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Les modèles à prix rigides et la persistance des chocs of monétaires

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Sommaire

Cette thèse est composée de trois essais. Le premier essai porte sur le problème de persistance soulevé par Chari, Kehoe et McGrattan (2000a). Ces derniers montrent que, dans les modèles standard dynamiques et stochastiques d'équilibre général à prix rigides, les chocs monétaires n'ont pas d'effets persistants sur la production. Plus précisément, leurs effets ne se prolongent pas au-delà de la durée des contrats fixant les prix, même si ces contrats sont imbriqués. Dans cet essai, nous élaborons et estimons un modèle dynamique et stochastique d'équilibre général à prix rigides où les consommateurs forment des habitudes et où l'ajustement du capital est coûteux. Le modèle est estimé par la méthode du maximum de vraisemblance à l'aide de données américaines sur la production (mesurée par le PIB réel), le stock de monnaie (mesuré par M2) et le taux d'intérêt nominal (mesuré par le taux des bons du Trésor américain à trois mois). Cette méthode génère des estimateurs plausibles des paramètres structurels. L'analyse de la réaction du PIB réel aux chocs monétaires indique que celle-ci est persistante et en forme de bosse. Des simulations numériques montrent que la formation d'habitudes ne peut expliquer à elle seule la persistance de la réaction. Elle amplifie plutôt la propagation des chocs de croissance monétaire par son interaction non linéaire avec les coûts d'ajustement du capital. Par rapport à un vecteur autorégressif non contraint, le modèle estimé décrit mieux le profil d'évolution de la production et du stock de monnaie, et à peine moins bien celui du taux d'intérêt nominal aux États-Unis. Il rend également compte de façon convenable du comportement de la consommation et de l'investissement, mais explique mal celui du taux d'inflation.

Le deuxième et le troisième essais s'intéressent à la dynamique du taux de change réel. Il est bien établi dans la littérature empirique que les taux de change réels sont extrêmement volatils et persistants. Après le travail précurseur d'Obstfeld et Rogoff (1995), beaucoup d'efforts ont été consacrés à l'explication des propriétés du taux de change réel au moyen de modèles dynamiques d'équilibre général à prix rigides. Les études actuelles utilisant

cette approche réussissent plutôt bien à générer une volatilité élevée du taux de change réel. Mais, à moins que l'on ne pose l'hypothèse d'une rigidité extrême des prix (en supposant, par exemple, une durée excessive des contrats de prix et de salaires), ces études sont incapables de reproduire la persistance observée du taux de change réel.

Dans le deuxième essai, nous étudions la possibilité de générer des taux de change réels persistants en introduisant la formation d'habitudes dans les préférences des consommateurs. Nous étendons le modèle dynamique d'équilibre général à deux pays de Betts et Devereux (2000) en supposant une rigidité des prix du type Calvo (1983) et une fonction d'utilité qui dépend des habitudes de consommation. Les résultats des simulations corroborent la conclusion de Betts et Devereux que le pricing-to-market amplifie la volatilité du taux de change réel. Nous trouvons, cependant, qu'il n'a aucun effet sur la persistance. Par ailleurs, nous montrons que la formation d'habitudes est complètement impertinente pour expliquer les propriétés du taux de change réel, bien qu'elle provoque une réaction plus persistante de la consommation et de la production suite à un choc monétaire.

Dans le troisième essai, nous développons et estimons un modèle dynamique d'équilibre général à prix rigides qui rend compte de la persistance du taux de change réel. S'éloignant de l'hypothèse classique d'élasticité constante, nous postulons que l'élasticité de la demande croît avec le prix relatif, ce qui revient à dire que le taux de marge désiré de l'entreprise monopolistique diminue lorsque celle-ci augmente son prix relatif. Les variations du taux de marge désiré amplifient la rigiditdé nominale qui résulte des frictions imposées de manière exogène. Le modèle est estimé par la méthode du maximum de vraisemblance à l'aide de données sur le taux de change réel du dollar canadien par rapport au dollar américain, l'écart de taux d'inflation entre le Canada et les États-Unis et le ratio des stocks de monnaie des deux pays. Le modèle explique remarquablement bien le comportement du taux de change réel sur la période d'estimation. En particulier, le coefficient d'autocorrélation prévu par le modèle correspond à celui qui est observé dans la série du taux de change réel du dollar

canadien par rapport au dollar américain. Ce résultat est d'autant plus important qu'il est obtenu avec une durée plausible des contrats de prix et une convexité modérée de la fonction de demande.

Mots clés: Modèles à prix rigides, persistance, production, taux de change réel, coûts d'ajustement du capital, formation d'habitudes, pricing-to-market, variations du taux de marge désiré, filtre de Kalman

Summary

This thesis consists of three essays. The first essay addresses the so-called persistence problem raised by Chari, Kehoe, and McGrattan (2000). These authors show that standard dynamic stochastic general-equilibrium (DSGE) models with sticky prices fail to generate persistent output effects to monetary shocks. More precisely, the response of output to a money-growth shock does not last beyond the duration of price contracts, even if these contracts are set in a staggered fashion. The essay develops and estimates a DSGE model with sticky prices, habit formation, and adjustment costs to capital. The model is estimated by the maximum-likelihood method using U.S. data on output (measured by real GDP), the real money stock (measured by M2), and the nominal interest rate (measured by the three-month U.S. Treasury bill rate). The maximum-likelihood procedure yields plausible estimates of the structural parameters. Impulse-response analysis indicates that monetary shocks lead to a persistent and hump-shaped output response. Numerical simulations show that habit formation, by itself, does not solve the persistence problem. Instead, it interacts in a non-linear way with costly capital adjustment to increase the propagation of money-growth shocks in the model. The estimated DSGE model fits U.S. output and real money stock better than an unrestricted VAR and does only slightly worse for the nominal interest rate. The model also tracks well the behaviour of consumption and investment, but it does poorly in explaining the U.S. inflation rate.

The second and third essays deal with real exchange rate dynamics. A well-documented empirical regularity is that real exchange rates are highly volatile and persistent. Following the seminal work of Obstfeld and Rogoff (1995), substantial effort has been devoted to explaining real exchange rate properties using dynamic general-equilibrium sticky-price models. Existing studies that use this approach are rather successful in generating high real exchange rate volatility. But, unless they assume an unreasonable level of price rigidity (for example, via excessively long nominal contracts), these studies fail to match observed real

exchange rate persistence.

In the second essay, we investigate the possibility of generating persistent real exchange rates by allowing consumer preferences to exhibit habit formation. We extend the dynamic two-country sticky-price model of Betts and Devereux (2000) by assuming Calvo-type price setting and habit-forming preferences. The model allows the proportion of firms that engage in pricing-to-market behaviour to range from 0 to 1. Simulation results corroborate the conclusion reached by Betts and Devereux that pricing-to-market magnifies real exchange rate persistence. We find, however, that it does not affect persistence. Moreover, we show that habit formation is completely irrelevant to real exchange rate properties, although it makes consumption and output respond more persistently to shocks.

The third essay constructs and estimates a dynamic general-equilibrium sticky-price model that accounts for real exchange rate persistence. Departing from the standard assumption of constant elasticity of demand, this elasticity is assumed to increase with the relative price. This is equivalent to assuming that the firm's desired markup is decreasing with its relative price. Desired markup variations exacerbate the nominal rigidity that results from the exogenously imposed frictions. The model is estimated by the maximum-likelihood method using data on the Can$/US$ real exchange rate, the inflation differential between Canada and the United States, and the relative real money stock between the two countries. The results show that the model performs remarkably well in explaining in-sample real exchange rate dynamics. In particular, the model predicts the same autocorrelation found in the Can$/US$ real exchange rate series. More importantly, this is achieved with a plausible duration of price contracts and a moderate convexity of the demand function.

Key words: Sticky-price models, persistence, output, real exchange rate, adjustement costs of capital, habit formation, pricing-to-market, desired markup variations, Kalman filter

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À la mémoire de mon père,

à ma femme, Julie

et à notre futur bébé

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Introduction générale

L'idée que la monnaie puisse avoir des effets réels importants sur l'économie remonte au début de la pensée Keynesienne. Jusque là, on supposait que l'économie ouvre dans un cadre Walrasien où il n'y a pas de frictions et où les prix et les salaires sont parfaitement flexibles. Keynes et ses disciples stipulent, au contraire, que les prix et les salaires sont rigides à court terme, si bien que la monnaie n'est pas neutre. Cette hypothèse est certes réaliste, mais le modèle Keynesien demeure un modèle statique qui manque de fondements microéconomiques rigoureux. Il n'est donc pas utile pour étudier certaines questions économiques fondamentales telles que la propagation des chocs et l'évaluation du bien-être.

Au cours de la dernière décennie, une nouvelle piste de recherche en macroéconomie s'est appliquée à reformuler l'essence des enseignements Keynesiens au sein de modèles dynamiques d'équilibre général (DEG). Dans ces modèles, qualifiés parfois de néo-Keynesiens, les agents économiques optimisent sur un horizon intertemporel et la rigidité des prix est justifiée par la présence de concurrence monopolistique combinée à des coûts de menu.

Bien que les premiers modèles néo-Keynesiens aient été des modèles d'économie fermée, leur extension au contexte d'économie ouverte ne devait pas tarder. Dans un cas comme dans l'autre, établir une revue exhaustive de la littérature serait sans doute une tâche ardue. Nous nous contentons de citer parmi les modèles d'économie fermée ceux développés pas Kimball (1995), Yun (1996), Jeanne (1998) Bergin et Feenstra (2000), Kim (2000), Ireland (2001), Chari, Kehoe et McGrattan (2000a) et Christiano, Eichenbaum et Evans (2001). La liste des auteurs qui ont élaboré des modèles néo-Keynesiens d'économie ouverte inclut Obstfeld et Rogoff (1995), Betts et Devereux (1996, 2000), Bergin et Feenstra (2001), Chari, Kehoe et McGrattan (2000b) et Kollmann (2001).

Dans deux travaux importants, Chari, Kehoe et McGrattan (2000a, 2000b) se sont interrogés sur la capacité des modèles DEG à prix rigides à reproduire certains faits empiriques bien établis. Plus précisément, ces auteurs ont vérifié si ces modèles peuvent expliquer (i) la persistance de la production (l'output) et (ii) la volatilité et la persistance du taux de change réel.

Dans la première étude, Chari, Kehoe et McGrattan (2000a) construisent un modèle DEG standard d'économie fermée où les prix sont fixés par des contrats d'une durée déterminée. L'étude démontre que le modèle est incapable de générer des fluctuations de la production aussi persistantes que celles observées dans les données, à moins de supposer que les contrats de prix sont fixés pour une période excessivement longue. Plus précisément, l'étude révèle que les chocs monétaires n'ont pas d'effets sur la production au-delà de la durée moyenne des contrats, même si ces derniers sont imbriqués. Ce résultat est désormais connu sous le nom de *problème de persistance.*

Dans la deuxième étude, Chari, Kehoe et McGrattan (2000b) développent un modèle DEG à deux pays, qui s'inspire du travail précurseur d'Obstfeld et Rogoff (1995). Le modèle suppose que les entreprises monopolistiques discriminent entre le marché domestique et le marché étranger. Cette pratique, appelée pricing-to-market, justifie les écarts par rapport à la loi du prix unique et, par conséquent, l'échec de la parité des pouvoirs d'achat. Moyennant une calibration appropriée de l'aversion au risque, le modèle génère une volatilité du taux de change réel comparable à celle observée empiriquement. Par contre, avec une durée plausible des contrats de prix, le modèle ne réussit pas à reproduire la persistance présente dans les données.

Le premier essai de cette thèse s'intéresse à l'incapacité des modèles DEG à prix rigides à engendrer des effets persistants des chocs monétaires sur la production. Dans cet essai, nous étendons le modèle standard en supposant que les consommateurs forment des habitudes et que l'ajustement du capital est coûteux. L'intuition est la suivante: étant donné que les agents économiques n'aiment pas modifier leurs habitudes de consommation, la réaction de la consommation aux chocs est plus progressive et plus persistante que dans le modèle standard. Puisque la consommation est la principale composante du produit intérieur brut, la formation d'habitudes pourrait expliquer la persistance de la réaction de la production aux chocs de politique monétaire.

Nous estimons le modèle par la méthode du maximum de vraisemblance en utilisant des données américaines sur la production, la monnaie et le taux d'intérêt nominal. Le modèle estimé prédit que la réaction de la production à un choc monétaire est persistante et en forme de bosse, comme celle obtenue à partir d'un vecteur autorégressif standard. Des simulations numériques montrent que l'interaction de la formation d'habitudes et des coûts d'ajustement du capital est cruciale pour amplifier la propagation des chocs monétaires dans le modèle.

Dans le deuxième essai, nous vérifions si la formation d'habitudes est également capable d'induire plus de persistance dans les mouvements du taux de change réel au sein d'un modèle néo-Keynesien d'économie ouverte. Pour cela, nous étendons le modèle à deux pays de Betts et Devereux (2000) en y incorporant la formation d'habitudes et un mécanisme d'ajustement des prix du type Calvo (1983). Nous retrouvons le résultat établi par Betts et Devereux que le pricing-to-market amplifie la volatilité du taux de change réel. Nous démontrons, cependant, qu'il n'en affecte pas la persistance. Par ailleurs, notre étude révèle que la formation d'habitudes n'a aucun effet sur le comportement du taux de change réel, bien qu'elle augmente la persistance de la consommation et de la production. La neutralité de la formation d'habitudes quant à la dynamique du taux de change est démontrée de manière analytique dans une version du modèle où les marchés financiers sont complets. Nous concluons cet essai en faisant la conjecture qu'une modélisation plus rigoureuse du côté de l'offre de l'économie n'aiderait pas à amplifier la persistance du taux de change réel. Une avenue plus prometteuse serait d'envisager un mécanisme qui génère de la rigidité nominale de manière endogène.

Cette idée est formalisée dans le troisième essai dans lequel nous développons un modèle DEG à prix rigides où nous nous écartons de l'hypothèse de la constance de l'élasticité de la demande, communément postulée. Nous supposons, au contraire, que l'élasticité de la demande croît avec le prix relatif ou, de manière équivalente, que le taux de marge désiré de l'entreprise décroît lorsque celle-ci augmente son prix relatif. Il s'ensuit que les hausses du prix relatif sont inférieures à ce qu'elles seraient dans un modèle standard où l'élasticité de la demande est constante. Les variations du taux de marge désiré amplifient la rigidité nominale qui résulte des frictions imposées de manière exogène dans le marché des biens. Au lieu d'avoir recours à la calibration, comme c'est souvent le cas dans les travaux antérieurs, nous dérivons un modèle empirique dont nous estimons les paramètres structurels. Le modèle est estimé par la méthode du maximum de vraisemblance en utilisant des données canadiennes et américaines sur le taux de change, le stock de monnaie et le taux d'inflation. Les résultats révèlent que le modèle estimé explique remarquablement bien le comportement du taux de change réel sur la période d'estimation. En particulier, le modèle reproduit la persistance observée du taux de change avec une durée plausible des contrats de prix et un degré modéré de convexité de la fonction de demande.

Essay 1

Habit Formation and the Persistence of Monetary Shocks

1. Introduction

In a recent paper, Chari, Kehoe, and McGrattan (2000a) show that standard dynamic stochastic general-equilibrium (DSGE) models with sticky prices fail to generate persistent output effects to monetary shocks. More precisely, the response of output to a money-growth shock does not last beyond the duration of price contracts, even if contracts are staggered. Hence, unless one assumes an implausibly large degree of price rigidity, this type of model cannot replicate the persistent output response obtained using, for example, a benchmark vector autoregression (VAR). Previous empirical studies based on VARs document a persistent, hump-shaped response of output to a monetary shock with a peak at around four to six quarters after the shock (see Bernanke and Mihov 1998 and Christiano, Eichenbaum, and Evans 1999). The failure of DSGE models to replicate this feature of the data is called "the persistence problem."

This paper studies the effects of monetary policy on output using a DSGE model with sticky prices, habit formation, and adjustment costs to capital. Price rigidity is modelled as in Calvo (1983), where each firm has a constant exogenous probability of changing its price in every period. Habit formation has been employed previously by (among others) Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999) to study the equity premium puzzle; by Carrol, Overland, and Weil (2000) to explain the growth-to-savings causality; and by Fuhrer (2000) to explain the excess smoothness of consumption and inflation inertia. Because habit-forming agents dislike large changes in consumption, the consumption response to shocks is smoother and more persistent than predicted by the permanent income hypothesis (PIH) with a time-separable utility. Since consumption is the largest component in GDP, habit formation is a plausible candidate to explain the persistent and hump-shaped output response to monetary policy shocks.

The model is estimated by the maximum-likelihood (ML) method using U.S. data on output, the real money stock, and the nominal interest rate. The ML procedure yields plausible estimates of the structural parameters. Impulse-response analysis indicates that monetary shocks lead to a persistent and hump-shaped output response. Up to 95 per cent of the initial effect of a money-growth shock on output persists beyond the average duration of price contracts. A comparison of impulse responses and persistence measures for different values of the habit formation and capital-adjustment cost parameters indicates that habit formation, by itself, does not solve the persistence problem. Instead, habit formation interacts non-linearly with costly capital adjustment to increase the propagation of monetary shocks in the model. When the fit of the estimated DSGE model is compared with that of an unrestricted VAR, the mean squared error (MSE) of the DSGE model is smaller than that of the VAR for output and the real money stock and only slightly larger for the nominal interest rate. Variance decomposition indicates that money growth explains more than 50 per cent of the (conditional) output variability at horizons of less than one year. In the long run, money growth explains only 27.1 per cent of the unconditional output variability, while 71.4 per cent is explained by technology shocks.

Related papers include those by Bergin and Feenstra (2000), Dotsey and King (2001), and Dib and Phaneuf (2001). Bergin and Feenstra construct a model where the interaction of materials inputs and translog preferences leads to endogenous output persistence. Translog preferences dissuade firms from charging higher prices by making the elasticity of demand that a given firm faces depend on the firm's relative price. Dotsey and King construct a model that incorporates variable capital utilization, and materials input and labour flexibility. Results indicate that these three features are mutually reinforcing and magnify output persistence. Dib and Phaneuf construct a DSGE model with sticky prices and costly adjustment to labour. Their results show that adding adjustment costs to the labour input generates endogenous output persistence to monetary shocks. After our research was completed, we found a closely related paper by Christiano, Eichenbaum, and Evans (2001). These authors examine both output and inflation persistence using a limited-participation model that incorporates price and wage rigidities, optimizing and non-optimizing price- and wage-setting, habit formation, adjustment costs in investment, and variable capital utilization. Their results suggest that wage rigidity and variable capital utilization are also important for explaining output persistence in response to monetary shocks. Although their modelling strategy is similar to ours, Christiano, Eichenbaum, and Evans obtain empirical estimates by minimizing the distance between the impulse responses in a VAR and the ones predicted by the model, whereas we estimate the model by full-information ML using the Kalman filter. The Kalman filter allows us to deal with poorly measured or unobserved variables (like the stock of capital), and yields the optimal solution to the problem of predicting and updating state-space models. Furthermore, we propose a different propagation mechanism than the one emphasized in the earlier models. In fact, although apparently distinct, the crucial features of these models work through the same channel to increase output persistence. They prevent a rapid change in the real marginal cost after a monetary shock, and lead to stronger nominal rigidity.

The rest of the paper is organized as follows. Section 2 presents the theoretical model. Section 3 describes the estimation procedure, reports empirical results, and discusses the impulse-response functions and variance decompositions implied by the estimated model. Section 4 concludes.

2. The Model

The economy consists of (i) an infinitely lived representative household, (ii) a representative final-good producer, (iii) a continuum of intermediate-good producers indexed by $i \in [0,1]$, and (iv) a government. Intermediate goods are used in the production of the final good.

The final good is perishable and can be used for either consumption or investment. There is no population growth. The population size is normalized to one.

2.1 Households

The representative household maximizes lifetime utility, defined by

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u_s(c_s, c_{s-1}, m_s, \ell_s),$$

where $\beta \in (0,1)$ is the subjective discount factor and $u(\cdot)$ is the instantaneous utility function. Households derive utility from the consumption of the final good (c_t), real money balances (m_t), and leisure (ℓ_t). The household's preferences exhibit internal habit formation. That is, utility depends on current consumption relative to a habit stock determined by the household's own past consumption. Thus, consumption levels in adjacent periods are complements. In particular, the instantaneous utility function is assumed to be

$$u_t(c_t, c_{t-1}, m_t, \ell_t) = \frac{(c_t/c_{t-1}^{\gamma})^{1-\eta_1}}{1-\eta_1} + \frac{b_t(m_t)^{1-\eta_2}}{1-\eta_2} + \frac{\psi(\ell_t)^{1-\eta_3}}{1-\eta_3}, \tag{1}$$

where $m_t = M_t/P_t$, M_t is the nominal money stock, P_t is the aggregate price index, b_t is a preference shock, $\psi > 0$ measures the weight of leisure in the utility function, and η_1, η_2, and η_3 are positive preference parameters different from one. In the special case where $\eta_j \to 1$ for all j, the logarithmic utility function is obtained. In the special case where $\gamma = 0$, there is no habit formation and households care only about the absolute level of current consumption. In principle, the habit stock could include consumption levels prior to time $t-1$. Fuhrer (2000) estimates a model where the habit stock is a weighted average of past consumption and finds that the habit-formation reference level is essentially the previous period's consumption level.

In addition to money, households can hold interest-bearing, one-period nominal bonds. The gross nominal interest rate on bonds due at time $t+1$ is denoted by R_t. The household's resources in period t consist of the principal and the return on bonds purchased at time $t-1$, money holdings set aside in period $t-1$, wages and rents received from selling labour and renting capital to firms, dividends, and lump-sum transfers from the government.

The household's income in period t is allocated to consumption, investment, money holdings, and the purchase of nominal bonds. Investment increases the household's stock of capital according to

$$k_{t+1} = (1-\delta)k_t + x_t, \tag{2}$$

where $\delta \in (0,1)$ is the depreciation rate of capital. The capital stock is costly to adjust. The adjustment-cost function is assumed to be quadratic in investment and strictly convex:

$$\Gamma(x_t, k_t) = (\chi/2)(x_t/k_t - \delta)^2 k_t, \tag{3}$$

where $\chi \geq 0$. Investment beyond that required to replace depreciated capital entails a positive quadratic cost that is proportional to the current capital stock.

The representative household's budget constraint (expressed in real terms) is

$$c_t + a_t + m_t + x_t \leq (R_{t-1}/\pi_t)a_{t-1} + (m_{t-1}/\pi_t) + w_t n_t + q_t k_t + d_t + \tau_t - (\chi/2)(x_t/k_t - \delta)^2 k_t, \tag{4}$$

where $a_t = A_t/P_t$ is the real value of nominal bond holdings, A_t are nominal bond holdings, π_t is the gross rate of inflation between $t-1$ and t, w_t is the real wage, n_t is the number of hours worked, q_t is the real rental rate of capital, d_t are dividends, and τ_t are lump-sum transfers or taxes. The household's total endowment of time is normalized to one. Thus

$$\ell_t + n_t = 1. \tag{5}$$

The representative household maximizes its lifetime utility subject to constraints (2), (4), (5), and the no-Ponzi-game condition. The first-order necessary conditions associated with the optimal choice of $c_t, M_t, \ell_t, k_{t+1}$, and A_t for this problem are

$$\lambda_t = (1/c_{t-1}^{\gamma})(c_t/c_{t-1}^{\gamma})^{-\eta_1} - \beta\gamma E_t[(c_{t+1}/c_t^{1+\gamma})(c_{t+1}/c_t)^{-\eta_1}], \tag{6}$$

$$b_t m_t^{-\eta_2} = \lambda_t[(R_t - 1)/R_t], \tag{7}$$

$$(1 - n_t)^{-\eta_3} = \lambda_t w_t/\psi, \tag{8}$$

$$\lambda_t = \frac{\beta E_t\{\lambda_{t+1}[1 + q_{t+1} - \delta + \chi(x_{t+1}/k_{t+1} - \delta) + (\chi/2)(x_{t+1}/k_{t+1} - \delta)^2]\}}{1 + \chi(x_{t+1}/k_{t+1} - \delta)}, \tag{9}$$

$$\lambda_t = \beta R_t E_t\left(\lambda_{t+1}/\pi_{t+1}\right), \tag{10}$$

where λ_t is the Lagrange multiplier associated with the household's budget constraint at time t and equals the marginal utility of consumption at time t. Condition (7) determines money demand by equating the marginal rate of substitution of money and consumption to $(R_t - 1)/R_t$, where R_t is the gross return of the nominal bond. The interest elasticity of money is equal to $-1/\eta_2$.[1] The preference shock, b_t, can be interpreted as a money-demand shock. Condition (8) determines the labour supply by equating the marginal rate of substitution between labour and consumption to the real wage. Condition (9) prices the (marginal unit of) capital. Condition (7) prices the nominal bond. Conditions (9) and (10) imply that the ex-ante real interest rate should be equal to the ex-ante real return on capital.

2.2 The final-good producer

Final-good producers are perfectly competitive and aggregate the intermediate goods into a single perishable commodity. Their technology is constant elasticity of substitution (CES):

$$y_t = \left[\int_0^1 y_t(i)^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}, \tag{11}$$

where $y(i)$ is the input of intermediate good i, and $\theta > 1$ is the elasticity of substitution between different goods. As $\theta \to \infty$, goods become perfect substitutes in production. The

[1] Strictly speaking, $-1/\eta_2$ is the elasticity with respect to $(R_t - 1)/R_t$, rather than $R_t - 1$.

final-good producer solves the static problem

$$\underset{\{y_t(i)\}}{Max} \quad P_t y_t - \int_0^1 P_t(i) y_t(i) di,$$

subject to (11). $P_t(i)$ is the price of the intermediate good i and P_t is the aggregate price index. The solution of this problem yields the input demand of good i:

$$y_t(i) = (P_t(i)/P_t)^{-\theta} y_t, \tag{12}$$

where the elasticity of demand is θ. The zero-profit condition implies that the aggregate price index is given by

$$P_t = \left[\int_0^1 P_t(i)^{(1-\theta)} di\right]^{1/(1-\theta)}. \tag{13}$$

2.3 The intermediate-good producer

The representative firm i produces its differentiated good using the Cobb-Douglas technology:

$$y_t(i) = z_t k_t(i)^{\alpha} n_t(i)^{1-\alpha}, \tag{14}$$

where $0 < \alpha < 1$ and z_t is a serially correlated technology shock. The technology shock is common to all intermediate-good producers. Unit-cost minimization determines the demands for labour and capital inputs. Formally,

$$\underset{\{n_t(i), k_t(i)\}}{Min} \qquad w_t n_t(i) + q_t k_t(i),$$

subject to $z_t k_t(i)^{\alpha} n_t(i)^{1-\alpha} = y_t(i) \geq 1$. First-order conditions are

$$w_t = (1-\alpha)\, \phi_t [y_t(i)/n_t(i)], \tag{15}$$

and

$$q_t = \alpha \phi_t [y_t(i)/k_t(i)], \tag{16}$$

where the real marginal cost (ϕ_t) is the Lagrange multiplier associated with the constraint. Since technology is common, and labour and capital are perfectly mobile across industries, conditions (15) and (16) imply that all firms must have the same capital/labour ratio.

Intermediate-good producers are monopolistically competitive. Each firm faces the downward-sloping demand curve (12) for its differentiated good. Firm i chooses its (nominal) price $P(i)$ taking as given the aggregate demand and the price level. Nominal prices are assumed to be sticky. Price stickiness is modelled *à la* Calvo (Calvo 1983): a firm changes its price with constant and exogenous probability $1-\varphi$ in every period.[2] Alternatively, one could assume explicit costs of changing prices or Taylor's staggered price-setting. Quadratic costs of price adjustments, as in Rotemberg (1982), can be shown to lead to an aggregate pricing equation similar to the one obtained using Calvo's model. Moreover, aggregation is somewhat easier using Calvo-type than Taylor-type price rigidity, because it is not necessary to keep track of heterogeneous price cohorts. From the viewpoint of estimating the average length of price contracts using ML, Calvo's model is also easier to implement because the log-likelihood function is continuous on φ. This follows from the fact that the probability of price changes is continuous in the interval $[0,1]$. On the other hand, the contract length in Taylor's model is an integer number and, consequently, the log-likelihood function is discontinuous on this parameter.

Let us denote by P_t^* the optimal price set by a typical firm at period t. It is not necessary to index P_t^* by firm, because all the firms that change their prices at a given time choose the same price (see Woodford 1996). The total demand facing this firm at time s for $s \geq t$ is $y_s^* = (P_t^*/P_s)^{-\theta} y_s$. The probability that P_t^* "survives" at least until period s, for $s \geq t$, is φ^{s-t}. Then, the intermediate-good producer chooses P_t^* to maximize

$$E_t \sum_{s=t}^{\infty} (\beta\varphi)^{s-t} \Lambda_{t,s} \left(P_t^* - \Phi_s\right) y_s^*,$$

[2]Hence, the average duration of price contracts is given by $1/(1-\varphi)$.

where $\Lambda_{t,s} = (\lambda_s/P_s)/(\lambda_t/P_t)$ and Φ_s is the nominal marginal cost at time s. Differentiating with respect to P_t^* and equating to 0 yields

$$P_t^* = \left(\frac{\theta}{\theta - 1}\right)\left(\frac{E_t \sum_{s=t}^{\infty} (\beta\varphi)^{s-t} \Lambda_{t,s} y_s^* \Phi_s}{E_t \sum_{s=t}^{\infty} (\beta\varphi)^{s-t} \Lambda_{t,s} y_s^*}\right). \tag{17}$$

Equation (17) shows that the optimal price depends on current and expected future demands and nominal marginal costs. Owing to price stickiness, the equilibrium markup is not constant, as it would be if prices were flexible.

Assuming that price changes are independent across firms, the law of large numbers implies that $1 - \varphi$ is also the proportion of firms that set a new price each period. The proportion of firms that set a new price at time s and have not changed it as of time t (for $s \leq t$) is given by the probability that a time-s price is still in effect in period t. It is easy to show that this probability is $\varphi^{t-s}(1 - \varphi)$. It follows that the aggregate price level can be written as

$$P_t = \left((1 - \varphi) \sum_{s=-\infty}^{t} \varphi^{t-s} (P_t^*)^{1-\theta}\right)^{\frac{1}{1-\theta}}.$$

This expression can be written in recursive form as

$$P_t^{1-\theta} = \varphi P_{t-1}^{1-\theta} + (1 - \varphi)(P_t^*)^{1-\theta}. \tag{18}$$

2.4 The government

The government comprises both fiscal and monetary authorities. There is no government spending or investment. The government makes lump-sum transfers to households each period. Transfers are financed by printing additional money in each period. Thus, the government budget constraint is

$$\tau_t = m_t - m_{t-1}/\pi_t, \tag{19}$$

where the term on the right-hand side is seigniorage revenue at time t. Money is supplied exogenously by the government according to $M_t = \mu_t M_{t-1}$, where μ_t is the (stochastic) gross rate of money growth.[3] In real terms, this process implies

$$m_t \pi_t = \mu_t m_{t-1}. \tag{20}$$

2.5 Stochastic shocks

The economy is subject to shocks to technology (z_t), money-supply growth (μ_t), and money demand (b_t). These shocks follow the exogenous stochastic processes

$$\ln z_{t+1} = (1-\rho^z)\ln z + \rho^z \ln z_t + \epsilon_{z,t}, \tag{21}$$

$$\ln \mu_{t+1} = (1-\rho^\mu)\ln \mu + \rho^\mu \ln \mu_t + \epsilon_{\mu,t}, \tag{22}$$

$$\ln b_{t+1} = (1-\rho^b)\ln b + \rho^b \ln b_t + \epsilon_{b,t}, \tag{23}$$

where ρ^z, ρ^μ, and ρ^b are strictly bounded between -1 and 1, and the innovations $\epsilon_t = (\epsilon_{z,t}, \epsilon_{\mu,t}, \epsilon_{b,t})'$ are assumed to be normally distributed with a zero mean and variance-covariance matrix:

$$\mathbf{V} = Var(\epsilon_t \epsilon_t') = \begin{bmatrix} \sigma^2_{\epsilon_z} & 0 & 0 \\ 0 & \sigma^2_{\epsilon_\mu} & 0 \\ 0 & 0 & \sigma^2_{\epsilon_b} \end{bmatrix}. \tag{24}$$

Since households are identical, the net supply of (private) bonds is zero. Goods-market clearing requires that aggregate output be equal to aggregate demand:

$$y_t = c_t + x_t. \tag{25}$$

A symmetric equilibrium for this economy is a collection of 13 sequences (c_t, m_t, n_t, x_t, k_{t+1}, y_t, λ_t, ϕ_t, P_t, P_t^*, q_t, w_t, and $R_t)_{t=0}^{\infty}$ satisfying (i) the accumulation equation (2), (ii) the household's maximization conditions (equations (6) to (10)), (iii) the production function (14), (iv) the cost-minimization conditions (equations (15) and (16)), (v) the pricing conditions (equations (17) and (18)), (vi) the market-clearing condition (25), and (vii) the

[3]It is easy to extend the model to allow an endogenous process for money supply whereby money growth (or the nominal interest rate) follows, for example, a Taylor-type rule. In such an extension of the model, the endogenous reaction of the government might also increase the persistence of monetary shocks.

money-supply process (20), given the initial stocks of habit, real money, and capital, and the exogenous stochastic processes $(z_t,\ \mu_t,\ b_t)$.

Since the model cannot be solved analytically, the equilibrium conditions are log-linearized around the deterministic steady state to obtain a system of linear difference equations. (Appendix A gives the log-linearized version of the model.) After some manipulations, the log-linearized version of the model can be written as

$$\begin{bmatrix} \mathbf{X}_{t+1} \\ E_t\mathbf{Y}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{X}_t \\ \mathbf{Y}_t \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \mathbf{Z}_{t+1}, \tag{26}$$

$$\mathbf{Z}_{t+1} = \rho \mathbf{Z}_t + \epsilon_{t+1}, \tag{27}$$

where $\mathbf{X}_t = (\hat{k}_t,\ \hat{m}_{t-1},\ \hat{c}_{t-1})'$ is a 3×1 vector that contains the predetermined variables of the system (the circumflex denotes percentage deviations from the deterministic steady state); $\mathbf{Y}_t = (\hat{c}_t,\ \hat{\pi}_t,\ \hat{\lambda}_t,\ \hat{q}_t)'$ is a 4×1 vector that contains the forward-looking variables; $\mathbf{Z}_t = (\hat{z}_t,\ \hat{\mu}_t,\ \hat{b}_t)'$ is a 3×1 vector that contains the exogenous shocks; $\epsilon_t = (\epsilon_{z,t},\ \epsilon_{\mu,t},\ \epsilon_{b,t})'$ is a 3×1 vector with the innovations of z_t, μ_t, and b_t, respectively; ρ is a 3×3 diagonal matrix with elements ρ^z, ρ^μ, and ρ^b; and $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{B}_1$, and $\mathbf{B}_2$ are submatrices of appropriate size that contain combinations of structural parameters. The Blanchard-Kahn (1980) forward-backward solution method can be applied to (26) to obtain

$$\mathbf{X}_{t+1} = \mathbf{A}_{11}\,\mathbf{X}_t + \mathbf{A}_{12}\mathbf{Y}_t + \mathbf{B}_1\mathbf{Z}_t, \tag{28}$$

$$\mathbf{Y}_t = \mathbf{F}_1\mathbf{X}_t + \mathbf{F}_2\mathbf{Z}_t,$$

where $\mathbf{F}_1$ and $\mathbf{F}_2$ are both 4×3 matrices that include non-linear combinations of the structural parameters contained in $\mathbf{A}_{11}$, $\mathbf{A}_{12}$, $\mathbf{A}_{21}$, $\mathbf{A}_{22}$, $\mathbf{B}_1$, and $\mathbf{B}_2$. For the precise form of these matrices and the conditions for a unique solution, see Blanchard and Kahn (1980). The remaining (static) variables of the model can be collected in the 6×1 vector $\mathbf{S}_t = (\hat{x}_t,\ \hat{n}_t,\ \hat{w}_t,\ \hat{\phi}_t,\ \hat{R}_t,\ \hat{y}_t)'$ that follows:

$$\mathbf{S}_t = \mathbf{C}\mathbf{X}_t + \mathbf{D}\mathbf{Z}_t, \tag{29}$$

where $\mathbf{C}$ and $\mathbf{D}$ are matrices of size 6×3 whose elements are also non-linear combinations of structural parameters.

3. Econometric Analysis

3.1 Estimation method and data

The model is estimated by ML using the Kalman filter. Earlier studies that use ML procedures to estimate DSGE models include Christiano (1988), Altug (1989), Bencivenga (1992), McGrattan (1994), Hall (1996), McGrattan, Rogerson, and Wright (1997), Kim (2000), and Ireland (2001). Our estimation strategy is closest to that used by Ireland (2001). The Kalman filter allows us to deal with unobserved or poorly measured predetermined variables (like the stock of capital) and yields the optimal solution to the problem of predicting and updating state-space models. Hansen and Sargent (1998) show that the ML estimator obtained by applying the Kalman filter to the state-space representation of DSGE models is consistent and asymptotically efficient.

For the Kalman-filter estimation procedure, the transition (or state) equation is constructed using equations (27) and (28) to collect the predetermined and exogenous variables of the system into the 6×1 vector $\mathbf{H}_t = (\mathbf{X}_t \ \mathbf{Z}_t)' = (k_t,\ m_{t-1}, c_{t-1},\ z_t,\ \mu_t,\ b_t)'$ that follows the process

$$\mathbf{H}_{t+1} = \mathbf{Q}\mathbf{H}_t + \mathbf{e}_{t+1}, \tag{30}$$

where

$$\mathbf{Q} = \begin{bmatrix} \mathbf{A}_{11} + \mathbf{A}_{12}\mathbf{F}_1 & \mathbf{A}_{12}\mathbf{F}_2 + \mathbf{B}_1 \\ \mathbf{0} & \rho \end{bmatrix}$$

is a 6×6 matrix and $\mathbf{e}_t = (0, 0, 0, \epsilon_t)' = (0, 0, 0, \epsilon_{zt}, \epsilon_{\mu t}, \epsilon_{bt},)'$ is a 6×1 vector.

The measurement equation consists of the processes for output, the real money stock, and the nominal interest rate. After some fairly straightforward transformations, these variables are written as functions of $\mathbf{H}_t$:

$$\xi_t = \mathbf{W}\mathbf{H}_t, \tag{31}$$

where $\xi_t = (m_t, y_t, R_t)'$ is a 3×1 vector and $\mathbf{W}$ is a 3×6 matrix.[4] The elements of $\mathbf{Q}$ and $\mathbf{W}$ are non-linear functions of the structural parameters of the model. These elements are computed from the Blanchard-Kahn solution of the DSGE model in each iteration of the optimization procedure. Note that the estimation procedure imposes all restrictions implied by the theoretical model. Standard errors are computed as the square root of the diagonal elements of the inverted Hessian of the (negative) log-likelihood function evaluated at the maximum. To assess the robustness of the results to deviations from the assumption of normality, robust quasi-maximum likelihood (QML) standard errors (White 1982) are also computed. At the estimated ML parameters, the condition for the existence of a unique model solution is satisfied. That is, the number of explosive characteristic roots of the system of linear difference equations equals the number of non-predetermined variables.

The model is estimated using quarterly U.S. data on output, real money, and the rate of nominal interest. The series are taken from the database of the Federal Reserve Bank of St. Louis. The sample period is from 1960Q1 to 2001Q2. Output is measured by real GDP per capita. The stock of nominal money is measured by M2 per capita. By measuring these two series in per-capita terms, we aim to make the data compatible with our model, where there is no population growth. Population is measured by the civilian, non-institutional population, 16 years old and over. The gross nominal interest rate is measured by the

[4] As is well known, estimating DSGE models using more observable variables than structural shocks leads to a singular variance-covariance matrix of the residuals. See Ingram, Kocherlakota, and Savin (1994) for a discussion in the special case where the only shock is a technology shock. One way to address this issue is to add measurement errors to the observable variables (as in McGrattan 1994). A possible drawback to this approach is that measurement errors lack structural interpretation and essentially capture specification errors. Still, in preliminary work, we considered this approach. When we added measurement errors to all observable variables, we found that not all variances were identified, or that some of them converged to zero. When we added only as many errors as needed to make the system non-singular, we found that the results were very sensitive to the variable that was assumed to be measured with noise.

three-month U.S. Treasury bill rate. Because the variables in the model are expressed in percentage deviations from the steady state, the output and real money series were logged and detrended linearly. The nominal interest rate series was logged and demeaned. We also estimated the model using Hodrick-Prescott (H-P) filtered data, obtaining very similar results to the ones reported below.[5]

3.2 Estimates of structural parameters

The structural parameters estimated are the preference parameters η_2 and η_3, the habit-persistence parameter (γ), the probability of a price change by an intermediate-good producer (φ), the parameter of the capital-adjustment-cost function (χ), and parameters of the shock processes (ρ^z, ρ^μ, ρ^b, σ^z, σ^μ, and σ^b). Remaining parameters were either poorly identified or additional evidence about their magnitude is available. Data on national income accounts suggest that a plausible value for the share of capital in production is 0.36. The subjective discount factor is fixed to 0.99, meaning that the steady-state quarterly gross real interest rate is approximately 1.01. The rate of depreciation is fixed to 0.025. The gross rate of money growth (and inflation) at the steady state is fixed to 1.017. This value corresponds to the average gross rate of money growth during the sample period. Two important structural parameters that are poorly identified are the curvature parameter of the consumption component in the utility function (η_1) and the elasticity of demand (θ). Markup estimates reported by Basu and Fernald (1994) for U.S. data indicate that θ is approximately 10. Estimates of the curvature of the utility function with respect to consumption range from 0.5 to 5. We assume that $\eta_1 = 2$, but sensitivity analysis indicates that the results do not depend crucially on the magnitudes of θ and η_1.[6] Finally, fixing

[5]Results using H-P filtered data are available upon request.

[6]We also performed single and joint Lagrange multiplier (LM) tests of the null hypothesis that the true values of β, δ, η_1, α, and θ are the ones assumed during estimation. In all cases, one cannot reject the null hypothesis. These results, however, should be interpreted with caution because they might also reflect low test power.

the proportion of time worked in steady state (n) amounts to fixing either the mean of the technology shock (z) or the weight of leisure in the utility function (ψ).[7] We do not assign particular values to these parameters during the estimation procedure. Instead, we adjust them so that, along with the ML estimate of η_3, $n = 0.31$. This means that the proportion of time worked in steady state is approximately one third.

ML estimates of the parameters and their asymptotic and QML standard errors are reported in Table 1.1. Since asymptotic and QML standard errors have very similar magnitudes, conclusions regarding the statistical significance of the parameters do not depend on the estimate of the standard error employed to construct the t-statistic. The ML estimate of the habit-formation parameter (γ) is 0.98 (0.016). The term in parenthesis is the asymptotic standard error of the estimate. This estimate is significantly different from zero, but is not significantly different from one, at standard levels. This estimate is larger than, but still consistent with, the values of 0.80 (0.19) and 0.90 (1.83), reported by Fuhrer (2000); 0.63 (0.14), reported by Christiano, Eichenbaum, and Evans (2001); 0.73, reported by Boldrin, Christiano, and Fisher (2001); and 0.938 (1.775), reported by Heaton (1995).

The estimated value of the adjustment-cost parameter (χ) is 85.19 (18.94). To give meaning to this estimate and to allow its comparison with estimates based on other functional forms, it is useful to compute the elasticity of investment with respect to the price of installed capital. The elasticity implied by the estimate of χ is 0.47. This value is higher than the point estimates of 0.34 and 0.28 reported by, respectively, Kim (2000) and Christiano, Eichenbaum, and Evans (2001), but it is considerably lower than the typical value used to calibrate standard real business cycle (RBC) models (see, for example, Baxter and Crucini 1993).

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[7] n is the solution to the non-linear equation:
$n^{(1-2\gamma)\eta_1}(1-n)^{-\eta_3} = \left(\frac{(1-\alpha)(\theta-1)(1-\beta\gamma)}{\psi\theta}\right)\left(\frac{\theta(1/\beta-1+\delta)}{\alpha(\theta-1)}\right)^{(\alpha+(2\gamma-1)\eta_1)/(\alpha-1)}\left(\frac{\theta(1/\beta-1+\delta)}{\alpha(\theta-1)}-\delta\right)^{(2\gamma-1)\eta_1} z^{1/(\alpha-1)}$.
(See Appendix B.)

The estimated probability of not changing price in a given quarter or, equivalently, the proportion of firms that do not change prices in a given quarter, is 0.847 (0.034). This implies that the average length of price contracts is $1/(1-0.847) = 6.56$ (1.44) quarters. Previous estimates of the average time between price adjustments vary substantially. Galí and Gertler (1999) find that θ is approximately 0.83. Their estimate implies that prices are fixed between five and six quarters. Cecchetti (1986) reports that the average number of years since the last price adjustment for U.S. magazines ranges from 1.8 to 14. Kashyap (1995) finds that the average time between price changes in 12 mail-order catalogue goods is approximately 4.9 quarters. Taylor (1999) surveys empirical studies on price-setting and finds that the average duration of price contracts is about four quarters in the United States. Bils and Klenow (2001) document substantial heterogeneity in the frequency of price adjustments among the goods surveyed by the U.S. Bureau of Labor Statistics and report a median price duration of only 1.66 quarters. Christiano, Eichenbaum, and Evans (2001) find that the average length of price contracts is about two quarters and that of wage contracts is roughly 3.3 quarters.

The parameter estimates imply that the interest elasticity of money is 0.32 and the consumption elasticity of money is 0.65. The former estimate is very close to that of 0.39 reported by Chari, Kehoe, and McGrattan (2000a), but larger than the estimates of 0.10 and 0.11 found by, respectively, Christiano, Eichenbaum, and Evans (2001) and Dib and Phaneuf (2001).

From the estimate of the curvature parameter of the leisure component in the utility function (η_3), we can compute an estimate of the elasticity of labour supply with respect to the real wage (for a given marginal utility of consumption) as $(1-n)/(\eta_3 n) = (1-0.31)/(1.591 \cdot 0.31) = 1.4$ (2.99). (See Appendix A.) This estimate is too imprecise, however, to allow reliable conclusions.

Estimates of the shock processes' autoregressive coefficients indicate that all shocks are very persistent. Very persistent technology and money-demand shocks are also reported by Kim (2000), Ireland (2001), and Dib and Phaneuf (2001). The estimate of ρ^{μ} is higher than values found when money growth is estimated using a univariate process (as in Chari, Kehoe, and McGrattan 2000a).

3.3 Fit and specification tests

This section evaluates the model's goodness of fit, compares it with that of an unrestricted VAR, and performs specification tests on the model's residuals. Figure 1.1 plots the actual and predicted series of U.S. real money stock, output, and the nominal interest rate. This figure indicates that the model tracks the dynamics of these variables very well. A standard measure of the goodness of fit is the R^2, which measures the proportion of the total variation in the dependent variable that is explained by the model. The R^2s for the real money stock, output, and the nominal interest rate are 0.945, 0.948, and 0.893, respectively. Thus, roughly 95 per cent of the total variation of the real money stock and output can be explained by the DSGE model with sticky prices, habit formation, and costly capital adjustment. The model does not explain as well the behaviour of the nominal interest rate, but it still can account for more than 89 per cent of the total variation of this series.

It is instructive to compare the fit of the model with the one of an unrestricted VAR. The VAR is of order one and contains the following U.S. variables: real money stock, output, and the nominal interest rate. The comparison is made in terms of the MSE, defined as[8]

$$MSE = \left(\sum_{t=2}^{T} (X_t - X_t^p)^2 \right) / (T-1),$$

where $T = 166$ is the number of observations, X_t is either output, real money stock, or the rate of nominal interest, and X_t^p is the value predicted by the model. Since the VAR

[8]Note that the state-space and VAR models are non-nested, and that therefore standard likelihood ratio, LM, and Wald tests would not be appropriate.

uses the first observation in the sample to construct the lag, the number of observations used to construct the MSE is $T - 1 = 165$. Table 1.2 reports the MSE from the estimated DSGE model and the VAR. The DSGE model outperforms the VAR when explaining the behaviour of U.S. output and real money stock in that its MSE is smaller. For the nominal interest rate, however, the MSE of the VAR is slightly less than that of the DSGE model.

Table 1.3 reports test results for serial correlation of the residuals (Panel A) and neglected autoregressive conditional heteroscedasticity (ARCH) (Panel B). Consider first the Durbin-Watson test for first-order autocorrelation. Comparing the test statistic with the 5 per cent critical value of its tabulated distribution indicates that (i) one cannot reject the null hypothesis of no autocorrelation for the real money stock and output residuals, but (ii) one can reject it for the nominal interest rate residuals. Similarly, results of Portmanteau tests for the first- to third-order autocorrelations of the residuals yield statistics that are below (above) their 5 per cent critical value for real money stock and output (nominal interest rate).[9]

The LM tests for neglected ARCH were computed as the product of the number of observations and the uncentred R^2 of the OLS regression of the squared residual on a constant and one to three of its lags. Under the null hypothesis of no conditional heteroscedasticity, the statistic is distributed chi-square with as many degrees of freedom as the number of lagged squared residuals included in the regression. Results in Panel B indicate that the null hypothesis of no conditional heteroscedasticity cannot be rejected at the 5 per cent level for output and the real money stock, but that it can be rejected for the nominal interest rate in some cases. All these results indicate that the DSGE model tracks well the behaviour of U.S. output and real money stock, but that it is somewhat less successful in explaining the nominal interest rate.

[9] Under the null hypothesis of no autocorrelation, the Portmanteau test statistic is distributed chi-square with as many degrees of freedom as autocorrelations are tested for.

The DSGE model also generates predictions regarding series whose actual data were not used in the estimation procedure; for example, consumption, investment, the rate of inflation, and the real marginal cost. The real marginal cost is not directly observable, but under certain conditions it can be proxied by the labour share in national income (see Galí and Gertler 1999 for a detailed discussion). Figure 1.2 plots the actual and predicted series of U.S. consumption, investment, inflation, and real marginal cost. The figure shows that the model generates consumption and investment dynamics that are similar to the ones of their detrended U.S. counterparts. Predicted investment, however, is much smoother than the data.

The DSGE model does poorly in explaining the behaviour of the real marginal cost and inflation. This result reflects a drawback of inflation models based on forward-looking pricing rules. It is possible to show that, under Calvo-type pricing, the inflation deviation from steady state equals the present discounted value of current and future expected real marginal cost deviations from steady state.[10] This means that inflation inherits the dynamic properties of the real marginal cost and that current inflation is not helpful in predicting future inflation. Because lagged inflation is absent from the inflation equation, forward-looking pricing rules imply that inflation is less persistent than usually found in the data. To address this shortcoming of the model, some authors (for example, Galí and Gertler 1999) assume the existence of rule-of-thumb firms that fix their prices as a function of past inflation. Another problem with our model is that the real marginal cost is more volatile than the labour share in national income would suggest. One possibility is that the labour share in national income is a poor empirical proxy for the real marginal cost. More likely, the real marginal cost in our model is excessively volatile because it abstracts from supply-side features like variable capital utilization and adjustment costs to labour input.

[10]This result can be easily derived by rewriting equation (A.1) in Appendix A with current inflation in the left-hand side and iterating forward.

3.4 Impulse-response analysis

This section examines the response of the economy to a shock to the money-supply growth rate, hereafter called a money-supply shock. Our intention is to (i) assess the ability of the model to match the persistent output effect of monetary policy shocks documented in the VAR literature, and (ii) investigate the role of habit formation and costly capital adjustment in solving the persistence problem. We compare the impulse-response functions calculated using the estimated parameters with those obtained using two polar, counterfactual versions of the model. The first version assumes adjustment costs of capital but no habit formation. The second version assumes habit formation but no adjustment costs of capital.

Figure 1.3 plots the impulse responses of output, investment, consumption, labour, inflation, and the nominal interest rate to a 1 per cent money-supply shock. Following the shock, there is an increase in aggregate demand that causes output and consumption to increase. The consumption response is hump-shaped because, under habit formation, agents smooth both the level and the change of consumption. The output response is also hump-shaped, as in previous VAR literature. The peak of the output (consumption) response takes place after two (four) quarters, rather than the four to six quarters usually found in VAR models.

Figure 1.3 shows that the dynamic path of output is quite persistent. As a measure of the endogenous persistence of output generated by the model, we compute the proportion of the impact effect that persists beyond the average length of price contracts. Recall that the estimated probability of price changes implies an average duration of price contracts of 6.56 quarters. Thus, the measure of endogenous persistence is

$$\zeta \equiv \kappa(7)/\kappa(0),$$

where $\kappa(j)$ is the impulse-response coefficient at lag j.[11] In this case, $\zeta = 0.95$, meaning

[11] Of course, this measure of persistence applies only if $\kappa(0)$ is different from 0. This condition is satisfied

that 95 per cent of the initial effect of the monetary shock on output persists beyond seven quarters. This indicates that the estimated model produces a substantial amount of endogenous persistence.

Figure 1.3 also shows that investment and labour increase following a (positive) monetary shock. This result is due to the fact that aggregate demand is expected to increase in subsequent periods because prices adjust slowly. The nominal interest rate also rises after a positive monetary shock. Thus, the model does not generate a liquidity effect. A more detailed explanation of this result is presented below.

Figure 1.4 plots the impulse responses generated from a model with price stickiness and adjustment costs to capital but no habit formation. The parameter γ is set to zero and the remaining parameters are set to their ML estimates. In contrast to the previous model, output and consumption responses are not hump-shaped. Both variables jump immediately after the monetary shock and return gradually to their steady-state levels. The output response is less persistent than the one in Figure 1.3. Since $\zeta = 0.30$, this version of the model with no habit formation delivers only 30 per cent of endogenous persistence. This suggests that habit formation might be important in explaining the persistence of output in response to monetary shocks.

Figure 1.5 shows the impulse responses corresponding to a model with price stickiness and habit formation but no adjustment costs to capital. The parameter χ is set to zero and the remaining parameters are set to their ML estimates. A positive monetary shock triggers a large initial increase in output, investment, hours worked, and inflation, but the variables drop sharply in the following period and return close to their steady-state levels. The output response is caused by the fact that investment must increase to accommodate the upward shift in future demand. Because capital is free to adjust, however, all the

in our case since output is a non-predetermined variable in our model.

required increase in investment takes place immediately after the shock. This version of the model does not generate any significant amount of endogenous persistence: $\zeta = 0.03$, meaning that only 3 per cent of the initial effect of the monetary shock persists after seven quarters. Kim (2000) and Dib and Phaneuf (2001) report a similar dynamic path of output using models with price stickiness only. This suggests that habit formation alone does not solve the persistence problem. Instead, habit formation plays the role of a catalyst that, combined with additional features, helps to spread out the effects of monetary shocks. In this model, the additional feature is the adjustment costs of capital.

The increase in the nominal interest rate following a positive monetary shock is larger in Figure 1.4 than in Figure 1.5. This result is consistent with Kim's (2000) finding that real rigidities help to generate a liquidity effect in DSGE models. As Figure 1.3 shows, however, adjustment costs to the capital stock are not enough to generate a liquidity effect in this model. The reason is that the estimated money-growth process is highly autocorrelated. Thus, after a positive money-supply shock, expected inflation increases by a magnitude that is larger in absolute value than the decrease in the real interest rate. As a result, the net effect of the money shock on the nominal interest rate is positive.

In summary, impulse-response analysis indicates that both habit formation and adjustment costs to capital are likely to be important features in a model that seeks to explain the persistent output response to monetary policy shocks. To further understand the relationship between endogenous output persistence and the parameters that control habit formation and capital adjustment costs, the persistence measure, ζ, is computed for different combinations of the parameters γ and χ. Figure 1.6 plots the resulting three-dimensional graph. In this figure, γ varies from 0 to 1, and χ varies between 0 and 100. The figure shows that habit formation increases the output persistence of monetary shocks only to the extent that capital adjustment costs are not in the neighbourhood of zero. The increase in persistence is bounded at fairly low levels unless γ is sufficiently large. Hence, habit formation and adjustment costs of capital interact in a non-linear way to increase the output persistence of monetary policy shocks. This finding parallels the one in Bergin and Feenstra (2000), where the non-linear interaction of materials inputs and translog preferences increases endogenous output persistence.

Although we are primarily concerned with the effects of monetary policy shocks, the estimated model generates predictions regarding the effect of technology and money-demand shocks. Figure 1.7 plots the response of output, investment, consumption, labour, inflation, and the nominal interest rate to a 1 per cent technology shock. Because prices are rigid, the aggregate supply curve is upward-sloping. A positive technology shock shifts the aggregate supply curve to the right. Consequently, output increases and prices decrease. The response of output and consumption is persistent and hump-shaped. Hours worked decrease in a persistent manner following a technology shock. The intuition of this result is as follows. After a positive technology shock, the firm is able to satisfy current demand with a lower level of inputs, so labour input will decrease on impact. Eventually, as demand increases and capital is adjusted, labour demand increases. A similar decline of labour in response to a technology shock is reported by Galí (1999) using a structural VAR, and by Dib and Phaneuf (2001) and Vigfusson (2002) using DSGE models.

Figure 1.8 plots the impulse response functions generated by a 1 per cent money-demand shock. Because money supply is unchanged and prices are rigid, this shock produces a downward shift of aggregate demand in current and subsequent periods. Consequently, output, consumption, labour, and investment decrease. As a result of habit formation, the response of consumption has an inverted hump shape, with a trough around three periods after the shock.

3.5 Variance decomposition

In this section, we study the relative importance of monetary shocks for the fluctuations of output, investment, consumption, labour, inflation, and the nominal interest rate. To that effect, we compute the fraction of the conditional variance of the forecasts at different horizons that is attributed to each of the model's shocks. This variance decomposition is plotted in Figure 1.9. As the horizon increases, the conditional variance of the forecast error of a given variable converges to the unconditional variance of that variable. Table 1.4 reports the decomposition of the unconditional variances. Recall that a money-supply shock is a shock to the growth rate of the money supply, while a money-demand shock is a shock to the preference parameter of money in the utility function. Several results are apparent from Figure 1.9 and Table 1.4. First, money-demand shocks play an important role in explaining the fluctuations of the nominal interest rate. At horizons of less than six quarters, money-demand shocks explain more than 50 per cent of the conditional variance of the nominal interest rate. In the long run, money-demand shocks explain roughly 45 per cent of the conditional variance of the nominal interest rate. Second, money-supply shocks explain most of the fluctuations of the rate of inflation at all horizons. Third, technology shocks explain most of the variation in hours worked at all horizons. Fourth, money-supply shocks account for the largest part of the conditional variance in forecasting investment in the short run. As the horizon increases, the contribution of technology shocks increases and that of money-supply shocks decreases, but, even in the long run, money-supply shocks explain half of the variance of investment. Fifth, money-supply shocks and technology shocks are equally important in explaining the conditional variance of consumption in the very short run. As the horizon increases, however, the contribution of technology shocks increases and that of money-supply shocks decreases. In the long run, 77.3 per cent of the variance of consumption is explained by technology shocks and only 21.6 per cent by money-supply shocks. Finally, money-supply shocks account for the largest part of the conditional variance in forecasting output in the short run (i.e., less than a year). At higher horizons, most of the conditional variance is due to technology shocks. In the long run, 27 per cent of the unconditional variance of output is attributed to money-supply shocks, 2 per cent to money-demand shocks, and 71 per cent to technology shocks.

4. Conclusion

This paper has constructed a DSGE model with sticky prices, habit formation, and costly capital adjustment that accounts for the persistent and hump-shaped response of output to monetary shocks. Although habit formation, by itself, does not solve the persistence problem, it interacts non-linearly with costly capital adjustment to increase the internal propagation mechanism of the model.

The model was estimated by the ML method using U.S. data on output, the real money stock, and the nominal interest rate. Econometric results indicate that U.S. prices are fixed, on average, for six-and-a-half quarters. Although the peak of the output response takes place after two quarters (that is, less than the four to six quarters usually found in VAR models), up to 95 per cent of the initial effect of a monetary shock on output persists beyond the average duration of price contracts. Variance decomposition indicates that money growth explains more than 50 per cent of the (conditional) output variability at horizons of less than one year. In the long run, money growth explains only 27.1 per cent of the unconditional output variability, while 71.4 per cent is explained by technology shocks.

The DSGE fits U.S. output and real money stock better than an unrestricted VAR and does only slightly worse for the nominal interest rate. The model also tracks well the behaviour of consumption and investment, but it does poorly in explaining the U.S. inflation rate. This is partly the result of the forward-looking pricing rule and the prediction that the real marginal cost is much more volatile than the data. The inclusion of additional features would allow DSGE models to capture other stylized facts, such as inflation persistence and, perhaps, the liquidity effect. In future work, we intend to extend this model to allow for a backward-looking component in the price rule that arises directly from first principles.

Appendix A: The Log-Linearized Model

In this appendix, variables without time subscripts denote steady-state values, and the circumflex denotes percentage deviation from steady state. For example, $\hat{x}_t = (x_t - x)/x$ is the percentage deviation of investment from its steady state at time t. Linearizing (2) and the first-order conditions (6)-(10) yields

$$\begin{aligned}
\hat{k}_{t+1} &= (1-\delta)\hat{k}_t + \delta\hat{x}_t, \\
E_t\hat{c}_{t+1} &= \left(\frac{\beta\gamma\left(\gamma\left(1-\eta_1\right)+1\right)-\eta_1}{\beta\gamma\left(1-\eta_1\right)}\right)\hat{c}_t - \left(\frac{1}{\beta}\right)\hat{c}_{t-1} + \left(\frac{\beta\gamma-1}{\beta\gamma\left(1-\eta_1\right)}\right)\hat{\lambda}_t, \\
\hat{R}_t &= \eta_2\left(\frac{\pi-\beta}{\beta}\right)(\hat{\pi}_t - \hat{m}_{t-1} - \hat{\mu}_t) - \left(\frac{\pi-\beta}{\beta}\right)\hat{\lambda}_t + \left(\frac{\pi-\beta}{\beta}\right)\hat{b}_t, \\
\hat{n}_t &= \left(\frac{1-n}{n\eta_3}\right)\hat{\lambda}_t + \left(\frac{1-n}{n\eta_3}\right)\hat{w}_t, \\
E_t\hat{q}_{t+1} &= \left(\frac{1}{\beta q}\right)(\hat{\lambda}_t - E_t\hat{\lambda}_{t+1}) + \left(\frac{\chi(1+\beta\delta)}{\beta q}\right)\hat{k}_{t+1} - \left(\frac{\chi}{\beta q}\right)\hat{k}_t - \left(\frac{\chi\delta}{q}\right)\hat{x}_{t+1}, \\
E_t\hat{\lambda}_{t+1} &= \hat{\lambda}_t + E_t\hat{\pi}_{t+1} - \hat{R}_t.
\end{aligned}$$

The production function (14) and first-order conditions for cost minimization by the intermediate-good producer (equations (15) and (16)) become

$$\begin{aligned}
\hat{y}_t &= \alpha\hat{k}_t + (1-\alpha)\,\hat{n}_t + \hat{z}_t, \\
\hat{w}_t &= \hat{\phi}_t + \hat{y}_t - \hat{n}_t, \\
\hat{\phi}_t &= \hat{q}_t - \hat{y}_t + \hat{k}_t.
\end{aligned}$$

The linearized versions of equations (17) and (18), together, imply

$$E_t\hat{\pi}_{t+1} = \frac{1}{\beta}\hat{\pi}_t - \left(\frac{(1-\varphi)\,(1-\varphi\beta)}{\varphi\beta}\right)\hat{\phi}_t. \qquad \text{(A.1)}$$

Linearizing the equation that defines money growth (20) and the market-clearing condition

(25) yields

$$\begin{aligned} \hat{m}_t &= \hat{m}_{t-1} - \hat{\pi}_t + \hat{\mu}_t, \\ \hat{x}_t &= \left(\frac{y}{\delta k}\right)\hat{y}_t - \left(\frac{c}{\delta k}\right)\hat{c}_t. \end{aligned}$$

Finally, the stochastic processes of the shocks (equations (21)-(23)) are linearized as

$$\begin{aligned} \hat{z}_{t+1} &= \rho^z \hat{z}_t + \epsilon_{z,t}, \\ \hat{\mu}_{t+1} &= \rho^\mu \hat{\mu}_t + \epsilon_{\mu,t}, \\ \hat{b}_{t+1} &= \rho^b \hat{b}_t + \epsilon_{b,t}. \end{aligned}$$

Appendix B: Derivation of the Expression for n

From equation (2) we have

$$x = \delta k, \tag{B.1}$$

which means that there are no adjustment costs of capital in the steady state (see eq. (3)). Therefore, equation (9) implies

$$q = \frac{1}{\beta} - 1 + \delta. \tag{B.2}$$

Prices are assumed to be perfectly flexible in the steady state. This means that the probability of not changing prices, φ, is equal to 0. Hence, equation (17) becomes

$$P = \left(\frac{\theta}{\theta - 1}\right)\Phi. \tag{B.3}$$

It follows that the real marginal cost is given by

$$\phi = \frac{\theta - 1}{\theta}. \tag{B.4}$$

Substituting (B.2) and (B.4) into equation (16), yields

$$\frac{y}{k} = \frac{\theta\,(1/\beta - 1 + \delta)}{\alpha(\theta - 1)}. \tag{B.5}$$

Dividing both sides of equation (14) by k gives

$$\frac{k}{n} = z^{1/(\alpha-1)}\left(\frac{y}{k}\right)^{1/(\alpha-1)}. \tag{B.6}$$

Using equation (B.6), it is easy to show that

$$\frac{y}{n} = z^{1/(\alpha-1)}\left(\frac{y}{k}\right)^{\alpha/(\alpha-1)}. \tag{B.7}$$

Substituting this expression into equation (15) results in

$$w = \left(\frac{(1-\alpha)\,(\theta-1)}{\theta}\right) z^{1/(\alpha-1)}\left(\frac{y}{k}\right)^{\alpha/(\alpha-1)}. \tag{B.8}$$

From equation (6), we have

$$\lambda = (1-\beta\gamma)c^{(2\gamma-1)\eta_1}. \tag{B.9}$$

Substituting equation (B.1) into equation (25) and dividing both sides by k gives:

$$\frac{c}{k} = \frac{y}{k} - \delta. \tag{B.10}$$

Using (B.8) and (B.9) to substitute for w and λ in equation (8) yields

$$\begin{aligned}(1-n)^{-\eta_3} &= \left(\frac{(1-\alpha)\,(\theta-1)(1-\beta\gamma)}{\psi\theta}\right) z^{1/(\alpha-1)} \left(\frac{y}{k}\right)^{\alpha/(\alpha-1)} c^{(2\gamma-1)\eta_1} \\ &= \left(\frac{(1-\alpha)\,(\theta-1)(1-\beta\gamma)}{\psi\theta}\right) z^{1/(\alpha-1)} \left(\frac{y}{k}\right)^{\alpha/(\alpha-1)} \left(n\frac{c}{k}\frac{k}{n}\right)^{(2\gamma-1)\eta_1}.\end{aligned}$$

Using (B.6) and (B.10) to substitute for k/n and c/k, we obtain

$$n^{(1-2\gamma)\eta_1}(1-n)^{-\eta_3} = \left(\frac{(1-\alpha)\,(\theta-1)(1-\beta\gamma)}{\psi\theta}\right) z^{1/(\alpha-1)} \left(\frac{y}{k}\right)^{(\alpha+(2\gamma-1)\eta_1)/(\alpha-1)} \left(\frac{y}{k}-\delta\right)^{(2\gamma-1)\eta_1}.$$

Finally, using (B.1) to substitute for y/k yields the following non-linear equation:

$$n^{(1-2\gamma)\eta_1}(1-n)^{-\eta_3} = \vartheta z^{1/(\alpha-1)}, \tag{B.11}$$

where $\vartheta \equiv \left(\frac{(1-\alpha)(\theta-1)(1-\beta\gamma)}{\psi\theta}\right)\left(\frac{\theta(1/\beta-1+\delta)}{\alpha(\theta-1)}\right)^{(\alpha+(2\gamma-1)\eta_1)/(\alpha-1)}\left(\frac{\theta(1/\beta-1+\delta)}{\alpha(\theta-1)}-\delta\right)^{(2\gamma-1)\eta_1}$.

Table 1.1: Maximum-Likelihood Estimates

Description	Parameter	Estimate	Asymptotic S.E.	Robust S.E.
Habit-formation parameter	γ	0.982	0.016	0.021
Probability of no price change	φ	0.847	0.037	0.023
Adjustment cost	χ	85.188	20.728	29.402
Preference parameter	η_2	3.089	0.827	1.462
Preference parameter	η_3	1.591	3.530	3.732
AR coefficient of technology shock	ρ^z	0.867	0.055	0.055
AR coefficient of money-supply shock	ρ^μ	0.879	0.035	0.053
AR coefficient of money-demand shock	ρ^b	0.924	0.019	0.029
S.D. of technology shock	σ_{ϵ_z}	0.040	0.027	0.032
S.D. of money-supply shock	σ_{ϵ_μ}	0.007	0.002	0.003
S.D. of money-demand shock	σ_{ϵ_b}	0.077	0.005	0.008

Notes: S.D. and S.E. are standard deviation and standard error, respectively. The restrictions imposed on the parameters are γ, $\varphi \in (0,1)$, ρ^z, ρ^μ, $\rho^b \in (-1,1)$, and η_2, η_3, χ, σ_{ϵ_z}, σ_{ϵ_μ}, $\sigma_{\epsilon_b} \in (0,\infty)$.

Table 1.2: Goodness of Fit

Variable	MSE ($\times 10^{-5}$)	
	DSGE model	Unrestricted VAR(1)
Output	6.073	6.562
Real money stock	4.599	7.841
Nominal interest rate	0.416	0.381

Notes: MSE is mean squared error.

Table 1.3: Test for Serial Correlation and Neglected ARCH

	Output	Real money stock	Nominal interest rate
Panel A. Test for Serial Correlation			
Durbin-Watson	2.15	1.98	1.50
Portmanteau			
One autocorrelation	1.11	0.002	9.38*
Up to two autocorrelations	1.11	1.86	14.27*
Up to three autocorrelations	2.21	4.18	17.32*
Panel B. LM Test for Neglected ARCH			
Number of squared lags			
One	0.90	3.62	1.82
Two	1.48	3.63	26.77*
Three	1.63	3.74	28.88*

Notes: The superscript * denotes the rejection of the null hypothesis that the parameter is zero at the 5 per cent significance level.

Table 1.4: Unconditional Variance Decomposition

	Fraction of the unconditional variance due to		
Variable	Technology shocks	Money-supply shocks	Money-demand shocks
Output	0.714	0.271	0.015
Investment	0.469	0.493	0.038
Consumption	0.773	0.216	0.011
Labour	0.872	0.120	0.008
Inflation rate	0.221	0.756	0.023
Nominal interest rate	0.163	0.389	0.448

Notes: The money-supply shock is a shock to the growth rate of the money supply. The money-demand shock is a shock to the preference parameter of money in the utility function.

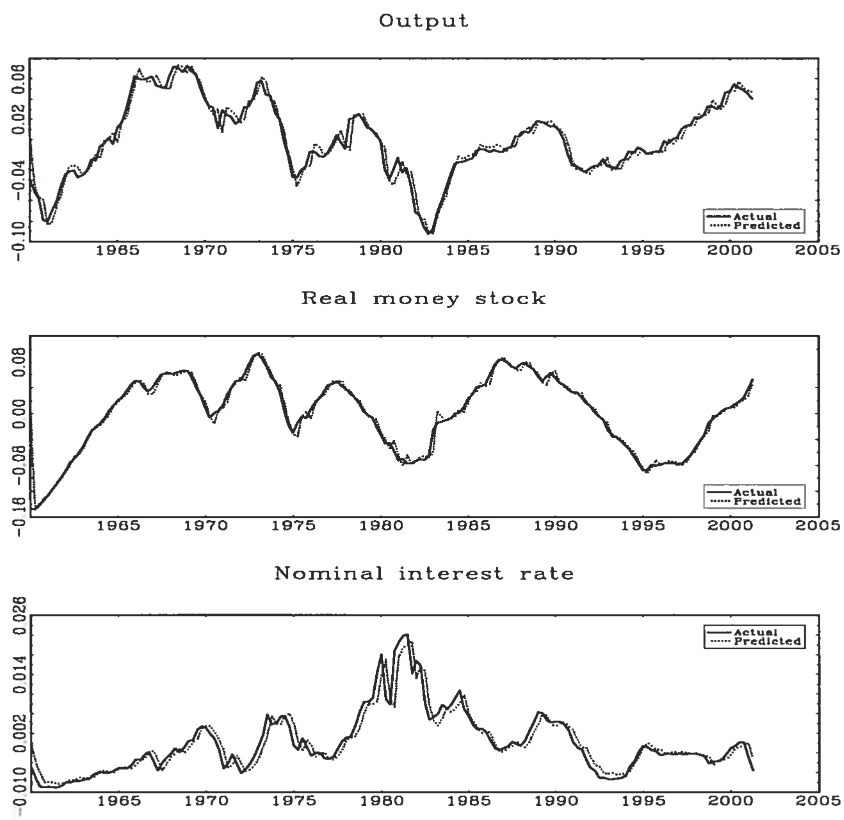

Figure 1.1: Actual and predicted values of variables in the measurement equation

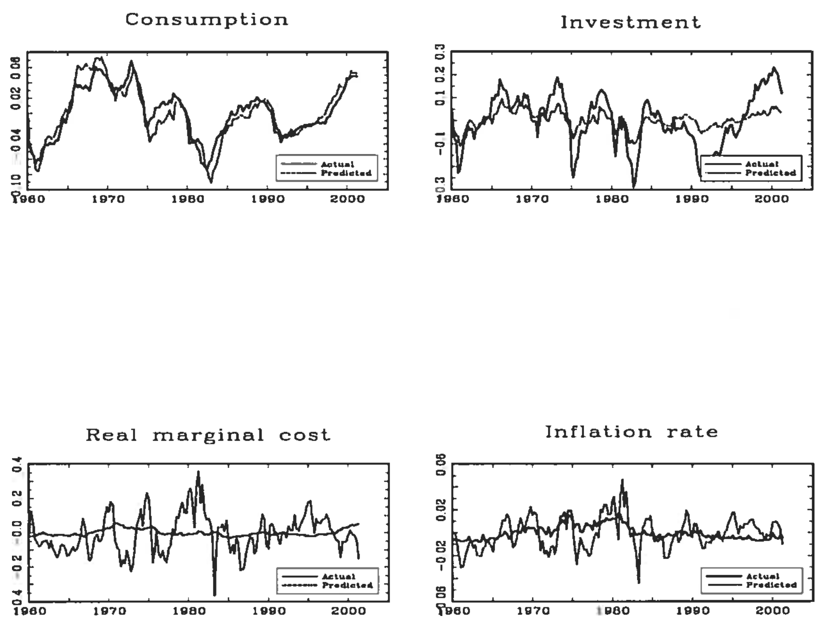

Figure 1.2: Actual and predicted values of other model variables

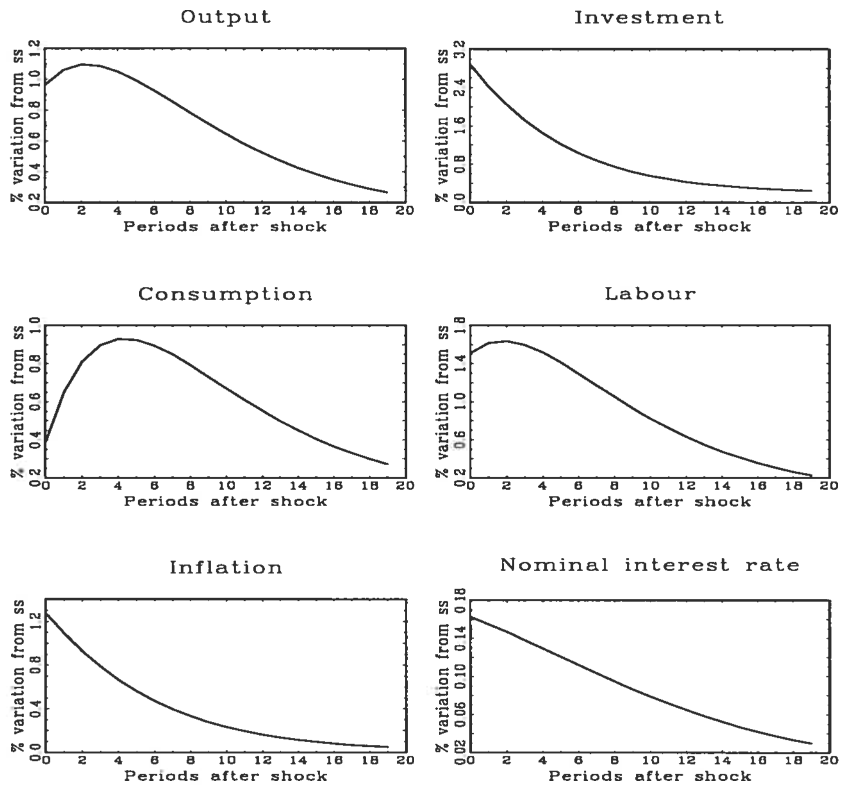

Figure 1.3: Impulse responses to a 1 per cent money-supply shock

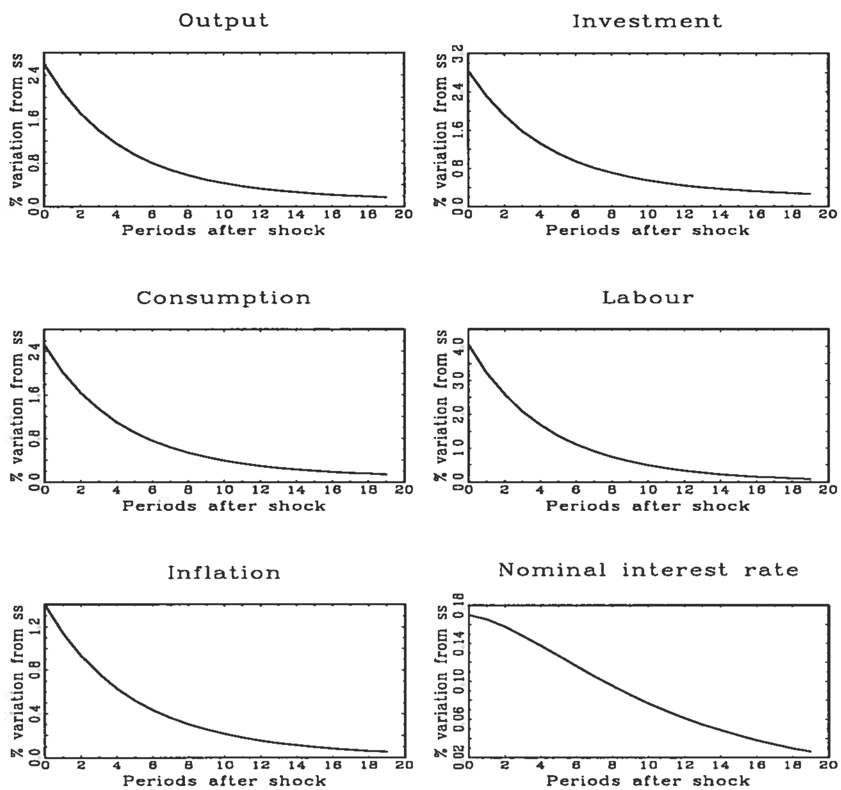

Figure 1.4: Impulse responses to a 1 per cent money-supply shock ($\gamma = 0$)

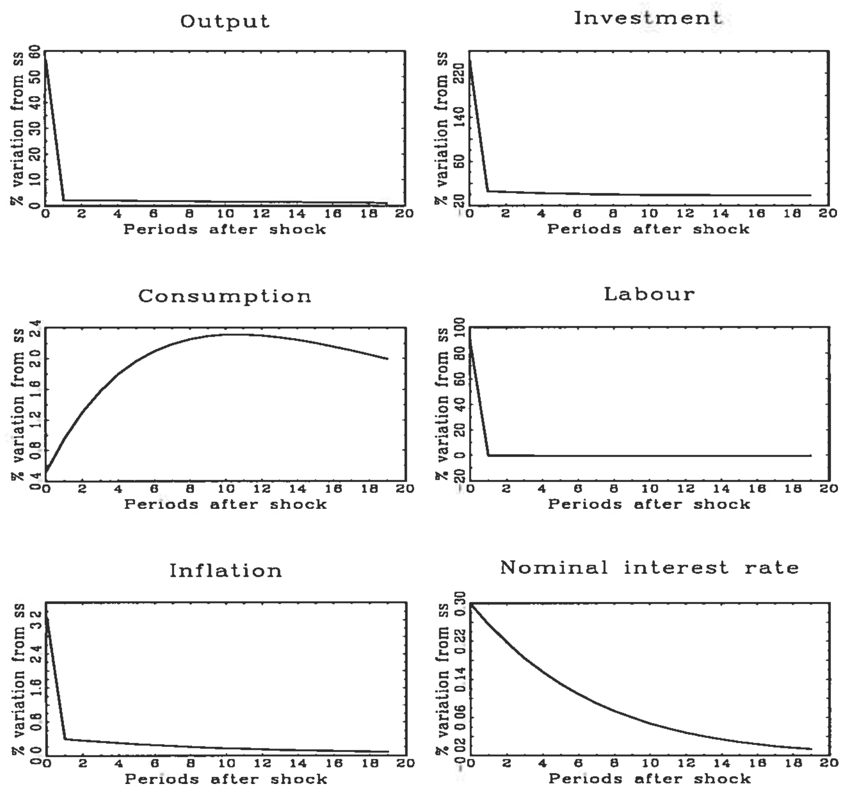

Figure 1.5: Impulse responses to a 1 per cent money-supply shock ($\chi = 0$)

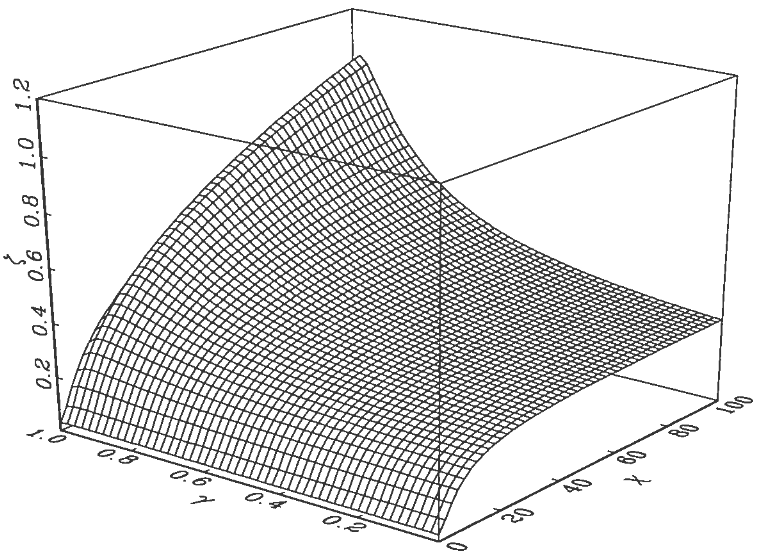

Figure 1.6: Endogenous persistence as a function of γ and χ

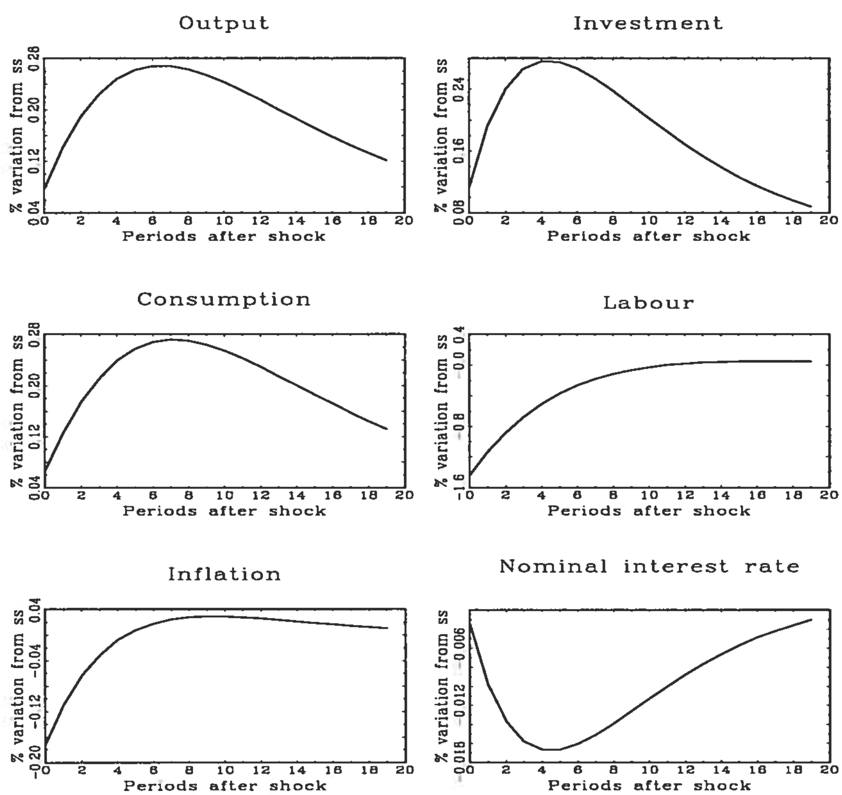

Figure 1.7: Impulse responses to a 1 per cent technology shock

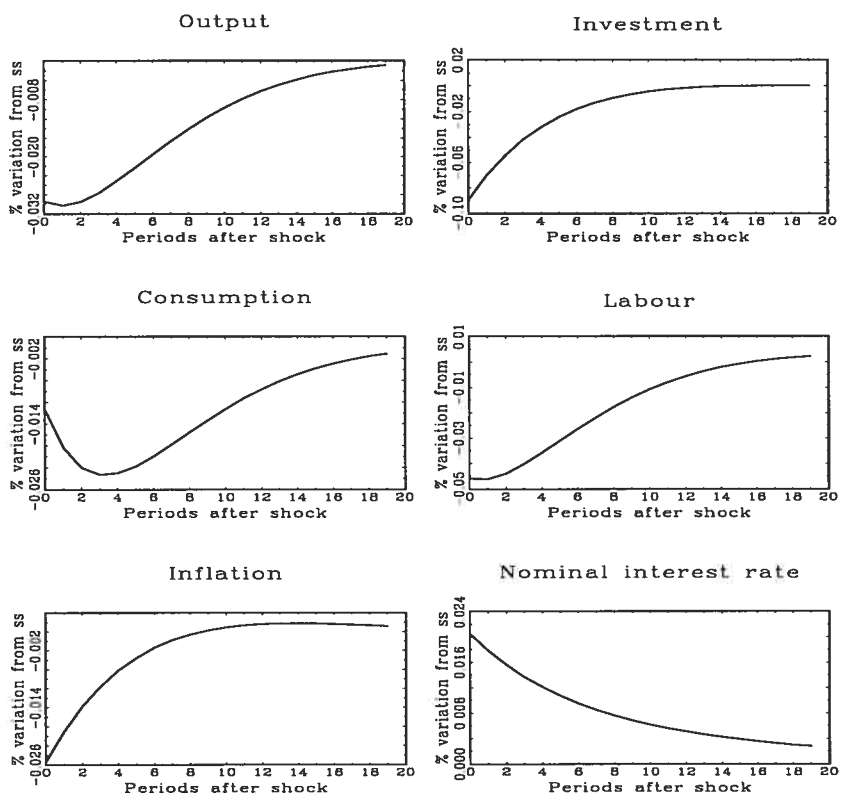

Figure 1.8: Impulse responses to a 1 per cent money-demand shock

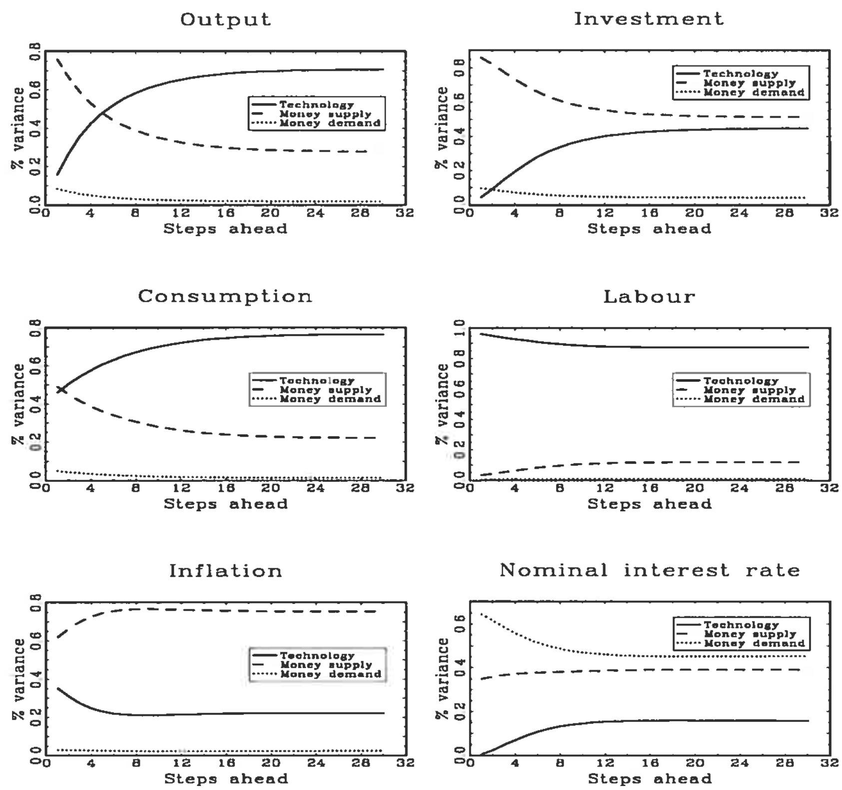

Figure 1.9: Variance decomposition

Essay 2

Habit Formation, Pricing-to-Market, and Real Exchange Rate Dynamics

1. Introduction

Real exchange rate fluctuations are a central issue in international macroeconomics. Aside from the theoretical interest, understanding the sources of these fluctuations can be useful in conducting macroeconomic policies. Today, there is substantial evidence that, in the short run, real exchange rates are highly volatile and persistent. The traditional explanation for short-term real exchange rate volatility emphasizes the interaction of monetary shocks and sluggish adjustment of nominal prices. This explanation is put forth mainly by Dornbusch (1976) who shows that, in a sticky-price setting, nominal exchange rates overshoot in response to a monetary shock, rising more in the short run than in the long run. Owing to price rigidity, nominal exchange rate volatility is inherited by the real exchange rate. This argument is very appealing but it was originally developed in a Keynesian partial-equilibrium framework that lacks microfoundations. In recent years, the literature on exchange rate determination focused on reformulating Dornbusch's argument into dynamic general-equilibrium sticky-price models. These models incorporate the realistic feature of price rigidity in a rigorous intertemporal optimizing framework. Early attempts to construct such models include Svensson and Van Wijnbergen (1989) and Stockman and Ohanian (1993). But perhaps the most influential contribution is made by to Obstfeld and Rogoff (1995) who develop a dynamic two-country sticky-price model that permits the study of exchange rate dynamics as well as international policy transmission. However, the basic version of their model is not appropriate to examine real exchange rate dynamics since all goods are tradable and the law of one price applies continuously so that purchasing power parity holds (i.e., the real exchange rate is always equal to unity). To generate real exchange rate fluctuations, they present an extended version of the model with nontradable goods. Because the law of one price does not hold for nontradables, purchasing power parity breaks down.

An alternative way to motivate departures from purchasing power parity is to assume that the law of one price does not hold even for tradable goods.[1] Typically, this is achieved by assuming the presence of pricing-to-market (PTM) in traded goods. PTM is the price discrimination by a monopolistic firm across segmented national markets. This segmentation is possible only if there are impediments to goods-market arbitrage.[2] The extension of Obstfeld and Rogoff's model to allow for PTM was first developed by Betts and Devereux (1996). In their study, as well as in that by Obstfeld and Rogoff, a closed-form solution of the model is made possible by a simple specification of nominal rigidity which assumes that prices are held fixed for only one period. But it is clear that this kind of rigidity is not capable of generating persistent effects of monetary shocks on real variables. For this reason, quantitative studies that build on these papers allow for a more staggered price setting. Most of these studies rationalize the violation of purchasing power parity via PTM.[3] Examples include Chari, Kehoe, and McGrattan (1997, 2000b), Bergin and Feenstra (2001) and Kollmann (2001).

Overall, these studies have been rather successful in replicating real exchange rate volatility. In particular, Chari, Kehoe, and McGrattan (2000b) show that, provided risk aversion is high, it is possible to match exactly the historical volatility of real exchange rates with a reasonable duration of price contracts. However, all of the aforementioned studies invariably fail to generate the observed persistence of real exchange rates, unless an extremely unrealistic amount of nominal rigidity is assumed.[4]

Typically, in two-country general-equilibrium models, first-order conditions link the real exchange rate to the ratio of marginal utilities of consumption in the two countries. This

[1] Engle (1993) and Engle and Rogers (1996) find strong empirical evidence against the law of one price.

[2] Empirical studies of pricing-to-market include Marston (1990) and Knetter (1989, 1993).

[3] To the best of my knowledge, Jung (2000) is the only quantitative work based on Obstfeld and Rogoff (1995) that motivates departures from purchasing power parity through the presence of nontradable goods.

[4] Assuming an average length of price contracts of one year, Chari, Kehoe, and McGrattan (2000b) could generate only about 75% of the historical autocorrelation of the real exchange rate.

led Chari, Kehoe, and McGrattan (2000b) to conclude regarding the failure of their model to generate enough persistence in real exchange rates:

> "...this is primarily because the model does not generate enough persistence in consumption. One avenue to generate more persistence in consumption is to include habit persistence or consumption durability in preferences."

It is well known that habit formation leads to a sluggish adjustment of consumption to shocks. This property has been useful in explaining several puzzles in the literature. For example, Abel (1990), Constantinides (1990) and Campbell and Cochrane (1999), among others, find that habit formation helps to solve the equity premium puzzle. Carrol, Overland and Weil (2000) include habit formation in a standard growth model and show that this feature can account for the documented growth-to-saving causation. Fuhrer (2000) demonstrates that habit-forming preferences are relevant in explaining the excess smoothness of consumption and inflation inertia. Finally, Uribe (1997) uses habit formation to account for consumption dynamics during exchange-rate-based stabilization programs.

In this paper, I investigate Chari, Kehoe, and McGrattan's (2000b) conjecture that habit formation potentially increases real exchange rate persistence. For this purpose, I extend the two-country model of Betts and Devereux (2000) by assuming habit-forming preferences. The model is also amended by introducing a Calvo-type price setting.

Numerical simulations show that habit formation is completely irrelevant to real exchange rate dynamics. This result is also established analytically within a version of the model that assumes complete financial markets. The robustness of the result to the structure of asset markets is due to the smallness of wealth effects in the model.

The rest of the paper is organized as follows. Section 2 presents the model and derives the equilibrium conditions. Section 3 describes the calibration of the model's parameters. Simulation results are reported and discussed in section 4. Section 5 concludes.

2. The Model

It is assumed that the world is populated by a continuum of households, indexed by $i \in [0,1]$. A fraction n of them live in the home country and the remaining $1-n$ live in the foreign country. Foreign-country variables are denoted by an asterisk. There is also a continuum of monopolistic firms distributed similarly to households between the two countries. Each monopolistic firm exclusively produces and sells a differentiated good. Firms and goods will also be indexed by $i \in [0,1]$. In Betts and Devereux (2000), PTM is introduced by assuming that the discriminating firms set their prices in the currency of the seller. The proportion of these firms is the same in both countries. Here, however, this proportion is allowed to differ across the two countries. The fraction of firms in the home country that are able to discriminate between the domestic and the foreign markets is denoted by α. The corresponding fraction in the foreign country is α^*. These firms will be referred to as PTM firms. The remaining firms (called non-PTM firms) sell their goods without market segmentation so that the law of one price holds continuously for these goods.

2.1 Households

The representative household in the home country maximizes

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u_s,$$

where E_t denotes the mathematical expectation conditional on the information available up to and including period t, β is the subjective discount factor $(0 < \beta < 1)$ and u is the instantaneous utility function. Households value real consumption relative to a habit stock determined by their own past consumption. This feature is referred to as internal habit formation. They also derive utility from holding real money balances and from leisure. The instantaneous utility function is assumed to be

$$u_t(C_t, H_t, M_t/P_t, L_t) = \frac{1}{1-\varepsilon_1}\left(\frac{C_t}{H_t^{\gamma}}\right)^{1-\varepsilon_1} + \frac{\chi_t}{1-\varepsilon_2}\left(\frac{M_t}{P_t}\right)^{1-\varepsilon_2} + \frac{\psi}{1-\varepsilon_3}(1-L_t)^{1-\varepsilon_3}, \quad (1)$$

where C_t is consumption, H_t is the stock of habit, M_t is nominal money holding, P_t is the aggregate price level, L_t is the number of hours worked, χ_t is a preference shock, $\psi > 0$ measures the weight of leisure in the utility function, and $\varepsilon_1, \varepsilon_2$, and ε_3 are positive parameters different from one.[5] The parameter $\gamma \in [0,1]$ measures the importance of habit formation in consumers' behaviour. If $\gamma = 0$, then the utility function reduces to the one used by Betts and Devereux (2000). In this case, there is no habit formation and consumers care only about the absolute level of consumption. The habit stock H_t evolves according to the following equation of motion:

$$H_t = \rho H_{t-1} + (1-\rho)C_{t-1}, \quad (2)$$

where the parameter $\rho \in [0,1[$ measures the persistence of habits. If $\rho = 0$, then only last period's consumption enters the current utility function. However, for strictly positive values of ρ, all past consumption levels pile up to form the habit stock.

The argument C_t in (1) is a consumption index given by

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \quad (3)$$

where $c(i)$ is the home resident's consumption of good i and $\theta > 1$ is the elasticity of substitution between goods. Following Betts and Devereux (2000), I let p denote home-currency prices of home produced goods, p^* denote home-currency prices of foreign PTM goods and q^* denote foreign-currency prices of foreign non-PTM goods. The consumption-based price index associated with (3) is given by:

$$P_t = \left[\int_0^n p_t(i)^{1-\theta} di + \int_n^{n+(1-n)\alpha^*} p_t^*(i)^{1-\theta} di + \int_{n+(1-n)\alpha^*}^1 \left(e_t q_t^*(i)\right)^{1-\theta} di\right]^{\frac{1}{1-\theta}}, \quad (4)$$

[5]The total endowment of time is normalized to unity so that leisure is equal to $1 - L$.

where e_t is the nominal exchange rate defined as the price of one unit of the foreign currency in terms of the home currency (an increase in e means a depreciation of the home currency). Home resident's demand for good i, $c_t(i)$ is given by

$$c_t(i) = \left(\frac{\upsilon_t\,(i)}{P_t}\right)^{-\theta} C_t, \tag{5}$$

where $\upsilon_t\,(i) = p_t(i)$, $p_t^*(i)$ or $e_t q_t^*(i)$ respectively for $i \in [0, n]$, $i \in]n, n + (1-n)\alpha^*]$ and $i \in]n + (1-n)\alpha^*, 1]$. Observing that

$$\int_0^n p_t(i)c_t(i)di + \int_n^{n+(1-n)\alpha^*} p_t^*(i)c_t(i)di + \int_{n+(1-n)\alpha^*}^1 e_t q_t^*(i)c_t(i)di = P_t C_t,$$

allows me to express the household's budget constraint in terms of the composite good only.

The only tradable assets are one-period non-state-contingent nominal bonds denominated in home country's currency. Foreign money cannot be held by home country residents and vice versa. Only bonds are interest bearing. The nominal interest rate on bonds due at $t+1$ is denoted by i_t. Bond holding entails quadratic costs, paid to the government. During period t, the household pays a lump-sum tax, T_t, to the government; sells $L_t(i)$ units of labour to each firm $i \in [0, n]$ at the nominal wage, W_t; and receives nominal dividends $\pi_t(i)$ from each firm $i \in [0, n]$. The household allocates some of its income to consumption and carries the remaining units of money into period $t+1$. Hence, its intertemporal budget constraint is

$$B_t + \frac{\varsigma}{2}\left(\frac{B_t^2}{P_t}\right) + M_t = (1 + i_{t-1})B_{t-1} + M_{t-1} + \pi_t + W_t L_t - P_t C_t - T_t, \tag{6}$$

where B_t and M_t denote, respectively, the stocks of bonds and domestic money held by a home household entering period $t+1$, $\frac{\varsigma}{2}\left(\frac{B_t^2}{P_t}\right)$ are bond holding costs, $L_t = \int_0^n L_t(i)di$, and $\pi_t = \int_0^n \pi_t(i)di$. The representative household in the home country maximizes its lifetime utility subject to (2), (6), and a no-Ponzi game condition. First-order necessary conditions

(FONC) for this problem are

$$\begin{aligned}
\Lambda_t &= \frac{1}{C_t}\left(\frac{C_t}{H_t^{\gamma}}\right)^{1-\varepsilon_1} + (1-\rho)\,\phi_t, && (7)\\
\beta\rho E_t\phi_{t+1} &= \phi_t + \beta\gamma E_t\frac{1}{H_{t+1}}\left(\frac{C_{t+1}}{H_{t+1}^{\gamma}}\right)^{1-\varepsilon_1}, && (8)\\
\Lambda_t\left(1+\zeta\frac{B_t}{P_t}\right) &= \beta(1+i_t)E_t\frac{P_t\Lambda_{t+1}}{P_{t+1}}, && (9)\\
\chi_t\left(\frac{M_t}{P_t}\right)^{-\varepsilon_2} &= \Lambda_t - \beta E_t\frac{P_t\Lambda_{t+1}}{P_{t+1}}, && (10)\\
L_t &= 1-\left(\frac{\Lambda_t w_t}{\psi}\right)^{-\frac{1}{\varepsilon_3}}, && (11)
\end{aligned}$$

where Λ_t is the Lagrange multiplier associated with the consumer's budget constraint at time t, expressed in real terms. It represents the marginal utility of consumption at time t. The variable ϕ_t is the Lagrange multiplier associated with (2) at time t, and w_t denotes the real wage. Equation (9) is the standard Euler equation for consumption. From (9) and (10), the following money demand equation can be obtained:

$$\chi_t\left(\frac{M_t}{P_t}\right)^{-\varepsilon_2} = \Lambda_t\left(\frac{i_t-\zeta\frac{B_t}{P_t}}{1+i_t}\right). \tag{12}$$

Taking the logarithm of both sides gives[6]

$$\ln\left(\frac{M_t}{P_t}\right) \simeq -\frac{1}{\varepsilon_2}\ln\Lambda_t - \frac{1}{\varepsilon_2}\ln i_t + \frac{1}{\varepsilon_2}\ln\chi_t. \tag{13}$$

Hence, $\frac{1}{\varepsilon_2}$ represents the interest elasticity of money and χ_t may be interpreted as a money demand shock. Finally, (11) is the labour supply equation which describes the optimal trade-off between consumption and leisure.

Both countries' residents have identical preferences. Thus, the foreign resident's problem will be analogous to that of the home resident. The representative foreign household

[6]The derivation of equation (13) takes into account the fact that ζ will be set to a very low value when the model is calibrated. See section 3 for details.

maximizes the same objective function subject to the following budget constraint:

$$\frac{B_t^*}{e_t} + \frac{\zeta}{2}\left(\frac{B_t^{*2}}{e_t^2 P_t^*}\right) + M_t^* = (1+i_{t-1})\frac{B_{t-1}^*}{e_t} + M_{t-1}^* + \pi_t^* + W_t^* L_t^* - P_t^* C_t^* - T_t^*, \tag{14}$$

where B_t^* represents the stock of domestic-currency-denominated bonds held by a foreign resident. FONC for the foreign household are written below for completeness

$$\Lambda_t^* = \frac{1}{C_t^*}\left(\frac{C_t^*}{H_t^{*\gamma}}\right)^{1-\varepsilon_1} + (1-\rho)\,\phi_t^*, \tag{15}$$

$$\beta\rho E_t \phi_{t+1}^* = \phi_t^* + \beta\gamma E_t \frac{1}{H_{t+1}^*}\left(\frac{C_{t+1}^*}{H_{t+1}^{*\gamma}}\right)^{1-\varepsilon_1}, \tag{16}$$

$$\Lambda_t^*\left(1+\zeta\frac{B_t^*}{e_t P_t^*}\right) = \beta(1+i_t)E_t\frac{e_t P_t^* \Lambda_{t+1}^*}{e_{t+1}P_{t+1}^*}, \tag{17}$$

$$\chi_t^*\left(\frac{M_t^*}{P_t^*}\right)^{-\varepsilon_2} = \Lambda_t^* - \beta E_t \frac{P_t^* \Lambda_{t+1}^*}{P_{t+1}^*}, \tag{18}$$

$$L_t^* = 1 - \left(\frac{\Lambda_t^* w_t^*}{\psi}\right)^{-\frac{1}{\varepsilon_3}}. \tag{19}$$

2.2 The government

Government spending is exogenous and is entirely financed by lump-sum taxes, seigniorage and bond holding costs.[7] Government consumption per capita in the home country, G, is assumed to be a composite index identical to the private consumption index. That is,

$$G_t = \left[\int_0^1 g_t(i)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}, \tag{20}$$

where $g(i)$ denotes home government consumption per capita of good i. By analogy to the consumer's demand, the government demand for good i is given by

$$g_t(i) = \left(\frac{v_t(i)}{P_t}\right)^{-\theta} G_t, \tag{21}$$

where $v_t(i)$ is defined as in equation (5). The government budget constraint, therefore, is:

$$P_t G_t = T_t + M_t - M_{t-1} + \frac{\zeta}{2}\left(\frac{B_t^2}{P_t}\right). \tag{22}$$

[7]The assumption that the government runs a balanced budget each period is innocuous since Ricardian equivalence holds in this model.

Money supply is given by

$$M_t = \mu_t M_{t-1}, \tag{23}$$

where the (gross) rate of money growth, μ_t, is a stochastic process. The foreign government is described in an analogous manner.

2.3 Firms

2.3.1 Production

The home firm hires labour domestically to produce its differentiated good using the following linear technology:

$$Y_t(i) = A_t l_t(i), \tag{24}$$

where $Y_t(i)$ is the production of good i, $l_t(i)$ is domestic labour input used by firm i and A_t is a technology shock that affects labour productivity symmetrically across firms in the home country. As in Betts and Devereux (2000), there is no capital in the model. I argue further (section 4) that this assumption is innocuous and should not affect the main results regarding exchange rates properties.[8]

2.3.2 Pricing

Firms in both countries are monopolistically competitive. Thus, each firm chooses its own price taking as given the aggregate demand and the price level in each country. Prices are assumed to be sticky. The assumption of firms being price makers is simply a convenient way to motivate price stickiness. In Betts and Devereux (2000), prices are held fixed for one period, after which the economy reaches its new steady state. Obviously, this kind of rigidity cannot give rise to persistent movements of real variables following a monetary disturbance. Instead, a more staggered price setting is required. In this paper, price rigidity

[8]Chari, Kehoe and McGrattan (1997) note that abstracting from capital in their model yields similar results for exchange rates to those obtained when capital is included.

is introduced along the lines of Calvo (1983). That is, it is assumed that each firm changes its price randomly from one period to the next, with a (constant) probability $1-\delta$.[9]

Non-PTM firms Non-PTM firms do not segment markets by country. That is, a typical non-PTM firm chooses the same price in home currency for the home and foreign markets to maximize profits. The foreign-currency price of a non-PTM good will simply be equal to the home-currency price divided by the nominal exchange rate so that the law of one price holds. Let $\widetilde{p}_{1t}$ denote the price set by a typical non-PTM firm at period t.[10] The total demand facing this firm in period s ($s \geq t$) is

$$Y_{1s}^{d} = (\frac{\widetilde{p}_{1t}}{P_s})^{-\theta} n(C_s + G_s) + (\frac{\widetilde{p}_{1t}}{e_s P_s^*})^{-\theta}(1-n)(C_s^* + G_s^*). \tag{25}$$

The probability that $\widetilde{p}_{1t}$ "survives" at least until period s ($s \geq t$) is equal to δ^{s-t}. Therefore, the firm chooses $\widetilde{p}_{1t}$ to maximize

$$E_t \sum_{s=t}^{\infty} (\beta\delta)^{s-t} \lambda_{t,s} (\widetilde{p}_{\alpha t} - MC_s) Y_{1s}^{d},$$

where $\lambda_{t,s} = \frac{\Lambda_s/P_s}{\Lambda_t/P_t}$ and $MC_s \equiv \frac{W_s}{A_s}$ is the nominal marginal cost at time s. Differentiating this expression with respect to $\widetilde{p}_{1t}$ and equating to zero yields

$$\widetilde{p}_{1t} = \left(\frac{\theta}{\theta-1}\right) \frac{E_t \sum_{s=t}^{\infty} (\beta\delta)^{s-t} \lambda_{t,s} Y_{1s}^{d} MC_s}{E_t \sum_{s=t}^{\infty} (\beta\delta)^{s-t} \lambda_{t,s} Y_{1s}^{d}}. \tag{26}$$

Since there is a continuum of non-PTM firms, and assuming that price changes are stochastically independent across them, the law of large numbers implies that $1-\delta$ is also the proportion of non-PTM firms that set a new price each period of time. Let p_{1t} denote the price index of non-PTM goods at time t, defined as the average of the $\widetilde{p}_1$'s still in effect at time t, weighted by the proportion of non-PTM firms with the same $\widetilde{p}_1$. Again, by the law

[9] Hence, the average length of a price quotation is equal to $\frac{1}{1-\delta}$.

[10] $\widetilde{p}_{1t}$ does not depend on the firm index because all the firms that have the opportunity to change their prices at a given time, choose the same price. As a consequence, these firms will be facing the same demand at a given time as long as they do not change their price.

of large numbers, the proportion of non-PTM firms that set a new price at time s and have not changed it as of time t $(s \leq t)$, is given by the probability that a given price is still in effect in period t, which is $\delta^{t-s}(1-\delta)$. It follows that

$$p_{1t} = \left[(1-\delta)\sum_{s=-\infty}^{t}\delta^{t-s}\widetilde{p}_{1s}^{1-\theta}\right]^{\frac{1}{1-\theta}}. \tag{27}$$

This expression can be rewritten in the following recursive form:

$$p_{1t}^{1-\theta} = \delta p_{1t-1}^{1-\theta} + (1-\delta)\widetilde{p}_{1t}^{1-\theta}. \tag{28}$$

PTM firms PTM firms are able to segment markets. They choose separately a home-currency price for the home country and a foreign-currency price for the foreign market in order to maximize profits. Let $\widetilde{p}_{2t}$ and $\widetilde{q}_t$ denote respectively the home-currency and the foreign-currency prices set by a typical PTM firm at time t. The domestic demand facing a PTM firm at time s $(s \geq t)$ is given by

$$Y_{2t}^d = \left(\frac{\widetilde{p}_{2t}}{P_t}\right)^{-\theta} n(C_t + G_t), \tag{29}$$

while the foreign demand is given by

$$Y_{3t}^d = \left(\frac{\widetilde{q}_t}{P_t^*}\right)^{-\theta}(1-n)(C_t^* + G_t^*). \tag{30}$$

The PTM firm chooses $\widetilde{p}_{2t}$ and $\widetilde{q}_t$ to maximize

$$E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\lambda_{t,s}\left\{\widetilde{p}_{2t}Y_{2s}^d(i) + e_s\widetilde{q}_tY_{3s}^d(i) - \frac{W_s}{A_s}\left(Y_{2s}^d + Y_{3s}^d\right)\right\}.$$

FONC for this maximization yield

$$\widetilde{p}_{2t} = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\lambda_{t,s}Y_{2s}^d MC_s}{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\lambda_{t,s}Y_{2s}^d}, \tag{31}$$

$$\widetilde{q}_t = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\lambda_{t,s}Y_{3s}^d MC_s}{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\lambda_{t,s}Y_{3s}^d e_s}. \tag{32}$$

The price index of PTM goods sold in the home market, denoted by p_{2t}, satisfies

$$p_{2t}^{1-\theta} = \delta p_{2t-1}^{1-\theta} + (1-\delta)\,\widetilde{p}_{2t}^{1-\theta}. \tag{33}$$

Likewise, the price index of PTM goods sold in the foreign market, denoted by q_t, satisfies:

$$q_t^{1-\theta} = \delta q_{t-1}^{1-\theta} + (1-\delta)\,\widetilde{q}_t^{1-\theta}. \tag{34}$$

As stated above, foreign-good prices are determined analogously to home-good prices. I still include them here for completeness. Denoting by $\widetilde{q}_{1t}^*$ and $\widetilde{q}_{2t}^*$ the prices set respectively by a foreign non-PTM and by a foreign PTM firm in the foreign market, and by $\widetilde{p}_t^*$ the price set by a foreign PTM firm in the home market, I have

$$\widetilde{q}_{1t}^* = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\,\lambda_{t,s}^* Y_{1s}^{*d} MC_s^*}{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\,\lambda_{t,s}^* Y_{2s}^{*d}}, \tag{35}$$

$$\widetilde{q}_{2t}^* = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\,\lambda_{t,s}^* Y_{1s}^{*d} MC_s^*}{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\,\lambda_{t,s}^* Y_{2s}^{*d}}, \tag{36}$$

$$\widetilde{p}_t^* = \left(\frac{\theta}{\theta-1}\right)\frac{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\,\lambda_{t,s}^* Y_{3s}^{*d} MC_s^*}{E_t\sum_{s=t}^{\infty}(\beta\delta)^{s-t}\,\lambda_{t,s}^* Y_{3s}^{*d}\frac{1}{e_s}}. \tag{37}$$

Moreover, let q_{1t}^*, q_{2t}^*, and p_t^* represent price indexes of foreign goods, defined similarly to p_{1t}, p_{2t}, and q_t. By analogy, these indexes satisfy, respectively

$$q_{1t}^{*1-\theta} = \delta q_{1t-1}^{*1-\theta} + (1-\delta)\,\widetilde{q}_{1t}^{*1-\theta}, \tag{38}$$

$$q_{2t}^{*1-\theta} = \delta q_{2t-1}^{*1-\theta} + (1-\delta)\widetilde{q}_{2t}^{*1-\theta}, \tag{39}$$

$$p_t^{*1-\theta} = \delta p_{t-1}^{*1-\theta} + (1-\delta)\widetilde{p}_t^{*1-\theta}. \tag{40}$$

When $\delta = 0$, i.e., when all firms change their prices each period, the pricing decisions above collapse to the following static conditions:

$$\begin{aligned} p_{1t} &= p_{2t} = e_t q_t = (\frac{\theta}{\theta-1})MC_t, \\ q_{1t}^* &= q_{2t}^* = \frac{p^*}{e_t} = (\frac{\theta}{\theta-1})MC_t^*, \end{aligned}$$

which state that (i) the optimal price chosen by a given firm is set as a constant markup over its marginal cost and (ii) the law of one price holds even for non-PTM goods. The latter result is due to the fact that both markets are characterized by the same elasticity of demand. Therefore, despite the presence of pricing-to-market, purchasing power parity must hold when prices are flexible.

Given the definitions of p_{1t}, p_{2t}, q_t, q_{1t}^*, q_{2t}^*, and p_t^*, the consumption-based price index in the home country becomes

$$P_t = \left[n\left\{(1-\alpha){p_{1t}}^{1-\theta} + \alpha {p_{2t}}^{1-\theta}\right\} + (1-n)\left\{\alpha^* {p_t^*}^{1-\theta} + (1-\alpha^*)(e_t q_{1t}^*)^{1-\theta}\right\}\right]^{\frac{1}{1-\theta}}, \quad (41)$$

while its foreign counterpart is equal to

$$P_t^* = \left[(1-n)\left\{(1-\alpha^*){q_{1t}^*}^{1-\theta} + \alpha^* {q_{2t}^*}^{1-\theta}\right\} + n\left\{\alpha {q_t}^{1-\theta} + (1-\alpha)\left(\frac{p_{1t}}{e_t}\right)^{1-\theta}\right\}\right]^{\frac{1}{1-\theta}}. \quad (42)$$

2.4 Closing the model

Let y_t be the aggregate output per capita in the home country. Then

$$y_t = \frac{1}{n}\int_0^n Y_t(i) = \frac{A_t}{n}\int_0^n l_t(i) = A_t L_t. \quad (43)$$

Goods market clearing requires that aggregate output must equal aggregate demand. That is,

$$\begin{aligned} A_t L_t \;=\; & \left[\int_0^{n(1-\alpha)} p_{1t}(i)^{-\theta} di\right]\left\{(\frac{1}{P_t})^{-\theta}(C_t + G_t) + (\frac{1}{e_t P_t^*})^{-\theta}(\frac{1-n}{n})(C_t^* + G_t^*)\right\} \\ & + \left[\frac{1}{n\alpha}\int_{n(1-\alpha)}^n p_{2t}(i)^{-\theta} di\right](\frac{1}{P_t})^{-\theta}(C_t + G_t) \\ & + \left[\frac{1}{n\alpha}\int_{n(1-\alpha)}^n q_t(i)^{-\theta} di\right](\frac{q_t}{P_t^*})^{-\theta}(\frac{1-n}{n})(C_t^* + G_t^*). \end{aligned}$$

Now, define the following alternative price indexes $p_{1t}' = \left[\frac{1}{n(1-\alpha)}\int_0^{n(1-\alpha)} p_{1t}(i)^{-\theta} di\right]^{-\frac{1}{\theta}}$, $p_{2t}' = \left[\frac{1}{n\alpha}\int_{n(1-\alpha)}^n p_{2t}(i)^{-\theta} di\right]^{-\frac{1}{\theta}}$, and $q_t' = \left[\frac{1}{n\alpha}\int_{n(1-\alpha)}^n q_t(i)^{-\theta} di\right]^{-\frac{1}{\theta}}$. Therefore, the market clearing condition above becomes

$$\begin{aligned} A_t L_t &= (1-\alpha)\left\{(\frac{p'_{1t}}{P_t})^{-\theta} n(C_t+G_t) + (\frac{p'_{1t}}{e_t P^*_t})^{-\theta}(1-n)(C^*_t+G^*_t)\right\} \\ &\quad +\alpha\left\{(\frac{p'_{2t}}{P_t})^{-\theta} n(C_t+G_t) + (\frac{q'_t}{P^*_t})^{-\theta}(1-n)(C^*_t+G^*_t)\right\}. \end{aligned} \tag{44}$$

The corresponding condition for the foreign country is

$$\begin{aligned} A^*_t L^*_t &= (1-\alpha)\left\{(\frac{q'^*_{1t}}{P^*_t})^{-\theta}(1-n)(C^*_t+G^*_t) + (\frac{e_t q'^*_{1t}}{P_t})^{-\theta} n(C_t+G_t)\right\} \\ &\quad +\alpha\left\{(\frac{q'^*_{2t}}{P^*_t})^{-\theta}(1-n)(C^*_t+G^*_t) + (\frac{p'^*_t}{P_t})^{-\theta} n(C_t+G_t)\right\}, \end{aligned} \tag{45}$$

where q'^*_{1t}, q'^*_{2t}, and p'^*_t are constructed analogously to p'_{1t}, p'_{2t}, and q'_t. In addition, the condition for bonds market clearing is

$$nB_t + (1-n)B^*_t = 0 \tag{46}$$

Finally, aggregate consistency implies the following balance of payments equations for both countries:

$$\begin{aligned} P_t C_t + P_t G_t + B_t &= (1+i_{t-1})B_{t-1} + \int_0^{n(1-\alpha)} p_{1t}(i) y_{1t}(i) di + \\ &\quad \int_{n(1-\alpha)}^{n} \left[p_{2t}(i) y_{2t}(i) + (e_t q_t(i))\, y_{3t}(i)\right] di, \end{aligned} \tag{47}$$

$$\begin{aligned} P^*_t C^*_t + P^*_t G^*_t + \frac{B^*_t}{e_t} &= (1+i_{t-1})\frac{B^*_{t-1}}{e_t} + \int_{n+(1-n)\alpha^*}^{1} q^*_{1t}(i) y^*_{1t}(i) di + \\ &\quad \int_{n}^{n+(1-n)\alpha^*} \left[q^*_{2t}(i) y^*_{2t}(i) + \frac{p^*_t(i)}{e_t} y^*_{3t}(i)\right] di, \end{aligned} \tag{48}$$

where $y_1(i)$ represents the output per capita of non-PTM firm i, $y_2(i)$ the output per capita of PTM firm i sold in the home market and $y_3(i)$ the output per capita of PTM firm i sold in the foreign market.

2.5 Stochastic processes

It is assumed that only technology shocks are correlated across countries. The stochastic processes describing money-demand, money-growth and government-spending shocks in the home country are given by:

$$\ln \chi_{t+1} = (1-\rho_\chi)\ln \chi + \rho_\chi \ln \chi_t + \epsilon_{\chi t}, \quad (49)$$

$$\ln G_{t+1} = (1-\rho_G)\ln G + \rho_G \ln G_t + \epsilon_{Gt}, \quad (50)$$

$$\ln \mu_{t+1} = (1-\rho_\mu)\ln \mu + \rho_\mu \ln \mu_t + \epsilon_{\mu t}, \quad (51)$$

where the parameters ρ_χ, ρ_G, and ρ_μ are strictly bounded between -1 and 1, and $\epsilon_{\chi t}, \epsilon_{Gt}$, and $\epsilon_{\mu t}$ are assumed to be normally distributed, non-correlated, zero-mean disturbances. The exogenous processes in the foreign country are defined analogously. It is assumed that these shocks are not correlated across countries. However, in conformity with the international real business cycle literature, I assume that technology shocks are correlated across countries. Specifically, these shocks follow the following joint process:

$$\ln(A_{t+1}, A^*_{t+1})' = (I-\Omega)\ln(A, A^*)' + \Omega \ln(A_t, A^*_t)' + (\epsilon_{At}, \epsilon_{A^*t})', \quad (52)$$

where ϵ_{At} and ϵ_{A^*t} are normally distributed positively cross-correlated zero-mean disturbances. Technology shocks are not correlated with any other shocks within or across countries.

2.6 Symmetric sticky-price equilibrium

A symmetric equilibrium for this economy is a collection of 25 sequences (C_t, C^*_t, H_t, H^*_t, M_t, M^*_t, L_t, L^*_t, Λ_t, Λ^*_t, ϕ_t, ϕ^*_t, P_t, P^*_t, W_t, W^*_t, i, $\widetilde{p}_t$, p_t, $\widetilde{q}_t$, q_t, $\widetilde{q}^*_t$, q^*_t, $\widetilde{p}^*_t$, p^*_t, B_t and e_t)[11] satisfying (i) the FONC for households' maximizations in both countries (equations 7-11

[11]Equation (46) is used to substitute for B^* in (48). Then (48) is dropped since, by Walras law, one equation is redundant. Moreover, in the linearized version of the model, it turns out that $\widetilde{p}_{1t} = \widetilde{p}_{2t}$ and $p'_{1t} = p_{1t} = p_{2t} = p'_{2t}$. This also applies to foreign prices.

and 15-19) (*ii*) the definitions of the home and foreign consumption-based price indexes (equations 41,42), (*iii*) the pricing conditions (equations 26, 28, 32, 34, 35, 37, 38 and 40), (*iv*) the market clearing conditions (equations 44 and 45), (*v*) the balance of payments equation (47) and (*vi*) the money supply equations (23 and its foreign counterpart), given p_{-1}, q_{-1}, q^*_{-1}, p^*_{-1}, H_{-1}, H^*_{-1}, M_{-1}, M^*_{-1}, B_{-1} and the exogenous stochastic processes (A_t, A^*_t, χ_t, χ^*_t, G_t, G^*_t, μ_t, μ^*_t).

The model does not have a closed-form solution. Instead, it is solved up to a first-order approximation around a deterministic symmetric initial steady state. That is, the model's equilibrium equations are log-linearized to obtain a system of difference equations; then the method of Blanchard and Kahn (1980) is applied.

3. Calibration

This section describes the calibration of a baseline model, in which there is no habit formation. That is, the parameter γ is set to zero (ρ becomes irrelevant). The remaining parameters are calibrated as follows. The subjective discount factor, β, is set to 0.99 so that the steady-state annual interest rate is about 4%. The curvature parameter, ε_1, is chosen to equal 2, as in standard real business cycle (RBC) literature. This implies that the intertemporal elasticity of substitution is equal to 0.5.[12] The parameter ε_2 is calibrated so that the interest elasticity of money is equal to 0.1. This value is consistent with the estimates reported in the empirical literature.[13] I set ε_3 to 1 and choose ψ so that the steady-state labour-leisure ratio, η, is equal to .45 as in RBC models.

Unfortunately, the elasticity of substitution between goods within the same country and across countries, and the steady-state markup cannot be parameterized independently here since they both depend on θ. Choosing a high value for θ implies a high elasticity of

[12]This is no longer the case, however, when γ is different from zero.
[13]See, among others, Mankiw and Summers (1986) and Goldfeld and Sichel (1990).

substitution but a low markup and vice versa. I set $\theta = 3$, which is a reasonable value for the elasticity of substitution and which, in turn, implies a steady-state markup equal to 1.5, the value used by Bergin and Feenstra (2001).

A recent survey by the ECU Institute (1995) shows that 92% of U.S. exports and 80% of U.S. imports are denominated in U.S. dollars. In terms of my notation, this implies that α and α^* should be equal to 0.08 and 0.8, respectively. The probability of changing prices, $1 - \delta$, is chosen so that, conditional on all shocks, price volatility generated by the model matches closely the observed volatility in U.S. price data. The value of δ that meets this requirements is found to be 0.2, meaning that the average duration of price contracts is five quarters.

Technology and money-growth processes are parameterized according to Backus, Kehoe, and Kydland (1995). For money-demand shocks, I use the estimate reported by Kim (2000) for ρ_{χ}.[14] Government-spending shocks are parameterized as in Chari, Kehoe, and McGrattan (2000b). The elasticity of bond holding costs, ζ, is set to 10^{-6}. With zero holding costs, the model possesses a unit root, as transitory shocks lead to permanent wealth reallocations between the two countries. That is, the bond holding process is stationary and so are the variables that depend on it. Therefore, the unconditional moments of these variables cannot be computed. By assigning a very low value to ζ, I aim to circumvent the unit root problem while allowing for minimal effects form holding costs.

In computing the steady state, I need to set initial values for bond holdings and government spending. The initial stock of bonds is assumed to be zero ($B = 0$) and the steady-state level of government spending ratio, $\frac{G}{y}$, is set to 0.2. Finally, the home country and the foreign country are assumed to be of equal size, which implies that $n = 0.5$. Except for pricing-to-market, both countries are treated symmetrically in every other respect. All

[14]Kim (2000) estimates a sticky-price general-equilibrium model that has a money-demand shock similar to the one in this model.

parameters values are summarized in Table 2.1.

4. Results

Simulations results are presented in Table 2.2. In order to assess the ability of the model to replicate the observed properties of the data, some selected moments of U.S. data are reported. These moments are based on quarterly data from IFS for the period 1974Q1–1999Q4. The data is logged and passed through Hodrick-Prescott (H-P) filter. We simulate the model 100 times over 100 quarters. This time span corresponds to the length of the sample used to calculate the historical moments. Regular-font numbers are average values of the statistics across simulations, while small-font numbers are their corresponding standard deviations.

4.1 Base case

First, consider the baseline model which assumes a realistic degree of PTM and no habit formation. The model generates standard deviations of output, consumption and the nominal exchange rate that could be reconciled with those of the data. The observed volatility of each variable lies within two standard deviations of its predicted one. However, the model fails to replicate real exchange rate volatility as it generates just about 25% of the actual standard deviation. In addition, the autocorrelation of each variable, except the price level, is below its empirical counterpart. The model produces nominal and real exchange rates that are procyclical while they are essentially acyclical in the data. On the other hand, the observed correlation between the two variables is higher than the value generated by the model, although it lies borderline in the 95% confidence region. Finally, the model fails to reproduce the cross-country correlations of output and consumption. The former has the wrong sign while the latter is over-predicted by the model. To summarize, the model calibrated to the U.S. economy seems unable to account for the "real" dollar fluctuations.

Furthermore, it performs poorly in replicating the comovements of output and consumption across countries.

4.2 The role of pricing-to-market

To understand the role of pricing-to-market on exchange rate movements, I consider a version of the model where I keep abstracting from habit formation but where I set $\alpha = \alpha^* = 1$. This means that all firms segment markets by country so that there is no pass-through from nominal exchange rates to import prices. Betts and Devereux (2000) show that the higher the degree of PTM, the greater is nominal (and real) exchange rate volatility in response to a monetary shock. They also show that the presence of PTM tends to increase the cross-country correlation of output and to reduce that of consumption.

Simulation results for this case show that the standard deviation of output is substantially decreased, while that of consumption is slightly increased. But more importantly, nominal exchange rate volatility improves to 4.6% and the real exchange rate is almost three times as volatile as in the base case, although it is still low compared to the data. These results corroborate those of Betts and Devereux (2000). However, the serial correlations of both exchange rates are essentially the same as in the baseline model. Thus, although PTM magnifies exchange rate volatility, it does not seem to affect persistence. To confirm this observation, Figure 2.1 plots real exchange rate volatility and persistence against the degree of PTM measured by α and α^*. The figure shows that the volatility increases monotonically as α and α^* increase, reaching its maximum when PTM is full ($\alpha = \alpha^* = 1$). On the other hand, the persistence seems to be independent of α and α^* as the autocorrelation remains virtually constant at 0.6.[15]

Besides generating high exchange rate volatility, the model with full PTM successfully

[15] In figure 1, α and α^* start at 0.1, because when $\alpha = \alpha^* = 0$ there is obviously no variation in the real exchange rate.

replicates the acyclicity of exchange rates and marginally improves the correlation between nominal and real exchange rates. In addition, the model seems to solve the so-called *quantity anomaly* as it predicts the same ordering for cross-country correlations of output and consumption as in the data.

In brief, pricing-to-market offers the potential to magnify real exchange rate volatility and to account for some puzzling features stressed by international business cycle literature. However, it proves to be irrelevant to explain real exchange rate persistence.

4.3 The role of habit formation

The purpose of this paper is to investigate whether habit formation increases real exchange rate persistence. Thus, I need to assign a value different from zero to the parameter γ.[16] Unfortunately, empirical studies do not provide much guidance about the magnitude of γ for two reasons. First, in many of these studies, the specification of habit formation is different from the one used in this study. Hence, one cannot obtain direct estimates of γ. Second, these studies yield mixed evidence on the importance of habit formation.[17] Therefore, I choose to examine the two cases $\gamma = 0.5$ and $\gamma = 1$. The results are shown in the last two columns of Table 2.2. These results are derived with $\rho = 0$. Unreported sensitivity analysis indicated that the main results regarding exchange rates were robust to alternative parameterizations of ρ.

When $\gamma = 0.5$, the most notable difference in comparison with the non-habit case is the increase in output and consumption persistence. To understand this, note that the instantaneous utility function can be rewritten in the following suggestive form:

[16]The parameter ρ must also be different from 1, otherwise the habit stock enters the utility function as a constant, which amounts to assuming non-habit preferences.

[17]Fuhrer (2000) reports estimates of γ approaching 1. In contrast, Heaton (1993) and Dynan (2000) find little evidence of habit formation.

$$u_t = \frac{1}{1-\varepsilon_1}\left(\frac{C_t}{C_{t-1}} C_{t-1}^{1-\gamma}\right)^{1-\varepsilon_1} + \ ...$$

Thus, when γ is greater than zero but less than one, consumers smooth both the level and the growth rate of consumption. Hence, consumption responds more sluggishly and more persistently to shocks than when consumers care only about the absolute level of consumption. Since there is no capital in the model, consumption persistence is imparted to output.

Aside from these effects, there are no significant differences in the moments generated by this model relative to the case where $\gamma = 0$. In particular, real exchange rate volatility and persistence remain unchanged. To check the robustness of these results to the choice of γ, I set $\gamma = 1$. Under this parameterization, consumers smooth only the growth rate of consumption. This suggests that consumption response to shocks will be even more gradual and more persistent than in the previous case. This prediction is confirmed by the quantitative results, which show that the serial correlations of output and consumption have indeed increased. In fact, output and consumption are now as highly autocorrelated as in the data. However, their standard deviations become very low compared to the data. Regarding real exchange rate properties, simulation results show that neither volatility nor persistence is affected by this alternative parameterization of γ. Thus, habit formation does not seem to make any difference when it comes to real exchange rate dynamics.

An alternative way to examine how the short run dynamics of the model are affected by habit formation is to look at the impulse response functions. In this model, as in Kollmann (2001) and Chari, Kehoe, and McGrattan (2000b), money-supply shocks are the major source of exchange rate fluctuations.[18] Hence, I restrict myself to examine the impulse responses resulting from a monetary shock. Figures 2.2 and 2.3 depict the responses of the

[18]Simulation results for the case where the economy is subject to monetary shocks only are not reported, but they are available from the author upon request.

main variables to a 1 per cent money-growth shock starting at the steady state with and without habit formation, respectively. In both cases, full pricing-to market ($\alpha = \alpha^* = 1$) is assumed.

Figure 2.2 shows that a permanent increase in money supply causes the nominal exchange rate to depreciate. The nominal interest rate decreases, which implies, by the uncovered interest parity, that the nominal exchange rate overshoots. The monetary shock raises the price level in the home country. Due to the staggered price setting in the model, the price level in the home country initially rises less than proportionally to money and converges to its new steady-state level gradually. As a result, home consumption increases. In contrast, since all prices are set in local currencies, the foreign price level remains almost unchanged as does foreign consumption. The presence of PTM and price rigidity cause the real exchange rate to depreciate following the shock. As prices adjust, the real exchange rate returns to its initial level. Because there is no pass-through from exchange rates to import prices, there is no *expenditure switching* from foreign to domestic goods. Thus, output rises in both countries.[19]

The impulse responses corresponding to the case $\gamma = 0.5$ are illustrated in Figure 2.3. In this case, consumption response is hump-shaped, and the initial impact of the monetary disturbance is smaller than in the previous case. The reason, as stated above, is that consumers smooth both the level and the growth rate of consumption when γ is strictly positive. Output also exhibits a hump-shaped response in both countries. Moreover, it is visually obvious that consumption and output responses are more persistent than those displayed in Figure 2.2. On the other hand, nominal and real exchange rates responses are virtually the same as when $\gamma = 0$. This is also the case for the terms of trade, the nominal interest rate and the price level. The invariability of nominal interest rate response under

[19]This point is explained in detail in Betts and Devereux (2000).

the two parameterizations $\gamma = 0$ and $\gamma = 0.5$ reflects the inability of habit formation to exacerbate the liquidity effect generated by the model.

4.4 Explanation

Why is habit formation completely irrelevant in explaining exchange rate fluctuations ? To answer this question, it is useful to examine a version of the model where financial markets are complete. That is, I assume that there exists a complete set of state-contingent bonds that allow households in both countries to pool risks perfectly. Technically, under this assumption, the maximization problems of the domestic and foreign representative households are equivalent to a social planner's problem of maximizing a weighted sum of the utilities of both households subject to the world resource constraint.[20] In this case, the first-order condition (17) is replaced by the following risk-sharing equation:

$$e_t = \frac{\Lambda_t^*/P_t^*}{\Lambda_t/P_t}, \tag{53}$$

or, equivalently,

$$e_t^r = \frac{\Lambda_t^*}{\Lambda_t}, \tag{54}$$

where e_t^r is the real exchange rate, and Λ and Λ^* are given by (7) and (15) respectively.

In the case where $\alpha = \alpha^* = 1$, it can be shown that nominal (and real) exchange rate dynamics are fully determined by the following three-equation system (see Appendix A for the derivation):[21]

$$\begin{aligned} \hat{P}_t - \hat{P}_t^* &= \delta(\hat{P}_{t-1} - \hat{P}_{t-1}^*) + (1-\delta)(\hat{\tilde{P}}_t - \hat{\tilde{P}}_t^*), & (55) \\ E_t(\hat{\tilde{P}}_{t+1} - \hat{\tilde{P}}_{t+1}^*) &= \frac{1}{\beta\delta}(\hat{\tilde{P}}_t - \hat{\tilde{P}}_t^*) - \frac{1-\beta\delta}{\beta\delta}\hat{e}_t, & (56) \\ E_t\hat{e}_{t+1} &= \frac{1}{\beta}\hat{e}_t + \frac{(1-\beta)(\varepsilon_2 - 1)}{\beta}(\hat{P}_t - \hat{P}_t^*) - \frac{\varepsilon_2(1-\beta)}{\beta}(\hat{M}_t - \hat{M}_t^*). & (57) \end{aligned}$$

[20]The weights correspond to the population size of each country.

[21]We abstract from money demand shock in deriving equation (57).

These equations show that when markets are complete, real shocks (such as technology and government-spending shocks) play no role in exchange rate determination. Only monetary shocks matter. It is also obvious that investment decisions would not affect exchange rate properties if capital was included in the model. In fact, nominal and real exchange rates turn out to be totally independent of the supply side of the model.[22] Moreover, the only relevant parameters in this case are β, δ, and ε_2. In particular, γ does not show up in the system above. Hence, in the complete markets environment, habit formation has no influence on exchange rate dynamics. Note that this result is robust to the way habit formation is introduced. Alternative functional forms of habit formation (such as Campbell and Cochrane's specification, for example) will not overturn the result.[23]

The structure of asset markets was shown by Chari, Kehoe, and McGrattan (2000b) and Betts and Devereux (2001) to make no difference to the behaviour of exchange rates. This result is confirmed by Figure 2.4 which depicts the impulse responses of real and nominal exchange rates under complete and incomplete markets.[24] In both cases, each variable displays almost exactly the same dynamic path in response to a money-growth shock.[25] Chari, Kehoe, and McGrattan (2000b) argue that this outcome is due to the smallness of wealth effects in this kind of models. Given this result, it is not surprising that the irrelevance of habit formation under complete markets persists when incomplete markets are assumed.

[22]This result has already been shown by Devereux (1997).

[23]Note, however, that if I depart from the assumption that both countries have identical preferences by introducing a home country bias (that is, each country has a preference for its own goods), the model with complete markets no longer exhibits the same separation between the determinants of the nominal (and real) exchange rate and the factors determining marginal costs. As a consequence, habit formation could affect the nominal (and real) exchange rate through its effect on marginal costs. However, given that the properties of domestic and foreign marginal costs tend to cancel out in this type of model, it is unlikely that the effect of habit formation on exchange rates would be strong.

[24]For the complete markets case, I only need to calibrate β, δ, and ε_2. These parameters are set to the same values as in the model with incomplete markets.

[25]This is not the case for technology and government-spending shocks. As stated earlier, these shocks have no impact on exchange rates in the complete markets environment. However, when markets are incomplete, real shocks do affect exchange rate dynamics.

5. Conclusion

Quantitative sticky-price models based on pricing-to-market proved to be somewhat successful in generating the observed volatility of real exchange rates. However, these models fail to replicate the persistence found in real exchange rate series. This paper has shown that for such models to generate enough volatility, the degree of PTM has to be unrealistically high. Moreover, while pricing-to-market magnifies real exchange rate volatility, it has no impact on persistence. The model by Betts and Devereux (2000) was extended to allow for habit formation in consumer preferences; an analysis was then conducted to determine whether this feature helps to increase real exchange rate persistence. Results show that habit formation is irrelevant to exchange rate properties—although it makes consumption respond more persistently to shocks—thereby reflecting its inability to exacerbate the liquidity effect generated by the model. The version of the model with complete financial markets suggests that, in a symmetric environment, any attempt to generate a liquidity effect by manipulating the supply side of the model will fail. Hence, future research on exchange rate determination should focus on considering alternative goods market structures that could endogenously increase price inertia. In particular, models where desired markups are allowed to vary seem to offer a promising avenue in generating endogenous rigidities.[26]

[26]See Rotemberg and Woodford (1999) for a survey on models of variable desired markups.

Appendix A: Derivation of Equations (55)–(57)

In what follows, the circumflex denotes the percentage deviation of a variable from its steady-state value ($\hat{x}_t = \frac{x_t - x}{x}$).

The linearization of equations (26) and (31) leads to the same expression for $\widehat{\tilde{p}}_{1t}$ and $\widehat{\tilde{p}}_{2t}$. Hence, both prices shall be denoted by $\widehat{\tilde{p}}_t$. Likewise, $\widehat{\tilde{q}}_{1t}$ and $\widehat{\tilde{q}}_{2t}$ reduce to the same expression and shall be denoted by $\widehat{\tilde{q}}_t$. The linearized pricing equations are

$$\hat{p}_t = \delta \hat{p}_{t-1} + (1-\delta)\widehat{\tilde{p}}_t, \tag{A.1}$$

$$\hat{q}_t = \delta \hat{q}_{t-1} + (1-\delta)\widehat{\tilde{q}}_t, \tag{A.2}$$

$$\hat{p}^*_t = \delta \hat{p}^*_{t-1} + (1-\delta)\widehat{\tilde{p}}^*_t, \tag{A.3}$$

$$\hat{q}^*_t = \delta \hat{q}^*_{t-1} + (1-\delta)\widehat{\tilde{q}}^*_t, \tag{A.4}$$

$$E_t\widehat{\tilde{p}}_{t+1} = \frac{1}{\beta\delta}\widehat{\tilde{p}}_t - \frac{1-\beta\delta}{\beta\delta}\widehat{MC}_t, \tag{A.5}$$

$$E_t\widehat{\tilde{q}}_{t+1} = \frac{1}{\beta\delta}\widehat{\tilde{q}} - \frac{1-\beta\delta}{\beta\delta}\widehat{MC}_t + \frac{1-\beta\delta}{\beta\delta}\hat{e}_t, \tag{A.6}$$

$$E_t\widehat{\tilde{p}}^*_{t+1} = \frac{1}{\beta\delta}\widehat{\tilde{p}}^*_t - \frac{1-\beta\delta}{\beta\delta}\widehat{MC}^*_t - \frac{1-\beta\delta}{\beta\delta}\hat{e}_t, \tag{A.7}$$

$$E_t\widehat{\tilde{q}}^*_{t+1} = \frac{1}{\beta\delta}\widehat{\tilde{q}}^*_t - \frac{1-\beta\delta}{\beta\delta}\widehat{MC}^*_t. \tag{A.8}$$

When $\alpha = \alpha^* = 1$, equations (41) and (42) become, respectively,

$$\hat{P}_t = n\hat{p}_t + (1-n)\hat{p}^*_t, \tag{A.9}$$

$$\hat{P}^*_t = (1-n)\hat{q}^*_t + n\hat{q}_t. \tag{A.10}$$

Define $\widehat{\tilde{P}}_t \equiv n\widehat{\tilde{p}}_t + (1-n)\widehat{\tilde{p}}^*_t$ and $\widehat{\tilde{P}}^*_t \equiv (1-n)\widehat{\tilde{q}}^*_t + n\widehat{\tilde{q}}_t$. Substituting (A.1) and (A.3) into (A.9), and (A.2) and (A.4) into (A.10), and taking the difference yields equation (55) in the main text. Equation (56) is straightforward to obtain from the definitions of $\widehat{\tilde{P}}_t$ and $\widehat{\tilde{P}}^*_t$.

Money demand equations (10) and (18), and the risk-sharing condition (53) are linearized as follows:

$$\hat{M}_t - \hat{P}_t = \frac{1}{\varepsilon_2(1-\beta)} \left[\beta E_t \hat{\Lambda}_{t+1} - \beta E_t \hat{P}_{t+1} - \hat{\Lambda}_t + \beta \hat{P}_t\right], \tag{A.11}$$

$$\hat{M}_t^* - \hat{P}_t^* = \frac{1}{\varepsilon_2(1-\beta)} \left[\beta E_t \hat{\Lambda}_{t+1}^* - \beta E_t \hat{P}_{t+1}^* - \hat{\Lambda}_t^* + \beta \hat{P}_t^*\right], \tag{A.12}$$

$$\hat{e}_t = (\hat{\Lambda}_t^* - \hat{P}_t^*) - (\hat{\Lambda}_t - \hat{P}_t). \tag{A.13}$$

Subtracting (A.12) from (A.11) and substituting (A.13) gives equation (57) in the main text.

Table 2.1: Base Case Parameters Values

Description	Parameter	Value
Preferences		
Discount factor	β	0.99
Importance of habit formation	γ	0
Persistence of habits	ρ	–
Curvature of the utility function	ε_1	2
(Inverse of) Interest elasticity of money	ε_2	10
(Inverse of) Leisure elasticity	ε_3	1
Elasticity of demand	θ	3
Pricing		
Fractions of local currency pricing	α	0.08
	α^*	0.80
Probability of changing prices	δ	1/5
Others		
Elasticity of bond holding costs	ζ	10^{-6}
Initial bond holdings	B	0
Initial level of government spending ratio	$\frac{G}{y}$	0.2
Relative country size	n	0.5
Stochastic processes		
Technology	Ω_A	$\begin{pmatrix} 0.906 & 0.08 \\ 0.08 & 0.906 \end{pmatrix}$
	$corr(\epsilon_A, \epsilon_A^*)$	0.258
	$\sigma_{\epsilon_A} = \sigma_{\epsilon_A^*}$	0.00852
Money demand	ρ_X	0.99
	σ_{ϵ_X}	0.05
Government spending	ρ_G	0.97
	σ_{ϵ_G}	0.01
Money growth	ρ_μ	0.57
	σ_{ϵ_μ}	0.0092

Table 2.2: Simulation Results

Statistic	**U.S. data**	**Baseline model**	$\alpha = \alpha^* = 1$		
			$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$
Std. deviation (%)					
y	1.88	2.45	0.89	0.93	0.54
		0.27	0.13	0.15	0.10
C	1.65	1.13	1.26	1.33	0.33
		0.14	0.15	0.19	0.07
P	1.51	1.59	1.51	1.52	1.50
		0.39	0.38	0.39	0.38
e	5.52	4.03	4.56	4.56	4.56
		0.94	1.05	1.05	1.05
e^r	5.00	1.27	3.40	3.40	3.40
		0.14	0.38	0.38	0.38

Table 2.2: Simulation Results (Cont.)

Statistic	U.S. data	Baseline model	$\alpha = \alpha^* = 1$		
			$\gamma = 0$	$\gamma = 0.5$	$\gamma = 1$
Autocorrelation					
y	0.88	0.57	0.69	0.78	0.87
		0.08	0.08	0.06	0.04
C	0.89	0.60	0.59	0.74	0.93
		0.08	0.08	0.05	0.02
P	0.94	0.89	0.92	0.92	0.92
		0.07	0.07	0.07	0.07
e	0.85	0.70	0.67	0.67	0.67
		0.07	0.07	0.07	0.07
e^r	0.78	0.61	0.59	0.59	0.59
		0.08	0.08	0.08	0.08
Correlation					
(y, C)	0.91	0.77	0.59	0.60	0.29
		0.06	0.12	0.14	0.23
(y, P)	-0.77	0.26	0.07	0.13	0.05
		0.12	0.20	0.21	0.27
(y, e)	-0.08	0.91	-0.03	-0.02	-0.01
		0.07	0.21	0.22	0.21
(y, e^r)	-0.06	0.70	-0.05	-0.05	-0.05
		0.03	0.20	0.20	0.18
(e, e^r)	0.90	0.85	0.87	0.87	0.87
		0.03	0.03	0.03	0.03
Cross-country correlation					
(y, y^*)	0.57	-0.76	0.44	0.49	-0.45
		0.06	0.17	0.17	0.18
(C, C^*)	0.37	0.79	0.07	0.08	0.61
		0.07	0.16	0.19	0.17

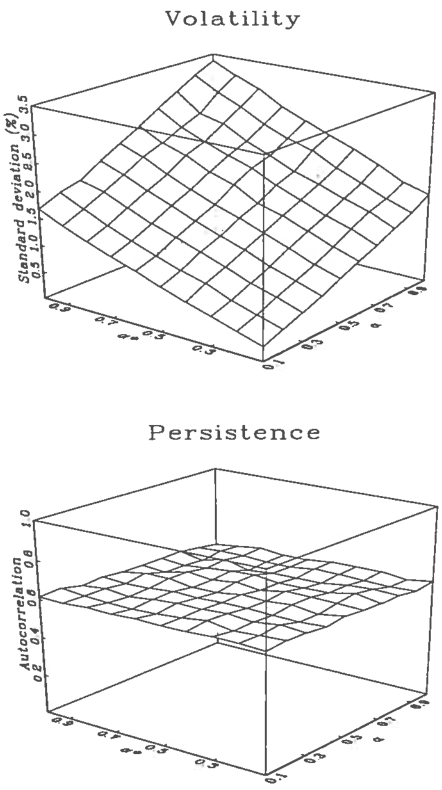

Figure 2.1: Real exchange rate volatility and persistence as functions of α and α^*

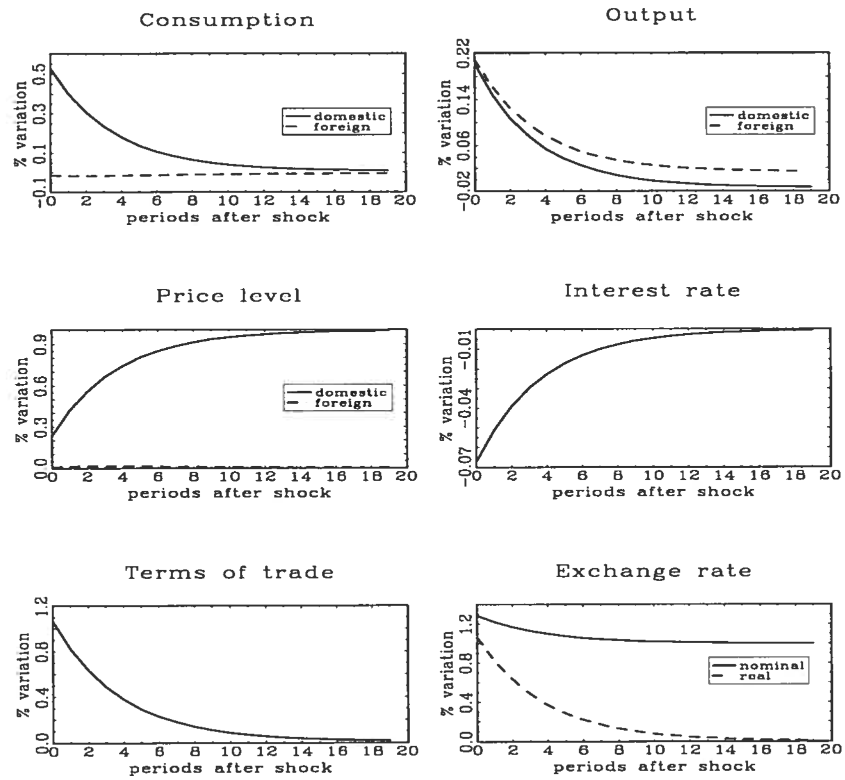

Figure 2.2: Impulse responses to a 1 per cent money-growth shock ($\gamma = 0$)

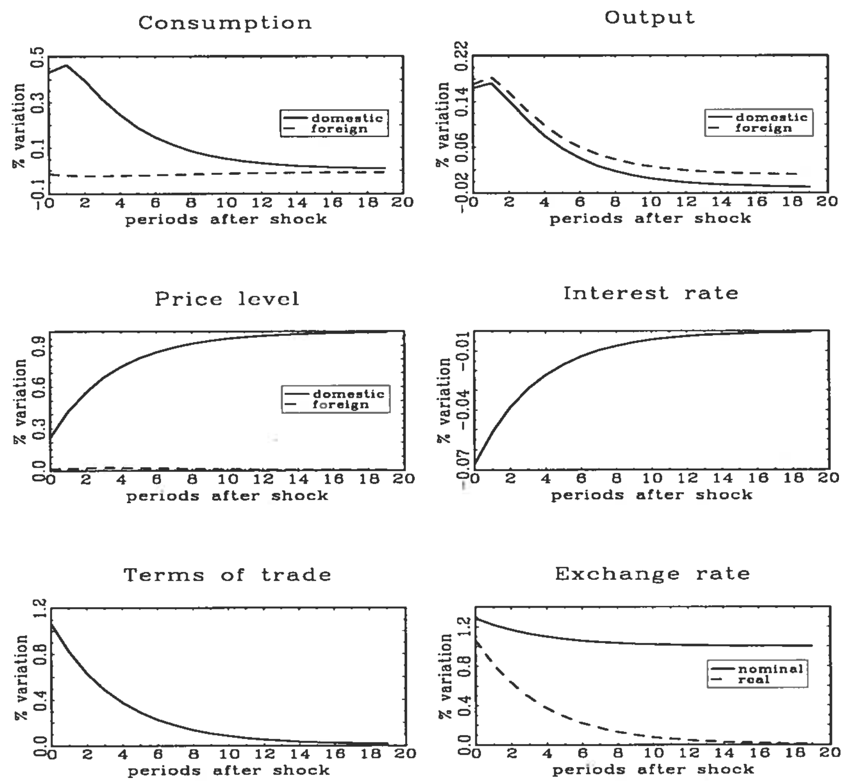

Figure 2.3: Impulse responses to a 1 per cent money-growth shock ($\gamma = 0.5$)

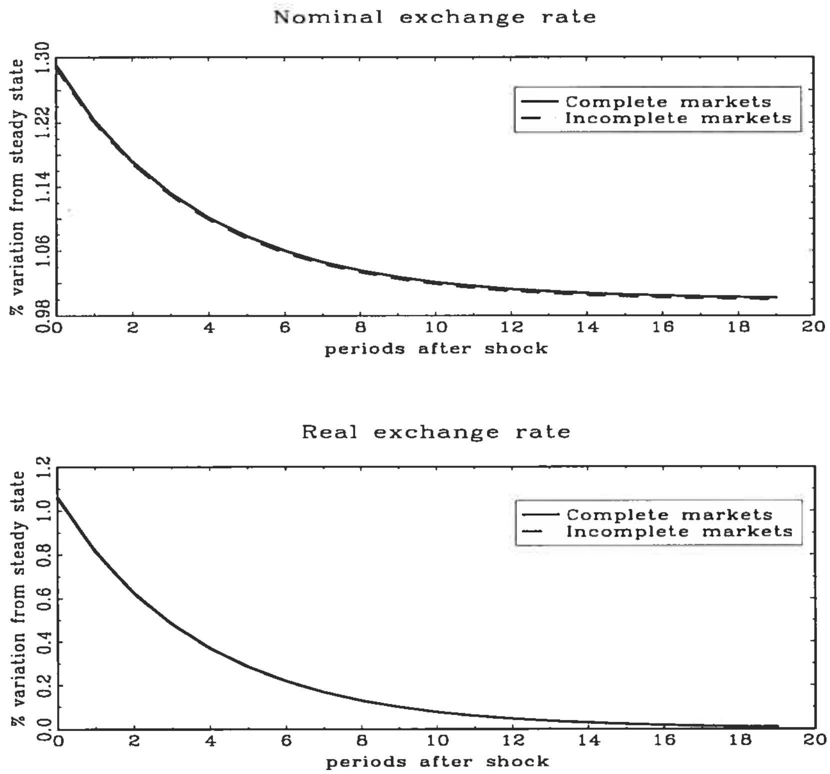

Figure 2.4: Impulse responses of nominal and real exchange rates to a 1 per cent money-growth shock under complete and incomplete markets

Essay 3

Nominal Rigidity, Desired Markup Variations, and Real Exchange Rate Persistence

1. Introduction

In recent years, a new line of research on exchange rate determination, pioneered by the seminal work of Obstfeld and Rogoff (1995), has developed. The new approach examines exchange rate dynamics within dynamic general-equilibrium (DGE) sticky-price models. Examples of studies that use this approach include Betts and Devereux (2000), Chari, Kehoe, and McGrattan (2000b), Bergin and Feenstra (2001), and Kollmann (2001). In each of these studies, price stickiness is motivated through monopolistic competition in the goods market, while departures from the purchasing-power parity (PPP) are due to the failure of the law of one price (LOP) in traded goods. The latter feature arises from pricing-to-market behaviour by monopolistic firms that segment markets by country.

A primary objective of the literature on exchange rate determination is to account for the well-documented volatility and persistence of the real exchange rate. Figure 3.1 illustrates these stylized facts in the case of the Can$/US$ real exchange rate. The logged and Hodrick-Prescott (H-P) filtered Can$/US$ real exchange rate has a relative standard deviation of 2.09 with respect to Canadian real GDP, and a serial correlation of 0.86.[1] Other bilateral real exchange rates with the U.S. dollar exhibit a similar degree of persistence and even higher volatility.[2] Overall, the above-noted studies have been successful in generating high real exchange rate variability. In particular, using a careful parameterization of risk aversion, Chari, Kehoe, and McGrattan (2000b) closely replicate the volatility observed in the data. But unless they assume an unreasonable level of price rigidity (for example, via excessively long nominal contracts), standard DGE sticky-price models fail to match real exchange rate persistence. Additional features such as the incompleteness of the financial

[1] These statistics are computed from quarterly data on the consumer price index (CPI)-based real exchange rate over the period 1975Q1–2001Q2.

[2] The average standard deviation (relative to that of output) of bilateral real exchange rates with the U.S. dollar for G-7 countries is about 4.8. See Bergin and Feenstra (2001).

market and labour market frictions are shown by Chari, Kehoe, and McGrattan (2000b) to be quantitatively ineffective in generating more persistence. Furthermore, Bouakez (2002) finds that habit formation in consumer preferences is irrelevant to exchange rate persistence.

In this paper, I construct a DGE sticky-price model in the spirit of Obstfeld and Rogoff (1995). Departing from their model where the elasticity of demand is assumed to be constant, I allow this elasticity to be time-varying. More specifically, I consider a variety aggregator that yields an elasticity of demand that is increasing in the relative price. This assumption may reflect search costs that cause a typical firm to lose more customers when it raises its price than it gains when it reduces its price by the same amount. As Stiglitz (1979), Woglom (1982), and Ball and Romer (1990) point out, this information imperfection leads to kinked (or bent) demand curves. Ball and Romer (1990), Kimball (1995), Bergin and Feenstra (2000), and Rotemberg and Woodford (1999) show that a demand function with a time-varying elasticity exacerbates the real effects of monetary shocks. Intuitively, an elasticity of demand that is increasing in the relative price means that the desired markup is decreasing in the relative price. Because a monopolistic firm will lower its desired markup whenever it raises its relative price, the increase in the relative price will be smaller than it would be if the elasticity of demand was constant. Hence, allowing for desired markup variations leads to additional price stickiness beyond that resulting from the exogenously imposed frictions. A corollary is that a large degree of nominal rigidity may be rationalized with a reasonable exogenous length of nominal contracts.

In a related work, Bergin and Feenstra (2001) construct a model that incorporates translog preferences and materials inputs. Their results show that these two features generate endogenous real exchange rate persistence, but not to the extent actually observed in the data. This paper differs from Bergin and Feenstra's in two main respects. First, the demand function considered in this paper is general and can exhibit any desirable degree of curvature. In contrast, the translog preferences that Bergin and Feenstra assume imply a limited curvature of the resulting demand function. Second, and more importantly, Bergin and Feenstra use calibration to assess the relevance of the key elements of their model. I, instead, derive an empirical model and obtain econometric estimates of the structural parameters. To the best of my knowledge, with the exception of a very recent paper by Bergin (2002), no previous studies have attempted to estimate DGE sticky-price models of exchange rate determination.[3]

The model is estimated via maximum likelihood (ML) using data on the Can$/US$ real exchange rate, the inflation differential between Canada and the United States, and the relative real money stock between the two countries. The results show that the model performs remarkably well in explaining in-sample real exchange rate dynamics. In particular, the model predicts the same autocorrelation found in the Can$/US$ real exchange rate series. Moreover, I find that, with a constant desired markup, prices have to be fixed for 18 quarters on average for the model to match real exchange rate persistence. If, however, one allows markups to vary by a reasonable amount, then the model generates the required persistence with a plausible duration of price contracts. Variance decomposition indicates that monetary shocks explain more than 40 per cent of real exchange rate variability in the short run and roughly 50 per cent of its unconditional variance.

The rest of this paper is organized as follows. Section 2 presents the theoretical model. Section 3 provides some intuition for the role of desired markup variations. Section 4 describes the estimation methodology and the data. Section 5 reports the empirical results and performs a robustness analysis. Section 6 concludes.

[3]Apart from using related estimation methodologies, this paper and the one by Bergin (2002) are distinctly different. In particular, Bergin (2002) estimates a small open economy model with price and wage rigidities and focuses on testing the theoretical model by comparing its likelihood with that of an unrestricted counterpart. In this study, I estimate a two-country sticky-price model with the aim of explaining real exchange rate persistence.

2. The Model

The model consists of two countries, each characterized by (i) a representative infinitely lived household, (ii) a representative final-good producer, (iii) a continuum of intermediate-good producers indexed by $i \in [0,1]$, and (iv) a government. A fraction n (respectively, $1-n$) of intermediate-good producers are located in the home (foreign) country. Intermediate goods are differentiated and are used to produce the final good in both countries. The final good is used exclusively for consumption and is not tradable between the two countries.

2.1 Households

The representative household in the home country has the following lifetime utility function:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s, m_s),$$

where E_t denotes the mathematical expectation conditional on the information available up to and including period t, β is the subjective discount factor $(0 < \beta < 1)$, and u is the instantaneous utility function. Households derive utility from consumption (c) and from holding real money balances (m).[4] The instantaneous utility function is assumed to be

$$u(c_t, m_t) = v(c_t) + \frac{\gamma}{1-\eta} m_t^{1-\eta},$$

where the function v satisfies $v'(c) > 0$ and $v''(c) < 0$, $m_t = M_t/P_t$, M_t is the nominal money stock, P_t is the aggregate price index, and γ is a positive parameter.[5]

Foreign money is not held by home-country residents and vice versa. Both countries' residents, however, can hold interest-bearing, one-period nominal bonds denominated in domestic currency. The nominal interest rate on bonds due at time $t+1$ is denoted by i_t.

[4] I assume that households do not derive utility from leisure. Hence, labour supply is perfectly inelastic in this model. I argue further (section 2.5) that this assumption is completely innocuous and does not affect the results of this study.

[5] The assumption of separability between consumption and money in the utility function is not crucial and could easily be relaxed.

The household's resources at the beginning of period t consist of money holdings set aside in period $t-1$ and the gross return on bonds purchased at time $t-1$. The household then receives a lump-sum transfer from the government. Next, the bonds market opens, allowing the household to purchase new nominal bonds. During period t, the household sells $l_t(i)$ units of labour to each intermediate-good producer $i \in [0, n]$ at the nominal wage, W_t. It also receives dividends $D_t(i)$ from each intermediate-good producer $i \in [0, n]$. The household allocates some of its income to consumption and carries the remaining units of money into period $t+1$.

The representative household's budget constraint, expressed in real terms, is

$$c_t + b_t + m_t \leq (1 + i_{t-1})b_{t-1}/\pi_t + m_{t-1}/\pi_t + w_t\bar{l} + d_t + \tau_t, \tag{1}$$

where $b_t = B_t/P_t$, B_t are nominal bond holdings, π_t is the gross inflation rate between $t-1$ and t, w_t is the real wage, $\bar{l} = \int_0^n l_t(i)di$ is the household's total endowment of time, $d_t = D_t/P_t$, $D_t = \int_0^n D_t(i)di$ are total dividends, and τ_t is a real lump-sum transfer.

The representative household in the foreign country has the following budget constraint:

$$c_t^* + b_t^*/e_t + m_t^* = (1 + i_{t-1}) \left(b_{t-1}^*/e_t\right)/\pi_t^* + m_{t-1}^*/\pi_t^* + w_t^*\bar{l}^* + d_t^* + \tau_t^*, \tag{2}$$

where the asterisk denotes variables in the foreign country and e_t is the nominal exchange rate, defined as the price of one unit of the foreign currency in terms of the home currency.

I assume that financial markets are complete, meaning that there exists a complete set of state-contingent Arrow-Debreu bonds that allow households in both countries to pool risks perfectly. Technically, under this assumption, the maximization problems of the domestic and foreign representative households are equivalent to a social planner's problem of maximizing a weighted sum of the utilities of both households subject to the world resources constraint expressed in domestic currency. Because households are identical within each country, the weight attached to the utility of each representative household corresponds to the population size in its country of origin.[6] The first-order necessary conditions associated with the optimal choice of $c_t, c_t^*, b_t, b_t^*, m_t$, and m_t^* for this problem are

$$\lambda_t = v'(c_t), \tag{3}$$

$$\lambda_t q_t = v'(c_t^*), \tag{4}$$

$$\lambda_t = \beta(1+i_t)E_t\left(\lambda_{t+1}/\pi_{t+1}\right), \tag{5}$$

$$\gamma m_t^{-\eta} = \lambda_t - \beta E_t\left(\lambda_{t+1}/\pi_{t+1}\right), \tag{6}$$

$$\gamma m_t^{*-\eta} = \lambda_t q_t - \beta E_t\left(\lambda_{t+1} q_{t+1}/\pi_{t+1}^*\right), \tag{7}$$

where λ_t is the Lagrange multiplier associated with the combined budget constraint and $q_t = e_t P_t^*/P_t$ is the real exchange rate.[7] Equations (3) and (4) imply the following risk-sharing condition:

$$v'(c_t^*)/v'(c_t) = q_t, \tag{8}$$

which states that, to the extent that the PPP holds, domestic and foreign households will enjoy the same level of consumption. Equation (5) is the standard Euler equation that prices nominal bonds. Equations (6) and (7) describe the optimal trade-off between consumption and money holdings. Equations (5) and (6) lead to the following money-demand equation:

$$\gamma m_t^{-\eta} = \lambda_t\left(\frac{i_t}{1+i_t}\right), \tag{9}$$

where the parameter η can be interpreted as the inverse of the interest elasticity of money demand.

2.2 The final-good producer

Final-good producers are perfectly competitive. They use the differentiated intermediate goods from both countries to produce a single, country-specific perishable commodity. I

[6]This weighting ignores initial wealth differences between the two countries.

[7]Differentiating the Lagrangian with respect to b_t^* leads to the same first-order condition as equation (5).

follow Kimball (1995) in assuming that the technology for producing the domestic final good is given implicitly by

$$1 = \int_0^n \psi\left(y_{ht}(i)/y_t\right) di + \int_n^1 \psi\left(y_{ft}(i)/y_t\right) di, \tag{10}$$

where y_t is the aggregate output, $y_{ht}(i)$ (respectively, $y_{ft}(i)$) is the input of intermediate good i produced in the home (foreign) country, and the function ψ satisfies $\psi(1) = 1$, $\psi'(x) > 0$ and $\psi''(x) < 0$, for all $x \geq 0$. It is assumed that exports are invoiced in the currency of the importing country. This assumption, often called local currency pricing (LCP), was introduced by Betts and Devereux (1996, 2000) into Obstfeld and Rogoff's (1995) model to characterize pricing-to-market behaviour by monopolistic firms. Pricing-to-market is the ability of a monopoly to set different prices in the home and foreign countries by somehow segmenting the market. Typically, this price discrimination leads to the violation the LOP among traded goods, and ultimately to a departure from the PPP. It is clear, though, that such behaviour is possible only if there are economic and/or institutional constraints that prevent consumers from taking advantage of international arbitrage opportunities in the goods market. Empirically, studies by Knetter (1989, 1993), Engel (1993), and Engel and Rogers (1996) seem to provide strong evidence in favour of pricing-to-market, as departures from PPP were found to reflect mainly the failure of the LOP between traded goods, rather than the presence of non-traded goods. Under the assumption of LCP, the final-good producer solves the following problem:

$$\underset{\{y_{ht}(i),\, y_{ft}(i)\}}{Min} \quad \textstyle\int_0^n P_{ht}(i) y_{ht}(i) di + \int_n^1 P_{ft}(i) y_{ft}(i) di,$$

subject to (10), where $P_{ht}(i)$ (respectively, $P_{ft}(i)$) is the price of intermediate-good i produced in the home (foreign) country. The solution of this problem yields the input demand of good i:

$$y_{jt}(i) = y_t \psi'^{-1}\left(\psi'(1) P_{jt}(i)/P_t\right), \tag{11}$$

where $j = h$ for $i \in [0, n]$ and $j = f$ for $i \in]n, 1]$. P_t is the aggregate price index given implicitly by

$$P_t = \int_0^n P_{ht}(i)\psi'^{-1}\left(\psi'(1)P_{ht}(i)/P_t\right) di + \int_n^1 P_{ft}(i)\psi'^{-1}\left(\psi'(1)P_{ft}(i)/P_t\right) di.$$

Let P_{ht} and P_{ft} denote, respectively, the price indexes of home and foreign intermediate goods sold in the home country.[8] Hence, the aggregate price index can be written as

$$P_t = nP_{ht} + (1-n)P_{ft}. \tag{12}$$

The problem of the representative foreign final-good producer is described in an analogous manner.

2.3 The intermediate-good producer

The representative firm i in the home country produces its differentiated good using the simple technology

$$y_t(i) \equiv y_{ht}(i) + y^*_{ht}(i) = h_t(i),$$

where $h_t(i)$ denotes labour input.[9] Intermediate-good producers are monopolistically competitive. Each firm faces a downward-sloping demand curve for its differentiated good in each country. Firm i chooses its (nominal) prices, $P_h(i)$ and $P^*_h(i)$, taking as given the aggregate demand and the price level in each country. Nominal prices are assumed to be sticky. Price stickiness is modeled *à la* Calvo (1983). That is, each period, some firms are randomly selected to set new prices for the home and foreign markets. The probability of being selected in any particular period is constant and is equal to $1 - \varphi$.

Let us denote by $\tilde{P}_{ht}$ and $\tilde{P}^*_{ht}$ the optimal prices set by a typical firm at period t in the home and foreign countries, respectively. It is not necessary to index $\tilde{P}_{ht}$ and $\tilde{P}^*_{ht}$ by firm,

[8] More precisely, P_{ht} and P_{ft} are defined as follows:
$P_{ht} \equiv \frac{1}{n}\int_0^n P_{ht}(i)\psi'^{-1}\left(\psi'(1)P_{ht}(i)/P_t\right) di$ and $P_{ft} \equiv \frac{1}{1-n}\int_n^1 P_{ft}(i)\psi'^{-1}\left(\psi'(1)P_{ft}(i)/P_t\right) di$.

[9] Labour market clearing requires that $\int_0^n h_t(i)di = n\bar{l}$.

because all of the firms that change their prices at a given time choose the same price (see Woodford 1996). The total domestic and foreign demands facing this firm at time s for $s \geq t$ are $\tilde{y}_{hs} = y_s \psi'^{-1}\left(\psi'(1)\tilde{P}_{ht}/P_s\right)$ and $\tilde{y}^*_{hs} = y^*_s \psi'^{-1}\left(\psi'(1)\tilde{P}^*_{ht}/P^*_s\right)$, respectively. The probability that $\tilde{P}_{ht}$ and $\tilde{P}^*_{ht}$ "survive" at least until period s, for $s \geq t$, is φ^{s-t}. Thus, the intermediate-good producer chooses $\tilde{P}_{ht}$ and $\tilde{P}^*_{ht}$ to maximize

$$E_t \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} \Lambda_{t,s} \left[\tilde{P}_{ht}\tilde{y}_{hs} + e_s \tilde{P}^*_{ht}\tilde{y}^*_{hs} - W_s(\tilde{y}_{hs} + \tilde{y}^*_{hs})\right],$$

where $\Lambda_{t,s}$ is the marginal utility of a dollar earned at time s relative to its marginal utility at time t. First-order conditions for this problem are

$$\tilde{P}_{ht} = \frac{E_t \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} \Lambda_{t,s}\theta_s(\tilde{y}_{hs}/y_s)W_s\tilde{y}_{hs}}{E_t \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} \Lambda_{t,s}\left(\theta_s(\tilde{y}_{hs}/y_s) - 1\right)\tilde{y}_{hs}}, \tag{13}$$

$$\tilde{P}^*_{ht} = \frac{E_t \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} \theta_s(\tilde{y}^*_{hs}/y^*_s)W_s\tilde{y}^*_{hs}}{E_t \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} \left(\theta_s(\tilde{y}^*_{hs}/y^*_s) - 1\right)e_s\tilde{y}^*_{hs}}, \tag{14}$$

where $\theta(.)$ is the elasticity of demand given by: $\theta(x) = -\frac{\psi'(x)}{x\psi''(x)}$.[10]

Note that, in the flexible-price case ($\varphi = 0$), the right-hand sides of equations (13) and (14) collapse to $\frac{\theta}{\theta-1}$, where $\theta \equiv \theta(1)$. That is, the optimal price in each country is set as a constant markup over the nominal marginal cost (the nominal wage, in this model). When prices are sticky ($0 < \varphi \leq 1$), however, the markup becomes dynamic, for two reasons. First, since the nominal wage is perfectly flexible while prices adjust only sluggishly, the markup unavoidably deviates from the flexible-price-equilibrium value of $\frac{\theta}{\theta-1}$. Second, price stickiness implies that the price set by a monopolistic firm in a given country at a given time is different from the aggregate price level in that country at that time. This means that the relative price of that firm is different from unity. Because the elasticity of demand depends on the firm's market share, or, equivalently, on its relative price, the desired markup, defined as $\nu(x) \equiv \frac{\theta(x)}{\theta(x)-1}$, varies whenever the economy deviates from the flexible-price equilibrium.

[10]To see this, note that from (11) we have $\frac{1}{\theta(y(i)/y)} \equiv -\frac{\partial \ln P(i)}{\partial y(i)}y(i) = -\frac{(y(i)/y)\psi''(y(i)/y)}{\psi'(y(i)/y)}$.

Clearly, this second source of markup variations (that is, variations in the desired markup) cannot arise in a model in which the elasticity of demand is constant.

Although it is fairly easy to construct a variety aggregator that leads to any desirable dependance of the elasticity of demand (and consequently the desired markup) on the firm's relative output (see Kimball 1995), I need not specify a functional form for ψ, since I will solve the model up to a first-order approximation. Instead, I need only specify the elasticity of the desired markup with respect to the firm's market share. This elasticity is assumed to be constant and is denoted by ξ ($\xi > 0$).

Assuming that price changes are independent across firms, the law of large numbers implies that $1 - \varphi$ is also the proportion of firms that set a new price each period. The proportion of firms that set a new price at time s and have not changed it as of time t (for $s \leq t$) is given by the probability that a time-s price is still in effect in period t. It is easy to show that this probability is $\varphi^{t-s}(1-\varphi)$. It follows that P_{ht} and P^*_{ht} can be written, respectively, as

$$P_{ht} = (1-\varphi) \sum_{s=-\infty}^{t} \varphi^{t-s} \tilde{P}_{hs} \psi'^{-1}\left(\psi'(1)\tilde{P}_{hs}/P_t\right), \tag{15}$$

$$P^*_{ht} = (1-\varphi) \sum_{s=-\infty}^{t} \varphi^{t-s} \tilde{P}^*_{hs} \psi'^{-1}\left(\psi'(1)\tilde{P}^*_{hs}/P^*_t\right). \tag{16}$$

2.4 The government

The government represents both the fiscal and monetary authorities in each country. There is no government spending or investment. Each period, the government makes lump-sum transfers to households. Transfers are financed by printing additional money in each period. Thus, the government budget constraint in the home country is

$$\tau_t = m_t - m_{t-1}/\pi_t. \tag{17}$$

Money is supplied exogenously by the government according to $M_t = \mu_t M_{t-1}$, where μ_t is the gross rate of money growth. In real terms, this process implies

$$m_t \pi_t = \mu_t m_{t-1}. \tag{18}$$

The rate of money growth, μ_t, is assumed to follow a first-order autoregressive process given by

$$\ln \mu_t = (1 - \rho^\mu) \ln \mu + \rho^\mu \ln \mu_{t-1} + \epsilon_{\mu,t}, \tag{19}$$

where ρ^μ is strictly bounded between -1 and 1, μ is the rate of money growth at the steady state, and $\epsilon_{\mu,t}$ is a normally distributed zero-mean disturbance with variance $\sigma^2_{\epsilon_\mu}$. Money-growth shocks are assumed to be non-correlated across countries. On the other hand, the first-order autocorrelation, ρ^μ, is assumed to be the same for both countries.

2.5 The log-linearized model

Since the model cannot be solved analytically, I follow the usual strategy of considering an approximate solution in the neighbourhood of the steady state. I do so by log-linearizing the equilibrium conditions around a zero-shock initial steady state in which all variables are constant. The steady state corresponds to a symmetric flexible-price equilibrium. From the log-linearized version of the model, it is easy to show that the real exchange rate (expressed as a percentage deviation from its steady-state value) is fully determined by the following four-equation system (see Appendix A for the derivation):

$$\hat{\mu}^d_t = \rho^\mu \hat{\mu}^d_{t-1} + \epsilon^d_{\mu,t}, \tag{20}$$

$$\hat{m}^d_t = \hat{m}^d_{t-1} - \hat{\pi}^d_t + \hat{\mu}^d_t, \tag{21}$$

$$E_t \hat{\pi}^d_{t+1} = \frac{1}{\beta} \hat{\pi}^d_t - \kappa \hat{q}_t, \tag{22}$$

$$E_t \hat{q}_{t+1} = \frac{1}{\beta} \hat{q}_t - E_t \hat{\pi}^d_{t+1} - \frac{\eta(1-\beta)}{\beta} \hat{m}^d_t, \tag{23}$$

where the circumflex denotes the percentage deviation of a variable from its steady-state value $[\hat{x}_t = (x_t - x)/x]$, the superscript d denotes the difference between home and foreign values of a given variable $[x_t^d = (x_t - x_t^*)]$, and $\kappa \equiv \frac{(1-\varphi)(1-\varphi\beta)}{\varphi\beta(1+\theta\xi)}$ is a positive parameter.

Equations (20) and (21) have straightforward interpretations: they are, respectively, the stochastic process for money growth and the money-supply equation, expressed in country differences. Equation (22) extends the standard closed-economy neo-Keynesian Phillips curve to a two-country framework. This equation, which might be interpreted as an international Phillips curve, stems from the combination of both countries' Phillips curves (equations (A.12) and (A.13)). Owing to openness, these curves depend not only on the domestic real marginal cost, as in a closed-economy set-up, but also on the foreign real marginal cost and the real exchange rate.[11] Because the domestic and foreign real marginal costs enter identically both countries' Phillips curves, these variables cancel each other out when the foreign Phillips curve is subtracted from the domestic one (or vice versa). The resulting equation is one that links the inflation differential to the real exchange rate. From the viewpoint of estimating the structural parameters β and κ within a single-equation model, equation (22) might be easier to estimate than the closed-economy Phillips curve, because a measure of the real exchange rate is more easily obtained than one for the real marginal cost. Finally, equation (23) ensues from the combination of money-demand equations in the two countries.[12] Note that equations (20)–(23) hold regardless of the degree of elasticity of labour supply by the households, which justifies our simplifying assumption of an inelastic labour supply. The model is also robust to the specification of the technology and the inputs used in the production of the intermediate goods. For example, allowing for capital accumulation in the model will not alter equations (20)–(23) in any way.

[11]See Razin and Yuen (2001) and Galí and Monacelli (2002) for a generalization of the neo-Keynesian Phillips curve in the context of an open economy.

[12]The key assumption in deriving equation (23) is the completeness of financial markets.

The log-linearized model (20)–(23) can be written as

$$\begin{bmatrix} \mathbf{x}_{t+1} \\ E_t\mathbf{p}_{t+1} \end{bmatrix} = \begin{bmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{x}_t \\ \mathbf{p}_t \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \otimes \epsilon_{t+1}, \tag{24}$$

where $\mathbf{x}_t = (\hat{\mu}_t^d, \hat{m}_{t-1}^d)'$ is a 2×1 vector that contains the state variables in the system, $\mathbf{p}_t = (\hat{\pi}_t^d, \hat{q}_t)'$ is a 2×1 vector that contains the forward-looking variables, $\epsilon_t = (\epsilon_{\mu,t}^d, 0)'$ is a 2×1 vector, and $\mathbf{a}_{ij}$ $i, j = 1, 2$ are 2×2 matrices whose elements are combinations of structural parameters. The Blanchard-Kahn (1980) method can be applied to (24) to obtain

$$\mathbf{x}_{t+1} = \mathbf{F}\mathbf{x}_t + \epsilon_{t+1}, \tag{25}$$

$$\mathbf{p}_t = \mathbf{Q}\mathbf{x}_t \tag{26}$$

where the matrices $\mathbf{F}$ and $\mathbf{Q}$ are 2×2 matrices that contain combinations of the elements of $\mathbf{a}_{ij}$, $i, j = 1, 2$.

3. Some Intuition

In this section, I provide some intuition about the role of desired markup variations and how they affect real exchange rate dynamics. For this purpose, I use impulse-response analysis to show how the response of the real exchange rate to a monetary shock depends on the parameter ξ. The first step is to assign plausible values to the remaining structural parameters. Hence, I set β to 0.99 so that the annual real interest rate in the steady state is about 4 per cent. The probability of not changing price in a given quarter, φ, is set to 0.75. This parameterization of φ is consistent with Taylor's (1999) conclusion, reached after he surveyed the empirical literature on price-setting, that prices are fixed for approximately four quarters on average in the United States. Following Kimball (1995), and in conformity with the empirical results of Basu and Fernald (1994), I choose $\theta = 11$.[13] I set η to 10,

[13]Basu and Fernald (1994) report markup estimates of about 10 per cent for U.S. data. This value implies that the elasticity of demand in the steady state (θ) is equal to $\frac{10}{10-1} = 11$.

which means that the interest elasticity of money demand is equal to 0.1, as estimated by Ireland (2001) and Christiano, Eichenbaum, and Evans (2001). This elasticity, however, has not been decisively estimated by previous empirical studies, as econometric estimates range from 0.05 in Mankiw and Summers (1986) to 0.39 in Chari, Kehoe, and McGrattan (2000a). Finally, the autocorrelation coefficient of the relative monetary shock, ρ^{μ}, is calibrated to 0.5.

The impulse-response functions generated by the model in response to a 1 per cent relative money-growth shock are depicted in Figure 3.2. These responses are computed for different values of ξ, ranging from 0 to 5. Figure 3.2 shows that, regardless of the magnitude of ξ, a positive money-growth shock triggers initial jumps in the inflation differential, real exchange rate, and relative real money stock. All three variables then return gradually to their steady-state values. As ξ increases, however, the dynamic paths of the variables become more persistent, as the initial effects of the shock take longer to die out. In addition, on impact, the real exchange rate depreciates more as ξ rises. Hence, allowing the desired markup to depend on the relative price seems to magnify the volatility and the persistence of the real exchange rate. To understand this result, it is useful to rewrite equation (22) as

$$\hat{\pi}_t^d = \beta E_t \hat{\pi}_{t+1}^d + \beta \kappa \hat{q}_t.$$

Because the parameter κ is decreasing in ξ, $\hat{\pi}_t^d$ is lower the higher is ξ, for any given values of $E_t \hat{\pi}_{t+1}^d$, $\hat{q}_t$, and the parameters β, φ, and θ. That is, the inflation differential jumps less (following a shock) as ξ increases, *ceteris paribus*. As expected, the top panel of Figure 3.2 clearly shows that rising ξ dampens the initial effect of the relative monetary shock on the inflation differential. To obtain the intuition for this result, it is instructive to examine the optimal pricing decisions that characterize the intermediate-good producers in the two countries (equations (A.4), (A.6), (A.7), and (A.8)). In each of these equations, the optimal relative price of a typical monopolistic firm is a decreasing function of ξ. Because the elasticity of demand is an increasing function of the firm's relative price (or, equivalently, the desired markup is an increasing function of the firm's market share), the re-optimizing firm is reluctant to charge a higher price following a positive monetary shock.[14] Since all monopolistic firms have less incentive to change prices by much with $\xi > 0$, the jumps in the aggregate price level are smaller and inflation is more inertial than in the case where $\xi = 0$. Thus, desired markup variations act as an additional source of price rigidity and lead to persistent effects of monetary shocks on real variables, including the real exchange rate. Note, however, that money neutrality still holds when prices are perfectly flexible, since the desired markup remains constant in this case. In other words, desired markup variations amplify the effects of monetary shocks only to the extent that prices are sticky. On the other hand, a given degree of price stickiness may be rationalized with a lower value of the probability of not changing price φ, once one allows for desired markup variations.

To gain further insight into how the interaction of price rigidity and desired markup variations increases real exchange rate persistence, I compute the autocorrelation of the simulated real exchange rate series, ϱ, for different combinations of φ and ξ. The resulting three-dimensional graph is plotted in Figure 3.3. This figure shows that ϱ increases non-monotonically with φ and ξ. In particular, holding φ constant, the gain in persistence from increasing ξ is larger when starting from relatively low values of this parameter. More importantly, Figure 3.3 suggests that, at least theoretically, it is possible to replicate any value of ϱ with an appropriate choice of φ and ξ. This is precisely what is illustrated in Figure 3.4, which depicts the combinations of φ and ξ that lead to the same value of ϱ. The resulting iso-persistence curves suggest that, eventually, the observed persistence of the real exchange rate can be replicated with reasonable values of φ, provided that ξ is sufficiently greater than zero.

[14]See Rotemberg and Woodford (1999) for further discussion.

4. Estimation Methodology and Data

The Blanchard-Kahn solution (25, 26) can be rewritten to collect the state variables into a *transition equation* and the observable variables into a *measurement equation*. This yields the following state-space representation of the model:

$$\mathbf{x}_{t+1} = \mathbf{F}\mathbf{x}_t + \epsilon_{t+1}, \tag{27}$$

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t, \tag{28}$$

where $\mathbf{y}_t = (\hat{\pi}_t^d, \hat{q}_t, \hat{m}_t^d)'$ and $\mathbf{H}$ is a 3×2 matrix that includes combinations of the structural parameters. Provided that there are at least as many shocks as observable variables, dynamic systems like (27, 28) can be estimated by ML using the Kalman filter to evaluate the likelihood function. The ML estimator obtained in this case would be consistent and asymptotically efficient. If the number of variables in the measurement equation exceeds the number of shocks, however, as it does here, the variance-covariance matrix of the residuals will be singular. One approach to circumvent this problem is to add measurement (non-structural) errors to the variables in the observation equation. Studies using this strategy include Altug (1989), McGrattan (1994), McGrattan, Rogerson, and Wright (1997) and Ireland (1999). Following these studies, I assume that the measurement errors are serially correlated. Hence, the model becomes

$$\mathbf{x}_{t+1} = \mathbf{F}\mathbf{x}_t + \epsilon_{t+1}, \tag{29}$$

$$\mathbf{y}_t = \mathbf{H}\mathbf{x}_t + \mathbf{u}_t, \tag{30}$$

$$\mathbf{u}_{t+1} = \mathbf{D}\mathbf{u}_t + \mathbf{e}_{t+1}, \tag{31}$$

where $\mathbf{u}_t = (u_{\pi,t}, u_{q,t}, u_{m,t})'$, $\mathbf{D}$ is a 3×3 diagonal matrix with elements ρ^π, ρ^q, and ρ^m (which are strictly bounded between -1 and 1), and the innovations $\mathbf{e}_t = (e_{\pi,t}, e_{q,t}, e_{m,t})'$ are assumed to be normally distributed with a zero mean and the following variance-covariance matrix:

$$\mathbf{V} = Var(\mathbf{e}_t\mathbf{e}_t') = \begin{bmatrix} \sigma^2_{e_\pi} & 0 & 0 \\ 0 & \sigma^2_{e_q} & 0 \\ 0 & 0 & \sigma^2_{em} \end{bmatrix}.$$

The model is estimated using Canadian and U.S. quarterly data ranging from 1975Q1 to 2001Q2.[15] The data are taken from International Financial Statistics, Statistics Canada's database, and the Federal Reserve Bank of St. Louis' database. The gross inflation rate is measured by the change in the CPI in each country. The real exchange rate is constructed by multiplying the nominal exchange rate, defined as the price of one U.S. dollar in terms of Canadian dollars, by the ratio of U.S. CPI to Canadian CPI. The real money stock in each country is measured by M2 divided by the CPI and expressed in per capita terms by dividing it by the civilian population age 16 and over. The real exchange rate and real money stock series are logged and H-P filtered, while the inflation series is logged and demeaned. The inflation differential ($\hat{\pi}^d$) and the relative real money stock ($\hat{m}^d$) are constructed by subtracting U.S. inflation and real money stock from their Canadian counterparts.

5. Empirical Results

5.1 Parameter estimates

From equations (20)–(23), it can readily be seen that the parameters φ, θ, and ξ cannot be identified. In addition, it turns out that the elements of the matrices $\mathbf{F}$ and $\mathbf{H}$ that are functions of β, η, and κ are such that it is impossible to identify these parameters. Because κ is our parameter of interest, and because there is a large consensus regarding the value of the subjective discount factor, on the one hand, and a range of empirical estimates of the interest elasticity of money on the other hand, I choose to fix β and η and to estimate

[15] One could argue that to consider Canada as part of a two-country framework might be inappropriate, because Canada is much smaller than the United States. Recall, however, that our theoretical model allows for country size asymmetries through the parameter n. The fact that n vanishes once the model is linearized and expressed in terms of country differences makes the empirical model (29)–(31) consistent with any country pair.

κ along with the parameters $\rho^{\mu}, \rho^{\pi}, \rho^{q}, \rho^{m}, \sigma_{\epsilon_{\mu}}, \sigma_{e_{\pi}}, \sigma_{e_{q}}, \sigma_{e_{m}}$. Based on the arguments made in section 3, I set β to 0.99 and η to 10. In section 5.4, however, I perform a sensitivity analysis to assess the robustness of the empirical results to the parameterization of η.

ML estimates and their corresponding standard errors are reported in Table 3.1. Standard errors are computed as the square root of the diagonal elements of the inverted Hessian of the (negative) log-likelihood function evaluated at the maximum. At the estimated parameters, the condition for the existence of a unique solution to the model is satisfied. That is, the number of explosive eigenvalues of the matrix $\mathbf{A} = [\mathbf{a}_{ij}]_{i,j=1,2}$ equals two, the number of non-predetermined variables.

The estimate of the parameter κ is equal to 0.0038. This value is an order of magnitude lower than the estimates found by Galí and Gertler (1999) for the case of a closed-economy Phillips curve. Using GMM and restricting β to be equal to unity, Galí and Gertler report estimates of κ of 0.035 and 0.007, depending on the way the orthogonality conditions are normalized. Although the estimate of κ has little informative value per se, it allows us to compute the combinations of φ and ξ that lead to the same likelihood function of the model, conditional on the value of θ. This is precisely what is reported in Table 3.2, where the pairs (φ, ξ) are computed assuming that $\theta = 11$. Table 3.2 shows that the estimated value of κ implies that the average duration of price contracts has to be about 4.5 years if $\xi = 0$.[16] This level of price rigidity is obviously highly implausible and cannot be reconciled with the empirical evidence on price-setting.

Assuming that prices are fixed for one year on average, the duration suggested by Taylor (1999), ξ must be equal to 1.97 for the model to generate the same level of real exchange rate persistence as when the elasticity of demand is constant and prices are fixed for 18 quarters on average. It remains to be seen whether this value of ξ is empirically plausible.

[16]The average length of price contracts is equal to $1/(1-\varphi)$.

Taking "a stab in the dark," Kimball (1995) suggests that ξ equals 4.28, implying that a 1 per cent rise in the firm's market share, which follows from a decline in its relative price, lowers the elasticity of demand from 11 to 8 (so that the desired markup increases from 1.1 to 1.1428). Kimball's parameterization of ξ has been criticized by Chari, Kehoe, and McGrattan (2000a), who argue that such a value of ξ implies an extremely convex demand function. To show this, they take a first-order approximation of the elasticity of demand in the neighbourhood of its steady-state value, θ. This yields (ignoring time and country subscripts)

$$\theta(y(i)/y) \simeq \theta - [1 + \theta - \chi]\,(y(i)/y - 1), \tag{32}$$

where $\chi = -h''(\psi'(1))\psi'(1)/h'(\psi'(1))$ is the curvature of the demand function evaluated at the steady state, and $h = \psi'^{-1}$. Simple calculation reveals that Kimball's parameterization implies that $\chi = 288$. To assess the implied convexity of the demand function, Chari, Kehoe, and McGrattan (2000a) take a second-order Taylor expansion series of the demand function at the steady state. The approximation results in

$$\psi'^{-1}\left(\psi'(1)P(i)/P\right) \simeq 1 - \theta(P(i)/P - 1) + \frac{\chi\theta}{2}(P(i)/P - 1)^2. \tag{33}$$

Equation (33) indicates that a value of -288 for the curvature parameter implies that a 2 per cent increase in the relative price leads to an 85 per cent decline in demand compared with a modest 22 per cent reduction that would occur if the elasticity of demand was constant. As Chari, Kehoe, and McGrattan point out, this level of convexity of the demand function is clearly unrealistically high.

In light of this result, it is natural to question whether my estimate of ξ results in a plausibly convex demand function. To answer this question, I go through the same steps described above. First, note that my estimated value of ξ means that the elasticity of demand decreases from 11 to 9.37 following a 1 per cent increase in the market share.

Using equation (32), it is easy to show that the resulting value of the curvature parameter χ is -153. Finally, equation (33) implies that a 2 per cent rise in the relative price yields a 55 per cent decline in demand. The latter value lies halfway between the lowest possible value of 22 per cent (which corresponds to the constant-elasticity-of-demand case) and the value of 85 per cent implied by Kimball's choice of ξ. While I do not claim that the convexity of the demand function implied by my estimate of ξ is indisputably plausible, it is certainly not as extreme as that suggested by Kimball's parameterization.

5.2 Fit of the model

This section assesses the ability of the estimated model to fit the data. In particular, I investigate whether the model can account for the dynamics of the Can$/US$ real exchange rate. Figure 3.5 plots the actual and predicted series of the model's endogenous variables. It shows that the model tracks the behaviour of the Can$/US$ real exchange rate and the relative real money stock remarkably well. The model does not explain as well, however, the movements of the inflation differential between Canada and the United States, as the predicted series looks smoother than the actual one. Going beyond the visual impression that Figure 3.5 provides, Table 3.3 reports the moments of the actual and predicted series of the model's variables, showing that the model matches exactly the autocorrelations of the real exchange rate and the relative real money stock. The predicted series of these variables, however, are slightly less volatile than the observed ones. As anticipated, the model is less successful in replicating the volatility of the inflation differential, although it still does a good job matching the historical autocorrelation of this series. Overall, the estimated model seems to fit the data considerably well. More importantly, the model is able to account for the persistence of the Can$/US$ real exchange rate. As stated in the previous section, if $\xi = 1.97$, then the model can replicate real exchange rate persistence using the assumption that prices are held fixed for four quarters on average.

5.3 Variance decomposition

Given that the fluctuations of the endogenous variables are driven by both structural and non-structural shocks, one might ask how much of these fluctuations is attributed to each type of shock. An assessment of the relative importance of monetary shocks in explaining real exchange rate movements is of primary interest. This issue has recently motivated a new line of research led by Clarida and Galí (1994), who use a structural vector autoregression to compute the variance decomposition of the real exchange rate. Clarida and Galí's identification strategy is based on long-run restrictions that are implied by a sticky-price two-country model inspired by Dornbusch (1976). Using data from Canada, Britain, Germany, and Japan, Clarida and Galí find that demand shocks explain most of the unconditional variance of the change in the real exchange rate. In the case of Canada, only 3 per cent of this variance is due to monetary shocks. Rogoff (1996) views Clarida and Galí's approach as promising. He criticizes their underlying theoretical model, however, stating that it "is based on the somewhat anachronistic Mundell-Fleming-Dornbusch IS-LM framework, rather than a modern sticky price intertemporal model." Because my estimated equations were derived within a dynamic optimizing general-equilibrium framework, Rogoff's criticism does not apply here.

Table 3.4 shows the variance decomposition of the forecast error of the real exchange rate. It indicates that, at horizons of less than one year, monetary shocks explain slightly more than 40 per cent of real exchange rate variability. This percentage rises steadily as the horizon increases. As the horizon approaches infinity, the conditional variance of the forecast error of a given variable converges to the unconditional variance of that variable. Table 3.4 shows that roughly 50 per cent of the unconditional variance of the Can$/US$ real exchange rate is attributed to monetary shocks. Overall, these results do not corroborate Clarida and Galí's findings.

5.4 Robustness analysis

Because, as stated earlier, there is no consensus on the precise magnitude of the interest elasticity of money demand, I check the sensitivity of my results to alternative calibrations of the parameter η. For this purpose, I estimate the model imposing values of η ranging from 1 to 20. Then, from each estimate of κ, I compute the implied value of ξ assuming that $\varphi = 0.75$, $\theta = 11$, and $\beta = 0.99$. The implied values of ξ are depicted in Figure 3.6. This figure shows that varying η in the range that yields plausible values of the interest elasticity of money demand has only minor effects on ξ. In fact, the implied value of ξ is significantly affected only for values of η that are lower than 2 (implying an interest elasticity of money demand above 0.5).

The robustness of the results can also be assessed by using an alternative measure of the aggregate price index. So far, the CPI has been used to compute the inflation rate and the real exchange rate, and to deflate the nominal money stock. As an alternative, I use the GDP deflator. Table 3.5 reports the estimation results based on data constructed using the GDP deflator. Overall, the results are similar to those reported in Table 3.1. Interestingly, however, the estimate of κ is higher than the one obtained using CPI-based data. This implies a lower value of ξ than the one implied by the estimate of κ in Table 3.1, for a given choice of the parameters φ, θ, and β. For example, assuming that $\varphi = 0.75$, $\theta = 11$, and $\beta = 0.99$, the estimate of κ implies that $\xi = 1.31$ (compared with 1.97). Thus, in this case, an average length of price contracts of one year can be rationalized with a smaller degree of curvature of the demand function. Therefore, one can conclude that the results are robust and, if anything, better when the GDP deflator is used as an alternative to measure the aggregate price index.

6. Conclusion

It is a well-established fact in international finance that real exchange rates are highly volatile and persistent. Standard DGE sticky-price models succeed in replicating the documented volatility, but fail to generate real exchange rates as persistent as in the data. This paper has constructed and estimated a DGE sticky-price model that allows for a time-varying elasticity of demand, which causes a firm's desired markup to vary whenever its relative price changes. Simulation results show that desired markup variations lead to additional nominal rigidity beyond that stemming from the exogenously imposed frictions in the goods market.

The model was estimated by the ML method using Canadian and U.S. data. The estimated model tracks the behaviour of the Can$/US$ real exchange rate remarkably well. In particular, the model is capable of matching exactly the persistence found in the real exchange rate series. More importantly, the model's success is achieved with a plausible duration of price contracts if one allows for a sufficiently convex demand function. Interestingly, I find that the level of convexity required to achieve enough persistence is not as extreme as in Kimball (1995). Yet, the fact that the model underpredicts inflation volatility may suggest that the endogenous rigidity that results from desired markup variations is too high. For this reason, one might suspect that the convexity of the demand function is still too high. Nonetheless, this study shows that allowing for desired markup variations in DGE sticky-price models is an important step towards a more complete model that could account for the joint behaviour of inflation and the real exchange rate.

Appendix A: Derivation of Equations (20)–(23)

Derivation of equation (20)

The gross rate of money growth being equal to unity in the steady state, equation (19) is linearized as

$$\hat{\mu}_t = \rho^{\mu}\hat{\mu}_{t-1} + \epsilon_{\mu,t}. \tag{A.1}$$

Subtracting from (A.1) its foreign counterpart yields

$$\hat{\mu}_t - \hat{\mu}_t^* = \rho^{\mu}\left(\hat{\mu}_{t-1} - \hat{\mu}_{t-1}^*\right) + \left(\epsilon_{\mu,t} - \epsilon_{\mu,t}^*\right),$$

which is equation (20) in the main text.

Derivation of equation (21)

Equation (18) is approximated as

$$\hat{m}_t = \hat{m}_{t-1} - \hat{\pi}_t + \hat{\mu}_t. \tag{A.2}$$

Taking the difference between (A.2) and its foreign counterpart results in

$$\hat{m}_t - \hat{m}_t^* = \left(\hat{m}_{t-1} - \hat{m}_{t-1}^*\right) - \left(\hat{\pi}_t - \hat{\pi}_t^*\right) + \left(\hat{\mu}_t - \hat{\mu}_t^*\right),$$

which is equation (21) in the main text.

Derivation of equation (22)

Dividing both sides of equation (13) by P_t and using the fact that $W_s/P_t = \left(\Pi_{k=t+1}^{s}\pi_k\right)w_s$, I obtain

$$\tilde{p}_{ht} = \frac{E_t\sum_{s=t}^{\infty}(\varphi\beta)^{s-t}\Lambda_{t,s}\theta_s(\tilde{y}_{hs}/y_s)\left(\Pi_{k=t+1}^{s}\pi_k\right)w_s\tilde{y}_{hs}}{E_t\sum_{s=t}^{\infty}(\varphi\beta)^{s-t}\Lambda_{t,s}\left(\theta_s(\tilde{y}_{hs}/y_s) - 1\right)\tilde{y}_{hs}}, \tag{A.3}$$

where $\tilde{p}_{ht} = \tilde{P}_{ht}/P_t$. This equation can be approximated as

$$\begin{aligned}
\widehat{\tilde{p}}_{ht} &= (1-\varphi\beta)\, E_t \sum\nolimits_{s=t}^{\infty} (\varphi\beta)^{s-t} \left(\hat{w}_s + \sum\nolimits_{k=t+1}^{s} \hat{\pi}_k + \frac{\hat{\theta}_s}{\theta - 1} \right) \\
&= (1-\varphi\beta)\, E_t \sum\nolimits_{s=t}^{\infty} (\varphi\beta)^{s-t} \left(\hat{w}_s + \sum\nolimits_{k=t+1}^{s} \hat{\pi}_k - \hat{\nu}_s \right) \\
&= (1-\varphi\beta)\, E_t \sum\nolimits_{s=t}^{\infty} (\varphi\beta)^{s-t} \left(\hat{w}_s + \sum\nolimits_{k=t+1}^{s} \hat{\pi}_k - \theta\xi \widehat{\tilde{p}}_{ht} \right) \\
&= \frac{1-\varphi\beta}{1+\theta\xi} E_t \sum\nolimits_{s=t}^{\infty} (\varphi\beta)^{s-t} \left(\hat{w}_s + \sum\nolimits_{k=t+1}^{s} \hat{\pi}_k \right),
\end{aligned}$$

which can be rewritten in the following recursive form

$$\widehat{\tilde{p}}_{ht} - \varphi\beta E_t \widehat{\tilde{p}}_{ht+1} = \frac{1-\varphi\beta}{1+\theta\xi} \hat{w}_t + \frac{\varphi\beta}{1+\theta\xi} E_t \hat{\pi}_{t+1}. \tag{A.4}$$

Similarly, dividing both sides of equation (14) by P_t^* and using the fact that $e_s P_t^*/P_t = \left(\Pi_{k=t+1}^{s} \pi_k\right)\left(\Pi_{k=t+1}^{s} \pi_k^*\right)^{-1} q_s$, yields

$$\tilde{p}_{ht}^* = \frac{E_t \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} \Lambda_{t,s} \theta_s (\tilde{y}_{hs}^*/y_s^*) \left(\Pi_{k=t+1}^{s} \pi_k\right) w_s \tilde{y}_{hs}^*}{E_t \sum_{s=t}^{\infty} (\varphi\beta)^{s-t} \Lambda_{t,s} \left(\theta_s (\tilde{y}_{hs}^*/y_s^*) - 1\right) \left(\Pi_{k=t+1}^{s} \pi_k\right) \left(\Pi_{k=t+1}^{s} \pi_k^*\right)^{-1} q_s \tilde{y}_{hs}^*}, \tag{A.5}$$

where $\tilde{p}_{ht}^* = \tilde{P}_{ht}^*/P_t^*$. Following the same steps involved in obtaining equation (A.4), it is easy to show that the approximation of equation (A.5) can be written as

$$\widehat{\tilde{p}}_{ht}^* - \varphi\beta E_t \widehat{\tilde{p}}_{ht+1}^* = \frac{1-\varphi\beta}{1+\theta\xi} (\hat{w}_t - \hat{q}_t) + \frac{\varphi\beta}{1+\theta\xi} E_t \hat{\pi}_{t+1}^*. \tag{A.6}$$

By analogy to (A.3) and (A.5), the pricing decisions by the foreign monopolistic firm are approximated by

$$\widehat{\tilde{p}}_{ft} - \varphi\beta E_t \widehat{\tilde{p}}_{ft+1} = \frac{1-\varphi\beta}{1+\theta\xi} (\hat{w}_t^* + \hat{q}_t) + \frac{\varphi\beta}{1+\theta\xi} E_t \hat{\pi}_{t+1}, \tag{A.7}$$

and

$$\widehat{\tilde{p}}_{ft}^* - \varphi\beta E_t \widehat{\tilde{p}}_{ft+1}^* = \frac{1-\varphi\beta}{1+\theta\xi} \hat{w}_t^* + \frac{\varphi\beta}{1+\theta\xi} E_t \hat{\pi}_{t+1}^*, \tag{A.8}$$

where $\tilde{p}_{ft} = \tilde{P}_{ft}/P_t$ and $\tilde{p}_{ft}^* = \tilde{P}_{ft}^*/P_t^*$.

Using equations (12), (15) and its foreign counterpart for P_{ft}, I obtain

$$P_t = (1-\varphi)\sum_{s=-\infty}^{t}\varphi^{t-s}\left[n\tilde{P}_{hs}\psi'^{-1}\left(\psi'(1)\tilde{P}_{hs}/P_t\right) + (1-n)\tilde{P}_{fs}\psi'^{-1}\left(\psi'(1)\tilde{P}_{fs}/P_t\right)\right]. \tag{A.9}$$

Dividing both sides of equation (A.9) by P_t results in

$$\begin{aligned} 1 &= (1-\varphi)\sum_{s=-\infty}^{t}\varphi^{t-s}\left[n\tilde{p}_{hs}\left(\Pi_{k=s+1}^{t}\pi_k\right)^{-1}\psi'^{-1}\left(\psi'(1)\tilde{p}_{hs}\left(\Pi_{k=s+1}^{t}\pi_k\right)^{-1}\right)\right. \\ &\left. +(1-n)\tilde{p}_{fs}\left(\Pi_{k=s+1}^{t}\pi_k\right)^{-1}\psi'^{-1}\left(\psi'(1)\tilde{p}_{fs}\left(\Pi_{k=s+1}^{t}\pi_k\right)^{-1}\right)\right]. \end{aligned}$$

The linearization of this equation yields

$$0 = (1-\varphi)\sum_{s=-\infty}^{t}\varphi^{t-s}\left[n\left(\widehat{\tilde{p}}_{hs} - \sum_{k=s+1}^{t}\hat{\pi}_k\right) + (1-n)\left(\widehat{\tilde{p}}_{fs} - \sum_{k=s+1}^{t}\hat{\pi}_k\right)\right],$$

or

$$\hat{\pi}_t = (1-\varphi)\sum_{s=-\infty}^{t}\varphi^{t-s-1}\left[n\left(\widehat{\tilde{p}}_{hs} - \sum_{k=s+1}^{t-1}\hat{\pi}_k\right) + (1-n)\left(\widehat{\tilde{p}}_{fs} - \sum_{k=s+1}^{t-1}\hat{\pi}_k\right)\right].$$

Subtracting $\varphi\hat{\pi}_t$ from both sides of this equation gives

$$\hat{\pi}_t = \frac{1-\varphi}{\varphi}\left[n\widehat{\tilde{p}}_{ht} + (1-n)\widehat{\tilde{p}}_{ft}\right]. \tag{A.10}$$

The foreign counterpart of equation (A.10) is

$$\hat{\pi}_t^* = \frac{1-\varphi}{\varphi}\left[n\widehat{\tilde{p}}_{ht}^* + (1-n)\widehat{\tilde{p}}_{ft}^*\right]. \tag{A.11}$$

Substituting (A.4) and (A.7) into (A.10) and rearranging, I obtain

$$E_t\hat{\pi}_{t+1} = \frac{1}{\beta}\hat{\pi}_t - \frac{(1-\varphi)(1-\varphi\beta)}{\varphi\beta(1+\theta\xi)}\left[n\hat{w}_t + (1-n)(\hat{w}_t^* + \hat{q}_t)\right]. \tag{A.12}$$

Similarly, equation (A.11), with (A.6) and (A.8) substituted in for $\widehat{\tilde{p}}_{ht}^*$ and $\widehat{\tilde{p}}_{ft}^*$, becomes

$$E_t\hat{\pi}_{t+1}^* = \frac{1}{\beta}\hat{\pi}_t^* - \frac{(1-\varphi)(1-\varphi\beta)}{\varphi\beta(1+\theta\xi)}\left[n(\hat{w}_t - \hat{q}_t) + (1-n)\hat{w}_t^*\right]. \tag{A.13}$$

Finally, subtracting (A.13) from (A.12) yields

$$E_t\left(\hat{\pi}_{t+1}-\hat{\pi}^*_{t+1}\right)=\frac{1}{\beta}\left(\hat{\pi}_t-\hat{\pi}^*_t\right)-\frac{(1-\varphi)\left(1-\varphi\beta\right)}{\varphi\beta(1+\theta\xi)}\hat{q}_t,$$

which is equation (22) in the main text.

Derivation of equation (23)

Linearizing the first-order conditions (6) and (7) yields, respectively,

$$\eta\hat{m}_t=\frac{\beta}{1-\beta}(E_t\hat{\lambda}_{t+1}-E_t\hat{\pi}_{t+1})-\frac{1}{1-\beta}\hat{\lambda}_t, \tag{A.14}$$

and

$$\eta\hat{m}^*_t=\frac{\beta}{1-\beta}(E\hat{\lambda}_{t+1}+E_t\hat{q}_{t+1}-E_t\hat{\pi}^*_{t+1})-\frac{1}{1-\beta}(\hat{\lambda}_t+\hat{q}_t). \tag{A.15}$$

Subtracting (A.14) from (A.15) and rearranging, I obtain

$$E_t\hat{q}_{t+1}=\frac{1}{\beta}\hat{q}_t-E_t\left(\hat{\pi}_{t+1}-\hat{\pi}^*_{t+1}\right)-\frac{\eta\left(1-\beta\right)}{\beta}\left(\hat{m}_t-\hat{m}^*_t\right),$$

which is equation (23) in the main text.

Table 3.1: Maximum-Likelihood Estimates

Parameter	Estimate	Standard error
κ	0.0038	0.0040
ρ^{μ}	0.2596	0.2406
ρ^{π}	0.4849	0.0886
ρ^{q}	0.8921	0.0536
ρ^{m}	0.9362	0.0323
σ_{ϵ_μ}	0.0041	0.0014
σ_{e_π}	0.0051	0.0003
σ_{e_q}	0.0119	0.0021
σ_{e_m}	0.0109	0.0008

Notes: The restrictions imposed on the parameters are $\kappa, \sigma_{\epsilon_\mu}, \sigma_{e_\pi}, \sigma_{e_q}, \sigma_{e_m} \in (0, \infty)$ and $\rho^{\mu}, \rho^{\pi}, \rho^{q}, \rho^{m} \in (-1, 1)$. Standard errors are the square root of the diagonal elements of the inverted Hessian of the (negative) log-likelihood function evaluated at the estimates.

Table 3.2: Combinations of φ and ξ that Yield the Same Value of the Likelihood Function

φ	$1/(1-\varphi)$	ξ
0.1	1.11	195.07
0.2	1.25	77.12
0.3	1.43	39.39
0.4	1.67	21.71
0.5	2	12.06
0.6	2.5	6.42
0.7	3.33	3.07
0.75	**4**	**1.97**
0.8	5	1.16
0.9	10	0.20
0.9448	**18.11**	**0**

Notes: $1/(1-\varphi)$ is the average length of price contracts in quarters. For each value of φ, the implied value of ξ is computed assuming that $\beta = 0.99$ and $\theta = 11$.

Table 3.3: Moments of Actual andPredicted Series of the Model's Variables

	Autocorrelation		Standard deviation (%)	
Variable	*Actual*	*Predicted*	*Actual*	*Predicted*
Inflation differential	0.49	0.52	0.59	0.29
Real exchange rate	0.86	0.86	3.18	2.88
Relative real money stock	0.92	0.92	3.39	3.15

Table 3.4: Variance Decomposition of the Real Exchange Rate

Horizon	Fraction of variance due to monetary shocks
1	0.4167
2	0.4268
3	0.4348
4	0.4417
8	0.4626
12	0.4723
24	0.4932
∞	0.4970

Table 3.5: Maximum-Likelihood Estimates using the GDP Deflator as a Measure of the Aggregate Price Index

Parameter	Estimate	Standard error
κ	0.0056	0.0049
ρ^{μ}	0.1766	0.1843
ρ^{π}	0.1681	0.1149
ρ^{q}	0.9006	0.0461
ρ^{m}	0.9299	0.0349
$\sigma_{\epsilon_{\mu}}$	0.0049	0.0012
$\sigma_{e_{\pi}}$	0.0048	0.0003
$\sigma_{e_{q}}$	0.0139	0.0016
$\sigma_{e_{m}}$	0.0089	0.0008

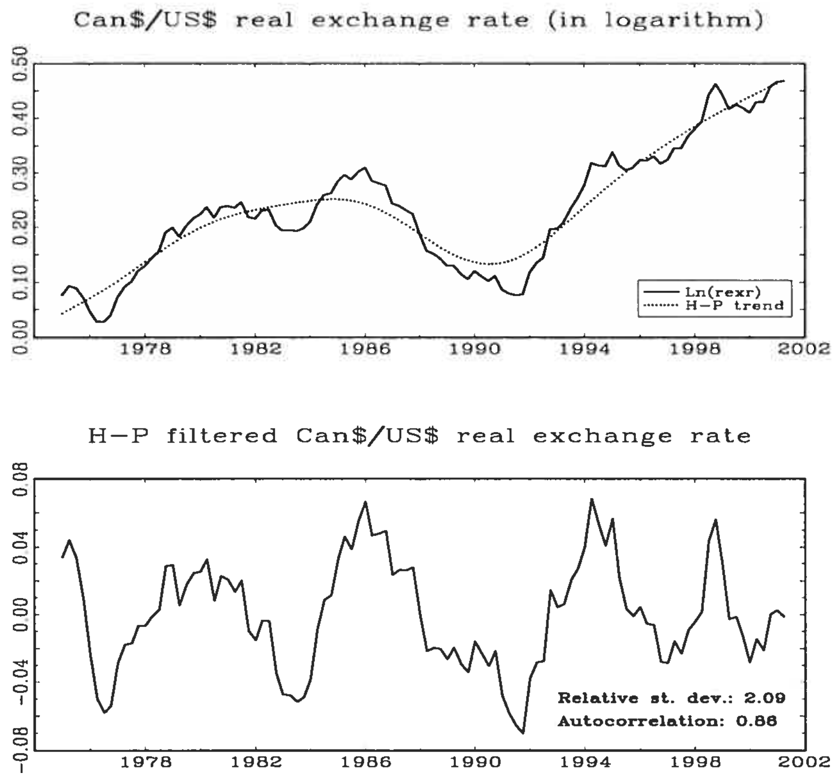

Figure 3.1: Can$/US$ real exchange rate properties

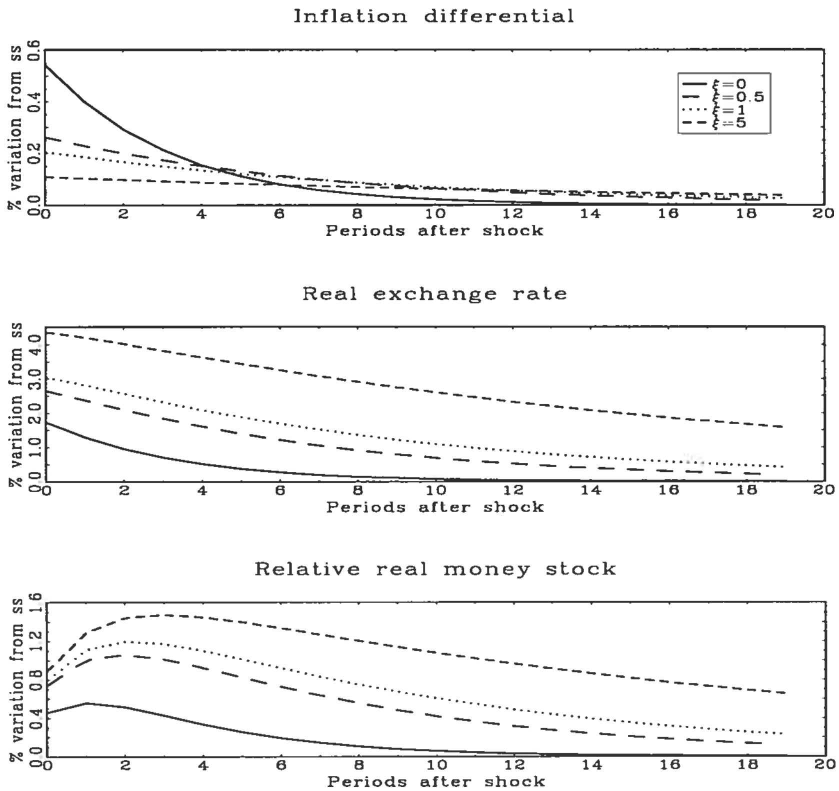

Figure 3.2: Impulse responses to a 1 per cent money-growth shock for different values of ξ

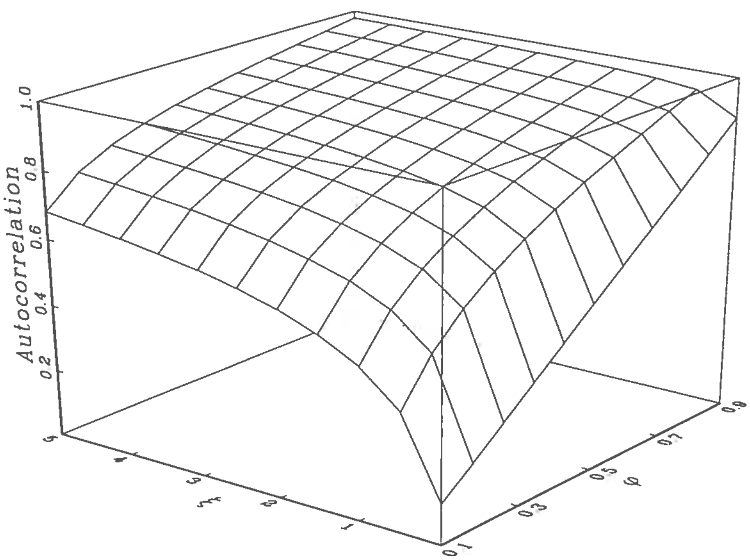

Figure 3.3: Real exchange rate persistence as a function of φ and ξ

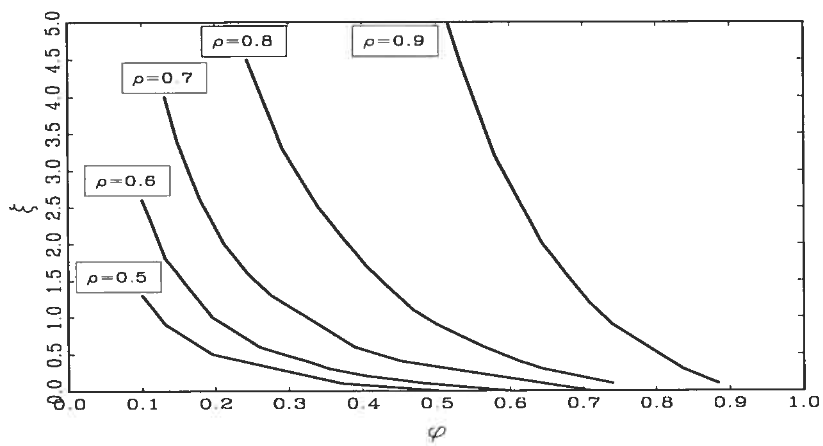

Figure 3.4: Iso-persistence curves

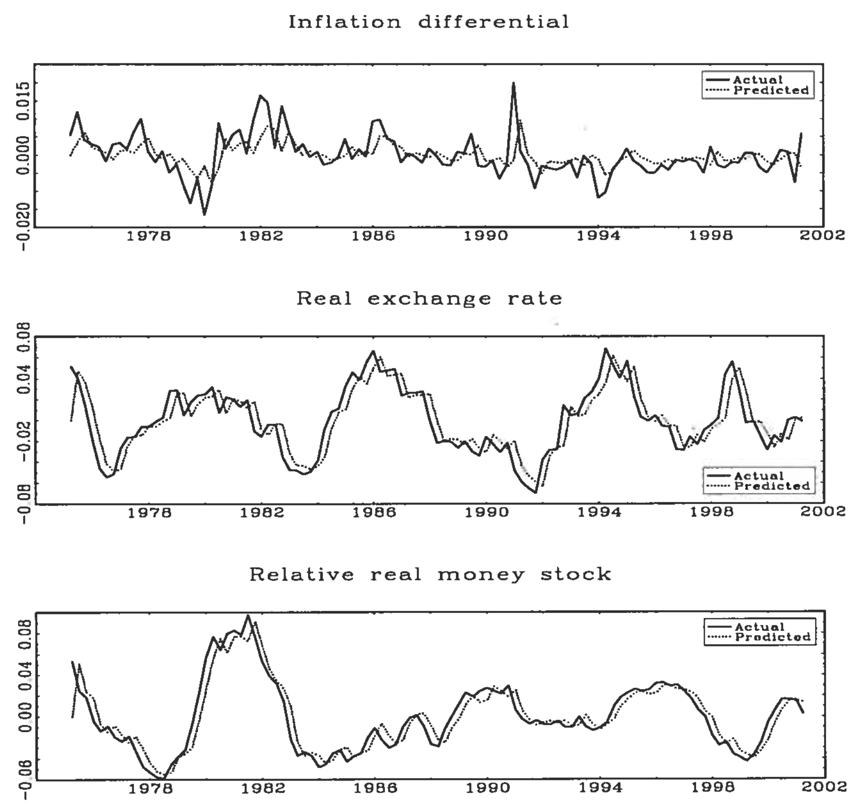

Figure 3.5: Actual vs. predicted values of endogenous variables

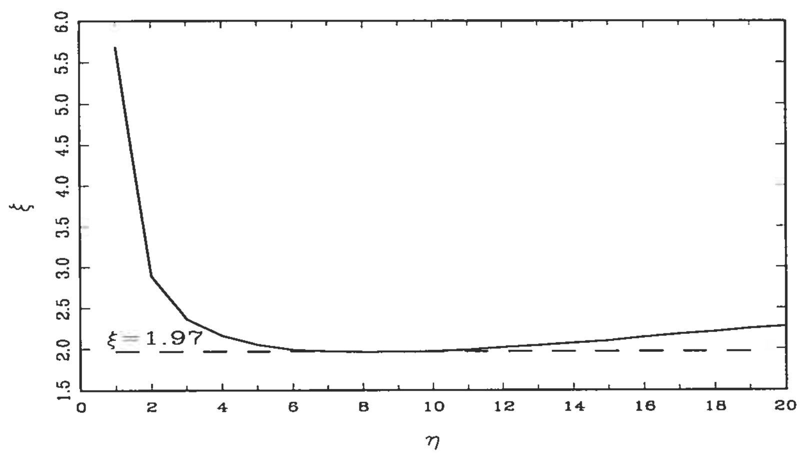

Figure 3.6: Sensitivity of the implied value of ξ to the parameter η (φ=0.75, θ=11, and β=0.99)

Conclusion générale

Dans cette thèse, nous nous sommes intéressés au problème de persistance; à savoir, l'incapacité des modèles DEG à prix rigides à générer des effets persistants des chocs monétaires sur la production et sur le taux de change réel. Cette anomalie, récemment soulevée par Chari, Kehoe et McGrattan (2000a, 2000b), empêche les modèles DEG à prix rigides de constituer un cadre adéquat d'analyse et d'évaluation des répercussions de la politique monétaire. Dans cette thèse, nous avons étudié certaines extensions du modèle standard à prix rigides, dont le but est d'en renforcer le mécanisme de propagation interne.

Dans le premier essai, nous avons incorporé la formation d'habitudes et les coûts d'ajustement du capital dans ce qui serait autrement un modèle standard d'économie fermée à prix rigides. Nous avons estimé le modèle par maximum de vraisemblance en utilisant des données américaines sur la production, la monnaie et le taux d'intérêt. Le modèle estimé reproduit fidèlement le profil du PIB américain sur la période d'estimation. L'analyse des fonctions de réponse montre que l'interaction de la formation d'habitudes et des coûts d'ajustement du capital est importante pour générer une réaction persistante et en forme de bosse de la production suite à un choc monétaire, comme celle obtenue à l'aide d'un vecteur autorégressif.

Néanmoins, si la formation d'habitudes s'est avérée pertinente pour rendre compte de la persistance de la production, nous avons montré dans le deuxième essai qu'elle ne peut remédier à l'incapacité des modèles à prix rigides d'économie ouverte à engendrer des taux de change réels persistants. Dans cet essai, nous avons étendu le modèle à deux pays de Betts et Devereux (2000) pour tenir compte de la formation d'habitudes. Les résultats des simulations ont montré que cette extension n'a aucun impact sur la dynamique du taux de change réel. Pour appuyer ce résultat et en saisir l'intuition, nous avons présenté une version légèrement modifiée du modèle, qui nous permettait d'établir la neutralité de la formation d'habitudes de manière analytique.

Dans le troisième essai, anous vons développé un modèle dynamique d'équilibre général à prix rigides où le taux de marge désiré de l'entreprise est une fonction croissante de son prix relatif. Cette caractéristique amplifie la rigidité nominale qui résulte des frictions imposées de manière exogène. Grâce à cette rigidité endogène, le modèle réussit à reproduire la persistance observée du taux de change réel avec une durée plausible des contrats de prix.

Par ailleurs, s'écartant de toutes les études antérieures ayant développé des modèles DEG à deux pays, nous n'avons pas calibré le modèle afin d'en évaluer la performance. Nous avons plutôt dérivé un modèle empirique dont nous avons estimé les paramètres. La prolifération des travaux théoriques portant sur les modèles DEG à prix rigides d'économie ouverte n'a pas été accompagnée par l'émergence d'une littérature parallèle sur l'estimation et la validation empirique de ces modèles. Le troisième essai de cette thèse constitue l'une des toutes premières tentatives de combler cette lacune.

En conclusion, cette thèse a fourni une résolution du problème de persistance qui handicape les modèles à prix rigides standard. Les modèles empiriques que nous avons développés expliquent bien le comportement de la production dans le cas d'une économie fermée, et celui du taux de change réel dans le cas d'une économie ouverte. Les modèles estimés ont, toutefois, quelques lacunes dont les plus sérieuses sont (i) leur insuccès à rendre compte des fluctuations de l'inflation, et (ii) leur incapacité à générer l'effet de liquidité dans le cas d'une économie fermée. Une extension naturelle des modèles présentés dans cette thèse serait de modéliser plus rigoureusement le mécanisme de détermination des prix, de manière à mieux décrire le comportement de l'inflation.

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