Université de Montréal

Essais sur les frictions financières dans les modèles d'équilibre général dynamique

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Thèse présentée à la Faculté des études supérieures
en vue de l'obtention du grade de Philosophiæ Doctor (Ph.D.)
en sciences économiques

Juin 2010

Université de Montréal
Faculté des études supérieures

Cette thèse intitulée :
Essais sur les frictions financières dans les modèles d'équilibre général dynamique

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Thèse acceptée le 13 août 2010

Résumé

Cette thèse examine les effets des imperfections des marchés financiers sur la macroéconomie. Plus particulièrement, elle se penche sur les conséquences de la faillite dans les contrats financiers dans une perspective d'équilibre général dynamique.

Le premier papier construit un modèle qui utilise l'avantage comparatif des banques dans la gestion des situations de détresse financière pour expliquer le choix des firmes entre les prêts bancaires et les prêts du marché financier. Le modèle réussit à expliquer pourquoi les firmes plus petites préfèrent le financement bancaire et pourquoi les prêts bancaires sont plus répandus en Europe. Le premier fait est expliqué par le lien négatif entre la valeur nette de l'entreprise et la probabilité de faire faillite. Le deuxième fait s'explique par le coût fixe d'émission de bons plus élevé en Europe.

Le deuxième papier examine l'interaction entre les contraintes de financement affectant les ménages et les firmes. Une interaction positive pourrait amplifier et augmenter la persistance de l'effet d'un choc agrégé sur l'économie. Je construis un nouveau modèle qui contient des primes de financement externes pour les firmes et les ménages. Dans le modèle de base avec prix et salaires flexibles, j'obtiens une faible interaction négative entre les coûts de financement des firmes et des ménages. Le facteur clé qui explique ce résultat est l'effet du changement contre cyclique du coût de financement des ménages sur leur offre de travail et leur demande de prêts. Dans une période d'expansion, cet effet augmente les taux d'intérêt, réduit l'investissement et augmente le coût de financement des entreprises.

Le troisième papier ajoute les contraintes de financement des banques dans un modèle macroéconomiques avec des prêts hypothécaires et des fluctuations dans les prix de l'immobilier. Les banques dans le modèle ne peuvent pas complètement diversifier leurs prêts, ce qui génère un lien entre les risques de faillite des ménages et des banques. Il y a deux effets contraires des cycles économiques qui affectent la prime de financement externe de la banque. Premièrement, il y a un lien positif entre le risque de faillite des banques et des emprunteurs qui contribue à rendre le coût de financement externe des banques contre cyclique. Deuxiément, le lissage de la consommation par les ménages rend la proportion de financement externe des banques pro cyclique, ce qui tend à rendre le coût de financement bancaire pro cyclique. En combinant ces deux effets, le modèle peut reproduire des profits bancaires et des ratios d'endettement bancaires pro cycliques comme dans les données, mais pour des chocs non-financiers les frictions de financement bancaire dans le modèle n'ont pas un effet quantitativement significatif sur les principales variables agrégées comme la consommation ou l'investissement.

Mots-clés : frictions financières, verification côuteuse de l'état, détresse financière, prime de financement externe, cycles économiques, contraintes de crédit,capital bancaire

Abstract

This Dissertation examines the effect of financial market imperfections on the Macroeconomy. More particularly, it focuses on the consequences of equilibrium default using a Dynamic General Equilibrium approach.

The first paper builds a dynamic general equilibrium model that emphasizes banks' comparative advantage in monitoring financial distress in order to explain firms' choice between bank loans and market debt. Banks can deal with financial distress more cheaply than bond holders, but this requires a higher initial expenditure proportional to the loan size. In contrast, bond issues may involve a small fixed cost. Entrepreneurs' choice of bank or bond financing depends on their net worth. The model can explain why smaller firms tend to use more bank financing and why bank financing is more prevalent in Europe than in the US. The first fact can be explained by the negative link between the net worth of a business and its default probability. Explaining the second fact requires taking into account the higehr fixed cost of issuing market debt in Europe.

The second paper examines the possibility of feedback effects between between the financing constraints of households and of firms. A positive interaction between the financial strength of household and firm balance sheets may amplify aggregate shocks and increase the persistence of aggregate fluctuations. I develop a new model that incorporates both firm and household external finance spreads and time varying leverage. Contrary to a common intuition, the baseline Real Business Cycle model with credit constraints produces a small negative interaction between the costs of external financing for firms and households. The key factor in this result is the effect of changes in the external finance premium on borrowers' labour supply and the demand for loans. The reduction in households' cost of borrowing in a boom decreases labour supply and raises houshold loan demand. This increases interest rates, crowds out investment, and raises borrowing costs for financially constrained firms.

The third paper integrates household financing frictions with bank financing frictions and house price fluctuations in a dynamic general equilibrium model. The key assumption in the model is that a bank cannot fully diversify shocks, leading to a link between household and bank sectors' default risks. The cyclical behaviour of banks' external funding cost is determined by two main factors. On one hand, booms improve the financial health of the banks' borrowers which tends to reduce the cost of bank funding. On the other hand, consumption smoothing by savers and borrowers during booms increases the proportion of external financing in the banks' balance

sheet which tends to increase the cost of bank funding. As a result of these opposing effects, the model matches procyclical profits and leverage in the financial sector, as observed in the data, but for non financial shocks the banking frictions in the model have an insignificant impact on the main macroeconomic aggregates such as output, consumption and investment.
Keywords : financial frictions,costly state verification, financial distress, external finance premium, business cycle ,credit constraints, bank capital

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à mes parents

Remerciements

J'aimerais d'abord remercier mes directeurs de recherche, Francisco Ruge-Murcia et Onur Ozgür, pour leur soutien et leurs commentaires tout au long de ma recherche.

J'aimerais ensuite remercier ma co-auteure Jing Yang pour m'avoir donné la chance de travailler sur un projet si intéressant avec elle.

J'aimerais aussi remercier le département de sciences économiques de l'université de Montréal et le CIREQ pour leur soutien académique et financier et pour tout ce que j'ai appris pendant mes études ici. Je dois également remercier la banque d'Angleterre et la banque centrale Européenne pour leur accueil chaleureux et stimulant durant les étés 2008, 2009, pour toutes les discussions et commentaires que j'ai eu a ces endroits et pour leur soutien financier.

Finalement je suis infiniment reconnaissant à mes amis, mes collègues du programme de doctorat et mes parents. Tout ceci aurait été impossible sans leur soutien.

Apport des coauteurs

Chapitre 1 : Banks As Better Monitors and Firms' Financing Choices in Dynamic General Equilibrium

Cet article a été redigé sans coauteur.

Chapitre 2 : Are There Any Spillovers between Household and Firm Financing Frictions ? A Dynamic General Equilibrium Analysis

Cet article a été redigé sans coauteur.

Chapitre 3 : Bank Capital, Housing and Credit Constraints

Cet article a été écrit en collaboration avec Jing Yang de la banque d'Angleterre. Bien que j'ai été maître d'oeuvre pour cet article Jing Yang a joué un rôle critique dans l'élaboration de la question du project ainsi que lors de certaines étapes importantes.

Liste des abbréviations

DSGE	Dynamic Stochastic General Equilibrium
BGP	Balanced Growth Path
LTV	Loan to Value Ratio
RBC	Real Business Cycle
TFP	Total Factor Productivity
IRF	Impulse Response Function
BGG	Bernanke Gertler and Gilchrist

Introduction

Cette thèse regarde de plus près les conséquences macroéconomiques du défaut sur les contrats financiers. Les imperfections des marchés financiers ont pris une place de plus en plus importante dans les débats et les modèles macroéconomiques. Les modèles d'équilibre général permettent de mieux structurer les débats et les décisions sur les politiques qui essaient de contrer les effets de ces imperfections financières. Généralement ces modèles se concentrent sur l'impact du choix entre le financement interne et l'émission de dette, ce qui se justifie par la dominance de ces types de financement (Damodaran (1999) [46]. Le niveau d'endettement des agents économiques est limité dans la plupart des modèles par une contrainte de collatéral qui exclut le défaut sur le contrat (voir Kiyotaki et Moore (1997) [83] et Carroll (2001) [35] pour des exemples représentatifs). Dans ce type de modèle un contrat de dette garantit un taux d'intérêt sans risque qui est le même pour le gouvernement ou pour une petite entreprise. En réalité, les entreprises et les ménages font occasionnellement défaut sur leurs paiements de dette et les taux d'intérêts sur la dette contiennent une prime qui compense ce risque. Il existe des modèles qui permettent le défaut (par exemple Bernanke et al. (1999) [19] pour les entreprises et Chaterjee et al. (2007) [38] pour les ménages), mais ces modèles doivent souvent simplifier en supposant des emprunteurs neutres au risque ou en se restreignant a l'étude d'états stationnaires sans fluctuations agrégées. Un des objectifs de cette thèse est de relâcher ces contraintes en proposant un modèle qui préserve au moins partiellement les conséquences de l'aversion au risque de la plupart des agents économiques tout en permettant facilement l'étude des fluctuations agrégées.

Le premier chapitre étudie le choix de l'entreprise entre les prêts bancaires et la dette de marché en équilibre général. Il existe déjà une vaste littérature théorique qui étudie ce choix en équilibre partiel (voir par exemple Diamond (1991) [49] et Rajan (1992) [103]. Il y a aussi des études empiriques en forme réduite qui étudient cette décision (par exemple Kashyap et al. (1993) [78] ou Cantillo et Wright (2000) [29]. Mais avec l'exception de De Fiore et Uhlig (2005) [55], les implications d'équilibre général de ce choix n'ont pas été étudiés. Ici je propose un modèle inspiré du fait stylisé que généralement les banques ont un meilleur contrôle sur les problèmes causés par la détresse financière des entreprises que les investisseurs du marché financier qui sont plus dispersés et qui ont souvent moins d'information que les banques sur les emprunteurs. Je généralise le modèle d'asymétrie d'information de Townsend (1979) [114] où le prêteur peut apprendre l'état financier de l'emprunteur en payant un coût de vérification. Je permets à l'entreprise un choix entre la banque qui a un coût de verification plus faible mais un coût ex-ante plus élevé et un contrat du marché financier. Tant qu'il existe un certain niveau de rendements décroissants à l'échelle dans la production (qui peut être pris comme une forme réduite pour de la compétition imparfaite ou un autre facteur de production difficile a ajuster comme l'apport spécifique du gérant de la firme) les entreprises avec une valeur nette plus faible dans ce modèle préfèrent le financement bancaire. Ceci reproduit le fait stylisé que les entreprises de plus petite taille utilisent plus fréquemment le financement bancaire. En tenant compte aussi du coût fixe plus élevé d'émission de bons en Europe le modèle peut aussi capter la dépendance beaucoup plus importante sur les prêts bancaires en Europe par rapport aux Etats Unis.

Le deuxième chapitre examine les liens entre le niveau d'endettement des entreprises et des ménages dans un cadre d'équilibre général dynamique. L'idée qu'un resserrement des contraintes financières des ménages peut empirer une récession est présente dans beaucoup des débats sur les politiques fiscales par exemple. La même idée est avancée pour les contraintes financières des entreprises. L'idée principale est celle de l'accélérateur financier (Bernanke et al (1999) [19] dans lequel un choc négatif réduit l'accès aux fonds externes des entreprise, ce qui réduit encore plus l'activité économique, qui contribue à un déclin encore plus fort de la capacité de financement des entreprises. Ceci suggère que l'interaction entre ces deux types de contraintes financières pourrait être positive et contribuer a aggraver les cycles économiques.

J'étudie cette possibilité dans un modèle où une partie des ménages et les entreprises font face à des contraintes financières.

Le modèle permet aux ménages et aux firmes de faire défaut sur leurs paiements quand la valeur de leur collatéral est relativement faible, tout en permettant l'aversion au risque des emprunteurs et les fluctuations agrégées. Ceci génère des primes de financement externes et des mouvements endogènes dans le ratio d'endettement des emprunteurs, ce qui n'est pas le cas des modèles sans défaut d'équilibre comme Kiyotaki et Moore (1997) [83].

Pour mieux isoler l'interaction entre les contraintes de financement des firmes et des ménages, j'utilise une modèle à prix et salaires flexibles. Ce modèle génère facilement des primes de financement externes contre cycliques, mais l'interaction entre les primes de financement des firmes et des ménages est faiblement négative. Même si le resserrement de contraintes financiers réduit la consommation des ménages, qui tend a réduire la production et serrer les contraintes financières des entreprises, il y a d'autres forces qui vont dans le sens contraire. L'empirement de la situation financière des ménages les pousse à travailler plus et réduit les salaires d'équilibre. L'augmentation de la prime de financement externe réduit la demande des prêts des ménages, ce qui réduit les taux d'intérêt auxquels font face les entreprises. Ces effets sur les salaires et les taux d'intérêt réduisent le coût de financement externe des entreprises.

Finalement, le dernier chapitre ajoute des imperfections dans le financement des banques au cadre d'analyse du deuxième chapitre. Ceci nous permet d'étudier l'interaction entre les risques du crédit des prêts aux ménages et des prêts aux banques qui financent les ménages. En comparaison, la supposition standard dans les modèles

d'équilibre général est que les banques peuvent complètement diversifier le risque de faillite de leurs prêts. Pour un ratio donné de financement externe des banques, il y a un lien positif entre la probabilité de défaut des banques et des ménages. Ceci tend à rendre la prime de financement externe des banques contre cyclique. Par contre, le lissage de consommation par les ménages tend à rendre le ratio de financement externe des banques pro cycliques. Ceci contribue à une prime de financement externe des banques pro cyclique. Pour toutes les calibrations essayées, on trouve que face à des chocs non financiers le deuxième effet domine.

Chapitre 1

Banks As Better Monitors and Firms' Financing Choices in Dynamic General Equilibrium

Abstract

This paper builds a dynamic general equilibrium model that emphasizes banks' comparative advantage in monitoring financial distress in order to explain firms' choice between bank loans and market debt. Banks can deal with financial distress more cheaply than bond holders, but this requires a higher initial expenditure proportional to the loan size. In contrast, bond issues may involve a small fixed cost. Entrepreneurs' choice of bank or bond financing depends on their net worth. The steady state of the model can explain why smaller firms tend to use more bank financing and why bank financing is more prevalent in Europe than in the US. We find that a higher fixed cost of issuing market debt is a key factor in replicating the higher use of bank financing relative to market debt in Europe. Finally, we find that for plausible calibrations one can predict aggregate quantities just as well using a model with only one type of loan with costs of financial distress that are an average of the costs for bank loans and market debt.

JEL classification : E4, G3
Key words : financial frictions, costly state verification, financial distress

1.1 Introduction

Debt financing is the most prevalent form of external financing in most developed countries(see for example Damodaran (1999) [46] and Gorton and Winton (2002)[64]). One of the most important characteristics of debt is whether it is issued by a bank(or a similar institution such as a finance company) or whether it is market debt. There is a vast theoretical litterature discussing the difference between these two types on debt(Diamond (1991)[49] and Rajan (1992)[103] are seminal contributions), emphasizing the trade off between the better loan monitoring or information gathering abilities of banks and the extra costs attached to borrowing from a bank. Because the level of financial frictions attached to the two types of loans is different, the composition of financing between them may matter for the overall level of financial frictions affecting firms and for macroeconomic outcomes. To complicate matters, the choice between bank and market debt almost certainly depends on aggregate macroeconomic conditions.

Most dynamic general equilibrium models with financial frictions ignore the distinction between bank loans and market debt[32][19]. Models that examine the effect of frictions between banks and their depositors assume that banks are responsible for all lending in the economy[91][42]. This assumption leads to a potential overestimate of the impact of bank lending on the transmission of shocks, since it eliminates the possibility of using other types of financing. Market debt accounts for 57.5% of total non financial sector debt in the US and for 12% of non financial sector debt in the Euro area (De Fiore and Uhlig (2005) [55]). Clearly assuming 100% bank or bond lending may be misleading in analyzing financial frictions in the US. For the Euro area such an assumption may seem like a good approximation, except that it is still possible that the low average proportion of bond financing hides important variation across the business cycle which may be relevant for the propagation of various shocks.

Investigation of the effect of the financing choice between bank and market debt on macroeconomic outcomes has been mostly based on reduced form models. In studies with aggregate level VAR's such as Kashyap, Stein and Wilcox (1993)[78] or Oliner and Rudebusch (1996) [96] it is difficult to distinguish between the hypothesis that financial frictions in general affect the transmission of economic shocks from the hypothesis that that the source of financing matters. Micro level data as in Cantillo and Wright (2000) [29] can provide stronger evidence on the importance of macroeconomic conditions for the choice between bank and market debt, but without a general equilibrium framework it is impossible to go in the other direction and judge the impact of financing choice on macroeconomic outcomes.

In this paper we develop a dynamic general equilibrium model of the firm's choice between bank and bond market financing based on the idea that banks are better loan monitors than markets in financial distress situations. Firms facing financial frictions can use either bank financing or bond financing. Lending to firms is subject to a costly state verification problem as in [32]. Banks are better monitors than debt markets, but their superior lending technology requires them to spend more resources per dollar of loans before the firm produces. Market debt has higher monitoring costs, and it may require a fixed under-writing cost. This paper studies the implications of the model for the steady state of such an economy.

The use of a dynamic general equilibrium framework allows us to make a more quantitative assessment of the magnitude of frictions required to generate realistic financing choice patterns. It also allows investigation of financing choice dynamics in reaction to structural changes in financing costs as well as in reaction to business cycles.

Our modeling of banks as better monitors in the costly state verification framework is based on empirical evidence suggesting that banks' key advantage is in dealing with financial distress situations. Bank loans are easier to renegotiate, and banks have a better understanding of the businesses they are dealing with than bondholders, for example by forcing borrowers to maintain a transactions account at the bank[93]. As a result banks are more capable of dealing with problems such as risk shifting in default, and they are less likely to engage in inefficient liquidation of firms[62][29][23].

In contrast to most papers that model costly state verification in dynamic general equilibrium, such as Carlstrom and Fuerst, (1998) [32], and Bernanke Gertler and Gilchrist (1999) [19], we assume that firms' production technology exhibits decreasing returns to scale. As a result, firms' financing choice depend on their net worth. Our analysis shows that differences in net worth may be important in accounting for the choices between bank and bond financing observed in the data. In particular, as in the data, higher net worth reduces the likelihood of choosing bank financing(see Cantillo and Wright (2000)[29] and Sufi (2005)[112] for evidence on this point). This occurs in our model despite the fact that all firms are equally productive ex-ante, before signing the financial contract. One could imagine that the link between net worth and financing choice is natural in the presence of a fixed cost of issuing bonds. However, the benchmark model produces a strong negative correlation between bank financing and net worth even without the fixed cost. Intuitively, higher net worth reduces the probability of financial distress, which makes the bank's comparative advantage in handling financial distress less valuable. The direct link between net worth and financing choice in the model is in contrast with most previous theoretical work that has explained why less productive firms may prefer bank loans, with the link between firm size and bank financing explained through a positive correlation between firm size and productivity(see for example Rajan (1992) [103]and Diamond (1991) [49]).

The benchmark model , where the only cost of bond financing is the cost of auditing distressed firms, cannot explain the high relative use of bank financing in Europe without unrealistically low costs of bank financing. [1] Therefore, we extend the model by assuming that issuing a bond in Europe also requires a small fixed cost. The extra cost is motivated by evidence that until recently bond financing was more expensive in Europe than in the US[107]. The fixed cost assumption is motivated by

[1]Europe refers to the Euro Area in this paper.

evidence of large economies of scale in market debt issue costs, which are not present for bank debt[46][37]. We find that a small fixed cost of bond issue (around 0.22% of the average value of issued bonds) can explain most of the discrepancy between the relative amount of bank financing in Europe and the US. To the degree that bond markets in Europe have become more competitive[107], the ratio of bond to bank financing in Europe may converge to that in the US.

Finally, we examine the importance of explicitly modeling the choice between bank and market debt for aggregate output and consumption. We find that for reasonable calibrations, the steady state aggregates of the model are virtually identical to those of a model with only one type of financial intermediary with monitoring costs that are an average of those of the bank and bond contracts. This suggests that at least for the analysis of the steady state, a researcher interested only in aggregates may choose to ignore the choice between bank and market financing.

Several papers have studied the choice between bank and market debt in partial equilibrium (prominent examples include Rajan (1992)[103], Diamond (1991)[49], Holmstrom and Tirole (1997) [69]and Bolton and Freixas (2000) [23]). The general message of most of these papers is that more profitable(for a fixed loan size) or higher quality firms tend to prefer market debt, while lower quality firms will prefer bank loans. Holmstrom and Tirole (1997)[69] distinguish between unmonitored lending(market bonds) and monitored lending(bank lending), where monitoring increases entrepreneur effort and the success probability of projects. Their model generates a negative link between bank finacncing and net worth, just like our model. Meh and Moran(2007) [91] incorporate the Holmstrom and Tirole model into a fully specified dynamic general equilibrium model with entrepreneur and bank capital dynamics.The model in this paper examines a mechanism for the link betwee net worth and financing choice which complements the moral hazard based mechanism in Holmstrom and Tirole[69], while being more focused on the role of banks in managing financial distress emphsized by the empirical evidence. Furthermore, Holmstrom and Tirole only explore the extreme assumptions of a fixed project size and variable project size with linear returns. Assuming linear returns as in Meh and Moran's paper eliminates any possible link between the size of firms and financing choice, while assuming only a fixed project size is usually unrealistic.

Perhaps the closest papers to this one are Cantillo and Wright (2000) [29] and De Fiore and Uhlig (2005) [55]. Cantillo and Wright's model generates a negative link between net worth and financing and bank financing due to a higher default rate for smaller firms and banks providing cheaper reorganisation in default. They use a partial equilibrium framework with a fixed project size. As a result their framework ignores the possibility that larger firms may prefer bank financing if they are allowed to undertake a larger project and (as in their model and in essentially all applications of the costly state verification framework) monitoring costs are increasing in the size of the project. This effect is eliminated by assumption in a fixed project size model. This paper explores the intermediate case of variable project size with nonlinear returns and provides conditions that generalize some of the insights from the fixed project size model to the more general setup.

De Fiore and Uhlig[55] model the firm's choice between bank and bond financing in a costly state verification framework and integrate this choice into a standard RBC framework. They model banks as offering better ex ante screening of projects at a cost, generating a tradeoff in which lower productivity entrepreneurs prefer bank financing. The realism of their focus on superior ex ante screening by banks is unclear. In fact, it may be more realistic to model bond market lenders as better screeners of projects due to the screening activities of bond rating agencies and investment banks[61]. Furthermore, the prediction of their model that more profitable firms prefer market debt is empirically controversial(see Sufi (2005)[112] and Cantillo and Wright (2000) [29]). Furthermore, their model does not capture the key stylized fact that the use of bank financing relative to market debt decreases with the size of firms. [2]

In the rest of the paper we proceed as follows : section 2 describes the model and provides a sufficient condition for the model to match the empirical link between firm size and the choice between bank and market debt. Section 3 discusses the results of numerical simulations of the model's steady state. Section 4 concludes.

1.2 The Model

The model features overlapping generations of risk neutral entrepreneurs, a representative risk averse worker and a continuum of pefectly competitive financial intermediaries. Entrepreneurs produce all the output in the economy using capital and labour. They accumulate net worth using capital. When their accumulated net worth is insufficient to fully fund their desired output level, they require loans from financial intermediaries. Due to information frictions, loans require using one of two types of financial intermediaries :banks and bond mutual funds. The differences between these intermediaries and the choice between them will be described in greater detail below.

1.2.1 Entrepreneurs

The heart of the model is the entrepreneur's intratemporal financing and production decision. Therefore we start by describing the financial contracting environment in any given period.

[2] A recent paper by Champonnois[37] estimates a structural model of bank versus bond financing that reproduces the empirical link between the use of bank loans and firm size, assuming that larger firms have systematically higher productivity levels than smaller firms. The two period nature of the model and the assumption that entrepreneurs do not have any net worth(no equity) makes integration of the model into standard dsge's difficult, and may miss important dynamics of financing choice. Also, it is not clear that larger firms are always more productive.

Entrepreneur Production and Financing Decisions for a Fixed Level of Net Worth

Entrepreneur j produces final output using the production function
$y_{jt} = z_t \omega_{jt} k_{jt}^{\alpha} l_{jt}^{\gamma}$, , $0 < \theta \equiv \alpha + \gamma < 1$,

The firm specific productivity shock $\omega_{jt} \in [0, \infty)$ is i.i.d across both entrepreneurs and time, has a CDF $\Phi(\omega)$, a PDF $\phi(\omega)$ and $E\omega_{jt} = 1$. ω_{jt} is unknown when signing the financial contract. We define $\bar{y}_{jt} \equiv Ey_{jt} = z_t k_{jt}^{\alpha} l_{jt}^{\gamma}$. The realisation of ω_{jt} is the private information of the entrepreneur, but can be observed by a type i intermediary at a cost of $\mu_i \bar{y}_{jt}$. z_t is an aggregate productivity shock with a mean of 1.

Entrepreneurs rent capital at a rate r_t and buy labour from households at a wage rate w_t. Production requires spending $x_{jt} = r_t k_{jt} + w_t l_{jt}$ before output is obtained. The entrepreneur has n_{jt} in internal funds available, of which he devotes $\bar{n}_{jt} \leq n_{jt}$ to the project. If the desired x_{jt} exceeds the entrepreneur's internal funds dedicated to the project, the entrepreneur will require external financing from a lender. Alternatively, we can think of the entrepreneur as being able to post a collateral of n_{jt} before output is realized. Any loan below n_{jt} does not involve any financing frictions or other costs. But any part of the loan above n_{jt} will be subject to information frictions.

There is a continuum of fully diversified financial intermediaries of two types : banks and mutual funds, indexed by $i \in \{b, m\}$. Both intermediaries collect funds from households, and use them to make loans to entrepreneurs. Competition for borrowers ensures that the financial intermediaries make zero profits. Because financial intermediaries can fully diversify the idiosyncratic risk of the entrepreneurs, households are risk neutral with respect to intermediaries' loan portfolios. Because the loans are intratemporal and risk free, the required gross rate of return is 1.

The difference between the two types of intermediaries is that banks are better informed about borrowers than mutual funds. This superior information makes banks better monitors in case of financial distress. In particular, banks can observe ω_{jt} at a lower cost than mutual funds.

Banks can learn ω_{jt} at a cost of $\mu_b \bar{y}_{jt}$. Mutual funds can learn ω_{jt} at a cost of $\mu_m \bar{y}_{jt}$,where $\mu_m > \mu_b$. With a more general interpretation of audit costs as financial distress costs one can imagine for example that auditing prevents entrepreneur from taking on risky projects that may benefit him but reduces the expected value of the assets obtained by lender, and banks are better at controlling this risk-shifting. In order to offer lower cost monitoring, banks must spend $\tau(x_{jt} - \bar{n}_{jt})$, where $\tau > 0$. These could be interpreted directly as costs of gathering more information on lenders. They could also be seen as a reduced form for costs related to frictions between banks and depositors in a model where banks cannot accumulate any capital. [3] At the same time, bond mutual funds may require a fixed cost C_m in order to issue a bond.

To simplify notation we can abstract from time and entrepreneur subscripts. It

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[3] For example, we can imagine 1 period bankers that can run away with and consume a proportion τ of the loan. In this case the loan contract must ensure that the expected repayments by entrepreneurs net of the audit costs and the deposit repayments exceed $\tau(x_{jt} - \bar{n}_{jt})$, which is exactly the bank's break-even constraint in the financial contract.

is convenient to first pick the optimal amounts of capital and labour given a total expenditure $x = rk + wl$ and characterize the solution to the contract in terms of x and the bankruptcy threshhold $\bar{\omega}$. For a given expenditure level the optimal expected output is $\bar{y} = Mx^{\theta}$, where $M = z\frac{\alpha^{\alpha}\gamma^{\gamma}}{\theta^{\theta}r^{\alpha}w^{\gamma}}$. Once we have solved the financial contract, factor demands are given by $k = \frac{\alpha}{\theta r}x$ and $l = \frac{\gamma}{\theta w}x$.We start by solving for the entrepreneur's production level when he is restricted to self financing.The expenditure level without access to external financing solves

$\pi_a^e = \max_x Mx^{\theta} + (n - x)$

subject to

$x \leq n.$

$x \geq 0.$

Depending on the availability of internal funds, the entrepreneur either picks the first-best interior solution $x_a = \hat{x} \equiv (\theta M)^{1/(1-\theta)}$, or he sets $x_a = n$. In fact since $\hat{x}$ is independent of n, any entrepreneur with n above a certain threshold value will prefer self-financing.

We now turn to the contract conditional on the entrepreneur requiring external financing and having chosen a type i intermediary. Let R_i be the required gross rate of return to the lender($R_b = 1 + \tau, R_m = 1$). Because the entrepreneur has access to a storage technology, his opportunity cost of funds is 1. The optimal contract specifies a state contingent repayment schedule and the set of audited states in order to maximize entrepreneur profits subject to incentive compatibility constraints for the entrepreneur and the lender's break-even constraint.The distribution of ω and the auditing costs satisfy the conditions in Gale and Hellwig (1985) [58] for the optimal contract to be a debt contract with a threshhold $\bar{\omega}$ such that the repayment is $b(\omega) = y$ for $\omega \leq \bar{\omega}$ and $b(\omega) = \bar{\omega}\bar{y}$ if $\omega > \bar{\omega}$.

We define the entrepreneur's and the lenders' expected shares of $\bar{y}$ as

$$
\begin{aligned}
f(\bar{\omega}) &= \int_{\bar{\omega}}^{\infty} \omega d\Phi - \bar{\omega}[1 - \Phi(\bar{\omega})] \text{ for the entrepreneur.} \\
m(\bar{\omega}) &= \int_{0}^{\bar{\omega}} \omega d\Phi + \bar{\omega}[1 - \Phi(\bar{\omega})] - \mu\Phi(\bar{\omega}) \text{ for the lender.}
\end{aligned}
$$

Like other papers(e.g Bernanke, Gertler and Gilchrist (1999) [19] and Covas and Den Haan (2007) [45]), we assume that the hazard rate of ω is increasing in ω at the optimal $\bar{\omega}$,

Assumption 1 :

$$
\frac{d}{d\bar{\omega}}\left(\frac{\phi(\bar{\omega})}{1 - \Phi(\bar{\omega})}\right) > 0, \ \phi(\bar{\omega}) > 0 \text{ when } \bar{\omega} > 0.
$$

This assumption is satisfied by commonly used distributions such as the lognormal or uniform distributions. Define $C_i \geq 0$ to be the fixed cost of issuing debt for type i intermediary, where $C_b = 0$. Define $1(x - \bar{n} > 0)$ as the indicator function for $x - \bar{n} > 0$.

The optimal contract with type $i \in \{b, m\}$ financial intermediary solves

$$\pi_i^e = \max_{x,\bar{\omega},\bar{n}} f(\bar{\omega}_i) M x_i^\theta + (n - \bar{n}_i)$$

subject to

$$\begin{aligned} m(\bar{\omega}_i)\bar{y}_i &\geq R_i(x_i - \bar{n}_i) + C_i 1(x - \bar{n} > 0) && (1.1) \\ \bar{n}_i &\leq n. && (1.2) \\ \bar{n}_i &\geq 0 && (1.3) \\ x_i - \bar{n} &\geq 0 && (1.4) \end{aligned}$$

If $\phi(0) = 0$,$R = 1$ and $C_m = 0$, then $x - \bar{n} > 0$ whenever $n < \hat{x}$, the first best level of output. Otherwise, when $C_m = 0$ we can guarantee that $x - \bar{n} > 0$ on an interval of $n's$ $(0, \bar{n})$, where $\bar{n}$ tends to $\hat{x}$ as μ and τ go to 0 (See the appendix for the proof). When $C_m > 0$, $x - \bar{n} > 0$ as long as C_m is not too large. In this section, we assume $x - \bar{n} > 0$. [4] The numerical algorithm in the next sections allows for the possibility of rationing. Let λ_i, ξ_i, ψ_i be the lagrange multipliers for constraints (1-3) respectively. Besides the complementary slackness conditions, the first order conditions are

$$\begin{aligned} x &: \theta M x_i^{\theta-1}[f(\bar{\omega}_i) + \lambda_i m(\bar{\omega}_i)] = \lambda_i R_i && (1.5) \\ \bar{\omega} &: f'(\bar{\omega}_i) + \lambda_i m'(\bar{\omega}_i) = 0 && (1.6) \\ \bar{n} &: \psi_i + \lambda_i R_i = \xi_i + 1. && (1.7) \end{aligned}$$

Finally, the entrepreneur picks $Ey^e = \max[\pi_a^e, \pi_m^e, \pi_b^e]$.

The following lemma collects some straightforward results that simplify the solution of the model and help us characterize the financing choice of entrepreneurs :

Lemma 1. *a) At the optimum, $m'(\bar{\omega}) > 0$ and the bank's break-even constraint is binding.b) Under the assumption that $\Phi'(\bar{\omega}) = \phi(\bar{\omega}) > 0$ the entrepreneur always chooses Maximal Equity Participation(MEP) : $\bar{n} = n$.* [5] *c)external financing is never optimal if the entrepreneur's wealth constraint does not bind. d)Under assumption 1,$\lambda'(\omega) > 0$.e)$\frac{d\bar{\omega}}{dn} < 0$ and $\frac{d\bar{\omega}}{dC_m} > 0$. f)$\frac{d\bar{\omega}}{d\tau} < 0$ as long as C_m is not too large.*

Démonstration. see the appendix. □

[4] Alternatively one can think of all the proofs in this section as applying only to those firms that are not rationed.

[5] The strict optimality of MEP contracts relies on our limited liability assumption concerning $n - \bar{n}$. Gale and Hellwig[58]assume $n - \bar{n}$ can be used as collateral. In that case the optimal level of equity participation is indeterminate, and the MEP contract only weakly dominates any other contract.

Due to the i.i.d nature of the productivity shock ω_{jt}, the only ex-ante heterogeneity among entrepreneurs is due to different levels of net worth n_{jt}. There are several effects that determine the desirability of bank or bond financing for a given level of net worth. The model emphasizes the role of banks in reducing costs of financial distress. Since the default rate is decreasing in net worth ($\frac{d\bar{\omega}}{dn} < 0$) for a fixed financing type, it seems natural then that holding $\bar{y}$ constant, smaller firms that are more likely to default for a given financing type will gravitate towards banks. At the same time a higher net worth increases the project's expected output $\bar{y}$, which increases expected default costs $\mu\Phi(\bar{\omega})\bar{y}$ for a given μ and $\bar{\omega}$. For a fixed project size x, the extra cost per dollar of bank loan $\tau(x-n)$ penalizes small firms with higher $x-n$. Things are less clear cut when firms can adjust the size of their project. Higher net worth reduces the desired leverage ratio $\frac{x-n}{n}$. [6] Decomposing the ex-ante cost of bank financing $\tau(x-n)$ as $\tau\left(\frac{x-n}{n}\right)n$, we see that it may actually decline in n, though this does not have to be the case. We cannot theoretically rule out that the last two effects overwhelms the effect of a lower $\bar{\omega}$ and makes bank financing more attractive for larger firms for general values of n and θ but, we will find conditions for larger firms to prefer bond financing conditions on certain range of $n's$ and $\theta's$. The numerical analysis in the next section will examine the plausibility of those conditions.

Before proceeding with the analysis for the decreasing returns to scale, we can verify that under the standard assumption in for example Carlstrom and Fuerst (1998) [32] or Bernanke, Gertler and Gilchrist (1999) [19], of constant returns to scale and no fixed cost of market finance the choice of financing type does not depend on net worth. Therefore, all firms in our setup would choose the same financial intermediary if $\theta = 1$. The only possibility for a link between net worth and financial intermediation choice in the constant returns to scale case arises when there is a fixed cost of market financig $C_m > 0$ in an environment in which market financing would dominate when $C_m = 0$ or when some firms would be rationed at $x - n = 0$ with market financing :

Proposition 2. *a)Suppose $\theta = 1$ and for each i $M > \frac{R_i}{1-\mu\phi(0)}$. If $C_m = 0$,then the optimal choice of financial intermediary is independent of n. Therefore, all firms choose the same financial intermediary type. All firms prefer bank financing when $\tau < \bar{\tau}$ and all firms prefer bond financing when $\tau > \bar{\tau}$, for some $\bar{\tau} > 0$. b) If $C_m > 0$, any firm rationed at $x - \bar{n}$ by the bond contract will pick bank financing. Among non-rationed firms, If the optimal form of intermediation is bank financing when $C_m = 0$ then this is also the optimal choice for all firms when $C_m > 0$. If the optimal financial intermediation is market financing when $C_m = 0$, then in the economy with $C_m > 0$ there exists a threshold $\hat{n}$ such that all non rationed firms with $n < \hat{n}$ prefer bank financing while all non rationed firms with $n \geq \hat{n}$ prefer market financing.*

[6] For the leverage ratio, note that combining the foc for x and the break even constraint we have $\theta(1 - n/x + \frac{C}{Rx})\left(\frac{f+\lambda m}{\lambda m}\right) = 1$. The second term in brackets is decreasing in $\bar{\omega}$. So x/n must be increasing in $\bar{\omega}$ if $C = 0$. The comparative statics with respect to n and M now follow from the relation between $\bar{\omega}$ and those parameters. With $C > 0$ its is possible for x/n to decrease in $\bar{\omega}$, but by continuity $\frac{d(x-n)/n}{dn} < 0$ should still hold for a small enough C.

Démonstration. a) Consider first the case when $C_m = 0$. Our assumption that $M > \frac{R_i}{1-\mu\phi(0)}$ ensures that $x - n > 0$ for all firms(see the appendix for the proof when $\theta < 1$. The proof for $\theta = 1$ is similar). Using the break-even constraint of the financial intermediary to solve for x, the entrepreneur's expected profit with type i intermediary is $\pi_i^e = f(\bar{\omega}_i)M\frac{nR_i}{R_i - m(\bar{\omega})M}$. $\pi_b^e - \pi_m^e = Mn\left(\frac{(1+\tau)f(\bar{\omega}_b)}{(1+\tau)-m(\bar{\omega}_b)M} - \frac{f(\bar{\omega}_m)}{1-m(\bar{\omega}_m)M}\right) \equiv Mn\Delta > 0$ iff $\Delta > 0$. From the first order conditions, $\frac{d\omega}{dn} = 0$ when $\theta = 1$. Since n does not directly affect $f(\bar{\omega})$ or $m(\bar{\omega})$ Δ is independent of n, making the sign of $\pi_b^e - \pi_m^e$ independent of n. Since $\frac{d\pi^e}{d\mu} < 0$, $\pi_b^e - \pi_m^e > 0$ when $\tau = 0$. $\frac{d\pi^e}{d\tau} < 0$, and therefore $\pi_b^e - \pi_m^e$ is decreasing in τ. Finally, $\lim_{\tau\to\infty} \pi_b^e - \pi_m^e < 0$. By the continuity of $\pi_b^e - \pi_m^e$ in τ, there exists a unique $\bar{\tau}$ such that $\pi_b^e - \pi_m^e > 0$ whenever $\tau < \bar{\tau}$ and $\pi_b^e - \pi_m^e < 0$ for $\tau > \bar{\tau}$.
b) Next, consider the case when $C_m > 0$. Any firm that would be rationed by the bond contract will obviously pick the bank contract, since by our assumption on M the optimal bank contract dominates choosing $x = n$. Now consider firms that are not rationed by the bond contract. The relative profit of bank financing versus bond financing is $\pi_b^e - \pi_m^e = M\left[n\Delta + \frac{f(\bar{\omega}_m)C_m}{1-m(\bar{\omega}_m)M}\right]$. If $\Delta > 0$ (all firms prefer bank financing when $C_m = 0$), then bank financing is preferred iff $n > -\frac{1}{\Delta}\frac{f(\bar{\omega}_m)C_m}{1-m(\bar{\omega})M}$. Since the last expression is negative and $n \geq 0$, this constraint never binds and all firms pick bank financing in this case. If $\Delta = 0$, then clearly all firms prefer bank financing. If $\Delta < 0$ (all firms prefer bond financing when $C_m = 0$), then a firm prefers bank financing iff $n < -\frac{1}{\Delta}\frac{f(\bar{\omega}_m)C_m}{1-m(\bar{\omega}_m)M} \equiv \hat{n}$. All other firms prefer bond financing. Note that $\bar{\omega}$ remains independent of n if $x - n > 0$ regardless of C_m. From the same first order conditions, $\bar{\omega}$ is also independent of C_m. As a result $\hat{n}$ can be computed as the product of C_m and a term that depends only on M. □

The empirical calibrations in the next section suggests that the extra intermediation fee of the bank τ is quite low, casting doubt on the ability of the constant returns to scale model to generate a negative relation between net worth and financing choice, except through the rationing of small firms by the bond contract due to the fixed cost. [7]

We would like to establish some conditions guaranteeing that the model with $\theta < 1$ reproduces the pattern observed in the data where smaller firms prefer bank financing and larger firms prefer market financing. In particular if the derivative of the relative bank versus bond contract profit with respect to wealth $\frac{d(\pi_b^e - \pi_m^e)}{dn} < 0$, and firms choose both bank and bond financing then it must be the case that for some $\bar{n}$ firms with $n < \bar{n}$ prefer bank financing while firms with $n \geq \bar{n}$ prefer bond financing.

The following proposition gives a sufficient condition guaranteeing the existence of an interval of values of $\tau \in (0, \tau^*)$ and an interval of $n \in (0, n^*)$ values for which bond financing becomes more attractive as n increases. The key requirement is that bank financing lowers the shadow cost of external finance λ :

[7] We did a few quick tests of the constant returns to scale model in a partial equilibrium setting, with the wage and interest rate fixed by the steady state of an open economy where there are no financial frictions in the rest of the world. Bank financing was optimal in those tests even for $\tau = 0.08$ which is much higher than the evidence presented in Erosa(2001)[53].

Proposition 3. *Suppose that* $\frac{d\lambda_m}{d\mu_m} > 0$ *at* $n = 0$. *Then there exists a neighbourhood* $N_\varepsilon \subset R^2_+$ *of* $n = 0 = \tau$ *such that* $\frac{d(\pi^e_b - \pi^e_m)}{dn} < 0$ *whenever* $(\tau, n) \in N_\varepsilon$. *Therefore there exist a* τ^* *and a* n^* *such that whenever* $\tau < \tau^*$ *and* $n < n^*$ *we have* $\frac{d(\pi^e_b - \pi^e_m)}{dn} < 0$.

Démonstration. By the envelope theorem $\frac{d(\pi^e_b - \pi^e_m)}{dn} = (1+\tau)\lambda(\bar{\omega}_b(\tau, n)) - \lambda(\bar{\omega}_m(0, n)) \equiv S(\tau, n) - \lambda_m(0, n)$. Since $\mu_b < \mu_m$, $\frac{d\lambda_m}{d\mu_m} > 0$ at $n = 0$ $\frac{d\bar{\omega}}{dC} > 0$ and $\lambda'_i(\bar{\omega}) > 0$, we have $S(0, 0) - \lambda_m(0, 0) = \lambda_b(0, 0) - \lambda_m(0, 0) < 0$. By the maximum theorem, for either $i = b$ or m, $\bar{\omega}_i(\tau, n)$ is continuous on R^2_+ at $\tau = n = 0$. Therefore $S(\tau, n) - \lambda_m(\tau, n)$ is continuous in (τ, n) at $(0, 0)$. Together with $S(0, 0) - \lambda_m(0, 0) < 0$ this implies the existence of the required N_ε neighbourhood. The existence of τ^* and n^* is immediate by taking any rectangle contained in N_ε. □

$\frac{d\lambda}{d\mu} = \frac{\partial\lambda}{\partial\mu} + \lambda'(\bar{\omega})\frac{d\bar{\omega}}{d\mu}$. Since $\frac{\partial\lambda}{\partial\mu} > 0$ and $\lambda'(\bar{\omega})$, $\frac{d\bar{\omega}}{d\mu} \geq 0$ is a sufficient condition for $\frac{d\lambda}{d\mu} > 0$. $\frac{d\bar{\omega}}{d\mu} \geq 0$ holds for the standard costly state verification model with a fixed project size. It may still hold with a variable project size as long as x does not decline too much when μ increases. [8] Intuitively, increasing μ raises the required repayments for a given expenditure x. If x does not react too strongly , this requires an increase in the coupon rate and hence an increase in the default rate. If we had constant returns to scale ($\theta = 1$), the entrepreneur would react to a higher μ by lowering x so much that $\bar{\omega}$ would decline. Intuition suggests that with sufficiently decreasing returns to scale the reaction of x is small enough that $\bar{\omega}$ may actually increase as in the model with an exogeneously fixed project size. At least, the decrease in $\bar{\omega}$ in response to a higher μ would be small enough to allow the positive direct effect on λ of a higher μ to dominate. More formally, we have :

Lemma 4. *Suppose that at* $n = \tau = 0$, $\lim_{\theta\to 0}\phi(\bar{\omega}) > 0$, $\lim_{\theta\to 0}\bar{\omega} < \infty$, *and* $\lim_{\theta\to 0}m'(\bar{\omega}) > 0$. *Then* $\lim_{\theta\to 0}\frac{d\omega_m}{d\mu} \geq 0$, *and there exists a* $\bar{\theta} > 0$ *such that* $\frac{d\lambda}{d\mu} > 0$ *at* $\tau = n = 0$ *whenever* $\theta < \bar{\theta}$.

Démonstration. See the appendix. □

The requirement that $\lim_{\theta\to 0}\phi(\bar{\omega}) > 0$ may be problematic. Certainly this condition holds for the uniform distribution. For the lognormal distribution the condition is always satisfied if the standard deviation σ of $\ln\bar{\omega}$ is high enough, but for typical calibrations(as well for the calibration in section 3), $\lim_{\theta\to 0}\phi(\bar{\omega}) > 0$ requires the presence of a fixed cost $C_m > 0$. The numerical calibration in section 3 shows that the model with decreasing returns to scale can generate a realistic negative relation between net worth and bank financing even without the fixed cost. This is in contrast to the

[8] x must be decreasing in μ : Let $A \equiv \theta M x_i^{\theta-1}\frac{f(\bar{\omega}_i)+\lambda_i m(\bar{\omega}_i)}{\lambda_i}$. From the first order condition for x, $\frac{dA(x(\mu),\bar{\omega}(\mu),\mu)}{d\mu} = \frac{\partial A}{\partial x}\frac{dx}{d\mu} + \frac{\partial A}{\partial\bar{\omega}}\frac{d\bar{\omega}}{d\mu} + \frac{\partial A}{\partial\mu} = 0$. $\frac{\partial A}{\partial x} < 0$ and $\frac{\partial A}{\partial\bar{\omega}} < 0$. Since $\frac{\partial\lambda(\bar{\omega},\mu)}{\partial\mu} > 0$ and $\frac{\partial m(\bar{\omega},\mu)}{\partial\mu} < 0$, $\frac{\partial A}{\partial\mu} < 0$. This implies that $\frac{\partial A}{\partial x}\frac{dx}{d\mu} + \frac{\partial A}{\partial\bar{\omega}}\frac{d\bar{\omega}}{d\mu} > 0$. Let $\Delta\mu > 0$. Since $\frac{d\pi^e}{d\mu} < 0$, $\Delta x \geq 0$ implies that $\Delta\bar{\omega} > 0$. But then $\frac{\partial A}{\partial x}\frac{dx}{d\mu} + \frac{\partial A}{\partial\bar{\omega}}\frac{d\bar{\omega}}{d\mu} < 0$, which is impossible. Therefore, $\frac{dx}{d\mu} < 0$.

constant returns to scale model which requires a fixed cost in order to have a non degenerate financing choice. Also, the net worth-financing choice link emerges in the numerical analysis for $\theta = 0.9$, suggesting that the degree of decreasing returns to scale required by the theoretical results above is plausible.

If we have the stronger condition that $\lim_{\theta \to 0} \frac{d\omega_m}{d\mu} > 0$, we can also derive an interesting result about the effect of a general improvement in the auditing technology for both banks and bond funds. Let $s \equiv \mu_m/\mu_b > 1$. We are interested in the consequences of a reduction in the cost of auditing μ_b for a fixed ratio of market to bank debt auditing efficiency s. If a reduction in μ_b leads more firms to prefer bond financing, then according to the model a general improvement in the cost of dealing with financial frictions causes a switch towards more market debt. Intuitively this should be the case : if auditing technology in general improves the importance of banks as better auditors should diminish. Thus the model also provides a potential explanation for the shift towards more market debt financing in the last 30 years(Samolyk,2004)[106]. [9]We can show that this is what happens if we have sufficiently strong decreasing returns to scale(low θ) :

Proposition 5. *Suppose* $\frac{d\bar{\omega}_m}{d\mu} \geq 0$ *and* $\lim_{\theta \to 0} \bar{\omega}_m > 0$. *Then for a fixed* $s > 1$ *, there exists a* $\theta_* > 0$ *such that* $\frac{d(\pi_b^e - \pi_m^e)}{d\mu_b} > 0$ *for* $\theta \in (0, \theta_*)$.

Démonstration. See the appendix. □

The sufficient condition in proposition 5 holds when $C_m > 0$ in a neighbourhood of $n = 0$, as long as $\bar{\omega}$ and $\frac{d\bar{\omega}}{d\mu}$ are continuous in θ at 0 :

Lemma 6. *Suppose that* $\bar{\omega}$, $\frac{d\bar{\omega}_m}{d\mu}$ *is right continous in* θ *at* $\theta = n = 0$ *and* $C_m > 0$. *Then* $\lim_{\theta \to 0} \bar{\omega}_m > 0$ *and there exists a* $\theta^* > 0$ *such that* $\frac{d\bar{\omega}_m}{d\mu} > 0$ *at* $n = 0$ *for any* $\theta < \theta^*$. *Furthermore, these properties continue to hold for* $n \in (0, n^{**})$, *where* n^{**} *is a positive number.*

Démonstration. See the appendix. □

The Dynamic Behaviour of Entrepreneurs

We model entrepreneur savings in a similar way to Bernanke,Gertler and Gilchrist [19]. There are overlapping generations of two-period lived entrepreneurs. There is a measure 1 of old entrepreneurs in each period that can operate a project and then exit the economy. Each period, they are replaced by a measure 1 of young entrepreneurs.

[9]Samolyk [106] finds that the proportion of short term nonfinancial business lending done by commercial banks in the US has declined from around 75% to about 50% between 1974 and 2004. This figure may overestimate the decline of bank-like lending to the degree that many finance company loans(that have increased significantly during this period) may be very similar to bank loans.

Young entrepreneurs cannot produce or work, and they are born without any endowment. Entrepreneurs are risk neutral and only care about consumption when old. An old entrepreneur begins the period with a stock of capital k^e_{jt}. The entrepreneur can rent out his capital to obtain $n_{jt} = k^e_{jt}(1 + r_t - \delta)$, where δ is the depreciation rate of capital. Then, the entrepreneur makes the financing decision in order to maximise expected income Ey^e_{jt}. Based on net worth n_{jt} and the aggregate state of the economy, the entrepreneur decides whether to produce with a bank loan, produce with a mutual fund loan or rely only on n_{jt} for funding. Next, if the entrepreneur has decided to contract with a financial intermediary the contract is signed for a loan of $r_t k_{jt} + w_t l_{jt} - n_{jt}$, and the entrepreneur rents capital and hires labour. If the entrepreneur prefers autarky he uses part of his net worth to finance his production, and he stores the remaining funds $n_{jt} - \bar{n}_{jt}$ till the end of the period.

Finally, the idiosyncratic shocks ω_{jt} are realised, entrepreneurs produce, pay for capital and labour and deliver the loan repayment $b(\omega_{jt})$. This leaves entrepreneurs with income y^e_{jt}. At this point old entrepreneurs get to consume all their income with probability π_e. In this case they consume $c_{jt} = y^e_{jt}$, leaving the young without any capital in the beginning of the next period. With a probability $1 - \pi_e$, the young get all the income as a bequest from the old entrepreneurs. In this case the young entrepreneur saves $k^e_{j,t+1} = y^e_{jt}$. [10]In the case of constant returns to scale this structure gives exactly the same saving function as the original Bernanke et al. [**?**] model with infinitely lived entrepreneurs with a constant death probability of π_e. With constant returns to scale, the risk neutrality of entrepreneurs would make it optimal for them to maximise expected profits from production inside each period, despite the infinite horizon. Therefore, we can use the previously derived financial contract to describe entrepreneur production decisions. The decreasing returns to scale assumption complicates matters. If the default rate were independent of net worth, then the value of the firm would still be increasing in expected current profits, and the static contract would still be optimal. Since the default rate is decreasing in net worth maximising expected current profits is no longer optimal. In particular the entrepreneur may prefer a lower project size relative to the static case in order to lower the default rate and reduce the chances of entering the next period with zero net worth. In combination with the discrete financing choice this makes the optimisation problem with decreasing returns to scale considerably more challenging, particularly if the goal is to eventually study financing choices in general equilibrium with aggregate shocks. The overlapping generations assumption sidesteps this issue. [11]

[10]This outcome can be derived from a negotiation between old and young entrepreneurs over $y^e_{j,t}$. The old make a take it or leave it offer to the young with probability π_e, while the young make a take it or leave it offer to the old with probability $1 - \pi_e$.

[11]Another assumption that would preserve the optimality of static expected profit maximisation would be allowing the entrepreneur to diversify away the risk of default by holding a continuum of projects, and generating differences among entrepreneurs through ex-ante idiosyncratic shocks(occuring before the financial contracting decision is made).

Workers

There is a measure 1 of risk averse workers. The representative worker chooses sequences of consumption and saving to maximise $E_0 \sum_{t=0}^{\infty} \beta^t \ln c_{h,t}$ subject to the sequence of budget constraints

$$c_{h,t} + k_{h,t+1} = k_{h,t}(1 + r_t - \delta) + w_t \tag{1.8}$$

where r_t and w_t are the rental rate and the real wage rate. Workers rent out capital and work. They can then use their income to lend to financial intermediaries $L_t \leq r_t k_{ht} + w_t l_{ht}$. Both banks and bond mutual funds are completely diversified with respect to entrepreneurs' idiosyncratic risk.This, in addition to their intratemporal nature, makes the gross rate of return on loans 1. At the end of the period all payments are made and workers consume and save. From the workers' optimisation problem, we get the standard Euler equation :

$$\frac{1}{c_{ht}} = \beta E_t \frac{1}{c_{h,t+1}} \left(1 + r_{t+1} - \delta\right) \tag{1.9}$$

In the steady state, this equation pins down the interest rate at $r = \frac{1}{\beta} - 1 + \delta$.

Timeline of the model during a period :
1. z_t and the value of 1_d^e are known to everyone.
2 Based on $n_{jt} = k_{jt}^e(1 + r_t - \delta)$ the entrepreneur picks contract type $i \in \{a, b, m\}$ and the desired amount of loans, capital and labour.
3. Entrepreneurs rent out their capital. Workers rent out their capital and work. Households extend the loans through financial intermediaries
4. Entrepreneur output is realized and all payments are made.
5. Entrepreneurs and households consume and save for the next period.

1.2.2 The Competitive Equilibrium

We are now in a position to define the steady state competitive equilibrium for this economy. The state of each entrepreneur is described by $S_{jt} = \left(\omega_{jt}, k_{jt}^e, 1_d^e\right)$, where 1_d^e indicates entrepreneur death, with a joint probability measure $F_t(S)$. Let $C_t = c_{h,t} + \int c_{jt} dF$, $n_t = \int n_{jt} dF$, $x_t = \int x_{jt} dF$, $l_t = \int l_{jt} dF$, and $i_t = K_{t+1} - (1-\delta)K_t$ where $K_t = k_{h,t} + \int k_{jt}^e dF$. Finally define 1_b, 1_m, 1_a and 1_s as the indicator functions respectively for bank financing, market debt financing, autarky and entrepreneur default.

A steady state competitive equilibrium consists of capital rental rate and wage rates (r, w) entrepreneur and worker consumption and saving policies $\left\{c^e(S_{jt}), k_{j,+1}^e(S_{jt}), c_h(k_t^h), k_{h,+1}(k_{h,t})\right\}$ and decisions $\left\{1_m(k_{jt}^e), 1_b(k_{jt}^e), 1_a(k_{jt}^e), 1_s(k_{jt}^e, \omega_{jt})\right\}$ such that

1. The capital market clears : $k_h + \int k_{jt}^e dF = \int k_{jt} dF$.
2. The labour market clears : $l_t = 1$.

2. The output market clears : $y = C + i + \int 1_s 1_b \mu_b \bar{y}_{jt} dF + \int 1_s 1_m \mu_m \bar{y}_{jt} dF + \tau \int 1_b (x_{jt} - n_{jt}) dF + C_m \int 1_m dF$.

3. The loan market clears : $rk + wl - n = x - n$.

4. Financial contracts are optimal, entrepreneurs maximise their expected income Ey^e_{jt} and pick $k^e_{j,t+1}$ optimally.

5. Households pick consumption and saving optimally.

7. F is an invariant distribution : given the conditional probability function $Q(S, A)$ and given any event A,

$F(A) = \int_A Q(S, S')F(dS')$.

1.3 Results

1.3.1 Calibration

The model period is one quarter. Following Carlstrom and Fuerst (1998) [32], we set $\beta = 0.99$, $\delta = 0.02$.We set aggregate productivity to $z = 1$. Following the discussion in Restuccia and Rogerson (2003) [104] and Jaimovich and Rebelo(08) we set $\theta = 0.9$, implying a profit share of around 10%. A θ of 0.9 is in the upper bound of empirical estimates. If anything, choosing a relatively high value of θ should make it more difficult for the model to generate a link between firm size and financing choice. We set the share of capital to one third, giving $\alpha = 0.3$. The entrepreneurs' death rate is 3%, based on Bernanke,Gertler and Gilchrist[19].

The calibration of μ_m and μ_b does not have any precedents in the litterature, requiring us to make some extra assumptions. Let $s = \frac{\mu_m}{\mu_b}$. We define the average audit cost,

$$\mu \equiv \mu_b \frac{\int 1_s 1_b \bar{y}_{jt} dF}{\int 1_s \bar{y}_{jt} dF} + \mu_m \frac{\int 1_s 1_m \bar{y}_{jt} dF}{\int 1_s \bar{y}_{jt} dF} = \mu_b \hat{p} + \mu_m (1 - \hat{p}), \text{ where } \hat{p} \equiv \frac{\int 1_s 1_b \bar{y}_{jt} dF}{\int 1_s \bar{y}_{jt} dF}.$$

To simplify the calibration we approximate $\hat{p}$ by $p \equiv \frac{\int 1_b (x_{jt} - n_{jt}) dF}{\int (1 - 1_a)(x_{jt-} n_{jt}) dF}$ and set $\mu = p\mu_b + (1 - p)\mu_m = \mu_b[p + (1 - p)s]$. [12] Given μ and p, we could solve for μ_b and μ_m if we knew s. We approximate p by the average ratio of bank finance to total debt finance over 1997-2003 in the US and in the Euro area, as reported in De Fiore and Uhlig (2005) [55]. This gives us $p = 0.425$ for the US and $p = 0.88$ for the Euro area. As there are several estimates of μ in the litterature, we take an intermediate estimate of $\mu = 0.15$ from Carlstrom and Fuerst (1998)[32]. This leaves s. In line with our focus on banks' lower cost of dealing with financial distress, we use evidence from Gilson,Kose and Lang (1990)[62] on the probability of private restructuring as opposed to formal bankruptcy and on the relative costs of these procedures to determine s. Let π_i be the probability of private restructuring for a debt of type i. Let $\bar{m}$ be the proportional cost of private restructuring and $\hat{m}$ be the proportional cost of formal

[12] The approximation is exact if we have constant returns to scale.

bankruptcy, where due to lack of evidence we assume these costs are the same for both bank and non-bank debt. Since our model does not distinguish between these two forms of financial distress, we assume that $\mu_i = \pi_m \bar{m} + (1-\pi_m)\hat{m}$.This implies that $s = \frac{\mu_m}{\mu_b} = \frac{\pi_m \bar{m} + (1-\pi_m)\hat{m}}{\pi_b \bar{m} + (1-\pi_b)\hat{m}} = \frac{\pi_m h + (1-\pi_m)}{\pi_b h + (1-\pi_b)}$, where $h = \bar{m}/\hat{m}$. Gilson,Kose and Lang [62] estimate $\pi_b = 0.9$ and $\pi_m = 0.375$. They also report that the average successful private restructuring takes 15.4 months, as opposed to 28.5 months for the average unsuccesful restructuring and formal bankruptcy. Assuming that the costs of these procedures is proportional to their duration, we get an estimate of $h = 15.4/28.5 = 0.54$. With these numbers we find $s = 1.412$, $\mu_b = 0.121$ and $\mu_m = 0.171$ for the US. For the Euro area, we assume the same s and the same μ, giving $\mu_b = 0.143$ and $\mu_m = 0.202$.[13]

In the US calibration we assume that there are no fixed costs of issuing bonds($C_m = 0$). In this case, we pick the other parameters to roughly match the quarterly default rate for the US of 0.974% reported in Carlstrom and Fuerst[32], the ratio of bank financing to market debt financing from De Fiore and Uhlig[55] and the costs of bank intermediation per dollar of loans in developed economies from Erosa (2001) [53]. [14] For the European calibration, we pick C_m to match the difference between bond issue costs in the US and Europe from Santos and Tsatsaronis (2003) [107].

We assume that ω follows a log-normal distribution where, $\ln \omega$ has a standard deviation σ and mean $-\sigma^2/2$.

We set $\sigma = 0.185$. We try several values of τ ranging from 0.005% to 2%. The most successful calibration has $\tau = 0.25\%$. In combination with the costs of auditing financially distressed firms, this value leads to a reasonable estimate of total bank intermediation costs.

1.3.2 General Equilibrium Results

For the US, the model provides a rough match to default rates, the ratio of bank to market debt and the costs of financial intermediation in the US with$\tau = 0.25\%$ (table 1). The 2.8% (10.74% at an annual rate) default rate obtained in our simulations is high relative to the 1% default rate used as a target. However, the 1% estimate is biased downwards due to underrepresentation of small unincorporated firms in the sample [56]. Such firms are included in our model, and they are potentially important in explaining the prevalence of bank financing. The higher default rate in the simulations may be quite compatible with the behaviour of those firms, particularly once one realises that default in our model does not just represent formal bankruptcy or liquidation but any failure to fully repay the promised coupon $\bar{\omega}\bar{y}$.The model's annual capital to output ratio of approximately 2.5 is not too far from the average ratio for

[13] The idea that costs of financial distress are higher in Europe is supported by Djankov et al's analysis of bankruptcy costs around the world[50]

[14] In the interpretation of the model where n_{jt} is partly used as collateral for loans that are not subject to frictions instead of just being directly used for self financing, we assume that lending not subject to frictions is allocated between banks and markets in the same proportion as lending subject to frictions.

the US of 3 reported in Cagetti and de Nardi (2006) [27], even though this ratio was not targetted in the calibration.

The model generates a negative correlation between firm size and bank financing. In particular, the correlation between the bank financing indicator (1 if $i = b$, 0 otherwise) and n (our measure of the value of equity) is significantly negative. As discussed in the theoretical analysis of the financial contract, the negative link between n and the default threshhold $\bar{\omega}$ is probably the key mechanism producing this effect.

The smallest firms in our model use bank loans. Next, intermediate size firms use market debt. Finally the largest firms are financially unconstrained. Note that this cross sectional pattern also holds for the time series evolution of a typical firm. So we can also interpret the results as a model of the life-cycle of firm financing choices. As the firm becomes older it evolves from bank debt to market debt to a regime where financing choices do not matter very much.

Comparing the strength of this effect in the model with actual data is difficult due to the limited availability of data on the division of debt between bank and market sources. Nevertheless, this qualification we compare the model's predicted correlation between net worth and market debt issues with the one indicator available in Compustat data- the existence of a bond rating. Cantillo and Wright (2000) [29] were able to obtain more precise data on the decomposition of debt between banks and markets for a subset of Compustat firms. For that subsample, they find a nearly perfect correlation between the existence of a debt rating and the existence of outstanding market debt in a given year. On the assumption that this strong correlation continues to hold in the general Compustat sample, as well as using the fact that firms in our model do not issue bank and market debt simultaneously and that debt in our model lasts for only one period, we can associate the existence of a bond rating in a given firm-year with the issuance of market debt. Therefore, we compare the correlation between using market debt and net worth in the model(calibrated at an annual frequency to match Compustat's ratings information) to the correlation between having a bond rating and net worth in Compustat between 1997 and 2006. From this perspective, our model is rejected : the model's correlation between net worth and issuance of market debt is around 0.77 while in Compustat the correlation is only around 0.25. There are several possible explanations for this failure. One possibility is that Compustat contains very large firms that are financially unconstrained in our model. To check this, we reexamined the correlation in the Compustat data excluding from the analysis the top 31% of firms by net worth that would be financially unconstrained according to the model. The correlation in the restricted Compustat sample is around 0.21, again, far from the model's prediction. [15]

The model without fixed bond issue costs cannot match the ratio of bank to market debt in Europe without assuming an extremely low bank administration cost

[15] There are several measurement issues that could also explain this discrepancy. The assumption of perfect correlation between having a bond rating and having outstanding market debt may be a bad approximation in our sample. Also, Compustat data is biased biased towards larger firms that can issue equity.

parameter τ(table 2). Even with $\tau = 0.005\%$, we can only get a ratio of bank financing to market financing of 2.43 in comparison to a ratio of 7.33 in the data. The problem is that lowering τ in Europe even further leads to an implausibly low estimate of the cost of bank intermediation per dollar of loan in Europe relative to the cost in the US. It is certainly possible that bank monitoring is cheaper in Europe. For example, we know that banks in many European countries can acquire equity stakes in firms that they lend to more easily than in the US. This may lower the cost of monitoring loans(showing up in reduced form in the model either as a lower μ_b/μ_m or as a lower τ for a given μ_b/μ_s). [16] An alternative interpretation of this result is that market financing is relatively more expensive in Europe. Santos and Tsatsaronis(2003)[107] find that until 2001 average bond underwriting fees in Europe exceeded American average underwriting fees by approximately $0.05\% - 0.8\%$. [17] The introduction of the Euro led to greater competition among investment banks in the Euro area, lowering bond underwriting fees. We can model the difference in underwriting fees by allowing for positive bond issuance costs in Europe. To capture this we specify a fixed cost of issuing market debt $C_m > 0$ in Europe in addition to the expected costs of financial distress. We assume that $\tau = 0.25\%$ in Europe as in our preferred specification for the US. Like Santos and Tsatsaronis, we use the loan size weighted average of issue costs per dollar of lending as our measure of average issue costs.

The addition of a fixed bond issue cost leads to a large increase in the relative desirability of bank financing (table 3). An average issue cost of $0.17\%(C_m = 0.25\%)$ almost triples the relative proportion of bank loans to market loans in Europe. With an average issue cost of $0.22\%(C_m = 0.35\%)$ we get a ratio of bank to bond financing of 5.54 in Europe. The cost of bank intermediation per dollar of loans in Europe is still estimated to be significantly lower than in the US (1.78% versus 2.72% in the US), but the difference is much more plausible than the one obtained trying to match the relative amount of bank financing in Europe without bond issue costs. This estimate is significantly lower than the estimates reported by Erosa[53] for European countries. However, his estimates are for the year 1985. It is quite possible that due to technological progress in the financial sector, costs of bank intermediation have declined significantly since 1985.

Finding a lower cost of bank intermediation in Europe despite having the same loan administration cost τ, a higher audit cost parameter μ and the same average default rate as in the US may seem counterintuitive at first. The explanation lies in the audit cost function used. Recall that the audit cost is $\mu M x^\theta$, which is concave

[16] In contrast American banks could not hold equity in firms that they lend to until 1999, with the repeal of the Glass-Steagall act. Even with the repeal of the Glass-Steagall act, European banks still have more freedom to own equity in borrowing firms(Barthet al 2000). Santos(97) argues that in practice European banks' equity holdings are small, though one cannot exclude the possibility that even small equity holdings translate into significant reductions in bank monitoring costs.

[17] Their sample covers only international bond issues, which includes almost all European corporate bond issues, but excludes many American bonds issued only domestically. One should also bear in mind that the market finance in our model is closer to commercial paper, while Santos and Tsatsaronis cover longer term bonds.

in x.The average x financed by the bank in Europe is 1.91. The average x financed by the bank in the US is 1.34. This difference occurs due to the larger number of high n firms using bank financing in Europe, reflected in a lower magnitude of the negative correlation between n and bank financing choice in Europe. The cost of bank intermediation per dollar of loans is $\tau + \frac{\mu M \int 1_b x_j^\theta dF}{\int 1_b (x_j - n_j) dF}$. The amount of bank loans in Europe is more than double the amount in the US(the denominator). At the same time, while the total amount of expenditure by bank financed firms is larger, due to the diminishing marginal cost of auditing and the higher average expenditure financed by a bank loan in Europe, the total auditing costs in Europe are smaller relative to the amount of loans. As a result, we get a lower cost of bank intermediation per dollar of loans in Europe.

The model has more limited success in matching some other stylized facts about the distribution of firms. The model predicts a negative relation between the debt to equity ratio $\frac{x-n}{n}$ and n, with a correlation of around -0.3 for the US, and -0.53 in Europe. This prediction is at odds with the empirical evidence for Compustat data reported in Frank and Goyal(2005)[119]. In fact the negative correlation between the leverage ratio(leverage is $\frac{x}{n}$, that is the debt to equity ratio minus 1) and net worth is so strong that it also leads to a negative correlation between loan sizes and n. This prediction is again probably unrealistic. There are several possible reactions to this problem. First, the evidence for a positive link between leverage and size is not conclusive. Arellano,Bai and Zhang(2007) [11] examine data from the UK and find a negative correlation between leverage and size for the whole sample. They only find a positive leverage and size correlation for the largest firms in the UK. To the degree that the compustat data oversamples the largest American firms, Frank and Goyal's conclusion on the correlation between size and leverage is consistent with Arellano et al. [18] Second, these correlations emerge in the model when restricting the analysis to firms that require external financing. For higher net worth firms, the Modigliani-Miller theorem applies in our model, and the financing choice is indeterminate. Those firms could for example prefer higher leverage due to unmodeled tax trade-offs between debt and equity. Therefore, our model is not necessarily at odds with a positive size and leverage relation for the largest firms. Third, the negative link between $x-n$ and n does not necessarily mean that the model predicts a negative correlation between n and overall lending. Recall the alternative interpretation of the model according to which $x - n$ is the part of lending subject to frictions, with the total amount of lending relative to self-financing being indeterminate. This interpretation is consistent with an economy where firms with higher net worth borrow more, but the amount of their borrowing subject to information frictions is lower than for low net worth firms. Finally, like virtually all implementations of the CSV model, we have assumed that the idiosyncratic shocks ω are i.i.d. Suppose instead that ω follows an AR(1) process. In this case M is increasing in $E(\omega_t|\omega_{t-1})$. A high sequence of ω's would raise M. Since

[18] Cooley and Quadrini(2001) also report that the leverage ratio is negatively correlated with firm equity when the sample is not restricted to corporations.

$\frac{d(x-n)/n}{dM} > 0$, the firm's leverage ratio would also increase for a given n. At the same time, the firm's net worth n should increase due to the higher recent profitability. In this case we may see a rise in n accompanied by a rise in $\frac{x-n}{n}$, despite the negative direct relation between these two variable for a fixed M. [19] Finally, we have followed the standard costly state verification framework in restricting entrepreneurs to using only internal equity. Allowing firms in the model to issue external equity subject to the typical quadratic issuance costs (see for example Henessy and Whited[68]) would increase the cost of equity disproportionately for the largest firms and encourage them to increase the proportion of debt financing.

Only around 40% of firms in our calibrations actually borrow. The result that most firms in our simulations are not financially constrained is unrealistic. For example Henessy and Whited (2006) [68] estimate a more quantitative model of investment with financial frictions and find the presence of moderate financing frictions even for large US corporations, though as in our model they find that financial frictions are considerably less important for larger firms. One result of the high proportion of self-financing firms in the model is the extremely low aggregate debt to equity ratio of around 0.07 predicted in our simulations. The actual aggregate debt to equity ratio for 1997-2003 was 0.41 in the US and 0.61 in Europe [55].The debt to equity ratio of borrowing firms is actually around 1.36 for our US calibrations, but these firms only hold around 20% of assets and even less of the aggregate equity. As a result, the total amount of debt to equity for all firms is low.

The key factor explaining the large number of financially unconstrained firms in our simulations is the aggressive saving behaviour of the entrepreneurs. This allows many firms to accumulate enough net worth to make financial frictions irrelevant. The largest firms in the model do not require any external financing. The simplest way to improve the model's performance in this respect is to increase the bargaining power of old entrepreneurs π_e. This would directly reduce the entrepreneur dynasties' ability to accumulate large net worth levels. For example, increasing π^e to 10% while keeping all other parameters at their level in the preferred US calibration increases the aggregate debt to equity ratio to 0.36, at the cost of an unrealistically high default rate of 6.25%. A more realistic but more challenging approach would be to model risk averse/consumption smoothing entrepreneurs as in Zha(2001)[120].

To summarize, the main factor explaining higher bank financing in Europe in our model is the higher cost of issuing market financing in Europe. The higher audit costs for a given loan size (higher μ) do not seem to play a large role in explaining European firms' stronger preference for bank financing. To the degree that bond underwriting costs have declined with the introduction of the Euro, our model predicts increased reliance on market financing in Europe. One should note though, that while the introduction of the Euro has lowered international bond issue costs, it is not clear

[19] One potential question is how would financing choice be affected by a model with persistent productivity shocks. Computing the derivative of $\pi_b^e - \pi_m^e$ at $n = 0$ and taking the limit as $\theta \to 0$, it can be shown that if n and θ are not too large and there is a fixed cost of issuing bonds, then bank financing becomes less profitable relative to bond financing as M increases.

that it had a similar effect on the costs of issuing commercial paper (which is a better description of the short term market debt in the model). If differences in the cost of issuing commercial papers persist, bank financing should remain more popular in Europe in the future.

Finally, we compare the model with bank versus market debt choice to the standard Carlstrom and Fuerst (1998) [32] model with only one intermediary type(table 4). This is a special case of the benchmark model with $\mu_b = \mu_m = 0.15$ and $\tau = 0$. The other parameter values are the same. We compare this model to our preferred calibration for the US with two types of financial intermediaries. The differences in the prediction of the two models for aggregate output and consumption are practically indistinguishable. Output and total consumption are higher by about 0.11% in the model with bank and market debt. Worker consumption is slightly higher than in the Carlstrom and Fuerst model, while entrepreneur consumption is slightly lower. In the context of this model, allowing for two types of financing does not seem to matter very much for predicting aggregate quantities. An economist could have done just as well in predicting aggregate output and consumption in this economy by assuming a single type of intermediary with audit costs being an average of bank and bond audit costs. This provides some evidence that, at least for some purposes, the usual practice of modeling a single type of financial intermediary may provide a good approximation as long as the lending technology of that financial intermediary is appropriately calibrated.

1.4 Conclusion

We have examined the ability of a simple extension of the standard costly state verification model of financing, based on the idea of banks as better monitors, to account for firms' choice between bank and market debt. The model successfuly captures the tendency of larger firms to use more market debt. The intuition for this result is that financial distress is a smaller problem for these firms, and this makes the advantages offered by banks in dealing with default less valuable. In order to capture the higher use of bank financing in Europe relative to the US we depart from the benchmark model by assuming higher fixed costs of issuing market debt in Europe. The fixed cost model can generate a realistic bank to market financing ratio in Europe.

There are several extensions of this paper that should be explored in future research. First, we have only examined the steady state behaviour of the model. Once we allow for aggregate shocks, we can study how financing choices change over the business cycle. Many authors(see for example Oliner and Rudebusch (1996), and Cantillo and Wright (2000)[96][29]) have argued that the impact of different financing types on business cycle dynamics depends on the degree to which firms switch from one mode of financing to another in reaction to aggregate shocks. Beyond the usual productivity and demand shocks, an interesting type of aggregate shock that may be worthwhile to investigate within this model is a shock to the cost of monitoring μ_b. At the end of section 2 we discussed the possibility of explaining the switch to more market based financing in the US as a decline in μ_b in recent decades. Such an explanation seems plausible. At the same time, it is sometimes argued that the switch away from standard bank based loans in recent years has been excessive and may have even contributed to the current financial problems in the US. One way to formalize such a story in this model is to model a news shock to μ_b, where agents in the economy overestimate the future declines in μ_b.

Another interesting extension is to endogenize part of the ex-ante cost of bank financing τ, by introducing bank capital effects. Reduced form empirical research has found significant effects of the strength of the bank's balance-sheet on interest rates and lending[98][99][70]. An extension of this model with bank capital would allow investigation of these effects in a more structural framework.This could be done in our model by limiting the ability of the bank to diversify part of the firms' risk across its loan portfolio. For example, we could introduce an industry level shock in an environment where the bank must specialize in a specific industry. In this case, the rate of return on bank deposits is no longer certain. Assuming that the realized return on the bank's loan portfolio is freely observable only to the banker, the depositors will have to audit the bank in case of low returns, just as the bank has to audit the entrepreneur when output is low. The financial friction between banks and depositors makes bank capital(the banker's net worth) valuable in this environment, just as the entrepreneur's net worth reduces frictions in the current model.

Finally, while we focus on bank and market financing in developped economies, in principle the framework that we present can be used to study any situation in which the financing options available to firms differ by the degree of monitoring. For example, we could also study the introduction of various intermediate forms of financing such as market debt backed by bank repayment guarantees to investors. Or we could examine the choice of farmers and entrepreneurs in a developing country between local lenders with better information on borrowers and outside financial intermediaries (for example foreign banks) with a more efficient lending infrastructure but less access to information on local borrowers.

Table 1,
US, benchmark calibration

τ	Pr(default)	bankcostratio	btomf#firms	btomfloans	Debt/Equity	Pr(extfin)
0.01	0.026	0.045	0.248	0.21	0.069	0.403
0.005	0.029	0.04	0.372	0.353	0.0697	0.401
0.0025	0.029	0.027	0.619	0.671	0.07	0.399

τ	corr(i=a,n)	corr(i=b,n)	Y	K
0.01	0.592	-0.543	2.631	26.314
0.005	0.591	-0.639	2.633	26.336
0.0025	0.589	-0.754	2.634	26.347

Bankcostratio is $\tau + \frac{\mu M \int 1_b x_j^\theta dF}{\int 1_b (x_j - n_j) dF}$. btomf#firms is the ratio of the number of firms choosing bank to those choosing market debt.

btomfloans is the ratio of bank loans to market loans. Debt/Equity is the aggregate debt to equity ratio.

Table 2,
Europe, benchmark calibration

τ	Pr(default)	bankcostratio	btomf#firms	btomfloans	Debt/Equity	Pr(extfin)
0.01	0.0263	0.052	0.247	0.188	0.067	0.404
0.005	0.0288	0.0481	0.386	0.337	0.068	0.404
0.0025	0.0288	0.03	0.643	0.656	0.068	0.403
0.0005	0.0288	0.021	1.221	1.624	0.069	0.403

τ	corr(i=a,n)	corr(i=b,n)	Y	K
0.01	0.592	-0.539	2.629	26.286
0.005	0.593	-0.644	2.63	26.301
0.0025	0.592	-0.759	2.631	26.312
0.0005	0.592	-0.871	2.632	26.327

Table 3
Fixed bond issue cost in Europe ($\tau = 0.0025$)

C_m	$E(\frac{1_m C_m}{x-n})$	Pr(default)	bankcostratio	btomf#firms	btomfloans	Debt/Equity
0.0025	0.174%	0.0288	0.022	2.602	1.855	0.068
0.003	0.199%	0.0288	0.019	4.466	3.138	0.068
0.0035	0.224%	0.0288	0.0178	7.81	5.544	0.068

C_m	Pr(extfin)	corr(i=a,n)	corr(i=b,n)	Y	K
0.0025	0.396	0.586	-0.196	2.6307	26.302
0.003	0.396	0.587	-0.122	2.6309	26.303
0.0035	0.396	0.586	-0.069	2.6306	26.299

Table 4,
Model with only 1 financial intermediary type.
US calibration uses τ=0.25%, μ_b=0.121,μ_m=0.171
Single financial intermediary model uses μ=0.15,τ=0

	Y	K	C_h	C_e
1FI	2.631	26.315	1.772	0.325
2FIs,US	2.634	26.347	1.775	0.326
1FI/2FIs	1.001	1.001	1.002	0.998

Appendix A :

Sufficient conditions for $x-\bar{n}>0$ (no rationing) :

Consider first the case when $C_m=0$. Define the relative profit between external financing and autarky as a function of x as $\Delta(x)\equiv\pi^e(x)-\pi_a^e(n)$. We will show that $\Delta'(n)>0$ when $n<\left(\frac{\theta M[1-\mu\phi(0)]}{R}\right)^{\frac{1}{1-\theta}}$. Therefore, by continuity and the mean value theorem, for small enough $\varepsilon=x-n>0$,$\Delta(n+\varepsilon)-\Delta(n)=\Delta(n+\varepsilon)>0$. Taking on some positive loan must lead to higher profits than those available with just $x=n$. To establish this result, we go through the following steps :

1)$m'(\bar{\omega})>0$. Otherwise, since $f'(\bar{\omega})<0$,for a given x one could always raise $\bar{\omega}$ and raise π^ewhile satisfying the lender's break-even constraint

2) $m(\bar{\omega})\bar{y}=R(x-\bar{n})$. Otherwise, by 1) and $f'(\bar{\omega})<0$ we could reduce $\bar{\omega}$ and tighten the lender's break even constraint without violating it, while increasing π^e for any given x.

3)$x=0$ is never optimal. If $x=\bar{n}=0$, then $\lim_{x\to 0}(1-\mu)\theta Mx^{\theta-1}-R=\infty$. This implies that we can always guarantee a positive loan surplus by picking a small $x'>0=\bar{n}$. Continuity allows us to find a finite $\bar{\omega}>0$ such that $m(\bar{\omega})Mx'^{\theta}\geq x'$ and $\pi^e=f(\bar{\omega})Mx'^{\theta}>0$. So $x=\bar{n}=0$ can never be optimal.

4) $x-\bar{n}=0$ if and only if $\bar{\omega}=0$. The only if comes from realizing that $f(\omega)=1-m(\bar{\omega})-\mu\Phi(\bar{\omega})$, and that 2) and 3) require that $m(\bar{\omega})=0$. This can be achieved without any monitoring by setting $\bar{\omega}=0$. For any given x, picking any other $\bar{\omega}>0$ such that $m(\bar{\omega})=0$ (if it exists) would generate positive expected monitoring costs. Therefore $\bar{\omega}=0$ is optimal. For the if, setting $\bar{\omega}=0$ when $x-\bar{n}>0$ would mean that the borrower could make expected repayments arbitrarily small while evading all auditing by always reporting that $\omega=\varepsilon>0$ whenver $\omega\geq\varepsilon$ for some small ε such that $\varepsilon Mx^{\theta}<R(x-\bar{n})$. But this violates the lender's break-even constraint.

Since $f(0)=1$, $\pi^e(n)=\pi_a^e$ and $\Delta(n)=0$.

5) $\bar{n}=n$. Therefore $\Delta(n)=0$. See the proof of lemma 1b) to see that this is optimal when $x-\bar{n}>0$. When $x-\bar{n}=0$, $\pi^e=M\bar{n}^{\theta}+n-\bar{n}=\pi_a^e$.Since $x_a=n$ whenever $n\leq\hat{x}$, it is also optimal for the financial intermediary to set $\bar{n}=n$.

6)Define $\Delta(x)\equiv\pi^e-\pi_a^e=f(\bar{\omega}(x))Mx^{\theta}-Mn^{\theta}$, where $\bar{\omega}(x)$ is implicitly defined by the break even constraint $m(\bar{\omega})Mx^{\theta}=R(x-\bar{n})$ (the function$\bar{\omega}(x)$ is well defined since $m'(\bar{\omega})>0$). Using the break-even constraint to find $\frac{d\bar{\omega}}{dx}$, $\Delta'(x)=f'(\bar{\omega})\frac{d\bar{\omega}}{dx}Mx^{\theta}+\theta f(\bar{\omega})Mx^{\theta-1}=-\left[1-\Phi(\bar{\omega})\right]\frac{R-\theta m(\bar{\omega})Mx^{\theta-1}}{m'(\bar{\omega})}+\theta f(\bar{\omega})Mx^{\theta-1}$ .Evaluating this at n, $\Delta'(n)=\theta Mn^{\theta-1}-\frac{R}{1-\mu\phi(0)}>0$ iff $n<\left(\frac{\theta M[1-\mu\phi(0)]}{R}\right)^{\frac{1}{1-\theta}}$. Note that if $\phi(0)=0$ and $R=1$ then the right hand side of this inequality is the first best level of output with no financial frictions $\hat{x}$. In this case, the condition holds for any $n<\hat{x}$ by the concavity of $\theta Mx^{\theta-1}$ and the first order condition defining $\hat{x}$.

Therefore, for any $n<\left(\frac{\theta M[1-\mu\phi(0)]}{R}\right)^{\frac{1}{1-\theta}}$ by the mean value theorem there exists an $\varepsilon>0$ such that $\Delta(n+\varepsilon)-\Delta(n)=\Delta(n+\varepsilon)>0$. This implies that $x-\bar{n}=x-n=0$ cannot be optimal when there are no fixed costs.

Furthermore, note that $\left(\frac{\theta M[1-\mu\phi(0)]}{R}\right)^{\frac{1}{1-\theta}}$ is decreasing in both μ and τ. As a consequence for any value of n for which $x-n>0$ with a given τ and μ, $x-n>0$ still holds for lower values of τ and μ.

When $C_m>0$, $\Delta'(n)>0$ continues to hold if C_m is low enough. By the mean value theorem, we can then once again find $x=n+\varepsilon>n$ such that $\Delta(n+\varepsilon)-\Delta(n)>0$.

Proof of lemma 1 :

1a) First note that $f'(\bar{\omega})=-[1-\Phi(\bar{\omega})]<0$ for any finite $\bar{\omega}$.From the f.o.c for $\bar{\omega}$,we see that this requires that $\lambda>0$ and $m'(\bar{\omega})>0$.

1b) If $\bar{n}=0$, we have $\psi+\lambda_l R=1$. Since $R\geq 1$, $\lambda>1$ is sufficient for $\psi=0$. If $0<\bar{n}<n$, the f.o.c for $\bar{n}$ implies that $\xi>0$ is equivalent to $\lambda_l R-1>0$.Since $R\geq 1$, a sufficient condition for this is again that $\lambda>1$. By (1a) and the f.o.c for $\bar{\omega}$, $\lambda=-\frac{f'(\bar{\omega})}{m'(\bar{\omega})}$. Note that $f(\bar{\omega})+m(\bar{\omega})=1-\mu\Phi(\bar{\omega})$, implying that $f'(\bar{\omega})+m'(\bar{\omega})=-\mu\phi(\bar{\omega})<0$. Since $m'(\bar{\omega})>0$, this is equivalent to $\lambda=-\frac{f'(\bar{\omega})}{m'(\bar{\omega})}>1$.

1c) Using the MEP result and the lender rationality constraint, for any financial intermediary $\pi^e=f(\bar{\omega}^*)Mx^{*\theta}=[1-\mu\Phi(\bar{\omega}^*)]Mx^{*\theta}-R(x^*-n)-C$, where starred variables indicate optimal choices. By the MEP, when the entrepreneur uses external financing $x\geq n$. If the first best level with no financial frictions $x_{nofrictions}\leq n$, then $Mx^{\theta}-(x_{nofrictions}-n)\geq Mx^{*\theta}-(x^*-n)\geq[1-\mu\Phi(\bar{\omega}^*)]Mx^{*\theta}-R(x^*-n)-C$.

i.e, whenever external financing may be optimal($x\geq n$), internal financing of $x_a=x_{nofrictions}$ is preferred.

1d)$\lambda'(\bar{\omega})=\frac{m''(\bar{\omega})f'(\bar{\omega})-f''(\bar{\omega})m'(\bar{\omega})}{m'(\bar{\omega})^2}=\frac{\mu\left[\phi'(\bar{\omega})(1-\Phi(\bar{\omega}))+\phi(\bar{\omega})^2\right]}{m'(\bar{\omega})^2}$. The denominator is clearly positive. The numerator is positive whenever $\frac{d}{d\bar{\omega}}\left(\frac{\phi(\bar{\omega})}{1-\Phi(\bar{\omega})}\right)=\frac{\phi'(\bar{\omega})(1-\Phi(\bar{\omega}))+\phi(\bar{\omega})^2}{[1-\Phi(\bar{\omega})]^2}>0$, which is assumption 1. Therefore, $\lambda'(\bar{\omega})>0$.

e) The proof technique is from Covas and Den Haan[45]. From the first order conditions, $\theta Mx_i^{\theta-1}\frac{f(\bar{\omega}_i)+\lambda_i m(\bar{\omega}_i)}{\lambda_i}=R_i$. Suppose C_m falls. Under assumption 1, $\Delta\bar{\omega}\geq 0\implies\Delta\frac{f(\bar{\omega}_i)+\lambda_i m(\bar{\omega}_i)}{\lambda_i}\leq 0\implies\Delta\theta Mx_i^{\theta-1}\geq 0\implies\Delta x\leq 0$. $\Delta\bar{\omega}\geq 0$ and $\Delta x\leq 0$ imply that $\Delta\pi^e\leq 0$, but this contradicts $\frac{d\pi^e}{dC}<0$.Therefore, $\frac{d\bar{\omega}}{dC}>0$. The proof that $\frac{d\bar{\omega}}{dn}<0$ follows the same argument, except that now $\frac{d\bar{\omega}}{dn}>0$ leads to a contradiction.

f) We will start by showing that $\frac{dx}{d\tau}<0$. This will imply that $\frac{d\bar{\omega}}{d\tau}<0$. From the first order condition $\theta Mx_i^{\theta-1}\frac{f(\bar{\omega}_i)+\lambda_i m(\bar{\omega}_i)}{\lambda_i}=1+\tau$, $\Delta\tau>0$ and $\Delta x\geq 0$ imply that $\Delta\frac{f(\bar{\omega}_i)+\lambda_i m(\bar{\omega}_i)}{\lambda_i}>0$ and $\Delta\bar{\omega}<0$. But this contradicts $\frac{d\pi^e}{d\tau}<0$. Therefore $\frac{dx}{d\tau}<0$. When $C_m=0$, substituting the financial intermediary's break-even constraint into the first order condition for x, we get $\theta(1-\frac{n}{x})(\frac{f}{\lambda m}+1)=1$. Since $\frac{dx}{d\tau}<0,\Delta\tau>0$ implies that $\Delta(1-\frac{n}{x})<0$ and $\Delta(\frac{f}{\lambda m}+1)>0$. Since $\frac{f}{\lambda m}$ is decreasing in ω, $\Delta\bar{\omega}<0$ when τ increases. By continuity, the result continues to hold if C_m is close to 0.

Proof of lemma 4 :

$\frac{d\lambda}{d\mu}=\frac{\partial\lambda}{\partial\mu}+\lambda'(\bar{\omega})\frac{d\bar{\omega}}{d\mu}$. $\frac{\partial\lambda}{\partial\mu}=\phi(\bar{\omega})\frac{1-\Phi(\bar{\omega})}{m'(\bar{\omega})^2}>0$. By our assumption that $\lim_{\theta\to 0}\bar{\omega}<\infty$ and $\lim_{\theta\to 0}\phi(\bar{\omega})>0$, the limit of the numerator as $\theta\to 0$ is positive. $\lim_{\theta\to 0}m'(\bar{\omega})>0$ implies that the limit of the denominator is also positive as $\theta\to 0$ (note that since

$m'(\bar{\omega}) > 0$ for any $\theta > 0$, $\lim_{\theta\to 0} m'(\bar{\omega}) \geq 0$. The only assumption is that the inequality is strict.). Therefore $\lim_{\theta\to 0}\frac{\partial\lambda}{\partial\mu} > 0$. Since $\lambda'(\bar{\omega}) = \frac{\mu\left[\phi'(\bar{\omega})(1-\Phi(\bar{\omega}))+\phi(\bar{\omega})^2\right]}{m'(\bar{\omega})^2} > 0$ whenever $\theta > 0, \lim_{\theta\to 0}\lambda'(\bar{\omega}) \geq 0$. Moreover, it is clear from examining the expression for $\lambda'(\bar{\omega})$ that its limit is bounded. Totally differentiating the first order conditions with respect to μ, we get $\frac{d\bar{\omega}}{d\mu} = -\frac{\frac{\phi(\bar{\omega})\lambda(\bar{\omega})}{m'(\bar{\omega})} - \frac{\lambda(\bar{\omega})^2}{f(\bar{\omega})+\lambda(\bar{\omega})m(\bar{\omega})}\left(\frac{\phi(\bar{\omega})m(\bar{\omega})}{m'(\bar{\omega})} - \Phi(\bar{\omega})\right)}{\lambda'(\bar{\omega})\frac{f(\bar{\omega})}{f(\bar{\omega})+\lambda(\bar{\omega})m(\bar{\omega})} + \frac{1-\theta}{\theta}\frac{\lambda(\bar{\omega})^2}{f(\bar{\omega})}m'(\bar{\omega})} - \frac{-\frac{\lambda(\bar{\omega})^2}{f(\bar{\omega})}\Phi(\bar{\omega})}{\frac{\theta}{1-\theta}\lambda'(\bar{\omega})\frac{f(\bar{\omega})}{f(\bar{\omega})+\lambda(\bar{\omega})m(\bar{\omega})} + \frac{\lambda(\bar{\omega})^2}{f(\bar{\omega})}m'(\bar{\omega})} \equiv -A(\theta) - B(\theta)$. $\lim_{\theta\to 0}A(\theta) = 0$ and $\lim_{\theta\to 0}B(\theta) \leq 0$. Therefore $\lim_{\theta\to 0}\frac{d\bar{\omega}}{d\mu} \geq 0$. Together with our results for the limits of $\frac{\partial\lambda}{\partial\mu}$ and $\lambda'(\bar{\omega})$, this implies that $\lim_{\theta\to 0}\frac{d\lambda}{d\mu} > 0$. But then there must be a $\bar{\theta} > 0$ such that $\frac{d\lambda}{d\mu} > 0$ still holds when $\theta < \bar{\theta}$.

Proof of proposition 5 :

By the envelope theorem, ,$\frac{d(\pi_b^e - \pi_m^e)}{d\mu_b} = s\lambda(\omega_m, \mu_m)\Phi(\bar{\omega}_m)Mx_m^\theta - \lambda(\omega_b, \mu_b)\Phi(\bar{\omega}_b)Mx_b^\theta$. Since $\frac{d\bar{\omega}_m}{d\mu} \geq 0$, and $\frac{d\bar{\omega}}{d\tau} < 0$ (from lemma 1f) $\omega_m > \omega_b$. This, together with $\frac{\partial\lambda(\bar{\omega},\mu)}{\partial\mu} > 0$ and $\lambda'(\bar{\omega}) > 0$ implies that $\lambda(\omega_m, \mu_m)\Phi(\bar{\omega}_m) > \lambda(\omega_b, \mu_b)\Phi(\bar{\omega}_b)$ for any $\theta > 0$. Since $\lim_{\theta\to 0}\bar{\omega}_m > 0$ and $s > 1$, this implies that $\lim_{\theta\to 0} s\lambda(\omega_m, \mu_m)\Phi(\bar{\omega}_m) > \lim_{\theta\to 0}\lambda(\omega_b, \mu_b)\Phi(\bar{\omega}_b)$. But then since $\lim_{\theta\to 0} Mx^\theta = M$, we have that $\lim_{\theta\to 0}\frac{d(\pi_b^e - \pi_m^e)}{d\mu_b} > 0$. Therefore, there exists a $\theta_* > 0$ such that $\frac{d(\pi_b^e - \pi_m^e)}{d\mu_b} > 0$ when $\theta < \theta_*$.

Proof of lemma 6 :

Suppose $\theta = n = 0$, and consider the (suboptimal) pseudo-bond contract where the fixed cost is paid even when $x = n = 0$. The lender's break even constraint is now $m(\bar{\omega})\bar{y} \geq C_m$. Note that now $\bar{y}$ is fixed and therefore the optimal $x = 0$. The optimal contract with external financing contract must have $m'(\bar{\omega}) > 0$ and $m(\bar{\omega}_m)\bar{y} = C_m$, otherwise we could increase $\bar{\omega}_m$ and improve the borrower's expected profits without violating the lender's break-even constraint. Since $m(0) = 0$ and $C_m > 0$, $\bar{\omega}_m > 0$. The right continuity of $\bar{\omega}_m$ at $\theta = 0$ then implies that $\lim_{\theta\to 0}\bar{\omega}_m > 0$. Since $\frac{\partial m(\bar{\omega}_m,\mu)}{\partial\mu} < 0$, $m'(\bar{\omega}_m) > 0$ and $\bar{y}$ is fixed, an increase in μ implies an increase in $\bar{\omega}_m$: $\frac{d\bar{\omega}_m}{d\mu} > 0$ when $\theta = 0$. The right continuity in θ of $\frac{d\bar{\omega}_m}{d\mu}$ at $\theta = n = 0$ imply the existence of a $\theta^* > 0$ such that $\frac{d\bar{\omega}_m}{d\mu} > 0$ whenever $\theta < \theta^*$. Since for any $n > 0$ and $\theta > 0$ $\frac{d\bar{\omega}_m}{d\mu}$ is a continuous function of n, we can find a $n^{**} > 0$ such that $\frac{d\bar{\omega}_m}{d\mu} > 0$ and $\lim_{\theta\to 0}\bar{\omega}_m > 0$ continue to hold whenever $\theta < \theta^*$ and $n < n^{**}$.

Model Solution :

Partial Equilibrium :

We can solve the financial contract recursively by first solving a nonlinear equation for $\bar{\omega}$ and then finding $x(\bar{\omega})$. We use the first order conditions for x and $\bar{\omega}$ to find $x(\bar{\omega}) = \left[\frac{\theta M(f(\bar{\omega})+\lambda_l m(\bar{\omega}))}{\lambda_l R}\right]^{1/(1-\theta)}$. We then replace x with this equation in the lender's break even constraint and solve for $\bar{\omega}$. We solve for $\bar{\omega}$ using Brent's algorithm, which is a refinement of the standard bisection algorithm.

As for the uniqueness of the the solution found by this method we have the following result :

Proposition 7. *$\lambda'(\bar{\omega}) > 0$ and $m'(\bar{\omega}) \geq 0$ imply a unique solution(if it exists).*

Démonstration. We're looking for $F(\bar{\omega}) = m(\bar{\omega})Mx^{\theta} - Rx + Rn - C = 0$. It is sufficient to prove the strict monotonicity of $F(\bar{\omega})$. $F'(\bar{\omega}) = Mx^{\theta}m'(\bar{\omega}) + x'(\bar{\omega})[\theta Mm(\bar{\omega})x^{\theta-1} - R]$. The first term is positive or zero if $m'(\bar{\omega}) \geq 0$. As for the second term, after substituting our expression for $x(\bar{\omega})$ and simplifying $\theta Mm(\bar{\omega})x^{\theta-1} - R = R(\frac{\lambda m(\bar{\omega})}{f(\bar{\omega})+\lambda m(\bar{\omega})} - 1) < 0$. $x'(\bar{\omega}) = -\frac{R\theta M}{1-\theta}x^{\theta}\frac{\lambda_l'(\bar{\omega})f(\bar{\omega})}{(\lambda R)^2} < 0$ if $\lambda'(\bar{\omega}) > 0$. Therefore under our assumptions the second term is also positive and $F'(\bar{\omega}) > 0$. □

We know from lemma 1 that $\lambda'(\bar{\omega}) > 0$ iff the hazard rate $h(\omega)$ satisfies $h'(\bar{\omega}) > 0$. For the lognormal distribution there exists a ω' such that the lognormal distribution's hazard rate is increasing for $\omega < \omega'$ and is decreasing for $\omega > \omega'$. Therefore we can restrict our search for a solution to $\bar{\omega} < \omega'$.The other condition is that $m'(\bar{\omega}) \geq 0$. This must be true at an optimum, but need not hold for all $\bar{\omega}$. However, once again restricting the search for a solution to the region where $h'(\bar{\omega}) > 0$ helps. $m'(\bar{\omega}) = (1-\Phi)(1-\mu h) \geq 0$ iff $1 - \mu h \geq 0$. Since $h'(\bar{\omega}) > 0$ there exists a unique ω'' such that for any $\bar{\omega} < \omega''$, $1 - \mu h(\bar{\omega}) \geq 0$ and $1 - \mu h(\bar{\omega}) < 0$ for any $\bar{\omega} > \omega''$. Therefore there is a unique region $[0, \min(\omega'', \omega')]$ on which the conditions of the proposition are satisfied. This implies that if our candidate solution satisfies $\lambda'(\bar{\omega}) > 0$ and $m'(\bar{\omega}) \geq 0$, then we have found the optimal $\bar{\omega}$.

General Equilibrium computation :

Our goal is to find fixed steady state values for capital stocks, prices, aggregate consumption and an invariant distribution of entrepreneur net worth levels. Since the workers' euler equation fixes $r = 1/\beta - 1 + \delta$, solving the GE model amounts to finding the labour market clearing wage w.

As long as for at least some positive measure of entrepreneurs x_j is lower in the presence of auditing costs relative to the frictionless economy(implying that labour demand is lower with financial frictions), we can bound the equilibrium $w_{frictions} \leq w_{nofrictions}$. In fact, define $\bar{l}(w)$ to be the labour demand in the economy where there are no financial frictions. We have $l(w) \leq \bar{l}(w)$[20]. Then, $l(w_{nofrictions}) - 1 \leq \bar{l}(w_{nofrictions}) - 1 = 0$.Furthermore, if $l(w) - 1$ is continuous, there exists a small enough $\varepsilon > 0$ such that $l(\varepsilon) > 1$. We can then guarantee that using $[\varepsilon, w_{nofrictions}]$ as the starting interval the bisection algorithm will converge to $w_{frictions}$. Absent switches between external financing types, firm and hence aggregate labour demands are continuous. The possibility of switching financing types makes showing continuity difficult. Even if the true $l(w)$ function is continuous, we must approximate it with

[20] In our model$x_{jfrictions} \leq x_{jno\ frictions}$ whenever $x_{jno\ frictions} \geq n$, since $f(\bar{\omega})Mx_j^{\theta}$=$(1 - \mu \Pr(s_j < \bar{\omega}_j))M_j x_j^{\theta} - R(x_j - n_j)$ is concave in x, holding $\bar{\omega}$ constant. In this case, by proposition 5 in Gale and Hellwig[58] and the concavity of the revenue function$x_{jfrictions} \leq x_{jno\ fricitons}$.$l_{jfrictions} \leq l_{jno\ fricitons}$ from the experssion for labour demand as a function of total expenditure.

a finite number N of entrepreneurs, and this approximation may be discontinous. In practice finding a market clearing wage was never a problem. In all of the model parametrisations tried the bisection algorithm did converge to a solution according to the stopping criterion $\left|z^l\right| = \left|\frac{1}{N}\sum_{t=1}^{N} l_{jt} - 1\right| < 10^{-4}$. As for uniqueness, we know that for each entrepreneur labour supply is still downward sloping in w in the economy with financial frictions, as long as the change in w does not make him switch between bank and bond financing[21]. If there were no such switches, aggregate labour demand would be clearly downward sloping in w, ensuring a unique equilibrium $w_{frictions}$. If the entrepreneur switches between external financing types, it is still clear that an increase in the wage must reduce profits, but this may occur through an increase in both x and $\bar{\omega}$. In practice, partial equilibrium analysis for our calibrations suggested that entrepreneur labour demand is still declining in the wage despite the possibility of switching financial contracts.

We start the algorithm with the initial interval $[w_{low}, w_{nofrictions}]$ for a small $w_{low} > 0$. We iterate on the following :

1. Given the interval $[w_l^i, w_h^i]$ and $w^i = \frac{w_l^i + w_h^i}{2}$ approximate the invariant distribution of $\{n_j, k_{jt}\}$. To do this we simulate a long time series of observations for a single entrepreneur starting from an initial net worth of n_0 and discard a subset of the observations to reduce the impact of the initial n_0. This leaves a sample of N observations. Appealing to a law of large numbers we then take the sampe averages of c_{jt}^e, k_{jt}^e n_{jt}, l_{jt} and k_{jt} to approximate aggregate entrepreneur consumption capital supply, net worth, labour demand and capital demand.

2. Use the capital,output and consumption markets clearing conditions and the estimates of aggregate$y$, c^e and k^e from the previous step to solve for c_h, c, k_h.

3. If the labour excess demand function satisfies $\left|z^l\right| = \left|\frac{1}{N}\sum_{t=1}^{N} l_{jt} - 1\right| < \varepsilon_l$ stop. Otherwise, update the bisection interval and proceed to the next iteration.

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[21] To see this it is sufficient to note that the labour demand function decreases in w as long as x decreases in w. But raising w decreases M, which lowers x(see Covas and Den Haan[**?**] for the argument that $\partial x/\partial M > 0$).

Chapitre 2

Are There Any Spillovers between Household and Firm Financing Frictions ? A Dynamic General Equilibrium Analysis

Abstract

Economic commentators frequently imply the existence of positive spillovers between the financing frictions affecting households and firms, suggesting another way in which credit constraints may amplify aggregate shocks and increase the persistence of aggregate fluctuations. To examine this possibility, I develop a new model that incorporates both firm and household external finance spreads while improving in several dimensions on existing frameworks. Contrary to a common intuition, the baseline Real Business Cycle model with credit constraints produces small negative spillovers between the costs of external financing for firms and households. A key factor in this result is the income effect of changes in the external finance premium on borrower labour supply. The reduction in households' cost of borrowing in a boom decreases labour supply, increases the risk free interest rate and crowds out investment, raising borrowing costs for financially constrained firms.

JEL classification : E3, E4, G3.

Key words : Financial frictions, external finance premium, business cycles.

2.1 Introduction

The goal of this paper is to examine possible feedback effects between the strength of credit constraints and external finance premia in the household and in the production sectors of the economy, and the impact of such effects on business cycles. To do this, it proposes a new model of financing frictions for firms and households that explicitly models the external finance spread faced by both types of agents, while improving in several other dimensions on existing models of credit constraints and aggregate fluctuations. Financing frictions are often suggested as a prime candidate for endogenously amplifying and increasing the persistence of even small transitory exogenous shocks. The basic idea, often called the financial accelerator, is that in the presence of credit constraints exogenous shocks can generate a positive feedback effect between the financial health of borrowing firms or households and output (Bernanke, Gertler and Gilchrist (1999) [19]). The standard approach to analysing credit constraints focuses either on households or on firms in isolation. Despite the conjecture that credit constraints can significantly amplify and increase the persistence of small shocks, a frequent finding is that the ability of credit frictions to amplify fluctuations is small or modest and in many circumstances they dampen the effect of shocks on output.[1]

This raises the following question : can allowing for financing frictions both for households and firms simultaneously enhance the ability of financing constraints to amplify shocks and increase the persistence of fluctuations ? If both household and firm level financing frictions create financial accelerators which on their own amplify output fluctuations, then intuitively there should be positive a positive interaction between them : if the household level financial constraints increase the sensitivity of output to shocks, then due to the firm level financial accelerator they should amplify the procyclicality of financially constrained firms' collateral values and further relax(tighten) their financing constraints in a boom(recession). Similarly, the firm level financial accelerator should increase the procyclicality of households' collateral values and increase(decrease) their borrowing ability in a boom (recession). To quote Bernanke et al (1999) [19] :

" By enforcing the standard consumption Euler equation (in the firm financial accelerator model), we are effectively assuming that financial market frictions do not impede household behavior... An interesting extension of this model would be to incorporate household borrowing and associated frictions. With some slight modification, the financial accelerator would then also apply to household spending, strengthening the overall effect."

The possibility of such an interaction is particularly relevant in the recession of 2007-2009. Mian and Sufi (2009) [94] rank US counties by the growth rate in household leverage (measured by debt to income) in 2002-2006. They find that in 2006-2008 the unemployment rate increased by about 2.5% more in the top 10% leverage growth

[1] For example see Christiano, Motto and Rostagno(07)[42], and Iacoviello and Neri(08)[74]

counties than in the bottom 10% leverage growth counties. This sort of evidence is suggestive of a positive feedback from the leverage level of households to a decline in aggregate economic activity, which can then affect the tightness of firms' borrowing constraint through a financial accelerator effect. The following scenario roughly matches the mechanism underlying many popular and business press reports. A negative aggregate shock reduces the net worth of credit constrained households, raising the cost of borrowing and decreasing spending. The decline in spending lowers the net worth of financially constrained firms, increasing their financing costs and decreasing their demand for labour and capital. This feeds back into further declines in households' net worth and spending, again reducing the net worth of credit constrained firms. In a world with perfect competition and no financing frictions this situation would be impossible : for a given level of productivity the lower salaries of workers in the scenario above would stimulate firms' labour demand and output. The presence of firm level financing frictions that depend in part on aggregate demand makes this a plausible chain of events even with perfect competition and price flexibility in all markets.

There are several related questions for which understanding the interaction between household and firm credit constraints matters. For example, should we expect a liberalisation in household borrowing conditions to create a general boom including output and investment increases ? Can fluctuations in housing prices that affect households' borrowing capacity amplify GDP fluctuations ? The joint examination of credit constraints affecting households and firms may also matter for analysing policy responses to financial crises. Recent initiatives by many central banks have focused on quantitative easing operations aimed at reducing private sector financing spreads relative to the risk free rate. For example the US Federal Reserve Board launched the Mortgage Backed Securities Purchase Programme in November 2008 with a mandate to purchase up to 1.25 trillion USD of mortgage backed securities during 2009. In parallel, the Fed also launched a commercial paper purchase program with 350 billion USD of purchases by January 2009. [2] This leads to the following question : is it more important to target the spreads on household debt such as mortgages or firm level debt such as commercial paper ? The answer may depend significantly on the macroeconomic interaction of the two types of credit constraints.

To examine these issues, I develop a new model of financing frictions with aggregate fluctuations. The model has a subset of firms that are financially constrained and can only borrow by using revenue and capital as collateral, and a subset of financially constrained households that use debt collateralised by housing and part of their wage income. Both firms and households are affected by idiosyncratic shocks to their collateral values. Firms and households default on their loans when the value of their collateral is below the repayment promised to the lender. I follow other

[2] See http ://www.newyorkfed.org/markets/mbs_faq.html and the supplemental report on the Commercial Paper Funding Facility athttp ://www.newyorkfed.org/aboutthefed/annualreports.html.

DSGE models of financial frictions in using differences in the level of impatience of agents to generate equilibrium borrowing and lending(e.g. Iacoviello (2005) [72]). In equilibrium more impatient agents (borrowers and entrepreneurs) will borrow from patient savers. In order to keep the model tractable, I assume that borrowers of each type (households and firms) have access to full insurance contracts against the idiosyncratic shocks affecting the value of their collateral. This allows them to diversify their idiosyncratic risk each period after all debt contracts are settled. The bank cannot seize the proceeds of the insurance payments when the borrower defaults. The combination of insurance and limited liability partially preserves the effects of risk averse/consumption-smoothing behaviour of agents despite the ex-ante heterogeneity among agents and the nonlinear default decision. [3] To isolate the effect of financing frictions, I focus on a real business cycle model with flexible wages and prices.

The model generates countercyclical external finance premia for both households and firms, a key feature observed in the data. If these external finance premia are linked, the financing frictions in both the production and household sectors may jointly amplify each other. My analysis shows that this outcome is far from obvious. While household level financial frictions can amplify the effect of shocks on consumption and housing investment, this does not necessarily translate into amplifying the response of output unless the response of investment also becomes more procyclical or its procyclicality is not significantly reduced.

Consider an economic boom (the same logic in reverse can be applied to a recession). Borrowing households see a relaxation of their credit constraints that stimulates their consumption to a greater extent than for savers. But the relaxation of the borrowing constraints also reduces the incentives of borrowers to work, both because with diminishing marginal utility the higher consumption of borrowers reduces the marginal value of labour income (the standard income effect on leisure) and because lower financing costs diminish the importance of wage income as collateral. The less procyclical labour supply of borrowers makes it harder to have a simultaneous expansion of consumption and investment. The reduction in financial frictions for households also raises their loan demand, which because of the link between household and firm debt markets raises interest rates for entrepreneurs. Overall, the reduction in labour supply and the increase in loan demand of borrowers depress investment and the price of capital and raise the risk free interest rate. These changes reduce the value of entrepreneur collateral and make entrepreneur borrowing more expensive. Therefore, a relaxation of households' borrowing constraints leads to a tightening of firms' borrowing constraints.

There is another effect going in the opposite direction. The higher interest rate encourages savers to reduce their current consumption relative to their future consumption and increase their labour supply. To a first order approximation the labour supply

[3] These insurance contracts are mathematically equivalent to the the large family insurance scheme assumed in Shi (1997) [109] and other monetary search models.

of savers is more sensitive to changes in consumption. It seems possible that the higher labour supply of savers dominates the lower labour supply of borrowers. In that case, we could get positive spillovers between the tightness of household and firm borrowing constraints. However, I show that for plausible assumptions on agents' preferences this cannot happen as long as household borrowing constraints make aggregate consumption more procyclical and the ability to use wage income as collateral is low. As a result, while households' credit constraints make their consumption more sensitive to shocks, there are no positive spillovers between the strength of household and firm credit constraints. I discuss several features that can reverse this result. These include limited production factor mobility across sectors and wage rigidities. Finally, because changes in household borrowing costs have opposing effects on the consumption and labour supply of financially unconstrained and financially constrained households, the overall interaction of household and firm financing frictions tends to be quantitatively insignificant unless capital adjustment costs are very low.

2.1.1 Litterature Review

Most existing models of household borrowing in a DSGE framework follow Iacoviello (2005) [72] and Kiyotaki and Moore (1997) [83] in using a hard borrowing constraint and assuming it always binds. The Kiyotaki and Moore model of credit constraints can be seen as a special case of the current model in which there is no uncertainty about the future value of the collateral when the loan is made. The assumption that the constraint always binds makes the leverage ratio in their model constant. Furthermore, they ignore any difference between borrowing rates and the risk free rate. The model proposed here can at least qualitatively match the countercyclical leverage ratio of households found in the US by Adrian and Shin (2008) [1]. The assumption of an always binding borrowing constraint is questionable for large shocks that may be of particular interest to policymakers, and it may severely distort the dynamics of borrowers and the rest of the economy in those circumstances. The soft borrowing constraint in my model(with interest rates rising smoothly as a function of borrowing) will always bind as long as it can be satisfied.

Bernanke, Gertler and Gilchrist(henceforth BGG) (1999) [19] as well as Carlstrom and Fuerst (1997) [31] introduced equilibrium default of firms into DSGE models. To facilitate aggregation, they assumed risk neutral entrepreneurs, and constant returns to scale production. Using a setup with equilibrium default as in those models allows me to examine the impact of endogenous time varying interest rate spreads and leverage ratios. Equilibrium default also increases the realism of the debt contract by creating a trade-off between the equity downpayment on the loan and the interest rate charged by the lender. At the same time my model of firms' financial constraints allows me to consider a more standard formulation of entrepreneur balance sheets than the less conventional balance sheets used by BGG or Carlstrom and Fuerst to make their models tractable. In particular, entrepreneurs in my model own their capital stock, as in more sophisticated heterogeneous agent models of financing constraints, and do not have to repurchase it or rent it each period as in BGG or Carlstrom and Fuerst. Furthermore, entrepreneurs are risk averse and make a meaningful consumption-saving choice. In contrast, BGG assume an exogenously fixed constant saving rate for entrepreneurs,while Carlstrom and Fuerst assume that they are risk neutral. The risk-aversion of borrowers in my model of equilibrium default also makes it more applicable to households, for which assuming risk neutrality or an exogenously fixed saving rate is undesirable. Finally, my model of firms allows the researcher to consider other nonlinearities in the budget constraint of financially constrained entrepreneurs, such as decreasing returns to scale, imperfect competition or labour adjustment costs.

To the best of my knowledge, the only other papers with aggregate fluctuations that have allowed for financing frictions affecting both households and firms are Iacoviello (2005) [72] and Gerali et al (2009) [60]. Both of these papers rely on hard borrowing constraints as in Kiyotaki and Moore[83] to model credit frictions and assume the borrowing constraints always bind. The analysis in this paper of an environment with default costs and actual lending spreads provides an alternative perspective. These papers do not explicitly examine the effect of modeling both types of financing frictions as opposed to just one type, choosing to focus on other issues. Finally, they only analyse models with nominal rigidities which mix real effects of financing frictions with other channels such as imperfect indexation of debt to inflation. In contrast, this paper's main goal is to isolate and understand the feedback between household and firm financing frictions in the flexible price equilibrium of the economy. This flexible price equilibrium defines an output gap relative to the equilibrium with sticky prices or wages that is often important in monetary policy models. Therefore, the analysis should also be relevant to models with nominal rigidities.

2.2 The Model

The model has two types of households distinguished by their discount factors. Patient households with a relatively high discount factor lend to other households and firms, as well as owning some of the firms in the economy. Impatient households with a lower discount factor borrow from the patient households to finance housing and consumption subject to financing frictions. Financially constrained entrepreneurs produce final output. They can borrow from patient households to finance consumption , investment and wages subject to credit constraints. These entrepreneurs are also more impatient than the lenders. Capital producers transform final output into capital subject to capital adjustment costs, and are owned by the patient households. Housing producers transform final output into housing subject to housing adjustment costs, and are owned by the patient households.[4]

[4]For other models using discount factor differences to generate equilibrium borrowing, see Iacoviello(2005)[72] and Krusell and Smith(1998)[88] for households, Carlstrom and

2.2.1 The Household Sector

Patient Households (savers)

There is a measure θ^s of patient households that have a relatively high discount factor, and access to complete financial markets without any financing constraints. They provide loans through banks to firms and households. Following BGG(1999) [19] and Iacoviello(2005) [72] I assume that the deposits are risk free in aggregate. The representative saver picks sequences of consumption, working hours, housing and deposits at the bank $\{c_{s,t}, n_{s,t}, h_{s,t}, d_t\}$ to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^t u_{s,t},$$

where

$$\begin{aligned} u_{s,t} &= \frac{(c_{s,t}^{\xi_c} h_{s,t}^{\xi_h} (1-n_{s,t})^{\xi_n})^{1-\sigma}}{1-\sigma}\ , \text{ for } \sigma \neq 1, \\ u_{s,t} &= \xi_c \ln c_{s,t} + \xi_h \ln h_{s,t} + (1-\xi_c-\xi_h)\ln(1-n_{s,t}), \text{ for } \sigma = 1, \\ \xi_n &= 1-\xi_c-\xi_h. \end{aligned}$$

subject to a sequence of constraints

$$c_{s,t} + q_t[h_t^s - (1-\delta_h)h_{s,t-1}] + d_t = R_t d_{t-1} + w_t n_{s,t} + \Pi_{s,t},$$

where Π_t^s are profits from housing and capital producers.

Impatient Households (borrowers)

There is a measure $\theta^{bo} \equiv 1-\theta^s$ of impatient households. They have the same intra-period preferences over housing,consumption and leisure as patient households, but they have a lower discount factor than lenders(patient households) : $\beta^{bo} < \beta$. The lower discount factor means that impatient households will be borrowers in a neighborhood of the steady state. In fact, absent any frictions their borrowing would be unbounded in the steady state. Financing frictions make borrowing $l_{bo,t}$ bounded. Borrowers' incomes and housing stock values are subject to common idiosyncratic shocks $\varepsilon_{bo,t}$ that are i.i.d across borrowers and across time. $\varepsilon_{bo,t}$ has a CDF $F(\varepsilon_{bo,t})$ with $F'(\varepsilon_{bo,t}) = f(\varepsilon_{bo,t})$, and a mean $E(\varepsilon_{bo}) = 1$. As in Bernanke, Gertler and Gilchrist(1999) [19], I assume that the derivative of $\frac{\bar{\varepsilon} f(\bar{\varepsilon})}{1-F(\bar{\varepsilon})}$ is positive. This will be true

Fuerst(1997,1998[31][32]) and Kiyotaki and Moore (1997) [83] for firms. Estimation of structural consumption models supports this heterogeneity in impatience levels (Cagetti(2003)[26]). Higher impatience is also a reduced form proxy for higher expected income growth (see Browning and Tobacman's(2007)[24]) , or Carroll (2000)[34]). Under this interpretation, the impatient agents roughly correspond to young homeowners with an upward sloping expected wage profile that use borrowing to enjoy some of their higher future salaries in terms of consumption today. The patient households can be thought of as middle aged couples with relatively low expected salary growth. The entrepreneur‘s relative impatience can also be interpreted as reflecting a higher expected future profit growth relative to more mature but financially unconstrained firms, or a higher death rate of entrepreneurial firms relative to households.

over the range that is relevant for the optimal debt contract when ε follows a lognormal distribution. It will also be useful to define the sum of the value of the borrower's house and his wage income

$$A_{bo,t} = \varepsilon_{bo,t}[q_t(1-\delta_h)h_{bo,t-1} + n_{bo,t}w_t] = \varepsilon_{bo,t}\bar{A}_{bo,t}.$$

Lending in this economy is only possible through 1-period debt contracts that require a constant repayment $R_t^l l_{bo,t-1}$ independent of $\varepsilon_{bo,t}$ if the borrower is to avoid costly loan monitoring or enforcement, where R_t^l is the loan rate. The borrower can default and refuse to repay the debt. Savers cannot force borrowers to repay. Instead lending must be intermediated by banks that have a loan enforcement technology allowing them to seize collateral

$$\varepsilon_{bo,t}\tilde{A}_{bo,t} = \varepsilon_{bo,t}[(1-s_h)q_t(1-\delta_h)h_{bo,t-1} + (1-s_w)n_{bo,t}w_t]$$

at a proportional cost $\mu_{bo}\varepsilon_{bo,t}\tilde{A}_{bo,t}$ when the borrower defaults. $\mu_{bo} \in (0,1)$ determines the deadweight cost of default, $0 < s_w \leq 1$ and $0 < s_h \leq 1$ represent define the maximum loan to collateral ratio (often called the Loan to Value ratio) that the bank is willing to grant against each component of the collateral (as in Iacoviello (2005) [72]). Conditional on enforcement, the law cannot prevent the bank from seizing $\varepsilon_{bo,t}\tilde{A}_{bo,t}$. Suppose first that the borrower does not have access to any insurance against the $\varepsilon_{bo,t}$ shock. Whenever $\varepsilon_{bo,t} < \bar{\varepsilon}_{bo,t}$ the borrower prefers to default and lose

$$\varepsilon_{bo,t}\tilde{A}_{bo,t} < R_t^l l_{bo,t-1} = \bar{\varepsilon}_{bo,t}\tilde{A}_{bo,t}$$

when the bank enforces the contract. On the other hand when $\varepsilon_{bo,t} \geq \bar{\varepsilon}_{bo,t}$ the borrower prefers to pay $R_t^l l_{bo,t-1}$ rather than lose $\varepsilon_{bo,t}\tilde{A}_{bo,t} \geq R_t^l l_{bo,t-1}$. The net worth of the borrower after any loan repayment or default is $A_{bo,t} - \min[\varepsilon_{bo,t}, \bar{\varepsilon}_{bo,t}]\tilde{A}_{bo,t}$.[5]

To be able to use a representative agent framework while maintaining the intuition of the default rule above, I make two assumptions. First, borrowers' labour supply is predetermined with respect to the idiosyncratic shock. Second, borrowers have access to insurance contracts providing them with payments conditional on the realisation of $\varepsilon_{bo,t}$,

$$p_{bo}[\bar{A}_{bo,t} - A_{bo,t} + (H(\bar{\varepsilon}_{bo,t}) + \min[\varepsilon_{bo,t}, \bar{\varepsilon}_{bo,t}])\tilde{A}_{bo,t}], \text{ where}$$

[5] We could allow for separate shock processes affecting housing and wages while preserving tractability as long there are two types of loans : one loan collateralised by housing and another collateralised by income. This would force us to keep track of two types of debt and two external finance premia, and at least as a start it isn't clear that this extra complication is important for aggregate dynamics.

My derivation of the default rule also assumes that deposits are not seizable by the bank. This is without loss of generality : Suppose borrowers' deposits $d_{bo,t-1}$(or another safe asset such as money) can be seized by lenders to repay debt at a cost $\mu_d R_t d_{bo,t-1}$ with $\mu_d \geq 0$. If $\mu_d > 0$, a borrower would never want to hold deposits and loans simultaneously. In the special case of $\mu_d = 0$ and $\bar{R}_t = R_t$(deposits are a perfect collateral), $l_{bo,t} - d_{bo,t}$ is indeterminate, and we can set $d_{bot} = 0$ without loss of generality.

$$H(\bar{\varepsilon}_{bo,t}) = -[(1 - F(\bar{\varepsilon}_{bo,t}))\bar{\varepsilon}_{bo,t} + \int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon dF].$$

with $0 \leq p_{bo} \leq 1$. The insurer offers only complete insurance with $p_{bo} = 1$. Since they are risk averse, borrowers willingly buy this insurance, which completely diversifies the risk related to $\varepsilon_{bo,t}$. The payments from the insurance scheme cannot be seized by the bank. As a result, despite the insurance the bank cannot force the borrower to repay $R_t^l l_{bo,t-1}$ when $\varepsilon_{bo,t} < \bar{\varepsilon}_{bo,t}$. The borrower cannot commit to always repay the loan (or make up for any lack of payment by an individual borrower), even though from an ex-ante perspective it is optimal to do so. For any given R_t^l, this makes the borrower default when $\varepsilon_t < \bar{\varepsilon}_t$ and repay $R_t^l l_{bo,t-1}$ when $\varepsilon_{bo,t} \geq \bar{\varepsilon}_{bo,t}$. With the insurance, the borrower is guaranteed total resources of

$$\bar{A}_{bo,t} + H(\bar{\varepsilon}_{bo,t})\tilde{A}_{bo,t}.$$

While this level of insurance is clearly exaggerated, it provides a significant gain in tractability. [6] The $fs = 1$ case of full insurance can also be used as a point around which to approximate more realistic partial insurance or the uninsurable risk case with $p_{bo} = 0$ through perturbation methods. [7] For these reasons, the full insurance assumption can be seen as a useful benchmark for starting the analysis. Even with full insurance, we can still have a nondegenerate distribution of default rates and loan positions for arbitrary initial resource distributions among borrowers. To further simplify the model, I assume a symmetric initial distribution of housing among borrowers. This leads to a symmetric distribution of assets and income across borrowers, and allows us to discuss their choices using a representative borrower (henceforth also called the borrower). Define the rate of return required on loans made at $t-1$ as $\bar{R}_t$ (to be distinguished from the loan rate R_t^l). Since $\varepsilon_{bo,t}$ is idiosyncratic the bank can diversify it by lending to a large number of borrowers. Therefore it only requires that the loan is profitable in expectation. Loan rates depend on the aggregate state of the economy as in BGG[19]. [8] In order to participate in the loan, the bank requires that

$$[1 - F(\bar{\varepsilon}_{bo,t})]\,\bar{\varepsilon}_{bo,t}\tilde{A}_{bo,t} + (1 - \mu_{bo}) \int_0^{\bar{\varepsilon}_{bo,t}} \tilde{A}_{bo,t}\varepsilon_{bo} dF \geq \bar{R}_t l_{bo,t-1}.$$

The bank participation constraint will act as a borrowing constraint in this model. The bank offers the borrower each period a sequence of default rates contingent on the aggregate state. Competition among banks will make the bank participation constraint bind. Defining

$$G(\bar{\varepsilon}_{bo,t}) = [1 - F(\bar{\varepsilon}_{bo,t})]\bar{\varepsilon}_{bo,t} + (1 - \mu_{bo}) \int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon_{bo} dF,$$

[6] The insurance arrangement is mathematically identical to an economy where each borrower belongs to a large family with a continuum of members that treats all its members symmetrically, as in Shi (1997) [109] and other models of monetary and credit frictions.

[7] See Den Haan et al (2009)[5] for an example of using a model without idiosyncratic shocks or with insurance against idiosyncratic shocks to approximate a model without insurance.

[8] See appendix C for a more realistic model with loan rates that are predetermined with respect to the aggregate state.

we can rewrite the participation constraint as

$$G(\bar{\varepsilon}_{bo,t})\tilde{A}_{bo,t} = \bar{R}_t l_{bo,t-1}.$$

Since deposits are required to be risk-free, $\bar{R}_t = R_t$, the risk free rate. Noting that $G'(\bar{\varepsilon}_{bo,t}) > 0$ at an optimum, we can solve for a function $\bar{\varepsilon}_{bo}(\frac{\bar{R}_t l_{bo,t-1}}{\tilde{A}_{bo,t}})$, with the default threshold $\bar{\varepsilon}_{bo,t}$ increasing in the ratio of debt repayment to collateral. For a given idiosyncratic risk distribution the default rate is increasing in the default threshold, so that a higher debt to collateral ratio also increases the probability of default. [9]

With the assumption of perfectly competitive banks we can represent the problem of borrowers as if they choose default thresholds as a function of the aggregate states directly, subject to the bank's participation constraints. The representative borrower picks sequences of consumption,housing, loans, deposit , labour supply and default threshold functions $\{c_{bo,t}, h_{bo,t}, l_{bo,t}, d_{bo,t}, n_{bo,t}, \bar{\varepsilon}_{bo,t}\}_{t=0}^{\infty}$

to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^{bo^t} u_{bo,t}$$

where

$$\begin{aligned} u_{bo,t} &= \frac{(c_{bo,t}^{\xi_c} h_{bo,t}^{\xi_h} (1-n_{bo,t})^{\xi_n})^{1-\sigma}}{1-\sigma} \text{ for } \sigma \neq 1 \\ u_{bo,t} &= \xi_c \ln c_{bo,t} + \xi_h \ln h_{bo,t} + \xi_n \ln(1-n_{bo,t}) \text{ for } \sigma = 1, \end{aligned}$$

subject to a sequence of budget constraints

$$\begin{aligned} c_{bo,t} + q_t h_{bo,t} + d_{bo,t} &= q_t(1-\delta_h)h_{bo,t-1} + n_{bo,t}w_t \\ &\quad + H(\bar{\varepsilon}_{bo,t})[(1-s_h)q_t(1-\delta_h)h_{bo,t-1} \\ &\quad + (1-s_w)n_{bo,t}w_t] + l_{bo.t} + d_{bo,t-1}R_t \end{aligned}$$

and the participation constraints of the bank

$$G(\bar{\varepsilon}_{bo,t})\tilde{A}_{bo,t} = G(\bar{\varepsilon}_{bo,t})[(1-s_h)q_t(1-\delta_h)h_{bo,t-1} + (1-s_w)n_{bo,t}w_t] = \bar{R}_t l_{bo,t-1} \ .$$

In a neighbourhood of the steady state impatient households will set $d_{bo,t} = 0$ and $l_{bo,t-1} > 0$ for all t.[10]

[9] The caveat is that if a new unexpected shock significantly lowers the value of $\tilde{A}_{bo,t}$ it may be impossible to find a default threshold that allows the bank to break even on the loan with the risk free rate. This should not be a major concern except for very low aggregate shock values. This problem can be completely eliminated in the version of the model in which loan rates are predetermined with respect to the aggregate state (see appendix C).See the appendix in BGG(1999)[19] for a discussion of the same issue in their model. This issue also occurs in the Kiyotaki and Moore [83] model if a shock is unexpected (i.e it is not a news shock). Iacoviello (2005) [72] gets around this by assuming that lenders can seize borrower resources up to a fraction of the expected value of collateral assets.

[10] It is true in general that one cannot simultaneously have $l_{bo,t} > 0$ $d_{bo,t} > 0$. What cannot be excluded completely is that impatient households may wish to become savers for large enough positive shocks. This would be particularly troublesome in a model with a fixed borrowing limit as in Carroll(2001).[35]Here, we have a procyclical borrowing limit, and an impatient household would want to increase its borrowing in response to a higher limit.

2.2.2 Production

Financially Constrained Entrepreneurs

There is a measure 1 continuum of risk averse entrepreneurs that use capital and labour to produce final output. Just like borrowing households, I assume that their discount factor is below that of savers to guarantee that they borrow in equilibrium in a neighborhood of the steady state. Entrepreneurs' capital and output are subject to common multiplicative idiosyncratic shocks ε_{e_t}. These shocks are independent and identically distributed across time and across entrepreneurs with $E(\bar{\varepsilon}_{e,t}) = 1$, and a CDF $F(\bar{\varepsilon}_{e,t})$. Production is also subject to an aggregate TFP shock $z_{e,t}$. $z_{e,t} = G^t_{z,e}\tilde{z}_{e,t}$, where

$$\ln \tilde{z}_{e,t+1} = \rho \ln \tilde{z}_{e,t} + \varepsilon_{z,t+1}.$$

It will be useful to define expected output conditional on the aggregate shock

$$\begin{aligned} y_{e,t} &= z_{e,t}((u_{e,t}k_{e,t})^{\alpha} n_{e,t}^{1-\alpha})^{\theta}, 0 < \theta < 1 \text{ and} \\ \bar{A}_{e,t} &= (1-\delta_{e,t})q^k_t k_{e,t} + y_{e,t}. \end{aligned}$$

[11] The entrepreneurs' financial contract is similar to that of borrowers. Entrepreneurs are restricted to using one-period debt contracts in which the loan rates can be made contingent on aggregate shocks z_t but not on the idiosyncratic shock $\varepsilon_{e,t}$.They have access to insurance contracts that completely diversify the idiosyncratic risk after loan contracts are settled, but cannot commit to sharing the proceeds of this insurance with banks. Banks can seize collateral $\varepsilon_{e,t}\tilde{A}_{e,t}$ when the entrepreneur refuses to pay at a cost of $\mu_e\varepsilon_{e,t}\tilde{A}_{e,t}$. As in other models of financial frictions such as CMR(2007) [42] and Carlstrom and Fuerst (1998) [32], a fraction a of the wage bill must be paid before production occurs, requiring an intratemporal loan from the bank. This sort of working capital financing friction has been found to play a potentially important role in helping to explain the effect of credit frictions in response to news and credit shocks(see Inaba and Kobayashi (2007) [85] and Jermann and Quadrini (2008) [76]). The wages in advance of production requirement implies that the entrepreneur must choose the amount of labour before knowing the value of $\varepsilon_{e,t}$. The new intratemporal

[11] We need $\theta < 1$ to get a solution to the non stochastic balanced growth path when trying to match data on the firm default rate, foreclosure costs and long run external finance premium. With $\theta = 1$, we would have an Euler equation for capital which is only a function of the volatility $\sigma_e$of ε_e and the entrepreneur default threshold $\bar{\varepsilon}_e$. Together, with the Euler equation for entrepreneur loans, for any given σ_e we would have two equations in one unknown $\bar{\varepsilon}_e$, a problem which (generically) has no solution. We could allow for $\theta = 1$ if we give up on trying to match either the entrepreneur default rate target or foreclosure cost proportion, and use the Euler equation for loans and capital to jointly determine $(\sigma_e, \bar{\varepsilon}_e)$ or $(\mu_e, \bar{\varepsilon}_e)$ .Still, wihout $\theta < 1$, the model's BGP does not exist for arbitrary exogeneous (σ_e, μ_e). The assumption that θ is below 1, but close to 1 is consistent with empirical evidence and can be interpreted as a reflection of limited span of control for entrepreneurs. A similar issue occurs in BGG's[19] model. Their steady state system of equations is overidentified unless the marginal return on the entpreneur's project is endogenous.

loan modifies the default rule : the entrepreneur defaults if and only if

$$\begin{aligned} \varepsilon_{e,t}\tilde{A}_{e,t} &< R^l_{e,t}l_{e,t-1} + R^w_t a w_t n_{e,t} = \bar{\varepsilon}_{e,t}\tilde{A}_{e,t}, \text{ where} \\ \tilde{A}_{e,t} &= (1-s_k)(1-\delta_{e,t})q^k_t k_{e,t} + (1-s_y)y_{e,t}. \end{aligned}$$

I assume that the capital utilisation rate is predetermined with respect to the idiosyncratic shock to facilitate aggregation. s_k and s_y reflect differences in the ability to collateralise capital and revenue. This specification nests two cases that are frequently examined in the literature : a model where only capital serves as collateral as in Gerali et al (2009) [60] or Kobayashi et al (2007) [85] , and a model where only revenue serves as collateral as in Carlstrom and Fuerst (1998) [32] or in BGG (1999) [19]. For now I will focus on the case with $s_k = s_y = 0$. Also, I will focus on the case in which all of wages must be paid in advance,$a = 1$ as in Carlstrom and Fuerst's model. This case fits most naturally with the assumption that the lender can seize all the entrepreneur's revenue. [12] As for borrowers, I define

$$\begin{aligned} H(\bar{\varepsilon}_{e,t}) &= -[(1-F_e(\bar{\varepsilon}_{e,t}))\bar{\varepsilon}_{e,t} + \int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF_e] \text{ and} \\ G(\bar{\varepsilon}_{e,t}) &= [1-F_e(\bar{\varepsilon}_{e,t})]\bar{\varepsilon}_{e,t} + (1-\mu_{e,t})\int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF_e. \end{aligned}$$

the representative entrepreneur picks sequences of consumption, capital, utilisation rates, labour demand, default thresholds, loans and deposits $\{c_{e,t}, k_{e,t+1}, u_{e,t}, n_{e,t}, \bar{\varepsilon}_{e,t}, l_{e,t}, d_{e,t}\}$ to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^{e^t} \ln c_{e,t}$$

subject to a sequence of constraints

$$c_{e,t} + (1-a)w_t n_{e,t} + q^k_t k^e_{t+1} + d_{e,t} = \bar{A}_{e,t} + H(\bar{\varepsilon}_{e,t})\tilde{A}_{e,t} + l_{e,t} + R_t d_{e,t-1}$$

and the bank's break-even constraints

$$G(\bar{\varepsilon}_{e,t})\tilde{A}_{e,t} = R_t l_{e,t-1} + a w_t n_{e,t}$$

In a neighbourhood of a balanced growth path $d_{e,t} = 0$ and $l_{e,t} > 0$ for all t.

[12] A more realistic assumption is that lenders can seize profits rather than revenue, with remaining wages owed to workers paid out before debt repayments. However, this would force me to keep track of 2 default thresshold : one default thresshold when the value of collateral is below the required loan repayment, and another one when profits turn negative and the lender can only seize capital. The importance of this complication isn't obvious. In the meantime, one can try to approximate the model where workers are paid first in default by calibrating a lower s_y to match the proportion of the remaining wage bill $(1-a)w_t n_{e,t}$ out of revenue absent financing constraints. I have examined more realistic cases such as $s_y = 0.5$ and $a = 0.5$. The results were similar.

Financially Unconstrained Firms

Latter, I will compare the model where firms are financially constrained with a model where firms are financially unconstrained. In this case, there is a representative firm owned by savers that picks sequences of capital, utilisation rates and labour $\{k_{u,t+1}, u_{u,t}, n_{u,t}\}$ to maximise its value

$$E_0\Sigma_{t=0}^{\infty}\beta^t\lambda_t^s[z_{u,t}(u_{u,t}k_{u,t})^{\alpha}n_{u,t}^{1-\alpha} - w_t n_{u,t} - q_t^k(k_{u,t+1} - (1-\delta_{u,t})k_{u,t})],$$

where $0 < \alpha < 1$, and $\delta_{u,t} = \delta_u u_{u,t}^{\phi_u}$.

$$z_{u,t} = G_{z,u}^t \tilde{z}_{u,t},\ \ \tilde{z}_{u,t} = \tilde{z}_{e,t}.$$ [13]

2.2.3 Housing Production

I use a standard convex adjustment cost model of housing. This is the simplest model that generates procyclical house prices. A representative firm owned by the savers produces housing. The firm purchases I_t^h units of the consumption good from savers and turns it into

$$I_t^h = h_t - (1-\delta_h)h_{t-1}$$

units of housing while paying an adjustment cost of

$$\frac{\gamma^h}{2}(\frac{I_t^h}{h_{t-1}} - \frac{I_h}{h}))^2 h_{t-1}.$$

Note that the firm takes the aggregate housing stock h_{t-1} as given when choosing I_t^h. With these assumptions, the housing producer's problem reduces to picking I_t^h each period to maximise profits

$$\theta^s\Pi_t^h = (q_t - 1)I_t^h - \frac{\gamma^h}{2}(\frac{I_t^h}{h_{t-1}} - \frac{I_h}{h})^2 h_{t-1},$$

where $\frac{I_h}{h}$ is the investment in housing to housing ratio in the balanced growth path.

The resulting housing supply curve is

$$q_t = 1 - \gamma^h\frac{I^h}{h} + \gamma^h\frac{I_t^h}{h_{t-1}}. \tag{2.1}$$

Along the balanced growth path, $q = 1$ and the housing producer makes no profits.

[13] $G_{z,u} \neq G_{z,e}$ due the difference in returns to scale between uconstrained firms and entrepreneurs. For our calibration of $\theta = 0.95$ the difference is small.

2.2.4 Capital Production

Capital producers face the same problem as housing producers. Savers own the capital producing firm. Profit maximisation yields the supply curve

$$q_t^k = 1 - \gamma \frac{I}{k} + \gamma \frac{I_t}{k_t}. \tag{2.2}$$

2.2.5 Equilibrium

Combining the budget constraints of the model's agents gives us the GDP identity (output market clearing condition) for the model :

$$Y_t = C_t + I_t + I_t^h + AC_t + FF_t \tag{2.3}$$

where C_t, I_t, I_t^h are aggregate consumption, investment in capital and investment in housing, AC_t are non financial adjustment costs of capital and housing and FF_t are the costs of default on financial contracts, and Y_t is aggregate final output.

In addition labour and credit market clearing require

$$N_t^s = \theta_s n_{s,t} + \theta_{bo} n_{bo,t} = n_{e,t} = N_t^d, \tag{2.4}$$

$$\theta_s d_t = \theta_{bo} l_{bo,t} + \theta_e l_{e,t} \tag{2.5}$$

Equilibrium consists of a set of prices and interest rates $\{q_t, q_t^k, R_{t+1}, w_t\}$ for all possible states and for all $t \geq 0$ such that all markets clear when households and firms maximise while taking prices as given.

2.2.6 Financial frictions in partial Equilibrium

Borrowers

Using the relation between the Lagrange multiplier on the bank participation constraint $\psi_{bo,t}$ and the marginal utility of consumption $\lambda_{bo,t}$, the first order conditions for the borrower's problem are :

$$c_{bo,t} \; : \; \lambda_{bo,t} = \xi_c \frac{(c_{bo,t}^{\xi_c} h_{bo,t}^{\xi_h} (1 - n_{bo,t})^{\xi_n})^{1-\sigma}}{c_{bo,t}}, \tag{2.6}$$

$$\bar{\varepsilon}_{bo,t} \; : \; \psi_{bo,t} = \lambda_{bo,t} \left[\frac{-H'(\bar{\varepsilon}_{bo,t})}{G'(\bar{\varepsilon}_{bo,t})} \right] \equiv \lambda_{bo,t} efp_{bo,t}, \tag{2.7}$$

$$l_{bot} \; : \; \lambda_{bo,t} = \beta^{bo} R_{t+1} E_t \lambda_{bo,t+1} efp_{bo,t+1} \tag{2.8}$$

$$h_{bo,t} \; : \; q_t \lambda_{bo,t} = \frac{\xi_h}{\xi_c} \frac{c_{bo,t}}{h_{bo,t}} \lambda_{bo,t} + \beta^{bo}(1 - \delta_h) E_t \lambda_{bo,t+1} q_{t+1} A_{hh}(\bar{\varepsilon}_{bo,t+1}). \tag{2.9}$$

$$n_{bot} \; : \; \frac{\xi_n}{1 - n_{bo,t}} = \xi_c \frac{w_t}{c_{bo,t}} A_{nw}(\bar{\varepsilon}_{bo,t}), \tag{2.10}$$

where

$$A_{nw}(\bar{\varepsilon}_{bo,t}) = 1 + (1 - s_w)[(H(\bar{\varepsilon}_{bo,t}) + efp_{bo,t}G(\bar{\varepsilon}_{bo,t})] \text{ and}$$
$$A_{hh}(\bar{\varepsilon}_{bo,t}) = 1 + (1 - s_h)[(H(\bar{\varepsilon}_{bo,t}) + efp_{bo,t}G(\bar{\varepsilon}_{bo,t})].$$

[14] The loan Euler equation has the same form as that of the saver except that the effective gross interest rate is now $R_{t+1}efp(\bar{\varepsilon}_{bo,t+1})_{bo,t+1}$. $efp_{bo,t} = -\frac{H'(\bar{\varepsilon}_{bo,t})}{G'(\bar{\varepsilon}_{bo,t})} > 1$ is the external finance premium faced by borrowers on loans relative to the risk free interest rate R_t. $\frac{d}{d\bar{\varepsilon}_{bo}}(\frac{\bar{\varepsilon}f(\bar{\varepsilon}_{bo})}{1-F(\bar{\varepsilon}_{bo})}) > 0$ at an optimum implies that $efp_{bo,t}$ is increasing in $\bar{\varepsilon}_{bo,t}$, and therefore in the default rate as well. The result that the external finance premium is increasing in the default rate is quite intuitive. It implies, that if we can reproduce the countercyclical default rates found in the data, the model will generate a countercyclical external finance premium. This result will also allow us to interchangeably interpret any result obtained for the effect of an increase in the default rate in terms of an increase in the external finance premium (as long as the distribution of idiosyncratic shock and the loan enforcement technology μ_{bo} are fixed across time). For simplicity, I will focus on the case with separable utility ($\sigma = 1$). An increase in the expected future external finance premium reduces current consumption relative to future consumption, just like an increase in the risk free interest rate. From the bank's participation constraint we know that the default rate is increasing in the debt to collateral ratio (a measure of leverage) : $\bar{\varepsilon}'_t(\frac{\bar{R}_t l_{t-1}}{\bar{A}_{bo,t}}) > 0$. On impact, with $l_{bo,t-1}$ and $h_{bo,t-1}$ predetermined, any positive shock which increases wages, labour supply and house prices reduces the loan to collateral ratio. This decreases the default rate, and lowers the external finance premium. The effect of the positive shock in the next periods depends on how borrowers adjust loan demand and housing in response to the shock. In a model with risk neutral households, loan demand may increase so much that a positive shock would ultimately lead to a higher loan to collateral ratio and a higher default rate. But with a diminishing marginal utility of consumption, this does not have to be the case. The existence of a soft borrowing constraint makes the impatient household behave more like the consumption smoothing patient household with a bias towards debt financed consumption instead of saving. For a fixed level of financial frictions and desired housing, a consumption smoothing borrower reacts to an increase in wealth by increasing savings. This increase in saving can be accomplished through a combination of reduced borrowing and increased accumulation of collateral in the form of housing. At the same time an increase in the value of collateral encourages higher borrowing. If the first effect dominates, then the loan to collateral ratio $\frac{\bar{R}_t l_{bo,t-1}}{\bar{A}_{bo,t}}$ is countercyclical and therefore the external finance premium is also countercyclical beyond the initial impact of a shock.

To gain further insight into the effect of financial frictions on consumption it will be useful to derive a consumption function for the log-utility case under perfect foresight for exogenous housing investment and labour supply. By combining the bank

[14]Here and throughout the paper, optimality also requires the transversality conditions to hold.

loan participation constraint with the budget constraint, and iterating forward on the resulting expression for $l_{bo,t-1}$, we obtain

$$\begin{aligned} c_{bo,t} &= \frac{R_{bo,t}}{1+\sum_{j=1}^{\infty}\beta^{bo^j}\Pi_{k=1}^{j}\frac{R_{t+k}efp_{bo,t+k}}{R_{bo,t+k}}}\sum_{j=0}\frac{n_{bo,t+j}w_{t+j}-q_{t+j}I_{bo,t+j}^{h}}{\Pi_{k=0}^{j}R_{bo,t+k}} \\ &\quad -\frac{R_{bo,t}l_{bo,t-1}}{1+\sum_{j=1}^{\infty}\beta^{bo^j}\Pi_{k=1}^{j}\frac{R_{t+k}efp_{bo,t+k}}{R_{bo,t+k}}},\text{ where} \\ R_{bo,t+k} &= -\frac{H(\bar{\varepsilon}_{bo,t+k})}{G(\bar{\varepsilon}_{bo,t+k})}R_{t+k}=\left(1+\frac{\mu_{bo}\tilde{A}_{bo,t+k}\int_0^{\bar{\varepsilon}_{bo,t+k}}\varepsilon dF_{bo}}{R_{t+k}l_{bo,t+k-1}}\right)R_{t+k}>R_{t+k}. \end{aligned}$$

Financial frictions affect consumption by modifying the effective interest rates facing the household through factors that depend on default thresholds $\bar{\varepsilon}_{bo,t+k}$ (or equivalently on default rates for fixed idiosyncratic risk distributions and enforcement cost parameter μ_{bo}). In fact the consumption function without financial frictions can be obtained by setting $efp_{bo,t+k} = 1$ and $R_{bo,t+k} = R_{t+k}$ for all periods. We have already established that $efp_{bo,t+k}$ is increasing in $\bar{\varepsilon}_{bo,t+k}$. $R_{bo,t+k}$ is expected rate of return on the loan including the compensation for the expected default costs per dollar of loan $\frac{\mu_{bo}\tilde{A}_{bo,t+k}\int_0^{\bar{\varepsilon}_{bo,t+k}}\varepsilon dF_{bo}}{R_{t+k}l_{bo,t+k-1}}$. This "credit risk" spread is clearly increasing in the default rate for a fixed leverage ratio. Allowing for the positive relation between the default rate and leverage complicates matters, but $R_{bo,t+k}$ is still increasing in $\bar{\varepsilon}_{bo,t+k}$ as long as the gross spread $1+\frac{\mu_{bo}\tilde{A}_{bo,t+k}\int_0^{\bar{\varepsilon}_{bo,t+k}}\varepsilon dF_{bo}}{R_{t+k}l_{bo,t+k-1}}$ is smaller than $efp_{bo,t+k}$.[15]. This condition holds in a neighbourhood of the steady state in all plausible calibrations (see the discussion in the calibration section). The effect of increasing external finance premia $efp_{bo,t+k}$ is clearly to reduce the present value of the household's wealth net of housing investments, which leads to a reduction in consumption. The effect of changes in future $R_{bo,t+k}$ is more ambiguous. On one hand higher $R_{bo,t+k}$ reduce the present value of future income net of housing investment. On the other hand they reduce the effect of higher $efp_{bo,t+k}$ on the present value of that future income. Finally an increase in $R_{bo,t}$ reduces consumption by raising the household's debt burden. Of course, in order to get a definite answer about the effect of expected default rates on non-durable consumption, we need to also solve for the behaviour of housing investment and labour supply. Doing this does not allow us to definitively sign the relation between financial frictions and consumption, but in all the calibrations I tried declining default rates led to an increase in borrowers' consumption.

[15] Omitting time subscripts,

$$\frac{d}{d\bar{\varepsilon}_{bo}}\left(\frac{R_{bo}}{R}\right)=-\frac{H'(\bar{\varepsilon}_{bo})G(\bar{\varepsilon}_{bo})-H(\bar{\varepsilon}_{bo})G'(\bar{\varepsilon}_{bo})}{G(\bar{\varepsilon}_{bo})^2}>0$$

iff $\frac{R_{bo}}{R}=\frac{-H(\bar{\varepsilon}_{bo})}{G(\bar{\varepsilon}_{bo})}<\frac{-H'(\bar{\varepsilon}_{bo})}{G'(\bar{\varepsilon}_{bo})}=efp_{bo}$.

The Euler equation for housing shows how financial frictions distort the usual housing investment equation. In a neighbourhood of the steady state, $A_{hh}(\bar{\varepsilon}_{bo,t+1})$ is greater than 1, meaning that the marginal value of investing in housing is more sensitive to the future expected value of housing than in the model without financing frictions. The effect of a change in the expected future external finance premium is less clear-cut. For a given expected house price appreciation an increase in the expected future default rate (and hence in the external finance premium) increases the marginal value of housing as collateral : $\frac{d}{d\bar{\varepsilon}_{bo}}(\frac{\bar{\varepsilon}f(\bar{\varepsilon}_{bo})}{1-F(\bar{\varepsilon}_{bo})}) > 0$ implies that $A_{hh}(\bar{\varepsilon}_{bo,t+1})$ is increasing in $\bar{\varepsilon}_{bo,t+1}$.[16] Fixing consumption across time, this would make the borrower's housing investment increasing in the expected external finance premium. At the same time, there is an indirect effect of financing frictions on housing investment through the effect of these frictions on non durable consumption. From the Euler equation for loans, a reduction in the future external finance premium increases $\frac{\lambda^{bo}_{t+1}}{\lambda^{bo}_t}$, which reduces the marginal cost relative to the marginal value of housing investment for borrowers. This effect may cause housing investment to be decreasing in the expected external finance premium. In the presence of aggregate uncertainty, we cannot determine analytically which effect dominates. However, the effect of the future default rate on the non durable consumption to housing ratio $\frac{c_{bo,t}}{h_{bo,t}}$ can be easily determined in the special case when agents have perfect foresight. Combining the loan and housing Euler equations under perfect foresight we obtain :

Proposition 8. *With perfect foresight,* $\frac{\partial(c_{bo,t}/h_{bo,t})}{\partial\bar{\varepsilon}_{bo,t+1}} > 0$.

Démonstration. See appendix A. □

This proposition implies that the borrower's housing investment must increase as long as consumption increases when the external finance premium falls, and vice versa if the external finance premium rises. Due to certainty equivalence, the proposition also holds in a linear approximation of the dynamics. For small levels of uncertainty, the effects of changes in the expected future default rates should be similar in a nonlinear approximation as well.

The labour supply equation shows how the financial friction distorts the borrower's labour supply decision. The level of financing frictions has a direct impact on labour supply through its effect on $A_{nw}(\bar{\varepsilon}_{bo,t})$ for a given $\frac{w_t}{c^{bo}_t}$. $A_{nw}(\bar{\varepsilon}_{bo,t})$ is increasing in the default rate, and therefore a similar analysis as for housing applies here as well : the direct impact of higher financing frictions is to increase the work effort of borrowers due to the higher value of wage income as loan collateral. A reduction in financing frictions will therefore lower borrowers' labour supply, holding everything else constant. This effect disappears when labour income is fully exempt. In that case, labour supply is determined by the same equation as patient households' labour supply. In addition, the effect of the external finance premium on $c_{bo,t}$ changes the strength of the income effect on labour supply. For example if the external finance premium

[16] $\frac{d(A_{hh}(\bar{\varepsilon}_{bo,t}))}{d\bar{\varepsilon}_{bo,t}} = [H'(\bar{\varepsilon}_{bo,t}) + G'(\bar{\varepsilon}_{bo,t})efp(\bar{\varepsilon}_{bo,t}) + G(\bar{\varepsilon}_{bo,t})efp'(\bar{\varepsilon}_{bo,t})] = G(\bar{\varepsilon}_{bo,t})efp'(\bar{\varepsilon}_{bo,t}) > 0.$

is countercyclical, then borrowers' nondurable consumption is more procyclical than savers' due to the relaxation in the financing constraint. In this case the income effect on labour supply is stronger for borrowers than for savers. As a result, even when wages cannot be seized by lenders, borrowers work hours will be less procyclical than savers' work hours in the model.

Entrepreneurs

As for the borrowers, we can use the relation between the Lagrange multiplier on the bank participation constraint and the marginal utility of consumption to obtain the following first order conditions for the entrepreneurs :

$$c_{e,t} : \frac{1}{c_{e,t}} = \lambda_{e,t} \tag{2.11}$$

$$\bar{\varepsilon}_{e,t} : \psi_{e,t} = \lambda_{e,t} \frac{-H'(\bar{\varepsilon}_{e,t})}{G'(\bar{\varepsilon}_{e,t})} \equiv \lambda_{e,t} efp_{et}. \tag{2.12}$$

$$l_{e,t} : \lambda_{e,t} = \beta^e R_{t+1} E_t \lambda_{e,t+1} efp_{e,t+1}. \tag{2.13}$$

$$k_{e,t+1} : \lambda_{e,t} q_t^k = \beta^e E_t \lambda_{e,t+1} [A_{kk}(\bar{\varepsilon}_{e,t+1}) q_{t+1}^k (1 - \delta_{e,t+1}) + A_{ky}(\bar{\varepsilon}_{e,t+1}) \alpha \theta \frac{y_{e,t+1}}{k_{e,t+1}}]. \tag{2.14}$$

$$n_{e,t} : A_{ky}(\bar{\varepsilon}_{e,t})(1-\alpha)\theta \frac{y_{e,t}}{n_{e,t}} = (1 - a + a efp_{e,t}) w_t \tag{2.15}$$

$$u_{e,t} : A_{ky}(\bar{\varepsilon}_{e,t}) \alpha \theta \frac{y_{e,t}}{k_{e,t}} = \phi_e A_{kk}(\bar{\varepsilon}_{e,t}) \delta_{e,t} q_t^k, \tag{2.16}$$

where

$$\begin{aligned} A_{kk}(\bar{\varepsilon}_{e,t}) &= 1 + (1 - s_k)[(H(\bar{\varepsilon}_{e,t}) + efp_{e,t} G(\bar{\varepsilon}_{e,t})] \text{ and} \\ A_{ky}(\bar{\varepsilon}_{e,t}) &= 1 + (1 - s_y)[(H(\bar{\varepsilon}_{e,t}) + efp_{e,t} G(\bar{\varepsilon}_{e,t})]. \end{aligned}$$

The analysis of these equations parallels in many respects the previous analysis for borrowing households. The key modification of entrepreneur behaviour relative to a standard financially unconstrained firm comes through the evolution of the external finance premium on the bank loan, $efp_{e,t+1}$ which is increasing in the default threshold $\bar{\varepsilon}_{e,t+1}$. The Euler equation for loans implies that an increase in the expected future external finance premium reduces current consumption relative to future consumption for the entrepreneur. In response to an exogenous increase in the value of collateral, on one hand the entrepreneur wants to expand his loan. This tends to raise $efp_{e,t+1}$. On the other hand the entrepreneur will smooth changes in his non-durable consumption by moderating the increase in the loan and by raising investment in the collateral asset, capital. To analyse the impact of financing frictions on investment and labour demand, start by assuming a fixed capital utilisation rate. Consider an increase in the expected future finance premium. Due to diminishing marginal utility of consumption, this is equivalent to a reduction in the marginal value of transferring resources into the future relative to the current cost. In combination with diminishing marginal productivity of capital, this reduces the entrepreneur's desired capital investment. At the same time, an increase in the expected future external finance premium also increases the marginal value of capital as collateral ($A_{kk}(\bar{\varepsilon}_{e,t})$ is increasing in $\bar{\varepsilon}_{e,t+1}$, and $\bar{\varepsilon}_{e,t+1}$ is increasing in $efp_{e,t+1}$). As a result, for a given ratio of marginal utilities $\frac{\lambda_{e,t+1}}{\lambda_{e,t}}$ and prices q_t^k, q_{t+1}^k, an increase in $\bar{\varepsilon}_{e,t+1}$ will increase $k_{e,t+1}$. The sign of the relation between the expected future external finance premium and investment is ambiguous with uncertainty, but we can obtain a sharper result under perfect foresight or linear approximation dynamics, using a similar proof to that for the housing to nondurable consumption ratio of borrowing households :

Proposition 9. *With perfect foresight,* $\frac{\partial k_{e,t+1}}{\partial \bar{\varepsilon}_{e,t+1}} < 0$ *for fixed employment. With variable employment* $\frac{\partial n_{e,t}}{d\bar{\varepsilon}_{e,t}} < 0$ *is a sufficient condition for* $\frac{\partial k_{e,t+1}}{\partial \bar{\varepsilon}_{e,t+1}} < 0$.

Démonstration. See appendix A. □

The labour demand decision of entrepreneurs is directly distorted by financing frictions as long as $s_y < 1$ or $a > 0$. In the presence of working capital requirements on wages ($a > 0$), a higher external finance premium acts as a tax on hiring and lowers labour demand. At the same time, if revenue can be used as collateral ($s_y < 1$), then an increase in the external finance premium raises the value of labour as an input into the collateral that can be offered by the entrepreneur. This would tend to stimulate labour demand in response to a higher external finance premium. If $a = 0$, labour demand is increasing in the default rate since $A_{ky}(\bar{\varepsilon}_{e,t})$ is increasing in $\bar{\varepsilon}_{e,t}$. When $a > 0$ the overall effect of $\bar{\varepsilon}_{e,t}$ on labour demand is ambiguous, but it is possible to give a sufficient condition on a guaranteeing that an increase in the default rate (and hence in the external finance premium) reduces entrepreneur labour demand :

Proposition 10. $a \geq 1 - s_y$ *implies that* $\frac{\partial n_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$.

Démonstration. See appendix A. □

The proposition says quite intuitively that labour demand is guaranteed to be decreasing in the level of financing frictions if the wages in advance requirement is strong enough or if revenue is hard to collateralize. Finally, given our benchmark of $s_y = s_k$, the capacity utilisation rate decision is identical to that of a financially unconstrained firm. However, the financing frictions will affect the entrepreneurs' utilisation rate decision indirectly. For example, if a decline in the external finance premium stimulates labour demand, this will increase the marginal product of capital which will in turn increase the desired capital utilisation rate.

2.3 General Equilibrium Results

I solve the model at a quarterly frequency using loglinear and second order perturbations around the deterministic Balanced Growth Path (henceforth the BGP). After detrending all variables by their deterministic growth rates, I use Klein's (2000) [84] QZ decomposition method to solve for the linear approximation and Schmitt-Grohe and Uribe's (2004) algorithm to solve for the second order approximation [67]. In the nonlinear case I examine the effect of a single initial shock, starting from the ergodic means of the variables. When performing simulations using second order approximations, there is always the possibility that the approximated dynamics are explosive even though the true dynamics are stable. I apply the standard solution of using pruned dynamics as in Kim et al (2008) [80].

2.3.1 Calibration

I calibrate the model to the balanced growth path of the US economy at a quarterly frequency. Some of the calibrations follow standard practice in other models without financial frictions (see table 1). The coefficient of relative risk aversion σ is set to 1.5, which is in line with the empirical upper bounds established in Chetty (2006) [39] and the evidence of consumption-labour complementarity in Basu and Kimball (2002) [16]. The results are similar for the separable log-utility case. The patient households' discount factor β is set to match an average annual real risk-free rate of 4%. This implies $\beta = 0.995$ for the nonseparable utility case. The housing and consumption share parameters in the utility function ξ_h and ξ_c are set to deliver an annual housing stock to output ratio of around 1.3 as in Davis and Heathcote (2005) [47] and an hours of work share of 0.32. This leads to a share of consumption in the utility function of $\xi_c = 0.3044$ and a share of housing of $\xi_h = 0.0375$.

The trend growth rate of output is set to an annual rate of 1.6%. The capital share α is set to 0.3. This is a common choice in other studies distinguishing housing from other forms of capital, and delivers an annual capital to output ratio of around 2.31. For the depreciation rates on housing and capital (excluding housing) I rely on the estimates in Davis and Heathcote (2005)[47]. Therefore, I set $\delta_h = 0.016/4$ and $\delta = 0.056/4$. The elasticity of the price of capital to the investment to capital ratio $\gamma I/K$ is set at 1, based on the estimates reported in Christiano and Fisher (1998) [41] . The elasticity of the housing price to investment is set to 1 as in Carlstrom and Fuerst (2006) [33]. [17] The entrepreneur returns to scale parameter θ is set to 0.95. This is in line with several estimates that report a degree of returns to scale close to but below 1 (see the discussion in Atkeson and Kehoe (2007)[14]). The benchmark model assumes a contemporaneous TFP shock as the only source of aggregate uncertainty. I assume $\rho_z = 0.95$ and $\sigma_z = 0.007$ as in Cooley and Prescott (1995) [43].

[17] We should note that they use the flow formulation of investment adjustment costs.

We now come to the parameters related to the financial frictions. I set the share of impatient households at 40%, in line with estimates reported in Scoccianti (2009) [108]of the proportion of US households in debt in the early 2000's. This number is somewhat higher than that used in models with rule of thumb households that consume all their income or in models using the Iacoviello framework, but our definition of being credit constrained is weaker than the quantity rationing imposed in those models.

The exemption levels control the maximum loan to value allowed by the bank against different types of collateral. As a baseline, I interpret the household's debt contract in the model as a mortgage in which the bank can seize the whole value of the house before the foreclosure costs captured by $\mu_{bo} > 0$. Therefore, I set $s_h = 1$ and $s_w = 1$. Allowing for plausibly low wage garnishment rates to account for informal bankruptcy (see Ausubel and Dawsey (2004) [15]) does not have a quantitatively significant impact on the results. For the entrepreneurs I start by assuming $s_y = s_k = 0$ and $a = 1$. More realistic combinations such as $s_y = 0.5$ and $a = 0.5$ do not lead to quantitatively significant changes.

I calibrate the borrowers' and entrepreneurs' discount factors and the idiosyncratic shock volatilities to match aggregate leverage ratios of firm and household sectors and default rates while maintaining plausible differences in discount factors. Covas and Den Haan (2007) [45] report an average debt to assets ratio for nonfinancial Compustat corporations of 0.587 over 1971-2004. This target leads to setting $efp_e = 1.015$, leading to a discount factor β^e of around 0.98. For the household sector, there is less information. [18] Campbell and Hercowitz (2006) [28]report an average ratio of aggregate mortgage debt to housing of around 0.32 over 1954-1982 and 0.42 over 1995-2005. If we interpret all debt to be literally held by the borrowers in the model, any attempt to match such a ratio would require implausibly high differences in discount factors between households. Instead, I assume in the baseline calibration a discount factor for borrowers of $\beta^{bo} = 0.97$ ($efp_{bo} = 1.025$), which is in the middle of plausible estimates (see Iacoviello (2005)[72] and Iacoviello and Neri (2008) [74]). This discount factor leads to a BGP loan to collateral value ratio for the borrowing households of 0.77, which is close to the average loan to value ratio on mortgages of 0.76 over 1973-2006 (Iacoviell and Neri (2008) [74]). [19] The results are quantitatively similar for moderate changes in β^{bo}, and qualitatively the same for very high or very low β^{bo}. The calibration generates a credit spread for the household loan of 0.3% per year and a credit spread for the firm loan of 0.42% per year. These numbers are considerably lower than the credit spreads of $2-3\%$ between for example prime loan rates or commercial paper and government T-bills used to calibrate other models with

[18] Other authors such as Frank and Goyal [119] compute significantly lower leverage ratios based on the market values of firms. Using the book value of assets as in Covas and Den Haan [45] is closer to the definition of leverage in my model.

[19] We could match a long-run household leverage ratios above, if we allow for the possibility that savers also owe some debt, but because they can be better monitored by the banks they are not subject to the same financial frictions as borrowers (in which case their debt holdings are indeterminate).

default such as BGG (1999) [19] or Carlstrom and Fuerst (1998) [32]. [20] Trying to generate higher and more realistic credit spreads requires implausibly high discount factor differences among agents, or a mixture of implausibly high steady states default rates and low recovery rates on collateral by lenders (very high μ_{bo}, μ_e). Bankruptcy costs in the model seem insufficient to explain the observed credit spreads between government and private debt. Other unmodelled factors such as loan processing and screening costs or ex-ante asymmetric information may be required to explain these spreads.

The monitoring parameter μ_i is a measure of the loss suffered by the bank in default. There are many estimates of the costs of default for firms, though they differ significantly in whether or not they include direct or indirect bankruptcy costs and the representativeness of the sample of firms they use. I set the loss given default parameter for entrepreneurs $\mu_e = 0.15$ as in Carlstrom and Fuerst (1998) [32]. This is in the middle of empirical estimates of this parameter. Information on μ_{bo} is scarce. Jeske and Krueger (2006) [77] use an estimate $\mu_{bo} = 0.22$ based on comparing the price of foreclosed homes to comparable non foreclosed homes. Both ε_{bo} and ε_e are assumed to follow a lognormal distribution with a standard deviation σ_i of $\ln \varepsilon_i$, $i \in \{e, bo\}$. I calibrate the the volatility of the idiosyncratic shocks σ_i to match estimates of default rates for borrowers and firms. For firms, I use a quarterly default rate of 0.75% as in BGG [19]. This is in the middle of the range used by most authors(e.g. Carlstrom and Fuerst (1998) [32] use an estimate for US corporations of 0.974% per quarter, while Mankart and Rodano (2008) [90] report a bankruptcy rate for entrepreneurs of 0.5% per quarter). For households, I interpret default in the model to be similar to foreclosure on a mortgage in the data. I use the average annual foreclosure rate of 1.4% in 1990-2004 from Garriga and Shlagenhauf (2009) [59] to calibrate the BGP default rate households.

[20] In BGG's model, the external finance premium defined as the effective discount rate on borrowing relative to the risk free rate ($efp_i - 1$ in my model) is equal to the expected cost of default per dollar of loans (e.g $\frac{\mu_{bo}\tilde{A}_{bo}\int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon dF_{bo}}{l_{bo,t-1}}$ for the borrowing household in my model). This is not the case in my model, where the external finance premium measured by the loan Euler equation efp_{bo} is much higher than $\frac{\mu_{bo}\tilde{A}_{bo}\int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon dF_{bo}}{l_{bo,t-1}}$. The expected cost of default per dollar of loans is closer to the observed interest rate spread due to default risk.Using the spread based on the loan rate $R_{i,t}^l$ yields a similarly low number(0.34% annually for borrowers and 0.62% annually for entrepreneurs in the baseline calibration).

2.3.2 Business Cycle Dynamics Using the Loglinear Approximation

I compare the responses of variables in four models to a positive 1 standard deviation 0.7% Total Factor Productivity (henceforth TFP) shock (figures 1-3). [21]Model 1 (light green IRFs) is the frictionless Real Business Cycle (henceforth RBC) model. Model 2 (green IRFs) is a classical financial accelerator model with financially constrained entrepreneurs. Model 3 (red IRF's) assumes only households are credit constrained and drops the entrepreneur sector. Model 4 (blue) is the full model with all financing frictions. I start the analysis with model 1, then examine the effect of adding one financial friction at a time. Finally, I analyse the effect of having both financing frictions in model 4. A key result in this section is that household borrowing frictions reduce the procyclicality of labour supply as long as they increase the procyclicality of aggregate household consumption. This result will play an important role in understanding the interaction between household and entrepreneur borrowing frictions.

Model 1 is a standard complete markets Real Business Cycle model with capital and housing adjustment costs. The increase in current and future TFP raises investment and labour demand, though by less than in a model without adjustment costs. The price of capital q_t^k increases because of the adjustment costs, especially in early periods when TFP is higher and investment is more profitable. Combining the savers' deposit Euler equation with the unconstrained firms' capital Euler equation under the simplifying assumptions of fixed capacity utilisation and perfect foresight, we get

$$R_{t+1} = \frac{(1-\delta_u)q_{t+1}^k + \alpha \frac{y_{u,t+1}}{k_{u,t+1}}}{q_t^k}. \tag{2.17}$$

From this equation, the increase in the marginal product of capital $\alpha \frac{y_{u,t+1}}{k_{u,t+1}}$ increases R_{t+1} and investment demand, but the front loaded rise in q_t^k reduces the capital gain $\frac{q_{t+1}^k}{q_t^k}$ (and possibly also the return on capital per unit of consumption $\alpha \frac{y_{u,t+1}}{q_t^k k_{u,t+1}}$). By the standard no-arbitrage argument, R_{t+1} declines. Because of the fall in R_{t+1} and the smaller increase in labour demand relative to a model without adjustment costs the income effect dominates the behaviour of labour supply in the short run, and hours of work decline.

Financially Unconstrained Household Sector With Financially Constrained Entrepreneurs

Model 2 is a classical firm level financial accelerator model, comparable to BGG's[19] or Kiyotaki and Moore's[83] models'. The entrepreneur's external finance premium

[21]Since this is a loglinear approximation, the responses in percentage deviations from the balanced growth path to a negative shock are approximately the same with all signs reversed.

declines strongly on impact because the loan is predetermined and the improvement in productivity raises the value of the entrepreneurs' collateral both by increasing revenue and by pushing up the value of the entrepreneur's capital stock (figure 3). In subsequent periods the external finance premium returns gradually to its long run value as the entrepreneur increases his borrowing and the price of capital starts declining. As highlighted in the partial equilibrium analysis, the entrepreneur uses the boom and the reduction in financing costs to build up his collateral through higher investment. On impact the entrepreneur financing frictions amplify the response of investment by 45% relative to the frictionless model, and by over 30% for several periods after the shock (figure 1). Finally, the entrepreneur also significantly increases his consumption. [22] Overall the entrepreneur financing frictions amplify the response of output relative to the frictionless model by 12.5-10% in the first 10 periods after a shock. The effect on output is modest because in general equilibrium entrepreneur financing frictions lead to a more procyclical risk free interest rate and price of capital. These moderate the increase in investment and make capacity utilisation more countercyclical. [23]

The effect of entrepreneur financing frictions on fluctuations depends significantly on the level of capital adjustment costs and the steady state strength of the financing frictions. For example setting the elasticity of the price of capital with respect to investment at 10^6 leads to output amplification relative to the frictionless benchmark of 20.2%-18.5% (figure 4). In contrast, setting this elasticity to 10^{-6} results in entrepreneur financing frictions dampening the response of output by about 23% during the first 10 periods (figure 5). The reason is that the lower capital adjustment costs reduce the volatility of the entrepreneur's collateral value and make the entrepreneur's external finance premium procyclical.

Increasing the steady state external finance premium of the entrepreneur strengthens the effect of the financing frictions on dynamics. For example setting $efp_e = 1.035$ ($\beta^e = 0.9605$) leads to an output amplification of 35% on impact relative to the frictionless model, while setting $efp_e = 1.005$ ($\beta^e = 0.989$) reduces amplification to less than 1% on impact for all periods.

[22] The strong reaction of entrepreneur consumption is in line with micro evidence that consumption of the wealthiest households in the US is much more procyclical than that of other households (Parker and Vissing Jorgensen (2009) [97]).

[23] capital utilisation is slightly countercyclical because higher capital prices make depreciation more costly. One way to make capacity utilisation procyclical while allowing for procyclical variations in the price of capital is by using the investment adjustment costs model in Christiano and Fisher (2003) [40]. Another way to see the strength of general equilibrium effects in moderating the output response of the entrepreneur to shocks is to examine a model where some production is done by financially unconstrained firms. In this version, entrepreneurs have a significantly smaller effect on prices, and their output response is amplified by around 10-100% during the first 10 periods relative to the frictionless model. However, this version of the model has the counterfactual implication that financially unconstrained firms' output is highly countercyclical.

Adding Financially Constrained Households

We now come to model 3, adding household borrowing frictions to the frictionless RBC model. Figure 2 shows the responses of borrowers and savers to the TFP shock. First, in response to the benchmark positive TFP shock, borrowers expand their loans, but by less than the increase in the value of their collateral. As a result, we get a countercyclical external finance premium $efp_{bo,t+1}$ even beyond the initial period when the loan is predetermined. A reduction in $efp_{bo,t+1}$ makes borrowing households expand their nondurable consumption and housing investment by more than savers or the representative household of the frictionless RBC model. For a given wage rate, borrowers will reduce their work hours, because the increase in nondurable consumption reduces the marginal value of working. [24] Unless savers' work hours expand sufficiently, the result is a decline in current and future labour supply. This reduces the marginal product of capital for firms and lowers desired investment.

Clearly, if savers' consumption also increases relative to their model 1 consumption, their labour supply curve also shifts to the left, and aggregate labour supply declines (for a given wage) relative to model 1. However, savers' consumption response decreases relative to the frictionless RBC model response. The higher future interest rates in model 3 encourage them to save more. [25]The reduction in savers' consumption relative to model 1 increases the marginal value of work through the standard income effect and encourages them to work more than in model 1. It would appear that the more procyclical nature of savers' labour supply could dominate the response of aggregate labour supply. In the BGP, financing frictions reduce borrowers' consumption relative to that of savers and increase their work effort relative to that of savers. As a result the absolute value of the elasticity of savers' labour supply to changes in their consumption $\frac{1-n_s}{n_s} > \frac{1-n_{bo}}{n_{bo}}$, the absolute value of the elasticity of borrowers' labour supply to changes in their consumption. The higher elasticity means that a given change in the consumption of savers relative to the BGP has more impact on aggregate labour supply than the a change in borrowers' consumption. Nevertheless, as long as household financing frictions increase the procyclicality of aggregate consumption and as long as the direct effect of the financing frictions on borrower labour supply through $A_{nw}(\bar{\varepsilon}_{bo,t})$ is not too large, the presence of borrowers will reduce the procyclicality of aggregate labour supply. In our baseline calibration $s_w = 1$. In this case, we can combine the first order conditions for labour supply of savers and borrowers to obtain

[24]With $s_w > 0$ the reduction in the external finance premium reduces the value of wages as collateral, which would further reduce the borrower's labour supply for a given wage.

[25]This conclusion may not seem so obvious in the model with low capital adjustment costs (figure 6) where interest rates are initially lower with household borrowing constraints. However, after a few periods interest rates with borrowing constraints are higher than in the frictionless RBC model. This reduces the present value of savers' future income sufficiently to encourage higher saving relative to the frictionless RBC model.

$$1 - N_t^s = \theta_s(1 - n_{s,t}) + \theta_{bo}(1 - n_{bo,t}) = \theta_s \frac{\xi_n c_{s,t}}{\xi_c w_t} + \theta_{bo} \frac{\xi_n c_{bo,t}}{\xi_c w_t} = \frac{\xi_n}{\xi_c w_t} C_t^{hholds}, \text{ where}$$
$$C_t^{hholds} = \theta_s c_{s,t} + \theta_{bo} c_{bo,t}.$$

This equation shows that labour supply will decline for a given wage if and only if aggregate consumption increases in model 3, unless s_w is large enough. For plausible calibrations, household borrowing frictions increase the procyclicality of aggregate household consumption. As a result, in response to a positive TFP shock model 3 labour supply declines relative to labour supply in the frictionless RBC model.

Without adjustment costs there are two opposing effects on the risk-free interest rate relative to the frictionless model (figure 5). First, the reduction in external financing frictions stimulates the demand for loans by borrowing households and raises the risk free interest rate. At the same time, the persistent decline in labour supply reduces the future marginal product of capital. By the Euler equation for capital (which is the same in this model as in model 1), this puts downwards pressure on interest rates. The overall result is that the risk-free interest rate is lower in model 3 relative to model 1 in the initial periods after the shock, but becomes higher than in model 1 afterwards. With adjustment costs, the addition of household borrowing constraints makes the risk free interest rate R_{t+1} more procyclical in all periods after a shock. The decline in future labour supply relative to the frictionless model reduces the future marginal product of capital $\alpha \frac{y_{u,t+1}}{k_{u,t+1}}$ for fixed $k_{u,t+1}$ as in the economy without adjustment costs. At the same time, the falling demand for investment caused by the lower labour supply reduces the price of capital. This effect is stronger in the initial periods due to the stronger initial reduction in $efp_{bo,t+1}$. As a result, the addition of household borrowing frictions increases $\frac{q_{t+1}^k}{q_t^k}$, (and possibly also $\alpha \frac{y_{u,t+1}}{q_t^k k_{u,t+1}}$). By no-arbitrage under perfect foresight or low uncertainty, R_{t+1} will have to increase relative to the model without household borrowing frictions if the change in q_t^k is large enough. For my calibrations, the effect of changes in q_t^k dominates and R_{t+1} becomes more procyclical when we add household borrowing constraints in the presence of capital adjustment costs.

After discussing models 1 to 3, it is straightforward to understand the effect of combining household and firm credit frictions in model 4. The dynamics of all agents are qualitatively the same. However, we now have a small negative interaction between the tightness of households' credit constraints to that of entrepreneurs' credit constraints measure by the external finance premium (figure 3). There are two key channels for this effect. First the higher loan demand by borrowers for a given R_{t+1} raises the risk free interest rate, making firm financing more expensive. Second, there is a negative effect of the improvement in household financing conditions on labour supply. The reduction in labour supply relative to the entrepreneur model without borrowers raises wages, depresses the price of capital and further contributes to a higher risk free interest rate. Recall the bank's break even constraint for the entrepreneurs

$$G(\bar{\varepsilon}_{e,t})[(1-s_k)(1-\delta_{e,t})q_t^k k_{e,t} + (1-s_y)y_{e,t}] = R_t l_{e,t-1} + a w_t n_{e,t}.$$

Using $G'(\bar{\varepsilon}_{e,t}) > 0$, a higher risk free rate, higher wages and lower capital prices all reduce the ability of entrepreneurs to borrow and discourage entrepreneur production. As a result, adding household financing frictions does not enhance the amplification effect of the firm level financing frictions on output. On the contrary, the model with both household and firm frictions has lower amplification.

For the baseline calibration, the opposing effects of household borrowing constraints on the consumption responses of borrowers and savers make aggregate household consumption only slightly more procyclical relative to the models without financially constrained households. This makes the effect on aggregate labour supply and the indirect effect on entrepreneurs' external financing costs small. As a result, adding household financing frictions to the model with entrepreneurs reduces the response of output to shocks by only around 2.5-4%. Household financing constraints make aggregate consumption and labour supply significantly more sensitive to shocks when capital adjustment costs are very low (figure 5). Intuitively, this occurs for two reasons. First, lower capital adjustment costs increase movements in output and housing demand that lead to more procyclical resources and collateral values of borrowers. This amplifies the movements in borrower consumption and labour supply that I've emphasized before. Second, eliminating the movements in the price of capital makes the risk-free interest rate less procyclical relative to the model without household borrowing constraints. This reduces the tendency of saver consumption and labour supply to move in the opposite direction to that of borrowers in response to changes in household external finance costs. The combination of these two factors means that the responses of borrower consumption and labour supply to changes in the household external finance premium dominate the responses of aggregate consumption and labour supply. Now, the change in aggregate consumption is amplified by almost 32.5% on impact relative to the model with only firm financing frictions and aggregate labour supply declines more significantly in response to a relaxation of borrowers' credit constraints.

2.3.3 Discussion

The labour market plays an important role in the results. The model predicts that if household borrowing constraints make aggregate consumption more responsive to shocks, they will make labour supply less responsive to shocks. This result should hold more generally for utility functions compatible with a balanced growth path (King, Plosser, Rebelo preferences [82]), if the constant elasticity of substitution between consumption and leisure of the model's Cobb-Douglas aggregator is a good approximation. A more robust conclusion is that household borrowing constraints decrease borrowers' labour supply procyclicality and increase savers' labour supply procyclicality. Further research is required to understand which effect dominates under what conditions. The model assumes a perfectly competitive labour market, divisible labour and insurance of households against the idiosyncratic shocks. However, the conclusion that countercyclical financing frictions may make the labour supply of more credit constrained agents less procyclical, which seems critical to our results, also holds in an incomplete markets model with matching frictions and endogenous separations (see Bils et al 2007) [21] : in that model, workers with a lower level of net financial assets are more willing to accept lower wages in order to avoid losing their job in a recession, and vice versa in a boom. Krusell et al (2007) [87] examine a model that combines the standard uninsured idiosyncratic income risk model of Aiyagari with the standard labour market matching framework of Mortensen and Pissarides. They also find that low wealth households negotiate higher (lower) wages as their asset position improves (deteriorates). [26]These models also feature indivisible hours (one can work in a full time job or not at all), suggesting that the effects highlighted in a divisible labour model are also relevant in the presence of indivisibilities through their effect on the reservation wages of workers. [27] However, the strength of the income effect from financing frictions on labour supply may be attenuated in more realistic labour market models than the perfectly competitive model. Also, in models with uninsured income risk, aggregation is more complicated even with the utility function that I have used. As a result, the relation between changes in aggregate consumption and aggregate labour supply with household borrowing constraints is more ambiguous.

It is also instructive to contrast the model of household borrowing here with currently popular models using hard borrowing constraints based on Iacoviello (2005) [72]. Due to the assumption of an always binding collateral constraint, that model has a fixed leverage ratio equal to the maximum Loan to Value ratio, but the borrower's decisions still affect the tightness of the collateral constraint through his choice of housing (the collateral in the model). In Iacoviello's model an increase in house prices relaxes the borrowing constraint for a given loan size and reduces the shadow cost of borrowing. This shadow cost is similar to the external finance premium in my model. To the degree that the shadow cost of borrowing in the hard borrowing constraint model is countercyclical, its borrower dynamics should be qualitatively similar to those in my analysis, while missing the endogenous adjustment in borrower leverage in a model with default risk.

[26]See Domeij and Floden(2006)[51] for a similar effect in the precautionnary saving model with variable labour supply and perfectly competitive labour markets. See Alexopoulous and Gladden(2006) [4] for evidence from an estimated search model of unemployment of significant effects of financial wealth on reservation wages and probabilities of transitioning into employment which are in line with the outcomes of my model. *[4]*

[27]Alternatively we can interpret $n_{bo,t}$ as including worker effort, following Bils and Chang (2003)[20]. In this case, the model says quite intuitively that even if a worker cannot smoothly adjust his working hours, a relaxation of his family's borrowing constraint reduces his effort at work and forces employers to pay more for a given amount of effort.

There are several modifications that may change the current results of household financing frictions dampening fluctuations(household financial decelerator) and having negative spillovers on the strength of firm financing frictions. One element deserving closer examination is limited production factor mobility between the consumption and investment goods sectors. Consider a decline in consumer borrowing and spending due to an increase in household financial frictions. Without any limits to moving production resources between consumption and investment goods production, the worse credit conditions for households increase labour supply, reduce the wage rate and stimulate investment (holding other factors constant). However, higher investment may not occur in the short run if it takes time for workers to find new jobs in the investment goods sector. As a result, a slump in the consumption sector due to tighter credit conditions for households may be not compensated for by relatively higher investment in the short run, leading to a bigger decline in output. If the firm's revenue is part of its loan collateral, this stronger output decline reduces the financial strength of firms and raises the external finance premia that they face, which through the financial accelerator mechanism can lead to a longer lasting decline in investment as well as consumption. The baseline model already includes some limited factor mobility through capital adjustment costs. These can be viewed as a reduced form for difficulties in moving workers and other production factors between the investment and consumption sectors. [28] In the scenario of a recession I've just described, the model predicts that worsening financial conditions for households increases labour supply relative to a model without household financing frictions, and makes investment relatively more attractive. Because of the adjustment costs, this puts upwards pressure on the price of capital and downwards pressure on the risk free interest rate, which improves the entrepreneur's financial position (relative to the model without household borrowing constraints) and increases entrepreneur output. The overall result is that higher adjustment costs of capital do not generate positive spillovers between the cost of external finance for households and firms, even in an environment with an almost fixed capital stock (see figure 4). There is another problem with the argument that limited factor mobility between the consumption and the investment goods sectors can generate a positive interaction between household and firm financing constraints. Once one takes into account the consumption of entrepreneurs is isn't clear that household credit constraints actually make aggregate consumption more sensitive to shocks. The financing constraints of borrowers increase the procyclicality of total (non entrepreneur) household consumption, but they reduce the procyclicality of entrepreneur consumption. As a result, aggregate consumption dynamics in the baseline calibration are virtually identical in the model with all financing frictions and in the model where only firms are financially constrained.

In light of the importance of income effects on labour supply for the results, we

[28] Kim(2003) [79]shows an equivalence between intertemporal and intratemporal adjustment costs in the standard real business cycle model under some conditions. This suggests that the results with convex capital adjustment costs are relevant more generally, though investigation of more realistic models of limited factor mobility such as Phelan and Trejos (2000)[100] may still be worthwhile.

could also examine modifications to households' preferences. First, if housing and consumption are complements (as suggested by micro estimates in Flavin and Nakagawa (2008)[57]) then more procyclical housing investment by borrowers makes their marginal utility of consumption more procyclical for a given amount of (non-durable)consumption, which weakens or may even reverse the countercyclical impact of the income effect on borrower labour supply. Similar effects may be achieved by habit formation(though the housing based mechanism gets much stronger empirical supports from micro data estimates in Flavin and Nakagawa (2008) [57]). As for the income effect on labour supply, the standard way of eliminating it is to replace the King, Plosser and Rebelo (1988) [82] preferences used here with the preferences in Greenwood Hercowitz and Huffman (GHH1988) [66] of the form

$$u_{it} = \frac{[v(c_{i,t}, h_{i,t}) - \xi_n n_{it}^{\theta_n}]^{1-\sigma}}{1-\sigma}.$$

These preferences are quite important in generating comovement and strong amplification in many recent RBC models(e.g. [75]), though the microfoundations of just assuming these preferences is debatable. In particular these preferences are incompatible with the evidence both across time and the cross section that the income and substitution effects approximately cancel each other out and that income effects are non negligible at least for large changes in income (unless one assumes unrealistically that the productivity of home production and market production of households is almost perfectly correlated in the time series, cross section and across countries(Kimball and Shapiro (2003) [81],Basu and Kimball (2002) [16]). [29] As long as the wealth effect on labor supply is present, we should get dampening of fluctuations in the baseline model due to countercyclical household credit frictions, though the effect may be smaller if labour and consumption complementarity are strong ($\sigma > 1$ is high enough).

Wage rigidities of various types also weaken the link between borrower consumption and labour supply and tend to enhance comovement across sectors (see for example Di Ceccio (2009) [48]). However, the evidence suggests that wage rigidity may not be important for new hires, and for workers with relatively rigid wages contract hours also tend to be more rigid(see Pissarides(2008) [101] and Amano et al (2008) [7]). To the degree that younger more recently hired workers with high wage growth expectations are more like the impatient borrowers in our model, while older workers with tenure are more like the patient savers, wage rigidity need not lead to positive spillovers between household and firm borrowing constraints.

2.3.4 Analysis of Second Order Approximation

As a robustness, check I also examine results from a second order approximation of the model's dynamics. I study the impulse response functions from the ergodic

[29] Jaimovich and Rebelo(2008)[75] develop a new class of preferences that allows for weak income effects in the short run while preserving the approximate cancellation of income and substitution effects in the long run, but with this new preference class leisure is an inferior good for several periods after an aggregate shock.

mean of the 2nd order approximation (figures 6-9). Overall, the impulse response functions are qualitatively the same as those emerging from the linear approximation. In particular, financial frictions do not seem to have significantly bigger impact on aggregate dynamics than was found with the loglinear approximation. The magnitudes of the amplification of output and investment responses to shocks through firm financing frictions is about the same as in the loglinear approximation, but there is slightly more persistence in the dynamics. The finding that adding household financing frictions dampens the effect of firm level financing frictions on aggregate dynamics continues to hold.

There is some evidence of moderate nonlinearity. For example, in the model with all financing frictions moving from a one to a two standard deviation shock increases the response of output by a factor of 1.79-1.89 for negative shocks and by a factor of 2.135- 2.35 for positive shocks. There is also some assymetry in the dynamics, with the response of output being larger by a factor of 1.24-1.7 (1.095-1.29) for negative shocks relative to positive shocks for a one (two) standard deviations shock. Similar patterns hold for other aggregates such as consumption and investment.

2.3.5 A Comparison of Model and US Statistics

For the sake of completeness, I also compare model based time series statistics with some time series statistics for the US from 1955 to 2004 (see table 2). While the fit is not very good, we should note that the current version only has TFP shocks. A model with more shocks (e.g investment shocks or financial shocks) should have a better fit. The model with household and firm financing frictions can match around 63% of the volatility of output and 89.5% of the volatility of consumption. At the same time, it can only match 7% of the volatility of housing investment and 23% of the volatility of investment. It clearly misses the ranking of the relative volatilities of output components, with investment being far too smooth relative to output and consumption being too volatile relative to output. Another major discrepancy between the model and the data is excessively low volatility of hours worked. Note that these discrepancies relative to the data are also a problem for the frictionless real business cycle model From the analysis of the IRF's, it is clear that the key culprits in these empirical failures of the model are the adjustment costs of housing and capital. These costs make it difficult for firms and households in the model to smooth consumption by adjusting investment. They also strengthen the importance of the income effect for labour supply relative to the substitution effect, reducing the overall movement of hours of work in response to shocks. The low volatility of investment and hours of work are problematic for any model of financing frictions relying on convex capital adjustment costs (or the special case of a fixed capital stock) to generate volatility in collateral values and costs of external financing, including the Kiyotaki and Moore model. To the degree that financing frictions are considered important this suggests that models where the whole value of the firm is part of collateral (because default involves the entrepreneur losing the firm as opposed to just the current capital) as in Jermann and Quadrini (2008) [76] are more promising. [30] A simpler way to improve the performance of the model in these dimensions is to use the investment adjustment costs in Christiano and Fisher (2003) [40], which penalises changes in the growth rate of investment. [31] Other extensions that may lead to an important role for firm financing frictions in business cycles with high investment volatility include investment specific technology shocks as in Christiano and Fisher [40], allowing for an almost fixed factor of production such as structures or land to be part of firm collateral as in Iacoviello (2005) [72] and Liu et al (2009) [89], or relying on financial shocks that directly affect the tightness of the borrowing constraint as in Christiano et al. (2007) [42]. [32]Finally, the low volatility in our measure of the labour input may not be such a concern if we interpret it as including time varying effort along the lines of Bils and Chang (2003) [20]. In that case low volatility in the aggregate labour input is perfectly compatible with stronger procyclicality of hours as long as effort per hour is countercyclical . Since labour hour choices are usually discrete, while effort or efficiency per hour is a continuous variable, this is arguably a more plausible interpretation of the divisible labour supply model in many standard DSGE models such as Cooley and Prescott (1995) [43].

All models also overestimate the contemporaneous correlation between output and its components and the correlation between the wage rate and output. In contrast to RBC models with home production or housing but without adjustment costs, the model with housing adjustment costs can generate a positive correlation between business and housing investment. However this correlation is far too high in the model. Finally, as one would expect from the IRFs analysis, the effect of financing frictions on volatilities and correlations is modest for output, but more significant for investment. Adding firm financing frictions to the basic RBC model increases the volatility of output by 12%, and investment by 44%. Adding household borrowing frictions to the model with firm financing frictions leads to a 3% reduction in the volatility of output and a 13% reduction in the volatility of investment. Adding firm financing frictions does generate a significant improvement the correlation between output and hours worked. This correlation is significantly negative in the frictionless RBC model, in contrast to the data, while with financing frictions it becomes significantly positive.

[30]Default never occurs in Jermann and Quadrini's model. A version of their model with equilibrium default may be tractable as long as the value of the firm is increasing in the idiosyncratic shock.

[31]Christiano and Fisher's specification of adjustment costs is also more realistic in allowing the elasticity of investment with respect to the price of capital to be larger for more persistent chnages in q_t^k than for transitory changes in q_t^k. It should be noted that their specification still suffers from a volatility of investment that is too low relative to the data.

[32]Iacoviello (2005) [72], and Liu et al (2009) [89] rely on a fixed supply of housing and structures to generate volatile house prices. This leads to the same problem we have with investment under TFP shocks : in the data housing investment is very volatile, not fixed. This criticism does not apply to financial shocks, which at least in theory can generate volatility in both collateral prices and investment in new collateral.

2.4 Conclusion

This paper has examined the possibility of spillovers between household and firm financing frictions in an otherwise standard Real Business Cycle model with housing. In the absence of other frictions, I find a small negative interaction between the external finance premia of households and firms. Workers in the model use a relaxation of their financing constraint in a boom to increase their leisure. This crowds out investment, raises the risk-free interest rate and reduces the value of capital. The higher loan demand by borrowers in a boom also pushes up the interest rates faced by entrepreneurs. All these elements tighten the financing constraints of firms in a boom, in comparison to a model where households are financially unconstrained. The opposite movements occur in a recession, as financially constrained households respond to higher costs of external financing by raising their labour supply and reducing their demand for loans. While household financing frictions increase the procyclicality of savers' labour supply, this is not enough to counter the effects of less procyclical borrower labour supply.

I have discussed several factors that may change the results. These include limited production factor mobility across sectors and labour market imperfections. In the context of limited factor mobility it would be interesting to explore the financing frictions in this paper in a more detailed multisector model which goes beyond the convex adjustment cost model of housing and capital supply. In the current model entrepreneurs produce final output which is then converted without other financing frictions into housing and capital. This implicitly assumes that entrepreneurs producing consumption goods can automatically switch to producing capital goods if tighter household financing constraints depress consumption. A more realistic assumption for most firms is that in the short run entrepreneurs cannot switch between producing consumption or investment goods, leading to separate external finance premia for different sectors. To the degree that tighter financing constraints for households lead to a slump in consumption, allowing for these separate external finance premia may matter. Other features that may be worth exploring include a separate role for commercial property in the entrepreneur's production function and collateral constraint as in Liu et al (2009) [89], a relatively fixed supply of land and differences in factor intensities across sectors as in Davis and Heathcote (2005) [47]. In particular, explicit modeling of the role of commercial property creates a potentially important channel for interaction between household and firm financing constraints to the degree that prices of commercial and residential property are linked.

Several other extensions or modifications of the current model deserve further consideration. To simplify the solution of the model and analysis, I have assumed that households and firms have access to insurance arrangements allowing them to diversify their idiosyncratic risk after settling debt contracts. This raises the question of the robustness of the results to a more realistic setup with uninsured idiosyncratic, where both households and firms engage in precautionary saving. Aggregating the labour supply of borrowers and savers is no longer so straightforward with uninsured idiosyncratic risk. This opens up the possibility that household financing frictions make aggregate labour supply more procyclical due to the behaviour of savers. The interaction in debt markets between household and firm loan demand is also more complex in an economy where the proportion of borrowing households and firms can change across the business cycle. In the current model all firms have negative net financial assets in each period, so that an increase in loan demand by households in a boom makes things worse for firms by raising their borrowing costs. In a model with heterogeneous entrepreneurs, in each period some entrepreneurs will have positive net financial assets. For these entrepreneurs, an increase in the risk-free interest rate due to higher loan demand by borrowers makes it easier to cumulate assets that may relax their future borrowing constraints. Clearly, in such an environment the interaction between the strength of financing frictions affecting households and firms is more subtle. Solving a model where both firms and households are affected by uninsurable idiosyncratic shocks is difficult (if not impossible) using the standard Krusell and Smith [88] algorithm for heterogenous agent models with aggregate uncertainty. However, this objective may be feasible with recent techniques that obtain aggregate laws of motion by explicitly aggregating individual policy functions (Den Haan et al. (2009) [5]). These techniques reduce the use of simulation in solving the model to a minimum, promising large improvements in solution speed or the complexity of models that can be handled. [33] Finally, I have followed the common practice of using differences in discount factors to generate equilibrium borrowing and lending. This is often justified as a shortcut for modeling differences in expected income growth between younger and older households (see Carroll (2000) [34]). It may be worthwhile to investigate the validity of this shortcut using a more explicit model of lifecycle effects. This may be tractable using the Blanchard-Yaari perpetual youth framework (e.g Cagetti and De Nardi (2006) [27]), generalised to take into account the distinction between young workers and middle aged workers.

[33] Krusell and Smith[88] find that idiosyncratic risk does not have quantitatively significant effects on macroeconomic aggregates. Key to this result is the high ability and incentives for households to self insure through holding capital (equity) in their baseline model. But this model cannot match the US wealth distribution and it cannot reproduce the riskiness of equity in the data. In models that match the wealth distribution and the riskiness of equity such as those in Carroll (2000) [34] or Favilukis (2007)[54], households do not have such strong incentives to self insure and the insurance that can be provided by equity or bonds is more limited. In these models, idiosyncratic risk matters for aggregate consumption dynamics.

2.5 Appendix A, proofs of propositions

Proposition1 : With perfect foresight, $\frac{\partial(c_{bo,t}/h_{bo,t})}{\partial\bar{\varepsilon}_{bo,t+1}} > 0$.

Proof : With perfect foresight we can combine the loan and the housing Euler equations to obtain

$$q_t = \frac{\xi_h}{\xi_c}\frac{c_{bo,t}}{h_{bo,t}} + (1-\delta_h)q_{t+1}\frac{A_{hh}(\bar{\varepsilon}_{bo,t+1})}{R_{t+1}efp(\bar{\varepsilon}_{bo,t+1})}.$$

$$\begin{aligned}\frac{d\left(\frac{A_{hh}(\bar{\varepsilon}_{bo,t+1})}{efp(\bar{\varepsilon}_{bo,t+1})}\right)}{d\bar{\varepsilon}_{bo,t+1}} &= \frac{-efp'(\bar{e}_{bo,t+1})}{efp^2_{bo,t+1}} + (1-s_h)[\frac{H'(\bar{\varepsilon}_{bo,t+1})}{efp_{bo,t+1}} + G'(\varepsilon_{bo,t+1}) - H(\bar{\varepsilon}_{bo,t+1})\frac{efp'(\bar{\varepsilon}_{bo,t+1})}{efp^2_{bo,t+1}}] \\ &= -\frac{efp'(\bar{\varepsilon}_{bo,t+1})}{efp^2_{bo,t+1}}[1 + (1-s_h)H(\bar{\varepsilon}_{bo,t+1})] < 0,\end{aligned}$$

where the last inequality follows from $efp'(\bar{\varepsilon}_{bo,t+1}) > 0$ and $H(\bar{\varepsilon}_{bo,t+1}) > -1$. Therefore, an increase in $\bar{\varepsilon}_{bo,t+1}$ must lead to a rise in $\frac{c_{bo,t}}{h_{bo,t}}$ for given house prices.

Proposition 2 :With perfect foresight, $\frac{\partial k_{e,t+1}}{\partial\bar{\varepsilon}_{e,t+1}} < 0$.

Proof : The proof is similar to that of proposition 1. Combining the loan and capital Euler equations, we have

$$q^k_t = \frac{1}{R_{t+1}efp_{e,t+1}}[A_{kk}(\bar{\varepsilon}_{e,t+1})q^k_{t+1}(1-\delta_e) + A_{ky}(\bar{\varepsilon}_{e,t+1})\alpha\theta\frac{y_{e,t+1}}{k_{e,t+1}}].$$

Using the fact that $A_{ky}(\bar{\varepsilon}_{e,t+1})/efp(\bar{\varepsilon}_{e,t+1})$ and $A_{kk}(\bar{\varepsilon}_{e,t+1})/efp(\bar{\varepsilon}_{e,t+1})$ are both decreasing in $\bar{\varepsilon}_{e,t+1}$ and the diminishing marginal productivity of capital we find $\frac{\partial k_{e,t+1}}{\partial\bar{\varepsilon}_{e,t+1}} < 0$ for fixed $n_{e,t+1}$. Now allow for variable labour supply with $\frac{\partial n_{e,t}}{\partial\bar{\varepsilon}_{e,t}} < 0$. Rewrite the first order condition for $n_{e,t}$ as

$$\frac{1+(1-s_y)[H(\bar{\varepsilon}_{e,t}) + efp_{e,t}G(\bar{\varepsilon}_{e,t})]}{(1-a+aefp_{e,t})}(1-\alpha)\theta\frac{y_{e,t}}{n_{e,t}} = w_t = B_{n_e}(\bar{\varepsilon}_{e,t})(1-\alpha)\theta\frac{y_{e,t}}{n_{e,t}} = w_{t.}$$

By diminishing marginal productivity of labour, $\frac{\partial n_{e,t}}{\partial\bar{\varepsilon}_{e,t}} < 0$ if and only if $\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} < 0$. Solving for n_{et} as a function of capital and substituting the result into the Euler equation for capital,

$$\begin{aligned}q^k_t &= \frac{1}{R_{t+1}efp_{e,t+1}}A_{kk}(\bar{\varepsilon}_{e,t+1})q^k_{t+1}(1-\delta_e) \\ &+\frac{1}{R_{t+1}efp_{e,t+1}}A_{ky}(\bar{\varepsilon}_{e,t+1})\left(\frac{(1-\alpha)\theta z_{e,t+1}}{w_{t+1}}B_{n,e}(\bar{\varepsilon}_{e,t+1})\right)^{\frac{(1-\alpha)\theta}{1-(1-\alpha)\theta}}k_{e,t+1}^{\frac{\alpha\theta}{1-(1-\alpha)\theta}-1}\end{aligned}$$

$\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} < 0$ implies that as before the right hand side of this equation is decreasing in $\bar{\varepsilon}_{e,t+1}$. Since $k_{e,t+1}^{\frac{\alpha\theta}{1-(1-\alpha)\theta}-1}$ is decreasing in $k_{e,t+1}$, a higher $\bar{\varepsilon}_{e,t+1}$ must lead to a lower $k_{e,t+1}$.

Proposition 3 : $a \geq 1 - s_y$ implies that $\frac{\partial n_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$.

Proof : From the previous proof, we know that $\frac{\partial n_{e,t}}{\partial \bar{\varepsilon}_{e,t}} < 0$ if and only if $\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} < 0$, where

$$\frac{1 + (1 - s_y)[H(\bar{\varepsilon}_{e,t}) + efp_{e,t}G(\bar{\varepsilon}_{e,t})]}{(1 - a + aefp_{e,t})} = B_{n_e}(\bar{\varepsilon}_{e,t}).$$

.

$$\begin{aligned}
\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} &= \frac{efp'(\bar{\varepsilon}_{e,t})}{(1 - a + aefp_{e,t})^2}(1 - s_y)[(1 - a + aefp_{e,t})G(\bar{\varepsilon}_{e,t}) \\
&\quad - \frac{efp'(\bar{\varepsilon}_{e,t})}{(1 - a + aefp_{e,t})^2}a(H(\bar{\varepsilon}_{e,t}) + efp_{e,t}G(\bar{\varepsilon}_{e,t}))] - a \\
&= \frac{efp'(\bar{\varepsilon}_{e,t})}{(1 - a + aefp_{e,t})^2}\{(1 - s_y)[G(\bar{\varepsilon}_{e,t})(1 - a) - a(H(\bar{\varepsilon}_{e,t})] - a\} \\
&= \frac{efp'(\bar{\varepsilon}_{e,t})}{(1 - a + aefp_{e,t})^2}\{(1 - s_y)[G(\bar{\varepsilon}_{e,t}) + a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF] - a\}
\end{aligned}$$

where the last equality uses $H(\bar{\varepsilon}_{e,t}) + G(\bar{\varepsilon}_{e,t}) = -\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF$. $\frac{efp'(\bar{\varepsilon}_{e,t})}{(1-a+aefp_{e,t})^2} > 0$, since $efp'(\bar{\varepsilon}_{e,t}) > 0$. Therefore, $\frac{dB_{n_e}(\bar{\varepsilon}_{e,t})}{d\bar{\varepsilon}_{e,t}} < 0$ if and only if

$$(1 - s_y)[G(\bar{\varepsilon}_{e,t}) + a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF] - a < 0$$

, that is $G(\bar{\varepsilon}_{e,t}) + a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF < \frac{a}{1-s_y}$. Since $G(\bar{\varepsilon}_{e,t}) + a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF < 1$, $a \geq 1 - s_y$ is a sufficient condition for $(1 - s_y)[G(\bar{\varepsilon}_{e,t}) + a\mu_{e,t}\int_0^{\bar{\varepsilon}_{e,t}} \varepsilon dF] - a < 0$.

2.6 Appendix B, the Balanced Growth Path :

I approximate the model around a Balanced Growth Path (BGP) where $k_{u,t}, k_{e,t}, I_{u,t}, I_{e,t}$,

$y_{u,t}, y_{e,t}, h_{s,t}, h_{bo,t}, c_{s,t}, c_{bo,t}, d_t, l_{e_t}, l_{bo,t}$ and w_t all grow at the same gross rate G_y. hours of work, capacity utilisation rates, default thresholds and the real interest rate on deposits are all constant in the BGP. This implies that the growth rates of total factor productivity in the BGP are $G_{z_e} = G_y^{1-\alpha\theta}$ and $G_{z_u} = G_y^{1-\alpha}$. Let $\tilde{x}_t = \frac{x_t}{G_x^t}$, where G_x is the gross growth rate of x in the BGP. $\hat{x}_t = \frac{x_t - x}{x}$ where x is the BGP value of $\tilde{x}_t$. I normalize $z_u = z_e = 1$. To find the BGP, first detrend all foc's and constraints.

1) We start by computing the default thresholds and idiosyncratic shock volatilities to match BGP default rates.

For $i = e, bo$ $\bar{\varepsilon}_i$ solves

$1 = \beta^i G_{\lambda_i} efp_i(\bar{\varepsilon}_i)R.$

Note that this solution ignores the constraint that entrepreneur loans $l_e \geq 0$ which may in theory be violated due to the wages in advance constraint affecting part of the wage bill. I solve the model conjencturing that this constraint does not bind. Afterwards, I can compute l_e and check that this is indeed the case. For my calibrations, this was never a problem with $\frac{Rl_e}{G_e\tilde{A}} > 0$ by a large margin.

There are two solutions to each of the above equations. The monotone hazard rate assumption implies that the optimal solution is the lower $\bar{\varepsilon}_i$ solution(this is just like in the BGG model- see Christiano et al's[42] appendix for a discussion of this). After picking the lower solution interval, a simple bisection finds each $\bar{\varepsilon}_i$.

We can then solve for various other expressions that are functions of $\bar{\varepsilon}_i$ such as $H_i, G_i, efp_{i,}$.

2) We need to compute the market clearing wage. This can be done by a bisection, in which we solve for the model quantities conditional on a given wage, check if the labour market equilibrium has converged and iterate if required. Since entrepreneurial financing frictions tend to lower the demand for labour relative to that of financially unconstrained firms, the wage in a model with financially unconstrained firms is a reliable upper bound for the solution of the model where all firms are financially constrained.

2.1) Conditional on the guess for the wage rate we compute entrepreneur variables :
We already have $\bar{\varepsilon}_e$ from 1). $u_e = 1$.
Next we use the labour demand foc to express n_e as a function of k_e :
$n_e = \left[\frac{(1-\alpha)\theta z_e(u_e k_e)^{\alpha\theta}}{A_{ne} w}\right]^{\frac{1}{1-\theta(1-\alpha)}}$, where $A_{ne} = \frac{1-a+aefp_e}{1+(1-s_y)(H_e+G_e efp_e)}$.
This allows us to use the euler equation for entrepreneur capital to solve for k_e.
$k_e = \frac{1}{G_y}\left[\frac{\frac{G_y}{\beta^e}-(1-\delta_e)A_{kk}}{\alpha\theta A_{yk}(G_{z_e}z_e)^{\frac{1}{1-\theta(1-\alpha)}}(\frac{(1-\alpha)\theta u_e}{G_y w A_{ne}})^{\frac{\theta(1-\alpha)}{1-\theta((1-\alpha)}}}\right]^{\frac{1-\theta(1-\alpha)}{\theta-1}}$, where $A_{yk} = 1+(1-s_y)(H_e+efp_e G_e)$ and $A_{kk} = 1 + (1 - s_k)(H_e + efp_e G_e)$.
It can be shown that the expressions above are always positive.
Next, we can find ϕ_e from $u_e = 1$ and the first order condition for u_e :
$\phi_e = \frac{A_u}{\delta_e}\left(\frac{\frac{G_y}{\beta^e}-(1-\delta_e)A_{kk}}{A_{yk}}\right)$, where $A_u = \frac{A_{yk}}{A_{kk}}$.
Given the variables above it is easy to find y_e, $\tilde{A}_e, \bar{A}_e$, $l_e = (\tilde{A}_e G_e - awn_e)/R$ and $c_e = \bar{A}_e + H_e\tilde{A}_e - (1-a)wn_e + l_e G_y - k_e G_y$.
2.2) Now we compute the borrower variables.
We already have the default threshold $\bar{\varepsilon}_{bo}$ and functions of this threshold from 1).
Using the first order conditions for housing and labour supply we can solve for
$h_{bo} = A_{hbo}c_{bo}$ where $A_{hbo} = \frac{\xi_h}{\xi c G_y(1-\beta^{bo}G_\lambda(1-\delta_h)[1+(1-s_h)(H_{bo}+efp_{bo}G_{bo})])}$, and
$n_{bo} = 1 - A_{nbo}\frac{c_{bo}}{w}$ where $A_{nbo} = \frac{\xi_n}{\xi_c[1+(1-s_w)(H_{bo}+efp_{bo}G_{bo})]}$.
We can then also solve for $\bar{A}_{bo} = (1-\delta_h)h_{bo} + wn_{bo}$, $\tilde{A}_{bo}$ and $l_{bo} = G_{bo}\tilde{A}_{bo}/R$ as functions of c_{bo}.

To compute c_{bo} we plug the expressions above into the budget constraint and solve for c_{bo} :

$c_{bo} = \frac{X_{nbo}}{1+A_{hbo}X_{hbo}+A_{nbo}X_{nbo}} w$ where $X_{nbo} = 1 + (1 - s_w)(\tilde{H}_{bo} + \frac{G_y}{R} G_{bo})$ and $X_{hbo} = G_y - (1 - \delta_h)[1 + (1 - s_h)(H_{bo} + G_{bo}\frac{G_y}{R})]$

Again, it can be shown that the expressions for c_{bo} and h_{bo} are always positive. $n_{bo} > 0$ for any calibration in which $n_s > 0$ since $n_{bo} > n_s$.

Given c_{bo}, we can now go back and solve for the other borrower variables using the expressions derived earlier.

2.3) Next we compute the variables for the savers.

For the savers, the solution is similar to that for the borrowers. We use the first order conditions for housing and labour supply to express $h_s = A_{hs} c_s$, where $A_{hs} = \frac{\xi_h}{\xi_c G_y [1 - \beta G_\lambda (1-\delta_h)]}$ and $n_s = 1 - \frac{\xi_n}{\xi_c}\frac{c_s}{w}$. We also determine deposits from the market clearing condition $d = \frac{\theta_e l_e + \theta_{bo} l_{bo}}{\theta_s}$.

We can then substitute the expressions above, using the results from all the previous steps, into the savers' budget constraint to solve for $c_s = \frac{(R-G_y)d+w}{1+(G_y-(1-\delta_h))A_{hs}+\frac{\xi_n}{\xi_c}}$.

3) We now check if the excess demand $|n_e - \theta_s n_s - \theta_{bo} n_{bo}| < \varepsilon^n$, and update the wage guess if necessary.

This concludes the solution of the BGP.

Alternatively, we can solve for the equilibrium wage analytically if we know the targeted proportion of hours worked in the BGP. Suppose we have a target in mind for n_e. The first order condition for capital can be solved for the equilibrium $\frac{y_e}{k_e}$ ratio as a function of $\bar{\varepsilon}_e$. $\frac{y_e}{k_e} = z_e \left(\frac{k_e}{n_e}\right)^{\alpha\theta - 1} n_e^{\theta - 1}$ allows us to solve for $\frac{k_e}{n_e}$. The labour demand function can be written as $\theta(1-\alpha)\frac{y_e}{n_e} = \theta(1-\alpha)\left(\frac{k_e}{n_e}\right)^{\alpha\theta} n_e^{\theta-1} = A_{n_e}(\bar{\varepsilon}_e) w$. Since we know $\frac{k_e}{n_e}, n_e$ and $\bar{\varepsilon}_e$ we can solve for w analytically. To make the labour market clear we adjust the labour supply through the ratio $\frac{\xi_n}{\xi_c}$ for a given ξ_h until $|\theta_s n_s + \theta_{bo} n_{bo} - n_e| < \varepsilon_N$ for a small ε_N. A good upper bound for the labour market clearing $\frac{\xi_n}{\xi_c}$ can be found by solving the economy where only borrowers work, while savers just earn income from deposits (this bound can be computed analytically for $G_y = 1$, and it is usually also an upper bound for $G_y > 1$ but close to 1).

2.7 Appendix C, the model with a predetermined loan rate with respect to aggregate shocks

For concreteness, I focus on the borrowing household. Similar derivations apply to the entrepreneur.

Suppose that now R_t^l must be determined before knowing the aggregate state of the economy at t. Since $l_{bo,t-1}$ is predetermined at t, This makes the repayment without default $R_t^l l_{bo,t-1} = \bar{\varepsilon}_{bo,t} \tilde{A}_{bo,t}$ predetermined.

As before, the bank can still diversify its exposure to the borrowers' idiosyncratic shocks. We can still write the bank's expected repayments as $G(\bar{\varepsilon}_{bo,t})\tilde{A}_{bo,t} = \bar{R}_{bo,t} l_{bo,t-1}$ for some $\bar{R}_{bo,t} \geq 0$, where $G(\bar{\varepsilon}_{bo,t}) = [1 - F(\bar{\varepsilon}_{bo,t})]\bar{\varepsilon}_{bo,t} + (1 - \mu_{bo,t}) \int_0^{\bar{\varepsilon}_{bo,t}} \varepsilon_{bo} dF$. $\bar{\varepsilon}_{bo,t}$

must satisfy two conditions : $\bar{\varepsilon}_{bo,t} = \frac{R_t^l l_{bo,t-1}}{\tilde{A}_{bo,t}}$ and $G(\bar{\varepsilon}_{bo,t}) = \frac{\bar{R}_{bo,t} l_{bo,t-1}}{\tilde{A}_{bo,t}}$. Before we assumed that $\bar{R}_{bo,t} = R_t$ and allowed $R_t^l l_{bo,t-1}$ to adjust as a function of aggregate conditions to make the first condition trivially hold. But now $R_t^l l_{bo,t-1}$ is independent of the aggregate state. As a result, if $\bar{R}_{bo,t}$ is independent of the aggregate state we have a system of 2 equations in one unknown $\bar{\varepsilon}_{bo,t}$ that (generically) has no solution. Therefore, $\bar{R}_{bo,t}$ must adjust as a function of the aggregate state. The contract can no longer be reduced to picking a schedule of default thresholds $\bar{\varepsilon}_{bo,t}$. Instead, the representative borrower now picks $\left\{c_{bo,t}, h_{bo,t}, l_{bo,t}, d_{bo,t}, R^l_{bo,t+1}, n_{bo,t}, \bar{\varepsilon}_{bo,t}\right\}_{t=0}^{\infty}$

to maximise

$E_0 \Sigma_{t=0}^{\infty} \beta^{bo^t} u_{bo,t}$

subject to a sequence of budget constraints

$c_{bo,t} + q_t h_{bo,t} + d_{bo,t} = q_t(1-\delta_h)h_{bo,t-1} + n_{bo,t}w_t + H(\bar{\varepsilon}_{bo,t})[(1-s_h)q_t(1-\delta_h)h_{bo,t-1} + (1-s_w)n_{bo,t}w_t] + l_{bo.t} + d_{bo,t-1}R_t$

,the participation constraints of the bank $G(\bar{\varepsilon}_{bo,t})\ [(1-s_h)q_t(1-\delta_h)h_{bo,t-1} + (1-s_w)n_{bo,t}w_t] = \bar{R}_{bo,t} l_{bo,t-1}$

and $\bar{\varepsilon}_{bo,t} = \frac{R^l_{bo,t} l_{bo,t-1}}{\tilde{A}_{bo,t}}$.

The expected rates of return conditional on the aggregate state $\bar{R}_{bo,t}$ must now satisfy

$\lambda_{s,t} = \beta E_t \lambda_{s,t+1} \bar{R}_{bo,t+1}$.

This can be shown using either a decentralisation where savers are assumed to directly make risky loans to borrowers, or an equivalent (by the Modigliani-Miller theorem) but more realistic decentralisation where savers provide banks that they own with risk free deposits and the banks lend out those funds at the risky rate $\bar{R}_{bo,t+1}$, ,repay the borrowers' deposits and distribute all profits(negative if they suffer a loss) to the savers.

Note that the deterministic balanced growth path solution, around which we approximate the dynamics, is the same as for the model where R_t^l is conditional on the aggregate state.

For the entrepreneur, we get the same results if $a = 0$, otherwise we need to think more carefully about the joint modeling of the intratemporal working capital loan and the intertemporal loan. The most direct generalisation of the previous setup is to assume that the predetermined loan rate $R^l_{e,t}$ is the same on both types of loans. That is $\bar{\varepsilon}_{e,t}\tilde{A}_{e,t} = R^l_{e,t}(l_{e,t-1} + a w_t n_{e,t})$. With this assumption, the analysis for the entrepreneur financial contract is similar to that of the of the borrower's contract. Otherwise, we will need to model these two types of loans separately.

2.8 Data Appendix

I use US aggregate data from the first quarter of 1955 to the 4th quarter of 2004. All time series are from the Federal Reserve Economic Data at http ://research.stlouisfed.org/fred2/ :

1) Y_t : Real GDP in chained 2005 dollars divided by civilian population over the age of 16.

2) C_t : Real Personal Consumption Expenditures in chained 2005 dollars divided by civilian population over the age of 16.

3) I_t^h : Real Private Residential Fixed Investment in chained 2005 dollars divided by civilian population over the age of 16.

4) I_t : Real Private Non Residential Fixed Investment in chained 2005 dollars divided by civilian population over the age of 16.

5) N_t : Non Farm Business Sector Hours all Persons divided by civilian population over the age of 16.

6) W_t : Non Farm Business Sector Real Compensation per Hour.

7) $R_{t+1} - 1$: I construct the real risk-free interest rate by taking the arithmetic average of nominal annualised monthly 3 month T-bill rates and subtracting the quarterly annualised inflation rate. I calculate the inflation rate as the growth rate in the Implicit GDP deflator.

Table 1
Calibration

Parameter	Description	Value/Target
β	Patient households' discount factor	$R-1=0.01$
β^e	Entrepreneurs' discount factor	$\beta^e=0.98$ leverage$\simeq 0.58$
β^{bo}	Borrowerd discount factor	$\beta^{bo}=0.97$
σ	relative risk aversion coefficient	$1, 1.5$
G_y-1	trend growth rate of GDP	0.4%
α	capital share	0.3
$\delta_e=\delta_u$	capital depreciation rate	1.4%
δ_h	housing depreciation rate	0.4%
$\gamma I/K$	elasticity of q_t^k to I_t/k_t	1
$\gamma_h I_h/h$	elasticity of q_t to I_t^h/h_{t-1}	1
ρ	persistence of TFP shock	0.95
σ_z	standard deviation of TFP innovation	0.007
θ	entrepreneur span of control	0.95
$\xi_{c,}\xi_n, \xi_h$	$u(.)$ weights of consumption, leisure, housing	housing/GDP$\simeq 1.3$, $N^s\simeq 0.32$
θ_{bo}	proportion of borrowers	0.4
μ_{bo}	monitoring cost, borrowers	0.22
μ_e	monitoring cost, entrepreneurs	0.15
σ_{bo}	idiosyncratic shock std. dev, borrowers	$\sigma_{bo}=0.0951$, 1.4% default rate
σ_e	idiosyncratic shock std. dev, entrepreneurs	$\sigma_e=0.2118$, 3% default rate
s_w	wage exemption	1
s_h	housing exemption	0
s_y	output exemption	0
s_k	capital exemption	0
a	wages in advance proportion	1

Table 2
Comparison of US time series statistics with model generated statistics

Statistic	US data	All frictions	Firm frictions	Household frictions	No frictions
σ_y	1.54	0.9747	1.0065	0.8881	0.8971
σ_c	1.21	1.0834	1.0914	0.9532	0.9026
σ_{I^h}	9.63	0.6565	0.7338	0.8204	0.9339
σ_I	4.75	1.0839	1.2518	0.7471	0.8709
σ_N	1.74	0.1417	0.2169	0.052	0.006
σ_w	0.91	0.9023	0.8724	0.933	0.9009
$\sigma_{R_{t+1}}$	2.36	0.2340	0.2983	0.3361	0.449
$\rho(c, y)$	0.87	0.9993	0.9995	0.9991	0.9999
$\rho(I^h, y)$	0.64	0.9885	0.9981	0.9917	0.9999
$\rho(I, y)$	0.77	0.9991	0.9985	0.9967	0.9998
$\rho(N, y)$	0.87	0.9938	0.9874	-0.8558	-0.6126
$\rho(w, y)$	0.24	0.9999	0.9997	0.9996	0.9999
$\rho(R_{t+1}, y)$	0.03	-0.2435	-0.496	-0.4084	-0.5931
$\rho(I_h, I)$	0.25	0.9878	0.9951	0.9947	0.9998

US data is from the first quarter of 1955, to the 4th quarter of 2004. σ_x is the standard deviation of x in percentages. $\rho(x, y)$ is the correlation of x and y. The second column reports statistics for the model with all financing frictions, the third column for the model with only firm frictions, the fourth column for the model with only household frictions and the last column for the frictionless model. All series except the interest rate were logged, linearly detrended and then HP filtered with $\lambda = 1600$. Interest rates are annualised. Model statistics are based on averages of 500 simulations of 1000 periods using the 2nd order approximation, with the first 800 periods of each simulation discarded to reduce the effect of initial conditions. Model time series were filtered in the same way as the data.

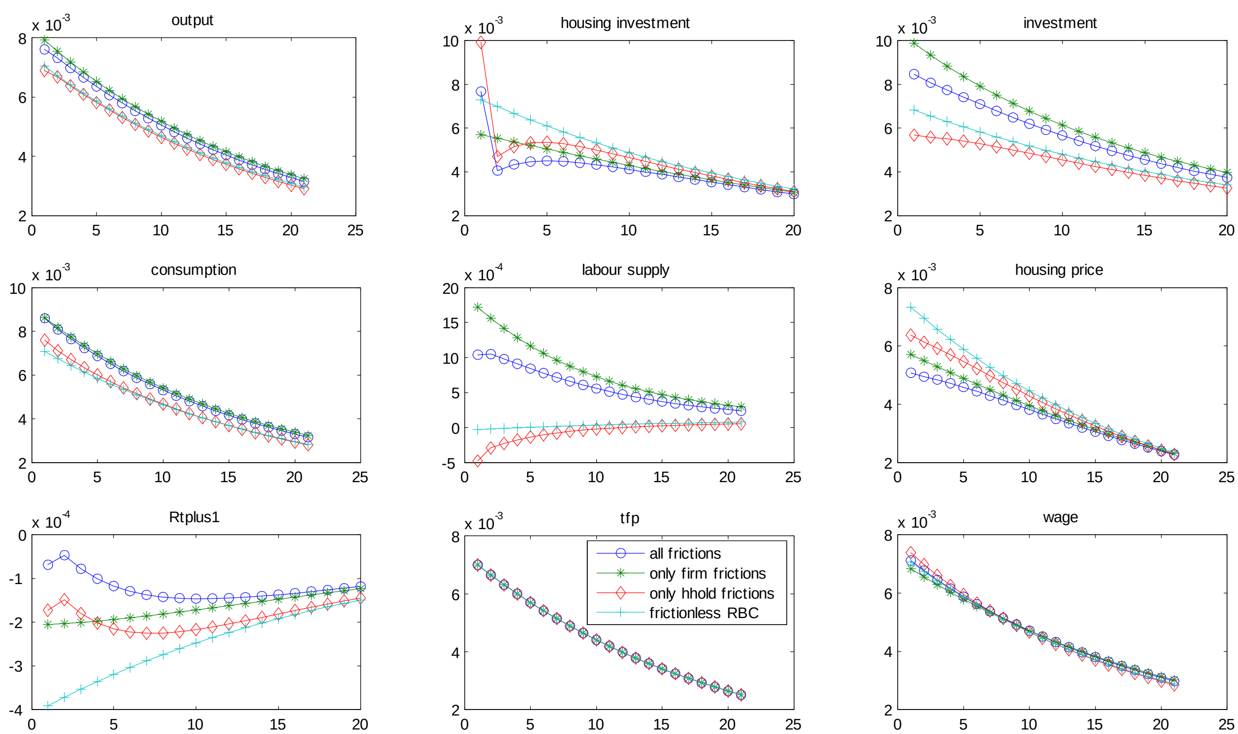

FIG. 2.1 – Baseline calibration, aggregates. All IRFs are in percentage deviations from the deterministic BGP.

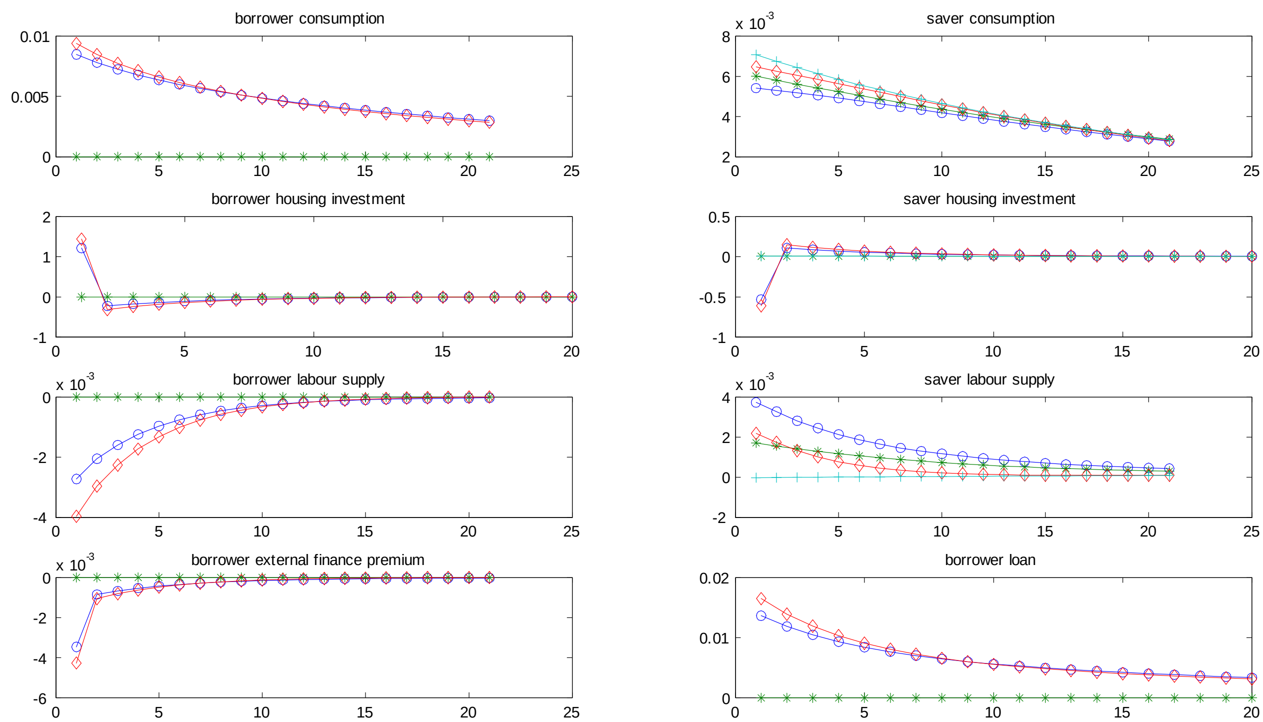

FIG. 2.2 – Baseline calibration, household sector. All IRFs are in percentage deviations from the deterministic BGP.

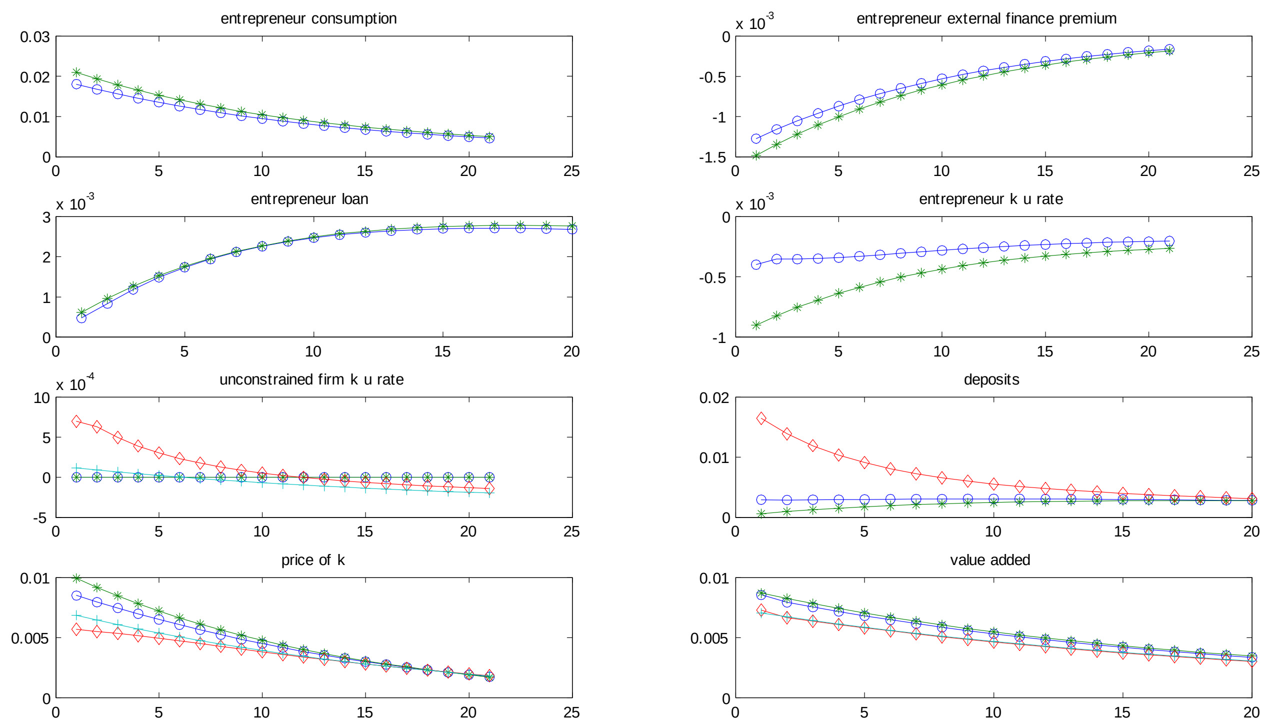

FIG. 2.3 – Baseline calibration, firm and other variables. All IRFs are in percentage deviations from the deterministic BGP.

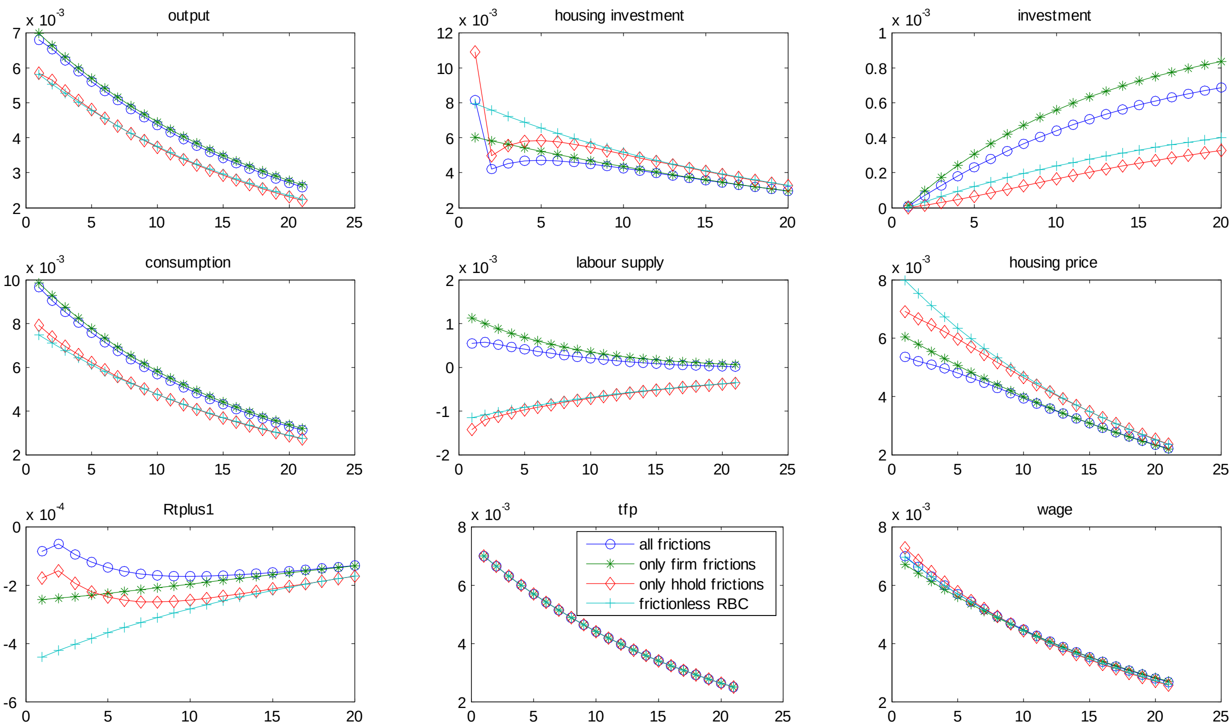

FIG. 2.4 – High K adjustment costs calibration, aggregates. All IRFs are in percentage deviations from the deterministic BGP.

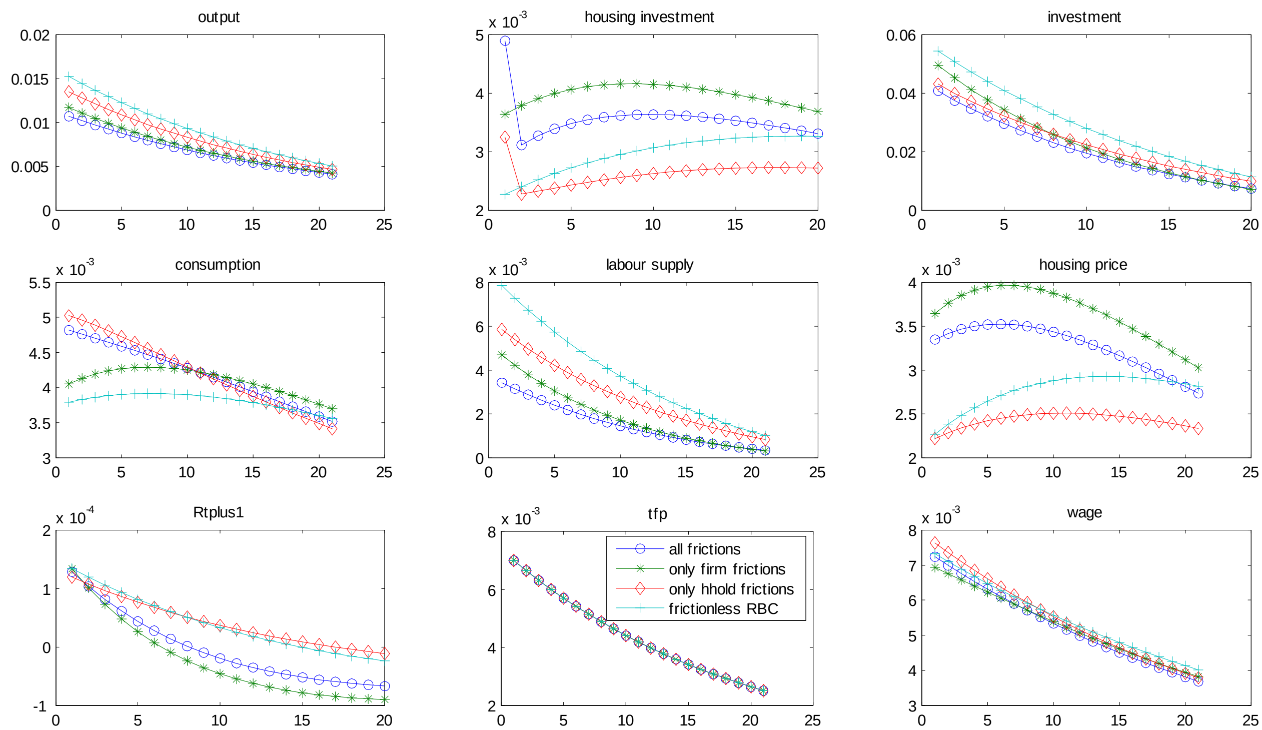

FIG. 2.5 – Low K adjustment costs calibration, aggregates. All IRFs are in percentage deviations from the deterministic BGP.

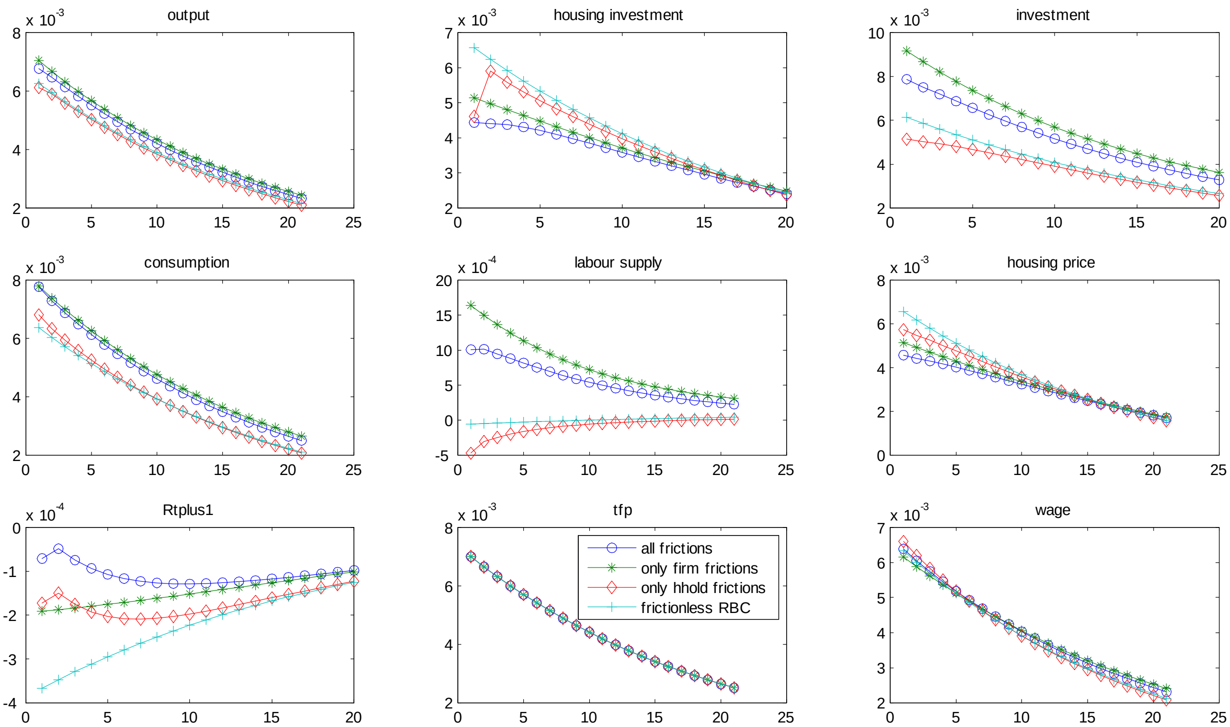

FIG. 2.6 – Baseline calibration, 2nd order approximation +1 standard deviation tfp shock. All IRFs are in percentage deviations from the ergodic means.

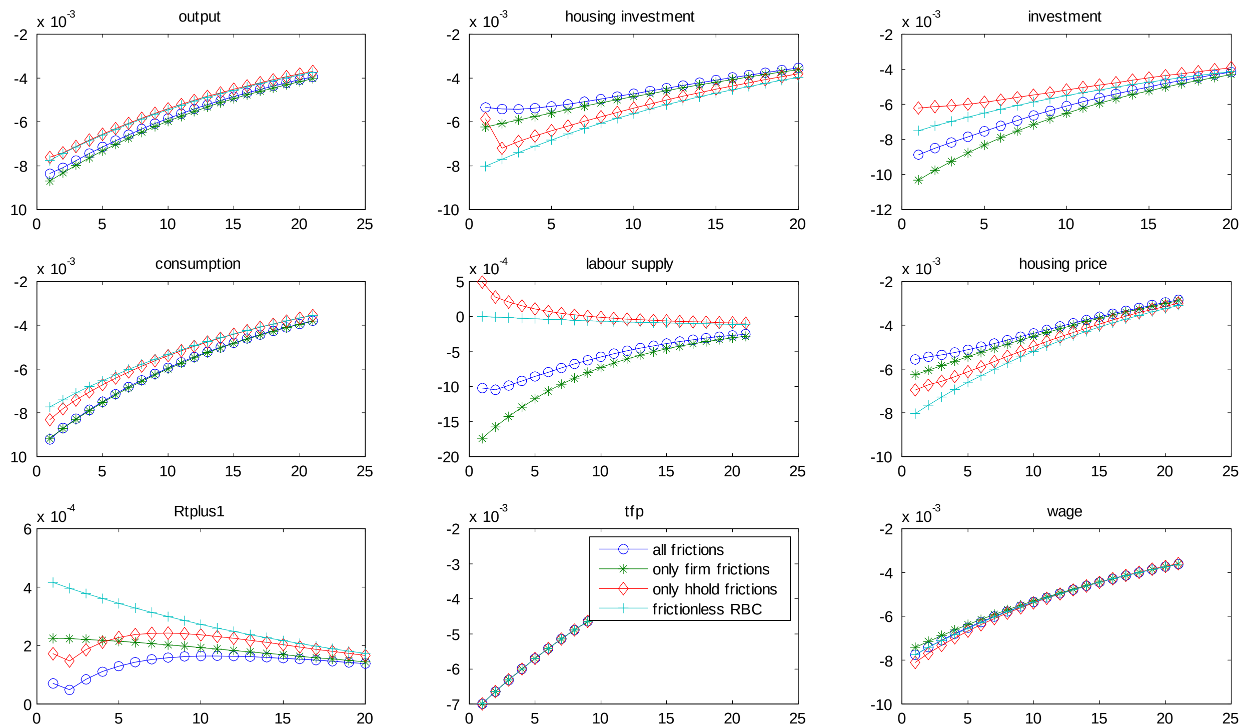

FIG. 2.7 – Baseline calibration, 2nd order approximation -1 standard deviation tfp shock. All IRFs are in percentage deviations from the ergodic means.

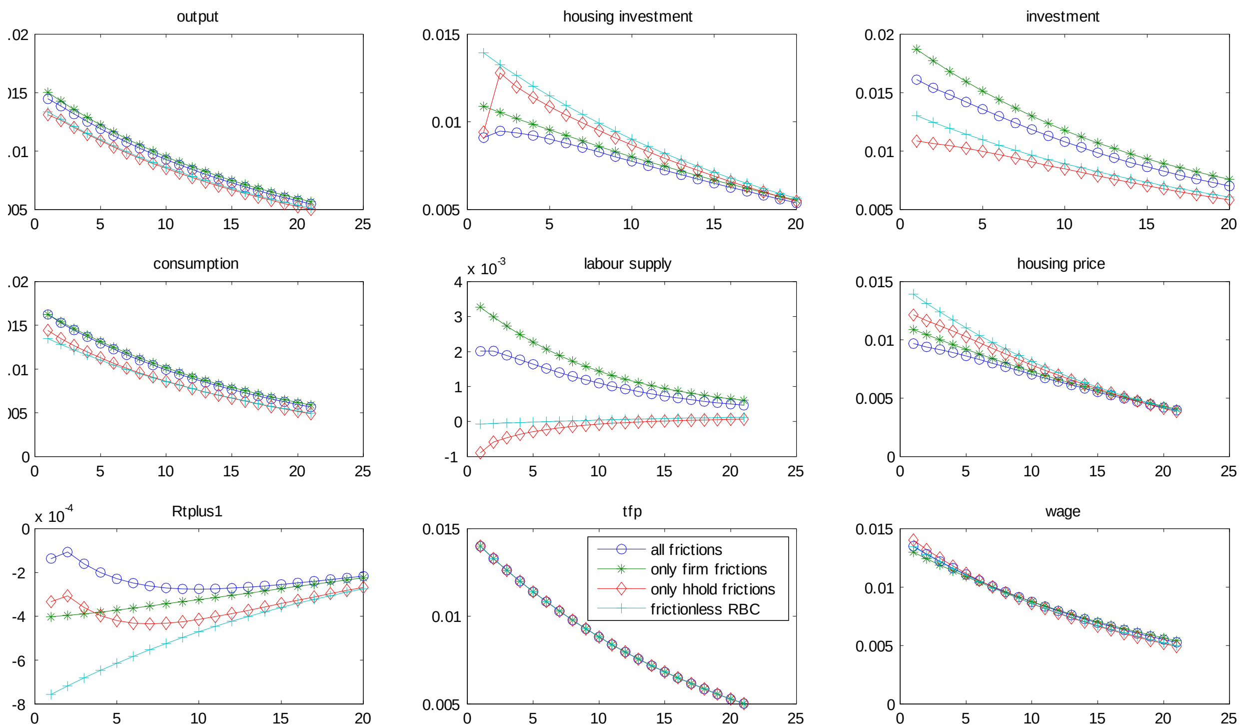

FIG. 2.8 – Baseline calibration, 2nd order approximation +2 standard deviation tfp shock. All IRFs are in percentage deviations from the ergodic means

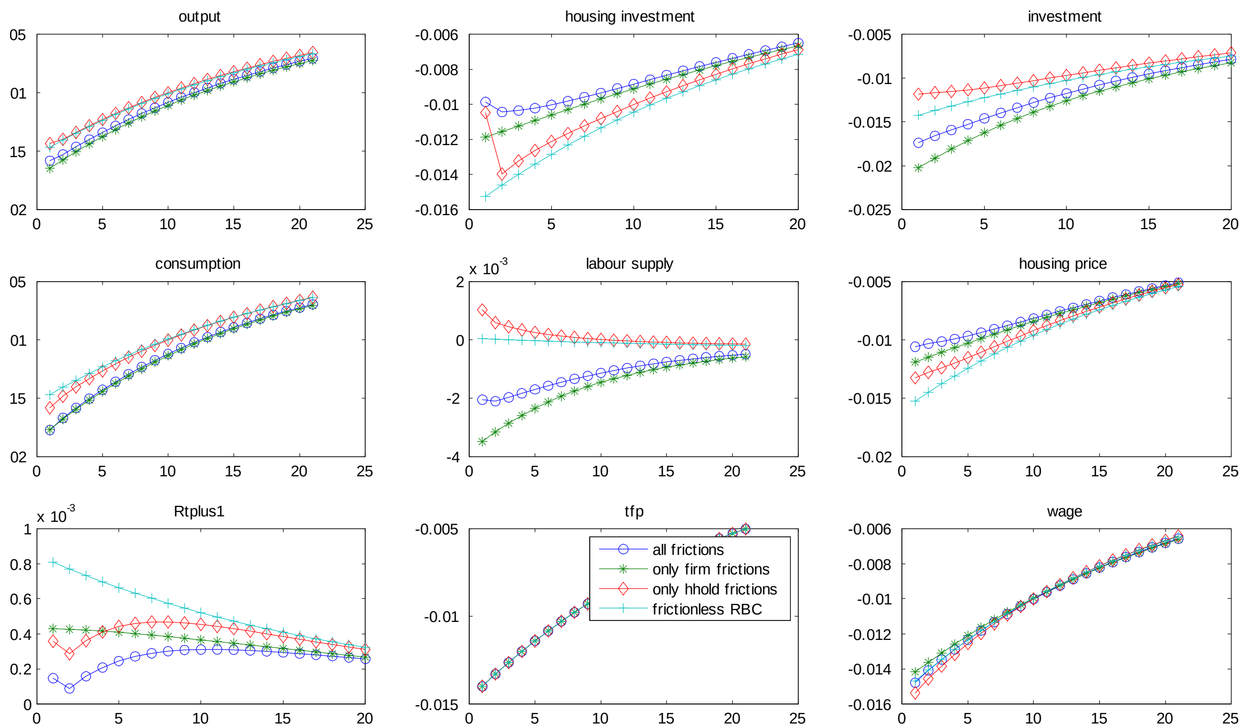

FIG. 2.9 – Baseline calibration, 2nd order approximation -2 standard deviation tfp shock. All IRFs are in percentage deviations from the ergodic means

Chapitre 3

Bank Capital, Housing and Credit Constraints

abstract

This paper integrates household financing frictions with bank financing frictions and housing price fluctuations in a dynamic stochastic general equilibrium model. We use a two-sided debt contract framework in which the bank cannot fully diversify shocks to its borrowers to study the link between household and bank sectors' default risks. The cyclical behaviour of the cost of the bank-depositors financing friction is determined by two main factors. On one hand, booms improve the financial health of the banks' borrowers which tends to reduce the cost of bank funding. On the other hand, consumption smoothing by savers and borrowers during booms increases the proportion of external financing in the banks' balance sheet which tends to increase the cost of bank funding. As a result of these opposing effects, the model matches procyclical profits and leverage in the financial sector, as observed in the data, but for non financial shocks the banking frictions in the model have an insignificant impact on the main macroeconomic aggregates such as output, consumption and investment.

JEL classification : E3, E4

Keywords : Financial frictions, bank capital, business cycles

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3.1 Introduction

This paper develops a DSGE model of the interaction between financing frictions facing households and financial intermediaries. The 2008 financial crisis in the US and the UK as well as earlier financial crisis in Scandinavia and Japan in the early 1990's suggest the potential usefulness of building a quantitative framework to study the links between the financial sector, real estate and household balance sheets. The usefulness of such a model is also supported by reduced form empirical evidence that loan rates may depend on the health of financial intermediaries' balance sheets and that housing wealth has a significant impact on consumption and debt levels[1]. Of course reduced form evidence may easily confound correlation and causality due to omitted variables bias. Because the financial health of banks and borrowers tends to be positively correlated, it is hard to tell for example if loans from banks in poor financial health are more expensive due to the state of the banks' balance sheets or the financial weakness of its borrowers. This makes it desirable to complement the existing evidence and ultimately base the analysis on a structural economic model. The main question is : what is the quantitative importance of the interaction between household and bank balance sheets ?

Following several recent DSGE models (starting with Kiyotaki and Moore 1997[83] and Iacoviello 2005[72]) our model has borrowers and savers distinguished by differences in their degree of impatience and limited enforcement frictions. This heterogeneity generates equilibrium lending with collateral. Our model adds bankers as the only agents in the economy capable of evaluating and monitoring loans to households. Banks finance impatient borrowers through debt contracts collateralised by borrower housing and wage income. We make two key assumptions. First, a borrower's collateral is subject to shocks. This provides borrowers with incentives to default on their loan repayments in bad states, requiring costly loan enforcement by the bank. Second, banks cannot fully diversify the shocks to their loan portfolios resulting from the shocks to borrower collateral, due to the need to specialize in a segment of borrowers. As a result they cannot guarantee depositors a safe rate of return. Because of asymmetric information about the realisation of bank loan revenues, the banks themselves would have an incentive to misreport their revenues and not repay depositors without some form of monitoring. Depositors can monitor and enforce deposit repayments. However these monitoring costs make deposits a relatively costly source of loan financing in comparison to the banks' own net worth (bank capital in the model). These assumptions create a positive link between the financing frictions affecting borrowers and banks. In combination with countercyclical borrower default rates, this can in theory amplify the effect of financial constraints on business cycles.

[1] For examples of evidence on the effect of bank capital on loans see Hubbard et al (2002)[71], Peek and Rosengren(2000)[98] and Carlson et al (2008)[30]. For empirical evidence suggesting a strong effect of house prices on consumption and debt see Dynan and Kohn(2007)[52], Iacoviello and Minetti(2008)[73] and Case et al (2005)[36].

At the same time the desire to smooth consumption by borrowers and savers tends to make the deposits to loans ratio procyclical. This raises (lowers) the bank's leverage in a boom (recession), which increases (decreases) the financing frictions between the bank and depositors and tends to dampen the effect of financial constraints on business cycles [2].

We solve the model and find that the effect of consumption smoothing by borrowers and savers dominates, and that in response to non-financial shocks the overall effect on the extra cost of bank funding is small. This allows us to reproduce the empirical evidence of a procyclical bank leverage ratio and procyclical bank profits, but the impact of bank capital fluctuations on non-bank aggregates is insignificant. Thus, the financial friction modeled in this paper cannot rationalise claims that procyclical financial sector leverage amplifies fluctuations or that bank capital matters in response to shocks that are exogenous to the banking sector. This does not exclude the possibility that bank financing frictions may matter for model dynamics in response to financial shocks directly affecting costs of monitoring banks or other financial parameters as in Christiano et al (2007) [42], but the exogeneity of such shocks is unclear.

There are several caveats to our results. First, this version of the paper only examines linear approximations to the model's dynamics. It is quite possible that the effect of banking on output fluctuations is nonlinear, with stronger effects occurring for bigger shocks. Given the tractability of the model, this should be easy to explore using higher order perturbation approximations as in Schmitt-Grohe and Uribe (2004)[67]. We intend to pursue this direction in a future version of the paper. Second, in keeping with most of the litterature on financing frictions in DSGE models we do not have any maturity mismatch in banks' balance sheets. Third, while we allow for asymmetric information about the outcome of loans, we assume perfect information on the stochastic process of the shocks affecting the loans. Both these omissions probably bias our estimate of the impact of bank financing frictions on business cycle fluctuations downwards. Finally, we assume that financing frictions only affect household sector borrowing. Meh and Moran (2008)[92] find significant effects of bank capital dynamics on output responses in a model where financing frictions affect capital producing firms. This suggests that results may be different in a version of the model where both firms and households face similar credit constraints.

As in Solomon (2009) [111], the paper uses a highly tractable alternative framework to study the effects of household credit frictions in DSGE models that allows for equilibrium default while preserving the assumption of risk averse/consumption

[2]This effect is similar to the banking attenuator effect found by Goodfriend and McCallum(2007)[63] in a DSGE model of banking with a reduced form loan production function. There, the attenuation effect of banking on aggregate fluctuations was produced by a deposits in advance constraint on consumption that increased demand for deposits in booms. The effect that we find does not rely on modeling the inside money role of bank deposits.

smoothing households and banks, in contrast to the risk neutral borrowers in Bernanke, Gertler and Gilchrist (henceforth BGG1999)[19] and in the Holmstrom and Tirole model (1997)[69]. This should make the analysis easier to compare to standard business cycle models, as well as providing potentially more realistic and less extreme borrower dynamics than those obtained in previous models with risk neutral agents. Furthermore, it may eventually allow us to examine the effects of precautionary savings on borrowers and banks by using higher order approximations of the model's dynamics. Naturally this tractability comes at a cost : we allow agents to insure themsevles against the idiosyncratic shocks that lead to default, to facilitate aggregation both at the level of borrowers and of banks. We see this as a useful assumption at least as a first step, while acknowledging the value of eventually studying these issues in a heterogenous agent model with a richer distribution of assets. At the same time, the idea that defaulting agents may benefit from some intra family financial or in kind support that is hard to track and seize by the banks, or that bank managers may be able insure part of their earnings against the risk that their bank defaults has some plausibility (though the insurance is likely to be much less comprehensive than assumed in this paper). By preserving the assumption of risk averse borrowers, the model provides a new mechanism for explaining the countercyclicality of external finance premia. Consumption smoothing by borrowers reduces fluctuations in their desired loan size relative to fluctuations in the value of collateral. The resulting countercyclicality in leverage translates into countercyclical default rates and external finance premia. [3]

Our model of banks is inspired by several theoretical papers that emphasize the effect of limited bank loan diversification and the ability of bank capital to reduce asymmetric information frictions between banks and depositors(See for example Williamson (1986) [117],Winton (1995)[118], Krasa and Villamil (1992)[86], Valencia (2006)[115]). Williamson (1986) shows how an intermediated lending arrangement using banks can reduce loan monitoring costs if the bank can diversify away idiosyncratic loan risks. Krasa and Villamil (1992) show that even with imperfect loan diversification and monitoring costs that are increasing in the size of the bank intermediated lending can be more efficient in terms of overall monitoring costs than direct lending. Winton (1995)[118] shows how in the presence of imperfect loan risk diversification, bank capital can reduce information frictions between the bank and depositors. The main competitor to the asymmetric information framework is Holmstrom and Tirole's bank capital model (1997)[69] and its extensions to a DSGE framework (Meh and Moran 2007[91],2008[92] and Aikman and Paustian 2006[3]), where bank capital plays a role of giving the bank incentives to monitor firms before their returns are realised. The Holmstrom and Tirole story may be relevant for firm level credit frictions but it is hard to apply to households : it is plausible that banks indirectly monitor the efforts of businesses to ensure they can repay the loans through the

[3]Note that a similar argument could be made for financially constrained firms : the desire to smooth dividends or entrepreneur consumption can help generate a countercyclical firm level external finance premium (see Solomon (2009) [111]).

terms of the loan for example or through monitoring of firms' transactions accounts held at the banks. However, one does not observe financial intermediaries monitoring the effort of households to improve their probability of repaying the loan. Instead, it seems more realistic to focus on borrower monitoring and contract enforcement when the borrower is in financial distress and cannot fully repay the loan. Models of bank capital that use the Holmstrom-Tirole model find that bank capital movements can amplify fluctuations, though the quantitative importance of the effect is sensitive to other features of the model[4]. These models also assume risk neutral banks and borrowers, which eliminates some of the consumption smoothing considerations that help determine the cost of bank loans in our model and may potentially bias the results on the effect of bank capital on fluctuations.

Our paper is also related to the literature on household financing frictions in DSGE models. Most of the literature follows Kiyotaki and Moore (1997)[83] and Iacoviello (2005)[72] in using a combination of differences in household impatience rates in combination with hard borrowing constraints using housing as collateral. These models have found an important role of household borrowing constraints in business cycle dynamics(see for example Iacoviello and Neri (2008)[74] and Monacelli (2008)[95]). In contrast to our framework, these papers assume an always binding hard borrowing constraints in their analysis of shocks. The soft borrowing constraint that we use, where interest rates increase smoothly in the size of the loan, is more likely to bind even for larger shocks than the hard borrowing constraint. In addition, it allows us to realistically incorporate time varying leverage ratios for households. [5]Recent extensions of Iacoviello's model such as Gerali et al(2009)[60] and Andres and Arce (2008)[9] include a non trivial role for banks, but they focus on imperfect competition in banking or model bank capital dynamics using a reduced form convex bank capital adjustment cost. Gerali et al (2009) [60] also find that the quantitative role of bank capital frictions is minor for non-financial shocks. This paper attempts to take a more micro-founded approach based on a well defined notion of financial distress. One advantage of this approach is that it allows for feedback in both directions between

[4]Aikman and Vlieghe(2004) [2]find a small contribution of bank capital movements to output fluctuations, while Meh and Moran (2008)[92] find a larger contriubtion.

[5]The only other framework allowing for equilibrium default of households in a DSGE model with aggregate shocks is the model of Aoki et al(2004)[10]. They adapt the BGG framework to housing by positing the existence of a special class of risk neutral home owners that rent housing to households. The financing frictions in their model apply to these risk neutral home owners (the equivalent of entrepreneurs in BGG) as opposed to the risk averse households. In order to model an effect of housing wealth on household consumption they are forced to adopt an ad-hoc dividend payment rule between home owners and households as well as assuming rule of thumb consumers that simply consume all their wealth each period.

Finally there is an emerging heterogeneous agent litterature modeling housing collateralised loans in general equilibrium(see for example Silos(2005)[110] and Iacoviello and Pavan(2008)[**?**]) using hard borrowing constraints. These models provide a much richer picture of the interaction of aggregate shocks and credit constraints by allowing for a non-degenerate distribution of assets, but they're much harder to solve and it may be difficult to extend them to allow for other modeling features.

external finance premia for borrowers and for banks. In contrast, the reduced form approach of Gerali et al (2009) [60]does not take into account that a higher default rate on the bank's loans usually also raises the bank's external finance premium.

3.2 The model

The model consists of the following agents. Patient households with a relatively high discount factor lend to banks and own all non financial firms in the economy. Impatient households with a lower discount factor borrow from banks to finance housing and consumption subject to financing frictions. Bankers take the deposits from patient households and lend to impatient households subject to financing frictions with respect to depositors. Finally, we close the model with a standard production sector without financing frictions. Perfectly competitive producers use labour and capital to produce final output, and are owned by the patient households. Housing producers transform final output into housing subject to housing investment adjustment costs, and are also owned by the patient households.

3.2.1 Household Sector

There is a measure 1 of households, with θ_s patient savers and $\theta_{bo} = 1 - \theta_s$ impatient borrowers.

Patient Agents (savers)

Patient households have a relatively high discount factor, and they have access to complete financial markets without any financing constraints. They provide deposits to banks. These savers have access to a deposit insurer that can diversify any repayment risk from an individual bank's deposits at no cost to the savers. In addition they own all firms in the economy. The representative saver picks sequences of consumption, working hours, housing and deposits at the bank $\{c_t^s, n_t^s, h_t^s, d_t\}$ to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^t \left[\ln c_t^s + \xi_h \ln h_t^s + \xi_n \ln(1 - n_t^s)\right]$$

subject to a sequence of constraints

$$c_t^s + q_t[h_t^s - (1 - \delta_h)h_{t-1}^s] + d_t = R_t d_{t-1} + w_t n_t^s + \Pi_t^h + \Pi_t.$$

where Π_t^h are profits from the housing producers and Π_t are profits from the final output producers.

The first order conditions for the saver are standard :

$$\begin{aligned}
d_t &: \quad \frac{1}{c_t^s} = \beta E_t \frac{1}{c_{t+1}^s} R_{t+1} \\
h_t^s &: \quad q_t \frac{1}{c_t^s} = \frac{\xi_h}{h_t^s} + \beta E_t \frac{1}{c_{t+1}^s} q_{t+1}(1-\delta_h) \\
n_t^s &: \quad \frac{\xi_n}{1-n_t^s} = \frac{1}{c_t^s} w_t.
\end{aligned}$$

Impatient Agent (borrowers)

Our model of the financial frictions affecting households is almost identical to the model in Solomon (2009) [111], except that we now also introduce bank funding frictions. Impatient households have the same intra-period preferences over housing,consumption and leisure as patient households. They are risk averse, and have a lower discount factor than lenders(patient households) : $\beta^{bo} < \beta$. The higher discount rate will imply that in equilibrium impatient households will be borrowers in a neighbourhood of the steady state. In fact, absent any frictions their borrowing would be unbounded in the steady state. Financing frictions make borrowing l_t bounded. Borrowers belong to one of a measure 1 continuum of population segments, each of measure $\theta^{bo} \equiv 1 - \theta^s$. Each segment is served by a continuum of banks. Each bank is specialized and can only lend to one segment.

Borrowers' incomes and housing stock values are subject to common segment specific shocks ε_t that are i.i.d across segments and across time. [6] Clearly, financial intermediaries are not so specialized in the real world. However, the assumption used here is a tractable starting point for capturing the fact that bank loan portfolios are imperfectly diversified-an assumption that will be essential in motivating financial frictions on the funding side of banks and linking them to borrower-bank financing frictions[7].We assume that ε_t has a CDF $F(\varepsilon_t)$ with $F'(\varepsilon_t) = f(\varepsilon_t)$.As in BGG (1999) [19], we assume that the derivative of $\frac{\bar{\varepsilon} f(\bar{\varepsilon})}{1-F(\bar{\varepsilon})}$ is positive. This will be true over the range that is relevant for the optimal $\bar{\varepsilon}$ when ε follows a lognormal distriubtion. Defining the total borrower resources relevant to the financial contract A_t as the sum of the

[6] The assumption that the shock is common to both the housing stock and and wage income simplifies the model substantially. See Jeske and Krueger(2005)[77] for evidence on regional shocks to the value of housing and another model where the stock of housing of borrowers is subject to shocks.

[7] To justify the use of banks in order to process loans to borrowers,we can assume that borrowers are also subject to idiosyncratic borrower specific shocks ω_t, but these shocks can be diversified across each segment, and banks have the ability to costlessly enforce a constant loan repayment regardless of the realisation of ω_t. Since borrowers inside the segment can in fact diversify away the differences in ω_t they will be able to make a fixed payment despite a low realisation of ω_t. The reduction in loan monitoring costs obtained by the diversification of this borrower specific shock can justify the use of banks instead of direct lending between savers and borrowers.

In contrast we assume that banks cannot force borrowers to make a constant repayment regardless of the value of the segment specific ε_t.

value of the borrower's house and his wage income, we have

$$A_t = \varepsilon_t[q_t(1-\delta_h)h_{t-1}^{bo} + n_t^{bo}w_t] = \varepsilon_t \bar{A}_t.$$

Lending in this economy is only possible through 1-period debt contracts that require a constant repayment $R_t^l l_{t-1}$ independent of ε_t. The borrower can default and refuse to repay the debt. Savers cannot force borrowers to repay. Instead lending must be intermediated by banks that have an enforcement technology allowing them to seize collateral

$$\varepsilon_t[(1-s_h)q_t(1-\delta_h)h_{t-1}^{bo} + (1-s_w)n_t^{bo}w_t] = \varepsilon_t\tilde{A}_t$$

at a cost $\mu\varepsilon_t\tilde{A}_t$ when the borrower defaults. $0 < \mu < 1$ determines the deadweight cost of default, $0 < s_w \leq 1$ and $0 < s_h \leq 1$ represent wage income and housing exemptions respectively. Realistic exemptions for housing are discontinuous and cannot be handled by standard perturbation methods, and it isn't clear yet how to set s_h to take them into account. Therefore, all the results that we report assume $s_h = 0$.In contrast we will vary s_w to reflect various wage garnishment rates. Conditional on enforcement, the law cannot prevent the bank from seizing all of $\varepsilon_t\tilde{A}_t$. Suppose first that the borrower does not have access to any insurance against the ε_t shock. Whenever $\varepsilon_t < \bar{\varepsilon}_t$ the borrower prefers to default and lose

$$\varepsilon_t\tilde{A}_t < R_t^l l_{t-1} = \bar{\varepsilon}_t\tilde{A}_t$$

when the bank enforces the contract. On the other hand when $\varepsilon_t \geq \bar{\varepsilon}_t$ the borrower prefers to pay $R_t^l l_t$ rather than lose $\varepsilon_t\tilde{A}_t \geq R_t^l l_t$. This implies that the net worth of the borrower after any loan repayment or default is $A_t - \min[\varepsilon_t, \bar{\varepsilon}_t]\tilde{A}_t$.

To be able to use a representative agent framework while maintaining the intuition of the default rule above, we make two assumptions. First, borrowers' labour supply is predetermined with respect to the ε_t shock. Second, borrowers have access to insurance contracts providing them with payments conditional on the realisation of $\varepsilon_{bo,t}$,

$$fs[\bar{A}_t - A_t + (H(\bar{\varepsilon}_t) + \min[\varepsilon_t, \bar{\varepsilon}_t])\tilde{A}_t], \text{ where}$$

$$H(\bar{\varepsilon}_{bo,t}) = -[(1-F(\bar{\varepsilon}_t))\bar{\varepsilon}_t + \int_0^{\bar{\varepsilon}_t} \varepsilon dF]\tilde{A}_t].$$

with $0 \leq fs \leq 1$. The insurer offers only complete insurance : $fs = 1$. The risk averse borrowers buy this insurance, which completely diversifies the risk related to ε_t. The payments from the insurance scheme cannot be seized by the bank. As a result, despite the insurance the bank cannot force the borrower to repay $R_t^l l_{t-1}$ when $\varepsilon_t < \bar{\varepsilon}_t$. The borrower cannot commit to always repay the loan, even though from an ex-ante perspective it is optimal to do so. Therefore, for a given R_t^l the borrower defaults when $\varepsilon_t < \bar{\varepsilon}_t$ and repays $R_t^l l_{t-1}$ when $\varepsilon_t \geq \bar{\varepsilon}_t$. With the insurance, the borrower is guaranteed total resources of

$$\bar{A}_t + H(\bar{\varepsilon}_t)\tilde{A}_t.$$

Finally we assume a symmetric initial distribution of housing among borrowers. This leads to a symmetric distribution of assets and income across borrowers, allowing us to reduce the model to a representative borrower (henceforth also called the borrower). Define the rate of return required by the bank on loans made at $t-1$ as $\bar{R}_t$. Banks have access to an inter-bank insurance scheme that allows them to diversify the segment specific shock ε_t among themselves. Therefore, each bank only requires that the loan is profitable in expectation. In order to participate in the loan, the bank requires that

$$[1-F(\bar{\varepsilon}_t)]\, R_t^l l_{t-1} + (1-\mu)\int_0^{\bar{\varepsilon}_t} \tilde{A}_t \varepsilon dF = [1-F(\bar{\varepsilon}_t)]\, \bar{\varepsilon}_t \tilde{A}_t + (1-\mu)\int_0^{\bar{\varepsilon}_t} \tilde{A}_t \varepsilon dF \geq \bar{R}_t l_{t-1}.$$ [8]

Competition among banks will make this constraint bind. The bank's break-even constraint will act as the borrowing constraint in this model. Defining

$$G(\bar{\varepsilon}_t) = [1-F(\bar{\varepsilon}_t)]\bar{\varepsilon}_t + (1-\mu)\int_0^{\bar{\varepsilon}_t} \varepsilon dF,$$

we can rewrite the bank participation constraint as

$$G(\bar{\varepsilon}_t)\tilde{A}_t = \bar{R}_t l_{t-1}.$$

The representative borrower picks sequences of consumption,housing, loans, deposits, labour supply and default threshold functions $\left\{c_t^{bo}, h_t^{bo}, l_t, d_{bo,t}, n_t^{bo}, \bar{\varepsilon}_t\right\}_{t=0}^{\infty}$ to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^{bo^t}\left[\ln c_t^{bo} + \xi_h \ln h_t^{bo} + \xi_n \ln(1-n_t^{bo})\right]$$

subject to a sequence of budget constraints

$$\begin{aligned} c_t^{bo} + q_t h_t^{bo} + d_t^{bo} \quad &= \quad q_t(1-\delta_h)h_{t-1}^{bo} + n_t^{bo} w_t \\ &\quad +H(\bar{\varepsilon}_t)[(1-s_h)q_t(1-\delta_h)h_{t-1}^{bo} + (1-s_w)n_t^{bo} w_t] + l_t + R_t d_{bo,t-1} \end{aligned}$$

and the participation constraints of the bank

$$G(\bar{\varepsilon}_t)[(1-s_h)q_t(1-\delta_h)h_{t-1}^{bo} + (1-s_w)n_t^{bo} w_t] = \bar{R}_t l_{t-1}$$

At an optimum $G'(\bar{\varepsilon}_t) > 0$. As a result, one can solve for a function $\bar{\varepsilon}_t(\frac{\bar{R}_t l_{t-1}}{\tilde{A}_t})$ with $\bar{\varepsilon}_t'(\frac{\bar{R}_t l_{t-1}}{\tilde{A}_t}) > 0$. Therefore, the default rate is increasing in the beginning of period household leverage ratio. In a neighbourhood of the steady state, impatient households

[8] With aggregate shocks, the borrower would prefer to get some insurance by having a lower $\bar{R}_t$ in a recession, in exchange for accepting a higher $\bar{R}_t$ in a boom. Computing the optimal risk sharing between risk averse depositors, bankers and borrowers is a difficult task. We follow the standard assumption in the litterature by ignoring these optimal risk sharing considerations and assuming that $\bar{R}_t$ is only a function of the risk free rate and the additional marginal cost of loans due to frictions between bankers and depositors.

will set $d_t^{bo} = 0$ and $l_t > 0$. We can use the relation between the Lagrange multiplier on the bank participation constraint $\psi_{bo,t}$ and the marginal utility of consumption $\lambda_{bo,t}$ to derive the following first order conditions for borrowers :

$$\begin{aligned}
\frac{1}{c_t^{bo}} &= \beta^{bo} E_t \frac{1}{c_{t+1}^{bo}} efp(\bar{\varepsilon}_{t+1}) \bar{R}_{t+1} \ , \\
\frac{1}{c_t^{bo}} q_t &= \frac{\xi_h}{h_t^{bo}} + \beta^{bo} E_t \frac{1}{c_{t+1}^{bo}} \left[1 + (1 - s_h)[H(\bar{\varepsilon}_{t+1}) + G(\bar{\varepsilon}_{t+1}) efp(\bar{\varepsilon}_{t+1})]\right] q_{t+1}(1 - \delta_h) \text{ and} \\
\frac{\xi_n}{1 - n_t^{bo}} &= \left[1 + (1 - s_w)[H(\bar{\varepsilon}_t) + efp(\bar{\varepsilon}_t)_t G(\bar{\varepsilon}_t)]\right] \frac{w_t}{c_t^{bo}} . \\
\bar{\varepsilon}_t &: \ \psi_t = \frac{1}{c_t^{bo}} \left(- \frac{H'(\bar{\varepsilon}_t)}{G'(\bar{\varepsilon}_t)} \right) = \frac{1}{c_t^{bo}} efp(\bar{\varepsilon}_t)
\end{aligned}$$

$efp_t = -\frac{H'(\bar{\varepsilon}_t)}{G'(\bar{\varepsilon}_t)} > 1$ can be interpreted as the the external finance premium faced by borrowers on loans. The monotone hazard rate $\frac{d}{d\bar{\varepsilon}}(\frac{\bar{\varepsilon} f(\bar{\varepsilon})}{1-F(\bar{\varepsilon})}) > 0$ means that $efp'(\bar{\varepsilon}_t) > 0$, which makes it countercyclical as long as the default threshold $\bar{\varepsilon}_t$ is countercyclical. The response of the default threshold to a shock is governed by $\bar{\varepsilon}_t'(\frac{\bar{R}_t l_{t-1}}{\bar{A}_t}) > 0$. On impact, with l_{t-1} and h_{t-1}^{bo} predetermined, any positive shock which increases the value of borrower collateral will reduce the default rate and lower the external finance premium. The effect of the positive shock in the next periods depends on how borrowers adjust loan demand and housing in response to the shock. In a neighbourhood of the steady state the existence of a soft borrowing constraint makes the impatient household behave more like the consumption smoothing patient household with a bias towards debt financed consumption instead of saving. For a fixed level of financial frictions and desired housing, a consumption smoothing borrower reacts to an increase in wealth by increasing savings. Since financial asset savings are negative, he reduces borrowing and increases his housing stock (the collateral asset). At the same time an increase in the value of collateral encourages higher borrowing. For plausible calibrations the first effect dominates, and the loan to assets ratio $\frac{\bar{R}_t l_{t-1}}{\bar{A}_t} = \frac{\bar{R}_t l_{t-1}}{q_t h_{t-1} + n_t^{bo} w_t}$ is countercyclical. This makes the external finance premium countercyclical beyond the initial impact of a shock [9].

The Euler equation for housing shows how financial frictions distort the usual housing investment equation. On one hand for a given expected house price appreciation an increase in the default rate (and hence in the external finance premium) increases the marginal value of housing as collateral : $\frac{d}{d\bar{\varepsilon}}(\frac{\bar{\varepsilon} f(\bar{\varepsilon})}{1-F(\bar{\varepsilon})}) > 0$ implies that $H(\bar{\varepsilon}_{t+1}) + efp_{t+1} G(\bar{\varepsilon}_{t+1})$ is increasing in $\bar{\varepsilon}_{t+1}$. On the other hand, there is an indirect effect of financing frictions on housing investment through the effect of these frictions

[9] With uncertainty, one also has to take into account precautionnary saving which generates movements in saving in the opposite direction to those motivated by consumption smoothing. But for the typical level of aggregate flucutations, the consumption smoothing motive should dominate(in fact the linear approximation of the model omits any precautionnary savings effect).

on non durable consumption. From the Euler equation for loans, a reduction in the external finance premium increases $\frac{c_t^{bo}}{c_{t+1}^{bo}}$ which lowers the effective discount rate applied to housing investment. In the special case of perfect foresight about aggregate shocks, we can combine the loan and housing Euler equations to conclude (holding house prices and the risk-free interest rate constant) that $\frac{\partial(h_{bo,t}/c_{bo,t})}{\partial\bar{\varepsilon}_{bo,t+1}} < 0$ (see Solomon (2009) [111] for the proof) . This result should also hold for small levels of uncertainty, as well as in a certainty-equivalent linear approximation. Finally, in a neighbourhood of the steady state,

$$1 + (1 - s_h)[H(\bar{\varepsilon}_t) + efp_t G(\bar{\varepsilon}_t)]$$

is greater than 1, meaning that the marginal value of investing in housing is more sensitive to the future expected value of housing than in the model without financing frictions. The labour supply equation shows how the financial friction distorts the borrower's labour supply decision. For a given default rate, the financing friction affects labour supply indirectly by changing the sensitivity of labour supply to $\frac{w_t}{c_t^{bo}}$ and by affecting c_t^{bo}. The level of financing frictions has a direct impact on labour supply through its effect on $H(\bar{\varepsilon}_t)+efp_t G(\bar{\varepsilon}_t)$ for a given $\frac{w_t}{c_t^{bo}}$. The same analysis as for housing applies here as well : the direct impact of higher financing frictions is to increase the work effort of borrowers, essentially in order to maintain their living standards with higher financing costs. A reduction in financing frictions will therefore lower borrowers' labour supply, holding everything else constant. This effect disappears when labour income is fully exempt. In that case, labour supply is determined by the same equation as patient households' labour supply. The other effects are again similar to the ones for housing. Financial frictions increase the sensitivity of borrower labour supply to the wage rate and non durable consumption(as long as wages are not fully exempt), and the stronger reaction of c_t^{bo} to shocks results in a larger income effect on labour supply.

3.2.2 Banks

We introduce bankers separately from households, since one cannot really have financing frictions between identical depositors and banks if those very same depositors own the banks. We restrict financial arrangements between banks and depositors to 1 period contracts. Our model of the financing friction between banks and depositors is similar to Townsend's (1979)[114] and Krasa and Villamil's (1992)[86] costly state verification framework. Without loss of generality, assume that there is a measure 1 of ex-ante identical banks at the beginning of period t. Bankers pick loan supply, deposit demand and consumption to maximise their lifetime expected utility. They take the expected rate of return on loans and the risk free rate earned on deposits as given and maximise expected life time utility by picking consumption(the dividend), loan supply and deposit demand in each period. We assume that deposits take the form of 1 period debt contracts that guarantee depositors a repayment of $R_t\theta^s d_t$.

In order be able to enforce loan contracts against borrowers, a bank must specialize in a specific segment of borrower, exposing it to a common shock to borrowers ε_t. Therefore, it cannot guarantee depositors a constant deposit repayment. The segment specific shock ε_t is the private information of banks serving the segment. Without any monitoring mechanism banks would have an incentive ex-post to claim that their revenues were too low and that they cannot repay deposits. Depositors can diversify the deposit repayment risk across many banks. They have access to a monitoring technology that allows them to verify a bank's assets A_t^b at a cost $\mu_b A_t^b$ and seize A_t^b. While endowing depositors with such a monitoring ability sounds implausible, it is equivalent to a more realistic but complicated model in which the risky banks in our model are financed by a large number of safe banks financed by depositors. The safe banks lend to the risky banks that have the capability to monitor household loans, and monitor the risky banks when they are financially distressed. We make similar assumptions regarding banks' ability to diversify the risk of default as for borrowers. Banks belong to an inter-bank insurance arrangement that can diversify the shocks to their profits and guarantee each bank the same ex-post profit at no cost. Banks can hide any gains from this insurance arrangement from the depositors. Define $s_t \equiv \min[\bar{\varepsilon}_t, \varepsilon_t]$. The bank's revenue is $\theta^{bo} s_t \tilde{A}_t$. In non audited states incentive compatibility requires the bank to make a fixed repayment to depositors

$$R_t^d \theta^s d_{t-1} \equiv \hat{s}_t \theta_{bo} \tilde{A}_{t-1}.$$

Our assumption of an inter-bank insurance arrangement whose benefits cannot be committed to repay depositors implies that, as in the standard costly state verification framework of Townsend (1979)[114], it is optimal to minimize the expected monitoring costs. To achieve this, the contract establishes a default threshold $\hat{s}_t$, such that the bank makes the fixed deposit repayment if $s_t > \hat{s}_t$ and repays $s_t \theta_{bo} \tilde{A}_t$ otherwise. In order for banks to make money on the loans and be willing to serve as intermediaries, $\hat{s}_t$ must be lower than $\bar{\varepsilon}_t$. As a result the contract can also be seen as specifying a threshold $\hat{\varepsilon}_t$ such that the bank defaults whenever $\varepsilon_t < \hat{\varepsilon}_t < \bar{\varepsilon}_t$.

With the insurance scheme, and using $G(\bar{\varepsilon}_t)\tilde{A}_t = \bar{R}_t l_{t-1}$ each bank gets profits from loans(net of deposit repayments) of

$$\theta^{bo}[G(\bar{\varepsilon}_t) + H(\hat{\varepsilon}_t)]\tilde{A}_t = \theta^{bo}\left[1 + \frac{H(\hat{\varepsilon}_t)}{G(\bar{\varepsilon}_t)}\right]\bar{R}_t l_{t-1} \equiv \theta^{bo} H^b(\hat{\varepsilon}_t, \bar{\varepsilon}_t)\bar{R}_t l_{t-1}$$

Finally, depositors expect to break even on deposits :

$$\theta^{bo}\frac{G(\hat{\varepsilon}_t)}{G(\bar{\varepsilon}_t)}\bar{R}_t l_{t-1} \equiv \theta^{bo} G^b(\hat{\varepsilon}_t, \bar{\varepsilon}_t)\bar{R}_t l_{t-1} \geq \theta^s R_t d_{t-1}$$

,which is binding at an optimum. Defining the bank's capital as $\theta^{bo} l_t - \theta_s d_t$ this constraint is the model's version of a market determined bank capital adequacy requirement.

The representative banker picks sequences of consumption c_t^b, loans l_t , deposits d_t and $\hat{\varepsilon}_t$ to maximise

$$E_0 \Sigma_{t=0}^{\infty} \beta^{b^t} \ln c_t^b$$

subject to a sequence of constraints

$$\begin{aligned} c_t^b + \theta^{bo} l_t &\leq \theta^{bo} H^b(\hat{\varepsilon}_t, \bar{\varepsilon}_t) \bar{R}_t l_{t-1} + \theta^s d_t \text{ and} \\ \theta^{bo} G^b(\hat{\varepsilon}_t, \bar{\varepsilon}_t) \bar{R}_t l_{t-1} &\geq \theta^s R_t d_{t-1} \end{aligned}$$

The first order conditions of the representative banker are :

$$\begin{aligned} d_t &: \frac{1}{c_t^b} = \beta^b R_{t+1} Eefp_{t+1}^b \\ l_t &: \frac{1}{c_t^b} = \beta^b \bar{R}_{t+1} E_t \left(\frac{1}{c_{t+1}^b} H^b(\hat{\varepsilon}_{t+1}, \bar{\varepsilon}_{t+1}) + \psi_{t+1}^b G^b(\hat{\varepsilon}_{t+1}, \bar{\varepsilon}_{t+1}) \right) \\ \hat{\varepsilon}_t &: \psi_t^b = \frac{1}{c_t^b} \left(-\frac{H'(\hat{\varepsilon}_t)}{G'(\hat{\varepsilon}_t)} \right) = \frac{1}{c_t^b} efp_t^b. \end{aligned}$$

where ψ_t^b is the multiplier on the depositor break-even constraint. The external finance premium on deposits $efp_t^b = \frac{-H'(\hat{\varepsilon}_t)}{G'(\hat{\varepsilon}_t)}$ is increasing in $\hat{\varepsilon}_t$ from the monotone hazard property. $\bar{R}_{t+1} - R_{t+1}$ represents the spread between the expected rate of return on the loan and on the deposits due to the bank financing frictions. Absent the enforcement problems between the bank and depositors we would have $\bar{R}_{t+1} = R_{t+1}$. By linearizing the first Euler equations for bank deposit demand and loan supply around the steady state, and equating the resulting expressions for the banker's expected consumption growth we obtain the following expression in percentage deviations from the steady state :

$$\begin{aligned} \frac{\bar{R}_{t+1} - \bar{R}}{\bar{R}} - \frac{R_{t+1} - R}{R} \simeq\ & R\beta^b efp_b'(\hat{\varepsilon})\hat{\varepsilon}[1 - \frac{\theta_s d}{\theta_{bo} l_{bo}}] E_t \frac{\hat{\varepsilon}_{t+1} - \hat{\varepsilon}}{\hat{\varepsilon}} \\ & + \bar{R}\beta^b \frac{G'(\hat{\varepsilon})}{G(\hat{\varepsilon})^2} \left[H(\hat{\varepsilon}) + \frac{G(\hat{\varepsilon})}{\beta^b R} \right] E_t \frac{\bar{\varepsilon}_{t+1} - \bar{\varepsilon}}{\bar{\varepsilon}}. \end{aligned}$$

The first coefficient is positive because the steady state deposits to loans ratio $\frac{\theta_s d}{\theta_{bo} l_{bo}}$ is less than 1. Therefore, $\bar{R}_{t+1} - R_{t+1}$ is increasing in $E_t \hat{\varepsilon}_{t+1}$ to a first order approximation. The second coefficient cannot be signed conclusively, but for all attempted calibrations $H(\hat{\varepsilon}) + \frac{G(\hat{\varepsilon})}{\beta^b R} > 0$, making $\bar{R}_{t+1} - R_{t+1}$ increasing in $E_t \bar{\varepsilon}_{t+1}$. This shows the potential for feedback between the default rates of banks and borrowers. Higher expected bank default rates raise $\bar{R}_{t+1} - R_{t+1}$ which for any given expectations on future borrower collateral values and any given loan raise expected borrower default rates, which in turn raise $\bar{R}_{t+1} - R_{t+1}$. However, this does not imply that $\bar{R}_{t+1} - R_{t+1}$ is countercyclical in general equilibrium. First, borrowers will respond to a higher

$\bar{R}_{t+1}$ by reducing their lending and increasing their work effort which alleviates the need for $\bar{\varepsilon}_{t+1}$ to rise. Second, consumption smoothing by borrowers and lenders will tend to raise $\frac{d_t}{l_t}$ in a boom. This increases the financing frictions between banks and depositors and raises $E_t\hat{\varepsilon}_{t+1}$. As a result $\bar{R}_{t+1} - R_{t+1}$ may be procyclical even if $E_t\bar{\varepsilon}_{t+1}$ is countercyclical.

3.2.3 Production

Housing is produced by a representative firm owned by the savers. The firm purchases I_t^h units of the consumption good from savers and turns it into $I_t^h = h_t - (1-\delta)h_{t-1}$ units of housing while paying an adjustment cost of $\frac{\gamma^h}{2}(\frac{I_t^h}{h_{t-1}} - \delta))^2 h_{t-1}$. Note that the firm takes the aggregate housing stock h_{t-1} as given when choosing I_t^h. With these assumptions, the housing producer's problem reduces to picking I_t^h each period to maximise profits

$$\theta^s \Pi_t^h = (q_t - 1)I_t^h - \frac{\gamma^h}{2}(\frac{I_t^h}{h_{t-1}} - \delta_h))^2 h_{t-1}.$$

The first order condition for housing supply is :

$$q_t = 1 - \delta_h \gamma^h + \gamma^h \frac{I_t^h}{h_{t-1}} = 1 - \gamma_h + \gamma_h \frac{h_t}{h_{t-1}}.$$

In the steady state $q = 1$ and the housing producer makes no profits.

Final output is produced by perfectly competitive financially unconstrained firms using capital and labour. The firms are owned by savers. The representative firm produces
$y_t = z_t k_t^{\alpha} n_t^{1-\alpha}$,
where z_t is an aggregate productivity shock, equal to 1 in the steady state.

$$\ln z_t = \rho \ln z_{t-1} + e_t^z, \ e_t^z \backsim N(0, \sigma_z).$$

Firms pick sequences of capital and labour $\{k_{t+1}, n_t\}$ to maximise their present value

$$E_0 \Sigma_{t=0}^{\infty} \beta^t \frac{1}{c_t^s}[z_t k_t^{\alpha} n_t^{1-\alpha} - w_t n_t - (k_{t+1} - (1-\delta)k_t)].$$

The first order conditions for this problem are :

$$\begin{aligned} 1 &= \beta E_t \frac{c_t^s}{c_{t+1}^s}\left[1 + \alpha \frac{y_{t+1}}{k_{t+1}} - \delta\right] \\ (1-\alpha)\frac{y_t}{n_t} &= w_t. \end{aligned}$$

Given the importance of the labour market for the model's dynamics, we also solve a version of the model with capital adjustment costs of the same form as the housing adjustment costs in order to examine the effect of a less elastic labour demand curve.

We assume that savers own the capital producing firm. Profit maximisation by capital producers yields the supply curve

$$q_t^k = 1 - \delta\gamma + \gamma\frac{I_t}{k_t}.$$

Finally we also examine extensions allowing for variable capital utilisation rates and labour adjustment costs. Our main interest in adding these features is to see how modifications in the factors shifting labour demand affect the impact of financing frictions on the transmission of shocks. Variable capital utilisation u_t modifies the production function $z_t(u_t k_t)^\alpha n_t^{1-\alpha}$. As in Burnside and Eichenbaum(1996)[25] increasing capacity utilisation raises the depreciation of capital and modifies the capital accumulation equation to $k_{t+1} = I_t + (1-\delta_0 u_t^\phi)k_t$.We model labour adjustment costs as a quadratic function $\gamma_n(\frac{n_t - n_{t-1}}{n_{t-1}})^2 n_{t-1}$ as in Cooper and Willis(2006)[44].

3.3 Results

3.3.1 Calibration

We solve the model at a quarterly frequency. Since our main focus is on the model's dynamics in response to stationary shocks we abstract from long run growth and set $z = 1$ in the steady state. Table 1 provides a summary of our calibration. We use standard values from the real business cycle literature for most parameters that are not related to the financing frictions. These include β, δ, δ_h,α and ρ. We set ξ_n and ξ_h to match a steady state work share of around $\frac{1}{3}$ and an annualized housing stock to output ratio of around 1.35, in line with the estimates for the US in Iacoviello and Neri (2008)[74].

γ is set to $0.25/\delta$ as in Bernanke, Gertler and Gilchrist (1999)[19]. There is much less evidence on the adjustment cost parameter for housing supply. We set $\gamma_h = 1$ based on the estimates in Topel and Rosen (1988)[113] on the short run elasticity of housing prices in the US. For the model with variable capital utilisation rates, the curvature of the cost of varying utilisation rates ϕ can be found from the steady state first order condition for u_t after setting $u = 1$ and $\delta = 0.02$ in the steady state. For the model with labour adjustment costs we use $\gamma_n = 2$ based on estimates in Cooper and Willis (2006)[44].

We set the share of patient agents $\theta_s = 0.65$ in line with the estimates in Iacoviello (2005)[72]. The impatient households' discount factor of 0.97 is in the middle of the range of estimates for poorer households who are more likely to be heavily in debt as reported in Iacoviello (2005)[72] and Cagetti (2003)[26]. There is little evidence on the cost of loan enforcement parameter μ for household loans. We use $\mu = 0.25$ as in Carlstrom and Fuerst (1997)[31], which is in the upper range of estimates used in firm level financial accelerator models, since we suspect that the losses in home foreclosures are higher than the costs for business default [10]. Since we have not found any direct evidence on μ_b we assume as a benchmark that $\mu_b = \mu$ and do sensitivity analysis with respect to μ_b/μ. Given μ and β^{bo} we calibrate σ to try to match a steady state annual household bankruptcy rate in the US of around 1% as reported in Athreya (2001)[13]. $\sigma = 0.12$ produces a steady state annual default rate of 1.27%. Again, the results reported below are not very sensitive to moderate changes in σ[11].

We set β^b to match a financial sector loans to bank capital ratio (the model's leverage ratio) of around 6 as reported in Greenlaw, Hatzius, Kashyap and Stein (2008)[65]. This is lower than the leverage ratio of around 7.15 used in Meh and Moran (2008)[92] or the ratio of 10 used in Aikman and Paustian (2006)[3]. We end up using a lower leverage ratio because we net out the assets and liabilities owed among different financial intermediaries. This is consistent with the model's definition of leverage which only includes the assets and liabilities of the banking sector with respect to the non-financial sector. Finally, we examine two scenarios for the wage garnishment rate. The benchmark scenario treats wages like housing and sets $s_w = 0$. The second scenario examines the other extreme and sets $s_w = 1$.

3.3.2 Impulse Response Function Analysis

We linearize the model's equations around the deterministic steady state and solve the approximate model using a standard linear rational expectations algorithm (Klein 2000)[84]. We compare impulse response functions for 3 models : a standard Real Business Cycle model (henceforth RBC model) with housing but without any financing frictions, a model with borrower-lender frictions but without any banking related financing frictions, and finally the full model with a role for bank capital.

We first consider the effect of a 1% increase in total factor productivity (henceforth TFP) relative to the steady state (figures 1-3). The responses of the RBC model without financing frictions to a temporary but persistent increase in total factor productivity are by now well documented. The only twist to the standard story is the addition of housing to the model. Without housing adjustment costs households at first reduce investment in housing in order to allow the firms they own to invest more in capital and to increase more valuable non durable consumption. Afterwards, as TFP converges back to the steady state value and the returns to investing in capital

[10]We have only recently become aware of a paper by Krueger and Jeske(2005) which reports a deadweight loss in foreclosure estimate of 0.22, based on comparing resale values of foreclosed properties with an estimate of their market value without foreclosure. Based on sensitivity analysis, we suspect that using $\mu = 0.22$ wouldn't have a significant impact on the results.

[11]We also report some results for $\beta^{bo} = 0.98$, in which case we recalibrate $\sigma = 0.2$.

decline households start increasing investment in housing relative to the steady state. With adjustment costs, it no longer makes sense to first reduce investment housing and increase it later. Instead households increase their investment in housing gradually with a peak around 15 quarters after the shock.

Adding financing frictions for households dampens the response of output to the shock in the short run. On impact the rise in output is smaller by 0.11% with household financing frictions. However, the increase in output is slightly more persistent in the model with borrowing constraints. In fact if we simulate the response to the shock for a longer horizon, the increase in output relative to the steady state is larger in the model with credit constraints than in the standard RBC model after 57 quarters, but by then the original TFP shock has almost completely died out and the difference is very small. The credit constraints create a difference in the composition of the response to the shock. Consumption relative to the steady state increases significantly more than in the RBC model initially, but is lower than in the RBC model after 11 quarters. In contrast both investment in housing and capital increase by less than in the RBC model. Together with the smaller rise in labour supply this explains why output does not respond as much as in the model without financing frictions.

To understand these differences in more detail, we now examine the dynamics of borrowers and savers separately. For borrowers, the increase in wages makes external financing for a given level of borrowing less expensive. Since borrowers are credit constrained , their steady state consumption and housing stock is significantly below that of savers. As a result, they take advantage of the reduction in borrowing costs to sharply increase their non durable consumption and their stock of housing. The increase in aggregate housing demand raises house prices which further relaxes the borrowing constraint of borrowers for a given wage. Note that borrowing costs decline because loan demand increases by less than the value of borrower collateral. This is a reflection of the borrower's desire to smooth his nondurable consumption. As productivity converges back towards its steady state level, wages and house prices decline . This increases the cost of financing for a given level of borrowing. As a result, the rise in non durable consumption and housing investment by borrowers is strongest on impact and declines slowly over time. Meanwhile, given the role of wage income as collateral, the relaxation of the borrowing constraint reduces the value of working for borrowers. In combination with the strong wealth effect generated by the rise in consumption, this encourages borrowers to reduce labour supply. The decline in borrower labour supply is strong enough to actually reduce aggregate labour supply. This lower labour supply also reduces the marginal value of capital and investment.

For savers, the key difference in comparison to the RBC model is the extra demand for loans from borrowers in a boom. Initially, interest rates are actually higher in the RBC model, due to the lower demand for capital, but eventually the effect of higher borrower loan demand dominates and the risk free rate in the borrowing constrained economy rises relative to the RBC economy risk free rate. Patient households increase their savings at the expense of lower housing investment and consumption, both because of the standard intertemporal substitution effect but also because higher future interest rates reduce the present value of their future income stream. The lower increase in saver consumption also raises the marginal value of savers' labour supply, and as a result they work more than in the RBC economy. However, this is not enough to prevent aggregate labour supply from being less procyclical relative to the RBC economy. The analysis so far highlights two difficulties in amplifying exogenous shocks using household level credit frictions. First, to the degree that borrowing ability is positively linked to labour income (either because it serves as collateral or more generally because there are limits on borrowing related to income) and to the degree that borrowers' consumption is more responsive to shocks than that of savers, there may be strong income effects on borrower labour supply that dampen the change in labour supply in response to shocks. Second, the greater increase in consumption and housing investment by borrowers must be financed in general equilibrium by more modest increases (or even declines)in the consumption, housing and capital investment of savers if production does not increase sufficiently. Overall, the effects of adding financing frictions on dynamics is modest if one only looks at aggregate consumption or output or investment. The financing frictions do have a large impact on the distribution of consumption and housing investment, with consumption and housing investment being significantly more procyclical for borrowers than for savers .

The addition of bank-depositor frictions has virtually no impact on aggregate output, investment, consumption and labour supply. The only perceptible effect is on bank and lending related variables. Note that if there were no spread between the expected rate of return for bank and for depositors ($\bar{R}_{t+1} = R_{t+1}$), then the model reduces to the standard case of direct intermediation between savers and borrowers. The movement in spreads in response to tfp shocks is minimal with a maximum change of $\frac{6}{10^7}$. Of course even if there were no deviation of the spread from the steady state value the existence of a spread in the steady state also affects dynamics. However this effect seems to be quite small. The inability to generate large variations in this spread explains why banking frictions have a small impact on real variables. With the addition of banks, we see a modest increase in the deposits to loans ratio accompanied by a rise in bank profits, in line with the empirical evidence of procyclical financial intermediary leverage(see Adrian and Shin (2008) [1], Meh and Moran (2007) [91]). On one hand, consumption smoothing by bankers and increasing bank profits would tend to decrease the deposits to loans ratio as bankers attempt to increase savings. At the same time consumption smoothing by savers tends to raise deposits, and consumption smoothing by borrowers means that despite the relaxation of the borrowing constraint borrowers don't increase their demand for loans by as much as savers increase their supply of deposits. The second effect dominates in equilibrium, and the bank's leverage increases. The banks' increasing reliance on external funding through deposits raises their external finance premium and the spread between $\bar{R}_{t+1}$ and R_{t+1} by a small amount. As a result, the strength of the bank financing frictions in the model is procyclical. One could argue that perhaps the result that bank depositor frictions don't matter for the main aggregate macroeconomic quantities is due to our assumption that the enforcement cost parameter of the bank $\mu_b = \mu$ of the borrower. But this result is robust to making $\mu_b > \mu$ (e.g. $\mu = 3\mu$) or $\mu_b < \mu$ (e.g. $\mu_b = \mu/3$)[12]. Finally one may argue that the weak effects of bank-depositor frictions is due to our calibration of the bank's discount factor which produces a very low steady state external finance premium for banks. To test this, we reset the bankers' discount factor to $\beta^b = 0.985$, generating a bank leverage ratio of 37 at the steady state. Even with this unrealistically high steady state difference between risk free rates and bank lending rates, the impulse responses for the model with bank-depositor frictions were practically indistinguishable from those of the model with only borrower-lender financing frictions. For high bank leverage ratios, we can obtain a countercyclical spread $\bar{R}_{t+1} - R_{t+1}$, while preserving the procyclical bank leverage ratio and profits. This occurs because the positive link between $\bar{\varepsilon}_{t+1}$ and $\bar{R}_{t+1} - R_{t+1}$ now dominates the effect of higher bank leverage on the spread (recall our linear approximation for this spread). However, movements in this spread in response to shocks are still quite small (on the order of 10^{-5}).

In an attempt to reverse the declining labour supply of the borrowers, we examine a version of the model in which labour income is fully exempt from seizure by the bank (figures 4-6). In this version of the model, labour income is no longer part of the loans's collateral. Therefore, a reduction in the costs of borrowing no longer directly encourages workers to reduce their labour supply. Borrowers' labour supply now increases for the first 3 periods . It then declines relative to the steady state due to the strong income effect generated by the boom in borrower consumption, but by less than in the case without an exemption. The response of output is a bit stronger than in the case without exemptions, but the IRF's are otherwise very similar. Once again, the impact of adding bank-depositor frictions is minimal. Another factor that affects the labour supply of borrowers is the degree of borrower impatience. Lower impatience reduce the optimal amount of borrowing and lowers the steady state external finance premium of borrowers. As a result the increase in borrowers' work effort due to financing frictions is smaller. We raised β^{bo} to 0.98 and recalibrated σ to match an annual default rate of around 1%. Now borrower labour supply increases in response to a positive tfp shock, but the increase is still much smaller than for savers. Finally, we experimented with version of the model incorporating other features such as variable capacity utilisation rates, capital adjustment costs and labour adjustment costs. These extensions did not change the main conclusions on the effect of borrower and bank financing frictions.

Next we analyse the effect of a demand shock in the model (figures 7-10). We use shocks to agents' discount factors to examine the model economy's response to

[12] We also did sensitivity analysis to changes in the cost of auditing μ, the idiosyncratic volatility σ, the adjustment cost parameters for housing and capital γ_h and γ . In all these cases bank funding frictions had a similarly small effect on non-bank related variables.

demand shocks. The discount factor is now βb_t. Justiniano et al. (2006)[102] have shown the importance of allowing for such shocks to the model's Euler equations. If taken literally as random changes in the level of impatience, it is hard to interpret these shocks as structural which may cast some doubt on analysing them as exogenous. A possibly more structural interpretation is suggested by Browning and Tobacman (2007)[24] who show that changes in the impatience level are isomorphic to changes in agents' expectation about future paths of income : in particular one cannot use observations on consumption or saving to disentangle higher impatience from more optimistic expectations on future income. Under this interpretation the demand shocks in the model can be seen as temporary deviations from rational expectations due for example to overoptimistic forecasts of future productivity levels or future asset prices. As such, they can provide insight into the impact of the financial frictions in the model on the response to news shocks as in Jaimovich and Rebelo (2008)[75] or to asset price bubbles as in Bernanke and Gertler(2000)[18]. Similar to the productivity shock, the discount factor shock b_t equals 1 in the steady state and its percentage deviation from the steady state follows $\hat{b}_t = \rho_b \hat{b}_{t-1} + \varepsilon_t^b$, where ε_t is i.i.d and $0 < \rho_b < 1$. Following the estimation results of Justiniano and Primicieri we set $\rho_b = 0.83$. We assume that the preference shock is the same for bankers, borrowers and savers.

Like the typical RBC model our model cannot generate demand driven booms. In the version without financing frictions, an increase in b_t encourages agents to increase non durable consumption. However due to the wealth effect on labour supply, households reduce labour supply. Labour demand does not move on impact since productivity has not changed and capital is predetermined. As a result output falls, and non durable consumption crowds out investment in housing and capital. Despite the fact that the preference shock also increases the relative utility of current housing, households prefer to increase non durable consumption at the expense of lower housing investment due to the lower utility weight of the housing stock. The reduction in investment sustains the decline in output despite the decline in b_t over time. Adding financial frictions on the household side reduces the decline in output significantly. The fall in housing investment relative to the steady state leads to a decline in house prices. This increases the external finance premium faced by borrowers despite the increase in wages. Holding the wealth effect from increasing non durable consumption constant, the increase in the cost of borrowing pushes impatient households to increase labour supply in the model without any labour income exemption. In addition, due to the increase in financing costs the increase in borrower consumption is smaller than that of the saver. This reduces the negative wealth effect on borrower labour supply, relative to savers' labour supply. Overall labour supply declines by less and therefore the decline in output is lower than in the RBC model without financing frictions.

The effect of financial frictions between banks and depositors is, as for TFP shocks, virtually non existent for non-banking related macroeconomic aggregates. The banks' deposits to loans ratio declines, mostly because savers reduce deposits to increase their consumption by more than the fall in borrowers loan demand. The higher external finance premium leads to a reduction in loans, but by less than the decline in deposits, due to the countervailing effect of borrowers' desire to consume more. The external finance premium on deposits rises on impact since the banks' leverage ratio is predetermined and the borrower's external finance premium has increased. In subsequent periods the external finance premium on deposits falls due to the fall in bank leverage. This leads to a fall in $\bar{R}_{t+1} - R_{t+1}$ and in banks' profits.

Comparison of model second moments to the data

This section examines the effect of financing frictions on various statistics of key economic aggregates (table 2). We compare the model with both household and bank funding frictions, the model with only household funding frictions and the Real Business Cycle model without any financing constraints to US data from 1955 to 2004 (see the data appendix for details). Since preference shocks do not generate realistic business cycles, we examine the benchmark calibration for models driven only by the productivity shock. For the volatility of the TFP innovation, we use the estimates in Cooley (1995) [43], setting $\sigma_z = 0.007$. As expected from the analysis of Impulse Response Functions, the time series moments for the model with bank financing constraints are virtually identical. Household credit constraints increase the volatility of consumption by around 17%, reduce the volatility of investment by around 12% and reduce the volatility of output by around 6%. At the same time, the correlations between various aggregates and output are very similar to those in the model without any credit constraints. While the models match the volatility of output, investment and wages reasonably well, they underestimate the volatility of consumption, housing investment and hours by a wide margin. Again, these mismatches are shared by the model with financing frictions and the frictionless model.

3.4 Conclusion

This paper has developed a model of the interaction between household credit frictions and bank capital based on imperfect diversification of bank loan portfolios and costly financial distress. The analysis so far has not found an important role for the bank capital channel for the transmission of non-financial shocks. Instead of amplifying the effect of the more standard household financing frictions, bank financing frictions in the model dampen their effect on business cycles, due the effects of consumption smoothing by borrowers and savers on bank leverage. In the current version of the model credit frictions only have a modest impact on aggregate output fluctuations though they have significant distributional consequences both in terms of the ratio of consumption to investment and in terms of the different responses of borrowers and lenders to shocks. In response to supply shocks, the credit frictions in the model dampen output fluctuations while increasing their persistence. This is due in large part to the income effect generated by changes in the costs of external financing on labour supply.

There are several issues that we have ignored in the current model and that could give bank financing frictions a more important role. We have abstracted from issues related to maturity mismatch and lending among financial intermediaries. At the same time while allowing for costs related to defaults in bank loan portfolios we have omitted any asymmetric information related to the quality of the loans before default. For example we have assumed perfect information about the volatility of the bank specific shocks σ. It is possible that asymmetric information about the uncertainty of loan outcomes could generate a more important role for bank financing frictions frictions. In theory incorporating this feature should not be difficult, at least if we start with the simple case in which σ can take two values. In the current model we have followed BGG (1999)[19] in assuming that loan rates could be made contingent on the aggregate state of the economy. A more realistic assumption is that loan rates are predetermined with respect to the aggregate state of the economy, and that they can only be partially renegotiated. Finally, we have assumed that firms are unaffected by financing frictions. Adding firm financing frictions to in a model where banks lend to both firms and households would allow us to analyse the possibility of contagion between the cost of bank financing for the household and firm sectors. We leave these extensions for future research.

Appendix :

Linearized equations :

n.b : all variables below are in either %deviations of in deviations from SS, whether they're hatted or no$t$

Savers :

Budget Constraint-

$c^s\hat{c}_t^s + \delta h^s \hat{q}_t + h^s \hat{h}_t^s + d\hat{d}_t = (1-\delta_h)h^s\hat{h}_{t-1}^s + Rd(\hat{R}_t + \hat{d}_{t-1}) + wn^s(\hat{n}_t + \hat{w}_t) + \Pi_t^h + \Pi_t$

deposits Euler equation-

$\hat{c}_t^s = E_t\hat{c}_{t+1}^s - \hat{R}_{t+1}$

Housing Euler equation-

$(\hat{c}_t^s - \hat{q}_t) = \frac{\xi^h c^s}{h^s}\hat{h}_t^s + \beta(1-\delta_h)E_t(\hat{c}_{t+1}^s - q_{t+1})$

Labour supply-

$\frac{n^s}{1-n^s}\hat{n}_t^s = \hat{w}_t - \hat{c}_t^s.$

Borrowers :

Budget constraint-

$c^{bo}\hat{c}_t^{bo} + h^{bo}\left[1-(1-\delta_h)(1+(1-s_h)H(\bar{\varepsilon}))\right]\hat{q}_t + h^{bo}h_t^{bo} = H'(\bar{\varepsilon})\bar{\varepsilon}\tilde{A}_{bo}\bar{\varepsilon}_t + (1-\delta_h)h^{bo}[1+(1-s_h)H(\bar{\varepsilon})]\hat{h}_{t-1}^{bo}$
$+[1+H(\bar{\varepsilon})(1-s_w)]n^{bo}w\left(\hat{w}_t + \hat{n}_t^{bo}\right) + l\hat{l}_t.$

Bank break-even constraint-

$G'(\bar{\varepsilon})\bar{\varepsilon}\tilde{A}_{bo}\bar{\varepsilon}_t + G(\bar{\varepsilon})(1-\delta_h)(1-s_h)h^{bo}(\hat{q}_t + h^{bo}_{t-1}) + G(\bar{\varepsilon})(1-s_w)wn^{bo}(\hat{w}_t + \hat{n}^{bo}_t) = \bar{R}l(\bar{R}_t + \hat{l}_{t-1})$.

Loan Euler equation-

$\hat{c}^{bo}_t = -E_t\hat{\psi}_{t+1} - \bar{R}_{t+1}$

Housing Euler equation-

$\frac{1}{c^{bo}}(\hat{c}^{bo}_t - \hat{q}_t) = \frac{\xi_h}{h^{bo}}\hat{h}^{bo}_t + \beta^{bo}(1-\delta_h)\frac{1+(1-s_h)H(\bar{\varepsilon})}{c^{bo}}E_t\hat{c}^{bo}_{t+1} - \beta^{bo}(1-\delta_h)\left[\frac{1+(1-s_h)H(\bar{\varepsilon})}{c^{bo}} + \psi G(\bar{\varepsilon})(1-s_h)\right]E_t\hat{q}_{t+1} - \beta^{bo}(1-s_h)(1-\delta_h)\psi G(\bar{\varepsilon})E_t\hat{\psi}_{t+1}$

Labour supply-

$\frac{\xi_n}{(1-n^{bo})^2}n^{bo}\hat{n}^{bo}_t = \left(\psi G(\bar{\varepsilon})(1-s_w) + \frac{1+(1-s_w)H(\bar{\varepsilon})}{c^{bo}}\right)w\hat{w}_t - \frac{1+(1-s_w)H(\bar{\varepsilon})}{c^{bo}}w\hat{c}^{bo}_t + \psi G(\bar{\varepsilon})(1-s_w)w\hat{\psi}_t$

Optimal default threshold

$\psi\hat{\psi}_t = \frac{H'(\bar{\varepsilon})}{G'(\bar{\varepsilon})c^{bo}}\hat{c}^{bo}_t + \left(\frac{H'(\bar{\varepsilon})G''(\bar{\varepsilon}) - H''(\bar{\varepsilon})G'(\bar{\varepsilon})}{G'(\bar{\varepsilon})^2}\right)\frac{\bar{\varepsilon}}{c^{bo}}\bar{\varepsilon}_t$

Note that using this expression for $\hat{\psi}_t$ and the steady state condition $\psi c^{bo} = -\frac{H'(\bar{\varepsilon})}{G'(\bar{\varepsilon})}$, we can get a borrower's counterpart to the saver's linearized Euler equation of the form : $\hat{c}^{bo}_t = E_t\hat{c}^{bo}_{t+1} - A_{\bar{\varepsilon}}E_t\bar{\varepsilon}_{t+1} - \bar{R}_{t+1}$, where $A_{\bar{\varepsilon}} = \left(\frac{H'(\bar{\varepsilon})G''(\bar{\varepsilon}) - H''(\bar{\varepsilon})G'(\bar{\varepsilon})}{G'(\bar{\varepsilon})^2}\right)\frac{\bar{\varepsilon}}{\psi c^{bo}} > 0$. This highlights the key modification of the borrower's linearized Euler equation relative to the standard one : in addition to $\bar{R}_{t+1}$, the borrower faces a linearized external finance premium $A_{\bar{\varepsilon}}E_t\bar{\varepsilon}_{t+1}$ which depends on changes in the expected future default rate.

Banks :

Bank's balance sheet/budget constraint :

$c^b\hat{c}^b_t + \theta^{bo}l\hat{l}_t = \theta^{bo}H^b(\hat{\varepsilon})\bar{R}l\left(\bar{R}_t + l_{t-1}\right) + \theta^{bo}\bar{R}l\frac{H'(\hat{\varepsilon})}{G(\bar{\varepsilon})}\hat{\varepsilon}\hat{\varepsilon}_t - \theta^{bo}\bar{R}l\frac{H(\hat{\varepsilon})}{G(\bar{\varepsilon})^2}G'(\bar{\varepsilon})\bar{\varepsilon}\bar{\varepsilon}_t + \theta^s dd_t$

Depositor break-even constraint :

$\theta^{bo}G^b(\hat{\varepsilon})\bar{R}l\left(l_{t-1} + \bar{R}_t\right) + \theta^{bo}\frac{G'(\hat{\varepsilon})}{G(\bar{\varepsilon})}\bar{R}l\hat{\varepsilon}\hat{\varepsilon}_t - \theta^{bo}\frac{G(\hat{\varepsilon})}{G(\bar{\varepsilon})^2}\bar{R}lG'(\bar{\varepsilon})\bar{\varepsilon}\bar{\varepsilon}_t = \theta^s Rd\left(d_{t-1} + R_t\right)$

Deposit Euler equation :

$c^b_t = -E_t\psi^b_{t+1} - R_{t+1}$

Loan Euler equation :

$\frac{1}{c^b}c^b_t = \beta^b\bar{R}\frac{H^b(\hat{\varepsilon})}{c^b}E_tc^b_{t+1} - \frac{1}{c^b}\bar{R}_{t+1} - \beta^b\bar{R}\psi^bG^b(\hat{\varepsilon})E_t\psi^b_{t+1} + \beta^b\bar{R}\frac{G'(\bar{\varepsilon})}{G(\bar{\varepsilon})^2}\left(\frac{H(\hat{\varepsilon})}{c^b} + \psi^bG(\hat{\varepsilon})\right)\bar{\varepsilon}E_t\bar{\varepsilon}_{t+1}$.

$\hat{\varepsilon}_t$ foc :

$\psi^b_t = -c^b_t + \frac{1}{c^b\psi^b}\frac{d}{d\hat{\varepsilon}}\left(\frac{-H'(\hat{\varepsilon})}{G'(\hat{\varepsilon})}\right)\hat{\varepsilon}\hat{\varepsilon}_t$

Housing supply :

$\hat{q}_t = \gamma^h(\hat{h}_t - \hat{h}_{t-1})$

$\Pi^h_t \approx \frac{h\delta_h}{\theta^s}\hat{q}_t$

$h\hat{h}_t = \theta^s h^s\hat{h}^s_t + \theta^{bo}h^{bo}\hat{h}^{bo}_t$

final output profit :

$\pi_t = \theta^s \Pi_t = (y - wn)y_t - k\left[k_{t+1} - (1-\delta)k_t\right]$

Labour demand :
$y_t - n_t = w_t.$
Labour adjustment costs modify this to
$(1-\alpha)\frac{y}{n}\hat{y}_t - [(1-\alpha)\frac{y}{n} + \gamma_n(1+\beta)]\hat{n}_t + \gamma_n\hat{n}_{t-1} + \beta\gamma_n E_t\hat{n}_{t+1} = w\hat{w}_t$
capital :

$c_t^s = E_t c_{t+1}^s - \alpha\beta\frac{y}{k}\left(E_t y_{t+1} - k_{t+1}\right)$
With capital adjustment costs we have
$c_t^s = \hat{q}_t^k + E_t c_{t+1}^s - \alpha\beta\frac{y}{k}\left(E_t y_{t+1} - k_{t+1}\right) - \beta(1-\delta)E_t\hat{q}_{t+1}^k$,
$\hat{q}_t^k = \gamma(\hat{k}_{t+1} - \hat{k}_t),$
$\hat{\pi}_t^k = \delta k\hat{q}_t^k.$
With variable capacity utilisation rates we have
$\hat{y}_t - \hat{k}_t = q_t^k + \phi\hat{u}_t,$
$c_t^s = \hat{q}_t^k + E_t c_{t+1}^s - \alpha\beta\frac{y}{k}\left(E_t y_{t+1} - k_{t+1}\right) - \beta(1-\delta)E_t\hat{q}_{t+1}^k + \beta\delta\phi E_t\hat{u}_{t+1}.$
production function

$y_t = z_t + \alpha k_t + (1-\alpha)n_t$
or
$y_t = z_t + \alpha(k_t + \hat{u}_t) + (1-\alpha)n_t$ with variable capacity utilisation.
TFP shock process : $\hat{z}_t = \rho\hat{z}_{t-1} + e_t^z$

3.5 Data Appendix

We use US aggregate data from the first quarter of 1955 to the 4th quarter of 2004. All time series are from the Federal Reserve Economic Data at http ://research.stlouisfed.org/fred2/ :

1) Y_t : Real GDP in chained 2005 dollars divided by civilian population over the age of 16.

2) C_t : Real Personal Consumption Expenditures in chained 2005 dollars divided by civilian population over the age of 16.

3) I_t^h : Real Private Residential Fixed Investment in chained 2005 dollars divided by civilian population over the age of 16.

4) I_t : Real Private Non Residential Fixed Investment in chained 2005 dollars divided by civilian population over the age of 16.

5) N_t : Non Farm Business Sector Hours all Persons divided by civilian population over the age of 16.

6) W_t : Non Farm Business Sector Real Compensation per Hour.

7) $R_{t+1}-1$: the real risk-free interest rate is approximated by taking the arithmetic average of nominal annualised monthly 3 month T-bill rates and subtracting the quarterly annualised inflation rate. We calculate the inflation rate as the growth rate in the Implicit GDP deflator.

Table 1
Calibration

Parameter	Description	Value
β	Patient households' discount factor	0.99
β^{bo}	Impatient households' discount factor	0.97
β^{b}	Bankers' discount factor	0.9899
α	capital share	0.34
δ	capital depreciation rate	0.02
δ_h	housing depreciation rate	0.005
$\gamma_h \delta_h$	elasticity of q_t^h to I_t^h/h_{t-1}	1
$\gamma\delta$	elasticity of q_t to I_t/k_t	0.25
γ_n	labour adjustment cost parameter	2
ρ	persistence of tfp shock	0.95
ϕ	curvature of capital depreciation function	1.5051
ξ_n	weight of leisure in utility function	1.8
ξ_h	housing weight in utility function	0.12
θ_s	proportion of patient households	0.65
μ	enforcement cost parameter for households	0.25
μ_b	enforcement cost parameter for banks	0.25
σ	standard deviation of borrower segment specific shock	0.12
s_w	exemption level for wage income	0 or 1

Table 2
Comparison of US time series statistics with model generated statistics

Statistic	US data	All frictions	Only household frictions	no frictions
σ_y	1.54	1.2717	1.2727	1.3385
σ_c	1.21	0.4549	0.4552	0.3895
σ_{I^h}	9.63	0.3033	0.3032	0.3233
σ_I	4.75	4.209	4.2121	4.768
σ_N	1.74	0.5703	0.5717	0.6714
σ_w	0.91	0.716	0.7155	0.6862
$\sigma_{R_{t+1}}$	2.36	0.2678	0.2679	0.2818
$\rho(c, y)$	0.87	0.9497	0.9497	0.9013
$\rho(I^h, y)$	0.64	0.8795	0.8793	0.8677
$\rho(I, y)$	0.77	0.993	0.993	0.9926
$\rho(N, y)$	0.87	0.9858	0.9859	0.9856
$\rho(w, y)$	0.24	0.9910	0.9915	0.9863
$\rho(R_{t+1}, y)$	0.03	0.6367	0.6368	0.6304
$\rho(I_h, I)$	0.25	0.8176	0.8173	0.801

US data is from the first quarter of 1955, to the 4th quarter of 2004. σ_x is the standard deviation of x in percentages. $\rho(x, y)$ is the correlation of x and y. All series except the interest rate were logged, linearly detrended and then HP filtered with $\lambda = 1600$. Interest rates are annualised. Model statistics are based on averages of 1000 simulations of 1000 periods using the first order approximation, with the first 800 periods of each simulation discarded to reduce the effect of initial conditions, and HP filtering just like for data time series.

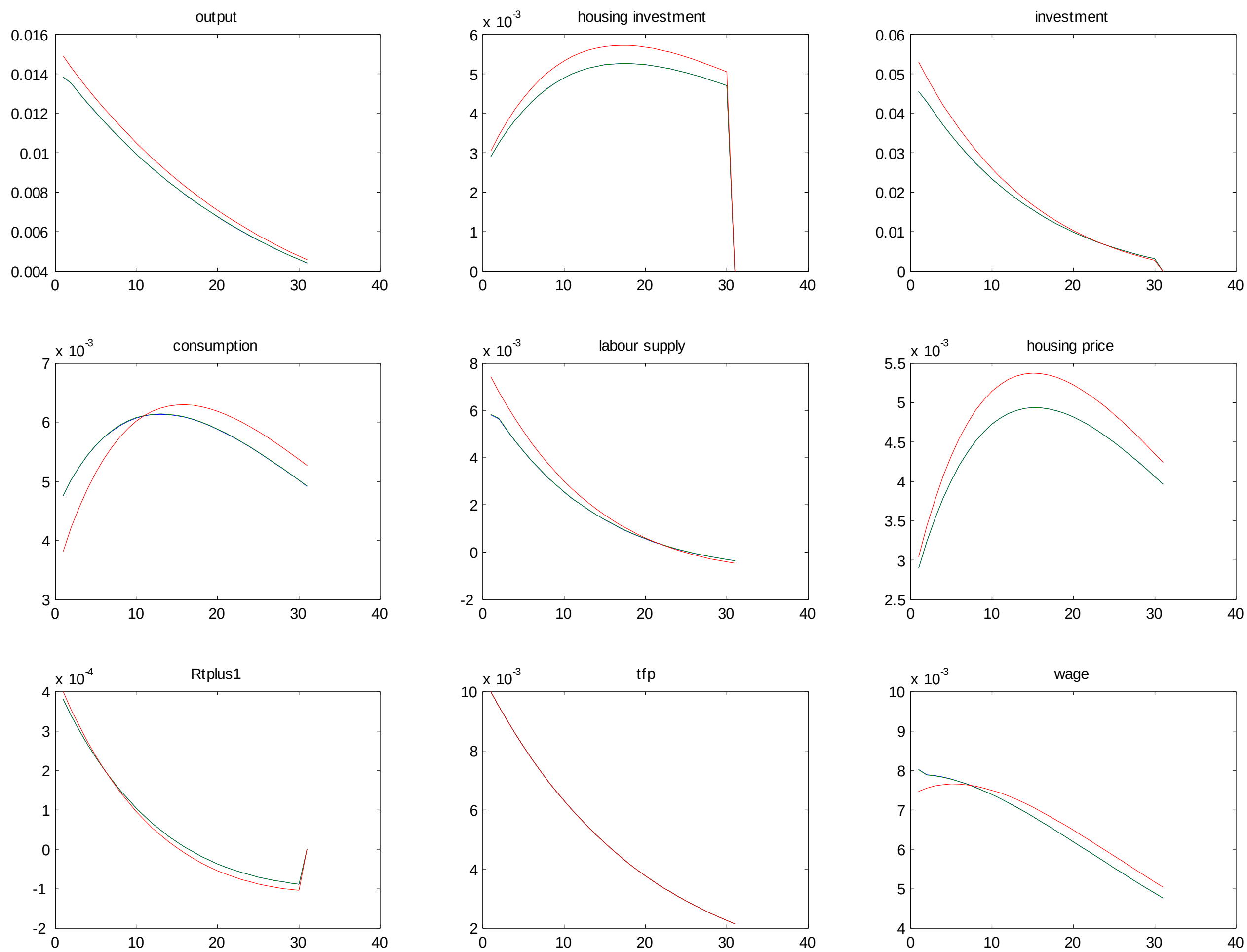

FIG. 3.1 – 1% positive tfp shock, sw=0, aggregates. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

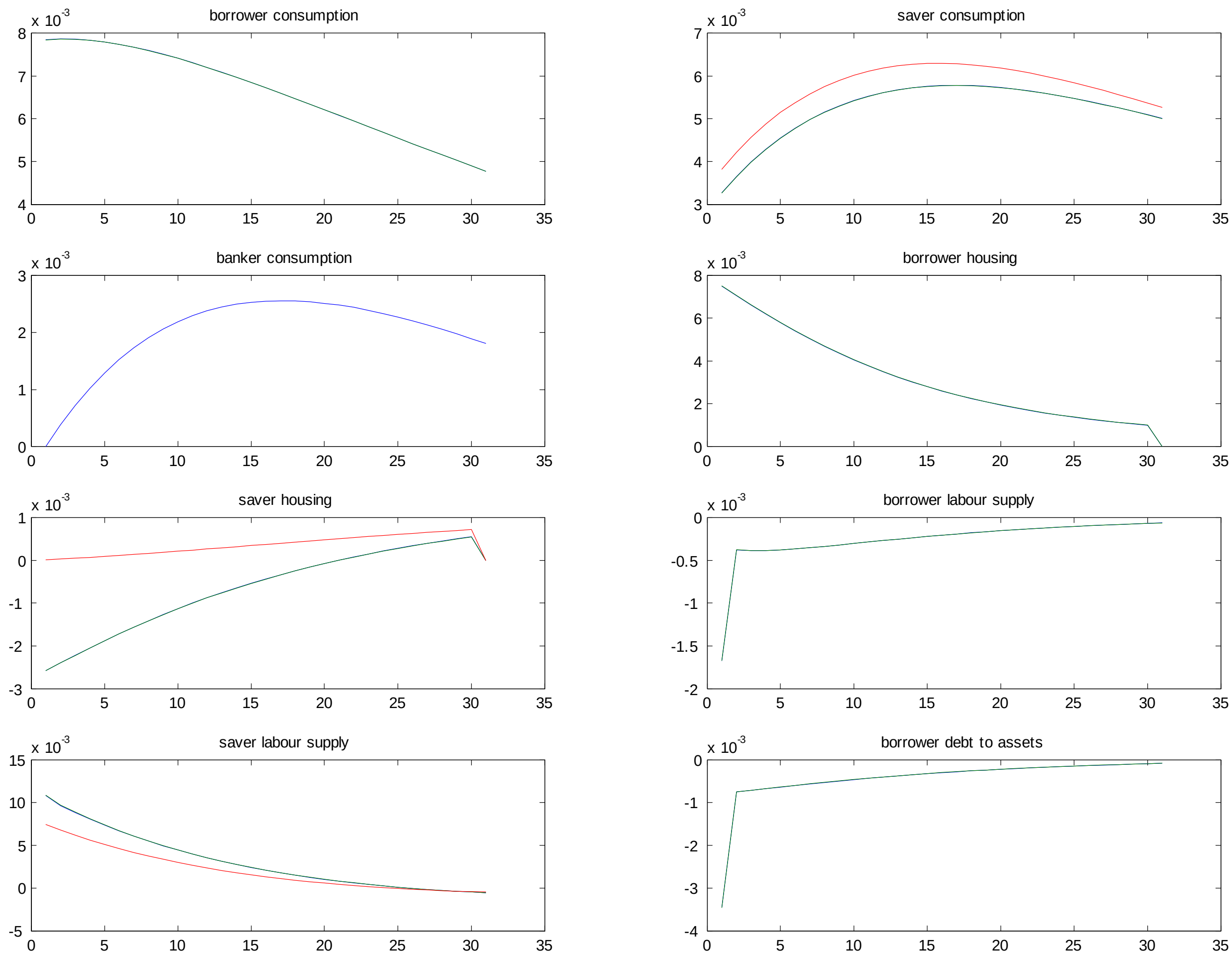

FIG. 3.2 – 1% positive tfp shock, sw=0, borrowers and savers. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

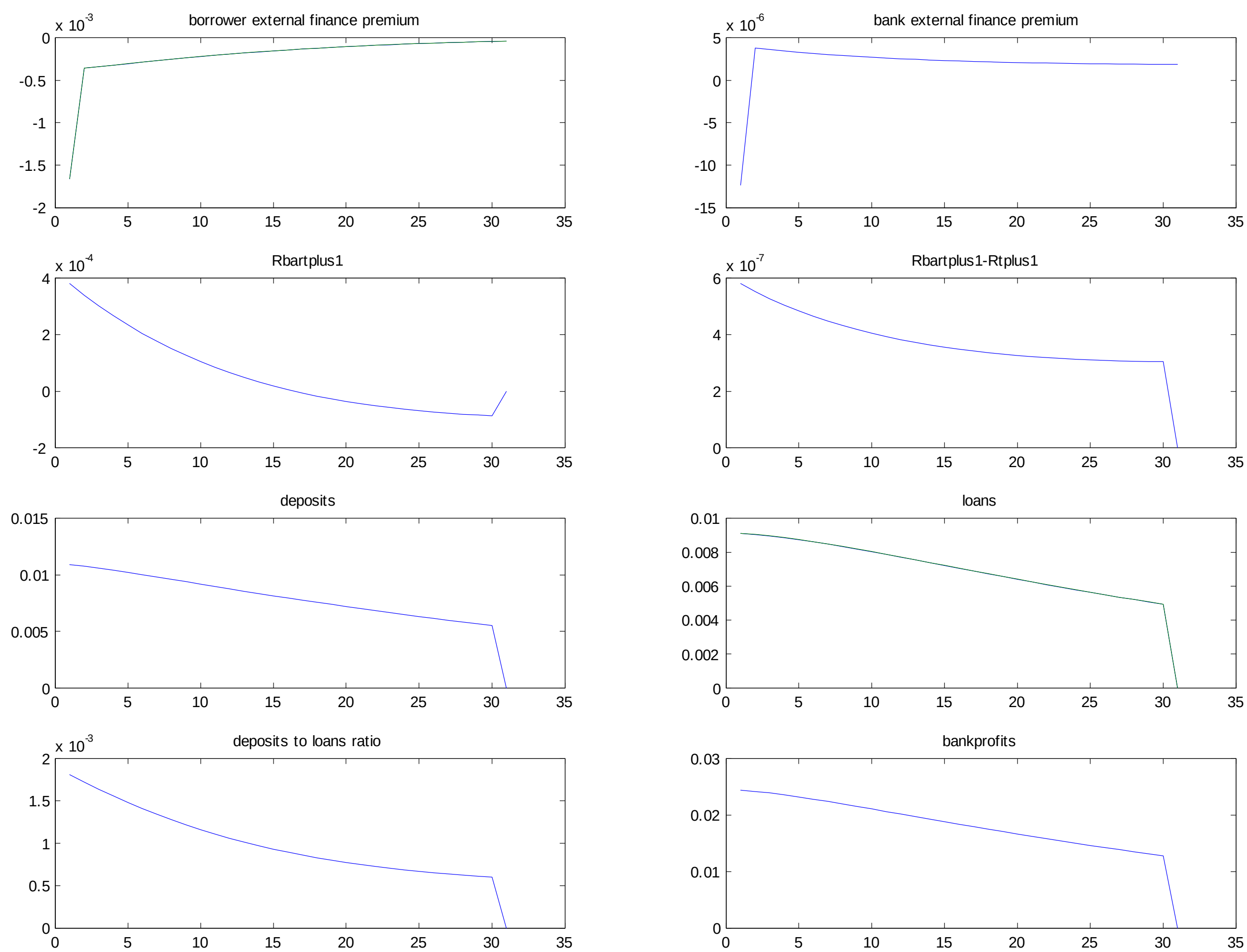

FIG. 3.3 – 1% positive tfp shock, sw=0, bank and other variables. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

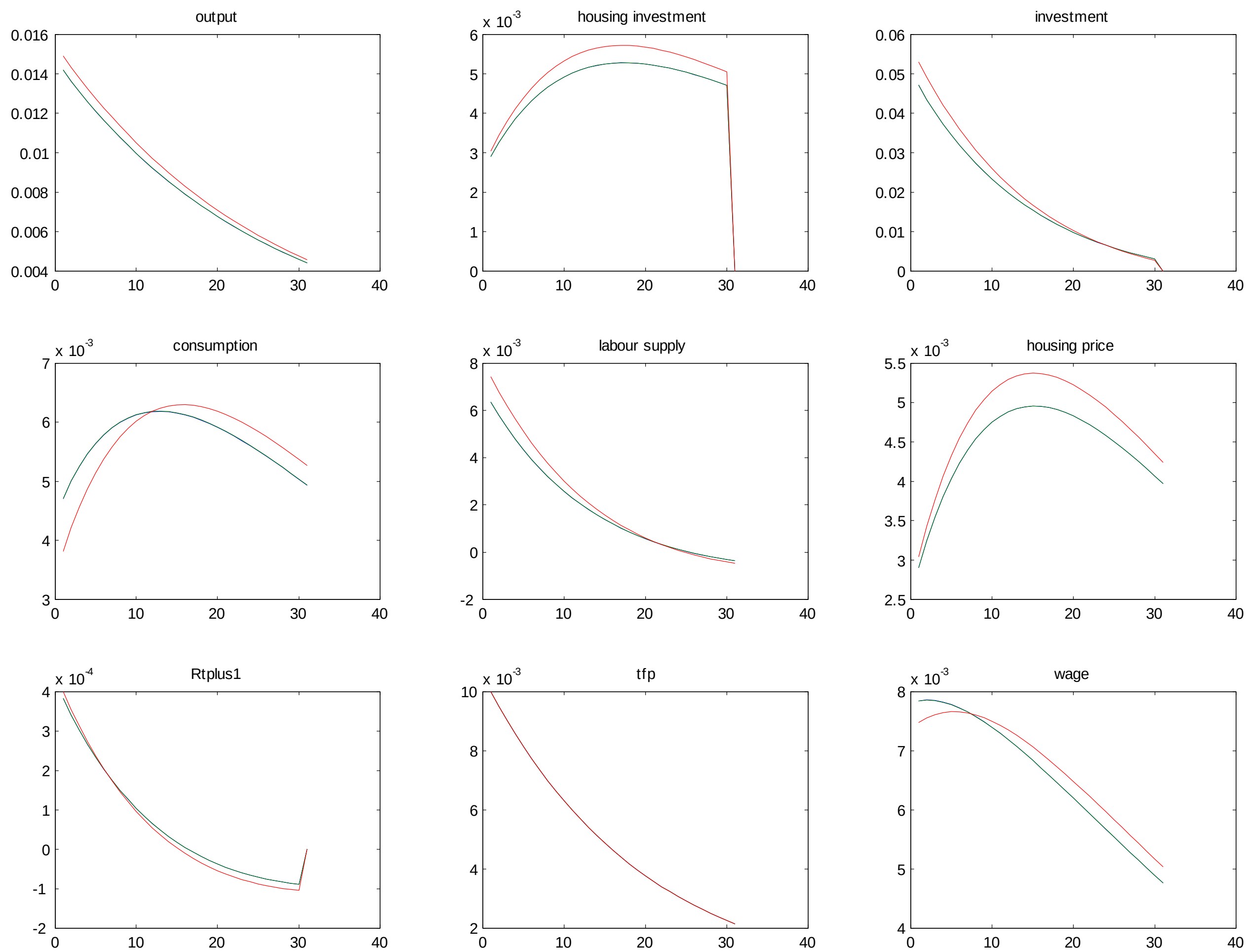

FIG. 3.4 – 1% positive tfp shock, sw=1, aggregates. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

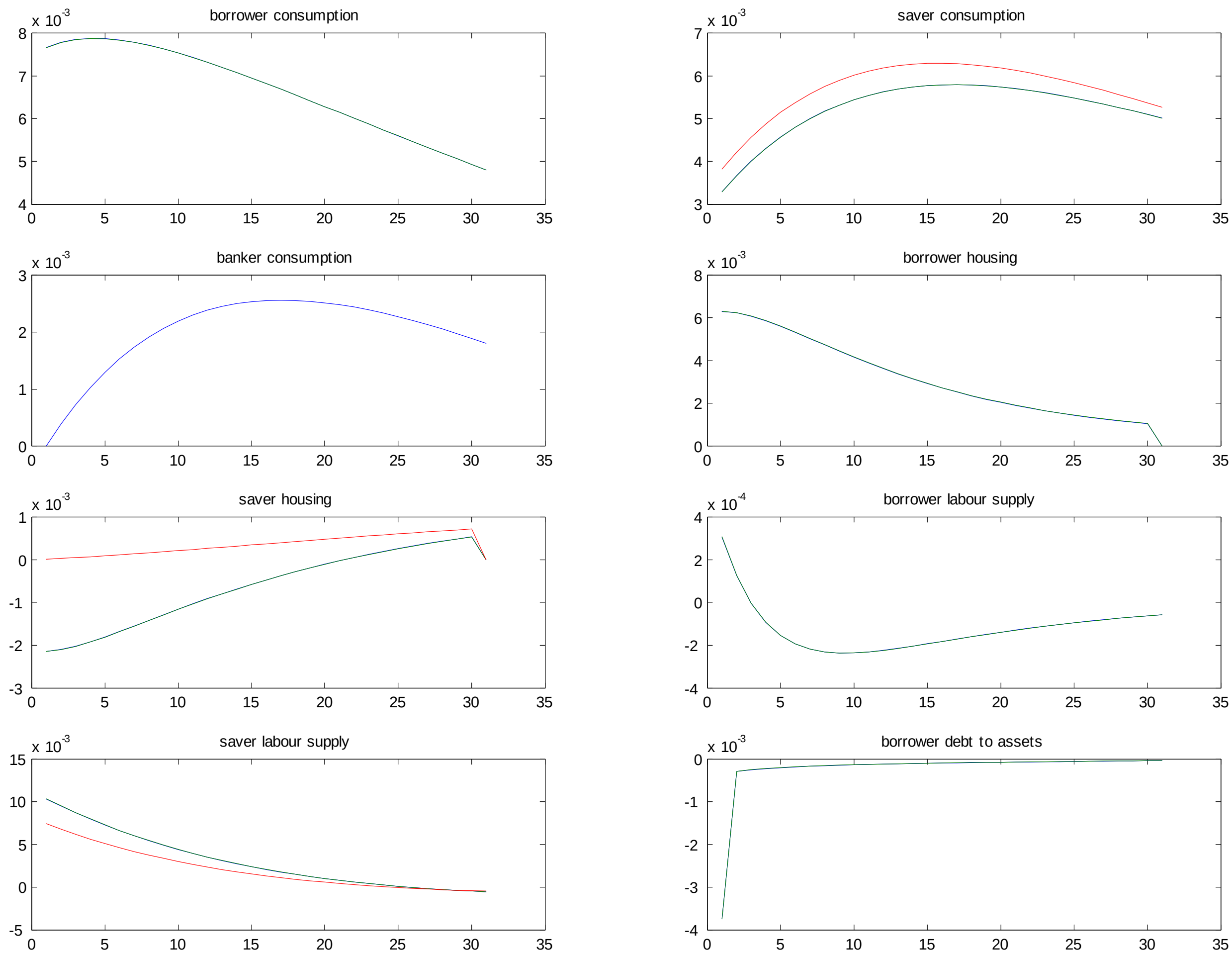

FIG. 3.5 – 1% positive tfp shock, sw=1, borrowers and savers. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

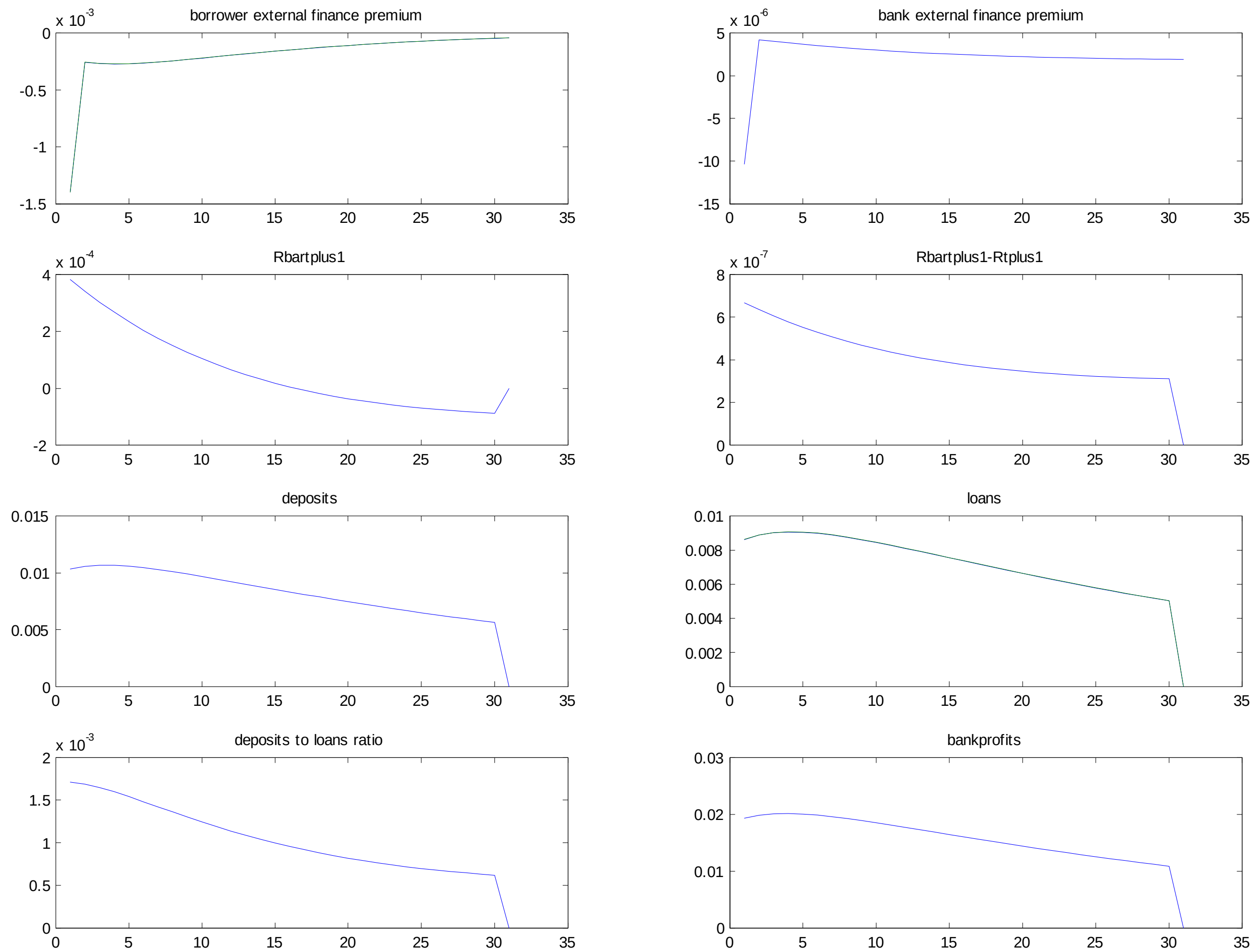

Fig. 3.6 – 1% positive tfp shock, sw=1, bank and other variables. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

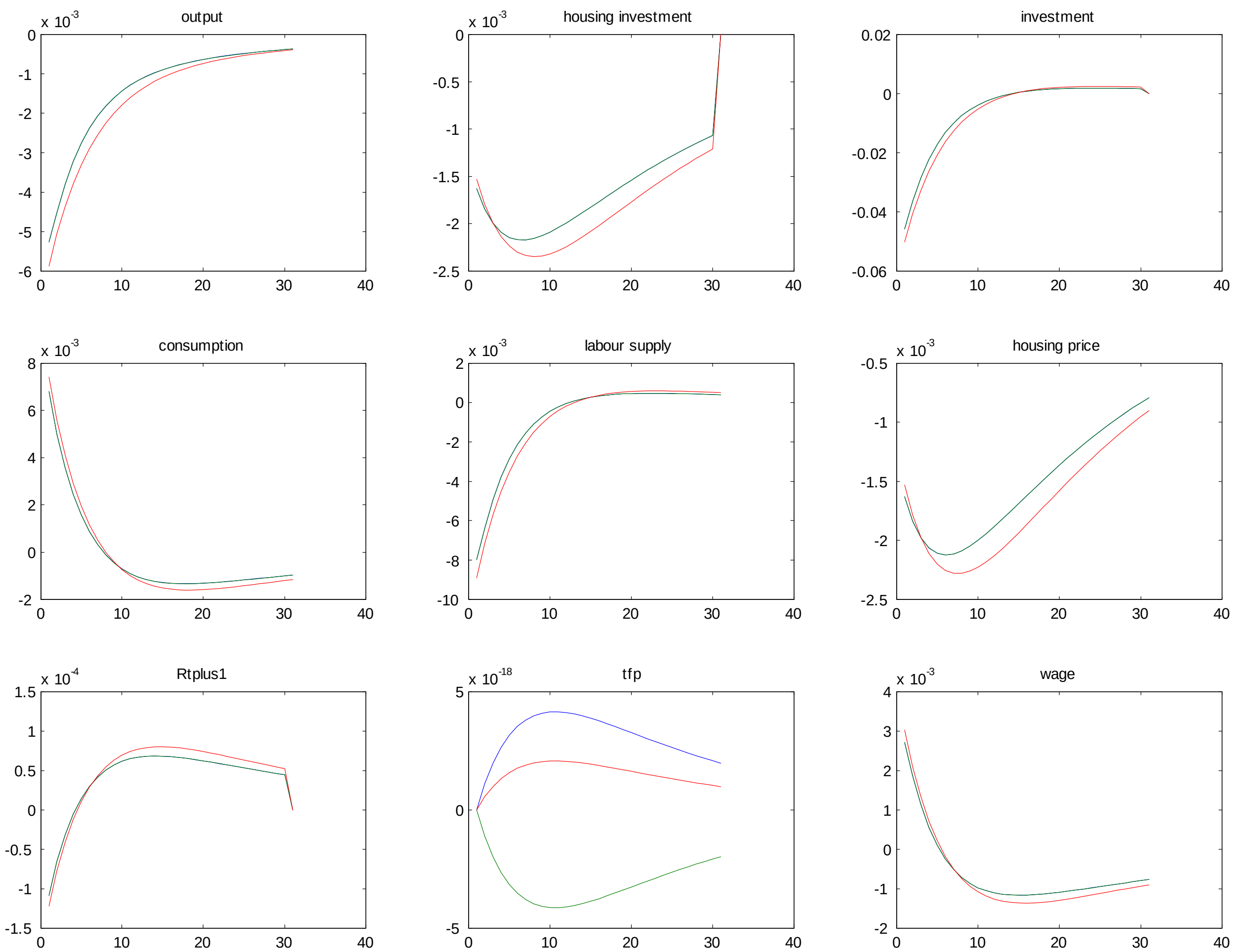

Fig. 3.7 – 1% positive preference shock, sw=0, aggregates. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

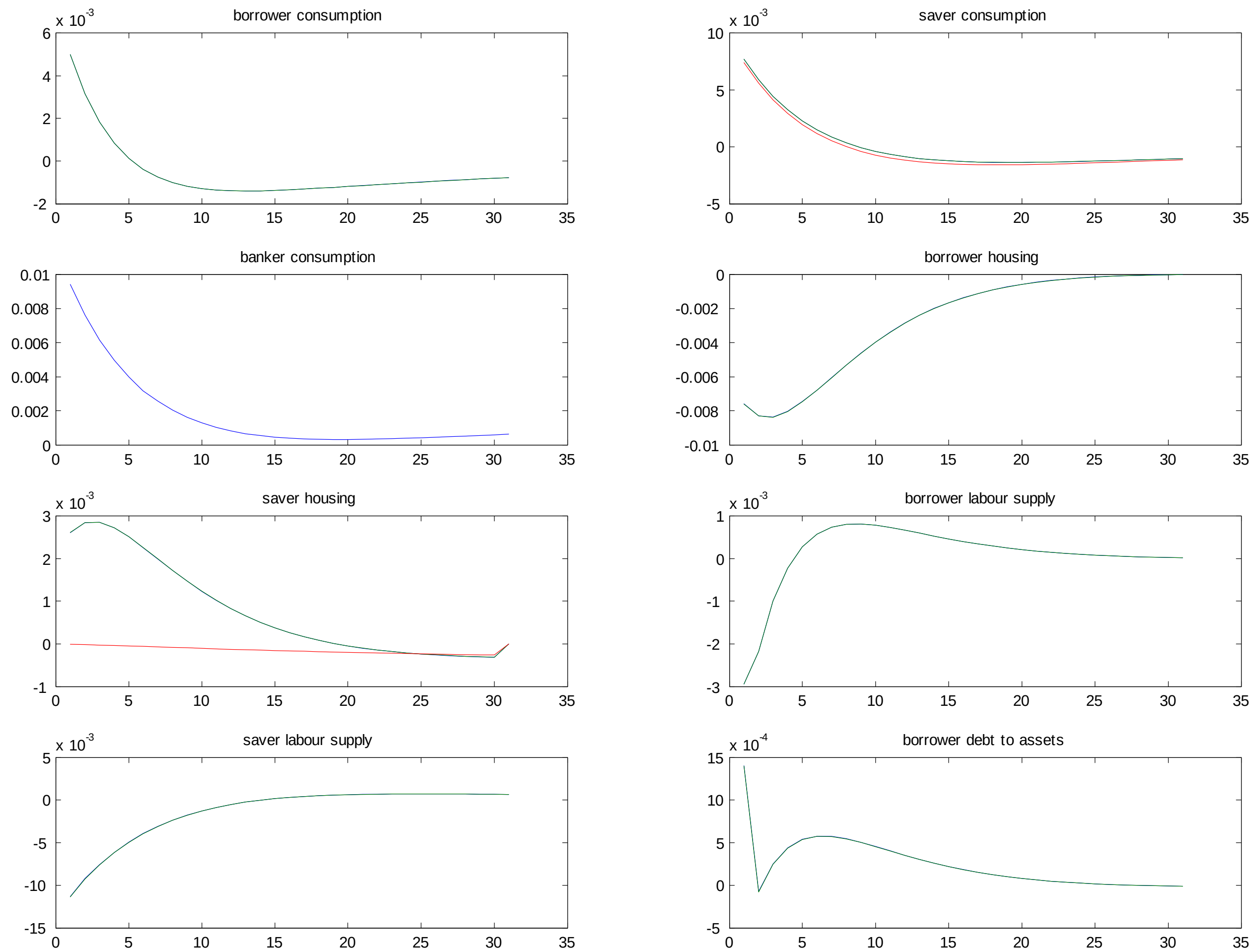

FIG. 3.8 – 1% positive preference shock, sw=0, borrowers and savers. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

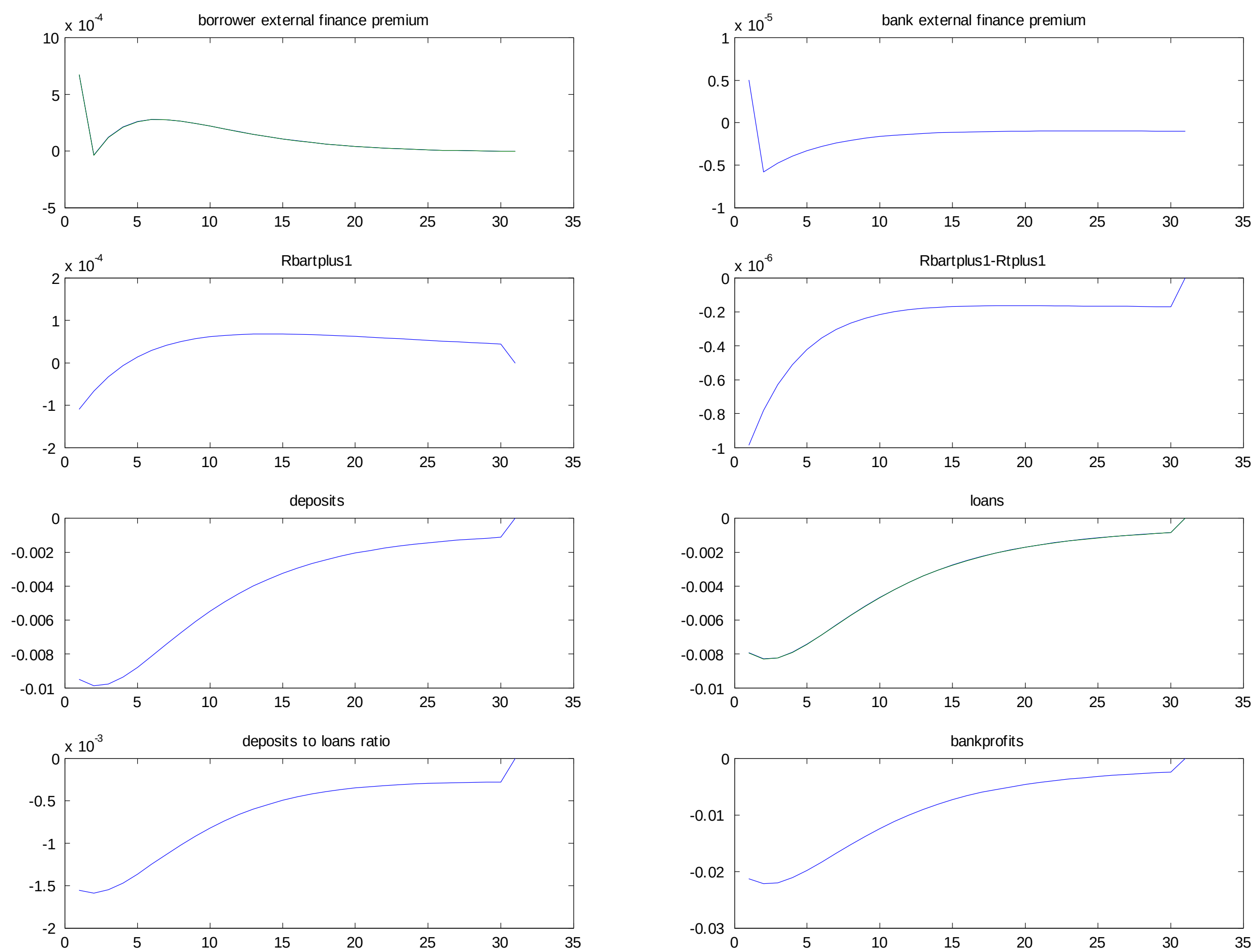

Fig. 3.9 – 1% positive preference shock, sw=0, bank and other variables. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

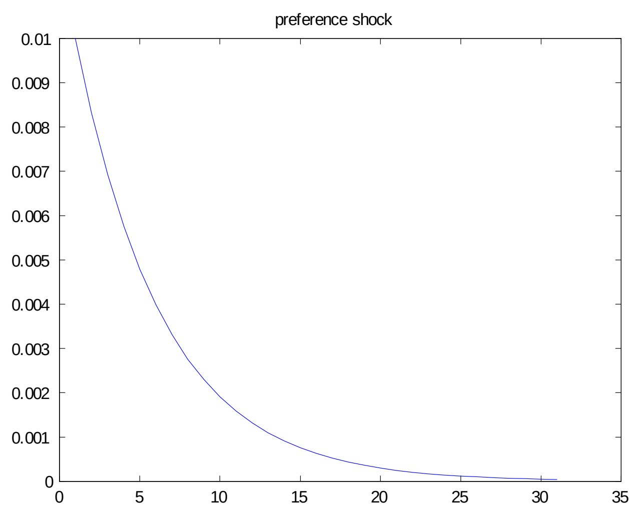

Fig. 3.10 – 1% positive preference shock, sw=0. Red is RBC model with no financial frictions, green is model with borrowing frictions but no banking frictions, blue is full model with banking frictions.

Conclusion

Dans cette thèse j'ai examiné plusieurs aspects de l'influence du défaut sur les contrats dans la macroéconomie. Dans le premier chapitre j'ai étudié une extension du modèle de dette avec défaut de Townsend (1979) [114] qui permet aux entreprises de choisir entre un prêt bancaire et un prêt du marché financier (un bon à court terme ou du papier commercial). Ce modèle a mis l'emphase sur l'avantage comparatif des banques dans la surveillance des compagnies en détresse pour reproduire le lien négatif entre la taille de l'entreprise (mesurée par la valeur nette de l'entreprise) et la dépendance sur les prêts bancaires. L'idée essentielle est que les entreprises plus petites ont une plus grande probabilité de faire défaut sur leur dette. Dans ce cas, ils accordent plus de valeur aux coûts de gestion de détresse plus faibles de la banque, même si ce bénéfice coûte plus cher ex-ante pour l'administration du prêt. Par contre, il est difficile d'expliquer la prépondérance si forte des prêts bancaires en Europe juste avec ce facteur. Pour cela, il faut tenir compte aussi des coûts fixes plus élevés de l'émission de bons en Europe, au moins jusqu'a l'établissement de l'Euro. Dans la mesure où l'arrivée de l'Euro a facilité une intégration et une plus grande efficacité des marchés de dette en Europe, il est possible que cette discrepance entre l'utilisation de la dette de marché entre l'Europe et les Etats Unis va se réduire significativement. Un autre résultat intéressant est qu'une baisse globale du coût de gestion de détresse pour tous les types de prêts réduit la proportion de prêts bancaire dans le modèle. Ceci fournit une explication possible pour le recours plus grand par les entreprises aux marchés de dette dans les 30 dernières années, dans la mesure où la technologie financière s'est améliorée.

Le deuxième chapitre examine la possibilité d'interactions entre les niveau d'endettement et les couts de financement externe des entreprises et des ménages en équilibre général. Dans un modèle avec prix et salaires flexibles, je trouve que l'interaction est négative, même en tenant compte des difficultés d'ajuster la production entre les secteurs de biens de consommation et d'investissement. Dans une récession, le coût plus grand de financement externes des ménages endettées tend à augmenter l'offre de travail et à réduire la demande de prêts des ménages. Ces deux effets réduisent indirectement le coût de financement externe des entreprises. J'ai discuté plusieurs extensions qui pourraient changer les résultats en donnant des conclusions plus Keynésiennes. Premièrement, on pourrait regarder un modèle plus détaillé des coûts d'ajustement entre les secteurs de consommation et d'investissement. Deuxiément, un modèle avec un niveau d'hétérogénéité plus réaliste dans la distribution de richesse pourrait changer les résultats. Troisièment, j'ai omis le rôle des immeubles commerciaux dans la production. Dans la mesure ou il y à un lien rapproché entre le prix de la propriété commerciale et résidentielle, et dans la mesure où on réussit à produire des fluctuations plus grandes dans la demande d'immeubles résidentiels à travers les contraintes de crédit sur les ménages, ceci donnerait un autre mécanisme d'interaction entre les entreprises et les ménages. L'autre contribution de ce chapitre est de fournir un modèle alternatif des contraintes financières qui tient compte des primes de défaut entre les taux d'intérêt sans risque et les taux d'interêt aux entreprises et aux ménages tout en permettant l'aversion au risque des emprunteurs, contrairement aux modèle plus fréquemment utilisé de Kiyotaki et Moore (1997) [83] et de Bernanke et al. (1999) [19] .

Le dernier chapitre construit un modèle permettant l'interaction entre la probabilité de faillites des ménages et des banques. Contrairement aux modèles standard de frictions financières en équilibre général (par exemple Bernanke et al. (1999) [19], les intermédiaires financiers dans notre modèle ne peuvent pas complètement diversifier le risque lié a leur prêts. Ceci les expose à un risque de défaut financier quand le rendement de leurs prêts est plus bas. Ce type de modèle peut expliquer la procyclicalité des profits et des ratios de dette des banques, mais tout au moins pour des chocs de productivité agrégés l'effet des frictions bancaires sur la production, la consommation et l'investissement est limité. Plusieurs éléments omis du modèle pourrait changer ce résultat. Premièrement, le modèle suppose une information parfaite sur le niveau de risque et la probabilité de défaut des prêts. Deuxiement, nous avons supposé que les taux d'emprunt pouvaient être conditionnés parfaitement sur l'état aggregé de l'économie, ce qui est rarement le cas. Finalement, nous avons ignoré l'interaction possible avec le risque de défaut des entreprises. Ces éléments pourraient servir de base pour des extensions importantes du modèle de ce chapitre.

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Annexes