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**MULTIVARIATE COINTEGRATION IN THE  
PRESENCE OF STRUCTURAL BREAKS :  
THE CASE OF MONEY DEMAND IN MEXICO**

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## RÉSUMÉ

Ce papier se consacre à l'étude des implications de la libéralisation financière sur la demande de monnaie, et ce, en s'attaquant à la stabilité et à la prévisibilité des agrégats monétaires mexicains. Pour réaliser ceci, nous faisons appel à des techniques avancées d'études de séries temporelles. La demande de monnaie mexicaine est alors analysée par le biais de la cointégration multivariée, et ce, dans un cas où il y a rupture de tendance.

Mots clés : cointégration multivariée, rupture de tendance, racines unitaires

## ABSTRACT

This paper investigates the implications of financial liberalization on money demand, focusing on the stability and predictability of Mexico's narrow and broad monetary aggregates. In order to do so, two advanced time series econometric techniques, multivariate cointegration and structural break analysis, were jointly applied.

Key words : multivariate cointegration, structural break, unit roots

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## INTRODUCTION

The purpose of this part of the paper is to investigate the implications of financial liberalization on money demand, focusing on stability and predictability of narrow and broad monetary aggregates. The study gravitates around the important econometric concept that most macroeconomic time series are characterized by a unit root; which implies that random shocks have a permanent effect on the system (Nelson and Plosser, 1982). Two influential concepts that followed from work on unit roots, and that are jointly used in this paper, are: first the concept of cointegration, whereby two or more time series move together in a long run equilibrium relationship (see for instance Engle and Granger (1987), for the univariate case; and Johansen (1988, 1989), for the multivariate case). Second the concept of structural breaks, first introduced by Perron (1989, 1993a), and whereby a stationary time series is characterized by a structural change in its mean level. The study at hands, therefore, combines cointegration analysis with variables that exhibit tendency breaks, or simply putting, the existence of a long run equilibrium relationship of the studied variables around a deterministic trend which is *de facto* broken. The investigation is carried out by the utilization of Johansen's maximum likelihood multivariate cointegration technique in the presence of structural changes. The paper is divided into four parts. The first section is dedicated to discuss the theoretical issues behind the estimation. The specification is addressed in section B and its implementation in section C. Section D concludes, with appendixes on data, critical values calculation and on results of the cointegration analysis.

## A. THEORETICAL ISSUES ON THE EFFECTS OF FINANCIAL LIBERALIZATION ON MONEY DEMAND

The existence of a stable and predictable relationship between monetary aggregates, economic activity, prices, and interest rates is a crucial element in the formulation of monetary policy. Financial liberalization that improves the quality of economic signals, alters the institutional environment, and expands the array of financial opportunities creates potential for instability in money demand (Tseng and Corker, 1991, p.11).

Because of the changes associated with the financial liberalization incurred by Mexico during the period 1982-1993<sup>1</sup>, one could expect to see shifts in the level of money holdings. These changes will occur basically as a result of fundamental changes in:

- a) interest rates, which will after the change, be liberalized and thus better define the degree of riskiness of the market and of the borrowers.
- b) improved money market and monetary policy instruments to influence interest rates. These measures make money more demand oriented, as opposed to be supply determined (by means of direct credit controls, for example).
- c) improved competition in the financial markets, which will lower the transaction costs and increase efficiency, what will affect the speed of adjustment of money demand in relation to

<sup>1</sup> For a complete study of the Mexican financial liberalization, see Grandmont (1994).

interest rates differentials (domestically and internationally) (Leite and Sundararajan, 1990a, p. 9; and Tseng and Corker, 1991, p. 11).

Mexico has, in recent years, undertaken an important structural adjustment of its entire economy. A significant part of the financial liberalization, took place after 1988, and it is around then that we can see a more discrete shift in the behaviour of money demand. As illustrated in figure 1, the ratio of broad money (defined here as currency plus demand and time deposits in the banking system) to income tended to decrease from 1979 until 1988<sup>2</sup> and to increase thereafter, reflecting rapid growth in quasi-money balances. The ratio of narrow money (currency plus demand deposits in the banking system) to income shows a downward trend from 1970 up to 1990 and an increasing trend thereafter. Figure 1 seems to indicate instability in the trend of the monetary aggregates during the period 1978 - 92. Thus, in order to estimate the magnitude of the instability, some econometric techniques, following on work done by Perron (1989 and 1993b) will be used.

<sup>2</sup> This is the period that comprises the debt crisis and the stabilization programs, which started in 1982, but were very slow up to 1988. After 1988 the liberalization process accelerated, especially due to the control of inflation that resulted from the Economic Solidarity Pact of 1987.

Figure 1: Mexico's Broad and Narrow Monetary Aggregates, 1970:1 1992:1  
in percentage of GDP

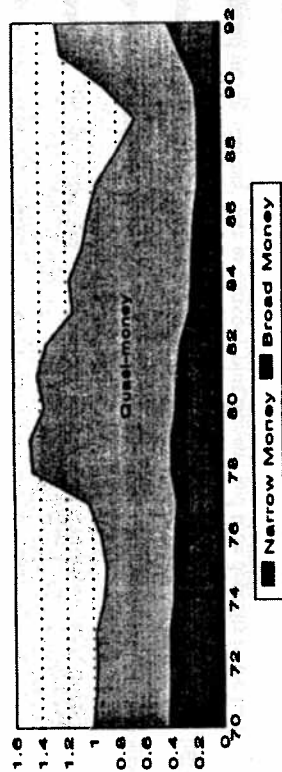
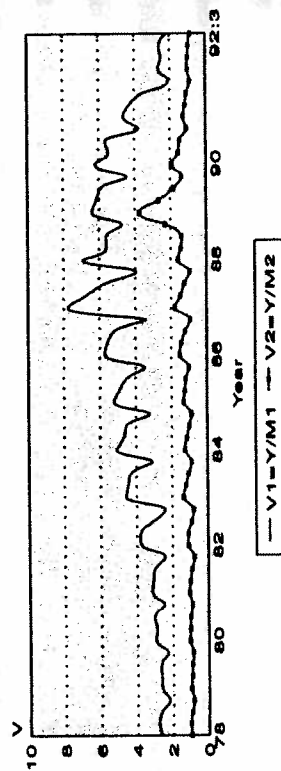


Figure 2: Mexico's Income Velocity of Money, 1978:1 1992:4



1. The Notion of Stability

Tseng and Corker (1991) explain that money demand will tend to be a stable function of income, prices and interest rates only in an unchanging institutional environment. Changes in the institutional environment are believed to cause long-run cycles in the income velocity of money. Thus, a country which is in its early stages of economic development and have immature financial

markets will typically have money growth exceeding income growth due to the monetization of the economy, hence money velocity will trend downwards. The opposite will also be true (upward trend in velocity of money) because as the degree of sophistication of the economy increases, financial market innovations permit agents to economize on their holdings of money. As we know, monetization is a phenomenon usually associated with narrow monetary aggregates, however, after the financial deregulation (mainly after 1988), Mexico seems to be in a stage of monetization for broad money, see figures 1 and 2. Tseng and Corker (1991) also found, for 7 of the 9 countries that they studied, that countries would tend to have rapid growth of quasi-money balances after financial liberalization. In general, then, these authors conclude that a rapid growth of quasi-money balances associated with a decrease in the income velocity of money reflect growth in wealth in excess of growth in income.

A financial liberalization will affect the institutional environment, and consequently possibly affect the stability of money demand relationships both in the long and short runs. Furthermore, as explained by Christiano (1991), positive changes in the growth rate of money supply will affect interest rates depending on which force is greater: the *liquidity effect*<sup>3</sup> or the *anticipated inflation effect*<sup>4</sup>. If the liquidity effect is greater than the anticipated inflation effect, the extra money in the economy will tend to push down interest rates which stimulates economic activity. As can be seen

<sup>3</sup> *Liquidity effect*: when an increase in money supply pushes down interest rates, which stimulates the economy (Christiano, 1991).  
<sup>4</sup> *Anticipated inflation effect*: when an increase in money supply leads people to expect further increases in money supply, and so more inflation. The expectation of higher inflation makes borrowers put a premium for inflation in interest rates, pushing interest rates up and maybe depress economic activity (Christiano, 1991).

from figures 3.a and 3.b, after 1988 there was indeed an increase in Mexico's money supply. From figures 3.e, 3.f and 3.g we can see the decrease in interest rates, what would illustrate the liquidity effect.

By using a multivariate cointegration estimation procedure we will, investigate the stability of the money demand functions. A rejection of cointegration means that money, income, price and interest rates have not shown a predictable and stable relationship over the span of the data sample. One of the reasons that may explain the rejection of cointegration is a change in institutional environment. Note, however, that the test is performed in a sample that goes from the first quarter of 1978 to the fourth quarter of 1992, being therefore an indicator of medium-term stability. It is relevant to point out that it is possible to have some instability which is not covered by the tests, and that it would be wrong to attribute this instability to the financial liberalization process. For instance, such instability may have occurred due to a misspecification of the demand function for money or because behavioral properties cannot be deducted using available data or even to inappropriate treatment of expectations. Thus one must be very careful, and sometimes even have to do some additional judgemental procedures to trace the source of the instability.

Figure 3.a: M1, In Levels

$$M1 = \ln(M1) - \ln(p)$$

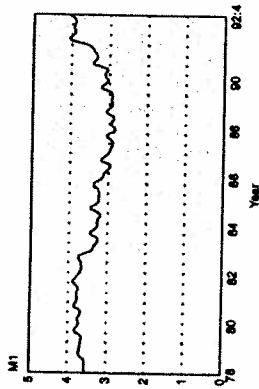


Figure 3.b: M2, In Levels

$$M2 = \ln(M2) - \ln(p)$$

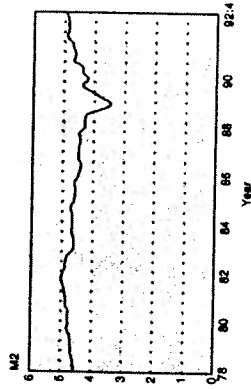


Figure 3.c: Real GDP, In Levels

$$y = \ln(gdp) - \ln(p)$$

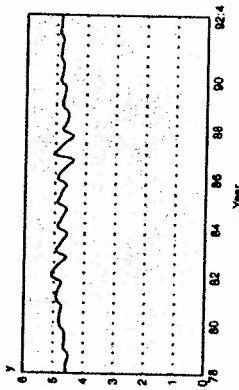


Figure 3.d: Prices, in Levels

$$p = \ln(cpi)$$

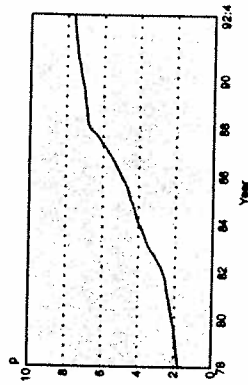


Figure 3.e: NRa, In Levels

NRa = deposit rates

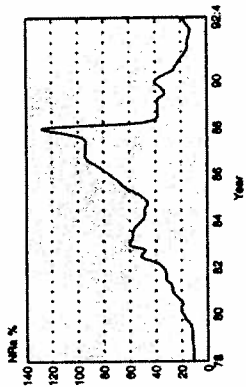


Figure 3.g: BRo, In Levels

BRo = deposit rates<sup>2</sup>/(M2-M1)/M2

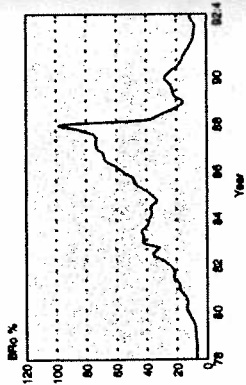


Figure 3.f: BRa, In Levels

BRa = (treasury bills' rates - deposit rates)<sup>2</sup>/(M2-M1)/M2

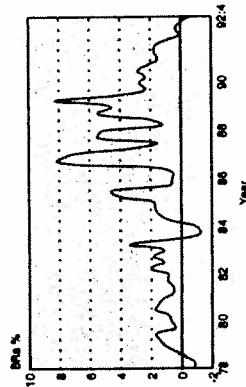


Figure 4.a: M1, in First Differences  
GM1 = M1 - M1(-1)

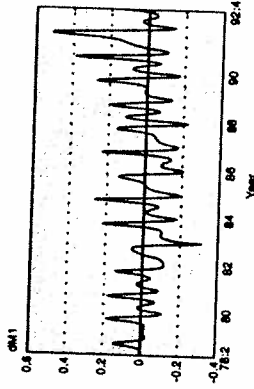


Figure 4.b: M2, in First Differences  
GM2 = M2 - M2(-1)

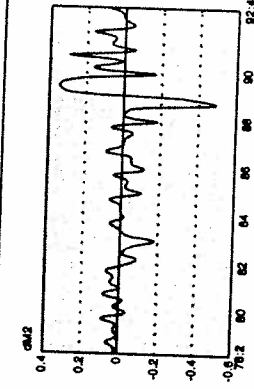


Figure 4.c: Real GDP, in First Differences  
dy = y - y(-1)

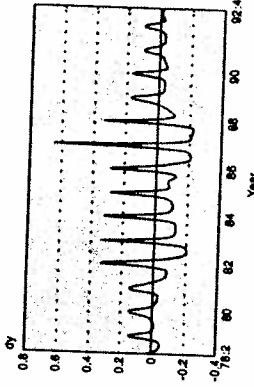


Figure 4.d: Prices, in First Differences  
dp = p - p(-1)

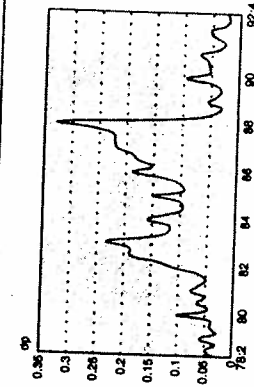


Figure 4.e: NRa, in First Differences

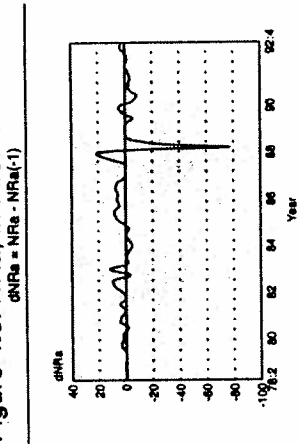


Figure 4.f: BRa, in First Differences

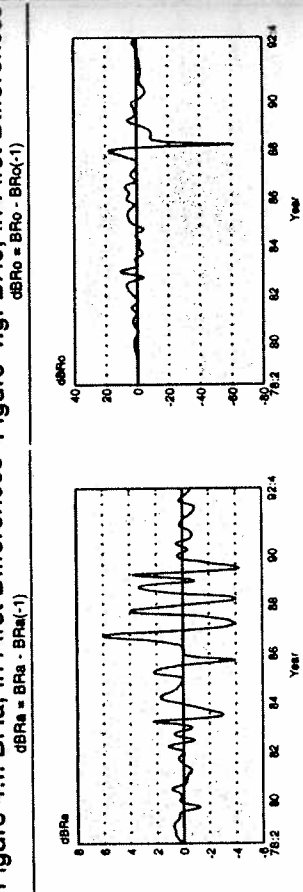
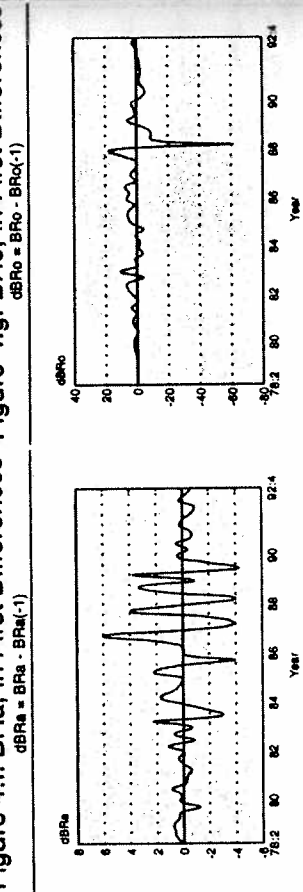


Figure 4.g: BRo, in First Differences



C. SPECIFICATION AND ESTIMATION ISSUES

1. Specification Issues

Many studies in the past years have dealt with the cointegration relationship, univariate or multivariate, of the demand for money equation. Johansen (1989); Johansen and Juselius (1990); Hendry and Ericsson (1991); Tseng and Corker (1991); King et al. (1991); Baba, Hendry and Starr (1992); *inter alia*, found a cointegration relationship for the demand for money. The money demand model to be used here, as in Tseng and Corker (1992), is in accordance with the general specification of money demand, in which it is a function of income (or wealth), prices, and interest rates. Specification which is in harmony with Blanchard and Fischer (1989, pp. 512-513) and with Dornbusch, Fischer and Sparks (1989, pp. 332-333). With this simple model we are interested in examining the stability and predictability of money demand.

Thus, our general cointegrating regression is defined as:

$$M = \alpha_0 + \alpha_1 P + \alpha_2 (Y/P) + \alpha_3 Ra + \alpha_4 Ro \quad (1)$$

Where<sup>3</sup>:

M: money (narrow, and broad), in natural logarithms

P: the general price level, in natural logarithms

Y: aggregate incomes, in natural logarithms

<sup>3</sup> Please refer to Appendix 1 for precise sources and definitions, as well as for data restrictions.



Ra: interest rate on alternative assets

for M1 (narrow money): NRa = deposit rates

for M2 (broad money): BRa = treasury bill rate minus deposit rates weighted by the share of quasi-money in broad money

Ro: return on money

for M1 (narrow money): NRo = zero

for M2 (broad money): BRo = deposit rate multiplied by the share of quasi-money in broad money

Price homogeneity is imposed ( $\alpha = 1$ ), and therefore, a real demand for money is the one being estimated. Variables, except for interest rates, are expressed in natural logarithms.

Equation (1) assumes that the level of money demanded adjusts instantaneously to changes in the variables. In reality, however, this is not the case as there is a time lag for the adjustment to take place. There are probably two reasons for this slowness: firstly there is a cost associated to adjusting financial portfolios and secondly the slow adjustment in expectations in response to new information. To attempt to capture the sluggish adjustment of money demand toward desired equilibrium holdings an error correction dynamic specification is used. The error correction equation "can be thought of as a more general, intertemporal version of partial adjustment in which expectations are based on available information" (Tseng and Corker, 1991, p. 13).

As our analysis is in the style of Johansen's multivariate cointegration, the vector autoregressive process (VAR), which includes a constant and independent Gaussian errors, has the following statistical model:

$$H_t: X_t = \Pi_1 X_{t-1} + \dots + \Pi_k X_{t-k} + \gamma_t + \varepsilon_t \quad (t = 1, \dots, T) \quad (2)$$

Where the sequence  $\{\varepsilon_t\}$  is an i.i.d.  $p$ -dimensional Gaussian random variables with mean zero and variance matrix  $\Lambda$ ,  $\varepsilon_t \sim \text{iid}(0, \Lambda)$ , and where  $X_{t-1}, \dots, X_{t-k}$  are fixed.

In the case where there is cointegration, the error correction model is specified as being equation (2) in first differences plus the term  $\Pi X_{t-1}$ , that is:

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \dots + \Gamma_{k-1} \Delta X_{t-k+1} + \Pi X_{t-1} + \gamma_t + \varepsilon_t \quad (3)$$

where,

$$\Gamma_i = -I + \Pi_1 + \dots + \Pi_i \quad (i = 1, \dots, k-1)$$

and

$$\Pi = -(I - \Pi_1 - \dots - \Pi_k) \quad (4)$$

Johansen (1989), and Johansen and Juselius (1990) explain that the coefficient matrix  $\Pi$  is the one which will convey information on a possible long-run equilibrium relationship among the variables in the data vector. The rank of  $\Pi$  will equal the dimension of the cointegrating space, so there are three possible cases:

- (i)  $\text{Rank}(\Pi) = p$  (the number of variables), i.e. the matrix  $\Pi$  has full rank, indicating that all variables are  $I(0)$  and that the vector process  $X_t$  is stationary; a cointegration investigation becomes, therefore, redundant.
- (ii)  $\text{Rank}(\Pi) = 0$ , i.e. the matrix  $\Pi$  is the null matrix and (3) corresponds to a traditional differenced vector time series model, without a stationary process.

(iii)  $0 < \text{Rank}(\Pi) = r < p$ , implying that there are  $p \times r$  matrices  $\alpha$  and  $\beta$  such that  $\Pi = \alpha\beta'$ . Here the linear combinations given by  $\beta'X_t$  are stationary (even though  $X_t$  itself is non-stationary), that is  $X_t$  is cointegrated with cointegrating vectors  $\beta$  and adjustment coefficients  $\alpha$ . So, in this case (3) can be interpreted as an error correction model.

Thus the main hypothesis to be considered in this study is the hypothesis of  $r$  cointegrating vectors:

$$H_1: \Pi = \alpha\beta' \quad (4)$$

## 2. Estimation Procedure

The maximum likelihood procedure suggested by Johansen (1989), will be used here to calculate the parameters of equation (3). The rationale behind the estimation are the notions of non-stationarity<sup>6</sup> integration<sup>7</sup> and cointegration<sup>8</sup>. In this paper, however, we follow the definition of cointegration suggested by Campbell and Perron (1991) and by Perron and Campbell (1992):

<sup>6</sup> Nelson and Plosser (1982) defined a time series with *non-stationarity* in the mean as being a series without fixed long term mean, "or put positively, [the series] has a tendency to move further away from any given initial state as time goes on." These authors also defined two important non-stationary processes:

*Trend-stationary process (TS)*: process consisting of a deterministic function of time, called a trend, plus a *stationary* stochastic process with mean zero. The TS process is fundamentally deterministic in nature.

*Difference-stationary process (DS)*: process in which the first or higher order differences is a stationary and invertible autoregressive moving-average (ARMA) process. The DS process is purely stochastic in nature. Furthermore, their empirical work demonstrates that macroeconomic time series are mainly DS processes.

A vector of variables [...] is said to be cointegrated if there exists at least one non zero  $n$ -element vector  $\beta$ , such that  $\beta'y_t$  is trend-stationary.  $\beta$  is called a cointegrating vector. If there exists  $r$  such linearly independent vectors,  $\beta_i$  ( $i = 1, \dots, r$ ), we say that  $\{y_t\}$  is cointegrated with cointegrating rank  $r$ . We then define the  $(n \times r)$  matrix of cointegrating vectors  $\beta = (\beta_1, \dots, \beta_r)$ . The  $r$  elements of the vector  $\beta'y_t$  are trend-stationary and  $\beta$  is called the cointegrating matrix (Campbell and Perron, 1991, p. 164).

As noted by these authors, two important consequences follow from this definition:

First, this definition allows the linear combinations of the variables that eliminate the unit roots to have non zero linear trends. This corresponds to the notion of "stochastic cointegration" in Ogaki and Park (1990). A stronger definition of cointegration, called "deterministic cointegration" by Ogaki and Park, would require that the same

<sup>7</sup> *Integration*: "A series with no deterministic component which has a stationary, invertible, ARMA representation after differentiating  $d$  times, is said to be *integrated* of order  $d$ , denoted  $x_t \sim I(d)$ " Engle and Granger (1987, p. 252).

<sup>8</sup> *Cointegration*: "The components of the vector  $x_t$  are said to be *co-integrated* of order  $d$ ,  $b$ , denoted  $x_t \sim CI(d,b)$ , if (i) all components of  $x_t$  are  $I(d)$ ; (ii) there exists a vector  $\alpha$  ( $\neq 0$ ) so that  $z_t = \alpha'x_t \sim I(d-b)$ ,  $b > 0$ . The vector  $\alpha$  is called the *co-integrating vector*" Engle and Granger (1987, p. 253). So, if two or more series are cointegrated, that means that they share a common stochastic trend (Stock and Watson, 1988, p. 164). Or simply, that integrated series may have stationary linear combination of their variables.

<sup>9</sup> According to Ogaki and Park's definitions, we have: stochastic cointegration:

$$y(t) = \theta_c + v_t + \gamma_t'X(t) + \varepsilon_t(t)$$

deterministic cointegration:

$$y(t) = \theta_c + \gamma_t'X(t) + \varepsilon_t(t)$$

Ogaki and Park (1989, pp. 12,13).

vectors  $\beta_i$  that eliminate the unit roots also eliminate the deterministic trend from the data. [...]

Second, the definition [1] does not require that each of the individual series be integrated of order one; some or all series can be trend-stationary. In this respect definition [1] differs from the definition given in Engle and Granger (1987) (Campbell and Perron, 1991, p. 165).

The second consequence of the definition has a special relevance, as it allows series with different orders of integration to be cointegrated.

In our analysis, the series will be first tested for their degree of integratedness, as for possible structural breaks in their trends. The investigation is however, centred in the evaluation and testing of deterministic cointegrating equilibrium vectors in our demand for money systems. The study follows Johansen's multivariate cointegration technique, as applied in Johansen (1989), and a brief summary of the procedure is as follows:

After regressing  $\Delta X_t$  and  $X_{t-k}$  on  $\Delta X_{t-1}, \dots, \Delta X_{t-k+1}, D_t$ , and 1, and taking the residuals ( $R_{it}$  and  $R_{it}$ , respectively), the matrices  $S_{op}, S_{w}, S_{w0}$  and  $S_{w0}$  are formed by:

$$S_{ij} = T^{-1} \sum_{t=1}^T R_{it} R_{jt}' \quad (i,j = 0, k) \quad (5)$$

The maximum likelihood estimator of  $\beta$  is then found by solving the following eigenvalue problem:

$$|\lambda S_{w0} - S_{w0} S_{w0}^{-1} S_{w0}| = 0 \quad (6)$$

which gives  $\lambda_1 > \dots > \lambda_p$  eigenvalues and  $V = (v_1, \dots, v_p)$  eigenvectors, which are normalized to have  $V'S_{w0}V = I$ . The choice of  $\beta$  is now:  $\beta = (v_1, \dots, v_p)$ , which gives:

$$L_{-2T}^{-1} \text{tr} \left( H_1 \right) = |S_{w0}| \prod_{i=1}^r (1 - \lambda_i) \quad (7)$$

The likelihood test statistic for the hypothesis  $H_1$  in  $H_1$ , since  $H_1$  is a special case of  $H_2$  for the choice  $r = p$ , is estimated by using the calculated eigenvalues in the following trace statistic test:

$$-2 \ln(Q; H(r) | H(p)) = -T \sum_{i=r+1}^p \ln(1 - \lambda_i) \quad (8)$$

The number of cointegrating vectors is then tested in a sequential way starting with  $r = 0$  and going up to  $r \leq p - 1$ .

The critical values with which to compare these trace statistics (table A.2) had to be computed by simulation. The reason being that in the study at hand, the cointegration analysis is carried out in the presence of structural breaks. Thus, the asymptotic distribution of the likelihood ratio test, the trace statistics, has to account for the rupture. The simulation<sup>10</sup> was done by using Nielsen's cointegration rank test statistical software, DisCo Version 1.1 (1993).

<sup>10</sup> Please refer to Appendix 2, for information regarding the simulation of the critical values for the trace statistics test, as well as for the table of critical values itself.

## D. EMPIRICAL ANALYSIS

### 1. Test for a Unit Root in the Series

In this first part, the individual series are tested for stationarity. Here is where we estimate whether the trend is stochastic, through the presence of a unit root, or deterministic, through the presence of a polynomial trend (Phillips and Perron, 1988).

The first results are from the tests for unit roots in the series. The tests that were used are: the Dickey-Fuller test (Dickey and Fuller, 1979); the Augmented Dickey-Fuller test (Said and Dickey, 1984) and the Phillips-Perron test (Phillips and Perron, 1988).

After a visual inspection of the series, see figures 3 and 4, we have decided to include in the tests for unit roots, a drift and a polynomial trend that goes up to order two, as well as to test for structural breaks. It is relevant to note that there are important differences between a second order polynomial trend series and a series characterized by a structural break. First, in series which are distinguished by a one tendency break, the rupture is much more severe than in series which have second order polynomial trends. Second, in the light of cointegration, the detrending of the series is done differently if there are structural breaks. When there is a break in trend for a certain series, which is used in conjunction with others in cointegration analysis, the break has to be taken into account when detrending all series, and this in order to have a uniform and consistent detrending of the series (this issue is further discussed in section C.2).

It is important for the Augmented Dickey-Fuller test as well as for the Phillips-Perron test, to have the proper truncation lag parameter  $k$  (see for instance Perron, 1992). Therefore, we follow the general to specific directives<sup>11</sup> suggested by Perron (1992), and Campbell and Perron (1991), for the selection of  $k$ ; the results are found in table 2.

Table 1: Results For The Truncation Lag Parameter  $k$   
 $k\text{-maximum} = 4$

A) Series in Levels						
Series	M1	M2	Y	P	NRa	BRa BRo
$k'$	4	4	4	1	0	2 0
B) Series in First Differences						
Series	dM1	dM2	dY	dP	dNRa	dBRa dBRo
$k'$	3	3	3	3	4 0	1 0

Results for the unit roots tests on the series, in levels and in first differences, are found in table 2. Summarizing, all tests demonstrated that NRa and BRo are integrated of order 1,  $I(1)$ . Y and BRa are stationary processes,  $I(0)$ , according to the DF tests and the PP tests; the ADF tests suggest, however, that these series in levels have a unit root,  $I(1)$ . M1 was characterized as an  $I(1)$  process by the DF and PP tests, and as an  $I(2)$  by the ADF tests. Two unit roots,  $I(2)$ , were found for consumer

<sup>11</sup> "Start with some upper bound on  $k$ , say  $k_{max}$ , chosen a priori. Estimate an autoregression of order  $k_{max}$ . If the last included lag is significant (using the standard normal asymptotic distribution), select  $k = k_{max}$ . If not, reduce the order of the estimated autoregression by one until the coefficient on the last included lag is significant. If none is significant, select  $k = 0$ ." (Campbell and Perron, 1991, p. 155)

prices under all tests, except for the ADF  $t^*$  who characterises the series as an I(1) process around a second order polynomial trend. ADF  $t^*$  also differentiated itself in the case of M2 where it found the series to be I(2), all other tests suggested a single unit root.

Underlined tests results are the ones that have been chosen, as we believe that they best characterise the series in question. Note that, even if the results are the same, the tests without trend were preferred to the ones with one trend, as the former represents a more parsimonious regression. The series for which no results are underlined, namely interest rates and inflation rates, are the ones we believe to have structural breaks (see figures 3 and 4). Thus a more detailed analysis of these series is required.

Series in Levels

Series	k	DF	DF $t^*$	ADF	ADF $t^*$	PP	PP $t^*$
M1	4	-1.2042	-0.7755	<u>-2.4246</u>	<u>-2.5219</u>	-1.0608	-0.3651
M2	4	-1.4256	-1.2741	-1.7469	<u>-1.9907</u>	-1.5580	-1.4312
P	1	-0.3581	-0.5064	-0.3829	<u>-1.0317</u>	-0.3687	-1.4312
Y	4	-5.6035	-5.6531	-5.5845	<u>-2.4782</u>	-5.7056	-5.8322
NRA	0	-1.6878	-1.4945	-2.7618	-2.4330	-1.6158	-1.4959
BRA	2	-3.6639	-3.6346	-4.1790	-2.6162	-3.5482	-3.5548
BRo	0	-1.6720	-1.4931	-2.6629	-2.7924	-1.8006	-1.5892

Series in First Differences

Series	k	DF	DF $t^*$	ADF	ADF $t^*$	PP	PP $t^*$
DM1	3	-10.6134	-10.9106	-11.8945	-1.7561	-10.521	-11.1462
DM2	3	-6.1115	-6.0985	-6.3554	<u>-2.9321</u>	-6.1220	-6.1641
DP	4	-2.4194	-2.4162	-3.3867	-2.4770	-3.3723	-2.4710
DY	3	-9.9197	-9.8354	-9.7729	<u>-3.8334</u>	-3.8763	-2.4939
DNA	0	-7.5745	-7.6710	-7.9157	-7.8964	-4.2786	-7.9375
DBRA	1	-7.9775	-7.9736	-7.9157	-7.9434	-13.907	-9.6502
DBRo	0	-7.7259	-7.8174	-7.7493	-7.9434	-10.521	-8.0309

Table 2: Unit Root Test Results,  $t^*$ -Statistic for the Series' Regression

(1) Dickey-Fuller's  $t$ -Test Results, Regressing with a constant:  $X_t = \alpha + \beta X_{t-1} + \xi_t$   
 (2) Dickey-Fuller's  $t$ -Test Results, Regressing with a constant and a trend:  $X_t = \alpha + \beta X_{t-1} + \gamma t + \xi_t$   
 (3) Dickey-Fuller's  $t$ -Test Results, Regressing with a constant:  $X_t = \alpha + \beta X_{t-1} + \sum_{i=1}^k \delta_i X_{t-i} + \xi_t$   
 (4) Augmented Dickey-Fuller's  $t$ -Test Results, Regressing with a constant and a second order polynomial trend:  $X_t = \alpha + \beta X_{t-1} + \gamma t^2 + \sum_{i=1}^k \delta_i X_{t-i} + \xi_t$   
 (5) Augmented Dickey-Fuller's  $t$ -Test Results, Regressing with a constant and a trend:  $X_t = \alpha + \beta X_{t-1} + \gamma t + \sum_{i=1}^k \delta_i X_{t-i} + \xi_t$   
 (6) Augmented Dickey-Fuller's  $t$ -Test Results, Regressing with a constant and a second order polynomial trend:  $X_t = \alpha + \beta X_{t-1} + \gamma t^2 + \sum_{i=1}^k \delta_i X_{t-i} + \xi_t$   
 (7) Phillips-Perron's  $t$ -test Results, Regressing with a constant  
 (8) Phillips-Perron's  $t$ -test Results, Regressing with a constant and a trend  
 Critical values ( $n=50$ ):  
 for (1), (4) and (7): -2.93 (Fuller, 1976).  
 for (2), (5) and (8): -3.50 (Fuller, 1976).  
 for (3) and (6): -3.82 (Ouliaris, Park and Phillips, 1989).  
 (\*) Denotes statistical significance, on at least a 5% level, for stationarity.  
 Note: for ADF  $k=4$ , for PP  $k=k_{max}$ , ( $k_{max}=4$ ). As  $k=0$  for some series, the ADF test becomes the DF test.

### 1.1 Unit Root Tests with Structural Changes in the Series

According to most of the results of the conventional stationarity tests (DF, ADF and PP) reported in table 2, the three interest rates series would be I(1), and prices would be I(2). A visual inspection of these series (see figures 3 and 4) indicates, however, that the structural changes that the Mexican economy has been undergoing, since the last decade, may have altered the long term economic fundamentals, and therefore the trends of the respective series. As demonstrated by the seminal work of Perron (1989), tests of the unit root hypothesis against trend stationary alternatives are biased towards non-rejecting the unit root when there are one-time changes (breaks) in the tendency of a trend-stationary series. These changes are, normally, the results of a "big shock" or of an unusual event that have permanent effects on the level of the series.

The literature on structural breaks has been showing the importance of the correct characterization of the economic time series when doing applied work<sup>12</sup> (see *inter alia* Zivot and Andrews, 1992; Perron and Vogelsang, 1992; Banerjee, Lumsdaine and Stock, 1992). New tests have been developed to estimate the precise moment of rupture, so to avoid the *a priori* date specification problem denounced by Christiano (1992). In this paper we follow Perron's (1993b) statistical procedures for testing for a unit root while allowing for the possible presence of a one-

<sup>12</sup> Note that on the paper by Tseng and Corker (1991), they seemed to face structural changes, due to financial liberalization, in the Philippines, Myanmar and Indonesia. Although no formal tests for structural breaks were performed, they tried to account for the changes by imposing up to two intercept shift dummies in the monetary equations. These dummies were introduced at pre-specified dates.

time change in trend, and this at an unknown date. In his article, two general models are suggested:

- a) the "additive outlier" model (AO); appropriate when the change is sudden,
- b) the "innovation outlier" model (IO); adequate when the change is gradual.

There are three qualifications for the AO model, (of which only 1 and 2 apply to the IO model):

1. The *crash* model, which allows for a one-time change in the intercept of the trend function.
2. The *sudden change in level followed by a different growth path* model, allowing for both a change in the intercept and the slope of the trend function to take place simultaneously.
3. The *changing growth* model, which allows for a change in the slope of the trend function without any sudden change in the level at the time of the break.

The second specification is the one being used in our estimations, as it has a more complete representation. The null hypothesis specifies the presence of a unit root, while the alternative hypothesis specifies trend-stationarity. "Note that the changes in the trend function are allowed to occur under both the null and the alternative" (Perron, 1993b, p. 5).

For the AO model, the procedure consists of a two step approach, where in the first, the trend function of the series is estimated and removed from the original series through the following regression:

$$y_t = \nu + \beta t + \theta DU_t + \gamma DT_t + \tilde{y}_t \quad (9)$$

Where  $DU_t = 1$  and  $DT_t = (t - T_b)$  if  $t > T_b$  (zero otherwise), and where  $\hat{y}_t$  is defined as the detrended series.  $DU_t$  accounts for the change in level while  $DT_t$  for the change in slope. For the second step, the test is based on the value of the  $t$ -statistic for testing that the sum of the autoregressive coefficients is equal to 1 ( $\alpha = 1$ ) in the following autoregression applied to the estimated noise component  $\hat{y}_t$ :

$$\hat{y}_t = \alpha \hat{y}_{t-1} + \sum_{i=0}^k \alpha_i D(T_b)_t + \sum_{i=1}^k \alpha_i \Delta_{t-i} + \epsilon_t \quad (10)$$

where  $D(T_b) = 1$  if  $t = T_b + 1$  and zero otherwise.

For the IO model, the test is based on the following regression:

$$\hat{y}_t = v + \beta t + \theta DU_t + \gamma DT_t + \delta D(T_b)_t + \alpha \hat{y}_{t-1} + \sum_{i=1}^k \alpha_i \Delta \hat{y}_{t-i} + \epsilon_t \quad (11)$$

where  $DU_t$ ,  $DT_t$  and  $D(T_b)$  follow the same pattern as in the AO model.

The construction of the test statistics of interest, namely the  $t$ -statistic for testing

$\alpha = 1$  in [9] - [10] and [11] depends on two parameters that are in general

unknown: the break date  $T_b$  and the truncation lag parameter  $k$ . (Perron, 1993b, pp. 14-15).

The time of the break is estimated endogenously through a procedure whereby  $T_b$  is selected as the value, over all possible break points, which minimizes the  $t$ -statistic for testing  $\alpha = 1$  in the appropriate autoregression. The truncation lag parameter  $k$  was set to 4 ( $k_{max} = 4$ ) for all series.

Table 3 presents the empirical results for the series in levels and in first differences. Columns 1 and 2 list the series and the model of the trend function that was used. Columns 3 and 4 give the date of the break in the trend function and the relative time of the break in relation to

the total sample size. Column 5 shows the value of the truncation lag parameter  $k$  being selected by the  $t$ -statistic to test for the significance of the last lag.  $\beta$ ,  $\theta$  and  $\gamma$  are the estimated values of the coefficients of equations (9) or (11), their corresponding  $t$ -statistics are in parenthesis.  $\beta$  is the estimate of the initial (pre-break) slope of the trend function,  $\theta$  is the estimate of the change in the intercept of the trend function and  $\gamma$  is the estimate of the change in the slope of the trend function. Columns 9 and 10 present the estimate of the sum of the autoregressive coefficients,  $\alpha$ , and its associated  $t$ -statistic for testing that  $\alpha = 1$ ,  $t_\alpha$ .

Note that only in the case where there is a rejection of the unit root, the  $t$ -statistic on the change in slope and/or intercept is asymptotically normally distributed. Moreover, stationarity makes the selected value of  $T_b$  be a consistent estimate of the time of the break (Perron, 1993b).

As the two models (AO and IO) were employed, one has to select the appropriate one for each series. Because M1, dM1, M2, dM2, Y, dY and P were already characterized (see table 2), the structural break analysis will primarily focus on the interest and inflation rates. It is relevant to note, however, that in general, under both models, the results obtained in table 2 are confirmed for these variables (M1, dM1, M2, dM2, Y, dY and P). The only exceptions may be M2 and Y for which one of the models implies stationarity at level of the series, while the other suggests a unit root. Due to the ambiguity of the results, and after a visual inspection of the series (see figures 3.b and 3.c), we have decided to characterize these series as  $I(1)$  processes without break. Our decision is based on the fact that, for both series, the break appears to be very shallow and may not be significant in a larger sample analysis.

Table 3: Results of the Unit Root Test with Structural Changes in the Series

Series	Model	T <sub>1</sub>	λ=T <sub>1</sub> /T	k	β	θ	γ	α	t <sub>α</sub>
MI	AO2	1986:2	0.57	4	-0.017 (-5.14)	-2.676 (-10.51)	0.061 (10.01)	0.679	-3.791
	IO2	1988:4	0.73	4	-0.01 (-3.57)	-1.869 (-2.97)	0.041 (3.11)	0.555	-4.024
PM1	AO2	1984:2	0.44	4	-0.004 (-1.01)	-0.219 (-1.60)	0.008 (-1.75)	-0.606	-3.429
	IO2	1982:2	0.30	4	-0.007 (-0.97)	-0.318 (-2.19)	0.013 (1.57)	-0.406	-3.228
M2	AO2	1987:2	0.63	4	-0.006 (-2.10)	-2.510 (-7.07)	0.049 (6.48)	0.528	-4.372
	IO2	1988:2	0.70	4	-0.008 (-4.89)	-3.097 (-6.39)	0.006 (6.47)	0.324	-6.589
DM2	AO2	1987:3	0.66	4	-0.001 (-0.88)	-0.393 (-1.66)	0.009 (1.89)	-0.239	-5.187
	IO2	1988:4	0.73	2	-0.004 (-3.30)	0.667 (2.09)	-0.007 (-1.15)	-0.695	-9.226
Y	AO2	1986:4	0.60	4	-0.001 (-2.54)	0.089 (1.59)	-0.000 (-0.36)	0.189	-5.477
	IO2	1987:3	0.65	4	-0.001 (-0.87)	-0.463 (-2.39)	0.009 (2.45)	0.490	-3.243
ΔY	AO2	1981:1	0.22	4	0.003 (0.23)	-0.025 (-0.18)	-0.002 (-0.19)	-1.960	-5.077
	IO2	1982:1	0.28	4	-0.002 (-0.26)	-0.229 (-2.08)	0.004 (0.60)	-2.144	-5.288

Table 3: Results of the Unit Root Test with Structural Changes in the Series

Table 3: Results of the Unit Root Test with Structural Changes in the Series (Continued)

Series	Model	T <sub>1</sub>	λ=T <sub>1</sub> /T	k	β	θ	γ	α	t <sub>α</sub>
P	AO2	1985:3	0.51	4	0.101 (18.91)	0.928 (3.14)	-0.004 (-0.566)	0.902	-3.256
	IO2	1986:3	0.58	1	0.011 (3.95)	0.394 (4.06)	-0.008 (-4.05)	0.914	-3.534
ΔP	AO2	1986:4	0.66	4	0.004 (4.86)	0.493 (6.70)	-0.129 (-7.72)	0.258	-5.240
	IO2	1987:4	0.67	4	0.003 (4.99)	0.104 (1.16)	-0.005 (-2.47)	0.245	-6.447
NRA	AO2	1987:3	0.65	0	2.277 (12.42)	215.407 (9.07)	-5.849 (-11.72)	0.558	-4.334
	IO2	1987:4	0.67	0	1.161 (9.357)	57.670 (3.10)	-2.392 (-5.29)	0.332	-10.016
ΔNRA	AO2	1987:4	0.67	0	0.096 (0.62)	-46.833 (-2.14)	0.736 (1.63)	-0.274	-10.953
	IO2	1988:1	0.68	0	0.145 (2.29)	-1.972 (-0.17)	-0.110 (-0.48)	0.158	-13.610
BRa	AO2	1986:1	0.55	1	0.050 (1.68)	10.752 (5.31)	-0.222 (-4.39)	-0.030	-7.399
	IO2	1986:2	0.57	1	0.035 (1.37)	13.889 (5.64)	-0.270 (-4.94)	-0.031	-7.408
ΔBRa	AO2	1989:1	0.76	1	0.000 (0.01)	0.268 (0.04)	-0.014 (-0.12)	-0.499	-9.311
	IO2	1989:1	0.76	2	-0.003 (-0.15)	-5.937 (-1.08)	0.091 (0.88)	-1.022	-8.260



In the case of the inflation rates, we have decided to choose the results of the IO model. Firstly because the critical value for the rejection of the unit root is much higher. Secondly because an IO model, that describes a gradual change, better portrays the wage and price controls of the *Economic Solidarity Pact*<sup>13</sup> that was signed on December 1987. Note how figure 5.d tends to indicate that the inflation rates seem to be returning to the pre oil-shock, debt-crisis level.

For the NRa, the judgment obviously favours the IO model as it depicts the notorious break in the series (see figure 3.e). It is relevant to note that the  $t_a$  on the AO model is somewhat ambiguous, not being high enough to reject the unit root but being close. The problem being, probably, with the power of the test. The IO model is also selected for BRa and BRo. Note that the IO model is preferred for the interest rates because of its nature of gradual transition to the new trend.

Therefore, table 4 tells us that by choosing  $T_b$  minimizing the  $t$ -statistic on  $\alpha$  and by choosing  $k$  using the recursive  $t$ -statistic on the last lag, we reject the null hypothesis of unit root in favour of the alternative hypothesis of stationary fluctuations around a broken trend function

<sup>13</sup> Accord signed on December 15, 1987, by the government, the industrial sector and union leaders, with the intention of reducing the inflation rate by controlling prices and wages.

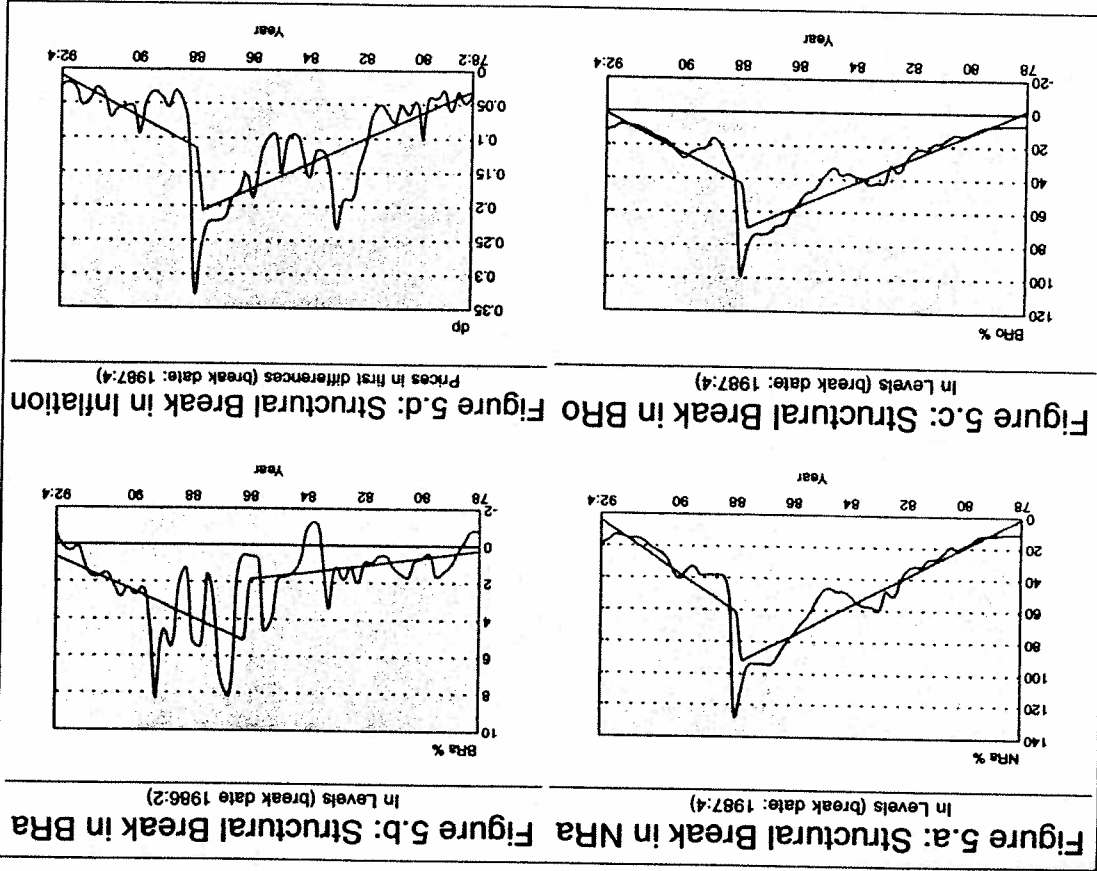
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*(This text is a mirror image of the text on page 28 and is therefore not included in this transcription.)*

Series	Model	$T_b$	$\lambda=T_b/T$	k	$\beta$	$\theta$	$\gamma$	$\alpha$	$t_a$
DBRo	AO2	1986:3	0.57	0	0.097 (0.612)	-7.684 (-0.60)	0.016 (0.05)	-0.096	-8.242
	IO2	1987:4	0.67	3	1.473 (9.532)	59.933 (3.32)	-2.271 (-5.33)	0.240	-9.755
BRo	AO2	1987:2	0.63	2	1.750 (10.77)	172.349 (9.18)	-4.632 (-11.49)	0.324	-6.326
	IO2	1987:3	0.65	0	0.075 (0.55)	-41.837 (-2.33)	0.663 (1.79)	-0.162	-8.749

Table 3: Results of the Unit Root Tests with Structural Changes in the Series (Continued)

for the interest rates series (NRa, BRa, BRo) and for the inflation rates<sup>14</sup> (dP). The *t*-statistics associated with the change in slope and intercept are, in general, highly significant, confirming the breaks and justifying the selection of specification 2.



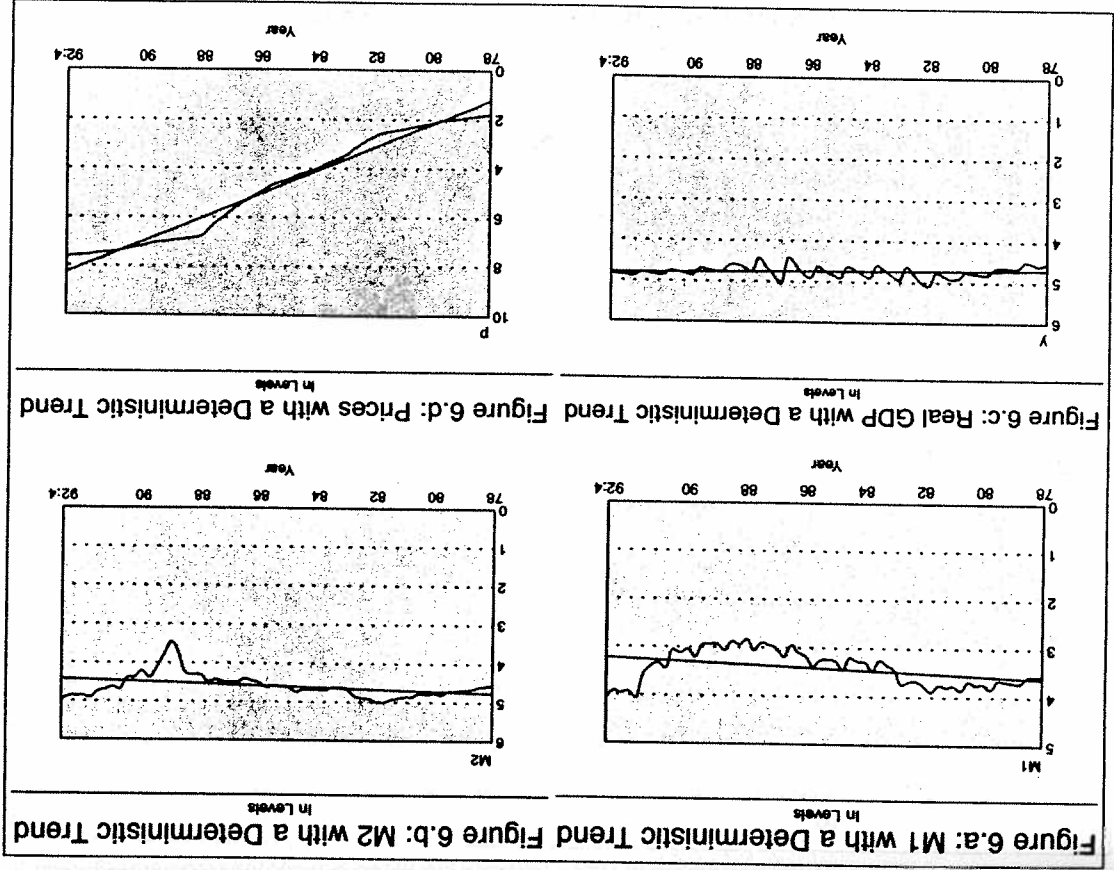
<sup>14</sup> Note that Haldrup (1991) found prices in Mexico to be integrated of order 2, I(2). It is relevant to mention that his tests did not take into account possible structural breaks; and that the tests were performed on quarterly data covering the period 1948:1 to 1988:4, which may be insufficient to detect the break of 1987:4.

Figure 5<sup>15</sup> shows the structural breaks in the series. It is very relevant to note that the IO model suggests a non-linear trend function, in order to show the gradual adjustment to the new path after the break date. In this study, however, we are analyzing the cointegration relationship of these variables, and as mentioned before, in the process, we have to account for the structural breaks. Therefore, critical values for the cointegration rank statistics test had to be calculated. These were computed with Johansen and Nielsen's DisCo software. This software is, however, limited to linear drift functions, what constrains us to impose linearity in the IO model. Thus, as can be seen in figures 6 and 7, where trends were fitted on the series, the series are characterized as follows:

Table 4: Characterization of the Series

Series	Degree of Integratedness	Structural Break in Trend
M1	I(1)	No
M2	I(1)	No
Y	I(1)	No
P	I(1)	Yes, in dP
NRa	I(0)	Yes
BRa	I(0)	Yes
BRo	I(0)	Yes

<sup>15</sup> The breaks in Figure 10, are constructed by regressing:  
 $Y_t = \text{constant} + \text{trend} + DU_t + DT_t$ ,  
 and by taking the fitted values. The fit becomes the broken trend.



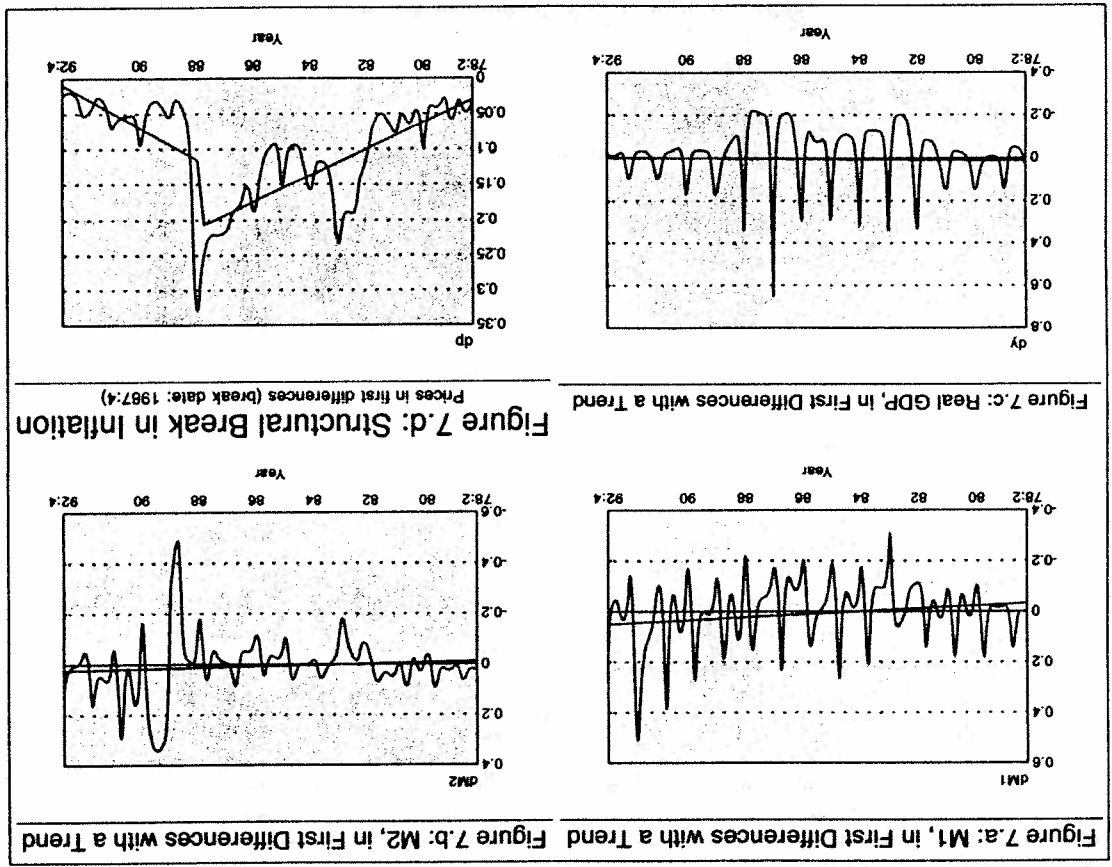


Figure 7.a: M1, in First Differences with a Trend  
 Figure 7.b: M2, in First Differences with a Trend

Figure 7.c: Real GDP, in First Differences with a Trend  
 Figure 7.d: Structural Break in Inflation (Prices in first differences (break date: 1987.4))

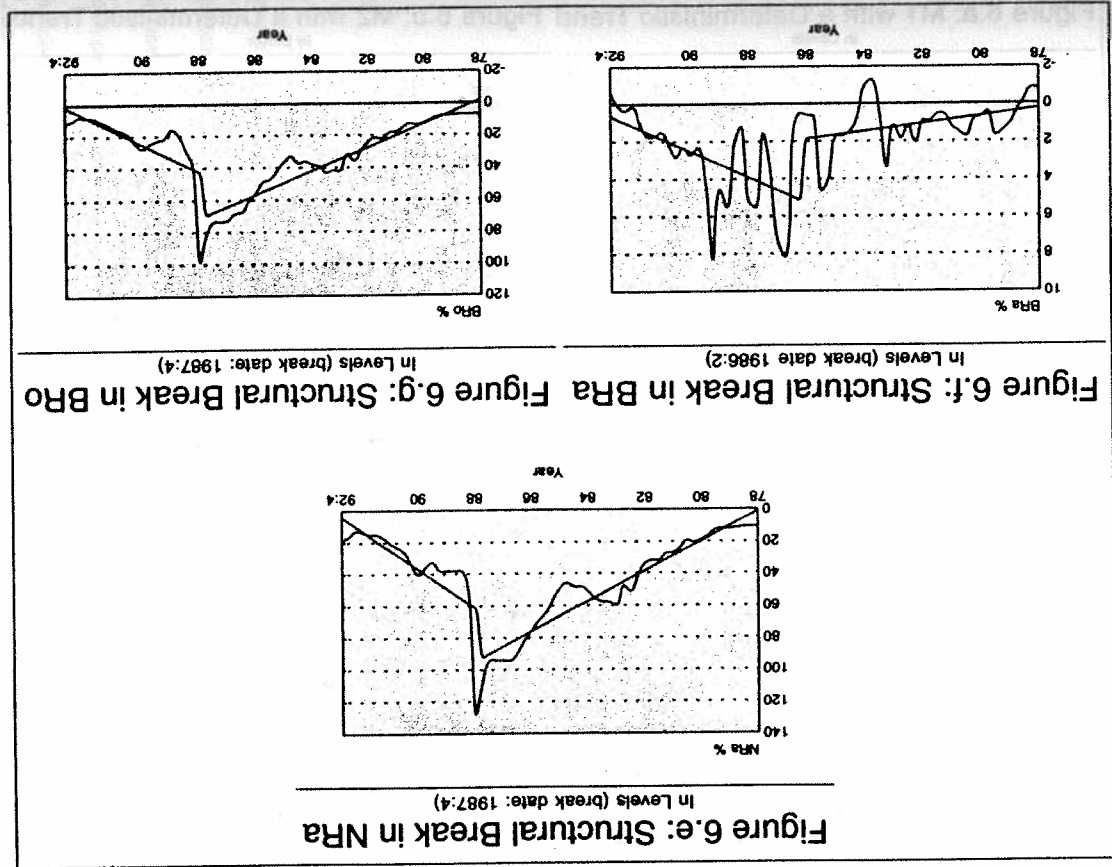
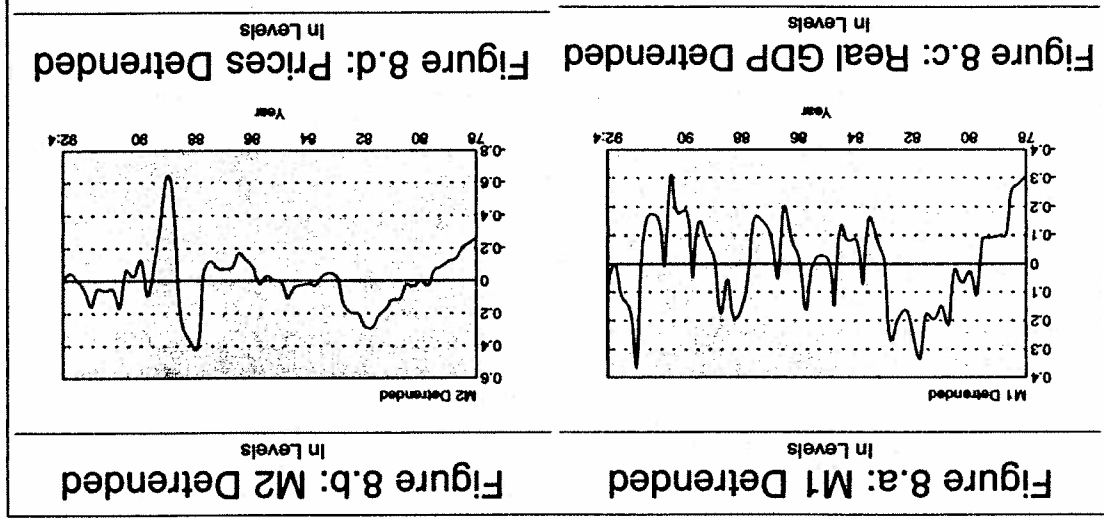
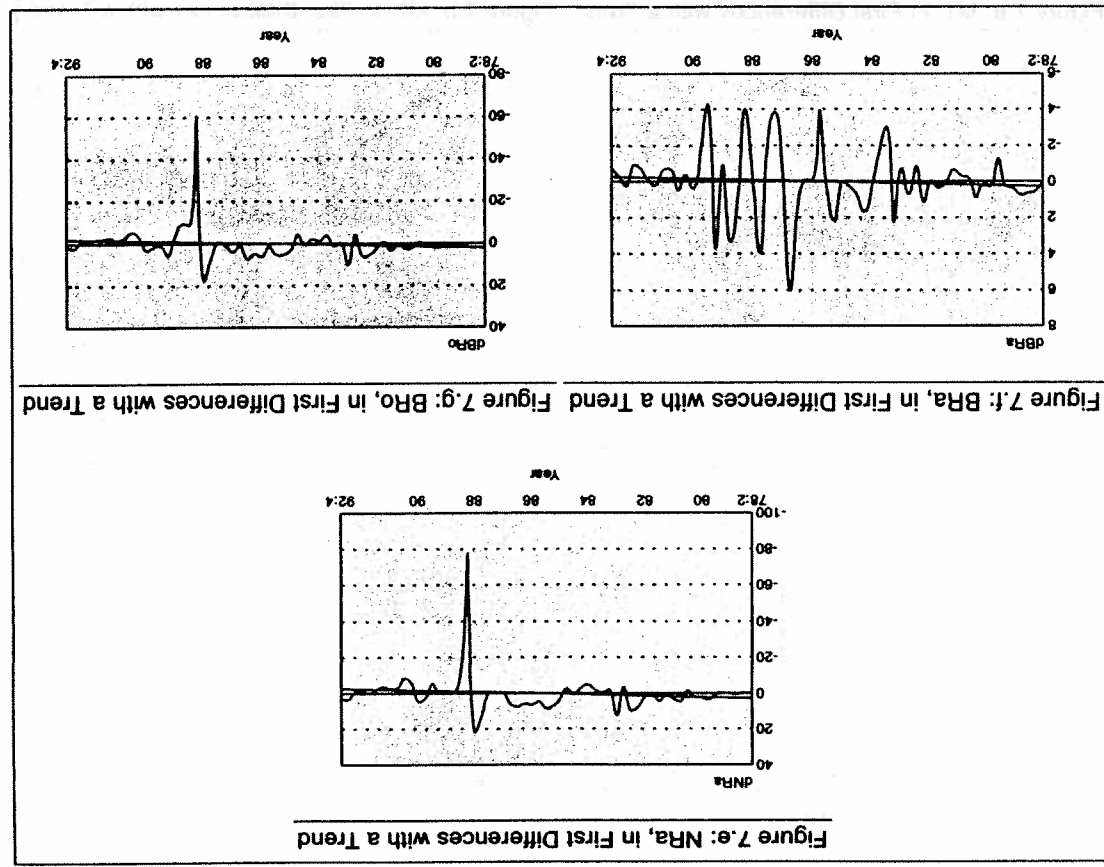


Figure 6.e: Structural Break in NRA (In Levels (break date: 1987.4))  
 Figure 6.f: Structural Break in BRA (In Levels (break date: 1986.2))  
 Figure 6.g: Structural Break in BRO (In Levels (break date: 1987.4))



All detrendings account for a break on 1987.4.

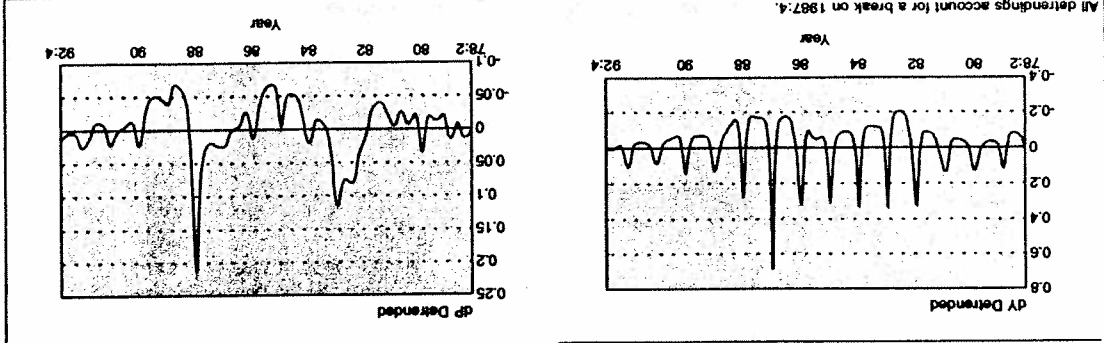


Figure 9.c: Real GDP Detrended  
In First Differences

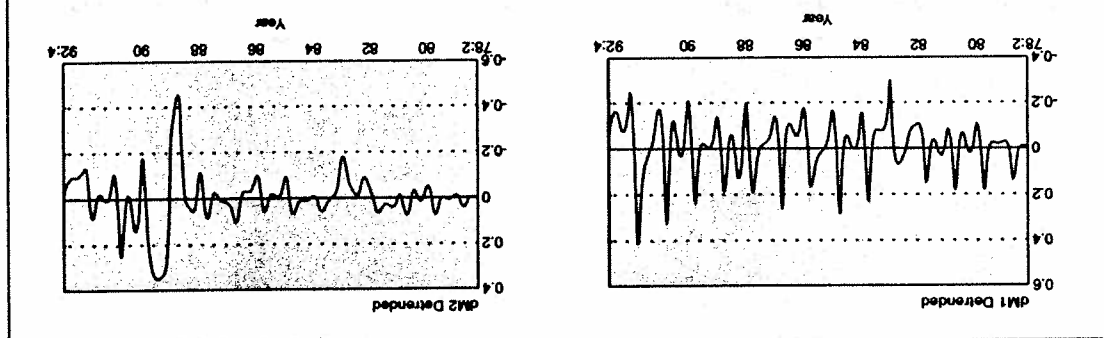


Figure 9.a: M1 Detrended  
In First Differences

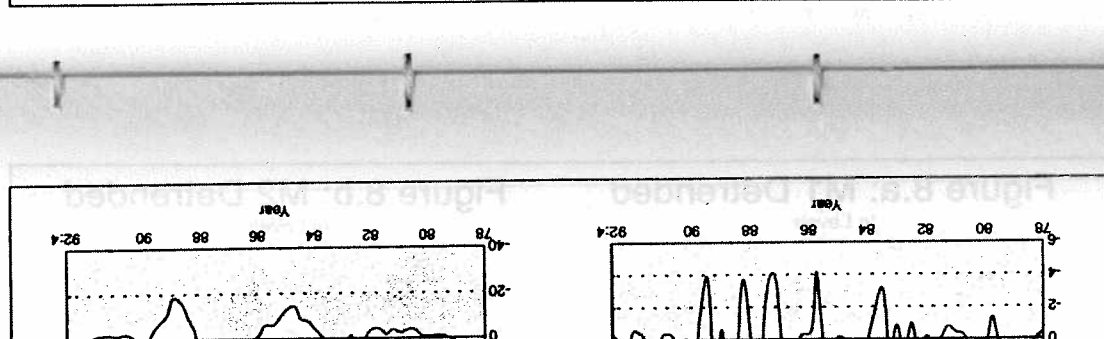


Figure 8.f: BRA Detrended  
In Levels

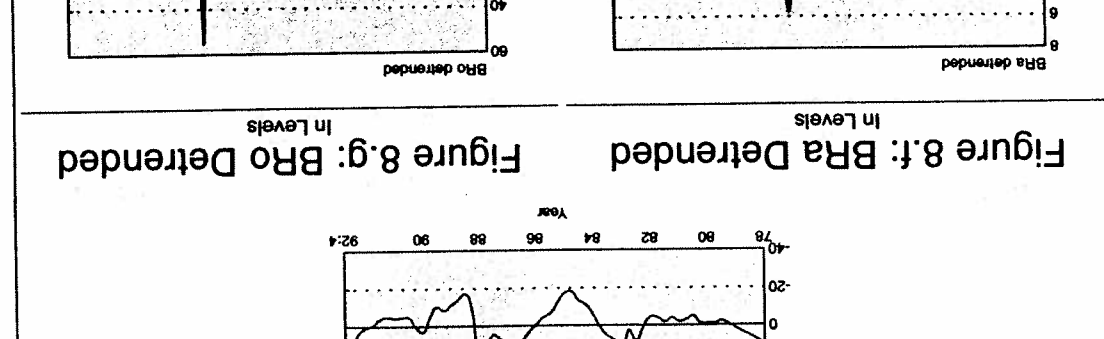


Figure 8.g: BRo Detrended  
In Levels

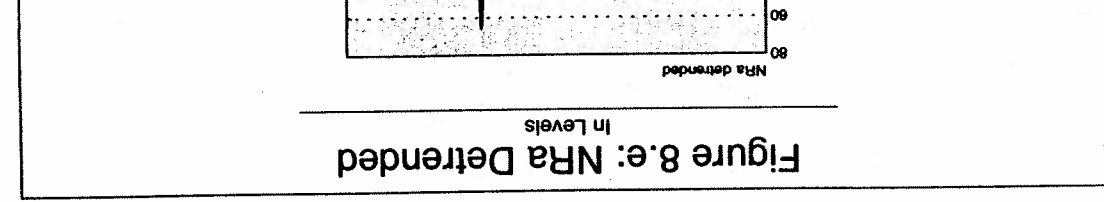


Figure 8.e: NRA Detrended  
In Levels

### 2. Detrending the Series

After having established the nature of the data series, we have found that some series are characterized by structural changes. It is therefore, important to properly detrend the series, by accounting for the breaks. Note that only after having detrended all series with respect to the same broken trend we can perform the deterministic cointegration tests. If the breaks were not considered in the detrending, deterministic cointegration could not be used. Stochastic multivariate cointegration would then have to be used, and the structural breaks would have to be modelled into the trend function, what would complicate considerably the estimation procedures.<sup>16</sup> If instead the breaks were completely disregarded, it would be like imposing deterministic cointegration where one should have stochastic cointegration. As presented by Perron and Campbell (1993), this improper model characterization would lead to incorrect inferences on the cointegrating relations and on the causality relations.

Following from the examination of the series when the correct trend is introduced, see figure 5, the break date of 1987:4 (which distinguishes NRa and BRo) was selected to be the one to be used in the detrending. There are some distinct idiosyncrasies that make the selection of the 1987:4 break date evident. Firstly, as we plot the trends, including the breaks (see figure 5), on the series, we can see that NRa and BRo are the ones that display the best fit. Secondly, the break date is the same for both series and for the inflation rates. Thirdly, it is very convenient to have a structural break with the same date for the M1 equation and for the M2 equation,

<sup>16</sup> See Perron and Campbell (1993) for stochastic multivariate cointegration estimation.

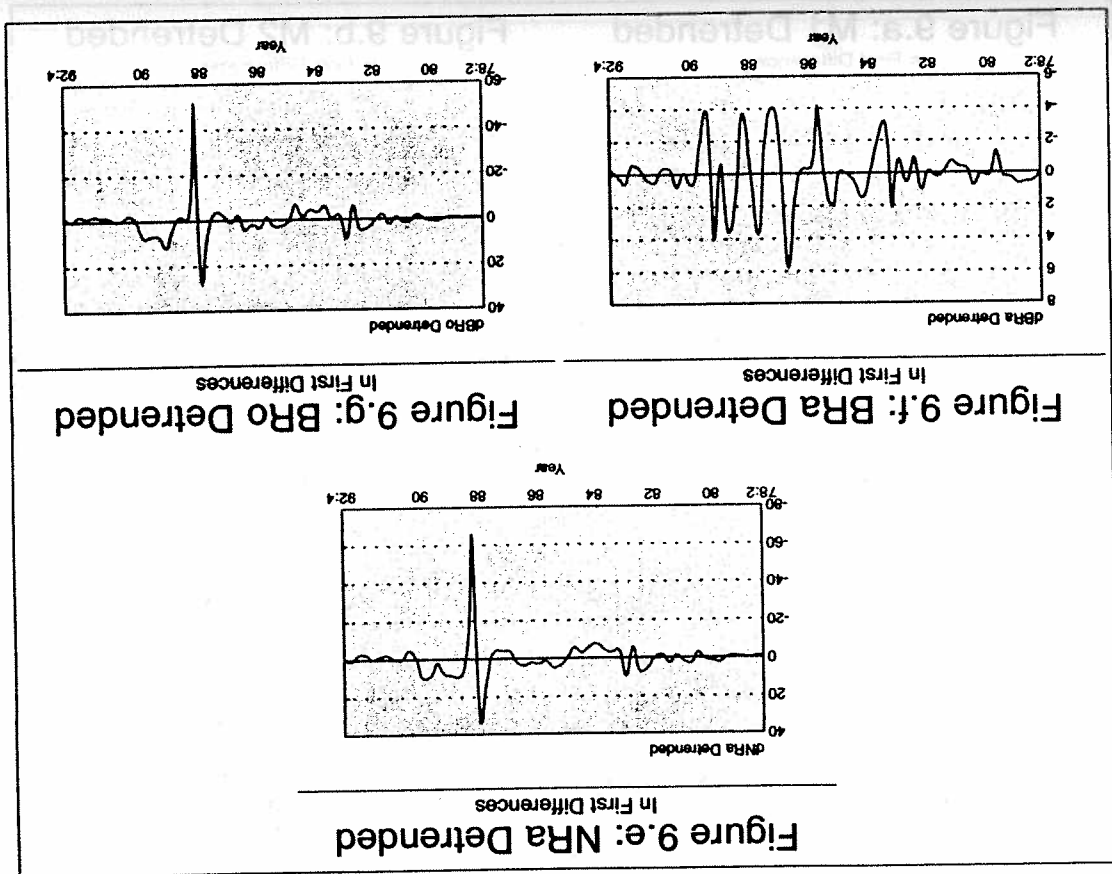


Figure 9.e: NRa Detrended  
In First Differences

Figure 9.f: BRa Detrended  
In First Differences

Figure 9.g: BRo Detrended  
In First Differences

although it was coincidental. This advantage is reflected in the calculation of the critical values for the cointegration rank test. Because the date of the break is the same, we can use the same critical values for both equations (see appendix 2 for details on the calculations). Finally, the break date truly confirms two major macroeconomic changes in the Mexican economy, the first being the control of inflation and its intrinsic economic and financial consequences. The second being the acceleration of the financial liberalization process.

The procedure of detrending is done by regressing every series on a constant, a linear trend and on  $DU_t$  and  $DT_t$ , where  $DU_t = 1$  and  $DT_t = t - T_b$  if  $t > T_b$  (zero otherwise), and  $T_b = 1987:4$ . The residuals of these regressions become the detrended series (see figures 13 and 14) to be used in the cointegration analysis. Throughout the rest of the paper we use an asterisk (\*) on the series name to represent the detrended series.

### 3. Estimation Results

Equation (3) was calculated for broad and narrow money, and by using the detrended series. Because of possible seasonality in  $dY^*$  (see figure 9.c), with periodicity in the fourth lag, the estimation was performed with 4 lags. It was found that with 4 lags, the residuals for both equations passed the Box-Pierce  $Q$  test for autocorrelation (see Appendix 3), so there is no need to increase the number of lags any further. The estimates of equation 3 are given in Appendix 3.

By using the detrended series, the eigenvalues and eigenvectors for Mexico's two demand for money equations were calculated following the procedure elaborated in section B.2. Results are reported in table 5.

**Table 5: The Eigenvalues  $\lambda$  and eigenvectors  $V$  for Mexico's Money Demand Equations**

	M1	M2	
Eigenvalues $\lambda$	(0.3892 0.1917 0.1196)	(0.6901 0.2773 0.1574 0.1027)	Eigenvalues $\lambda$
Eigenvectors $V$	$\begin{bmatrix} -10.178 & 7.873 & -2.161 \\ 12.677 & -4.239 & 16.145 \\ 0.112 & 0.090 & 0.003 \end{bmatrix}$	$\begin{bmatrix} M2^* \\ Y^* \\ BRa^* \\ BRo^* \end{bmatrix}$	$\begin{bmatrix} -0.976 & 11.585 & 12.032 & -0.944 \\ 1.650 & -10.728 & -22.439 & -11.876 \\ -1.116 & 0.866 & -0.047 & 0.195 \\ 0.116 & 0.002 & -0.205 & 0.019 \end{bmatrix}$

Note that because the equations have a different number of parameters, the narrow money equation may have at most 2 cointegrating vectors, while the broad money equation may have 3 cointegrating vectors. Recall from Johansen (1989), that there may be as many as  $(p - 1)$  linear combinations of  $p$  data series that are cointegrated.

Table 6 has information on the calculated likelihood ratio test statistics and on the 95% quantiles of the appropriate limiting distribution. Note that the critical values have to account for



the structural break in the series, thus, the asymptotic distributions of the statistics were calculated and are tabulated in table A.1, in Appendix 2.

**Table 6: Trace Test Statistics for the Hypothesis  $H_1$  and the General Alternative  $H_0$  for Mexico's Demand For Money Equations**

Narrow Money (M1)		Broad Money(M2)			
$H_1$	Trace	Trace(95%)	$H_0$	Trace	Trace(95%)
$r \leq 2$	7.132	3.953	$r \leq 3$	6.071	3.953
$r \leq 1$	19.057	12.834	$r \leq 2$	15.665	12.834
$r = 0$	46.666	25.040	$r \leq 1$	33.858	25.040
			$r = 0$	99.464	41.824

The eigenvalues reported in table 6 were used to calculate trace statistics through equation (8).

### 3.1 Multivariate Cointegration Results and Discussion

To estimate the dimensions of the cointegrating vector, the sequential approach, is used. The null hypotheses, of  $r$  cointegrating vectors, are tested sequentially, from  $r = 0$  to  $r \leq p - 1$ .  $r+1$  cointegrating vectors are selected if  $r$  is the last significant statistic.

*Narrow Money Demand Equation (M1)*: the hypothesis of zero cointegrating vectors was first tested and rejected as the comparison with the 95% quantile in the asymptotic distribution for  $r =$

0 shows that this value is significant. If we test the hypothesis that  $r \leq 1$ , we get a test value of 19.057, which is greater than the critical value of 12.834, making us reject the hypothesis of only one cointegration relation. We then test for  $r \leq 2$ , where we get a value of 7.132, which is much greater than the critical value of 3.953, telling us that there are 3 cointegrating vectors.

*The Broad Money Demand Equation (M2)*: here, as well, after performing the sequential approach to determine the dimension of the cointegrating space, we notice that there are 4 cointegrating vectors, as the hypothesis of  $r \leq 3$  is accepted.

The results of having all vectors cointegrating in both equations is somewhat perplexing. For both equations we have fallen in the extreme case of having a full rank matrix  $\Pi$ . Remember from section B.1 that the matrix  $\Pi$  is the one which conveys information on the long-run equilibrium relationship among the variables in the data vector. Having the matrix  $\Pi$  being full rank means that all elements of  $X_t$  are stationary around a trend vector (Campbell and Perron, 1991). In our particular case, this trend vector has a break, taking the form of  $\beta t + \Theta D U_t + \gamma D T_t$ .

The interesting insinuation of having the vector process  $X_t$  stationary is that it implies that all variables are  $I(0)$  around a trend that has a break at 1987:4. Now, as we have seen by the battery of unit root tests performed in the series, interest rates were  $I(0)$  around a broken trend, but M1, M2 and  $Y$  seemed to be integrated processes of order one without structural breaks.

Recall that, even though the structural break tests suggested contradictory results for  $Y$  and M2 (see table 3), none of them implied that the break was at 1987:4, as was "imposed" in the detrending. And for the M1 case, both AO and IO tests did not find any structural break (not even at the 10% level).

What seems to be happening here is that when the detrending (including the break at 1987:4) was imposed, the variables became stationary. As there was already an ambiguous result for M2 and Y, the imposition of the break intrinsically selected stationarity for these variables. Note that according to the tests presented in table 3, the breaks on M2 and Y would be separated from the 1987:4 break by a maximum of one year, which is not much. The surprise is really M1. Although we did not find any indication of a possible break in the unit root tests, when the rupture was imposed in the series it was accepted, changing the characterization of M1 to an  $I(0)$  process with a break. Therefore, we are probably facing a *type II error* (not rejecting the null hypothesis of unit root with break, when in fact it is false) in the test for unit roots in the presence of structural changes.

We conclude by saying that we do believe the series were correctly characterized in section C.1 (see table 4). However, the imposition of the break in the detrending, necessary for the multivariate cointegration analysis, changed the structure of the series. This helped the series, which were already prone to have a rupture, to exhibit such a break. In the multivariate cointegration analysis, we have found that, for both equations, the cointegrating matrix is of full rank. Campbell and Perron (1991) note that in a situation where the matrix  $\Pi$  is of full rank, no restrictions are imposed on the reduced form representation. It is, thus, a case for the standard VAR model, where the "standard VAR analysis applied to the level of the series"<sup>17</sup> is the appropriate estimation strategy" (Campbell and Perron, 1991, p. 168).

<sup>17</sup> Note that here we are referring to the detrended series, including the break of 1987:4.

## D. CONCLUSION

In this paper, extensive use of econometric techniques were applied to investigate whether or not the financial liberalization in Mexico has affected the long run equilibrium relationship in the demand for narrow and broad money. Tendency breaks were present for some series, notably for inflation and interest rates, reflecting the impact of the macroeconomic structural changes undergone in the last decade. Although it is difficult to link any instability to precise financial reforms, as these were taken in various fronts, it is reasonable to say that the structural breaks in interest rates are largely due to the reduction of inflation and to the improvement of the financial system that followed. The inflation rates fell mainly because of the wage and price freezes stipulated by the Economic Solidarity Pact of 1987. Private sector expectations about lower inflation and economic optimism not only permitted but contributed to the success of the reforms.

Accounting for the structural breaks, the stability of the money demand equations were studied by means of Johansen's multivariate cointegration analysis. All possible cointegration relationships were found for each equation, implying that both monetary aggregates (M1 and M2) have been related in a stable long-run fashion to real income and interest rates. As the matrix  $\Pi$  was found to be of full rank, a cointegration analysis would redundant and one can estimate Mexico's broad and narrow monetary equations directly by using the break detrended series in levels.

The interesting factor here is that these results were obtained only after a tendency break was imposed in the detrending of series. This revealed that some series, which were characterized as being integrated processes of order 1,  $I(1)$ , were in fact stationary processes around a broken trend. This outcome shows the relevance of working with series which are correctly characterized. Therefore, the importance of the unit root tests that account for structural breaks cannot be stressed enough.

In sum, we can say that the stabilization program undergone by Mexico and the financial liberalization that took place in the last few years did not disrupt the equilibrium demand for money equations. The results may be due to the fact that structural breaks were permitted and accounted for in the cointegration analysis. However, it is our view that the correct characterization of the series has to be emphasized and that only then results become meaningful. Furthermore, the breaks that were found are due to important macroeconomic reforms. These changes are so notorious, that we are convinced that they will be significantly apparent in any long term analysis that takes into account a larger sample size.

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#### DATA SOURCES

- \* International Financial Statistics, CD-ROM Database, International Monetary Fund.
- \* International Financial Statistics, books, International Monetary Fund, various issues.

#### STATISTICAL SOFTWARES

- \* RATS (Regression Analysis Time Series), VAR Econometrics, Version 4.0.  
Procedures: PPUNIT.SRC  
DFUNIT.SRC  
AOM2ALPH.SRC  
IOM2ALPH.SRC  
JOHANSEN.SRC
- \* Program on structural breaks that runs on RATS, by Pierre Perron and Serena Ng, Department of Economics, Université de Montréal, September 1993.
- \* DisCo Version 1.0, by Bent Nielson (1993), Institute of Mathematical Statistics, University of Copenhagen.

## APPENDIX 1: DATA SOURCES AND

### DEFINITIONS

Except for interest rates, all series are transformed to their natural logarithms, so to benefit from the near equivalence between differences in logarithms and percentage rates of growth in absolute levels.

$$M1 = \ln(M1/CPI), M2 = \ln(M2/CPI), Y = \ln(Y/CPI), P = \ln(CPI), NRa, NRo, BRo.$$

### I. MONETARY AGGREGATES

*Narrow Money* (M1): Currency held by the public plus demand deposits (checking accounts).

*Broad Money* (M2): Narrow money plus time and savings deposits (quasi-money)

Data for both narrow and quasi-money are in quarters and were taken from lines 27334 and 27335, respectively, of the IMF's International Financial Statistics CD-Rom database. Data for narrow and broad money are in millions of New Pesos.

### I. INCOME AND PRICES

*Income* was proxied by nominal GDP.

Data were taken from line 27399B of the IMF's International Financial Statistics CD-Rom database. As the IFS CD-Rom only has GDP per year, quarterly data for nominal GDP were calculated from annual observations according to the pattern of quarterly movements in

industrial production plus oil production (lines 27366, and 27366AA respectively). Nominal GDP data are in millions of New Pesos. Industrial production and oil production data are index numbers (1985 = 100) of period averages.  
Prices were proxied by consumer prices.

Data were taken from line 27364, of the IMF's International Financial Statistics CD-Rom database. Consumer prices data are index numbers (1985 = 100) of period averages.

### III. INTEREST RATES

#### A. Rate of return on money (Ro):

\* *For Narrow money:* NRo = zero, as narrow money is assumed to be non-interest bearing. (to avoid multicollinearity between interest rates, only one rate is used to calculate the opportunity cost of narrow money, Ra).

\* *For Broad money:* BRo was approximated by the deposit rate multiplied by the share of quasi-money in broad money.

#### B. Interest rates on alternative assets (Ra):

\* *For Narrow money:* NRa was proxied by deposit rates.

\* *For Broad money:* BRa was proxied by the treasury bill rate minus the deposit rate weighted by the share of quasi-money in broad money.

Data for Deposit Rates were taken from line 27360L, and data for the Treasury Bill Rates were taken from line 27360C, of the IMF's International Financial Statistics CD-Rom database. Interest rates data are in percent per annum, reflecting the average for the quarter.

Note that because the Treasury Bill Rates are reported only from 1978:1, we are constrained to work with data starting on this date.

Note: Data for the Deposit Rate for the second quarter of 1983 was missing from the IMF's International Financial Statistics CD-Rom database; for what it was taken from the IMF's International Financial Statistics book for December 1985.

Data for the Treasury Bill Rate for the third quarter of 1986 was missing from the IMF's International Financial Statistics CD-Rom database, for what it was taken from the IMF's International Financial Statistics book for December 1988.

## APPENDIX 2: NOTES ON THE

### SIMULATION OF TRACE STATISTICS'

#### CRITICAL VALUES IN THE PRESENCE OF

#### A BROKEN TREND

As explained by Johansen and Nielson (1993, p. 3), "... the asymptotic distribution of the likelihood ratio test for the cointegration rank is not  $\chi^2$ , but generalised Dickey-Fuller distributed, with a structure depending on the problem." Their statistical software DisCo, was created to calculate asymptotic critical values for the cointegration rank test.

In our analysis, we are confronted with the presence of structural breaks in the trend function of the data. These breaks were founded after the series were tested for unit roots, allowing for the possibility of a one time structural change. According to the results that we have obtained after using Perron's Innovation Outlier Model 2 and Additive Outlier Model 2 (Perron, 1993b), IOM2ALPH.SRC and AOM2ALPH.SRC in Perron's statistical procedures for testing for unit roots allowing for a structural break at an unknown time, in RATS Version 4.0, we have found that there are 4 series with breaks (see figure 5). After analysis of the breaks and of the fit of the changing trend, we have decided to work with the breaks occurring on the fourth quarter of 1987, in BRo and NRa. The break means that a change in the intercept and in the slope of the trend function occurred simultaneously, which is equal to having a change in level followed by a different growth path (Perron, 1993b).

In Perron's AO and IO Models 2, the change in slope,  $DT_1^*$ , and the change in intercept,  $DU_1$ , (see equations 9 and 11) take the values:  $DU_1 = 1$ ,  $DT_1^* = t - T_1$ , if  $t > T_1$ , and 0 otherwise. These values have then to be incorporated into the simulation to find the asymptotic critical values of the trace statistics, as intervention dummies.

In Johansen and Nielson's Disco software, the system is constructed for the VAR model analyzed by likelihood inference, as the one we are using in this study (equation 3). They assume that the drift term  $\gamma_t$  is decomposed as

$$\gamma_t = \sum_{j=1}^m \gamma_j k_j(t) \quad t = 1, \dots, T$$

where the drift functions  $k_j(t)$  are some deterministic functions of  $t$ , chosen such that the matrix

$$\begin{bmatrix} k_1(1) & \dots & k_1(T) \\ \vdots & & \vdots \\ k_m(1) & \dots & k_m(T) \end{bmatrix}$$

is of full rank.

Johansen and Nielson (1993) show that the asymptotic analysis has to distinguish the drift parameters into unrestricted and into  $\alpha$  restricted ones. For such, they introduce the notation  $H_{m-m}(r)$ , such that the model has  $m+n-m=n$  drift functions. "The  $m$  first of these correspond to unrestricted parameters, while the other  $n-m$  of these has  $\alpha$ -restricted parameters" (p. 5). In our case we have  $m = 1$  and  $n = 1$ , specifically we have one unrestricted parameter composed of a constant, a time trend and two intervention dummies  $DU_1$  and  $DT_1^*$ , that account for the break.

The asymptotic analysis follows from the stochastic matrix:

$$-2\log Q(H(r) | H(p)) \Rightarrow tr \left\{ \int_0^1 (dB) F \left[ \int_0^1 FF' du \right]^{-1} \int_0^1 F(dB)' \right\} \quad (A.1)$$

where  $B$  is a  $p$ - $r$  dimensional Brownian motion and  $F$  depends on the tested hypothesis, which, in our case is:

$$H_{1,0}(r): \gamma_i = \alpha + \beta t + \Theta DU_i + \delta DT_i,$$

Thus  $F$  is equal to:

$$(G_j(u), B_j(u), \dots, B_{p-r}(u))' \text{ corrected for } (g_j(u)).$$

Albeit it is not our interest to go in deep detail on the asymptotic analysis, the interested reader is referred to Johansen and Juselius (1990) and Johansen and Nielson (1993) for specifics on the asymptotic distribution statistical analysis.

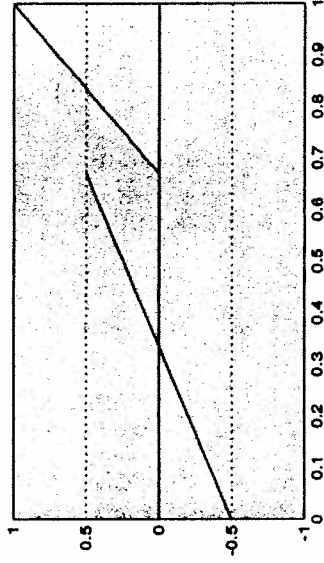
"The asymptotic analysis of the likelihood ratio test is performed by the theory of weak convergence on the space  $D[0,1]$ . So the process is considered on the interval  $[0,1]$  instead of  $\{0,1, \dots, T\}$  by the operation on  $t \in [0,1]$  which finds the integer part of  $tT$ " (Johansen and Nielson, 1993, p. 3). Therefore, in our case the simulations were performed with one drift function with unrestricted parameter ( $\gamma_i = \alpha + \beta t + \Theta DU_i + \delta DT_i$ ).

As the break date was estimated to be in the third quarter of 1987,  $DU_i = 1$  and  $DT_i = t - T_0$  at  $T_0 + 1$ , or 67%, our drift was determined as follows:

$$\begin{aligned} (0.00, -0.50) &\rightarrow (0.67, 0.50) \\ (0.67, 0.00) &\rightarrow (1.00, 1.00) \end{aligned}$$

Or graphically:

Figure A.2: Plot of the Drift Function



I. Simulation Principles:

The asymptotic distribution of the test statistic  $(-2\log Q(H_{\alpha,\beta,\beta}(r) | H_{\alpha,\beta,\beta}(p)))$  has a limit distribution that depends only on the parameters  $\alpha, \beta, \beta, \dots, \beta_m$  through the rank of the dimensions  $p$  and  $r$ , and only through the difference  $(p-r)$ , Johansen (1991). So the same limit distribution would be obtained by the  $(p-r)$  dimensional model:

$$H_{\alpha,\beta,\beta}(p-r): \Delta Y_t = \Pi Y_{t-1} + \sum_{i=1}^m \epsilon_i k_i(t) + \sum_{i=1}^m \Pi k_i(t) + \epsilon_t,$$

and test the hypothesis:  $H_{\alpha,\beta,\beta}(0): \Pi = 0.$

Here  $\epsilon_1, \dots, \epsilon_p$  are the columns of the unit matrix. This model corresponds to restrict the parameters of the drift functions  $k_{m+1}, \dots, k_p$  to be in the  $\alpha$ -space and the parameters of  $k_1, \dots, k_m$  to be unrestricted.

Under the null hypothesis the process  $Y_t$  fulfils the equation:



$$\Delta Y_i = \begin{bmatrix} \epsilon_i(t) + k_1(t) \\ \vdots \\ \epsilon_n(t) + k_n(t) \\ \epsilon_{n+1}(t) \\ \vdots \\ \epsilon_{p+1}(t) \end{bmatrix}$$

The process  $Y_i$  is simulated by generating data from this equation. Together with the drift functions with  $\alpha_L$ -restricted parameters the vector process:

$$Y_i^* = \begin{bmatrix} \Sigma_{i+1}^T \Delta Y_i \\ k_{i+1}(t) \\ \vdots \\ k_n(t) \end{bmatrix}$$

is obtained. The vectors  $\Delta Y_i$  and  $Y_i^*$  are then regressed on the variables  $k_1, \dots, k_n$  which is done by some orthogonal projection with matrix representation  $P$ . This gives the residuals:

$$R_{0i}(t) = P \Delta Y_i = \epsilon_i \quad \text{and} \quad R_i(t) = P Y_i^*$$

From this the matrices  $TS_{0i} = \Sigma_{i+1}^T R_{0i}(t) R_{0i}'(t)$  and  $TS_{ii} = \Sigma_{i+1}^T R_i(t) R_i'(t)$  are calculated. Further the trace distribution ([A.2]) is asymptotically equivalent to and therefore calculated by:

$$\text{tr}(S_{0i} (S_{ii} T)^{-1} S_{0i}) = \text{tr}(\Sigma R_{0i} R_{0i}' (\Sigma R_i R_i')^{-1} \Sigma R_i R_{0i}')$$

(Johansen and Nielsen, 1993, p.12).

To calculate the trace statistic critical values, the Brownian motion is approximated by a random walk with  $T = 400$  steps. The process was repeated 5,000 times and results are tabulated in table A.1.

Table A.1: Distribution of the Critical Values for the Trace Statistics Test

Distribution of the Trace of the Stochastic Matrix

$$\{I_0^*(dB) F [I_0^* FFdu]^* [I_0^* F(dB)]\}$$

Where B is a  $p-r$  dimensional Brownian motion and F depends on the structure of the hypothesis which is

$$H_0(r): \gamma = \alpha + \beta I + \theta DU + \delta DT,$$

tested, which is:

Thus F is equal to:

$$(G_1^*(u), B_1^*(u), \dots, B_{p-r}^*(u))' \text{ corrected for } (g_1^*(u)).$$

Dim (I)	50%	80%	85%	90%	92.5%	95%	97.5%	99%	Mean	Variance
1.	0.473	1.669	2.122	2.752	3.219	3.953	5.169	6.436	1.021	2.099
2.	4.984	8.468	9.340	10.738	11.607	12.834	14.743	16.992	5.681	13.729
3.	14.463	19.440	20.744	22.447	23.451	25.040	27.356	30.806	15.149	29.712
4.	28.071	34.498	36.430	38.469	39.820	41.824	44.976	47.876	28.786	52.492

(1) Dimension  $p - r$ , i.e. the number of non-stationary components under the hypothesis.

Simulations were performed using the simulation program *DisCo* Version 1.0 by Nielson (1993). Johansen and Nielson (1993). Simulations were executed using a discretisation of the Brownian Motion equal to 400 and the process was simulated 5,000 times.

### APPENDIX 3: RESULTS OF THE MULTIVARIATE COINTEGRATION ANALYSIS

Analysis done with the RATS Version 4.0 Procedure *JOHANSEN.SRC*. Program written by J. Johansen, K. Juselius and H. Hansen, and converted into a procedure by Thomas Doan, Var Econometrics, July 1990.

#### PROGRAM RESULTS

This are results from a cointegration analysis using

MI\*  
Y\*  
NRA\*

Statistics for the error-processes

Autocorrelations in the error-process

Lag	1	2	3	4	5	6	7	8
0.036	-0.092	-0.039	0.123	0.165	-0.008	-0.108	-0.075	
0.171	0.144	0.085	0.198	0.026	0.102	0.043	0.009	
0.102	0.037	0.162	-0.032	0.011	-0.065	-0.049	0.034	

mean var skewness kurtosis normality B.P-Q(12)  
0.00000 0.00440 0.69109 0.51029 5.06530 5.72670\*  
0.00000 0.00390 0.36587 1.76352 8.50605 7.18416\*  
0.00000 79.52049 1.46282 6.84378 129.25908 6.88185\*

The critical values for the Box-Pierce Q statistic are those of the Chi-Squared distribution. For 12 degrees of freedom, the critical value at the 5% level is 21.03 (Greene, 1990). The asterisk, therefore denotes the acceptance of the null hypothesis of no autocorrelation.

Partial autocorrelations in the cointegration-relations

Lag	1	2	3	4	5
-0.028	-0.148	-0.152	0.534	-0.544	
0.356	-0.035	0.009	0.321	-0.309	
0.104	-0.231	0.083	0.729	-0.436	

eigenvalues, lambda-max- and tracetest  
0.38922 0.19179 0.11959  
27.60893 11.92451 7.13261  
7.13261 19.05712 46.66605

Eigenvectors  
 -10.178 7.873 -2.161  
 12.677 -4.239 16.145  
 0.112 0.090 0.003

beta  
 1.000 -1.857 -834.059  
 -1.246 1.000 6230.399  
 -0.011 -0.021 1.000

alpha  
 -0.259 0.104 0.000  
 0.240 0.029 0.000  
 20.350 15.011 0.004

pi  
 -0.432 0.265 0.001  
 0.227 -0.582 -0.003  
 -11.000 15.595 -0.540

sigma  
 0.00432 0.00121 0.16885  
 0.00121 0.00383 -0.05125  
 0.16885 -0.05125 78.10048

corr(eps)  
 1.00000 0.29659 0.29067  
 0.29659 1.00000 -0.09375  
 0.29067 -0.09375 1.00000

gamma  
 -0.144 0.029 -0.302  
 0.237 0.153 0.188  
 -48.492 0.833 -5.253

fi and Ao  
 -0.036 0.098 0.067 0.002 0.000 0.000

0.003 -0.386 -0.309 -0.252  
 -0.795 -0.975 -1.076 -0.003 -0.003 -0.002  
 0.008 0.094 0.036 0.015  
 -36.999 -20.609 -12.072 -0.487 -0.536 -0.491  
 0.235 7.839 -4.209 2.167

The unit-matrix (control)

1.00000  
 0.00000 1.00000  
 0.00000 0.00000 1.00000

End of session

This are results from a cointegration analysis using

MZ'  
 Y'  
 BRa'  
 BRo'

Statistics for the error-processes

Autocorrelations in the error-process  
 Lag 1 2 3 4 5 6 7 8  
 0.087 0.226 0.030 -0.191 -0.022 -0.141 -0.296 -0.121  
 0.075 0.080 0.030 0.073 -0.119 0.028 -0.072 0.080  
 0.094 -0.162 -0.034 0.078 0.117 0.045 -0.069 -0.061  
 0.152 0.184 0.106 -0.202 0.129 -0.106 -0.175 0.019

mean var skewness kurtosis normality B.P-Q(12)  
 0.00000 0.00694 0.28784 0.96005 2.92391 16.30935\*  
 0.00000 0.00281 0.17636 0.31430 0.52080 5.52521\*  
 0.00000 1.20741 -0.25460 2.04941 10.40521 8.37269\*  
 0.00000 29.18521 0.71114 3.67950 36.31037 15.02150\*

The critical values for the Box-Pierce Q statistic are those of the Chi-Squared distribution. For 12 degrees of freedom, the critical value at the 5% level is 21.03 (Greene, 1990). The asterisk, therefore denotes the acceptance of the null hypothesis of no autocorrelation.

Partial autocorrelations in the cointegration-relations

Lag 1 2 3 4 5

0.372 -0.135 -0.288 -0.073 -0.333  
 0.189 -0.247 -0.187 0.295 -0.290  
 -0.041 -0.369 -0.175 0.546 -0.488  
 0.300 -0.132 0.202 0.635 -0.441

eigenvalues, lambda-max- and tracetest  
 0.69011 0.27738 0.15745 0.10275  
 65.60568 18.19276 9.59397 6.07172  
 6.07172 15.66569 33.85845 99.46413

Eigenvectors

-0.976 11.585 12.032 -0.944  
 1.650 -10.728 -22.439 -11.876  
 -1.116 0.866 -0.047 0.195  
 0.116 0.002 -0.205 0.019

beta

1.000 -1.080 -257.611 -48.930  
 -1.690 1.000 480.437 -615.419  
 1.143 -0.081 1.000 10.107  
 -0.118 0.000 4.396 1.000

alpha

0.039 0.253 0.001 0.000  
 0.024 -0.088 -0.001 0.000  
 -0.420 6.132 -0.010 0.000  
 5.304 21.070 0.037 -0.009

pi

-0.468 0.373 0.029 -0.001  
 0.267 -0.575 0.037 -0.005  
 -4.429 2.022 -0.986 0.004  
 -26.418 35.485 4.301 -0.480

sigma

0.00682 0.00066 0.00589 0.21541  
 0.00066 0.00276 -0.00201 -0.09017  
 0.00589 -0.00201 1.18585 0.98124  
 0.21541 -0.09017 0.98124 28.66405

corr(epsil)

1.00000 0.15291 0.06553 0.48728  
 0.15291 1.00000 -0.03514 -0.32051  
 0.06553 -0.03514 1.00000 0.16830  
 0.48728 -0.32051 0.16830 1.00000

gamma

0.046 -0.550 -0.610 -0.153  
 0.320 0.183 0.275 -0.865  
 -4.327 -4.459 0.015 -0.861  
 -26.926 -20.285 -25.837 -17.028

fi and Ao

0.216 0.030 0.020 0.013 0.001 -0.003  
 0.004 0.001 0.007 -0.171 0.014 -0.156  
 -0.959 -1.067 0.024 0.025 0.029 -0.006  
 -0.006 -0.004 0.005 0.100 0.062 -0.001  
 0.042 5.675 -0.492 -0.692 -0.799 -0.018  
 0.014 0.021 0.030 0.674 1.663 0.703  
 1.777 -4.213 1.885 1.568 1.916 -0.651  
 -0.451 -0.186 0.128 -1.861 -1.487 -1.973

The unit-matrix (control)

1.00000  
 0.00000 1.00000  
 0.00000 0.00000 1.00000  
 0.00000 0.00000 0.00000 1.00000

End of session