earning comes from financial assets (7.4%) when compared to borrowers with a fixed-rate loan.[7] A finer look is devoted to the mortgage selection within the income distribution (unweighted). We show the proportion of mortgage instruments by quintile of income; the cutoff levels are \$37,098, \$51,043, \$66,497, \$92,995, respectively. The highest fractions of ARM contracts appear in the fifth (22.07%) and first (21.94%) quintiles, respectively.

In Table 4, we successively consider the sub-samples of original mortgage loans, and those originated after 1995. Loans that have been not refinanced yet, represent 60.7% of the sample, with 24.1% of them being adjustable-rate mortgages. Borrowers in this sample present below-average values in all categories (column 3). Recently originated loans represent only 21.43% of the sample, with 19.79% of them being adjustable-rate mortgages. Furthermore, except for education, outstanding debt and fraction of income coming out of financial assets, all other variables present below-average values as well. It is not surprising that heads of those households are younger, and have spent less time in their current job. When the distinction between mortgage types is made, borrowers in this group depart from the sample averages in many ways. However, they represent less than 5% of the working sample.

Finally, we look at mortgage selection along the main occupation of the household's head (Table 5). In the same table we also consider the sub-samples of original mortgage loans, and recently

[7]Only 352 households earn revenues from financial assets. Part of this low number can be explained by the "participation puzzle", that is, the stylized fact that the vast majority of households in most countries hold only safe assets, despite positive excess returns expected on risky assets (Guiso *et al.* (2001)).

TAB. 2.3 – Mortgage selection and sociodemographic characteristics, all (average)

Variables	ARM (1)	FRM (2)	ARM+FRM (3)
Age	42.13	42.03	42.05
Family size	3.20	3.23	3.22
Education	16.14	15.17	15.37
Income	72,717.2	71,008.8	71,356.8
from financial assets (% of income)	7.4	3.8	4.54
quintiles			
lowest	21.94	78.06	
second	21.0	79.0	
third	19.22	80.78	
fourth	17.6	82.4	
highest	22.07	77.93	
House value	136,852.18	137,182.06	137,114.86
Outstanding debt	77,305.0	85,772.51	79,029.7
Time in current job	10.1	10.98	10.81

NOTES : Data definitions and sources for all variables are reported in the Appendix. All dollar amounts are 1996 dollars. Only 352 households earn revenues from financial assets.

TAB. 2.4 – Mortgage selection and sociodemographic characteristics, by sub-samples

Variables	ARM (1)	FRM (2)	ARM+FRM (3)
Original	24.1	75.9	60.67
Age	40.99	39.95	40.2
Family size	3.19	3.18	3.19
Education	15.94	15.17	15.36
Income	67,877.91	62,083.63	63,480.23
from financial assets (% of income)	5.95	3.21	3.83
House value	129,443.44	126,099.75	126,905.68
Outstanding debt	84,525.58	74,365.75	76,814.58
Time in current job	9.69	9.73	9.72
Originated after 1995	19.79	80.21	21.43
Age	37.01	40.35	39.69
Family size	3.14	3.08	3.09
Education	16.5	15.43	15.64
Income	77,378.78	67,166.96	69,188.052
from financial assets (% of income)	4.3	4.87	4.76
House value	148,972.37	122,379.22	127,642.45
Outstanding debt	111,051.57	83,229.08	88,735.61
Time in current job	7.46	9.15	8.81

NOTES : Data definitions and sources for all variables are reported in the Appendix. All dollar amounts are 1996 dollars. Only 352 households earn revenues from financial assets.

TAB. 2.5 – Mortgage selection by occupation (proportion)

Variables	ARM (1)	FRM (2)	ARM+FRM (3)
All			
Professional, Technical	20.9	79.1	23.21
Managers and Administrators	21.07	78.93	20.93
Sales Workers	16.67	83.33	5.4
Clerical	23.58	76.42	6.86
Craftsman	18.77	81.23	17.24
Operatives and Transport	20.18	79.82	6.1
Original			
Professional, Technical	26.9	73.1	22.91
Managers and Administrators	25.25	74.75	18.58
Sales Workers	21.57	78.43	4.69
Clerical	25	75	7.36
Craftsman	23.23	76.77	18.22
Operatives and Transport	20.55	79.45	6.72
Originated after 1995			
Professional, Technical	25	75	18.75
Managers and Administrators	24	76	19.53
Sales Workers	26.1	73.9	6.0
Clerical	26.67	73.33	7.81
Craftsman	16.18	83.82	17.71
Operatives and Transport	16	84	6.5

NOTE : Data definitions and sources for all variables are reported in the Appendix.

originated loans. The most represented occupations are professional and technical workers (23.21%), followed by managers and administrators (20.93%), and craftsman (17.24%). In all three samples of Table 5, it appears that managers and administrators tends to have above-average fractions of ARMs.

2.4 Preliminary analysis

In this section, we introduce the key variables that we will be using to explain the mortgage choice based on the findings in the Chapter 1. We first consider the interest-rate differential between the two type of mortgages as proxied by the FRM-ARM rate differential. We use this measure to verify the conclusion that : the higher the interest-rate under an FRM loan, the more likely households choose ARM loans because it becomes increasingly attractive to bet on a lower interest-rate in the future.

Second, we consider the level of wealth. From the previous chapter : households with enough wealth are more likely to select adjustable-rate loans over fixed-rate loans because they enjoy similar advantages to investing in a risky asset while being able to self-insure against interest-rate increases. Ideally, we would like to classify households by their level of wealth specially prior to the mortgage acquisition. However, the 1996 PSID does not record households' balance sheets. As an operational proxy we rely instead on borrower credit history. Our reasoning is that among mortgage borrowers in the sample, liquidity-constrained borrowers were the ones with less liquidity at the time they sought their mortgage loan (and even beyond), irrespective of the timing of the binding constraint.[8] We classify as liquidity-constrained households that confessed having had "money problems" since January 1991 (see the Appendix for details). Figure 3 displays the number of cases each year in the sample. They represent 27.73% of the sample, among them 95.0% confessed having "found themselves unable to pay your bills when they were due" (293 households) or "obtained a loan to consolidate or pay off your debts" (171 households).[9]

In Table 6, we compute sociodemographic characteristics of borrowers by credit status and mortgage selection. In line with their credit status, liquidity-constrained borrowers are younger,

[8] The constraint is probably less binding compared to the average household surveyed in the 1996 PSID.

[9] Both are minor credit blunders, but they can rightfully be identified as "default" as well.

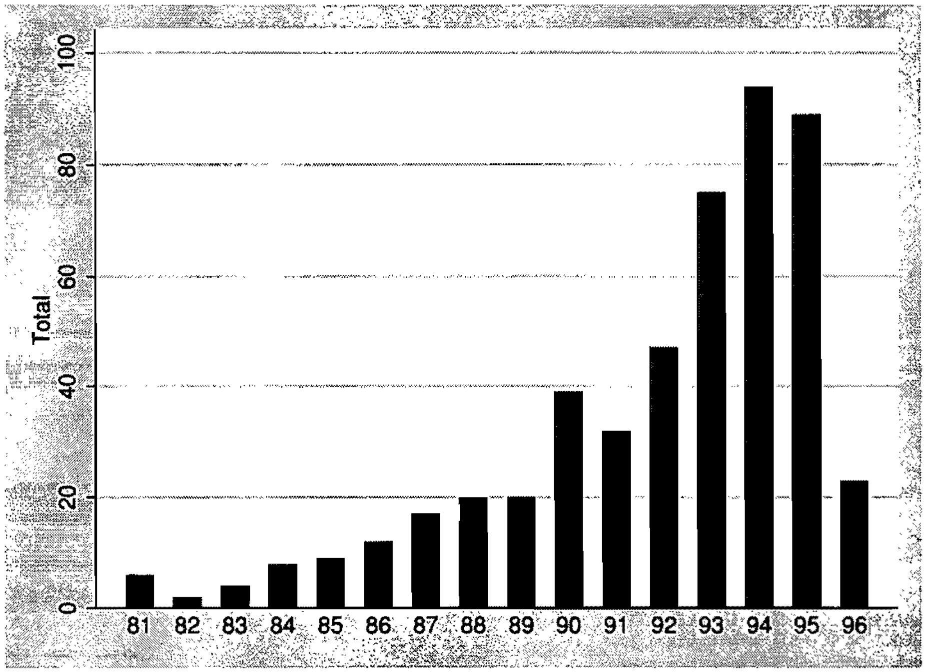


FIG. 2.4 – Mortgage loans for liquidity-constrained borrowers (total)

belong to a larger family, earn a lower annual income and have a house, i.e. real asset, worth less than the unconstrained borrowers.[10] It is in line with the intuition that individuals with a high level of wealth have high income as well. On these grounds, this proxy is appropriate to recount borrowers with less liquidity. Contrary to what our prediction would imply, a bigger fraction of them hold an adjustable-rate loan compared to the group of less constrained. Obviously, we need to control several other variables, especially the interest-rate levels.

Third, we consider risk aversion. More risk averse households are more likely to select FRM loans over ARM loans because they tend to shy away from fluctuating consumption paths as those implied by repayment schedules under ARM loans. A singularity of the 1996 PSID is that questions about households' risk aversion were included in the annual survey. It asked how willing respondents are to

[10]Using a different definition of "liquidity constrained", Jappelli (1990) arrived to a similar set of characteristics for liquidity-constrained households.

TAB. 2.6 – Mortgage selection and sociodemographic characteristics, by liquidity status

Variables	ARM (1)	FRM (2)	ARM+FRM (3)
Unconstrained	18.25	81.75	72.77
Age	43.23	42.36	42.5
Family size	3.12	3.17	3.16
Education	15.94	15.40	15.5
Risk tolerance	.307	.276	.281
Income	80,509.9	74,267.0	75,406.4
from financial assets (% of income)	8.42	3.92	4.72
House value	145,366.58	147,540.14	147,143.4
Outstanding debt	87,391.9	80,728.7	81,944.82
Time in current job	10.6	11.48	11.32
Constrained	26.0	74.0	27.23
Age	40.08	41.07	40.95
Family size	3.34	3.41	3.38
Education	16.5	14.5	15.2
Risk tolerance	.306	.293	.296
Income	58,113.45	67,195.22	60,276.3
from financial assets (% of income)	4.44	3.27	3.67
House value	120,896.06	106,595.57	110,232
Outstanding debt	82,737.8	67,195.2	71,131.34
Time in current job	9.16	9.5	9.41

NOTES : Data definitions and sources for all variables are reported in the Appendix. All dollar amounts are 1996 dollars. Only 352 households earn revenues from financial assets.

switch to a job that is "equally good" except for income prospects. Answers to the latter questions were in turn converted into a single, quantitative index of risk tolerance – the reciprocal of relative risk aversion, i.e., $1/\nu$ – by Barsky, Juster, Kimball and Shapiro (1997). The resulting estimates appear in the 1996 Supplemental PSID data under the labels RT and $RT1$ (see Appendix A.3 for details). We rely on this variable to proxy risk aversion.

From Table 6, borrowers with ARM contracts have higher risk tolerance, irrespective of their credit status. It is important to stress that other proxies for risk aversion based on households' reported characteristics –such as presence of children in the household– may capture effects that have little to do with aversion to risk. Therefore, difficulties in pinning down the effect of risk aversion can be circumvented by drawing upon a self-reported indicator of the behavior towards a particular risk. However, even this measure has some caveats : there is no guaranty that people's responses to income-risk situations translate into similar behavior towards interest-rate risk. It should also be pointed out that this measure limits the sample to households in the labor force.

Finally, we consider household income growth. In this respect we showed that : for households with similar wealth, those with a faster income growth are more likely to choose FRM loans because a higher share of their discounted lifetime income is then subject to uncertainty. Our proxy for income growth is education. In fact, vast literature in labor economics has established that higher schooling signals higher future earnings.

We need to control for borrower's age, family size, geographic region, race, sex, as well as insurance program because all these variable are indirect screening variables at different stages of the mortgage application and thus influence the mortgage choice. We also control for borrower's mobility. In Table 7, we break up mortgage choice in terms of borrower's likelihood to move. People were asked to answer the question "Do you think you might move in the next couple of years ?" A

TAB. 2.7 – Mortgage selection and sociodemographic characteristics, by mobility status

Variables	ARM (1)	FRM (2)	ARM+FRM (3)
Will not move	20	80	77.0
Age	42.82	42.46	42.54
Family size	3.22	3.26	3.25
Education	16.64	15.30	15.57
Risk tolerance	.288	.271	.275
Income	68,718.2	68,074.4	68,203.2
from financial assets (% of income)	8.61	3.97	4.78
House value	126,833.5	130,280.0	129,590.8
Outstanding debt	78363.5	76,360.2	76,760.8
Time in current job	10.14	11.30	11.07
Will move (definitely, probably or uncertain)	21.6	78.4	23
Age	39.97	40.57	40.47
Family size	3.11	3.15	3.14
Education	14.61	14.73	14.7
Risk tolerance	.359	.309	.321
Income	85,118.4	81,038.4	81,919.8
from financial assets (% of income)	5.23	3.47	3.92
House value	167,921.3	160,772.7	162,317.0
Outstanding debt	108,748.8	80,534.54	86,629.4
Time in current job	9.95	9.89	9.9

NOTES : Data definitions and sources for all variables are reported in the Appendix. All dollar amounts are 1996 dollars.

proportion of 23% did not exclude the idea of moving; they are younger, have less time in their current job, a smaller family, than those who planned not to move. What stands out is that they are less educated, have higher earnings, higher valued housing and are more risk tolerant. Among more mobile borrowers, 21.6% chose adjustable-rate loans, which is more than their counterparts.

2.5 Empirical methodology

According to our theoretical model, a household's choice of mortgage is a function of its initial net-worth, its degree of aversion to risk and its income growth. A more complete model would include controls for life cycle factors, contracts mechanics, and credit markets environment as well.

The problem of the utility-maximizing household selecting either an ARM or an FRM is estimated with a binomial probit given by :

$$\Omega_j^* = \beta_0 + \beta_l LC_j + \beta_r RA_j + \beta_e ED_j + \gamma(r_j^f - r_j^a) + \beta_b r_b + \beta_c' C_j + \epsilon_j \tag{2.1}$$

where Ω_j^* is an unobservable index indicating the propensity to choose an ARM over an FRM loan for household j ; LC equals 1 if the borrower is liquidity constrained (i.e., had low initial net worth) and zero otherwise; RA denotes the inverse of the risk tolerance; ED is education; r_j^f and r_j^a are the contract rates on the FRM and the ARM, respectively; r_b is a benchmark interest-rate to control for the level of mortgage cost (see Appendix 2.2 for a description of r_b); C_j denotes a vector of control variables described above that are meant to capture other determinants of the mortgage choice; βs and γ are coefficient vectors (the expected signs are $\beta_l > 0$, $\beta_r < 0$, $\beta_l < 0$, and $\gamma > 0$); and ϵ_j is a standard normal error term. The observable variable is Ω_j which takes the value 1 if $\Omega_j^* > 0$, i.e. household j chooses an ARM contract, and zero otherwise.

The interest-rate differential between FRM and ARM in $Eq.(1)$ is unknown. For household k, for instance, which answered "fixed" to the question "Is the interest rate on that loan fixed or variable (adjustable) ?", only the interest-rate of the selected contract (i.e. r_k^f) is reported in the 1996 PSID; while the interest-rate r_k^a of the rejected ARM contract is unobservable. Following Brueckner and Follain (1988), we predict r_k^a from the ARM sub-sample. We proceed in the same manner for rejected FRM rates. OLS estimation would produce inconsistent coefficients because the sub-samples that we use to estimate interest-rate comprise only borrowers who find those rates attractive. To address this issue, we ran the Heckman selection procedure, with the decision whether to choose ARM or FRM instruments given by $Eq.(1)$. More specifically, we first regress the interest over set of explanatory variables X on the sub-sample of ARM, and FRM, respectively.

$$r_j^i = \gamma_i' X_j + \nu_i \tag{2.2}$$

where ν_i is the error term, $j = a, f$. Then, the inverse Mills ratios are added as explanatory variables in the ARM and FRM rate equations. Finally, we estimate the following equation :

$$\Omega_j^* = \beta_0 + \beta_l LC_j + \beta_r RA_j + \beta_e ED_j + \gamma(\hat{r}_j^f - \hat{r}_j^a) + \beta_c' C_j + \epsilon_j \tag{1'}$$

where $\hat{r}_j^f$ and $\hat{r}_j^a$ are the predicted FRM and ARM loans rate, respectively.

2.6 Empirical results

In this section, we present and comment on the results of several regressions. Columns 1-6 of Table 8 report the estimation of different variants of $Eq.(1')$; we included controls in every regressions unless we did not report all of them. In Column 2, we show the baseline regression with the correction for the selectivity bias. The comparison with Column 1, which does not correct for the bias, shows that changes are fairly small.

Not surprisingly, the prices of mortgage loans influence the demand for mortgage, that is, the more expensive FRMs are relative to ARMs, the more likely borrowers will choose ARMs. In Column 2, the FRM-ARM differential appears strongly significant as a determinant of the mortgage choice. One can also interpret the rate differential as a measure of expectations over future rates. In this respect, the higher the probability to choose an ARM contracts, the higher is the expected increase in the interest-rate for any general level of mortgage rates.

The proxy for the wealth level is also strongly significant with a t-statistic equal to 3. But, contrary to our predictions the probability to choose an ARM loan is higher for less liquid borrowers. To verify whether this effect plays through the degree of riskiness of the mortgage contracts, we interact the dummy for liquidity-constraint with the interest-rate differential (see Column 5). The interaction term is not significant, which suggests that the wealth level plays through another channel. In this respect, the positive coefficient on LC is in line with Sa-Adu and Sirmans (1995) findings that mortgage affordability has a positive influence on the choice of ARM loans. The underlining assumption is that ARM loans are "cheaper". This finding is robust to the introduction of controls for the period of origination, the borrower's income or the ratio of mortgage payment to income.

We also control for government programs that facilitate access to homeownership for some categories of the population. The coefficient for the dummy of government insured loans is significant and negative. As it turns out, the propensity to select an ARM loan increases for homeowners that reject government insurance programs. This finding is true despite the fact that government programs are available irrespective of the type of instruments selected. It is a little puzzling since

TAB. 2.8 – PROBIT ESTIMATES OF THE MORTGAGE CHOICE

SELECTIVITY CORRECTION	NO		YES				
			Original	95-96		*FHA/VA*	
Variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	-0.60	-0.57	-0.92	-0.57	-0.58	-0.58	-52.18
	(-1.85)	(-1.48)	(-2.53)	(-1.35)	(-1.81)	(-1.79)	(-0.96)
Proxy for wealth level (LC)	0.26	0.25	0.34	0.26	0.30	0.26	0.214
	(3.09)	2.76)	(3.14)	(2.77)	(2.92)	(2.99)	(2.12)
Risk aversion (RA)	-0.03	-0.036	-0.04	-0.036	-0.03	-0.036	-0.036
	(-1.97)	(-1.96)	(-1.76)	(-1.98)	(-2.01)	(-1.92)	(-1.95)
Proxy for income growth (ED)	-0.006	0.004	0.008	0.004	0.003	0.004	0.002
	(1.45)	(0.97)	(1.59)	(0.93)	(0.94)	(0.98)	(0.64)
FRM-ARM *rate differential* ($r^f - r^a$)	0.368	0.43	0.27	0.377	0.41	0.37	0.570
	(3.55)	(2.76)	(2.12)	(2.14)	(3.54)	(3.62)	(2.46)
Benchmark interest-rate	-0.023	-0.02	-0.17	-0.021	-0.022	-0.076	-0.005
	(-0.66)	(-0.51)	(-0.38)	(-0.51)	(-0.54)	(-0.89)	(-0.10)
Dummy for mobility	0.04	0.044	-0.026	0.045	0.04	0.046	0.044
	(1.07)	(1.03)	(-0.48)	(1.06)	(1.04)	(1.09)	(1.06)
Dummy for goverment insured	-0.25	-0.25	-0.32	-0.26	-0.25	–	-0.248
	(-2.88)	(-2.86)	(-3.16)	(-2.93)	(-2.93)		(-2.80)
LC x ($r^f - r^a$)	–	–	–	–	-0.10	–	–
					(-0.74)		
FHA	–	–	–	–	–	-0.41	–
						(-2.45)	
VA	–	–	–	–	–	-0.18	–
						(-1.77)	
Dummy recent issues	–	–	–	0.004	–	–	–
				(0.02)			
Yeargot	–	–	–	–	–	–	0.026
							(0.95)
Age	0.0007	0.0004	0.007	0.0004	0.0003	0.0009	0.0007
	(0.16)	(0.1)	(1.23)	(0.08)	(0.07)	(0.19)	(0.15)
Marital status	0.004	0.01	-0.008	0.018	0.020	0.024	0.026
	(0.09)	(0.38)	(-0.14)	(0.38)	(0.41)	(0.51)	(0.55)
Sex	-.020	-0.06	-0.067	-0.058	-0.053	-0.10	-0.072
	(0.13)	(-0.41)	(-0.38)	(-0.38)	(-0.35)	(-0.63)	(-0.47)
j	14	14	13	15	14	15	15
$\chi^2(j)$	79.26	79.83	44.67	79.57	68.51	79.23	80.49
Number of observations	1503	1503	906	1503	1503	1503	1503

NOTES : The dependent variable takes the value 1 if the household chooses an ARM loan, zero otherwise. In a sample of 1,792 households, estimates of the risk tolerance where available for 1,503 households. Those estimates are computed for households in labor force only. All regressions includes as demographics and regional controls. Data definition and sources for all variables are reported in the Appendix. t-statistics are in parenthesis.

people that used government programs are motivated by mortgage affordability without necessarily being liquidity-constrained according to the definition above. In Column 6, we break up the dummy for government insured into one dummy that takes the value 1 if the insurance is with the FHA, and zero otherwise, and one dummy that takes the value 1 if the insurance is with the VA, and zero otherwise. It still stands that the propensity to select an FRM loan is higher for households that select government insurance programs.

The risk aversion measure has predictive power for mortgage choice with a t-statistic of -1.98, and the correct sign. The probability to choose an adjustable-rate mortgage decreases with the degree of risk aversion. This measure seems to capture attitude towards financial risks, at least for the sub-sample of homeowners in the labor force. Although not reported, we did not have the same success with proxies such as the age of the youngest child or the number of children in the household.[11] We can give additional support for the relationship between households' attitude towards risk and their mortgage choice, by a re-interpretation of dummy of government insured loan. We speculate that the dummy of government insured loan proxy households have a need for "security". Thus, a positive sign on this coefficient means that the propensity to choose a fixed-rate mortgage is higher for households that need more security.

The proxy for income growth turns out to be insignificant as a determinant of mortgage demand. This lack of explanatory power carries to demographic variables in general, and sex, age or marital status in particular. Other controls such as the dummy for mobility are not significant. We also estimate $Eq.(1)$ on the sub-sample of original and recently originated mortgage loans, respectively. Both the degree of risk aversion and the level of wealth lose significance in recently originated loans. This is due to the fact that mortgage interest-rates are poorly estimated once we consider

[11]"Child" is defined as person under 18 years old.

only sub-samples. When we estimate FRM-ARM rate differentials on the appropriate sub-sample predictive powers of both RA an LC increase, the dummy for government insured programs stays insignificant.

One issue with our analysis is the potential difference in timing between the mortgage selection and the explanatory variables. In Column 4 and Column 5, we control for the recently issued mortgage loans in the baseline regression (by a dummy that takes the value 1 if the mortgage was issued before after 1995, and 0 otherwise), and the year the loan were issued, respectively. In both cases, the control is not significant, while both the sign and the significance of the coefficients of the key variables are pretty much unchanged. However, we observe a rise in the coefficient of the interest-rate differential; it suggests that the weight of the rate differential in explaining the mortgage selection is even more important. In Column 3, we run $Eq.(1)$ on the sub-sample of original loans for additional robustness check. The conclusion is the same.

2.7 Conclusion

In this paper, we follow Chapter 1 findings to identify the determinants of the mortgage choice. More specifically, using the 1996 PSID we try to find the role of mortgage interest-rates, borrowers' wealth level, degree of risk aversion and income growth in determining the mortgage choice. One important feature of the underlying theory is that choosing between an ARM loan and a FRM loan is no different than choosing between a risk-free and a risky asset.

We found that : FRM-ARM rate differentials, borrowers' wealth level and degree of risk aversion are important determinants of the mortgage choice. As it turns out, affordability considerations play a key role in the mortgage choice, that is, less wealthy households are more likely to choose *ARM* loans. Except for the degree of risk aversion few demographics variables influence the choice between an adjustable-rate mortgage and a fixed-rate mortgage. But some of these conclusions do not carry on for recently originated loans. Clearly, a theory of the mortgage choice should include both liquidity constraints and the dynamic feature of the mortgage which allow borrowers to self-insure against the interest-rate risk.

Chapitre 3

On the Maturity Structure of Foreign Debt Contracts in Emerging Countries

3.1 Introduction

Lengthening the maturity structure of external debt in emerging countries is a widespread policy recommendation. Usually, such a recommendation is aimed at preventing volatile capital, in search of highest returns, to flow to emerging economies (Tirole, 2002). As shown in different case studies, short-term capital flows exacerbate financial turmoil in debtor countries by forcing domestic firms to sell off illiquid assets, or local government and international organizations into expensive rescue packages (for instance, South Korea and Malaysia in 1997-98, as discussed by Krugman, 1998 ; and Aguiar and Gopinath, 2005) ; especially when the country is unable to roll-over large volumes of maturing debt. In fact, unlike other developing countries, emerging economies have relatively high shares of short-term external debt. For instance, between 1988-97, approximatively 27% of foreign debt in Asia/Pacific and 21% for Latin America were of maturity of one year or less, compared to 17% for Europe.[1] With both Asia/Pacific and Latin America at the center of the debate on the maturity structure of foreign debt, after they suffer a severe crisis during the same period. So, the question is why do emerging countries borrow short-term ?

In a frictionless world, a debtor country that enters into a multi-period relationship with a foreign investor must be indifferent between signing a long-term contract or a succession of short-term contracts covering the whole relationship. However, if it is unfeasible to sign contracts fully contingent on all states of the nature, then a long-term contract and a succession of short-term contracts are no longer equivalent. This restriction binds when the economy is subject to external shocks, as developing countries usually are. In this case, long-term and short-term contracts differ in the degree that they expose the debtor to those shocks. Since they do not accommodate for

[1] Data are from the Institute of International Finance, it includes debt owed to foreign banks and others. Our calculations are based on average using the composition of foreign debt by region in Rodrik and Velasco (1999). See also Dadush, Dasgupta and Ratha (2000).

contingencies, long-term contracts are risk-free for the debtor; hence the lender bears the risk. Whereas the sequential feature of short-term contracts induces the debtor to take into account all possible contingencies; hence the debtor bears the risk in this case. In this paper, we investigate the idea that a motive for choosing short-term debt contract is to benefit from favorable contingencies, that is, better borrowing conditions in the future.[2]

We opt for a three-period model, where a small open economy must choose how to transfer resources over time, given the uncertainty over the future interest rate; and given available contract types : long-term and short-term. Long-term contracts provide more insurance, but preclude borrowers from profiting from future interest rate decreases. Short term contracts, on the other hand, allow for this possibility, at the cost of less insurance. For a debtor, better borrowing conditions –than what a long-term contract offers– come with a low interest rate, through a positive income effect. Hence, it benefits from interest rate volatility if the gains from low interest rate realizations outweigh the losses from high realizations. The net gain is not zero because the income effect is asymmetric for a risk averse debtor. We model the maturity choice of debt as a trade-off between those contractual options. The purpose of this paper is to understand the reasons that lead a debtor country to prefer one form of contract over another.

The model has three implications. First, a country's short-term indebtedness varies negatively with the interest rate volatility. As the gain from volatility decreases with its magnitude, so does the willingness to choose the short-term debt. Second, a country's short-term indebtedness varies negatively with its growth. Among two countries with different growth rates, the country with the lower rate faces relatively less uncertainty, because it has less resources concentrated in the future. Hence, for a given level of volatility, this country is more willing to choose the short-term debt. Third,

[2] Myers (1977) pioneered the distinction between long-term and short-term debt as one of adjustability to contingencies when it is too costly to sign contracts fully contingent on all states of the nature.

the effect of growth on short-term indebtedness is exacerbated by the interest rate volatility. That is, high volatility increases uncertainty over future resources; hence makes countries less willing to choose the short-term debt. We test these findings on a panel of 31 emerging countries over the period 1988 to 1998. We find that the effect of growth on the maturity choice plays through volatility; and despite the fact that the independent effects of volatility and growth are not confirmed by the data.

The literature on international capital flows considers the question : why do emerging countries borrow short-term despite its associate risk? Roughly, the leading answer is that investors' concerns explain emerging economies' short-term indebtedness (Tirole, 2002).[3] Since emerging markets are perceived as risky, foreign investors select early-maturing contracts when they lend to emerging countries : either to monitor debtors' actions when comes refinancing time, because opportunistic governments are tempted not to implement sound reforms (Jeanne, 2003; and Rodrik and Velasco, 1999); or to quickly recover the proceeds of invested-funds, by charging a high risk premium on long-term debt (Broner, Lorenzoni and Schmukler, 2005). In this paper, we look at borrowers' willingness instead. That country-specific factors (e.g. borrowers' willingness, or policies) may have been important is suggested by the fact that investors' concerns alone cannot explain the maturity composition of capital inflows during periods of lending booms for instance. In fact, lending booms are period of enthusiasm towards emerging economies, characterized by a wave of private capital inflows with no apparent restrictions on their maturity composition (see Gourinchas, Valdès and Landerretche, 1999).

The rest of the paper is organized as follows. Section 2 presents the environment. Section 3 carries out some preliminary analysis. In section 4, we study the link between the maturity choice and the interest rate risk. In section 5, we consider how economic growth affects the maturity structure. In

[3] Following the "original sin" view, emerging market countries are obliged to borrow short-term, and in foreign currency because of their financial fragility and sovereign risk (Eichengreen and Hausmann, 1999).

section 6, we conduct an empirical investigation of the model implications. Conclusions and policy recommendations are presented in section 7.

3.2 A simple contracting game

3.2.1 The setup

We consider a small open economy in a one-good world, where the good is non-storable. The economy is inhabited by a representative consumer, referred to as the debtor country. The country can only borrow or lend through foreign investors, since they have access to the international capital market. Agents live for three periods, $t = 1, 2, 3$.

Debtor country

At each period t it is endowed with Y_t units of the good, where $Y_t > 0$ and deterministic. The country ranks consumption streams according to :

$$\boldsymbol{E} \sum_{t=1}^{3} \beta^{t-1} \log (C_t), \qquad 0 < \beta < 1,$$

where C_t represents period-t consumption, β is the discount factor and $\boldsymbol{E}$ is the expectation operator conditional on information available before any trade takes place.

Foreign investors

Foreign investors are risk-neutral, and offer the country a means to transfer resources over time, through contracts. Competition among investors ensures that terms on which these contracts are offered maximize the country's expected utility, while providing zero-profit in equilibrium. Moreover, foreign investors have "deep pockets."

International capital market

Interest rates are determined in world markets. Denote R_t the (gross) interest rate between periods t and $t+1$ (with $t = 1, 2$), and R is the two-period interest rate between periods 1 and 3. The future interest rate is subject to random disturbances; hence from period 1 standpoint, R_2 is risky. We assume that $R_2 = \bar{R}_2 + \epsilon$, where ϵ is a random variable with zero mean and finite variance σ^2, and $\bar{R}_2$ is the expected value of the future interest rate.

3.2.2 Contracting options

A *short-term contract* is a pair $\langle D_t, D_{t+1} \rangle$, where D_t represents the net transfer from the contracting partner to the country in period t. A *long-term contract* is a triplet $\langle D_1, D_2, D_3 \rangle$.

We assume that long-term contracts, fully contingent to the realization of the interest rate shock, are unfeasible. The lack of a third party (supranational) to settle disputes over contractual matters, is characteristic of international lending to emerging countries. We assume that this limitation makes it impossible to sign contingent contracts *ex-ante*.[4] In addition, contracts that start in period 2 are signed after the market draws the interest rate R_2. Finally, all parties can fully commit to transfer part of future endowments, and only borrowing-lending schemes are considered.

On account of the latter assumptions, the country can reach an agreement with a foreign investor in two ways : (1) by signing one long-term contract at the beginning of period 1 ; or (2) by signing two short-term contracts, at the beginning of period 1 and period 2, successively. The main difference between the two agreements lies in the following : if the country foregoes signing a long-term contract, then it has the option to sign another contract, namely short-term, after the shock is realized. However, the country must decide from the beginning, once-and-for-all, whether or not to exercise this option. We summarize the two options in Figure 1.

Short-term agreement

[4]Despite the fact that whether the relevant contingency occurs or not is *observed* by (and is common knowledge among) the contracting parties.

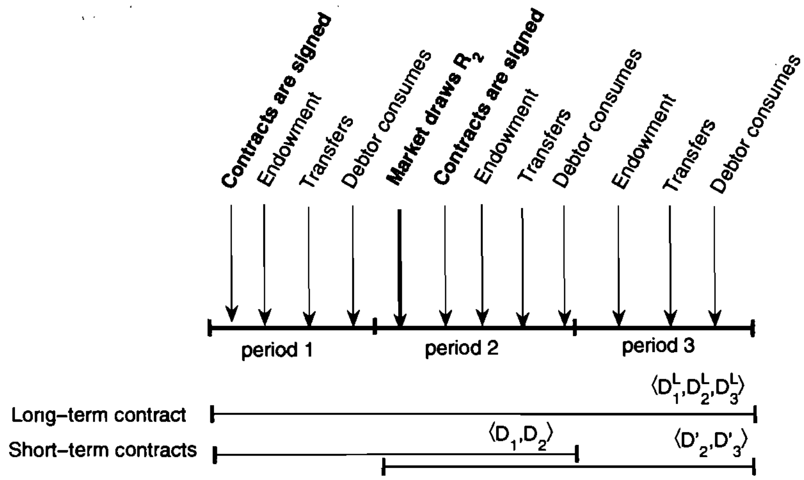


FIG. 3.1 – Timing of the events

The optimal short-term agreement is the solution to the following maximization problem :

$$W^{ST} \equiv \max_{D_1,D_2} \{\log(Y_1 + D_1) + \beta \boldsymbol{E} V(D_2; R_2)\}$$

$$(P^{ST})$$

$$s.t. \qquad D_1 + \frac{D_2}{R_1} = 0,$$

where for each R_2

$$V(D_2; R_2) \equiv \max_{D_2',D_3'} \{\log(Y_2 + D_2 + D_2') + \beta \log(Y_3 + D_3')\}$$

$$s.t. \qquad D_2' + \frac{D_3'}{R_2} = 0.$$

Problem P^{ST} mimics the sequential structure of a short-term agreement (see Figure 1), with V

representing the continuation utility from signing a short-term contract at the beginning of period 2.[5] Each period, the debtor consumes its endowment plus all the transfers received during the ongoing period. For instance, in period 2 the country consumes its endowment Y_2 plus the net sum of last and first transfers it had agreed upon in the first and second short-term contracts, respectively (i.e D_2 and D'_2). Throughout the agreement, the (counterpart) foreign investor faces a zero-profit condition in each short-term contract. Finally, the expectation operator materializes the fact that the second short-term contract is signed after the market draws the interest rate ; while the decision to exercise the option is made before this draw is known to the parties (see Figure 1).

Long-term agreement

The optimal long-term agreement is the solution to the following maximization problem :

$$W^{LT} \equiv \max_{D_1^L, D_2^L, D_3^L} \left\{ \log(Y_1 + D_1^L) + \beta \log(Y_2 + D_2^L) + \beta^2 \log(Y_3 + D_3^L) \right\} \tag{P^{LT}}$$

$$s.t. \qquad D_1^L + \frac{D_2^L}{R_1} + \frac{D_3^L}{R} = 0.$$

Only one zero-profit constraint must be satisfied throughout a long-term agreement, since it comprised of just one (long-term) contract.

The country will choose the short-term agreement over the long-term agreement if its lifetime utility, when it can adjust to the interest rate shock, is greater than or equal to its lifetime utility when it cannot, i.e., $W^{ST} > W^{LT}$. Alternatively, the country will choose the long-term agreement

[5] This overlapping structure of short-term contracts also appears in Rey and Salanié (1990). Rey and Salanié (1990) are concerned with conditions under which multi-period contracts are equivalent to sequences of short-term contracts. They identify three conditions : no private information, surjectivity and conflicting objectives. The comparison is not straightforward in our set up because we assume away fully contingent contracts. However if we allow fully contingent contracts, the condition most likely to be violated is the surjectivity because the debtor country cannot borrow or lend outside of the relationship with the foreign investor (its only outside option is autarky). For more on this literature see Townsend, 1982 ; Malcomson and Spinnewyn, 1988 ; and Fudenberg, Holmström and Milgrom, 1990.

if $W^{ST} < W^{LT}$. When $W^{ST} = W^{LT}$, the country is indifferent between the two agreements.

3.3 Preliminary analysis

In this section, we aim at clarifying what makes an agreement to be preferred to the other. We start by rewriting the long-term problem P^{LT} in a way that facilitates comparison with P^{ST}.

3.3.1 Rewriting the long-term problem

Let us assume that the term structure of interest rates is such that $R = R_1 \bar{R}_2$. This assumption guarantees that investors will be indifferent between either agreement type.[6] Thus, we can rewrite problem P^{LT} as follows :

$$W^{LT} = \max_{D_1, D_2} \{\log(Y_1 + D_1) + \beta V(D_2; \bar{R}_2)\} \qquad (P^{LT\prime})$$

$$s.t. \qquad D_1 + \frac{D_2}{R_1} = 0,$$

where

$$V(D_2; \bar{R}_2) \equiv \max_{D_2', D_3'} \{\log(Y_2 + D_2 + D_2') + \beta \log(Y_3 + D_3')\}$$

$$s.t. \qquad D_2' + \frac{D_3'}{\bar{R}_2} = 0.$$

At the optimum, we go from $P^{LT\prime}$ to P^{LT} by taking $D_1^L = D_1$, $D_2^L = D_2 + D_2'$ and $D_3^L = D_3'$. The virtue of working with $P^{LT\prime}$ instead of P^{LT} is to make the long-term agreement resemble the problem of signing a sequence of short-term contracts. In $P^{LT\prime}$ the last-period interest rate, i.e., $\bar{R}_2$ is known with certainty as though the second short-term contract were signed at the known are

[6] It could be derived from a general-equilibrium model (see Campbell, 1986).

$R_2 = \bar{R}_2$. From now on, we consider problems $P^{LT\prime}$ and P^{ST} alone.

3.3.2 Unfeasibility of contingent contracts

Let us consider the situation when there is no uncertainty, i.e., the distribution of the future interest rate is degenerate. It follows that problem P^{ST} collapses to become $P^{LT\prime}$, making the country indifferent between the two agreements. Basically, in this case contingent contracts would have played no role, just like in a frictionless world.

In general, however, the assumption that contingent contracts are not feasible needs not be innocuous. For instance, consider the situation when signing a long-term agreement brings the same outcome as remained in autarky, i.e., $R_1 = \beta^{-1}$ and $R = \beta^{-2}$. Thus, any contingency R_2, different than the mean $\bar{R}_2$, is a (intertemporal) trading opportunity because it also means that the market interest rate R_2 is different than the discount factor. Therefore, having the option to adjust to ex post realizations of the interest rate is valuable for the country.[7] In this case, the short-term agreement is always preferred to the long-term agreement because the latter is not contingent to interest rate changes.[8]

3.3.3 Country's attitude toward interest rate volatility

Notice that P^{ST} features the expected continuation utility, while $P^{LT\prime}$ features the continuation utility evaluated at the expected interest rate. Thus, the long-term agreement only accounts for the mean of the interest rate shock, while the short-term agreement accounts for the mean and variance

[7] Notice that this result holds regardless of the country's degree of risk aversion, and the size of the risk. It has the same savior as Ruffin's (1974) "non-autarky theorem" of international trade theory, namely : under uncertainty on terms-of-trade, autarky is never optimal regardless of the prospect of world prices.

[8] It follows from the convexity of the maximum operator. A similar result appears in Drèze and Modigliani (1972), "Proposition 2.1 : A temporal uncertain prospect is never preferred to the timeless uncertain prospect described by the same mass or density function, no matter what the consumer's utility function may be." The difference here is that we assume additive utilities over time, but Ahsan (1977) showed that it is as restrictive as Drèze and Modigliani's assumption 3.

of the shock. Heuristically, a short-term agreement is like a long-term agreement plus interest rate risk, without risk both agreements are equivalent.[9]

Denote V_{22} as the second derivative of V with respect to the interest rate. Thus, we expect the country's choice of agreement to vary according to its attitude toward interest rate volatility as measured by V_{22}. If $V_{22} > 0$, then it will choose the short-term agreement; if instead $V_{22} < 0$, then it will choose the long-term agreement. This intuition can be supported using Jensen's inequality. Assume V_{22} is negative over the interval of admissible interest rates and for any D_2. From Jensen's inequality :

$$\boldsymbol{E}V\left(D_2; R_2\right) < V\left(D_2; \bar{R}_2\right). \tag{3.1}$$

Multiplying both sides of the inequality by β and adding up $\log\left(Y_1 - \frac{D_2}{R_1}\right)$ yields :

$$\log\left(Y_1 - \frac{D_2}{R_1}\right) + \beta \boldsymbol{E}V\left(D_2; R_2\right) < \log\left(Y_1 - \frac{D_2}{R_1}\right) + \beta V\left(D_2; \bar{R}_2\right).$$

This last inequality holds for any D_2, therefore it must hold when taking the maximum operator on both sides :

$$\max_{D_2} \log\left(Y_1 - \frac{D_2}{R_1}\right) + \beta \boldsymbol{E}V\left(D_2; R_2\right) < \max_{D_2} \log\left(Y_1 - \frac{D_2}{R_1}\right) + \beta V\left(D_2; \bar{R}_2\right).$$

It follows that :

$$\text{if} \qquad V_{22} < 0 \qquad \text{, then} \qquad W^{LT} > W^{ST}.$$

The opposite can be shown when $V_{22} > 0$, i.e $W^{LT} < W^{ST}$.

To summarize, knowing the curvature of the indirect utility with respect to R_2 is a sufficient

[9] In a different environment "Long term debt is like short term debt plus rollover insurance" (Caballero, 2000).

condition for characterizing the optimal choice of agreement. For this matter, V_{22} needs not keep the same sign over the set of interest rates ; all it requires is an unambiguous comparison between the expected indirect utility and the indirect utility at the expected interest rate (see (1)). From now on we will focus on the properties of V.

3.4 Debt maturity choice and interest rate volatility

In this section, we characterize the optimal choice of agreement with respect to the interest rate risk.

Consider the continuation problem solely. For a given D_2 and $\bar{R}_2$, V is similar to the indirect utility of an agent who receives the income $Y_2 + D_2$ today, Y_3 tomorrow, and must decide how much to borrow given the interest rate $\bar{R}_2$, subject to its intertemporal budget constraint. We want to understand how changes in $\bar{R}_2$ affect the curvature of V. We derive the first-order and envelope conditions of the continuation problem :

$$D_2' : \quad (Y_2 + D_2 + D_2')^{-1} - \beta \bar{R}_2 \left(Y_3 - \bar{R}_2 D_2'\right)^{-1} = 0. \tag{3.2}$$

and

$$V_2(D_2; \bar{R}_2) = -\beta \left(Y_3 + \bar{R}_2 D_2'\right)^{-1} D_2'. \tag{3.3}$$

Combining (2) and the zero-profit constraint, we obtain :

$$D_2' = \frac{y_3 - \beta \bar{R}_2 (Y_2 + D_2)}{\bar{R}_2 \left(1 + \beta\right)}. \tag{3.4}$$

Denote R_2^A as the interest rate such that the country does not obtain (or make) a transfer from a foreign investor, that is $D_2' = 0$; from (2) :

$$R_2^A = \frac{Y_3}{\beta(Y_2 + D_2)}. \tag{3.5}$$

By construction, at the optimum, when R_2^A is strictly lower than the interest rate $\bar{R}_2$, the country lends part of period 2's income in order to consume more last period, i.e., $D_2' < 0$.[10] Conversely, when $R_2^A > \bar{R}_2$ the country borrows to consume more in period 2, i.e., $D_2' > 0$.

Combining (3) and (4) to obtain an expression for V_2, and deriving it with respect to $\bar{R}_2$ (details of calculations appears in the Appendix A.1) we can now state *Lemma* 1.

Lemma 1. *The function V has the following properties :*

(i) There exists a unique R_2^A such that $V_2(D_2; R_2^A) = 0$;

(ii) There exists a unique $R_2^I > 0$ such that $V_{22}(D_2; R_2^I) = 0$;

(iii) When $\bar{R}_2$ is greater than R_2^I, $V(., \bar{R}_2)$ is concave; and convex otherwise;

(iv) R_2^I is greater than R_2^A.

Proof See Appendix A.2

Lemma 1 implies that the indirect utility function will typically resemble the curve depicted in Figure 2. First, the indirect utility decreases when R_2^A is higher than the world interest rate and increases otherwise. Second, the indirect utility function is convex for interest rates lower than R_2^I and concave otherwise.

Based on (3), an increase in the interest rate makes the country better-off as long as $D_2' < 0$; and worse-off when $D_2' > 0$. That is, the income effect dictates the behavior of the indirect utility with respect to interest rate variations.[11]

[10] We are concerned only with interior solutions.

[11] The benefit of using a logarithm utility function is that we do not need to look at the interplay between income and substitution effect, that is only the income effect matters (Obstfeld and Rogoff, 1995).

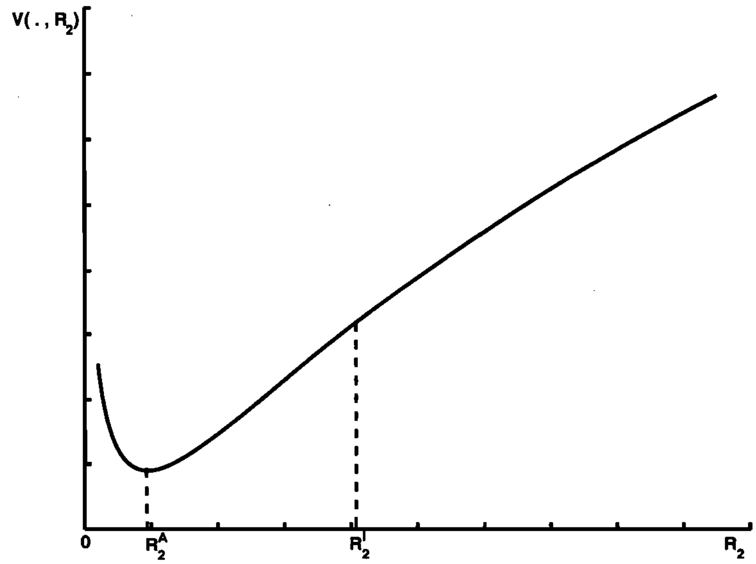


FIG. 3.2 – Effects of the interest rate on the indirect utility.

It follows that a slower increase of the indirect utility for high interest rates portrays a weakening of the income effect. Indeed, as we move away from low values of $\bar{R}_2$ above R_2^A, C_3 is getting high relative to the autarkic consumption Y_3 ; while more and more units of consumption C_2 are foregone. So, the equilibrating marginal rate of substitution is increasing in order to equate $\bar{R}_2$. Therefore it is increasingly costly to forego extra units of C_2 ; a further increase in the interest rate will have a relatively small effect on welfare. That is, the welfare is still increasing but at a weaker rate, which is in fact a fall of the strength of the income effect.

In the light of *Lemma* 1, whether $\boldsymbol{E}V(D_2; R_2) \geq V(D_2; \bar{R}_2)$ or not depends on where R_2^A lies relative to the interest rate $\bar{R}_2$. We expect the country to accept the long-term agreement when the gap between the two rates is high. Otherwise, it will choose the short-term agreement when the gap is sufficiently low.

Figures 3 and 4 give the intuition. Let the distribution of future interest rates be $\{\bar{R}_2 \pm \varepsilon, 1/2\}$.

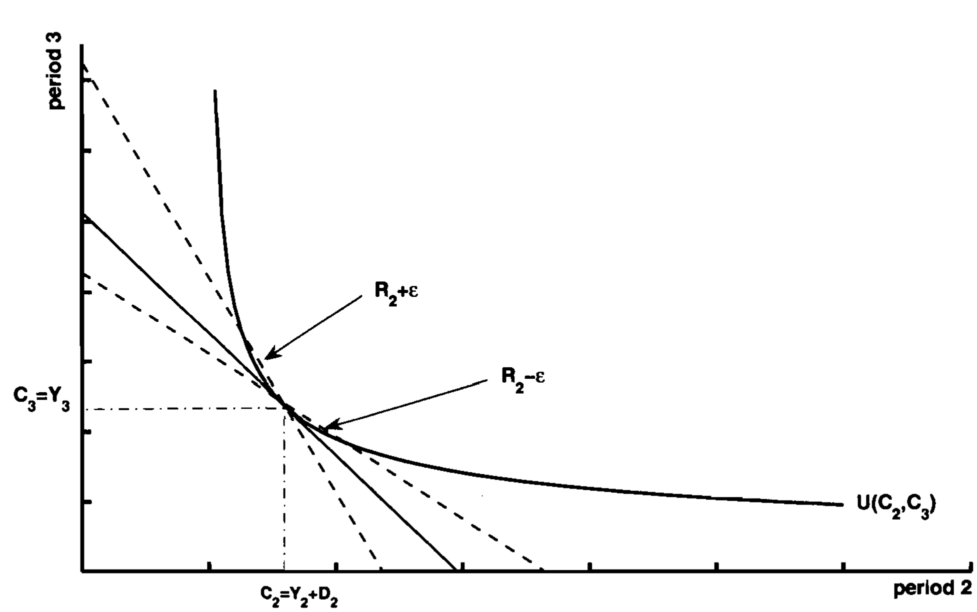


FIG. 3.3 – Interest rate risk is welfare-enhancing

In Figure 3, the tangency between the indifference curve and the budget constraint (the solid line) is represented for an interest rate $\bar{R}_2 = R_2^A$. A higher interest rate $\bar{R}_2 + \varepsilon$, necessarily leads to an upper indifference curve; as does a lower interest rate $\bar{R}_2 - \varepsilon$. Both interest rate changes make the country better off because they create trading opportunities, i.e the income effect is high around autarky. Moreover, the higher ε the higher those gains. That is what is happening in the convex portion of the indirect utility function (i.e when the gap between R_2^A and $\bar{R}_2$ is low), and that should lead the country to prefer the short-term agreement over the long-term one.

In Figure 4, the tangency occurs away from autarky. In this configuration, if the country is presented with such an interest rate $\bar{R}_2$, then it will save part of period 2 income. Therefore, a higher interest rate $\bar{R}_2 + \varepsilon$ makes him better off, because it creates more trading opportunities. On the contrary, it will be worse off at the interest rate $\bar{R}_2 - \varepsilon$. The gain from a higher interest rate does

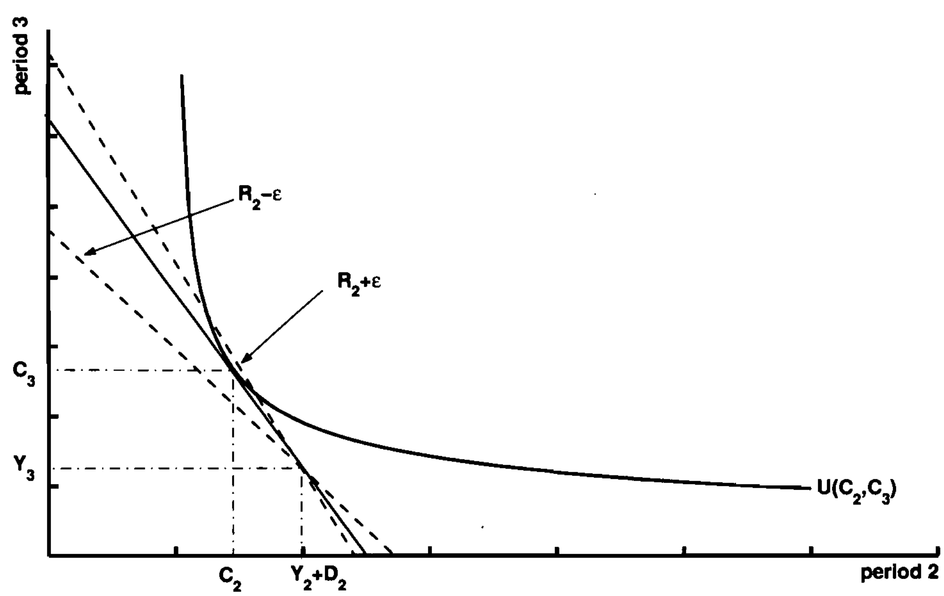


FIG. 3.4 – Interest rate risk is not welfare-enhancing

not necessarily outweigh the loss from a lower interest rate; as the income effect is getting weaker it is more likely to be a net loss. Moreover, the higher ε the higher that net loss. Hence, leading the country to prefer the long-term agreement over the short-term one, when the gap between R_2^A and $\bar{R}_2$ is high. We summarize our findings as follow,

Proposition 1. *A country is more likely to choose the short-term agreement :*

(i) the lower the interest rate offered by the long-term agreement,i.e., $\bar{R}$ (for a given level of risk);

(ii) the higher the risk, when $\bar{R}$ is low;

– the lower the risk, when $\bar{R}$ is high.

Specific to the country is the implicit R_2^A, which from (5) is a complex combination of its discount factor β and its income profile. We will now turn to how the income profile determine the maturity choice.

3.5 Debt maturity choice and economic growth

In this section, we study the effect of growth on the choice of debt maturity. More specifically, let us define g such that $g = Y_3/Y_2 > 1$; we are interested in how an increase in g affects the difference $W^{LT} - W^{ST}$. We consider non-stochastic growth.

3.5.1 The effect of growth : intuition

All the intuition over the effect of growth can be gleaned using Figure 5. In this configuration, the last period income is high enough so that the country consumes more than its period 2 income. As Y_3 increases, everything else being equal, the tangency between the budget constraint and the indifferent curve occurs in flatter portions of the indifference curve. That is, the income effect becomes weak. So basically, a high period 3 income makes the autarcic interest rate so high that at the extreme the debtor always ends up in the concave portion of the indirect utility function. That is, the more resources that comes in the future, the higher the share of lifetime income subject to risk hence the higher the risk overall. To summarize, growth worsens the existing effect of uncertainty away from the autarcic point.

3.5.2 The effect of growth : analytic

It is useful to distinguish two channels when thinking about growth in the model. Denoted ω the lifetime income where $\omega = Y_1 + \dfrac{Y_2}{R_1} + \dfrac{Y_3}{R_1 \bar{R}_2}$; and without loss of generality assume $R_1 = \beta^{-1}$. Higher g means more resources throughout the lifetime, i.e., higher ω (first channel). It also has the effect of tilting the income profile (second channel). In what follows we derive how growth impacts both agreements through these two channels.

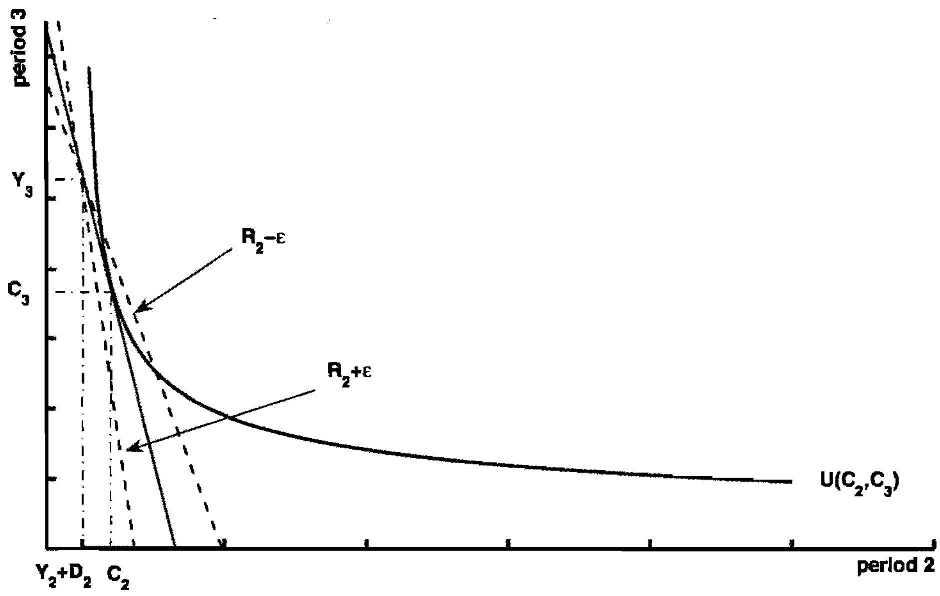


FIG. 3.5 – Interest rate risk is not welfare-enhancing for fast-growth

We compute the optimal consumption stream derived from the long-term agreement :

$$C_1^{lt} = \left[\frac{1}{1+\beta\,(1+\beta)}\right]\omega \tag{3.6}$$

$$C_2^{lt} = \left[\frac{1}{(1+\beta)\beta}\right]\left(\omega - C_1^{lt}\right) \tag{3.7}$$

$$C_3^{lt} = \left[\frac{\bar{R}_2}{1+\beta}\right]\left(\omega - C_1^{lt}\right) \tag{3.8}$$

From (6)-(8), it follows that fixed shares of lifetime income are allocated to consumption in each period, and those shares do not depend on the parameter g. Thus, faster-growth affects the long-term agreement only by raising the lifetime income (first channel). Or say differently, two countries with different gs and similar lifetime incomes will have the same utility within the long-term agreement.

The effect of growth in presence of uncertainty is hard to derive analytically for a non specified distribution. We can, however, recover the intuition of the previous section using an approximation to obtain closed-forms solutions of the consumption stream. We derive the analogue of (6) within the short-term agreement, by using a second-order Taylor expansion of the first-order equation on C_1 around optimal "long-term" values,

$$C_1^{st} = \left[\frac{1}{1+\beta\,(1+\beta)\,\phi\,(g,\sigma^2)}\right]\omega \tag{3.9}$$

where $\phi\left(g,\sigma^2\right) = 1 + \kappa g^2 \sigma^2$, and $\kappa > 0$ is a constant.[12] Both C_2^{st} and C_3^{st} keep the same form as in (7) and (8) respectively, with C_1 given by (9) and $\bar{R}_2$ replaced by R_2.

Not surprisingly uncertainty affects the consumption profile within the short-term agreement. Since $\phi(.,.) \geq 1$, the country consumes less in the first-period within the short-term agreement than within the long-term agreement, i.e., $C_1^{st} \leq C_1^{lt}$. Uncertainty on the present value of Y_3 creates an extra motive for saving, namely precautionary saving. The higher the risk, the higher the saving, i.e., $\phi_\sigma > 0$; without risk $\phi(g,0) = 1$ there is no precautionary saving. Second, a change in ω (everything else being equals, especially g) enters consumption streams in both agreements in the same manner. Hence, a shift in the lifetime income (first channel) does not affect the trade-off between agreements, that is

$$\frac{\partial W^{LT}}{\partial \omega} = \frac{\partial W^{ST}}{\partial \omega} = \frac{1+\beta\,(1+\beta)}{\omega}$$

Therefore, faster growth does not affect the maturity choice through an increase in the *level* of lifetime income.[13]

Finally, the slope of the income profile matters within the short-term agreement (second channel). More precisely, the higher the share of resources that comes in period 3, the higher the pre-

[12]We extend Blundell and Stocker's (1999) procedure to the case of interest rate risk, see Appendix A.3. Blundell and Stocker considered income risk (and log utility).

[13]It follows from the homotheticity of logarithm utility function (see the Appendix A.4).

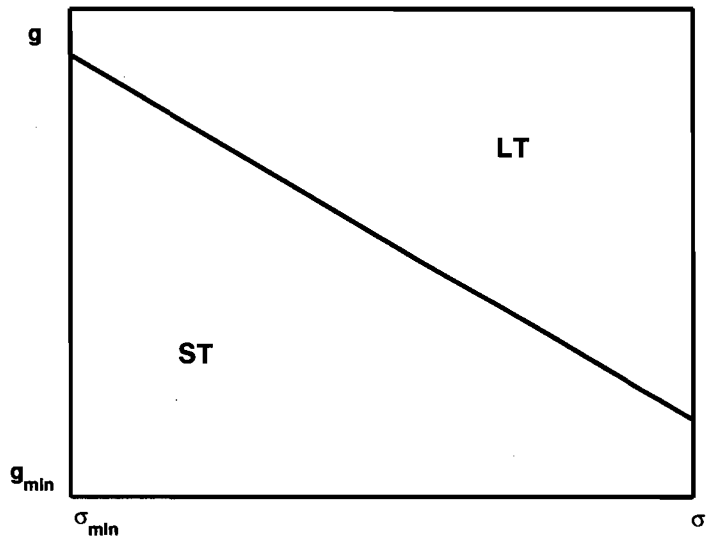


FIG. 3.6 – Debt maturity and growth

cautionary saving, i.e., $\phi_g > 0$. That is, g and σ have the same effect on the short-term agreement. Therefore, $\partial W^{ST}/\partial\sigma^2$ and $\partial W^{ST}/\partial g$ have the same sign, and

$$\frac{dg}{d\sigma^2} = -\frac{\frac{\partial W^{ST}}{\partial\sigma^2}}{\frac{\partial W^{ST}}{\partial g}} < 0$$

We summarize this section finding in the following proposition :

Proposition 2. *Consider two countries that differ only in terms of growth. The country with the lower growth is more likely to choose the short-term agreement. Moreover, countries choice of contracts are reinforced by the magnitude of the interest risk.*

We illustrate this result in Figure 6. Starting from a point where $W^{LT} = W^{ST}$, a higher risk (or growth) raises the likelihood that the country will choose the long-term agreement.

3.6 Empirical analysis

From the analysis carried out in sections 4 and 5, the model has three implications. First, a country's short-term indebtedness varies negatively with the interest rate volatility. As the gain from volatility decreases with its magnitude, so does the willingness to choose the short-term debt contract. Second, a country's short-term indebtedness varies negatively with its growth. Among two countries with different growth rates, the country with the lower rate faces relatively less uncertainty, because it has less resources concentrated in the future. Hence, for a given level of interest rate volatility, this country is more willing to choose the short-term debt contract. Third, the effect of growth on short-term indebtedness is magnified by the interest rate volatility. That is, high volatility increases the uncertainty over future resources; hence makes countries less willing to choose the short-term debt. We test these implications across countries, given the level of wealth.

The model is well-suited to describe emerging economies. In fact, because of underdeveloped domestic financial markets and a narrow economic base, developing countries are more vulnerable to external shocks than developed countries. Moreover, among developing countries the policy question of increasing the debt maturity structure is more acute for emerging economies, because they are most susceptible to tap foreign capital.[14] Our sample covers a panel of 31 emerging countries, as listed by the Institute of International Finance, from 1988 to 1998. Israel and South Korea are excluded because of missing observations for the total external debt. Except for the measure of short-term indebtedness, all variables come from the World Bank databases (World Development Indicators and Global Development Finance) and the IMF International Finance Statistics. The components of the measure of short-term indebtedness come from the BIS International Financial

[14]Interest rates are not sufficient to determine investment flows, country-specific factors such as country default risk and credit worthiness are important as well (albeit the high concentration of foreign investment in Latin America and East Asia relative to sub-Saharan Africa).

Statistics.

The sample period belongs to a decade of capital surge to emerging countries (Gourinchas, Valdès and Landerretche, 1999). The relative attractiveness of placing funds in emerging countries, vis-à-vis developed countries, is at the heart of this wave of capital flows. Conceptually, the corresponding literature addresses this phenomenon by considering capital flows as influenced by *push* and by *pull* factors (Fernandez-Arias and Montiel, 1996). Pull factors are those that induce an improvement on the risk-return characteristics of assets issued by emerging countries, effectively pulling capital into the domestic economy. Domestic policies or creditor-friendly economic reforms are examples of pull factors. Push factors, on the other hand, operate by lowering the attractiveness of lending to developed economies, so as to push funds away from these economies. For instance, deterioration in the risk-return characteristics of assets issued in developed countries or the increasing role of institutional investors. Fernandez-Arias (1996) found that for the typical country –a country creditworthy enough to receive portfolio flows–, 86% of the surge in inflows of the 1990s, can be explained by movements in international interest rates (push factor), which in turn influences the creditworthiness of receiver countries.

This period of lending booms ended with the financial crisis of 1997-1998. At the onset of the crisis, international capital markets were more volatile for several reasons : political uncertainty, risk of depreciation/devaluation and fire sale of foreign direct investments (Eichengreen, 2003). Thus, compared to 1988-1995, we expect the sub-period 1996-1998 to embed more volatility. To capture the market volatility (interest rate volatility in the model), we use a dummy variable that takes the value of zero for the years in the 1988-1995 sub-period and one otherwise. We estimate a pooled regression with thirty-one times ten observations. To control for country's wealth, as required by the model, we include the following regressors the financial depth (M2 over GDP), trade openness (trade over GDP) and the ratio of indebtedness (total external debt/GDP). For a given country i, at time t, we measure short-term indebtedness as the ratio of short-term debt owed to foreign banks to the total external claims (ST_{it}).

The regressions in rows I to IV in Table 1 add real GDP growth at $t+1$ (Gr_{it+1}), the volatility's dummy ($Volty_t$) or growth times the volatility's dummy ($Gr_{it+1} * Volty_t$), to the regressors cited above :

$$ST_{it} = \beta_0 + \beta_1 M2_{it} + \beta_2 Trade_{it} + \beta_3 Debt_{it} + \beta_4 Gr_{it+1} + \beta_5 Volty_t + \beta_6 Gr_{it+1} * Volty_t + \varepsilon_{it}$$

where the β_js are parameters, and ε_{it} is an error term. Each regression also includes country dummies, unless we do not report their coefficient. The t-statistics in regressions I and III have been estimated using robust standard errors, computed with the Huber-White methodology. Regression I reveals a positive relation between short-term indebtedness and financial depth. That is, as financial markets deepen, the debt maturity structure shorten. The coefficient of debt to GDP is positive, and marginally significant. Unlike Rodrik and Velasco's (1999), the debt burden explains cross-country differences in short-term indebtedness. We find trade openness to be insignificant in determining short-term borrowing. The coefficient of real GDP growth is negative and precisely estimated, in line with the predictions : a 10 percent increase in growth reduces the share of short-term debt in total debt by almost twenty percentage points. Finally, the coefficient of volatility is marginally significant with the correct sign, and a quite high magnitude. As predicted by the model, market volatility and growth have negative effects on countries short-term indebtedness.

The negative relationship between growth and short-term indebtedness is at odds with the common wisdom. In fact, one expects a country reliance on short-term debt to increase as it develops

TAB. 3.1 – Short-term external debt, growth and market volatility[a,b]

Dependent variable : *Ratio of short-term debt to total debt*

Estimation method	M2	Trade	Debt	Growth	Volatility	Growth x Volatility	Adj. R^2
I.[c] (OLS)	2.004 (3.72)	−0.112 (−0.35)	0.271 (1.54)	−1.931 (−2.26)	−9.433 (−1.48)		0.17
II.[d] (IV)	2.24 (5.05)	−0.25 (−0.72)	0.267 (1.52)	0.34 (0.13)	−6.676 (−0.37)		0.13
III. (OLS)	1.929 (4.21)	−0.10 (−0.33)	0.23 (1.66)	−1.09 (−1.51)	−3.8 (−0.45)	−3.497 (−2.65)	0.17
IV. (IV)	2.205 (4.52)	−0.25 (−0.72)	0.171 (0.94)	1.178 (0.41)	6.24 (0.48)	−5.052 (−2.20)	0.13

a. All regressions includes country dummies. Data definition and sources for all variables are reported in Appendix 3.5.
b. *t*-statistics are in parenthesis.
c. OLS regression are robust using the Huber-White methodology.
d. The instrument for growth is lagged growth.

(Dadush, Dasgupta and Ratha, 2000). Such prediction is not contradictory with our findings, because a good measure of development as considered by the common wisdom should be the financial depth (M2/GDP). So, what we show is that once we control for this effect, it is no longer true that fast-growing economies have a larger share of short-term foreign debt. However, the coefficient may be biased if growth is positively correlated with the error term, especially if short-term borrowing today influences tomorrow's GDP growth. To take this potential bias into account, in row II we perform the same regression using the instrumental variables estimation ; the instrument for growth is lagged growth. Our results do not stand as both growth and volatility loose all significance. Thus, suggesting that a country's short-term indebtedness determines its future growth. On these grounds, the model first two predictions are not robust to endogeneity bias.

In rows III to IV, we test the third prediction of the model, namely that the effect of growth on short-term borrowing depends on the level of market volatility. We replace the volatility's dummy with the product of the latter and growth. Since, endogeneity is an issue we turn to regression IV.

Contrary to the model's prediction, growth now has a positive effect on countries' short-term indebtedness. Recall that our setup is essentially a partial equilibrium framework (borrowers' willingness). The opposite effect than what we expected can arise from the supply side (investors' concerns). So that it can be due to an increase of investors' risk aversion as argued by Broner, Lorenzoni and Schmukler (2005). However, this coefficient is not significant.

Finally, the coefficient of the interaction term is significant with the correct sign, regardless of the method of estimations. Hence, the data do not reject the prediction that the uncertainty over future resources, created by the market volatility explains emerging countries short-term borrowing. Countries with lower growth prospects end up having higher shares of short-term debt, everything else being equals.

3.7 Conclusions and policy implications

We have argued that there were important theoretical reasons why one would expect country-specific factors' to influence the maturity structure of foreign debt to emerging countries. In this paper we have focused on the idea that emerging countries rely on short-term debt in order to benefit from better borrowing conditions in the future. We model long-term and short-term debt as different instruments to deal with uncertainty over future shocks. We find that a country's short-term indebtedness varies negatively with the market volatility. Second, a country short-term indebtedness varies negatively with its growth. Third, the effect of growth on short-term indebtedness is magnified by the market volatility.

The evidence reported in section 6 does not reject the effect of the interaction between country's growth and market volatility on the maturity composition of external debt. That is, emerging countries with lower growth prospects have, on average, higher shares of short-term debt in presence of market volatility. However, we found little support for the independent effects of growth, or market volatility, specially once we control for endogeneity issues.

The main feature of our setup is that short-term and long-term debt contracts are imperfect substitutes. Thus, any policy aiming at discouraging short-term indebtedness will also limit emerging economies' instruments to cope with external shocks. In this paper, we focused on cost to emerging countries, however, discouraging short-term indebtedness incur costs for foreign investors as well, and more generally for all "agents operating in a global environment" (see De la Torre and Schmukler, 2004). Regarding emerging countries, our framework suggests that such policy must be accompanied with alternatives solutions to deal with uncertainty over future shocks. This paper open an aera for future research in this direction.

Conclusion

Cette thèse contribue à la littérature sur la durée des contrats en identifiant les bénéfices et les coûts de l'engagement de long terme dans le cas des contrats de dette internationale, et dans le cas des contrats d'emprunt hypothécaire ; notamment du point de vue d'un emprunteur. L'élément important de mon modèle est de ramener le choix entre un contrat de dette à court terme et un contrat de dette à long terme à un choix entre instrument sans risque et instrument risqué. Ainsi, le degré d'aversion de l'emprunteur, et son niveau de revenu déterminent sont besoin d'assurance et sa capacité d'auto assurance.

Dans le chapitre 1, ce modèle de permet d'expliquer le recours aux contrats d'hypothèque à taux flexibles par des agents averses au risque. Ma contribution, à la littérature sur le choix de contrats d'hypothèque, est de montrer que les contrats d'hypothèque à taux variable permettent aux agents de s'auto assurer contre le risque de taux, en ajustant leurs paiements en conséquence.

Dans le chapitre 2, j'estime une forme réduite du modèle de choix de contrats d'hypothèque, fondée sur les résultats du chapitre 1. Je compare les estimations aux résultats de la littérature empirique. J'ai recours aux des données de l'enquête des ménages du Panel Survey of Income Dynamic (PSID). Exceptionnellement, l'enquête conduite en 1996 inclus des informations sur le type de contrats d'hypothèques que possèdent les propriétaires résidentielles. Elle inclut aussi un index qualitatif du degré d'aversion relative au risque des ménages. Mes estimations permettent de conclure que le degré d'aversion au risque et le niveau de richesse du ménage sont des facteurs importants du choix de contrats d'hypothèque.

Dans le chapitre 3, ce modèle de choix de la durée de contrat permet d'expliquer le recours excessif des pays émergents à la dette extérieure de court terme dans les années 1990. Ma contribution, à la littérature sur les mouvements de capitaux internationaux, est de montrer que la dette de long terme et la dette de court terme sont des substituts imparfaits. Aussi, toute politique visant la réduction de l'endettement à court terme devrait proposer des instruments alternatifs.

Certains approfondissement sont néanmoins envisageables pour rendre le modèle plus réaliste. Une extension possible serait de considérer un profil de revenu stochastique, ou encore de considérer une économie de production. Dans le cas de la l'endettement international, par exemple, les chocs sur la productivité sont en partie responsables des besoins en capitaux étrangers. La degré de corrélation de ces chocs avec le taux d'intérêt mondial pourrait affecter mes conclusions sur le choix de contrats de dette.

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Annexes

.1 Chapter one

.1.1 Sketch of the mortgage problem

Consider a three-period mortgage describes as follows : the borrower receives cash $\bar{H}$ to purchase the housing; in the subsequent periods, he makes interest-payment at rate r_t, and prepayment of the principal K_t, out of his period-income Y_t ; by the last-period he must have paid off the mortgage loan. An optimal fixed-rate mortgage contract (FRM) is the solution to the following maximization problem :

$$\max_{\{C_t,K_t\}_{t=1}^{3}} u(C_1) + \beta u(C_2) + \beta^2 u(C_3)$$

$$\begin{aligned}
\text{subject to :} \quad & C_1 = Y_1 - r_1\bar{H} - K_1 > 0 \\
& C_2 = Y_2 - r_2\left(\bar{H} - K_1\right) - K_2 > 0 \\
& C_3 = Y_3 - (1+r_3)\left(\bar{H} - K_1 - K_2\right) > 0
\end{aligned}$$

$$-\bar{H} + \frac{r_1\bar{H} - K_1}{1+r_1} + \frac{r_2\left(\bar{H} - K_1\right) + K_2}{(1+r_1)(1+r_2)} + \frac{(1+r_3)\left(\bar{H} - K_1 - K_2\right)}{(1+r_1)(1+r_2)(1+r_3)} = 0$$

In practice, interest-payments are computed with the compounded interest-rate r_c such that $(1+r_2)(1+r_3) = (1+r_c)^2$. Here, period-payments are not necessarily equal because prepayment of the principal is endogenous.

Finally, denote $H = \bar{H}R_0$. With the following change of variables we obtain the FRM problem as in pp. 13 :

$$\begin{aligned}
R_{j-1} &= 1 + r_j \\
D_1 &= r_1\bar{H} + K_1 \\
D_2 &= r_2\left(\bar{H} - K_1\right) + K_2 \\
D_3 &= (1+r_3)\left(\bar{H} - K_1 - K_2\right)
\end{aligned}$$

Hence the problem becomes :

$$\max_{\{C_t,D_t\}_{t=1}^{3}} u(C_1) + \beta u(C_2) + \beta^2 u(C_3)$$

$$\text{subject to :} \quad C_t = Y_t - D_t > 0, \qquad \text{for all } t$$
$$-\bar{H}R_0 + D_1 + \frac{D_2}{R_1} + \frac{D_3}{R_1 R_2} = 0$$

.1.2 A value of information perspective of the ARM problem

Consider the three-period problem introduced in the paper with the following twist in the information structure. Uncertainty on the future interest-rate is described by X possible realized values of R_2, indexed by $x = 1, ..., X$. Vector $\Phi = (\phi_1, ..., \phi_M)$ measures the probability of occurrence of these states of the world. Uncertainty on the future interest-rate is resolved in period 2, after payments D_2 and D_3 are decided. Prior to period 2, parties observe a signal indexed by $m = 1, ..., M$. Vector $Q = (q_1, ..., q_M)$ denotes the vector of unconditional probabilities of the different signals. Observing the signal allows the decision maker to revise the probability distribution of the future interest-rate using the Bayes's rule. This relationship is characterized by the matrix of posterior probabilities $P = [p_{mx}]$ with $m = 1, ..., M$ and $x = 1, ..., M$. The decision problem is written

$$\max_{D_1} u(Y_1 - D_1) + \beta \sum_{m=1}^{M} q_m \left[\max_{D_2^m} u(Y_2 - D_2^m) + \beta \sum_{x=1}^{X} p_{mx} u(Y_3 - R_2^x (R_1 (H - D_1) - D_2^m)) \right]$$

Two polar cases of this problem are :

(*a*) where m provides no information about x, so that m may be ignored completely since it doesn't enter the utility function ;

$$W^{no} \equiv \max_{D_1} u(Y_1 - D_1) + \beta \left[\max_{D_2} u(Y_2 - D_2) + \beta \sum_{x=1}^{X} \phi_x u(Y_3 - R_2^x (R_1 (H - D_1) - D_2)) \right]$$

(*b*) and where m provides perfect information about x, so that x is essentially observed before period 2.

$$W^{full} \equiv \max_{D_1} u(Y_1 - D_1) + \beta \sum_{x=1}^{X} \phi_x \left[\max_{D_2^x} u(Y_2 - D_2^x) + \beta u(Y_3 - R_2^x (R_1 (H - D_1) - D_2^x)) \right]$$

Basically, the ARM problem is similar to case (b), i.e., $W^A \equiv W^{full}$. These two cases feature different timing of the resolution of uncertainty on R_2 : case (a) corresponds to a late resolution while case (b) is an early resolution. From the convexity of the maximum operator (see Marschak (1954), pp.61) it follows that $W^{full} > W^{no}$, irrespective of the utility function u. In words, the individual cannot be worse off with an early resolution of uncertainty provided that he can modify his decision from the information that he gets (i.e., flexibility is valuable). Notice that, **increasing risk is the same as increasing the informativeness of the signal** in case (b) ; so, more information can increase the optimal exposure to risk.

.1.3 Curvature of the indifference curve

This section is based on Drèze and Modigliani (1972). Consider the general two-period utility index $U(C_2, C_3)$ and the price ratio $R_2 (= p_2/p_3)$. Let (C_2, C_3) be a point of tangency between an indifference curve $U(C_2, C_3) = \bar{U}$ and the budget line $C_2 + \dfrac{C_3}{R_2} = C^{te}$, so that the marginal rate of substitution of C_2 for C_3, i.e., $MRS_{C_2,C_3} = -\left.\dfrac{dC_3}{dC_2}\right|_{\bar{U}} = \dfrac{U_1}{U_2} = R_2$. It follows that :

$$\begin{aligned}
\frac{dMRS_{C_2,C_3}}{dC_3} &= \frac{\left(U_{11}\left.\frac{dC_2}{dC_3}\right|_{\bar{U}} + U_{12}\right)U_2 - \left(U_{22} + U_{21}\left.\frac{dC_2}{dC_3}\right|_{\bar{U}}\right)U_1}{U_2^2} \\
&= \frac{\left(-U_{11}R_2^{-1} + U_{12}\right)U_2 - \left(U_{22} - U_{21}R_2^{-1}\right)U_2R_2}{U_2^2} \\
&= \frac{-U_{11} + 2R_2U_{12} - R_2^2U_{22}}{R_2U_2}
\end{aligned}$$

The numerator is positive, and is simply the opposite of the second-order condition of the above problem. It measures the curvature of the indifference loci at the optimal bundle (C_2, C_3).

In our framework, $U_1 = u'(C_2)$, $U_2 = \beta u'(C_3)$, $U_{11} = u''(C_2)$, $U_{22} = \beta u''(C_3)$ and $U_{12} = 0$ (since utility is assumed additively separable in time), thus

$$\begin{aligned}
\frac{dMRS_{C_2,C_3}}{dC_3} &= -\frac{u''(C_2) + \beta R_2^2 u''(C_3)}{R_2\beta u'(C_3)} \\
&= \frac{\gamma_2 + R_2\gamma_3}{\gamma_3}
\end{aligned}$$

.1.4 Some calculations

1.1 The Euler equation of problem (IU) can be written as follows

$$u'(C_2) - \beta R_2 u'\left(\left(Y_2 - C_2 - (H - D_1)R_1\right) R_2 + Y_3\right) = 0$$

Total differentiation the Euler equation yields

$$\left(u''(C_2) + \beta R_2^2 u''(C_3)\right) dC_2 - \left(\beta u'(C_3) + \beta R_2 \left(Y_2 - C_2 - (H - D_1)R_1\right) u''(C_3)\right) dR_2 = 0$$

notice that $D_3 = -R_2\left(Y_2 - C_2 - (H - D_1)R_1\right)$, hence

$$\frac{dC_2}{dR_2} = \frac{\beta u'(C_3) - \beta D_3 u''(C_3)}{u''(C_2) + \beta R_2^2 u''(C_3)}$$

To obtain dC_3/dR_2, take the total differentiation of the two-period budget constraint of problem (IU), that is

$$dC_2 + \frac{1}{R_2} dC_3 = -\frac{Y_3 - C_3}{R_2^2} dR_2$$

Hence

$$\frac{dC_3}{dR_2} = -\left(R_2 \frac{dC_2}{dR_2} + \frac{D_3}{R_2}\right)$$

Let $\gamma_t = -u''(C_t)/u'(C_t)$ denote the coefficient of absolute risk aversion. One can rewrite dC_2/dR_2 as follows

$$\frac{dC_2}{dR_2} = \frac{\beta u'(C_3)\left(1 + D_3\gamma_3\right)}{-u'(C_2)\left(\gamma_2 - \beta R_2^2 \frac{u''(C_3)}{u'(C_2)}\right)}$$

Using $Eq.(8)$, we replace $u'(C_2)$ by $\beta R_2 u'(C_3)$ to obtain

$$\frac{dC_2}{dR_2} = -\frac{1 + D_3\gamma_3}{R_2\left(\gamma_2 + R_2\gamma_3\right)}$$

While,

$$\frac{dC_3}{dR_2} = \frac{u'(C_2)\left(-R_2 + D_3\gamma_2\right)}{-u'(C_2) R_2\left(\gamma_2 - \beta R_2^2 \frac{u''(C_3)}{u'(C_2)}\right)}$$

Using $Eq.(8)$, we replace $u'(C_2)$ by $\beta R_2 u'(C_3)$ to obtain

$$\frac{dC_3}{dR_2} = \frac{R_2 - D_3\gamma_2}{R_2\left(\gamma_2 + R_2\gamma_3\right)}$$

Let $\nu_t = -C_t u''\left(C_t\right)/u'\left(C_t\right)$ denote the coefficient of relative risk aversion, and replace D_3 by $R_2\left(R_1\left(H - D_1\right) - Y_2 + C_2\right)$. Then, we can rewrite dC_3/dR_2 as follows

$$\begin{aligned}\frac{dC_3}{dR_2} &= \frac{R_2 - R_2\left(R_1\left(H - D_1\right) - Y_2 + C_2\right)\gamma_2}{R_2\left(\gamma_2 + R_2\gamma_3\right)} \\ &= \frac{1 - \nu_2 - \left(R_1\left(H - D_1\right) - Y_2\right)\gamma_2}{\gamma_2 + R_2\gamma_3}\end{aligned}$$

1.2 From the envelope theorem

$$v_2\left(D_1, R_2\right) = \beta u'\left(C_3\right)\left(-\frac{D_3}{R_2}\right)$$

Derive v_2 with respect to its second argument

$$v_{22}\left(D_1, R_2\right) = \beta u''\left(C_3\right)\left(-\frac{D_3}{R_2}\right)\frac{\partial C_3}{\partial R_2} - \beta u'\left(C_3\right)\left(-\frac{D_3}{R_2^2} + \frac{1}{R_2}\frac{\partial D_3}{\partial R_2}\right)$$

Replace $\partial D_3/\partial R_2$ by $-\partial C_3/\partial R_2 = R_2\ \partial C_2/\partial R_2 + D_3/R_2$ and we get

$$v_{22}\left(D_1, R_2\right) = \beta u''\left(C_3\right)\left(-\frac{D_3}{R_2}\right)\frac{\partial C_3}{\partial R_2} - \beta u'\left(C_3\right)\frac{\partial C_2}{\partial R_2}$$

1.3 From the envelope theorem

$$v_1\left(D_1, R_2\right) = \beta R_1 R_2 u'\left(C_3\right)$$

Derive v_1 with respect to its second argument

$$v_{12}\left(D_1, R_2\right) = \beta R_1 u'\left(C_3\right) + \beta R_1 R_2 \frac{\partial C_3}{\partial R_2} u''\left(C_3\right)$$

A further derivation of v_{12} with respect to R_2 yields

$$v_{122}\left(D_1, R_2\right) = \beta R_1 \frac{\partial C_3}{\partial R_2}\left(2 + R_2\frac{\partial C_3}{\partial R_2}\right) u''\left(C_3\right) + \beta R_1 R_2 \frac{\partial^2 C_3}{\partial R_2^2} u'''\left(C_3\right)$$

1.3′ From the envelope theorem

$$v_1\left(D_1, R_2\right) = \beta R_1 u'\left(C_2\right)$$

Derive v_1 with respect to its second argument

$$v_{12}(D_1,R_2)=\beta R_1\frac{\partial C_2}{\partial R_2}u''(C_2)$$

A further derivation of v_{12} with respect to R_2 yields

$$v_{122}(D_1,R_2)=\beta R_1\frac{\partial^2 C_2}{\partial R_2^2}u''(C_2)+\beta R_1\left(\frac{\partial C_2}{\partial R_2}\right)^2 u'''(C_2)$$

1.4 Computing $\partial^2 C_3/\partial R_2^2$ for a CRRA utility function

$$\frac{\partial^2 C_3}{\partial R_2}=\Phi(\nu-1)\left[R_2\left((2\nu-1)(\beta R_2)^{1/\nu}+R_2\right)(R_1H-R_1D_1-Y_2)+\left((\beta R_2)^{1/\nu}+(2\nu-1)R_2\right)Y_3\right]$$

where $\Phi=\dfrac{(\beta R_2)^{1/\nu}}{\nu^2 R_2\left(R_2+(\beta R_2)^{1/\nu}\right)^3}>0$. Notice that both $\partial^2 C_3/\partial R_2^2$ and $\partial^2 C_3/\partial\eta\varepsilon$ have the same sign.

$$\frac{\partial^2 C_3}{\partial\eta\partial\varepsilon}\begin{cases}<0 & \text{when} \quad 1/2<\nu<1,\\ >0 & \text{when} \quad \nu>1\end{cases}$$

provided that the borrower is not rich enough to pay off its loan within the first-two periods (which implies $R_1H-R_1D_1-Y_2>0$).

.1.5 Proof

Lemma 1. *Under Condition 1, $\partial C_3/\partial R_2\leqslant 0$ everywhere.*

Proof of Lemma 1. In *Eq.*(8), let us replace D_3 by $R_2(R_1(H-D_1)-Y_2+C_2))$ and rewrite it as

$$\frac{\partial C_3}{\partial R_2}=\frac{1-\nu_2-(R_1H-R_1z-Y_2)\gamma_2}{\gamma_2+R_2\gamma_3}$$

The term in parenthesis represents the lower bound of the outstanding loan balance at the end of period 2. Since $0\leqslant z<Y_1$, Lemma 1 follows. ■

Lemma 2. *A sufficient condition for first-period payment to increase with respect to risk is $v_{122}\geqslant 0$ uniformly.*

Proof of Lemma 2. Consider the function J such that :

$$J(D_1) = u(Y_1 - D_1) + \beta \mathbf{E} v(D_1, R_2)$$

with
$$J'(D_1) = -u'(Y_1 - D_1) + \beta \mathbf{E} v_1(D_1, R_2)$$

From Assumption 1, $J(D_1)$ is concave in D_1; and rewriting *Eq.*(4) leads to $J'(D_1^{\mathbf{A}}) = 0$. Consequently, $D_1^{\mathbf{F}} \leqslant D_1^{\mathbf{A}}$ if $J'(D_1^{\mathbf{F}}) \geqslant 0$. Noticing from *Eq.*(2) that

$$u'(Y_1 - D_1^{\mathbf{F}}) = \beta v_1(D_1^{\mathbf{F}}, \bar{R}_2).$$

In turn, $D_1^{\mathbf{F}} \leqslant D_1^{\mathbf{A}}$ if

$$\mathbf{E} v_1(D_1^{\mathbf{F}}, R_2) \geqslant v_1(D_1^{\mathbf{F}}, \bar{R}_2)$$

where, from the envelope theorem,

$$v_1(D_1^{\mathbf{F}}, R_2) = \beta R_1 R_2 u'\left(Y_3 + R_2 D_2'(D_1^{\mathbf{F}}, R_2)\right).$$

Using Jensen's inequality (and following Rothschild and Stiglitz (1970) and Hadar and Seo (1990)), this condition is met whenever $v_{122} \geqslant 0$ over the interval of R_2. Moreover, *Eq.*(9) can be rewritten as

$$\frac{\partial C_1}{\partial \eta} = \beta \frac{E(\varepsilon v_{12}(D_1, R_2))}{D}$$

so that $v_{122} \geqslant 0$ also guarantees that $\partial C_1 / \partial \eta \leqslant 0$ ■

.1.6 Mortgage choice

TAB. 2 – Mortgage choice, housing value (H) and relative risk aversion (ν)

H \ ν	0.7	0.9	1.1	1.3	1.5	1.7	1.9	2.1	2.3	2.5
3.95	1	1	1	1	1	1	1	1	1	1
3.8	0	1	1	1	1	1	1	1	1	1
3.65	0	0	1	1	1	1	1	1	1	1
3.5	0	0	0	1	1	1	1	1	1	1
3.35	0	0	0	0	1	1	1	1	1	1
3.2	0	0	0	0	0	1	1	1	1	1
3.05	0	0	0	0	0	0	1	1	1	1
2.9	0	0	0	0	0	0	0	1	1	1
2.75	0	0	0	0	0	0	0	0	1	1
2.6	0	0	0	0	0	0	0	0	0	0

Optimal choice : FRM = 1 ; ARM = 0.

.2 Chapter two

.2.1 Data definitions and sources

Housing

Homeowner : a household who answered "Yes" to ER7035, and "Mortgage" to ER7036. ER7035 asked "Do you have a mortgage on this property ?", and ER7036 asked "Is that a mortgage, a land contract, a home equity loan, or what ?".

Year got the mortgage (ER7052)

Current interest-rate (ER7048+ER7049)

Outstanding loan balance (ER7042) : the principal currently owed from all mortgages contracts on the home.

Number of years remaining on the mortgage (ER7054) : the number of years left on the longest-term mortgage that the family unit has.

Monthly mortgage payment (ER7044) :

Whether the interest rate on the home mortgage were fixed or adjustable (ER7046)

Whether hold a second-mortgage (ER7110)

Annual property taxes (ER7033) : current annual property tax liability.

Annual owner insurance premium (ER7034) : annual dollar amount of homeowner's insurance premiums.

Whether mortgage payment include property taxes (ER7112)

Whether mortgage payment include owner insurance premium (ER7113)

House value (ER7032) : value of the home in whole dollars.

Dummy for dwelling (ER7014) : takes value 1 if the dwelling is a one-family house, and 0 other wise. Type of the dwelling are one-family house, two-family house, an apartment, a mobile home, or other)

Dummy for government insured (ER7038) : takes value 1 if insured by a government agency, and 0 otherwise. The main government agencies are the Federal Housing Administration (FHA), or the Veterans Administration (VA).

Dummy for original (ER7040) : takes value 1 if original loan and terms, and 0 otherwise.

Demographics

Marital status (ER7013)

Sex of the head (ER7007)

Dummy for mobility (ER7162) : takes the value 1 if the family unit will move (definitely, probably or uncertain), and 0 otherwise.

Education (ER33315) : the actual grade of school completed; e.g., a value of 08 indicates that this individual completed the eighth grade by the time of the 1996 interview. A code value of 17 indicates that this individual has completed at least some postgraduate work.

Dummy for liquidity-constrained (ER8841) : takes the value 1 if the head found himself unable to pay your bills when they were due, or obtained a loan to consolidate or pay off your debts, or had a creditor call or come to see you to demand payment, or had your wages attached or garnished by a creditor, or had a lien filed against your property because you could not pay a bill Had your home, car, or other property repossessed; and 0 otherwise.

Family size (ER7005) : number of persons currently in the family unit.

occupation (ER9108) : The 3-digit occupation code from 1970 Census of Population.

Race (ER9060)

Other

Risk tolerance (RT1)

Time in current job (ER) :

Income (ER9244) : Total 1995 Family Money Income

Income from financial assets : sum of earnings from dividend (ER8335, ER8642), interest (ER8350, ER8657), trust fund (ER8365, ER8672) of head and wife.

State of Residence (ER9247).

.2.2 Benchmark interest-rate

The data for the benchmark interest-rate come from historical summary data from the Finance Board's Monthly Survey of Rates and Terms on Conventional Single-Family Non-farm Mortgage Loans edited by the Federal Housing Finance Board (Table 15. Annual by State). This survey is the U.S. most comprehensive source of information on conventional mortgage rates and terms. The reported information is based on fully amortized mortgage loans used to purchase single-family non-farm homes. Loans used to refinance houses are excluded, as are non-amortized and balloon loans. The survey reports only conventional mortgages, and thus excludes mortgage loans insured by the Federal Housing Administration (FHA) or guaranteed by the Veterans Administration (VA).

We use the contract interest-rate (as opposed to the effective, the difference between the two rates comes from initial fees and charges)

We build an algorithm that assigns an interest rate to each borrower based on the state where he lives and the date of his mortgage origination.

.2.3 Estimating Risk Tolerance from the 1996 PSID by Ming-Ching Luoh and Frank Stafford

In questions M1-M5 (page 138 on the paper mock-up of the questionnaire), employed respondents are asked how willing they are to take jobs with different income prospects. These questions are similar to ones used in the Health and Retirement Study, but here they indicate that the new job will be equally good, having the same non-monetary attributes as their current job. All answers to these questions offer a 50-50 chance to double income or to cut income in different proportions.

If the respondent is willing to take a chance and answers yes, the next question branches into a query about their willingness to accept a doubling on the up side, combined with a cut to one half. If the respondent answers yes again, they are asked how willing they would be to accept a cut of 75 percent. If, however, the respondent answers no to the initial double or one-third option, the question branches into a query about their willingness to accept a 20 percent cut on the downside. If they answer no here, they are asked about their willingness to accept only a 10 percent cut on the downside.

Based on responses to these questions, we can arrange people into six groups with an exact risk tolerance range, and four groups with larger ranges due to item non-response along some of the branches.

To convert these answers into a single, quantitative index of risk tolerance (see Barsky, Juster, Kimball and Shapiro, Quarterly Journal of Economics, May 1997), suppose the utility function is $U(c) = (1/(1-1/q))c^{1-1/q}$. Our task is to estimate q. Assume that q is log-normally distributed, then $X = ln(q)$ is a normal distribution. However, X is unobservable. What we observe is X^*, which is in one of the ten groups according to the following process : X^* in group i, if $B_i - 1 < x < B_i$ where B_i is the cutoff point on X determined by the design of the survey questions, indicated above. The likelihood function then is the product of each individual's probability of being in that particular group. We can estimate mean m and standard deviation s_x by maximizing the likelihood function. We can then recover q for each group by computing the expected e^x conditional on being in that group. The estimated m is -1.27, while the estimated $s_x = 1.579$. The conditional means of q are as follows for each group.

A risk tolerance file is available which shows the difference between the risk tolerance estimates and the tolerances after measurement errors have been corrected. The first variable in the comparison file is the 1996 Family ID. The second variable, Risk Tolerance, is estimated from 1996 PSID questions M1-M5 without correcting for measurement error. In the third variable, Risk Tolerance 1, measurement errors are corrected based on both HRS waves I and II. Data for Risk Tolerance 1 are taken from the last column of Table 1 in Barsky, Juster, Kimball and Shapiro (Quarterly Journal of Economics, May 1997). Fields without numbers indicate that the head was not in the labor force, or that the question was unanswered.

.3 Chapter three

.3.1 V's curvature

We can compute V_2 at the optimum,

$$V_2(d_2;\overline{R}_2) = -\frac{Y_3 - \beta\overline{R}_2(Y_2+D_2)}{\overline{R}_2\left(y_3+\overline{R}_2(Y_2+D_2)\right)}. \tag{10}$$

Thus, we derive V_{22},

$$V_{22}(D_2;\bar{R}_2) = \frac{-\beta(Y_2+D_2)^2\bar{R}_2^2 + 2Y_3(Y_2+D_2)\bar{R}_2 + Y_3^2}{\bar{R}_2^2\left(\bar{R}_2(Y_2+D_2)+Y_3\right)^2}$$

Since by definition $\bar{R}_2 > 0$ and $(Y_2+D_2) + \dfrac{Y_3}{\bar{R}_2} > 0$ (resource available for period 2 and 3), the denominator of V_{22} is strictly positive, thus

$$Sign\left(V_{22}(D_2;\bar{R}_2)\right) = Sign\left(-\beta(Y_2+D_2)^2\bar{R}_2^2 + 2Y_3(Y_2+D_2)\bar{R}_2 + Y_3^2\right). \tag{11}$$

.3.2 Proof

Proof of Lemma 1. (*i*) From (3), $V_2(D_2;R_2^A) = 0$ iff $D_2' = 0$, which from (5) holds iff $R_2^A = \frac{Y_3}{\beta(Y_2+D_2)}$; thus (*i*) follows.

(ii) From (11) the sign of V_{22} is given by a second order polynom in $\bar{R}_2$. The discriminant of the latter is $\Delta = 4Y_3^2(Y_2 + D_2)^2\,(1+\beta)$ which is always positive, so that there are two roots R_2^- and R_2^+. Moreover, we can easily show that $R_2^- R_2^+ = \frac{Y_3^2}{-\beta(Y_2+D_2)^2} < 0$; it implies that the two roots have opposite signs. Denoted by R_2^I the positive root R_2^+, thus (ii) follows.

(iii) Since the coefficient of $\bar{R}_2^2$ in polynom of (11) is negative, thus (iii) follows.

(iv) Let compute R_2^I :

$$
\begin{aligned}
R_2^I &= \frac{-2Y_3(Y_2+D_2) - \sqrt{\Delta}}{-2\beta(Y_2+D_2)^2} \\
&= R_2^A + \frac{\sqrt{\Delta}}{2\beta(Y_2+D_2)^2} > R_2^A
\end{aligned}
$$

thus (iv) follows.

Lemma 1 follows. ■

.3.3 Approximation of C_1 in the short-term agreement

Consider the continuation problem, when solving for C_1 and C_2 we obtain

$$
\begin{aligned}
C_2 &= \frac{1}{(1+\beta)}\left((Y_2+D_2) + \frac{Y_3}{R_2}\right) \\
&= \frac{1}{(1+\beta)}(\tilde{\omega} - C_1) \\
C_3 &= \beta R_2 C_2
\end{aligned}
$$

with

$$
\tilde{\omega} = \omega_1 + \frac{Y_3}{R_1 R_2}
$$

and

$$\omega_1 = Y_1 + \frac{Y_2}{R_1}$$

we can rewrite W^{ST} as,

$$W^{ST} = \max_{C_1} \left\{ \log(C_1) + \beta \boldsymbol{E} \left[(1+\beta) \log\left(\widetilde{\omega} - C_1\right) + C^{te} \right] \right\}$$

First-order condition on C_1 leads to :

$$\begin{aligned}
\frac{1}{C_1} &= \beta\,(1+\beta)\,\boldsymbol{E}\left[\frac{1}{\widetilde{\omega} - C_1}\right] \\
&= \frac{\beta\,(1+\beta)}{\omega - C_1}\boldsymbol{E}\left[\frac{\omega - c_1}{\omega_1 + \frac{Y_3}{R_2} - C_1}\right] \\
&= \frac{\beta\,(1+\beta)}{\omega - C_1}\boldsymbol{E}\left[\frac{\bar{R}_2\,(\omega - C_1)}{\bar{R}_2\,(\omega_1 - C_1) + \frac{\bar{R}_2}{R_2} Y_3}\right]
\end{aligned}$$

$$\frac{1}{C_1} = \frac{\beta\,(1+\beta)}{\omega - C_1}\boldsymbol{E}\left[\frac{\bar{R}_2\,(\omega - C_1)}{\bar{R}_2\,(\omega - C_1) + \left(\frac{\bar{R}_2}{R_2} - 1\right) Y_3}\right] \tag{12}$$

We turn our focus to the function within the expectation operator in (12). Let make the following change of variables

$$\begin{aligned}
\Omega &= \bar{R}_2\,(\omega - C_1) \\
\rho &= \left(\frac{\bar{R}_2}{R_2} - 1\right) Y_3 = -\frac{\varepsilon}{R_2} Y_3
\end{aligned}$$

moreover let

$$\begin{aligned}
\Omega_{LT} &= \bar{R}_2\left(\omega - C_1^{lt}\right) \\
&= \bar{R}_2\kappa^{-2}\omega \\
\rho_{LT} &= 0
\end{aligned}$$

Consider the second-order Taylor expansion of the function $\frac{\Omega}{\Omega+\rho}$ around (Ω_{LT}, ρ_{LT})

$$\begin{aligned}
\frac{\Omega}{\Omega+\rho} &= \frac{\Omega_{LT}}{\Omega_{LT}+\rho_{LT}} + \left[\frac{1}{\Omega_{LT}+\rho_{LT}} - \frac{\Omega_{LT}}{(\Omega_{LT}+\rho_{LT})^2}\right](\Omega-\Omega_{LT}) + \left[-\frac{\Omega_{LT}}{(\Omega_{LT}+\rho_{LT})^2}\right](\rho-\rho_{LT}) \\
&+\frac{1}{2}\left[-\frac{2}{(\Omega_{LT}+\rho_{LT})^2} + \frac{2\Omega_{LT}}{(\Omega_{LT}+\rho_{LT})^3}\right](\Omega-\Omega_{LT})^2 \\
&+\left[-\frac{1}{(\Omega_{LT}+\rho_{LT})^2} + \frac{2\Omega_{LT}}{(\Omega_{LT}+\rho_{LT})^3}\right](\Omega-\Omega_{LT})(\rho-\rho_{LT}) \\
&+\frac{1}{2}\left[\frac{2\Omega_{LT}}{(\Omega_{LT}+\rho_{LT})^3}\right](\rho-\rho_{LT})^2 + C^{te}
\end{aligned}$$

When taking the expectation, all terms vanish except the first and the last terms, thus

$$\begin{aligned}
\boldsymbol{E}\left[\frac{\Omega}{\Omega+\rho}\right] &\simeq 1+\frac{1}{\Omega_{LT}^{2}}\boldsymbol{E}\left[\rho^{2}\right] \\
&= 1+\frac{1}{\left(\bar{R}_{2}\kappa^{-2}\omega\right)^{2}}Y_{3}^{2}\boldsymbol{E}\left(\frac{\varepsilon^{2}}{R_{2}^{2}}\right) \\
&= 1+\kappa\left(\frac{Y_{3}}{\bar{R}_{2}\omega}\right)^{2}\boldsymbol{E}\left(\frac{\varepsilon^{2}}{R_{2}^{2}}\right) \\
&= 1+\kappa g^{2}\sigma^{2} \\
&= \phi\left(g,\sigma^{2}\right)
\end{aligned}$$

where $\boldsymbol{E}\left(\frac{\varepsilon^{2}}{R_{2}^{2}}\right)$ is obtained from an approximation

$$\begin{aligned}
\boldsymbol{E}\left(\frac{\varepsilon^{2}}{R_{2}^{2}}\right) &= \boldsymbol{E}\left(\frac{\varepsilon^{2}}{\left(\bar{R}_{2}-\varepsilon\right)^{2}}\right) \\
&\simeq \frac{1}{\bar{R}_{2}^{2}}\boldsymbol{E}\left(\varepsilon^{2}\right) \\
&= \frac{1}{\bar{R}_{2}^{2}}\sigma^{2}
\end{aligned}$$

After replacing $\phi\left(g,\sigma^{2}\right)$ in (12) we get

$$C_{1}^{st}=\left[\frac{1}{1+\beta\left(1+\beta\right)\phi\left(g_{2},\sigma^{2}\right)}\right]\omega$$

.3.4 Partial derivative of W^{ST} and W^{LT} wrt to ω

Using the envelop theorem, we have that

$$\begin{aligned} \frac{\partial W^{ST}}{\partial \omega} &= \frac{\partial C_1}{\partial \omega}\frac{1}{C_1} + \boldsymbol{E}\left[\beta \frac{\partial C_2}{\partial \omega}\frac{1}{C_2} + \beta^2 \frac{\partial C_3}{\partial \omega}\frac{1}{C_3}\right] \\ &= \frac{1}{\omega}\left[\frac{\partial C_1}{\partial \omega}\frac{\omega}{C_1} + \boldsymbol{E}\left[\beta \frac{\partial C_2}{\partial \omega}\frac{\omega}{C_2} + \beta^2 \frac{\partial C_3}{\partial \omega}\frac{\omega}{C_3}\right]\right] \end{aligned}$$

the log utility function is homothetic therefore $\frac{\partial C_j}{\partial \omega}\frac{\omega}{C_j} = 1$ for $j = 1, 2, 3$, and

$$\frac{\partial W^{ST}}{\partial \omega} = \frac{1 + \beta(1 + \beta)}{\omega}$$

The result does not depend on $\boldsymbol{E}$, thus the same computation carry on for $\frac{\partial W^{LT}}{\partial \omega}$.

.3.5 Data definitions and sources

All variables are denoted in US dollars.

Short-term debt is defined along the remaining maturity criteria. That is, liabilities with an original maturity of one year or less, plus repayments due within the next 12 months on liabilities with an original maturity of over a year, plus arrears (both locational and consolidated). Source : BIS International Financial Statistics.

Total external debt is the sum of public and private external obligations with original or extended maturity of a year or less. Source : World Development Indicators.

M2 ; *Trade* is the sum of exports and imports, Source : World Development Indicators.