ISSN 0709-9231

CAHIER 8559

Dealing with Moral Hazard and Adverse Selection Simultaneously

by

Georges Dionne
and
Pierre Lasserre

December 1985

Cette étude a été publiée grâce à une subvention du fonds F.C.A.R. pour l'aide et le soutien à la recherche. Ce cahier est aussi publié par :

Centre de recherche sur les transports - Publication 446

ABSTRACT

Although insurers face adverse selection and moral hazard when they set insurance contracts, these two types of asymmetrical information have been given separate treatments sofar in the economic literature. This paper is a first attempt to integrate both problems into a single model. We show how it is possible to use time in order to achieve a first-best allocation of risks when both problems are present simultaneously.

RESUME

Même si les assureurs doivent tenir compte du risque moral et de la sélection adverse lorsqu'ils préparent leurs contrats d'assurance, ces deux formes d'asymétrie d'information ont toujours été traitées séparément dans la littérature économique. Cet article constitue une première tentative d'intégration de ces deux problèmes dans un seul modèle. Nous montrons comment il est possible d'utiliser le temps pour obtenir une allocation optimale des risques lorsque les deux problèmes sont présents simultanément.

Introduction

Although insurers face adverse selection and moral hazard problems simultaneously when they set insurance contracts, these two types of asymmetrical information have been given separate treatments sofar in the economic literature[0]. In the case of moral hazard, the principal knows the characteristics of the agent but cannot observe whether the latter indeed carries out the actions which would have been taken under perfect information. In the case of adverse selection, while the agent cannot affect the states of nature, a problem arises from the fact that the principal does not know the probability which characterizes the agent.

This paper is a first attempt to integrate both information problems into a single model[1]. While such an extension has a theoretical interest in itself, it is also warranted on empirical grounds. Applied studies are still few in this area, but they will not be able to dispense from considering both kinds of information asymmetries in the future. In the insurance literature, the sole significant contribution is that by Dalhby (1983) who studied the relationship between statistical discrimination and adverse selection. He concluded to the presence of adverse selection in the market studied, on the basis of differences in accident frequencies between groups or insurance types. However it is not clear in this study that adverse selection is the only source of resource misallocation and moral hazard might also explain part of the frequency differences. It would clearly be

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[0] See among those who have contributed on single-period contracts in both literatures Pauly (1974), Rothschild and Stiglitz (1976), Stiglitz (1977), Riley (1979), Shavell (1979), Dionne (1982), Arnott and Stiglitz (1983).

[1] After this paper was written, it was pointed out to us that Laffont and Tirole (1984) deal with both problems in a context of public firm regulation. However, their model does not cover insurance because of their assumption of risk neutrality for all agents.

desirable to identify the role of each information problem in future empirical studies, as the solutions proposed usually differ in the case of adverse selection from the case of moral hazard.

Individuals' past experience is a source of statistical information permitting to estimate probabilities of accidents. Recently, many authors have analysed the nature of long-term contracts and have presented conditions permitting long-term contracts to be Pareto superior to a series of short-term (one-period) contracts. Indeed, Radner (1981) and Rubinstein and Yaari (1983) have shown that it is possible to achieve a first-best allocation of risk under moral hazard if there is no discounting and if the number of periods is large[2]. Dionne (1983) and Dionne and Lasserre (1985) have obtained a similar result for the adverse selection problem.

The model presented in this paper is a combination and an extension of the contributions by Rubinstein and Yaari (1983) and Dionne and Lasserre (1985). Indeed we show how it is possible to achieve a first-best allocation of risks when moral hazard and adverse selection problems are present simultaneously. While we draw heavily on the contributions of Rubinstein and Yaari and Dionne and Lasserre, the integration of the two types of information problems turns out to be more than a straightforward exercice.

The characteristics of the model are discussed in Section 1 and the formal model is presented in Section 2. Section 3 gives the main results. In the conclusion we suggest extensions for future research.

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[2]On repeated insurance contracts see also Allen (1985), Hosios and Peters (1985), Rogerson (1985), Radner (1985), Dionne and Lasserre (1984) and Henriet and Rochet (1984).

1. Characteristics of the model

Some important features of the model are underlined here. Let us consider a simple economy with two states of the world ($j = 1, 2$) and two types of individuals ($i = a, b$). In an adverse selection model without moral hazard, since the agents cannot affect their probabilities of accident, each agent is fully described by a single number, his probability of accident p^i. With moral hazard, however, the probability of accident is a function of the level of care $p^i(c^i)$; c^i is not observed by the insurer; neither are the values taken by the function p^i. The set of possible functions, $\{p^a(\cdot), p^b(\cdot)\}$ in our two agent types example, may or may not be known to the insurer. In Rubinstein and Yaari (1983) the insurer knows his customer's probability function, although he does not observe c nor, as a result, the values $p(c)$. In this paper, all that the insurer has to know about $p^i(\cdot)$ is that it is decreasing and convex.

Another issue has to do with the convenient and widely used distinction between low-risk, and high-risk, individuals. It is no longer straightforward here. In fact the best endowed individual b ($p^b(0) < p^a(0)$) may be less effective than a in the production of care so that we may have $p^b(c^b) > p^a(c^a)$ for some range(s) of c as we observe in Figure 1 :

INSERT FIGURE 1 HERE

For example, a skilful driver who does not spend in care activities (repair of defective brakes or lights, use of seatbelts ...) may be a worse risk than a poor driver who does. Furthermore, although identical individuals will behave identically in our model, it is clear that different individuals will not produce the same level of care in general. For reasons which will become clear as the paper proceeds, we choose to characterize individuals according to their full information equilibrium.

Section 3 presents a strategy which achieves a first-best allocation of risk under both information problems. It uses time to provide an incentive for the potential customer (1) to reveal his risk type in the first period and (2) to produce the efficient level of care in all periods. It is shown that this strategy is enforceable and optimal.

2. The model

A monopolistic insurer[3] faces customers who, for simplicity, are assumed to fall into two risk types only ($i \in \{a, b\}$). Each risk type is characterized by a probability of accident $p^i(c^i)$ ($0 < p^i(c^i) < 1$) which depends on the individual's level of prevention activity c^i, with $dp^i/dc < 0$, and $d^2p^i/dc^2 > 0$. This formulation admits as special cases the moral hazard model of Rubinstein and Yaari (1983) for $a = b$, as well as the adverse selection model of Dionne and Lasserre (1985) for c^i given and $p^a > p^b$. Suppose for simplicity that the insurer offers insurance at the actuarial premium[4], $P^i(c^i) = p^i(c^i)q^i$, where q^i is insurance coverage. Then, under perfect information q^i and c^i must be selected so as to

$$\text{Max } p^i(c^i)\ U(S - D + q^i - c^i - p^i(c^i)q^i) + (1 - p^i(c^i))\ U(S - c^i - p^i(c^i)q^i)$$

where U is a strictly increasing and strictly concave function of wealth, D is the loss in case of accident, S is initial wealth and c^i has been expressed in monetary units.

Under the above assumptions, it is well known that the optimum for q^i is full coverage, $q^{i*} = D$. Using this result, c^{i*} is the solution of $-(dp^i/dc)D = 1$, which determines the optimum levels of c^i, c^{a*} and c^{b*}. In general, $c^{a*} \neq c^{b*}$.

[3]For the case of competition, see Kunreuther and Pauly (1985).

[4]This assumption implies zero profits for the monopoly. All our results remain valid under the assumption of profit maximization.

Under private information i and c^i are not observable. Therefore agents of each type have no incentive to produce any care ($c^i = 0$) under the above contract and the less endowed individual has no incentive to reveal this characteristic. In the next section we propose a contract which deals with those problems.

Since we will be concerned with the first-best allocation of risks, we shall use the solution of the perfect information case as a reference to define high and low risks individuals. Therefore the high risk individuals will be those having the highest probability of accident at c^{i*}. We shall assume that type a individuals are the high-risk individuals, so that $p^a(c^{*a}) > p^b(c^{*b})$, which may involve $c^{*a} \gtreqless c^{*b}$, and $U(S - p^a(c^{a*}) - c^{a*}) \gtreqless U(S - P^b(c^{b*}) - c^{b*})$.

3. A contract with announcement and statistical inference

As in Rubinstein and Yaari for the case of moral hazard and in Dionne and Lasserre for the case of adverse selection, time can be used to eliminate inefficiencies associated with private information, even when moral hazard and adverse selection coexist. Both the insurer's offer and the insured's response now take the form of long term strategies. The agents follow a Stackelberg game with the insurer announcing his strategy and the insureds giving a best response. The insurer is committed not to alter the contract in the future; however the contract provides for adjustments in insurance premiums over time as new information becomes available. Let $\tilde{f}$ ang $\tilde{g}^i$ respectively be the strategies, to be specified below, which yield the full information optimum in each period if they are adopted by the insurer ($\tilde{f}$) and the insureds ($\tilde{g}^i$). Any strategy which deviates from $\tilde{f}$ (or $\tilde{g}^i$) will be referred to as f (or g^i). Those strategies are row vectors which specify, for each period, the action(s) to be taken, according to current information :

$$\tilde{f} = (\tilde{f}^1, \tilde{f}^2, \tilde{f}^3, \ldots)$$

$$\tilde{g}^i = (\tilde{g}^{i1}, \tilde{g}^{i2}, \tilde{g}^{i3}, \ldots)$$

where $\tilde{f}^t$ ($\tilde{g}^{it}$) defines the action(s) taken at date t under strategy $\tilde{f}$ ($\tilde{g}^i$).

For the insureds, strategies $\tilde{g}^i$ (i = a, b) are defined as follows. In period 1 (the first period) the insured (1) announces his true probability of accident in full information equilibrium; (2) buys insurance; (3) produces the full information level of prevention c^{i*}. In all subsequent periods, the insured (1) buys insurance; and (2) produces c^{i*}.

For the insurer, strategy $\tilde{f}$ is defined as follows. In period 1, he offers any customer to select, within the set of all possible premiums, the actuarial premium corresponding to that customer in the full information equilibrium. We shall assume that the insurer knows only, about any customer, the signs of the first and second derivatives of $p^i(c^i)$. As a result, any choice within the interval [0, D] is acceptable[5] : $\tilde{f}^1 = [0, D]$.

By choosing a particular premium, the insured is informed that he announces his risk type and that this will have consequences in future periods. In fact, in subsequent periods if the insured has declared to be a risk p^d (chosen a premium of P^d),

$$\tilde{f}^{t+1} \begin{cases} = P^d & \text{if} \quad \sum_{s=1}^{t} \theta_D^s / N(t) < \overline{D}(p^d) + \alpha^{d,N(t)} \\ = P_K \text{ , otherwise,} & \end{cases}$$

[5]As it appears, the announcement allows us to dispense from the assumption that the insurer knows the individual's full information level of care (and risk) equilibrium (as in Rubinstein and Yaari) or that he knows both the full information level of risk and the function p(c) (as in Shavell, 1979).

where θ_D^s is the claim filed in period s $(\theta_D^s \in \{0, D\})$, $N(t) \leq t$ is the number of times the customer has taken insurance from period 1 to t, $\overline{D}(p^d)$ is the expected value of the claim, in any period, by an individual whose probability of accident is p^d, $\alpha^{d,N(t)}$ is a statistical margin of error which depends on the announcement p^d and on N(t), and P_K is a penalty premium. According to this rule, a customer who claims to be a low risk $(p^d = p^b(c^{*b}))$ will pay the actuarial premium corresponding to the optimal (full information) prevention level of a true low risk, unless his average loss exceeds by more than a "reasonable" amount (more than $\alpha^{b,N(t)}$) the expected loss $\overline{D}[p^b(c^{b*})]$. Otherwise he is offered the penalty premium P_K, which involves a lower utility level than under self insurance $(U(S - P_K) < \overline{U}^i$, where $\overline{U}^i$ is the maximum expected utility which can be achieved by choice of c^i under self-insurance).

$\alpha^{d,n}$ is the statistics which underlies the Law of the Iterated Logarithm (see Rubinstein and Yaari for details on its use in this context) :

$$\alpha^{d,n} = ((2\lambda\sigma_d^2 \log\log n)/n)^{\frac{1}{2}}\ , \quad \lambda > 1\ ,$$

where σ_d^2 is the variance of the random variable δ_t^d $(\delta_t^d = 0$, no accident over period t; $\delta_t^d = D$, accident over period t) for an individual whose probability of accident is p^d. In particular, for a low risk individual who produces the full information optimal level of care c^{b*}, and tells the truth, $p^d = p^b(c^{b*})$ and σ_d^2 is computed accordingly. As used in the contracts offered by the insurer under strategy $\tilde{f}$, this statistical instrument has the property of enabling the insurer to detect non optimal levels of care $(c^i < c^{i*})$ and (or) lies, i.e. wrong risk announcement in period 1 $(p^d \neq p^i(c^{i*}))$, quickly enough to make such underproduction or lying unattractive, while not being too hard on customers who, although they announced their true risk and produced adequate prevention, have been unlucky. This will be shown in the process of proving the theorem below, which states that it is in the interest of the insurer and his customers to use strategies $\tilde{f}$ and $\tilde{g}^i$ respectively.

Theorem :

Let $P^i(c^{i*})$ and U be such that

$$P^i(c^{i*}) = \overline{D}[p^i(c^{*i})] \text{ and } U(S - P^i(c^{i*}) - c^{i*}) > \overline{U}^i ,$$

where $\overline{U}^i = (1 - p^i(\overline{c}^i))\, U(S - \overline{c}^i) + p^i(\overline{c}^i)\, U(S - D - \overline{c}^i)$, $\overline{c}^i$ being the optimal prevention level under self-insurance. Then, if $T \to \infty$, $(\tilde{f}, \tilde{g}^i)$ is such that :

(1) $H_1(\tilde{f}, \tilde{g}^i) = 0$, $i = a, b$

$H_2^i(\tilde{f}, \tilde{g}^i) = U(S - P^i(c^{i*}) - c^{i*})$, where H_1 and H_2^i are the average welfare levels achieved by the insurer and the insured respectively.

(2) $(\tilde{f}, \tilde{g}^i)$ is enforceable, i.e. $\tilde{f}$ and $\tilde{g}^i$ are absolutely best responses to each other.

Proof[6] :

A. Proof of (1) :

The proof of (1), in each case (i = a, b), is identical to that provided by Rubinstein and Yaari (p. 90) for the same property under moral hazard. It hinges on an implication of the Law of the Iterated Logarithm that $\sum_{s=1}^{T} \theta_D^s/N(t) < \overline{D}^i(c^{i*}) + \alpha^{i,N(t)}$ for all but finitely many values of T. As a result, for all but finitely many values of T, the customer is offered insurance at the actuarial premium (see the definition of $\tilde{f}^{t+1}$); hence he buys insurance and achieves $U(S - P^i(c^{i*}) - c^{i*})$. When the average utility is taken over an infinite number of periods this occurence dominates the finitely many exceptions so that the customer achieves $H_2^i = U(S - P^i(c^{i*}) - c^{i*})$. Clearly, since the premium is actuarial except for finitely many exceptions, the insurer achieves zero profits ($H_1 = 0$) on both types of customers.

[6]We present here the main developments of the proof without repeating the parts that are already in the literature. Details are available on request.

B. Proof of (2)

B.1. $\tilde{f}$ is a best response to $\tilde{g}^i$

In order to focus on the behavior of the customers, we have not provided for any profit margin for the insurer in $\tilde{f}$, except in periods when the insured pays P_K. For a profit maximizing monopoly working under the constraint of a non positive profit margin, the maximum average profit that can be achieved is clearly zero. Since $H_1(\tilde{f}, \tilde{g}^i) = 0$ by (1) no other strategy dominates $\tilde{f}$ under that constraint. More generally, for an unconstrained monopoly, the premiums in $\tilde{f}$ must be redefined to include a profit margin $\overline{Z}$ which exactly exhausts the customers' rent at the full information level of care. For that modified contract it is easily shown that $H_1 = \overline{Z}$. Again no alternative strategy f can do better because, if it did (if $H_1(f, \tilde{g}) > \overline{Z}$) the customer would rather be self insured in all periods so that the average profit of the insurer would be 0, contradicting the original proposition.

B.2. $\tilde{g}^i$ is a best response to $\tilde{f}$

In order to prove this result, we divide the possible alternative strategies to $\tilde{g}^i$ into two categories, truth telling in the first period, or lying in the first period.

B.2.1. Truth telling strategies

Consider truth telling first ($P^d = P^i(c^{i*})$). Then the only remaining problem is moral hazard. In that case the proof of Rubinstein and Yaari can be used (p. 92) to show that the best customer strategy is to apply the prevention level c^{i*}. Hence there is no strategy in the truth telling category that dominates $\tilde{g}^i$.

B.2.2. Strategies involving a false announcement

Now consider customer strategies which involve lying in the first period. In order to show that it is not in any customer's interest to lie, we first find a maximum to the average expected utility which can be reached by a customer who makes a false announcement, and then we show that this maximum is smaller than the expected utility under $\tilde{g}^i$.

The optimal strategy of a customer of type i who has made a false announcement in response to $\tilde{f}$ consists in choosing a level of prevention c^{id} which makes him statistically look like he announced : $P^i(c^{id}) = P^d$ (if such a level of c^i exists, which we shall assume[7]). The proof of this result is provided in the appendix. The average expected utility that can be so achieved is U^{id} where $U^{id} = U(S - P^d - c^{id})$ is the utility level[8] of a customer of type i who has announced a probability of accident p^d, who has not been detected (i.e. who pays a premium P^d), and who produces the prevention level c^{id}.

To complete the proof, we have to show that $U^{id} \leq U^{i*}$ where $U^{i*} = U(S - P^i(c^{i*}) - c^{i*})$. We do it here for the case of a high risk a who pretends to be a lower risk and produces a level of care $c^{ad} > c^{a*}$ in order to be credible ($P^a(c^{ad}) = P^b(c^{b*})$). Suppose that $U^{a*} < U^{ad}$, i.e. $U(S - P^a(c^{a*}) - c^{a*}) < U(S - P^a(c^{ad})) - c^{ad})$. It follows that $P^a(c^{a*}) + c^{a*} > P^a(c^{ad}) + c^{ad}$ and, taking a Taylor-Lagrange expansion of the right hand side around c^{a*},

[7] If there does not exist any c^{id} such that $P^i(c^{id}) = P^d$, it is trivial to show that, by the Law of the Iterated logarithm, the customer will pay the penalty premium P_K infinitely many times with probability one so that lying is not optimal in that case.

[8] We assume $U^{id} \geq \overline{U}^i$. Otherwise self insurance is optimal after a false announcement.

$$P^a(c^{a*}) + c^{a*} > P^a(c^{a*}) - \frac{dP^a(c^{a*})}{dc}(c^{a*} - c^{ad}) + \frac{1}{2}\frac{d^2P^a(\hat{c}^a)}{dc^2}(c^{a*} - \hat{c}^a)^2 + c^{ad},$$

where $c^{a*} < \hat{c}^a < c^{ad}$ and, by the first order condition for the full information optimization problem, $dP^a(c^{a*})/dc = (dp^a(c^{a*})/dc)D = -1$. After simplification, one gets $d^2P^a(\hat{c}^a)/dc^2 < 0$ which contradicts the convexity of $P^a(c)$.

The same argument applies in the (odd) case of customer pretending to be a higher risk in order to reduce his care level.

This completes the proof that $\tilde{g}^i$ is a best response to $\tilde{f}$.

3. Conclusions

We have presented an integration of moral hazard and adverse selection in a single model, and we have shown how to achieve a first-best allocation of risks under both informational problems. Let us note that the result applies whatever the number of individual types and whether the monopolistic insurer is regulated (zero profits) or not. Many extensions of the above analysis can be considered.

Two of them consist in introducing contracts with either discounting or a finite horizon, or with both characteristics at the same time. While such extensions are discussed in the respective literature on moral hazard and adverse selection (see footnote 2 above for references), it is not clear that straightforward combinaisons of existing models are feasible or optimal. In this paper we have shown that the analysis of both problems simultaneously involves some synergetic effects. Indeed, in the model, presented above, the insurer needs less information on the individuals' probability function than in a model obtained from a juxtaposition of the contributions by Rubinstein and Yaari, and Dionne and Lasserre.

Another investigation concerns the efficient use of past information, and the allocation of instruments, toward the solution of each particular information problem. For a long time self-selection mechanisms have been proposed in response to adverse selection while nonlinear pricing was advocated against moral hazard. In one-period contracts both procedures involve inefficiency (partial insurance) which can be reduced by the introduction of time in the contracts. Our paper shows that self selection may help solve moral hazard problems, as well as adverse selection problems. Thus there may not exist any one to one correspondance between instruments and targets. There is also no denial that it might be desirable to use time more sparingly than is done in our paper (infinite horizon with no discounting). In comparing alternative ways to alleviate asymmetric information problems,procedures should not only be judged on the basis of the welfare improvement achieved but also according to their effectiveness in using scarce information.

APPENDIX

Optimal strategy under false announcement in response to $\tilde{f}$

1. Assertions

We shall need the following assertions (Rubinstein and Yaari, 1983).

Assertion A. Let X be a random variable, bounded by some real number B. Let A be an event with $Pr(A) = 1 - \varepsilon$. Then

$$|E(X) - E(X \mid A)| \leq \frac{2B\varepsilon}{1 - \varepsilon} .$$

Assertion B. (Consequence of Egoroff's Theorem). Let $\{X^t\}$ be a sequence of random vectors with values in $\mathbb{R}^n$. Let $A \subset \mathbb{R}^n$ be a closed set with the property that $\rho(X^t, A) \to 0$ a.s., where ρ denotes Euclidean distance. Then, for every $\delta > 0$, there exists an event E with $Pr(E) \geq 1 - \delta$ such that $\rho(X^t, A) \to 0$ uniformly on E.

Assertion C. Let $A \subset \mathbb{R}^n$ be a convex compact set, and let $\{X^t\}$ be a sequence of random vectors, with values in $\mathbb{R}^n$, such that $E(X^t \mid X^1, \ldots, X^{t-1}) \in A$ for all t. Let $\frac{1}{T}\sum_{t=1}^{T} X^t$ be denoted $\bar{X}^T$. Then $\rho(\bar{X}^t, A) \to 0$, a.s., where ρ is Euclidean distance.

Assertion D. Let K be a real number. Let $\{X^t\}$ be a sequence of random variables. Let x^t denote a value of X^t and for every $x^1, \ldots, x^T$, let $(1/T)\sum_{t=1}^{T} x^t$ be denoted $\bar{x}^T$. If $\bar{x}^T > K$ and $E(x^{T+1} \mid x^1, \ldots, x^T) \leq K$, then almost surely, $\limsup \bar{x}^T \leq K$.

2. Definitions

Let $d^{id}(v)$ be such that

$$d^{id}(v) = \overline{D}^{i}[-P^{d} - U^{-1}(v)] .$$

$d^{id}(v)$ gives the expected loss corresponding to the level of c^i selected by a customer of type i who pays a premium of P^d and wants to achieve a utility level of v. Since, with full insurance, $v = U(S - P^d - c^i)$, this customer can increase his utility only by reducing c^i, which raises the chance of accident. Consequently, $d^{id}(v)$ is rising. From the curvatures of U and $p^i(c^i)$, it can also be shown that $d^{id}(v)$ is convex.

Similarly, let $d^i_d(v)$ be such that

$$d^i_d(v) = \overline{D}^{i}[-P_K - U^{-1}(v)] .$$

$d^i_d(v)$ has the same interpretation and properties as $d^{id}(v)$ except that it applies to a customer who pays a premium of P_K rather than P^d. Since no one will make any announcement which entails paying more than the penalty premium P_K, $P^d < P_K$ from which it follows that $d^i_d(v)$ is everywhere on the left of $d^{id}(v)$, as in Figure 1-A. Indeed for any given levels of both c^i and expected loss, v is higher when P^d is paid rather than P_K. Also, d^{id} and d^i_d have the same range, contained in [0, D] and are defined on closed intervals.

INSERT FIGURE A-1 HERE

3. Implications

From Assertion A for any $\varepsilon_0 > 0$, we can select an event $\tilde{E}$ whose probability $\tilde{\varepsilon} < \varepsilon_0$ is so small that for every c^i and for every t

$$|E\{U(S - c^i - \delta_t^i)\} - E\{U(S - c^i - \delta_t^i) \mid \tilde{E}\}| \leq 2B\tilde{\varepsilon}/(1-\tilde{\varepsilon}) \leq \varepsilon_0/4$$

where B is an upper bound to $U(S - c^i - \delta_t^i)$ and δ_t^i is a random variable ($\delta_t^i = 0$, no accident over period t; $\delta_t^i = D$, accident over period t). It follows that

$$E\{U(S - c^i - \delta_t^i) \mid \tilde{E}\} \leq \overline{U}^i + \varepsilon_0/4 \qquad \text{(A1)}$$

Let A be the convex hull of graphs $d_d^i(v)$ and $d^{id}(v)$. From Assertion C,

$$\rho[(\overline{U}^{iT}, \overline{D}^{iT}), A] \to 0 , \qquad \text{a.s.,} \qquad \text{(A2)}$$

where ρ denotes Euclidean distance; $\overline{U}^{iT} = \frac{1}{T} \sum_{t=1}^{T} h_i^t(\tilde{f}, g^i)$ is the average utility derived over T periods by an agent i who plays g^i, a strategy involving $p^d < p^i(c^{i*})$, in response to $\tilde{f}$; $\overline{D}^{iT} = \frac{1}{T} \sum_{t=1}^{T} \theta_D^t(\tilde{f}, g^i)$ is the average loss experienced by that individual; and, since the expected value of θ_D^t is either d_d^i or d^{id}, the condition $E[(\theta_D^t, h_i^t(\tilde{f}, g^i)) \mid (\theta_D^{t-1}, h_i^{t-1}(\tilde{f}, g^i)), \ldots, (\theta_D^1, h_i^1(\tilde{f}, g^i))] \in A$ is met.

Given (A2), Assertion B implies that for any $\hat{\varepsilon} > 0$, there exists an event $\hat{E}$, with $\text{prob}(\hat{E}) > 1 - \hat{\varepsilon}$, such that $\rho[(\overline{U}^{iT}, \overline{D}^{iT}), A] \to 0$ uniformly on $\hat{E}$. As a result, in particular, there exists T such that, for every $t \geq T$, the average utility in $\hat{E}$ does not exceed by more than $\varepsilon_0/2$ the image of the average loss by $(d^{id})^{-1}$:

$$\overline{U}^{it} - \left(d^{it}\right)^{-1}(\overline{D}^{it}) < \varepsilon_0/2 \qquad \text{(A3)}$$

Since $\alpha^{d,t} \to 0$, there exists T_0 such that for $t \geq T_0$,

$$\overline{D}(p^d) + \alpha^{d,t} < d^{id}(U^{id} + \varepsilon_0/2) \qquad \text{(A4)}$$

The expected loss associated with a probability of accident of p^d, plus $\alpha^{d,t}$, is smaller than the expected loss corresponding to a level of care yielding a utility of $U^{id} + \varepsilon_0/2$ when the premium is P^d (i.e. undetected low level of c^i).

4. The optimal strategy after a false announcement

We are now ready to show that the optimal strategy of a customer who chose a premium of P^d in period 1 is to select $c^i = c^{id}$ and to buy insurance in all subsequent periods, thus achieving an average expected utility of U^{id}.

That U^{id} can be obtained under that strategy is a direct implication of the Law of the Iterated Logarithm. What needs to be shown is that there is no better alternative. We define a better alternative as a strategy g^i having at least the property that the event $\{H_2^i(\tilde{f}, g^i) \geq U^{id} + \varepsilon$ for infinitely many values of T$\}$ has positive probability. This implies

$$\limsup \overline{U}^{it} \geq U^{id} + \varepsilon \text{ with positive probability} \qquad \text{(A5)}$$

which in turn implies that, for high values of t, $\overline{U}^{it} \geq U^{id} + \varepsilon$ with positive probability.

Thus we assume $\overline{U}^{it} \geq U^{id} + \varepsilon_0$, with $t > \max\{T^0, T\}$, and we show this to imply that (A5) is violated. By (A3), $\overline{U}^{it} < (d^{id})^{-1}(\overline{D}^{it}) + \varepsilon_0/2$. Since $\overline{U}^{it} \geq U^{id} + \varepsilon_0$, one has $U^{id} + \varepsilon_0 < (d^{id})^{-1}(\overline{D}^{it}) + \varepsilon_0/2$, so that $d^{id}(U^{id} + \varepsilon_0/2) < \overline{D}^{it}$. But from (A4), $d^{id}(U^{id} + \varepsilon_0/2) > \overline{D}(p^d) + \alpha^{d,t}$ which by transitivity implies $\overline{D}^{it} > \overline{D}(p^d) + \alpha^{d,t}$. Thus a high premium P_K is being charged under the provisions of $\tilde{f}$. The customer may choose to pay the high premium; if he does his utility will be smaller than $\overline{U}^i$ in t+1. Else, the customer may choose to be self-insured in t+1 and to select c^i so as to maximise $E\{U(S - c^i - \delta^i_{t+1}) \mid \hat{E}\}$, which is smaller than $\overline{U}^i + \varepsilon_0$ by (A1). By Assertion D, it follows in both cases that $\limsup \overline{U}^{it} < \overline{U}^i + \varepsilon_0/4$, which is smaller than $U^{id} + \varepsilon_0/4$ since $\overline{U}^i < U^{id}$ by assumption. Since $\limsup \overline{U}^{it}$ cannot exceed $U^{id} + \varepsilon_0/4$ unless $\overline{U}^{it} > U^{id} + \varepsilon_0/4$ for some $t > \max[T, T_0]$, this proves that

$$\limsup \overline{U}^{it} \leq U^{id} + \varepsilon_0/4 \quad \text{a.s. on } \hat{E}\text{, contradicting (A5), for } \varepsilon = \varepsilon_0.$$

But since ε_0 can be chosen to be arbitrarily small, there does not exist any $\varepsilon > 0$ such that (A5) is not violated.

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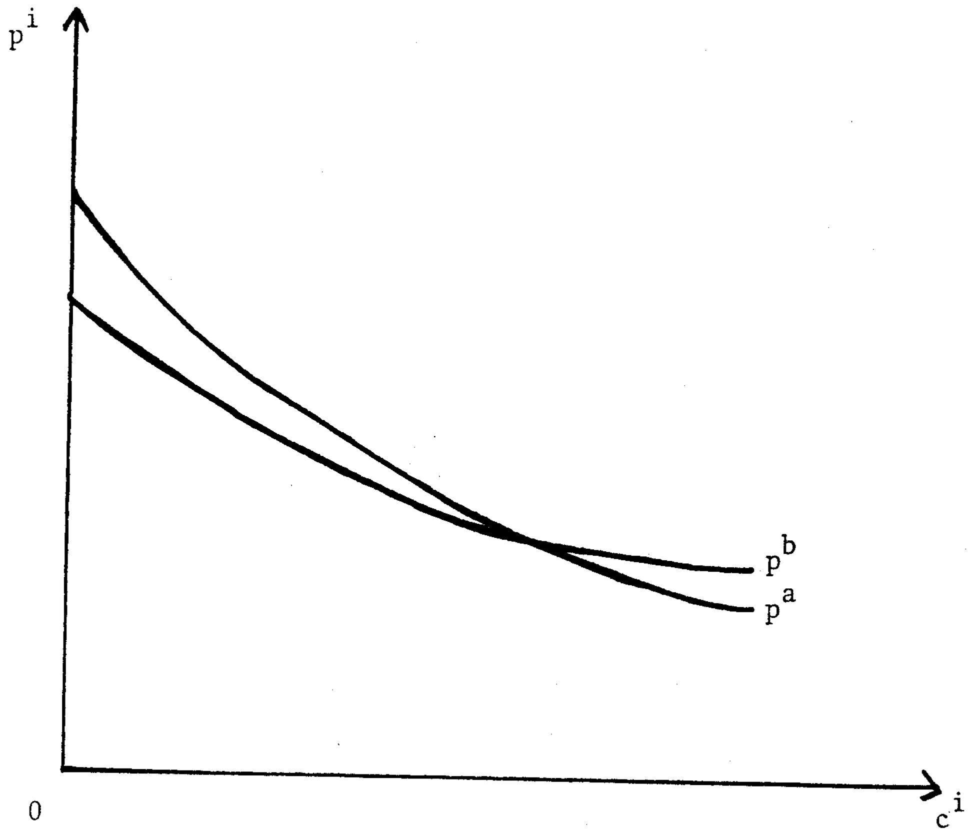
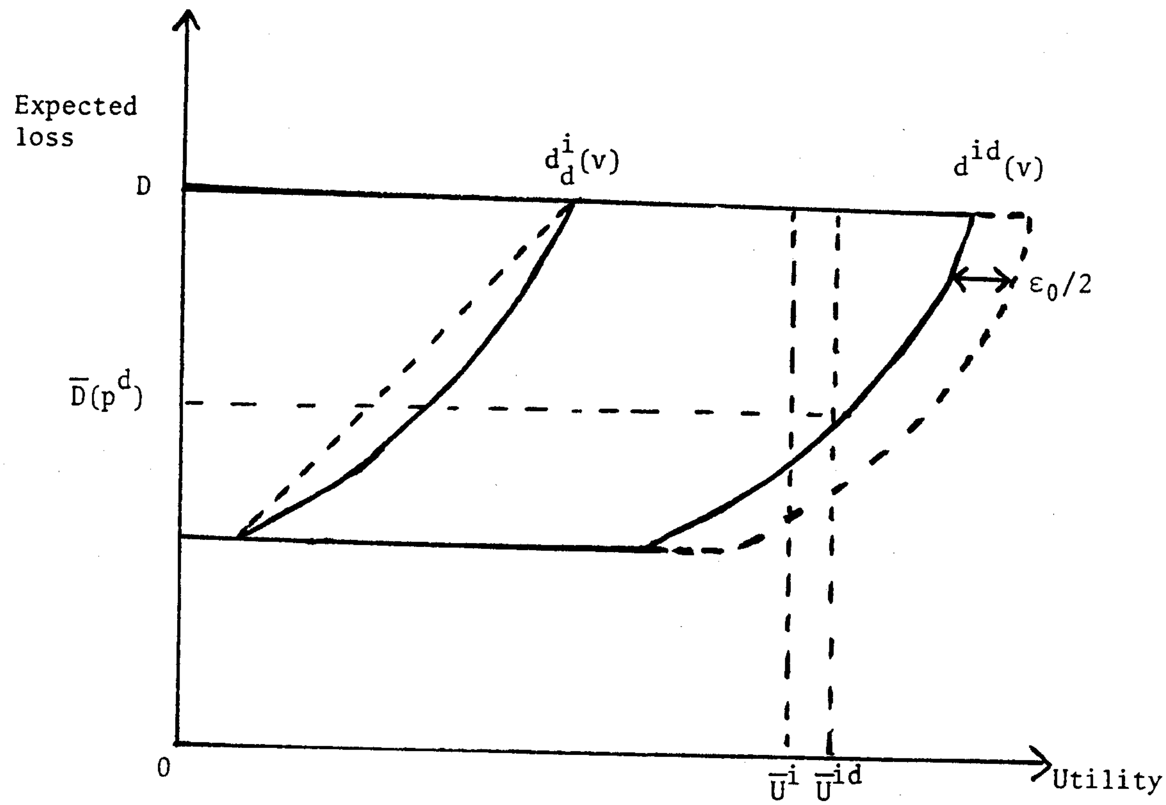


Figure 1



Expected loss
$d^i_d(v)$
$d^{id}(v)$
D
$\varepsilon_0/2$
$\overline{D}(p^d)$
0
$\overline{U}^i$ $\overline{U}^{id}$
Utility

Figure 1A