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DETERMINANTS OF GOVERNMENT EXPENDITURES:
AN APPLICATION OF THE LINEAR EXPENDITURE SYSTEM

BY

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Résumé

Cette étude applique le système de dépenses linéaire dans une tentative de mesurer l'importance des économies d'échelle dans la consommation des dépenses publiques non militaires pour un groupe de pays avancés. La présence de telles économies, combinées avec une élasticité-"prix" non unitaire de la demande des biens publics, impliquerait une relation fonctionnelle entre la taille relative du secteur public d'un pays et de sa population. Nous constatons que si l'effet d'échelle est spécifié selon la méthode proposée par Borcharding et Deacon, la fonction de vraisemblance de notre modèle aura deux maxima. Cependant, quand le paramètre d'échelle est contraint à une gamme de valeurs qui est acceptable sur le plan théorique, nous trouvons des économies d'échelle significatives et possiblement croissantes dans le temps. Puisque la demande des biens publics semble inélastique, nos résultats suggèrent une relation négative entre la population d'un pays et la part du secteur public. Enfin, il n'est pas possible de confirmer la loi de Wagner, selon laquelle la part publique serait une fonction croissante du revenu par habitant.

Are the goods and services purchased by governments private goods or public goods? The question is an important one, since models of public choice developed by Borcharding and Deacon (1972) and Bergstrom and Goodman (1973) suggest that if a publicly-provided good is subject to consumption economies of scale, its price as perceived by the median voter-taxpayer will be a decreasing function of the number of taxpayers. However, with the exception of Orr (1976), empirical estimates of the demand for individual categories of public expenditures have generally failed to reveal any significant degree of publicness. As a result, the tax price remains largely a theoretical curiosity. One problem in these studies may be the level of disaggregation. Browning (1975) has suggested theoretical problems in the application of voting models when two or more categories of public expenditure are financed from a common fund. Moreover, many of these studies have used a constant-elasticity-of-demand model. As Brown and Deaton (1972, p. 1193) have shown, whenever income changes this model violates the constraint that income shares add up to unity¹.

The overall size of the public sector has been the subject of numerous studies, most of which have focused on the level of income as an explanatory variable. Time-series studies for individual countries have generally shown an increase in the relative size of the public sector as income levels rise², in accord with Wagner's Law³. However, the results of cross-section analysis have been less conclusive⁴. Almost without exception, however, the empirical relationships examined in these studies have been based on ad hoc reasoning.

In this paper, we propose to explain the size of aggregate (non-military) public spending by means of the theory of public choice. We develop an

ABSTRACT

This paper applies the linear expenditure system in an attempt to measure the degree of consumption economies of scale in non-military public expenditures for a group of developed countries. In the presence of such scale economies and a non-unitary "price"-elasticity of demand for public goods, the size of a country's public sector will be related to its population. Use of the constant-elasticity specification for the publicness effect suggested by Borcharding and Deacon is shown to yield dual maxima for the likelihood function of the model. However, when the publicness parameter is constrained to a theoretically-admissible range, there is evidence of a significant and possibly increasing degree of publicness in non-military government expenditures. Since the demand for such expenditures is found to be price-inelastic, the results suggest a negative relationship between a country's population and its public share. Furthermore, the results failed to confirm the positive relationship between per-capita income and the public share implied by Wagner's Law.

integrated model of public and private expenditure in which the adding-up constraint is respected. By assuming that the model takes the form of the linear expenditure system, we are able to estimate price and income elasticities of the demand for public spending. However, in this revised specification, non-linear and constrained parameters complicate the empirical analysis.

I. THE DEMAND FOR PRIVATE AND PUBLIC EXPENDITURE

Consider the situation of an individual in a particular nation who is asked to vote on whether or not he favours the purchase of H units of a quasi-public good. This expenditure is to be financed by a tax to be levied on all individuals in the society. Let this person's preferences be represented by a strictly quasi-concave monotone increasing utility function whose arguments are the quantities of n private goods, q_1, q_2, \dots, q_n , and his own consumption of the quasi-public good. If the latter good is subject to crowding, the amount of it which is actually consumed by the individual may be represented by H/N^α , where N is the total number of (consuming and taxpaying) individuals in the society. The utility function is then:

$$U = U(q_1, q_2, \dots, q_n, H/N^\alpha) \quad \alpha > 0 \quad (1)$$

It is assumed that U is twice differentiable and that the second-order derivatives with respect to q_i and H are continuous in these variables. Note that if the publicness parameter $\alpha = 0$, H is a pure public good while if $\alpha = 1$ H is a private good, there being no economies of scale in its consumption.

The individual is also subject to a budget constraint

$$\sum_{i=1}^n p_i q_i + t p_h H = m \quad (2)$$

where m is his income, t is his share of the total taxes needed to finance H , p_i is the price of the i^{th} private good and p_h the price of H . If there

is a single tax proportional to income⁵, the individual's tax share will be equal to his share of total income:

$$t = m/(yN) \quad (3)$$

where y is mean income.

Let μ be the ratio of the individual's income to mean income.

$$\mu = m/y \quad t = \mu N \quad (4)$$

After substitution from (3) and (4), the budget constraint (2) becomes

$$\sum_{i=1}^n p_i q_i + \mu p_h H/N = \mu y \quad (5)$$

Maximization of U subject to (5) yields, along with (5) the following first-order conditions:

$$\partial U / \partial q_i = \lambda p_i \quad i = 1, \dots, n \quad (6)$$

$$\partial U / \partial \left(\frac{H}{N^\alpha} \right) = \gamma \mu p_h / N^{1-\alpha} \quad (7)$$

where λ is a lagrangien multiplier which may be interpreted as the marginal utility of income.

It is evident that the above problem is analogous to the familiar problem of consumer demand theory. The only change is the addition of an $(n + 1)^{th}$ good whose price as perceived by the taxpayer under a system of a single tax proportional to income is $\mu p_h / N^{1-\alpha}$. In short, the perceived price of a quasi-public good will generally depend on the size of the collectivity or "nation" as measured by the number of taxpayers.

The first order conditions (5), (6) and (7) may be solved to yield a set of (n+1) demand equations:

$$q_i = q_i (p_1, p_2, \dots, p_n, \mu p_H / N^\alpha, \mu y), \quad i = 1, \dots, n \quad (8)$$

$$H = H (p_1, p_2, \dots, p_n, \mu p_H / N^\alpha, \mu y) \quad (9)$$

Provided that U exhibits what Katzner (1968) has defined as strong quasi-concavity, the solution will be unique.

If H is assumed to be a superior good, all individuals with higher incomes than the person who is being considered (i.e. a higher μ) will prefer larger amounts of the public good than he does, while all those with lower incomes will prefer less public spending. A majority decision in the country in question will be determined by the preferences of the median voter according to equation (9), with μ measuring the ratio of the median voter's income to mean income. Any larger amount would be opposed by a majority of voters while any smaller amount could be increased without being defeated in a majority vote.

II. THE FUNCTIONAL FORM OF THE DEMAND FOR PUBLIC EXPENDITURE

As Barten (1977) has observed, the number of possible functional forms for demand equations is infinite. One of the simplest and most frequently used is the linear expenditure system, based on the Stone-Geary utility function. With n private goods and one public good, H , the utility function takes the following form:

$$U = \sum_{i=1}^n \beta_i \ln (q_i - \gamma_i) + \theta \ln (H/N^\alpha - \eta) \quad (10)$$

where $\gamma_i < q_i$ and $\eta < H$ are constants, and $\sum_i \beta_i = 1 - \theta$

It should be noted that this functional form implies that all goods are superior and that there is no complementarity.

Maximization of U from equation (10) subject to the budget constraint (5) followed by solution of the resulting first-order conditions yields the following demand functions:

$$\begin{aligned} q_i &= \gamma_i + (\beta_i/p_i) (\mu y - \sum_k p_k \gamma_k - \mu p_h \eta / N^{1-\alpha}) & i = 1, \dots, n \\ H &= \eta + [\theta N / (\mu p_h)] (\mu y - \sum_k p_k \gamma_k - \mu p_h \eta / N^{1-\alpha}) \end{aligned} \quad (11)$$

A special case (to be tested empirically in the following section) involves a single private good and a composite public good. Let the private good be selected as numeraire. Assuming that H is produced under constant costs, one may choose units of measurement for the public good such that its price is a unit of the private good. Let G represent public expenditures.

Then

$$G \equiv p_h H = H$$

Equations (11) then become

$$q = \gamma + \beta (\mu y - \gamma - \mu \eta / N^{1-\alpha}) \quad (12)$$

$$G = \eta + (\theta N / \mu) (\mu y - \gamma - \mu \eta / N^{1-\alpha}) \quad (13)$$

where the subscripts have been dropped for convenience.

Under Walras's Law, system (12) and (13) comprises a single independent equation, and may thus be reduced to the single equation (13) expressing the demand for the public good. If μ is the relative income of the median voter, G will be the quantity of public expenditures which, under the argument of the preceding section, will obtain the support of a majority of voters.

Divide equation (13) by the nation's total income, $Y = Ny$

$$G/Y = \theta - (\theta \gamma / \mu) (1/y) + (1 - \theta) \eta / (y N^{1-\alpha}) \quad (14)$$

After a regrouping of terms, this equation becomes

$$y (G/Y - \theta) = -[\theta \gamma / \mu - (1 - \theta) \eta / N^{1-\alpha}] \quad (15)$$

Equation (15) expresses a relationship between the two variables involved in the most frequently-used test of Wagner's Law; namely, the share of public expenditures in GNP (G/Y) and per-capita income (y). Assume that

there is some positive minimum level of the public good which is demanded (i.e. $\eta > 0$) and examine the effect of a change in per-capita income on the public share. For a given population, N , the relationship will be a hyperbola such as the curves presented in Figure 1. Provided that the country's population is greater than a certain critical level

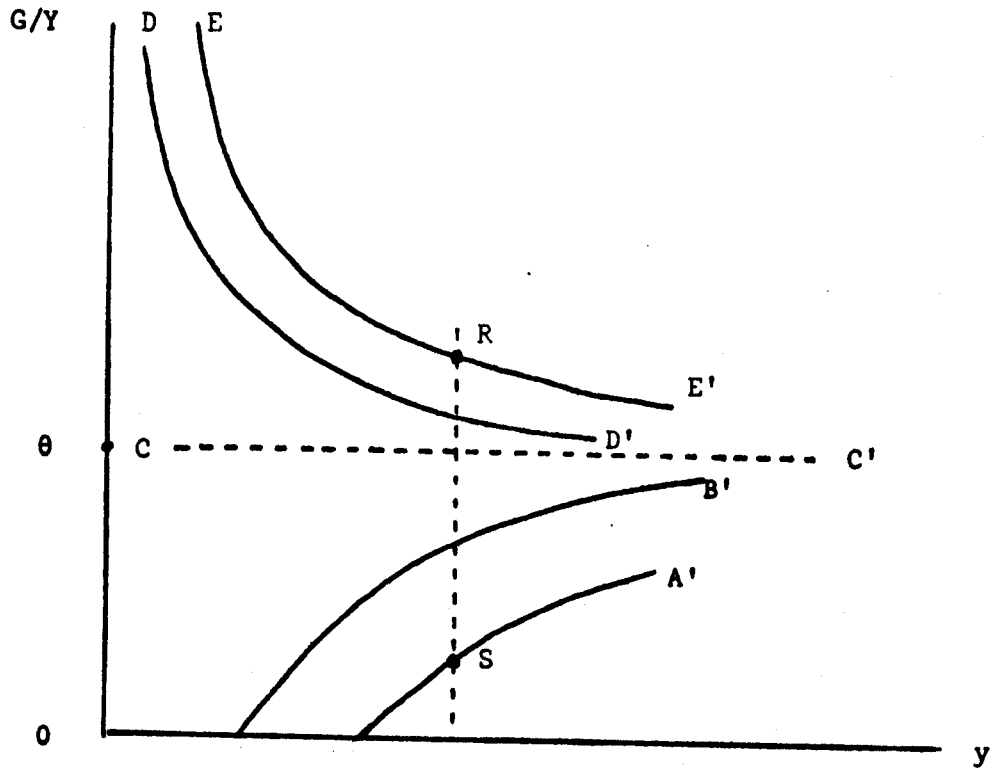
$$N^* = [(1 - \theta)\eta\mu/(\theta\gamma)]^{1/(1-\alpha)} \quad (16)$$

the relationship will be monotonically increasing, as depicted by curve AA', in accord with Wagner's Law. Note, however, that for a country with a smaller population but still greater than N^* , the curve shifts upward and inward to position BB'. If the population is equal to N^* , differences in per-capita income will have no effect on the relative size of the public sector, as indicated by curve CC'.

For countries with a population smaller than the critical level, however, the relationship between the public share and per-capita income will be monotonically decreasing - the opposite of what Wagner predicted. The curves DD' and EE' show the postulated relationship for successively smaller levels of population.

How might these results be interpreted? Consider first the negative relationship between population size and the share of the public sector (as indicated by a declining public share as one moves from R toward S along RS). One may conclude from Section I that the reciprocal of the population represents the perceived price of the public good to the voter with mean income. A fall in the public share as population increases implies that the price-elasticity of demand for the public good is less than unity (in

Figure 1



absolute value). This elasticity (see Appendix) may easily be derived from equation (15).

$$\xi(G, 1/N) \equiv \frac{\partial \ln G}{\partial \ln (1/N)} = -[1 - \eta(1 - \theta)(1 - \alpha)N^\alpha/G] \quad (17)$$

For the particular case where $\eta > 0$, it is evident that this elasticity is greater than -1.

Turn now to an explanation of the effect of per-capita income on the public share. Consider several countries with the same level of per-capita income (for example the points along RS). The total amount of public expenditure will be an increasing function of population; it follows that the country represented by point R will have lower total public expenditures than that represented by S. Now let per-capita income increase slightly, holding population constant in each country. The decrease in the relative size of the public sector in the smallest country along RE', indicates that for low levels of government expenditure, public services are a necessity. Once the basic services have been provided, taxpayers prefer to spend an increasing part of each dollar of income on private goods. However, in a country with a large population, an increase in income will generate proportionally greater demands on the public purse. As a result, in passing from S toward A', the economy will experience a rise in the share of public expenditures in G.N.P.; that is, public services are a luxury.

If equation (15) is the correct specification of the relationship between the public share and per-capita income, it becomes apparent why most cross-section studies of countries with countries of similar structure have failed

to provide support for Wagner's Law: the relationship between these two variables is neither linear nor log-linear. If one wishes simply to explain government expenditures, however, there is an alternative specification which might be used. Divide equation (13) by N:

$$G/N = - \theta\gamma/\mu + \theta y + (1 - \theta)\eta/N^{1-\alpha} \quad (18)$$

Equation (18) expresses per-capita public expenditures as a function of per-capita income and the reciprocal of population. Except for the latter term involving population, this equation corresponds to the per-capita specification of Wagner's Law which has been used by some authors as an alternative to the public share specification analysed above.

Finally, there is an elasticity measure which has been examined in the discussion of Wagner's Law, the elasticity of the public share with respect to per-capita income, which may be calculated from equation (15):

$$\xi(G/Y, y) \equiv \frac{\partial \ln(G/Y)}{\partial \ln y} = \frac{\theta}{G/Y} - 1 \quad (19)$$

A positive value would be consistent with Wagner's Law as some have interpreted it. Note that this elasticity will be a decreasing function of the size of the public share itself. Thus if Wagner's Law is correct in more highly developed economies there should be a tendency towards a leveling off of the relative size of the public sector as per-capita income increases.

III. EMPIRICAL RESULTS

In this section, we present estimates of the parameters of the demand for public expenditures as expressed by equation (18) and then use these results to calculate the price and income elasticities of equations (17) and (19). Consider first the data. The difficulties of obtaining reliable and comparable national accounts data for international comparisons are well known. For this reason, we limited our sample to 15 O.E.C.D. countries for which all the main components of government expenditures were readily available over the periods considered⁶. For each country, we examined non-military public expenditure, equal to the sum of consumption, transfers and gross capital formation less spending on defence. All variables in national monetary units were expressed in 1970 prices and converted into U.S. dollars at 1970 exchange rates. In order to minimize the importance of year-to-year disturbances we used a three-year average of values for 1962 to 1964 and for 1972 to 1974⁷.

The regression analysis⁸ is based on the per-capita version of the reduced form demand for non-military public expenditures (equation 18):

$$G/N = \beta_0 + \beta_1 y + \beta_2 N^{(-1+\alpha)} + \varepsilon \quad (20)$$

with $\beta_0 = -\theta\gamma/\mu$, $\beta_1 = \theta$, $\beta_2 = (1 - \theta)$ and ε an error term⁹.

There are two points which must be taken into consideration in the estimation of equation (20). First, this equation is non-linear in α , the measure of the publicness of public spending. Second, we have extraneous information concerning two of the parameters of the original model; namely, μ , the ratio of median to mean income, and γ , the minimum level of private-goods consumption. As may be seen from equation (20), this extraneous information implies a constraint on two of the reduced-form parameters:

$$\beta_0 = -\gamma^* \beta_1 / \mu^* \quad (21)$$

where the star indicates the known value. Accordingly, we tried two alternative maximum-likelihood (ML) methods, the first assuming that γ was deterministic, the second that it was stochastic with known variance.

A) Maximum likelihood estimates under a deterministic constraint

On the assumption that the error term of equation (20) is distributed normally with mean zero and with variance σ^2 , the likelihood function of the parameters β , α and σ^2 is:

$$L(G/N, \beta, \alpha, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp \left[-\frac{1}{2\sigma^2} (G/N - x^{(\alpha)}\beta)' (G/N - x^{(\alpha)}\beta)\right] \quad (22)$$

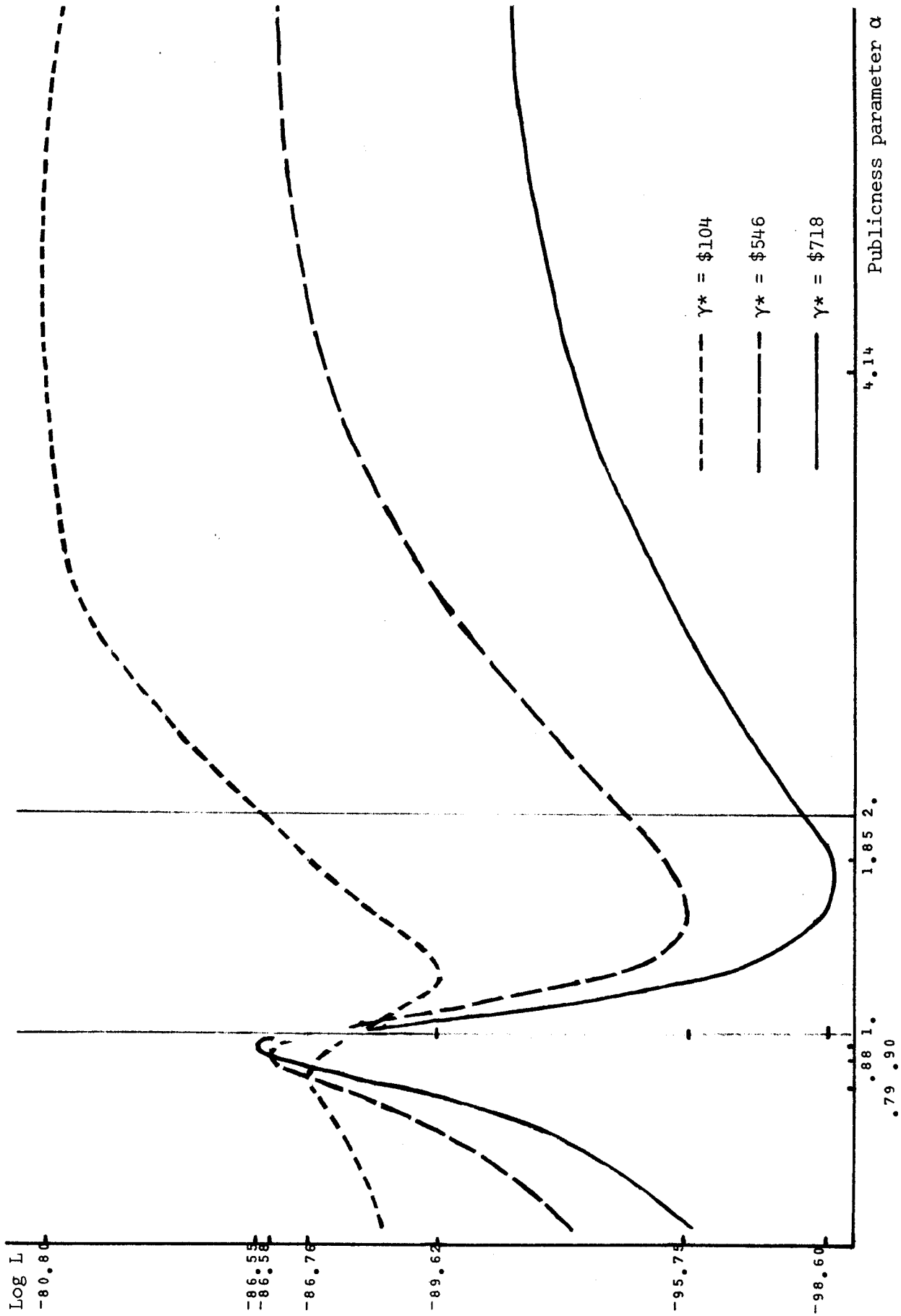
where $x^{(\alpha)} = (1, y, N^{-1+\alpha})$
 $\beta' = (\beta_0, \beta_1, \beta_2)$
 $n =$ number of observations

To explore the properties of the logarithm of this function, one may first estimate the parameters of equation (20) for different values of α by ordinary least squares and then use these results to calculate the log-likelihood.

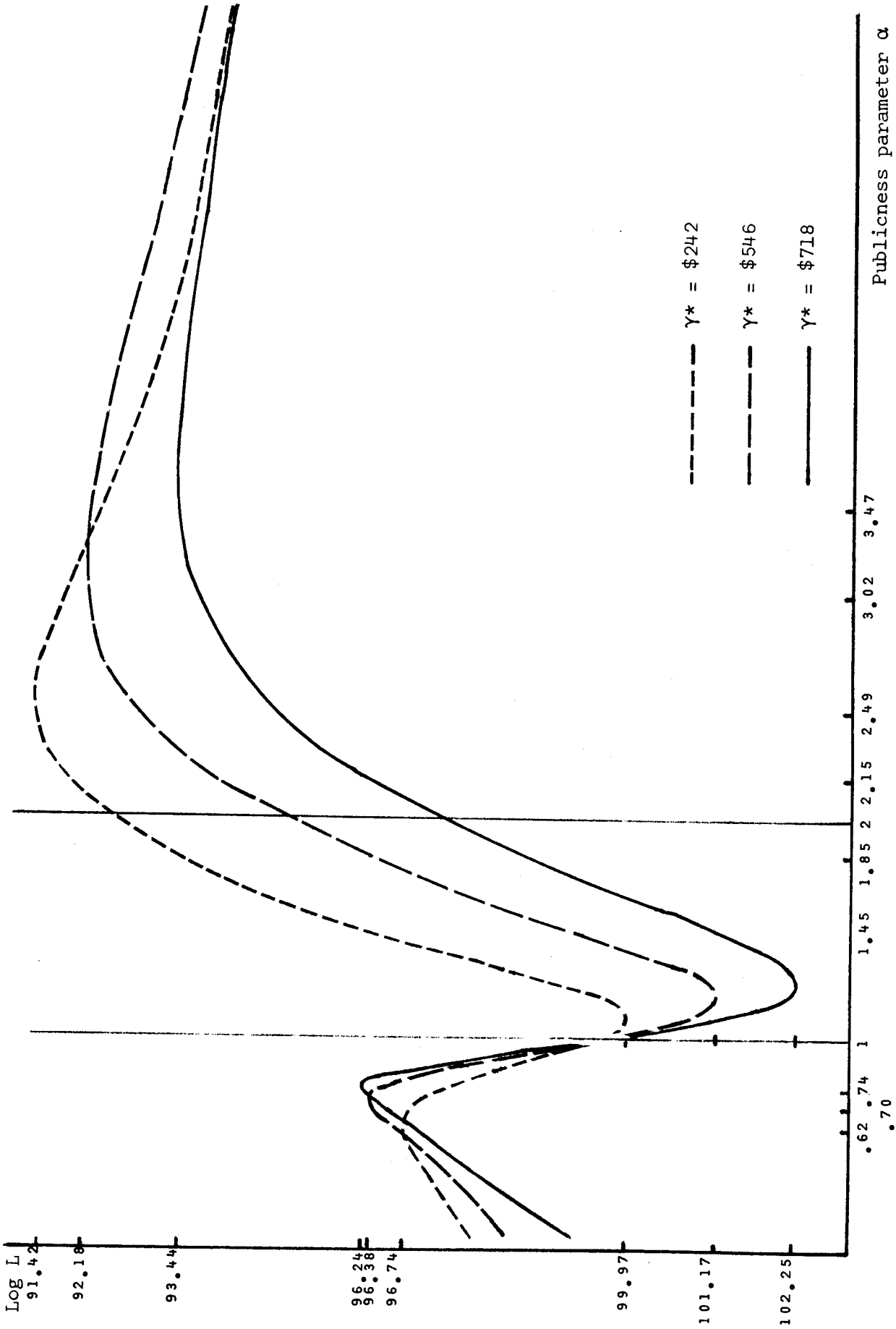
Consider first, however, the values chosen for the extraneous parameters in (21). Sawyer (1976, table 6)¹⁰ has calculated the Gini coefficient for 10 countries in our sample over the period 1966-1973¹¹. By assuming a lognormal income distribution, we were able to use his results to estimate an average value for μ^* , the ratio of median to mean income; namely, 0.8185.

In the case of γ^* , the minimum level of per-capita private-goods consumption, we selected several values in order to observe the sensitivity of our estimates to the value chosen. In our sample, the smallest observed level of per-capita private-goods consumption was that of Italy, while the lowest for all OECD member countries was that of Portugal. We chose the average of these two levels as a rough measure of γ . For 1962-1964, the resulting figure was \$546, while for 1972-1974, it was \$718, both expressed in 1970 U.S. dollars. For each period, a third value for γ^* was obtained by application of the Davidson-Fletcher-Powell method¹² directly to the logarithm of equation (22) without constraints on the parameters. The resulting estimates of γ^* were \$104 and \$242 for 1962-1964 and 1972-1974 respectively.

Given the values of γ^* and μ^* , we may then proceed to the estimation of the log-likelihood function in terms of α , the publicness parameter. Figure 2 presents three versions of the log likelihood function for 1962-1964, two corresponding to the constrained value for γ , and the third to the unconstrained value. Figure 3 presents similar results for 1972-1974.



1972-74 period



Publicness parameter α

A quick look at these figures will indicate that in each case this function has two peaks, one for α in the range between zero and one and a second with α greater than two. In four of the six cases, the global maximum is in the upper range. Moreover, even in the two cases for the earlier period in which the constrained maximum corresponds to a value for α lower than unity, the likelihood-ratio test for the hypothesis that γ is equal to the constrained value is rejected at the 0.01 significance level. In short, these results suggest a publicness parameter whose value is greater than two.

What are we to make of these results? On theoretical grounds, an α of two (or greater) makes little sense. It would imply that a citizen of Japan, which has roughly ten times the population of Belgium, would receive (over) 10 times less in "public" goods for dollar of public spending per capita than his Belgian counterpart! We are therefore led to question the specification of the model, in particular, the way in which a country's population affects the availability of public goods per dollar of spending. The specification used in equation (1) of this model, and in earlier studies such as Borchering and Deacon (1972), with population raised to the exponent α , tends to exaggerate the effects of small populations on per-capita public spending when α is less than unity -- an effect which is avoided when α is close to two. We may see this point by differentiating equation (20) partially with respect to population:

$$\frac{\partial(G/N)}{\partial N} = -\eta(1-\theta) (1-\alpha) N^{-(2-\alpha)}$$

As N decreases at relatively small values, with α less than one and η positive, the estimated level of per-capita public spending rises sharply. With α equal to unity, population has no effect on per-capita public spending.

However, with α close to two, population affects per-capita spending without an explosive effect at small populations: the first derivative in equation (23) is close to being constant.

Rather than experiment with alternative specifications of the publicness effect, we chose for the purposes of this paper to focus on those values of α which make sense from a theoretical point of view; that is, the range from zero (pure public good) through one (pure private good) to 1.4 (substantial diseconomies of scale in consumption). Under this new restriction, as is apparent from the graphs, the log-likelihood function reaches a maximum at values less than unity. For 1962-1964, the selected values of γ^* imply values for α ranging from 0.79 to 0.90. For 1972-1974, the corresponding range is from 0.62 to 0.74. In both periods, the highest value of the log-likelihood function is reached for $\gamma^* = \$718$. If we are prepared to limit α to values that are theoretically acceptable, therefore, these results suggest that government expenditures finance neither public nor private goods, but rather an intermediate type of quasi-public goods.

Finally, it is important to note that in all cases, the minimum of the log-likelihood function was obtained for α equal to one, the situation in which population has no effect on per-capita public spending. It would therefore be unacceptable to drop the population variable from the model.

B. Maximum likelihood estimates under a stochastic constraint

In order to obtain more precise estimates of equation (20) than those provided by the ML - deterministic - constraint method used in the preceding section, we may derive alternative ML estimates under the assumption of a

stochastic constraint on the parameter γ . Specifically let us define:

$$\gamma = \gamma^* + v \quad (23)$$

where v is an error term with mean 0 and standard error σ_v . After substitution from (23) the constraint (22) becomes

$$\beta_0 = - \frac{\gamma^*}{\mu^*} \beta_1 - \frac{\beta_1}{\mu^*} v$$

or $0 = \frac{1}{\beta_1} \left(\frac{\mu^* \beta_0}{\sigma_v} - \frac{\gamma^* \beta_1}{\sigma_v} \right) + h \quad (24)$

where $h = \frac{v}{\sigma_v}$ with zero mean and unit variance.

In matrix notation, we may express equation (24) as:

$$0 = \frac{1}{\beta_1} R\beta + h \quad (25)$$

with $R = \left(-\frac{\mu^*}{\sigma_v}, -\frac{\gamma^*}{\sigma_v}, 0 \right)$; $\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$

We now consider equation (24) as an additional observation (Theil (1971, p. 349)) to our system. In matrix notation:

$$\begin{pmatrix} G/N \\ 0 \end{pmatrix} = \begin{pmatrix} X^{(\alpha)} \\ \frac{1}{\beta_1} R \end{pmatrix} \beta + \begin{pmatrix} \varepsilon \\ h \end{pmatrix} \quad (26)$$

where $X^{(\alpha)} = (1, y, N^{(-1+\alpha)})$; $E \begin{pmatrix} \varepsilon \\ h \end{pmatrix} = 0$ and $E \begin{pmatrix} \varepsilon \\ h \end{pmatrix} \begin{pmatrix} \varepsilon \\ h \end{pmatrix}' = \begin{pmatrix} \sigma^2 I & 0 \\ 0 & I \end{pmatrix}$

The likelihood function of the parameters β, α, σ^2 is then:

$$f(N, 0; \beta, \alpha, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp \left[-\frac{1}{2\sigma^2} (G/N - X^{(\alpha)}\beta)' (G/N - X^{(\alpha)}\beta) - \frac{1}{2\beta_1^2} (RB)' (RB) \right]$$

Setting $\gamma^* = 546$, we assumed that σ_v was equal to \$107.50; that is one-half the difference between this value and the upper and lower limits represented by Italy and Portugal respectively, as described above. With $\mu^* = 0.8185$ from our earlier discussion we applied the Davidson-Fletcher-Powell method¹² of maximisation of the logarithm of the likelihood function, constraining α to the range ($0 < \alpha < 1.4$). The following ML point estimates with their corresponding asymptotic standard errors (in parentheses) were found:

$$1962-64: \quad G/N = -168.0 + .250y + 360.08N^{(-1+.884)}$$

$$\quad\quad\quad (38.6) \quad (.029) \quad (85.9) \quad (.073)$$

$$\text{Log L} = -86.583; \quad \sigma_{\epsilon}^2 = 6863.0$$

$$1972-74: \quad G/N = -217.0 + .319y + 705.72N^{(-1+.703)}$$

$$\quad\quad\quad (52.2) \quad (.042) \quad (226.8) \quad (.151)$$

$$\text{Log L} = -96.378; \quad \sigma_{\epsilon}^2 = 25377.0$$

All the coefficients are relatively large with respect to their standard errors while the values of the log likelihood and the relevant asymptotic variances confirm the goodness of fit.

The hypothesis that the publicness parameter α is equal to zero (the pure public good hypothesis) is rejected for both periods. However in each period the same parameter is also significantly less than unity at respectively

the .07 and .037 levels, under a one-tailed test. It would appear, therefore, that there are appreciable economies of scale in non-military government purchases.

It is interesting to note that the degree of publicness of government non-military expenditures rose between the two periods, with values for α moving from .884 for 1962-64 to .703 in the recent period. Since it is primarily transfer payments which have increased between these periods¹³, these results support Orr's (1976, 1978) contention that income transfers should be considered to be public goods.

C. Income and price elasticities of the demand for public spending

To test Wagner's Law, we must determine the critical population level (equation 16) at which the elasticity of the share of public expenditure in G.N.P. with respect to per-capita G.N.P. (equation 19) changes sign from negative to positive. For the earlier period this value is 733 million and therefore, Wagner's Law is rejected for this sample. The elasticity of the public share with respect to per-capita income (equation 19) computed at mean values is negative (-.15). This result is consistent with the relatively low degree of publicness of non-military government expenditures realised over that period as mentioned above. For the 1972-74 period, the critical population value is 56.5 million. Therefore, as a group, Japan and the United States satisfied Wagner's Law with a per-capita income elasticity of the public share equal to 0.35. However, for a second group of countries composed of France, Italy, West Germany and the United Kingdom, with populations close to the critical value this elasticity became negative, at -0.09.

For the countries with populations under 50 million¹⁴, the per-capita income elasticity of the public share was even lower, at -0.14. In short, with the exception of the U.S.A. and Japan in the more recent period, these results fail to confirm Wagner's Law.

Finally, for a given publicness parameter α , the regression results indicate a fall in per-capita public expenditure as population increases. Accordingly the demand for quasi-public goods (equation 17) appears to be inelastic, with price elasticities of -.96 and -.93 computed at mean values for the two periods.

IV. CONCLUSION

The use of the theory of consumer demand and the median voter model rather than ad hoc specifications to explain the relative size of the government sector among countries was shown to affect not only the specification of the relationship to be estimated but also the method of estimation. The study suggested the importance of a nation's size, as measured by its population, in determining its non-military public expenditures.

If there are significant scale economies in the consumption of the goods purchased with tax revenues, a country's population determines the amount of a public (or quasi-public) good available to each taxpayer for a given amount of total expenditure. One important finding of this study was that specifications which omitted this publicness effect minimized the log-likelihood function of our model. However, since the specification of this publicness effect that has been used in other studies produced theoretically unacceptable results when applied to our data, we chose to constrain the corresponding parameter within an acceptable range. Under this restriction, maximum-likelihood estimates suggest a significant, and possibly increasing, degree of publicness in government non-military expenditures.

Population was also shown to have a significant effect on the way per-capita income influences the public share. For the smaller countries which make up the bulk of our sample, the elasticity of the public share with respect to per-capita income was negative. Only for the two largest countries in the second of our two sub-periods was this elasticity positive. These results fail to confirm what has become known as Wagner's Law.

Finally, a country's population determines the tax share required from the median voter if a given level of availability of the public good is to be financed. We found the elasticity of demand with respect to this "perceived price" to be less than unity.

How might these somewhat surprising cross-section results - in particular the negative per-capita-income elasticity of the public share for countries with a small population - be reconciled with time-series evidence from other studies? Such studies have yielded positive public share income elasticities, even for small countries. The answer would appear to lie in the difference between estimation with a constant structure and estimation with a changing structure. Over time, there are many factors other than levels of per capita income which may affect the demand for public spending; for example, the degree of urbanization and education, or the concentration of power in the private sector. It is perhaps for this reason that Wagner himself did not limit his Law to the simple effects of changing per-capita income. If so, the time-series verification of Wagner's Law has yet to be properly specified.

APPENDIX

DERIVATION OF ELASTICITY MEASURES USED IN TEXT

1. Elasticity of the public share with respect to per-capita income

From equation (18) $\partial(G/N)/\partial y = \theta$

$$\frac{y}{G/N} \frac{\partial(G/N)}{\partial y} = \frac{y\theta}{G/N}$$

$$\xi(G/N), y \equiv \frac{\partial \ln(G/N)}{\partial \ln y} = \frac{\theta}{G/Y} \quad (A1)$$

By definition $G/Y = \frac{G}{N} \cdot \frac{1}{y}$

Take natural logarithms

$$\ln(G/Y) = \ln(G/N) - \ln y$$

$$\frac{\partial \ln(G/Y)}{\partial \ln y} = \frac{\partial \ln(G/N)}{\partial \ln y} - 1$$

From equation (A1) $\xi(G/Y, y) \equiv \partial \ln(G/Y)/\partial \ln y = \frac{\theta}{G/Y} - 1 \quad (19)$

2. Elasticity of the demand for public expenditure with respect to its perceived price

$$\ln G + \ln(1/N) = \ln(G/N)$$

$$\frac{\partial \ln G}{\partial \ln(1/N)} + 1 = \frac{\partial \ln(G/N)}{\partial \ln(1/N)}$$

From equation (18) $\frac{\partial \ln G/N}{\partial \ln (1/N)} = \eta(1 - \theta) (1 - \alpha)N^\alpha/G$

$$\xi(G, 1/N) \equiv \partial \ln G / \partial \ln (1/N) = -[1 - \eta(1 - \theta) (1 - \alpha)N^\alpha/G] \quad (17)$$

FOOTNOTES

- 1 Deacon (1978) avoids this problem by assuming a Rotterdam system of consumer demand equations.
- 2 Wagner's ideas, which with one exception are not available in English, are reviewed by Peacock and Wiseman (1967) and Bird (1971).
- 3 For a review of the individual country studies, see Bird (1971). The results of a recent study of non-defence public expenditures in the United States by Henning and Tussing (1974) are also consistent with Wagner's Law.
- 4 Lall (1969) discovered no relationship between per capita G.N.P. and the G.N.P. share of various public expenditure categories for 46 developing countries, while Musgrave (1969, 110-124) found no support for such a relationship in countries at the upper end of the per capita income scale. However, Chester (1977) regressed the public share on per capita income and variables measuring union density and the proportion of Catholics in the population for 16 developed countries. The income coefficient was significantly different from zero and positive at the .05 level with a one-tailed test.
- 5 This hypothesis is fairly realistic when the total incidence of the tax system is taken into account. See, for example, Musgrave, Case and Leonard (1974) and Gillespie (1976).
- 6 Spain and Portugal were dropped from the sample as the median voter model was inconsistent with the political regime of those countries.
- 7 Main sources of data are: i) National Accounts O.E.C.D., 1960-1971, 1962-1973, 1975; ii) O.E.C.D. Development Cooperation, 1976; iii) International Monetary Fund, International Financial Statistics, supplement to 1966/67; U.N. Statistical Yearbooks, 1966, 1976. Complete details on sources and construction of variables are available from the authors.
- 8 Ad hoc specifications in terms of non defence public spending in G.N.P. found in previous cross section studies, for example in Lall (1969) and Williamson (1961), were tried with our sample data and showed no significant relationship between per capita G.N.P. and the public share. The specification of Gupta (1968) yielded a near singular matrix.
- 9 Since the dependent variable per capita public expenditure is a component of one of the explanatory variables, per capita income, there is a problem of simultaneity in this specification. The result is a possible upward bias in the estimate of β_1 .

- 10 Based on post-tax income data which are generally more reliable than pre-tax income data. With the latter set of data over the same countries, $\mu^* = .7946$. Note that under our assumption of a proportional income tax system, measures of income inequality should be considered the same for the before and after tax income distributions (Montmarquette, 1974).
- 11 The Gini relative mean difference measure used by Sawyer is equal to the Lorenz measure following Dalton (1920, 353-354). Aitchison and Brown (1963, 154) relates theoretical Lorenz measure to the ratio of the mean to the median of a lognormal income distribution.
- 12 See Fletcher and Powell (1963).
- 13 On average per-capita transfer expenditures have increased over the periods considered by 125.6% compare to 41.7% for consumption expenditures.
- 14 The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, Norway and Sweden.

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