

UNIVERSITE DE MONTREAL

**Modeling Induced Technological Change:
Taxes vs. Cap-and-Trade Climate Change Policies**

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1. Introduction

Since the conception of the Kyoto Protocol there has been much debate about the best way to achieve a level of greenhouse gas emissions compatible with a sustainable economy. It is imprudent at best to introduce a public policy without first evaluating its relative effectiveness with respect to other policy options. The objectives of climate change policy should be threefold:

- 1) Reduce greenhouse gas emissions to 'sustainable levels'
- 2) Incite investment in green technology
- 3) Incite research and development into low carbon, carbon neutral or carbon sequestering technologies.

There exist a variety of policies that may be used jointly or in isolation, some of which impose a price on carbon and others which regulate technologies. Within the scope of carbon pricing policies are cap-and-trade, carbon taxes, and subsidies. Non-carbon pricing policies sometimes referred to as command-and-control include, technological regulation, vehicle emission standards, and minimum efficiency requirements (Energy Star), among others. It has become apparent that command-and-control, although effective to some degree, does not create the necessary incentives for technological innovation required to mitigate climate change and thus carbon pricing has become the tool of choice among policy makers.

Our objective is to establish a model that will aid in comparing a carbon tax to a cap-and-trade system based on investment in current technologies, technological innovation through research and development and effectiveness with respect to total emission reduction.

We will present a simplified theoretical model under certainty of a firm's profit maximization function under various carbon restricting policies. Ideal greenhouse gas emission levels or, equivalently, taxation levels are assumed to be known and given the imposed restriction we characterise optimal investment in currently available 'green' technologies and optimal research and development expenditure in new technology.

Research and development is assumed to be firm specific in order to circumvent the issue of free-ridership with respect to new technologies. Investment in current technology is assumed to have constant returns in emission abatement specific to each firm which saturate at a given

level of emission intensity. We then complement the theoretical analysis by running a simulation of the model that allows us to more easily observe the impacts of the different policies.

Section 2 sets the theoretical and political stage for climate change policies. Section 3 presents the model and discusses the resulting necessary conditions for profit maximization. Section 4 presents the results of a simulation of our model, under a number of simplifying assumptions, which aids in contrasting the policies. Finally section 5 concludes and comments on potential extensions to the model.

2. Economic instruments and Policy Options

There are two main alternative climate policy approaches; 1) direct emissions policies which include carbon taxes, carbon quotas, tradable CO₂ emission permits (cap-and-trade), and subsidies to CO₂ emissions abatement, and 2) technology push policies, which include subsidies to R&D in low-carbon technologies, public-sector R&D in low-carbon technologies, government-financed technology competitions, and strengthened patent rules. We will focus on market-based tools, particularly those that impose a positive price on externalities. This excludes subsidies (negative prices) and any type of command and control approach. We will deal only with taxes and cap-and-trade within our model. Taxes are used as a price-based tool to limit greenhouse gases emitted into the atmosphere by providing a signal about the environmental costs of greenhouse gas emissions, while cap-and-trade directly limits emissions. The price associated with cap-and-trade (with grand-fathered permits) provides insight into the variation of marginal abatement costs within the participating firms or countries.

2.1. Social Welfare and Marginal Abatement Costs

The typical approach for establishing optimal taxes or cap levels when faced with negative externalities is to maximize social welfare. Policy mechanisms (e.g., taxes, regulations, fiscal incentives) are required to provide the correct signal about the full social and environmental costs of greenhouse gas emissions. Tools that define explicit prices (taxes) or those that define maximum allowable levels of pollution (caps), both under perfect information, may achieve similar results.

“In the neat world of economic theory, carbon reduction makes sense until the marginal cost of cutting carbon emissions is equal to the marginal benefit of cutting carbon emissions. If policymakers knew the exact shape of these cost and benefit curves, it would matter little whether they reached this optimal level by targeting the quantity of emissions (through a cap) or setting the price (through a tax).”

(‘Tradable emissions permits are a popular, but inferior, way to tackle global warming’, *The Economist*, June 14, 2007)

Social welfare maximization requires that abatement cost functions and social damages functions be either known or estimated. Should total industry marginal abatement costs and social marginal damages be known, it is theoretically possible to force a socially optimal level (or price) of emissions by equating the cost along the marginal abatement cost function to the desired emission level (price)¹. Under cap-and-trade, a 'first-best' solution may be achieved since MAC for each firm will equal marginal damages if the cap is chosen appropriately, while a corrective tax will achieve a 'second best' solution. However, given that marginal abatement costs are not static but vary with investment and innovation, both taxes and cap would not achieve optimal solutions if they were also static.

The issue of greenhouse gases abatement is particularly complex due to the persistence of gases in the atmosphere and their global nature and, consequently, due to the intrinsic difficulties in establishing a social cost function through time (intergenerational equity) and across nations (geographic equity). According to James Hansen, director of NASA's Goddard Institute for Space Studies, scientific evidence suggests that atmospheric carbon dioxide levels below 350 ppm are sustainable while those above are deemed unacceptably dangerous. This translates into a horizontal (0) marginal social cost function up to emission levels resulting in CO₂ level of 350 and vertical (infinite costs) beyond this point. We will not attempt to estimate the exact shape of the societal damage function, but instead rely on scientific evidence to establish a sustainable and thus optimal level of atmospheric CO₂. The concentration of carbon in the atmosphere is now about 380 parts per million and increasing by 2 parts per million each year.

2.2. Cap-and-Trade

Emission trading or cap-and-trade is based primarily on the allocation of property rights. The theoretical articulation of a quantity instrument was first introduced by Coase and emerged as Coase's theorem. In practice, a central authority imposes a cap and emitters are required to hold a number of rights (credits) to pollute a specific amount. Credits may be allotted by the central authority (grand-fathered) or auctioned or sold. Emitters who do not

¹ See Pigou (1920)

hold the amount of emission permits that maximize their profit function may choose to buy (sell) additional (excess) permits but the total amount of permits must remain constant. In theory, companies with the lowest marginal abatement cost functions will reduce emissions, achieving the emission reduction at the lowest possible cost to society.

Within the system of a cap there exist a number of possible sub-categories;

- 1) auctioned permits: emission permits are sold by auction and firms are thus forced to reveal their marginal abatement costs;
- 2) sold permits: permits are sold at a specified price, much like a tax, but may be exchanged during the period should the amount purchased no longer be optimal;
- 3) 'grandfathered' permits: permits are distributed freely according to a baseline period;
- 4) falling caps: caps are reduced in each period in order to compensate for falling marginal abatement costs and to maintain the prices high enough to encourage continued investment;
- 5) banking of permits: permits may be saved for use in future periods;
- 6) carbon intensity: emission intensity (per unit produced) is capped and selling of emissions not produced at a certain production level as a result of below-cap intensity is permitted.

The first and most successful emission trading program in place is the acid rain program created by Title IV of the Clean Air Act Amendments of 1990. Allowances were 'grand-fathered' and banking (saving permits for use in subsequent years) was allowed. Emissions quickly dropped to lower than those allotted, within 3 years reaching 33% below total allowances (Schmalensee et al. 1998). This behaviour is primarily attributed to faster than expected decreasing marginal abatement costs as a result of overinvestment (Schmalensee et al. 1998).

A concern with respect to cap-and-trade that became apparent as a result of the acid rain program is the rewarding of past high emitters with greater allowances and the penalizing of low emitters with smaller initial allowances. This concern may be addressed by auctioning

permits or limiting emission intensity, however neither alternative seems to have been applied yet.

2.3. Taxes versus Cap-and-Trade

The issue of the superiority of price over quantity instruments has been the area of much research. An emission cap and permit trading system fixes the overall emission level (quantity) and allows the price to vary. One problem with the cap and trade system is the uncertainty of the cost of compliance as the price of a permit is not known in advance and will vary over time according to market conditions. In contrast, an emissions tax fixes the price while the emission level is allowed to vary according to economic activity. A major drawback of an emission tax is that the environmental outcome (e.g. the amount of emissions) is not guaranteed. Wietzman (1974) shows that, when faced with uncertain abatement costs, taxes are superior to quantity tools when marginal costs are steeper than marginal damages. This is the case for emissions resulting in less than 350 ppm. In contrast, a cap would be preferable for the case where emissions are significantly above the critical threshold (as they appear to be). Kaplow and Shavell (2002) argue the general superiority of non-linear corrective taxes although they do discuss the possibility that under damage uncertainty (as is the case for climate change) a cap may be superior. Mendelsohn (1984) considers quantity-based instruments more advantageous because price-based regulation leads to excessive output variations. Kahn and Franceschi (2006) list advantages of a tax among which are, a continuous incentive to reduce emissions and greater incentives for technological innovation, both of which are supported by our model. Winkler (2008) presents an inter-temporal optimization problem to analyze the medium-term perspective on climate change mitigation.

Another fundamental difference between taxes and cap-and-trade lies in government revenues from taxation which may be used to fund environmental research and development or to reduce other forms of taxation. However, both taxes and cap-and-trade have the ability to raise public revenues if one considers auctioned permits.

A third option is a hybrid of the price and quantity instruments. The system is an emission cap-and-trade system but with a limited maximum (and/or minimum) permit price.

Emitters have the choice of either trading with other firms or permit holders or purchasing them from the issuing body at the trigger price. The system is recommended as a way of overcoming the fundamental disadvantages of both systems, namely rigidity of price or quantity, by giving governments the flexibility to adjust the system as new information comes to light. It can be shown that by setting the trigger price high enough or the number of permits low enough, the safety valve can be used to mimic a tax or a cap-and-trade mechanism. The problem of unstable prices can be dealt with by the creation of forward markets in caps such as the one in the Montreal Climate Exchange.

2.4. Canadian Context

In 2005, Canada's total GHG emissions were estimated to be 747 megatons of CO₂ equivalent, up 25.3 per cent from 1990 and 32.7 per cent above the nation's Kyoto Protocol target of reducing emissions by at least six per cent below 1990 levels by 2008–2012 (Environment Canada, 2007). Of all Kyoto Annex I Parties, Canada was the fifth largest emitter of GHGs in 2004, behind the United States, the Russian Federation, Japan and Germany (Government of Canada, 2006).

In Canada's *National Round Table on Energy and the Environment* 2008 publication on climate change, six principal recommendations were made to the federal government for the development of a policy framework that would stimulate technological innovation. These were:

- 1) Implementing a clear and consistent GHG emission price signal;
- 2) Instituting an emission tax or cap-and-trade system, or a combination of the two;
- 3) Developing regulatory policies for sectors that do not respond to price signals;
- 4) Supporting R&D technologies and strategic investments in infrastructure;
- 5) Establish a Canada-wide plan to better coordinate federal, provincial and territorial mitigation actions;
- 6) Apply mitigation policies that incorporate adaptive management practices.

2007's *Turning the Corner* is the present federal government's regulatory framework for greenhouse gas emissions. It is an intensity-based system, where emitters must reduce their emission intensity in tons per unit produced by 18% by 2010 and by 2% per year as of 2010. This does not necessarily result in a reduction in total emissions as production could increase offsetting the efficiency reduction. There are also a number of mechanisms which permit polluters to exceed these levels, primarily an effective cap of 15\$/ton since emitters can purchase carbon offsets at this price. As well, newly installed polluters have a 3-year pollute-free-of-charge card.

The Liberal's proposed *The Green Shift* is a tax-based system beginning at \$10 per ton of greenhouse gas emissions and steadily rising by an additional \$10 per ton each year, reaching \$40 per ton within four years.

At the provincial level, British Columbia introduced a revenue-neutral destination based carbon tax (10\$/ton rising to 30\$/ton by 2012) on all fuels. British Columbia's *Greenhouse Gas Reduction Targets Act* establishes emission reduction targets of 33 per cent by 2020 from 2007 levels, and 80 per cent reductions by 2050 from 2007 levels. The province is also expected to introduce a cap-and-trade system for large emitters. Alberta is the first Canadian jurisdiction to regulate large emitters. As of July 1, 2007, regulations set a 12 per cent emissions intensity reduction target for all large facilities with a price cap of 15\$/ton and grand-fathered permits. Nova Scotia's *2007 Environmental Goals and Sustainable Prosperity Act* establishes a goal of reducing GHG emissions to 10 per cent below 1990 levels by 2020. In 2007, Quebec became the first North American jurisdiction to impose a carbon tax (0.8 cents/liter of gas and 0.9 cents/liter of diesel) levied against large emitters and distributors (origin based). Revenues from the tax are allocated to a Green Fund to implement Quebec's climate change plan. In June of 2008, Quebec and Ontario announced their intention to launch a joint cap-and-trade system based on the original Kyoto baseline of 1990.

2.5. Investment and Innovation

A low-carbon society is an inevitable outcome as carbon-based energy is a non-renewable resource and therefore some level of environmental innovation will occur without additional incentives. The externality of climate change has created the need to accelerate this transition and we seek to identify the tool that will create the greatest incentive for innovation and green investment at the lowest cost. Price signals that reflect the negative environmental and social impacts related greenhouse gas emissions are useful in accelerating the implementation of climate-friendly technologies and innovation and distribution of new technologies. The benefits of greener technologies, in the form of fewer environmental externalities, are not fully appreciated when compared with conventional technologies without an explicit emission market price. As well, new technologies do not have the benefit of “learning by doing” that existing technologies have.

The presence of induced technological change lowers the costs of achieving emissions reductions and justifies more extensive reductions in GHGs than would otherwise be called for. Moreover, announcing climate policies in advance can reduce policy costs by encouraging investment. Goulder and Mathai (2000) estimate that when a stringent target of 350 ppm is imposed, induced technological change reduces costs by 51 percent.

Empirically-based simulation models consistently yield rising abatement through time for both R&D based and learning-by-doing based technological change, a result which is both intuitive and consistent with our model.

Carbon taxes and cap-and-trade are often assumed to induce environmental R&D and green investments (Nordhaus, 2002; Goulder and Mathai, 2000), however Farzin and Kork (2000) show that optimal R&D is non-monotonic in an emissions tax. Baker and Shittu (2006) find that optimal R&D can decrease with increases in an emissions tax while investment will first increase then decrease unless carbon capture and storage is optimal. They and many others also deal with the issue of uncertainty; however there is no consensus as to the optimal tool.). Krysiak (2008) finds that only quantity based regulation can lead to induced technology which is socially optimal. One of the principal issues modeled in the relevant literature are

input substitution, output reduction and impacts on marginal abatement cost function by means of static or dynamic models. The results are not consistent since much depends on the assumptions regarding firm, R&D and uncertainty characteristics. Goulder and Mathai, 2000 find that if technical change is a result of R&D expenditure, then lower abatement will result in the near term since R&D and abatement expenditures are substitutes. Our model is also consistent with this result.

3. The Model

3.1. Nomenclature

3.1.1. Variables

- t $t \in \{0,1,2\}$, index of period
- $q_{i,t}$ Quantity of widgets (w) produced by firm i during period t
- $q_{i,bl}$ Baseline quantity produced ($q_{i,bau}$ or $q_{i,0}$)
- $e_{i,t}$ Total emissions in tons produced by firm i during period t
- $I_{i,t}$ Investment (in \$ per widget) in available emission-intensity-reducing technology by firm i during period t (begins to have impact during period $t+1$); $I_{i,0}$; $I_{i,1}$
- $CI_{i,t}$ Cumulative investment in available emission-intensity-reducing technology by firm i up to and including period $t-1$; $CI_{i,1} = q_{i,0}I_{i,0}$; $CI_{i,2} = q_{i,0}I_{i,0} + q_{i,1}I_{i,1}$
- $RD_{i,t}$ Investment in R&D (in \$ per widget) towards emission-reducing technology by firm i during period t (begins to have impact during period $t+2$); $RD_{i,0}$
- $\varepsilon_{i,t}$ Emission intensity of firm i per widget produced (ton/ w)
- $\varepsilon_{i,bl}$ Baseline emission intensity ($\varepsilon_{i,bau}$ or $\varepsilon_{i,0}$)
- $C_{i,t}$ Production cost in \$/ w by firm i during period t where $C_{i,t} = a_{i,t}q_{i,t} + \frac{b_{i,t}}{2}q_{i,t}^2$
- $R_{i,t}$ Revenue in \$ of firm i during period t
- $w_{i,t}$ Emission permit in tons bought by firm i during period t
- z_t Price in \$ per ton of emission permit during period t

3.1.2. Parameters

- p_t^* Perfectly competitive equilibrium market-clearing price during period t in $\$/w$
- δ Discount factor
- T Emission tax in $\$$ per ton
- α_i Emission intensity rate of reduction for firm i per amount invested in available technology (tons/widget)/ $\$$
- β_i Emission intensity rate of reduction for firm i per amount invested in R&D (tons/w)/ $\$$
- ε_i^{\min} Minimum emission intensity in tons/widget achievable by firm i through available technology
- RD_i^{\min} Minimum R&D investment in $\$/widget$ needed by firm i
- $\Delta RD_{i,0} = RD_{i,0} - RD_i^{\min}$ Incremental R&D investment in $\$/widget$ by firm i beyond minimum

3.2. Modeling Assumptions

We consider a three period model ($t=0,1,2$) in which emission-reducing policies, such as cap-and-trade or carbon tax, are announced at $t=0$ to be implemented during periods 1 and 2.

Under either policy, firms may choose to invest towards emission reduction through either currently available technology, sometimes referred to as 'learning-by-doing' in the literature, or through new technologies developed through R&D expenditures in order to maximize the net present value of its profit. Kammen and Margolis (1999) estimate private returns from economy-wide R&D at 20-30%, we however, assume a linear return only above a minimal level of R&D investment. Investment in currently available technology is assumed to have an effect on emissions at the start of the period following the investment, while the impact of R&D on emission reduction is slower-acting, being felt at the beginning of the second

period following the investment. The three time periods considered are analogous to the classical economic concepts of short-run (fixed capital), medium-run (variable capital) and long-run (variable technology) time frames. This time frame separation is useful in comparing the two types of investment of which R&D is a long-run investment involving changes in technology (hence the delay between expenditure and impact on emissions). As well, the simplification that R&D is firm specific allows us to avoid any game theory with respect to this variable.

We now define the cumulative investment at time t by firm i in available technology by,

$$CI_{i,t} = \sum_{\tau=0}^{t-1} q_{i,\tau} I_{i,\tau} \quad (1)$$

From (1) it is evident that the only relevant cumulative investment variables are $CI_{i,t} \in \{CI_{i,1}, CI_{i,2}\}$. This is consistent with the fact that during period $t=0$ we are in the short-run and capital is a fixed production factor. Similarly, because of the inherent delay in deployment, the only relevant variable in R&D expenditures is $RD_{i,t} \in \{RD_{i,0}\}$. From (1), we can state,

$$CI_{i,t} = \begin{cases} q_{i,0}I_{i,0} + q_{i,1}I_{i,1} & ; t = 2 \\ q_{i,0}I_{i,0} & ; t = 1 \end{cases} \quad (2)$$

Thus, the impact of investment and R&D can be modeled through the functional relationships between the costs $C_{i,t}$ and the emission intensities $\varepsilon_{i,t}$ and the three investment decisions variables $CI_{i,1}, CI_{i,2}$ and $RD_{i,0}$. In this study, these relationships take the form,

$$C_{i,t} = \begin{cases} C_{i,2}(CI_{i,2}, RD_{i,0}, q_{i,2}); & t = 2 \\ C_{i,1}(CI_{i,1}, q_{i,1}); & t = 1 \\ C_{i,0}(q_{i,0}); & t = 0 \end{cases} \quad (3)$$

The $C_{i,t}$ functions in (3) are assumed to be twice-differentiable and convex in the quantities produced. With respect to the investment variables, the relationship depends on the technology. For example, switching to a cleaner fuel would increase the marginal cost; however changing to a renewable resource would likely decrease the marginal cost.

The emissions produced by firm i during period t , $e_{i,t}$, are assumed to be proportional to the quantity produced, where the proportionality factor $\varepsilon_{i,t}$ is termed the emission intensity,

$$e_{i,t} = \varepsilon_{i,t} q_{i,t} \quad (4)$$

The functional relation between the emission intensities, $\varepsilon_{i,t}$, and the cumulative investments $CI_{i,1}, CI_{i,2}$ and $RD_{i,0}$ is assumed to be piece-wise linear, with the intensity decreasing at a constant rate in terms of the investment. This piece-wise behaviour is dictated by the basic assumptions that investment in R&D by firm i does not have an effect until $RD_{i,0} \geq RD_i^{\min}$ and that investment in available technology by firm i cannot reduce emission intensity below ε_i^{\min} . Thus,

$$e_{i,t} = \varepsilon_{i,t} q_{i,t}$$

Where,

$$\varepsilon_{i,1} = \begin{cases} \varepsilon_{i,0} - \alpha_i I_{i,0} q_{i,0} & ; \text{ if } CI_{i,1} \leq \frac{\varepsilon_{i,0} - \varepsilon_i^{\min}}{\alpha_i} \\ \varepsilon_i^{\min} & ; \text{ if } CI_{i,1} > \frac{\varepsilon_{i,0} - \varepsilon_i^{\min}}{\alpha_i} \end{cases} \quad (5)$$

$$\varepsilon_{i,2} = \begin{cases} \varepsilon_{i,0} - \alpha_i (I_{i,0} q_{i,0} + I_{i,1} q_{i,1}) - \beta_i \Delta RD_{i,0} q_{i,0} & ; \text{ if } RD_{i,0} \geq RD^{\min}, CI_{i,2} \leq \frac{\varepsilon_{i,0} - \varepsilon_i^{\min}}{\alpha_i} \\ \varepsilon_i^{\min} - \beta_i \Delta RD_{i,0} q_{i,0} & ; \text{ if } RD_{i,0} \geq RD^{\min}, CI_{i,2} > \frac{\varepsilon_{i,0} - \varepsilon_i^{\min}}{\alpha_i} \\ \varepsilon_{i,0} - \alpha_i (I_{i,0} q_{i,0} + I_{i,1} q_{i,1}) & ; \text{ if } RD_{i,0} < RD^{\min}, CI_{i,2} \leq \frac{\varepsilon_{i,0} - \varepsilon_i^{\min}}{\alpha_i} \\ \varepsilon_i^{\min} & ; \text{ if } RD_{i,0} < RD^{\min}, CI_{i,2} > \frac{\varepsilon_{i,0} - \varepsilon_i^{\min}}{\alpha_i} \end{cases} \quad (6)$$

The relation described by equations (5) and (6) and depicted in Figures 1(a) and 1(b) says that $\varepsilon_{i,0}$, the emission intensity of any firm at time $t=0$, cannot be affected by any type of

investment made during any period beyond and including $t=0$. Firm i can however reduce its emission intensity during period $t=1$ at a rate of α_i but only by investing in current technologies during period $t=0$. Moreover, the reduction in emission intensity by investing in current technologies is assumed to be limited, saturating at a non-zero level, that is, $\varepsilon_{i,t} \geq \varepsilon_i^{\min}$. This means that any cumulative investment in current technology beyond $\frac{(\varepsilon_{i,0} - \varepsilon_i^{\min})}{\alpha_i}$ will not reduce emission intensity. Because of their delay in having an effect, R&D expenditures may only affect emission intensity in period $t=2$. A minimum amount of such R&D expenditures, RD_i^{\min} , is however required before emission intensity can be reduced at a rate of β_i . This models the expectation that low levels of R&D expenditures will not lead to new technologies capable of reducing emission intensities in a more efficient manner. Thus, we also assume that once R&D expenditures are high enough to impact emission intensities, they do so at a greater rate than current technology investments, which implies that $\beta_i > \alpha_i$. Were this condition not be satisfied, there would no incentive for R&D expenditures since available technologies would produce equal or greater returns. Furthermore, whereas investments in current technology are assumed not be able to achieve zero emission intensity, such a restriction is not imposed for new technologies derived from finite R&D investments.

Figure 1(a)

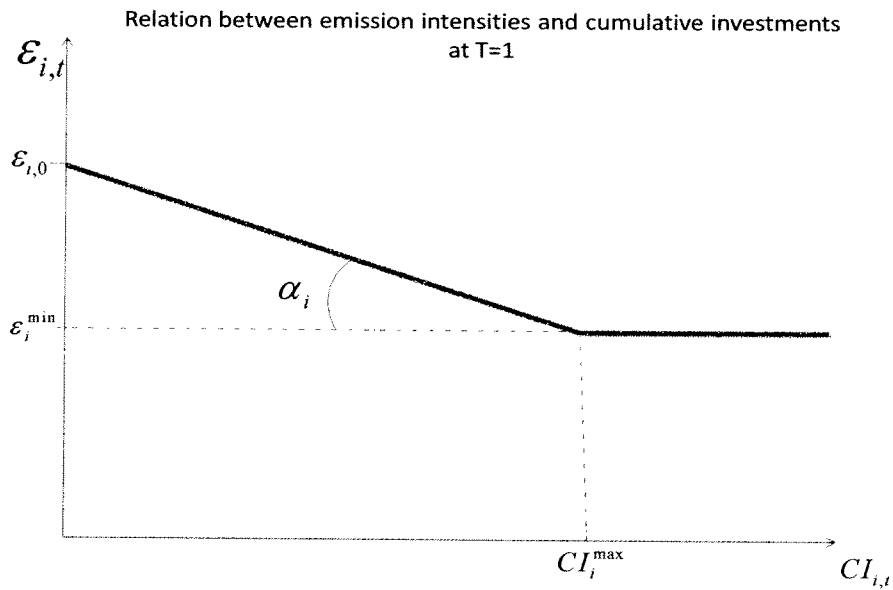
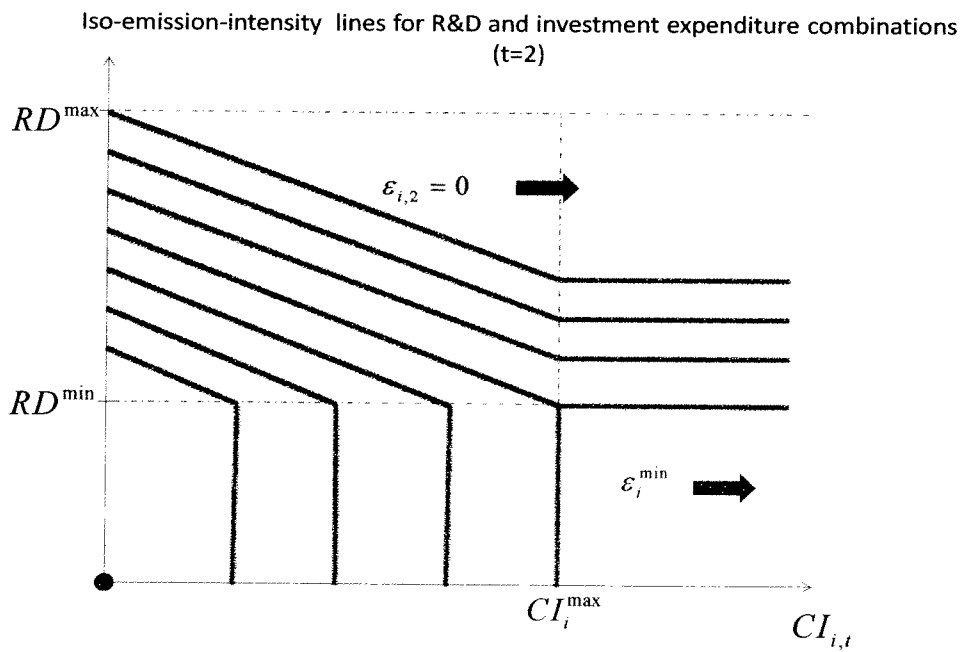


Figure 1(b)



3.3. Profit-Maximization in Perfectly Competitive Markets

3.3.1 Individual Firm Profit Maximization under Emissions Tax

Taking the prices p_t^* as exogenously given parameters, each firm determines its production levels so as to maximize its overall profit. The total profit of firm i for the three periods without emission tax is given by,

$$\begin{aligned}\Pi_i &= p_0^* q_{i,0} - C_{i,0} - q_{i,0} I_{i,0} - q_{i,0} RD_{i,0} \\ &\quad + \delta (p_1^* q_{i,1} - C_{i,1} - q_{i,1} I_{i,1}) \\ &\quad + \delta^2 (p_2^* q_{i,2} - C_{i,2})\end{aligned}\tag{7}$$

With a tax T on the emissions in \$/ton, since $e_{i,t} = \varepsilon_{i,t} q_{i,t}$, the profit takes the form²,

$$\begin{aligned}\Pi_i &= (p_0^* - I_{i,0} - RD_{i,0}) q_{i,0} - C_{i,0} \\ &\quad + \delta ((p_1^* - T\varepsilon_{i,1} - I_{i,1}) q_{i,1} - C_{i,1}) \\ &\quad + \delta^2 ((p_2^* - T\varepsilon_{i,2}) q_{i,2} - C_{i,2})\end{aligned}\tag{8}$$

With emissions tax, the profit maximizing problem for firm i is of the form:

$$\begin{aligned}\max &\left\{ \begin{aligned} \Pi_i &= (p_0^* - I_{i,0} - RD_{i,0}) q_{i,0} - C_{i,0} \\ &\quad + \delta ((p_1^* - I_{i,1} - T\varepsilon_{i,1}) q_{i,1} - C_{i,1}) \\ &\quad + \delta^2 ((p_2^* - T\varepsilon_{i,2}) q_{i,2} - C_{i,2}) \end{aligned} \right\} \\ \text{s.t.} & \\ q_{i,t} &\geq 0; \forall i, t \\ I_{i,t} &\geq 0; \forall i, t = 0, 1 \\ RD_{i,0} &\geq 0\end{aligned}\tag{9}$$

Where since $e_{i,t} = \varepsilon_{i,t} q_{i,t}$,

² Note that the tax is announced in period $t=0$ and only becomes active in periods 1 and 2.

$$\frac{\partial e_{i,1}}{\partial I_{i,0}} = \frac{\partial \varepsilon_{i,1}}{\partial I_{i,0}} q_{i,1} \quad \frac{\partial e_{i,2}}{\partial I_{i,1}} = \frac{\partial \varepsilon_{i,2}}{\partial I_{i,1}} q_{i,2} \quad (10)$$

$$\frac{\partial e_{i,2}}{\partial I_{i,0}} = \frac{\partial \varepsilon_{i,2}}{\partial I_{i,0}} q_{i,2} \quad \frac{\partial e_{i,2}}{\partial RD_{i,0}} = \frac{\partial \varepsilon_{i,2}}{\partial RD_{i,0}} q_{i,2}$$

The necessary conditions for maximum profit are:

1)

$$\frac{\partial \Pi_i}{\partial q_{i,0}} = p_0^* - I_{i,0} - RD_{i,0} - \frac{\partial C_{i,0}}{\partial q_{i,0}} - \delta T \frac{\partial e_{i,1}}{\partial q_{i,0}} - \delta^2 T \frac{\partial e_{i,2}}{\partial q_{i,0}} \geq 0$$

$$\frac{\partial \Pi_i}{\partial q_{i,1}} = \delta \left(p_1^* - I_{i,1} - T \varepsilon_{i,1} - \frac{\partial C_{i,1}}{\partial q_{i,1}} \right) - \delta^2 T \frac{\partial e_{i,2}}{\partial q_{i,1}} \geq 0$$

$$\frac{\partial \Pi_i}{\partial q_{i,2}} = \delta^2 \left(p_2^* - T \varepsilon_{i,2} - \frac{\partial C_{i,2}}{\partial q_{i,2}} \right) \geq 0$$

Which can be re-written under the assumptions that investment is not saturated and R&D is active as;

$$\frac{\partial \Pi_i}{\partial q_{i,0}} = p_0^* - a_i - b_i q_{i,0} - I_{i,0} - RD_{i,0} - \delta T \alpha_i I_{i,1} q_{i,1} - \delta^2 T (\alpha_i I_{i,0} q_{i,2} + \beta_i \Delta RD_{i,0} q_{i,2}) \geq 0$$

$$\frac{\partial \Pi_i}{\partial q_{i,1}} = p_1^* - a_i - b_i q_{i,1} - I_{i,1} - T (\varepsilon_{i,0} - \alpha_i I_{i,0} q_{i,0}) - \delta T \alpha_i I_{i,1} q_{i,2} \geq 0$$

$$\frac{\partial \Pi_i}{\partial q_{i,2}} = p_2^* - a_i - b_i q_{i,2} - T (\varepsilon_{i,0} - \alpha_i (I_{i,0} q_{i,0} + I_{i,1} q_{i,1}) - \beta_i \Delta RD_{i,0} q_{i,0}) \geq 0$$

These conditions imply that production in each period occurs at a point such that marginal cost of production (including tax) minus the economies as a result of investment must be equal to the international price.³

2)

$$\frac{\partial \Pi_i}{\partial I_{i,0}} = -q_{i,0} - \delta \left(T \frac{\partial e_{i,1}}{\partial I_{i,0}} + \frac{\partial C_{i,1}}{\partial I_{i,0}} \right) - \delta^2 \left(T \frac{\partial e_{i,2}}{\partial I_{i,0}} + \frac{\partial C_{i,2}}{\partial I_{i,0}} \right) \geq 0$$

Which becomes if investments are not saturated;

$$q_{i,0} = \delta \left(T \alpha_i q_{i,0} q_{i,1} + \frac{\partial C_{i,1}}{\partial I_{i,0}} \right) + \delta^2 \left(T \alpha_i q_{i,0} q_{i,2} + \frac{\partial C_{i,2}}{\partial I_{i,0}} \right)$$

This condition characterizes period zero production as a function of production in periods 1 and 2 as well as the tax. We do not consider the impact of investment or R&D on the cost function in our simulation since it may increase marginal production costs, as the case of switching to cleaner coal, or decrease costs, as the case of installing solar panels.

The first period investment first order condition is;

$$\frac{\partial \Pi_i}{\partial I_{i,1}} = -\delta q_{i,1} - \delta^2 \left(T \frac{\partial e_{i,2}}{\partial I_{i,1}} + \frac{\partial C_{i,2}}{\partial I_{i,1}} \right) \geq 0$$

which becomes, if investments are not saturated;

$$q_{i,1} = \delta \left(T \alpha_i q_{i,1} q_{i,2} + \frac{\partial C_{i,2}}{\partial I_{i,1}} \right)$$

The left hand side is the marginal cost investment in period one. The two terms on the right hand side describe the discounted marginal benefit of investment. The first of these is the

³ These conditions are subject to the limitation imposed on investments and R&D described in the emission-intensity investment relationship.

emission tax savings in period 2 as a result of investment in period 1, and the second is as discussed before positive or negative depending on the type of technology. Thus, if we disregard investment's effect on the cost function, period 2 production is a constant, inversely related to the tax and investment effectiveness.

3) The first order condition for R&D is;

$$\frac{\partial \Pi_i}{\partial RD_{i,0}} = -q_{i,0} - \delta^2 \left(T \frac{\partial e_{i,2}}{\partial RD_{i,0}} + \frac{\partial C_{i,2}}{\partial RD_{i,0}} \right) \geq 0$$

which becomes, if investment in R&D is greater than the minimum required;

$$q_{i,0} = \delta^2 \left(T \beta_i q_{i,0} q_{i,2} + \frac{\partial C_{i,2}}{\partial RD_{i,0}} \right)$$

The left hand side is the marginal cost R&D in period zero. The two terms on the right hand side describe the discounted marginal benefit of R&D. The first of these is the emission tax savings as a result of R&D in period 2, and the second is as discussed before positive or negative depending on the type of technology. Thus, if we disregard R&D's effect on the cost function, period 2 production is a constant, inversely related to the tax and R&D effectiveness.

It is interesting to note that if the product is perishable good (i.e. can only be sold in the period it is produced) even if a tax level is chosen such that emissions are reduced below a chosen baseline, first order conditions under a tax are independent of the baseline period chosen.⁴ We have established that period zero production under a tax depends only on the tax level and the impact of R&D this will be compared to the case of an emissions cap in the following section.

⁴ This is not the case for cap-and-trade as will be proven the section 4.4.3.

3.3.2. Individual Firm and Global Profit Maximization under Cap and Trade

The objective of the government in imposing a cap is for the global emissions per period for all firms $\sum_i e_{i,t}; t=1,2$ to be lower than the total baseline emissions, $\sum_i e_{i,bl}; t=1,2$, by a factor less than one, $(\rho < 1)$, that is,

$$\sum_i e_{i,t} \leq \rho \sum_i e_{i,bl}; t=1,2$$

The upper bound $\sum_i e_{i,bl}; t=1,2$ can be a parameter such as some historical emission level or it can be a variable, namely the level of emissions during period 0, $\sum_i e_{i,0}; t=1,2$. The case for a baseline of $t=0$ is comparable to updating in existing cap-and-trade schemes, since occasionally new baselines are set and these may be anticipated by the participating firms. This global amount is initially allocated among individual firms, that is, $e_{i,t} \leq \rho e_{i,bl}; t=1,2; \forall i$. However since this individual allocation does not consider marginal abatement costs of individual firms, emissions trading among each other is introduced through the variables $w_{i,t}$. These variables could be either positive, in the case where the firm needs to buy emission credits (high abatement costs) or negative in the case where the firm can reduce emissions at a low cost and can therefore sell its excess allocation, in other words,

$$e_{i,t} \leq \rho e_{i,bl} + w_{i,t}; \forall i, t \quad (11)$$

In addition, since firms trade among each other only, during each period, the amount sold by the selling firms must exactly balance the amount bought by the buying firms,

$$\sum_i w_{i,t} = 0; t=1,2$$

With this condition, if each individual firm operates under the constraint, $e_{i,t} \leq \rho e_{i,bl} + w_{i,t}$, the total emissions satisfy,

$$\sum_i e_{i,t} \leq \sum_i (\rho e_{i,bl} + w_{i,t}) = \rho \sum_i e_{i,bl}; t=1,2$$

Which is consistent with the total cap defined in (11).

Under cap and trade, the profit maximizing problem of the combined profits of all firms takes the form,

$$\max \left\{ \begin{array}{l} \sum_i (p_0^* - I_{i,0} - RD_{i,0}) q_{i,0} - C_{i,0} \\ + \delta \sum_i ((p_1^* - I_{i,1}) q_{i,1} - C_{i,1} - z_1 w_{i,1}) \\ + \delta^2 \sum_i (p_2^* q_{i,2} - C_{i,2} - z_2 w_{i,2}) \end{array} \right\}$$

st.

$$e_{i,t} \leq \rho e_{i,bl} + w_{i,t}; \forall i, t$$

$$q_{i,t} \geq 0; \forall i, t$$

$$I_{i,t} \geq 0; \forall i, t = 0, 1$$

$$RD_{i,0} \geq 0$$

$$z_t \geq 0; \forall t$$

$$\sum_i w_{i,t} = 0; \forall t$$

Note that, at the equilibrium of this global profit maximization, the emission exchanges $w_{i,t}$ are sold at the same price z_t for all firms i . Moreover, this global profit maximization is equivalent to maximizing the profit of each individual firm for a given z_t assuming that some “invisible hand” is ensuring that the emissions exchanges satisfy the zero emissions balance constraints $\sum_i w_{i,t} = 0; t = 1, 2$. It can be shown that the first order conditions of the combined industry problem are identical to the individual firm maximization problem if we impose the required emission market clearing condition (see appendix 2). In this thesis, therefore, the individual firm profit maximization with cap and trade is analyzed and solved through the above global profit maximization. The Lagrange optimization framework is used to determine the required reductions for each firm, based on their marginal abatement cost, so that the total cost of reduction is minimized.

Thus, letting $\lambda_{i,t}$ be the Lagrange multiplier associated with the constraint $e_{i,t} \leq \rho e_{i,bl} + w_{i,t}$, the Lagrangean takes the form,

$$\begin{aligned} \mathcal{L} = & \sum_i (p_0^* - I_{i,0} - RD_{i,0}) q_{i,0} - C_{i,0} \\ & + \delta \sum_i \left((p_1^* - I_{i,1}) q_{i,1} - C_{i,1} - z_1 w_{i,1} - \lambda_{i,1} (e_{i,1} - \rho e_{i,bl} - w_{i,1}) \right) \\ & + \delta^2 \sum_i \left(p_2^* q_{i,2} - C_{i,2} - z_2 w_{i,2} - \lambda_{i,2} (e_{i,2} - \rho e_{i,bl} - w_{i,2}) \right) \end{aligned}$$

The first order conditions for maximum profit are:

1.

$$\left. \begin{aligned} \frac{\partial \mathcal{L}}{\partial w_{i,1}} = \delta (-z_1 + \lambda_{i,1}) = 0 \\ \frac{\partial \mathcal{L}}{\partial w_{i,2}} = \delta^2 (-z_2 + \lambda_{i,2}) = 0 \end{aligned} \right\} \forall i$$

This condition requires that $\lambda_{i,t} = z_t; \forall i, t$, which is the market-clearing condition for emission permits. Thus the Lagrange Multiplier represents the current market allowance price of emissions.

In addition,

2.

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial q_{i,0}} = & p_0^* - I_{i,0} - RD_{i,0} - \frac{\partial C_{i,0}}{\partial q_{i,0}} \\ & - \delta z_1 \left(\frac{\partial (e_{i,1} - \rho e_{i,bl})}{\partial q_{i,0}} \right) - \delta^2 z_2 \left(\frac{\partial (e_{i,2} - \rho e_{i,bl})}{\partial q_{i,0}} \right) \\ & \geq 0 \end{aligned}$$

This condition requires that marginal cost plus marginal investment plus the impact of the cap be equal to the exogenous price. The impact of the cap in the first period consists of two opposing terms. The first, depending on period one emissions reduces period zero production. The second, depending on the baseline period, has no impact if the baseline is historical but increases production if the baseline is $t=0$.

If the baseline is $t=0$ and assuming non-saturated investment and active R&D, we have;

$$q_{i,0} \leq \frac{p - a_i}{b_i} - \frac{I_{i,0}}{b_i} - \frac{RD}{b_i} + \frac{\delta z_1 \alpha_i I_{i,0} q_{i,1} + \delta^2 z_2 (\alpha_i I_{i,0} q_{i,2} + \beta_i \Delta RD_{i,0} q_{i,2})}{b_i} + \rho \varepsilon_{i,0} (\delta z_1 + \delta^2 z_2)$$

This implies that during period 0, the production of firm i has three terms other than its marginal cost production affecting production. One term, depending on marginal investment tends to decrease production since this investment although cost lowering in future periods is an additional cost to the firm in period zero. The second, depending on z_1 , z_2 , and the cumulative investment's ability to reduce future emissions tends to increase production in order to increase the cap the firm will face in the future. The third further increases production as a result of anticipated emission prices, this is intuitive as future emission prices are a cost and thus it is beneficial to produce more during the lower cost period. Even though this is not statically profit-maximizing during period 0, the result is expected from the point of view of the inter-temporal profit maximization since it allows the firm to raise the cap it will have to meet during periods 1 and 2 while investing to reduce future emission intensity.

“If there is an expectation that the baseline year upon which free allocations are based will be updated, participants have incentives to invest in dirty infrastructure and emit more now to get more free allowances in the future.”
(Stern, N., *The Economics of Climate Change*, p.379)

3.

$$\frac{\partial \mathcal{L}}{\partial q_{i,1}} = \delta [p_1^* - I_{i,1} - \frac{\partial C_{i,1}}{\partial q_{i,1}} - z_1 \varepsilon_{i,1}] - \delta^2 \frac{\partial e_{i,2}}{\partial q_{i,1}} \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial q_{i,2}} = \delta^2 [p_2^* - \frac{\partial C_{i,2}}{\partial q_{i,2}} - z_2 \varepsilon_2] \geq 0$$

These conditions imply that production in each period occurs at a point such that marginal cost of production (including expenditures on emissions) minus the economies as a

result of investment must be equal to the international price. The last term $z_2\varepsilon_2$ is simply the marginal cost of emission permits needed and so lowers production as does any extra cost.

The derivative with respect to period one production may be written as;

$$q_{i,1} \leq \frac{p_1^* - a_i - I_{i,1} - z_1(\varepsilon_{i,0} - \alpha_i I_{i,0} q_{i,0}) - \delta \alpha_i I_{i,1} q_{i,0}}{b_i}$$

During period 1, the production of firm i , has two opposing effects other than its marginal cost production. One effect, depending on first period investment, tends to lower production, while the second, depending on the period zero investment, tends to increase production though a reduction of period 1 emission intensity. As well, first period production is inversely related to its contemporaneous emissions price.

Second period production may be written as;

$$q_{i,2} \leq \frac{p_2^* - a_i - z_2 \left[\varepsilon_{i,0} - \alpha_i (I_{i,0} q_{i,0} + I_{i,1} q_{i,1}) - \beta_i \Delta RD_{i,0} q_{i,0} \right]}{b_i}$$

Thus second period production is positively related to all previous investments and negatively related to its contemporaneous emissions price. Here we see that all first and second period investments allow the firm to increase period 2 production. This cost of these investments must be offset by the additional profit generated in period 2 by this additional production.

4.

$$\frac{\partial \mathcal{L}_i}{\partial I_{i,0}} = -q_{i,0} - \delta \left(z_1 \frac{\partial e_{i,1}}{\partial I_{i,0}} + \frac{\partial C_{i,1}}{\partial I_{i,0}} \right) - \delta^2 \left(z_2 \frac{\partial e_{i,2}}{\partial I_{i,0}} + \frac{\partial C_{i,2}}{\partial I_{i,0}} \right) \geq 0$$

Assuming investments are not saturated and R&D is active;

$$q_{i,0} \leq \delta \left(z_1 \alpha_i q_{i,0} q_{i,1} + \frac{\partial C_{i,1}}{\partial I_{i,0}} \right) + \delta^2 \left(z_2 \alpha_i q_{i,0} q_{i,2} + \frac{\partial C_{i,2}}{\partial I_{i,0}} \right) \quad (12)$$

This condition characterizes period zero production as a function of production in periods 1 and 2 as well as the emissions prices.

5.

$$\frac{\partial \mathcal{L}_i}{\partial I_{i,1}} = -\delta q_{i,1} - \delta^2 \left(z_2 \frac{\partial e_{i,2}}{\partial I_{i,1}} + \frac{\partial C_{i,2}}{\partial I_{i,1}} \right) \geq 0$$

Becomes;

$$q_{i,1} \leq \delta \left(z_2 \alpha_i q_{i,1} q_{i,2} + \frac{\partial C_{i,2}}{\partial I_{i,1}} \right) \quad (13)$$

Thus period one production is characterized as a function of emission price and period 2 production. The left hand side is the marginal cost investment in period one. The two terms on the right hand side describe the discounted marginal benefit of investment. The first of these is the emission permit purchases savings in period 2 as a result of investment in period 1, and the second is as discussed before positive or negative depending on the type of technology.

6.

$$\frac{\partial \mathcal{L}_i}{\partial RD_{i,0}} = -q_{i,0} - \delta^2 \left(z_2 \frac{\partial e_{i,2}}{\partial RD_{i,0}} + \frac{\partial C_{i,2}}{\partial RD_{i,0}} \right) \geq 0$$

which becomes, if investment in R&D optimal;

$$q_{i,0} = \delta^2 \left(z_2 \beta_i q_{i,0} q_{i,2} + \frac{\partial C_{i,2}}{\partial RD_{i,0}} \right) \quad (14)$$

Thus period zero production is characterized as a function of emission prices and period 0 and 2 production. The left hand side is the marginal cost R&D in period zero. The two terms on the right hand side describe the discounted marginal benefit of R&D. The first of these is the emission permit price savings as a result of R&D in period 2, and the second is as discussed before positive or negative depending on the type of technology.

The derivatives with respect to first period investment (13) and R&D (14) are clearly incompatible since particular investment and R&D efficiency relationships would have to always hold true. This is clearly not the case generally and so possible solutions must be combination of either $\{I_0, I_1\}$ or $\{I_0, RD_0\}$.

3.3.3 Comparison of Tax and Cap-and-Trade Profit Maximization FOC

Emission Tax	Cap-and-Trade
$\frac{\partial \Pi_i}{\partial q_{i,0}} = p_0^* - I_{i,0} - RD_{i,0} - \frac{\partial C_{i,0}}{\partial q_{i,0}} - \delta T \frac{\partial e_{i,1}}{\partial q_{i,0}} - \delta^2 T \frac{\partial e_{i,2}}{\partial q_{i,0}} \geq 0$	$\frac{\partial \mathcal{L}}{\partial q_{i,0}} = p_0^* - I_{i,0} - RD_{i,0} - \frac{\partial C_{i,0}}{\partial q_{i,0}} - \delta z_1 \frac{\partial w_{i,1}}{\partial q_{i,0}} - \delta^2 z_2 \frac{\partial w_{i,2}}{\partial q_{i,0}} \geq 0$
$\frac{\partial \Pi_i}{\partial q_{i,1}} = p_1^* - I_{i,1} - T \varepsilon_{i,1} - \frac{\partial C_{i,1}}{\partial q_{i,1}} - \delta T \frac{\partial e_{i,2}}{\partial q_{i,1}} \geq 0$	$\frac{\partial \mathcal{L}}{\partial q_{i,1}} = p_1^* - I_{i,1} - \frac{\partial C_{i,1}}{\partial q_{i,1}} - z_1 \varepsilon_{i,1} - \delta \frac{\partial e_{i,2}}{\partial q_{i,1}} \geq 0$
$\frac{\partial \Pi_i}{\partial q_{i,2}} = p_2^* - T \varepsilon_{i,2} - \frac{\partial C_{i,2}}{\partial q_{i,2}} \geq 0$	$\frac{\partial \mathcal{L}}{\partial q_{i,2}} = p_2^* - \frac{\partial C_{i,2}}{\partial q_{i,2}} - z_2 \varepsilon_{i,2} \geq 0$
$\frac{\partial \Pi_i}{\partial I_{i,0}} = -q_{i,0} - \delta \left(T \frac{\partial e_{i,1}}{\partial I_{i,0}} + \frac{\partial C_{i,1}}{\partial I_{i,0}} \right) - \delta^2 \left(T \frac{\partial e_{i,2}}{\partial I_{i,0}} + \frac{\partial C_{i,2}}{\partial I_{i,0}} \right) \geq 0$	$\frac{\partial \mathcal{L}}{\partial I_{i,0}} = -q_{i,0} - \delta \left(z_1 \frac{\partial e_{i,1}}{\partial I_{i,0}} + \frac{\partial C_{i,1}}{\partial I_{i,0}} \right) - \delta^2 \left(z_2 \frac{\partial e_{i,2}}{\partial I_{i,0}} + \frac{\partial C_{i,2}}{\partial I_{i,0}} \right) \geq 0$
$\frac{\partial \Pi_i}{\partial I_{i,1}} = -\delta q_{i,1} - \delta^2 \left(T \frac{\partial e_{i,2}}{\partial I_{i,1}} + \frac{\partial C_{i,2}}{\partial I_{i,1}} \right) \geq 0$	$\frac{\partial \mathcal{L}}{\partial I_{i,1}} = -q_{i,1} - \delta \left(z_2 \frac{\partial e_{i,2}}{\partial I_{i,1}} + \frac{\partial C_{i,2}}{\partial I_{i,1}} \right) \geq 0$
$\frac{\partial \Pi_i}{\partial RD_{i,0}} = -q_{i,0} - \delta^2 \left(T \frac{\partial e_{i,2}}{\partial RD_{i,0}} + \frac{\partial C_{i,2}}{\partial RD_{i,0}} \right) \geq 0$	$\frac{\partial \mathcal{L}}{\partial RD_{i,0}} = -q_{i,0} - \delta^2 \left(z_2 \frac{\partial e_{i,2}}{\partial RD_{i,0}} + \frac{\partial C_{i,2}}{\partial RD_{i,0}} \right) \geq 0$

The equations above differ only in the price variable T (in the case of a tax policy) and z (in the case of a cap-and-trade policy). However, T is a fixed parameter, while z is a variable which varies from period to period. Only in situations where variable taxes or variable caps were implemented would solutions be identical. One difference that is immediately apparent in our case we can imagine that emission prices should be much smaller than taxes for a same level of emission reduction. This is due to the fact that taxes are paid on all units of emissions whereas emission trading (under grand-fathered permits) represents only a small portion of total emissions. As well since firms will invest to reduce emission intensity (if it is optimal) it seems probable that the demand for permits would decrease in future periods and thus prices

would be decreasing. It is possible however that it would be optimal for firms to increase production due to the lower emission intensity and thus demand for permits would not be decreasing. Indeed, since the effects are non-linear, intuition may lead us astray. Let us examine these potential effects by running a simulation.

4. Simulation

The solution of this problem with n firms, involves $(6*n+2)$ unknowns, namely the 3 investment variables for each firm, the three production variables for each firm, and the two emission price variables. Since the associated first order conditions are non-linear we run a simulation to quantitatively demonstrate the impact of different taxes or cap levels on the firm's variables of choice.

The three models of profit maximization under an emission tax, cap-and-trade with a historical business-as-usual (BAU) baseline⁵ and cap-and-trade with a $t=0$ baseline are simulated using Matlab's optimization toolbox (see appendix 1, 2, and 3 for respective M-files).

In these simulations, we make the following simplifying assumptions,

- a. The cost functions are independent of investments and R&D.

$$\text{That is } \frac{\partial C_i}{\partial I_{i,t}} = 0; \forall i,t \text{ and } \frac{\partial C_i}{\partial RD_{i,t}} = 0; \forall i,t;$$

We do not consider the impact of investment or R&D on the cost function in our simulation since it may increase marginal production costs, as the case of switching to cleaner coal, or decrease costs, as the case of installing solar panels.

- b. The product prices are constant over the three periods, $p_t^* = p; \forall t = 0,1,2;$
- c. The firms marginal costs of production are linear in the outputs and invariant over all periods, $\frac{\partial C_i}{\partial q_{i,t}} = a_i + b_i q_{i,t}$ where $a_i, b_i \geq 0;$
- d. Two firms are considered, $i=1,2;$
- e. No R&D investments are considered;
- f. The baseline emissions are defined by a constant historical amount;
- g. The investments do not exceed the maximum amount defined by the minimum possible emission intensity.

⁵ BAU production is defined here as marginal cost production. In this case $q=(p-a)/b$.

h. All variables are within their bounds.

We contrast a cap-and-trade system with a business-as-usual baseline to a tax based system using three different cases. The first case is of a perfectly homogenous industry where both firms have identical cost functions, emission intensities and abatement costs. The second case sets firm 1's costs lower than firm 2's but sets its emission intensity higher, both firms have identical marginal abatement of intensity costs. We can compare this case to a power plant (firm 1) producing energy with cheaper, dirtier coal than its competitor (firm 2). Case 3 is similar to case 2 except that firm 1 also has greater intensity abatement costs. This case is comparable to a firm that is more polluting having more difficulty transitioning to clean technology than a firm that has already begun the transition, perhaps a 'learning-by-doing' effect.

Table 1: Parameter choices for BAU cap-and-trade and tax simulations.

Parameter	Case 1		Case 2		Case 3	
	Marginal cost - $C'(q)$	$20+0.1q$	$20+0.1q$	$10+0.1q$	$30+0.1q$	$10+0.1q$
Marginal cost of emission intensity reduction - (α)	0.001	0.001	0.001	0.001	0.001	0.003
Initial emission intensity - (ε)	1	1	3	1	3	1

For all three cases:

- 1) $p = 60$ the price is constant
- 2) the discount factor is 1

- 3) maximum efficiency attainable through investment in current technologies is 0.7 for all firms
- 4) In the cap-and-trade business-as-usual quantity is defined as marginal cost production $(p-a)/b$

4.1. Simulation Results: Case 1 (BAU baseline)

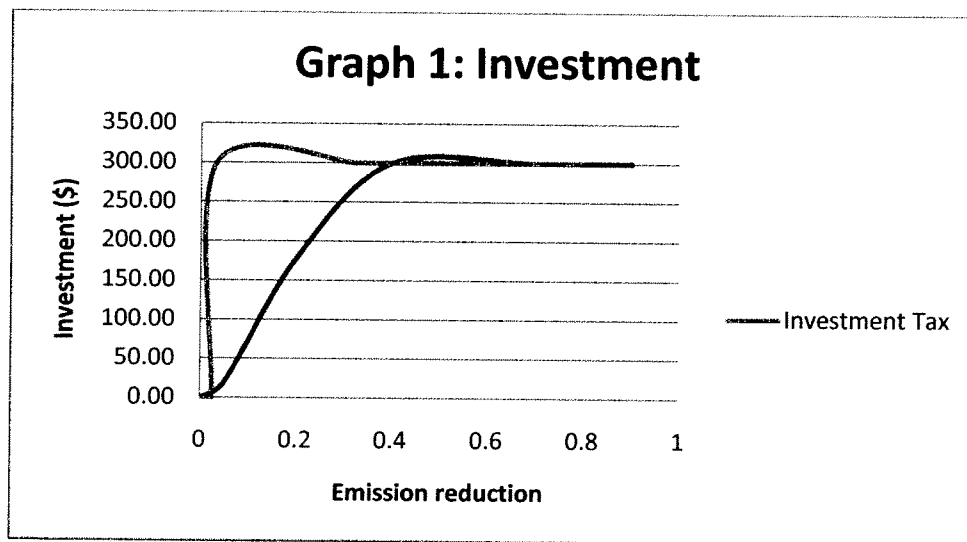
Table 2a: Case 1 - Cap-and-Trade

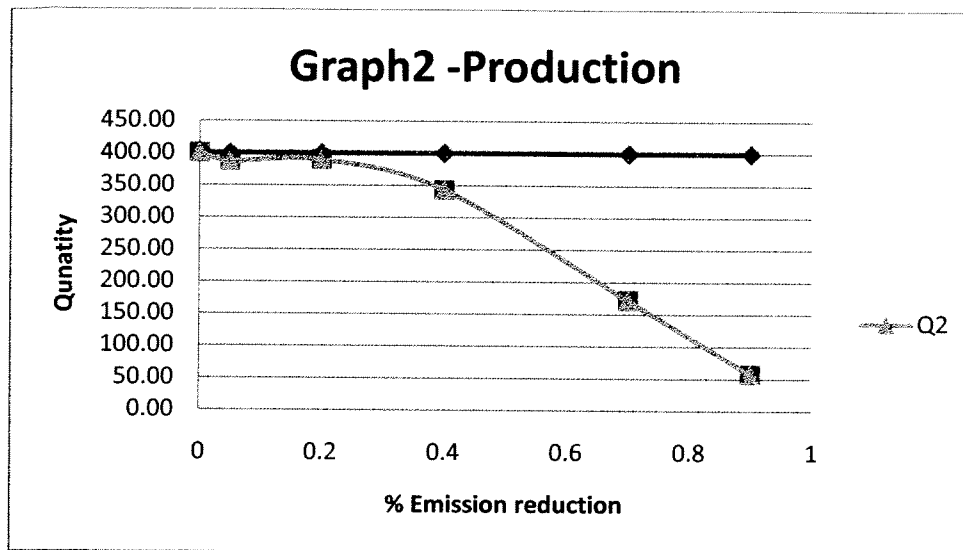
Cap (% of BAU)	emission reduction	Q @ t=0	Q @ t=1	IO	IOQ0	emission intensity	E1	industry profit
100	0.00	400.00	400.00	0.00	0.00	1.00	400.00	48000.00
95	0.05	400.00	387.34	0.05	18.94	0.98	380.00	47930.04
80	0.20	400.00	389.45	0.45	178.33	0.82	320.00	47621.08
60	0.40	400.00	342.86	0.75	300.00	0.70	240.00	46746.94
30	0.70	400.00	171.43	0.75	300.00	0.70	120.00	36951.02
10	0.90	400.00	57.14	0.75	300.00	0.70	40.00	23889.79

Table 2b: Case 1 – Emissions Tax

tax	emission reduction	Q @ t=0	Q @ t=1	IO	IOQ0	emission intensity	E1	industry profit
0	0	400	400	0	0	1	400	48000
1	0.025	400	390	0	0	1	390	46420
2	0.3245	400	386	0.75	300	0.7	270.2	45199.2
3	0.33675	400	379	0.75	300	0.7	265.3	44128.2
5	0.36125	400	365	0.75	300	0.7	255.5	42045
10	0.4225	400	330	0.75	300	0.7	231	37180

In this case, the results for both firm 1 and 2 are identical and so only one firm is presented for each policy. There is no supply or demand of emission rights and therefore no resulting emission price. It is clear from graph 1 that investment under a tax policy is initially much greater. This may be explained by the fact that, under a cap-and-trade system there exists a pair of investment and emission price combinations which are optimal under the imposed cap whereas under a tax, investment is always optimal if it reduces emissions more cheaply than the cost of the tax. Thus if marginal abatement costs are cheaper than the tax it is optimal to achieve maximal allowable efficiency. Investment under both policies increases monotonically to a level of 300\$, up to approximately a 30% emission reduction, and remains constant from there on. This limitation is imposed by the maximum efficiency capabilities of investment. Industry profits are greater for all levels of emissions reductions under cap-and-trade than under a tax since permits are grandfathered as opposed to auctioned.





From graph 2 we see that production is not affected by a cap of less than 20% emission reduction for this level of investment efficiency.

4.2. Simulation Results: Case 2 (BAU Baseline)

Table 3a: case 2 BAU firm 1

Emission reduction	Q0	Q1	Q2	I0	I0Q0	emission intensity	E1	W1	W2
0	500	499.9969	499.9972	0	0	3	1499.991	-0.009	-0.008
0.05	500	473.0001	472.9999	0	0	3	1419	-6.000	-6.000
0.2	500	474.4475	474.4384	1.15	575	2.43	1150.553	-49.449	-49.471
0.4	500	483.0653	483.0654	2.73	1364	1.64	790.3507	-109.65	-109.65
0.7	500	485.7138	485.7148	4.6	2300	0.7	339.9997	-110.00	-110.00
0.9	500	228.5714	228.5714	4.6	2300	0.7	160	10.000	10.000

Table 3b: case 2 Tax firm 1

t	E reduction	Q0	Q1	Q2	I0	I0Q0	intensity	e1
0	0	500	500	500	0	0	3	1500
1	0.06	500	470	470	0	0	3	1410
1.1	0.77	500	492.30	492.3	4.6	2300	0.7	344.61
1.3	0.77	500	490.9	490.9	4.6	2300	0.7	343.63
2	0.773	500	486	486	4.6	2300	0.7	340.2
3	0.776	500	479	478.9999	4.6	2300	0.7	335.3
5	0.783	500	465	464.9997	4.6	2300	0.7	325.4998

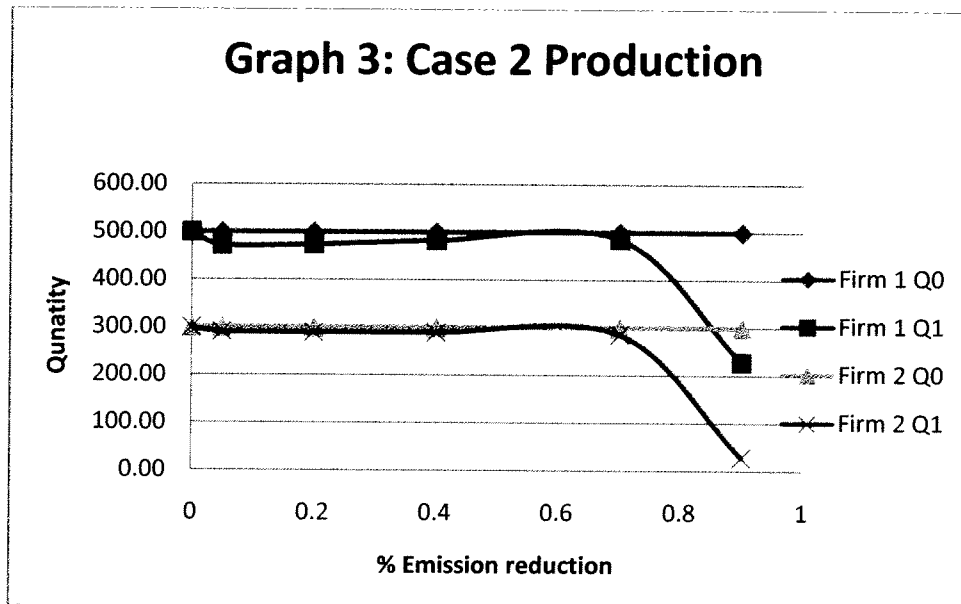
Table 3b case 2 BAU firm 2

Emission reduction	Q0	Q1	Q2	I0	I0Q0	E1	emission intensity	W1	W2
0	300	300	300	0	0	300	1	0.009	0.008
0.05	300	291	291.00	0	0	291.00	1	6.000	6.000
0.2	300	289.45	289.47	0	0	289.45	1	49.449	49.471
0.4	299	289.65	289.65	0	0	289.65	1	109.649	109.649
0.7	300	285.7148	285.71	1	300	200	0.7	110.000	110.000
0.9	300	28.57149	28.57	1	300	20	0.7	-10.000	-10.000

Table 3c case 2 BAU

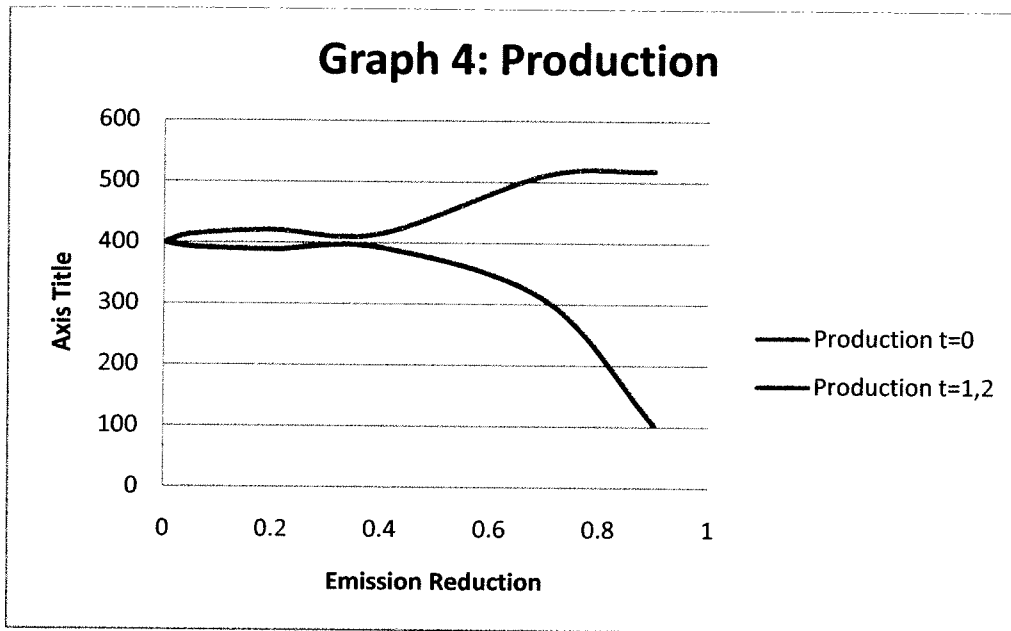
Emission reduction	Z1	Z2	Industry profit
0	0.050695	0.055423	51000
0.05	0	0.012605	50918.99
0.2	0.000238	0	50348.61
0.4	0.009058	0.042837	49596.72
0.7	0.004669	4.05E-05	48359.18
0.9	0.00097	0	33665.3

We see here that a small change in the tax rate causes a large change in investment level, maximal investment is achieved by increasing the tax from 1 to 1.1. A very large cap, approximately a 70% reduction in emissions is needed in order to achieve the maximal investment.



It is interesting to note that if as in case one, we have a uniform industry, production begins to decrease at a cap of ~20% while in this case, it is possible to impose a much greater cap (~70%) without decreasing productivity (compare graph 2 to graph 3).

4.3. Simulation Results: Case 2 (t=0 Baseline)



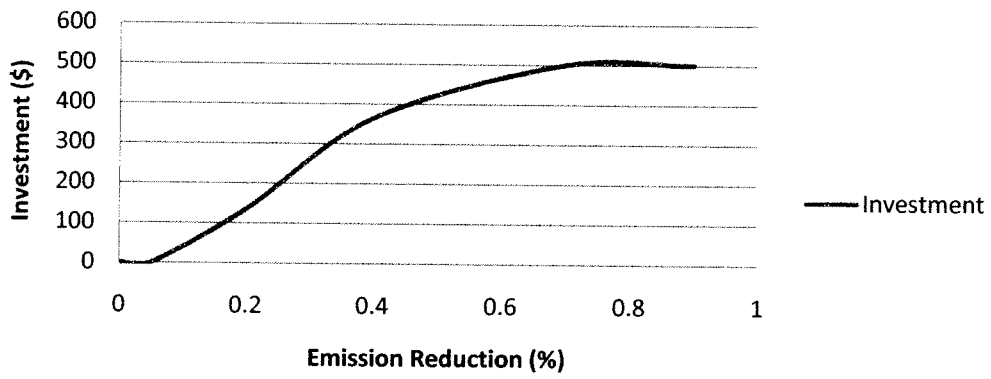
What is most noteworthy in the case of a t=0 baseline is the fact that firms increase production in period 0 to benefit from a greater cap in the future. We see in graph 4 that period zero production increases much more above BAU beyond the 40% emission reduction level.

Returning to the associated equation;

$$q_{i,0} \leq \frac{p - a_i}{b_i} - \frac{I_{i,0}}{b_i} - \frac{RD}{b_i} + \frac{\delta z_1 \alpha_i I_{i,0} q_{i,1} + \delta^2 z_2 (\alpha_i I_{i,0} q_{i,2} + \beta_i \Delta RD_{i,0} q_{i,2})}{b_i} + \rho \varepsilon_{i,0} (\delta z_1 + \delta^2 z_2)$$

It is also at this point (~40% reduction) that investment (graph 5) becomes concave in emission reduction.

Graph 5: Investment



5. Conclusion

In this paper we compare two greenhouse gas emission limiting policies, namely taxes and cap-and-trade by presenting a dynamic model of firm profit maximization with investment and R&D. Despite the simplified nature of the model it allows us to comment on interesting issues that may have been traditionally overlooked when attempting to establish a preferred climate change policy. We have shown that intra-industry variations play an important role in induced technological innovation resulting from different policies. "It is impossible to attain the technology induced by one instrument with another instruments, regardless of the instruments' designs, like the tax rate or the number of permits" (Krysiak 2008). The type of industry heterogeneity should be a consideration for policymakers attempting to create a framework for climate change mitigation.

Further work should evaluate the effects of an increase in the exogenous price, as this reflects the tendency of other producing countries to implement caps or taxes and thus affect global prices. Uncertainty, with respect to prices and investment returns are also potential extensions to the model as well as sharing of research and development. This would imply the application of game theory to establish optimal R&D and is certainly much more realistic.

6. Appendix 1

6.1 Sample Simulation M-files

```
function [x,fval,exitflag,output,lambda,grad,hessian] = optimizer(x0,lb)
% This is an auto generated M-file from Optimization Tool.
```

```
% Start with the default options
options = optimset;
% Modify options setting
options = optimset(options,'Display','iter');
options = optimset(options,'Algorithm','active-set');
options = optimset(options,'Diagnostics','off');
options = optimset(options,'GradConstr','off');
[x,fval,exitflag,output,lambda,grad,hessian] = ...
fmincon(@objfun3,x0,[],[],[],[],lb,[],@confun2,options);
```

```
function [c, ceq] = confun2(x)
% Nonlinear inequality constraints:
global ga al rho ep be rdmin
c = [ep(1)*x(2)-al(1)*x(4)*x(1)*x(2)-rho*ep(1)*x(1)-x(7);
     ep(1)*x(3)-al(1)*(x(4)*x(1)+x(5)*x(2))*x(3)-be(1)*(x(6)-rdmin)*x(1)*x(3)-
rho*ep(1)*x(1)-x(8);
     ep(2)*x(10)-al(2)*x(12)*x(9)*x(10)-rho*ep(2)*x(9)-x(15);
     ep(2)*x(11)-al(2)*(x(12)*x(9)+x(13)*x(10))*x(11)-be(2)*(x(14)-
rdmin)*x(9)*x(11)-rho*ep(2)*x(9)-x(16)];
     ga-ep(1)+al(1)*x(4)*x(1);
     ga-ep(2)+al(2)*x(12)*x(9);
     ga-ep(1)+al(1)*(x(4)*x(1)+x(5)*x(2));
     ga-ep(2)+al(2)*(x(12)*x(9)+x(13)*x(10))];

ceq = [x(7)+x(15);
       x(8)+x(16);
       x(6);
       x(14)];
```

```
function [c, ceq] = confunbau2(x)
% Nonlinear inequality constraints:
global al rho p b a
c = [x(1)-al(1)*x(4)-rho*((p-a(1)/b(1)))-x(6);
     x(1)-al(1)*(x(4)+x(5))-rho*((p-a(1)/b(1)))-x(7);
     x(8)-al(2)*x(11)-rho*((p-a(2)/b(2)))-x(13);
     x(8)-al(2)*(x(11)+x(12))-rho*((p-a(2)/b(2)))-x(14);
     x(1)-al(1)*x(4)+x(8)-al(2)*x(11)-rho*((p-a(1)/b(1))+((p-a(2)/b(2)))));
     x(1)-al(1)*(x(4)+x(5))+x(8)-al(2)*(x(11)+x(12))-rho*((p-
a(1)/b(1))+((p-a(2)/b(2))))];
ceq=[x(6)+x(13);
     x(7)+x(14)];
```

```

function f = objfun3(x)
% Objective function
global a b del p;
f1 = (p-a(1))*(x(1)+del*x(2)+del^2*x(3))-
0.5*b(1)*(x(1)^2+del*x(2)^2+del^2*x(3)^2)-(x(4)+x(6))*x(1)-del*x(5)*x(2)-
del*x(17)*x(7)-del^2*x(18)*x(8);
f2 = (p-a(2))*(x(9)+del*x(10)+del^2*x(11))-
0.5*b(2)*(x(9)^2+del*x(10)^2+del^2*x(11)^2)-(x(12)+x(14))*x(9)-
del*x(13)*x(10)-del*x(17)*x(15)-del^2*x(18)*x(16);
f=-f1-f2;

```

```

function f = objfuntx(x)
% Objective function
global a b del p T al ep;
f1 = (p-a(1))*(x(1)+del*x(2)+del^2*x(3))-
0.5*b(1)*(x(1)^2+del*x(2)^2+del^2*x(3)^2)-(x(4)+x(6))*x(1)-del*x(5)*x(2)-
del*T*(ep(1)*x(2)-al(1)*x(4)*x(1)*x(2))-del^2*T*(ep(1)*x(3)-
al(1)*(x(4)*x(1)+x(5)*x(2))*x(3));
f2 = (p-a(2))*(x(7)+del*x(8)+del^2*x(9))-
0.5*b(2)*(x(7)^2+del*x(8)^2+del^2*x(9)^2)-(x(10)+x(12))*x(7)-del*x(11)*x(8)-
del*T*(ep(2)*x(8)-al(2)*x(10)*x(7)*x(8))-del^2*T*(ep(2)*x(9)-
al(2)*(x(10)*x(7)+x(11)*x(8))*x(9));
f=-f1-f2

```

7. Appendix 2

Individual firm maximization problem

$$\text{max} \left\{ \begin{array}{l} (p_0^* - I_{i,0} - RD_{i,0})q_{i,0} - C_{i,0} \\ + \delta \left((p_1^* - I_{i,1})q_{i,1} - C_{i,1} - z_1 w_{i,1} \right) \\ + \delta^2 \left(p_2^* q_{i,2} - C_{i,2} - z_2 w_{i,2} \right) \end{array} \right\}$$

s.t.

$$e_{i,t} \leq \rho e_{i,t-1} + w_{i,t}; \forall i, t$$

$$q_{i,t} \geq 0; \forall i, t$$

$$I_{i,t} \geq 0; \forall i, t = 0, 1$$

$$RD_{i,0} \geq 0$$

$$z_t \geq 0; \forall t$$

By imposing the emission market clearing condition $\sum_i w_{i,t} = 0; \forall t$, we obtain the same first order conditions as the combined industry maximization. In this case the Lagrange multiplier is effectively the shadow price of emission permits which must be equal for all firms. By using the combined problem we eliminate the need to assume an 'invisible hand' clearing the markets, while maintaining the same solutions. The first order conditions are;

$$\frac{\partial \mathcal{L}}{\partial w_{i,1}} = \delta(-z_1 + \lambda_{i,1}) = 0$$

$$\frac{\partial \mathcal{L}}{\partial w_{i,2}} = \delta^2(-z_2 + \lambda_{i,2}) = 0$$

$$\frac{\partial \mathcal{L}}{\partial q_{i,0}} = p_0^* - I_{i,0} - RD_{i,0} \frac{\partial C_{i,0}}{\partial q_{i,0}} - \delta z_1 \frac{\partial w_{i,1}}{\partial q_{i,0}} - \delta^2 z_2 \frac{\partial w_{i,2}}{\partial q_{i,0}} \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial q_{i,1}} = p_1^* - I_{i,1} - \frac{\partial C_{i,1}}{\partial q_{i,1}} - z_1 \varepsilon_{i,1} - \delta \frac{\partial e_{i,2}}{\partial q_{i,1}} \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial q_{i,2}} = p_2^* - \frac{\partial C_{i,2}}{\partial q_{i,2}} - z_2 \varepsilon_{i,2} \geq 0$$

$$\frac{\partial \mathcal{L}}{\partial I_{i,0}} = -q_{i,0} - \delta \left(z_1 \frac{\partial e_{i,1}}{\partial I_{i,0}} + \frac{\partial C_{i,1}}{\partial I_{i,0}} \right) - \delta^2 \left(z_2 \frac{\partial e_{i,2}}{\partial I_{i,0}} + \frac{\partial C_{i,2}}{\partial I_{i,0}} \right) \geq 0$$

$$\left(\frac{\partial \mathcal{L}_i}{\partial I_{i,1}} = -q_{i,1} - \delta \left(z_2 \frac{\partial e_{i,2}}{\partial I_{i,1}} + \frac{\partial C_{i,2}}{\partial I_{i,1}} \right) \geq 0 \right.$$

$$\left. \frac{\partial \mathcal{L}_i}{\partial RD_{i,0}} = -q_{i,0} - \delta^2 \left(z_2 \frac{\partial e_{i,2}}{\partial RD_{i,0}} + \frac{\partial C_{i,2}}{\partial RD_{i,0}} \right) \geq 0 \right)$$

The above are identical to first order conditions of the combined maximization problem.

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