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Ce mémoire intitulé

The load planning problem for double-stack intermodal trains

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Résumé et mots-clés

Les trains qui transportent des conteneurs empilés (en deux niveaux) sont un élément important du réseau de transport nord-américain. Le problème de chargement des wagons correspond un problème opérationnel couramment rencontré dans les terminaux ferroviaires. Elle consiste optimiser l'affectation des conteneurs des emplacements spécifiques sur les wagons.

Ce mémoire est centré sur un article scientifique traitant le chargement optimal publié dans le European Journal of Operational Research (Volume 267, Numéro 1, Pages 107-119, 2018). Nous avons formulé un modèle linéaire en nombres entiers (ILP) et apporté un certain nombre de contributions. Premièrement, nous avons proposé une méthodologie générale qui peut traiter des wagons double ou simple empilement avec des "patrons" de chargement arbitraires. Les patrons tiennent compte des dépendances de chargement entre les plateformes sur un wagon donné. Deuxièmement, nous avons modélisé les restrictions du centre de gravité (COG), les règles d'empilement et un nombre de restrictions techniques de chargement associées certains types de conteneurs et/ou de marchandises. Les résultats montrent que nous pouvons résoudre des instances de taille réaliste dans un délai raisonnable en utilisant un solveur ILP commercial et nous illustrons que le fait de ne pas tenir compte de la correspondance conteneurs-wagons ainsi que des restrictions COG peut conduire une surestimation de la capacité disponible.

Mots-clés: transport; marchandises; conteneurs, planification de chargement, chargement de trains deux niveaux; terminaux ferroviaires intermodaux

Summary and keywords

Double-stack trains are an important component of the railroad transport network for containerized cargo in specific markets such as North America. The load planning problem embodies an operational problem commonly faced in rail terminals by operators. It consists in optimizing the assignment of containers to specific locations on the train.

The work in this thesis is centered around a scientific paper on the optimization on load planning problem for double stack-trains, published in the European Journal of Operation Research (Volume 267, Issue 1, Pages 107-119) on 16 May 2018. In the paper, we formulated an ILP model and made a number of contributions. First, we proposed a general methodology that can deal with double- or single-stack railcars with arbitrary loading patterns. The patterns account for loading dependencies between the platforms on a given railcar. Second, we modeled Center of gravity (COG) restrictions, stacking rules and a number of technical loading restrictions associated with certain types of containers and/or goods. Results show that we can solve realistic size instances in reasonable time using a commercial ILP solver and we illustrate that failing to account for containers-to-cars matching as well as COG restrictions may lead to an overestimation of the available train capacity.

Keywords: Transportation; Freight; Containers, Load Planning, double-stack train loading; Intermodal Rail Terminals

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Chapter 1

Introduction

The demand for goods has grown strongly over the past half century so that today an essential ingredient of a thriving national economy is a cost effective freight transportation system. This involves the use of multimodal, including intermodal, transportation options. Intermodal movements are those in which two or more different transportation modes are linked end-to-end in order to move freight and/or people from a point of origin to a point of destination. Intermodal freight transportation relies heavily on containerization due its several advantages: Containerization offers safety by reducing loss and damages of the product, by ensuring a faster exchange of modes and by decreasing transportation cost due to a smaller effort in moving the freight itself.

In the context of intermodal transportation, terminals play an important role by providing an interface between different transportation modes such as trains, trucks and vessels, in order to manage the sustained flow of containers from their origin to the final destination. Within a terminal there are several components to be managed, including the maritime side, the land side, the yard and the equipment. The activities in each component impact directly the performance of the others.

From a planning perspective, there are strategic, tactical and operational problems that arise at intermodal terminal. Especially at the operational level, typically the experience of planners plays an important role in the decision making process, and in many cases the policies are based on simple rules of thumb.

The need for optimization using methods of operations research in container terminal operations is becoming increasingly important. This is because the logistics, especially of large container terminals, is increasing complex when operating close to capacity and with quickly changing market requirements. Decision-aid tools based on optimization methods can provide terminal managers necessary information to take better decisions. This thesis focuses on an operational planning problem arising in intermodal rail terminals.

Trains are a widely used transportation mode for containerized cargo. Intermodal trains are composed of sequences of railcars, designed to carry single- or double-stacked containers. Railcars differ on attributes such as the number and the length of platforms, in turn composed of different slots, and the weight holding limit. Loading rules may not be derived regardless those differences. By nature, they can carry many more containers inland than trucks while being cost effective compared to airplanes. Relative to single-stack trains, double-stack trains have an increased carrying capacity as they allow the placement of two containers, one on top of the other, rather than a single one. Double-stack trains are more rare, but they are extensively used in some large markets like North America.

In this context, the Load Planning Problem (LPP) addresses the problem of loading the containers on the railcars that form the departing train. Load planning aims to find an assignment of stored containers to specific railcars. With this thesis, the goal is to present a general methodology that addresses the load planning problem for intermodal trains that can deal with single- or double-stack railcars as well as arbitrary containers-to-cars matching rules. Terminals represent the backbone of the entire international chain, and thus we aim at improving their efficiency through a general methodology that can be used by terminal operators as a decision-support-tool for addressing the load planning problem.

In the literature, most of the studies focus on the single-stack load planning problem, where the loading is simple. The authors aim at deriving loading plans such that handling costs (e.g. Corry and Kozan, 2008) in the yard or train set-up costs (Bruns and Knust, 2012) are minimized. To the best of our knowledge, Lai, Barkan and Önal (2008) is the only study on the double-stack load planning problem. A number of crucial aspects are however ignored, which make the problem challenging. First, they address the matching among containers and railcar types without considering platform dependencies, ignoring the cases when loading may be constrained by the railcar sequence. Moreover, the problem is studied without accounting for center-of-gravity restrictions and stacking rules.

We contribute to the existing literature by providing a general methodology that can address the load planning problem in all its crucial operational constraints and restrictions. Given a set of containers stored in a terminal and a departing train, the problem is to select the optimal set of containers to load and the optimal way of loading them, using the maximum of the available capacity. In our research we address the problem for double-stack trains, where the load planning problem is more challenging because of a number of loading rules that depend on container and railcar characteristics, and on the way they match together. The size, the location of the load-bearing along the length of the container, the type (e.g., tanker and dangerous containers have restrictions with respect to the position in the stack they may occupy) of containers determine how containers can be stacked.

The remainder of the thesis is articulated as follows: In Chapter 2, we present the extensive framework of intermodal freight transportation where terminal fits, to the seaports and the railway yards. We describe the container terminals in 2.3, giving an overview of what terminals are and which equipment are needed. Section 2.3.1 provides an overview of the sea ports and section 2.3.2 of the yard terminals. In section 2.3.3 we discuss the problems that arise at container terminal, together with the reasons that encourage this research. Chapter 3 presents the research paper on the load planning problem for double-stack intermodal trains published in the European Journal of Operational Research. Chapter 4 concludes on the thesis' findings and suggests future research direction for the LPP for double-stack trains.

Chapter 2

Background on intermodal freight transport systems

2.1 Intermodal and multimodal transportation

In this section, we broadly discuss the intermodal freight transportation. Terminals, which are the principal entities of our study, play indeed an important role in a larger and complex intermodal freight transportation network.

The demand for goods has grown strongly over the past half century so that today an essential ingredient of a prosperous national economy is a cost-effective freight transportation system. This involves the use of multimodal, including intermodal, transportation. Even if the transportation could be of passengers or of freight, we focus on the latter.

Intermodal transportation relies heavily on containerization due its several advantages, such as the increase of the safety by significantly reducing loss and damage, the gain in the speed in performing transfer operations at terminal, the flexibility in the transport of products of various types and dimension and so on. The result is a most profitable flow of cargo, which is not damaged easily but it is simple to control and to schedule.

A container, as defined by the European Conference of Ministers of Transport (2001), is a *"generic term for a box to carry freight, strong enough for repeated use, usually stackable and fitted with devices for transfer between modes"*. Containers are large, metal and uniform boxes, that are used to transport goods from one destination to another one and with a standardized dimensions: A standard container is the 20-foot box, which is 20 feet long, 8'6" high and 8 wide. It is referred to as a twenty-foot equivalent unit (TEU). Containers are either made of steel, for maritime transport, or aluminum, for domestic transport. In the recent years, they have got a great importance especially in international maritime freight transportation.

Intermodal movements are those in which two or more different transportation modes are linked end-to-end in order to move freight and/or people from point of origin to point of destination. Intermodal freight transport is the term used to describe the combination of at least two modes of transport into a single transport chain, without a change of container for the goods, with most of the route traveled by rail, inland waterway or ocean-going vessel, and with the shortest possible initial and final journeys by road. Despite the growing emphasis being placed on intermodal transportation by government and industry, a consensus definition of intermodal transportation does not exist. According to Jones et al. (2000), *"a large number of definitions are present in the research literature, suggesting that a fundamental interpretation of this term does not currently exist"*.

European Conference of Ministers of Transport on 1993 described Intermodal freight transport such as the movement of goods in one and the same loading unit or vehicle which uses successive, various modes of transport (road, rail, water) without any handling of the goods themselves during transfers between modes.

Contrary to conventional transportation systems, intermodal transportation aims at integrating various modes and services of transportation to improve the efficiency of the whole distribution process, in order to create a seamless journey, where transitions between modes occur smoothly with minimal delay. In fact, according to the U.S. Department of Transportation (2006), the value of the multimodal shipments, increased from about $662 billion to about $1.1 trillion in a period of nine years (1993 - 2003). Because of its importance, intermodal transportation forms the backbone of world trade, and it exhibits significant growth.

It is important to underline the analogies and the dissimilarities between intermodal and multimodal transportation, as two different types of freight transportation. In fact, Intermodal and Multimodal are two terms often used loosely and interchangeably, but they have discernible differences:

- The International Multimodal Transport Association defines **multimodal transport** as the chain that interconnects different links or modes of transport air, sea, and land into one complete process that ensures an efficient and cost-effective door-to-door movement of goods under the responsibility of a single transport operator, known as a Multimodal Transport Operator, on one transport document. A multimodal transport contract is a single contract for carriage of goods by at least two different modes of transport, where a multimodal transport operator (MTO) assumes responsibility for the performance thereof as a carrier.

- **Intermodal transport** is a particular type of multimodal transport, wherein the goods are moved in one and the same loading unit, for example: containers. Intermodal transport uses more than one mode of transports, where each of these modes has a different transport provider or entity responsible with independent contracts, and since the loading unit remains the same, the goods being transported, are themselves not handled each time there is a change of mode.

2.2 Transportation systems

Almost all types of freight carriers and terminal operators may be involved in intermodal transportation by operating an intermodal transportation system. Because of the interplay between producers and consumers and the distance that often divide them, demand for freight transportation comes up. In fact, the one who produces raw materials requires transportation services in order to distribute final goods to satisfy customers demand.

A supply network for freight consists not only of nodes and links but also of terminals nodes (freight hubs, logistic centers, shunting yards, warehouses) with specific characteristics concerning capacity and transfer delay time. Figure 2.1 shows the process steps and decision levels in freight transport as defined in the terminology on combined transport. At each decision level specific decisions concerning the movement of goods are required:

- The sender (shipper, consignor) demands the transport of goods-units and puts these goods-units in the care of others (freight forwarder, carrier) to be delivered to a consignee. The sender will decide on a freight forwarder based on price and other factors like temporal constraints or reliability and generate the demand for transportation.

- The freight forwarder organizes the shipping process. It provides and schedules unimodal or intermodal transport chains for shipping the goods. For this it may subcontract carriers or provide its own carrier service.

- The carrier is responsible for the carriage of goods, supplying transportation service. The carrier answers the demand of transportation and provides the vehicles required for the transport along a unimodal section of the transport chain. The vehicles operate on the link infrastructure connection origin, hubs and destination.

- The driver steers the transport vehicle along a predefined tour. In case of road transport the driver may decide on the route between two points of the tour.

- The consignee is entitled to take delivery of the goods.

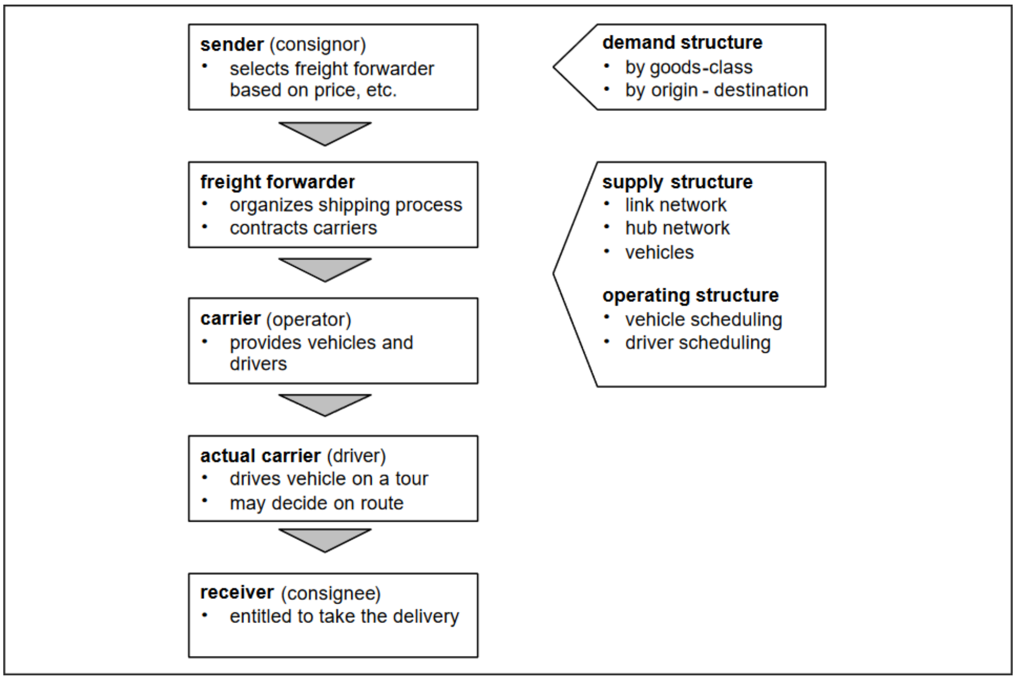

Figure 2.1: Process steps and decision levels in freight transport Source: Friedrich (2003)

Considering the type of service they provide, intermodal terminals and such facilities may be described as carriers as well. Terminals are, in this context, important for the operations of loading, unloading, transfer operations and consolidation, and their efficiency is vital for the performance of the entire transportation chain. The complex interactions between the actors is one reason why it is more complicated to model the decision processes in freight transport compared to passenger transport. The intermodal transport system can be viewed according to the different actors:

- We can look at it from the firm, that offers the service and that has problems related to the organization of activities. This is the point of view of the carriers (railway companies, shipping line, trucking companies and so on) and of the terminal itself, which needs to allocate resource to tasks in order to be as efficient as possible.

- We can look at it from the point of view of a individual container or more in general from the point of view of shippers, which want to ship containers in the best way as possible. Here, we find the carriers on the other side, providing services to shippers, who obviously want the service to be as cheap as possible, while having some guarantees on the service quality.

This combination of issues makes the intermodal transportation an hard topic. In our study, we take the point of view of a carrier, who offers the service at the best quality as possible, by minimizing its own cost. As states in the Global Logistics Management book [Voortman (2004)], *"Logistics involves getting the right product to the right customer at the right time, in the right condition, at the right place, at the right price."*.

In an intermodal transport chain we have, on the one hand, consolidated transport where one vehicle or convoy serves to move freight for different customers with possibly different origins and final destinations, and, on the other hand, customized transportation carriers that provide dedicate service to each particular customers with possibly different origins and destinations. Full-load trucking is a classic example of customized transportation, that travels to the customer location, and once loaded, it moves to the destination, where it is unloaded. At the end of this process, the truck is repositioned. Customized transportation is not always the appropriate answer to shipper's need, because of the relations and the trade-off between volume and frequency of shipping and the cost, the frequency and delivery time of transportation, that may dictate the use of consolidation services.

Consolidation can be a more attractive alternative. Freight consolidation transportation is performed by less-than-truckload (LTL) motor carriers, railways, ocean shipping lines, regular and express postal services, etc. Consolidation transportation carriers and most intermodal transportation systems are organized as so-called hub and spoke network, where shipments for a number of origin-destination points may be transferred via intermediate consolidation facilities, or hubs, such as airports, seaport container terminals, rail yards, truck break-bulk terminals, and intermodal platforms. An example of an hub and spoke networks follows in Figure 2.2. In such systems, services are offered between a certain number of origins and destination points (local/regional terminal), represented by the numbered nodes. Taking advantage of economies of scale, low volume demands are moved first to an intermediate point, a consolidation terminal or hub, such an airport, seaport container terminal, rail yard, or intermodal platform. So, in hub and spoke networks, low volume demands are firstly moved from their origins to a hub where traffic is sorted and grouped, namely classified and consolidated.

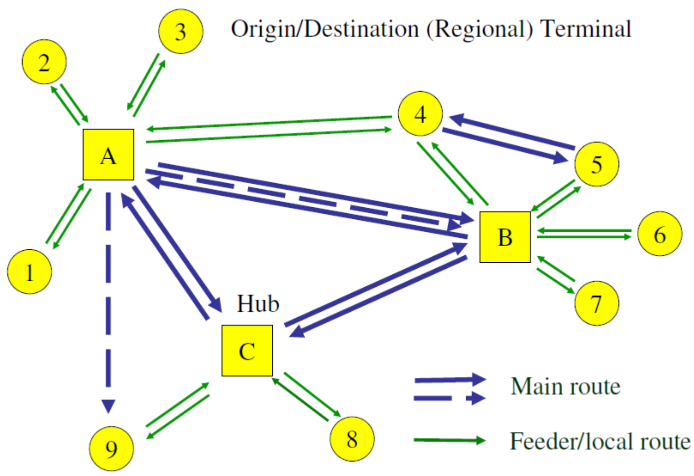

Figure 2.2: Network with consolidation terminals/hubs. Source: Crainic and Kim (2007)

The aggregated traffic is then moved in between hubs by services and loads are transferred to their destination points from the hub by lower frequency services often utilizing smaller vehicles. When the level of demand is sufficiently high, direct services may be run between a hub and a regional terminal. This way to move freight between origins and destinations is an efficient utilization of resources and it is lower cost for shippers, but it can bring a higher amount of delays and a lower reliability due to the longer routes and the additional operations performed at terminals. All consolidation-based transportation modes involved in intermodal transportation must provide efficient, reliable, and cost-effective services. In this context, carriers face a number of challenges which may be examined according to classical categorization of planning decisions, namely strategic (long-term), tactical (medium-term), and operational (short-term) level of planning and management of operations.

2.3 Container terminals

In this section we describe various types of container terminals. First, we give an overview of what they are and which equipment is needed. Second, we describe sea ports and rail yard terminals. Third, we present the class of planning problems which arise there, to provide the full context of our research.

An intermodal terminal provides an interface between different transportation modes such as trains, trucks and vessel in order to manage the sustained flow of containers from their origin to the final destination. It can be defined as any facility where passengers and/or freight are consolidated and deconsolidated. The focus of our study is only the freight terminals. Terminals may also be points of interchange involving the same mode of transport, which ensure a continuity of the flows but they are also important points of transfer between modes.

Three major attributes affect the performance of terminals:

- **Location:** The key factor of a transport terminal is obviously to serve a large concentration of population and/or industrial activities, representing a terminal's market area. Specific terminals have specific location constraints, such as port and airport sites. New transport terminals tend to be located outside central areas to avoid high land costs and congestion.

- **Accessibility:** Accessibility to other terminals (at the local, regional and global scale) as well as how well the terminal is linked to the regional transport system. For instance, a maritime terminal has little relevance if it is efficiently handling maritime traffic but is poorly connected to its market areas through an inland transport system (rail, road or barge).

- **Infrastructure:** The main function of a terminal is to handle and transship freight or passengers since modes and passengers or cargo are physically separated. They have a physical capacity which is related to the amount of land they occupy and their level of technological, labor and managerial intensity. Infrastructure considerations are consequently important as they must accommodate current traffic and anticipate future trends and also technological and logistical changes.

Terminal costs represent an important component of total transport costs. They are fixed costs that are incurred regardless of the length of the eventual trip, and vary significantly between modes. They can be considered as: (i) **Infrastructure costs:** Include construction and maintenance costs of structures such as piers, runways, cranes and facilities (warehouses, offices, etc.); (ii) **Transshipment costs:** The costs of loading and unloading passengers or freight; (iii) **Administration costs:** Many terminals are managed by institutions such as port or airport authorities or by private companies (e.g., terminal operators). In both cases administration costs are incurred.

Terminal costs play an important role in determining the competitive position between the modes. Because of the high freight terminal costs, ships and rail are generally unsuitable for short-haul trips. Competition between the modes is frequently measured by cost comparisons. Reduced terminal costs would have a major impact on transportation system cost and the whole international trade. Due to that fact, the need for optimization using methods of operations research in container terminal operations has become more and more important in recent years. This is because the logistics especially of large container terminals has already reached a degree of complexity that further improvements require scientific methods.

2.3.1 Container port terminals

A container port terminal provides transfer facility for containers among sea vessels and land transportation modes. Two interfaces make it up in order to guarantee a smooth flow of containers: The first one is the **quayside** where loading and unloading of ships take place, and the **land side** where containers are loaded and unloaded on/off trucks and trains.
Containers could be transshipped directly or can be stored in a storage area for facilitating the decoupling of quayside and land side operation. Figure 2.3 shows the operations areas in a seaport. Three main types of handling operations are performed at port terminal:

- Ship operations associated with berthing, loading and unloading container ships.
- Receiving/delivery operations from outside trucks and trains.
- Container handling and storage operations in the yard.

After its arrival in the port, the ship is assigned to a berth equipped with quay cranes able to load and unload containers. Different types of ships have to be served at the quayside, such as deep sea vessels and feeder vessels. Berth space is a very important resource in a container terminal and berth scheduling determines the berthing time and position of a container ship at a given quay. Quay-crane allocation is the process of determining the vessel that each quay crane will serve and the associate service time. Stowage sequencing determines the sequence of unloading and loading containers, as well as the precise position each container being loaded into the ship is to be placed.

During the unloading operation, a Quay-crane transfers a container from a ship to a transporter. Then, the transporter delivers the inbound (unloaded) container to a yard crane that picks it up and stacks it into a given position in the yard. This sequence of operations is called indirect transfer. Some terminals use a direct transfer system where the equipment used to move containers between the quay and the yard will also stack them.

For export (loading) operation, the process is carried out in the opposite direction. On the land-side, the receiving and delivery operations provide the interface between the container terminal activities and the external movements. A receiving operation starts when containers arrive at the gate of the terminal carried by one or several outside trucks or a train. When the outside truck arrives at the indicated transfer point, a yard crane lifts a container from the truck and stacks it according to the plan.

When containers arrive by rail, the rail cars are brought in the rail area where containers and documents are examined. Containers are then transferred by a gantry crane to a transporter, which delivers them to the yard and stacks them. The sea-side and land-side operations interact with the yard container handling and storage operations through the information on where the containers are or must be stacked within the yard. How containers are stored in the yard is one of the important factors that affect the turnaround time of ships and land vehicles. The space-allocation problem is concerned with determining storage locations for containers either individually or as a group.

Yard storage space is pre-assigned to containers of each ship arriving in the near future to maximize the productivity of the loading and unloading operations. Container handling and storage operations include the management and the handling of containers while they are in the storage space of the yard and thus occur between the receiving and delivery operations and the ship operations. Container-handling equipment performs the placement of containers

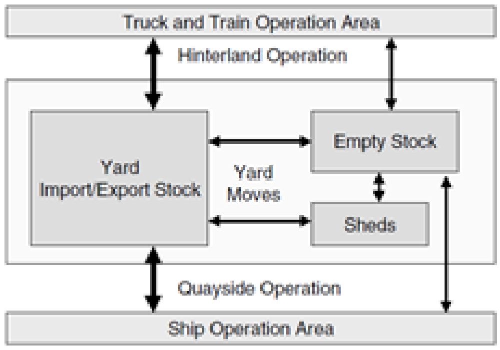

Figure 2.3: Operation areas of a seaport container terminal and flow of transport. Source: Steenken et al. (2004).

into storage and their retrieval when needed. Yard cranes move along blocks of containers to yard bays to perform these operations. Planning these operations is part of the equipment-assignment process, which allocates tasks to container-handling equipment. Based on the quay crane schedule, one or two yard cranes are assigned to each quay crane for loading and unloading. The remaining yard cranes are allocated to receive and delivery operations. Terminal operators aim to assign and operate yard cranes in such a way that inefficient moves and interference among yard cranes are minimized. Figure 2.4 shows an example of container terminal.

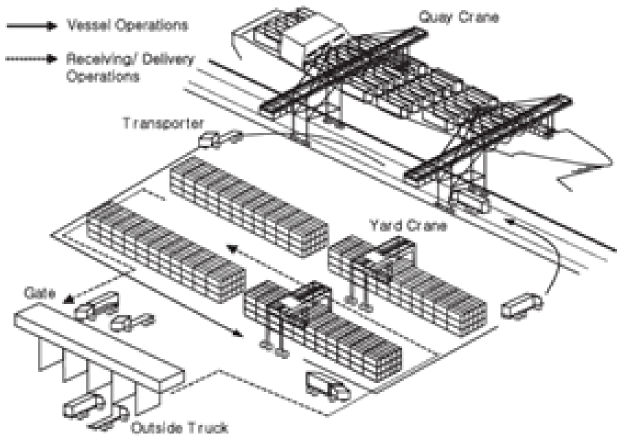

Figure 2.4: Example of a container terminal with an indirect transfer system. Source: Crainic and Kim (2007)

For the planning issues proper only of a maritime terminal, a fully description of the terminal operations and a wide literature review we refer to Steenken et al. (2004), Vis and De Koster (2003), and Crainic and Kim (2007), while for the planning issues of the land side of a maritime terminal, we refer to the ones that we cite in the following section.

2.3.2 Railway terminals

A railway yard is a special transshipment node in a rail network where loads for trains are processed (collected, rearranged, unloaded, stored, loaded, picked up, etc.). It can be a terminal itself or it can be located in (or nearby) a seaport for moving freight with the hinterland. In this section we describe the basic structure of rail yards. For a broader literature review, we refer to Boysen et al. (2013).

Usually, an intermodal freight rail yard serves at least one of two main purposes in the railway system:

- A terminal can serve as an interface in intermodal transport, so that shipment can be interchanged between the rail system and another alternative mode of transportation (trucks, ships). Typically trains operate long-haul routes, while trucks act as link with the end nodes of an intermodal network, and then act on a short-haul transportation. So, a container that, for instance, must be moved from an origin to a destination where there is not a direct rail system, will be moved before by truck, that in general serve customers on the last mile, and then, once a railway terminal has been reached, it will be loaded into a train that will bring it to its destination.

- A terminal can be a hub node in a hub and-spoke network, so that containers are exchanged between different trains. Economies of scale are generated because of the consolidations of several blocks, that represent short groups of railcars having loads with different destinations grouped into few very long trains, which share part of the intermodal trip. Again, railway transport oversees the long haul routes because of very high fixed costs.

According to Boysen et al. (2013) three different kinds of yards have been established, depending on their generation (here we include yard types that are not limited to intermodal traffic): In traditional classification yards (first generation shunting yards, see Figure 2.5) trains arrive onto a receiving tracks, railcars are typically pushed over a ramp (so-called hump), which redirects them toward classification tracks, where they are switched and addressed to departure tracks to outbound trains.

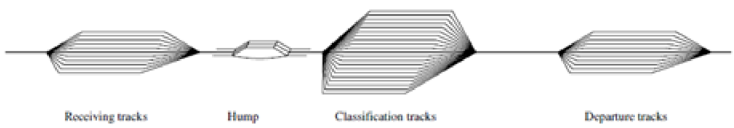

Figure 2.5: Classification yard. Source: Boysen et al. (2013)

Second generation railway yards are the traditional rail-roads terminals, where trains usually keep the railcars and only containers are moved by means of huge gantry cranes, that span several parallel tracks. In this type of yards, often, additional elements are present, such as areas for intermediate storage of containers and adjacent truck lanes for immediate transshipment from trains to trucks and vice versa. The main purpose of such yards is to serve as an interface between different modes of transportation. Lastly, third generation railway yard, which have a similar layout compared to second generation railway terminal, but are typical in a modern rail-rail transshipment, where a fully automated sorting system is employed, instead of conventional floor storage. Those kinds of terminal are still in a design phase, and hence we mainly focus on second generation railway yards with some mentions to the third generation terminal issues, since, instead, shunting yards are fundamentally different in structure and operations, and by the way are rarely part of modern container-based rail network.

A rail terminal, as already underlined, serves as interface of exchange between different transportation modes. Such terminals often feature a holding zone for trucks, a gate, and an office area for controlling access into and out the terminal. Trucks connecting the terminal with the hinterland arrive on parallel truck lanes, divided into driving and parking lanes, bringing into the terminal or out of the terminal, containers which have been moved by train. Trucks may arrive respecting some schedules or certain booking windows. For a given container, if its outbound train is not being loaded as the container arrives, it is moved to a storage area.

Trains are parked into parallel transshipment tracks of the terminal that cranes or reach stacker trucks can reach for loading and off-loading activities.

A terminal manager coordinates the activities in the three main interacting components of the terminal: rail track operations, storage yard operations and gate operations. Since a delay related to any type of operation affects the others, the objective of a terminal manager is to ensure that all the operations run as effectively as possible. For instance, a problem with storage yard operations can create delays both at the rail track and gate operations and have an impact on the terminal productivity and the quality of its services.

A real world example of a large-scale intermodal terminal is the BNSF Logistics Park Terminal, in the suburbs of Chicago (60 km southwest), which represents the largest intermodal terminal in North America (Figure 2.6). It is a typical example of contemporary intermodal terminal which includes all the components of a modern intermodal site in a typical rectangular shape. In addition to the common elements that are the intermodal yard, namely the storage area, chassis depots and the access gates (separate entry and exit locations), it also include classification yards and a car terminal.

Figure 2.6: Chicago rail terminal. Source: Google Earth.

2.3.3 Planning problems

Planning problems occurring at intermodal rail terminals can be divided into two categories: infrastructure planning (e.g., design) and operational planning (e.g., resource management) (Boysen et al., 2013).

Design. The first type of problems concern the long-term strategic planning intended to answer questions such as which elements should compose the infrastructure, and how to integrate them into the system, in order to make the existing operations more efficient (Boysen et al., 2013).

During the design phase of a terminal, critical decisions in regard to the dimensions of each terminal elements are made. For instance, the number of holding and transshipment tracks, the capacity of the storage and parking area, the number of technology and gantry cranes need to be determined. All these choices are interdependent and influence the yard performance.

Some planning decisions connected with railway terminals are:

- *Infrastructure layout:* It is the problem of configuring terminal infrastructure. Reconstruction activities may comprise the reduction, exchange or amplification of tracks, retarders or safety equipment, exchange or amplification of storage yard.

- *Resource type and dimensioning:* The problem is to find the type of equipment and the optimal number of resources to be used in the terminal. This is a strategic level decision, because the horizon in which the decision has an impact is about few years.

The existing literature on layout planning mainly consists of simulation studies, in order to anticipate yard performance for different terminal layouts. Within the existing literature on the broad range of issues regarding the layout of the terminal, refer to Boysen et al. (2013).

Operational problems. Once the layout of the terminal is fixed, several problems at operational level arise. Timetable of the trains is supposed to be fixed, and chosen at tactical level considering the whole network, hence it will not be accounted in the study of a single terminal. But the terminal manager, considering trains that arrive and leave the yard following a given schedule, needs to manage the flow of containers and railcars.

An operational problem worth of study is the assignment of an inbound train to a parking position in the yard. Any parking position in the yard is characterized, according to Boysen et al. (2013), by a vertical and an horizontal coordinate, and it must be assigned to any train that enters the transshipment yard. Firstly, the train is assigned to a vertical parking position of the yard, which relates to the actual track on which the train enters the yard, and then, an horizontal parking position of a train is assigned too, which refers to the slot in which the traction vehicle is positioned.

As soon as a train is parked, the unloading of all inbound containers can begin. Normally, some of the trucks that must take inbound containers are already in the yard, waiting for the unloading operations. In such cases, it can be preferable to transship directly a container from the train to the truck, rather than incurring in a double handling movement of moving it before to the intermediate storage area of the yard.

If a truck is waiting in the holding area, it is assigned to a parking position. This is normally a parking lane just beside the respective target railcar. This allows crane (or reach stacker) to move a container from the railcar to the truck directly.

If a container must be subject to a double handling, which is normally referred as a split move, it requires that a storage location close to the railcar is assigned to it, with the goal to reduce the crane operating time. The objective in this case, is to make the retrieval of that container as easy as possible once the truck arrives to the terminal. This could be done by organizing the containers based on updated information of the arrival times of the trucks, which could avoid future additional handlings of any blocking containers.

The processing of outbound operations is carried out similarly to the inbound operations but in a reversed order. Whenever a truck that is transporting an outbound container reaches the terminal, depending if the target train is already on the track or not, the truck is directed either to the storage area or besides one of the railcars of the target train, ready for a direct transfer.

An operational planning issue in this case is to determine, for each outbound container that reaches the terminal, which is its ideal position in the yard, in order to minimize the reshuffles of whole set of containers presented in the yard.

The aforementioned operational problems related to outbound containers are intrinsically connected to the load planning problem. The latter defines a plan identifying where to place each container on the available (double-stack) outbound railcars. This should be done such that crane handling costs are minimized while maximizing the utilization of available capacity and respecting a number of constraints. This thesis focuses on the load planning problem, central to effective operations at intermodal rail terminals.

Chapter 3

The load planning problem for double-stacked intermodal trains

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Author contribution

This chapter is based on a project in collaboration with the Canadian National Railway Company. The work involved everything from mapping the decision processes at a terminal, documenting the operational constraints and objectives to mathematical modelling the problem and validation of the solutions with the operations at the company. Given the extent of the work, several people were involved over a quite long period of time. I was involved in all aspects of the scientific work: literature review, mathematical modelling, the design of the experimental results, the generation of results and the documentation of the work.

Abstract

This paper presents a general methodology that addresses the load planning problem for intermodal trains. We propose a model that can deal with single- or double-stack railcars as well as arbitrary containers-to-cars matching rules. Moreover, we model weight and center of gravity constraints, stacking rules and technical loading restrictions associated with specific container types and/or contents. We propose an integer linear programming (ILP) formulation whose objective is to choose the optimal subset of containers and the optimal way of loading them on outbound railcars so as to minimize the resulting loading cost. An extensive numerical study is conducted. It shows that ignoring center of gravity constraints and containers-to-cars matching rules may lead to an overestimation of the train capacity and to select load plans that are not feasible in practice. We also show that we can solve realistic instances to optimality in reasonable computational time using a commercial ILP solver.

3.1 Introduction

Nowadays, an essential ingredient of a competitive economy is a cost-effective freight transportation system. Intermodal transportation is an important component of this system in which different transport modes are linked in order to move freight from a point of origin to a point of destination. Taking advantage of economies of scale, low volume demands are first shipped to an intermediate point, a consolidation terminal or hub, where traffic is sorted (classified) and grouped (consolidated). Then, the consolidated traffic is moved between hubs by efficient transport modes. In this paper we deal with intermodal railway transportation where containers are consolidated and transported by trains on the long-haul part of their trip. We focus on the North American market and on double-stack trains.

Intermodal transportation relies heavily on containerization because, in addition to decreasing transportation cost, it ensures faster and safer handling as well as transfer between transport modes. Intermodal containers are steel frame boxes designed to move goods across the world using different transport modes without any re-handling of the cargo. Since 2005, the containerized worldwide traffic has increased from 382 to 684 million of TEU (Twenty Foot Equivalent Unit) (CBRE Research, 2015) and, since 1990, North American ports have seen container traffic grow by an annual average of 5.3% (International Association of Ports and Harbors, 2015). This growth is placing a heavy burden on the entire consolidation-based transportation system, which must provide efficient, reliable and cost-effective services.

Terminals are major components of any intermodal transportation system and thus are critical to the entire international trade. They are special transshipment nodes that provide equipment and space where containers are processed, loaded, unloaded and stored to ensure a seamless transfer between different modes. Carriers, in our case railways, face a number of challenging planning issues, which may be examined according to the classical categorization with respect to the planning horizon, that is strategic, tactical, operational. In this study, we focus on the *load planning problem*, which is an operational problem arising at intermodal railway terminals.

Given a set of containers stored in a terminal and a sequence of railcars, the problem is to determine the optimal subset of containers to load and the exact way of loading them on an optimal subset of railcars while minimizing cost. We address this problem for double-stack trains. This is a challenging problem because the load plan must satisfy a number of complex loading rules that depend on specific container and railcar characteristics. For example, stacking rules depend on container sizes, weights, and contents and on Center Of Gravity (COG) restrictions. While the methodology expounded in this paper is general, the North American market is the main focus of our attention because it is particularly challenging. Indeed, there are in North America a large number of railcar types and several more container types and containers than the standard 20 ft (feet) and 40 ft.

As we detail in Section 3.3, with one exception, the existing literature does not address the load planning problem for double-stack trains. Moreover, the simplifying assumptions that are adopted may lead to load plans that violate important loading rules and hence cannot be used in practice. For example, none of the studies model the COG restrictions. The objective of this paper is to propose a general methodology that addresses the load planning problem of double-stack trains taking into account all the different loading rules encountered in actuality.

There are a large number of possible ways – so-called *loading patterns* – in which containers of different sizes may be loaded onto a railcar of a particular standardized type. The multitude of railcar types and the very large cardinalities of several of the associated sets of loading patterns is a key issue. We refer to this problem as *containers-to-car matching*. In connection with this problem, we make a number of contributions. First, we propose a general model that can deal with single- and double-stack railcars that can be of different types and subject to different loading rules. Second, our model accounts for additional loading constraints related to the specific container types, contents and weights as well as to COG restrictions. Third, we present an extensive set of numerical results based on a case study focusing on the North American market.

The numerical results indicate that our model provides an appropriate framework for solving very large instances of the load planning problem in reasonable time using a commercial solver.

They also demonstrate that failing to account for containers-to-cars matching as well as COG and stacking restrictions may lead to overestimations of the usable capacity and to suggesting load plans that are not applicable in practice.

The remainder of the paper is structured as follows. Section 3.2 describes the load planning problem in detail. Section 3.3 is dedicated to a review of the existing literature on the assignment of containers to railcars and to highlight our main contributions. Section 3.4 presents the ILP formulation of the load planning problem. Section 3.5 describes the content of the empirical study and examines its results. Finally, Section 3.6 draws conclusions and discusses possible directions for future research.

3.2 The double-stack train loading problem

This section presents a detailed description of the load planning problem for double-stack trains. We examine the ways in which containers and railcars can be physically matched together and explain how these loading possibilities depend on the exact characteristics of the containers and railcars. We start by successively describing the intermodal containers and the rules for stacking them as well as the intermodal railcars. We then present the rules governing the loading of containers onto railcars.

3.2.1 Intermodal containers

Intermodal containers are characterized by (i) their size (length and height) (ii) their type (iii) their contents and (iv) their weight, filling level and weight distribution. In order to facilitate their handling, sizes are standardized. There are four ISO standard sizes used worldwide: 20 ft high cube, 40 ft low and high cube, 45 ft high cube (the height of low cube containers is 8 ft 6 in / 2.6 m whereas it is 9 ft 6 in / 2.9 m for high cubes). This paper focuses on the North American market, where there are two additional sizes of high cubes: 48 ft and 53 ft.

Figure 3.1: Examples of container types

For each size, containers are available in several standardized types. Some are illustrated in Figure 3.1. Ninety percent of the global fleet consists of general purpose containers, called “dry containers”, that are steel frame boxes with 6 solid sides (upper left in the figure). Several other types of containers are designed to transport goods for which dry containers are not suitable. For instance, reefers (refrigerated containers) or heated containers are designed to carry goods needing temperature control (bottom left in the figure). During transport, the reefers can either be connected to a genset (power generator set) supplying electrical power to a number of them or can have individual power units. Figure 3.1 also shows an open/soft top container without a roof (upper right), an open-side container and a tank container for the transportation of liquids (bottom right). While the designs of these containers are different, their sizes remain standard. Containers can carry hazardous materials in which case special restrictions usually govern their storage and transport.

Containers can be stacked one on top of another. In addition to rules governing the weights and the positioning of containers loaded onto railcars, the stacking of containers must conform to rules prescribing their relative position. In essence, the containers must be positioned so as to ensure that their load is transferred in accordance with the design of their steel frames. Specifically, the container above can be connected to the container(s) below with four Inter Box Connectors (IBC) designed for this purpose and the standard lengthwise distance between the connecting points where these couplings can be installed is 40 ft. This is illustrated in Figure 3.2 where the thick lines indicate this 40 ft distance. Hence, a 40 ft container can be loaded on top of two 20 ft but a 20 ft container cannot be loaded on top of a 40 ft one. Since the connecting points are symmetrically located from the mid-length of the containers, a longer container (45, 48, 53 ft) must be centered on top of a shorter one (40, 45, 48 ft) or on top of a pair of 20 ft containers.

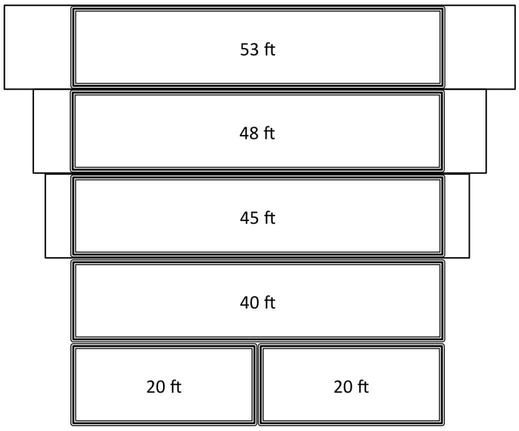

Figure 3.2: Container stacking at 40 ft distance

Lastly, we assume that there exists a per container cost associated with the failure to load an available container standing for, e.g., customer penalties for late arrival and storage costs in the terminal.

3.2.2 Intermodal railcars

Intermodal trains consist of a sequence of railcars designed to carry single- or double-stacked containers. Intermodal railcars are characterized by their number of platforms and by the length, weight-carrying capacity and tare weight of each one.

Figure 3.3 illustrates a five-platform double-stack railcar. In accordance with the North American industry standard, the front platform is named A, the rear B and the other platforms C to E from front to rear. Similarly, the platforms of a three-platform railcar are named A, C, B from front to rear, and so forth. Each double-stack platform has two slots: bottom and top.

Costs are associated with the operation of a train and are incurred in the acquisition and maintenance of the locomotives and railcars, in purchasing fuel and employing crews (see e.g. Bouzaiene-Ayari et al., 2014). We hence assume that there is a cost associated with leaving slots empty on outbound railcars.

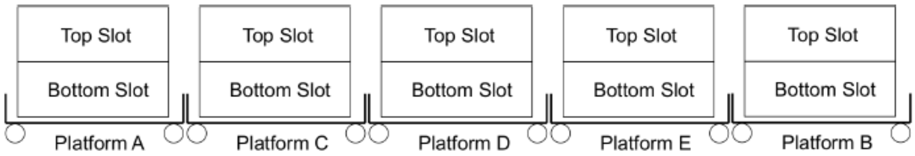

Figure 3.3: Five-platform double-stack railcar

3.2.3 Loading containers on railcars

Trains are composed of blocks where a block in this context is a group of railcars that move between an Origin and Destination (OD) pair of terminals without being reclassified. The purpose of grouping railcars, with different OD terminal pairs, into blocks is to minimize the transfer from one train to another or the classification of individual railcars at intermediate terminals. The block plan is a tactical decision problem and the operational load planning problem is solved separately for each block.

Containers arrive to an intermodal rail terminal by trucks or by vessels. Upon their arrival, the containers are either classified according to the block on which they will travel and stored in the yard, or directly loaded on outbound railcars of this block. Since containers can arrive shortly before, or even during the loading operations, load plans must be computable in a short time (preferably within a few minutes).

The assignment of containers to slots must conform to a number of rules that depend on the characteristics of the railcars and the containers. We start by describing the rules that pertain to *container size* only. We refer to them as *containers-to-cars matching rules*. For the North American market, the *AAR Guide* (Association American Railroads, 2014) provides for each listed series of railcars a complete description of the combinations of container lengths that can be loaded in the bottom and top slots of each platform. Except for the bottom slots that can generally accommodate up to a pair of 20 ft containers placed end-to-end, slots can receive at most one container. Table 3.1 provides an illustration for a five-platform railcar series. The second block of rows is excerpted from the AAR Guide. It prescribes for each one of the five platforms (A-E) which container sizes can be loaded in the bottom and top slots respectively. Each row in the third block states one particular loading possibility, i.e. a *loading pattern*, satisfying the prescriptions of the second block.

The platform length for this series of railcars is 40 ft whence the bottom slots can accommodate one or two 20 ft containers ($2-20'$ in the table) or one 40 ft ($1-40'$ in the table). The load in the top slot must conform to the stacking rules and to the space available. The space between platforms can in some cases be sufficient to allow the loading in the top slot of a container exceeding the length of the platform. This is examplified in Table 3.1 where each top slot can accommodate a 40 ft, a 45 ft or a 48 ft container and where the space between the platforms allows to load 53 ft containers in the top slot of platforms A, D and B, provided there is a 40 ft container or no container at all in top slots of platforms C and E (see table footnote). Crucially, these joint requirements imply that the containers-to-cars matching rules cannot be described for each platform separately.

Slots may also be left empty and an upper slot can be filled only provided the slot below is filled. Moreover, a top slot cannot be filled if there is a single 20 ft container loaded in the slot below. There are clearly several different ways in which to load a five-platform railcar so as to satisfy the loading capabilities stipulated in the AAR Guide and the lower rows of Table 3.1 exemplify a very small number of them. For example, the last row describes a loading pattern where a 40 ft container is placed in every slot except in the top one of platform E. The latter is left empty.

Bottom slot					Top slot				
A	*C*	*D*	*E*	*B*	*A*	*C*	*D*	*E*	*B*
AAR Guide									
$2-20'$	$2-20'$	$2-20'$	$2-20'$	$2-20'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$
$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-45'$	$1-45'$	$1-45'$	$1-45'$	$1-45'$
					$1-48'$	$1-48'$	$1-48'$	$1-48'$	$1-48'$
					$1-53'^{(*)}$		$1-53'^{(*)}$		$1-53'^{(*)}$
Some examples satisfying AAR Guide									
$2-20'$	$2-20'$	$2-20'$	$2-20'$	$2-20'$	$1-48'$		$1-40'$		$1-45'$
$1-40'$	$2-20'$	$1-40'$	$2-20'$	$1-40'$	$1-45'$	$1-40'$	$1-53'$	$1-40'$	$1-53'$
$2-20'$	$1-40'$	$2-20'$	$1-40'$	$1-40'$	$1-48'$	$1-45'$	$1-48'$	$1-45'$	$1-48'$
$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$	$1-40'$		$1-40'$

Table 3.1: Example of AAR Guide railcar series BN 63900 - 63909 type IBC 100 tons (*): 53 ft containers in top slot of platforms A, D and B only when a 40 ft container or none is loaded in top slot of platforms C and E.

The assignment of the containers to the slots of the railcars is conditioned by the weights of the containers and the weight-carrying characteristics of the railcars. There are two main loading restrictions with respect to the weight of the load on a platform. First, the total weight of the containers loaded on a platform must be smaller than the weight capacity of the platform. Second, a condition pertaining to the height of the center of gravity must be satisfied. This expression is used in the North American railway industry with a meaning identical to that of the expression *center of mass*. Although it designates more generally the mean location of a distribution of mass in space, it is defined in the context of railway operations as the mean location of mass along the vertical axis of a platform. The AAR Guide states "*…The COG for a double-stack car and the load in the platform must be less than or equal to 98 inches at top of rail. Reference Rule 89, Section C.2.e. in the AAR Field Manual*". A failure to obey this rule would imply practically that the container placed in the top slot of the platform is too heavy in comparison with the weight of the container(s) placed in the bottom slot. This situation would be viewed as a risk factor to a derailment. While the actual COG depends on the filling level and the load distribution in the containers, the COG restriction stated in the AAR Guide relies on the assumption of a uniform weight distribution. In the case of a solid body with uniform weight distribution, the center of mass is the same as the centroid of the body. In this paper we follow the AAR definition of COG.

The COG restriction is expressed as an upper bound on the weight of the container in the top slot, given the characteristics of the container in the bottom slot. Figure 3.4 provides an illustration for a single platform. There are three solid bodies: the platform p, the bottom container i and the top container i'. Their centroids are illustrated with black dots and the associated heights from the top of the rail are denoted m_p, m_i and $m_{i'}$, respectively. The bottom and top containers are connected with IBCs. Under the assumption of a uniform weight distribution, the height of the COG m for the three solid bodies is

$$m = \frac{m_p g_p + m_i g_i + m_{i'} g_{i'}}{g_p + g_i + g_{i'}} \tag{3.1}$$

where g_p is the platform tare weight and g_i and $g_{i'}$ are the weights of the bottom and top containers respectively. According to the AAR Guide, $m \leq M$ where M equals 98 in (2.5 m). Using (3.1) and M it is possible to compute a maximum weight c for the top container. By rearranging (3.1) and using M instead of m and c instead of $g_{i'}$ we obtain

$$c = \frac{g_p(M - m_p) + g_i(M - m_i)}{m_{i'} - M}. \tag{3.2}$$

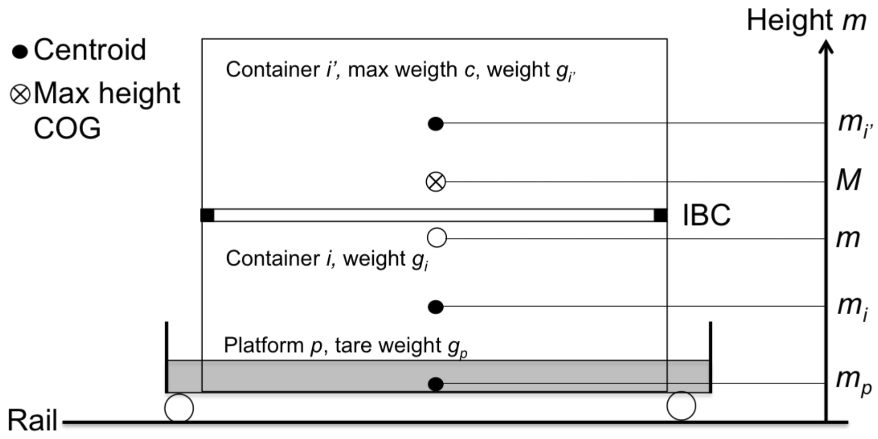

Figure 3.4: COG restriction

Containers exist in a diversity of types carrying a diversity of contents and rules are attached to particular combinations of types and contents. These rules give rise to a number of additional constraints in the container loading problem. For the North American market we have identified six *technical loading restrictions* that apply to certain types of containers and contents or to combinations thereof:

1. Loading is restricted to railcars having a given minimum weight-carrying capacity (independently of railcar series). This restriction applies to containers whose weight is above a certain threshold and needs to comply with additional restrictions not captured by the weight-carrying capacity of a platform.

2. Loading is restricted to certain positions in the sequence of railcars (e.g., hazardous material).

3. Loading is restricted to high weight capacity railcars (only certain railcar series).

4. Loading in top slot is forbidden.

5. Loading in top slot and double stacking is forbidden.

6. Loading must be on a platform within a maximum distance from a specific container (e.g., for the reefers that must be connected to a genset).

This set of technical loading restrictions is sufficiently general to cover the specificities that we have identified thus far in relation with the North American railways. Additional technical loading restrictions might have to be defined in order to reflect new or presently unknown railway policies or country regulations.

In summary, we focus on the load planning problem: Given a set of containers stored in a terminal, a sequence of railcars, and the relevant constraints, determine the subset of containers to load and the exact way of loading them. The objective is to minimize the cost of unloaded containers and the cost of empty slots. A key performance indicator currently used to measure the efficiency of a load plan is the *slot utilization*, which measures the percentage of the available slots on the railcars that are occupied in the load plan (Burriss, 2003). We note that we focus on a deterministic setting, and that we do not model the different handling costs associated with retrieving containers in the terminal. Our goal is to develop a general methodology, which can be used within a decision support tool that provides load plans to decision makers. We deal with all the loading rules and restrictions that arise for double-stack trains, by taking into account the multitude of containers and railcars types that exist in the North American market.

3.3 Literature review

The load planning problem may be viewed as a special case of the packing-cutting-knapsack problems (Martello and Toth, 1990; Dowsland and Dowsland, 1992; Dyckhoff et al., 1997; Wäscher et al., 2007). The goals and the associated models are different in most cases, however. For example, in two- (Lodi et al., 2002) and three-dimensional packing (Crainic et al., 2008) one faces a much larger number of items than the number of available (or desirable) loading units (bins) and the dimensions of the items span a broad range of values from tiny to almost as large as the bin. One then focuses on identifying the "best" bin and the "best" position in the bin to load all items in as few bins as possible. In the rail load planning problem, on the other hand, bins - the railcar platforms - and the items - the containers - are fundamentally of similar dimensions, the positioning being determined by the physical configurations of both. The goal is then changed from packing as many items as possible into as few bins as possible to identifying the best combination (assignment) of given container dimensions and weights to the available railcars given technical loading constraints (e.g., total weight and COG). The cutting/packing setting closest to the problem we address is identified as the multiple identical large object placement problem by Wäscher et al. (2007), where the multi-platfom railcars would be the more or less identical large bins, while the heterogeneous fleet of containers would correspond to the set of large items. There were no contributions to this problem class when the classification of Wäscher et al. (2007) was published and we are not aware of any more recent ones either.

COG and load balancing concerns also arise when planning the loading of vehicles for other freight transportation modes, e.g., trucking, sea and air transportation. Each transportation mode has its own vehicle and operation characteristics, resulting in particular forms of these general restrictions. For example, the axle weight restriction for trucks may result in particular requirements for weight distribution when loading the containers before even the ocean segment of their trip (Lim et al., 2013). The distribution of weight, and thus of containers, is of capital importance for the stability of ships and airplanes. The COG of the vehicle thus becomes a hard safety constraint in ship stowage (Steenken et al., 2004; Stahlbock and Voß, 2008) and airplane (Mongeau and Bes, 2003), but while the number of container re-handles (at intermediate stops) is generally not relevant in the latter case, it is an element to be taken into account in the former case (Imai et al., 2006).

We open this overview of the literature relevant to the rail load planning problem by pointing to two surveys whose scopes extend to intermodal freight transportation activities in general: See Crainic and Kim (2007) for the planning of intermodal carrier and terminal operations, and Carlo et al. (2014) for transportation activities in container terminals. Several studies focus on the train blocking problem (e.g., Bodin et al., 1980; Newton et al., 1998; Barnhart et al., 2000). For general views on the rail load planning problem per se, see Heggen et al. (2016) for a recent classification of the existing literature and Boysen et al. (2013) for a comprehensive overview of the planning issues that arise specifically in railway yards, including the load planning problem.

Specifically in connection with the rail load planning problem, Feo and Gonzalez-Velarde (1995) made the first contribution and, later on, Powell and Carvalho (1998) dealt with the problem of balancing the flat cars over a network from a load planning perspective. Similarly to these two studies, most contributions in the literature addressed the simpler single-stack load planning problem where the set of matches between container and railcar combinations is smaller than the double-stack one. The existing literature examined simpler settings than that of this paper, accounting mainly for limits on axial and total train weight. It has generally focused on objectives related to, e.g., minimizing handling costs in the yard (e.g., Corry and Kozan, 2006, 2008; Ambrosino et al., 2011; Ambrosino and Siri, 2015) or train set-up costs (Bruns and Knust, 2012; Bruns et al., 2014), rather than optimizing the capacity made available by a given train or block as in this paper.

The authors focusing on single-stack loading deal with load planning at different degrees of detail. For example, Corry and Kozan (2008) consider matching different container and railcar types, while Corry and Kozan (2006) do not. Bruns and Knust (2012) extend the former work by considering both the matching problem between containers and railcars and the weight constraints. Heggen et al. (2016) build on the latter and integrate a number of practical loading constraints. Ambrosino et al. (2011) and Ambrosino and Siri (2015) minimize re-handling in the yard and unproductive movements of cranes. Anghinolfi et al. (2014) consider several container lengths and possible railcar (platform) loading combinations to accommodate them, combined to axial and train weight restrictions. For single-stack loading problems, Dotoli et al. (2015) consider issues often addressed during previous planning processes (e.g., block planning) such as the positioning of the loaded cars within the train and their transfer from one train to another. Besides the weight-related restrictions, the authors also address the so-called commercial value of the train measured by the priority and, possibly, the value of the containers. Dotoli et al. (2017) extends this work to a decision-support system, which also addresses the management of the containers in the yard. Finally, Bruns et al. (2014) consider several sources of uncertainty (regarding, e.g., weights, lengths and equipment failures) in a robust optimization approach.

The aforementioned studies focus on single-stack trains, and on the main challenges associated with optimizing yard or transport operations rather than on train loading. We study the operational problem of loading double-stack trains in a context where the tactical train and block plan, i.e., what trains are operated, what sequence of trains moves each block, and what block moves each container (loaded on a car) from its origin to its destination, were previously constructed. Loading double-stack trains is a difficult problem taking place in a complex setting. On the one hand, it requires considering the matching between a multitude of different railcar (platform types and configurations) and container types, while putting containers on top of other containers. On the other hand, we enforce a good number of technical constraints, in particular the COG restrictions and stacking rules. We therefore focus on the train (block) load planning problem, assuming the cars making up the train (block) are given, as well as the containers to load.

To the best of our knowledge, the first contributions to the double-stack loading literature aimed for automatic heuristic rules and procedures. Pacanovsky et al. (1995) embedded such procedures into a simulation-based decision-support system. Lai, Barkan and Önal (2008) is the first optimization study on the double-stack load planning problem. Similarly to this study, they also ignore handling costs. Their focus is on minimizing the aerodynamic drag of double-stack trains that depends on the gaps between containers and the location of these gaps along the train. They present an integer linear programming formulation, but they make a number of simplifying assumptions. First, they address the matching among containers and railcars types, deriving the loading patterns without considering the possible platform dependencies. This implies that loading rules can be defined for platforms independently (Table 3.1 shows an example where this assumption is invalid). Second, they ignore the possible dependencies between the loadings of the individual railcars in the sequence (such dependencies are introduced by technical restrictions, e.g., the requirement that reefers must be loaded in close proximity of the genset supplying the required power). These dependencies make it inappropriate to define loading rules over each platform or each railcar independently. Third, they study the problem without accounting for COG and technical restrictions. The authors extend the model to a rolling horizon setting and show that one could improve the loading by considering several trains at a time. Lai, Ouyang and Barkan (2008), Lang et al. (2020) consider COG concerns within the study of a limited number of containers-to-cars configurations based on the case of Chinese rail. Detailed formulas are developed for each configuration and are embedded into a multi-objective formulation.

As this literature survey illustrates, there currently does not exist a comprehensive optimization model for the double-stack train loading problem considering a realistic set of constraints and a broad range of container and railcars types. We present such a model in the next section.

3.4 Mathematical formulation

A realistic load plan must comply with the applicable set of loading patterns, weight and COG restrictions, stacking rules and technical restrictions. This section presents an Integer Linear Programming (ILP) formulation of the load planning problem whose objective is to maximize slot utilization. This is accomplished by minimizing an appropriately weighted sum of the cost of containers that are not loaded and the cost of railcars that are used for loading. We open this section with detailed explanations, first, of the mathematical structure describing the containers-to-cars matching rules and, second, of the COG constraints. Next, we provide a detailed description of the full ILP formulation.

3.4.1 Modeling containers-to-cars matching

We model the containers-to-cars matching through *loading patterns*. A loading pattern describes a feasible assignment of container lengths to the slots of a railcar. Whereas Corry and Kozan (2008) and Lai, Ouyang and Barkan (2008) also use loading patterns, the main difference here lies in the fact that we account for dependencies between the loadings of the platforms on a railcar. As illustrated by the example shown in Table 3.1, accounting for these dependencies is important but leads to an exponential growth in the number of loading patterns as the number of platforms increases.

Let H be the set of standard container lengths in feet. In our case, $H = \{20, 40, 45, 48, 53\}$. A loading pattern $k \in K_j$ is a n-tuple that specifies the total number of containers of each length $h \in H$ that can be loaded on each platform of a given railcar $j \in J$. We show an example of a one-platform railcar in Table 3.2. Each row corresponds to a loading pattern and there is a total of 11 possible patterns $|K_j| = 11$, including empty slots but excluding an empty railcar. The first row shows, for example, that the platform can hold one 20 ft container, the second, two 20 ft containers, the third, one 40 ft container and so forth. Notice that the loading patterns do not indicate the slots in which the containers can be loaded, only the number per platform. This information may be inferred from the content of the platform based on stacking rules. When a railcar consists of several platforms the n-tuples are concatenated from left to right. For example, a particular pattern for a railcar comprising three platforms is described by a 3-n-tuples concatenation. The set of loading patterns K_j is composed of all feasible loadings as described in Association American Railroads (2014) and discussed in depth in Section 3.2.3. The set of railcars J can be divided into subsets J_t ($\bigcup_{t \in T} J_t = J$) where each type $t \in T$ has a unique set of loading patterns K_t. The loading patterns can be generated and stored a priori for all railcar types.

k / h	*20*	*40*	*45*	*48*	*53*
1	1	0	0	0	0
2	2	0	0	0	0
3	0	1	0	0	0
4	2	1	0	0	0
5	2	0	1	0	0
6	2	0	0	1	0
7	2	0	0	0	1
8	0	2	0	0	0
9	0	1	1	0	0
10	0	1	0	1	0
11	0	1	0	0	1

Table 3.2: Example set of loading patterns K_j for a one-platform railcar j (the rows correspond to one pattern $k \in K_j$ and columns to container lengths $h \in H$)

The number of loading patterns increases exponentially with the number of platforms. However, in rather general circumstances, there may exist redundancies among them, in the sense that, for a given railcar, a number of distinct loading patterns may accommodate exactly the same set of containers. As an example of redundancy that justifies a reduction in the number of loading patterns without loss of generality, consider the three distinct loading patterns assigning one 40 ft container to a three-platform railcar: the 40 ft container could be placed on platform A, B or C. However, in view of the independence of the platforms it would be sufficient to consider only one of these three patterns.

We remove redundancy between loading patterns by defining equivalence classes over the loading patterns and selecting a single representative loading pattern for the class. We define the equivalence classes as follows: loading patterns for a given railcar type are deemed equivalent if they can be obtained one from the other through a permutation of the individual n-tuples describing the loadings of each platform. We call this *equivalence with respect to platform permutations*. Proceeding in this fashion, we can achieve important reductions in the cardinalities of the sets of loading patterns without loss of generality. We present descriptive statistics for the North American railcar fleet in Section 3.5.

It is important to note that the classes of equivalence for the loading patterns must be defined in accordance with the characteristics of the load planning problem at hand, if their use is not to cause a loss of generality in the description of the loading possibilities. An example where equivalence with respect to platform permutations may not hold is the model presented in Lai, Barkan and Önal (2008) where the aerodynamic efficiency of the load plan is optimized and where the longitudinal position of the containers is of importance. Notice, however, that the key aspect of optimizing aerodynamic efficiency resides in choosing the location of empty slots/platforms, and that this aspect becomes significantly less important in situations with excess demand. In this case, the equivalence classes may not impact the quality of the solution.

3.4.2 Modeling the COG restriction

We described the COG restriction in Section 3.2.3 as an upper limit on the weight of container i', $g_{i'}$, loaded in the top position on a platform p. Stated as an inequality the weight limit (3.2) is

$$g_{i'} \leq c = \frac{g_p(M - m_p) + g_i(M - m_i)}{m_{i'} - M}. \tag{3.3}$$

Notice that it depends non-linearly on the characteristics of the container loaded in the bottom slot. In the following we indicate how we can express the COG restriction using linear

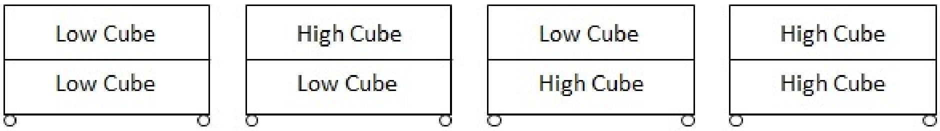

Figure 3.5: Four height configurations relevant to the COG constraints

constraints.

While (3.3) depends on the size of the containers through the height of their centroids (m_i and $m_{i'}$), we only need to consider four height configurations. Indeed, as illustrated in Figure 3.5, containers are either Low Cube (LC) or High Cube (HC). Hence, for a given container in the bottom slot i, $m_{i'}$ can take two values depending on i' being HC or LC. Let m_i^{LC} and m_i^{HC} denote these two possible values of $m_{i'}$. The platform height and tare weight (m_p and g_p) are constants. We can now write (3.3) with the following two inequalities:

$$g_{i'} \leq c_i^{\text{LC}} = \frac{g_p(M - m_p) + g_i(M - m_i)}{m_i^{\text{LC}} - M} \quad \text{if } i' \text{ is LC,} \tag{3.4}$$

$$g_{i'} \leq c_i^{\text{HC}} = \frac{g_p(M - m_p) + g_i(M - m_i)}{m_i^{\text{HC}} - M} \quad \text{if } i' \text{ is HC.} \tag{3.5}$$

The COG restriction is always satisfied when the container in the bottom slot has the same weight or is heavier than the one in the top slot ($g_i \geq g_{i'}$). Finally we note that we can compute c_i^{LC} and c_i^{HC} for all containers $i \in N$ a priori. We take into account that two 20 ft containers can be loaded in the same bottom slot by considering this pair of 20 ft containers as the bottom load and compute the weight limit on the upper container accordingly, based on the total weight of the pair.

3.4.3 ILP formulation

A container of length $h \in H$, $i \in N_h$, $N = \bigcup_{h \in H} N_h$, is characterized by its weight g_i, length l_i, cost if left on the ground π_i, and, possibly, by a particular technical loading restriction. Let N_{LC} and N_{HC} denote the sets of low-cube and high-cube containers. Let $\widetilde{N}_s \subseteq N$ be the set of containers affected by technical loading restriction $s \in S$. Then, for the $s = 1, \ldots, 6$ classes of technical loading restrictions identified for the North American market (Section 3.2.3), we have: D_W, the minimum weight-carrying capacity of a railcar that can receive container $i \in \widetilde{N}_{s_1}$; pre-processed parameter $F_j = 1$ when, given the sequence of railcars, one cannot load on railcar j containers $i \in \widetilde{N}_{s_2}$ (0, when one can); indicator $\alpha_j = 1$ when railcar $j \in J$ with high-weight capacity U_j may receive containers $i \in \widetilde{N}_{s_3}$; R, the maximum number of consecutive platforms on the train between a refrigerated container and the source of electric power.

A railcar $j \in J$ is characterized by its weight-carrying capacity G_j and a utilization (by at least one container) cost τ_j. Let P represent the set of platforms of all railcars, and P_j the set of platforms of railcar $j \in J$. Each platform $p \in P$ is characterized by its length L_p, its weight-carrying capacity G_p, and a sequence number γ_p, numbered from head to tail of the train. Let Q be the set of all slots, Q_p the set of slots of a given platform p, and μ_q be a binary parameter equal to 1 if $q \in Q$ is a bottom slot, 0 otherwise. Furthermore, let $c_i^{\text{LC}\ p}$ and $c_i^{\text{HC}\ p}$ be the low cube and high cube weight limit, respectively, of the top slot for container $i \in N$ loaded in the bottom slot of platform $p \in P$, calculated using (3.4) and (3.5).

Railcars are defined by their type as presented in Section 3.4.1. For the sake of notational simplicity we let K_j be the set of loading patterns for railcar $j \in J$, with n_{kp}^h, the number of containers of length $h \in H$ on platform p in loading pattern $k \in K$.

We define two main sets of *decision variables*. First, $v_{iq} = 1$, if container $i \in N$ is assigned to slot $q \in Q$, and zero otherwise. Second, $w_{jk} = 1$ if railcar $j \in J$ is assigned loading pattern $k \in K_j$, and zero otherwise. We also define two sets of auxiliary binary variables linking the container assignment variables v_{iq} to platforms and railcars. More precisely, let $y_{ip} = 1$, if container $i \in N$ is loaded on platform $p \in P$, 0 otherwise, and $x_{ij} = 1$, if container $i \in N$ is loaded on railcar $j \in J$, 0 otherwise. The model then becomes:

$$\min \quad \sum_{i\in N} \pi_i (1 - \sum_{q\in Q} v_{iq}) + \sum_{j\in J} \tau_j (\sum_{k\in K_j} w_{jk}) \tag{3.6}$$

$$\text{s.t} \quad \sum_{q\in Q} v_{iq} \leq 1 \qquad \forall i \in N \tag{3.7}$$

$$y_{ip} = \sum_{q\in Q_p} v_{iq} \qquad \forall i \in N,\ \forall p \in P \tag{3.8}$$

$$x_{ij} = \sum_{p\in P_j} y_{ip} \qquad \forall i \in N,\ \forall j \in J \tag{3.9}$$

$$\sum_{k\in K_j} w_{jk} \leq 1 \qquad \forall j \in J \tag{3.10}$$

$$\sum_{k\in K_j} n^h_{kp} w_{jk} = \sum_{i\in N_h} y_{ip} \qquad \forall p \in P_j,\ \forall j \in J,\ \forall h \in H \tag{3.11}$$

$$\sum_{i\in N} \sum_{q\in Q_p} \mu_q v_{iq} l_i \leq L_p \qquad \forall p \in P \tag{3.12}$$

$$\sum_{i\in N} y_{ip} g_i \leq G_p \qquad \forall p \in P \tag{3.13}$$

$$\sum_{i\in N_{\text{LC}}} \sum_{q\in Q_p} (1-\mu_q) v_{iq} g_i \leq \sum_{i\in N} \sum_{q\in Q_p} \mu_q v_{iq} c_i^{\text{LC}\ p} \qquad \forall p \in P \tag{3.14}$$

$$\sum_{i\in N_{\text{HC}}} \sum_{q\in Q_p} (1-\mu_q) v_{iq} g_i \leq \sum_{i\in N} \sum_{q\in Q_p} \mu_q v_{iq} c_i^{\text{HC}\ p} \qquad \forall p \in P \tag{3.15}$$

$$\sum_{j\in J} x_{ij} (G_j - D_W) \geq 0 \qquad \forall i \in \widetilde{N}_{s_1} \tag{3.16}$$

$$\sum_{j\in J} x_{ij}\ F_j = 0 \qquad \forall i \in \widetilde{N}_{s_2} \tag{3.17}$$

$$\sum_{j\in J} x_{ij} (\alpha_j U_j - g_i) \geq 0 \qquad \forall i \in \widetilde{N}_{s_3} \tag{3.18}$$

$$\sum_{q\in Q} v_{iq} (1-\mu_q) = 0 \qquad \forall i \in \widetilde{N}_{s_4} \tag{3.19}$$

$$y_{ip} + \sum_{q\in Q_p} v_{i'q} (1-\mu_q) \leq 1 \qquad \forall i \in \widetilde{N}_{s_5},\ \forall i' \in N \setminus i,\ \forall p \in P \tag{3.20}$$

$$\sum_{p\in P} \gamma_p y_{ip} - \sum_{p\in P} \gamma_p y_{i'p} \leq R + (|P| - R)(1 - \sum_{p\in P} y_{i'p}) \qquad \forall i, i' \in \widetilde{N}_{s_6}, i \neq i' \tag{3.21}$$

$$v_{iq} \in \{0,1\} \qquad \forall i \in N,\ \forall q \in Q \tag{3.22}$$

$$y_{ip} \in \{0,1\} \qquad \forall i \in N,\ \forall p \in P \tag{3.23}$$

$$x_{ij} \in \{0,1\} \qquad \forall i \in N,\ \forall j \in J \tag{3.24}$$

$$w_{jk} \in \{0,1\} \qquad \forall j \in J,\ \forall k \in K \tag{3.25}$$

The objective (3.6) of the ILP model seeks to minimize the total of the cost of the containers left on the ground and the cost of the railcars used to load at least one container. We note that $\sum_{k\in K_j} w_{jk} = 0$ when railcar j is not used. Under certain cost parameterizations this generalized cost leads to the maximization of the slot utilization.

There are five sets of loading constraints in the model. The *assignment constraints* (3.7) ensure that each container $i \in N$ can be assigned to at most one slot $q \in Q$. For a given container $i \in N$, $\sum_{q\in Q} v_{iq} = 0$ implies that the container is not assigned to any slot and thus is left on the ground. Constraints (3.8) and (3.9) define the auxiliary assignment variables y_{ip} and x_{ij} of containers to slots and platforms, respectively.

The *loading pattern constraints* (3.10) ensure that exactly one loading pattern $k \in K_j$ is assigned to each railcar $j \in J$. Constraints (3.11) link variables w_{jk} and y_{ip}, enforcing that the number of loaded containers of length $h \in H$ on platform $p \in P$ equals n^h_{kp}. Constraints (3.12) ensure that the length of the container(s) loaded in the bottom slot of platform $p \in P$ does not exceed the length of the platform.

The *weight capacity constraints* (3.13) ensure that the total weight of the loaded containers does not exceed the maximum allowable weight limit of the platform. The *COG restrictions* are modeled by constraints (3.14) and (3.15).

In addition to dimensional and weight restrictions, there are also a variety of *technical loading restrictions* imposing or forbidding the loading of certain types of containers on specific railcars or slots. Constraints (3.16) - (3.21) correspond to the six classes of technical loading restrictions we identified for the North American market. Notice, however, that these can be easily extended to describe other company policies or country regulations.

Constraints (3.16) state that containers $i \in \widetilde{N}_{s_1}$ can only be loaded on railcars that have the minimum weight-carrying capacity D_W. Constraints (3.17) restrict the loading of containers $i \in \widetilde{N}_{s_2}$ to a particular railcar in the given sequence of railcars, while constraints (3.18) restrict the loading of containers $i \in \widetilde{N}_{s_3}$ to railcars with a suficiently high weight capacity.

There are two types of stacking constraints. First, containers $i \in \widetilde{N}_{s_4}$ cannot be loaded in the top slot (3.19). Second, containers $i \in \widetilde{N}_{s_5} \subseteq \widetilde{N}_{s_4}$ cannot be loaded in a top slot and cannot be double stacked (3.20).

Constraints (3.21) concern the storage of refrigerated containers that need a source of electrical power (the genset is a container $i \in \widetilde{N}_{s_6}$), limiting the distance between any two loaded containers belonging to the set $\widetilde{N}_{s_6}$. Finally, expressions (3.22)-(3.25) define the domain of the decision variables.

3.5 Numerical results

Two numerical studies have been conducted. The first one assesses the effects of containers-to-cars matching and COG restrictions on load planning solutions. The second one examines the relationship between the particular sets of characteristics presented by the load planning problems and the computation times required for their solution. Without loss of generality, and to simplify the discussion, we assume for all instances that container costs $\pi_i = \pi$, $\forall i \in N$ and railcar cost $\tau_j = \tau$, $\forall j \in J$.

The Java programming language was used for processing the data and for running and post processing the solutions on an Intel(R) Core(TM) i5-5300U, 2.30 GHz CPU processor equipped with 24 GB of RAM. The ILP optimization model was solved using a 32-bit version of the IBM ILOG CPLEX 12.6 solver, with a preset computational time limit of 10 hours. The reported computational times only account for the solver's CPU time, since the computations associated with pre processing the data and post processing the solutions required negligible time.

The sets of loading patterns were generated one time, a priori, using the Python programming language. The generation for every railcar series found in the North American fleet required less than 45 minutes of CPU time.

Table 3.3 reports descriptive statistics for the sets of loading patterns associated with all railcar types in the North American fleet. Each row corresponds to single- (S) or double-stack (D) railcars with a given number of platforms. The third column gives the number of unique sets of loading patterns (there is a total of 60 sets). The next block of three columns reports the average, minimum and maximum cardinalities of the original sets of loading patterns and the last three columns the same figures for the reduced sets. The latter are obtained by defining equivalence classes as we describe in Section 3.4.1. Whereas the cardinalities of the sets of loading patterns increase with the number of platforms and become very large, the use of equivalence classes results in important reductions.

# platforms	Single-/ double-stack	# sets	Cardinality original set			Cardinality reduced set		
			Avg.	Min	Max	Avg.	Min	Max
1	S	1	7	7	7	7	7	7
1	D	6	18	6	27	15	6	21
2	S	1	25	25	25	15	15	15
3	S	3	171	125	245	56	35	80
3	D	10	4,741	1,000	9,261	940	220	1,771
4	D	2	106921	83,521	130,321	6,080	4,845	7,315
5	S	6	3,803	32	7,776	194	6	371
5	D	31	485,664	1,024	4,084,101	10,915	56	53,130

Table 3.3: Descriptive statistics for the sets of loading patterns for railcars in the North American fleet

3.5.1 Effects of containers-to-cars matching and COG restrictions

In order to isolate and measure the effects of containers-to-cars matching and COG restrictions on load planning solutions, we designed a stylized experiment. It is based on 396 generated instances, differentiated with respect to the main container and railcar characteristics, namely length and weight of containers and length and weight capacity of railcar platforms. The goal is to examine the changes in the use of block capacity resulting from changes in the characteristics of the containers and railcars. In the following, we first describe the instance generation, and then we present the results.

In all generated instances, we keep the length of the railcar sequence as fixed. Yet, the capacity in terms of number of slots can still vary since platforms have different lengths. We define four railcar scenarios that involve either one- or five-platform cars with either 40 ft or 53 ft platforms. We assume that it is possible to include 25% more 40 ft slots than 53 ft ones and fix the capacity to 250 40 ft slots or 200 53 ft slots. For each railcar scenario, we choose one railcar type whence the set of loading patterns is the same for all railcars in a given scenario.

We consider 18 different scenarios for the container sets. The number of containers in each set is equal to the number of slots in the block. They have different characteristics in terms of mix of container lengths and weights. There are five different length mixes: 50% 40 ft containers and 50% 53 ft containers, 75% 40 ft containers and 25% 53 ft containers and vice versa, 100% 40 ft and 100% 53 ft. The containers are assigned weights in three different ways, two deterministic and one random. The deterministic cases present favorable weight distributions, i.e. where one can use the maximum capacity because there are no restrictions related to the COG. This holds when either all containers have equal weight, or half of the containers are light and half heavy. We draw weights at random for 40 ft and 53 ft containers from uniform distributions, respectively in [8,000;62,000] lb (equivalent to [4;31] tons) and [11,000;72,000] lb (equivalent to [5.5;36] tons), and we generate 20 instances for each length mix. We define light and heavy to be the first and third quartile, respectively. The 18 different scenarios defining the container sets are denoted S1–S18 and each scenario comprises 22 instances (20 random and 2 deterministic) whence a total of 396 instances are solved.

Tables 3.4 and 3.5 report the results for the scenarios with one- and five-platform railcars, respectively. In both tables, the first two columns show the number of loaded containers and the number of used railcars in the optimal solution. The third column shows CPLEX solution time. The gap is not reported because all the instances are solved to optimality. Note that in case of random weights, we report an average over the 20 instances.

The results show that the solution time is less than 200 seconds for one-platform railcars, while it increases to a maximum of 935 seconds for five-platform railcars. This results from the increased cardinality of the sets of loading patterns K_j. In the case of 40 ft one-platform railcars, the maximum number of containers that can be loaded is equal to the number of slots, that is 250. However, 53 ft containers can only be loaded in the top slot since their length exceeds the platform length. Hence, as long as there are less than 125 53 ft containers in the instances (S1-S4 in Table 3.4), all slots can be used under the favorable weight values of the deterministic instances. In Table 3.4, S5 is an example of a scenario where the number of 53 ft containers exceeds the number of top slots, and where, as a result, even though the weight values are favorable, some of the containers cannot be loaded.

Loading patterns may impose additional restrictions for five-platform railcars. In particular, as seen in Section 3.4.1, 53 ft containers cannot be loaded in the top slots of contiguous 40 ft platforms. This is illustrated in scenarios S13 and S14 of Table 3.5 where regardless of the favorable deterministic weight values, some slots must be left empty because of the high proportion of 53 ft containers.

The results obtained with differing container weights clearly illustrate that the maximum capacity of 40 ft railcars can only be reached under favorable weight settings. In the case of random container weights where COG restrictions play a role, the results indicate a decrease in the average number of loaded containers, even when their lengths are well-matched to the railcars. The 53 ft platform railcars are more flexible because they can accommodate 53 ft containers also in the bottom position. For every set of weight values, it is possible to load all containers. However, since 53 ft railcars are longer, there are only 200 slots compared to 250 for the 40 ft platform railcars.

In order to load as many containers as possible, there is a trade-off between using 53 ft and 40 ft platforms and this trade-off depends on both the size of the containers and their weights. For example, the 250 slots on the 40 ft platforms can only be used under the most favorable settings. As the share of 53 ft containers increases (in particular for the random weight setting), the number of containers loaded decreases towards 200 (and might possibly reach less than 200 if weights are unfavorable as in S14).

This stylized numerical study demonstrates that ignoring COG restrictions and containers-to-cars matching (i.e., assuming favorable container length and weight settings) may lead to an overestimation of the usable capacity offered by the railcars in a block. Indeed, in the case of 40 ft platforms, the number of loaded containers varies between 175 and 250 while the capacity is 250.

250 CONTAINERS	125 ONE 40ft PLATFORM RAILCARS		
INSTANCE DESCRIPTION	**LOADED CONTAINERS**	**USED RAILCARS**	**TIME [s]**
S1 : 250 40ft containers			
1) Containers same weights	250	125	17.8
2) Containers half low and half high weights	250	125	22.07
3) Containers random weights	250	125	46.30
S2: 200 40ft containers and 50 53ft containers			
1) Containers same weights	250	125	14.61
2) Containers half low and half high weights	250	125	22.02
3) Containers random weights	244	123	51.54
S3: 150 40ft containers and 100 53ft containers			
1) Containers same weights	250	125	11.21
2) Containers half low and half high weights	250	125	14.49
3) Containers random weights	238	120	76.0
S4: 125 40ft containers and 125 53ft containers			
1) Containers same weights	250	125	10.22
2) Containers half low and half high weights	250	125	11.49
3) Containers random weights	234	118	200.82
S5: 100 40ft containers and 150 53ft containers			
1) Containers same weights	200	100	9.28
2) Containers half low and half high weights	200	100	12.81
3) Containers random weights	200	100	35.62
200 CONTAINERS	**100 ONE 53ft PLATFORM RAILCARS**		
INSTANCE DESCRIPTION	**LOADED CONTAINERS**	**USED RAILCARS**	**TIME [s]**
S6: 200 40ft containers			
1) Containers same weights	200	100	8.10
2) Containers low and high weights	200	100	16.22
3) Containers random weights	200	100	15.63
S7: 125 40ft containers and 75 53ft containers			
1) Containers same weights	200	100	9.69
2) Containers low and high weights	200	100	13.55
3) Containers random weights	200	100	23.47
S8: 75 40ft containers and 125 53ft containers			
1) Containers same weights	200	100	9.45
2) Containers low and high weights	200	100	15.77
3) Containers random weights	200	100	28.07
S9: 0 40ft containers and 200 53ft containers			
1) Containers same weights	200	100	9.84
2) Containers low and high weights	200	100	8.74
3) Containers random weights	200	100	27.09

Table 3.4: Effects of matching problem and COG restrictions: number of loaded containers, number of used railcars and solution time for one-platform railcars

250 CONTAINERS	25 FIVE 40ft PLATFORM RAILCARS		
INSTANCE DESCRIPTION	**LOADED CONTAINERS**	**USED RAILCARS**	**TIME [s]**
S10: 250 40ft containers			
1) Containers same weights	250	25	111.69
2) Containers half low and half high weights	250	25	167.99
3) Containers random weights	250	25	178.14
S11: 200 40ft containers and 50 53ft containers			
1) Containers same weights	250	25	126.59
2) Containers half low and half high weights	250	25	132.83
3) Containers random weights	233	24	935.88
S12: 150 40ft containers and 100 53ft containers			
1) Containers same weights	250	25	119.32
2) Containers half low and half high weights	250	25	120.38
3) Containers random weights	219	24	764.72
S13: 125 40ft containers and 125 53ft containers			
1) Containers same weights	200	25	116.75
2) Containers half low and half high weights	200	25	127.73
3) Containers random weights	200	25	331.79
S14: 100 40ft containers and 150 53ft containers			
1) Containers same weights	175	25	113.67
2) Containers half low and half high weights	175	25	125.89
3) Containers random weights	175	25	329.88

200 CONTAINERS	20 FIVE 53ft PLATFORM RAILCARS		
INSTANCE DESCRIPTION	**LOADED CONTAINERS**	**USED RAILCARS**	**TIME [s]**
S15: 200 40ft containers			
1) Containers same weights	200	20	514.81
2) Containers low and high weights	200	20	539.24
3) Containers random weights	200	20	733.83
S16: 125 40ft containers and 75 53ft containers			
1) Containers same weights	200	20	574.13
2) Containers low and high weights	200	20	636.86
3) Containers random weights	200	20	799.33
S17: 75 40ft containers and 125 53ft containers			
1) Containers same weights	200	20	471.43
2) Containers low and high weights	200	20	844.02
3) Containers random weights	200	20	932.13
S18: 0 40ft containers and 200 53ft containers			
1) Containers same weights	200	20	429.88
2) Containers low and high weights	200	20	513.44
3) Containers random weights	200	20	859.54

Table 3.5: Effects of matching problem and COG restrictions: number of loaded containers, number of used railcars and solution time for five-platform railcars

3.5.2 Computational times

This section examines the computation times required to solve realistic instances of the load planning problem. For this purpose, we generate instances of diverse sizes and characteristics. Sets of railcars are sampled at random from the types available in the North American fleet. We also generate sets of containers with different cardinalities comprising containers with diverse characteristics.

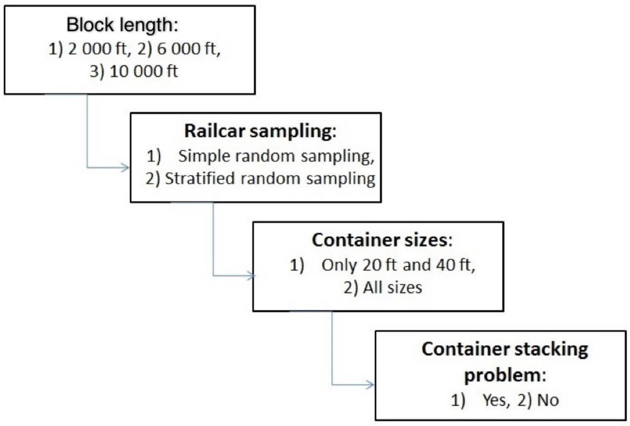

Figure 3.6: Overview of the instance generation process

Figure 3.6 overviews the generation process. Four block lengths are considered: 2,000 ft (0.6 km), 6,000 ft (1.8 km), 10,000 ft (3 km) and 14,000 ft (4.3 km). Loading and unloading a block is highly dependent on the layout of the terminals, in particular on the length of the tracks and the location of the container storage in relation to the tracks. In practice, block lengths of 10,000 or 14,000 ft may therefore be considered unreasonably long. We include them for the sake of comparison. For the same reason, 6,000 ft is considered very large and 2,000 ft a realistic length.

For each block length we generate 20 sequences of railcars by sampling the North American distribution of railcar types. We apply two different sampling protocols: simple random (10 sequences) and stratified random (10 sequences). We classify the railcars in the North American fleet according to their flexibility in accommodating a diversity of load patterns. This flexibility is indexed over the railcar types by calculating for each one the average number of loading patterns per platform. Figure 3.7 shows a histogram of the share of railcar types over the values of the flexibility index. Since a large proportion of the railcar types exhibit high flexibility indices, the railcar sequences generated by simple random sampling present a greater share of railcars with high flexibility index values than the sequences generated by stratifying the sample over the values of the flexibility index.

Containers are selected as follows. For each block length, and for each railcar sequence, we consider four sets of containers. The sizes of these sets are equal to 1.5 times the number of slots in the railcar sequence. This ensures that solutions achieving a slot utilization close to 100% are possible. There are two mixes of container lengths: one with only 20 ft and 40 ft containers and one with all lengths (20,40,45,48 and 53 ft). In a manner similar to that of the previous section, we assign weights to containers by drawing from a weight distribution that is conditional on container size. In order to assess the influence of the technical loading restrictions on computational time, some instances include containers affected by stacking restrictions. In these instances, each container is assigned a stacking restriction at random which results in a share of 3%.

We solve a total of 240 instances, that is, six railcar sequence scenarios with 10 sequences

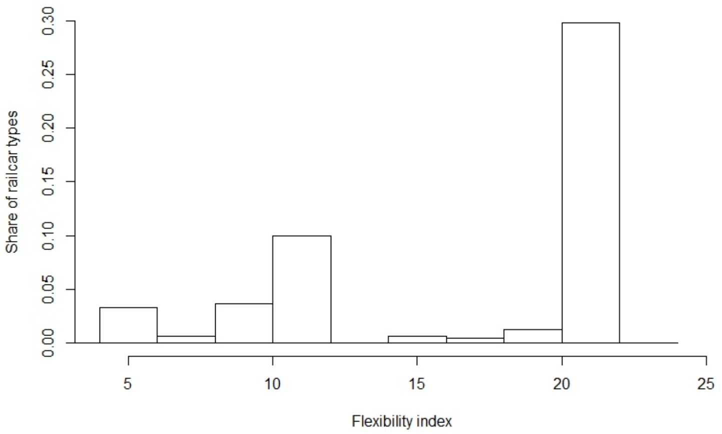

Figure 3.7: Share of North American railcar types over flexibility index values

each, and four container sets per railcar sequence. Table 3.6 displays the average computational time. The results show that we can find an optimal solution for the instances of 2,000 ft in less than 24 seconds on average, for all scenarios. The stratified random sampling contains a higher share of railcars with low flexibility index than the other, which results in longer computational time. The average computational time is longer for instances of 6,000 ft. Still, all instances can be solved in less than 17 minutes on average. We note that the instances of 10,000 ft can be solved to optimality but in the most complex case (i.e., including containers of all sizes and featuring technical restrictions) requires on average 3.5 hours which is not reasonable for the problem at hand. The cardinality of the sets of loading patterns has an important impact on computational time. This can clearly be seen by comparing the computational time for the instances including only 20/40 ft containers with those including all container sizes. The number of variables ranges from some 45,000 for the simplest instances (2,000 ft and 20/40 ft container) to some 1.9M in the more complex settings (14,000 ft and all container sizes with stacking restrictions). Similarly, the number of constraints ranges from some 19,000 to 2.6M.

Summing up, this numerical study shows that we can solve realistic instances in short computational time and very large instances (6,000 ft) in reasonable time. It is also possible to solve to optimality instances of 10,000 ft and the simpler settings of the 14,000-ft instances. However, in the most complex settings, this cannot be accomplished in reasonable time on the single core machine used here. In the most complex setting (14,000 ft, all container sizes and stratified sampling) the optimal solution is not found for 6 out of 10 instances in 10 hours. The average gap for those instances is 0.43.

	Containers without technical loading restrictions				Containers with technical loading restrictions			
Block length /	20 & 40 ft		All sizes		20 & 40 ft		All sizes	
Sampling protocol	Time [s]	Slot U [%]	Time [s]	Slot U [%]	Time [s]	Slot U [%]	Time [s]	Slot U [%]
2,000 ft								
Simple random	7.11	100.00	13.10	100.00	7.95	100.00	14.36	100.00
Stratified random	11.97	99.54	21.20	99.78	12.92	99.54	24.74	99.78
6,000 ft								
Simple random	184.59	100.00	450.96	100.00	661.05	100.00	639.37	100.00
Stratified random	209.15	99.58	576.12	99.54	377.63	99.50	1,077.98	99.58
10,000 ft								
Simple random	967.42	100.00	4,010.52	100.00	1,963.78	100.00	8,266.35	100.00
Stratified random	1,653.16	99.50	4,217.13	99.54	2,755.56	99.50	13,254.41	99.54
14,000 ft								
Simple random	4,714.59	100.00	17,510.18	100.00	18,220.68	100	22,860.32	98.78
Stratified random	5,677.45	99.64	22,828.71	99.67	15,295.58	99.64	**27,508.56**	**99.59**

In bold: statistics based on 4/10 instances that were solved to optimality in a time limit of 36,000 s.

Table 3.6: Average computational time and average slot utilization for instances with diverse characteristics

3.6 Conclusions and directions for future research

In this chapter we studied the load planning problem for double-stack intermodal trains. Given a set of containers stored in a terminal, a sequence of railcars, and the relevant constraints, determine the subset of containers to load and the exact way of loading them. The objective is to minimize the cost of unloaded containers and the cost of empty slots. Under certain cost parameterizations this generalized cost leads to the maximization of the slot utilization, a key performance indicator in the industry. Previous studies in the literature either do not address the load planning problem for double-stack trains or make simplifying assumptions that may lead to load plans that violate important loading rules. The problem related to double-stack trains is challenging because the load plan must respect a number of loading rules that depend on container and railcars characteristics such as containers-to-cars matching and COG restrictions.

We formulated an ILP model and made a number of contributions. First, we proposed a general methodology that can deal with double- or single-stack railcars with arbitrary loading patterns. The patterns account for loading dependencies between the platforms on a given railcar. Second, we modeled COG restrictions, stacking rules and a number of technical loading restrictions associated with certain types of containers and/or goods.

We presented two numerical studies. We show that we can solve realistic size instances in reasonable time using a commercial ILP solver and we illustrate that failing to account for containers-to-cars matching as well as COG restrictions may lead to an overestimation of the available train capacity. The results showed that the computational time varies with the size and characteristics of the instances. For example, it is more time consuming to solve instances with five-platform railcars and several container sizes compared to fewer platforms and only 20 and 40 ft containers. This is due to the cardinality of the sets of loading patterns. It is also more time consuming to solve instances with containers having technical loading restrictions than those without.

On the one hand one may use the proposed methodology in decision-aid tools for terminal managers in charge of the load planning. On the other hand, in a more tactical or strategic planning setting to assess railcar fleet management decisions. We also note that we can extend the model to plan several trains ahead under perfect information, similar to Lai, Ouyang and Barkan (2008).

There are several possible directions for future research. One could extend the model in order to consider handling costs in the yard or the potential penalties of not delivering on time, for example, by selecting containers according to their location in stacks or their priority and time left before the due date, respectively. Furthermore, several aspects of the problem may be subject to uncertainty, for example, the availability and characteristics of containers and railcars. Modeling this uncertainty is another topic of future research.

Chapter 4

Conclusions

In this thesis we provided a broad context on intermodal transportation with a particular focus on the main entities of the intermodal transportation chain: the terminals. We presented some planning problems that arise at terminals, emphasizing that decision-aid tools based on optimization methods can provide terminal managers necessary information to take better decisions. This need increased in recent years because the logistics especially at large container terminals is complex.

The focus of the thesis is the freight transportation, using containers as a key loading devices. In this context, we focused specifically on rail transportation, and we addressed the Load Planning Problem (LPP) for double-stack intermodal trains. The work of the thesis was based on a scientific article, published in the European Journal of Operational Research in 2018.

The LPP consists in finding an optimal assignment of stored containers of different sizes and with different characteristics to slots on railcars of a departing train. The goal was to work on a general methodology solving realistic instances with reasonable amount of time, which would have been used by terminal managers as a decision aid tools. Studies in the literature either do not address the LPP for double-stack trains or make simplifying assumptions that may lead to load plans that violate important loading rules.

We formulated an ILP model and made a number of contributions. We model the matching among types of containers and railcars through loading patterns, with the same logic as the ones we found in the literature (i.e., Corry and Kozan, 2008; Lai, Barkan and Önal, 2008), but accounting for dependencies between the loading of the platforms of the same railcars for the first time. For the North American market, the matching among container and railcar types is covered by the AAR guide (Association American Railroads, 2014), which provides information on which container sizes that can be loaded in the bottom and top slot of each platform of the existing railcars. We refer to them as *containers-to-cars matching rules*. The guide reports the loading capabilities but it does not show all the possible ways of matching them. Plus the loading patterns for certain platforms may depend on the loading of the others and thus one may not decompose the railcar loading problem by platform. We generated all the combination of containers-to-cars matching rules but reduced the number by defining equivalent classes, solving for any double- or single-stack railcars with arbitrary loading patterns.

In deriving load plans, we accounted for the weight holding restrictions. First, the total weight of the containers loaded on a given platform could not exceed its weight holding capacity. The second type of restriction concerned the vertical center of gravity (COG restrictions), that is the unique point where the weighted relative position of the distributed mass sums to zero. Finally, we addressed technical loading restrictions associated with certain types of containers and/or goods.

We performed two numerical studies, to assess the importance and the accuracy of the model. Results showed that ignoring containers-to-cars matching and center-of-gravity restrictions may lead to an overestimation of the train capacity and to infeasible load plans. Moreover, we showed that, using a commercial solver, we find an optimal solution for realistic instances in a reasonable amount of time.

To conclude, the proposed methodology presented in the thesis can be used as a decision-aid tool for terminal managers in charge of the load planning. In the long term, as the problems become larger and more complex, the solutions provided by an optimization model do not only contribute to significantly reduce cost, but also make the operations of the railway company less dependent on the unique expertise of a limited number of individuals, making a decision-aid tool more and more valuable to the terminal planners.

There are several possible directions for future research. First, we could extend the model to plan several trains ahead under perfect information, similar to Lai, Ouyang and Barkan (2008). One could extend the model to consider handling costs in the yard or the potential penalties of not delivering on time, for example, by selecting containers according to their location in stacks or their priority and time left before the due date, respectively.

A big challenge for a future research, will be to develop decision aid tools to manage the terminal in a real time. The goal being not only the loading of the current train, but also incorporating equipment and resources management in the study. At the time of the planning, the containers which are not physically in the terminal, that are about to arrive (before the cut off time of the train), could be taken into account in deciding where to load each unit of cargo on the first leaving train. Given that containers continue to arrive after the loading of a train has been started, one challenge will be to consider the sequence in which the containers are loaded. One idea is to formulate a dynamic model to derive the optimal loading sequence.

Furthermore, several aspects of the problem may be subject to uncertainty, for example, the availability and characteristics of containers and railcars. Modeling this uncertainty is another topic of future research.

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