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**The transformation problem : an alternative solution  
with an identical aggregate profit rate in the labor value space  
and the monetary space**

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## RÉSUMÉ

Le but de cet article est de démontrer que, même si la solution de Marx au problème de la transformation peut être modifiée, ses conclusions restent valables. La nouvelle solution qui est proposée est fondée sur la contrainte d'un taux de profit moyen commun aux deux espaces de valeur et un taux de salaire nominal qui est déterminé simultanément avec les prix. Notre solution diverge de celle de Duménil-Foley-Lipietz quant à l'hypothèse d'un taux de salaire nominal supposé ici endogène et déterminé par la concurrence sur le marché du travail. On ne suppose plus que le salaire est fixé par la valeur d'un panier de subsistance. Comme on peut le constater, cette solution diffère aussi nettement de celle de Morishima et des néo-ricardiens. Notre solution est une alternative à celle de Marx, car elle transforme tous les coûts et maintient les deux contraintes macro et un taux général de profit commun aux deux espaces de valeur.

**Mots clés:** transformation, valeur, prix, plus value, profit, salaire, capital, travail

**Classification JEL:** B-14, B-24, D-33, D-46, D-57, E-11, P-16.

## Abstract

The aim of this paper is to demonstrate that, even if Marx's solution to the transformation problem can be modified, his basic conclusions remain valid. The proposed alternative solution which is presented here is based on the constraint of a common general profit rate in both spaces and a money wage level which will be determined simultaneously with prices. This solution diverges from the Duménil-Foley-Lipietz solution on the assumption of the money wage rate which is assumed here as an endogenous variable. The money wage level is determined by competition on the labor market. It is no more assumed to be fixed by the value of a subsistence basket. This is also quite different from the Morishima or the neo-ricardian solution. This solution is an alternative to the Marx solution because it fully transforms the cost of production and maintains the two macro constraints and a general profit rate between the monetary and the social spaces.

**Key words:** Transformation, value, price, surplus value, profit, wage, capital, labor.

**JEL classification:** B-14, B-24, D-33, D-46, D-57, E-11, P-16.

## Introduction

The debate around the transformation problem is more than a century old and it continues to fuel passionate discussions among marxian and neo-ricardian as well as mainstream economists'. The latter, like Samuelson (1971), view it as a good illustration of the redundancy of the old ricardian and marxian labor value theory while radical economists, who still believe in the validity of an alternative approach to marginal utility theory, periodically attempt to give an adequate answer to valid objections formulated against Marx's way of establishing the correspondence between labor values and prices. As was pointed out accurately by L. Gill (1996, p.532) in quoting M. Desal ... "the debate does not seem to end and is viewed as a rare example of a problem which continues to produce new solutions or reformulation of old solutions in a new mathematical language.... [perhaps] there is more than a simple technical question."

Marx's solution to the transformation problem<sup>1</sup> is based on the hypothesis of the non-transformation of the cost of production (constant and variable costs). It is a simplifying assumption which allows him to concentrate on the distribution of the surplus value and the profit rate in the labor value space and the monetary space. His main interest was to show that, although the profit rate can be different from one sector to another in the value space, the monetary profit rate in a competitive world must be the same in all sectors. The reason why the profit rate varies from one sector to another in the value space is caused by the differences in organic composition of capital due to unequal development between industries. It is also because some sectors do not produce any surplus value, such as the circulation sphere, in particular, the financial sector. Marx was well aware that the non transformation of the production costs was a simplifying assumption and did not consider that his conclusion would be substantially changed if that assumption were modified<sup>2</sup>.

<sup>1</sup> For a review of this debate, see in particular Dostaler (1978), Beaud and Dostaler (1993), Estwell, Milgate and Newman (1987-90), Gill (1996), Labman (1973), Mandel and Freeman (1984), Samuelson (1971), Steedman (1981), Sweezy (1949). The authors who have contributed with new solutions to the debate are Dumentil (1980,1983), Foley (1982,1986), Lipietz (1982,1983), Morishima and Seton (1981), Morishima (1973), Naples (1989), Salton (1957), Shaikh (1977), Stanfa (1960).

<sup>2</sup> See in particular Marx (1967) Book III, part 1 and 2.

<sup>3</sup> Marx (1967), in Book III, chapter 11, p. 165, says ... Since the price of production may differ from the value of a commodity, it follows that the cost-price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it.... Our present analysis does not necessitate a closer examination of this point.

dear to Marx and his followers. This implies that, even if the macro constraint of value conservation is preserved, some monetary surplus value would be created or lost during the transformation process, an awkward situation, since the core of the labor theory of exploitation is to explain how the monetary surplus value is created in the value space. Samuelson's sharp criticism of Marx's theory of labor value would still remain valid since he pretends that there is no logical connection between the two spaces: the monetary values would be independent of the labor values and, hence, there is no point in having two value spaces to explain what is going on in the real world.

The aim of this paper is to demonstrate that, even if Marx's solution can be modified, his basic conclusions remain valid. It will be shown in particular by the three following points:

- i) Marx's solution requires the specification of one macro constraint despite that his results satisfy the two macro equalities and a common general profit rate. This conclusion is already well known in the literature, but needs to be restated since it is a very crucial point.
- ii) The Dumenil-Foley-Lipietz solution requires one macro constraint and satisfies the two macro equalities if the value added is chosen, although the general profit rate is different between the two spaces. The result is not valid anymore if the gross value is chosen as the macro constraint. The Morishima solution yields a similar result, even with the value added.
- iii) The new alternative solution which is presented here is based on the constraint of a common general profit rate in both spaces and a money wage level which will be determined simultaneously with prices and will produce results which satisfy the two macro constraints. This solution contradicts the neo-ricardian approach based on a composite commodity as a standard of value, since neo-ricardians argue that there exists a linear relation between profit and wage and, once one of them is specified, the other is automatically determined.

His main conclusion is twofold: on a micro basis, prices differ from values - and this is why a labor value theory is relevant as an explanation- but on a macro basis, there are two macro constraints which ought to be satisfied if the principle of value conservation is to be preserved. The first constraint is that the sum of values must be equal to the sum of prices, i.e. the aggregate gross production must be equal in both monetary and abstract labor spaces. The second constraint is that the sum of profits must be equal to the sum of surplus values. Since the transformation problem is a static analysis similar to a general calculable equilibrium approach<sup>4</sup>, the same amount of value must be preserved in both spaces when one chooses a general equivalent form for measuring prices. There is more than one way of choosing that price standard:

- i) Choose a level of expenditure common to both spaces and one macro constraint. This is the Marx solution with the hypothesis of no-cost transformation.
- ii) Choose a level of wage common to both spaces and one macro constraint. This is the Dumenil-Foley-Lipietz solution (1980, 1982, 1982) of the money wage determined exogenously and fixed to the nominal wage in the value space.
- iii) Choose a level of real wage common to both spaces and one macro constraint. This is the Morishima solution (1973) of money wages determined simultaneously with prices. It is also the neo-ricardian solution based on Sraffa's composite commodity (1960) as a standard of value<sup>5</sup>.

Only the Marx solution preserves the same general profit rate between the two spaces and satisfies the two macro constraints, although only one constraint is effective. The other two solutions generate a monetary profit rate which is not equal to the general profit rate in the abstract space. Worse still, the other two solutions cannot satisfy the two macro constraints so

<sup>4</sup> The dynamic approach put forward by M. Naples (1989) is an interesting approach which could be pursued. After all, even neoclassical economists have developed a dynamic general equilibrium approach; but I still consider that Marx's static equilibrium approach has to be addressed adequately in this debate.  
<sup>5</sup> For a survey of the neo-ricardian debate, see Mandel and Freeman (1984) and Steedman (1981). Also, a good criticism of the neo-ricardian approach is found in Naples (1989) and Shaikh (1982).

**Definition of a two sector economy**

In order to illustrate the various solutions, a two sector economy will be chosen as an example. One assumes the following value table measured into the abstract value space, i.e. one unit is one hour of social or abstract labor<sup>6</sup>.

Table 1

sector	constant capital c	variable capital v	surplus value	pl	total ox
I	3 240	2160	1 080		6 480
II	2 780	1380	2 070		6 210
Total	6 000	3540	3 150		12 880

It is assumed that the turnover of capital is unity for all types of capital, that the exchange rate between one dollar and one unit of social labor is unity, that sector I produces 4000 units of consumption goods and sector II 2000 units of production goods. It is also assumed that output of sector II is chosen as an input for both sectors and that output of sector I is chosen as an input of that sector in the proportion of 1/5. By inspecting the figures contained in the table, the following results can easily be deducted.

Table 2

sector	quantity x	unit value	expropriation rate	organic rate	profit rate	wage rate
I	4 000	1.62	0.5	1.5	0.20	.667
II	2 000	3.105	1.5	2.0	0.50	.40

<sup>6</sup> This example was part of a final exam given to my students in Winter 1986.

<sup>7</sup> Since the turnover of capital is assumed equal to unity for all types of capital,  $r=e/(r+1)$ .

Following Gleicher's idea of labor specialization (Gleicher, 1989), a specific rate of exploitation for each industry is assumed here and, therefore, a specific wage rate in the social labor space. The necessary time to reproduce specialized labor differs from one sector to another. This can also be justified by historical conditions of unequal development of productivity between industrial branches. Moreover, one hour of direct or living labor in each sector can be decomposed into a fraction of necessary time ( $\mu$ ) and a fraction of non necessary time for reproduction of specialized labor ( $1-\mu$ ). Hence, the specific exploitation rate to each sector is  $e=(1-\mu)/\mu$ . Therefore,  $\mu = 1/(1+e)$ .

The real wage for each sector can therefore be calculated by dividing the wage rate by the value of the consumption goods  $\theta_i=1.62$ . Hence, the real wage in the value space is  $\mu_i/\theta_i=.4115$  for sector I and  $\mu_{II}/\theta_{II}=.2469$  for sector II. The value figures contained in table 1 are generated by the following value equations:

$$\theta_1 X_1 = c_1 + v_1 + pl_1$$

$$\theta_2 X_2 = c_2 + v_2 + pl_2$$

The transposition of these value equations into Leontief equations is:

$$\theta_1 X_1 = a_{11}\theta_1 X_1 + a_{21}\theta_2 X_2 + l_1 X_1$$

$$\theta_2 X_2 = a_{12}\theta_1 X_1 + a_{22}\theta_2 X_2 + l_2 X_2$$

The identification of each component is based on the definition of gross value = value of constant capital + value added:

$$c_1 = a_{11}\theta_1 X_1 + a_{21}\theta_2 X_2 = 3240$$

$$c_2 = a_{12}\theta_1 X_1 + a_{22}\theta_2 X_2 = 2760$$

$$v_1 + pl_1 = l_1 X_1 = 3240$$

$$v_2 + pl_2 = l_2 X_2 = 3450.$$

Therefore,

$$l_1 = 3240/4000 = 0.81$$

$$l_2 = 3450/2000 = 1.725$$

In order to calculate the value of the technical coefficient  $a_{ij}$ , one has to remember the a priori hypothesis, i.e. the quantity of input of sector  $i$  which enters the production of one unit of output of sector  $j$  is  $1/5$ . Hence,  $a_{11} = 0.20$ . On the other hand, since the input of sector  $i$  does not enter into the output of sector  $ii$ ,  $a_{22} = 0$ . Hence,

$$a_{21} = [3240 - 20(1.62)4000] / (3.105)4000 = 0.1565$$

$$a_{22} = 2760 / (3.105)2000 = 0.4348$$

In order to calculate and write the monetary values, one has to write the corresponding price equations, based on Marx's assumption of an equal profit rate  $r$  for both sectors.

$$p_1x_1 = (1+r)(.20p_1x_1 + .1565p_2x_1 + .81w_1x_1)$$

$$p_2x_2 = (1+r)(.4348p_1x_2 + 1.725w_2x_2)$$

The analogue of the price equations in marxian components is

$$p_1x_1 = c'_1 + v_1 + \pi_1$$

$$p_2x_2 = c'_2 + v_2 + \pi_2$$

The identification term by term is

$$c'_1 = .20p_1x_1 + .1565p_2x_1$$

$$c'_2 = .4348p_1x_2$$

$$v_1 = .81w_1x_1$$

$$v_2 = 1.725w_2x_2$$

$$\pi_1 = r(.20p_1x_1 + .1565p_2x_1 + .81w_1x_1)$$

$$\pi_2 = r(.4348p_1x_2 + 1.725w_2x_2)$$

### Marx's solution

Marx's solution is based on the hypothesis of the non transformation of costs and on the macro constraint that the gross value in the social space must equal the gross value in the monetary space. Translating these hypotheses in the mathematical model, the implication is:

$$c_1 = c'_1 = 3240, \quad v_1 = v'_1 = 2160$$

$$c_2 = c'_2 = 2760, \quad v_2 = v'_2 = 1380$$

$$p_1x_1 + p_2x_2 = \theta_1x_1 + \theta_2x_2 = 12690$$

It is immediately observed that Marx's second macro constraint, -the sum of profits is equal to the sum of surplus values- is already implied by the previous constraints. Indeed,

$$c_1 + v_1 + \pi_1 + c_2 + v_2 + \pi_2 = c_1 + v_1 + p_1 + c_2 + v_2 + p_2$$

Hence,

$$\pi_1 + \pi_2 = p_1 + p_2 = 3150$$

Therefore, Marx's solution does not imply two independent macro constraints, but only one. In that respect, his solution is not different from the other "algebraic" solutions proposed by neo-ricardians, Morishima or by Dumenil-Foley-Lipietz.

What is interesting, however, is that Marx's particular choice of assumption, -the non-transformation of costs- yields two remarkable results:

- i) the sum of profits equals the sum of surplus values;
- ii) the general or average rate of profit is the same in both spaces, although it is different on a sectoral basis in the social labor space.

This last result is easily deduced from the definition of the general profit rate in both spaces:

$$\begin{array}{ll} \text{value space} & \text{monetary space} \\ r = \Sigma p_i / \Sigma(c_i + v_i) & r' = \Sigma \pi_i / \Sigma(c'_i + v'_i) \end{array}$$

Since, by hypothesis,  $\Sigma(c_i + v_i) = \Sigma(c'_i + v'_i)$  and, by deduction,  $\Sigma p_i = \Sigma \pi_i$ , it follows that  $r = r'$ .

The monetary value table associated to Marx's solution is easily calculated after solving a system of three simultaneous equations:

$$\begin{aligned} (1) \quad p_1 X_1 &= (1+r)(c_1 + v_1) = (1+r)5400 \\ (2) \quad p_2 X_2 &= (1+r)(c_2 + v_2) = (1+r)4140 \\ (3) \quad p_1 X_1 + p_2 X_2 &= 12690 \end{aligned}$$

Replacing the quantities by their respective number and solving,

$$\begin{aligned} (1+r) &= 6.345/4.77 = 1.3302 \\ p_1 &= 1.35(1.3302) = 1.7958 \\ p_2 &= 2.07(1.3302) = 2.7535 \end{aligned}$$

Using these results, the table of monetary values can be constructed\*

$$\begin{aligned} p_1 X_1 &= 1.7958(4000) = 7183 \\ p_2 X_2 &= 2.7535(2000) = 5507 \\ p_1 X_1 + p_2 X_2 &= 12690 \\ \pi_1 &= p_1 X_1 - (c_1 + v_1) = 7183 - 5400 = 1783 \\ \pi_2 &= p_2 X_2 - (c_2 + v_2) = 5507 - 4140 = 1367 \\ \pi_1 + \pi_2 &= 3150 \end{aligned}$$

Table 3 (monetary values)

sector	constant capital c'	variable capital v'	profit $\pi$	total px
I	3 240	2 160	1 783	7 183
II	2 760	1 380	1 367	5 507
Total	6 000	3 540	3 150	12 690

### The other algebraic solutions

The rejection of the non transformation of costs assumption was judged by some marxists as a betrayal of Marx's insistence on the principle of conservation of value and that no monetary surplus value can be created or destroyed in the transformation process as long as it is analysed in a static framework. The demonstration that the simultaneous solution put forward by the other algebraic transformers would require only one macro constraint did not please all Marx's disciples since it lead to the rejection of one important constraint. For instance, if the Dumenil-Foley-Lipietz solution is calculated by standardizing with respect to the gross value instead of the value added, the sum of profits is no more equal to the sum of surplus values and their solution is not qualitatively different from Morishima's solution or from other neo-ricardian solutions. We can easily show that by choosing the value added constraint, the non-transformation of one component will impose equality on the other component between the two spaces. Indeed, the definition of value added is

$$\begin{array}{ll} \text{abstract space} & \text{monetary space} \\ va = v_1 + p l_1 + v_2 + p l_2 & va = v'_1 + \pi_1 + v'_2 + \pi_2 \end{array}$$

The non transformation of variable capital implies  $v_1 = v'_1$  and  $v_2 = v'_2$ . Hence,  $p l_1 + p l_2 = \pi_1 + \pi_2$ . In the Leontief form, these macro constraints are written as

$$va = l_1 X_1 + l_2 X_2 = w_1 l_1 X_1 + \pi_1 + w_2 l_2 X_2 + \pi_2$$

Hence,

$$l_1 X_1 - w_1 l_1 X_1 + l_2 X_2 - w_2 l_2 X_2 = \pi_1 + \pi_2$$

Since  $\mu_i = w_i$  by assumption,

$$(1-\mu_1)x_1 + (1-\mu_2)x_2 = \pi_1 + \pi_2$$

or

$$p_1 + p_2 = \pi_1 + \pi_2$$

Therefore, the assumption of equality of value added between the two spaces is identical to the assumption of equality between the sum of surplus values and the sum of profits. It follows that if the gross value is chosen as the macro constraint between the two spaces, the sum of surplus values is not equal anymore to the sum of profits. Indeed,

$$\begin{array}{ll} \text{abstract space} & \text{monetary space} \\ gv = c_1 + v_1 + p_1 + c_2 + v_2 + p_2 & gv = c_1 + v_1 + \pi_1 + c_2 + v_2 + \pi_2 \end{array}$$

Assuming that  $v_1 = V_1$  and  $v_2 = V_2$ , the macro constraint is reduced to

$$c_1 + c_2 + p_1 + p_2 = c_1 + c_2 + \pi_1 + \pi_2$$

Unless  $(c_1 + c_2)$  is assumed equal to  $(c_1' + c_2')$ ,  $(p_1 + p_2)$  will be different from  $(\pi_1 + \pi_2)$ . It necessarily follows that the measurement of the average monetary profit rate will diverge from the general profit rate calculated in the social space, because both the numerator and the denominator of the money profit rate will be different from those already established in the social space. But what would be the effect of imposing the same general profit rate on the now familiar algebraic solutions? This will be the topic of the next section.

#### The general profit rate constraint

Let us first determine the Dumenil-Foley-Lipietz solution with the macro constraint of equalizing the gross value between the two spaces. The price equations are

$$\begin{aligned} p_1 &= (1+r)(.20p_1 + .1565p_2 + .81w_1) \\ p_2 &= (1+r)(.4348p_2 + 1.725w_2) \end{aligned}$$

Since it is assumed that wages are exogenously determined and fixed at the same level as in the abstract space,  $w_1 = .667$  and  $w_2 = .40$ . Hence,

$$\begin{aligned} p_1 &= (1+r)(.20p_1 + .1565p_2 + .54) \\ p_2 &= (1+r)(.4348p_2 + .69) \end{aligned}$$

The macro constraint of gross value is

$$p_1(4000) + p_2(2000) = 12690$$

or

$$2p_1 + p_2 = 6.345$$

The solution of this non-homogeneous system of three equations is

$$p_1 = 1.89 \quad p_2 = 2.57 \quad r = .426^{10}$$

It can be checked that the sum of profits diverges from the sum of surplus values<sup>11</sup>.

The introduction of a general profit constraint in both spaces -by imposing  $r = .3302$ - would reduce the system to two equations only. The macro constraint would become redundant!

$$\begin{aligned} p_1 &= 1.3302(.20p_1 + .1565p_2 + .54) \\ p_2 &= 1.3302(.4348p_2 + .69) \end{aligned}$$

The solution to this system is

$$p_1 = 1.596 \quad p_2 = 2.177$$

<sup>9</sup> A quick check of this is  $v_1 = .54(4000) = 2160$  and  $v_2 = .69(2000) = 1380$ .

<sup>10</sup> It should be noted by the way that these results are precise only with two decimal figures because of rounding off errors.

<sup>11</sup> From the price equations we calculate  $\pi_1 = .426(3120 + 2160) = 2250$  and  $\pi_2 = .426(2235 + 1380) = 1540$ , hence,  $\pi_1 + \pi_2 = 3790$  which is different from  $p_1 + p_2 = 3150$ .



It can be checked that this system does not satisfy any macro constraint<sup>12</sup>. In order to make effective the two macro constraints, the assumption about exogeneous wages has to be modified. Wages will be assumed to be determined in a competitive world as prices and again, the theory of labor value will be relevant in explaining the divergence between the money sphere and the social sphere. Therefore, in the monetary space, the price of both factor services (capital and labor) will be common to both sectors while they are different in the social space.

One now needs to specify a non homogeneous system of three equations. The first two equations will be the price equations and the last will be the equality of the gross value between the two spaces. We do not need to specify the other macro constraint because it is already implicitly contained in the equality of the general profit rate between the two spaces. Indeed,

$$\begin{aligned} \text{abstract space} \quad \text{monetary space} \\ r = \Sigma p_i / \Sigma (\theta_i X_i - p_i) &= r' = \Sigma \pi_i / \Sigma (p_i X_i - \pi_i) \\ 1/r = \Sigma \theta_i X_i / \Sigma p_i &= 1/r' = \Sigma p_i X_i / \Sigma \pi_i \end{aligned}$$

If, by hypothesis,  $\Sigma \theta_i X_i = \Sigma p_i X_i$ , then  $\Sigma p_i = \Sigma \pi_i$ . Therefore, the general profit rate constraint, combined with one of the two macro constraints, gives an analogous result to Marx's no cost transformation assumption. Moreover, it is seen that a unique solution requires the same monetary wage level for all sectors, although in the abstract space, there are different wage levels, as it is the case for the profit rate. It is as if in the monetary space there is a unique exploitation rate for all sectors.

Applying this new set of constraints to the chosen example, there is a set of three simultaneous equations to be solved:

$$\begin{aligned} (1) \quad p_1 &= 1.3302(20p_1 + .1565p_2 + .81w) \\ (2) \quad p_2 &= 1.3302(4348p_1 + 1.725w) \end{aligned}$$

<sup>12</sup> For instance, the gross value is  $1.596(4000) + 2.177(2000) = 10738$  which is far below 12690. Similarly for profits:  $\pi_1 = .3302(2640 + 2160) = 1555$  and  $\pi_2 = .3302(1893 + 1380) = 1051$ . The sum is 2606 which is again far below 3150.

$$(3) \quad 2p_1 + p_2 = 6.345$$

Rearranging,

$$\begin{aligned} (1) \quad .7340p_1 - .2082p_2 - 1.0775w &= 0 \\ (2) \quad .4216p_2 - 2.2946w &= 0 \\ (3) \quad 2p_1 + p_2 &= 6.345 \end{aligned}$$

The solution of this non-homogeneous system is<sup>13</sup>

$$p_1 = 1.667 \quad p_2 = 3.012 \quad w = .553$$

Let us now construct the monetary value table and test for the two macro constraints<sup>14</sup>.

Table 4 (monetary values)

sector	constant capital c'	variable capital v	profit $\pi$	total px
I	3 219	1 703	1 655	6 687
II	2 619	1 809	1 465	6 023
Total	5 838	3 702	3 150	12 690

It is verified that the sum of prices is equal to the sum of social values 12690 and that the sum of profits is equal to the sum of surplus values 3150. The general profit rate is the same in both spaces and equal to .3302. Money prices and wages differ from their values on a sectorial basis:

<sup>13</sup> From (2):  $p_2 = 5.4426w$ . Substituting this result into (3):  $p_1 = 3.1725 - 2.7213w$ . Substituting these two results into (1):  $2.3286 = 1.9974w + 1.1331w + 1.0775w$ . Hence,  $w = 2.3286 / 4.2081 = .5534$ .

<sup>14</sup> Here is the detailed calculus for the various entries of the table.

$$c'_1 = [20(1.667) + .1565(3.012)] / 2000 = 2619$$

$$v'_1 = .81(5534) / 4000 = 1703$$

$$v'_2 = 1.725(5534) / 2000 = 1909$$

$$\pi_1 = .3302(3219 + 1703) = 1655$$

$$\pi_2 = .3302(2619 + 1809) = 1465$$

abstract value	monetary value
$\theta_1 = 1.620$	$P_1 = 1.667$
$\theta_2 = 3.105$	$P_2 = 3.012$
$\mu_1 = .567$	$w_1 = .553$
$\mu_2 = .40$	$w_2 = .553$

#### Conclusion

The proposed new solution diverges from the Duménil-Foley-Lipietz solution on the hypothesis of the money wage rate which is assumed here as an endogenous variable. The money wage level is determined by competition on the labor market. It is no more assumed to be fixed by the value of a subsistence basket of goods, although it may be so in the social labor space. Moreover, the profit rate is predetermined in the social sector and, hence, is exogenous to the monetary space.

This solution also diverges from Morishima's solution (or neo-ricardian solutions) in other aspects.

- i) Morishima's constraint on the real wage is replaced by an exogenous profit rate equal to the profit rate already determined in the social sector. This result is fully compatible with the Frobenius-Perron theorem which states that there is a unique profit rate associated with the dominant characteristic root of a matrix formed by all the technical coefficients of the system<sup>15</sup>.

<sup>15</sup> This statement may appear ambiguous since the Frobenius root comes from a matrix usually constructed in the monetary space with a unique profit rate for all sectors. It can be shown that it is possible to construct a different matrix in the labor space which would include an exploitation rate in addition to the other technical coefficients.

ii) The money wage level, as well as the money profit rate, is unique for all sectors under the competitive hypothesis, although in the abstract space, these rates may be different between the sectors. Moreover, the money wage level is compatible with a flexible demand of consumption goods, and is not necessarily related to a real wage determined at the subsistence level by a basket of goods. The general money wage level could fluctuate around the wage rates determined in the social sector. There is no linear connection between the profit rate and the wage rate in the monetary space. The nominal wage rate is determined simultaneously with prices in the monetary space.

This solution is an alternative to the Marx solution because it fully transforms the cost of production and maintains the two macro constraints and a general profit rate between the monetary and the social spaces.

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