Université de Montréal
Faculté des arts et des sciences
Département de sciences économiques

CAHIER 9549

EXACT TESTS IN SINGLE EQUATION AUTOREGRESSIVE DISTRIBUTED LAG MODELS

Jan F. KIVIET[1] and Jean-Marie DUFOUR[2]

[1] Tinbergen Institute and Faculty of Economics & Econometrics, University of Amsterdam

[2] Département de sciences économiques and Centre de recherche et développement en économique (C.R.D.E.), Université de Montréal

October 1995

This work was supported by grants from the Social Sciences and Humanities Research Council of Canada, the Natural Sciences and Engineering Research Council of Canada and the Government of Québec (Fonds FCAR).

C.P. 6128, succursale Centre-ville
Montréal (Québec) H3C 3J7

Télécopieur (FAX): (514) 343-5831
Courrier électronique (E-Mail): econo@tornade.ERE.Umontreal.CA

Ce cahier a également été publié au Centre de recherche et développement en économique (C.R.D.E.) (publication no 4495).

Dépôt légal - 1995
Bibliothèque nationale du Québec
Bibliothèque nationale du Canada

ISSN 0709-9231

RÉSUMÉ

Étant donné les facilités actuelles de calcul, il n'est plus guère important que la distribution d'une statistique de test puisse être "tabulée". Le problème pertinent est de calculer des niveaux de signification marginaux ("p-values") tels que l'on puisse contrôler le niveau d'un test sur des échantillons finis. En économétrie, c'est rarement le cas pour les tests asymptotiques usuels. Dans ce texte, nous développons des procédures de tests applicables dans le cadre de modèles autorégressifs dynamiques avec régresseurs exogènes (modèles ARX). Sous l'hypothèse nulle, les statistiques de test proposés sont pivotales et permettent d'obtenir des inférences exactes pour toute distribution d'erreurs connue à un facteur d'échelle près. Les tests considérés comprennent des tests exacts sur l'ordre d'un modèle autorégressif, contre la présence de changement structurel et sur des hypothèses de racines unitaires possiblement saisonnières. Les procédures proposées sont appliquées à un modèle des mises en chantier au Canada. Nous concluons par une discussion critique de nos résultats.

Mots clés : modèle ARX, inférence exacte, test de Monte Carlo, ordre d'un modèle dynamique, randomisation, test α-semblable, changement structurel, racines unitaires

ABSTRACT

It is argued in this paper that, given present-day computer facilities, it is no longer important whether or not test statistics have null distributions that can be tabulated. A more relevant issue is whether *p*-values are exactly computable so that test sizes can be controlled in finite samples. In econometrics, this is seldom the case for the standard asymptotic test statistics. However, in this study, alternative test procedures are developed for hypotheses on the coefficient values of the lagged-dependent variables in the ARX class of dynamic regression models. Under the null hypothesis, the proposed test statistics are pivotal and yield exact inference for given (up to an unknown scale factor) distribution of the innovation errors. They include exact versions of tests on the maximum lag length, for structural change and on the presence of (seasonal) unit roots, i.e., they cover situations which are usually approached by asymptotic and non-exact *t*, *F*, ADF or HEGY tests. The various procedures are applied and compared in illustrative empirical models. Finally, the approach is critically discussed.

Key words : ARX model, exact inference, Monte Carlo test, order of dynamics, randomization, similar test, structural change, unit roots

1. Introduction

Maximum-likelihood based inference procedures are generally very popular. As a rule, this popularity is mainly based on asymptotic optimality properties and on computational convenience. However, given present-day computer speed and facilities, which allow the on-line almost instantaneous application of elaborate and highly sophisticated techniques, practitioners get into the situation where they may (and should) bring more aspects into their statistical utility function than just ease of computation and behaviour in infinitely large samples. Nowadays a more challenging and appropriate objective is to employ or develop procedures which optimize the actual efficiency and accuracy of inference from the finite set of sample data at hand.

As far as test procedures are concerned, it is no longer a serious requirement today for the userfriendliness of a test procedure that it has a null distribution (if only asymptotically) which can be tabulated. Instead, these days, the profession can be much better served if provided with testing techniques accompanied with software for producing (exact) p-values of the relevant statistics. Whether or not the null distributions of the test statistics involved are invariant to given design characteristics is no longer a relevant issue. The prime objectives now for test procedures should be: (i) control over level in finite samples, the most central problem for that purpose being the elimination of nuisance parameters; (ii) optimality properties under conceivable relevant and verifiable circumstances, or at least a good record in controlled experiments; and (iii) computational feasibility. Note also that a test procedure is meaningful only if the model under the null hypothesis is testable, i.e. if it is sufficiently restrictive to make the probability of certain non-trivial events (defined in terms of the available observations) boundable by a small probability (the level of the test) under all data generating processes compatible with the null hypothesis. This is really a logical prerequisite of any testing exercise.

In Dufour and Kiviet (1993 a,b) these goals are pursued in the context of the simple first-order dynamic regression model by combining procedures put forward in Dufour (1989, 1990) and Kiviet and Phillips (1990, 1992). In the classic format of econometrics teaching this model usually generates the first encounter where the student is taught that (s)he may stick to the usual inference procedures (since they are still asymptotically valid under certain regularity conditions) although one has to accept that the magnitude of the approximation errors is unknown, and in principle is unknowable, because it depends on the unknown values of the parameters of the model. Generally, bounds on such errors have not been established. Indeed, for certain problems (such as inference about long-run multipliers), the approximation error associated with any nuisance-parameter free asymptotic approximation must be arbitrarily bad on certain subsets of the parameter space; see Dufour (1994). Students are (still) taught just to live with this unpleasant situation and to find comfort in the thought that the larger the sample is, the smaller the committed errors will be. The relevant question how large a sample ought to be in order to feel confident is usually left unanswered, or receives an answer that has no consequences for actual practice.

In this paper a considerably more general model than the first-order autoregressive case is considered. We examine exact inference procedures for the one-equation higher-order dynamic model represented by:

$$(1.1) \qquad y_t = \sum_{i=1}^{p} \lambda_i y_{t-i} + \sum_{j=1}^{J} \sum_{i=0}^{p_j} \delta_{ji}\, z_{t-i}^{(j)} + \varepsilon_t , \qquad t = 1,\ldots,T$$

where the regressors are finite-order distributed lags of the (lagged-) dependent variable and of J linearly independent regressors $z^{(j)}$, $j = 1,..,J$. Using standard notation on polynomials in the lag- or backward-shift operator B, the model may also be expressed as:

$$(1.2) \qquad \lambda(B)y_t = \sum_{j=1}^{J} \delta_j(B) z_t^{(j)} + \varepsilon_t , \qquad t = 1,\ldots,T$$

where the order of the polynomial $\lambda(B)$ is p, and the order of the polynomials $\delta_j(B)$ is p_j, $j = 1,\ldots,J$. We examine inference procedures concerning the lagged-dependent regressor variable coefficients $\lambda_1,\ldots,\lambda_p$ only.

The general outline of this study is as follows. In Section 2 we highlight the problem to be tackled with and we state the assumptions which will make the above model genuinely testable. In Section 3 we derive an exact test for a joint hypothesis on all p elements of the vector $\lambda = (\lambda_1,\ldots,\lambda_p)'$. In Section 4 we examine tests on fewer than p restrictions on λ; we focus on tests for the order of the lag-polynomial $\lambda(B)$ and on tests for (multiple and/or seasonal) unit-roots of $\lambda(B)$. In Section 5 we develop an exact test for the occurrence of structural change(s) in the values of the elements of the vector λ. Section 6 provides empirical illustrations of the various tests and makes comparisons with standard asymptotic results. Section 7 concludes and discusses the practical relevance of the results.

2. Framework

By stacking the T observations on the variables of model (1.1) in columns and assembling all observations on the p lagged-dependent variables in rows, and so for all the other current and lagged explanatory variables, we may rewrite model (1.1) in matrix notation as:

$$(2.1) \qquad y = Y\lambda + X\beta + \varepsilon ,$$

where $Y = [\bar{y}_{-1}|\ldots|\bar{y}_{-p}]$ is a $T \times p$ matrix with $\bar{y}_{-i} = (y_{1-i},\ldots, y_{T-i})'$ for $i = 1,\ldots,p$, and X is a $T \times k$ matrix with $k = J + \Sigma_{j=1}^{J} p_j$. The $k \times 1$ coefficient vector β contains all coefficients δ_{ji} in the appropriate order.

Below we will be more precise about the assumptions actually made on the coefficients, regressor variables and disturbance vector of this model. Before we do that, we first examine some special characteristics of relationship (2.1), which stem from its linear dynamic structure. To this end, it will be helpful to introduce some further notation.

From the particular temporal structure of the matrix Y it easily follows that we may write

$$(2.2) \qquad y - Y\lambda = \Gamma y - Y_0\lambda ,$$

where Γ is the $T \times T$ lower-triangular matrix

$$(2.3) \qquad \Gamma = \begin{bmatrix} 1 & 0 & . & . & . & . & . & . & . & 0 \\ -\lambda_1 & 1 & 0 & & & & & & & . \\ -\lambda_2 & -\lambda_1 & 1 & . & & & & & & . \\ . & . & . & . & . & & & & & . \\ . & & . & . & . & . & & & & . \\ -\lambda_p & & & . & . & . & . & & & . \\ 0 & -\lambda_p & & & . & . & . & . & & . \\ . & . & . & & & . & . & . & . & . \\ . & & . & . & & & . & . & 1 & 0 \\ 0 & . & . & 0 & -\lambda_p & . & . & -\lambda_2 & -\lambda_1 & 1 \end{bmatrix},$$

and Y is the $T \times p$ upper-triangular matrix

$$\text{(2.4)} \qquad Y_0 = \begin{bmatrix} y_0 & y_{-1} & \cdot & \cdot & y_{1-p} \\ 0 & y_0 & \cdot & & y_{2-p} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & & \cdot & \cdot & y_{-1} \\ \cdot & & & \cdot & y_0 \\ \cdot & & & & 0 \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 \end{bmatrix} .$$

Now we can rewrite the model as

$$\text{(2.5)} \qquad \Gamma y = Y_0\lambda + X\beta + \varepsilon .$$

Note that Γ is a lower-triangular matrix, which is non-singular irrespective of the actual value of λ. So Γ^{-1} exists and is lower-triangular too, hence

$$\text{(2.6)} \qquad y = \Gamma^{-1}Y_0\lambda + \Gamma^{-1}X\beta + \Gamma^{-1}\varepsilon$$

which, unlike (2.1), gives an explicit (reduced form) expression for all elements of the vector y.

The precise assumptions we make are as follows.

ASSUMPTION A: *For the regressor matrices Y and X of (2.1), where Y is $T \times p$ and X is $T \times k$, we have $\text{rank}([Y:X]) = p + k$ with probability 1. The matrix X and the $T \times p$ matrix Y_0, given in (2.4), are stochastically independent of the $T \times 1$ vector ε. The disturbance vector ε is such that $\varepsilon = \sigma\eta$, where σ is a scalar scale factor and η is a $T \times 1$ stochastic vector which has a known distribution. The parameters λ, β and σ are constant but unknown, with $\sigma \in \mathbb{R}^1_+$, $\beta \in \mathbb{R}^k$ and $\lambda \in \mathcal{D}_\lambda \subseteq \mathbb{R}^p$, where $\mathcal{D}_\lambda$ is a priori specified.*

This assumption implies that we may condition on both X and Y_0, so that we can treat X and Y_0 as if they were in fact fixed.

Upon using $\eta = \varepsilon/\sigma$, we may express (2.6) as

$$(2.7) \qquad y = \Gamma^{-1}Y_0\lambda + \Gamma^{-1}X\beta + \sigma\Gamma^{-1}\eta \; .$$

This shows how the dependent variable y is determined by "fixed" variables (Y_0 and X), an unobserved random variable (η) with known distribution, and by unknown parameters (λ, β and σ). Note that the stochastic nature of y stems from the third component only.

Formula (2.7) also enables one to represent easily the dependence of the lagged-dependent regressor variables $\bar{y}_{-i}$ (and the matrix of regressors Y) on fixed and random (observed and unobserved) elements. We have

$$(2.8) \qquad \bar{y}_{-i} = L^i y + Y_0\iota_i \; , \qquad\qquad i = 1,\ldots,p$$

where L is the T × T matrix

$$(2.9) \qquad L = \begin{bmatrix} 0 & . & . & . & . & . & 0 \\ 1 & 0 & & & & & . \\ 0 & 1 & 0 & & & & . \\ . & . & . & . & & & . \\ . & & . & . & . & & . \\ . & & & . & . & . & . \\ 0 & . & . & . & 0 & 1 & 0 \end{bmatrix} ,$$

and ι_i denotes the p × 1 unit vector with a unit element in the i-th position, all others being zero. Substitution of (2.7) into (2.8) then yields

$$(2.10) \qquad \bar{y}_{-i} = L^i\Gamma^{-1}Y_0\lambda + Y_0\iota_i + L^i\Gamma^{-1}X\beta + \sigma L^i\Gamma^{-1}\eta$$

$$= d_i(\lambda) + C_i(\lambda)X\beta + \sigma C_i(\lambda)\eta \; , \qquad\qquad i = 1,\ldots,p$$

where we introduced the notation:

$$(2.11) \qquad d_i(\lambda) = C_i(\lambda)Y_0\lambda + Y_0\iota_i \quad , \quad C_i(\lambda) = L^i\Gamma^{-1} \; .$$

From the decomposition of $\bar{y}_{-i}$ in three terms given in the final expression of (2.10) the predetermined nature of $\bar{y}_{-i}$ straightforwardly emerges; the first two components are fixed, and the third stochastic component is a linear transformation of the vector $\varepsilon = \sigma\eta$, where the transformation matrix $C_i(\lambda)$ is lower triangular with zeros on the main diagonal and on i-1 lower sub-diagonals.

As will become clear below, it is the presence of the two fixed components in (2.10) in particular that cause problems in finite samples when standard least-squares or (pseudo-)maximum-likelihood inference methods are applied to model (2.1). In this paper we focus on the problems that crop up when hypotheses are tested on the values of the lagged-dependent variable coefficients λ only, and we produce some feasible solutions.

We shall illustrate the problem with the standard procedure only for the particular case of testing a joint hypothesis on all p coefficients in λ:

$$(2.12) \qquad H_0(\lambda):\ \lambda = \lambda_0 \quad \text{against} \quad H_1(\lambda):\ \lambda \neq \lambda_0 \ ,$$

where λ_0 is a p-element vector of known real numbers. Proceeding in the standard way, ordinary least-squares estimation is performed, and to test hypothesis (2.12) the F-statistic is used. This statistic is

$$(2.13) \qquad \mathcal{F}_\lambda = \frac{T-p-k}{p}\left[\frac{(y - Y\lambda_0)'M[X](y - Y\lambda_0)}{y'M[Y|X]y} - 1\right] ,$$

where, for any $T \times m$ matrix A of full column rank, $M[A] = I_T - A(A'A)^{-1}A'$.

Under $H_0(\lambda)$ and usual regularity conditions, we have

$$(2.14) \qquad \mathcal{F}_\lambda \overset{a}{\sim} F(p,T-p-k) \ .$$

Here $F(p,T-p-k)$ denotes the F-distribution with p and T-p-k degrees of freedom respectively, and " $\overset{a}{\sim}$ " indicates the approximate (asymptotic) validity of statement (2.14). Even if η is multivariate standard normally distributed, the actual finite-sample distribution of statistic $\mathcal{F}_\lambda$ under $H_0(\lambda)$ is rather complicated and depends *inter alia* on the nuisance parameters β and σ, which in practice are unknown. This non-similarity of the test can be seen as follows.

Under $H_0(\lambda)$, we have:

$$(2.15) \qquad (y - Y\lambda_0)'M[X](y - Y\lambda_0) = \sigma^2\eta'M[X]\eta \ ,$$

where $\eta' M[X]\eta$ follows a $\chi^2(T-k)$ distribution when $\eta \sim N[0,I_T]$. Complications arise, however, if in the minimization of the sum of squared residuals not all elements of λ are restricted. This is seen when no constraints are imposed, as in the denominator in (2.13). Using well-known results from partitioned regression, [see Dufour and Kiviet (1993a)], we find that

$$(2.16) \qquad y' M[Y \vdots X] y = \sigma^2 \eta' M[Y \vdots X] \eta$$

$$= \sigma^2 \eta' M[X]\eta - \sigma^2 \eta' M[X] Y \{Y' M[X] Y\}^{-1} Y' M[X]\eta \ ,$$

hence, under $H_0(\lambda)$,

$$(2.17) \qquad \mathcal{F}_\lambda = \frac{T-p-k}{p} \left[\frac{\eta' M[X]\eta}{\eta' M[X]\eta - \eta' M[X] Y \{Y' M[X] Y\}^{-1} Y' M[X]\eta} - 1 \right] .$$

Exploiting (2.10) we find that under $H_0(\lambda)$

$$(2.18) \qquad Y = \sum_{i=1}^{p} \bar{y}_{-i}\iota_i' = \sum_{i=1}^{p} [d_i(\lambda_0) + C_i(\lambda_0) X\beta + \sigma C_i(\lambda_0)\eta]\iota_i' \ ,$$

from which it becomes obvious that $\eta' M[X] Y \{Y' M[X] Y\}^{-1} Y' M[X]\eta$, even under normality of η, has a distribution which is fairly complicated and depends on nuisance parameters. Y is stochastic, as it is determined by η, and (2.17) does not reduce to a simple ratio of quadratic forms in η, as in the standard case. Because of the presence of the two fixed components in (2.10) and (2.18) the scale factor σ does not drop out. Moreover, the distribution of (2.17) depends on the value of β. Both β and σ are unknown nuisance parameters when testing $H_0(\lambda)$. Below, we consider an alternative to the test statistic $\mathcal{F}_\lambda$. Under $\lambda = \lambda_0$ this alternative will still have a complicated distribution function which is determined by λ_0 and X, but its distribution will be invariant to the unknown values β and σ, and hence enables one to produce exact inference on λ.

3. An Exact Joint Test for the Complete λ Vector

The nuisance components revealed above can be removed by extending the model, and testing $\lambda = \lambda_0$ (or an extension of this null) in an appropriately augmented model. This is seen as follows. In statistic $\mathcal{F}_\lambda$ the two fixed nuisance components of matrix Y will do no harm if it happens that they are located in the subspace spanned by the columns of the matrix X. This is easily seen upon rewriting the expression involving Y in (2.17) as follows:

$$\eta' M[X]Y\{Y'M[X]Y\}^{-1}Y'M[X]\eta = \eta' M[X]Y\{(M[X]Y)'M[X]Y\}^{-1}(M[X]Y)'\eta \; .$$

Hence, in statistic (2.17), Y is always premultiplied by the symmetric and idempotent matrix $M[X]$, which is the orthogonal projection matrix which projects on the orthogonal complement of the space spanned by the columns of X. Hence, by extending the set of strongly exogenous regressors X by some matrix $\bar{X}$, where $\bar{X}$ is chosen such that $[X \vdots \bar{X}]$ has full column rank, while $M[X \vdots \bar{X}]$ annihilates all the nuisance components from Y, we can obtain a pivotal statistic. The *p*-values of such a statistic can be obtained by Monte Carlo methods. Such *p*-values are exact, even in a simulation analysis with a finite number of replications, when the test statistic is not considered in the usual fashion, but as a so-called Monte Carlo test, where the stochastic nature of both the test statistic and the sample of simulation drawings from its null distribution are taken into account. In the remainder of this section we show how to derive $\bar{X}$ and how to obtain exact *p*-values.

From (2.18) and using (2.11), we find that the first nuisance component of Y is given by the $T \times p$ matrix

$$(3.1) \qquad D_p(\lambda_0) = \sum_{i=1}^{p} d_i(\lambda_0)\iota_i' = \sum_{i=1}^{p} [C_i(\lambda_0)Y_0\lambda_0 + Y_0\iota_i]\iota_i'$$

$$= Y_0 + \sum_{i=1}^{p} C_i(\lambda_0)Y_0\lambda_0\iota_i' = Y_0 + \bar{C}_p(\lambda_0)[I_p \otimes Y_0\lambda_0] \; ,$$

where

$$(3.2) \qquad \bar{C}_p(\lambda_0) = [\; C_1(\lambda_0) \;\vdots\; \ldots\ldots \;\vdots\; C_p(\lambda_0) \;]$$

is a $T \times pT$ matrix. The dependence of the matrix $D_p(\lambda_0)$ on λ_0 is, of course, no problem since λ_0 is known. Extending the set of regressors $[Y \vdots X]$

of the model by including $D_p(\lambda_0)$ leads to removing the first nuisance component from the relevant test statistic.

It is less straightforward how to remove the second nuisance component of Y, because of its dependence on the unknown vector β. It equals:

$$(3.3)\qquad \sum_{i=1}^{p} C_i(\lambda_0)X\beta\iota_i' = [\ C_1(\lambda_0)X\beta \mid \ldots\ldots \mid C_p(\lambda_0)X\beta\ ]$$

$$= \bar{C}_p(\lambda_0)[I_p\otimes X][I_p\otimes\beta]\ .$$

In order to annihilate the $T \times p$ component (3.3) we have to project off the space spanned by the columns of the $T \times pk$ matrix

$$(3.4)\qquad X_p(\lambda_0) = \bar{C}_p(\lambda_0)[I_p\otimes X] = [\ C_1(\lambda_0)X \mid \ldots\ldots \mid C_p(\lambda_0)X\ ]\ .$$

Note that the $T \times (k + p + pk)$ matrix $[X|D_p(\lambda_0)|X_p(\lambda_0)]$ does not necessarily have full column rank. We therefore define the $T \times \tilde{m}$ matrix $\tilde{X}_p(\lambda_0)$ such that $[X|\tilde{X}_p(\lambda_0)]$ has full column rank $m = k + \tilde{m}$, whilst the columns of the latter matrix span the same space as is spanned by the columns of the matrix $[X|D_p(\lambda_0)|X_p(\lambda_0)]$. Consequently, if $\lambda = \lambda_0$, we have:

$$(3.5)\qquad M[X|\tilde{X}_p(\lambda_0)]Y = \sigma\sum_{i=1}^{p} M[X|\tilde{X}_p(\lambda_0)]C_i(\lambda_0)\eta\iota_i'$$

$$= \sigma M[X|\tilde{X}_p(\lambda_0)]\bar{C}_p(\lambda_0)[I_p\otimes\eta]\ .$$

In order to keep the notation in what follows relatively simple we introduce the $T \times p$ matrix:

$$(3.6)\qquad Y(\eta,\lambda_0) = \frac{1}{\sigma}M[X|\tilde{X}_p(\lambda_0)]Y = M[X|\tilde{X}_p(\lambda_0)]\bar{C}_p(\lambda_0)[I_p\otimes\eta]\ .$$

Consider now testing $\lambda = \lambda_0$ in the extended model

$$(3.7)\qquad y = Y\lambda + X\beta + \tilde{X}_p(\lambda_0)\tilde{\beta} + \eta\ ,$$

where the $\tilde{m}$ regressors $\tilde{X}_p(\lambda_0)$ are redundant and hence actually $\tilde{\beta} = 0$. We can proceed in two ways. We can either test

$$(3.8)\qquad \tilde{H}_0(\lambda) : \lambda = \lambda_0 \quad \text{(leaving } \beta \text{ and } \tilde{\beta} \text{ unconstrained)}$$

or

$$(3.9)\qquad \bar{H}_0(\lambda,\bar{\beta}) : \lambda = \lambda_0 \ , \ \bar{\beta} = 0 \ .$$

For testing the p restrictions $\bar{H}_0(\lambda)$ in model (3.7) we use statistic

$$(3.10)\qquad \mathcal{F}_\lambda = \frac{T-p-m}{p}\left[\frac{(y - Y\lambda_0)'M[X\,|\,\bar{X}_p(\lambda_0)](y - Y\lambda_0)}{y'M[Y\,|\,X\,|\,\bar{X}_p(\lambda_0)]y} - 1\right] .$$

Under the null hypothesis $\bar{H}_0(\lambda)$ the first term in square brackets of (3.10) simplifies to

$$(3.11)\qquad \frac{\eta'M[X\,|\,\bar{X}_p(\lambda_0)]\eta}{\eta'M[X\,|\,\bar{X}(\lambda_0)]\eta - \eta'Y(\eta,\lambda_0)\{Y(\eta,\lambda_0)'Y(\eta,\lambda_0)\}^{-1}Y(\eta,\lambda_0)'\eta} .$$

This is pivotal (it only depends on the fixed and known Y_0, X and λ_0, and on the stochastic vector η, which has a known distribution). For testing the p+$\bar{m}$ restrictions $\bar{H}_0(\lambda,\bar{\beta})$ in (3.7) we use

$$(3.12)\qquad \mathcal{F}_{\lambda,\bar{\beta}} = \frac{T-p-k-\bar{m}}{p+\bar{m}}\left[\frac{(y - Y\lambda_0)'M[X](y - Y\lambda_0)}{y'M[Y\,|\,X\,|\,\bar{X}_p(\lambda_0)]y} - 1\right] .$$

Under $\bar{H}_0(\lambda,\bar{\beta})$ the first term in square brackets of (3.12) equals:

$$(3.13)\qquad \frac{\eta'M[X]\eta}{\eta'M[X\,|\,\bar{X}(\lambda_0)]\eta - \eta'Y(\eta,\lambda_0)\{Y(\eta,\lambda_0)'Y(\eta,\lambda_0)\}^{-1}Y(\eta,\lambda_0)'\eta} ,$$

which is also pivotal (invariant with respect to β and σ, but dependent on X, Y_0 and λ_0).

Critical- or *p*-values of tests (3.10) and (3.12) can be obtained approximately from Monte Carlo experiments. However, exact *p*-values can easily be obtained from a finite number of simulation experiments for a closely related test procedure, see Dufour and Kiviet (1993 a,b). We illustrate this for test (3.12).

First, generate N-1 (where N = 100 or N = 1000 , for instance) independent T-element vectors η_j (j = 1,...,N-1) which follow the same known distribution as $\eta = \varepsilon/\sigma$. From each vector η_j , a drawing $\mathcal{F}_{\lambda,\bar{\beta}}(j)$ from the true null distribution of the test statistic can be obtained, i.e.

$$(3.14) \qquad \mathcal{F}_{\lambda,\bar{\beta}}(j) = \frac{T-p-k-\bar{m}}{p+\bar{m}} \left[\frac{\eta_j' M[X]\eta_j}{\eta_j'\{M[X\,|\,\bar{X}(\lambda_0)] + M[Y(\eta_j,\lambda_0)] - I_T\}\eta_j} - 1 \right] .$$

To the series $\{ \mathcal{F}_{\lambda,\bar{\beta}}(1) , \ldots , \mathcal{F}_{\lambda,\bar{\beta}}(N-1) \}$, we add

$$(3.15) \qquad \mathcal{F}_{\lambda,\bar{\beta}}(N) = \mathcal{F}_{\lambda,\bar{\beta}} ,$$

which represents the realization of the test statistic from the actual sample data. Next, we order the values $\mathcal{F}_{\lambda,\bar{\beta}}(1),\ldots,\mathcal{F}_{\lambda,\bar{\beta}}(N)$ in increasing order. We indicate the position of $\mathcal{F}_{\lambda,\bar{\beta}}(N)$ after this ordering by $\mathcal{R}(N)$. Of course, $\mathcal{R}(N)$ is a discrete random variable; the positive integers 1 through N are its domain, and since under the null all $\mathcal{F}_{\lambda,\bar{\beta}}(j)$ are i.i.d., each one of the possible outcomes has probability 1/N. Instead of $\mathcal{F}_{\lambda,\bar{\beta}}$, we use $\mathcal{R}(N)$ in the simulation or randomization test procedure as our test statistic. This is done in the following way. From our one and only realization of model (3.1) (i.e. one realization of ε) we obtain one realization of statistic $\mathcal{F}_{\lambda,\bar{\beta}}$, and, after generating a random series of N-1 Monte Carlo realizations of the null distribution (3.14), we have obtained one realization of the simulation test statistic, which we indicate by $\hat{\mathcal{R}}(N)$. Then the p-value of the simulation test statistic $\hat{\mathcal{R}}(N)$, which is based on $\mathcal{F}_{\lambda,\bar{\beta}}$, is given by

$$(3.16) \qquad \mathcal{P}(\lambda_0,N,\mathcal{F}_{\lambda,\bar{\beta}}) = P[\ \mathcal{R}(N) \geq \hat{\mathcal{R}}(N) \ |\ \bar{H}_0(\lambda,\bar{\beta})\ ] = \frac{N - \hat{\mathcal{R}}(N) + 1}{N} .$$

Note that asymptotically, for $N \to \infty$, this test procedure coincides with the procedure where statistic $\mathcal{F}_{\lambda,\bar{\beta}}$ is used in combination with its exact null distribution. For finite N, we have a different procedure; its power will be related to the power of test $\mathcal{F}_{\lambda,\bar{\beta}}$, but is also characterized by N. However, even for finite N the randomized test is perfectly exact, since the probability to commit a type I error never exceeds the chosen significance level when we reject $\bar{H}_0(\lambda,\bar{\beta})$ if and only if $\mathcal{P}(\lambda_0,N,\mathcal{F}_{\lambda,\bar{\beta}})$ is smaller than this level.

Because of the validity of $\bar{\beta} = 0$, the test $\mathcal{F}_{\lambda,\bar{\beta}}$ seems logically the most attractive, and therefore may in general be more powerful than test $\mathcal{F}_{\lambda}$. Below, however, we will see that there is no uniform power difference between the two procedures. For the case $p = 1$ (first-order dynamics) the procedures developed above all simplify to variants of those presented in Dufour and Kiviet (1993a), where, for two particular data sets, experimental evidence is provided that sustains the conjectured "frequent superiority" of procedures that exploit $\tilde{\beta} = 0$.

In the study just mentioned, it is also shown that exact procedures such as developed here may even be valuable in circumstances where the distribution of the disturbances is unknown or misspecified and where the regressors are weakly- instead of strongly-exogenous, since, under usual regularity assumptions, the suggested procedures will nonetheless be asymptotically valid and, although nonexact now, may often involve less serious finite-sample inaccuracies than the crude standard asymptotic inference techniques.

4. Exact Tests on Special Characteristics of the Lag-Polynomial $\lambda(B)$

In models with higher-order dynamics ($p > 1$), we seldom want to test a hypothesis such as (2.12), i.e. $H_0(\lambda)$: $\lambda = \lambda_0$, where the complete vector λ is specified. The most relevant example of this case is probably represented by $\lambda_0 = 0$ for testing the presence of any lagged-dependent variables at all. Usually, however, we want to test less specific general characteristics of the lag-polynomial $\lambda(B)$, expressed by fewer than p linear restrictions.

A particularly relevant case is a test for the actual order of the lag-polynomial, where the null hypothesis involves:

$$(4.1) \qquad \lambda_p = \lambda_{p-1} = \ldots\ldots = \lambda_{p-r+1} = 0 \ .$$

Here r zero restrictions ($1 \leq r \leq p$) are tested in order to check whether the order of the polynomial can be reduced from p to p-r. Another most relevant case is the test for the presence of characteristic roots of the lag-polynomial of a particular value. Here the case of one or more (seasonal) unit roots deserves special attention. The single restriction that represents at least one unit root is:

$$(4.2) \qquad \lambda_1 + \lambda_2 + \ldots\ldots + \lambda_p = 1 \ .$$

The presence of a second unit root implies the validity of the hypothesis:

$$(4.3) \qquad \lambda_1 + 2\lambda_2 + \ldots\ldots + p\lambda_p = 0 \ .$$

Our framework also extends to the seasonal unit root case. The assumption that the lag polynomial $\lambda(B)$ can be factorized as

$$(4.4) \qquad \lambda(B) = (1 - B^4)\lambda^*(B) = (1 - B)(1 + B)(1 + B^2)\lambda^*(B)$$

implies four linear restrictions on the λ coefficients. These are given by (4.2) and:

$$(4.5a) \qquad -\lambda_1 + \lambda_2 - \lambda_3 + \ldots\ldots + (-1)^p\lambda_p = 1 \ ,$$

$$(4.5b) \qquad \lambda_1 - \lambda_3 + \lambda_5 - \ldots\ldots\ldots = 0 \ ,$$

$$(4.5c) \qquad -\lambda_2 + \lambda_4 - \lambda_6 + \ldots\ldots\ldots = 1 \ .$$

The latter two correspond to the two complex unit roots of (4.4), and (4.5a) follows from the minus unity root. Testing the four restrictions jointly or some (combination) of them separately using asymptotic methods is discussed in Dickey et al. (1984) and Hylleberg et al. (1990). Our procedures allow to produce exact tests for these various hypotheses.

Exact test procedures for r linear restrictions on the p elements of the coefficient vector λ can be obtained from the exact tests on the complete vector λ. Here we derive a general procedure, and we return to the various particular cases of interest given above in the illustrations in Section 6.

As a building block for dealing with the case $r < p$ we need an exact confidence set with confidence coefficient $1-\alpha$ for the complete coefficient vector λ. This is given by

$$(4.6) \qquad \mathfrak{C}_\lambda(\alpha, N, \mathfrak{F}_{\lambda,\bar{\beta}}) = \left\{ \lambda_0 \in \mathcal{D}_\lambda : \mathcal{P}(\lambda_0, N, \mathfrak{F}_{\lambda,\bar{\beta}}) \geq \alpha \right\},$$

where, for the sake of brevity, we again only deal with the case where the test statistic $\mathfrak{F}_{\lambda,\bar{\beta}}$ is being used. The dependence of the set on the value of N should be understood in the following broad sense: the calculation of the *p*-values $\mathcal{P}(\lambda_0, N, \mathfrak{F}_{\lambda,\bar{\beta}})$ is for each and every value λ_0 performed on the basis of the same set of randomly generated T-element vectors $(\eta_1, \ldots, \eta_{N-1})$.

The actual construction of set (4.6) for an empirical example requires extensive computations in order to establish to a prescribed degree of precision the boundaries of the confidence region. However, as it turns out, the test procedure developed below does not require the explicit construction of the p-dimensional set (4.6).

We define a general representation of the r linear restrictions to be tested. Let R be a known $r \times p$ matrix with $\text{rank}(R) = r$, where $1 \leq r \leq p-1$, and let

$$(4.7) \qquad \theta = R\lambda .$$

Hence, θ is an $r \times 1$ vector of linearly independent linear transformations of λ. We want to devise an exact test procedure for

$$(4.8) \qquad H_0(\theta): \theta = \theta_0 \quad \text{against} \quad H_1(\theta): \theta \neq \theta_0 ,$$

where θ_0 is a known r-element vector (for the case $r = 1$, we may also consider one-sided alternatives). For that purpose, we define the following set:

$$(4.9) \qquad \mathcal{C}_\theta(\alpha,N,\mathcal{F}_{\lambda,\bar{\beta}}) = \left\{ \theta_0 \in R^r : \exists\ \lambda_0 \in \mathcal{C}_\lambda(\alpha,N,\mathcal{F}_{\lambda,\bar{\beta}}) \text{ such that } R\lambda_0 = \theta_0 \right\} .$$

Obviously, we have

$$(4.10) \qquad 1 - \alpha = P[\lambda \in \mathcal{C}_\lambda(\alpha,N,\mathcal{F}_{\lambda,\bar{\beta}})] \le P[\theta \in \mathcal{C}_\theta(\alpha,N,\mathcal{F}_{\lambda,\bar{\beta}})] ,$$

so that (4.9) establishes a conservative confidence set for θ. In order to test $H_0(\theta)$ of (4.8) for a specific known θ_0 value at level α one only has to check whether a $\lambda_0 \in R^p$ exists which obeys the two requirements: (i) $\lambda_0 \in \mathcal{C}_\lambda(\alpha,N,\mathcal{F}_{\lambda,\bar{\beta}})$ and (ii) $R\lambda_0 = \theta_0$. $H_0(\theta)$ is rejected when such a λ_0 value cannot be found; as soon as the search process establishes one λ_0 value that obeys the two requirements "acceptance" of $H_0(\theta)$ is legitimate. If $H_0(\theta)$ and Assumption A are both true then the probability of rejection does not exceed the level α.

In practice this search is a computer-intensive problem. The procedure is relatively simple if r is large. For $r = p-1$, the search is only over a straight line. If $r \ll p-1$ [as it will often be the case, see (4.1), (4.2), (4.3) and (4.5abc)] more substantial numerical problems arise. Our approach employed in the empirical examples of Section 6 is as follows.

Given the matrix R, we can find a $(p-r) \times p$ matrix $\bar{R}$ such that $R^+ = [R' \,|\, \bar{R}']'$ is non-singular. Then we consider the reparametrization of λ to θ^+, where θ^+ is a $p \times 1$ vector:

$$(4.11) \qquad \theta^+ = \left(\frac{\theta}{\bar{\theta}}\right) = \left[\frac{R}{\bar{R}}\right]\lambda = R^+\lambda .$$

This reparametrization facilitates the search process since it enables one to impose the above mentioned requirement (ii) directly, and makes the search solely over the space $\mathcal{D}_{\bar{\theta}}$ of the parameter $\bar{\theta} \in \mathcal{D}_{\bar{\theta}} \subseteq R^{p-r}$, where $\mathcal{D}_{\bar{\theta}}$ is actually determined by $\mathcal{D}_\lambda$ and R, and may even be of smaller dimension than p-r.

Let the $(p - r) \times 1$ vector $\bar{\theta}_0^{(i)}$ be the i-th value of $\bar{\theta}$ that is to be checked. This check is performed by verifying the (in)validity of

$\mathcal{P}(\lambda_0^{(i)}, N, \mathcal{F}_{\lambda,\bar{\beta}}) \geq \alpha$, where

$$(4.12) \qquad \lambda_0^{(i)} = \left(R^+ \right)^{-1} \left(\frac{\theta_0}{\bar{\theta}_0^{(i)}} \right) .$$

Now the only problem left for fully executing the test is to devise a strategy for starting and updating the series $\{ \bar{\theta}_0^{(i)} \; ; \; i=1,\ldots\}$, in combination with the design of a stopping criterion. The stopping criterion is obvious for the case where an acceptable $\lambda_0^{(i)}$ value has been found [this implies "acceptance" of $H_0(\theta)$], but less so when all values of $\lambda_0^{(i)}$ according to (4.12) checked so far gave p-values smaller than α, and it looks rather unlikely that an acceptable value of $\lambda_0^{(i)}$, obeying the two requirements, will be found. In our implementation of this procedure used in the empirical section below, we enhance the probability of finding an acceptable test value (if there are such in the $\mathcal{D}_{\bar{\theta}}$ space) straight-away, by taking for $\bar{\theta}_0^{(i)}$ the value obtained as the estimate of $\bar{\theta}$ from estimating the model

$$(4.13) \qquad y = Y(R^+)^{-1}(R^+\lambda) + X\beta + \varepsilon = Y(R^+)^{-1}\theta^+ + X\beta + \varepsilon$$

under the r constraints $\theta = \theta_0 = R\lambda_0$.

5. Exact Tests on Structural Change in the Polynomial Coefficients

Several finite-sample tests of parameter constancy against the presence of structural change for a linear regression model with one lagged-dependent variable are proposed in Dufour and Kiviet (1993b). In that study we considered two distinct cases, depending on whether λ is assumed to be constant under the alternative or not. In the first case (λ fixed), we studied two categories of tests: analysis-of-covariance (AOC) type tests against alternatives where β may change at g known breakpoints, and CUSUM-type tests which are built against less specific alternatives. In the second case, we studied again two types of tests: predictive tests and AOC-type tests against alternatives where λ may change at one known breakpoint. For all these four types of tests we found exact procedures. They work very satisfactory, except for the last situation mentioned i.e. the AOC change in λ test. All four structural change test procedures can now be generalized for the higher-order dynamic model case by employing the results obtained in Section 3 of the present study. Since the focus of this paper is on inference regarding λ, and because the procedures developed for the higher-order model here suggest a simple alternative to the unsatisfactory (because of low power) change in λ test developed earlier (whilst the generalization to the $p > 1$ case is rather trivial for the other structural change tests), we shall only consider here another and more promising AOC-type test on structural change in the p-element vector λ.

In Dufour and Kiviet (1993b) we obtained a test of the constancy of λ for the case of one known breakpoint under the alternative by considering the difference between two particular (viz. invariant with respect to β and σ) estimates of λ, obtained from the two separate sub-samples respectively. Thus, it was avoided to obtain the test statistic from estimating the model from the full sample under the alternative hypothesis, which involves two regressors both including particular lagged-dependent variable observations. Here we have managed to deal with comparable multiple dynamic models, and hence it should now be possible to develop an AOC-type test following the basic principles more closely.

We have to introduce some new notation that will facilitate the representation of the null hypothesis and the alternative model. We consider the case where there may by g (≥ 1) changes in the value of the parameter vector λ at g given breakpoints:

(5.1) $\quad 1 < T_1 < T_2 < \ldots\ldots < T_g < T$.

We define

(5.2) $$h(i) = \begin{cases} T_i & \text{for } i = 1,\ldots, g \\ T+1 & \text{for } i = g+1 \end{cases}$$

and the $T \times T$ matrices H_t such that $H_1 = I_T$, $H_{T+1} = 0$ and

(5.3) $$H_t = \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & I_{T-t+1} \end{array}\right] , \qquad \text{for } t = 2,\ldots, T .$$

We also define the $T \times T$ matrices

(5.4) $$G_i = H_{h(i)} - H_{h(i+1)} , \qquad \text{for } i = 1,\ldots, g ,$$

the $T \times T(g+1)$ matrix

(5.5) $$G = [\, I_T \mid G_1 \mid \ldots \mid G_g \,] ,$$

and the $p(g+1) \times 1$ coefficient vector

(5.6) $$\bar{\lambda} = \begin{pmatrix} \lambda \\ \hline \lambda^{(1)} \\ \hline \vdots \\ \hline \lambda^{(g)} \end{pmatrix} ,$$

where $\lambda^{(i)}$, $i = 1,\ldots, g$, are $p \times 1$ vectors.

Now, we generalize model (2.1) to

(5.7) $$y = G(I_{g+1} \otimes Y)\bar{\lambda} + X\beta + \varepsilon .$$

This parametrization entails that over the period from observation 1 through $(T_1 - 1)$ the coefficient vector relating to the regressors Y is λ, from observation T_g through T this coefficient vector is $\lambda + \lambda^{(g)}$, whilst it is $\lambda + \lambda^{(i)}$ from T_i through $(T_{i+1} - 1)$ for $i = 1,\ldots, g-1$.

For model (5.7) we shall consider tests of the hypothesis

$$(5.8)\qquad H_0(\lambda^{(1)},\ldots,\lambda^{(s)}):\ \lambda^{(1)} = \ldots = \lambda^{(s)} = 0\ ,$$

which involves pg restrictions, and of

$$(5.9)\qquad H_0(\bar\lambda):\ \bar\lambda = \bar\lambda_0\ ,\quad \text{where}\quad \bar\lambda_0 = (\lambda_0'\ ,\ 0')'\ ,$$

which involves $p(g+1)$ restrictions, i.e. $\lambda = \lambda_0$ and $\lambda^{(1)} = \ldots = \lambda^{(s)} = 0$. This set-up concerns structural changes in the lagged-dependent variable coefficients only, but it is straightforward to allow for changes in (elements of) the β vector as well (under both the null and the alternative) by appropriately redefining X and β. For the sake of simplicity, we assume that the values T_i and the matrix $[Y|X]$ are such that all elements of the coefficient vector $\bar\lambda$ are estimable, i.e. $[G(I_{g+1}\otimes Y)|X]$ has full column rank. Handling the more general case is feasible, but not pursued here.

$H_0(\bar\lambda)$, given in (5.9), is again a hypothesis which restricts all (transformed) lagged-dependent variable coefficients, so that we should be able to develop an exact test procedure by augmenting the model with the appropriate redundant regressors, and then test the corresponding hypothesis $\bar H_0(\bar\lambda)$ or $\bar H_0(\bar\lambda,\bar\beta)$, as in (3.8) or (3.9). Analogous to what we did in Section 3, we now consider for testing $\bar H_0(\bar\lambda)$ the statistic

$$(5.10)\qquad \mathcal{F}_{\bar\lambda} = \frac{T-p(g+1)-\bar m}{p(g+1)}\left[\frac{(y - Y\lambda_0)'M[X|\bar X_p(\bar\lambda_0)](y - Y\lambda_0)}{y'M[G(I_{g+1}\otimes Y)|X|\bar X_p(\bar\lambda_0)]y} - 1\right],$$

which does not constrain the redundant regressor coefficients under the null, while for testing $\bar H_0(\bar\lambda,\bar\beta)$ we use

$$(5.11)\qquad \mathcal{F}_{\bar\lambda,\bar\beta} = \frac{T-p(g+1)-\bar m}{p(g+1)+\bar m-k}\left[\frac{(y - Y\lambda_0)'M[X](y - Y\lambda_0)}{y'M[G(I_{g+1}\otimes Y)|X|\bar X_p(\bar\lambda_0)]y} - 1\right],$$

which involves $\bar m - k$ extra zero restrictions on the redundant regressors. Here $\bar X_p(\bar\lambda_0)$ is a $T\times(\bar m - k)$ matrix which is built as follows. As in Section 3, under $\bar\lambda = (\lambda_0',\ 0')'$ we have

$$(5.12)\qquad Y = D_p(\lambda_0) + X_p(\lambda_0)[I_p\otimes\beta] + \sigma\bar C_p(\lambda_0)[I_p\otimes\eta]\ ,$$

with the matrices $D_p(\lambda_0)$, $X_p(\lambda_0)$ and $\bar C_p(\lambda_0)$ as defined in (3.1), (3.4) and

(3.2) respectively. Hence, it easily follows that, in order to obtain similarity of both test statistics, $\bar{\bar{X}}_p(\bar{\lambda}_0)$ has to be such that $[X \vdots \bar{\bar{X}}_p(\bar{\lambda}_0)]$ has full column rank $\bar{m}$, and spans the same column space as $[X \vdots G(I_{g+1} \otimes D_p(\lambda_0)) \vdots G(I_{g+1} \otimes X_p(\lambda_0))]$. Writing

$$(5.13) \qquad Y(\eta,G,\lambda_0) = \frac{1}{\sigma} M[X \vdots \bar{\bar{X}}_p(\bar{\lambda}_0)]G(I_{g+1} \otimes Y)$$

$$= M[X \vdots \bar{\bar{X}}_p(\bar{\lambda}_0)]G\Big[I_{g+1} \otimes [\ \bar{C}_p(\lambda_0)[I_p \otimes \eta]\ ]\Big] ,$$

the denominator of the first term in square brackets in (5.10) and (5.11) can be written (after division by σ^2) as

$$(5.14) \qquad \eta' M[X \vdots \bar{\bar{X}}_p(\bar{\lambda}_0)]\eta - \eta' Y(\eta,G,\lambda_0)\{Y(\eta,G,\lambda_0)' Y(\eta,G,\lambda_0)\}^{-1} Y(\eta,G,\lambda_0)' \eta ,$$

and it easily follows that under $\bar{\lambda} = (\lambda_0', 0')'$ the two test statistics are pivotal indeed, and can be used in a simulation test procedure to yield exact inference.

Testing $\bar{H}_0(\bar{\lambda})$ or $\bar{H}_0(\bar{\lambda},\bar{\beta})$ can be interesting, for instance, for the case $p = 1$, $g = 1$ and $\lambda_0 = 1$ (to check whether a unit root is valid also over the second part of a sample), but more generally – and similar to the procedures developed in the foregoing section – these tests are the building blocks for exact (conservative) inference on $H_0(\lambda^{(1)},\ldots,\lambda^{(s)})$ of (5.8). The latter hypothesis is "acceptable" if for some value λ_0 the hypothesis $\bar{H}_0(\bar{\lambda})$ or $\bar{H}_0(\bar{\lambda},\bar{\beta})$ is not rejected. If $\bar{H}_0(\bar{\lambda})$ or $\bar{H}_0(\bar{\lambda},\bar{\beta})$ is rejected for all λ_0 values located in a confidence set for $\bar{\lambda}$ then $H_0(\lambda^{(1)},\ldots,\lambda^{(s)})$ should be rejected too.

6. Illustrations

For illustrative purposes we will make use of a data set analyzed and published in Davidson and MacKinnon (1985). It concerns Canadian quarterly data on housing starts (HS), gross national expenditure in 1971$ (Y) and real interest rates (RR); the series were seasonally adjusted by a regression technique that allowed for time-varying seasonality. Davidson and MacKinnon (henceforth DM) focus on testing the most appropriate functional form of the relationship between these variables, upon allowing for a very simple dynamic specification. Here we shall simply adopt a particular functional form of the long-run equilibrium relationship [we use hs=ln(HS) as the dependent variable, and RR and y=ln(Y) as the explanatory variables; see graphs in Picture 1], but focus on the most appropriate specification of the short-run dynamics. We shall use and compare (as far as possible) both the results of the usual asymptotic and the here presented finite-sample techniques.

< Here Picture 1 >

In Table 1 we give results on the model specification as preferred by DM (they remark, however, that it does not provide grounds for complacency!). Note that all three right-hand side variables are lagged one period. DM give an obvious economic explanation for that, and also remark that this makes the model much more useful for forecasting purposes. We add to this that if an adequate specification does not require contemporaneous explanatory variables indeed, this enhances the strength of the exogeneity characteristics of the regressor variables other than hs_{-1}, and, although it does not imply the strong exogeneity of RR_{-1} and y_{-1}, it will help to uphold (or approximate more closely) the validity of our Assumption A. On the other hand, however, we realize that the (to us unknown) seasonal filter that has been applied to the variables, may undermine strong exogeneity. Since we use a smaller sample than Davidson and MacKinnon our results are slightly different (we withhold more initial observations in order to examine in a next stage higher-order dynamics). All standard asymptotic empirical results presented here have been obtained by version 7 of PcGive [see Doornik and Hendry (1992)]. Estimates of the asymptotic standard errors of coefficient estimates are given in parentheses, and *p*-values based on asymptotic distributions are given in brackets. When these are smaller than the nominal level of 1% this is marked with two asterisks, and with one when smaller than 5%.

< Here Table 1 >

Although it seems that the fit leaves room for further improvement, all four coefficients appear highly significant. Most diagnostics approve this specification. Only tests for high-order forms (four or higher) of serial correlation in the disturbances expose possible weaknesses in the specification (of the dynamics) of this model. $AR(v_1,v_2)$ is a LM test against serial correlation of order v_1, transformed such that its null distribution is approximately $F(v_1,v_2)$. $ARCH(v_1,v_2)$ is a statistic which under the null hypothesis of no autoregressive conditional heteroscedasticity of order v_1 is approximately distributed as $F(v_1,v_2)$. $H(v_1,v_2)$ tests for correlation between the squared residuals and the regressors and their squares (and eventually their cross-products), and under unconditional homoscedasticity it is approximately $F(v_1,v_2)$ distributed. $R(1,v_2)$ is the RESET test for significance of the squared fitted values which should be compared with the $F(1,v_2)$ distribution, and N(2) is a statistic that is asymptotically distributed as $\chi^2(2)$ when the disturbances are normally distributed; SK is the skewness, and EK the excess curtosis.

< Here Table 2 >

The dynamic <u>mis</u>specification of the model presented in Table 1 is apparent from estimation of the higher-order dynamic model in Table 2. Although the fit is still not spectacular, all diagnostics – except N(2) – are found to have high p-values, and the restrictions imposed in Table 1 relative to this fourth-order dynamic ARX-model are clearly rejected (by the asymptotic methodology). It is noteworthy that (in congruence with the DM findings) the two current explanatory variables RR and y are both insignificant (when tested by asymptotic tests under the maintained hypothesis of weak-exogeneity of all regressors), and that despite the serious multicollinearity among these fifteen regressors, eleven of them have absolute t-ratio's above unity and six above two. Note that the last four hypotheses tested in the Table fit precisely into the format of Section 4. At the end of this Section we shall examine what inference can be obtained on the order of the polynomial $\lambda(B)$ from exact procedures.

A highly significant static long-run relationship is established among these three variables from the results of Table 2, viz.

$$(6.1) \qquad hs = \underset{(0.844)}{+5.341} \; \underset{(0.018)}{-0.0812}\, RR \; \underset{(0.081)}{+0.554}\, y \, .$$

[For further details on the status and derivation of (6.1), see Doornik and Hendry (1992).] Therefore, in achieving a more parsimonious model specification, we preserve the levels of the variables but omit the less significant lag-polynomial coefficients. We obtain the results given in Table 3; this includes a few tests on the order of the polynomial $\lambda(B)$ that will be "exactified" below.

< Here Table 3 >

From a conventional augmented Dickey-Fuller analysis all three variables show a univariate behaviour that does not differ significantly from nonstationary processes with one unit root. Focusing on variable hs only, we obtain the results presented in Tables 4, 5 and 6. From Table 4 we see that an 8-th order autoregression (in the usual differenced form with one lagged regressor in levels) with a linear trend and without seasonal dummies yields an acceptable statistical representation of the dynamic structure.

< Here Table 4 >

In Table 5 we have omitted the linear trend. The F(2,93) statistic tests the joint significance of the variables Δhs_{-6} and Δhs_{-7}, hence checking whether the dynamics can be brought back from 8th to 6th order. This is again a test that fits into the framework of the exact tests developed in Section 4, and so do the series of tests mentioned in Table 6 which check for much further reduction of the order of the dynamics. The last two tests mentioned in Table 6 involve the significance of regressor hs_{-1}. Therefore, the *p*-value presented for the F(6,95) test is unfounded, since the approximation of the asymptotic null distribution of this test by the F distribution is improper. For testing the significance of hs_{-1} only [which is equivalent with testing (4.2)], the augmented Dickey-Fuller unit root test is asymptotically valid.

< Here Tables 5 and 6 >

The procedures developed in the foregoing sections, which are to be employed now, produce p-values on any type of linear hypothesis on λ, including unit root cases, that are valid asymptotically [see Dufour and Kiviet (1993)], and are even exactly correct in finite samples when the distribution of the disturbances is specified properly. To obtain empirical results the exact procedures have been programmed in the GAUSS 386 System. We first focus on tests on the dynamic specification and the presence of unit roots in the univariate model for hs. The first two blocks of results in Table 7 concern five different test procedures on a series of various null hypotheses which fully specify ($r = 6 = p$) the $\lambda(B)$ polynomial of the model analyzed in Table 6. F refers to the standard test of p restrictions on $\lambda(B)$; the F^* and F^{**} columns refer to the statistics (3.10) and (3.12) of the same restrictions in an augmented model, where F^{**} also incorporates the zero restrictions on the redundant regressors. The Table gives p-values which, for the asymptotic tests have been obtained with respect to the F distribution (NB: this is only valid when the null hypothesis specifies the coefficients λ_0 such that all roots of $\lambda(B)$ are located outside the unit circle). For the exact tests the p-values are obtained by confronting the test statistics with a series of 499 simulated drawings from their true null distribution. Unless stated otherwise, we assume normality of the disturbances in the exact procedures. In the first block of the Table we present results, labelled (a) through (k), for sample size $T = 102$ (as in Tables 4 through 6) and in the second block we analyze the same null hypotheses (a) through (k) for a sub-sample of size $T = 52$. The last three results in the third block regard structural change tests.

< Here Table 7 >

Just for curiosity we test in result (a) the 6 restrictions where λ_0, as given in the Table, equals the least-squares estimate of λ. As is to be expected, all p-values are equal or close to unity, especially for the larger sample. Result (b) fully relates to the test in Table 6, where the reduction of the model to a simple random walk with drift is examined, i.e. $\lambda_{0,1} = 1$, and $\lambda_{0,j} = 0$, for $j = 2,\ldots,6$. Like the standard F test also the asymptotic F^* and F^{**} tests seem to reject this hypothesis, but note that due to the unit root the asymptotic null distributions differ from F. We observe that the actual finite-sample p-values of the F^* and F^{**} tests lead to conflicting inferences at the 5% level. Contrary to our intuition the p-values of F^* are found to be smaller than those of F^{**} in the first block of the Table. We first discuss the results for the full sample only and neglect, for the moment, the F^{**} results.

From (b) we see that the random walk with drift specification is rejected by the exact F^{*} test. However, according to results (c) and (d), a stationary non-zero mean AR(1) process with an autocorrelation coefficient of 0.9 or 0.8, although rejected by the asymptotic F, is acceptable on basis of the exact procedure. The results (c) and (d), where $r = 6$, imply that conservative tests for the hypotheses $\lambda_6 = \cdots = \lambda_{6-r+1} = 0$, for $r = 1,\ldots,5 < p$, do not lead to rejection either. For $r = 1$ and $r = 2$ this is in agreement with the corresponding asymptotic results given in Table 6, but we find conflict between the exact and asymptotic procedures for $r = 3$, 4 and 5. On the whole we will see that the rejection probability of the asymptotic tests seems too high (i.e. the *p*-values are too low).

Results (e) and (f) are again meant to enable inferences on the order of the polynomial $\lambda(B)$, hence on fewer than p restrictions, although the null hypotheses actually tested do involve p restrictions. Here we used a λ_0 value obtained from (4.12), where θ_0 is chosen such that the r restrictions are imposed, and the $p - r$ elements of $\bar{\theta}_0$ are chosen equal to their least-squares estimate under these r restrictions, see (4.13). We argued in Section 4 that in that way rejection of the p restrictions will be less likely than for most other possible values of $\bar{\theta}_0$. In (e) this strategy is followed to test the AR(1) specification and (f) tests the AR(2) model. We find a somewhat more marked acceptance of the AR(1) and the AR(2) specifications indeed, whereas the asymptotic procedures again produce rejections. In (g) we find an unanimous overall rejection of the omission of all lagged hs regressors.

Next we perform unit root tests. In (h) initially the one unit root restriction (4.2) has been imposed, and the remaining $p - 1$ restrictions resulted from constrained estimation. We see that the AR(6) model with one unit root is not rejected, which is in agreement with the asymptotic ADF result given in Table 6. In (i) restriction (4.3) is imposed as well. Hence, here we test for two unit roots. Hypothesis (i) is rejected, and we are inclined to believe that a further search through the appropriate $\mathcal{D}_{\bar{\theta}}$ space will not lead to acceptance of the I(2) hypothesis. Results (j) and (k) relate to the so-called HEGY seasonal unit roots tests, see Hylleberg et al. (1990). In (j) the λ_0 vector obeys the four restrictions (4.2) and (4.5abc). We find a clear rejection of the occurrence of a factor $(1 - B^4)$ in $\lambda(B)$ [which is no real surprise, given the fact that hs is seasonally adjusted].

In (i) we test the two restrictions (4.2) and (4.5a) jointly. These are also rejected by the exact F^* test.

In the second block of Table 7 we perform the same type of analysis, omitting the first 50 of the 102 sample data. We now see that the F^{**} p-values are lower [except in (k)] than the F^* values, as they usually are according to our experience. As far as we can see, the atypical results in this respect for the present full data set are just a rather exceptional case, due to the (accidental) relatively small residuals obtained for the first observations in this sample. This aspect is relevant because of the following. In all tests performed in the first two blocks of Table 7 we found $m = 7$. Hence, of the 12 redundant regressors only 6 had to be taken into account [we use a singular value decomposition with a tolerance of 10^{-8}]. It can be made plausible that in this simple model, where X consists of one column of unit elements only, the appropriate matrix $\bar{X}$ consists of columns which are determined by a (linearly or nonlinearly) trended variable (heavily determined by the values of the elements of λ_0) and by the columns of the matrix $[I_p \vdots 0]'$. Hence, since the redundant regressors are closely related to the dummy variables that would annihilate the contribution of the initial observations, they happen to have a minor effect in the present model on the obtained residual sum of squares, and therefore the F^{**} test is less powerful in the full sample. In contrast we find in the sub-sample that the F^{**} exact p-values are not only smaller than the F^* values, but that they are often also smaller than the asymptotic F p-values. This boosts our expectations regarding the relative power of the exact procedures.

In order to try the exact structural change tests developed in Section 5 we tested the constancy of the λ vector over the two separate parts of the sample. Note that $1 \leq \bar{m} \leq 25$. In result (a') we consider the unrestricted AR(6) model. The chosen λ_0 value is the λ estimate obtained under the alternative specification. We see that constancy of λ is acceptable ($\bar{m} = 11$). In (b') the null model is a simple random walk with drift over the full sample, and the alternative model is an AR(6) with a break in all λ coefficients at T_1. Here the two exact tests give conflicting results ($\bar{m} = 9$). In (k') the λ_0 value is equal to (k) for $T = 102$. Hence, roots +1 and -1 are imposed. These were accepted for the second sub-sample and gave conflicting results over the full sample. Now, imposing them and testing for constancy, we find p-values above 5% ($\bar{m} = 14$).

To check the sensitivity of our exact results to the normality assumption, we also performed the simulation procedures under two alternative distributional assumptions. We examined the changes in the p-values when it is assumed that the η vector consists of independent elements obtained from either the Cauchy distribution (which has no finite moments) or from a normalized and sign-changed χ^2 distribution. In the latter case we took $\eta_t = -(w_t - 8)/4$, where $w_t \sim \chi^2(8)$. It can be shown that this yields disturbances with $SK = -1$ and $EK = 1.5$, which is not too far from the values actually obtained in the Tables 4 through 6. Results are collected in Table 8. For the transformed χ^2 disturbances we find slightly lower p-values for the smaller sample size; for $T = 102$ the results are very close to the normal case. The effects of Cauchy disturbances are more pronounced. Often the F^* and F^{**} results are affected in an opposite way.

Finally we employed various of the exact procedures in the context of the econometric model on housing starts. For the model in Table 3 we examined tests on the order of the $\lambda(B)$ polynomial and a few other hypotheses, see Table 9. Here the maximum number of redundant regressors that may be required is 36, whereas actually only 12 had to be used. In result (A) we test the least-squares estimate of Table 3, and again find a plausible result. In (B) we find conflicting evidence on the hypothesis that the polynomial $\lambda(B)$ can be reduced to $(1 - B)$. From (C) we see that we cannot omit all lagged-dependent variables, but (D), (E) and (F) show that reduction of the order of the $\lambda(B)$ polynomial to 1 is acceptable, which is in conflict with the asymptotic findings for $r < 4$ in Table 3. Result (G) indicates that a $(1 - B)$ factor in $\lambda(B)$ while maintaining $p = 4$ is acceptable. In (H) we test for structural change. Now the maximum number of redundant regressors is 72, whereas we actually had to use only 20. The chosen λ_0 value is the λ estimate obtained when a break is allowed for; constancy is not rejected.

In the original DM specification of Table 1, which seems dynamically misspecified, we did not employ our procedures for further inference on $\lambda(B)$. We only checked whether structural change tests would expose the inadequacy of this model. This is not the case. Taking $T_1 = 57$ the relevant exact tests have p-values slightly above 50%, whilst the asymptotic test on the significance of $\lambda^{(1)}$ has a p-value of 45%. This completes our extensive use of these data on house building starts. The reader should realize that these computations were not undertaken to throw new light on the building industry in Canada, but only serve to illustrate the performance of particular techniques for the econometric model building industry.

7. Conclusion and Discussion

Standard test procedures used in econometrics to find an adequate specification of the short-run dynamics and long-run relationships in linear autoregressive distributed lag models typically only have an asymptotic justification and are nonexact in finite samples. Actual sample sizes are finite, and usually their size is fixed in the short-run. Also, asymptotic large sample arguments, which may be very useful for the (dis)qualification of particular techniques in particular circumstances, cannot make actual sample sizes larger. To contend oneself with the empirical results of asymptotically valid techniques just like that, usually implies that one accepts risks to draw false inferences that cannot be kept under control. In science in general, and in econometric statistical methodology in particular, such situations should be suppressed where possible.

Here we develop alternative exact procedures in the context of single (reduced form) dynamic linear regression models and we demonstrate their feasibility in an illustrative empirical model. We only considered exact procedures for inference on the coefficients of the lagged-dependent variables. These, however, will allow to obtain exact inference results on almost any type of linear or nonlinear restriction on any of the parameters of such models, as is demonstrated in Dufour and Kiviet (1993a) for the first-order dynamic model. The procedures make use of redundant regressor variables which annihilate nuisance parameter dependence and yield similarity of the test statistics. This allows to use them as "randomized tests", which implies that exact p-values can be obtained from a limited number of Monte Carlo replications. Critics on these exact techniques that may be put forward are, among other things, that:

(i) the addition of redundant regressors to the model, although accomplishing full control over the level of the tests, will lead to power loss;

(ii) the requirement to adequately specify the distribution function of the disturbances cannot be fulfilled in practice since we lack (economic) theory on how to specify disturbance terms;

(iii) the requirement that the regressors, apart from the lagged-dependent variables, have to be strongly exogenous will not be fulfilled in most models of practical interest;

(iv) the simulation tests involve too much computational efforts.

Our response is as follows:

(i) Indeed we find that the standard asymptotic tests usually have smaller p-values than the exact tests, but that does not mean that they have more power. Power can only be discussed if we know the size, and that is the basic weakness of the asymptotic methods: their actual finite-sample size depends on unknown parameters and the chosen nominal significance level is as firm on the actual size as the level of a pitfall. The p-values of the standard tests can only be judged after size correction. However, using the same sort of tests after the addition of redundant regressors, and assessment of their true finite-sample null distribution via simulation works in fact as a size correction. It certainly seems likely that more redundant regressors leads to less power. Therefore it is comforting that we found that the actual number of redundant regressors required was much less than the theoretical number, due to linear dependencies between them. Moreover, from the results in Dufour and Kiviet (1993a) we know that when more restrictions are tested less redundant regressors are required. Hence, if we do not test just λ but also elements of β, even fewer redundant regressors are required.

(ii) If the actual distribution of the disturbances is (under the usual regularity assumptions) very non-normal then the accuracy of the asymptotic methods, although asymptotically still correct, will be very poor in finite samples. Our methods easily allow one to perform a sensitivity analysis of the exact results under various distributional assumptions. Moreover, they possess the same asymptotic validity properties as the standard procedures (although the latter are even incorrect when the true distribution of the disturbances is known).

(iii) In case of weakly exogenous regressors we can again invoke the same asymptotic arguments as are used for the standard procedures to uphold the validity of our procedures which, indeed, are only fully exact when the regressors are strongly exogenous. Hence, although we agree that this is one of the weaker elements in the methods proposed here, the standard procedures suffer from the same weakness. Further research on the seriousness of this aspect in empirically relevant cases should be done (also including size and power comparisons). If the inaccuracies in inference which find their origin in conditioning are really serious, then it could be worthwhile to abandon conditioning and to model not only the endogenous, but also the weakly exogenous variables. Next, the present approach could be followed again to develop test statistics which are invariant with respect to the nuisance parameters in these enlarged models, thus allowing to control the size. The other extreme case are the higher-order univariate AR models, possibly with an intercept, seasonal dummies and trends, which are used to characterize stationary and non-stationary (periodic) stochastic processes. These fit completely into our framework and therefore we could produce exact versions of the ADF and HEGY tests, and also exact tests for integration of higher orders. Their relative performance should be further examined in controlled experiments.

(iv) The exact test statistics are, like the standard tests, simple ratios of residual sums of squares. To obtain the exact *p*-value a relatively short series of simulated independent realizations of the test statistic has to be computed. Nowadays this requires only a few lines of computer code and each application takes just a few seconds or less.

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TABLES

TABLE 1: Simple forecasting model for hs: 1956(1) to 1982(4)

$$hs = \underset{(0.5380)}{2.4768} + \underset{(0.0700)}{0.6097}\, hs_{-1} - \underset{(0.0087)}{0.0411}\, RR_{-1} + \underset{(0.0547)}{0.1827}\, y_{-1}$$

$R^2 = 0.795$, $\hat{\sigma} = 0.1420$, RSS = 2.0987 , T = 108

AR(1,103)	= 0.0940	[75.98%]		
AR(2,102)	= 1.4453	[24.05%]		
AR(3,101)	= 2.1487	[9.88%]		
AR(4,100)	= 2.4660	[4.97%]	*	
ARCH(4,96)	= 0.8228	[51.38%]		
H(9,94)	= 0.6228	[77.49%]		
R(1,103)	= 0.0158	[90.03%]		
N(2)	= 3.0495	[21.77%]		<SK = -0.32 ; EK = 0.55>

TABLE 2: Autoregressive distributed lag model for hs: 1956(1) to 1982(4)

$$\begin{aligned} hs = {} & \underset{(0.6743)}{2.7888} + \underset{(0.1004)}{0.6544}\, hs_{-1} - \underset{(0.0252)}{0.0660}\, RR_{-1} + \underset{(1.0653)}{1.0744}\, y_{-1} \\ & - \underset{(0.1142)}{0.1643}\, hs_{-2} + \underset{(0.1172)}{0.2328}\, hs_{-3} - \underset{(0.0978)}{0.2452}\, hs_{-4} \\ & + \underset{(0.0166)}{0.0086}\, RR + \underset{(0.0270)}{0.0507}\, RR_{-2} - \underset{(0.0264)}{0.0645}\, RR_{-3} + \underset{(0.0186)}{0.0289}\, RR_{-4} \\ & + \underset{(0.9020)}{0.4941}\, y + \underset{(1.0343)}{0.8558}\, y_{-2} - \underset{(0.9963)}{2.3794}\, y_{-3} + \underset{(0.8625)}{0.2444}\, y_{-4} \end{aligned}$$

$R^2 = 0.842$, $\hat{\sigma} = 0.1321$, RSS = 1.6251 , T = 108

AR(1,92)	= 0.4303	[51.35%]		
AR(4,89)	= 0.2581	[90.40%]		
N(2)	= 5.8766	[5.30%]		<SK = -0.52 ; EK = 0.65>
F(3,90)	= 0.1411	[93.51%]		<addition of seasonal dummies>
F(3,90)	= 0.4974	[68.50%]		<addition of fifth-order lags>
F(11,93)	= 2.4637	[0.94%]	**	<reduction to the model of Table 1>
F(3,93)	= 0.2128	[88.73%]		<reduction to the model of Table 3>
F(1,93)	= 6.2873	[1.39%]	*	<reduction of polynomial λ(B) to order 3>
F(2,93)	= 3.4992	[3.43%]	*	<reduction of polynomial λ(B) to order 2>
F(3,93)	= 2.7415	[4.76%]	*	<reduction of polynomial λ(B) to order 1>
F(4,93)	= 13.723	[0.00%]	**	<reduction of polynomial λ(B) to order 0>

<u>TABLE 3</u>: Autoregressive distributed lag model for hs: 1956(1) to 1982(4)

$$hs = \underset{(0.6590)}{2.8492} \; \underset{(0.0945)}{+0.6649\, hs_{-1}} \; \underset{(0.0169)}{-0.0567\, RR_{-1}} \; \underset{(0.8726)}{+1.4582\, y_{-1}}$$

$$\underset{(0.1097)}{-0.1641\, hs_{-2}} \; \underset{(0.1153)}{+0.2255\, hs_{-3}} \; \underset{(0.0950)}{-0.2556\, hs_{-4}}$$

$$\underset{(0.0256)}{+0.0457\, RR_{-2}} \; \underset{(0.0257)}{-0.0625\, RR_{-3}} \; \underset{(0.0183)}{+0.0290\, RR_{-4}}$$

$$\underset{(0.9598)}{+1.0491\, y_{-2}} \; \underset{(0.8308)}{-2.2156\, y_{-3}}$$

$R^2 = 0.840$, $\hat{\sigma} = 0.1306$, RSS = 1.6363 , T = 108

AR(1,95)	= 0.2727	[60.27%]		
AR(4,92)	= 0.1763	[95.01%]		
N(2)	= 5.6846	[5.83%]		<SK = -0.52 ; EK = 0.59>
F(3,93)	= 0.2128	[88.73%]		<extension to the model of Table 2>
F(8,96)	= 3.3912	[0.18%]	**	<reduction to the model of Table 1>
F(1,96)	= 7.2380	[0.84%]	**	<reduction of polynomial λ(B) to order 3>
F(2,96)	= 3.8246	[2.52%]	*	<reduction of polynomial λ(B) to order 2>
F(3,96)	= 3.1124	[2.99%]	*	<reduction of polynomial λ(B) to order 1>
F(4,96)	= 16.076	[0.00%]	**	<reduction of polynomial λ(B) to order 0>

<u>TABLE 4</u>: Univariate model for hs (with Trend): 1957(3) to 1982(4)

$$\Delta hs = \underset{(0.9137)}{0.7018} \; \underset{(0.00083)}{-0.00033\, Trend} \; \underset{(0.0896)}{-0.0636\, hs_{-1}}$$

$$\underset{(0.1308)}{-0.1285\, \Delta hs_{-1}} \; \underset{(0.1259)}{-0.2229\, \Delta hs_{-2}} \; \underset{(0.1193)}{+0.0810\, \Delta hs_{-3}} \; \underset{(0.1133)}{-0.2200\, \Delta hs_{-4}}$$

$$\underset{(0.1146)}{-0.1915\, \Delta hs_{-5}} \; \underset{(0.1082)}{-0.0870\, \Delta hs_{-6}} \; \underset{(0.1066)}{-0.0016\, \Delta hs_{-7}}$$

$R^2 = 0.197$, $\hat{\sigma} = 0.1453$, RSS = 1.94280 , T = 102

AR(1,91)	= 0.0386	[84.48%]		
AR(4,88)	= 0.5212	[72.04%]		
ARCH(4,84)	= 0.8551	[49.45%]		
N(2)	= 8.3914	[1.51%]	*	<SK = -0.61 ; EK = 0.85>
H(18,73)	= 0.6832	[81.65%]		
R(1,91)	= 0.4206	[51.83%]		
F(3,89)	= 0.0437	[98.78%]		<addition of seasonal dummies>
F(3,89)	= 0.3515	[78.82%]		<addition of 3 extra lags>
ADF	= -0.710	[>10%]		

TABLE 5: **Univariate model for hs (without Trend): 1957(3) to 1982(4)**

$$\Delta hs = \underset{(0.5806)}{0.9775} \; \underset{(0.0545)}{-0.0914}\, hs_{-1}$$

$$\underset{(0.1041)}{-0.0977}\,\Delta hs_{-1} \; \underset{(0.1029)}{-0.1947}\,\Delta hs_{-2} \; \underset{(0.1040)}{+0.1036}\,\Delta hs_{-3} \; \underset{(0.1015)}{-0.2006}\,\Delta hs_{-4}$$

$$\underset{(0.1054)}{-0.1744}\,\Delta hs_{-5} \; \underset{(0.1023)}{-0.0737}\,\Delta hs_{-6} \; \underset{(0.1015)}{+0.0106}\,\Delta hs_{-7}$$

$R^2 = 0.196$, $\hat{\sigma} = 0.1447$, RSS = 1.94605 , T = 102

AR(1,92)	= 0.0341	[85.40%]		
AR(4,89)	= 0.3921	[81.38%]		
N(2)	= 8.2469	[1.62%]	*	<SK = -0.61 ; EK = 0.80>
F(2,93)	= 0.2842	[75.32%]		<omission of 2 highest lags>
ADF	= -1.678	[>10%]		

TABLE 6: **Univariate model for hs (without Trend): 1957(3) to 1982(4)**

$$\Delta hs = \underset{(0.5697)}{1.0164} \; \underset{(0.0534)}{-0.0951}\, hs_{-1}$$

$$\underset{(0.0998)}{-0.0830}\,\Delta hs_{-1} \; \underset{(0.0967)}{-0.1781}\,\Delta hs_{-2} \; \underset{(0.1004)}{+0.1034}\,\Delta hs_{-3} \; \underset{(0.0957)}{-0.1773}\,\Delta hs_{-4}$$

$$\underset{(0.0988)}{-0.1664}\,\Delta hs_{-5}$$

$R^2 = 0.191$, $\hat{\sigma} = 0.1436$, RSS = 1.95794 , T = 102

AR(1,94)	= 0.0781	[78.05%]		
AR(4,91)	= 0.4983	[73.70%]		
N(2)	= 7.5412	[2.30%]	*	<SK = -0.57 ; EK = 0.78>
AR(1,94)	= 0.0781	[78.05%]		
F(1,95)	= 2.8396	[9.53%]		<reduction of polynomial λ(B) to order 5>
F(2,95)	= 2.8165	[6.48%]		<reduction of polynomial λ(B) to order 4>
F(3,95)	= 3.1541	[2.84%]	*	<reduction of polynomial λ(B) to order 3>
F(4,95)	= 3.3309	[1.34%]	*	<reduction of polynomial λ(B) to order 2>
F(5,95)	= 2.7848	[2.16%]	*	<reduction of polynomial λ(B) to order 1>
F(6,95)	= 54.513	[0.00%]	**	<reduction of polynomial λ(B) to order 0>
F(6,95)	= 3.7402	[0.22%]	**	<reduction to random walk with drift>
ADF	= -1.780	[>10%]		

TABLE 7: Exact inference in the univariate model for hs of Table 6

$k = 1$ (intercept) ; $p = 6$; $r = 6$; $N = 500$; $\eta \sim N[0,I]$

null hypothesis: elements of vector λ_0'							p-values (%): asymptotic			exact	
	$\lambda_{0,1}$	$\lambda_{0,2}$	$\lambda_{0,3}$	$\lambda_{0,4}$	$\lambda_{0,5}$	$\lambda_{0,6}$	F	F^*	F^{**}	F^*	F^{**}
T = 102 : 1957(3) to 1982(4)											
(a)	0.822	-0.095	0.282	-0.281	0.011	0.166	100.0	99.6	99.7	99.8	99.6
(b)	1.0	0.0	0.0	0.0	0.0	0.0	0.2	0.5	3.6	3.6	15.0
(c)	0.9	0.0	0.0	0.0	0.0	0.0	3.1	3.6	11.3	5.8	16.4
(d)	0.8	0.0	0.0	0.0	0.0	0.0	2.4	5.8	12.7	6.6	16.0
(e)	0.858	0.0	0.0	0.0	0.0	0.0	3.9	6.7	15.0	8.0	19.2
(f)	0.797	0.071	0.0	0.0	0.0	0.0	4.8	8.0	16.5	10.0	20.4
(g)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.2
(h)	0.865	-0.090	0.287	-0.281	0.024	0.194	78.6	97.6	96.9	99.8	99.6
(i)	1.205	-0.122	0.413	-0.436	0.093	-0.154	0.0	0.0	0.0	0.2	1.2
(j)	0.665	-0.169	0.000	1.000	-0.665	0.169	0.0	0.0	0.0	0.2	0.2
(k)	0.595	0.349	-0.135	0.166	-0.460	0.485	0.0	0.2	1.7	4.4	11.6
T = 52 : 1970(1) to 1982(4)											
(a)	0.822	-0.050	0.267	-0.407	0.092	0.190	100.0	99.4	78.6	100.0	89.6
(b)	1.0	0.0	0.0	0.0	0.0	0.0	7.4	2.3	1.5	12.4	6.2
(c)	0.9	0.0	0.0	0.0	0.0	0.0	20.9	23.4	9.6	34.0	16.2
(d)	0.8	0.0	0.0	0.0	0.0	0.0	27.7	33.7	15.4	39.6	17.8
(e)	0.810	0.0	0.0	0.0	0.0	0.0	27.9	34.0	15.1	40.0	18.0
(f)	0.810	0.000	0.0	0.0	0.0	0.0	27.9	34.0	15.1	40.0	18.0
(g)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.2
(h)	0.835	-0.038	0.277	-0.401	0.106	0.221	99.9	81.1	60.4	96.6	86.0
(i)	1.222	-0.123	0.419	-0.583	0.291	-0.226	0.3	0.0	0.0	1.0	1.4
(j)	0.728	-0.265	0.000	1.000	-0.728	0.265	0.0	0.0	0.0	0.2	0.2
(k)	1.000	1.000	-0.121	0.038	-0.426	0.558	10.1	2.9	6.5	21.6	27.0
T = 102 ; T_1 = 51 : break at 1970(1)											
(a')	0.795	-0.107	0.293	-0.182	-0.060	0.100	94.2	93.9	77.5	96.2	85.4
(b')	1.0	0.0	0.0	0.0	0.0	0.0	3.5	0.2	1.5	4.0	15.2
(k')	0.595	0.349	-0.135	0.166	-0.460	0.485	1.0	0.1	1.7	5.2	19.0

TABLE 8: Exact p-values (%) in the univariate model for hs of Table 6
k = 1 (intercept) ; p = 6 ; r = 6 ; N = 500

	T = 52 : 1970(1) to 1982(4)				T = 102 : 1957(3) to 1982(4)			
	SK= -1 ; EK= 1.5		Cauchy		SK= -1 ; EK= 1.5		Cauchy	
	F^*	F^{**}	F^*	F^{**}	F^*	F^{**}	F^*	F^{**}
(a)	99.8	88.2	98.4	46.4	100.0	99.8	89.2	67.0
(b)	10.4	5.4	7.4	14.2	5.6	14.0	5.8	13.0
(c)	31.8	14.2	17.2	17.6	5.8	15.8	6.0	13.2
(d)	34.8	16.6	18.2	17.8	6.8	16.0	5.2	12.8
(e)	35.6	16.8	18.4	17.8	7.8	18.2	5.8	13.4
(f)	35.6	16.8	18.4	17.8	9.2	20.0	6.8	13.6
(g)	0.2	0.2	0.4	6.4	0.2	0.2	0.2	2.2
(h)	96.8	84.4	85.6	44.2	99.8	100.0	99.8	77.2
(i)	0.8	2.2	6.6	12.2	0.6	2.4	4.4	9.0
(j)	0.2	0.4	5.6	8.2	0.2	0.2	2.4	3.4
(k)	19.8	26.2	16.4	19.8	4.4	12.6	10.0	13.2

TABLE 9: Exact inference in the model for hs of Table 3
k = 8 ; p = 4 ; r = 4 ; N = 500 ; $\eta \sim N[0,I]$
T = 108 : 1956(1) to 1982(4) ; T_1 = 57 : break at 1970(1)

	null hypothesis				*p*-values (%)				
	elements of vector λ_0'				asymptotic			exact	
	$\lambda_{0,1}$	$\lambda_{0,2}$	$\lambda_{0,3}$	$\lambda_{0,4}$	F	F^*	F^{**}	F^*	F^{**}
(A)	0.6649	-0.1641	0.2255	-0.2556	100.0	98.0	87.3	99.2	89.4
(B)	1.0000	0.0	0.0	0.0	0.0	0.0	0.2	0.8	5.8
(C)	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.2	0.2
(D)	0.6463	-0.1576	0.0630	0.0	13.3	18.2	46.7	29.0	51.8
(E)	0.6407	-0.1187	0.0	0.0	11.5	13.3	43.0	20.4	49.4
(F)	0.5711	0.0	0.0	0.0	6.1	10.8	40.4	18.6	49.4
(G)	0.8735	-0.1275	0.3642	-0.1102	0.0	0.1	2.4	17.0	27.0
(H)	0.7103	-0.2782	0.3111	-0.3462	56.0	61.0	30.5	71.2	37.4

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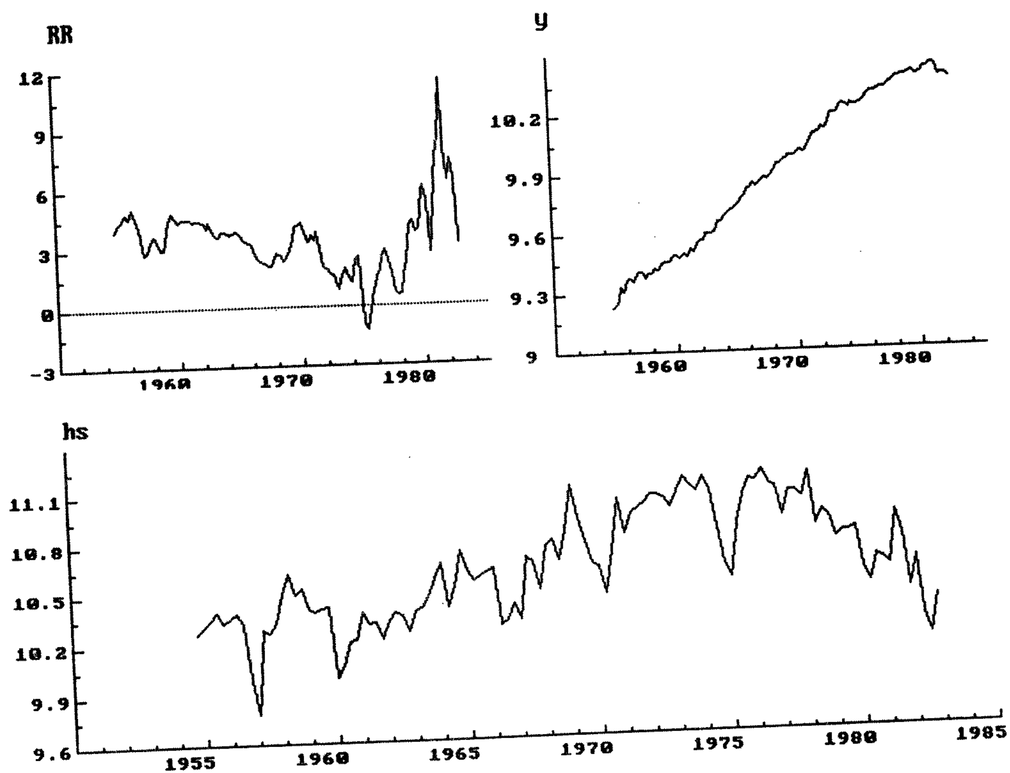
PICTURE 1



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