The Impact of Recycling on the Long-Run Stock of Trees[1]

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Abstract

Interest in recycling of forest products has grown in recent years, one of the goals being to conserve the stock of trees or possibly increase it to compensate for positive externalities generated by the forest and neglected by the market. This paper explores the issue as to whether recycling is an appropriate measure to attain such a goal. We do this by considering the problem of the private owner of an area of land, who, acting as a price taker, decides how to allocate his land over time between forestry and some other use, and at what age to harvest the forest area chosen. Once the forest is cut, he makes a new land allocation decision and replants. He does so indefinitely, in a Faustmann-like framework. The wood from the harvest is transformed into a final product which is partly recycled into a substitute for the virgin wood, so that past output affects the current price. We show that in such a context, increasing the rate of recycling will result in less area being devoted to forestry. It will also have the effect of increasing the harvest age of the forest, as long as the planting cost is positive. The net effect on the flow of virgin wood being harvested to supply the market will as a result be ambiguous. The main point however is that recycling will result in a smaller, not a larger, stock of trees in the long run. It would therefore be best to resort to other means if the goal is to increase the stock of trees.

Keywords: Recycling; Forestry; Faustmann Rule; Land Use.
J.E.L. classification: Q15, Q23, Q53.

Résumé

L'un des principaux arguments utilisé pour favoriser le recyclage des produits de la forêt est que cela augmenterait le stock d'arbre, compensant ainsi pour les effets externes positifs de la forêt que négligerait le marché. Le but de ce papier est d'étudier dans quelle mesure la promotion du recyclage est un instrument approprié pour atteindre un tel objectif. Pour ce faire, nous modélisons le comportement d'un propriétaire privé représentatif qui doit décider de l'allocation de sa terre entre la forêt et une autre utilisation, et de l'âge de coupe de la partie allouée à la forêt. Une fois les arbres coupés, il prend une nouvelle décision d'allocation de sa terre et reboise. Le bois récolté est transformé en un produit final qui est en partie recyclé en un substitut au produit primaire de la coupe, de sorte que le prix courant dépend de la production passée. Nous montrons qu'accroître le taux de recyclage a pour résultat qu'une plus faible surface sera allouée à la forêt à long terme. Ceci aura aussi pour effet d'accroître l'âge de coupe de la forêt. L'effet net sur le flux de bois coupé pour approvisionner le marché sera ambigu. Le point important, cependant, est que le recyclage résultera en un plus petit et non un plus grand stock d'arbres à long terme. Il vaudrait donc mieux recourir à d'autres moyens si l'objectif est d'accroître le stock d'arbre.

Mots-clés : Recyclage ; Forêt ; Règle de Faustmann ; Utilisation des sols.
Classification J.E.L. : Q15, Q23, Q53.

1 Introduction

In recent years, it has become fashionable to promote recycling of forest products, in particular paper. The main argument in favor of encouraging recycling is that it saves trees, the implicit objective therefore being to end up with a stock of trees which is greater than what it would be in the absence of recycling. The reason for wanting to increase the standing stock of trees is that it generates externalities: it procures direct amenities, it protects against soil erosion and it serves as a carbon sink. To the extent that positive externalities are involved, the market equilibrium will result in an insufficient stock of trees, which may justify policies meant to increase it. The purpose of this paper is to consider to what extent the promotion of recycling is an appropriate means of attaining such a goal.

To do this, we specify a simple dynamic model of land allocation by a private owner between forestry activities and alternative uses, such as agriculture. The model takes into account that the product of the forest can be partly recycled and it allows for two decision variables on the part of the land owner, namely the area of land allocated to forestry at any time and the age at which the forest is cut and replanted. This enables us to examine how recycling affects both the long-run equilibrium quantity of forest land and the long-run cutting age of the forest.

The question of the allocation of land between competing uses is of course, in itself, not new. For instance, in the recent literature involving use of land for forestry, Barbier and Burgess (1997) propose an intertemporal model to analyze the optimal conversion of land from timber to agriculture and use it to estimate the demand relationship for converted land. McConnell (1989) and Lopez, Shah and Altobello (1994) examine the optimal allocation of land between agricultural land, park and public land and urban land in the United States, using a static model. Ehui, Hertel, and Preckel (1990) use a two-sector dynamic model to study the optimal allocation of land between agriculture and forestry in a developing country. This model was also used by Ehui and Hertel (1989) to estimate the optimal steady-state forest stock in Ivory Coast. Hartwick, Long and Tian (2001) use a two-sector dynamic model

to analyze land clearing in a small open economy with a large endowment in forestry and a small endowment in agriculture, and facing given world prices for both agricultural and forest products. All of those papers treat the output of the forest as being totally consumed in a single usage and consequently exclude recycling.

Furthermore, whereas the original Faustmann rule (Faustmann, 1849), which is the basic intertemporal arbitrage rule for determining the optimal forest rotation, takes the land area planted in forest as given, all of those papers ignore the optimal rotation question and consider only the land allocation decision, in some cases treating the forest as a nonrenewable resource. None of them makes use of the Faustmann rule to determine the optimal harvest age and replanting decisions. The same is true of Darby (1973), who, in a short note, makes a stylized argument to the effect that recycling paper, by reducing the demand for wood, will result in less trees being planted. In a sense, this paper formalizes Darby's argument, by explicitly setting it into the optimal forest rotation model *à la* Faustmann and taking into account the effect on both the rotation over time and the area devoted to forestry.

We show that increasing the rate of recycling reduces the equilibrium area of land allocated to forestry in steady state and hence reduces the standing stock of trees. On the other hand, as long as the planting cost is positive, it leads to an increase in the harvest age of those trees. As a consequence, the effect of increasing the recycling rate on the equilibrium volume of virgin wood being supplied to the market is ambiguous, being more likely to be negative the smaller is the planting cost.

The next section serves to describe the model. In Section 3 we solve it and derive the comparative static results of varying the recycling rate on the steady-state equilibrium. We end with a few concluding remarks in Section 4.

2 The model

Consider a piece of land of fixed area A, to be allocated by its owner between forestry and some alternative use, say agriculture. Once the forest is cut, a new allocation is determined

and the area devoted to forest is immediately planted. This process is repeated indefinitely. Let i denote the ith such rotation. Then the forest cut at date t_i will have been planted at date t_{i-1}, which is the harvest date of the previous rotation. The cutting age for rotation i will therefore be $T_i = t_i - t_{i-1}$.

Let $X(T_i)$ denote the volume of wood per unit of area devoted to forestry obtained at age T_i. The growth function $X(T_i)$ is assumed to be an increasing and strictly concave function of T_i. If areas f_{i-1} and a_{i-1} were assigned respectively to forestry and to agriculture at planting date t_{i-1}, then the total volume of wood harvested at date t_i will be $h_i = f_{i-1}X(T_i)$. Since the total land area will be devoted either to forestry or agriculture, we will have:

$$f_{i-1} + a_{i-1} = A, \quad i = 1, ..., \infty. \tag{1}$$

Any use of the forest other than for the production of wood is neglected by this representative land owner. Once harvested, the wood is transformed into some recyclable final product, say paper. The final product can be equally well produced from virgin wood or from the recycled product.[1] We will assume that this is the only use for the wood being harvested.

Now let $S(t_i)$ denote the total quantity of input available for transformation into final product at date t. If a fraction δ of the stock available at date t_{i-1} is recycled, we will have:

$$S(t_i) = f_{i-1}X(T_i) + \delta S(t_{i-1}), \quad i = 1, ..., \infty \tag{2}$$

with $S(t_0) = S_0$, the given stock available at the initial planting date t_0. For simplicity, it will be assumed that one unit of this input can be transformed into one unit of the final product. If we let p_{t_i} denote the price of this input, then the inverse demand can be written:

$$p_{t_i} = P(S_{t_i}), \quad \text{with} \quad P(S_{t_i}) \geq 0 \quad P'(S_{t_i}) < 0 \quad \text{and} \quad \lim_{S_{t_i} \to \infty} P(S_{t_i}) = 0. \tag{3}$$

Note that since virgin wood and the recycled product are perfect substitutes, p_{t_i} is also the price of wood.

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[1]That virgin wood and the recycled product are perfect substitutes is a simplifying assumption. Assuming otherwise does not yield any additional insight towards the issue addressed in this paper.

Let $c \geq 0$ denote the cutting cost per unit of wood and $k \geq 0$ denote the planting cost per unit of area planted. It will be assumed that $P(0) > c$, so that it is profitable to exploit the forest to begin with. The present value at t_{i-1} of the net benefits from rotation i if an area f_{i-1} is planted and it is cut at age T_i will be:

$$\Pi_f(T_i, f_{i-1}; S(t_i)) = [P(S(t_i)) - c]f_{i-1}X(T_i)e^{-rT_i} - kf_{i-1},$$

where r is the discount rate. The semi-colon (;) in front of $S(t_i)$ is meant to reflect the fact that the representative land owner will, as a price taker, neglect the effect of his individual decisions on $S(t_i)$.

If an area f_{i-1} is devoted to forestry, then an area $a_{i-1} = A - f_{i-1}$ is devoted to agriculture. Let $g(a_{i-1})$ represent the net instantaneous benefit function from agriculture, with $g'(a_{i-1}) > 0$ and $g''(a_{i-1}) < 0$. The present value at t_{i-1} of the net benefits from agriculture over the same interval of time T_i will be:

$$\Pi_a(T_i, a_{i-1})) = \int_{t_{i-1}}^{t_i} g(a_{i-1})e^{-r(\tau - t_{i-1})}d\tau = \frac{g(a_{i-1})}{r}(1 - e^{-rT_i}).$$

The value at t_{i-1} of the net discounted benefits from total land use over the interval T_i is therefore:

$$\pi_i = \Pi(T_i, f_{i-1}, a_{i-1}; S(t_i)) = \Pi_f(T_i, f_{i-1}; S(t_i)) + \Pi_a(T_i, a_{i-1}). \tag{4}$$

3   The equilibrium land allocation and harvesting age

The representative land owner's decision problem at t_0, acting as a price taker, is to choose the sequence $\{T_i, f_{i-1}, a_{i-1}\}_{i=1}^{\infty}$ so as to maximize:

$$V(S_0) = \sum_{i=1}^{\infty} \Pi(T_i, f_{i-1}, a_{i-1}; S(t_i))e^{-r(t_{i-1}-t_0)} \tag{5}$$

subject to (1) and to $f_{i-1} \geq 0$, $a_{i-1} \geq 0$, $i = 1, ..., \infty$. We will hereafter consider only interior solutions for f_{i-1} and a_{i-1}, so that the nonnegativity constraints can be ignored. Substituting for a_{i-1} from (1) into (5), the problem can then be reformulated as choosing

the sequence $\{T_i, f_{i-1}\}_{i=1}^{\infty}$ to maximize:

$$
\begin{aligned}
V(S_0) &= \sum_{i=1}^{\infty} \Pi(T_i, f_{i-1}, A - f_{i-1}; S(t_i)) e^{-r(t_{i-1} - t_0)} \\
&= \Pi(T_1, f_0, A - f_0; S(t_1)) + \sum_{i=2}^{\infty} \Pi(T_i, f_{i-1}, A - f_{i-1}; S(t_i)) e^{-r \sum_{j=1}^{i-1} T_j} \qquad (6)
\end{aligned}
$$

where we have used the fact that $t_{i-1} - t_0 = \sum_{j=1}^{i-1} T_j$ for $i > 1$.

The first-order necessary conditions for interior solutions will be given by:

$$
\frac{\partial V(S_0)}{\partial f_0} = \frac{\partial \pi_1}{\partial f_0} - \frac{\partial \pi_1}{\partial a_0} = 0 \qquad (7)
$$

$$
\frac{\partial V(S_0)}{\partial f_{i-1}} = e^{-r \sum_{j=1}^{i-1} T_j} \left\{ \frac{\partial \pi_i}{\partial f_{i-1}} - \frac{\partial \pi_i}{\partial a_{i-1}} \right\} = 0, \qquad i = 2, \dots, \infty \qquad (8)
$$

$$
\begin{aligned}
\frac{\partial V(S_0)}{\partial T_1} &= \frac{\partial \pi_1}{\partial T_1} - r \sum_{k=2}^{\infty} \pi_k e^{-r \sum_{j=1}^{k-1} T_j} \\
&= \frac{\partial \pi_1}{\partial T_1} - r e^{-r T_1} \left[\pi_2 + \sum_{j=2}^{\infty} \pi_{i+j} e^{-r \sum_{k=1}^{j-1} T_{i+k}} \right] \\
&= \frac{\partial \pi_1}{\partial T_1} - r e^{-r T_1} V(S(t_1)) = 0 \qquad (9)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial V(S_0)}{\partial T_i} &= \frac{\partial \pi_i}{\partial T_i} e^{-r \sum_{j=1}^{i-1} T_j} - r \sum_{k=i+1}^{\infty} \pi_k e^{-r \sum_{j=1}^{k-1} T_j} \\
&= e^{-r \sum_{j=1}^{i-1} T_j} \left\{ \frac{\partial \pi_i}{\partial T_i} - r e^{-r T_i} \left[\pi_{i+1} + \sum_{j=2}^{\infty} \pi_{i+j} e^{-r \sum_{k=1}^{j-1} T_{i+k}} \right] \right\} \\
&= e^{-r \sum_{j=1}^{i-1} T_j} \left\{ \frac{\partial \pi_i}{\partial T_i} - r e^{-r T_i} V(S(t_i)) \right\} = 0, \qquad i = 2, \dots, \infty, \qquad (10)
\end{aligned}
$$

where the partial derivatives of π_i are obtained from (4). After substituting for these, we find that conditions (7) to (10) will be satisfied if and only if, for all $i = 1, \dots, \infty$:

$$
[P(S(t_i)) - c] X(T_i) e^{-r T_i} - k = \frac{g'(A - f_{i-1})}{r} (1 - e^{-r T_i}) \qquad (11)
$$

$$
[P(S(t_i)) - c] f_{i-1} X'(T_i) + g(A - f_{i-1}) = r \left\{ [P(S(t_i)) - c] f_{i-1} X(T_i) \right\} + r V(S(t_i)). \qquad (12)
$$

The left-hand side of (11) is the net marginal benefit of allocating land to forestry, while the right-hand side is the net marginal benefit of allocating land to agriculture. The condition simply says that the net benefit from the marginal unit of land must be the same in the two allocations.

Condition (12) says that the net benefit from delaying the cutting age marginally, which is given by the increment in the volume of wood resulting from forest growth valued at its net price plus the net benefit from agriculture obtained on area a_{i-1} during that marginal delay in cutting the forest, must be equal to the interest on the net benefit foregone from not harvesting the forest immediately, plus the interest foregone from delaying all future rotations.

Consider now a steady state, such that $T_i = T_{i-1} = T$, $f_i = f_{i-1} = f$ and $S(t_i) = S(t_{i-1}) = S$. We then find that, for all i:

$$V(S(t_i)) = \frac{[P(S) - c]fX(t)e^{-rT} - kf}{1 - e^{-rT}} + \frac{g(A - f)}{r},$$

from which it follows that (11) and (12) become:

$$F(f, T, S) = [P(S) - c]X(T)e^{-rT} - k - \frac{g'(A - f)}{r}(1 - e^{-rT}) = 0 \qquad (13)$$

$$G(f, T, S) = [P(S) - c]X'(T) - r\frac{(P(S) - c)X(T) - k}{1 - e^{-rT}} = 0. \qquad (14)$$

The steady-state stock of input S available for transformation into the final product is given by:

$$S = fX(T)/(1 - \delta). \qquad (15)$$

Condition (13) says that, given the harvesting age, the area devoted to forestry must be such as to equate the net marginal benefit between the two possible uses of the land. Condition (14) says that, given the area devoted to forestry, the harvesting age must be chosen so as to satisfy the Faustmann rule (Faustmann, 1849). Conditions (13) and (14), together with (15), determine the steady-state equilibrium area devoted to forestry, f, and harvesting age, T.

Notice that an interior solution for f is possible only if $P(S) - c > 0$. For if $P(S) - c \leq 0$, then $F(f, T, S) < 0$ and no land would be devoted to forestry ($f = 0$). Notice also that, if $P(S) - c > 0$, it follows from (13) and (14) that $X'(T) > 0$. It also follows from (14) that

$$X'(T) - \frac{rX(T)}{1 - e^{-rT}} = \frac{-rk}{(1 - e^{-rT})(P(S) - c)} \leq 0.$$

We show in the Appendix that a steady state defined by (13) and (14) will be locally stable if and only if

$$\triangle = F_f G_T + \frac{1}{1-\delta}[G_T F_S X(T) + F_f G_S f X'(T)] > 0, \qquad (16)$$

where[2]

$$F_f = \frac{g''(A-f)}{r}\left(1 - e^{-rT}\right) < 0 \qquad (17)$$

$$G_T = [P(S) - c][X''(T) - rX'(T)] < 0 \qquad (18)$$

$$F_S = P'(S)X(T)e^{-rT} < 0 \qquad (19)$$

$$G_S = P'(S)\left[X'(T) - \frac{rX(T)}{1 - e^{-rT}}\right] = \frac{-rkP'(S)}{(1 - e^{-rT})(P(S) - c)} \geq 0. \qquad (20)$$

To see the impact of the rate of recycling on the long-run equilibrium land allocation and harvesting age, differentiate totally (13) and (14) taking into account (15), to get:

$$\frac{df}{d\delta} = -\frac{1}{\triangle}\left[\frac{fX(T)}{(1-\delta)^2}F_S G_T\right] < 0$$

$$\frac{dT}{d\delta} = -\frac{1}{\triangle}\left[\frac{fX(T)}{(1-\delta)^2}F_f G_S\right] \geq 0.$$

Therefore, increasing the rate of recycling (from any admissible level, including $\delta = 0$) results unambiguously in less land being allocated to forestry in the long run and hence a smaller stock of trees. At the same time, the long run harvest age of those trees will either increase or stay the same. This means that the overall effect on $h = fX(T)$, the volume of virgin wood being supplied to the market, will be ambiguous. This effect is given by

$$\frac{dh}{d\delta} = fX'(T)\frac{dT}{d\delta} + X(T)\frac{df}{d\delta}$$

$$= -\frac{1}{\triangle}\frac{fX(T)}{(1-\delta)^2}[G_T F_S X(T) + F_f G_S f X'(T)].$$

The sign of this expression is indeterminate. It will depend on whether the longer growth allowed before harvesting compensates for the smaller area devoted to the forest. One of the

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[2]The second-order conditions for a maximum require $F_f \leq 0$, $G_T \leq 0$ and $F_f G_T \geq 0$, which are all satisfied given (17) to (20). Notice that $G_f \equiv 0$ and $F_T = [P(S) - c][X'(T) - rX(T)]e^{-rT} - g'(A-f)e^{-rT} = 0$ when (13) and (14) are satisfied.

important parameters is k, the planting cost. Since G_S is increasing in k, so is the marginal effect of δ on the harvest age T. The reason is that letting the trees grow is a way for the owner to delay the replanting cost. In particular, if the planting cost is zero, changing the recycling rate has no effect on the harvest age and the net effect on the volume of virgin wood supplied to the market is negative. The same holds for small planting costs. But if the planting cost is sufficiently large, increasing the recycling rate can result in a greater volume of virgin wood being produced.

4 Concluding remarks

One result that clearly comes out of our analysis is that increasing the rate of recycling will result in less, not more, land being allocated to forestry in the long run. If the only goal is to increase the stock of trees in order to compensate for external benefits that are neglected by the market, it would seem that to encourage recycling is not an appropriate measure. Measures aimed directly at the land allocation decision are more appropriate, whether they be incentive mechanisms, such as taxes or subsidies, or regulation aimed at maintaining the forest area or increasing it. There may of course be other reasons to pursue a recycling policy, but from the strict standpoint of protecting the forest area it is likely to have the reverse effect in the long run.

Appendix

Existence and stability of a steady state

Conditions (11) and (12) yield $f_{i-1}(S(t_i))$ and $T_i(S(t_i))$, the solution for the forest owner's decisions for f_{i-1} and T_i as a function of $S(t_i)$, from which we may write $h_i(S(t_i)) = f_{i-1}(S(t_i))X(T_i(S(t_i)))$, the solution for the total volume of wood harvested as a function of $S(t_i)$. Equation (2) therefore becomes:

$$S(t_i) = h_i(S(t_i)) + \delta S(t_{i-1}).$$

In steady state, $S(t_i) = S(t_{i-1}) = S$ and hence:

$$h(S) = (1 - \delta)S.$$

The function $h(S)$ takes a positive value at $S = 0$. This follows directly from (13), considering that $P(0) > c$. It also goes to zero as S goes to infinity. This last property holds because, in view of (3), as S increases it will eventually reach a value $\bar{S}$ such that $P(\bar{S}) = c$ and beyond which $P(S) - c < 0$, with the result that $F(f, T, S) < 0$ in (13). The value of f could then not be interior and must be zero: no land would be allocated to forestry by the land owner and hence $h = fX(T) = 0$. The function $h(S)$ being continuous and the right-hand side of the equation being a monotone increasing function of S that goes through the origin, it follows that there exists at least one steady state. Such a steady state will be locally stable if and only if, in its neighborhood,

$$h'(S) = -\frac{G_T F_S X(T) + f F_f G_S X'(T)}{F_f G_T} < 1 - \delta,$$

or, since $F_f G_T > 0$ (see (17) and (18)),

$$\triangle = F_f G_T + \frac{1}{1 - \delta}[G_T F_S X(T) + F_f G_S f X'(T)] > 0.$$

Note $h'(S)$ was obtained by applying Cramer's rule to the system (13)-(14).

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