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**OPTIMAL WATER QUALITY:
A DYNAMIC MODEL IN GENERAL EQUILIBRIUM**

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ABSTRACT

The purpose of this paper is to explore how the optimal level of water quality might be established in an economic context. A conventional Arrow-Debreu type model of Pareto optimality is developed in static equilibrium. It is modified to show how pollution caused by industry (in this example) can lead to market distortions. The model is compared with several others that have appeared in the literature.

The standard result is then established, that government intervention, either in the form of taxes or quantitative controls, might be appropriate. A number of weaknesses in the model, such as the assumptions of pure competition and the availability of information, are then discussed in the context of current literature.

In the final part, the model is extended into the dynamic framework, which has not to my knowledge been done with a model specifically oriented towards environmental problems (and the particular relationship between externalities and public goods that this entails). The dynamic model, which is developed in goods space, is shown to be balanced throughout the procedure.

SOMMAIRE

L'objet de ce rapport est d'étudier comment le niveau optimal de la qualité de l'eau pourrait être établi dans un contexte économique. Un modèle conventionnel de l'optimum de Pareto du type Arrow-Debreu est développé en équilibre statique. Dans cet exemple, il est démontré que la pollution peut créer des distortions dans le marché. Par la suite, le modèle est comparé avec quelques autres déjà parus dans la littérature.

Il est établi que le résultat standard (intervention gouvernementale sous forme de taxe ou de rationnement quantitatif) pourrait être approprié. Quelques faiblesses dans le modèle, comme les hypothèses de concurrence parfaite et la disponibilité des informations nécessaires, sont discutées par la suite, dans le contexte de la littérature courante.

Dans la dernière partie du rapport, le modèle a été amené à l'état dynamique. A ma connaissance, il n'a pas été fait avec un modèle spécifiquement orienté vers les problèmes de l'environnement (et les relations particulières entre les externalités et les biens publics que cela implique). Il est démontré que le modèle dynamique, qui est développé dans l'espace des biens, est balancé en tout temps.

I - INTRODUCTION

Pollution first became a major issue about fifteen years ago. In surviving the tests of time, it has proved to be more than a fad, in fact, a question of continuing importance for citizens and policy makers alike.

Surprisingly, economics, often seen as the villain of the piece, has contributed to the durability of this issue, in particular as it relates to air and water pollution. In part, this is because pollution fitted neatly into neo-classical economic theory in the context of externalities. It has been demonstrated that pollution is bound to cause "economic inefficiency", which economists generally would like to see purged from the earth.

Definition of terms

Before pursuing the analysis, it is essential to be clear about what we mean by various terms, such as economic efficiency, externalities and public goods, as it is my intention to present a fairly rigorous exposition of these issues. A very brief historical synopsis of the development of the concepts is also presented.

Economic Efficiency

Economic efficiency describes a situation in which resources are allocated in an "optimal" way. The most common definition of optimality is one developed by Pareto, which states that an optimal allocation is one for which no

one can be made better off without someone else being made worse off. In effect every individual has a veto on any change.

This has led to the development of the compensation principle, introduced by Kaldor and Hicks. By this principle, the gainers must be able to compensate losers (even if they do not). Otherwise the basic definition is respected.

The definition of optimality and economic efficiency has a major weakness in that it is incapable of evaluating the merit of questions involving the distribution of income. If 99% of the world's resources were controlled by one person, no Pareto optimal reallocation could leave him worse off than before. Whenever applied to a real world situation, therefore, this system tends to support the status quo.

In the context of pollution of the environment, distributional issues have been much discussed in the context of property rights. Is there a right to pollute, or a right to a clean environment? Are there acquired rights to pollute? While these topics go beyond the scope of this paper, property rights have been a major topic of discussion in the economic literature, in particular in the comparison of taxation and subsidy schemes. I will make clear at what points in the discussion different assumptions about initial distribution of rights could yield different results.

Externalities

Externalities in economics are of two basic types, pecuniary and technological. This distinction goes back to Marshall <1920>, though he did not use these exact terms. (cf. Laffont <1977> pp. 14-18. He credits Meade <1952> for the clear cut distinction between the two.) Pecuniary externalities are those which manifest themselves through the price system. The standard example is scarce resource inputs in an industry, such that expansion causes the price of the input to rise. As a result the whole industry supply curve is not equal to the simple sum of individual firms' supply curves (which are constructed holding other things equal, a condition which does not hold when several or all firms expand at the same time). Since pecuniary externalities work through the market, they do not distort the allocation of resources.

The concern of this paper is with externalities which do cause a distortion in the allocation of resources, thereby causing inefficiency, in that a different allocation can be shown to produce greater satisfaction. In the case of pollution, this is one of the principal demonstrations of the next chapter, where efficiency is analyzed in a static Arrow-Debreu type model.

An externality may be said to exist when the utility (productivity) of one consumer (producer) is affected by the activity of another agent. This may give

rise to endless possibilities: Arrow <1969>, in a much cited article, develops a model in which each consumer's utility may be affected by every other consumer's consumption of each good.

He draws two implications from this: first that the pricing mechanism requires at least the possibility of excluding non-buyers from the use of the product, which may be prohibitively expensive. He cites pollution as his main example. The second implication is that the market in effect dissolves: each "commodity" has only one buyer and one seller, and one cannot rely upon competitive equilibrium to establish optimal pricing.

Laffont <1977> has an even more complex model in which every producer is also affected by the production of the others. It is also possible to imagine consumption affecting production and production affecting consumption. The model that will be developed here however is relatively confined: consumers' utility is affected by externalities associated with production. The rationale for its exact formulation is discussed anon.

Monopoly is the standard example of Pareto inefficiency. The study of the allocative impact of monopoly antedates Pareto's work, going back to at least Edgeworth (cf. Arrow <1969>). Knight <1924> was one of those responsible for producing a more rigorous analysis. My interest

externalities analysis, relates to the question of what happens when there is pollution in an economy with monopoly. Faced with two distortions, it is no longer a simple matter to determine whether intervention is warranted. While not mathematically analyzed with an extension of the model, this question is discussed in the context of the current literature in chapter III.

Pigou <1932> may not have been the first to use environmental examples in his discussion of what are now termed externalities, but he is generally credited with proposing state intervention to rectify the resulting distortions. As Coase <1960> points out in detail, Pigou's analysis is not rigorous, but has led nevertheless to a "Pigovian tradition" and "Pigovian taxes", by which the state restores an optimal allocation. Besides Meade <1952>, Plott <1966> added considerable precision to the discussion. The form that these taxes might take is presented in the model towards the end of chapter II; however as there are many misgivings, a discussion (in narrative form) is presented with regard to several principal problems that have emerged in the literature. In this regard the impact of certain assumptions, presented with the formal model, should also not be taken lightly.

Public Goods

A certain amount of confusion arises in the discussion of public goods relative to externalities, at

least as far as environmental quality and externalities are concerned. In his discussion of externalities Arrow <1969>, for example, called on Musgrave's exclusion principle <1959>, which is normally seen to apply to public goods.

There has been much discussion in the literature concerning the essential nature of a public good (Samuelson <1954, 1955>, Davis and Winston <1967>, Bradford <1970>, Milleron <1972> and Head <1977>). Two characteristics stand out: a public good is one for which the consumption by one individual does not reduce the amount available to another (non-rivalness), and the impossibility of price exclusion, which induces the "free-rider" problem. Head introduces a third criterion as well, non-rejectability, which he attributes to Shoup. This is in sense the corollary of the last. Each individual is obliged to consume the good at the same level as everyone else.

There has been some attempt to rescue the reputation of Lindahl, who published in German on the subject in 1919 (see, for example, Johansen <1963> and Foley <1970>). Nevertheless, Samuelson's rigorous analysis may be said to have put the analysis of public goods on a new footing with the publication of his articles in 1954 and 1955. In these he set up the conditions of optimality in the context of the Arrow-Debreu model, and clearly presented the problem of the free rider (the non-paying user). This model is easily recognizable in current microeconomic texts.

Characterization of the water quality variables

There is abundant choice for the structure of a model involving environmental variables, as environmental economics has virtually become a field of its own. The confusion as to whether the model relates to externalities or public goods is compounded here. Just as examples, Plott <1966>, Baumol and Oates <1971, 1975> and Teitenberg <1973a, 1973b> prefer to talk about externalities; Suchanek <1977, 1979> uses public goods; meanwhile Montgomery <1972>, Tulkens and Schoumaker <1975>, and Hamlen <1977> hedge their bets and use both.

In fact these models show striking similarities. If, for the sake of comparability, we restrict the discussion to the four general equilibrium models included above (Teitenberg <1973a, 1973b>, Baumol and Oates <1975>, Hamlen <1976> and Suchanek <1977 and 1979>), we find the following:

- i) All treat an environmental quality variable as a factor affecting consumers' utility;
- ii) Two (Teitenberg and Baumol and Oates) consider environmental quality also as an input factor in production, while the other two do not;
- iii) All consider pollution generation to be exclusively the result of productive activity;
- iv) All see environmental quality as the simple sum of pollutant emissions, except Teitenberg who introduces an exogenous vector of "nature-determined inputs" as well.

The confusion over terminology, to my mind, revolves around the fact that environmental quality itself is basically a public good by the definition above, whereas the pollutant emissions are characteristic of externalities. They are externalities of a special type in that it is producer activity that affects consumers or (in two of the above cases, at least) consumers and other producers. There are also numerous examples in the literature of pollution externalities (generated by firms) which affect only other firms, such as Davis and Winston <1962>, Flott <1966> and Mestelman <1982>.

In the light of the above, my formulation of the problem is fairly conventional. Pollution generation is associated with production, but it is only consumers that suffer a loss of benefits. My environmental quality equation is not very different either, in that pollution detracts from some pre-existing state-of-nature to yield present quality. This hardly impinges on the analysis mathematically, but it is perhaps a little more realistic.

An important difference between the usual models involving public goods and ones involving the environment is that the public good is normally produced, whereas environmental quality is diminished by productive activity. Maximization of environmental quality would require the shutting down of all industries for which total pollution control would be too costly to consider. This structure is

perhaps realistic, if maximizing environmental quality were the objective, and explains some of the rhetoric in the debate over "environment versus jobs".

It should be clear, however, that the approach taken in this paper is intended to help establish optimal water quality, rather than to maximize it. It is implicit that some pollution would persist: but social utility or satisfaction would be maximized. I must make clear however that the analysis (both static and dynamic) is short run only, and therefore cannot be applied to any substance which the natural environment cannot absorb and simply accumulates, such as certain man-made substances like polychlorinated biphenyls (PCBs).

In chapter II, I present the basic model in static equilibrium. Chapter III is a narrative discussion of some of the limitations of Pigovian taxes (or their dual, quantitative controls). In chapter IV, I develop a dynamic version of the model, which I have never seen done despite the abundant work in static analysis. Finally, in chapter V, I bring together a number of conclusions based on this work. All the works cited are listed in chapter VI.

II - THE STATIC MODEL

This part is divided into three sections. In the first section the conditions of a Pareto optimum are established in an economy in which water pollution (effluent) is a normal "by-product" of the process of producing private goods. Private goods are such that the person consuming them receives the full benefits of consumption. However the effluents degrade water quality, which is, together with the level of private goods consumption, a determinant of individual welfare.

The less the water is polluted, the more satisfaction people get. The quality of the water may be considered a "public good". The "use" of water by one person has no effect on the quality available to another, unless the user pollutes the water. Price exclusion is not possible, and there is one level of water quality which everybody receives.

At the end of the first section the marginal substitution rates between goods that would result in a Pareto optimum in the economy are established for consumers and producers. This is done for all goods in the economy, including water quality and effluent levels. As a Pareto optimum is a state for which no other state can be found that would be preferable to it, given initial resources, technology and distribution. It must also be an equilibrium. An equilibrium is a situation in which no agent is

interested in altering his consumption or production choices, given the decisions of others.

In the second section, the impact of a price system based only on private goods is studied. This might be seen as the "pre-control" situation, where firms are permitted to dump effluents into the environment without costs or limitations. It is shown that an equilibrium may be established, but one that is non-optimal, in that it is different from the one in which all goods figured in the calculation.

In the third section there is an analysis of what the government would have to do to restore the optimum conditions in an otherwise private market economy. Systems of ("Pigovian") effluent taxes or quantitative limits suggest themselves.

Optimal conditions

An economy consists of I consumers and J producing firms. The firms produce K varieties of private goods, while generating L different wastes. The consumers may be individually identified by the index i ($i = 1, 2, \dots, I$); the producers by the index j ($j = 1, 2, \dots, J$); the private goods by the index k ($k = 1, 2, \dots, K$); and the effluents by the index l ($l = 1, 2, \dots, L$).

Consumers receive satisfaction from their consumption of private goods and the quality of the water. For any individual, this may be characterized as:

$$u^i = u^i(x^i, z) \quad \text{where } x^i = (x_1^i, x_2^i, \dots, x_k^i, \dots, x_k^i)$$

$$\text{and } z = (z_1, z_2, \dots, z_1, \dots, z_L) \quad \langle 1 \rangle$$

x_k^i is the amount of good k consumed by individual i , and z is a vector representing the quality of the environment in L dimensions. Every type of waste defines a parameter of water quality, e.g. mercury content.

Producers are assumed to be technically efficient, and are constrained by the production function:

$$f^j(y^j, -r^j) = 0 \quad \text{where } y^j = (y_1^j, y_2^j, \dots, y_k^j, \dots, y_k^j)$$

$$\text{and } r^j = (r_1^j, r_2^j, \dots, r_1^j, \dots, r_L^j) \quad \langle 2 \rangle$$

y_k^j defines net outputs of producer j of each good k , as in conventional neoclassical nomenclature. Negative quantities indicate inputs. This explains the negative sign of the second term r^j , which is the amount of the various L effluents produced by firm j . In the literature, effluents are sometimes considered as inputs, sometimes as joint outputs. As inputs they carry a negative sign (as a quantity), but as outputs they have a negative value. The former treatment is perhaps more appropriate, in that they are not a desired output, but a (perhaps) necessary input. This approach also permits a simple treatment of the question of environmental quality.

The vector e ($e = (e_1, e_2, \dots, e_1, \dots, e_L)$) represents the "state of nature" in the L dimensions of the various effluents. (Thus there may be naturally existing concentrations of mercury in the water.) Hence:

$$z_l = e_l - \sum_{j=1}^J r_l^j \quad \text{where } l = 1, 2, \dots, L \quad \langle 3 \rangle$$

z is a vector representing the actual level of water quality. It depends on the pre-existing state of nature and the sum of all the effluents of the J producers.

Finally, to complete our characterization of a simple economy, we must suppose that total consumption can never exceed production plus initial resources:

$$\sum_{i=1}^I x_k^i = \sum_{j=1}^J y_k^j + \omega_k \quad \text{where } k = 1, 2, \dots, K \quad \langle 4 \rangle$$

ω represents the initial resources of the K private goods.

Before going ahead it is worth summarizing the more important assumptions implicit in the model. One is the assumption of convexity in the production and consumption sets. For the firm convexity implies that if a given output can be produced by either of two sets of inputs, then it must be possible to produce the same output with an average of the two sets of inputs. For the consumer, the same reasoning applies to his utility with regard to two bundles of goods.

While the convexity of the production and consumption sets is a widely used assumption in economics, there is considerable concern over whether it should be assumed to apply where externalities are involved. Starrett <1972>, Baumol and Bradford <1972>, and Slater <1975> in particular demonstrate the weakness of this proposition. A

simple example is thresholds of water quality above which new water uses are possible, such as fishing or swimming. If there is more than one such threshold, the convexity (established by the second order conditions) is violated. This assumption is, however, retained in the model developed here.

Another assumption is that there are no consumption or production externalities (except for the pollution externalities under study). For example the pollution generated by one firm is assumed to have no impact on other producers. Davis and Whinston <1962> explore some of the implications of this assumption, whereas Beavis and Walker <1979> study the impact of synergistic effects between pollutants.

Another important assumption is that transaction costs are nil. Initially, no institutional framework is introduced, thus the nature of the process that might lead to the optimum is unconstrained but unknown. Any institutions, such as the market or a centralized planning system, are bound to manifest transaction costs. As these may vary with the type of transaction, one would suppose that the optimum defined in a model with these costs considered would be different from one where they are not.

Returning to the model, the conditions for an optimum are established by manipulating the partial derivatives of the Lagrangian expression. Setting these equal to

zero defines the maximum, given the constraints specified in the equation:

$$L = \sum_{i=1}^I \alpha^i u^i(x^i, z) - \sum_{j=1}^J \phi^j \{f^j(y^j, -r^j)\} \\ + \sum_{l=1}^L \lambda_l \{e_l - z_l - \sum_{j=1}^J r_l^j\} - \sum_{k=1}^K \psi_k \{ \sum_{i=1}^I x_k^i - \sum_{j=1}^J y_k^j - \omega_k \}$$

where $x^i = (x_1^i, x_2^i, \dots, x_k^i, \dots, x_L^i)$, $z = (z_1, z_2, \dots, z_1, \dots, z_L)$ and $r^j = (r_1^j, r_2^j, \dots, r_1^j, \dots, r_L^j)$ <5>

The first order conditions are:

$$\frac{\partial L}{\partial x_k^i} = \alpha^i \frac{\partial u^i}{\partial x_k^i} - \psi_k = 0 \quad <6>$$

$$\frac{\partial L}{\partial y_k^j} = -\phi^j \frac{\partial f^j}{\partial y_k^j} + \psi_k = 0 \quad <7>$$

$$\frac{\partial L}{\partial z_1} = \sum_{i=1}^I \alpha^i \frac{\partial u^i}{\partial z_1} - \lambda_1 = 0 \quad <8>$$

$$\frac{\partial L}{\partial (-r_1^j)} = \phi^j \frac{\partial f^j}{\partial (-r_1^j)} - \lambda_1 = 0 \quad <9>$$

These basic relationships can be transformed into the well known marginal rates of substitution (MRS) of consumers and rates of technical substitution (RTS) of producers between different goods. These substitution rates characterize the optimum. Setting $k=1$ as the numeraire,

$$\text{from } <6> \quad \alpha^i \frac{\partial u^i}{\partial x_1^i} = \psi_1 \quad <6'>$$

$$<6> \div <6'> \quad \frac{\alpha^i \frac{\partial u^i}{\partial x_k^i}}{\alpha^i \frac{\partial u^i}{\partial x_1^i}} = \frac{\psi_k}{\psi_1} = MRS_k^i, \quad <10>$$

$$\text{From } <7> \quad \phi^j \frac{\partial f^j}{\partial y_1^j} = \psi_1 \quad <7'>$$

$$\langle 7 \rangle + \langle 7' \rangle \quad \frac{\phi^j \frac{\partial f^j}{\partial y_k^j}}{\frac{\partial f^j}{\partial y_k^j}} = \frac{\psi_k}{\psi_j} = RTS_k^j, \quad \langle 11 \rangle$$

$$\text{From } \langle 6' \rangle \quad \alpha^k = \frac{-\psi_k}{\frac{\partial u^i}{\partial x_k^i}}$$

$$\text{Substituting into } \langle 8 \rangle \quad \frac{I}{\sum_{i=1}^I} \frac{-\psi_k}{\frac{\partial u^i}{\partial x_k^i}} \cdot \frac{\partial u^i}{\partial z_1} = \lambda_1$$

$$\text{Therefore } \frac{I}{\sum_{i=1}^I} \frac{-\frac{\partial u^i}{\partial z_1}}{\frac{\partial u^i}{\partial x_k^i}} = \frac{\lambda_1}{\psi_j} = \frac{I}{\sum_{i=1}^I} MRS_{k1}^i, \quad \langle 12 \rangle$$

$$\text{From } \langle 7' \rangle \quad \phi^j = \frac{-\psi_j}{\frac{\partial f^j}{\partial y_k^j}}$$

$$\text{Substituting into } \langle 9 \rangle \quad \frac{-\psi_j}{\frac{\partial f^j}{\partial y_k^j}} \cdot \frac{\partial f^j}{\partial (-r_1^j)} = \lambda_1$$

$$\text{Therefore } \frac{\frac{\partial f^j}{\partial (-r_1^j)}}{\frac{\partial f^j}{\partial y_k^j}} = \frac{\lambda_1}{\psi_j} = RTS_{k1}^j, \quad \langle 13 \rangle$$

Between the various K private goods, the results are strictly conventional: $MRS_{k1}^i = RTS_{k1}^i$, for all I consumers and J producers. However there is a distinct difference in the results for water quality and effluents. Optimal conditions are met only when the sum of all the consumers' MRS relative to a given environmental parameter e_1 equal the RTS for each producer.

If the system is normalized by setting $\psi_k = 1$, the interpretation of the lagrangian multipliers ψ_k and λ_1 in $\langle 10 \rangle$, $\langle 11 \rangle$, $\langle 12 \rangle$ and $\langle 13 \rangle$ becomes straightforward. They are the implicit prices (in terms of $k = 1$) of the remaining private goods and all of the pollutants. Thus λ_1 is the

value of a marginal change in z_1 (and r_1) to consumers (and producers).

With the Lagrangian multipliers λ_1 interpreted as prices, Teitenberg <1973>, and Dasgupta and Heal <1979>, among others, have shown that the solution to a similar model represents a stable equilibrium, given competitive behaviour. I will not make that proof here.

The apparent simplicity of the mathematical formulations hides a number of problems, such as how these rates are to be discovered and applied. The latter question will be dealt with first. The former question is essentially the subject of chapter IV.

A price system for private goods only

At this point no institutional structures such as markets, prices, planning agencies or whatever have been used in the model. Nevertheless the terms of substitution between any two goods, public or private, for consumers and producers, have been established for a Pareto optimum.

In the world, two principal methods of economic organization are used, which are commonly known as capitalism and socialism. A capitalist economy uses primarily markets and private ownership to allocate resources, while the socialist model uses planning and public ownership.

The next question is, does there exist a system of

prices (for private goods) within the context of a capitalist economy which would be an equilibrium? The interest of such a question lies in the evolution of the market economies. Private goods were priced; externalities (such as effluents) and public goods (such as water quality) were not. The impact of such a situation will now be demonstrated.

The economy retains its basic structure (equations <1> to <4> above). An additional assumption must be introduced, and that is that firms behave competitively and accept market prices. It is clear that a non-optimal allocation would result if this were not the case (Buchanan <1969>, Asch and Seneca <1975>, and Lee <1975>).

We now suppose that the consumer has an income of R^1 (without worrying about its source). His problem is to maximize his utility u , given a vector of prices p ($p = (p_1, p_2, \dots, p_k, \dots, p_K)$), with a price corresponding to each of the K goods. This may be expressed as:

$$L = u(x) - \mu \left(\sum_{k=1}^K p_k x_k - R \right)$$

where $x = x_1, x_2, \dots, x_k, \dots, x_K$ <14>

The second term constrains his expenditures to his level of revenues.

As far as the consumer is concerned environmental quality is an exogenous variable, hence he is unable to adjust it to improve his utility. Thus the remaining first order (maximizing) conditions are:

$$\frac{\partial L}{\partial x_k} = \frac{\partial u}{\partial x_k} - \mu p_k = 0 \quad \langle 15 \rangle$$

Therefore $\frac{\partial L}{\partial x_1} = \frac{\partial u}{\partial x_1} = \mu p_1 \quad \langle 16 \rangle$

$$\langle 15 \rangle \div \langle 16 \rangle \quad \frac{\frac{\partial u}{\partial x_k}}{\frac{\partial u}{\partial x_1}} = \frac{p_k}{p_1} = MRS_{k1} \quad \langle 17 \rangle$$

For his part the producer attempts to maximize profits and is constrained by his production function:

$$L = \sum_{k=1}^K p_k y_k - \phi\{f(y, -r)\} \quad \text{where } y = (y_1, y_2, \dots, y_k, \dots, y_K)$$

and $r = (r_1, r_2, \dots, r_1, \dots, r_L) \quad \langle 18 \rangle$

The first order conditions are:

$$\frac{\partial L}{\partial y_k} = p_k - \phi \frac{\partial f}{\partial y_k} = 0 \quad \langle 19 \rangle$$

$$\frac{\partial L}{\partial (-r_1)} = \phi \frac{\partial f}{\partial (-r_1)} = 0 \quad \langle 20 \rangle$$

From $\langle 19 \rangle \quad \phi \frac{\partial f}{\partial y_1} = p_1 \quad \langle 21 \rangle$

$$\langle 19 \rangle \div \langle 21 \rangle \quad \frac{\phi \frac{\partial f}{\partial y_k}}{\phi \frac{\partial f}{\partial y_1}} = \frac{p_k}{p_1} = RTS_{k1} \quad \langle 22 \rangle$$

But $\langle 20 \rangle \div \langle 21 \rangle \quad \phi \frac{\frac{\partial f}{\partial (-r_1)}}{\frac{\partial f}{\partial y_1}} = \frac{0}{p_1} = RTS_{11}$ $\langle 23 \rangle$

A price system defined by the vector p will establish equilibrium conditions throughout the economy, however from $\langle 23 \rangle$ it is evident that this equilibrium will not be optimal, unless $\lambda_1 = 0$ in equation $\langle 13 \rangle$. However this possibility is excluded by the form of $\langle 1 \rangle$ and $\langle 2 \rangle$.

It is implicit in the model that the producers suffer none of the loss caused by their pollution: thus they will pollute up to the point where there is no further profit to be gained. Such a point will not be infinite, since there will be decreasing returns on other inputs. Meanwhile consumers will be exposed to extensive degradation of environmental quality. As an equilibrium, it is not very enticing.

Government intervention

We now suppose that our market oriented society has recognized the pollution problem and found it expedient to create a governmental Authority, empowered to impose taxes or output constraints on consumers or producers.

There are a variety of ways in which the Authority could respond to the above situation: it could tax the producers directly, or it could tax consumers and use the proceeds to bribe the producers. There is considerable debate about the symmetry of these two solutions: see Kamien, Schwartz and Dolbear <1966>; and Porter <1974>; Dewees and Sims <1976>; Polinsky <1979>, and Mestelman <1982>.

I will only elaborate on a tax on effluents here, as well as its corollary, a permit market. Naturally the tax will in fact be a vector q ($q = q_1, q_2, \dots, q_1, \dots, q_L$), with a different tax rate for each of the L effluents.

The producer's problem may now be characterized as:

$$L = \sum_{k=1}^K p_k y_k - \phi\{f(y, -r)\} - \sum_{l=1}^L q_l r_l$$

where $y = y_1, y_2, \dots, y_k, \dots, y_K$ and $r = r_1, r_2, \dots, r_1, \dots, r_L$ <24>

With respect to the first order conditions, <19>, <21> and <22> are unchanged. Let us focus on the effluents:

$$\frac{\partial L}{\partial (-r_1)} = q_1 - \phi \frac{\partial f}{\partial (-r_1)} = 0 \quad \text{<25>}$$

$$\text{<25>} + \text{<21>} \quad \phi \frac{\partial f}{\partial (-r_1)} + \frac{\partial f}{\partial y_1} = q_1 + p_1 \quad \text{<26>}$$

Now if q_1 is chosen such that:

$$\frac{q_1}{p_1} = a \frac{\lambda_1}{\psi_1} \quad \text{<27>}$$

where "a" is some constant.

We have, as required, re-established the conditions necessary for a Pareto optimum (see equation <13>). It must also be that the consumer's utility is maximized through his choice of x and the z resulting from the Authority's choice of q , since r and z are codetermined by equation <4>. Any r that respects <13> can only exist with a z that will respect <12>.

Rather than set a tax level q_1 , the Authority might choose to fix the total allowable quantity of emis-

sions $\sum_{j=1}^J r_j$ and establish a market for effluent permits.

Some of the reasons it might choose to do this are discussed in section IV.

With a permit market, the problem for the producer will remain the same as expressed in <24>. It must still choose its effluent level based on the cost, which is no longer in the form of a tax, but a permit price. A constraint has been introduced, however, and that is that:

$$\sum_{j=1}^J r_j^i(q_1) = \bar{F}_1 \quad i = 1, 2, \dots, L \quad \langle 28 \rangle$$

For any given q_1 , each producer will choose its optimal level of effluent production. The sum of all producers' discharges must equal the level established by the gover-

ment. Since $\sum_{j=1}^J r_j^i$ varies with q_1 , there is only one q_1 that can result in \bar{F}_1 . If \bar{F}_1 is set at its optimal level, then the q_1 which results must also be optimal.

We have assumed that the cost of acquiring information is nil. Since q_1 and $\sum_{j=1}^J r_j^i$ depend on each other in a typical demand relationship, the same information is required to reach the optimal solution regardless which one is fixed and which is left to the market to establish. This information is considerable, as all outputs depend indirectly on the cost of generating effluents. More-or-less complete production functions of all firms must be known. At this point there is no saving in information costs associated with letting the market "do the work" of allo-

cating the permits. The government might just as well assign all the r_i , as a number of authors propose (Weitzman <1974>, Rose-Ackerman <1977> and Tohe <1981>).

We will see in the next chapter how, under more realistic conditions concerning information availability, market mechanisms might offer substantial economies.

III - CRITICISMS OF GOVERNMENT INTERVENTION

While nearly all economists agree that, within the confines of a simple model such as the one presented in the previous section, some form of government intervention may be required, many have delighted in finding situations in which intervention would be ill-advised. A number of other economists such as Baumol and Oates <1975> recognize the value of the contributions of these authors, and have responded by improving simple models to take account of these situations.

In this chapter three issues that tend to limit the application of the corrective taxes approach are discussed: spontaneous negotiation, monopoly, and information problems. They are presented more or less in the order that they emerged in the literature.

Spontaneous negotiation

In 1960 Coase published a much cited article challenging Pigou's conclusion that wherever there is evidence of externalities, the state should intervene to correct the resulting misallocation. What has come to be known as the Coase theorem resolves into two basic points: under the critical constraint of zero transaction costs, Coase argues that voluntary exchange will result in the elimination of externalities and achievement of Pareto-optimality, regardless of property rights distribution; and secondly,

even in the presence of transaction costs Pareto-optimality may still be achieved without government intervention.

While the first issue has been much discussed in the literature, the heroic nature of the assumption renders it uninteresting on a practical level. As Baumol <1972> points out, even where there is a small number of polluters, the number of pollutees is frequently very large (in the millions of people). Thus the hypothesis that transaction costs are zero where the externality is a pollutant is unsustainable. However, it may be noted in passing that this solution continues to be a relevant possibility in many experiments on public goods and externalities which deal with only a small number of subjects, (e.g. Polinsky <1980>, Prudencio <1982>).

Much more important in the real world setting is the suggestion that the optimum solution may be non-intervention even where negotiation is impossible. It is readily understood that government intervention is also costly: it is easy to imagine a case where a small amount of pollution causes a small damage, and where the costs of government intervention would greatly exceed the welfare gain. There is another issue as well, however. As Baumol puts it:

Coase's central argument appears to be the following: Every social cost is inherently reciprocal in nature. The nearby residents who breathe smoke spewn by a factory must share with the management of the factory the responsibility for the resulting social cost. True, if the factory were closed up the social cost

would disappear. But the same holds for its neighbors --were they to move away no one would suffer smoke nuisance. (<1972> p. 308)

This approach is reminiscent of Tiebout <1956>, who argued that people would prefer to reside in communities that offered an appropriate mix of municipal services. Such an analysis was first extended to environmental amenities by Anderson and Crocker <1971>, though they credit Lancaster <1966> and his derived demand using "characteristics" more directly. They studied land values as a measure of air pollution, the hypothesis being:

That a portion of air pollution damage to artifacts and organisms is capitalised negatively into the value of land and immobile durable improvements thereon. It is readily demonstrated that if air pollution is itself a source of disutility or if it negatively modulates the utility obtainable from other goods, and if dosages of pollution vary over space, then land rents will vary inversely with air pollutant dosages. (Anderson and Crocker <1971>, p. 171.)

In such an analysis it is implicit that consumers adjust to the damaging externality, thereby avoiding some of the damage. Coase's argument is that this may be the least cost solution to the externality problem. Baumol's <1972> response was twofold: continuing damages should still be subject to tax, and, furthermore, he asks who should be paying the cost of pollution control, the polluter or the pollutee?

Shibata and Winrich <1983> have attempted to resuscitate Coase's argument in the context of a conventional model. They elaborate on the point that the least

cost control method may not be control at source (by the polluter), but by those offended. As Baumol pointed out, this does not obviate government intervention, but given the cost of intervention, it probably reduces the number of cases when a net welfare gain may be expected.

Monopoly

Monopoly, or other deviations in market structure from perfect competition, also gives rise to misallocation of resources. Buchanan <1969> is generally credited with highlighting the fundamental "second best" nature of this problem. If there is more than one departure from optimal conditions, correction for one such departure may or may not lead to an overall welfare improvement.

Asch and Seneca <1975> elaborated on the Buchanan model, evaluating empirically the welfare costs associated with monopolistic power and environmental externalities in the automobile industry. They find that:

It is always desirable to correct for both monopoly and externalities, but to treat only one of these characteristics may worsen welfare. Indeed, the relevant comparisons suggest that if either "antitrust" or "environmental" policy is advantageous alone, the other may well be inadvisable. (<1975>, p.78.)

Misiolek <1980> expands upon their work, first defining the optimal tax formula, which may incorporate a subsidy depending on demand elasticity (but not market concentration per se). He goes further, noting that Asch and Seneca and Buchanan all posited a fixed relationship between external cost and output.

Using a model with two different production functions (with different external effects) he concludes:

As long as the optimal emissions tax can be calculated for each productive process available to each firm and political factors do not interfere with the payment of subsidies where appropriate, this approach results in the attainment of efficient output levels for affected industries. (<1980>, p.106)

Thus, while the existence of monopolies should not prevent any attempt at correction for the misallocation of resources resulting from either the monopoly or the pollution, it is clear that the calculation of benefits and appropriate taxes is somewhat more complicated.

Information

It was noted at the end of the previous chapter that the cost (to the Authority) of acquiring information relating to consumers' preferences and firms' production functions was assumed to be nil. Meanwhile this information was essential to enable it to establish the optimal tax rates or quantitative effluent controls.

In fact, masses of information would have to be gathered and processed, an argument which has often been used against "market mechanisms" of control as opposed to direct regulation (Rose-Ackerman <1973>, Suchanek <1979>). Problems of information collection and manipulation have plagued all centrally planned economic systems, and economists proposing partial central planning (in the current case only for environmental commodities) must also confront

this issue. (cf. Malinvaud <1967>, and Arrow and Radner <1979>.)

With public goods, the problem is complicated by the fact that, in order to achieve Pareto optimality, those who are to be taxed must declare truthfully their preferences. This was demonstrated by Samuelson <1954, 1955>. For private goods this does not matter: by definition the purchaser captures the full benefits of consumption, therefore he who is unwilling to pay does not receive. But with public goods, the consumer still receives the benefit of what others purchase, introducing an incentive to understate one's valuation of the good.

Ignoring this for the moment, let us assume that agents are strictly honest. It would still be very difficult to evaluate consumer preferences. Since our model relies on the market for the distribution of private goods, the Authority is only really concerned about the valuation of the public goods, perhaps in terms of some numeraire good.

Surveys have in fact often been used to evaluate consumer "willingness to pay" to save or re-establish some environmental amenity. Davis <1963> was perhaps the first to do so, but Randall, Ives and Eastman <1974> established a stronger theoretical analysis, relying on earlier work (in other fields involving public goods) by Bohm <1970, 1971> and Bradford <1970>.

Studies have been made on willingness to pay, willingness to accept compensation, behavioural or expenditure adjustments, or combinations of the above. No single standard has yet emerged. The purely hypothetical nature of the problem has often been offered as an explanation for the differences between estimated WTP and WTA. Knetsch and Sinden summarize a typical argument:

(T)hese survey results have all been based on hypothetical scenarios. Respondents may consequently have been free, and even encouraged by the nature of the questions and interviews, to engage in deceptive response strategies in the hope of possible personal benefit or otherwise to give unreliable evidence of what their actual behaviour might be if confronted with real payment or compensation options. (Knetsch and Sinden <1984>, p. 509.)

Knetsch and Sinden themselves are inclined to accept the experimental data, even though it throws into question such basic economic dogma as the reversability of movements along indifference curves, but we will not go into that here.

In short, the data generated are not usually very consistent, whether because the theory is flawed or simply because the selfish interest of agents is at play, or both. What chance is there then for the Authority to learn the true nature of all consumers' and producers' preference and cost functions? In the face of this problem economists have tried a variety of approaches:

i) Abandon the problem and search out alternative techniques such as direct regulation, based on whatever information the Authority has;

ii) Set an arbitrary standard of environmental quality, and attempt to minimize firms' costs of meeting this standard;
iii) Attempt to develop tax plans which are cheat-proof; and
iv) Construct a regulatory framework such that firms' interests will coincide with those of the collectivity.
These are not all mutually exclusive, as, for example, (ii) and (iv) could be used together.

Direct regulation

The first is not of much interest in the context of this paper, though it may be noted in passing that Buchanan and Tullock <1975>, Rose-Ackerman <1977>, Suchanek <1979> and Yohe <1981> have all attempted to defend direct regulation as being, in effect, as efficient in its results as any market based scheme.

Standards and charges

The second approach, usually referred to as standards and charges, is credited to Baumol and Oates <1971>, though Hass <1970>, building on earlier work by Deninger <1965>, used the same approach for a simulation based on the Miami River in Ohio.

This approach could lend itself easily to the problem as stated in the model in this paper. Instead of relying on consumers and producers to provide data for calculation of the true optimum levels of various water quality parameters, the Authority dictates a standard.

Needless to say, this could only by chance be optimal, but the Authority is not prevented from using whatever information it can get its hands on.

The method distinguishes itself from direct regulation in that it relies upon a market system to divide up the available effluents among producers. In the Baumol and Oates scheme, producers are assumed to be competitive. It will be seen below (re point iv) that this hypothesis need not be maintained.

This approach continues to find favour, as a number of authors have helped to refine the model. Teitenberg <1973b> introduces zones so that the marginal contribution to pollutant concentration need only be the same for all emitters in a given zone. Where the zone coincides with a centralized sewer system, this would in fact be the case. Revesz and Marks <1982> also introduce river parameters, specifically temperature and flow rate, to make it more realistic.

Cheat-proof tax schemes

The third approach is to develop cheat-proof or incentive-based tax schemes (mainly aimed at consumers). This burgeoning literature is one of the main foci of the current public goods literature. While it is for the most part in partial equilibrium, there have been a number of significant advances. It is also, notably, set in a dynamic

context. This is necessary, as it is impossible for the Authority to know the location of the optimal solution if it does not know consumers' preference functions. It is not likely either that consumers will know each other's preference functions. Thus the optimum can only be achieved through a dynamic learning process.

The development of incentive mechanisms goes back to Vickrey <1961>, but his work was not followed up for nearly a decade. Malinvaud <1971> and Drèze and de la Vallée Poussin <1971> are credited with development of a class of dynamic procedures now known as MDP processes. (These are in fact used extensively in the next chapter, in which the dynamic model is developed, although truthful reporting is assumed in my model.) In these models consumers are assumed to follow a minimax strategy, that is, each consumer assumes others will make decisions that are the least favourable to him. The stopping point, however, also represents a Nash equilibrium, that is, knowing the outcome, agents are not interested in changing their decisions.

Groves and Ledyard <1977> produce a general equilibrium model which induces truthful revelation. However they assume competitive behaviour by consumers, which Hurwicz <1972> has shown is not individually rational (not in the individuals' best interest). The basic Groves and Ledyard model is very similar to the one presented above, though theirs was not specifically conceived with environmental quality in mind.

More recent work has attempted to find procedures by which truthful revelation is the consumer's dominant strategy. Laffont and Maskin <1983> prove (in partial equilibrium) that, although no MDP procedure meets these requirements, there are nevertheless a large class of such processes that are Pareto optimal, balanced, and individually rational.

In all, there has been considerable progress in weakening the necessary assumptions concerning consumer strategies in public goods models. This all depends, however, on the idea that consumers know, or even want to know their own MRS.

Taxes and licences

The last approach bears a certain similarity to the previous one, except that the orientation is towards firms rather than consumers. Initially the two approaches started out on different footings, although Dasgupta, Hammond and Maskin <1980> have integrated them to some extent.

The equivalence of effluent taxes and licenses as means of achieving optimal environmental quality was discussed (albeit briefly) at the end of chapter II. This equivalence is however based on certainty of information. Suppose the Authority incorrectly estimates the cost functions: as Roberts and Spence point out

Effluent charges bring about too little cleanup when cleanup costs turn out to be higher than expected, and they induce excessive cleanup when the costs of cleanup

turn out to be low. Licenses have the opposite failing. Since the level of cleanup is predetermined, it will be too high when cleanup costs are high and too low when costs are low. (<1976>, p.194.)

Which system is better depends on the curvature of the cost and damage functions (in the area of the optimum). As noted by Dasgupta and Heal, a mixed scheme may not merely be better in general, but can never be worse than either scheme used in isolation (<1979>, p.407).

While Roberts and Spence were working in partial equilibrium, Kwerel <1977> extends their work by developing it in a general equilibrium analysis and by demonstrating that it works as well even when it is recognized that firms may wish to deceive the Authority.

Dasgupta, Hammond and Maskin <1980> are quite critical of the Kwerel model. (This is a little surprising since the same Dasgupta with Heal <1979> make abundant use of the original Roberts and Spence model, presenting it more clearly in fact than the original authors.)

In particular they are concerned with the limitations of perfect competition, separability of pollution effects and convexity of the damage function. They claim to do away with, or at least weaken, all of these assumptions. They introduce a complex tax formula which is not developed in their paper, but is a "simple adaptation" of the mechanism developed by Groves <1973>. Their tax system is "sufficient to guarantee the existences of taxes for which truth-telling is (locally) optimal for firms and

leads to optimum pollution levels". (<1980>, p.860).

In his two papers <1977 and 1979>, Suchanek compares tax and quota schemes, presenting these as "duals". Care should be taken however, as the models are not symmetrical. The quotas are not sold to the firms but are assigned, as one is likely to find in most conventional regulations today. Thus they are not identical counterparts. This may account for Suchanek's conclusion that "in general, mechanisms designed to compute emissions' charges dual to optimal quotas are not individually incentive compatible" <1979, p.112>, which appears to be in direct contradiction to the quotation above from Dasgupta, Hammond and Maskin.

In this debate the original tax/license scheme proposed by Roberts and Spence seems to have been forgotten in favour of an analysis of incentive tax schemes designed for industry. Either way, the purpose is essentially the same, to produce a regulatory framework by which firms would be induced to be honest, thereby helping the Authority to achieve its goals.

It may be noticed that the property rights question might affect the Authority's preferred approach, among the four outlined above. Those who are going to be asked to pay for the cleanup on the basis of the information they give have a much higher incentive to lie. While information from the other side, the "owners" of the

environment, is still necessary in order that a true optimum be obtained, each individual agent has only a minimal impact on his own welfare.

Information from consumers, who are much more numerous, will be much more costly than that concerning producers. Thus a system like that of Baumol and Oates (ii, above) has a certain administrative convenience about it, although it does presuppose that it is the consumers that have the right to a clean environment.

IV — THE DYNAMIC MODEL

In the previous chapter I introduced certain aspects of the information problem, in particular its costliness and the unlikelihood that agents will both know and want to give the Authority correct information concerning their substitution rates between goods. In this chapter those problems will be once again forgotten ("assumed away") while we focus on a third and critical aspect of the information problem.

In the model developed in chapter II, it is expected that the use of various inputs and outputs will vary with the level of the tax applied in order to restore the optimal conditions. The "optimal tax" will thus be associated with optimal levels of output and consumption as well, and many of these will be different from their original levels.

The problem is summarized by Baumol and Oates:

The optimal tax level on an externality generating activity is not equal to the marginal net damage it generates initially, but rather to the damage it would cause if the level of the activity had been adjusted to its optimal level.

If there is little hope of estimating the damage that is currently generated, how much less likely it is that we can evaluate the damage that would occur in an optimal world which we have never experienced or even described in quantitative terms. (<1971> p. 43.)

This highlights one of the valuable aspects of the general equilibrium approach: the model permits prices and quantities of all goods to vary, difficult as it may be to predict

the outcome. Intuitively, it is obvious that that the prices and outputs of those goods that are most inextricably tied to polluting processes will be the most heavily affected when compensating taxes or effluent rationing is introduced. In fact, the extent of this impact depends on various elasticities. (See for example Yohe <1976>).

The problem is not merely that the final outcome is not known:

Much of the investment that will be made in any pollution control program will take several years to plan and complete and will be largely irreversible once in place. Thus the response to all subsequent policies will be heavily dependent on previous history. Indeed the cycle time may be so great as to prevent convergence, since the "correct" solution will be constantly changing. (Roberts and Spence <1976>, p. 193.)

It is important, therefore, to arrive as quickly and painlessly as possible to the optimum level.

The MDF model, mentioned at the end of the last chapter, has been extensively developed by Malinvaud <1970-71, 1971, 1972>, Drèze and de la Vallée Poussin <1971> (referred to subsequently as DVP), and Champsaur, Drèze and Henry <1976> (referred to as CDH). The fundamental importance of this model is the dynamic processes involving public goods which it illustrates. Malinvaud et al. use a tâtonnement model, an approach first suggested by Walras, in which the demands and supplies of all goods are gradually adjusted, permitting the attainment of the true optimum. No exchange actually takes place until the solution is found.

No one, to my knowledge, has adapted such an approach

to externalities, per se, although Tulkens and Schoumaker <1975> have studied externalities in a "non-tâtonnement" model. A non-tâtonnement model is essentially one in which exchange is permitted during the search for the equilibrium. This may result in a redistribution of income during the process as a result of changing prices, and therefore it generally leads to a result different from the tâtonnement procedure (Negishi <1962>).

The Tulkens and Schoumaker model is limited by the fact that it reduces to a two person/two good exchange economy (no production). Their primary goal is to analyze distributional impacts of a Pigovian tax, a subject that goes beyond the scope of this paper.

There are certain differences between the various tâtonnement-type models developed in the articles mentioned above. Malinvaud <1970-71>, DVP and CDH have only one producer. When only public goods are concerned, this allows for direct comparison between the firm's RTS and the sum of the consumers MRS to establish "distance" from the optimum, at which point the two are equal. This is the basic approach of the latter two: CDH use a two part procedure, while DVP have only one private good. Malinvaud <1970-71, 1982> resorts to averages where private goods are concerned, as I do below for both private and public goods.

Another difference lies in whether the model

maintains feasible outcomes at every stage of the process. On the whole the above authors are able to achieve this only by working exclusively in goods space. Malinvaud, in the second (tax) model presented in <1970-71>, and also in <1972>, ventures into price space at the cost of balance between outputs and allocations during the process, though of course feasibility is re-established in the end.

Malinvaud's 1972 approach is particularly appealing, in that he relies on prices for private goods and quantitative controls for the public goods, much as I did in the static model. In the model presented below, however, I have opted for a well-behaved model that respects feasibility throughout. To do this I have limited the analysis to goods space. The Authority assumes a greatly expanded role, therefore, becoming in effect the "Bureau du plan", responding to consumers' and producers' preferences with consumption and production targets for every good and every agent. This a far cry from the free market, but it illustrates the essential characteristics of a dynamic adjustment process. I have also taken inspiration from Malinvaud <1982>, though in that dynamic model he has neither production nor public goods.

The same basic model is used, as defined by equations <1> to <4>. In addition the question of distribution of income must be resolved. To Malinvaud this question is of critical importance (Malinvaud <1967> and <1970-71>), though he cannot control it directly. Real income, as

perceived by the consumers themselves, is not known in advance, as it depends on their personal preferences. The Authority nevertheless has three tools with which it affects the distribution. They are the initial distribution, the weighting of marginal utilities, and the shares of the gains in welfare made at each stage of the tâtonnement process. Malinvaud contents himself with the following:

assume that the procedure starts with an initial situation considered as satisfactory from the distribution point of view, and verify that the revisions progressively made in the situation treat the various individuals equitably. (<1970-71> p. 189.)

In the context of the water quality model, it might be useful to think of the initial allocation as the status quo ante, i.e. the market equilibrium established above in the subsection "A price system for private goods only". As for the weighting of preferences and shares of the welfare gain, I have, for simplicity, made the three vectors of weights the same, all defined by R^+ . This implies that the owners of the polluting firms will experience an increase in welfare, along with everybody else. This need not be the case, but concerns relating to the distribution of income, or "property rights", go beyond the scope of this paper.

With regard to the weighting of preferences, it is also necessary to involve the producers in the collective decision making. The conditions of Pareto optimality are only met when (for private goods) all MRS_k and all RTS_k for each k are equal, and (for environmental goods) when the

sum of all MRS_i^j is equal to each RTS_i^j . Producers are assigned weights R^j . A fifth equation now completes the definition of the economy.

$$\sum_{i=1}^I R^i + \sum_{j=1}^J R^j = 1 \quad \langle 29 \rangle$$

Starting the procedure

The (Planning) Authority must start the procedure by suggesting initial consumption and production levels for all private goods for each consumer and producer, and the level of public goods (water quality), as well as dividing effluent rights among the producers. Malinvaud calls these suggestions "initial indicators". These indicators must be feasible, that is they must satisfy $\langle 2 \rangle$, $\langle 3 \rangle$ and $\langle 4 \rangle$. We have:

$$x_k^i = \frac{R^i}{1 - \sum_{j=1}^J R^j} \left(\sum_{j=1}^J y_k^j + \omega_k \right) \quad \begin{array}{l} i = 1, 2, \dots, I \\ k = 1, 2, \dots, K \end{array} \quad \langle 30 \rangle$$

$$y_k^j = \frac{R^j}{1 - \sum_{i=1}^I R^i} \left(\sum_{i=1}^I x_k^i - \omega_k \right) \quad \begin{array}{l} j = 1, 2, \dots, J \\ k = 1, 2, \dots, K \end{array} \quad \langle 31 \rangle$$

$$-r_l^j = \frac{R^j}{1 - \sum_{i=1}^I R^i} (e_l - z_l) \quad \begin{array}{l} j = 1, 2, \dots, J \\ l = 1, 2, \dots, L \end{array} \quad \langle 32 \rangle$$

These indicators are likely to be inspired by the existing market equilibrium, but the Authority may wish to anticipate increases in the desired levels of public goods, by implementing a certain reduction in effluents.

It is worth noting at this point that a relative-

ly strong assumption is required with respect to the production function <2>. This is that it is convex and is continuously differentiable. Its derivative must be positive with respect to the numeraire, that is, $\frac{\partial f}{\partial y_1} > 0$.

This assumption is necessary in order to maintain feasibility anywhere in the production set. Malinvaud <1970-71> demonstrates that one could alternatively require that all goods be necessities (restricting thereby the utility function).

Consumers and producers respond with "proposals". The consumer expresses his marginal rates of substitution (MRS) for the various private goods and water quality parameters in terms of the numeraire (the first private good, $k=1$). For simplicity, we define marginal rates of substitution as π_k^i and λ_l^i .

$$\pi_k^i = \frac{\frac{\partial U^i}{\partial x_k^i}}{\frac{\partial U^i}{\partial x_1^i}} \quad k = 1, 2, \dots, K \quad \langle 33 \rangle$$

$$\lambda_l^i = \frac{\frac{\partial U^i}{\partial Z_l}}{\frac{\partial U^i}{\partial x_1^i}} \quad l = 1, 2, \dots, L \quad \langle 34 \rangle$$

When <33> and <34> are compared with <10> and <12>, it will be remembered that at the optimum π_k^i is not only the $RTS_{k,1}^i$, but the price of any k relative to the numeraire. It is only the sum of the λ_l^i that will give the relative price of any l . This will have a bearing on the subsequent development.

The producers's proposal consists of his rates of technical substitution (RTS) for the various private goods and the effluents, all in terms of the numeraire. Again, we define π_k^j as the rate of technical substitution between good k and the numeraire, and λ_l^j as the rate of technical substitution between effluent input l and the numeraire, both for a given producer j .

$$\pi_k^j = \frac{\frac{\partial f^j}{\partial Y_k}}{\frac{\partial f^j}{\partial y_l^j}} \quad k = 1, 2, \dots, K \quad \langle 35 \rangle$$

$$\lambda_l^j = \frac{\frac{\partial f^j}{\partial (-r_l^j)}}{\frac{\partial f^j}{\partial y_l^j}} \quad l = 1, 2, \dots, L \quad \langle 36 \rangle$$

This completes the first exchange of information. The Authority calculates new indicators before initiating a second exchange. Each exchange of information occurs at a given point in time t ($t = 1, 2, \dots, T$). It is only at the end of the procedure, T , that the Authority defines the final program, and the goods are traded or distributed. This aspect characterizes the "tâtonnement" process. The final program identifies itself when no further changes can improve the welfare of the society. This will be developed presently.

All the variables should now be expressed as derivatives with respect to the dynamic variable t . Although the real variable (time) will be notationally suppressed in what follows, it is important to remember that all variables are now defined for a given t .

The adjustment process

Consumers and producers have responded to the Authority's initial indicators. Now it must revise them. In its search for a Pareto optimum, the Authority will want to do several things. It will increase allocations of particular goods to those consumers that have shown the highest marginal utility for them, by taking some away from those who value these goods least. To do this it must determine whether the individual consumer or producer values the good above or below the average.

The Authority will also attempt to induce a higher production of those goods showing the greatest difference between average MRS and average RTS, and discouraging the production of those that are the least valued (relative to their cost).

Revision of private goods indicators (other than numeraire)

The Authority must first establish the weighted average of the MRS and RTS with which it will compare each of the individual consumer's and producer's proposals. The weighted average $\bar{\pi}_k$ depends in part on the weights arbitrarily fixed in <29>, as well as the individual MRS and RTS themselves. For the private goods other than the numeraire this would give the following results:

$$\bar{\pi} = \sum_{i=1}^I R^i \pi_k^i + \sum_{j=1}^J R^j \pi_k^j \quad k = 2, 3, \dots, K \quad \langle 37 \rangle$$

The interpretation of this is relatively straightforward. Individual consumers or producers that have been awarded larger R^1 or R^2 will have a greater impact on the average.

The next step is the actual comparison of the individual MRS and RTS with the average, in order to determine whether the allocations should increase or decrease. The heavier the weight, the greater the change.

$$dx_k^i = b_k R^i (\pi_k^i - \bar{\pi}_k)$$

where $i = 1, 2, \dots, I$ and $k = 2, 3, \dots, K$ <38>

$$dy_k^j = b_k R^j (\bar{\pi}_k - \pi_k^j)$$

where $j = 1, 2, \dots, J$ and $k = 2, 3, \dots, K$ <39>

b is a vector of positive coefficients, one corresponding to each good.

To determine the total change in x and y , the Authority sums <38> and <39>, for all i and all j , respectively. The results are shown in <40> and <41>.

$$dx_k = b_k \left\{ \sum_{i=1}^I R^i (\pi_k^i - \bar{\pi}_k) \right\}$$

$$\text{where } dx_k = \sum_{i=1}^I dx_k^i \text{ and } k = 2, 3, \dots, K$$
 <40>

$$dy_k = b_k \left\{ \sum_{j=1}^J (\bar{\pi}_k - \pi_k^j) \right\}$$

$$\text{where } dy_k = \sum_{j=1}^J dy_k^j \text{ and } k = 2, 3, \dots, K$$
 <41>

To determine whether the procedure is balanced, that is, if the transfers sum to zero, we may subtract <41> from <40>, getting:

$$dx_k - dy_k = b_k \left(\sum_{i=1}^I R^i \pi_k^i - \sum_{i=1}^I R^i \bar{\pi}_k + \sum_{j=1}^J R^j \pi_k^j - \sum_{j=1}^J R^j \bar{\pi}_k \right)$$

$$\text{where } k = 2, 3, \dots, K \quad \langle 42 \rangle$$

Using <29> and <37> this easily reduces to zero, therefore:

$$dx_k = dy_k \quad k = 2, 3, \dots, K \quad \langle 43 \rangle$$

Thus, for each private good other than the numeraire, the procedure has the property of conservation of resources: the net change in consumption will be equal to the net change in production.

Adjustment of the public goods (water quality parameters)

The Authority must also revise the determination of level of the public goods, z , and divide up the effluent rights. As might be expected, the Authority's formulation of the problem is somewhat different than that for private goods.

Before calculating the weighted average of the MRS and RTS, we observe that it was the difference between this weighted average and the individual MRS and RTS that told the Authority whether the allocation of a particular private good should be increased or decreased for that agent. This was modified by the individual's weighting factor to establish the change in his allocation. (See <39> and <40>.) So it is as well for the firms' effluents, which behave much like any other input:

$$dr_l^i = c_l R^j (\bar{\pi}_l^i - \pi_l^i) \quad l = 1, 2, \dots, L \quad \langle 44 \rangle$$

Summed across all producers, this gives:

$$dr_1 = c_1 \sum_{j=1}^J R^j (\bar{x}_1 - x_1^j)$$

$$\text{where } dr_1 = \sum_{j=1}^J dr_1^j \quad \text{and } l=1,2,\dots,L \quad \langle 45 \rangle$$

It is evident however that the same approach cannot be used for the water quality parameters, which, first of all, are the same for all individuals and cannot be summed. Secondly, it is the sum of the individual MRS that represents the implicit price of the good, and it is this sum that must be compared to the weighted average (which we have yet to define). However the change in water quality still depends on the total weight of the consumers in the

economy, represented by $(\sum_{i=1}^I R^i)$. This is also true, in effect, for private goods. Thus the change in a given water quality parameter is defined by:

$$dz_1 = c_1 \left(\sum_{i=1}^I R^i \right) \left(\sum_{i=1}^I \bar{x}_1^i - \bar{x}_1 \right) \quad l = 1,2,\dots,L \quad \langle 46 \rangle$$

Now let us look at \bar{x}_1 , the weighted average. The question arises as to what weights should be attached to the

terms $\sum_{i=1}^I \bar{x}_1^i$ and $\sum_{j=1}^J R^j \bar{x}_1^j$. Again it is clear that the first

term must be modified by the total weight of consumers in the economy, if $\langle 29 \rangle$ is to be respected. Therefore:

$$\bar{x}_1 = \left(\sum_{i=1}^I R^i \right) \sum_{i=1}^I \bar{x}_1^i + \sum_{j=1}^J R^j \bar{x}_1^j \quad l = 1,2,\dots,L \quad \langle 47 \rangle$$

We may now compare the net change in z with that of r , as they are directly related by <4>.

$$\begin{aligned} dz_1 - dr_1 &= c_1 \left(\sum_{i=1}^I R^i \right) \left(\sum_{i=1}^I \bar{x}_i^t - \bar{r}_1 \right) - c_1 \sum_{j=1}^J R^j (\bar{r}_1 - \bar{x}_j^t) \\ &= c_1 \left\{ \left(\sum_{i=1}^I R^i \right) \sum_{i=1}^I \bar{x}_i^t - \left(\sum_{i=1}^I R^i \right) \bar{r}_1 - \sum_{j=1}^J R^j \bar{r}_1 + \sum_{j=1}^J R^j \bar{x}_j^t \right\} \quad \langle 48 \rangle \end{aligned}$$

reorganizing and using <47>:

$$dz_1 - dr_1 = c_1 \left\{ \bar{r}_1 - \left(\sum_{i=1}^I R^i + \sum_{j=1}^J R^j \right) \bar{r}_1 \right\} \quad \langle 48' \rangle$$

which by <29> is equal to zero. Thus:

$$dz_1 = dr_1 \quad \langle 49 \rangle$$

This proves that the procedure has the property of conservation of resources on the public goods and externalities accounts as well.

Welfare gain

Before it can distribute the numeraire, the Authority must first calculate the total welfare gain (dw) for a given round t .

The welfare gain for a single individual is:

$$\frac{du_i^t}{\partial x_i^t} = \sum_{k=2}^K \pi_k^t dx_k^t + \sum_{l=1}^L \bar{x}_l^t dz_l + dx_i^t \quad i = 1, 2, \dots, I \quad \langle 50 \rangle$$

Replacing π_k^t and \bar{x}_l^t with $(\pi_k^t - \pi_k^t) + \pi_k^t$ and $(\bar{x}_l^t - \bar{x}_l^t) + \bar{x}_l^t$, respectively, summing for all i and j , and remembering the special meaning of \bar{x}_l^t when summed, we have:

$$\begin{aligned}
dw = & \sum_{i=1}^I \sum_{j=1}^J \sum_{k=2}^K (\pi_k^i - \pi_k^j) dx_k^i + \sum_{j=1}^J \sum_{l=1}^L \sum_{i=1}^I (\sum \lambda_l^i - \lambda_l^j) dz_1 \\
& + \sum_{j=1}^J \sum_{k=2}^K \pi_k^j dx_k^j + \sum_{j=1}^J \sum_{l=1}^L \lambda_l^j dz_1 + \sum_{i=1}^I dx_i^i \quad \langle 51 \rangle
\end{aligned}$$

Taking into account <3> and <4>, any feasible outcome will be defined by:

$$\begin{aligned}
dw = & \sum_{i=1}^I \sum_{j=1}^J \sum_{k=2}^K (\pi_k^i - \pi_k^j) dx_k^i + \sum_{j=1}^J \sum_{l=1}^L \sum_{i=1}^I (\sum \pi_l^i - \pi_l^j) dz_1 \\
& + \sum_{j=1}^J \sum_{k=2}^K \pi_k^j dy_k^j + \sum_{j=1}^J \sum_{l=1}^L \lambda_l^j dr_l^j + \sum_{j=1}^J dy_j^j \quad \langle 52 \rangle
\end{aligned}$$

Noting that the total output of the numeraire is defined by:

$$\sum_{j=1}^J dy_j^j = - \sum_{j=1}^J \sum_{k=2}^K \pi_k^j dy_k^j - \sum_{j=1}^J \sum_{l=1}^L \lambda_l^j dr_l^j \quad \langle 53 \rangle$$

this reduces to:

$$dw = \sum_{i=1}^I \sum_{j=1}^J \sum_{k=2}^K (\pi_k^i - \pi_k^j) dx_k^i + \sum_{j=1}^J \sum_{l=1}^L \sum_{i=1}^I (\lambda_l^i - \lambda_l^j) dz_1 \quad \langle 54 \rangle$$

This equation defines the welfare gain for the whole economy. When $\pi_k^i = \pi_k^j$ and $\sum_{i=1}^I \lambda_l^i = \lambda_l^j$, these terms both reduce to zero. At such a point it is obvious that the welfare gain should be zero, because a Pareto optimum has been reached (MRS = RTS). This being the case, the current indicators become the final program, and the Authority may actually distribute the goods.

Distribution of the numeraire

The Authority is now ready to calculate the changes in the allocation of the numeraire. For each

individual, this may be described as his share of the welfare gain, net of the gains he has already received in the redistribution of the other private goods and the changing levels in the water quality parameters. He may as a result receive more or less of the numeraire than before.

This may be characterized as follows:

$$dx_i = R^i dw - \sum_{k=2}^K \pi_k^i dx_k - \sum_{l=1}^L \bar{x}_l^i dz_l \quad \langle 55 \rangle$$

However, the numeraire account must also be balanced. Summing across i ,

$$\sum_{i=1}^I dx_i = dw - \sum_{i=1}^I \sum_{k=2}^K \pi_k^i dx_k - \sum_{i=1}^I \sum_{l=1}^L \bar{x}_l^i dz_l \quad \langle 56 \rangle$$

Using <54>,

$$\begin{aligned} \sum_{i=1}^I dx_i &= \sum_{i=1}^I \sum_{j=1}^J \sum_{k=2}^K \{ (\pi_k^j - \pi_k^i) dx_k - \pi_k^i dx_k \} \\ &\quad + \sum_{i=1}^I \sum_{j=1}^J \sum_{l=1}^L \{ (\bar{x}_l^j - \bar{x}_l^i) dz_l - \bar{x}_l^i dz_l \} \\ &= - \sum_{j=1}^J \sum_{k=2}^K \pi_k^j dx_k - \sum_{j=1}^J \sum_{l=1}^L \bar{x}_l^j dz_l \end{aligned} \quad \langle 57 \rangle$$

By <43> and <49> this can be written:

$$\sum_{i=1}^I dx_i = - \sum_{j=1}^J \sum_{k=2}^K \pi_k^j dy_k - \sum_{j=1}^J \sum_{l=1}^L \bar{x}_l^j dr_l \quad \langle 58 \rangle$$

But this is the same as <53>. Therefore,

$$dx_i = dy_i \quad \langle 59 \rangle$$

The Authority has indeed balanced the production and consumption of the numeraire as well.

Malinvaud <1970-71> and <1972> goes on to establish the convergence of his model. Champsaur, Drèze and Henry <1976> also demonstrate the stability of similar models involving public goods.

To summarize briefly, the Authority would be able to direct production and consumption of all private goods, using what Drèze and de la Vallée Poussin call "minimal" information from the producers and consumers <1971, p. 139>. The same cannot be said for the cost of communication and of the computations that this information would require.

The tâtonnement process, it will be remembered, is designed to balance demand and supply at a single point in time. Changing tastes, new production processes, not to mention a changing population, would require a new process at frequent intervals. For this reason, a more decentralized process might be desirable.

It was pointed out at the beginning of this section, however, that a dynamic model defined in the goods space demonstrates fairly clearly the nature of the adjustments that must occur, though only offering a single example of how this might happen.

V - CONCLUSION

My basic objectives in this paper have been threefold: to present a relatively standard Arrow-Debreu type model dealing with environmental variables in static equilibrium; to discuss some of the limitations of the model; and, finally, to extend the model into the dynamic context.

The overwhelming majority of models presented in the literature that address pollution and environmental quality are restricted to static equilibrium analysis. The standard conclusion is that pollution causes a non-optimal allocation of resources (in the Pareto sense).

This conclusion is replicated in the simple model that I developed in chapter II. The quality of the environment affects consumers' satisfaction, but in a traditional market economy, producers have no incentive to reduce their effluents even where the social cost is greater than the benefits of the production.

It is often argued that this situation justifies government intervention, usually in the form of taxes, as proposed by Pigou. It is demonstrated that either such action or quantitative controls could restore the economy to a Pareto optimum.

A number of important simplifications render this conclusion questionable. Several of these are discussed in

the context of current economic literature, namely transaction costs, monopoly, and information problems. In the model it is assumed that transaction costs are nil, there is perfect competition, and the necessary information is available and forthcoming.

In the first two cases, while the point is well taken, these problems do not necessarily obviate any intervention, but they do probably reduce the number of situations where it would be appropriate. In the case of transaction costs, no models have been developed to account formally for the problem. In the case of monopoly, however, the conditions which would have to be met, if intervention were going to increase rather than decrease welfare, have been fairly well identified.

The information problem is complex. The model uses a substantial amount of information about consumers' and producers' marginal substitution rates for various goods. It is not at all certain that consumers have this information, although efficient producers can at least be assumed to want to know their own substitution rates for goods in relation to their production functions. Neither group, on the other hand, should be counted on to reveal these preferences honestly, if they know they will be taxed on the basis of this information.

In the face of these problems, I have identified four different approaches in the economic literature:

firstly, forget about optimality and go to direct regulation; secondly, adopt a system of standards and charges, whereby information relating to consumer preference functions is no longer required; thirdly, develop cheat-proof tax schemes (for consumers); and fourthly, attempt to induce producers to provide correct information through a combined effluent tax/permit system. To some extent the fourth group overlaps with the third, only producers are the target group. A system combining the second and fourth approaches might turn out to be the most practical in a real world situation.

There is another aspect of the information problem which is completely ignored in the static context: how the economy is supposed to move from a non-optimal situation to an optimum. The effluent taxes that bring about the optimum are after all dependent not on original outputs and prices, but on those that have themselves also been adjusted to optimal levels.

Walras hypothesized the market worked by a sort of "tâtonnement" process: this has been modelled for economies with public goods by Malinvaud and others. No one, to my knowledge, has published such a model for an economy specifically designed to portray the environmental problem, as I have done. It is interesting that, as the government is expected to intervene to restore optimal conditions, this analysis highlights the enormous role the government must

play as an information manager. While this realization need not discourage intervention, it certainly suggests that such plans be carefully thought out to demonstrate they will actually result in an improvement in welfare, and not in costly blunders.

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