Université de Montréal
Faculté des arts et des sciences
Département de sciences économiques

CAHIER 9531

NON-CONVEXITIES AND EFFICIENCY OF EQUILIBRIA IN INSURANCE MARKETS WITH ASYMMETRIC INFORMATION

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June 1995

This research was realized while G. Dionne was on sabbatical leave at the Paris X - Nanterre - CEPREMAP chair of insurance in Paris and was financed by the Fédération Française des Sociétés d'Assurance, CREST, CRSH Canada and FCAR Québec. A first version was presented at a Nanterre-CEPREMAP-DELTA workshop on insurance in Chantilly. We thank R. Guesnerie and J.C Rochet for their comments, J. Leblanc and J. Plante for their contribution in the preparation of the manuscript, and L. Cournoyer for the graphical representations.

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Ce cahier a également été publié au Centre de recherche sur les transports (C.R.T.) (publication no 95-30).

Dépôt légal - 1995
Bibliothèque nationale du Québec
Bibliothèque nationale du Canada

ISSN 0709-9231

RÉSUMÉ

Dans deux articles importants, Crocker and Snow (1985, 1986) ont étudié l'efficacité des équilibres des marchés d'assurance avec antisélection. Dans les deux articles, pour obtenir leurs résultats, ils ont utilisé le fait que la frontière d'efficacité dans l'espace des richesses est strictement concave. Dans cet article nous montrons que la frontière est strictement convexe pour des fonctions d'utilité connues et, plus important encore, nous montrons qu'elle ne peut pas être toujours strictement concave. Par contre, nous obtenons aussi que la frontière a des propriétés qui nous permettent de préserver les résultats initiaux de Crocker et Snow.

Mots-clés : sélection adverse, antisélection, non-convexités, efficacité, équilibre, catégorisation des risques

ABSTRACT

In two stimulating articles, Crocker and Snow (1985, 1986) studied the efficiency of equilibria in insurance markets with adverse selection. In both articles, in order to show the correspondence between equilibrium and efficiency, Crocker and Snow used the fact that the frontier of efficiency points in the income-states space is always strictly concave. In this paper we first show that, for well known von Neumann-Morgenstern utility functions, the efficient frontier is always strictly convex in the income-states space. More importantly, we also obtain that when this frontier is not (strictly) convex, it cannot be always (strictly) concave. In other words it must necessary have a convex portion under risk aversion. We show, however, that the efficiency frontier in the income-states space has a particularity that preserves the initial results of Crocker and Snow.

Keywords : Adverse selection, non-convexities, efficiency, equilibrium, risk categorization.

1. INTRODUCTION

In two stimulating articles, Crocker and Snow (1985, 1986) studied the efficiency of equilibria in insurance markets with adverse selection. By using the approach proposed by Harris and Townsend (1981) to characterize outcomes in presence of asymmetric information, they (1985) showed that a Miyazaki-Wilson (M.W.) equilibrium always results in an efficient allocation (second-best efficiency). They also obtained that when the proportion of high risk individuals (λ) is sufficiently high, then a Rothschild-Stiglitz (1976) equilibrium always exists and is second-best efficient. Otherwise, a Rothschild-Stiglitz equilibrium is not second-best efficient. In their 1986 article, they demonstrated how their general framework can be applied to derive the efficiency of risk categorization in presence of adverse selection.

In both articles, in order to show the correspondence between equilibrium and efficiency, Crocker and Snow use the fact that the frontier of efficiency points in the income-states space is always strictly concave. Consequently, when they derive their results, a local tangency between the efficient frontier and a risk type indifference curve is necessary and sufficient to obtain the desired results (unicity and direct correspondence between equilibrium and efficiency).

In this paper we first show that, for well known von Neumann-Morgenstern utility functions, this efficient frontier is always strictly convex in the income-states space. More importantly, we also obtain that when the frontier is not (strictly) convex, it cannot be always (strictly) concave. In other words it must necessary have a convex portion under risk aversion. When the utility function is cubic, for example, the frontier is both convex and concave which rises the issue of the one-to-one correspondence between an equilibrium and second-best efficiency for some concave utility functions.

We show, however, that the efficiency frontier in the income-states space has a particularity that preserves the initial results of Crocker and Snow. Contrary to other frontiers in standard allocation problems (with non convexities) that are determined by factors exogenous to those explaining the shape of the objective function [Guesnerie (1975)], the efficient frontier in the income-states space under adverse selection is function of the risk type indifference curve that matters. Consequently we are able to show that there is always a one-to-one correspondence between market equilibrium and second-best efficiency whatever the shape of the efficiency frontier.

The paper is organized as follows. In the next section we propose a sufficient condition to obtain a strictly convex efficiency frontier in the income-states space. Two well accepted utility functions (logarithmic and power) are consistent with a strictly convex efficiency frontier. We also show that the efficiency frontier must have a convex portion for all concave utility functions.

In Section 3, we present our main result which states that the one-to-one relationship between efficiency and market equilibrium is always maintained since the efficiency frontier is endogenously determined by the parameters of the risk type utility function considered. Finally, we demonstrate that the second-best efficiency frontier in the expected utility space remains strictly concave whatever the shape of the efficiency frontier in the income space. A short conclusion summarizes the main results and discusses how our results may affect the conclusions about the efficiency of risk categorization.

2. A SUFFICIENT CONDITION TO OBTAIN A CONVEX LOCUS IN THE INCOME-STATES SPACE

The material of this section is more concerned with the article in the *Journal of Public Economics* (1985), although, as we will see in the conclusion, the results will be significant for the risk categorization model.

We use the basic (two risk-types and two states) framework introduced in the literature by Rothschild and Stiglitz (1976)[1]. There are two types of individuals that differ only by their probability of accident : $0 < p^j < 1$, $j \in \{H,L\}$ for high and low risk, and $p^H > p^L$. Each individual has a von Neumann-Morgenstern utility function $U(W_i)$ with W_i being his wealth in the loss (i=2) and no-loss (i=1) state. The function $U(\cdot)$ is strictly increasing and concave in wealth (W) which means that $U'(W) > 0$, and $U''(W) < 0$. We can write a consumer's j expected utility facing a contingent wealth $W^j = (W_1^j, W_2^j)$ as $V(p^j, W^j) = (1-p^j)\ U(W_1^j) + p^j\ U(W_2^j)$ and his best welfare opportunity as ∇^j.

[1] See Dionne and Doherty (1992) and Hellwig (1987) for more details.

The no-loss state is characterized by the individual wealth $\overline{W}$ while the loss state implies a monetary loss ($d > 0$). Consequently, the individual's j expected wealth can be written as $\overline{W} - p^j d$ and the average per capita wealth as $\overline{W} - \overline{p}d$ where $\overline{p} = \lambda p^H + (1-\lambda)p^L$ and λ ($0 < \lambda < 1$) is the proportion of high risks in the population.

Using the framework of Crocker and Snow (1985, 1986) and, for the moment, limiting ourselves to the case $\overline{V}^H \leq V(p^H, F)$ where F is the equal per capita full insurance allocation ($F = \overline{W} - \overline{p}d$), we can write the problem of efficient allocation as :

$$\underset{W^H, W^L}{\text{Max}}\ V(p^L, W^L)$$

subject to :

- a resource constraint (γ) $\lambda\rho(p^H, W^H) + (1-\lambda)\rho(p^L, W^L) - \overline{W} \geq 0$

  where $\rho(p^j, W^j) = (1-p^j)W_1 + p^j(W_2 + d)$,

- self-selection constraints (μ_j)

$$V(p^j, W^j) \geq V(p^j, W^k) \qquad \text{for} \quad j,k \in \{H,L\} \quad \text{and} \quad j \neq k,$$

- and a constraint on the welfare of the type H individual (δ)

$$V(p^H, W^H) \geq \overline{V}^H.$$

We can summarize the main result of Crocker and Snow by the following theorem :

THEOREM 1 (Crocker and Snow, 1986) : A solution to the efficient allocation problem satisfies the following necessary conditions :

$$W_1^H = W_2^H \tag{a}$$

$$\begin{aligned} V(p^H,W^H) &= V(p^H,W^L) \\ V(p^L,W^L) &> V(p^L,W^H) \end{aligned} \tag{b}$$

$$\frac{(1-p^L)U'(W_1^L)}{p^L U'(W_2^L)} = \frac{\lambda(1-p^H)U'(W_1^L)+(1-\lambda)(1-p^L)U'(W_2^H)[1+\delta/\mu_H]}{\lambda p^H U'(W_2^L)+(1-\lambda)p^L U'(W_2^H)[1+\delta/\mu_H]} \tag{c}$$

$$\lambda\, p(p^H,W^H) + (1-\lambda)\, p(p^L,W^L) = \overline{W} \tag{d}$$

Condition (a) states that the high-risk individuals receive full insurance and condition (b) says that they are indifferent between this allocation and that provided to the low-risk individuals. The latters strictly prefer their risk-type allocation. Condition (c) determines a particular optimal allocation on the FL locus in Figure 1 while condition (d) indicates that the per capita resource constraint is binding.

When the utility constraint of the H-risks is binding, the solution is of the type (H,L) in Figure 1. Otherwise, $\delta = 0$, and a solution is represented by, both, a tangency between locus FL and the L-risks indifference curve (W^L) and by (W^H).

The problem with the above analysis relies on the conditions that characterizes the efficient solution. First, we will show that, in general, the locus FL is not strictly concave as shown in Figure 1. In fact, we will provide a sufficient condition on the utility function to obtain that the locus is strictly convex. In that particular case, which rules out, for example, constant absolute risk aversion, we will show that the above analysis is adequate. However, the graphical representation of the results in the contingent incomes space will have to be modified.

More generally, or when the sufficient condition is not imposed, the frontier can be concave but cannot be strictly concave on all the FL locus which implies that we have to verify if there still

exists a one-to-one correspondence between market equilibrium and second-best efficiency. We will also verify the consequence of the different configurations on the second-best efficiency frontier in the expected utility space.

As pointed out by Crocker and Snow (1986), the slope of the FL locus is given by the right hand side of condition (c) with $\delta = 0$. This can be rewritten as :

$$-\frac{dW_2^L}{dW_1^L} = \frac{\lambda(1-p^H)U'(W_1^L) + (1-\lambda)(1-p^L)U'(W_2^H)}{\lambda p^H U'(W_2^L) + (1-\lambda)p^L U'(W_2^H)} = \frac{B}{C} > 0. \qquad (1)$$

Therefore the curvature of FL has the sign of (see the proof of Proposition 1 for details) :

$$\frac{d^2W_2^L}{dW_1^{L2}} = \frac{\lambda(1-p^H)U'(W_1^L)A(W_1^L) + (1-\lambda)(1-p^L)U'(W_2^H)A(W_2^H)(dW_2^H/dW_1^L)}{C}$$
$$+ \frac{B(\lambda p^H U'(W_2^L)A(W_2^L)(A/B) - (1-\lambda)p^L U'(W_2^H)A(W_2^H)(dW_2^H/dW_1^L)}{C^2} \qquad (2)$$

where $$A(W_i^j) = -\frac{U''(W_i^j)}{U'(W_i^j)}, \qquad j = \{H,L\}, \quad i = \{1,2\}$$

is the measure of absolute risk aversion.

We now present our first result.

Proposition 1 : A sufficient condition to obtain $\frac{d^2W_2^L}{dW_1^{L2}} \geq 0$ for all (W_1^L, W_2^L) is that

$$\frac{A(W_2^L)}{U'(W_2^L)} \cdot \frac{U'(W_1^H)}{A(W_1^H)} \geq p^H \text{ with } W_1^H \geq W_2^L$$

Proof : See Appendix.

The sufficient condition implies that the measure of absolute risk aversion is strictly decreasing. However, strictly decreasing risk aversion is not sufficient. A stronger sufficient condition to obtain the desired result is that $\frac{A(W_2^L)}{A(W_1^H)} \geq \frac{U'(W_2^L)}{U'(W_1^H)} \geq 1$ for all $W_1^H \geq W_2^L$. The following corollary proposes three necessary and sufficient conditions on U that are equivalent to satisfy the above condition and consequently the sufficient condition in Proposition 1.

Corollary 1 : The following equivalent conditions are necessary and sufficient to obtain that

$$\frac{A(W_2^L)}{A(W_1^H)} \geq \frac{U'(W_2^L)}{U'(W_1^H)} \geq 1 \text{ for all } W_1^H \geq W_2^L.$$

i) $\frac{A(W)}{U'(W)}$ is non increasing.

ii) $P(W) - 2A(W) \geq 0$ where P(W) is the measure of absolute prudence (Kimball, 1990)[2].

iii) $\frac{1}{U'(W)}$ is convex.

Proof : See Appendix.

It is straightforward to verify that both the logarithmic and the power utility functions satisfy the conditions to obtain a strictly convex efficiency frontier. However the exponential function does not satisfy the above condition nor the quadratic utility.

Moreover, by evaluation of (1) at point F of Figure I, we obtain that the frontier is necessary linear at that point whatever the nature of the utility function. The slope of FL evaluated at F where $W_1^H = W_2^H = W_1^L = W_2^L$ is given by :

$$-\frac{dW_2^L}{dW_1^L} = \frac{\lambda(1-p^H) + (1-\lambda)(1-p^L)}{\lambda p^H + (1-\lambda)p^L} = \frac{1-\overline{p}}{\overline{p}}$$

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2 On other applications of the concept of prudence see Eeckhoudt and Kimball (1992) and Gollier and Pratt (1993). On the correspondence between risk aversion and prudence see Eeckhoudt and Schlesinger (1994).

which is the slope of the pooling budget line[3]. From the analysis in the Appendix, we can show that the FL curve is necessary convex around F since the pooling budget line is necessary below the efficiency frontier. This implies that the efficiency frontier cannot be strictly concave everywhere as shown in Crocker and Snow. For example, the cubic utility function generates a frontier that has the following configuration :

The specific example of Figure II raises the conventional question on the possibility to achieve a second-best efficient allocation by a decentralized economy. To be more precise, is it possible to construct a counterexample where two allocations, such as points A and B in Figure II, satisfy the necessary conditions of competitive equilibrium but correspond to two different levels of welfare? In other words, is it possible to draw two indifference curves that are tangent to the efficient frontier at points A and B respectively (necessary conditions) but correspond to two different levels of welfare for both the low risk and the high risk individuals?

In the next section we will show that such counterexample cannot be set up and we will show that an indifference curve that is tangent to the efficiency frontier cannot cross the frontier at another point. This proof will also be sufficient to show that, when the frontier is strictly convex, the correspondence between second-best optimality and market equilibrium is maintained.

3. CORRESPONDENCE BETWEEN SECOND-BEST PARETO OPTIMALITY AND MARKET EQUILIBRIUM

As we discussed in the previous section the existence of multiple equilibria would imply that the indifference curve (tangent to the efficiency frontier) of the low risk individual has more than one contact point with the frontier. In the next figure we provide such a configuration where the low risk indifference curves have two tangency points with the efficiency frontier. These points are now only necessary conditions (not sufficient) to determine the optimal solution since point A is prefered to point B by the low risk individual.

We now show that such situation is not possible since an indifference curve cannot have two contacts points (like B and C) with the efficiency frontier. Therefore, if B is a tangent point, A cannot be prefered by the low risk individual or even cannot exist because it would imply that the two indifference curves intersect.

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[3] See Pannequin (1992) for a similar observation.

Proposition 2 : An indifference curve tangent to the efficiency frontier cannot have another contact point with the frontier (tangent or not).

Proof : See Appendix.

We can also show that the second-best frontier in the expected utility space is always strictly concave whatever the shape of the efficiency frontier in the income-states space which confirms the above analysis on the one-to-one correspondence between equilibrium and efficiency whatever the shape of the efficiency frontier in the income-states space.

Proposition 3 : The second-best frontier in the expected utility space is always strictly concave.

Proof : See Appendix.

Now the question is : Why non-convexities in the efficiency frontier do not introduce problems of correspondence between equilibrium and efficiency as in any other resource allocation problem with non-convexities? The answer to this question lies in the composition of the ingredients that compose the frontier. This frontier is not completely exogenous of the expected utility of both individual types. When $\lambda = 1$ the frontier is always strictly convex whatever the utility function and, in fact, corresponds to the indifference curve of the high risk individuals evaluated at the low risk individual wealth levels (W_1^L, W_2^L) which has always a slope lower than that of the low risk individuals. When $\lambda < 1$, it is a combinaison of both the high and low risk indifference curves and, when $\lambda = 0$, it is equal to the zero profit line of the low risk individuals. Then, when the sufficient condition of Section 2 is met to obtain a strictly convex frontier or for any convex portion of a general efficiency frontier, its curvature is lower than that of the low risk individuals indifference curve at each point. When the risk aversion functions are such that the frontier can have concave portions, the curvature of the frontier in these portions is even lower to that of the low risk indifference curve since it is now of opposite sign (negative).

CONCLUSION

In this paper we have shown that the presence of non-convexities of the efficiency frontier in th incomes space does not affect the correspondence between market equilibrium and efficienc in insurance markets with asymmetric information. This implies that all second best allocatior in presence of adverse selection can be achieved by decentralized markets without any furth restrictions on the market behaviour of the participants than those already in the differer models. The result is explained by the fact that, in this environment, the shape of the efficienc frontier is endogenously determined by the nature of the insureds utility functions.

One may question whether these non-convexities can affect the conclusions about ris categorization [Crocker and Snow (1986), Henriet and Rochet (1987), Bond and Crocker (1991 and Puelz and Snow (1994)]. In this conclusion we shall limit our comments to costles categorization variables (age, gender, type of car, ...) which correspond to some applications ir the insurance industry. The main result of Crocker and Snow (1986) is to demonstrate tha costless categorization introduces a Pareto improvement over non-categorization for many equilibrium concepts. In particular, they show that with two categorization variables A and B, any pair of contracts (A^H,A^L) and (B^H,B^L) can be sustained as a Miyazaki-Spence-Wilson equilibrium. Categorization with appropriate taxes[4] improves the initial solution (without categorization) yielding a categorization efficiency frontier above the frontier without categorization, that represents a potential Pareto improvement. Since it is easy to show that the categorization efficiency frontier with appropriate taxes on contracts has the same properties than the efficiency frontier studied in this article, the categorization equilibrium remains unique and efficient.

[4] See Henriet and Rochet (1988) for another form of public intervention based on the quantity of insurance instead of one on price or taxe. They obtain equivalent results about the correspondence between equilibrium and optimality.

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APPENDIX

Proofs of different results

Proof of Proposition 1 : We present the detailed proof for the case below the 45 degrees line. By symmetry, the proof is identical for the case above. (A1) and (A2) are the binding constraints to the program in Crocker and Snow :

$$U(W_1^H)=(1-p^H)U(W_1^L)+p^H U(W_2^L) \tag{A1}$$

$$\lambda W_1^H+(1-\lambda)[(1-p^L)W_1^L+p^L W_2^L]=\overline{W}-\overline{p}D \tag{A2}$$

The total differentiation of (A1) and (A2) yields :

$$U'(W_1^H)dW_1^H = (1-p^H)U'(W_1^L)dW_1^L+p^H U'(W_2^L)dW_2^L \tag{A1^1}$$

$$\lambda dW_1^H+(1-\lambda)[(1-p^L)dW_1^L+p^L dW_2^L]=0 \tag{A2^1}$$

(A1^1) and (A2^1) can be rewritten as :

$$[\lambda(1-p^H)U'(W_1^L)+(1-\lambda)(1-p^L)U'(W_1^H)]dW_1^L= -[\lambda p^H U'(W_2^L)+(1-\lambda)p^L U'(W_1^L)]dW_2^L \tag{A3}$$

From (A3) we derive the expression of the slope of the efficiency frontier denoted as P_{inf} :

$$P_{inf} = -\frac{dW_2^L}{dW_1^L} = \frac{\lambda(1-p^H)U'(W_1^L)+(1-\lambda)(1-p^L)U'(W_2^H)}{\lambda p^H U'(W_2^L)+(1-\lambda)p^L U'(W_2^H)} > 0 \tag{A4}$$

In order to simplify the notation we write $P_{inf} = \frac{B}{C}$ with

$$B=\lambda(1-p^H)U'(W_1^L)+(1-\lambda)(1-p^L)U'(W_2^H)$$

and

$$C=\lambda p^H U'(W_2^L)+(1-\lambda)p^L U'(W_2^H)$$

The curvature of the frontier in the wealths space is given by :

$$\frac{d^2W_2^L}{dW_1^{L2}} = -\frac{1}{C}[\lambda(1-p^H)U''(W_1^L) + (1-\lambda)(1-p^L)U''(W_2^H)(dW_2^H/dW_1^L)]$$

$$+\frac{B}{C^2}[\lambda p^H U''(W_2^L)(dW_2^L/dW_1^L) + (1-\lambda)p^L U''(W_2^H)(dW_2^H/dW_1^L)]$$

which can be rewritten by using the measure of absolute risk aversion

$$A(W_1^j) = -\frac{U''(W_1^j)}{U'(W_1^j)} > 0 \ \forall j = H,L \quad \text{and} \quad i = 1,2$$

$$\frac{d^2W_2^L}{dW_1^{L2}} = \frac{1}{C}[\lambda(1-p^H)U'(W_1^L)A(W_1^L) + (1-\lambda)(1-p^L)U'(W_2^H)A(W_2^H)(dW_2^H/dW_1^L)] \qquad (A5)$$
$$+\frac{B}{C^2}[\lambda p^H U'(W_2^L)A(W_2^L)(\frac{B}{C}) - (1-\lambda)p^L U'(W_2^H)A(W_2^H)(dW_2^H/dW_1^L)]$$

From the total differentiation of the profits constraint we obtain the value of :

$$\frac{dW_2^H}{dW_1^L} = -\frac{(1-\lambda)}{\lambda}(1-p^L) - \frac{(1-\lambda)}{\lambda}p^L \frac{dW_2^L}{dW_1^L} = -\frac{(1-\lambda)}{\lambda}\left[(1-p^L) - p^L\frac{B}{C}\right] \qquad (A6)$$

with $(1-p^L) - p^L\frac{B}{C} = \frac{\lambda}{C}[p^H(1-p^L)U'(W_2^L) - p^L(1-p^H)U'(W_1^L)]$.

Since $U''(.) < 0$ and $p^H > p^L$ we have : $(1-p^L) - p^L\frac{B}{C} > 0 \Rightarrow \frac{dW_2^H}{dW_1^L} < 0$.

Substituting (A6) in (A5) we obtain :

$$\begin{aligned}\frac{d^2W_2^L}{dW_1^{L2}} = & \frac{1}{C}\left[\lambda(1-p^H)U'(W_1^L)A(W_1^L)\right] + \\ & \frac{1}{C}\left[-\frac{(1-\lambda)^2}{\lambda}(1-p^L)U'(W_2^H)A(W_1^L)\left((1-p^L) - p^L\frac{B}{C}\right)\right] + \\ & \frac{B^2}{C^3}\left[\lambda p^H U'(W_2^L)A(W_2^L)\right] + \\ & \frac{B}{C^2}\left[\frac{(1-\lambda)^2}{\lambda^2}p^L U'(W_2^H)A(W_2^H)\left((1-p^L) - p^L\frac{B}{C}\right)\right],\end{aligned} \qquad (A7)$$

or after some manipulations :

$$\begin{aligned}\frac{d^2W_2^L}{dW_1^{L2}} = \frac{\lambda}{V} & [(1-p^H)U'(W_1^L)A(W_1^L) \\ & - [(1-\lambda)/\lambda]^2 U'(W_2^H)A(W_2^H)\left((1-p^L) - p^L\frac{B}{C}\right)^2 \\ & + p^H U'(W_2^L)A(W_2^L)\frac{B^2}{C^2}] .\end{aligned} \qquad (A8)$$

The first and the third line are positive while the second line is negative.

After further manipulations,(A8) can be rewritten as :

$$\frac{d^2W_2^L}{dW_1^{L2}} = \frac{\lambda}{C^3}\Big\{U'(W_1^L)U'(W_2^L)U'(W_2^H)\Big[2(1-p^H)(1-\lambda)p^H p^L A(W_1^L) + 2p^H p^L(1-p^H)(1-p^L)(1-\lambda)^2 A(W_2^H) + 2p^H\lambda(1-\lambda)(1-p^H)(1-p^L)A(W_2^L)\Big] + U'(W_1^L)U'(W_2^L)\Big[\lambda^2 p^{H2}(1-p^H)U'(W_2^L)A(W_1^L) + p^H\lambda^2(1-p^H)^2U'(W_1^L)A(W_2^L)\Big] + U'(W_2^H)U'(W_1^L)\Big[(1-p^H)p^{L2}(1-\lambda)^2U'(W_2^H)A(W_1^L) - p^{L2}(1-p^H)^2(1-\lambda)^2U'(W_1^L)A(W_2^H)\Big] + U'(W_2^H)U'(W_2^L)\Big[(1-\lambda^H)^2p^H(1-p^L)^2U'(W_2^H)A(W_2^L) - (1-\lambda)^2p^{H2}(1-p^L)^2U'(W_2^L)A(W_2^H)\Big]\Big\} \quad (A9)$$

The first two expressions in square brackets are positive while the last two are ambiguous. Then a sufficient condition to obtain that (A9) is positive equals to :

$$p^L(1-p^H)\frac{U'(W_1^L)}{U'(W_2^L)}\Big[2p^H(1-p^L)U'(W_2^L) - p^L(1-p^H)U'(W_1^L)\Big] > p^H(1-p^L)^2U'(W_2^L)\left[p^H - \frac{U'(W_2^H)A(W_2^L)}{U'(W_2^L)A(W_2^H)}\right] \quad (A10)$$

Under risk aversion and since $p^H > p^L$, the left hand side of (A10) is positive. Consequently if the right hand side is negative the sufficient condition is verified. Then a sufficient condition to obtain $\frac{d^2W_2^L}{dW_1^{L2}} > 0$ is $\frac{U'(W_1^H)A(W_2^L)}{U'(W_2^L)A(W_1^H)} > p^H \quad \forall \quad W_1^H,\ W_2^L$.□

Proof of corollary 1 :

i) The frontier is strictly convex if $\frac{U'(W_1^H)A(W_2^L)}{U'(W_2^L)A(W_1^H)} \geq 1$ (A11)

(A11) $\Leftrightarrow \frac{A(W_2^L)}{U'(W_2^L)} \geq \frac{A(W_1^H)}{U'(W_1^H)}$, which implies that the ratio $\frac{A(W)}{U'(W)}$ is non increasing in W since $W_2^L < W_1^H$;

ii) The above condition that the ratio $\frac{A(W)}{U'(W)}$ is non-increasing can be interpreted in terms of prudence and absolute risk aversion :

By differentiation of the logarithm of $\frac{A(W)}{U'(W)}$ we have

$$\frac{U'(W)}{A(W)}\frac{d\frac{A(W)}{U'(W)}}{dW} = \frac{U'(W)}{A(W)}\frac{d\ln\frac{A(W)}{U'(W)}}{dW} = -\frac{[U'(W)]^2}{U''(W)}\frac{d\ln\left(-\frac{U'(W)}{[U'(W)]^2}\right)}{dW}$$

$$= -\frac{[U'(W)]^2}{U''(W)}\frac{\frac{-U'''W)[U'(W)]^2+2U''(W)U'(W)}{[U'(W)]^4}}{-\frac{U''(W)}{[U'(W)]^2}} = \frac{2U''(W)U'(W)-U'''(W)[U'(W)]^2}{[U''(W)]^2} \leq 0$$

$$\Leftrightarrow \frac{2U''(W)}{U'(W)} \geq \frac{U'''(W)}{U''(W)} \Leftrightarrow 2A(W) \leq P(W);$$

iii) Finally the correspondence between $\frac{d\frac{A(W)}{U'(W)}}{dW} \leq 0$ and the convexity of $\frac{1}{U'(W)}$ is direct. In fact,

$$\frac{d\frac{A(W)}{U'(W)}}{dW} \leq 0 \Leftrightarrow \frac{d\frac{U''(W)}{[U'(W)]^2}}{dW} \leq 0 \Leftrightarrow \frac{d^2\left(-\frac{1}{U'(W)}\right)}{dW^2} \leq 0 \Leftrightarrow \frac{d^2\frac{1}{U'(W)}}{dW^2} \geq 0.\square$$

Proof of Proposition 2 :

Consider B a tangency point between the low risk indifference curve and the efficiency frontier (see Figure III in the text).

At B, the two slopes are equal :

$P_{VL}|_B = P_F|_B$ where $P_{VL}|_B$ is -(slope) of the low risk indifference curve and $P_F|_B$ is -(slope) of the frontier. Consequently :

$$\frac{(1-p^L)}{p^L}\frac{U'(W_{1B}^L)}{U'(W_{2B}^L)} = \frac{\lambda(1-p^H)U'(W_{1B}^L)+(1-\lambda)(1-p^L)U'(W_{1B}^H)}{\lambda p^H U'(W_{2B}^L)+(1-\lambda)p^L U'(W_{1B}^H)} \qquad \text{(A12)}$$

$$\Leftrightarrow \frac{\lambda}{1-\lambda}\frac{(p^H-p^L)}{p^L(1-p^L)} = U'(W_{1B}^H)\left[\frac{1}{U'(W_{1B}^L)} - \frac{1}{U'(W_{2B}^L)}\right] \qquad \text{(A13)}$$

Suppose now that there exists a point C (as on Figure III in the text) such that the indifferent curve $V(p^L, B)$ has another contact point with the frontier FL. At this point we have

$$P_{VL}|_C > P_F|_C$$

$$\Leftrightarrow \frac{\lambda}{1-\lambda}\frac{(p^H-p^L)}{p^L(1-p^L)} > U'(W_{1C}^H)\left[\frac{1}{U'(W_{1C}^L)} - \frac{1}{U'(W_{2C}^L)}\right] \qquad \text{(A14)}$$

$$\text{(A13) and (A14)} \quad \Rightarrow U'(W_{1B}^H)\left[\frac{1}{U'(W_{1B}^L)} - \frac{1}{U'(W_{2B}^L)}\right] > U'(W_{1C}^H)\left[\frac{1}{U'(W_{1C}^L)} - \frac{1}{U'(W_{2C}^L)}\right] \qquad \text{(A15)}$$

Given the relative positions of B and C on the frontier, the following inequalities

$U'(W_{1B}^L) > U'(W_{1C}^L)$
$U'(W_{2B}^L) < U'(W_{2C}^L)$
$U'(W_{1B}^H) < U'(W_{1C}^H)$

yield :

$$U'(W_{1B}^H)\left[\frac{1}{U'(W_{1B}^L)} - \frac{1}{U'(W_{2B}^L)}\right] < U'(W_{1C}^H)\left[\frac{1}{U'(W_{1C}^L)} - \frac{1}{U'(W_{2C}^L)}\right] \qquad \text{(A16)}$$

(A16) is the desired contradiction of (A15). Consequently, if B is an optimum, a contact point like C cannot exist and, a fortiori, another tangent point such A in figure III cannot exist since it would imply that the indifference curves intersect.□

Proof of Proposition 3 :

The self-selection constraint of the H-risk individual can be written as :

$$(1-p^H)U(W_1^L) + p^H U(W_2^L) = U(W_1^H) = V^H \qquad \text{(A17)}$$

while the expected utility of the L-risk individual and the zero profits constraint are respectively equal to :

$$(1-p^L)U(W_1^L) + p^L U(W_2^L) = V^L \qquad \text{(A18)}$$

$$\lambda W_1^H + (1-\lambda)\left[(1-p^L)W_1^L + p^L W_2^L\right] = \overline{W} - \overline{p}D. \qquad \text{(A19)}$$

Let us define $x_1^L = U(W_1^L)$, $x_2^L = U(W_2^L)$, $x_1^H = U(W_1^H)$

$$\Rightarrow W_1^L = U^{-1}(x_1^L), \quad W_2^L = U^{-1}(x_2^L), \quad W_1^H = U^{-1}(x_1^H).$$

The equations (A17), (A18), (A19) can be rewritten as :

$$(1-p^H)x_1^L + p^H x_2^L = V^H \qquad (A17')$$

$$(1-p^L)x_1^L + p^L x_2^L = V^L \qquad (A18')$$

$$\lambda U^{-1}(x_1^H) + (1-\lambda)\left[(1-p^L)U^{-1}(x_1^L) + p^L U^{-1}(x_2^L)\right] = \overline{W} - \overline{p}D. \qquad (A20)$$

$$(A17') \text{ and } (A18') \Rightarrow x_1^L = \frac{p^H V^L - p^L V^H}{p^H - p^L} \text{ et } x_2^L = \frac{(1-p^L)V^H - (1-p^H)V^L}{p^H - p^L}$$

By substitution :

$$\lambda U^{-1}(V^H) + (1-\lambda)\left[(1-p^L)U^{-1}\left(\frac{p^H V^L - p^L V^H}{p^H - p^L}\right) + p^L U^{-1}\left(\frac{(1-p^H)V^L - (1-p^L)V^H}{p^H - p^L}\right)\right] = \overline{W} - \overline{p}D \quad (A20')$$

The total differentiation of (A20') yields :

$$\left[\lambda U^{-1'}(V^H) - (1-\lambda)\frac{p^L(1-p^L)}{(p^H-p^L)}U^{-1'}(x_1^L) + (1-\lambda)\frac{p^L(1-p^L)}{p^H-p^L}U^{-1'}(x_2^L)\right]dV^H =$$
$$\left[(1-\lambda)\frac{p^L(1-p^H)}{(p^H-p^L)}U^{-1'}(x_2^L) - (1-\lambda)\frac{p^H(1-p^L)}{(p^H-p^L)}U^{-1'}(x_1^L)\right]dV^L$$

$\Leftrightarrow$

$$\frac{dV^H}{dV^L} = \frac{p^L(1-p^H)U^{-1'}(x_2^L) - p^H(1-p^L)U^{-1'}(x_1^L)}{\frac{\lambda}{(1-\lambda)}(p^H-p^L)U^{-1'}(x_1^H) - p^L(1-p^L)\left[U^{-1'}(x_1^L) - U^{-1'}(x_2^L)\right]} = \frac{G}{H} \qquad (A21)$$

which is the slope of the desired frontier. Since U(.) is a strictly increasing and concave function, $U^{-1}(.)$ is strictly increasing and convex :

$U^{-1'}(.) > 0$ et $U^{-1''}(.) > 0$.

Then $x_1^L > x_1^H > x_2^L \Rightarrow U^{-1'}(x_1^L) > U^{-1'}(x_1^H) > U^{-1'}(x_2^L)$

and $p^H > p^L \Rightarrow p^H(1-p^L) > p^L(1-p^H)$.

G in (A21) is then negative. Therefore the sign of $\frac{dV^H}{dV^L}$ is given by the sign of H.

In the utilities space (V^H, V^L), the second best frontier is between B^H and B^L (see figure IV in the text). From F to B^L, $\frac{dV^H}{dV^L} < 0$ and from J^L to B^L, $\frac{dV^H}{dV^L} > 0$.

Showing that the frontier [F, J^L][1] is concave is equivalent to show :

i) when $\frac{dV^H}{dV^L} < 0, \frac{d^2V^H}{dV^{L2}} < 0$

ii) when $\frac{dV^H}{dV^L} > 0, \frac{d^2V^H}{dV^{L2}} > 0.$

We are interested to the sign of the second derivatives :

$$\frac{d^2V^H}{dV^{L2}} = \frac{G'H - GH'}{H^2}$$

$$\text{with } G' = -\frac{p^L(1-p^H)}{(p^H-p^L)}U^{-1'}(x_2^L)\left[(1-p^H) - (1-p^L)\frac{dV^H}{dV^L}\right] - \frac{p^H(1-p^L)}{(p^H-p^L)}U^{-1'}(x_1^L)\left[p^H - p^L\frac{dV^H}{dV^L}\right]$$

$$\text{and } H' = \frac{\lambda}{(1-\lambda)}(p^H-p^L)U^{-1'}(x_1^H)\frac{dV^H}{dV^L} - \frac{p^L(1-p^L)}{(p^H-p^L)}\left[\left(p^H-p^L\frac{dV^H}{dV^L}\right)U^{-1'}(x_1^L) + \left((1-p^H)-(1-p^L)\frac{dV^H}{dV^L}\right)U^{-1'}(x_2^L)\right]$$

The proof of i) is direct :

when $\frac{dV^H}{dV^L}<0$, $\left((1-p^H)-(1-p^L)\frac{dV^H}{dV^L}\right)$ and $\left(p^H-p^L\frac{dV^H}{dV^L}\right)$ are positive such that $G' < 0$ and $H' < 0$.

On the other hand, H must be positive when $\frac{dV^H}{dV^L} < 0$. Finally, $G < 0 \Rightarrow \frac{d^2V^H}{dV^{L2}} = \frac{G'H - GH'}{H^2} < 0.$

---

[1] A symmetric proof shows that the portion [F; J^H] of the frontier is also concave.

Proof of ii) :

We know that $G < 0$ and $H < 0$ implies $\frac{dV^H}{dV^L} > 0$. The signs of $(p^H - p^L\frac{G}{H})$ and $\left((1-p^H)-(1-p^L)\frac{G}{H}\right)$ cannot be obtained directly.

$$Sg\left(p^H - p^L\frac{G}{H}\right):$$

$$p^H - p^L\frac{G}{H} = \frac{1}{H}(p^H H - p^L G) = \frac{1}{H}\left[\lambda p^H(p^H - p^L)U^{-1'}(x_1^H) + (1-\lambda)p^L(p^H - p^L)U^{-1'}(x_2^L)\right] < 0 \text{ since } H < 0 \text{ and } U^{-1'}(.) > 0.$$

$$Sg\left[(1-p^H)-(1-p^L)\frac{G}{H}\right]:$$

$$\left[(1-p^H)-(1-p^L)\frac{G}{H}\right] = \frac{1}{H}\left[(1-p^H)H-(1-p^L)G\right] =$$

$$\frac{1}{H}\left[\lambda(1-p^H)(p^H-p^L)U^{-1'}(x_1^H) + (1-\lambda)(1-p^L)(p^H-p^L)U^{-1'}(x_1^L)\right] < 0$$

since $H < 0$ and $U^{-1'}(.) > 0$.

Consequently $G' > 0$ et $H' > 0$.

$$\begin{aligned} &Sg(G'H-GH') : \\ &G'H-GH' = \frac{1}{(1-\lambda)}\{-(1-\lambda)\frac{p^L(1-p^H)}{(p^H-p^L)}U^{-1''}(x_2^L)\left[(1-p^H)H-(1-p^L)G\right] \\ &-(1-\lambda)\frac{p^H(1-p^L)}{(p^H-p^L)}U^{-1''}(x_1^L)\left[p^H H-p^L G\right] - \lambda(p^H-p^L)\frac{G^2}{H}U^{-1''}(x_1^H) \\ &+(1-\lambda)\frac{p^L(1-p^L)}{(p^H-p^L)}\frac{G}{H}\left[U^{-1''}(x_1^L)(p^H H-p^L G) + U^{-1''}(x_2^L)\left[(1-p^H)H-(1-p^L)G\right]\right\} \end{aligned} \qquad \text{(A22)}$$

After some algebraic manipulations (A22) becomes :

$$\begin{aligned} G'H-GH' = &\frac{1}{(1-\lambda)}\{-\lambda(p^H-p^L)U^{-1''}(x_1^H)\frac{G^2}{H} \\ &+\lambda(1-\lambda)p^L(1-p^H)\left[(1-p^L)\frac{G}{H} - (1-p^H)\right]U^{-1'}(x_1^H)U^{-1''}(x_2^L) \\ &+\lambda(1-\lambda)p^H(1-p^L)(p^L\frac{G}{H}-p^H)U^{-1'}(x_1^H)U^{-1''}(x_1^L) \\ &+(1-\lambda)^2p^L(1-p^L)\left[(1-p^L)\frac{G}{H} - (1-p^H)\right]U^{-1'}(x_1^L)U^{-1''}(x_2^L)\} \end{aligned} \qquad \text{(A23)}$$

$G'H - GH'$ is positive since each line of (A23) is positive.

$$\Rightarrow \frac{d^2V^H}{dV^{L2}} = \frac{G'H-GH'}{H^2} > 0.\square$$

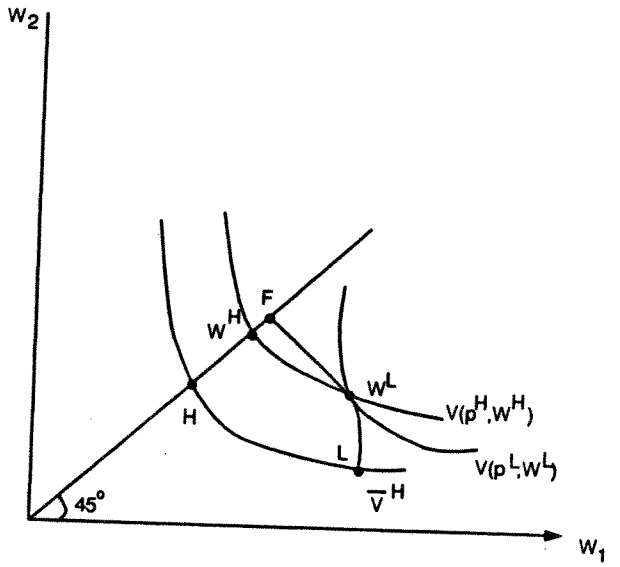
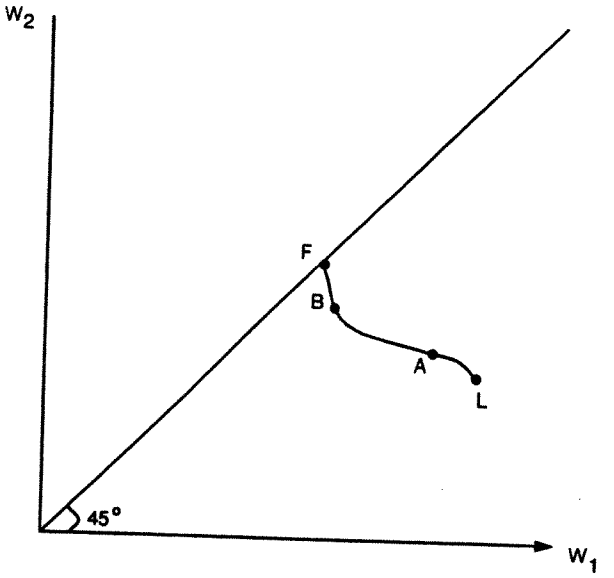


Figure I



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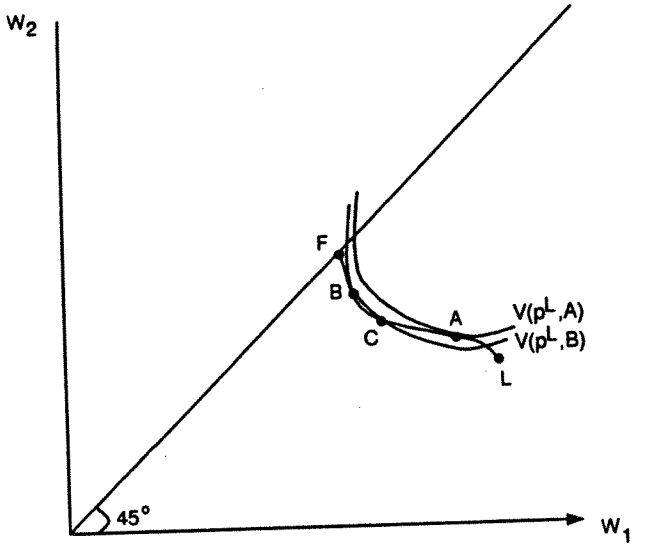

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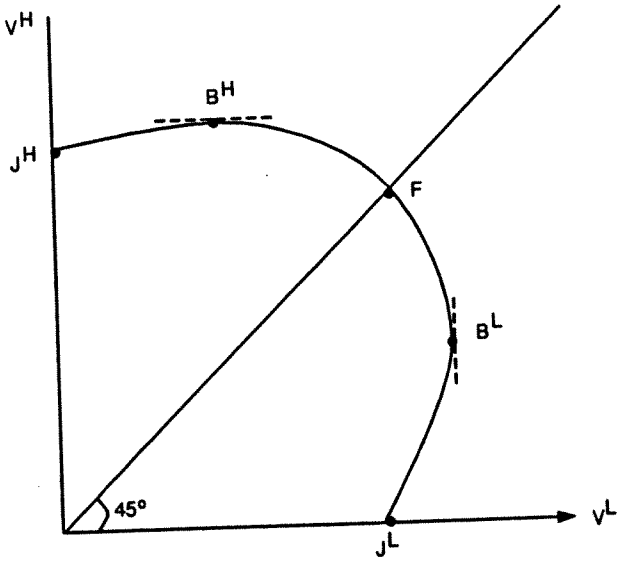

Figure IV

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