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Capital Income Taxation, Depletion  
Allowances, and Non-Renewable  
Resource Extraction

by

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### Résumé

Dans cet article, nous étudions les distorsions que cause l'impôt sur le revenu des sociétés dans le profil de production des firmes extractives et dans l'allocation des ressources entre les secteurs d'extraction et les autres secteurs soumis à l'impôt sur les sociétés. Nous étudions en particulier l'allocation d'épuisement, dont nous montrons qu'elle peut trouver sa justification, non pas à assurer la neutralité de l'impôt, mais en permettant l'établissement de taux effectifs d'imposition identiques dans les secteurs d'extraction et dans les autres secteurs.

### Abstract

This paper pays particular attention to the depletion allowance in examining some of the possible distortionary effects of the corporate income tax on the output path of the non-renewable resource sector and on the allocation of resources between the mining and non-mining corporate sectors. It is shown that the depletion allowance is justifiable not so much on the grounds of assuring an undistorted output path in the non-renewable resource sector as on the grounds of assuring equal effective capital income tax rates between the mining and non-mining corporate sectors.

The recent non-renewable resource literature has dealt with the effects of various types of taxes on the non-renewable resource extraction path (see for example Burness [4], Dasgupta and Heal [5], or Dasgupta, Heal and Stiglitz [6]). It includes analysis of such taxes as an ad valorem sales tax, a per unit production tax, a franchise tax, a present value tax, and an economic profit tax. The corporate income tax, as it is usually applied, does not really fall under any of those categories. It remains however an important means of taxation of the non-renewable resource extraction sector. The purpose of this paper is to analyse some of the possible distortionary effects of the corporate income tax on the output path of the resource sector and on the allocation of resources between the mining and non-mining corporate sectors.

In the next section we characterise the after tax optimal output path of the non-renewable resource extraction firm (the mining firm for short). The following two sections deal respectively with the effect of the corporate income tax structure on the extraction path and with the resulting effective tax rate on capital income for the mining and non-mining corporate sectors. In both cases we pay particular attention to the role played by the depletion allowance, which is an important element of the corporate tax as applied to the resource sector. We will show that the depletion allowance is justifiable not so much on the grounds of pure neutrality of the corporate tax as on the grounds of assuring equal effective capital income tax rates between the mining and non-mining sectors.

In a last section, we briefly evaluate the present U.S. corporate tax in the light of our results.

The corporate income tax and optimal resource extraction

The main features of the corporate income tax apply to any incorporated firm, and are therefore common to both the mining and non-mining firm. Among those, we retain: the corporate tax rate,  $u$ ; the rate of depreciation for tax purposes,  $\theta$  (we assume declining balance depreciation throughout the life of the asset); the investment tax credit,  $k$ . Some provisions are however specific to the mining firm, the most important being the depletion allowance. We will assume that a fraction  $\beta$  of gross revenues can be deducted as depletion allowance.<sup>1</sup>

We will simplify the extraction process by ignoring all variable factors and by supposing only one type of capital asset. Thus the rate of extraction is given by  $F(K(t))$ . The firm is assumed to be a price taker in both the output and factor markets. Under those conditions the net present value of the mine at time  $t_0 = 0$  is given by

$$\int_0^T e^{-rt} \{ pF(K) - qI - u[pF(K) - \theta C - \beta pF(K)] + kqI \} dt \\ + e^{-rT} V(K(T), C(T); q(T)) - \pi R(0)$$

where  $r$ ,  $p$  and  $q$  are respectively the rate of interest, the price of output and the price of capital goods.  $I$  is the gross rate of investment (assumed unbounded), and  $C$  is the value of undepreciated capital stock for tax purposes. The after tax scrap value of the mine, at exhaustion date  $T$ , is given by  $V(K(T), C(T); q(T))$ , which remains to be specified. The cost of acquisition, through either purchase or discovery, of a unit of initial reserves,  $R(0)$ , is given by  $\pi$ . We will in what follows assume for simplicity that  $p = 1$ . All other prices can thus be considered as real prices.

The firm is assumed to choose its investment path, and thus  $K(t)$  and its rate of extraction,  $F(K(t))$ , so as to maximise the net present value of the mine subject to<sup>2</sup>

$$\dot{K} = I - \delta K ; K(0) = K_0 \quad (1)$$

$$\dot{C} = (1-k)qI - \theta C ; C(0) = C_0 \quad (2)$$

$$\dot{R} = -F(K) ; R(T) = 0 \quad (3)$$

The corresponding current value hamiltonian is

$$\begin{aligned} H = F(K) - qI - u[F(K) - \theta C - \beta F(K)] + \kappa qI \\ + \gamma[I - \delta K] + \eta[(1-k)qI - \theta C] - \lambda F(K) \end{aligned} \quad (4)$$

Since the hamiltonian is linear in the control variable  $I$ , which is unbounded, we have as singular solution

$$(1-k)(1-\eta)q = \gamma \quad (5)$$

Equation (5) must be satisfied for all  $t \in [0, T]$ . The optimal control consists in adjusting instantaneously to the stock of capital which satisfies the equality given in (6) and maintaining that equality thereafter.

Other necessary conditions, in addition to (1), (2) and (3) are:

$$\dot{\gamma} = (r+\delta)\gamma - (1-(1-\beta)u-\lambda)F'(K) \quad (6)$$

$$\dot{\eta} = (r+\theta)\eta - u\theta \quad (7)$$

$$\dot{\lambda} = r\lambda \quad (8)$$

and the transversality conditions

$$\lambda(0) = \pi \quad (T.0)$$

$$\gamma(T) = \frac{\partial V}{\partial K(T)} \quad (T.1)$$

$$\eta(T) = \frac{\partial V}{\partial C(T)} \quad (T.2)$$

$$H(\dots) \Big|_T = r V(K(T), C(T); q(T)) + \frac{\partial V}{\partial q(T)} \dot{q}(T) \quad (T.3)$$

The auxiliary variables  $\gamma(t)$ ,  $\eta(t)$  and  $\lambda(t)$  are to be interpreted respectively as the current shadow values of a unit of productive capital, a unit of undepreciated accounting capital for tax purposes, a unit of in situ resource.

In order to better compare the results with the analysis of the traditional non-mining firm we will assume that there is no discontinuity in the tax treatment at the date of exhaustion.<sup>3</sup> We therefore assume that the firm will, after exhaustion of the deposit, continue a declining balance depreciation against other income, at rate  $\theta$ , of the remaining stock of capital, or have to "recuperate" at the same rate if the resale value of the

remaining stock of capital exceeds the accounting value of undepreciated capital for tax purposes. Under such a tax regime, the after tax scrap value will be given by

$$V(K(T), C(T); q(T)) = (1-k)q(T)K(T) + uz[C(T)-(1-k)q(T)K(T)] \quad (9)$$

where  $z = \frac{\theta}{r+\theta}$  is the present value of a dollar of undepreciated capital

which can be depreciated indefinitely at rate  $\theta$ . The scrap value is thus the resale value of the remaining capital stock plus the tax saving from the depreciation of the undepreciated value of the remaining stock of capital, or minus the tax recuperation if fiscal depreciation has exceeded true depreciation. Notice that the resale value of a unit of capital is  $(1-k)q$ . Indeed, given that a unit of new capital, and only new capital, is eligible to a tax credit of  $k$ , the effective purchase cost of a unit of new capital is  $(1-k)q$  and the market resale value of used capital cannot exceed this.

Conditions (T.1), (T.2) and (T.3) therefore become

$$\gamma(T) = (1-uz)(1-k)q(T) \quad (T.1')$$

$$\eta(T) = \dot{u} z \quad (T.2')$$

$$H(\dots)\Big|_T = r V(K(T), C(T); q(T)) - (1-uz)(1-k)\dot{q}(T)K(T) = 0 \quad (T.3')$$

Now consider equation (5), which must be satisfied for all  $t$ . Differentiating both sides with respect to  $t$ , and substituting for  $\dot{\gamma}$  from (6), we get

$$(1-(1-\beta)u-\lambda)F'(K) = (r+\delta-\dot{q}/q)q(1-k)(1-\eta)+(1-k)q\dot{\eta} \quad (10)$$

But the first order differential equation in  $\eta$  given by (7) has as general solution

$$\eta(t) = uz + (\eta(T) - uz)e^{-(r+\theta)(T-t)} \quad (11)$$

which is the discounted value of the flow of tax savings resulting from one dollar of undepreciated capital at time  $t$ . In order to satisfy condition (T.2') the specific solution reduces to

$$\eta(t) = uz$$

which implies  $\dot{\eta} = 0$ . Therefore  $\eta$  is independent of the time at which investment is undertaken, a result which depends crucially on the assumption of continuity of the tax treatment at the time of exhaustion, as is implied by equation (9) (see Gaudet and Lasserre [7]).

With  $\eta$  constant, equation (10) can be rewritten

$$F'(K) = c \frac{(1-k)(1-uz)}{1-(1-\beta)u-\lambda} \quad (12)$$



where  $c = (r + \delta - \dot{q}/q)q$ , that is the before tax real rental cost of capital of the traditional non-mining corporate firm. The term on the right hand side of equation (12) measures the after tax implicit real rental cost of capital to the corporate mining firm. The well known equivalent for the traditional corporate firm is<sup>4</sup>

$$F'(K) = c \frac{(1-k)(1-uz)}{1-u} \quad (13)$$

which can be obtained directly from (12) by simply setting  $\lambda = 0$  (the non-renewable resource constraint is not binding) and  $\beta = 0$ .

The expression for the after tax rental cost of capital to the mining firm therefore differs from that of the traditional firm for two reasons. The first is due to the depletion allowance which, ceteris paribus, reduces the after tax cost of capital. The second is due to the term  $\lambda$ , which is the share of the after tax in situ shadow value of the resource in the market price (set here equal to one). This term appears in the cost of capital of the mining firm because the marginal unit of capital is employed to reduce the reserves of the non-renewable resource by a quantity which will not be available for extraction at a future date. This opportunity cost has the effect of increasing both the before tax and after tax rental cost of capital to the mining firm relative to that of the traditional firm. Since  $\lambda$  increases at the rate of interest (condition (8)) it also follows that the cost of capital to the mining firm will increase as the date of exhaustion approaches.

Two interesting questions may be raised at this point. The first concerns the effect of corporate taxation on the extraction path: does the

corporate tax system distort the extraction path, and if so in what direction? The second question concerns the effect of the corporate tax on the real rental cost of capital for the mining firm relative to that for the traditional firm: does the corporate tax result in a higher or lower effective tax on capital income for the mining sector, thus favoring a shift of capital from one sector to the other?

#### Corporate taxation and the extraction path

Consider first the effect of the corporate tax structure on the extraction path. Substituting from (T.1') and (T.2') into (T.3'), and given that (12) must be satisfied at all  $t$ , (T.3') implies that at  $T$  we must have

$$\frac{F(K(T))}{K(T)} = F'(K(T)).$$

It follows that the terminal capital stock, and therefore the terminal extraction rate, are independent of the tax parameters.<sup>5</sup>

This does not mean however that the extraction path is left unchanged by the tax. Differentiating both sides of equation (12) with respect to  $t$  and using condition (8) we get (assuming  $c$  constant for simplicity):

$$\begin{aligned} \dot{K} &= \frac{r \lambda F'}{(1-(1-\beta)u-\lambda)F''} \\ &= \dot{K}^* + \frac{r[1-(1-\beta)u-(1-k)(1-uz)]F'}{(1-k)(1-uz) \frac{c}{F'}} F'' \end{aligned} \quad (14)$$

where

$$\dot{K}^* = \frac{r \left[ 1 - \frac{c}{F'} \right] F'}{\frac{c}{F'} F''} \quad (15)$$

The asterisk refers to the optimal path in the absence of taxes. Since  $\lambda > 0$ ,  $F' > 0$ , and since we must have  $F'' < 0$  along the optimal path, both  $\dot{K}$  and  $\dot{K}^*$  are negative. But we will have<sup>6</sup>

$$\dot{K} \begin{matrix} > \\ < \end{matrix} \dot{K}^* \text{ as } \beta \begin{matrix} < \\ > \end{matrix} \frac{(1-k)(1-uz)-(1-u)}{u}$$

The same relations of course hold for the extraction path, since if  $Q = F(K)$  is the extraction rate, then  $\dot{Q} = F'(K)\dot{K}$ , and  $F' > 0$ .

Assume for the moment that the initial stock of resources is given, which is equivalent to replacing condition (T.0) by  $R(0) = R_0$ , and consider the case where

$$\beta < \frac{(1-k)(1-uz)-(1-u)}{u} . \quad (16)$$

Since  $\dot{K}$  and  $\dot{K}^*$  are both negative, and since  $R^*(T^*) = R(T) = 0$ , the  $\{K(t)\}$  and  $\{K^*(t)\}$  paths will have to cut each other at least once. Suppose they cross at  $t = \hat{t}$ . We will then have  $K(\hat{t}) = K^*(\hat{t})$  and  $\dot{K}(\hat{t}) > \dot{K}^*(\hat{t})$ , which means that  $\{K(t)\}$  and  $\{K^*(t)\}$  will cross only once, with  $\{K(t)\}$  cutting  $\{K^*(t)\}$  from below. Since  $K(T) = K^*(T^*)$ , as established above, we must therefore necessarily have  $T > T^*$ . Corporate taxation would increase the

life of the mine in this case. Notice that this case includes the possibility of no depletion allowance ( $\beta = 0$ ). By the same reasoning the reverse of course holds ( $T < T^*$ ) if we have

$$\beta > \frac{(1-k)(1-uz)-(1-u)}{u} . \quad (17)$$

We therefore observe the not unexpected result that the higher the rate of depletion allowance the faster the exhaustion of a given non-renewable resource stock. Exhaustion may however be slower or faster than in the no tax situation, depending on the rate of depletion allowance chosen. The tax structure can even be designed so as not to distort the path of extraction of the given initial resource stock by setting

$$\beta = \frac{(1-k)(1-uz)-(1-u)}{u} . \quad (18)$$

Notice that although it does not distort the extraction path for a given resource stock, a tax structure satisfying (18) will not usually leave the output path of the non-mining firm undistorted. As can be seen directly from equation (13), this would require that we also have  $(1-k)(1-uz) = 1-u$ , so as to leave unchanged the rental cost of capital to the non-mining firm.

The reason why setting  $\beta$  as in (18) leaves the extraction path of the mining firm unchanged is of course that, with given initial resource stock, this makes the corporate tax a pure rent tax for the mining firm. We can verify this by substituting for this value of  $\beta$  into equation (12) and solving for  $\lambda$ , which gives

$$\lambda = (1-(1-\beta)u)\lambda^*$$

where  $\lambda^* = 1 - \frac{C}{F^T}$  is the before tax rent. After tax rent therefore becomes strictly proportional to before tax rent, leaving no incentive to modify the extraction path.

This is no longer true however when the mining firm is allowed to choose the initial stock of reserves which it intends to exploit. The optimal  $R(0)$  must then be chosen so as to satisfy condition (T.0). By the concavity of the value function it follows that if  $\lambda(0) < \pi$ ,  $R(0)$  is too high, and vice versa. Consequently, one can analyze the effect of the tax on the optimal initial stock of the resource by looking at the sign of  $\lambda(0) - \lambda^*(0)$ , for a given  $R(0)$ .

Assume then  $R(0)$  to be the optimal before tax initial stock of the resource, so that  $\lambda^*(0) = \pi$ . Since by condition (8),  $\lambda(0) = e^{-rT}\lambda(T)$ , and since  $K(T) = K^*(T^*)$  as already established, we get, by solving for  $\lambda(T)$  from (12):

$$\begin{aligned} \lambda(0) - \lambda^*(0) &= [-(1-\beta)ue^{-rT} + e^{-rT} - e^{-rT^*}]\lambda^*(T^*) \\ &+ e^{-rT}[1-(1-\beta)u-(1-k)(1-uz)] \frac{C}{F'(K(T))}. \end{aligned}$$

In both cases (16) and (18), for which we respectively have  $T > T^*$  and  $T = T^*$  for given  $R(0)$ , the right hand side of this expression is unambiguously negative. In both these cases the tax therefore creates an incentive to reduce the initial stock of resource to be exploited. The effective

after tax extraction path will therefore either lie everywhere below the before tax extraction path or cut it from below, with the effect on the effective terminal date being ambiguous. Thus even if the tax structure satisfies (18), so as to result in an after tax rent which is strictly proportional to before tax rent, the fact that the tax reduces the present value of rent will result in a distortion of the extraction path through its effect on the initial stock of resources to be exploited.

If the rate of depletion allowance is high enough to make inequality (17) hold, then the effect of the tax on the initial resource stock becomes ambiguous. For one cannot then exclude the possibility of  $\beta$  being sufficiently high so as to actually result in  $\lambda(0) > \lambda^*(0)$  (an increase in the present value of rent), in which case there is an incentive to exploit a greater initial stock. The value of  $\beta$  could even be chosen so as to leave the optimal  $R(0)$  unchanged. As already shown this would result in a faster exhaustion of the same stock ( $T < T^*$ ). Therefore there exists no constant value of  $\beta$  which will leave the effective extraction path undistorted.

#### Effective tax rates on capital income and the depletion allowance

Consider now replacing the corporate tax regime just described by a one parameter tax structure of  $\tau$  percent of true capital income, and  $\tau_m$  percent in the case of the non-renewable resource sector. Capital income in the present context is simply gross revenues (net of in situ shadow value for the resource sector) minus true economic depreciation. Solving, mutatis mutandis, the net present value maximisation problem already described with this alternative tax regime, we find that the equivalent to condition (12) becomes <sup>7</sup>

$$F'(K) = \frac{c - \tau_m \delta q}{(1 - \tau_m)(1 - \lambda)} \quad (19)$$

Similarly, the equivalent to equation (13) becomes<sup>7</sup>

$$F'(K) = \frac{c - \tau \delta q}{1 - \tau} \quad (20)$$

The right hand sides of equations (19) and 20) are the real implicit rental rates of capital under such a tax system for respectively the mining and non-mining sectors.

We may now calculate the effective tax rate on capital income resulting from the corporate tax system as applied to the mining sector by solving for the value of  $\tau_m$  which equates the right hand sides of (12) and (19). This gives

$$\tau_m = \frac{\left[ \frac{(1-k)(1-uz)}{1-u} - \frac{1-(1-\beta)u-\lambda}{(1-u)(1-\lambda)} \right] c}{\frac{c(1-k)(1-uz)}{1-u} - \frac{\delta q}{(1-u)(1-\lambda)}} \quad (21)$$

If  $\tau_m$  were to be chosen as in (21), the two tax systems would result in the same after tax cost of capital, and therefore the same paths for the stock of capital and the extraction rate (as well as the same path for  $\lambda$ ). Notice that  $\tau_m$  is not constant since the optimality conditions require that  $\lambda$  grow at the rate of interest.

Applying the same reasoning to equations (13) and (20), we find that the effective capital income tax rate for the non-mining corporate sector is given by

$$\tau = \frac{\left[ \frac{(1-k)(1-uz)}{1-u} - 1 \right] c}{\frac{c(1-k)(1-uz)}{1-u} - \delta q} \quad (22)$$

From equations (21) and (22) we can now easily compare the effective tax rates on capital income. We find that

$$\tau_m \begin{matrix} > \\ \equiv \\ < \end{matrix} \tau \quad \text{as} \quad \frac{1-(1-\beta)u-\lambda}{(1-\lambda)(1-u)} \begin{matrix} < \\ \equiv \\ > \end{matrix} 1$$

which implies

$$\tau_m(t) \begin{matrix} > \\ \equiv \\ < \end{matrix} \tau \quad \text{as} \quad \beta(t) \begin{matrix} < \\ \equiv \\ > \end{matrix} \lambda(t).$$

The corporate income tax therefore results in equal effective tax rates on capital income if and only if we have  $\beta = \lambda$ , that is if the rate of depletion allowance is equal to the share of rent in the market price of the resource.

This can be equivalently expressed in terms of market parameters instead of shadow prices by noting that optimality conditions require  $\dot{\lambda} = r\lambda$  and  $\lambda(0) = \pi$  (conditions (8) and (T.0)). Therefore



$$\tau_m(t) \begin{matrix} > \\ = \\ < \end{matrix} \tau \quad \text{as} \quad \beta(t) \begin{matrix} < \\ = \\ > \end{matrix} \pi e^{rt}.$$

In order to maintain equality between the two effective tax rates, the rate of depletion allowance should therefore be set for each period equal to the real capitalized value of the initial acquisition cost of a unit of in situ resource. If the stock equilibrium condition can be counted on to have the market value of a unit of in situ resource growing at the rate of interest, so as to make the non-renewable resource stock a no more or no less interesting investment than any alternative asset (see Solow [9]), then this could be attained by setting  $\beta(t)$  equal to the real current market value of a unit of in situ resource.<sup>8</sup>

It is easily verified that unless the tax leaves the rental cost of capital to the non-mining sector unchanged to begin with, it is impossible to design the depletion allowance so as to attain both an undistorted extraction path and equal effective tax rates on capital income. For suppose that we set  $\beta = \lambda = \pi e^{rt}$ , so as to have  $\tau_m(t) = \tau$ . Substituting for this value of  $\beta$  into equation (12) and differentiating with respect to  $t$ , we now get

$$\dot{K} = \dot{K}^* + \frac{r[(1-u)-(1-k)(1-uz)]F'}{(1-k)(1-uz) \frac{c}{F'}} F''$$

Thus unless  $(1-k)(1-uz) = (1-u)$  to begin with, in which case  $\tau_m = \tau = 0$  and the tax becomes a pure rent tax, the before tax and after tax extraction paths cannot coincide. In fact if  $(1-k)(1-uz) - (1-u) > 0$ , the corporate income tax would then increase the exhaustion date ( $T > T^*$ ) relative to the no tax situation for a given initial resource stock, and the analysis of the previous section applies.

Depletion allowance in the present U.S. corporate tax

It is interesting at this point to compare the treatment of the depletion allowance in the present U.S. corporate income tax with our results. The U.S. depletion allowance is so designed as to allow as deduction from the tax base in each period a fraction of the original acquisition cost, through either purchase or discovery, of estimated reserves. The fraction is calculated as the quantity of mineral extracted and sold during the tax period over the estimated quantity of mineral recoverable from the mine. In our notation, and under our assumptions, this amounts to calculating the depletion allowance as

$$\frac{F(K)}{R} \cdot \pi R = \pi F(K)$$

which is equivalent to setting  $\beta = \pi$ . Although the spirit of the depletion allowance seems to be equal treatment of the mining and non-mining sectors in terms of effective capital income tax rates, it fails to attain this by using the original acquisition cost rather than the current value of the unit of in situ resource. Our results imply that if we set  $\beta = \pi$ , we will have  $\tau_m(0) = \tau$  and  $\tau_m(t) > \tau$  for  $t > 0$ .

As for the over all effect on the extraction path, it depends on whether

$$\pi \begin{matrix} < \\ = \\ > \end{matrix} \frac{(1-k)(1-uz)-(1-u)}{u}$$

It can therefore go either way, depending on the actual values of the various tax parameters. Furthermore, since the parameters on the right side apply to all of the various types of resources, while  $\pi$  varies from one type of resource to the other, we can expect the effect to differ in size and direction among the different types of resources.

### Conclusion

Our analysis has shown that the qualitative as well as the quantitative effects of the corporate income tax on the output path of the non-renewable resource extraction firm depends on the relative values of the various tax parameters, and in particular on the rate of depletion allowance. In a second best context where the corporate income tax is not neutral for the non-mining corporate sector to begin with, it appears that the argument for the depletion allowance can be made not on the grounds of attaining an undistorted extraction path, but on the grounds of maintaining equal effective tax rates on capital income between the mining and non-mining corporate sectors. In fact the two criteria are mutually exclusive, unless the tax is already neutral for the non-mining sector. The present U.S. depletion allowance scheme fails to attain equal effective tax rates, since its computation is based on the original cost of acquisition of the in situ resource rather than its current market value.

## FOOTNOTES

- <sup>1</sup> As we will show later, by properly defining  $\beta$  we can capture the essence of the present U.S. depletion allowance scheme.
- <sup>2</sup> For simplicity, we assume throughout, as is implicit in equation (2), that the portion of the investment allowed as tax credit cannot be depreciated for tax purposes. The practice varies on this matter over countries and over time. Our analysis could easily accommodate the several treatments encountered by replacing  $k$  in equation (2) by  $\alpha k$ , with  $\alpha$  varying between zero and one. The Canadian tax law, for example, sets  $\alpha = 1$ . The U.S. tax law sets  $\alpha = \frac{1}{2}$  since 1982, whereas it set  $\alpha = 0$  before 1982, except for the 1962-1963 period during which the depreciation base had to be written down by the full value of the credit ( $\alpha = 1$ ).
- <sup>3</sup> For an analysis of the case where there is discontinuity, with immediate depletion being allowed at the date of exhaustion, see Gaudet and Lasserre [7].
- <sup>4</sup> See for example Hall and Jorgenson [8], or for excellent synthesis of the effect of the corporate tax on the marginal investment decision in the traditional firm, Boadway [3] and Auerbach [2].
- <sup>5</sup> This is not the case if the tax treatment is modified at the terminal date. See Gaudet and Lasserre [7].

- 6 We will assume here that  $(1-k)(1-uz) \geq (1-u)$ . In practice however, one should not exclude the possibility that this inequality may be reversed if the investment tax credit is large enough. The corporate tax would then actually reduce the rental cost of capital for the corporate firm (see equation (13)).
- 7 For similar calculations, see Auerbach [1].
- 8 This may be more appropriately interpreted as the share of the current value of a unit of in situ resource in the market price of the extracted resource,  $p$ , set here equal to one.

## REFERENCES

- [1] Auerbach, A.J. (1983) "Corporate Taxation in the United States", Brookings Papers on Economic Activity, 451-505.
- [2] Auerbach, A.J. (1983) "Taxation, Corporate Financial Policy and the Cost of Capital", Journal of Economic Literature, 21, 905-940.
- [3] Boadway, R. (1980) "Corporate Taxation and Investment: A Synthesis of the Neoclassical Theory", Canadian Journal of Economics, 13, 250-267.
- [4] Burness, H.S. (1976) "On the Taxation of Nonreplenishable Natural Resources", Journal of Environmental Economics and Management, 3, 289-311.
- [5] Dasgupta, P.S. and Heal, G.M. (1979) Economic Theory and Exhaustible Resources (Wolyn: Cambridge University Press).

- [6] Dasgupta, P.S., Heal, G.M. and Stiglitz, J.E. (1980) "The Taxation of Exhaustible Resources", in Public Policy and the Tax System, G.A. Hughes and G.M. Heal (eds), (London: George Allen and Unwin).
- [7] Gaudet G. and Lasserre, P. (1984) "L'impôt sur le revenu des sociétés et le coût du capital pour l'entreprise minière", Canadian Journal of Economics (forthcoming).
- [8] Hall, R.E. and Jorgenson, D.W. (1971) "Application of the Theory of Optimum Capital Accumulation", in Tax Incentives and Capital Spending, G. Fromm (ed.), (Washington: The Brookings Institution).
- [9] Solow, R.M. (1974) "The Economics of Resources or the Resources of Economics", American Economic Review, 64, No 2, 1-14.