LOCAL INTERACTIONS*

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Abstract

Local interactions refer to social and economic phenomena where individuals' choices are influenced by the choices of others who are 'close' to them socially or geographically. This represents a fairly accurate picture of human experience. Furthermore, since local interactions imply particular forms of externalities, their presence typically suggests government action. I survey and discuss existing theoretical work on economies with local interactions and point to areas for further research.

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Contents

1 Introduction

Social scientists discovered not so long ago that seemingly unrelated phenomena such as criminal activity, school attendance, out-of-wedlock pregnancy, substance use, adoption of new technologies, fashion and fads, panics and mania display similar empirical features.[1] Some of these features are

- Too much variation across space and time in the observable variables of interest relative to the variation in the observed fundamentals.
- S-shaped adoption (frequencies) of new technologies, behavior, fashion and norms.
- Presence of direct social (non-market) influences on individual behavior.

The response in the economics science has been to build model economies that can generate these empirical features as equilibrium properties. Economists call these phenomena *social interactions*, i.e., particular socio-economic events in which markets do not fully mediate individuals' choices, and each individual's choice might be in part determined by choices of other individuals in his *reference group*. The underlying idea is that individuals do not exist as isolated atoms but rather are embedded within networks of relationships, e.g. peer groups, families, colleagues, neighbors, or more generally any socio-economic group.

In most of the socioeconomic phenomena cited above and in many others, behavior and characteristics of agents who are 'close' to each other in some social or geographical sense, seem to be correlated: Adolescent pregnancy and school drop-out rates are correlated with neighborhood composition in inner city ghettoes (Catz(1991)); teenagers whose closest friends smoke are more likely to smoke (Nakajima (2007)); Coleman, Katz, and Mendel (1996) show how doctors' willingness to prescribe a new drug diffuses through local contacts; Topa (2001) finds, using Census Tract data for Chicago, that agents are more likely to find a job if their social contacts are employed and that these local spillovers are defined by neighborhood boundaries and ethnic dividing lines. Essentially, most of human interaction that we experience in our daily lives seems to be of similar nature.

The term **local interactions** is coined to refer to such environments where individuals interact with a group of agents close to them in an otherwise large economy. Therefore, in a general

[1] Glaeser, Sacerdote, and Scheinkman (1996) argue that they can explain the high variance of crime rates across space using local interaction. Crane (1991) finds that both high school drop-out and teenage childbearing rates are related to the local neighborhood characteristics; Haveman and Wolfe (1994) find similar results for drop-out rates. See Nakajima (2007) and Kremer and Levy (2008) for the existence of peer effects in smoking and drinking in teenagers and college students respectively. For technology adoption and local complementarities see Brock and Durlauf (2010), Durlauf (1993), Ellison and Fudenberg (1993). For threshold and herd behavior, multiple equilibria and cycles in fads and fashion see Bikhchandani, Hirshleifer, and Welch (1992) and Pesendorfer (1995). For similar behaviors in market crashes, panics and manias see Shiller (2000).

economy with local interactions, each agent's ability to interact with others depends on the position of the agent in a predetermined network of relationships, e.g., a family, a peer group, or more generally any socio-economic group. The origin of the term might be traced back to the Physics and Probability of Interacting Particles, where the fundamental question of interest is whether specification of a system at the particle level (local) can determine its global characteristics. In economics, the analogous question is whether social and economic interaction observed at the individual level can determine the properties of economic aggregates of interest.

My main objective in this chapter is to present and discuss existing theoretical work on economies with local interactions. Consequently, this is a review of the methodological contributions and I do not venture to survey the rapidly growing body of applications of local interaction methods. Interested reader should consult Brock and Durlauf (2001b), Durlauf (2004), Glaeser and Scheinkman (2001), Durlauf and Young (2001) and Manski (2000) for excellent surveys of the literature and more.

There does not yet exist what one may call a 'canonical' model of local interactions. Accordingly, there are rough dividing lines that partition the literature. The most important of these is the static vs. dynamic divide. Majority of the existing models are static, consequently static environments are the ones we best understood so far. Having said that, there is a plethora of questions that beg for and a number of theoretical questions that needs to be answered with dynamic models. Another division is along the binary vs. continuous choice line. Mathematical and econometric techniques currently used in each category are quite different. One final division is along the rational vs. myopic modeling choice. Early models of local interactions in economics have been built with myopic agents and under particular behavioral assumptions. This is changing recently. Thus, although I touch upon models with myopic best-responders, my focus is on models with rational agents. For all these reasons, I chose to follow similar division lines in this article.

2 Static Models

I start with a review of the existing static literature for two main reasons. Firstly, most of the important features of economies with local interactions we know of have been discovered originally in static environments, e.g., cross-sectional correlation of behavior, multiple equilibria, social multiplier. Secondly, this is clearly the most natural order to proceed in and once the reader has the necessary understanding of the aforementioned features, it is simpler to appreciate the delicate aspects of dynamic models and their equilibrium properties.

2.1 Baseline Static Model

In this section, I present a baseline model that will prevail throughout the chapter. I will use the same notation throughout although the original notation used in the articles that I present might be different. The framework is flexible enough to accommodate a variety of different economies of interest. The theoretical object of study is a class of local interaction economies, represented by the tuple $\mathcal{E} = (\mathbb{A}, X, \Theta, N, P, u)$. I describe below what each of these elements is.

Agents are represented by a countable set $\mathbb{A}$ and the letters $a, b, c \ldots$ are used for generic agents. In most of the literature, $\mathbb{A}$ is assumed to be a finite set.[2] Each agent $a \in \mathbb{A}$ makes his choices from a common **action set** X. Depending on the question at hand, structure will be given to X; for example it might be an interval of the real line as in Glaeser and Scheinkman (2003) and Bisin and Özgür (2009a,b) (*continuous choice*) or a binary choice set as in Brock and Durlauf (2001), and Glaeser, Scheinkman, and Sacerdote (1996) (*discrete choice*).

Any exogenous heterogeneity at the individual level (such as family background, observed or unobserved role model or peer group characteristics, individual ability and traits) will be captured by the common **type space** Θ. We will let θ^a be agent a's type, a random variable with support on the set Θ and $\theta := (\theta^a)_{a \in \mathbb{A}}$ be the vector of types for all individuals. At this point, no restriction is made on the admissible probabilistic structure on this set. Yet, the baseline model is general enough to incorporate economies where individual characteristics are correlated (observably or in a hidden way) across agents and time.

When all agents observe the realization of θ, we call the economy one with **complete information**. Otherwise, we say that the economy is with **incomplete information**. Typically, all results for complete information economies I will report will also apply to economies with incomplete information, unless it is mentioned otherwise.

There might be exogenous determinants of individual behavior affecting all agents. These latter will be presented by the **parameter** $p \in P$. When one is interested in modeling aggregate influences (e.g., global interactions, general equilibrium effects) one can extend the notion of equilibrium to allow for an endogenous p. Typically in those cases, p will be an aggregator of some sort.

Now that the underlying physical setup and choice sets are in place, I can introduce preferences. One novelty of the local interaction models is the local structure that allows agents' preferences to be affected by the choices of 'close' (geographically or socially) agents they care about. Consequently, in order to introduce individual preferences on the choice sets, one needs to be precise about who cares about whom. For an agent $a \in \mathbb{A}$, his **reference group** is given by $N(a) \subset \mathbb{A}$. Thus,

[2] Notable exceptions are Föllmer (1974), Durlauf (1993), Bisin, Horst and Özgür (2006), and Horst and Scheinkman (2006).

$$N : \mathbb{A} \to 2^{\mathbb{A}}$$

is a "neighborhood" operator that maps each agent $a \in \mathbb{A}$ to his reference group, $N(a) \subset \mathbb{A}$, the set of agents whose choices affect a's utility directly. Since the baseline model of this section is static, no time index appears. With dynamic models of Section 3, one can allow for intertemporal changes in the reference group of an agent a, i.e., $N : \mathbb{A} \times \{1, 2, \ldots\} \to 2^{\mathbb{A}}$.

Given the neighborhood structure, the preferences of an agent $a \in \mathbb{A}$ are represented by a **utility function** u^a of the form

$$\left(x^a, \{x^b\}_{b\in N(a)}, \theta^a, p\right) \to u^a \left(x^a, \{x^b\}_{b\in N(a)}, \theta^a, p\right)$$

Typical assumptions made in the literature on the utility function are: it is sufficiently smooth with respect to arguments and cross-arguments; that it is concave with respect to agent a's (own) choice. I will be more precise about these when I discuss particular models. Finally, one needs an equilibrium concept to close the model. The one that will be used throughout Section 2 is the following.

Definition 1 *An equilibrium for a static economy with local interactions and complete information, $\mathcal{E} = (\mathbb{A}, X, \Theta, N, P, u)$, is a family of choice maps $\{g^a\}_{a\in\mathbb{A}}$ such that, for each agent $a \in \mathbb{A}$, given θ and p,*

$$g^a(\theta, p) \in \arg\max_{x^a\in X} u^a \left(x^a, \{g^b(\theta, p)\}_{b\in N(a)}, \theta^a, p\right)$$

Notice that this definition assumes that agents, before making their choices, observe the characteristics of other agents and the value of the parameter p. More importantly, each agent a anticipates that any other agent b's choice will be dictated by the behavioral rule (strategy) $g^b : \Theta^{\mathbb{A}} \times P \to X$. For static environments, observing characteristics only of a smaller number of agents (say of one's peers only) is not a fundamental problem as long as the probabilistic structure is common knowledge. The equilibrium concept can be extended in a straightforward manner to incomplete information scenarios. However, in dynamic contexts, the nature of the restrictions that one imposes on the probabilistic structure becomes an important issue, as we will see in Section 3.

Remark 1 (Global interactions) *One might want to model phenomena where agents' preferences depend on some aggregate of individual choices, e.g., increase in average achievement in the classroom might have a positive effect on individual achievements; or the fact that a majority of the population behaves according to a particular social norm might affect behavior at the individual level. In other words, one might want to model the direct dependence of p on x, the action profile, such that $p(x)$ enters into the utility function of an agent. With a finite number of agents, this is a straightforward extension of the local interaction models. It is in that sense* *that global interaction is a special case of local interaction. However, with an infinite number of agents, one needs to be careful about continuity issues as we will see in Section 2.2 when we look at Horst and Scheinkman (2006).*

Remark 2 (Social Space) *To introduce the notion of reference groups means to endow the set of agents with the structure of a graph. Some in the literature stop at that point and use a binary relation and the properties of this latter to model interactions (Morris (2000)); some others look at mean-field interactions only (Brock and Durlauf (2001)). However, one may go further and model the interaction on a lattice and interpret it as a social space and the associated norm as representing social proximity, e.g. Akerlof (1997), Föllmer (1974), Bisin and Özgur (2010). The advantage of the lattice structure is that the mathematical theory of Markovian interaction on lattices is well developed.*

The methods used to study economies with discrete and continuous choices being quite different, there is a rough division in the literature along that line. On each side of the line, there exists a sufficient number of social and economic phenomena that justifies the the respective modeling choice. I start in the next section with the continuous choice models.

2.2 Continuous Choice Models

Some socio-economic phenomena have been naturally modelled using continuous choice in economics. Education is one such phenomenon (Bénabou (1993, 1996), Durlauf (1996a, 1996b)); since its quantity and frequency matters, addiction to substance use is another (Becker and Murphy (1988), Gul and Pesendorfer (2007)). Moreover, models with continuous actions are mathematically simpler to analyze since they yield themselves to differentiable methods. I survey in this section some of the mostly cited methodological contributions to the literature.

Föllmer (1974)

In the early 70s, general equilibrium economists (see Hildenbrand (1971), Malinvaud (1972), and Bhattacharya and Majumdar (1973)) took an interest in the following questions: How should the demand theory and the general equilibrium analysis, as we know them, be modified if individuals' preferences are allowed to be random? Can one always find prices that clear the markets? In particular, does the randomness die out at the aggregate when we look at large economies or limits of finite economies, so that one can use standard results from classical general equilibrium theory?

Hildenbrand (1971) formulated answers to the above questions under the hypothesis that the probability laws governing individual preferences and endowments are random but *independent* across agents. Consider the following class of economies. The set of agents $\mathbb{A}$ is countable.

For an agent $a \in \mathbb{A}$, $\preceq(a)$ denotes his *preferences*, an element in the set $\mathcal{P}$ of continuous complete preorderings on the *commodity space* $\mathbb{R}^l_+$, and $e(a) \in \mathbb{R}^l_+$ his *initial endowment*. Let $w(a) := (\preceq(a), e(a)) \in \mathcal{S} := \mathcal{P} \times \mathbb{R}^l_+$ be agent a's *state* and $\mathcal{S}$ the *set of possible states*. To avoid measure theoretical technicalities, let $\mathcal{S}$ be a finite set[3] and the individual preferences be monotonic and strongly convex (regularity conditions). In this environment, the map

$$w : \mathbb{A} \to \mathcal{S}$$

is called the *state of the economy*. Let Ω be the set $\mathcal{S}^{\mathbb{A}}$ of all possible states and $\mathcal{F}$ the σ-field generated by the individual states $w \to w(a)$, $a \in \mathbb{A}$. Hildenbrand shows that given some regularity conditions, one can choose a price system p such that

$$\lim_{|\mathbb{A}|\uparrow\infty} \frac{1}{|\mathbb{A}|} \sum_{a\in\mathbb{A}} \zeta\left(w(a), p\right) = 0, \text{ in probability,} \tag{1}$$

where $|\mathbb{A}|$ is the number of agents and $\zeta\left(w(a), p\right)$ is the excess demand of agent $a \in \mathbb{A}$ at prices p and individual state $w(a)$. It is not very surprising that randomness alone does not seriously affect the existence of price equilibria. Malinvaud (1972), and Bhattacharya and Majumdar (1973)) take the analysis one step further by dropping independence but imposing conditions on the underlying probability space $(\Omega, \mathcal{F})$ (e.g. strong mixing) that guarantee a suitable law of large numbers. Any conditions on the underlying stochastic structure of the economy are then encoded in to the probability law μ on the probability space $(\Omega, \mathcal{F})$.

Föllmer (1974) argues that conditions imposed directly on μ cease to be purely *microeconomic*, since local knowledge on individual laws is not enough to determine the aggregate μ; one needs to know the probabilities governing the joint behavior of all sub-populations in the economy. He rather asks '*Can one always find prices that clear the markets along with an aggregate probability law for a large economy just on the basis of microeconomic data (local specifications)*?

To that end, let $\eta : \mathbb{A} \setminus \{a\} \to \mathcal{S}$ be the *environment* of an agent $a \in \mathbb{A}$. The **local characteristics** of agent a are given by a probability kernel $\pi_a(\cdot|\,\eta)$, i.e., $\pi_a(s|\,\eta)$ is the probability that agent a's state is s given his environment η. Let Π be the collection of local (*microeconomic*) characteristics of the economy. Call any probability measure μ on $(\Omega, \mathcal{F})$ which is compatible with Π, i.e.,

$$\mu\left[w(a) = s \mid \eta\right] = \pi_a\left(s|\,\eta\right), \; \mu - a.s. \qquad (a \in \mathbb{A}, s \in \mathcal{S})$$

a global (macroeconomic) **phase** of the economy. We say that the local characteristics are **consistent** if they admit at least one global phase.[4]

[3]This is generalizable and the general version would require some compactness assumption.

[4]If $\mathbb{A}$ is finite then the macroeconomic phase is uniquely determined by the local characteristics; see Spitzer (1971). Similarly, under the independence assumption, as in Hildenbrand (1971), there exists a unique global phase μ given by the product measure on $(\Omega, \mathcal{F})$ with marginals $\mu_a(\cdot) = \pi_a\left(\cdot|\,\eta\right)$, $a \in \mathbb{A}$.

Definition 2 *A price p is said to* **stabilize** *the phase μ of an economy ℰ if*

$$\lim_{n\to\infty} \frac{1}{|\mathbb{A}_n|} \sum_{a \in \mathbb{A}_n} \zeta\left(w(a), p\right) = 0, \quad \mu\text{-almost surely}$$

whenever $(\mathbb{A}_n)$ *is an increasing sequence of subsets of agents which exhausts* $\mathbb{A}$[5]*. We say that p* stabilizes the economy $\mathcal{E}$ *if p stabilizes each phase μ of ℰ.*

Markovian Interaction. Föllmer uses the following class of economies to show that (i) even short range interaction may propagate through the economy and may indeed 'become an important source of uncertainty' and (ii) if the local interaction is 'strong' enough, the microeconomic characteristics may no longer determine the global probability law which governs the joint behavior of all economic agents; and in that case a given global phase typically will not satisfy a law of large numbers like in equation (1). Let

$$\mathbb{A} := \mathbb{Z}^d := \{a = (a_1, \dots, a_d)|\ a_i \text{ is integer}\}$$

for some $d \geq 1$ and the reference group of an agent a is given by

$$N(a) := \{b \in \mathbb{A}|\ ||b - a|| = 1\}$$

where $||\cdot||$ is the usual Euclidean norm. Thus each agent has $2d$ *immediate neighbors.* Call this economy *Markovian* if local characteristics are consistent and they satisfy

$$\pi_a(\cdot|\ \eta) = \pi_a(\cdot|\ \eta'), \ \text{ if } \eta \text{ and } \eta' \text{ coincide on } N(a)$$

that is, each agent a's state is influenced by the states only of those agents in his *reference group.* The economy is *homogeneous* if Π is translation invariant. A phase μ is called *homogeneous* if μ is a translation invariant measure[6] Let $\Phi(\mathcal{E})$ be the set of all phases of the Markovian economy $\mathcal{E}$; similarly, let $\Phi_0(\mathcal{E})$ be the set of all homogeneous phases of $\mathcal{E}$. Consistency of the local characteristics imply (Spitzer (1971)) that

$$|\Phi(\mathcal{E})| \ \geq \ |\Phi_0(\mathcal{E})| \ \geq \ 1$$

and both inequalities might be strict. This means that although the underlying structure is homogeneous, the global probability measure might not be ($|\Phi(\mathcal{E})| > |\Phi_0(\mathcal{E})|$) and in particular the

[5]This does not only mean that $\cup\mathbb{A}_n = \mathbb{A}$ but also that the subsets $\mathbb{A}_n$, are 'good representatives' of $\mathbb{A}$. That is, that they expand to $\mathbb{A}$ in approximately the same manner as the subsets $\mathbb{B}_n = \{a \in \mathbb{A} :\ ||a|| \leq n\}$. To be precise, it requires $\mathbb{A}_n \subset \mathbb{B}_n$ and the existence of some integer N and some $\delta > 0$ such that $\mathbb{A}_n$ is the disjoint union of at most N boxes parallel to the axes of the lattice $\mathbb{A}$ and satisfies $|\mathbb{A}_n||\mathbb{B}_n|^{-1} \geq \delta$.

[6]For $a \in \mathbb{A}$, consider the shift operator $T^a : \Omega \to \Omega$ defined by $T^a w(b) = w(a+b)$. Translation invariance of Π means $\pi_{a+b}(\cdot|\ \eta) = \pi_a(\cdot|\ \eta \circ T^b)$ where $\eta \circ T^b(c) = \eta(b+c),\ (a,b,c \in \mathbb{A})$. Translation invariance of μ means that $\mu \circ T^a = \mu,\ a \in \mathbb{A}$.

individual measures μ_a might be different (*symmetry breakdown*). Moreover, multiple consistent global phases are possible ($|\Phi(\mathcal{E})| > 1$) for the same local characteristics (*phase transition*). Let us call each extreme point[7] of $\Phi_0(\mathcal{E})$ a *pure phase*.[8] The following theorem states that given a pure phase for the economy, there always exists a price vector p such that aggregate excess demand vanishes when the economy gets large.

Theorem 1 *Any pure phase can be stabilized. In particular one can equilibrate the economy as soon as it admits only one phase.*

This is an affirmative answer to only one part of the question that Föllmer asked. The most important second part is not answered yet: do local characteristics determine the global phase? To this end, assume that local conditional probabilities $\pi_a(\cdot|\ \eta)$ are all strictly positive. Then thanks to a theorem by Averintzev (1970), the local characteristics are consistent if and only if they can be written in the following form

$$\pi_a(s|\ \eta) = Z(a,\eta)^{-1} \exp\left(\gamma(a,s) + \sum_{b\in N(a)} U\left(a,b,s,\eta(b)\right)\right) \tag{2}$$

where $Z(a,\eta)$ is a normalization factor to guarantee that $\sum_s \pi_a(s|\eta) = 1$. The function U satisfies

$$U(a,b,\cdot,\cdot) = 0 \quad \text{if} \quad b \notin N(a)$$

which corresponds to the Markov property [9] and homogeneity of Π is equivalent to

$$U(a+c,b+c,\cdot,\cdot) = U(a,b,\cdot,\cdot), \quad \gamma(a+c,\cdot) = \gamma(a,\cdot)$$

One may interpret this as γ representing the own-effect and the coupling factors $U(a,b,s,s')$ representing the *intensity of interaction* between the agents a and b when their respective states are s and s'. The representation in (2) is unique if one lets

$$\gamma(\cdot,s_0) = U(\cdot,\cdot,s_0,\cdot) = U(\cdot,\cdot,\cdot,s_0) = 0$$

for some reference state s_0. With this normalization one has $U = 0$ if and only if there is no interaction at all, in which case there is no phase transition. A much weaker condition is given by the following

[7] An extreme point of a convex set $\Phi(\mathcal{E})$ in a real vector space is a point in $\Phi(\mathcal{E})$ which does not lie in any open line segment joining two points of $\Phi(\mathcal{E})$.

[8] Föllmer argues that both $\Phi(\mathcal{E})$ and $\Phi_0(\mathcal{E})$ are metrizable simplices with respect to the weak topology on the space of measures over the compact space Ω (Choquet (1969), Georgii (1972)). Thus, by Choquet's integral representation theorem, each phase (resp. each homogeneous phase) can be written as a mixture of extreme points in $\Phi(\mathcal{E})$ (respectively $\Phi_0(\mathcal{E})$).

[9] Follmer argues that if we replace this condition by $\sum_b \max_{s,s'} U(a,b,s,s') < \infty$, we get an economy with infinite range interactions where interactions 'decay at infinity' and thanks to Georgii (1972), the results of this section remain valid.

Theorem 2 (Spitzer (1971), Dobrushin (1968)) *There exists a* unique *(no phase transition) global probability measure (phase) consistent with local conditional probabilities if either*

(i) $\max |U(\cdot,\cdot,\cdot,\cdot)|$ *is small enough, i.e., if the local interaction among economic agents are sufficiently* weak*, or*

(ii) $d = 1$*, i.e., the local interaction structure is one-dimensional.*

So, one should expect multiple phases when the local interaction is strong and complex enough. Moreover, when multiple phases exist, there is an infinity of non-pure phases due to the convexity of $\Phi(\mathcal{E})$; hence Theorem 1 is of no great use either. Finally, Föllmer demonstrates through an economic reinterpretation of well known example in Statistical Mechanics what sort of complications might arise when the conditions in Theorem 2 are violated.

Example 1 (Ising Economies) *Let $\mathcal{E}$ be a homogeneous Markov economy with two goods and $\mathbb{A} = \mathbb{Z}^2$. Let $e(a) = e := (e_1, e_2) \in \mathbb{R}^2_{++}$ (endowments are not random) and assume that π_a is rotation invariant, i.e., each agent $a \in \mathbb{A}$ reacts in the same way to neighbors in any direction. Moreover, assume that an agent either wants to consume as much as good 1 and does not care about good 2 (type $w(a) = +1$) or the other way around (type $w(a) = -1$).*

Due to rotation invariance, representation in (2) takes the form

$$\pi_a(\pm 1|\,\eta) = Z(\eta)^{-1}\exp\left(\pm\left(\gamma + J\sum_{b\in N(a)}\eta(b)\right)\right)$$

Follmer calls the case $J > 0$ *cyclic* (*conformity*) and $J < 0$ *acyclic* (*nonconformist, against the trend*). Consider a $\mu \in \Phi_0(\mathcal{E})$. At price p, agent a's excess demand is

$$\zeta(+1,p) = \left(\frac{p_2}{p_1}e_2, -e_2\right) \text{ respectively } \zeta(-1,p) = \left(-e_1, \frac{p_1}{p_2}e_1\right)$$

so his expected excess demand (given $\mu_1 = \mu[w(a) = +1]$ and $\mu_2 = \mu[w(a) = -1]$) vanishes if

$$\mu_1\left(\frac{p_2}{p_1}e_2, -e_2\right) + \mu_2\left(-e_1, \frac{p_1}{p_2}e_1\right) = (0,0)$$

which implies the necessary condition

$$\frac{p_2}{p_1} = \frac{e_1}{e_2}\frac{\mu_2}{\mu_1} \tag{3}$$

Due to a result in Spitzer (1971), when $J > 0$ and $\gamma \neq 0$, there is a unique phase which can be stabilized by Theorem 1. Now, assume that $\gamma = 0$. By a result in Georgii (1972), there is a critical value J_0 (that depends on the dimension of interaction d) such that for $J > J_0$, there are exactly two phases, say μ^1 and μ^2 that satisfy

$$\frac{\mu^1_1}{\mu^1_2} = \frac{\mu^2_1}{\mu^2_2} > 1, \tag{4}$$

Denoting expectation with respect to μ^i by E^i ($i = 1, 2$), we have by Theorem 1

$$\frac{1}{|\mathbb{A}_n|} \sum_{a \in \mathbb{A}_n} \zeta(w(a), p) \to E^i \left[\zeta(w(0), p)\right] \quad \mu^i - \text{almost surely}, \quad i = 1, 2.$$

Unfortunately, equations (3) and (4) combined imply that there does not exist a price $p \in \mathbb{R}^2_+$ which makes the right side of the above equation vanish simultaneously for μ^1 and μ^2. Hence, we cannot stabilize the economy. Follmer shows that actually the situation is even worse than that as summarized in

Theorem 3 *A cyclic Ising economy where $\gamma = 0$ and with strong and complex interaction can almost never be stabilized.*

Overall, apart from being a contribution to the general equilibrium theory of random economies, the most important impact of Föllmer (1974) on the economics science has been the introduction and reinterpretation of mathematical methods used in Statistical Mechanics (Probability and Physics of Interacting Particles) in economies with local interactions. Interested reader should consult the standard reference in Mathematics for Interacting Particle Systems Liggett (1985). Durlauf (2008) is a nice reading with many more references.

Glaeser and Scheinkman (2003)

The main contribution of Glaeser and Scheinkman (GS henceforth) is the exploration of the common mathematical structure in existing models of static social interactions. They provide conditions under which equilibria exist and are unique. They give sufficient conditions for the existence of multiple equilibria and social multiplier effects, and ergodicity of the large economy limits. Finally, they discuss possible approaches to measurement and estimation of interaction effects. With the exception of a small section on 'mean field' interaction (average population action as an argument in the utility) with binary choice, all results are obtained for **continuous choice**.

Formally, they study economies with a finite number of agents $\mathbb{A} = \{1, \ldots, n\}$, each of whom is subject to a taste shock θ^a with support on a set Θ. The common action set, X, is an interval of the real line. Although they allow for multiple reference groups for each agent a, i.e., $N^a_k \subset \mathbb{A}\backslash\{a\}$, $k = 1, \ldots, K > 1$ s.t. $N(a) = \cup_k N^a_k$, to accomodate some examples in the literature, their results are presented for a single reference group ($K = 1$). The utility function of agent a is defined as

$$u^a \left(x^a, \{x^b\}_{b \in N(a)}, \theta^a, p\right) := u^a \left(x^a, \bar{x}^a_1, \ldots, \bar{x}^a_K, \theta^a, p\right)$$

where

$$\bar{x}^a_k := \sum_{b \in \mathbb{A}} \gamma^{ab}_k x^b$$

with $\gamma_k^{ab} \geq 0$, $\gamma_k^{ab} = 0$, if $b \notin N_k^a$, $\sum_b \gamma_k^{ab} = 1$, and $p \in P$ is a vector of parameters.

Agents, when making choices, observe $\bar{x}^a$, the summary statistics of other agents' actions ($K = 1$). Given u^a that is twice continuously differentiable with $u_{11}^a < 0$, agent a's optimal interior choice is given by

$$u_1^a \left(x^a, \bar{x}^a, \theta^a, p\right) = 0 \tag{5}$$

Since $u_{11}^a < 0$, $x^a = g^a \left(\bar{x}^a, \theta^a, p\right)$ is well defined and

$$g_1^a \left(\bar{x}^a, \theta^a, p\right) = -\frac{u_{12}^a \left(x^a, \bar{x}^a, \theta^a, p\right)}{u_{11}^a \left(x^a, \bar{x}^a, \theta^a, p\right)} \tag{6}$$

Given this structure, an equilibrium always exists if the following holds.

Proposition 1 *Given a pair $(\theta, p) \in \Theta \times P$, suppose that for each a, $g^a \left(\bar{x}^a, \theta^a, p\right) \in I \subset X$, whenever $\bar{x}^a \in I$, where I is a closed and bounded interval. Then, there exists at least one equilibrium.*

One commonly used practice in the literature to generate multiple equilibria, e.g. Cooper and John (1988), is to introduce strategic complementarity into the utility functions. GS show, through an example, that strategic complementarity is *not* necessary for multiplicity. They also prove that, under standard regularity conditions, existence of a continuum of equilibria, such as in Diamond (1982), is non-generic in their economies[10]. A sufficient condition for a unique equilibrium, in these economies, is what they call the **Moderate Social Influence** (MSI) condition: The effect of a change in own action on own marginal utility is greater (in absolute value terms) than the effect on the latter of a change in average reference group action, i.e.,

$$\left| \frac{u_{12}^a \left(x^a, \bar{x}^a, \theta^a, p\right)}{u_{11}^a \left(x^a, \bar{x}^a, \theta^a, p\right)} \right| < 1 \tag{7}$$

This latter implies (although it is stronger than) from equation (6) that at the equilibrium profile, $\left|g_1^a \left(\bar{x}^a, \theta^a, p\right)\right| < 1$ for each agent a, which in turn implies uniqueness.

Proposition 2 *If for a given (θ, p), MSI holds for all $a \in \mathbb{A}$, then there exists at most one equilibrium.*

If, in addition to MSI, one assumes strategic complementarity ($u_{12}^a > 0$), one can show that there is a **social multiplier**: a change in the value of a parameter, say p^1 will have a *direct* effect going through the optimal choice g^a and an *indirect* effect going through the average reference group choice, $\bar{x}^a$. If each g^a has a positive partial derivative with respect to p^1, this will be

[10] The issue of multiplicity is studied in more detail in Section 2.3 along with the construction of the particular example in GS.

amplified through the increased averages that increase the marginal utility of each agent for any (θ, p), due to strategic complementarity[11]

The next interesting question they ask is: Can individual shocks determine aggregate outcomes for large groups? Generically, **ergodicity** depends on the details of the interaction structure unlike the other results that they obtain. Nevertheless, economies with i.i.d shocks and local interactions tend to behave ergodically. GS provide sufficient (but not necessary) conditions for the average action of a large population to be independent of the particular realization of the individual shocks.

Proposition 3 *Suppose that the following conditions hold*

1. *θ^a is i.i.d across agents.*
2. *u^a (hence g^a) is idependent of a (ex ante homogeneous preferences).*
3. *$N(a) := \mathbb{A} \setminus \{a\}$.*
4. *The interaction weights $\gamma^{a,b} := \frac{1}{n-1}$.*
5. *Action set X is bounded.*
6. *MSI holds uniformly, that is*
$$\sup_{\bar{x}^a, \theta^a} |g_1(\bar{x}^a, \theta^a, p)| < 1.$$

Let $x_n(\theta, p)$ denote the equilibrium when the population size is n and agent a's shock realization is θ^a. Then there exists an $\bar{x}(p)$ such that, with probability one,

$$\lim_{n\to\infty} \sum_{a=1}^{n} \frac{x_n^a(\theta, p)}{n} = \bar{x}(p)$$

One problem that is at the heart of empirical work in the literature is the empirical description of reference groups. GS touch upon the existing approaches to that question in the literature, namely: (i) models that take as an agent's reference group other individuals who are close to him geographically, e.g. Bénabou (1993), Glaeser, Sacerdote, and Scheinkman (1996); (ii) models that use random graph theory to treat particular reference groups as realizations of a random process, e.g. Kirman (1983), Ioannides (1990); (iii) models that treat individual incentives for the formation of reference groups, e.g. Jackson and Wolinsky (1996), and Bala and Goyal (2000).

Finally, GS give a tour of the empirical approaches that have been and that might be used to detect, measure, and estimate social interactions empirically. The three methods they consider are: (i) using the variance of group averages; (ii) regressing individual outcomes on group averages; and (iii) using the social multiplier.

[11] I will formulate this argument in Section 2.3 and compare it with similar results in other cited work.

Bisin, Horst, and Özgür (2006)

Bisin, Horst, and Özgür (BHÖ henceforth) consider general economies with static as well as dynamic local and global interactions. Here, I will present their study of static economies. Their most important contribution, that is, the study of the rational expectations equilibria of dynamic local and global interaction economies with rational forward looking agents is studied in section 3.2. Here is their contribution in a nutshell:

(i) For the static complete and incomplete information economies with local interactions, they provide conditions for existence, uniqueness, and Lipschitz continuity of equilibrium. Moreover, in their setup of the complete information economies with local interactions, BHÖ show that the law of the configuration of the endogenous choices of agents is a **Gibbs measure**[12] specified by a family of conditional probability distributions (agents' behavioral rules) given neighbors' equilibrium choices.

(ii) For the dyamic economies with forward looking rational agents with both local and global interactions, they show existence and Lipschitz continuity of stationary Markov equilibria. To do that, they use a novel separation argument to treat local and global equilibrium dynamics as independent processes and give conditions for these economies to converge to a unique probability law independent of initial conditions.

(iii) Finally, for a class of local conformity and habit formation economies, they characterize equilibria in closed form and study the effects of rationality, information, and dynamics on the existence (or suppresion) of social multiplier effects.

Formally, they consider economies with a large number of agents; $\mathbb{A}$ is countably infinite to be precise[13]. Hence each agent is 'insignificant' compared to the rest of the economy in the spirit of common general equilibrium abstraction. Types, θ^a, are i.i.d. across agents, with law ν, and support Θ. For each agent a, $N(a) = \{a+1\}$, i.e., the local interaction structure is one-sided. BHO use this particular form to study in an abstract way economies where interactions are directed (e.g. hierarchical interactions in organizations, local conformity and role model interactions)[14] The preferences of each agent a are represented by the utility function

$$\left(x^a, x^{a+1}, \theta^a\right) \to u\left(x^a, x^{a+1}, \theta^a\right)$$

which is assumed to be continuous and strictly concave in its first argument. Prior to his choice, each agent $a \in \mathbb{A}$ observes the realization of his own type θ^a as well as the realizations of the types

[12]Please see Georgii (1989), Liggett (1985), or Kindermann and Snell (1980)

[13]Their results apply to economies with a finite number of agents with straightforward modifications. Evidently, existence results are easier to prove in that case.

[14]See the discussion at the end of this section for how to extend their ideas to more general interaction structures.

θ^b of the agents $b \in \{a+1, a+2, ..., a+N\}$. The vector of types whose realization is observed by the agent $a = 0$ is denoted $\theta_N := \{\theta^0, \theta^1, \ldots, \theta^N\}$; by analogy $T^a\theta_N := \left(\theta^a, \ldots, \theta^{a+N}\right)$ denotes the vector of types whose realization is observed by the agent $a \in \mathbb{A}$.[15] If $N = \infty$ each agent has *complete information* about the current configuration of types when choosing his action. When instead $N \in \mathbb{N}$, an agent only has *incomplete information* about the types of the other agents. By convention, if $N = 0$, agents only observe their own types. Finally, the set of possible configurations of types of all agents $a \geq 0$ is given by $\mathbf{\Theta^0} := \{(\theta^a)_{a\geq 0} \ : \ \theta^a \in \Theta\}$.

The infinite number of agents assumption makes the standard existence results for finite economies unusable. Hence, in order to guarantee the existence and uniqueness of an equilibrium for static economies with local interactions, BHÖ impose a form of *strong concavity* on the agents' utility functions.

Definition 3 *Let $\alpha \geq 0$. A real-valued function $f : X \to \mathbb{R}$ is α-concave on X if the map $x \mapsto f(x) + \frac{1}{2}\alpha|x|^2$ from X to $\mathbb{R}$ is concave.*

This definition is first due to Rockafellar (1976), and is used for related purposes in Montrucchio (1987) and Santos (1991). Observe that a twice continuously differentiable map $f : X \to \mathbb{R}$ is α-concave, if and only if the second derivative is uniformly bounded from above by $-\alpha$.

In order to obtain parametric continuity of the equilibrium map, BHÖ require any agent's marginal utility with respect to his own action to depend in a Lipschitz continuous manner on the action taken by his neighbor. In this sense they impose a qualitative bound on the strength of local interactions between different agents.

Assumption 1 *The utility function $u : X \times X \times \Theta \to \mathbb{R}$ satisfies the following conditions:*

(i) The map $x \mapsto u(x, y, \theta)$ is continuous and uniformly α-concave for some $\alpha > 0$.

(ii) The map u is differentiable with respect to its first argument, and there exists a map $L : \Theta \to \mathbb{R}$ such that

$$\left|\frac{\partial}{\partial x}u(x, y, \theta^0) - \frac{\partial}{\partial x}u(x, \hat{y}, \theta^0)\right| \leq L(\theta^0)|\hat{y} - y| \quad \textit{and such that} \quad \mathbb{E}L(\theta^0) < \alpha. \tag{8}$$

The quantity $L(\theta^0)$ puts a bound on $\frac{\partial^2 u(x,y,\theta)}{\partial x \partial y}$, whereas α may be viewed as a bound on $\frac{\partial^2 u(x,y,\theta)}{\partial x^2}$. Thus, $\mathbb{E}L(\theta^0) < \alpha$ means that, *on average, the marginal effect of the neighbor's action on an agent's marginal utility is smaller than the marginal effect of the agent's own choice.* It is in this sense that (8) imposes a bound on the strength of the interactions between different agents. Notice that the *Moderate Social Influence* condition in Glaeser and Scheinkman (2003) corresponds to the stronger contraction condition $L(\theta^0) < \alpha$. Assumption 1 can easily be verified for the following example.

[15] Formally, $T^a : \Omega \mapsto \Omega$ ($a \in \mathbb{A}$) is the a-fold iteration of the canonical right shift operator T on Ω; that is, $T^a((\omega_b)_{b\in\mathbb{A}}) = (\omega_{b+a})_{b\in\mathbb{A}}$; furthermore, $T^a\theta_N := \left(\theta^0(T^a\omega), \ldots, \theta^N(T^a\omega)\right) = \left(\theta^a, \ldots, \theta^{a+N}\right)$.

Example 2 (Local Conformity) *Let α_1 $\alpha_2 \geq 0$ and consider a utility function of the form*

$$u(x^a, x^{a+1}, \theta^a) := -\alpha_1(x^a - \theta^a)^2 - \alpha_2(x^a - x^{a+1})^2. \tag{9}$$

Quadratic utility functions of the form (9) describe preferences in which agents face a trade-off between the utility they receive from matching their own idiosyncratic shocks and the utility they receive from conforming to the action of their peers. The higher the ratio $\frac{\alpha_2}{\alpha_1}$, the more intense is the agent's desire for conformity. It is easy to see that the map $x^a \mapsto u(x^a, x^{a+1}, \theta^a)$ is α-concave for all $\alpha \leq 2(\alpha_1 + \alpha_2)$. Moreover, Assumption 1 is satisfied with $L(\theta^0) = L := 2\max\{\alpha_1, \alpha_2\}$ and with $\alpha := 2(\alpha_1 + \alpha_2)$.

BHÖ study symmetric equilibria. Establishing the existence of a symmetric equilibrium is equivalent to proving the existence of a measurable function $g^* : \mathbf{\Theta^0} \to X$ which satisfies

$$g^*(\theta) = \arg\max_{x^a \in X} u(x^a, g^* \circ T(\theta), \theta^0) \qquad \mathbb{P}\text{-a.s.} \tag{10}$$

Each such map is a fixed point of the operator $V : B(\mathbf{\Theta^0}, X) \to B(\mathbf{\Theta^0}, X)$ which acts on the class $B(\mathbf{\Theta^0}, X)$ of bounded measurable functions $f : \mathbf{\Theta^0} \to X$ according to

$$Vg(\theta) = \arg\max_{x^a \in X} u(x^a, g \circ T(\theta), \theta^0). \tag{11}$$

On the other hand, each fixed point of V is a symmetric equilibrium. It is therefore enough to show that V has an almost surely uniquely defined fixed point.

BHÖ are also interested in deriving conditions which guarantee that the economy admits a *Lipschitz continuous equilibrium map.* Lipschitz continuity of the equilibrium map may be viewed as a minimal robustness requirement on equilibrium analysis. In particular it justifies comparative statics analysis. They metrize the product space $\mathbf{\Theta^0}$ in a way that allows them to parametrize the bound on the variation of the equilibrium policy. For an arbitrary constant $\eta > 0$ define a metric d_η on the product space $\mathbf{\Theta^0}$ by

$$d_\eta(\theta, \hat{\theta}) := \sum_{a \geq 0} 2^{-\eta|a|}|\theta^a - \hat{\theta}^a| \qquad (\theta = (\theta^a)_{a \in \mathbb{N}},\ \hat{\theta} = (\hat{\theta}^a)_{a \in \mathbb{N}}) \tag{12}$$

and denote by $\mathrm{Lip}_\eta(1)$ the class of all continuous functions $f : \mathbf{\Theta^0} \to X$ which are non-expanding with respect to the metric d_η, i.e.,

$$\mathrm{Lip}_\eta(1) := \{f : \mathbf{\Theta^0} \to X : |f(\theta) - f(\hat{\theta})| \leq d_\eta(\theta, \hat{\theta})\}$$

Their main result in this section is

Theorem 4 *Let $\mathcal{S}$ be a static economy with local interactions and complete information.*

(i) If the utility function $u : X^2 \times \Theta \to \mathbb{R}$ satisfies Assumption 1, then $\mathcal{S}$ admits a unique symmetric equilibrium g^*.

(ii) If, instead of (8), the utility function u satisfies the stronger condition,

$$\left|\frac{\partial}{\partial x}u(x,y,\theta)-\frac{\partial}{\partial x}u(x,\hat{y},\hat{\theta})\right|\leq L\left(|\hat{y}-y|+|\theta-\hat{\theta}|\right)\quad \textit{with}\quad L<\alpha, \tag{13}$$

then there exists $\eta^* > 0$ such that the unique symmetric equilibrium g^* is almost surely Lipschitz continuous with respect to the metric d_{η^*}:

$$|g^*(\theta)-g^*(\hat{\theta})|\leq \frac{L}{\alpha}d_{\eta^*}(\theta,\hat{\theta})\qquad \mathbb{P}\text{-}a.s..$$

An analogous result obtains for economies with incomplete information, where an individual agent only observes a finite number $N<\infty$ of types.

Theorem 5 *Let $\mathcal{S}$ be a static economy with local interaction and incomplete information, that is with $N \in N$.*

(i) If the utility function $u : X^2 \times \Theta \to \mathbb{R}$ satisfies Assumption 1 and if it is continuously *differentiable with respect to its first argument, then $\mathcal{S}$ admits a unique symmetric equilibrium g^*.*

(ii) If u satisfies condition (13), then g^* is almost surely Lipschitz continuous:

$$|g^*(\theta_N)-g^*(\hat{\theta}_N)|\leq \frac{L}{\alpha}|\theta_N-\hat{\theta}_N|\qquad \mathbb{P}\text{-}a.s..$$

Example 2 cont. (Local Conformity) *For the local conformity preferences described in (9), the equilibrium policy can be solved for in closed form. Let $\beta_1 := \frac{\alpha_1}{\alpha_1+\alpha_2}$ and $\beta_2 := \frac{\alpha_2}{\alpha_1+\alpha_2}$. If the agents have complete information, i.e., if $N=\infty$, then the equilibrium takes the form*

$$g^*\left(T^a\theta_N\right)=\beta_1\sum_{i=a}^{\infty}\beta_2^{i-a}\theta^i.$$

Observe that $\beta_1 \sum_{i=a}^{\infty} \beta_2^{i-a} = 1$. Thus, in equilibrium, the action of an agent $a \in \mathbb{A}$ is given by a convex combination of the types θ^b of the agents $b \in \{a, a+1, a+2, \ldots\}$. If the agents only have incomplete information, that is, if $N < \infty$, then

$$g^*\left(T^a\theta_N\right)=\beta_1\left(\sum_{i=a}^{a+N}\beta_2^{i-a}\theta^i+\beta_2^{N+1}\mathbb{E}\theta^a\right).$$

BHÖ study the statistical properties of the equilibrium for an economy with the above specification. In particular, they characterize the effects of local conformity on the variance and the correlation structure of individual actions in the population as well as on the variance of the mean action across different economies. When the variance of the mean action across economies is larger than the variance of each action in the population, they say that social interactions generate a *social multiplier* effect. I postpone the discussion of this part to Section 2.3.

Economies with more general interaction structures. While the *Moderate Social Influence* assumption is generally not enough to obtain existence and uniqueness of equilibrium in economies with more general interaction structures, a stronger condition, like condition (13), in fact suffices for existence, uniqueness, and Lipschitz continuity. This is the case for both complete and incomplete information economies. Consider the case in which agents are located on the d-dimensional integer lattice $\mathbb{Z}^d$, and the preferences of the agent $a \in \mathbb{Z}^d$ are described by a utility function of the form

$$\left(x^a, \{x^b\}_{b\in N(a)}, \theta^a\right) \mapsto \hat{u}\left(x^a, \{x^b\}_{b\in N(a)}, \theta^a\right)$$

where $N(a) := \{b \in \mathbb{Z}^d : \|a-b\| = 1\}$ denotes the set of the agent's nearest neighbors. In such a more general model, each symmetric equilibrium is given by a fixed point of the operator

$$Vg(\theta) = \arg\max_{x^0\in X} \hat{u}\left(x^0, \{g\circ T^a(\theta)\}_{a\in N(0)}, \theta^0\right).$$

BHO show that, if the utility function satisfies the contraction condition

$$\left|\frac{\partial}{\partial x^a}\hat{u}\left(x^a, \{x^b\}_{b\in N(a)}, \theta\right) - \frac{\partial}{\partial x^a}\hat{u}\left(x^a, \{\hat{x}^b\}_{b\in N(a)}, \hat{\theta}\right)\right| \leq L\max\{|\hat{x}^b - x^b|, |\theta-\hat{\theta}| : b\in N(a)\},$$

then V satisfies the contraction condition

$$|Vg - V\hat{g}| \leq \frac{L}{\alpha}\max\{|g\circ T^b - \hat{g}\circ T^b| : b\in N(a)\}.$$

Hence, V becomes a contraction that maps a set of Lipschitz continuous functions continuously into itself. Two-sided interactions are simply a special case of this general model.

Finally, BHÖ also show that the results they obtain for static economies can be reinterpreted (mathematically and economically) in two interesting ways:

(i) Equilibria in static economies can be characterized as *stationary solutions to a stochastic difference equation* derived from optimality conditions and as such a mathematical structure common to their environment and that of macroeconomic rational expectations models, e.g. Blanchard and Kahn (1980), can be unearthed;

(ii) Föllmer (1974) considers an economy where the law of the configuration of agents' exogenous types is a *Gibbs measure*. In their setup of the complete information economies with local interactions, BHÖ show that it is instead the law of the configuration of the endogenous choices of agents that is a Gibbs measure specified by a family of conditional probability distributions (agents' behavioral rules) given neighbors' equilibrium choices.

Horst and Scheinkman (2006)

Horst and Scheinkman (HS henceforth) are interested in equilibrium existence and uniqueness results in fairly general systems of static local and global interactions with an infinite number of agents. They also examine the structure of the equilibrium distribution and derive a "Markov" property for the equilibrium distribution of a class of spatially homogeneous systems.

Formally, the set of agents $\mathbb{A} \subset \mathbb{Z}^d$. Each agent $a \in \mathbb{A}$ makes a choice x^a from a common compact and convex set $X \subset \mathbb{R}^l$. The configuration space $S := \{x = (x^b)_{b\in\mathbb{A}} : \ x^b \in X\}$ of all action profiles is equipped with the product topology, and hence it is compact. Agent a's utility is affected by neighboring agents in varying degrees. To that end, let (J^a, θ^a) be a random variable where $J^a = (J^{a,b})_{b\neq a}$ with support $\Xi := \mathbb{R}^{\mathbb{A}\setminus\{0\}}$ capturing bilateral strength of interactions and θ^a with support Θ, agent a's taste shock. Agent a's reference group $N(a)$ is defined by the values of the realized interaction strength variable, i.e.,

$$N(a) := \Big\{ b \in \mathbb{A} : \ J^{a,b} \neq 0 \Big\}$$

These are the agents who interact with agent a *locally.* The agents who are not in a's reference group possibly affect his utility through a *global interaction* variable (empirical distribution) $p(x)$ associated with each action profile x. However, this way of modeling the global effect is not always appropriate for topological difficulties.[16] GS uses a two-step method to separate local (micro) and global (macro) interactions.

To that end, let $(\Omega, \mathcal{F}, \mathbb{P}) := ((\Xi \times \Theta)^{\mathbb{A}}, \mathcal{B}(\Xi \times \Theta)^{\mathbb{A}}, \mathbb{P})$ be the canonical probability space and let p be a probability measure on the action set X, and let $\mathcal{M}(X)$ be the set of such measures.[17] This way, a given aggregate belief $p \in \mathcal{M}(X)$ will simply be a parameter of the utility function without any explicit link between x and p. Thus, the preferences of agent a are represented by a utility function $U^a : S \times \mathcal{M}(X) \times \Xi \times \Theta \to \mathbb{R}$ such that

$$U^a(x^a, \{x^b\}_{b\neq a}, p, J^a, \theta^a) := u^a(x^a, \{J^{a,b}x^b\}_{b\neq a}, p, \theta^a)$$

They call the equilibrium that comes out of this structure given a common exogenous aggregate belief for all agents, a **microscopic equilibrium**, namely

Definition 4 *Given $p \in \mathcal{M}(X)$, an action profile $g(p, J, \theta) = \{g^a(p, J, \theta)\}_{a\in\mathbb{A}}$ is a* microscopic equilibrium *associated with p if*

$$g^a(p, J, \theta) \in \arg\max_{x^a \in X} U^a(x^a, \{g^b(p, J, \theta)_{b\neq a}, p, J^a, \theta^a\}) \qquad \mathbb{P} - a.s.$$

[16] Utility functions might not be continuous w.r.t product topology if x enters in a non-trivial fashion. In addition, the configuration x does not have to have an empirical distribution. Hence, the continuity of the utility functions already imposes a decay rate on the strength of interactions.

[17] $\mathcal{M}(X)$ is compact with respect to the topology of weak convergence.

which they show to exist(not necessarily homogeneous) for any random social interactions system (purely local ones in particular). When they talk about the full-fledged general equilibrium, they require the aggregate belief p to be *consistent* with the empirical distribution of equilibrium actions, say $p(J, \theta)$.

Definition 5 *A random variable* $g(J, \theta) = \{g^a(J, \theta)\}_{a \in \mathbb{A}}$ *is an* **equilibrium** *for* $\mathcal{E}$ *if*

(i) When $\mathcal{E}$ *is not purely local, the empirical distribution associated with the action profile* $g(J, \theta)$ *exists almost surely, i.e., the weak limit*

$$\lim_{n \to \infty} \frac{1}{|\mathbb{A}_n|} \sum_{a \in \mathbb{A}} \delta_{g^a(J,\theta)}(\cdot) = p(J, \theta)$$

exists almost surely for some random variable $p(J, \theta) \in \mathcal{M}(X)$ *along the increasing sequence of finite sets* $\mathbb{A}_n := [-n, n]^d \cap \mathbb{A} \uparrow \mathbb{A}$ *and*

(ii) No agent wants to deviate, i.e.,

$$g^a(J, \theta) \in \arg\max_{x^a \in X} U^a(x^a, \{g^b(J, \theta)_{b \neq a}, p(J, \theta), J^a, \theta^a\}) \qquad \mathbb{P} - a.s. \quad (a \in \mathbb{A}).$$

Unfortunately, unless some form of spatial homogeneity prevails, there is no reason to expect that the empirical distribution associated with the equilibrium actions exists (condition (i) above). For this reason, when global interactions are present, HS restrict themselves to **homogeneous** systems, i.e.,

Definition 6 *An economy* $\mathcal{E}$ *is homogeneous if* $\mathbb{A} = \mathbb{Z}^d$ *and*

(i) There exists a measurable mapping $U : S \times \mathcal{M}(X) \times \Xi \times \Theta \to \mathbb{R}$ *such that for all* $a \in \mathbb{A}$

$$U^a(x^a, \{x^b\}_{b \neq a}, p, J^a, \theta^a) = U(x^a, \{x^b\}_{b \neq a}, p, (T^a J)^0, (T^a \theta)^0)^{18}$$

(ii) The distribution of the random variable $(J, \theta) = \{(J^a, \theta^a)\}_{a \in \mathbb{A}}$ *is* stationary*, i.e.,*

$$\mathbb{P}[(J, \theta) \in B] = \mathbb{P}[T^a(J, \theta) \in B]$$

for all $a \in \mathbb{A}$ *and any measurable set* $B \in \mathcal{F}$*.*

The nice thing about the spatially homogenous systems, as they show, is that they can be viewed as convex combinations of ergodic systems.[19] In particular, a system where $(J^a, \theta^a)_{a \in \mathbb{A}}$ are i.i.d

[18] T^a is simply a shift operator that individualizes a random variable to agent a as before.

[19] A homogeneous system $\mathcal{E}$ is called **ergodic** if, the probability measure $\mathbb{P}$ is ergodic, i.e., it satisfies a 0-1 law on the σ-field of all shift invariant events. See for example Fristedt and Gray (1997), section 28.5 or Billingsley (1995), section 24.

is ergodic. Given a homogeneous system $\mathcal{E}$, there exists a set $\mathcal{M}_0$ of ergodic probability measures on $(\Omega, \mathcal{F})$ and a mixing measure π such that

$$\mathbb{P}(\cdot) = \int_{\mathcal{M}_0} \nu(\cdot)\pi(d\nu)$$

where the measures ν are mutually singular, i.e., there exists (a.s.) mutually disjoint sets Ω_ν such that

$$\nu(\Omega_\nu) = 1 \quad \text{and} \quad \nu(\Omega_{\hat{\nu}}) = 0 \quad \text{for} \quad \nu \neq \hat{\nu}.$$

Thus one can think of a homogeneous interaction economy in two steps. Nature first picks an ergodic system using a distribution π, and then chooses an interaction pattern and a taste shock according to the distribution of the selected ergodic system. Given this description of course the equilibrium of the homogeneous system can be written as a family of equilibria of the associated ergodic decomposition, i.e.,

Proposition 4 *Let $\mathcal{E}$ be a homogeneous system of random social interactions with an associated ergodic decomposition $(\mathcal{E}_\nu)_{\nu \in \mathcal{M}_0}$.*

(i) If g is a homogeneous equilibrium for $\mathcal{E}$, then g coincides a.s. with a homogeneous equilibrium g_ν for $\mathcal{E}_\nu$ on Ω_ν.

(ii) If for every ν, g_ν is a homogeneous equilibrium for $\mathcal{E}_\nu$, then the random variable g given by

$$g(J, \theta) = g_\nu(J, \theta) \quad \textit{if} \quad (J, \theta) \in \Omega_\nu$$

defines a homogeneous equilibrium for $\mathcal{E}$.

HS argue that to show the existence and uniqueness of homogeneous microscopic equilibria in ergodic systems, they need to bound the strengths of interactions between agents and the effect of the global interactions on the marginal utility. They say that **MSI** (Moderate Social Interactions) holds if the best response function (unique optimum due to their strict concavity of the utility function assumption) of agents, say agent 0, h^0, is Lipschitz continuous and if the Lipschitz constants can be chosen to satisfy

$$\sum_{a \neq 0} L^a(\cdot) \leq \alpha < 1$$

Furhermore, **MSI** holds in **strong** form if one can choose L^a and L^p such that

$$\sup L^p + \sum_{a \neq 0} L^a(\cdot) \leq \alpha < 1.$$

If MSI holds, they prove that an economy $\mathcal{E}$ that is ergodic has a unique homogeneous microscopic equilibrium $g(p, \cdot)$ with respect to every empirical distribution p, which prepares the background for their main existence result.

Theorem 6 *If $\mathcal{E}$ is ergodic and has a homogeneous microscopic equilibrium $g(p, \cdot)$ with respect to every $p \in \mathcal{M}(X)$, then*

(i) The empirical distribution associated to the equilibrium action profile $g(p, \cdot)$ exists and is a.s. equal to $\mu(p)$, the distribution of the random variable $g^0(p, \cdot)$. That is,

$$\lim_{n \to \infty} \frac{1}{|\mathbb{A}_n|} \sum_{a \in \mathbb{A}_n} \delta_{g^a(p,J,\theta)}(\cdot) = \mu(p) \quad \mathbb{P} - a.s.$$

(ii) If $\mathcal{E}$ satisfies MSI, then it has a homogeneous equilibrium whose associated empirical distribution is a.s. independent of (J, θ).

(iii) If MSI holds in strong form, the equilibrium is unique.

The power of the ergodic structure is exploited fully in (ii) which says that the empirical distribution which is basically the aggregation of agents' local equilibrium behavior is independent of the realizations of local data. Given the equilibrium map, the behavior of the aggregates is not dependent of a particular interaction structure. This is a very nice result. If a system is homogeneous but not ergodic, then the empirical distribution would of course vary with (J, θ) but would still be constant in each Ω_ν.

For one-sided systems, HS obtain existence from the weaker assumption of average moderate social interactions, AMSI, which basically says that the Lipschitz bounds hold on average. Uniqueness follows when they assume strict concavity and a stronger version of AMSI (similar to strong MSI but in expectations). Finally, HS also derive a spatial Markov property for the equilibrium distribution of a class of homogeneous systems.

2.3 Multiple Equilibria and Social Multiplier

One of the most appealing aspects of local interaction models is their ability to generate excess variation at the aggregate relative to the variation in exogenous data hence explain large differences in outcomes across populations and time with small differences in exogenous variables. Economists call this the **social multiplier** effect. The relevance of the social multiplier for policy analysis stems from the fact that when interactions are quantitatively important, policy interventions on single agents might have large aggregate effects.

The social multiplier concept is inherently related to two other issues: multiplicity of equilibria and identifiability of sources of variations. Typically, the forces that lead to multiple equilibrium also lead to large social multipliers. However, the former is not necessary for the latter as we will see below. I would like to argue in this section that local interaction models provide a natural outlet to tackle these issues; in particular, they suggest methods to obtain multiple equilibria and generate aggregate variation in a systematic way.

Cooper and John (1988) unearth the common features of Keynesian macroeconomic models. They ask what properties the economy should possess at the microeconomic level so that one obtains multiple equilibria at the aggregate. In particular, they are interested in coordination failures, that is, Pareto ranked multiple equilibria, and multiplier effects. They argue that the answer lies in **strategic complementarities**[20] at the individual level if the nature of the interaction is global.

They consider economies with $\mathbb{A} = \{1, 2, \ldots, I\}$, $X = [0, E]$ where E is finite. The interaction is through the average choice (global), i.e., $N(a) = \mathbb{A} \setminus \{a\}$ and agent a's utility from choosing x^a when everyone else chooses $\bar{x}$ is given by $V(x^a, \bar{x})$. They call $x^* \in X$ a symmetric Nash equilibrium choice if $V_1(x^*, x^*) = 0$. Their most important findings can be summarized as in this

Proposition 5 (Cooper and John (1988)) *In an economy with pure global interactions as described above, (i) strategic complementarity is necessary for multiple equilibria; (ii) strategic complementarity is necessary and sufficient for multipliers; (iii) given multiple equilibria and global positive spillovers ($V_2(x^a, \bar{x}) > 0$), equilibria can be Pareto ranked by the equilibrium action choice.*

This is a nice result for static games with purely global interactions. It suggests a way to generate multiplicity by focusing only on microeconomic fundamentals. However, Glaeser and Scheinkman (2003) show through the following example that the necessity of strategic complementarity is not robust in richer local interaction structures.

Example 3 (Glaeser and Scheinkman (2003)) *There are two sets of agents $\{\mathbb{A}_1\}$ and $\{\mathbb{A}_2\}$, with n agents in each set. For agents of a given set, the reference group consists of all agents of the other set. For $a \in \mathbb{A}_k$,*

$$\bar{x}^a = \frac{1}{n} \sum_{b \in \mathbb{A}_l} x^b$$

There are two goods, and the relative price is normalized to one. Each agent has an initial income of one unit, and his objective is to maximize

$$u^a(x^a, \bar{x}^a) = \log x^a + \log(1 - x^a) + \frac{\lambda}{2}(x^a - \bar{x}^a)$$

Only the first good exhibits social interactions, and agents of each set want to differentiate from the agents of the other set ($\lambda > 0$). There is NO strategic complementarity; an increase in the action of others (weakly) decreases the marginal utility of an agent's own action. An equilibrium

[20] The term strategic complements was introduced by Bulow, Geanokoplos, and Klemperer (1985) in the context of multimarket oligopoly. Following BGK, Cooper and John say that strategic complementarities arise if an increase in one player's strategy increases the optimal strategy of the other players. More precisely, if $V_{12}(x^a, \bar{x}) > 0$ which in turn implies that $\frac{\partial x^*(\bar{x})}{\partial \bar{x}} > 0$.

(same choice for agents of the same set) is described by a pair (x, y) *of actions for each set such that*

$$1 - 2x + \lambda x(1-x)(x-y) = 0$$
$$1 - 2y + \lambda y(1-y)(y-x) = 0$$

Clearly $x = y = 1/2$ *is always a symmetric equilibrium. If* $\lambda < 4$*, it is the unique equilibrium. For* $\lambda > 4$*, there are other equilibria too, e.g., for* $\lambda = 4.040404$*,* $(x, y) = (.55, .45)$ *is an equilibrium. Consequently, so is* $(x, y) = (.45, .55)$*. Hence existence of multiple equilibria does not imply strategic complementarity.*

Glaeser and Scheinkman argue further that one can have a unique equilibrium (thanks to their MSI condition) in the presence of strategic complementarities ($u^a_{12}(x^a, \bar{x}^a, \theta^a, p) > 0$) and obtain multiplier effects. Consider the effect of a change in the first component, p_1, of the parameter vector p. They show that if the partial of each agent's best response w.r.t p_1 is positive, one can write the impact of that effect on optimal choices as

$$\frac{\partial x}{\partial p_1} = (I + H)\left(\frac{\partial g^1}{\partial p_1}, \ldots, \frac{\partial g^n}{\partial p_1}\right)'$$

where H is a matrix with non-negative elements. This is equivalent to saying that there is a **social multiplier**. Holding all other choices constant

$$dx^a = \frac{\partial g^a(\bar{x}^a, \theta^a, p)}{\partial p_1} dp_1$$

whereas in equilibrium it becomes

$$\left[\frac{\partial g^a(\bar{x}^a, \theta^a, p)}{\partial p_1} + \sum_b H_{ab} \frac{\partial g^b(\bar{x}^b, \theta^a, p)}{\partial p_1}\right] dp_1$$

Then,

$$d\bar{x} = \frac{1}{n}\left[\sum_a \frac{\partial g^a(\bar{x}^a, \theta^a, p)}{\partial p_1} + \sum_{a,b} H_{ab} \frac{\partial g^b(\bar{x}^b, \theta^a, p)}{\partial p_1}\right] dp_1$$

which says that, average action changes not only because of the direct change in individual best responses (first sum inside the brackets), but also because of the interactive change (second sum inside brackets) in the behavior of all agents, of the same sign ($H_{ab} \geq 0$). The multiplier effect through shocks works in a similar fashion. The size of the social multiplier depends on the slope of the best response functions with respect to average choice. If this slope gets close to unity, one can generate arbitrarily large social multiplier effects. This is a serious concern, as argued in Glaeser, Sacerdote, and Scheinkman (2003), since it is common practice in empirical work in the social sciences to infer individual behavior from aggregate data.

Jovanovic (1987) is a critique along the same lines. He shows that any amount of aggregate variation can be generated by ‘unique’ equilibria of games where shocks are independent across agents. He argues that this is in stark contrast to standard macroeconomic ‘aggregate shocks’ methodology, either with intrinsic aggregate shocks (see Kydland and Prescott (1982)) or with extrinsic aggregate shocks (see Cass and Shell (1983)). Hence the modeling choice, just on theoretical grounds, in favor of aggregate shocks approach rather than the local interactions approach is moot.

Bisin, Horst and Özgür (2006) show through their pure conformity economies that the presence of local interactions is not sufficient for the existence of social multiplier effects. Consequently, social multiplier effects might not be robust to changes in the nature of interactions. When agents are rational and interact locally, multiplier effects may disappear and that the magnitude of social multipliers (in both static and dynamic settings) depends on the amount of local information people possess about the types of other individuals. For an interesting survey on the existence of social multipliers and their dependence on the nature of interactions see Burke (2008).

Jovanovic (1987) argues that no model is perfect and left-out variables (unobserved) might appear as aggregate shocks. A related point is in Glaeser and Scheinkman (2003), who argue that in the presence of unobserved heterogeneity, it may be impossible to distinguish between a large multiplier and multiple equilibria. It might be that either (i) within the same parameter regime, small differences in fundamentals across areas are amplified by strong social multiplier effects; or (ii) there are unaccounted influences (latent variables) that affect the aggregates in different ways in two different geographical areas.

One last important remark for this section is that, in the presence of multiple equilibria, the general framework of structural inference as presented in Koopmans (1949) (see also Koopmans and Reiersøl (1950)) is inadequate since it assumes that once the exogenous data is specified, the endogenous variables can be uniquely determined. Jovanovic (1989) warns that the set of distributions on observable outcomes that are consistent with a given structure can be quite large and consequently the model might be hard (if not impossible) to identify. For recent progress on this issue in the literature, see Bisin et al. (2009) and Galichon and Henry (2009).

A different kind of identification problem arises when one asks the question: Does one observe similar behavior by people within a group due to local interaction or due to the fact that people with similar characteristics choose to be part of the same group? (see e.g., Manski (1993)). This is an incredibly important question that permeates the social sciences. I will talk a little about how recent advances in the dynamic theory of local interactions might help in Section 3.6

2.4 Discrete Choice Models

There exists a number of social phenomena for which the discrete choice framework has been considered as a natural outlet, e.g., teenage pregnancy, technology adoption decisions, decision to enter or exit a market, staying in or dropping out of school, etc. Moreover, data sets rich in quantitative individual information did not exist before, and data on individual behavior have generally been categorized in a coarse yes-no, 0-1 fashion. Although this is changing now due to the advances in survey and collection technologies and availability of micro-level data, discrete choice methodology is widely used. For all these reasons, I will present two of the mostly cited studies in the literature on social interactions with discrete choice, namely Brock and Durlauf (2001) and Glaeser, Sacerdote, and Scheinkman (1996).

Brock and Durlauf (2001a)

Brock and Durlauf's (BD henceforth) framework is the basic machinery behind many models of binary choice with social interactions in the literature. I follow here their journal article closely although they present their theoretical and econometric methods in numerous other review and survey articles, e.g. Brock and Durlauf (2001b, 2002, 2007), Durlauf (1997, 2004, 2008). Their contribution is a framework to study economies with **global** (mean-field) **interactions** where agents interact through the population mean action. Their model being mathematically equivalent to logistic models of discrete choice (Blume (1993), Brock (1993)) is easily amenable to econometric analysis using the tools of the logistic models (see McFadden (1984) for the latter). This being a survey of theoretical contributions, I will not go into the details of their econometric analysis, although I will provide references for readers interested in further reading.

BD consider economies with a finite number of agents, each making a one-time choice x^a (simultaneously) from the common binary choice set $X = \{-1, 1\}$. Let $x := (x^b)_{b \in \mathbb{A}}$ and $x^{-a} := (x^b)_{b \neq a}$. Agent a's preferences are represented by

$$V(x^a) = u(x^a) + S(x^a, \mu^a(x^{-a})) + \theta(x^a)$$

where u is what they call the *private utility*, S the *social utility*, and θ a random utility term, i.i.d. across agents whose realization is known to agent a at the time of his decision. Let $m^{a,b} := E^a[x^b]$ be the expected value of agent b's choice with respect to agent a's subjective belief μ^a and $\bar{m}^a := (|\mathbb{A}| - 1)^{-1} \sum_{b \neq a} m^{a,b}$ be the average expected choice among agents other than a with respect to a's subjective belief of their likelihood. They impose a particular form of strategic complementarity on social utility, i.e.,

$$\frac{\partial^2 S(x^a, \bar{m}^a)}{\partial x^a \partial \bar{m}^a} = J > 0$$

which means that the marginal social utility to agent a's of choosing any action increases by an increase in the average expected action (from his point of view) in the rest of the population. They consider two classes of preferences depending on their parametric choice of the social utility. First, what they call the *proportional spillovers* case

$$S(x^a, \bar{m}^a) = J x^a \bar{m}^a$$

and second, the *pure conformity* case (as in Akerlof (1997) and Bernheim (1994))

$$S(x^a, \bar{m}^a) = -\frac{J}{2}(x^a - \bar{m}^a)^2$$

Finally, they assume that the error terms $\theta(-1)$ and $\theta(1)$ are independent and extreme-value distributed, so that the differences are logistically distributed

$$Prob\,(\theta(-1) - \theta(1) \leq x) = \frac{1}{1 + exp(-\beta x)}$$

Equilibrium analysis. They first study the equilibrium of the model under the proportional spillovers assumption and claim later that the same results apply under the pure conformity case. They argue that it is well known that under the extreme values hypothesis for $\theta(x^a)$, x^a will obey

$$Prob(x^a) = \frac{\exp\left(\beta(u(x^a) + Jx^a\bar{m}^a)\right)}{\sum_{\hat{x}^a \in \{-1,1\}} \exp\left(\beta(u(\hat{x}^a) + J\hat{x}^a\bar{m}^a)\right)}$$

As $\beta \to \infty$, the effect of the error term on agent a's choice vanishes; as $\beta \to 0$, the above probability goes to .5 regardless of anything else. Under the i.i.d assumption, the joint probability of the choice profile can be written

$$Prob(x) = \frac{\exp\left(\beta \sum_{a \in \mathbb{A}}(u(x^a) + Jx^a\bar{m}^a)\right)}{\prod_{a \in \mathbb{A}} \sum_{\hat{x}^a \in \{-1,1\}} \exp\left(\beta \sum_{a \in \mathbb{A}}(u(\hat{x}^a) + J\hat{x}^a\bar{m}^a)\right)}$$ [21]

Since choices are binary, one can write $u(x^a) = hx^a + k$ where h and k are chosen such that $k + k = u(1)$ and $-h + k = u(-1)$ and this way linearize the expression of the joint distribution above to get

$$\begin{aligned} E(x^a) &= 1 \cdot \frac{\exp\left(\beta h + \beta J(|\mathbb{A}| - 1)^{-1} \sum_{b \neq a} m^{a,b}\right)}{\exp\left(\beta h + \beta J(|\mathbb{A}| - 1)^{-1} \sum_{b \neq a} m^{a,b}\right) + \exp\left(-\beta h - \beta J(|\mathbb{A}| - 1)^{-1} \sum_{b \neq a} m^{a,b}\right)} \\ &\quad -1 \cdot \frac{\exp\left(\beta h + \beta J(|\mathbb{A}| - 1)^{-1} \sum_{b \neq a} m^{a,b}\right)}{\exp\left(\beta h + \beta J(|\mathbb{A}| - 1)^{-1} \sum_{b \neq a} m^{a,b}\right) + \exp\left(-\beta h - \beta J(|\mathbb{A}| - 1)^{-1} \sum_{b \neq a} m^{a,b}\right)} \\ &= \tanh(\beta h + \beta J(|\mathbb{A}| - 1)^{-1} \sum_{b \neq a} m^{a,b}). \end{aligned} \tag{14}$$

Finally, impose rational expectations, i.e., for all $a, b \in \mathbb{A}$, $m^{a,b} = E(x^b)$. Since the tanh function is continuous and the support of choices is $\{-1, 1\}^{\mathbb{A}}$, an **equilibrium** exists, in particular it is unique if $\beta J < 1$, i.e.,

$$m^* = \tanh\left(\beta h + \beta J m^*\right) \tag{15}$$

In the rest of the paper, they study the behavior of the above fixed point equation under different regimes for the parameters. In particular, they give conditions under which there are multiple equilibria

[21] BD argue that their structure is equivalent to the mean field version of the Curie-Weiss model of statistical mechanics, presented in Ellis (1985).

Proposition 6 *(i) If $\beta J > 1$ and $h = 0$, there exist three roots: one positive, one equal to zero, one negative.*

(ii) If $\beta J > 1$ and $h \neq 0$, there exists a threshold H such that

(a) for $|\beta h| < H$, there exist three roots, one of which has the same sign as h, the others possessing opposite sign;

(b) for $|\beta h| > H$, there exists a unique root with the same sign as h.

Letting m^*_- be the mean choice level in which the largest percentage of agents choose -1, m^*_+ as the one where they choose +1, and m^* as the root between the two, they can characterize the limiting percentage of positive and negative choices as a function of the parameters $\beta, h,$ and J. They then argue that if one reinterprets the equation (15) as a difference equation with m_t as a function of m_{t-1}, one can show that, if there is a unique fixed point to that equation, that fixed point is locally stable. However, if there are three roots, the fixed points m^*_- and m^*_+ are locally stable but the third one is locally unstable. For the rest, they focus on stable equilibria solely.

Since for any equilibrium, with positive probability there are agents who like the other equilibrium better and those who like the current one better, they cannot Pareto rank equilibria ex-post. However, using the ex-ante symmetry of the agents, they show that when $h > 0$ (< 0), the equilibrium associated with $m^*_+(m^*_-)$ gives a higher level of expected utility than the one associated with $m^*_-(m^*_+)$. Moreover, when $h = 0$, the two equilibria give the same level of expected utility. Note that their analysis so far was based on expected average choice and expected individual choices. However, they show that as the economy gets large ($|\mathcal{A}| \to \infty$), the sample average population choice weakly converges to the expected population choice.

Local Interactions. BD argue that their global interaction model is nested into a class of local interaction models where each agent interacts directly with only a finite number of others in the population. In other words, global interaction models are simply special cases of local interaction models.[22] They study a symmetric local interaction model where each neighborhood has the same size and each individual puts equal weights on his neighbors' choices. They find that

Theorem 7 *Any equilibrium expected individual and average choice level m for the global interactions model is also an equilibrium expected individual and average choice in a homogeneous local interactions model.*

To be clear, they add that local interactions model being more general, can exhibit a variety of other equilibria that one does not obtain in the global case.

[22] I discuss this issue carefully in dynamic environments in Section 3.3. When the population is finite, the claim is true. When the population is infinite, one should take care of some mathematical difficulties. Please see Section 3.3 for more details. Also see Sec 2.2 for a similar analysis in static models of continuous choice.

Multinomial Choice. Concerned with the limitations of the binary choice setting in theoretical and econometric studies, BD extend their model to a multiple discrete choice environment; see Brock and Durlauf (2002). They find similar existence and multiplicity results and provide conditions under which the interactions effects can be identified.

Social Planner's Problem. One would expect a planner to make choices on behalf of the population to maximize the sum of individual utilities, as it is done in economics. Unfortunately, the sum of extreme-value distributed random variables is not extreme-value distributed. To resolve this issue, BD assume that the error term for the planner's problem, $\theta(x)$ is itself independent and extreme-value distributed across all possible configurations of x. Given this assumption however, it is the planner's error term that will determine x rather than the original individual terms. BD remark that one can interpret this as noise in planner's ability to calculate tradeoffs between individual utilities. They look at the limit behavior of the joint law for planner's allocation under proportional spillovers and conformity effects. They find that under the first, equilibria are inefficient and can be Pareto ranked. Under the second though, equilibrium m^* with the same signs as h is efficient. BD argue that this is due to the fact that utility specification under pure conformity punishes large deviations from the mean in a harsher way than the proportional spillovers case does.

Finally, BD discuss some extensions of their model where social utility might depend on past society behavior, might be asymmetric around the mean level, and private utility might be heterogeneous. Most importantly, they study **identification** of their model's parameters, provide sufficient conditions for identification and discuss why their conclusions are different than the ones in Manski's (1993) analysis of identification in linear models with social interactions. Interested reader should look at their section 6. Moreover, for good reviews of identification of social interactions in general, see Blume et al. (2010, chapter 23), Blume and Durlauf (2005), Brock and Durlauf (2007), Graham (2010, chapter 29), and Manski (1993, 2007).

Glaeser, Sacerdote, and Scheinkman (1996)

Glaeser, Sacerdote, and Scheinkman (GSS henceforth) are after an explanation for the excess variation in crime rates across time and geography relative to the observable heterogeneity in individual and area characteristics. To that end, they build a model of local interactions and empirically test it using data on crime rates across US provided by FBI (six time points between 1970 and 1994), and crime rates across New York City, by precinct, from the 1990 Census. They find that less than 30% of variation in cross-city or cross-precinct crime rates can be explained by observable differences in local area attributes. Moreover, they argue that positive covariance across agents' decisions is the only explanation for the discrepancy between the variance in crime rates observed and the variance predicted by local characteristics (*social multiplier*). They then show that their empirical findings are consistent with the existence of such local interactions. Finally, they build an interaction index (strength of local interaction) for different categories of crime and show that the value of the index is decreasing in the severity of crime.

This being a theory survey, I will present their baseline model which is inspired by the voter models in statistical mechanics. There are $2n+1$ agents, $\mathbb{A} = \{-n, \ldots, 0, \ldots, n\}$, placed on a circle. Common action set is $X = \{0, 1\}$, 1 denoting committing a crime. The interaction structure is one-sided, i.e., $N(a) = a - 1$. Type set is $\Theta = \{0, 1, 2\}$. Type 1 and 0 agents are fixed. They are *criminal* and *non-criminal* types, respectively. Their choices are their types. Type 2 agents are *marginals* who are affected by the choice of their neighbors. Their choices are equal to the choices of their neighbors. The probabilities of being of type 0 and 1 are p_0 and p_1 respectively and are i.i.d across agents. The proportion of agents who are of fixed types in a city is $\pi = p_0 + p_1$.

Conditional on the realization and perfect observation of the types in the economy, there is a unique Nash equilibrium: one observes sequences of 1s and 0s of varying sizes depending on the realization of fixed agents' locations. Then, each agent's action x^a can be thought of as a binary random variable and the process $\{x^a, -\infty < a < \infty\}$ as stationary, with expected value $p := p_1/(p_0 + p_1)$. GSS argue that the presence of fixed types create enough mixing in the system so that a central limit behavior arises.[23] Let

$$S_n := \sum_{|a| \leq n} \left(\frac{x^a - p}{2n+1} \right)$$

be the empirical average of the deviations from the mean crime rate for a sample of $2n+1$ agents. Then, as the population gets large, we have

$$\begin{aligned} \lim_{n \to \infty} \mathbb{E}\left[(S_n \sqrt{2n+1})^2 \right] &= \lim_{n \to \infty} (2n+1)\, \mathbb{E}[S_n^2] \\ &= \operatorname{var}(x^0) + 2 \lim_{n \to \infty} \sum_{a=1}^{n} \operatorname{cov}(x^0, x^a) \end{aligned} \tag{16}$$

The choices of 0 and a are perfectly correlated conditional on the event that there does not exist a fixed type in the segment $[1, a]$. The probability of this event is $(1 - p_0 - p_1)^a$. If the complement of that event occurs, the covariance between these two agents is zero since they become independent. Since x^0 follows a binomial process, its variance is $\operatorname{var}(x^0) = p(1-p)$. Hence, (16) can be written

[23] Choices of any two agents $a > b$ are independent conditional on the existence of a fixed type between them. The probability of that type nonexisting goes to zero exponentially as $b - a \to \infty$. Consequently, the process $\{x^a, -\infty < a < \infty\}$ satisfies a *strong mixing* condition with exponentially declining bounds and central limit theorem obtains. See for example Fristedt and Gray (1997), p. 563.

as

$$\begin{aligned}\mathrm{var}(x^0) + 2\lim_{n\to\infty}\sum_{a=1}^{n}\mathrm{cov}(x^0, x^a) &= p(1-p) + 2\lim_{n\to\infty}\sum_{a=1}^{n} p(1-p)(1-p_0-p_1)^a \\ &= p(1-p) + 2p(1-p)\frac{(1-p_0-p_1)}{(p_0+p_1)} \\ &= p(1-p)\frac{(2-\pi)}{\pi} =: \sigma^2\end{aligned}$$

Since $\pi > 0$, $0 < \sigma^2 < \infty$ and central limit behavior obtains

$$S_n\sqrt{2n+1} \to N(0, \sigma^2)$$

and they have a very clean expression of how the average crime rates will be distributed in a largely populated area. They interpret $(2-\pi)/\pi$ as the *degree of imitation*. They estimate this latter using their data to measure the proportion of the population that is immune to social influences, π, which in turn provides and index of the degree of social interaction across cities and across crimes.

GSS also provide a dynamic extension of their framework with two-sided interactions $N(a) = \{a-1, a+1\}$, in order to motivate their analysis of the variance of the distribution of crime as the stationary distribution of a myopic infinite horizon dynamic local interaction process. At time $t = 0$, each agent chooses the action 1 independently with probability $p > 0$. Then, each agent is determined either to be "frozen" or not with probability $\pi > 0$. Frozen agents are stuck in a set S with their time 0 choices. Pick an agent $a \notin S$. Associated with a is an independent Poisson process with mean time 1. At each arrival, a will choose from among the actions of his neighbors with equal probability. This defines the stochastic process $\{x_t^a\}_{a\in\mathbb{A}}$. They show that for given parameters (p, π), for any n, there exists a limit probability measure $\mu_n(p, \pi)$ defined over choices $\{x^a : |a| \leq n\}$. Moreover, for $m > n$, $\mu_m(p, \pi)$ agrees with $\mu_n(p, \pi)$ on $\{x^a : |a| \leq n\}$.

GSS then consider the normalized sum $1/\sqrt{2n+1}\sum_{|a|\leq n}(x^a - p)$ as before. They show that the presence of frozen agents, as before, provides enough mixing to obtain central limit behavior for the normalized sum, and the asymptotic qualitative behavior of the variance matrix is exactly as in the model in the text.

3 Dynamic Models

The theoretical literature studying local interactions is not yet fully integrated into the standard dynamic economic analysis of equilibrium. Economists using the tools of the mainstream equilibrium analysis have predominantly built static models of local interactions until very recently.[24]

[24] The literature on dynamics modeled as population games and the later developed local interaction games with adaptive, myopically best-responding agents is discussed in Section 3.5.1.

The reason for this choice is the complexities involved in dynamic models with forward looking agents forming rational expectations: interaction structures embody complicated non-convexities to render standard fixed point arguments invalid (see Durlauf (1997)).

In many social phenomena of economic significance, static modeling leads to mis-specification or underestimation of social effects. For example, Binder and Pesaran (2001) study life-cycle consumption of agents who interact globally, through average consumption within local group they belong to. They consider conformism, altruism, and jealousy as forms of interaction and conclude that analyzing decisions of agents in static rather than dynamic settings is misleading. Moreover, they argue that dynamic social interactions coupled with habit formation or prudence might help solve the excess smoothness and excess sensitivity of consumption puzzles.

Recent empirical literature shifted attention to dynamic models, e.g. Kremer and Levy (2008) on the dynamically persistent detrimental effect of having drinking roommates on student GPAs; Carrell, Fullerton, and West (2008) on persistent group effects among randomly assigned students at the United States Air Force Academy; Cutler and Glaeser (2007) on the dynamic effects of smoking bans in the work place; DeCicca, Kenkel, and Mathios (2008) on the effect of cigarette taxes on smoking initiation and cessation cycles.

The theoretical counterpart of this body of work is in its infancy. There is a ton of questions to study and proper modeling to be done. In this section, I will first touch upon the early models of interactions with myopic dynamics. Then, I will present and study economies with forward looking rational agents and the implied rational expectations dynamics. As I mentioned in the Introduction and since I know more about them, I will focus my attention more on the latter, forward looking rational expectations economies.

3.1 Baseline Dynamic Model

The physical environment is the same as in the baseline model of Section 2.1 with the following additions: evolution of preferences, neighborhood structure, and individual and reference group characteristics. Similar to before, our theoretical object of study is a class of local interaction economies, represented by the tuple $\mathcal{E} = (\mathbb{A}, X, \Theta, N, P, u, \beta, T)$.

Interaction horizon is represented by T and can be finite or infinite. $\beta > 0$ is the common discount factor agents use to discount future utilities. With the dynamic specification, one can allow for interactions in a 'changing environment', that is

$$N : \mathbb{A} \times \{1, 2, \ldots, T\} \to 2^{\mathbb{A}}$$

meaning that the reference group $N_t(a)$ of agent a can change from one period to another. It is important to notice that even then, this is not about group formation but about a commonly known and exogenously given law that governs the changes in the environment of agents.[25]

[25] I will mention a few things on group formation along the lines of the selection and sorting in Section 4

Given the neighborhood structure, the contemporaneous preferences of an agent $a \in \mathbb{A}$ are represented by a **utility function** u^a of the form

$$\left(x_{t-1}^a, x_t^a, \{x_t^b\}_{b \in N_t(a)}, \theta_t^a, p(x_t)\right) \to u^a \left(x_{t-1}^a, x_t^a, \{x_t^b\}_{b \in N_t(a)}, \theta_t^a, p(x_t)\right)$$

Last period choice x_{t-1}^a is introduced as an argument to study endogenous preference formation (e.g., habits, addiction, norms) due to social interactions. As it is clear from the representation, the type of an agent a is a stochastic process. The most common assumption is to assume that it is i.i.d across agents and time. In principal, one can allow for intertemporal exogenous persistence, in which case the information structure becomes very important.

3.2 Rational Forward-looking Interactions

This body of work argues that the study of equilibrium dynamics of economies with local interactions, by allowing for rational expectations of forward looking agents, may elucidate several important aspects of social interactions. An example of a specific socio-economic environment might be helpful to illustrate the usefulness of the proper forward looking equilibrium analysis of dynamic economies in the presence of local interactions[26]: Consider a teenager evaluating the opportunity of dropping out of high school. His decision will depend on the conditions of the labor market, and in particular on the relevant wage differentials, which requires him to form expectations about the wage and labor conditions he will face if he graduates from high school. The teenager's decision might depend also on the school attendance of a restricted circle of friends and acquaintances: dropping-out is generally made simpler if one's friends also drop-out (local interactions). But as the decision of dropping out depends on the teenager's expectations of the wage differential, it will also in part depend on his consideration of the possibility that, for instance, while his friends have not yet dropped out of school, they soon will, perhaps even motivated by his own decision of dropping out. Similarly, our teenager might decide to stay in school even if most of his friends dropped out, if he has reason to expect their decision to be soon reversed. The teenager will form expectations about his friends' future behavior as well as about the future wage rate.

In the rest of this section, I will present two recent models of local interactions with forward looking rational agents, namely Bisin, Horst and Özgür (2006) and Bisin and Özgür (2010). They are both important methodological contributions in the direction of integrating local interactions models into the standard dynamic economic analysis of equilibrium. I presented BHO's study of static economies with local interactions in Section 2.2. Here I will present their analysis of infinite horizon economies with local interactions.

[26] The example comes from Bisin, Horst, and Özgür (2006).

Bisin, Horst and Özgür (2006)

BHÖ study infinite-horizon economies with local interactions and with infinitely-lived agents. While agents may interact locally, they are forward looking, and their choices are optimally based on the past actions of the agents in their neighborhood, as well as on their anticipation of the future actions of their neighbors. Their major contributions might be summarized as

(i) This is the first formal study in the literature, of rational expectations equilibria of infinite horizon economies with local interactions. They provide conditions under which such economies have rational expectations equilibria which depend in a Lipschitz continuous manner on the parameters. They show that such conditions impose an appropriate bound on the strength of the interactions across agents.

(ii) For a class of dynamic economies with *Conformity Preferences* (see e.g. Akerlof (1997), Brock and Durlauf (2001a), Bernheim (1994)), they consider local as well as global (e.g., global externalities, general equilibrium effects) equilibrium dynamics and characterize long run behavior of those joint processes. Moreover, they show formally that when agents have rational expectations, the effect of the local conformity component of their preferences on their equilibrium actions is reduced significantly with respect to the case in which agents are myopic.

Formally, BHÖ study the following class of economies: a countably infinite number of agents $\mathbb{A} = \mathbb{Z}$, common compact and convex action and type spaces X and Θ. Let $\mathbf{X^0} := \{x = (x^a)_{a \geq 0}\}$. Each agent $a \in \mathbb{A}$ interacts with his immediate neighbor $N(a) = a + 1$ only (*local interactions*). Information is incomplete, that is, each agent observes only his own type and the history of past choices in the economy before making a choice[27] They focus attention on Markov perfect equilibrium in pure strategies as the equilibrium concept.[28] Each agent $a \in \mathbb{A}$ believes that everyone else in the economy at any period t makes choices according to a given choice function $g : \mathbf{X^0} \times \Theta \to X$ in the sense that

$$x_t^a = g(T^a x_{t-1}, \theta_t^a) \quad \text{where} \quad T^a x_{t-1} = \{x_{t-1}^b\}_{b \geq a}.$$

Denote by $\pi_g(T^a x_{t-1}, \theta_t^a)$ the conditional law of the action x_t^a, given the previous configuration x_{t-1}. This latter induces a Feller kernel (a law of motion) for the system in the sense that

[27] BHÖ argue that this is not restrictive and that all the results they obtain apply in a straightforward fashion to the complete information economies.

[28] This is for reasons of parsimony and clarity of the message delivered. Moreover, by choosing to focus on MPEs, they actually make their task more difficult since there are no generally accepted conditions that guarantee the existence of pure strategy MPEs in any game. More generally, one can of course, consider more sophisticated punishment strategies, and coordination devices to achieve particular behaviors.

$$\Pi_g(x;\cdot) := \prod_{a=1}^{\infty} \pi_g(T^a x;\cdot). \tag{17}$$

The kernel Π_g describes the stochastic evolution of the process of individual states $\{(x_t^a)_{a>0}\}_{t\in\mathbb{N}}$. In this case, for any initial configuration of individual states $x \in \mathbf{X^0}$ and for each initial type θ_1^0, agent 0's optimization problem is given by

$$\max_{\{x_t^0\}_{t\in\mathbb{N}}} \left\{ \int u(x_1^0, x^0, x_1^1, \theta_1^0)\pi_g(Tx; dx^1) + \sum_{t\geq 2} \beta^{t-1} \int u(x_t^0, x_{t-1}^0, x_t^1, \theta_t^0)\Pi_g^t(Tx; dx_t)\nu(d\theta_t^0) \right\} \tag{18}$$

The value function associated with this dynamic choice problem is defined by the fixed point of the functional equation

$$\begin{aligned} V_g(x_{t-1},\theta_t^0) = V_g(x_{t-1}^0, Tx_{t-1},\theta_t^0) \;&=\; \max_{x_t^0\in X} \left\{ \int u\left(x_{t-1}^0, x_t^0, y_t^1, \theta_t^0\right) \pi_g(Tx_{t-1}; dy_t^1) \right. \\ &\left. + \beta \int_{\mathbf{X^0}\times\Theta} V_g(x_t^0, \hat{x}_t, \theta^1)\Pi_g(Tx_{t-1}; d\hat{x}_t)\nu(d\theta^1) \right\} \end{aligned} \tag{19}$$

and the maximizer of this problem is denoted

$$\begin{aligned} \hat{g}_g\left(x_{t-1},\theta_t^0\right) \;&=\; \arg\max_{x_t^0\in X} \left\{ \int u\left(x_{t-1}^0, x_t^0, y_t, \theta_t^0\right) \pi_g(Tx_{t-1}; dy_t) \right. \\ &\left. + \beta \int V_g(x_t^0, \hat{x}_t, \theta^1)\Pi_g(Tx_{t-1}; d\hat{x}_t)\nu(d\theta^1) \right\}. \end{aligned} \tag{20}$$

Finally, what they mean by *equilibrium* is stated in the following

Definition 7 *A symmetric Markov perfect equilibrium of a dynamic economy with forward looking and locally interacting agents is a map $g^* : \mathbf{X^0}\times\Theta \to X$ such that*

$$\begin{aligned} g^*\left(x_{t-1},\theta_t^0\right) \;&=\; \arg\max_{x_t^0\in X} \left\{ \int u\left(x_{t-1}^0, x_t^0, y_t, \theta_t^0\right) \pi_{g^*}(Tx_{t-1}; dy_t) \right. \\ &\left. + \beta \int V_{g^*}(x_t^0, \hat{x}_t, \theta^1)\Pi_{g^*}(Tx_{t-1}; d\hat{x}_t)\nu(d\theta^1) \right\}. \end{aligned} \tag{21}$$

BHÖ establish a series of results on the existence and the convergence of the equilibrium process. Such results require conditions on the policy function, and hence are not directly formulated as conditions on the fundamentals of the economy. They then introduce an economy with conformity preferences which is amenable to study. For this economy they show that their general conditions are satisfied, and hence are not vacuous.

In order to state a general existence result for equilibria in dynamic random economies with forward looking interacting agents, they introduce the notion of a **correlation pattern**.

Definition 8 *For $C > 0$, let*

$$L_+^C := \{\mathbf{c} = (c_a)_{a\in\mathbb{N}} \;:\; c_a \geq 0, \; \sum_{a\in\mathbb{A}} c_a \leq C\}$$

denote the class of all non-negative sequences whose sum is bounded from above by C. A sequence $\mathbf{c} \in L_+^C$ will be called a correlation pattern with total impact C.

Each correlation pattern $\mathbf{c} \in L_+^C$ gives rise to a metric

$$d_{\mathbf{c}}(x, y) := \sum_{a\in\mathbb{N}} c_a |x^a - y^a|$$

that induces the product topology on $\mathbf{X^0}$. Thus, $(d_{\mathbf{c}}, \mathbf{X^0})$ is a compact metric space. In particular, the class

$$\mathrm{Lip}_{\mathbf{c}}^C := \{f : \mathbf{X^0} \to \mathbb{R} : |f(x) - f(y)| \leq d_{\mathbf{c}}(x, y)\}$$

of all functions $f : \mathbf{X^0} \to \mathbb{R}$ which are Lipschitz continuous with constant 1 with respect to the metric $d_{\mathbf{c}}$ is compact in the topology of uniform convergence.

The constant c_a may be viewed as a measure for the total impact the current action x^a of the agent $a \geq 0$ has on the optimal action of agent $0 \in \mathbb{A}$. Since $C < \infty$, we have $\lim_{a\to\infty} c_a = 0$. Thus, the impact of an agent $a \in \mathbb{A}$ on the agent $0 \in \mathbb{A}$ tends to zero as $a \to \infty$. In this sense they consider *economies with weak social interactions*. The quantity C provides an upper bound for the total impact of the configuration $x = (x^a)_{a\geq 0}$ on the current choice of the agent $0 \in \mathbb{A}$. Given this structure, a general existence result for symmetric Markov perfect equilibria in dynamic economies with local interaction is given in the following

Theorem 8 (Existence and Lipschitz continuity) *Assume that there exists $C < \infty$ such that the following holds:*

(i) For any $\mathbf{c} \in L_+^C$, for all $\theta^0 \in \Theta$ and for each choice function $g(\cdot, \theta^0) \in \mathrm{Lip}_{\mathbf{c}}^C$, there exists $F(\mathbf{c}) \in L_+^C$ such that the unique policy function $\hat{g}_g(\cdot, \theta^0)$ which solves (20), is Lipschitz continuous with respect to the metric $d_{F(\mathbf{c})}$ uniformly in $\theta^0 \in \Theta$.

(ii) The map $F : L_+^C \to L_+^C$ is continuous.

(iii) We have $\lim_{n\to\infty} \|\hat{g}_{g_n}(\cdot, \theta^0) - \hat{g}_g(\cdot, \theta^0)\|_\infty = 0$ if $\lim_{n\to\infty} \|g_n - g\|_\infty = 0$.

Then the dynamic economy with local interactions has a symmetric Markov perfect equilibrium g^* and the function $g^*(\cdot, \theta^0)$ is Lipschitz continuous uniformly in θ^0.

Once the existence of an MPE is obtained, a natural question to ask is how the economy behaves in the long run given that individuals make choices according to the choice function whose existence it is shown. To that effect, BHÖ study the limit properties of the t-fold iteration of the stochastic kernel $\Pi_{g^*}(\mathbf{x};\cdot)$. To that end, they introduce the vector $r^* = (r^*_a)_{a\in\mathbb{A}}$ defined as

$$r^*_a := \sup\{\|\pi_{g^*}(x;\cdot) - \pi_{g^*}(y;\cdot)\| : x = y \text{ off } a\}. \tag{22}$$

Here, $\|\pi_{g^*}(x;\cdot) - \pi_{g^*}(y;\cdot)\|$ denotes the total variation of the signed measure $\pi_{g^*}(x;\cdot) - \pi_{g^*}(y;\cdot)$, and $x = y$ off a means that $x^b = y^b$ for all $b \neq a$. The next theorem gives sufficient conditions for convergence of the equilibrium process to a steady state. Its proof follows from a fundamental convergence theorem by Vasserstein (1969).

Theorem 9 (Ergodicity) *If $\sum_{a\in\mathbb{A}} r^a_{g^*} < 1$, then there exists a unique probability measure μ^* on the infinite configuration space $\mathbf{X}$ such that, for any initial configuration $\mathbf{x} \in \mathbf{X}$, the sequence $\Pi^t_{g^*}(\mathbf{x};\cdot)$ converges to μ^* in the topology of weak convergence for probability measures.*

Example 4 (Conformity Economies) *These are dynamic extensions of economies with local interactions that we saw in example 2. Let $X = \Theta = [-1,1]$, and assume that $\mathbb{E}\theta^0_t = 0$, and that an agent $a \in \mathbb{A}$ only observes his own type θ^a. If the instantaneous utility function takes the quadratic form*

$$u\left(x^a_{t-1}, x^a_t, x^{a+1}_t, \theta^a_t\right) = -\alpha_1\left(x^a_{t-1} - x^a_t\right)^2 - \alpha_2\left(\theta^a_t - x^a_t\right)^2 - \alpha_3\left(x^{a+1}_t - x^a_t\right)^2 \tag{23}$$

for positive constants α_1, α_2 and α_3, then BHÖ show that the hypotheses of Theorem 8 are satisfied hence the economy has a symmetric Markov perfect equilibrium g^*. Moreover, the policy function g^* can be chosen to be of the linear form

$$g^*(x,\theta^0) = c^*_0 x^0 + \gamma\theta^0 + \sum_{b\geq 1} c^*_b x^b$$

for some positive sequence $\mathbf{c}^* = (c^*_a)_{a\geq 0}$ and some constant $\gamma > 0$. For the same class of economies, one can also show convergence to a unique steady state. Consider the representation

$$g^*(x;\theta^0) = c^*_0 x^0 + \gamma_2\theta^0 + \sum_{a\geq 1} c^*_a x^a.$$

of the policy function g^*. For any two configurations $x, y \in \mathbf{X^0}$ which differ only at site $a \in \mathbb{A}$ we have

$$|g^*(x,\theta^0) - g^*(y,\theta^0)| \leq c^*_a|x^a - y^a|,$$

Thus, assuming that the taste shocks are uniformly distributed on $[-1,1]$ we obtain

$$|\pi_{g^*}(x;A) - \pi_{g*}(y;A)| \leq 2c^*_a$$

for all $A \in \mathcal{B}([-1,1])$, and so $\sum_{a\geq 0} r^a_{g^*} < 1$ if $\sum_{a\geq 0} c^*_a < \frac{1}{2}$. Hence the conditions of Theorem 9 are satisfied, which means that we obtain convergence to a steady state whenever α_1 is big enough and if α_3 is small enough, i.e., if the interaction between different agents is not too strong.

I mentioned in the beginning of this section that BHÖ also study local and global equilibrium dynamics together. I reserved this for Section 3.3. Finally, for their comparison of equilibria generated by myopic and forward looking behavior, see Section 3.5.2

Bisin and Özgür (2010)

Bisin and Özgür (BÖ henceforth) take up the study of dynamic economies from where they left and fill out many of the gaps they left for future research in Bisin, Horst, and Özgür (2006). Their major contribution is twofolds:

(i) Existence, uniqueness, parametric continuity, ergodicity, and welfare properties of equilibria of dynamic conformity economies with general interaction structures.

(ii) Most importantly, the **identification** of local interaction effects (from hidden correlated effects) at the population, exploiting in a novel way the dynamic equilibrium behavior.

BÖ focus their attention on economies with **conformity preferences**. These are environments in which each agent's preferences incorporate the desire to conform to the choices of agents in his reference group. They argue that in many relevant social phenomena, in fact, the effects of preferences for conformity are amplified by the presence of limits to the reversibility of dynamic choices. This is of course the case for smoking, alcohol abuse and other risky teen behavior, which are hard to reverse because they might lead to chemical addictions. In other instances, while addiction per se is not at issue, nonetheless behavioral choices are hardly freely reversible because of various social and economic constraints, as is the case, for instance, of engaging in criminal activity. Finally, exogenous and predictable changes in the composition of groups, as e.g., in the case of school peers at the end of a school cycle, introduce important non-stationarities in the agents' choice. These non-stationarity also call for a formal analysis of dynamic social interactions. In order to provide a clean and simple analysis of dynamic social interactions in a conformity economy, they impose strong(er than required) but natural assumptions. Namely

1. Time is discrete and is denoted by $t = 1, \dots, K$. They allow both for infinite economies ($K = \infty$) and economies with an end period ($K < \infty$).

2. Let $\mathbb{A} := \mathbb{Z}$ represent a general *social space*. Each agent interacts with his immediate neighbors, i.e., for all $a \in \mathbb{A}$, $N(a) := \{a-1, a+1\}$.[29]

[29]BÖ argue that the method of proof does not rely on the dimensionality of the social space. Hence, social space can be represented, at an abstract level, by any d-dimensional integer lattice. Similarly for the action and type spaces. The only thing that they cannot dispense with for their analysis is the convexity of the choice problem and the interiority of the optimal trajectories.

3. The contemporaneous preferences of an agent $a \in \mathbb{A}$ are represented by the utility function

$$u(x_{t-1}^a, x_t^a, x_t^{a+1}, x_t^{a-1}, \theta_t^a) \;:=\; -\alpha_1 (x_{t-1}^a - x_t^a)^2 - \alpha_2 (\theta_t^a - x_t^a)^2 \\ -\alpha_3 (x_t^{a-1} - x_t^a)^2 - \alpha_3 (x_t^{a+1} - x_t^a)^2$$

   where α_1, α_2, and α_3, are positive constants.

4. Let $X = \Theta = [\underline{x}, \bar{x}] \subset \mathbb{R}$, where $\underline{x} < \bar{x}$, $E[\theta] = \int \theta d\nu =: \bar{\theta} \in (\underline{x}, \bar{x})$.

Let $\mathbf{X} := \{x = (x^a)_{a \in \mathbb{A}} : x^a \in X\}$ and $\mathbf{\Theta} := \{(\theta^a)_{a \in \mathbb{A}} : \theta^a \in \Theta\}$. The timing of the type process and agents' choices are as in Bisin, Horst, and Özgür (2006). Each agent $a \in \mathbb{A}$ believes that everyone else in the economy makes choices according to a given choice function $g : \mathbf{X} \times \mathbf{\Theta} \times \{1, \ldots, K\} \to X$. Similar to BHÖ, they are after

Definition 9 *A symmetric Markov Perfect Equilibrium of a dynamic economy with social interactions is a measurable map $g^* : \mathbf{X} \times \mathbf{\Theta} \times \{1, \ldots, K\} \to X$ such that for all $a \in \mathbb{A}$ and for all $t = 1, \ldots, K$*

$$g^*_{K-(t-1)}(T^a x_{t-1}, T^a \theta_t) = \arg \max_{x_t^a \in X} E\Bigg[u\left(x_{t-1}^a, x_t^a, \{g^*_{K-(t-1)}(T^b x_{t-1}, T^b \theta_t)\}_{b \in N(a)}, \theta_t^a\right) \\ + \beta V^{g^*}_{K-t}\left(\{g^*_{K-(t-1)}(T^b x_{t-1}, T^b \theta_t\}_{b \in \mathbb{A}}, \theta^I_{t+1})\right)\Bigg] \quad (24)$$

Their first result shows that for finite horizon economies, there exists a unique MPE, which is characterized in a simple and intuitive way: agent a's optimal choice each period is a convex combination of last period's observed choices, today's observed type realizations, and the average type in the economy. Moreover, those weights capture an important phenomenon: Although fundamentally, agent's preferences are affected only by their immediate friends, in equilibrium their optimal choices are affected by (hence correlated with) choices of everyone in the economy in a decaying fashion, that is, farther an agent b is from an agent a, lesser weight agent a puts on the last choice of agent b, as can be seen in Figure 1 for strong (high α_3) and mild (low α_3) interactions. For an infinite horizon economy, the existence of a stationary MPE that behaves similarly is guaranteed. All this is summarized formally in

Theorem 10 (Existence - Complete Information) *Consider an economy with conformity preferences and complete information.*

1. *If the time horizon is finite ($K < \infty$), then the economy admits an a.s. unique symmetric Markov Perfect Equilibrium $g^* : \mathbf{X} \times \mathbf{\Theta} \times \{1, \cdots, K\} \mapsto X$ such that for all t, for all $(x_{t-1}, \theta_t) \in \mathbf{X} \times \mathbf{\Theta}$*

$$g^*_{K-(t-1)}(x_{t-1},\ \theta_t) = \sum_{a \in \mathbb{A}} c^a_{T-t+1}\, x^a_{t-1} + \sum_{a \in \mathbb{A}} d^a_{K-(t-1)}\, \theta^a_t + e_{T-t+1}\, \bar{\theta} \qquad \mathbb{P} - a.s.$$

   where $c^a_\tau, d^a_\tau, e_\tau \geq 0$, $a \in \mathbb{A}$, and $e_\tau + \sum_{a \in \mathbb{A}} (c^a_\tau + d^a_\tau) = 1$, $0 \leq \tau \leq K$.

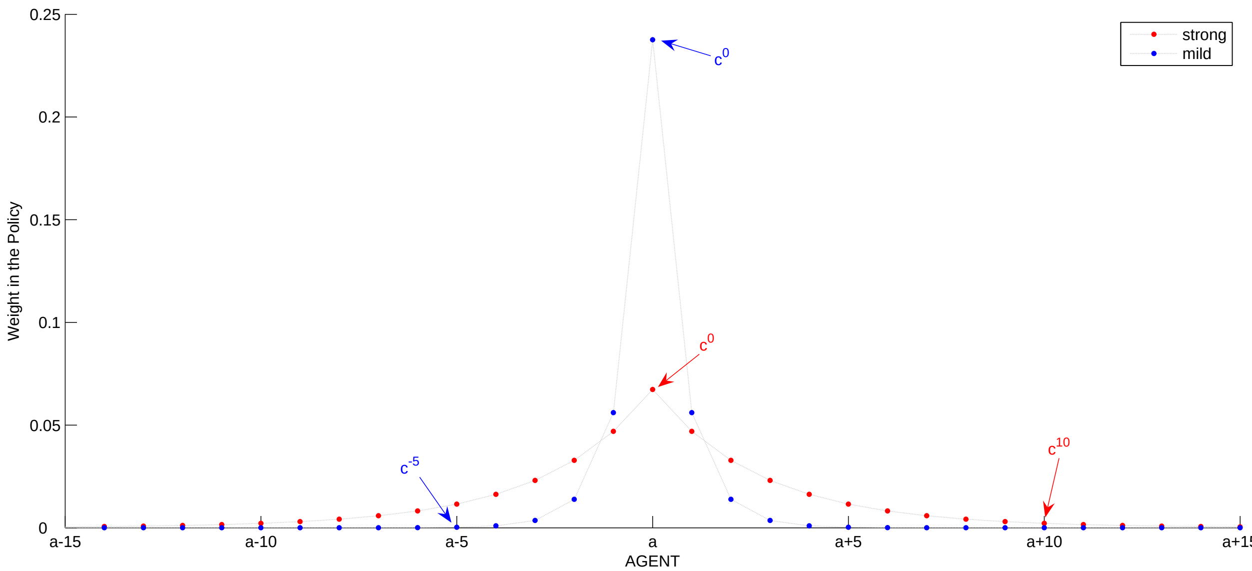

Figure 1: Weights on past history in the stationary policy function.

2. *If the time horizon is infinite* ($K = \infty$), *then the economy admits a symmetric Markov Perfect Equilibrium* $g^* : \mathbf{X} \times \boldsymbol{\Theta} \mapsto X$ *such that*

$$g^*(x_{t-1},\ \theta_t) = \sum_{a \in \mathbb{A}} c^a\, x^a_{t-1} + \sum_{a \in \mathbb{A}} d^a\, \theta^a_t + e\,\overline{\theta}$$

where $c^a, d^a, e \geq 0$, *for* $a \in \mathbb{A}$, *and* $e + \sum_{a \in \mathbb{A}} (c^a + d^a) = 1$.[30]

Their method of proof is constructive and the recursive map which induces the symmetric policy function at equilibrium provides a direct and useful computation method which they repeatedly exploit to characterize equilibria and to produce comparative dynamics exercises. All these are summarized in the following

Theorem 11 (Recursive Computability) *Consider a* $K(<\infty)$*-period economy with conformity preferences* ($\alpha_i > 0,\ i = 1, 2, 3$) *and complete information. The coefficients* $(c^*_s, d^*_s, e^*_s)_{s=1}^K$ *of the sequence of Markov polices whose existence is guaranteed by Theorem 10 are* computable recursively *as the unique fixed points of the the recursive maps* $T_s : L_c \to L_c$, $s = 1, \ldots, K$, *i.e.,*

[30]The theorems in this section can be extended with straightforward modifications to the case of incomplete information. Moreover, several assumptions can be relaxed while guaranteeing existence. In particular, the symmetry of the neighborhood structure can be substantially relaxed, adapting the analysis of Horst and Scheinkman (2006) to our dynamic environment.

for each $a \in \mathbb{A}$

$$\begin{aligned} c_s^{*a} &= \Delta_s^{-1}\left(\alpha_1 \mathbf{1}_{\{a=0\}} + \sum_{b\neq 0}\gamma_s^b c_s^{*a-b}\right) \\ d_s^{*a} &= \Delta_s^{-1}\left(\alpha_2 \mathbf{1}_{\{a=0\}} + \sum_{b\neq 0}\gamma_s^b d_s^{*a-b}\right) \\ e_s^{*a} &= \Delta_s^{-1}\left(\mu_s + e_s^* \sum_{b\neq 0}\gamma_s^b\right) \end{aligned} .$$

where $\Delta_K, (\gamma_K^a)_{a\neq 0}, \mu_K$ are the total effects on agent 0's marginal utility of an infinitesimal change in x_1^0, $(x_1^a)_{a\neq 0}$, and $\bar{\theta}$ respectively evaluated at the equilibrium path. Moreover, $\lim_{K\to\infty}(c_K^*, d_K^*, e_K^*) = (c_\infty^*, d_\infty^*, e_\infty^*)$ exists and is the coefficient sequence of an equilibrium policy function for the infinite horizon economy.

Before closing this section, I would like to mention the welfare effects of local interactions. BÖ argue that the equilibrium allocations of conformity economies are generally Pareto inefficient. Individuals do not internalize the impact of their choices on other agents today and in the future. The presence of social interactions might call for **policy interventions**. Most interventions (Medicaid, Food Stamps, Social Security Act) are thought to work on the fundamentals but generated social norms, e.g., welfare stigma. Well targeted policy interventions on a few agents might spill over other agents (multiplier effect); see Moffit (2001).

BÖ study the problem of a social planner whose objective is to maximize the ex-ante expected well-being of a generic agent, by restricting the planner to the same class of symmetric choice rules, treating individuals equally. They show that, in his optimal choice, in order to internalize the externalities generated by individual choices on other individuals, the planner puts more weight on an agent's neighbors' type realizations and past choices than the generic agent does in a laissez-faire equilibrium. Hence

Theorem 12 (Inefficiency of equilibrium) *Equilibrium of an economy with conformity preferences (finite or infinite horizon) is generically inefficient.*

One of the most important contributions of Bisin and Özgür (2010) is their study of the identification of social determinants of individual choice behavior. BÖ argue, in a novel way, that rational expectations dynamics might help the social scientist disentangle interaction effects from correlated effects. This is material for Section 3.6.

3.3 Local vs. Global Dynamics

This section extends the analysis of dynamic economies with local interactions to economies in which interactions have an additional **global** component. In particular, I present the methodology proposed in Bisin, Horst, and Özgür (2006) to study economies in which each agent's preferences depend on the average action of all agents. They argue that such dependence might occur, for instance, if agents have preferences for *social status*. Similarly, preferences to adhere to aggregate norms of behavior, such as specific group cultures, give rise to global interactions. More generally, global interactions could capture other externality as well as price effects. When the population is finite, global interactions are nested straightforwardly in local interaction models. When the number of agents is infinite, there are technical subtleties.

Consider a class of dynamic conformity economies, in which the preferences of each agent $a \in \mathbb{A}$ also depend on the average action of the agents in the economy,

$$p(x) := \lim_{n\to\infty} \frac{1}{2n+1} \sum_{a=-n}^{n} x^a,$$

when the limit exists. Let $\mathbf{X_e}$ denote the set of all configurations such that the associated average action exists:

$$\mathbf{X_e} := \left\{ x \in \mathbf{X} : \exists\ p(x) := \lim_{n\to\infty} \frac{1}{2n+1} \sum_{a=-n}^{n} x^a \right\}.$$

The preferences of the agent $a \in \mathbb{A}$ in period t are described by the instantaneous utility function $u : \mathbf{X_e} \times \Theta \to \mathbb{R}$ of the conformity class

$$\begin{aligned} & u\left(x_{t-1}^a, x_t^a, x_t^{a+1}, \theta_t^a, p(x_t)\right) \\ = \ & -\alpha_1 \left(x_{t-1}^a - x_t^a\right)^2 - \alpha_2 \left(\theta_t^a - x_t^a\right)^2 - \alpha_3 \left(x_t^{a+1} - x_t^a\right)^2 - \alpha_4 \left(p(x_t) - x_t^a\right)^2 \end{aligned}$$

for some positive constants α_i, $i = 1, 2, 3, 4$. As before, assume that $X = \Theta = [-1, 1]$ and that $\mathbb{E}\theta^0 = 0$. Assume also that information is incomplete so that an agent $a \in \mathbb{A}$ at time t only observes his own type θ_t^a, and all agents' past actions. Similar to before, a symmetric Markov perfect equilibrium of this economy is defined as in

Definition 10 *Let $\mathbf{x} \in \mathbf{X_e}$ be the initial configuration of actions. A symmetric Markov perfect equilibrium of a dynamic economy with local and global interactions is a map $g^* : \mathbf{X}^0 \times \Theta \times X \to X$ and a map $F^* : X \to X$ such that:*

$$\begin{aligned} g^*\left(x_{t-1}, \theta_t^0, p_t\right) = \ & \arg\max_{x_t^0 \in X} \left\{ \int u\left(x_{t-1}^0, x_t^0, y_t^1, \theta_t^0, p_t\right) \pi_{g^*}(Tx_{t-1}; dy_t^1) \right. \\ & \left. + \beta \int V_{g^*}\left(x_t^0, \hat{x}_t, \theta^1, p_{t+1}\right) \Pi_{g^*}(Tx_{t-1}; d\hat{x}_t) \nu(d\theta^1) \right\} \end{aligned} \tag{25}$$

and

$$p_{t+1} = F^*(p_t),$$

and

$$p_1 = p(x) \quad \text{and} \quad p_t = p(x_t) \ \text{almost surely.}$$

At a symmetric Markov perfect equilibrium, apart from anticipating play according to the policy function g^*, all agents rationally expect the sequence of average actions $\{p(x_t)\}_{t\in\mathbb{N}}$ to be determined recursively via the map F^*. BHÖ argue that two fundamental difficulties arise in studying existence of an equilibrium of a dynamic economy with local and global interactions

(i) The endogenous sequence of average actions $\{p(x_t)\}_{t\in\mathbb{N}}$ might not be well-defined for all t (that is, $\mathbf{x}_t$ might not lie in $\mathbf{X_e}$ for some t).

(ii) Even when $x_t \in \mathbf{X_e}$, an agent's utility function depending on the action profile $\mathbf{x_t}$ in a global manner through the average action $p(\mathbf{x}_t)$ will typically not be continuous in the product topology. Thus, standard results from the theory of discounted dynamic programming cannot be applied to solve the agent's dynamic optimization problem in (25).

In order to circumvent these difficulties, BHÖ use a two-step approach in which each agent treats the global dynamic process as exogenous and independent of choices, and makes optimal choices using a stationary policy that depends on last period choices, current type realizations, and the current value of the exogenous global process. They then show that the mean choice dynamics in the economy is independent of particular choice configurations and agrees with the exogenous global dynamics.[31] To be able to do that, they show that

(i) The endogenous sequence of average actions $\{p(\mathbf{x}_t)\}_{t\in\mathbb{N}}$ exists almost surely if the exogenous initial configuration $\mathbf{x}$ belongs to $\mathbf{X_e}$, and that

(ii) It follows a deterministic recursive relation.

More specifically, they first consider an economy where the agents' utility depends on some *exogenous* quantity p, constant over time and show that agents behave optimally according to a symmetric policy function g^* that has the following linear form

$$g^*(x, \theta^0, p) = e_0^* x^0 + \epsilon\theta^0 + \sum_{b\geq 1} e_b^* x^b + A(p) \tag{26}$$

where the correlation pattern $\mathbf{e}^* = (e_a^*)_{a\geq 0}$, and the constant $\epsilon > 0$ are independent of p. So, a change in p has only a direct effect on the chosen action but does not affect the dependency of the action on the realized agent's type nor on the neighbors' actions. It is this independence property that allows BHÖ to separate the local and global equilibrium dynamics. To that effect, they extend the analysis to the case in which the agents' utility depends on some *exogenous* but time-varying quantity $\{p_t\}_{t\in\mathbb{N}}$ described in terms of a possibly non-linear recursive relation of the form

$$p_{t+1} = F(p_t) \quad \text{for some continuous function} \quad F: X \to X. \tag{27}$$

Since F is continuous, an agent's optimization problem can again be solved using standard results from the theory of discounted dynamic programming. They show that, in this case, the optimal

[31] For similar separation arguments applied in the context of static economies with locally and globally interacting agents, see Horst and Scheinkman (2006) in Section 2.2. See also Föllmer and Horst (2001) for another application to interacting Markov chains.

symmetric policy function that each agent uses takes the form

$$g(x,\theta^0,p_1)=e_0^*x^0+\epsilon\theta^0+\sum_{b\geq 1}e_b^*x^b+\sum_{t\geq 1}h_t^*p_t$$

for some correlation pattern $e^*=(e_a^*)_{a\geq 0}$ and a positive sequence $h^*=(h_t^*)_{t\geq 1}$. These sequences can be chosen independently of F and satisfy

$$\sum_{a\geq 0}e_a^*+\sum_{t\geq 1}h_t^*\leq 1.$$

Finally, BHÖ show that the recursive structure of $\{p_t\}_{t\in\mathbb{N}}$ is preserved when each element of the sequence is required to be endogenously determined as the average equilibrium action: $p_t=p\left(\mathbf{x}_t\right)$, for any t, at the equilibrium configuration $\mathbf{x}_t$. To that effect, take a continuous function $F:X\to X$ that determines recursively the sequence $\{p_t\}_{t\in\mathbb{N}}$ as in (27). Assume that the exogenous initial configuration $\mathbf{x}$ has a well defined average $p:=p(\mathbf{x})$, that is, assume that $\mathbf{x}\in\mathbf{X_e}$. Let $F^{(t)}$ denote the t-fold iteration of F so that $p_t=F^{(t)}(p)$. Since the agents' types are independent and identically distributed, it follows from the law of large numbers that the average equilibrium action in the following period is almost surely given by

$$\lim_{n\to\infty}\frac{1}{2n+1}\sum_{a=-n}^{n}g(T^ax,\theta^a,p)=C^*p+\sum_{t\geq 1}h_t^*F^{(t)}(p)=:G(F)(p).$$

Thus, the average action in period $t=2$ exists almost surely if the average action in period $t=1$ exists, and an induction argument shows that the average action exists almost surely for all $t\in\mathbb{N}$. In order to establish the existence of an equilibrium, they first show that there exists a continuous function F^* such that, with $p_1:=p(\mathbf{x})$ we have

$$p_2:=F^*(p_1)=G(F^*)(p_1).$$

Finally, their main result can be summarized in

Theorem 13 *For the dynamic economy with local and global interactions introduced in this section, the following hold:*

1. *The economy has a symmetric Markov perfect equilibrium* (g^*,F^*) *where* $g^*:\mathbf{X}^0\times\Theta\times X\to X$ *and* $F^*:X\to X$.

2. *In equilibrium, the sequence of average actions* $\{p(x_t)\}_{t\in\mathbb{N}}$ *exists almost surely.*

3. *The policy function* g^* *can be chosen of the linear form*

$$g^*(x,\theta^0)=e_0^*x^0+\epsilon\theta^0+\sum_{b\geq 1}e_b^*x^b+B^*(p(x))\tag{28}$$

for some positive sequence $\mathbf{e}^*=(e_a^*)_{a\geq 0}$, *a constant* $\epsilon>0$, *some constant* $B^*(p(x))$ *that depends only on the initial average action.*

One note of precaution: It is important for their analysis in this section that the policy functions are linear. Only in this case, in fact, can the dynamics of average actions $\{p(x_t)\}_{t\in\mathbb{N}}$ can be described in terms of a recursive relation. In models with more general local interactions, the average action typically is not a sufficient statistic for the aggregate behavior of the configuration x; hence a recursive relation typically fails to hold, as shown e.g., by Föllmer and Horst (2001). In such more general cases, the analysis must be pursued in terms of empirical fields. Interested reader should see Föllmer and Horst (2001). I also found the book by

3.4 Ergodicity

Ergodicity is the mathematical study of measure-preserving transformations in general and long-term average behavior of systems in particular. Economists are especially interested in the long-run properties of equilibrium distributions of dynamic economies and games. In this section I will present existing results on the (non-)ergodicity of equilibria of economies with social interactions. Readers interested in general discussions of ergodicity should consult Halmos (1956), Petersen (1989) (ch. 1 is a gentle introduction to the kind of questions ergodic theory is concerned with), Nadkarni (1998), and Walters (2000). I also found the book by Meyn and Tweedie (1993) extremely helpful, especially when one deals with Markov processes with uncountable state spaces. For random field models, see Kindermann and Snell (1980), Liggett (1985), and Spitzer (1971).

Durlauf (1993) studies the dynamics of local interlinkages between sectors in an economy and the possibility of multiple long-run aggregate behavior emerging from the same local interactions between sectors. He uses the mathematics of random field theory to formulize his approach. Formally, at the local level, equilibrium technology, production, and capital accumulation choices give rise to

$$\mu\left(x_t^a = 1 \mid x_{t-1}^b = 1, \forall b \in N(a) \cup \{a\}\right)$$

a system of local conditional probabilities of choosing a particular technology (either 0 or 1) given last period technology choices of neighboring sectors (sectors that have linkages with sector a). Using a result by Dobrushin (1968), he shows that there exists at least one joint probability distribution on overall technology choices consistent with the local rules. The major economic questions Durlauf are after come from the theory of economic growth: do economies with identical technologies and preferences converge to the same long run average output? Can leading sectors tip off the economy from a low level equilibrium to a high level equilibrium due to strong interlinkages, as proposed by Hirschman (1958)? Durlauf argue that although previous models of increasing returns to scale and imperfect competition (e.g. Diamond (1982), Cooper and John (1988), Romer (1986), Lucas (1988)) have generated multiple equilibria, these latter are constant steady states entirely determined by initial conditions. Durlauf show that one can incorporate meaningful stochastic dynamics, interesting cyclic behaviour, volatility of output at the cross-section of industries into the model and still characterize conditions under which the economy is ergodic with a unique invariant distribution, independent of the initial conditions. He argues that these conditions are: (i) positive and non-degenerate conditional probabilities, and (ii) not too strong local spillovers.

Durlauf's dynamics are backward looking because periodic production choices can be solved independently due to the one-lagged Markov assumption on the dependence of current production on past technology choices. Nevertheless, the analysis using random field theoretical tools to obtain aggregate probability laws consistent with sectoral stochastic linkages is novel. Bisin, Horst, and Özgür (2006) are interested in a similar issue but with fully rational forward-looking agents. At an abstract level, agents interact only with their immediate neighbours, but anticipate the future choices of these latter. Equilibrium conditions give rise to a system of conditional laws that depend on past choices on the equilibrium path. Given the conditionally independent nature of these rules, there is a unique consistent global phase. BHÖ show that under relatively mild local interactions, there exists a unique long-run joint probability distribution on the space of individual configurations to which the sequence of finite horizon global phases converge, independent of the initial conditions of the economy.

For the class of conformity economies that they study, Bisin and Özgür (2010) show that no matter how strong the strength of local interactions can be, given a stationary equilibrium policy, the Markov process jointly induced by that policy and the sequence of individual shocks converges to a unique long run probability distribution on the space of configurations. This is due to the fact that the optimal policy is a stationary trade-off between dependence on the past, adaptation to the stochastic shocks, and co-ordination on the mean shock. In the long run, iteration of the same policy makes the dependence on the initial conditions die off. Consequently, it is the path of realized shocks that determine the state of the economy. Since the system is ergodic, the empirical distribution on all such paths converge to the same probability distribution in the long run.

In this section, I focused my attention on the ergodicity of dynamic economies with local interactions and its implications on the uniqueness of long-run limit distributions. A local interaction system can be ergodic at the cross-section (space) too. We saw the implications of this on the existence of consistent aggregate laws, as presented in Horst and Scheinkman (2006), in Section 2.2. For similar ideas in the context of population games, see Blume (1993) for a study of stochastic strategy revision processes and their long run properties and see Anderlini (1998) for an application to path dependence in local learning. A quick survey of (non)-ergodicity in economics is Horst (2008).

3.5 Myopia vs. Rationality

3.5.1 Myopic Interactions

Early models of dynamic social interactions mostly subscribe to the **evolutionary** point of view. What distinguish evolutionary from the classical point of view in economics, according to Young (1998), are the concepts of **equilibrium** and **rationality**. In the mainstream economic equilibrium analysis, individual behavior is assumed to be optimal given expectations and expectations are correct, justified by the statistical evidence (**rational expectations**). Agents know their environments, use all information they have to anticipate changes in them and act accordingly. Evolutionary approach treats individuals as **low-rationality agents**. They still **adapt** to changes in their environment. However, they account for neither their actions' impact on the evolution of their environment, nor the repercussions of this latter on their own future well-being. Young argues that they too are interested in equilibrium but that equilibrium can be understood only within a dynamic framework that explains how it comes about, by observing how things look on average over long periods of time. Good surveys of this approach exist. Interested reader should consult Blume (1997), Young (1998 and 2009), Sandholm (2010), and also Burke and Young (2008, chapter 9) for applications to the study of social norms. I am going to give a quick tour of the most cited articles in the literature.

One of the earliest models of local interactions in the social sciences is Schelling (1971). He argues that **segregation** (or separation, or sorting) might happen along many lines: income, sex, education, race, language, color, historical accidents; it might be the result of organizations, communication systems, or correlation with other modes of segregation. He is interested in segregation that results from *discriminatory individual choices*. He assumes that individuals, when making choices, are not capable of generating (often not even conscious of) changes on the aggregate dynamics of the system. **Evolutionary processes** stemming from individual actions bring about those changes in the long run. He first studies a *Spatial Proximity Model*, on a line and on a two-dimensional space. Population (finite) is divided into two permanent and recognizable groups according to color. Individuals are concerned about the proportion of their local neighbors of the opposite color. They each have a particular location at any time and can move if they are not happy with the particular color composition of their current neighborhood. Schelling uses different **behavioral rules** to represent individual choices. In one treatment, everybody wants at least half his neighbors to be of the same color and moves to another location otherwise. The rule about how agents move is deterministic and arbitrary. Nobody anticipates the movements of others (**myopic**) and agents continue moving until there is no dissatisfied agent in the system (**equilibrium**). When modeled on a two-dimensional space, agents move to the most preferred empty spaces available when dissatisfied. Once again, the dynamics come to an end when no one is dissatisfied with their neighborhood composition. In its essence, this is a

local interaction model, with **myopically best-responding** agents. Schelling looks at the segregation (or *clustering*, or *sorting*, or *concentration*) patterns that arise once the dynamics settle: One observes clusters of same color agents living together separated from other groups along well-defined boundaries. One interesting result is that minority tends to become more segregated from majority, as its relative size diminishes. Another is that segregation is more striking as the local demand for like-colored neighbors increases.

He then studies a *Bounded-Neighborhood Model*. This is a **global interaction** model, where each agent's utility is affected by the overall color composition of the neighborhood. Given a distribution of 'tolerance' levels, each agent stays in the neighborhood if the relative proportion of people of opposite color is less than his tolerance level; otherwise, he leaves. At each moment in time, the agents with the lowest tolerance levels leave and new agents with tolerance levels higher than the current composition enter. Schelling looks at the steady state of the induced deterministic dynamic processes. There always exist two stable states involving complete segregation along with a mixed (co-habitation of blacks and whites) state whose stability depends on the tolerance distribution and the relative proportions of blacks and whites. Some interesting results are: co-habitation is more likely with similar tolerance distributions for blacks and whites; in general, for mixed equilibria to emerge, minority must be the more tolerant group. Schelling applies his analysis to *neighborhood tipping* (the inflow of a recognizable new minority into a neighborhood in sufficient numbers to cause the earlier residents to begin evacuating). He argues that main determinants of a tipping phenomenon are whether the neighborhood size is fixed, whether the new entrants are identifiable as a group, the relative sizes of the entrants with respect to the size of the neighborhood, and the availability of alternative neighborhoods for evacuating people.

A large literature using evolutionary methodology as in Schelling (1971), but more formally, studies social interaction in large populations. The common hypothesis is that individuals need not know the total structure of the game but need information on the empirical distribution of strategy choices in the population. Two pillars of this approach are a **population game**, the structure of the global interaction to occur repeatedly, and a **revision protocol**, a myopic procedure that describes who chooses when and how previous choices are revised. A population game and a revision protocol jointly induce **evolutionary game dynamics** that describe how the aggregate behavior in the population changes over time. When the resulting process is ergodic, its long run behavior will focus on a subset of states called the **stochastically stable set**.

Most of the literature focuses on the relation between **risk dominance** (Harsanyi and Selten (1988)) and **stochastic stability**. Kandori, Mailath, and Rob (1993) are the first ones to have established that link. Essentially, they argue that the periodic shocks (mutations or mistakes that are part of the revision protocol) in a 2×2 game reduce the set of long run equilibria by acting as a selection mechanism. Provided that the population is sufficiently large, the risk dominant equilibrium is stochastically stable. Young (1993) shows, using different techniques but the same equilibrium concept, that the connection between risk dominance and stochastic stability is not robust to an increase in the number of strategies in the population game; the resulting stochastically stable equilibrium may be neither risk dominant nor Pareto optimal.

One criticism of this approach is the speed at which an equilibrium is selected in the long run. This process might take too long. Ellison (1993) shows that if agents respond to their immediate neighbors (*local interactions*), the time to reach a stochastically stable state is reduced greatly. Moreover, in large populations with uniform matching, play is determined largely by historical factors; whereas where agents are matched with a small set of agents only, it is more likely that the evolutionary forces determine the long run outcome. Blume (1993) studies local interaction dynamics on integer lattices. He characterizes stationary distributions and the limit behavior of these dynamic systems. He relates his results to equilibrium selection as in the rest of the literature and also introduces statistical mechanics techniques to study this kind of strategic interaction. Blume (1995) extends these results to $K \times K$ games when players update using a myopic best response rule. Finally, Morris (2000) looks at the possibility of spread of a behavior initially played by a small subset of the population to the whole population through local interactions. He shows that maximal **contagion** happens in the presence of sufficiently uniform local interactions and when the number of agents one can reach in k steps is not exponential in k.

3.5.2 Does it matter?

Does it matter to model interactions myopically rather than rationally? Does the modeling choice (rational vs. myopic) affect the results that one gets significantly? The answer that Bisin, Horst, and Özgür (2006) and Bisin and Özgür (2010) give is that myopic models have the general tendency to **overestimate** the local interaction effects relative to the rational models. The main idea is that a myopic agent is unable to anticipate the effect of his current action change on others' behavior, on the evolution of the system, and the repercussions of these latter on his future well-being, whereas a rational agent anticipates and incorporates these effects into his optimal choice. Consequently, a rational agent is more immune to local behavioral and environmental changes than a myopic agent.

This idea is nicely presented in Bisin, Horst, and Özgür (2006) using their example in Section 3.2. BHÖ study a simple two-period version of their conformity economies under two distinct hypotheses: **myopia** and **full rationality** (see Section 4.3, p. 98 of their paper). They find two differences between the behavior of myopic and rational agents: (i) whereas the myopic agent is backward-looking by basing his choice on the past choices of his immediate neighbors only, the fully rational agent's choice is based on the past choices of all agents. This creates long cross-sectional correlation terms. But most importantly (ii) the fully rational choice is more weighted on the the mean shock than the past actions: a rationally anticipating agent will try to smooth out local behavioral dependencies by anticipating that other agents will get a chance to change their actions next period. This further limits the component of local conformity in the choice of agents in the economy.

A similar criticism is found in Blume (1997). Blume argues that one of the most important barriers to the application of population game techniques to serious economic models is the assumption of **myopia**. The separation between choice and dynamics due to myopia makes the analysis of population games models particularly simple. But economic decision makers are typically concerned about the future as well as the present. Consequently, they try to forecast the strategy revision process, and take account of these forecasts when searching for the best response at a strategy revision opportunity. If there is any connection between the forecasts and the actual behavior of the strategy revision process, such as the hypothesis that expectations are rational, then the dynamic behavior of the strategy revision process cannot be simply computed from the choice rule. The framework used in the population games literature, to study stigma and enforcement of social norms, subscribe to the myopic formulation; hence it misses the richness of the account of individual choice that standard dynamic economic analysis offers. For instance, Blume argues, it would be hard to formulate a question about the effect of punishment duration in that framework. He points out that dispensing with the myopia hypothesis and recognizing players as intertemporal decision makers models would allow evolutionary game theory to be applicable to serious problems in the social sciences.

Blume (2003)

To exemplify such applicability, Blume (2003) models **stigma** and **social control**. Blume notes that stigma is in essence a dynamic phenomenon. Its costs are born in the future, and the magnitude of those costs are determined by the future actions of others. Hence, he **rejects myopia** and he models dynamic stigma costs as a population game (*global interactions*) with **forward looking** agents. This is a very nice and novel paper. Individuals in the model can entertain random criminal opportunities. There are two types of costs: a one-time utility cost if caught and a flow cost of stigma, when 'marked' as a criminal, that is increasing in the relative ratio of the unmarked population (*imposers of stigma*). Stigma ends at a random time when the agent gets 'unmarked'. Blume's agents perceive not only the immediate and current cost flow effects of their actions on themselves but also the externalities they generate on others and their repercussions on themselves in the future through the evolution of the marking and unmarking processes. Blume shows that apart from the *neoclassical effect* (Becker (1968)) of decreasing criminal activity, an increase in the arrest probability has a *social interaction effect*: it increases the number of tagged individuals which in turn reduces the stigma effects of being tagged as a criminal. Similar reasoning applies to the probability of getting untagged. Consequently, stigma costs of long duration will lead to increased crime rates!

3.6 Rational Dynamics and Identification

There are statistical problems that arise in the estimation of social interactions. Firstly, it is difficult to correctly identify individuals' reference groups. Moreover, one should distinguish between three effects in understanding group behavior (Manski (1993)): (1) correlation of individual characteristics, (2) influence of group characteristics on individuals, and (3) the influence of group behavior on individual behavior. The equilibrium allocations of economies with local interactions are in general Pareto inefficient because local interactions are a form of direct preference externalities. As a consequence, the presence of local interactions might call for policy interventions. Most policy interventions such as Medicaid, Food Stamps, Social Security Act are thought to operate on the fundamentals. However, there is documented evidence that responses of welfare recipients generated norms, and unexpected community responses due to social interactions (Moffitt (2001)). Thus, identifying the existence and nature of social interactions are of utmost importance for efficient policy implementation.

The question of identification goes back, in economics, to Pigou (1910), Schultz (1938), Frisch (1928, 1931, 1935, 1934, and 1938), Marschak (1942), Haavelmo (1944), Koopmans (1949), Koopmans, Rubin, and Leipnik (1950), Wald (1950), Hurwicz (1950). The standard definitons of identification that we still use are owed to Koopmans (1949) and Koopmans and Reiersøl (1950), both of which are very beautiful articles providing clear exhibitions of the main idea. More recent surveys on the topic exist of course; see Rothenberg (1971), Hausman and Taylor (1983), Hsiao (1983), Matzkin (2007), and Dufour and Hsiao (2008). Moreover, Blume and Durlauf (2005), Brock and Durlauf (2007), Manski (2007, 2000), Blume et al. (2010, chapter 23) and Graham (2010, chapter 29) in this volume, and Manski (1993, 2007) are good guides to the main questions pertaining to social interactions. Since the pessimistic view expressed in Manski (1993), there has been progress in the identification literature. Conley and Topa (2002, 2003) compare predictive power of different neighborhood structures to identify the reference groups. Graham (2008) uses excess variance across groups for identification. Davezies et al (2006) use size variation across groups; Bramoullé et al (2009) uses reference group heterogeneity for identification. Other recent contributions include Glaeser and Scheinkman (2001), Graham and Hahn (2005); De Paula (2009), Evans, Oates and Schwab (1992), Ioannides and Zabel (2008), and Zanella (2007).

The main question is easy to state. A **structure** is a specification of both the distribution of variables unobserved by the econometrician and the relationship connecting these latter to the observed variables, which implies a unique probability distribution. A **model** is simply a collection of admissible structures. One says that an admissible structure S is **identifiable** by the model (or that the model **identifies** a given structure) if there exists no other structure S' that induces the same probability distribution on the observable variables.

Bisin and Özgür (2010) argue that dynamic equilibrium processes generated by the actions of rational agents might help identify certain interaction effects. In particular, they are interested in identifying correlated effects (unobserved to the econometrician) from local interactions. They first argue that as suspected by Manski (1993) too, dynamic specification does not necessarily solve the identification problem and the necessary support for a particular intertemporal specification should come from data. They show that in **static** as well as **stationary dynamic** models, reflection problem presented in Manski (1993) kicks back. One interesting specification, Bisin and Özgür argue, is environments where correlated effects follow a stationary law through time whereas observed behavior is non-stationary. Take the question of whether adolescents' substance use is affected by their peers and if there is variation in their propensity to consume addictive substances across grades. If, as it is argued in Hoxby (2000a,b), for instance, the school composition is stationary (with no significant trend), in the short run, and that the core friendship groups have been formed already, any significant variation in adolescent behavior through time must be due to local interactions. This simple observation is due to the **rationality** of the agents in this dynamic environment. A rational agent, if his choice is affected by the choices of his peers, will take into account how much longer he will interact with them. In particular, his propensity to consume due to his peers' consumption must be the lowest in the final year and monotonically higher as one considers earlier years. This is exactly the equilibrium behavior Bisin and Özgür obtain from a finite-horizon dynamic model with local interactions. Consequently, the probability distributions on the observed adolescent behavior generated by the correlated structure and the local interaction structure are different.

4 Concluding Remarks

This paper has presented the current state of affairs in the theoretical literature on local social determinants of individual choice behavior. I discussed a variety of models on each side of many division lines that the literature subscribes to: discrete vs. continuous choice, static vs. dynamic interactions, rational vs. myopic behavior. For all the models I surveyed, I presented findings on equilibrium existence and uniqueness, long run behavior, social multiplier effects and multiple equilibria and identification of interaction effects. There is a lot more to be done on the theoretical front combined with a better understanding of empirical social processes.

One very important issue is the **determination of individual reference groups**. Most of the literature that I surveyed takes the assumption that when interactions are modeled, the relevant reference groups and the nature of the interactions is known to the agents and to the outsiders (read econometrician). However, when doing empirical work, it is not clear whether these assumptions stand up to criticisms. This will probably be a joint effort between better survey data collection and related theorizing.

Another future area of investigation is the use of **proper dynamic models** in empirical work rather than one-shot myopic models. Needless to say, this goes once alongside the availability of rich panel data sets. These latter began to appear and although most of them initially collected by sociologists, economists started to take an interest in them.[32] The proper modeling of dynamics might help identification of interaction effects as I argued in Section 3.6.

One last, but not least, future research area is the joint modeling of **self-selection** (or sorting) and **social interactions**. There already exists a literature on network formation whose dynamic counterpart is in the development stage. The joint modeling of these two phenomena would most probably help the researcher disentangle the interactions part of individual choice behavior by correctly accounting for behavior due to *equilibrium* self-selection or sorting. This latter is due to the fact that sorting behavior of *rational* agents carry information about their attitudes towards particular interaction processes that might follow.

[32]One interesting such data set is the National Longitudinal Study of Adolescent Health (**Add Health**). See http://www.cpc.unc.edu/projects/addhealth for more info.

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