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EQUAL FACTOR EQUIVALENCE IN ECONOMIES WITH MULTIPLE PUBLIC GOODS

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RÉSUMÉ

Nous réexaminons le problème de la provision et du financement des biens publics. Nous proposons de rendre chaque agent indifférent entre l'allocation recommandée et la possibilité de déterminer la production des biens publics sous la contrainte d'en financer *r* fois le coût, *r* étant le plus petit nombre assurant la réalisabilité. Nous caractérisons cette règle d'allocation dans les économies possédant un continuum d'agents par la propriété d'efficacité, une borne supérieure au bien-être de chacun et une exigence de solidarité face aux changements dans la population et les préférences.

Mots clés : biens publics, équivalence égalitaire

ABSTRACT

We consider the problem of provision and cost-sharing of multiple public goods. The efficient equal factor equivalent allocation rule makes every agent indifferent between what he receives and the opportunity of choosing the bundle of public goods subject to the constraint of paying *r* times its cost, where *r* is set as low as possible. We show that this rule is characterized in economies with a continuum of agents by efficiency, a natural upper bound on everyone's welfare, and a property of solidarity with respect to changes in population and preferences.

Key words : public goods, egalitarian equivalence

1 INTRODUCTION

This note reconsiders the problem of provision and cost-sharing of public goods : how much of each good should be produced and how should the corresponding cost be shared among the participating agents? Our concern is to find an efficient and equitable solution to this problem. If the technology for producing the public goods is the common property of the agents, it is natural to demand that the solution belong to the free-access core in the sense of Foley (1970) : no coalition of agents should be able to make all its members better off by producing a different bundle of public goods and fully covering its cost. The core interpretation of the ideal of fairness thus emphasizes the need to carefully take into account all cooperative opportunities available to the agents. When only one public good can be produced, the free-access core is nonempty. Several interesting selections of it have been proposed and analyzed in the literature. Among the best known are the ratio equilibrium [Kaneko (1977)] and the public good egalitarian-equivalent solution [Mas-Colell (1980) and Moulin (1987)].

Unfortunately, the core may easily be empty in problems involving multiple public goods. Moulin (1995) offers a simple example with quasi-linear convex preferences and a linear technology. On the other hand, the more basic requirement that no *individual* agent object to the recommended solution can still be met and disqualifies naïve solutions such as equal cost-sharing. The problem, then, is to suggest principles that are compatible with the individual stand alone constraints and can be met in the multiple goods problem.

The key principle proposed and analyzed in this note is a variant of the general idea of solidarity. Solidarity offers a view of equity that is markedly different from the core interpretation. Loosely speaking, it requires that all relevant agents be affected in the same direction when changes occur in variables over which they have no control [Thomson (1990)]. These variables may be the population under consideration, the technology itself, on even the preferences of the other agents. The reason for demanding solidarity is always the same : since none of the relevant agents has any particular responsibility in these changes, it would be unfair to reward one of them by a welfare gain while punishing another through a welfare loss.

The particular form of solidarity that is considered in this note pertains to (possibly joint) changes in population and preferences. Quite interestingly, this type of solidarity is compatible with the individual stand alone constraints *and* with another principle of fairness proposed by Moulin (1992) under the name All Sorry to Disagree. To understand Moulin's idea, observe that if everybody's preferences could be represented by the same utility function, the solution to our public goods problem would follow directly from the uncontroversial requirements of efficiency and equal treatment of equals : everyone should enjoy the same utility level, which in turn should be as high as possible. Let us now define an agent's unanimity (utility) level to be the level he would reach if all other agents shared his own preferences. In public goods problems, the vector of unanimity utility levels never lies below the Pareto frontier; when preferences differ, it will generally be strictly above the frontier : disagreeing constitutes a burden. Moulin's principle asks that everyone take a share of this burden : formally, no agent should end up above his unanimity utility level. This is the All Sorry to Disagree axiom.

All Sorry to Disagree, efficiency, and solidarity (with respect to population and preferences) are all satisfied by the (efficient) "equal factor equivalent" solution rule discussed in Moulin (1992, 1995).[1] This rule gives to each agent the utility that he would obtain if he could freely decide how much to produce of each public good subject to the constraint of paying r times the corresponding cost, where r is the smallest number that ensures feasibility. We will show that, in well-behaved large economies, the efficient equal factor equivalent rule is in fact the only rule satisfying

1 The term "equal ratio equivalent" suggested by Moulin (1992) is appropriate when the number of agents is finite since the parameter r to be defined shortly varies between 0 and 1 in that case. The phrase "equal factor equivalent" is better suited to the continuous case that we shall consider.

the three principles just mentioned. This implies, in particular, that the individual stand alone constraints are implied by our three principles since the efficient equal factor equivalent rule obviously satisfies them.

The reader familiar with the idea of egalitarian equivalence may have recognized that the efficient equal factor equivalent rule for the public goods problem closely resembles Pazner and Schmeidler's (1978) classical rule for the problem of fair division of private goods. Pazner and Schmeidler's rule makes every agent indifferent between what he gets and r times the mean endowment, r being chosen as high as possible. Sprumont and Zhou (1995) have characterized that rule in large economies by the properties of efficiency, solidarity (with respect to population and preferences), and the so-called Equal Split Lower Bound axiom : each agent should be as well off as if he consumed the mean endowment. Our characterization of the efficient equal factor equivalent rule is the public goods counterpart of Sprumont and Zhou's result. In the fair division problem, the differences in preferences create exchange opportunities. Equal Split Lower Bound requires, in effect, that everyone benefit from these opportunities. Thus, it expresses precisely the same general idea as All Sorry to Disagree in the public goods problem : differences in preferences should affect everyone in the same direction. This general principle was first articulated by Moulin (1990).

This note is organized as follows. In the next section, we set up our model of the public goods problem. In Section 3, we prove our characterization result under a simplifying technical assumption on the cost function. Section 4 and the Appendix discuss the robustness of the result and contain some additional comments.

2 THE PUBLIC GOODS PROBLEM

Consider an economy with m public goods and one private good. Each of the public goods $i = 1, \ldots, m$ can be produced at any non-negative level up to some maximum $\bar{y}_i$.

It costs $x = c(y)$ units of the private good to produce the vector of public goods $y = (y_1, \ldots, y_m) \in Y := \Pi_{i=1}^m [0, \bar{y}_i]$. Throughout this note, the *cost function* $c : Y \to \mathbb{R}_+$ is fixed and assumed to be continuous, strictly increasing, strictly convex, and to satisfy $c(0) = 0$. In the current and the next sections, we furthermore assume that the cost function is *regular* in the following sense :

$$\lim_{y_i \to 0} c(0, \ldots, 0, y_i, 0, \ldots, 0) / y_i = 0 . \tag{1}$$

A *preference* is a binary relation $\succsim$ over $\mathbb{R} \times Y$ that is complete, transitive, continuous, convex, strictly increasing in y, strictly decreasing in x (recall that x measures a quantity of the private good that is destroyed in the production process) and satisfies the following condition :

$$\exists \bar{x} \in \mathbb{R}_+ : (-\bar{x}, 0) \succsim (0, \bar{y}) . \tag{2}$$

This last assumption implies that no bundle of public goods in Y is so desirable that it is better than any consumption of the private good. The strict preference relation associated with $\succsim$ is $\succ$. The set of all preferences is P.

A technical but important point should be explained at this stage. Because of the regularity assumption (1), any agent who could choose the bundle of public goods subject to paying r times its cost would select a strictly positive bundle for any positive value of r. This simplifying feature will allow us to prove in Section 3 a basic version of our characterization result in a fairly straightforward manner. Assumption (1) however, is neither realistic nor necessary. It will be dropped in Section 4 and in the Appendix at the cost of some technical complications.

The economies we consider have a continuum of agents. Because we do not want to discriminate between agents with identical preferences, we follow Hart and Kohlberg (1974) and model an economy in anonymous terms. For simplicity, we only consider economies with finitely many preferences. Formally, then, an *economy* is a mapping $\mu : P \to \mathbb{R}_+$ such that $\{\succsim \in P | \mu(\succsim) > 0\}$ is a finite (nonempty) set which we call the *support* of μ and denote by supp(μ). The number $\mu(\succsim)$ represents the mass of agents having preference $\succsim$ in the economy μ. We will slightly abuse our notations and write

$$\mu(A) := \sum_{\succsim \in A \cap \mathrm{supp}(\mu)} \mu(\succsim)$$

for any $A \subset P$. An allocation for the economy μ, or a μ*-allocation*, is a pair (ξ, y) where ξ is a mapping from P to $\mathbb{R}, y \in Y$, and

$$\sum_{\succsim \in \mathrm{supp}(\mu)} \mu(\succsim)\xi(\succsim) = c(y). \tag{3}$$

Here, $\xi(\succsim)$ is the quantity of the private good that (each agent with) preference $\succsim$ contributes to the production process. The μ-allocation (ξ, y) is *efficient* if there is no other μ-allocation (ξ', y') such that $(\xi'(\succsim), y') \succ (\xi(\succsim), y)$ for all $\succsim \in \mathrm{supp}(\mu)$. This weak version of efficiency is equivalent to the usual strong version because preferences are continuous and strictly decreasing in x and because there is no bound on the private good transfers across agents.

If A is a subset of P, E_A denotes the set of economies whose support is included in A.[2] An *(allocation) rule (on E_A)* is a mapping F from $P \times E_A$ to $\mathbb{R} \times Y$ such that $F(\cdot, \mu)$ is a μ-allocation whenever $\mu \in E_A$.[3]

3 EQUAL FACTOR EQUIVALENCE

It is now time to give a precise definition of equal factor equivalence. For each $a \in \mathbb{R}$, define

[2] Clearly, E_A includes the economies whose entire mass is concentrated on a single preference in A. As pointed out by a referee, such economies are unrealisitic. In the variable-preference framework of this paper, however, an allocation rule should be regarded as a "constitution" known to the agents "before" they express their preferences. If everyone is to be given the same "freedom of speech", then even the most unrealistic economies should be included in the domain of a well-defined rule.

[3] It bears repeating that the names of the agents are not a part of the definition of an economy in the present model. Every allocation rule is thus implicitly constrained to be anonymous.

$$C(a) = \begin{cases} \{(x,0) \in \mathbb{R} \times Y \mid x \geq -a\} & \text{if } a \leq 0, \\ \{(x,y) \in \mathbb{R} \times Y \mid x \geq c(y)/a\} & \text{if } a > 0. \end{cases}$$

For any $\succsim \in P$, let $Z(\succsim) = \{(x,y) \in \mathbb{R} \times Y \mid (0,\bar{y}) \succ (x,y)\}$. There is a unique (continuous) utility function $u_{\succsim} : Z(\succsim) \to \mathbb{R}$ that represents the restriction of $\succsim$ to $Z(\succsim)$ and satisfies the normalization condition :

$$\max_{C(a)} u_{\succsim}(x,y) = a \text{ for all } a \in \mathbb{R}. \tag{4}$$

Note that $u_{\succsim}(0,0) = 0$ and $\lim_{y \to \bar{y}} u_{\succsim}(0,y) = \infty$. Observe also that condition (1) is crucial to guarantee that $u_{\succsim}$ exists. If (1) were violated, the best bundle of $\succsim$ in $C(a)$ could be $(0,0)$ for a whole range of (sufficiently small) positive values of a. In such a case, no function $u_{\succsim}$ satisfying (4) would represent the restriction of $\succsim$ to $Z(\succsim)$.

From now on and until the end of the section, fix an arbitrary subset A of P. Let μ be an economy in E_A. We say that a μ-allocation (ξ, y) is *equal factor equivalent* if $(\xi(\succsim), y) \in Z(\succsim)$ for all $\succsim \in \text{supp}(\mu)$ and there exists some $a \geq 0$ such that

$$u_{\succsim}(\xi(\succsim), y) = a \text{ for all } \succsim \in \text{supp}(\mu).$$

The latter condition means that everyone in the economy μ is indifferent between what he receives and choosing the bundle of public goods subject to paying $1/a$ times its cost. Many μ-allocations are equal factor equivalent but only one is efficient because of our (strict) convexity assumptions. The *efficient equal factor equivalent rule* on E_A associates with each economy in E_A its efficient equal factor equivalent allocation.

We now turn to the formal definitions of our axioms. In what follows, F is an allocation rule on E_A. The first property is standard.

Efficiency. For each $\mu \in E_A, F(\cdot, \mu)$ is an efficient μ-allocation.

For each $\mu \in E_A$ and each $\succsim \in \text{supp}(\mu)$, define the economy $\mu_{\succsim}$ by the conditions that $\mu_{\succsim}(\succsim) = \mu(A)$ and $\mu_{\succsim}(\succsim') = 0$ for every preference $\succsim'$ other than $\succsim$. All Sorry to Disagree forbids $\succsim$ to be better off in μ than in the unanimous economy of identical mass $\mu_{\succsim}$.

All Sorry to Disagree. For each $\mu \in E_A$ and every $\succsim \in \text{supp}(\mu)$, $F(\succsim, \mu_{\succsim}) \succsim F(\succsim, \mu)$.

A first consequence of this axiom is that $F(\succsim, \mu)$ belongs to $Z(\succsim)$ for all $\mu \in E_A$ and $\succsim \in \text{supp}(\mu)$, so that $u_{\succsim}(F(\succsim, \mu))$ is well defined. Furthermore, notice that if F is efficient, $u_{\succsim}\left(F\left(\succsim, \mu_{\succsim}\right)\right) = \mu(A)$. In that case, All Sorry to Disagree requires that $u_{\succsim}(F(\succsim, \mu)) \leq \mu(A)$ for each $\mu \in E_A$ and every $\succsim \in \text{supp}(\mu)$.

Finally, we state our condition of solidarity with respect to changes that may occur, perhaps simultaneously, in population and preferences. If $\mu_1, \mu_2 \in E_A$, let $\mu_1 \wedge \mu_2$ denote the economy where $(\mu_1 \wedge \mu_2)(\succsim) := \min\{\mu_1(\succsim), \mu_2(\succsim)\}$ for each $\succsim$ in P.

Solidarity. For any $\mu_1, \mu_2 \in E_A$, one of the following statements holds :

(i) $F(\succsim, \mu_1) \succsim F(\succsim, \mu_2)$ for all $\succsim \in \text{supp}(\mu_1 \wedge \mu_2)$,

(ii) $F(\succsim, \mu_2) \succsim F(\succsim, \mu_1)$ for all $\succsim \in \text{supp}(\mu_1 \wedge \mu_2)$.

Solidarity implies the familiar property of *Population Solidarity* by taking $\mu_1 \leq \mu_2$: everybody must be affected in the same direction by an increase in population. Taking μ_1 and μ_2 of identical masses, Solidarity implies *Preference Solidarity* : when the preferences of some agents are modified, all the remaining agents are affected in the same direction. A strict version of the latter property is analyzed in a more abstract framework by Sprumont (1996). See also Thomson (1993a, b) for an application to single-peaked economies.

We are now ready to prove the following version of our characterization result.

Theorem 1 *Suppose that the cost function c is regular and let A be an arbitrary subset of P. An allocation rule on E_A satisfies Efficiency, All Sorry to Disagree, and Solidarity if and only if it is the efficient equal factor equivalent rule.*

Proof. Suppose that c is regular and fix $A \subset P$. It is easy to check that the efficient equal factor equivalent rule on E_A satisfies the three axioms of Theorem 1. Conversely, let F be an allocation rule on E_A that satisfies these axioms. Fix $\mu \in E_A$. It will be convenient to write supp$(\mu) = \{\succsim_t \mid t \in T\}$. By assumption, T is finite. To alleviate notations, we write t instead of $\succsim_t$ whenever this causes no confusion. All economies in the rest of the proof are assumed to have the same support as μ unless mentioned otherwise. For an economy λ and $t \in T$, we write $\alpha(t,\lambda) = u_t(F(t,\lambda))$. Since $F(\cdot,\mu)$ is by assumption efficient, we need only show that

$$\alpha(t,\mu) = \alpha(q,\mu) \text{ for all } t,q \in T.$$

For the rest of the proof, we fix an arbitrary $q \in T$. For Steps 1 and 3, we also fix an arbitrary $t \in T$.

Step 1. Let R_t and R_q denote the respective ranges of $\alpha(t,\cdot)$ and $\alpha(q,\cdot)$. Define the correspondence ϕ as follows. For each $a \in R_q$,

$$\phi(a) = \{b \mid \text{ there is an economy } \lambda \text{ such that } \alpha(q,\lambda) = a \text{ and } \alpha(t,\lambda) = b\}.$$

By definition, the range of ϕ is R_t and

$$\alpha(t,\lambda) \in \phi(\alpha(q,\lambda)) \text{ for every economy } \lambda. \tag{5}$$

By Solidarity, ϕ is *nondecreasing* in the following sense :

$$\sup\phi(a) \le \inf\phi(a') \text{ for all } a,a' \in R_q \text{ such that } a < a'.$$

We will show that ϕ is in fact single-valued and that $\phi(a) = \{a\}$ for each $a \in R_q$. To this end, we first establish in the following step a useful lemma. It says that when an economy gains mass and converges to one where all preferences are $\succsim_q$, q's utility level converges to its level in this limiting unanimous economy.

Step 2. Let $a \in (0, \infty)$. Let $\{a_n\}$ be a sequence of numbers in $(0, a)$ converging to a and let $\{\lambda_n\}$ be a sequence of economies such that $\lambda_n(A) = a_n$ for all n and $\lambda_n(q) \to a$. We claim that $\alpha(q, \lambda_n) \to a$.

Suppose this claim is false. By All Sorry to Disagree, there exists some $\varepsilon > 0$ such that, for all n,

$$\alpha(s, \lambda_n) \leq a_n \text{ for all } s \in T \text{ and } \alpha(q, \lambda_n) < a - \varepsilon. \tag{6}$$

Since T is finite, assumption (2) allows us to find some $\bar{x} \in \mathbb{R}_+$ such that

$$(-\bar{x}, 0) \succ_s (0, \bar{y}) \text{ for all } s \in T. \tag{7}$$

For each n, let y^n be the (unique) maximizer of $u_q(c(y)/a_n, y)$ over Y and define

$$\begin{aligned} F^n(s, \lambda_n) &= (-\bar{x}, y^n) \text{ for all } s \in T \setminus \{q\}, \\ F^n(q, \lambda_n) &= \left(\frac{c(y^n) + (\lambda_n(A) - \lambda_n(q))\bar{x}}{\lambda_n(q)}, y^n \right). \end{aligned}$$

Clearly, $F^n(\cdot, \lambda_n)$ is a λ_n-allocation and, by (6) and (7), $F^n(s, \lambda_n) \succ_s F(s, \lambda_n)$ for all $s \in T \setminus \{q\}$ and all n. Letting y^∞ denote the maximizer of $u_q(c(y)/a, y)$, we observe that $F^n(q, \lambda_n) \to (c(y^\infty)/a, y^\infty)$ as $n \to \infty$. Since $F^n(q, \lambda_n) \in Z(q)$ for each n and u_q is continuous, $u_q(F^n(q, \lambda_n)) \to a$. For n large enough, therefore, $u_q(F^n(q, \lambda_n)) > \alpha(q, \lambda_n)$, making $F(\cdot, \lambda_n)$ inefficient. This is the desired contradiction.

Step 3. We now claim that

$$a \leq b \text{ for all } a \in R_q \cap (0, \infty) \text{ and all } b \in \phi(a). \tag{8}$$

To see why this is true, fix $a \in R_q \cap (0, \infty)$ and $b \in \phi(a)$. Let $\{a_n\}$ be a sequence in $(0, a)$ converging to a and $\{\lambda_n\}$ be a sequence of economies such that $\lambda_n(A) = a_n$ for all n and $\lambda_n(t) \to a$. By (5),

$$\alpha(t, \lambda_n) \in \phi(\alpha(q, \lambda_n)) \text{ for all } n. \tag{9}$$

By All Sorry to Disagree, $\alpha(q, \lambda_n) \le a_n < a$ for all n. Because of (9) and since ϕ is nondecreasing, $\alpha(t, \lambda_n) \le b$. On the other hand, $\alpha(t, \lambda_n) \to a$ by Step 2 since q was chosen arbitrarily and could be set equal to t. Hence $a \le b$.

This argument is easily adapted to show that

$$a \ge b \text{ for all } a \in R_q \text{ and all } b \in \phi(a) \cap (0, \infty). \tag{10}$$

Indeed, fix a and b as required in (10) and consider now a sequence $\{b_n\}$ in $(0, b)$ converging to b and a sequence $\{\lambda_n\}$ of economies such that $\lambda_n(A) = b_n$ for all n and $\lambda_n(q) \to b$. Because of (5), (9) still holds. By All Sorry to Disagree, $\alpha(t, \lambda_n) \le b_n < b$ for all n. Because of (9) and since ϕ is nondecreasing, $\alpha(q, \lambda_n) \le a$ for each n. But we have $\alpha(q, \lambda_n) \to b$ by Step 2. Hence $a \ge b$.

It is now easy to see that (8) and (10) imply

$$\phi(a) = \{a\} \text{ for all } a \in R_q \cap (0, \infty). \tag{11}$$

To check (11), fix $a \in R_q \cap (0, \infty)$. By (8), $a \le \inf \phi(a)$. Let us now verify that $a \ge \sup \phi(a)$. If this inequality were false, $b > a$ for some $b \in \phi(a)$. Since $a > 0$ by assumption and $b < \infty$, we know that $b \in (0, \infty)$. Thus (10) implies $a \ge b$, a contradiction.

Step 4. As t was chosen arbitrarily in Steps 1 and 3, we have established that for each $t \in T$, there is a nondecreasing correspondence ϕ_t from R_q onto R_t such that $\phi_t(a) = \{a\}$ whenever $a \in (0, \infty)$. To complete the proof, it only remains to be shown that $R_q \subset (0, \infty)$. All Sorry to Disagree guarantees that $\alpha(q, \lambda) < \infty$ for each λ. Suppose now that $\alpha(q, \lambda) \le 0$ for some λ. By Step 2, we know that $R_q \cap (0, \infty)$ is dense in $(0, \infty)$. Since every ϕ_t is nondecreasing on the whole of R_q, we conclude that $\alpha(t, \lambda) \le 0$ for each $t \in T$. This contradicts Efficiency. Q.E.D.

4 DISCUSSION

4.1 Comparing the efficient equal factor equivalent rule with Pazner-Schmeidler's rule

As mentioned in the introduction, our characterization of the efficient equal factor equivalent rule is the public goods counterpart of Sprumont and Zhou's characterization of the Pazner-Schmeidler rule in the fair division problem. In spite of this symmetry, important differences between the two results - and indeed between the two rules - should be kept in mind.

(1) Efficiency plays a much more important role in Theorem 1 than in Sprumont and Zhou's characterization. In the fair division problem, Solidarity and Equal Split Lower Bound are enough to obtain egalitarian equivalence along the ray through the mean endowment. The role of Efficiency is merely to force the indifference surfaces to intersect as high as possible on that ray. In the public goods problem, by contrast, Solidarity and All Sorry to Disagree do not require, in the absence of Efficiency, to equalize the utility representations of the preferences defined by (4). This is most easily seen when there is only one public good and preferences are strictly convex. For each economy μ, let $a^*(\mu)$ be the common utility level reached by all preferences in the support of μ at the efficient equal factor equivalent μ-allocation. (Utility representations are still chosen according to (4)). Consider now the following rule : in each economy μ, give to each preference a bundle indifferent to its maximizer under the smallest *linear* function *above* the cost function $c/a^*(\mu)$. This rule meets the requirements of Solidarity and All Sorry to Disagree, but fails to equalize the utility representations defined by (4). It is inefficient.

(2) While the efficient equal factor equivalent rule satisfies Population Solidarity, it generally fails to be population *monotonic* in the usual sense of that property in

the public goods problem. This fact is no surprise in the presence of multiple goods (since population monotonicity implies the core constraints which may themselves be impossible to respect), but also holds when there is only one public good [Moulin (1992)]. By contrast, Pazner-Schmeidler's rule *is* population monotonic in the usual sense given to this property in the context of fair division.

4.2 Dropping the regularity assumption

The efficient equal factor equivalent rule makes every preference indifferent between what it gets and choosing the bundle of public goods subject to the constraint of paying r times its cost, r being as small as feasible. Under restriction (1) on the cost function, this amounts to equalizing the utilities normalized according to (4) at the highest possible level. Thus, the efficient equal factor equivalent rule actually satisfies the following strict version of Solidarity : for any two economies μ_1 and μ_2, either all $\succsim$ in the support of $\mu_1 \wedge \mu_2$ strictly prefer μ_1 to μ_2, or they all strictly prefer μ_2 to μ_1, or they all are indifferent. This *Strict Solidarity* condition is implied by our three axioms essentially because the unanimity utility level associated with any given preference strictly increases with the total mass of the economy. Condition (1) is crucial to guarantee this strict monotonicity.

If (1) fails, the efficient equal factor equivalent rule no longer equalizes any utility representations of the preferences. If it did, it would satisfy Strict Solidarity, which it does not[4] : maximizing under the cost function rc may be strictly better than under $r'c$ for some preference but only just as good for another (if its best bundle is zero in both cases). It turns out, however, that Theorem 1 does carry over if we drop the regularity assumption on the cost function. A proof of this fact is given in the Appendix.

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[4] This is yet another difference with Pazner-Schmeidler's rule for the fair division problem.

4.3 Relaxing the solidarity axiom

It is easy to see that Efficiency, All Sorry to Disagree, and Solidarity are independent properties. As Solidarity is a strong axiom, it is natural to try and weaken it. Our characterization theorem no longer holds if Solidarity is replaced with the combination of Population Solidarity and Preference Solidarity as defined in Section 3, just before Theorem 1. To see this, suppose $m = 1$ and $c(y) = \frac{1}{2}y^2$ for $y \in Y := [0,6]$. Let $\succsim_1, \succsim_2$ be the preferences in P that admit the following utility representations w_1, w_2 :

$$w_1(x,y) = y - x, \quad w_2(x,y) = 2y - x \quad \text{for all } (x,y) \in \mathbb{R} \times Y.$$

The fact that w_1 and w_2 violate the normalization condition (4) is of no relevance to the present argument. Let $A = \{\succsim_1, \succsim_2\}$. Check that in every economy λ in E_A, $\succsim_2$'s w_2-unanimity level is four times $\succsim_1$'s w_1-unanimity level. For this reason, the efficient equal factor equivalent rule amounts to choosing the highest feasible utility vector at which the w_2-utility received by $\succsim_2$ is four times the w_1-utility received by $\succsim_1$. Writing λ_i instead of $\lambda(\succsim_i)$ to alleviate notations, this yields

$$w_1^*(\lambda) = \frac{\lambda_1^2 + 4\lambda_1\lambda_2 + 4\lambda_2^2}{2(\lambda_1 + 4\lambda_2)}, \quad w_2^*(\lambda) = 4w_1^*(\lambda).$$

Notice that the w_2-utility *loss* incurred by $\succsim_2$ from its w_2-unanimity level is also four times the w_1-utility loss incurred by $\succsim_1$ from its w_1-unanimity level.

Consider now the efficient rule under which $\succsim_2$'s loss is a times $\succsim_1$'s loss, where $1 \leq a \leq 4$. Check that it yields the utility distribution.

$$w_1^a(\lambda) = \frac{\lambda_1^2 + a\lambda_1\lambda_2 + a\lambda_2^2}{2(\lambda_1 + a\lambda_2)}, \quad w_2^a(\lambda) = aw_1^a(\lambda) + \frac{4-a}{2}(\lambda_1 + \lambda_2).$$

Straightforward computations show that w_1^a and w_2^a are nondecreasing in both $\lambda_1$and λ_2 : the rule satisfies Population Solidarity. (It is in fact population *monotonic* in the classical sense.) All Sorry to Disagree and Preference Solidarity are also satisfied. The rule, however, differs from the efficient equal factor equivalent rule whenever $a \neq 4$.

4.4 Replacing the continuum assumption

Fix a finite set A of preferences. If an economy μ in E_A takes on *integer* values only, $\mu(\succsim)$ may be interpreted as the *number* of agents with preference $\succsim$: we call μ a *finite* economy. While such an economy is of course admissible in the framework used in this paper, every real-valued mapping with support in A also qualifies as an admissible economy. This "continuum approximation" amounts to regarding agents as perfectly "divisible". It is a technical device which can only be used for convenience. The question therefore arises whether such an approximation is really innocuous.

Strictly speaking, our results no longer hold if economies must be finite. This is most easily seen under the assumption that the cost function is regular. Reconsider the family of sets $C(a)$, $a \in \mathbb{R}$, defined at the start of Section 3. Let $\partial C(a)$ be the south-east frontier of $C(a)$, i.e., $\partial C(a) = \left\{ \begin{array}{c} (x,y) \in C(a) \mid (x',y') = (x,y) \\ \text{if } x' \leq x,\ y' \leq y \text{ and } (x',y') \in C(a) \end{array} \right\}$. The family $C(a)$, $a \in \mathbb{R}$ is strictly monotonic in the sense that $a < b$ implies that i) $C(a) \subset C(b)$ and ii) $\partial C(a) \cap \partial C(b)$ contains at most the single point $(0,0)$. Let us now construct a *different* strictly monotonic family of closed sets $\tilde{C}(a)$, $a \in \mathbb{R}$, such that $\tilde{C}(a) = C(a)$ for every real $a \leq 1$ and every integer $a \geq 1$. For each preference $\succsim$, let us define the utility function $\tilde{\mu}_{\succsim}$ by the normalization condition

$$\max_{\tilde{C}(a)} \tilde{\mu}_{\succsim}(x,y) = a \text{ for all } a \in \mathbb{R}.$$

Equalizing such utilities at the highest feasible level defines a rule which satisfies our axioms on the subset of finite economies but differs from the efficient equal factor equivalent rule.

Efficient equal factor equivalence can nevertheless be characterized without resorting to the continuum approximation if the condition of Replication Invariance is imposed. Formally, this requires to let the cost function vary. Denote by $\mathcal{C}$ the set of cost functions meeting all the assumptions of Section 2, except possibly the regularity condition, and let $\tilde{E}_A$ be the set of finite economies in E_A. Define a *finite* *(allocation) rule* to be a mapping $\tilde{F}$ which assigns to each $(\succsim,\ \mu,\ c)$ in $P\times\ \tilde{E}_A \times C$ a bundle $(y,\ \xi\,(\succsim))$ in a way that is compatible with the feasibility condition (3). Say that $\tilde{F}$ satisfies *Replication Invariance* if

$$\tilde{F}\,(\succsim,\ \mu,\ c) = \tilde{F}\,(\succsim,\ n\mu,\ nc)$$

for every $(\mu,\ c)$ in $\tilde{E}_A \times C$, every $\succsim$ in $\text{supp}(\mu)$, and every positive integer n. (Here $n\mu$ is the finite economy where $(n\mu)\,(\succsim) = n\mu\,(\succsim)$ for each $\succsim$ and nc is the cost function given by $(nc)\,(y) = nc\,(y)$ for each y.) It is straightforward to redefine our three original axioms and equal factor equivalence in this modified setup.

We claim that *a finite allocation rule satisfies Efficiency, All Sorry to Disagree, Solidarity, and Replication Invariance if and only if it is the efficient equal factor equivalent rule.* To see why this is true, fix a finite rule $\tilde{F}$: $P\times\ \tilde{E}_A \times C \to \mathbb{R}\times Y$ satisfying all four axioms. Fix c in C. By Replication Invariance, there exists a rule $F : P \times E_A \to \mathbb{R} \times Y$ - in the sense of Section 2 - such that

$$\tilde{F}\,(\succsim,\ n\mu,\ pc) = F\left(\succsim,\ \frac{n}{p}\mu\right)$$

for every $\mu \in \tilde{E}_A$, $\succsim \in \text{supp}(\mu)$, and all positive integers $n,\ p$. Moreover, the restriction of F to those economies in E_A which take on *rational* values satisfies Efficiency, All Sorry to Disagree, and Solidarity. The arguments in the proof of Theorems 1 and 2 carry over and yield that F must be the efficient equal factor equivalent rule on the rational-valued economies in E_A. Since c was chosen arbitrarily, the "only if" part of the claim follows. The "if" part poses no difficulty.

4.5 Allowing for multiple private goods

Perhaps the most restrictive of our assumptions is that of a single private good. Allowing for several private goods is essential to obtain results of a general equilibrium nature.[5]

[5] In this perspective, the assumption of unbounded private good contributions should also be dropped.

The principle of equal factor equivalence can be extended in the following way. If n is the number of private goods and y is a bundle of public goods, let $\Gamma(y)$ contain those aggregate vectors of private good contributions $x = (x^1, \dots, x^n)$ which suffice to produce y. Under efficient equal factor equivalence, each agent would receive a bundle equivalent to his most preferred bundle in the set $\{(x, y) \mid x \in r\Gamma(y)\}$, where r is the smallest number that ensures feasibility.

In this multiple private good framework, however, it is no longer generally true that "disagreeing is a burden". As a consequence, the All Sorry to Disagree axiom may become infeasible and our characterization result does not carry over in any obvious way. Consider for instance an economy with one public good, two private goods, and two agents whose preferences are represented by the utility functions

$$\begin{aligned} w_1\left(x_1^1, x_1^2 \,;\, y\right) &= \ell n\,(1+y) - 2x_1' - \frac{1}{2}x_1^2 \,, \\ w_2\left(x_2^1, x_2^2 \,;\, y\right) &= \ell n\,(1+y) - \frac{1}{2}x_2^1 - 2x_2^2 \,, \end{aligned}$$

where x_i^j is the quantity of the j^{th} private good that agent i contributes. The feasibility constraint is

$$y \leq x^1\, x^2 \,,$$

where $x^j = x_1^j + x_2^j$ for $j = 1, 2$.

At the symmetric efficient allocation of an economy composed of two copies of agent 1, everyone would receive the bundle $\left(\frac{1}{4}, 1; 1\right)$: agent 1's unanimity utility level is therefore $\ell n\,(2) - 1$. A similar argument holds for agent 2. Clearly, the unanimity utility vector $(\ell n\,(2) - 1,\ \ell n\,(2) - 1)$ is strictly *below* the Pareto frontier of our original economy : indeed, it can be reached through the allocation $y = 1$, $x_1 = \left(\frac{1}{4}, 1\right)$, $x_2 = \left(1, \frac{1}{4}\right)$, at which the feasibility inequality is strict. The reason for this configuration is that the agents agree on how much public good should be produced but disagree on how it should be financed. Since the private goods are not perfectly substitutable inputs in the production of the public good, each agent gains by contributing more of the private good he values less and less of the good he values more.

Of course, the configuration where the unanimity utility vector is above the Pareto frontier remains possible as well. It could arise, for instance, if the agent's preferences over the private goods coincide (at every given bundle of public goods) or if the private goods are perfectly substitutable inputs.

This suggests that All Sorry to Disagree could be replaced with the axiom of Uniform Preference Externalities : when disagreeing is a burden, everyone should take a share of this burden; when disagreeing generates a surplus, everyone ought to get a share of that surplus. Whether this axiom, together with Efficiency and Solidarity, characterizes the efficient equal factor equivalent rule, is an open question.

APPENDIX

Our purpose here is to show that the regularity assumption made in the statement of Theorem 1 is not necessary. Throughout this Appendix, c is a fixed, not necessarily regular, cost function.

Our first task is to redefine equal factor equivalence. For each $\succsim \in P$, let $r_0(\succsim)$ denote the smallest r for which $\succsim$'s best bundle under rc involves no production, i.e.,

$$r_0(\succsim) := \inf \{r \mid (0,0) \succsim (rc(y), y) \text{ for all } y \in Y\}.$$

If $\succsim$'s best bundle is never $(0,0)$, set $r_0(\succsim) := \infty$. Let $a_0(\succsim) := 1/r_0(\succsim)$. For each $a \in \mathbb{R}$, define

$$C_{\succsim}(a) := \begin{cases} \{(x,0) \in \mathbb{R} \times Y \mid x \geq a_0(\succsim) - a\} & \text{if } a \leq a_0(\succsim), \\ \{(x,y) \in \mathbb{R} \times Y \mid x \geq c(y)/a\} & \text{if } a > a_0(\succsim). \end{cases}$$

Using the convention $1/\infty = 0$, we note that this set coincides with the set $C(a)$ defined in Section 3 if $a_0(\succsim) = 0$, i.e., if c is regular. There is again a unique (continuous) utility function $v_{\succsim} : Z(\succsim) \to \mathbb{R}$ that represents the restriction of $\succsim$ to $Z(\succsim)$ and satisfies the condition :

$$\max_{C_{\succsim}(a)} v_{\succsim}(x,y) = a \text{ for all } a \in \mathbb{R}.$$

Observe that $v_{\succsim}(0,0) = a_0(\succsim)$ and $\lim_{y \to \bar{y}} v_{\succsim}(0,y) = \infty$.

Throughout this Appendix, A is an arbitrary subset of P. If μ is an economy in E_A, we call a μ-allocation (ξ, y) *equal factor equivalent* if $(\xi(\succsim), y) \in Z(\succsim)$ for all $\succsim \in \text{supp}(\mu)$ and there exists some $a \geq \min\{a_0(\succsim) \mid \succsim \in \text{supp}(\mu)\}$ such that

$$v_{\succsim}(\xi(\succsim), y) = \max\{a, a_0(\succsim)\} \text{ for all } \succsim \in \text{supp}(\mu). \tag{12}$$

If $a > a_0(\succsim)$, it is clear that $\succsim$ is assigned the utility it would reach by choosing the bundle of public goods y and paying $c(y)/a$. If $a \leq a_0(\succsim)$, $\succsim$ reaches the utility level $a_0(\succsim)$. Since $a_0(\succsim) = v_{\succsim}(0,0)$, this is again the maximal utility $\succsim$ could obtain under the cost function c/a. Condition (12) is thus a proper expression of equal factor equivalence when c is arbitrary. The *efficient equal factor equivalent rule* on E_A is now defined in the obvious way; it coincides with the rule defined in Section 3 when c is regular.

While the definition of All Sorry to Disagree remains the same as in Section 3, its implications are a bit different. If F is an efficient rule, the utility reached by $\succsim$ in the unanimous economy $\mu_{\succsim}$ of total mass $\mu(A)$ is now

$$v_{\succsim}\left(F\left(\succsim,\mu_{\succsim}\right)\right) = \max\left\{\mu(A), a_0(\succsim)\right\}$$

rather than merely $\mu(A)$. Under Efficiency, All Sorry to Disagree thus requires that $v_{\succsim}(F(\succsim,\mu)) \leq \max\{\mu(A), a_0(\succsim)\}$ for each $\mu \in E_A$ and $\succsim \in \text{supp}(\mu)$.

We are now in a position to prove the following generalization of Theorem 1.

Theorem 2 *Let c be an arbitrary cost function and A an arbitrary subset of P. An allocation rule on E_A satisfies Efficiency, All Sorry to Disagree and Solidarity if and only if it is the efficient equal factor equivalent rule.*

Proof. Fix c and A. Only the only if part of the theorem needs a proof. Let thus F be an allocation rule on E_A satisfying our three axioms and fix $\mu \in E_A$ until the end of the proof. We adopt the same conventions and notations as in the proof of Theorem 1, except that $v_{\succsim}$ now replaces $u_{\succsim}$ for each $\succsim$. The proof is divided into six steps consecutively numbered 1, 2, 2*, 3, 3* and 4. Steps 1, 2, 3, 4 correspond to the identically numbered steps of the proof of Theorem 1 while Steps 2* and 3* deal with new issues. An arbitrary $q \in T$ is fixed form Step 1 to Step 3*. For Steps 1, 3 and 3*, we also fix an arbitrary $t \in T$.

Step 1. Define ϕ as in the proof of Theorem 1. The range of ϕ is R_t, (5) continues to hold, and ϕ is nondecreasing.

Step 2. Let $a \in (a_0(q), \infty)$ and let $\{a_n\}$ be a sequence of numbers in $(0, a)$ converging to a. Let $\{\lambda_n\}$ be a sequence of economies such that $\lambda_n(A) = a_n$ and $\lambda_n(q) \to a$. We claim that $\alpha(q, \lambda_n) \to a$.

Suppose the claim is false. By All Sorry to Disagree, there exists some $\varepsilon > 0$ such that, for all n,

$$\alpha(s, \lambda_n) \leq \max\{a_n, a_0(s)\} \text{ for all } s \in T \text{ and } \alpha(q, \lambda_n) < a - \varepsilon. \tag{13}$$

Let $\bar{x}$ satisfy (7) and define for each n the λ_n-allocation $F^n(\cdot, \lambda_n)$ as in the proof of Theorem 1. To check that this allocation Pareto-dominates $F(\cdot, \lambda_n)$ for n large enough, consider first $s \in T\backslash\{q\}$. If $a_n \geq a_0(s)$, (7) and (13) yield that $F^n(s, \lambda_n) \succ_s F(s, \lambda_n)$. If $a_n < a_0(s)$, the same conclusion holds because $a_0(s) = v_s(0,0)$. Turning now to q, we have, as in the proof of Theorem 1, that $\{v_q(F^n(q, \lambda_n))\}$ converges to the maximal value of $v_q(x, y)$ subject to the constraint $x \geq c(y)/a$. This maximal value is a by definition of $C_{\succsim}(a)$ and because $a > a_0(q)$. It follows from (13) that $F^n(\cdot, \lambda_n)$ dominates $F(\cdot, \lambda_n)$ when n is sufficiently large.

Step 2.* Fix $k \in (0,1)$. Let $\{\lambda_n\}$ be a sequence of economies such that $\lambda_n(q)/\lambda_n(A) = k$ for all n and $\lambda_n(A) \to 0$. We claim that $\alpha(q, \lambda_n) \to a_0(q)$.

Suppose this claim is false. Without loss of generality, we need only consider two cases. Suppose, first, that there exists some $\varepsilon > 0$ such that

$$\alpha(q, \lambda_n) > a_0(q) + \varepsilon \text{ for all } n.$$

Since q's unanimity utility level $\max\{\lambda_n(A), a_0(q)\}$ goes to $a_0(q)$, All Sorry to Disagree is violated when n is large enough. Suppose, next, that there exists $\varepsilon > 0$ such that

$$\alpha(q, \lambda_n) < a_0(q) - \varepsilon \text{ for all } n.$$

Since $a_0(q) = v_q(0,0)$ and q's preference is continuous, there exists some $x_0 > 0$ such that

$$\alpha(q, \lambda_n) < v_q(x_0, 0) \text{ for all } n.$$

For each $s \in T$, All Sorry to Disagree implies

$$\alpha(s, \lambda_n) \leq \max\{\lambda_n(A), a_0(s)\} \text{ for all } n$$

and the right-hand side of this inequality converges to $a_0(s) = v_s(0,0)$. The λ_n-allocation $F^n(\cdot, \lambda_n)$ defined by

$$\begin{aligned} F^n(q, \lambda_n) &= (x_0, 0), \\ F^n(s, \lambda_n) &= \left(\frac{kx_0}{k-1}, 0\right) \text{ for all } s \in T\backslash\{q\} \end{aligned}$$

therefore Pareto-dominates $F(\cdot, \lambda_n)$ when n is large enough, a contradiction.

Step 3. Suppose $a_0(q) \leq a_0(t)$. Essentially the same argument as in the proof of Theorem 1 shows that

$$\phi(a) = \{a\} \text{ for all } a \in R_q \cap (a_0(t), \infty). \tag{14}$$

Step 3.* Suppose $a_0(q) \leq a_0(t)$. We claim that

$$\phi(a) = \{a_0(t)\} \text{ for all } a \in R_q \cap (a_0(q), a_0(t)].$$

First of all, it is clear that $b \leq a_0(t)$ for all $b \in \phi(a)$ and $a \in R_q \cap (a_0(q), a_0(t)]$. This follows from (14), the fact that ϕ si nondecreasing over R_q, and the fact that $R_q \cap (a_0(t), \infty)$ is dense in $(a_0(t), \infty)$ because of Step 2. Suppose now that $b < a_0(t)$ for some $b \in \phi(a)$ and $a \in R_q \cap (a_0(q), a_0(t)]$. Pick a sequence of economies $\{\lambda_n\}$ such that $\lambda_n(q)/\lambda_n(A)$ and $\lambda_n(t)/\lambda_n(A)$ are positive constants and $\lambda_n(A) \to 0$. By Step 2*, $\alpha(q, \lambda_n) \to a_0(q)$ and $\alpha(t, \lambda_n) \to a_0(t)$. For n large enough, therefore, $\alpha(q, \lambda_n) < a$ and $\alpha(t, \lambda_n) > b$. This contradicts the fact that ϕ is nondecreasing.

Step 4. Choose now $q \in T$ such that $a_0(q) \leq a_0(t)$ for all $t \in T$. We have so far established that for each $t \in T$, the correspondence ϕ_t defined on R_q by $\phi_t(a) = \{b | \exists\lambda : \alpha(q, \lambda) = a \text{ and } \alpha(t, \lambda) = b\}$ is nondecreasing and

$$\phi_t(a) = \{\max\{a, a_0(t)\}\} \text{ for all } a \in R_q \cap (a_0(q), \infty).$$

This means that

$$\alpha(t,\lambda) = \max\{\alpha(q,\lambda), a_0(t)\} \text{ for each } t \in T$$

if $\alpha(q,\lambda) > a_0(q)$. This formula extends to the case where $\alpha(q,\lambda) = a_0(q)$: since $R_q \cap (a_0(q), \infty)$ is dense in $(a_0(q), \infty)$ and every ϕ_t is nondecreasing, $\alpha(t,\lambda) \leq a_0(t)$ for all $t \in T$ and Efficiency forces these inequalities to be equalities since $a_0(t) = v_t(0,0)$. In order to conclude that F is equal factor equivalent, it only remains to check that $\alpha(q,\lambda) \geq a_0(q)$ for every society λ. But this follows again from Efficiency.

Q.E.D.

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