

CAHIER 8053MORAL HAZARD AND STATE-DEPENDENT
UTILITY FUNCTION

by

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Abstract

The first purpose of this paper is to present the choice of the optimal insurance policy under a state-dependent utility function (S.D.U.F.) when the probabilities of loss are dependent on the actions of the insured. It is demonstrated that moral hazard is still an important problem under the S.D.U.F. .

The second purpose of the paper consists of verifying whether the problem of moral hazard is more or less important under a S.D.U.F.. A measure of moral hazard in terms of insurance coverage is put forward. This measure is a function of two factors : 1) an accident or illness reduces the utility of the individual and 2) the marginal utility of income is a function of the states of the world. The first factor always has a negative effect on moral hazard, but the second factor may affect moral hazard in different ways. Finally an analysis of the variation of the "deductible" is presented and it is found that the optimal "deductible" doesn't necessarily vary in the same way that moral hazard varies.

Résumé

Le premier objectif de ce cahier consiste à présenter le choix optimal de couverture d'assurance sous le risque moral lorsque les fonctions d'utilités sont dépendantes des états de la nature. Il est démontré que le risque moral demeure un problème important d'allocation des ressources.

Dans un deuxième temps, l'importance relative du risque moral est étudiée. Une mesure du risque moral en terme de couverture d'assurance est présentée. Cette mesure est fonction de deux facteurs : 1) un accident ou une maladie réduit l'utilité des individus et 2) l'utilité marginale du revenu est fonction des états de la nature. Le premier facteur a toujours un effet négatif sur le risque moral alors que le deuxième peut ne pas affecter, réduire ou augmenter le risque moral. Finalement une analyse de la variation du "déductible" est présentée et il est démontré que le "déductible" optimal ne varie pas nécessairement dans la même direction que le risque moral.

Introduction

In the economic literature on insurance, moral hazard is defined as the effect of insurance protection on the incentives of the insured to take care of himself/herself and of his/her belongings. Since the insurer cannot observe without cost the activities of care, the insured has less monetary incentives to do these activities. This results in an increase in the probabilities (or in the amounts) of loss and then, in an increase in the premiums. Two kinds of solutions have been proposed to reduce the misallocation of resources related to this problem of information¹: 1) partial insurance coverage and 2) the use, by the insurer of costly mechanisms to acquire imperfect information about the care activities of the insured. However, both are second-best solutions.

This literature does not take into account the fact that an individual may have other incentives to take care that are not monetary. An accident² or illness may be undesirable not only because it is associated with monetary losses but also because it procures direct disutility to the individuals affected. Taking into account the latter possibility, it increases the marginal benefit of taking care and the optimal amount of care for a given insurance coverage. But this is only part of the story.

The choice of the optimal insurance policy is also related to the fact that the utility function (and consequently the marginal utility of income) is a function of the states of the world. The optimal coverage is no longer limited to the monetary losses but is also related to the possibility of making a person whole, that is, of restoring the person to his/her initial level of utility. Since the amount of insurance coverage also affects the optimal amount of care, it is not immediately clear whether moral hazard is equally, more or less important under a state-dependent utility function (S.D.U.F.) than it is under a state-independent utility function (S.I.U.F.).

The first purpose of this paper is to present the choice of the optimal insurance policy under a S.D.U.F. when the probabilities of loss are dependent on the actions of the insured. It is demonstrated that moral hazard is still an important problem under the S.D.U.F.. In order to motivate the individual to take care and to be better off, optimal insurance coverages under moral hazard correspond to less insurance benefit than in the case of no moral hazard. But when the optimal amount of insurance coverage without moral hazard is greater than the amount which makes the person whole, it is shown that an upper limit of insurance coverage is necessary to avoid deceptive activities³ which may increase the probabilities of loss.

The second purpose of the paper consists of verifying whether the problem of moral hazard is more or less important under a S.D.U.F.. A measure of moral hazard in terms of insurance coverage is put forward. This measure is a function of the two factors above-mentioned: 1) an accident or illness reduces the utility of the individual and 2) the marginal utility of income is a function of the states of the world. The first factor always has a negative effect on moral hazard, but the second factor may affect moral hazard in different ways. Moral hazard is found to be less important under a S.D.U.F. when the effect of the second factor is negative or nul, that is when in State II (accident or illness) the marginal utility of income is less or equal to the marginal utility of income in State I (no accident or good health) for all levels of income. But moral hazard may be more important when the marginal utility of income is greater in State II for all levels of income. Finally an analysis of the variation of the "deductible" is presented and it is found that the optimal "deductible" doesn't necessarily vary in the same way that moral hazard varies, since the optimal levels of insurance coverage are also functions of the marginal utility of income.

The formal notation and the assumptions of the model specified in Section I are followed by a brief review of the main results presented in the literature concerning the choice of insurance under a S.D.U.F. without moral hazard and the choice of insurance under a S.I.U.F. with moral hazard. Section III presents the optimal choice of insurance under a S.D.U.F. with moral hazard and Section IV analyzes both the relative importance of moral hazard under a S.D.U.F. and the variation of the optimal "deductible". The concluding section of the paper summarizes the results obtained, and a new avenue of research is proposed.

I. Notation and assumptions of the model⁴

There are two possible states of the world in this model: State I corresponding to good health or no accident, and State II corresponding to illness or accident. The probability of the State II is $p(x)$ and $(1 - p(x))$ is equal to the probability of the State I. It is assumed that $p(x)$ is decreasing and strictly convex in x which is the level of care activities, therefore $p_x < 0$ and $p_{xx} > 0$.

The insured is assumed to maximize the expected value of his utility which is a function of wealth and of care activities. Defining S as the initial wealth in both states of the world, ℓ as the monetary loss in State II, P as the insurance premium, q as the amount of insurance payment, $U(S - P)$ as the utility function of wealth in State I, $V(S - P - \ell + q)$ as the utility function of wealth in State II and $C(x)$ as the utility cost of taking care, the expected utility is then assumed to be the following:

$$(1) \quad Z(x, q) = [1 - p(x)]U(S - P) + p(x)V(S - P - \ell + q) - C(x)$$

Where U and V are twice differentiable, increasing and strictly concave utility functions of wealth and C is a twice differentiable, increasing and strictly convex disutility function of taking care;

$$U_1 > 0, U_{11} < 0, V_2 > 0, V_{22} < 0,$$

$$C_x > 0, C_{xx} < 0, C_x(0) = 0, C_x(\infty) = \infty, \text{ where } U(1) \equiv U(S - P) \text{ and}$$

$$V(2) \equiv V(S - P - \ell + q).$$

Illness or accident is assumed not desirable; $U(S) > V(S)$ for all S . The marginal utility of income is also a function of the states of the world: three cases will be considered in this paper: (1) $U_S = V_S$ for all S , (2) $U_S > V_S$ for all S , (3) $U_S < V_S$ for all S . The notation " $U_S \gtrless V_S$ for all S " will be used to sum up the three cases. Finally it is assumed that all the insured represent identical risk to avoid the problem of adverse selection.

The insurer is risk neutral and the provision of insurance is without cost. In the case of perfect information about x , the insurance premium will be actuarial and linear in q , that is, $P(x, q) = p(x)q$. Under moral hazard, the nonlinear pricing rule proposed by Pauly (1974) will be used, that is $P(x(q), q) = p(x(q))q$.

II. Brief review of the main findings in the literature

A. The optimal choice of insurance under a S.D.U.F. without moral hazard

Since the insurer can observe the level of care activities, the actuarial premium is linear in q , that is $P(x, q) = p(x)q$, and $P_q = p(x) > 0$. It is also assumed that illness or accident creates direct disutility. The choice of the optimal insurance coverage and of the optimal amount of care activities by the consumer consists of maximizing the expected utility over q and x subject to the constraint that the insurer is in equilibrium:

$$(2) \quad \text{Max}_{x,q} [1 - p(x)]U(S - P(x,q)) + p(x)V(S - P(x,q) - \ell + q) - C(x)$$

$$\text{S.T.} \quad P(x,q) = p(x)q.$$

The first order conditions are:

$$(3.1) \quad p_x[V(2) - U(1)] - p_x q [[1 - p(x)]U_1 + p(x)V_2] = C_x$$

(3) {

$$(3.2) \quad p(x)[1 - p(x)][V_2 - U_1] = 0$$

The left-hand side of (3.1) represents the marginal benefit of taking care in terms of reducing both the probability of loss and the insurance premium. The right-hand side corresponds to the marginal cost of taking care in utility terms. Under perfect information, the insured may still have incentive to take care with full insurance ($V(2) = U(1)$) in order to reduce the insurance premium.

The optimal amount of insurance is given by (3.2). This implies that the marginal utilities of income are equal in both states, i.e., $V_2 = U_1$. The optimal amount of insurance is then less than, equal to or greater than the monetary loss ($q^* \lesseqgtr \ell$) as long as $V_S \lesseqgtr U_S$ for all S . The first case may correspond to the state of complete invalidity without responsibility for relatives, the second to accident or illness which does not cause permanent disutility, and finally, the third case may correspond to permanent disutility in which the individual may or may not have responsibility for relatives and/or may still have utility from consumption^{5,6}.

In the first two situations the insured would not be whole since q^* is less than the monetary amount which equalizes the utilities in both states, \bar{q} such that

$U(S - p\bar{q}) = V(S - p\bar{q} - \ell + \bar{q})$. In the last case, it is obtained that

$$q^* \geq \bar{q} \text{ when } \bar{V}_2 \geq \bar{U}_1, \text{ where } \bar{V}_2 \equiv V_2 \Big|_{q = \bar{q}} \text{ and } \bar{U}_1 \equiv U_1 \Big|_{q = \bar{q}}.$$

B. The optimal choice of insurance under a S.I.U.F. with moral hazard⁷

Now the insurer cannot observe the level of care activities and it is assumed implicitly that accident or illness does not cause direct disutility, therefore $U(S) = V(S)$ for all S . Following the methodology first proposed by Pauly (1974), the choice of the optimal amount of insurance under moral hazard will be presented in two steps. The first step consists of maximizing $Z(x, q)$ over x for a given q . The first order condition is equal to:

$$(4) \quad p_x [U(2) - U(1)] = C_x$$

where $U(2)$ is now the utility function of wealth in State II.

The right-hand side of (4) still corresponds to the marginal cost of taking care in utility terms; C_x is independent of q . The marginal benefit of taking care ($MB(q)$) is reduced to the marginal benefit of reducing the probability of loss and it is nul when $q = \ell$ ($MB(\ell) = 0$) since $U(2) = U(1)$. Moral hazard is possible when there is a positive solution to this problem at $q = 0$. Then $x^*(0) > 0$ is assumed. The optimal insurance coverage under moral hazard is calculated by maximizing $Z(x, q)$ over q , such that the consumer is in equilibrium in care activities for a given insurance coverage and that the insurer is in equilibrium in setting up a nonlinear insurance premium which takes into account the change in the expected insurance payment resulting from a reduction in care activities under moral hazard, that is $P(x(q), q) = p(x(q))q$ and $P_q = p_x x_q q + p(x(q))$. The first order condition of this problem may be written as follows:

$$(5) \quad \hat{q}^* = \frac{p(x(q))[1 - p(x(q))][U_2 - U_1]}{x_q p_x [[1 - p(x(q))]U_1 + p(x(q))U_2]}$$

We know that under a S.I.U.F. without moral hazard,⁸ the numerator is equal to zero when $q = \ell$ and greater than zero when $q < \ell$ since $Z(x, q)$ is increasing and strictly concave in q by assumption.⁹ The denominator is greater than zero under moral hazard, since $x_q < 0$, $p_x < 0$ and the terms in the brackets are greater than zero. Therefore it is obtained that $0 < \hat{q}^* < \ell$ under moral hazard since it cannot be equal either to zero or to ℓ : if $q = 0$, the numerator is greater than zero which is impossible and if $q = \ell$ the numerator is equal to zero which is also impossible.¹⁰

In using the nonlinear pricing rule, the insurer induces the insured to carry out care activities. This is a second-best solution but this is an improvement compared to the solution of full insurance under moral hazard ($x^*(\ell) = 0$).

III. The optimal choice of insurance under a S.D.U.F. with moral hazard

In this section, the two preceding cases are integrated; the insurer cannot observe the level of care activities of the insured and the utility function depends on non monetary factors.

Following the methodology presented in the preceding section, the choice of the optimal amount of insurance under moral hazard will be presented in two steps. The first order condition corresponding to the choice of the optimal level of care activities is now equal to:

$$(4') \quad p_x [V(2) - U(1)] = c_x$$

It is verified that $MB(\ell) > 0$ contrary to the case of the S.I.U.F.. The insured has greater utility to take care under a S.D.U.F. when the level of insurance coverage is equal to the monetary loss, since illness or accident procures

direct disutility. Of course, this situation is true for all $0 < q < \ell$. But as we saw in the previous section, the marginal utility of income is also a function of the states of the world and the insurance coverage may now be greater than ℓ . Then it is also verified that $MB(\bar{q}) = 0$ and $MB(q > \bar{q}) < 0$. The former now corresponds to the case of full insurance with the insured having no incentive (monetary or other) to take care ($x^*(\bar{q}) = 0$). The latter represents the situation where the insured chooses a level of insurance that would make him/her more than whole should the event, accident or illness occur.

A possible interpretation of this behavior is that the insured chooses a level of insurance to improve his welfare or the welfare of his relatives instead of being only compensated in whole or in part by the insurance when the event occurs. This individual is not motivated to take care under this level of insurance, ($x^*(q > \bar{q}) = 0$) and moreover, he/she may be motivated to increase the probability of loss by deceptive activities.⁴ This unusual possibility won't be formalized here but it has to be considered by the insurer to avoid losses.

Figure I shows $x^*(0)$, $x^*(\ell)$, and $x^*(\bar{q})$

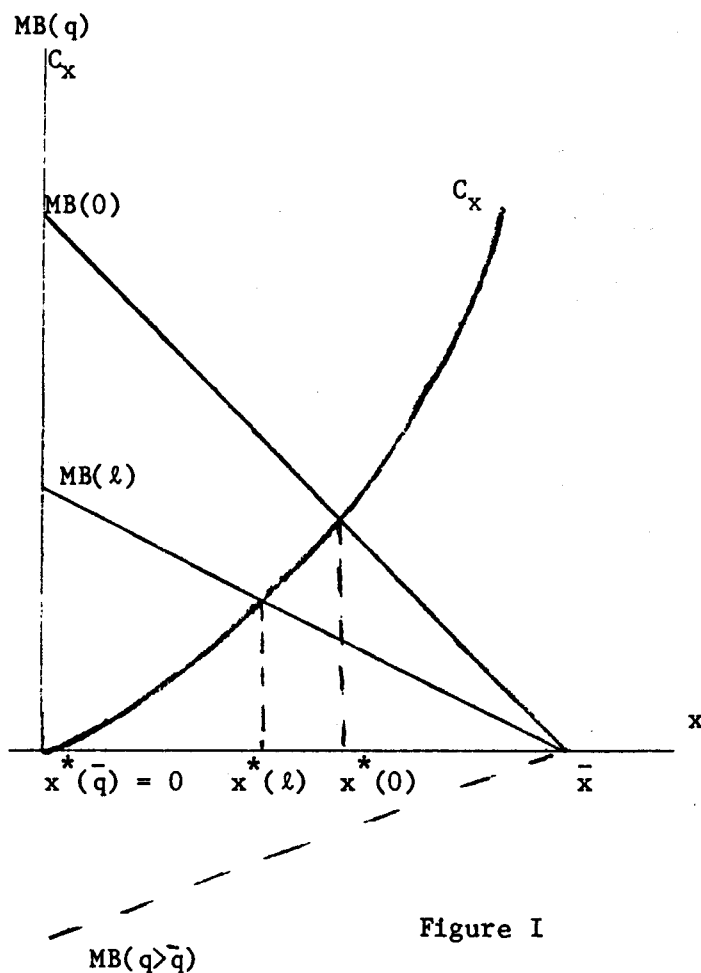


Figure I

In this figure, the marginal benefit curves are assumed linearly decreasing in x , that is, $p_{xxx} = 0$, and the marginal cost function is assumed increasing in x , at an increasing rate. As usual, the optimal solutions are obtained at the intersection of the curves except when $x^*(\bar{q}) = 0$, where the optimal solution is a corner solution since $MB(\bar{q})$ corresponds to the horizontal axis.

The sign of the variation of x with respect to q is formally verified by differentiating equation (4'):

$$(6) \quad x_q = - \frac{p_x \{V_2[1 - P_q] + U_1 P_q\}}{p_{xx} [V(2) - U(1)] - C_{xx}}$$

The denominator of (6) is equal to the second order condition of the choice of the optimal level of care activities; it is then less than zero at the optimum. The terms in the braces in the numerator are positive, therefore the sign of x_q is less than zero since the sign of p_x is less than zero. An increase in insurance coverage reduces the optimal level of care activities since the insurer cannot observe x . Moral hazard is still important under a S.D.U.F..

The second step finding the optimal insurance coverage under moral hazard, consists of maximizing $Z(q, x(q))$ over q , such that the consumer is in equilibrium in care activities and that the insurer is also in equilibrium in offering the following policy: $P(x(q), q) = p(x(q))q$ such that $q < \bar{q}$ to avoid deceptive activities.¹²

Proposition I: Under a S.D.U.F. with moral hazard,

- (a) the optimal amount of insurance coverage is always less than the optimal amount of insurance without moral hazard.
- (b) the optimal amount of insurance coverage with moral hazard is always greater than zero.
- (c) the optimal amount of insurance coverage with moral hazard is always less than \bar{q} .¹³

Proof: When the constraint $q < \bar{q}$ is not effective, the solution is similar to that obtained in (5), with the exception that the marginal utility of income in State II is now equal to $V_2 \gtrless U_2$.

By using (5) as modified as well as the results of the choice of the optimal amount of insurance under a S.D.U.F. without moral hazard, it is obtained that:

Case (1): when $U_S = V_S$ for all S

$$0 < \hat{q}^* < \ell = q^*$$

where \hat{q}^* represents the optimal choice of insurance with moral hazard and q^* , without moral hazard.

Case (2): When $U_S > V_S$ for all S

$$0 < \hat{q}^* < q^* < \ell$$

Case (3): When $U_S < V_S$ for all S

$$(3.1) \quad \text{if } \bar{V}_2 < \bar{U}_1 \\ 0 < \hat{q}^* < q^* < \bar{q}$$

$$(3.2) \quad \text{if } \bar{V}_2 = \bar{U}_1 \\ 0 < \hat{q}^* < q^* = \bar{q}$$

$$(3.3) \quad \text{if } \bar{V}_2 > \bar{U}_1 \\ 0 < \hat{q}^* < \bar{q} = \tilde{q}^* < q^*$$

where \tilde{q}^* is the optimal amount of insurance coverage under the constraint $q < \bar{q}$ without moral hazard.

This last result (3.3) needs some explanation. Only here is the constraint, $q < \bar{q}$, effective, therefore (5) is now equal to:

$$(7) \quad \hat{q}^* = \frac{p(x(q)) [1 - p(x(q))] [V_2 - U_1] - \lambda}{x_q p_x [[1 - p(x(q))] U_1 + p(x(q)) V_2]}$$

Where λ is a Lagrange multiplier reflecting the constraint.

The numerator is equal to zero when $q = \bar{q}$.¹⁴ Thus $\hat{q}^* < \bar{q}$ under moral hazard.

Another proof that \hat{q}^* cannot be greater than or equal to \bar{q} is presented in Appendix I.

In this section it has been shown that moral hazard is still an important problem of resource allocation under a S.D.U.F. contrary to the usual assumption that the individual will be necessarily more careful since illness or accident procure direct disutility. The important issue now is to verify whether moral hazard is less or more important under a S.D.U.F.. This is the main purpose of the next section which will also present a discussion of adjustment of the optimal insurance policy.

IV. The relative importance of moral hazard under a S.D.U.F. and the variation of the optimal "deductible"

As we saw in the previous section, two factors may influence the variation of care in relation to the variation of insurance under a S.D.U.F.: illness is not desirable and the marginal utility of income is a function of the states of the world. The first factor affects negatively the possibilities of moral hazard since the individual has a greater utility cost in order to reduce his self-protection under insurance. The second factor may not affect or may affect positively or negatively moral hazard. If the marginal utility of income is greater (equal or less) in State II than in State I, $V_S \begin{matrix} \geq \\ < \end{matrix} U_S$ for all S, the increase of insurance coverage will have a greater (equal or lesser) negative effect on the marginal utility of taking care, and will affect positively (will not affect or affect negatively) the possibilities of moral hazard, ceteris paribus.

Therefore the following results are anticipated:

- (1) If the marginal utility of income is the same in both states under the S.D.U.F., $U_S = V_S$ for all S, moral hazard will be less important than in the case of the S.I.U.F.. The first factor has a negative effect on moral hazard and the second factor has no effect.

- (2) If the marginal utility of income is less, for all S, in State II than in State I, moral hazard will be less important. Both factors have a negative effect on moral hazard.
- (3) If the marginal utility of income is greater for all S, in State II than in State I, moral hazard will be more, less or equally important than in the case of no S.D.U.F.. The first factor still has a negative effect, but the second factor now has a positive effect. The net effect depends on the relative magnitude of each factor.

The variation of moral hazard does not necessarily mean that the variation of the optimal insurance coverage will be affected in the same way since the latter is, itself, a function of the variation of the marginal utility of income. This implies that there may be a gap between the result about the variation of moral hazard and the adjustment of the related optimal insurance policy.

The following method will be used to prove the anticipated results. The marginal effect of introducing a S.D.U.F. on the level of utility and on the marginal utility of income is measured by the variation of a parameter (α). If U^1 and U^2 are defined as follows:

$$(8) \quad \begin{aligned} U^1 &= U(S) \\ U^2(\alpha) &= [1 - \alpha]U(S) + \alpha V(S) \quad \text{with } 0 < \alpha < 1 \end{aligned}$$

$$\text{Therefore } U_{\alpha}^2 = V(S) - U(S) < 0 \quad \text{and } U_{S\alpha}^2 = (V_S - U_S) > 0.$$

Proposition II: Under a S.D.U.F. moral hazard:

- (1) is less important than under a S.I.U.F. if $U_S = V_S$ for all S.
- (2) is less important than under a S.I.U.F. if $U_S > V_S$ for all S.
- (3) is more important than under a S.I.U.F. if $U_S < V_S$ for all S and if the positive effect of the second factor (the increase in the marginal utility of income) is greater than the negative effect of the first factor (bad events are not desirable).

Proof: Combining (6) and (8)

$$(9) \quad x_q(\alpha) = - \frac{p_x [U_2^2(\alpha) [1 - P_q] + U_1^1 P_q]}{p_{xx} [U^2(\alpha) - U^1] - C_{xx}}$$

and differentiating (9) with respect to α :

$$(10) \quad x_{q\alpha} = - \left\{ \frac{\Delta(\alpha) p_x U_{2\alpha}^2 [1 - P_q] - p_x [U_2^2(\alpha) [1 - P_q] + U_1^1 P_q] \Delta_\alpha}{[\Delta(\alpha)]^2} \right\}$$

where: $\Delta(\alpha) \equiv p_{xx} [U^2(\alpha) - U^1] - C_{xx} < 0$ for all α .

and $\Delta_\alpha = p_{xx} U_{2\alpha}^2 < 0$. Since $U_\alpha^2 = V(2) - U(2) < 0$ and $p_{xx} > 0$.

Δ_α corresponds to the first factor that is illness or accident are not desirable, and $U_{2\alpha}^2$ to the second factor that is the marginal utility of income is a function of the states of the world. The sign of $x_{q\alpha}$ depends on the assumptions made on $U_{2\alpha}^2$:

Case (1): when $V_S = U_S$ for all S

$$(11) \quad U_{2\alpha}^2 = V_2 - U_2 = 0 \quad \text{and}$$

$$(12) \quad x_{q\alpha} = \frac{p_x [U_2^2(\alpha) [1 - P_q] + U_1^1 P_q] \Delta_\alpha}{[\Delta(\alpha)]^2} = \frac{< 0 [> 0 > 0 + > 0 > 0] < 0}{> 0} > 0$$

Moral hazard is less important since $x_q < 0$: The first factor has a negative effect and the second factor has no effect.

Case (2): when $V_S < U_S$ for all S

$$(13) \quad U_{2\alpha}^2 < 0.$$

Using (10), (12) and (13)

$$(14) \quad x_{q\alpha} = - \frac{[< 0 - > 0]}{> 0} > 0$$

Moral hazard is less important: Both factors work in the same direction.

Case (3): when $V_S > U_S$ for all S

$$(15) \quad U_{2\alpha}^2 > 0$$

Using (10), (12) and (15)

$$(16) \quad x_{q\alpha} = - \frac{[> - >]}{> 0} \geq 0$$

Here moral hazard is less, equally or more important, depending on the relative magnitude of each factor: The first factor has still a negative effect but the second factor now has a positive effect on moral hazard. Therefore moral

hazard is more important when the effect of the second factor is greater than the effect of the first factor.

Proposition 3: The optimal "deductible" does not necessarily vary in the same way that moral hazard varies since the optimal amounts of insurance coverage are also functions of the marginal utility of income.

Proof: Under moral hazard the optimal amount of insurance is less than without moral hazard ($\hat{q}^* < q^*$). Define k as a measure of the difference between \hat{q}^* and q^* . This measure is a function of the variation of the insurance coverage explained by moral hazard (β) which is, itself, a function of the variation of care activities in relation to the level of insurance coverage (x_q). Thus:

$$(17) \quad k(\beta(x_q)) = q^* - \hat{q}^*(\beta(x_q)) \text{ and } k(0) = 0 \text{ since } \beta(0) = 0 \text{ and } \hat{q}^*(0) = q^*$$

Combining (8) and (17):

$$(18) \quad g(\alpha) = k(\alpha, \beta(\alpha, x_q(\alpha))) = q^*(\alpha) - \hat{q}^*(\alpha, \beta(\alpha, x_q(\alpha)))$$

where $g(\alpha)$ has the same definition as k .

The derivative of g in relation to α (g_α) measures the variation of the difference between the two optimal levels of insurance from a S.I.U.F. to a S.D.U.F. In fact, g_α is a measure of the variation of the optimal "deductible" when α varies. By definition, g_α is a function of the variation of moral hazard ($x_{q\alpha}$) but, since β , q^* and \hat{q}^* are also a function of the parameter α , the variation of the optimal "deductible" is not necessarily directly proportional to the variation of moral hazard. Differentiating (18) in relation to α :

$$(19) \quad g_\alpha = q_\alpha^* - \hat{q}_\alpha^* - \hat{q}_\beta^* [\beta_\alpha + \beta_{x_q} x_{q\alpha}]$$

To obtain the sign of the variation of the optimal "deductible", the detailed values of q_α^* , \hat{q}_α^* , $\hat{q}_\beta^* [\beta_\alpha + \beta_{x_q} x_{q\alpha}]$ must be specified. $q^*(\alpha)$ solves:

$$(20) \quad Z_q(x, q, \alpha) = p(x)[1 - p(x)][U_2^2(\alpha) - U_1^1] = 0$$

and, by totally differentiating (20):

$$(21) \quad q_\alpha^* = - \frac{Z_{q\alpha}(x, q, \alpha)}{Z_{qq}(x, q, \alpha)}$$

on the other hand, $\hat{q}^*(\alpha, \beta(\alpha, x_q(\alpha)))$ solves:

$$(22) \quad Z_q(x(q), q, \alpha) = p(x(q))[1 - p(x(q))][U_2^2(\alpha) - U_1^1] - p_{x_q}(\alpha)q [[1 - p(x(q))]U_1^1 + p(x(q))U_2^2(\alpha)] = 0$$

if $p(x(0)) = p(x)$, therefore (22) becomes:

$$(23) \quad Z_q(x, q, \alpha) \Big|_{q = \hat{q}^*} + \beta(\alpha, x_q(\alpha)) = 0$$

and by totally differentiating (23):

$$(24) \quad \hat{q}_\alpha^* + \hat{q}_\beta^* [\beta_\alpha + \beta_{x_q} x_{q\alpha}] = - \frac{Z_{q\alpha}(x(q), q, \alpha)}{Z_{qq}(x(q), q, \alpha)} = \frac{Z_{q\alpha}(x, q, \alpha) \Big|_{q = \hat{q}^*} - p_{x_q}(\alpha)q [pU_2^2] - p_{x_q}(\alpha)q [[1 - p]U_1^1 + pU_2^2(\alpha)]}{Z_{qq}(x(q), q, \alpha)}$$

where p represents $p(x(q))$.

substituting (21) and (24) in (19):

$$(25) \quad g_{\alpha} = - \frac{Z_{q\alpha}(x, q, \alpha)}{Z_{qq}(x, q, \alpha)} + \frac{Z_{q\alpha}(x, q, \alpha) \Big|_{q = \hat{q}^*}}{Z_{qq}(x(q), q, \alpha)} + W_{\alpha}$$

where

$$(26) \quad W_{\alpha} = - \frac{p x_{q\alpha}(\alpha) q [p U_{2\alpha}^2] + p x_{q\alpha} q [(1-p) U_1^1 + p U_2^2(\alpha)]}{Z_{qq}(x(q), q, \alpha)}$$

Using the signs of $x_{q\alpha}$ obtained in (12), (14) and (16), the signs of g_{α} are equal to:

Case (1): when $U_S = V_S$ for all S,

Using (11), (12), (26), $g_{\alpha} < 0$ since

$$(27) \quad Z_{q\alpha}(x, q, \alpha) = Z_{q\alpha}(x, q, \alpha) \Big|_{q = \hat{q}^*} = 0$$

and

$$(28) \quad W_{\alpha} = - \frac{p x_{q\alpha} q [(1-p) U_1^1 + p U_2^2(\alpha)]}{Z_{qq}(x(q), q, \alpha)} = - \frac{<0 >0 >0 [>0]}{<0} < 0$$

Here the optimal "deductible" varies in the same way that moral hazard varies.

Case (2) :when $U_S > V_S$ for all S,

Using (13), (14), (26), (28)

and assuming¹⁵ that $\frac{Z_{q\alpha}(x, q, \alpha)}{Z_{qq}(x, q, \alpha)} = \frac{Z_{q\alpha}(x, q, \alpha) \Big|_{q = \hat{q}^*}}{Z_{qq}(x(q), q, \alpha)}$

then $g_\alpha < 0$ since:

$$(29) \quad W_\alpha = - \frac{[<0 + <0]}{< 0} < 0$$

Therefore the optimal "deductible" varies in the same way that moral hazard varies. Without the assumption, $g_\alpha \begin{matrix} \geq \\ < \end{matrix} 0$.

Case (3): when $U_S < V_S$ for all S,

Using (15), (16), (26), (28) and given the same assumption as in the previous case, the sign of g_α will depend on $x_{q\alpha}$ as follows:

$$(30) \quad W_\alpha = - \frac{[>0 + <0 \begin{matrix} \geq \\ < \end{matrix} 0]}{< 0}$$

$$(30.1) \quad \text{if } x_{q\alpha} = 0, W_\alpha > 0 \text{ and } g_\alpha > 0$$

$$(30.2) \quad \text{if } x_{q\alpha} < 0, W_\alpha > 0 \text{ and } g_\alpha > 0$$

$$(30.3) \quad \text{if } x_{q\alpha} > 0, W_\alpha \begin{matrix} \geq \\ < \end{matrix} 0 \text{ and } g_\alpha \begin{matrix} \geq \\ < \end{matrix} 0.$$

Even under this restrictive assumption, the optimal "deductible" doesn't vary in the same way that moral hazard varies. Without the assumption, g_α cannot be determined.

Summary and conclusion

Table I is a summary of the main results of the paper:

Table I

Reference case, Results and Symbols	Assumptions on the marginal utility of income for all levels of income		
	$U_S = V_S$	$U_S > V_S$	$U_S < V_S$
Optimal insurance coverage under a S.D.U.F. without moral hazard (as reference), q^*	$q^* = \ell < \bar{q}$	$q^* < \ell < \bar{q}$	$q^* \geq \bar{q} > \ell$
1) Optimal insurance coverage under a S.D.U.F.,* with moral hazard hazard, q	$\hat{q}^* < \ell$	$\hat{q}^* < q^*$	$\hat{q}^* < \bar{q}$
2) Variation of the importance of moral hazard from a S.I.U.F. to a S.D.U.F., $-x_{q\alpha}$	< 0	< 0	$\begin{matrix} \geq \\ < \end{matrix} 0$
3) Variation of the optimal "deductible" from a S.I.U.F. to a S.D.U.F., g_α	< 0	$\begin{matrix} \geq \\ < \end{matrix} 0$	$\begin{matrix} \geq \\ < \end{matrix} 0$

The first result shows that moral hazard is still an important problem under a S.D.U.F. and that partial coverage has to be used to motivate the insured to take care even if an accident or illness incurs direct disutility. Moreover an upper limit of coverage is shown to be necessary to avoid deceptive activities in the case characterized by a level of insurance coverage that would make the insured more than whole.

It is also found that moral hazard may be more, less or equally important under a S.D.U.F. according to the relative magnitude and the direction of the variation of the marginal utility of income, for a given variation of the direct disutility corresponding to a bad event. But the variation of the optimal "deductible" doesn't

necessarily follow the variation of moral hazard, since the optimal levels of insurance coverage are themselves functions of the variation of the marginal utility of income.

To obtain these results, the author used the standard approach to moral hazard first proposed by Pauly (1974) and followed by many economists. This approach is characterized by a non-linear pricing rule used by the insurers, since they cannot observe the activities of the insured. Other second-best solutions have also been presented in the literature in order to minimize this problem of resource allocation: court action [Laffont, 1976], search of imperfect information [Shavell, 1979], the involvement of the insurer in the insured's activities [Dionne, 1980] and premium discrimination [Doherty, 1980]. One avenue of research would be to compare these solutions and to define a rule of choice in order to determine the best way to reduce the losses created by the problem of moral hazard.

Appendix I

Another proof that \hat{q}^* cannot be greater or equal than \bar{q} when $V_S > U_S$ for all S .

This proof is similar to that presented by Shavell (1979), to show that $\hat{q}^* < \ell$ under the S.I.U.F. with moral hazard. But now the proof has two steps. The first one consists to show that $\hat{q}^* = \bar{q}$ is not optimal when the cost of care activity is sufficiently low. This part of the proof is the same as the Shavell's proof and it will not be reproduced here.

The second step shows that the insurer will make losses selling an insurance policy characterized by a level of insurance coverage greater than \bar{q} if the cost of deceptive activities is sufficiently low. To show that, choose $\hat{q} > \bar{q}$, $\hat{p} > p(0)$ and $\hat{C} > 0$ such that:

$$[1 - \hat{p}]U(S - \hat{p}\hat{q}) + \hat{p}V(S - \hat{p}\hat{q} - \ell + \hat{q}) - \hat{C} > [1 - p(x)]U(S - p(x)\bar{q}) + p(x)$$

$$V(S - p(x)\bar{q} - \ell + \bar{q}) \equiv U(S - p(x)\bar{q})$$

If the cost of deceptive activities, $C(x)$, is sufficiently low, the insured will choose, under that insurance policy, a level of deceptive activities x^0 such that $p(x^0) > \hat{p}$ and the insurer will make losses. On the other hand, \tilde{C} may be chosen low enough so that if \tilde{x} is such that $p(\tilde{x}) = \hat{p}$, then $\tilde{C}(\tilde{x}) < \hat{C}$. Then it is obtained that:

$$[1 - p(x^0)]U(S - \hat{p}\hat{q}) + p(x^0)V(S - \hat{p}\hat{q} - \ell + \hat{q}) - C(x^0) >$$

$$[1 - p(\tilde{x})]U(S - \hat{p}\hat{q}) + p(\tilde{x})V(S - \hat{p}\hat{q} - \ell + \hat{q}) - \tilde{C}(\tilde{x}) >$$

$$[1 - \hat{p}]U(S - \hat{p}\hat{q}) + \hat{p}V(S - \hat{p}\hat{q} - \ell + \hat{q}) - \hat{C} > U(S - p(x)\bar{q}).$$

The insurer will make losses selling that policy. On the other hand, it may be shown (in step one not presented here) that a policy involving $\hat{q}^* < \bar{q}$ will give greater expected utility to the insured and that the insurer can make profits under that policy. Therefore $\hat{q}^* > \bar{q}$ cannot be optimal since it is possible to increase the welfare of both the insured and the insurer with $\hat{q}^* < \bar{q}$.

Appendix II

An example

If it is supposed that:

$$U^1 = a + bs - cs^2 \quad \text{where } s < \frac{b}{2c}, a, b, c > 0$$

$$U^2(\alpha) = [1-\alpha][a + bs - cs^2] + \alpha[a_1 + b_1s - cs^2] \quad \text{where } a_1 < a,$$
$$b_1 \gtrless b \text{ and } [a - a_1] > |[b - b_1]s| \text{ since}$$
$$U(s) > V(s) \text{ for all } s \text{ by assumption.}$$

$$\text{Therefore } U_{\alpha}^2 = [a_1 - a] + (b_1 - b)s < 0$$

and

$$U_{s\alpha}^2 = (b_1 - b) \gtrless 0$$

$$Z_q(x, q, \alpha) = p(x) [1 - p(x)] [(b_1 - b)\alpha - 2c(q - \ell)] = 0$$

$$\text{which implies that } q^*(\alpha) = \frac{\alpha[b_1 - b]}{2c} + \ell \gtrless \ell \text{ as } [b_1 - b] \gtrless 0$$

$$Z_{qq}(x, q, \alpha) = p(x) [1 - p(x)] [-2c] < 0$$

$$Z_{q\alpha}(x, q, \alpha) = p(x) [1 - p(x)] [b_1 - b] \gtrless 0$$

and

$$-\frac{Z_{q\alpha}(x, q, \alpha)}{Z_{qq}(x, q, \alpha)} = \frac{[b_1 - b]}{2c} \gtrless 0$$

moreover $Z_q(x(q), q, \alpha) = p(x)[1 - p(x)][b_1 - b]\alpha - 2c[q - \ell] + \beta(\alpha, x_q(\alpha)) = 0$

where $p(x(o))$ is assumed equal to $p(x)$

$$q^*(\alpha) = \ell + \frac{\alpha[b_1 - b]}{2c} + \frac{\beta(\alpha, x_q(\alpha))}{2c(p(x)[1 - p(x)])}$$

which implies that $q^* < \ell$ when $b_1 = b$
 $q^* < q^* < \ell$ when $b_1 < b$
 $q^* < q^*$ when $b_1 > b$

and $Z_{qq}(x(q), q, \alpha) = p(x)[1 - p(x)][-2c] + \beta_q(\alpha, x_q(\alpha)) < 0$

$$Z_{q\alpha}(x, q, \alpha) \Big|_{q=q^*} = p(x)[1 - p(x)][b_1 - b] \frac{\partial}{\partial \alpha} > 0$$

Therefore $\frac{\partial}{\partial \alpha} [a - p] + (d - p) = \frac{\partial}{\partial \alpha} [a - p]$ and $\frac{\partial}{\partial \alpha} (d - p) = \frac{\partial}{\partial \alpha} [d - p]$

therefore

$$g_\alpha = W_\alpha + \frac{[b_1 - b]}{2c} + \frac{0 = [(1 - p)\alpha - \alpha(d - p)] [p(x) - 1] (x)_q = (a, p, x)_{pp}}{[b_1 - b] [p(x)[1 - p(x)]]} + \frac{Z}{-2c [p(x)[1 - p(x)]]}$$

which implies that $\frac{\partial}{\partial \alpha} [d - p] = \frac{\partial}{\partial \alpha} [d - p] + \frac{\partial}{\partial \alpha} [d - p]$

where $Z \equiv B_q(\alpha, x_q(\alpha)) < 0$ if $x_{qq}(\alpha) < 0$

which implies that

$$0 < \frac{\partial}{\partial \alpha} [d - p] [p(x) - 1] (x)_q = (a, p, x)_{pp}$$

Case (1): $g_\alpha = W_\alpha < 0$ when $b_1 = b$

hence

Case (2): $g_\alpha = W_\alpha + < 0 + > 0 = < 0$ when $b_1 < b$

$$0 < \frac{\partial}{\partial \alpha} [d - p] = \frac{(a, p, x)_{pp}}{(a, p, x)_{pp}}$$

Case (3): $g_\alpha = W_\alpha + >0 + <0 = \begin{matrix} > \\ \equiv \\ < \end{matrix} 0$

when $W_\alpha > 0$, $g_\alpha > 0$

when $W_\alpha = 0$, $g_\alpha > 0$

when $W_\alpha < 0$, $g_\alpha \begin{matrix} > \\ \equiv \\ < \end{matrix} 0$

Footnotes

- (1) The content of this paper is limited to the first kind of solution.
- (2) The term accident is used in a general meaning: automobile collision, flight accident, fire and theft.
- (3) Deceptive activities are those activities which are voluntarily used to increase the probabilities of loss such as arson, complicity in theft, use of narcotics, etc.
- (4) () are function brackets, [] and { } are multiplicative brackets, * means optimal value and ^ is used to identify the optimal values under moral hazard.
- (5) For further interpretation about illness see Zeckhauser [1973], Arrow [1974] and Shavell [1978], about death see Jones-Lee [1976] and about irreplacable commodities see Cook and Graham [1977].
- (6) Shavell [1978] showed that the optimal compensation does not depend on the cause of the medical accident. This result is not necessarily true if different causes of the accident are considered as different states of the world corresponding to different utility levels. Using the Shavell's example, suppose that a particular accident can occur in one of the two ways A and B. The optimal insurance coverages are obtained in maximizing the following expected utility over q^A and q^B .
- $$[1 - p^A(x) - p^B(x)]U(S - P) + p^A(x)V^A(S - p - \ell + q^A) + p^B(x)V^B(S - P - \ell + q^B)$$
- subject to $P = p^A(x)q^A + p^B(x)q^B$.
- $q^A = q^B$ only if $V_S^A = V_S^B$ for all S.
- (7) This section is based on the papers of Arrow [1963], Pauly [1968, 1974] Ehrlich and Becker [1972], Marshall [1976], Shavell [1979], Dionne [1980]. Helpman and Laffont [1975] and Laffont [1976] present an analysis of moral hazard with S.D.U.F. but in a general equilibrium framework.
- (8) Under a S.I.U.F. without moral hazard, (3.2) becomes $p(x)[1 - p(x)][U_2 - U_1] = 0$ which implies that $U_2 = U_1$ and $q^* = \ell$.
- (9) From (3.2) it may be verified that under a state independent utility function, q^* cannot be greater than ℓ at the optimum.
- (10) For another proof that $q^* = \ell$ cannot be an optimal solution under moral hazard, see Shavell (1979).
- (11) The possibility of deceptive activities is not considered in the literature on moral hazard with the exception of Arrow [1971] which mentioned the possibility of arson: "An insurance company may easily observe that a fire has occurred but cannot, without special investigation, know whether the fire was due to causes exogenous to the insured or to decision of his (arson or at least carelessness)." See also Shavell [1979] footnote (11).

- (12) Under moral hazard and without this supplementary constraint, the insurer would incur losses selling a policy with an amount of insurance coverage greater than q .
- (13) This proposition is an extension of the Shavell's (1979) proposition number I. The main difference is in the part C of the preceding proposition. The proof is also different.
- (14) This may be verified in maximizing $Z(x, q)$ over q subject to:
 (1) $P(x, q) = p(x)q$ and (2) $q < \bar{q}$.
- (15) This is a strong assumption; see the example in Appendix II. In this example, it is shown that even if the marginal utility of income is a constant, the

assumption that $\frac{Z_{q\alpha}(x, q, \alpha)}{Z_{qq}(x, q, \alpha)} = \frac{Z_{q\alpha}(x, q, \alpha)}{Z_{qq}(x(q), q, \alpha)}$ is not verified. Moreover

$$\left. \vphantom{\frac{Z_{q\alpha}(x, q, \alpha)}{Z_{qq}(x, q, \alpha)}}} \right|_{q = \hat{q}^*}$$

the signs of g_α are calculated and it is shown that the variation of the optimal "deductible" is not directly proportional to the variation of moral hazard.

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