Université de Montréal

On the Use of an Exclusive Territory Clause

Par

Armando PRIEGUE-FREIRE

Département de Sciences Economiques
Faculté des Arts et des Sciences

Mémoire Présentée à la Faculté des études Supérieures
En vue de l'obtention du grade Maître ès Sciences (M.Sc)
en Sciences Economiques
option Economie Générale

Février, 2002

Université de Montréal
Faculté des études supérieures

Ce mémoire intitulé :

On the Use of an Exclusive Territory Clause

présenté par :

Armando PRIEGUE-FREIRE

A été évalué par un jury composé des personnes suivantes :

....
Président-rapporteur

....
Directeur de recherche

....
Codirecteur

....
Membre du jury

....
Examinateur Externe

....
Représentant du Doyen de la FES

Résumé

L'innovation est incontestablement un des facteurs les plus importants pour la croissance économique des pays. Dans les pays où la majorité des innovations sont réalisées par des agents économiques privés, comprendre les incitations de ces agents privés requiert que l'on mesure les gains que ces agents peuvent obtenir grâce à ces innovations. Cette thématique est évidemment extrêmement large et complexe. Nous allons donc nous concentrer dans ce mémoire sur un aspect particulier de la mesure des bénéfices privés d'une innovation, à savoir, le revenu qu'un innovateur qui détient un brevet sur une nouvelle technologie peut obtenir en octroyant des contrats de licence à des producteurs.

Notre analyse se situe dans un cadre théorique qui a pris naissance dans Arrow (1962). Il a été reformulé en termes de concepts de théorie des jeux dans Kamien et Tauman (1984) et (1986) ce qui a permis d'étendre la réflexion aux industries oligopolistiques. Ce cadre est le suivant. Un innovateur détient un brevet sur une nouvelle technolopgie qui permet de réduire le coût marginal de production d'un bien. Ce bien est produit par une industrie en concurrence imparfaite et l'innovateur ne fait pas partie de cette industrie. La seule manière pour l'innovateur d'obtenir un revenu est donc de vendre des licences d'utilisation de son invention aux producteurs du bien. Donc, typiquement, les interactions stratégiques entre l'innovateur et les producteurs sont modélisées par un jeux en trois étapes:

- à la première étape, l'innovateur propose un contrat aux producteurs[1]

- à la deuxième étape, après avoir observé les termes du contrat proposé par l'innovateur, les

[1]Dans certaines études, voir Kamien et Tauman (1986) ou Katz et Shapiro (1986), l'innovateur organise une enchère qui se tiendra à la deuxième étape. Dans ce cas, l'innovateur choisit, à la première étape, le nombre de licences qu'il mettra en vente et, à la deuxième étape, les producteurs soumettront un prix pour une licence. Ceux qui obtiendront une licence sont ceux qui auront soumis les prix les plus élevés.

producteurs décident simultanément d'accepter ou de refuser le contrat de l'innovateur;

- à la troisième étape, après avoir observé le nombre de producteurs ayant accepté le contrat à la deuxième étape, les producteurs décident simultanément de la quantité qu'ils produiront[2].

Les conclusions générales de ces études sont les suivantes:

- le revenu de l'innovateur dépend de la forme du contrat qu'il utilise et, évidemment de la qualité de l'innovation dont il détient le brevet;
- le revenu de l'innovateur sera plus grand s'il utilise un contrat qui spécifie uniquement le payement d'une charge fixe d'une valeur appropriée que s'il utilise un contrat qui spécifie le payement de redevances uniquement;
- le revenu de l'innovateur qui utilise un contrat spécifiant uniquement une charge fixe est inférieur au profit que ferait un monopoleur qui utiliserait l'innovation sauf dans le cas où, d'une part, l'innovation est drastique au sens d'Arrow (1962)[3] et, d'autre part, l'industrie à laquelle fait face l'innovateur est parfaitement compétitive;
- le revenu de l'innovateur est plus important s'il utilise un contrat qui spécifie le payement d'une charge fixe et de redevances et, comme démontré dans Erutku et Richelle (2000), ce revenu devient égal à celui que ferait un monopoleur utilisant l'innovation quelle que soit la qualité de l'innovation et quel que soit le nombre de producteurs pourvu que celui-ci dépasse deux.

---

[2]Il faut noter que certaines études, comme Kamien et Tauman (1986) ou Muto (1993), considèrent que les producteurs choisissent simultanément le prix auquel ils vendront leur bien.

[3]Un innovation est drastique au sens d'Arrow (1962) lorsqu'elle permet au prix qui serait fixé par un monopoleur utilisant la nouvelle technologie d'être inférieur au coût marginal prévalant avant l'innovation.

Ce dernier résultat est intéresant car il fournit une explication à l'utilisation, fréquemment observée (voir Rostocker (1983)), de contrats basés sur le payement d'une charge fixe et de redevances[4]. Toutefois, ce n'est pas le seul type de contrats observé. Dans l'étude de Rostocker (1983), par exemple, 13% des contrats stipulent le payement d'une charge fixe uniquement. D'autre part, il est bien documenté (voir Anand et Khanna (2000)) que les contrats contiennent aussi d'autres clauses que celles qui définissent la méthode de payement. La clause de ce type qui retiendra notre attention dans ce travail est *la clause d'exclusivité territoriale*. Plus précisément, nous allons considérer les contrats qui stipulent, d'une part, le payement d'une charge fixe uniquement et, d'autre part, une clause d'exclusivité territoriale. La question que nous nous poserons est de savoir si ce genre de contrat peut permettre à l'innovateur d'obtenir un revenu égal à celui qui serait obtenu par un monopoleur utilisant l'innovation et, si cela est effectivement le cas, dans quelles circonstances.

Pour examiner cette question, nous allons considérer une situation où deux producteurs vendent un produit différencié sur deux marchés différents. Chacun de ces producteurs est localisé sur un marché. Les interactions stratégiques entre les deux producteurs et l'innovateur sont décrites comme dans le jeu à trois étapes présentés ci-dessus. Dans ce contexte, une clause d'exclusivité n'est effective que si les deux producteurs ont accepté le contrat et empêche chacun de ceux-ci de vendre leur produit sur le marché de leur concurrent. Par contre, si un seul de ces producteurs à accepté le contrat, la clause d'exclusivité n'est pas contraignante et les deux producteurs peuvent vendre sur les deux marchés s'ils trouvent cela profitable.

[4]Une explication alternative à cette forme de contrat est la présence d'une information asymétrique entre l'innovateur et les producteurs. Par exemple, Gallini et Wright (1990) suppose que les producteurs ne connaissent pas avec exactitude la qualité de l'innovation et que l'innovateur ne peut transmettre les informations relatives à la qualité de son innovation sans transmettre aux producteurs les moyens de réduire leur coût tout en n'achetant pas de licence.

Dans ce contexte, les résultats principaux que nous avons obtenus sont les suivants:

- il existe une valeur critique pour la qualité de l'innovation telle que si l'innovation est d'une qualité supérieure ou égale à cette valeur critique, l'innovateur peut obtenir un revenu égal à la somme des profits qui seraient réalisés par un monopoleur sur chacun des marchés en choisissant de manière appropriée la charge fixe stipulée dans le contrat;

- cette valeur critique de la qualité de l'innovation est négativement corrélée au degré de substituabilité entre les produits vendus par les procucteurs;

- il existe une valeur critique pour le degré de substituabilité entre les produits telle que, si le degré de substituabilité dépasse cette valeur critique, l'innovateur est capable d'obtenir un revenu équivalent à la somme des profits de monopole alors que son innovation n'est pas drastique au sens d'Arrow (1962);

- pour n'importe quelle qualité de l'innovation, il existe un degré de substituabilité telle que l'innovateur est capable d'obtenir un revenu égal à la somme des profits de monopole sur les marchés en utilisant un contrat stipulant le payement d'une charge fixe et une clause d'exclusivité territoriale.

Ces résultats montrent donc que l'utilisation de contrats stipulant une charge fixe et une clause d'exclusivité peuvent être parfaitement justifiés lorsque les produits vendus par les producteurs sont suffisamment substituts et/ou lorsque la qualité de l'innovation est suffisamment élevée.

Mots-Clés: Contrat de licence, clause d'exclusivité territoriale, valeur d'un brevet.

Abstract

In this paper, we examine a framework where two differentiated products are sold in two different marketplaces. An innovator owns a patent on an innovation that allows its user to decrease its marginal cost of production. The innovator does not belong to the industry and seeks therefore to license his patent to the producers. We assume that the licensing contract stipulates the payment of a fixed fee together with an exclusive territory clause. We show that, whenever the innovation quality is sufficiently large, there exists such a contract that allows the innovator to obtain a licensing revenue equal to the sum of the profits a monopolist using the cost-reducing innovation will make in each marketplace. Such a kind of contract is called "an optimal contract." Moreover, if the two products are not too differentiated, then an optimal fixed fee plus exclusive territory clause contract exists even if the innovation is not drastic in the sense of Arrow (1962). Our results will therefore provide some insight on the frequent use of fixed fee only payment as well as of exclusive territory clause that has been observed in Rostocker (1983) and Anand and Khanna (2000).

Keywords: Licensing contracts, value of a patent, fixed-fee contract, exclusive territory clause.

Contents

List of Figures

In memory of

Armando Manuel Priegue Guerra

(1921 – 1995)

Aknowledgements

This work would not be possible without the help and support of my parents. I offer them all my gratitude and love. Thanks for loving me so much to let me go.

My most high recognition goes to Dr Yves Richelle. Thank you very much for the proposition you made to me in July 2000. It was indeed worth it. It was a pleasure to listen to your explanations throughout all this months, to learn from you the true value of economics, and the passion for this discipline. Thank you for your dedication, patience and implication which went beyond the call of duty. I am eternally grateful to you. Last but not least, thank you for your friendship.

To my family, my sisters Dolores and Monica and my niece and nephew, Laura y Nicolas because they are a true source of inspiration and motivation. To them I give all my love.

1 Introduction

Licensing of patents and technologies is an important phenomenon for two main reasons. On the one hand, it is one of the few significant mechanisms of technology transfer between firms. On the other hand, it is one of the most commonly observed inter-firm contractual agreements. Taking into account the increasing importance of licensing as a tool for managing the intellectual property of firms, it becomes more and more important to understand the economics of licensing since the way licenses are priced and sold may have a major impact on market structure, on the total revenue of the inventor and therefore, on the incentive to innovate.

In licensing a patented cost-reducing innovation, an innovator may use different policies. For instance, the innovator can organize an auction to sell a given number of licenses or he can impose a fixed-fee or a royalty to be paid and let firms decide whether or not they want to acquire a license. In the theoretical literature, the properties of most of these licensing policies have been established in a variety of contexts (see Kamien (1992) for a survey). For instance, Kamien and Tauman (1986) have looked at the revenue that an innovator can obtain if he uses a licensing contract based on a royalty or on a fixed fee in a model where, on the one hand, the innovation allows members of an imperfectly competitive industry to reduce their marginal cost and, on the other hand, the innovator does not belong to the industry. These authors have shown that the innovator is able to obtain a larger revenue by using an appropriate fixed-fee contract than the one he can obtain by using a royalty contract. Moreover, this revenue is shown to be less that the profit a monopolist using the innovation will make on the market. There is a noticeable exception to this result, namely, the case where perfect competition prevails on the market, the innovator uses a fixed fee licensing contract and the innovation is "drastic" in the sense of Arrow (1962). Recently, Erutku and Richelle (2000) have shown that if one allows the innovator to use a contract based on both a fixed fee and a royalty, then the innovator can, for any number of firms and for any innovation quality, obtain a revenue equal to the profit a monopolist will obtain by using the innovation. It follows from the analysis of Erutku and Richelle (2000) that actual contracts should specify the payment of a fixed fee as well as a royalty. According to Rostocker (1983)'s study, this is indeed the case for 46% of the contracts under review. However, there is also 13% of the reviewed contract that are based only on a fixed fee. Moreover, Anand and Khanna (2000) show that licensing contracts often use qualitative clauses such as an exclusive territory clause in addition to a specified payment scheme. The purpose of the present work is to establish that, whenever the innovation quality is sufficiently large, contracts that specify a fixed fee together with an exclusive territory clause allow the innovator to obtain a revenue equal to the profit a monopolist will obtain by using the innovation. Our study will therefore provide an explanation to the frequent use of a fixed fee payment and an exclusive territory clause in licensing contracts.

The paper is organised as follows. Section 2 describes the model. The set of parameter values for which there exists a fixed fee plus exclusive territory clause such that the innovator revenue is equal to the profit a monopolist will achieve by using the innovation are given in Section 3 and Section 4 concludes.

2 The model

We consider a situation where two differentiated products are sold on two different marketplaces. Each product is produced by only one firm and we assume, for simplicity, that the marginal production cost of each commodity is constant and equal to c. The demand for products i and j in marketplace k, denoted by x_{ik} and x_{jk} respectively, comes from the maximization of the utility function of a representative consumer under the usual budget constraint. For simplicity, we assume that this utility function is given by:

$$U(x_{ik}, x_{jk}, m_k) = a(x_{ik} + x_{jk}) - \frac{x_{ik}^2 + x_{jk}^2}{2} - \gamma x_{ik} x_{jk} + m_k \tag{1}$$

where a is strictly greater than the marginal production cost c; γ, stands for the degree of product differentiation and belongs to $[0, 1)$; and m_k stands for the consumption of an aggregate commodity. The maximization problem of the representative consumer is the following:

$$\max_{x_{ik}, x_{jk}, m_k} U(x_{ik}, x_{jk}, m_k) \tag{2}$$

$$s.t.\;\; p_{ik}x_{ik} + p_{jk}x_{jk} + m_k = R_k$$

$$x_{ik} \geq 0; x_{jk} \geq 0$$

It is then quite easy to verify that the demand functions in marketplace k are given by

$$x_{ik} = \begin{cases} a - p_{ik} & \text{for } p_{ik} \text{ and } p_{jk} \text{ such that } a(1-\gamma) - p_{jk} + \gamma p_{ik} \leq 0; \\ \dfrac{a(1-\gamma) - p_{ik} + \gamma p_{jk}}{1-\gamma^2} & \text{for } p_{ik} \text{ and } p_{jk} \text{ such that } a(1-\gamma) - p_{ik} + \gamma p_{jk} \geq 0 \\ & \text{and } a(1-\gamma) - p_{jk} + \gamma p_{ik} \geq 0; \\ 0 & \text{for } p_{ik} \text{ and } p_{jk} \text{ such that } a(1-\gamma) - p_{ik} + \gamma p_{jk} \leq 0. \end{cases} \tag{3}$$

$$x_{jk} = \begin{cases} a - p_{jk} & \text{for } p_{ik} \text{ and } p_{jk} \text{ such that } a(1-\gamma) - p_{ik} + \gamma p_{jk} \leq 0; \\ \dfrac{a(1-\gamma) - p_{jk} + \gamma p_{ik}}{1-\gamma^2} & \text{for } p_{ik} \text{ and } p_{jk} \text{ such that } a(1-\gamma) - p_{jk} + \gamma p_{ik} \geq 0 \\ & \text{and } a(1-\gamma) - p_{ik} + \gamma p_{jk} \geq 0; \\ 0 & \text{for } p_{ik} \text{ and } p_{jk} \text{ such that } a(1-\gamma) - p_{jk} + \gamma p_{ik} \leq 0. \end{cases} \tag{4}$$

We assume, for simplicity, that the utility function of the representative consumer in marketplace l takes the same form as the one given in (1) i.e.

$$U(x_{il}, x_{jl}, m_l) = a(x_{il} + x_{jl}) - \frac{x_{il}^2 + x_{jl}^2}{2} - \gamma x_{il} x_{jl} + m_l.$$

The demand for product i and j in marketplace l are therefore given by:

$$x_{il} = \begin{cases} a - p_{il} & \text{for } p_{il} \text{ and } p_{jl} \text{ such that } a(1-\gamma) - p_{jl} + \gamma p_{il} \leq 0; \\ \\ \dfrac{a(1-\gamma) - p_{il} + \gamma p_{jl}}{1-\gamma^2} & \text{for } p_{il} \text{ and } p_{jl} \text{ such that } a(1-\gamma) - p_{il} + \gamma p_{jl} \geq 0 \\ & \qquad \text{and } a(1-\gamma) - p_{jl} + \gamma p_{il} \geq 0; \\ \\ 0 & \text{for } p_{il} \text{ and } p_{jl} \text{ such that } a(1-\gamma) - p_{il} + \gamma p_{jl} \leq 0. \end{cases} \tag{5}$$

$$x_{jl} = \begin{cases} a - p_{jl} & \text{for } p_{il} \text{ and } p_{jl} \text{ such that } a(1-\gamma) - p_{il} + \gamma p_{jl} \leq 0; \\ \\ \dfrac{a(1-\gamma) - p_{jl} + \gamma p_{il}}{1-\gamma^2} & \text{for } p_{il} \text{ and } p_{jl} \text{ such that } a(1-\gamma) - p_{jl} + \gamma p_{il} \geq 0 \\ & \qquad \text{and } a(1-\gamma) - p_{il} + \gamma p_{jl} \geq 0; \\ \\ 0 & \text{for } p_{il} \text{ and } p_{jl} \text{ such that } a(1-\gamma) - p_{jl} + \gamma p_{il} \leq 0. \end{cases} \tag{6}$$

Now, suppose that an innovator owns a patent on a technology that reduces the marginal cost of production of these two differentiated commodities. More precisely, we assume that, with the adoption of the "new" technology, the marginal production cost of each commodity becomes equal to $c - \epsilon$. ϵ, which is assumed to belong to $[o, c)$, stands therefore for the innovation quality.

The innovator does not produce any of the two products. It therefore seeks to license its innovation to the producers of these goods. The innovator has the opportunity to choose between many kinds of licensing contracts. In the present work, we shall restrict our attention to contracts that specify a *fixed fee* together with an *exclusive territory clause*. In our setting, such a clause garantees to a licensee that no one else will sell its product in the same marketplace. This means that *if both firms accept the contract proposed by the innovator then, in each marketplace, only one product will be available.* We shall adopt the convention that, whenever both firms accept the licensing contract, product i cannot be sold in marketplace l while product j cannot be sold in marketplace k. In such cases, the demand for the available product in a marketplace is obtained by solving the maximization problem of the representative consumer under the additional restriction that the consumption of the good that is not available in the considered marketplace must be equal to zero. Consequently, whenever both producers are licensee, we obtain the following demand functions:

$$\bullet\, x_{ik} = \begin{cases} a - p_{ik} & \text{if } p_{ik} \leq a, \\ 0 & \text{otherwise} \end{cases} ; \qquad \bullet\, x_{jk} = 0 \tag{7}$$

$$\bullet\, x_{il} = 0; \qquad \bullet\, x_{jl} = \begin{cases} a - p_{jl} & \text{if } p_{jl} \leq a, \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

On the other hand, whenever only one producer or no producer accepts the proposed licensing contract, demand functions in marketplace k, resp. l, are given by (3) and (4), resp. by (5) and (6). Indeed, when only one producer becomes a licensee, the exclusive territory clause does not prevent the licensee as well as the nonlicensee to sell in both marketplaces so that both products will be avalaible in both marketplaces.

The interactions between these three players are described by a three-stage game where (i), at the first stage, the innovator proposes a contract among the set of fixed fee plus exclusive territory clause contracts; (ii) at the second stage, the two producers, having observed the quality of the innovation ϵ as well as the characteristics of the proposed contract, decide simultaneously to accept or to reject the contract; and, (iii) at the third stage, the two producers decide simultaneously the price they will set for their product in each marketplace where their product will be available.

Before we begin the analysis of this game, let us introduce the following definitions:

Definition 1. *An innovation is said drastic if and only if the price a monopolist using the innovation will charge in a marketplace, P_ϵ^M, does not exceed the pre-innovation marginal cost, c. An innovation is then considered to be non-drastic if $P_\epsilon^M > c$. Accordingly, with the specification of the present model, an innovation will be drastic whenever $\epsilon \geq (a - c)$ and it will be non-drastic whenever $\epsilon < (a - c)$.*

Definition 2. *Let π^M be the profit that a monopolist with marginal cost $c - \epsilon$ achieves on a marketplace. An equilibrium[5] contract is said to be optimal if it leads to a revenue equal to $2\pi^M$ for the innovator. An optimal contract is said to be strongly optimal whenever the subgame that follows the proposal of this contract has a unique equilibrium outcome.*

Our main purpose, in the next section, is to determine the set of parameter values for which there exists a (strongly) optimal fixed fee plus exclusive territory clause contract. As discussed in the introduction, our interest in optimal contracts is twofold. First, whenever an optimal contract exists, the private value of a patent is easy to determine since it is simply the profit a monopoly will obtain if it uses the patented innovation and, second, whenever an optimal contract exists, the contractual surplus is maximized and there is thus no contractual inefficiency in the relationship between the innovator and the licensees. Note also that we restrict ourselves to a very particular form of contracts, namely, those that specify the payement of a fixed fee as well as an exclusivity clause. As already noted, this form is frequently observed. In the next section, we provide a theoretical support for the use of such kind of contracts since it allows the innovator to obtain the highest benefit for its innovation i.e. the monopoly profit on each marketplace.

[5]The equilibrium concept we use throughout this work is the subgame perfect equilibrium one.

3 Optimal fixed fee plus exclusive territory clause contracts

As we shall see, there exists some values for the parameters of the model such that the equilibrium contract proposed by the innovator is also optimal in the sense given in Definition 2. The main purpose of this section is to characterize this set of parameter values. In order to do that, we must first determine, for a given contract, the Nash equilibrium that will prevail at the third stage of the game for three possible scenarii, i.e., (i) whenever both firms have decided to accept the proposed contract; (ii) whenever only one firm has decided to accept the proposed contract; and, (iii) whenever the contract is rejected by both firms. With these equilibria in hand, we will be able to determine, for a given contract, the equilibrium decisions at the second stage of the game. This will then allow us to determine the contract that the innovator must propose at equilibrium and, by looking at the revenue that this contract induces, we can characterize the set of parameters of the model for which the equilibrium contract is strongly optimal.

3.1 Equilibria in the marketplaces

Our first step in the characterization of marketplaces equilibria will be to determine the best-response of a firm whenever at least one firm rejects the proposed licensing contract. In this case, demand functions are given by equations (3), (5), (4), and (6). On the other hand, we shall denote producer i's marginal production by c_i in order to consider simultaneously the case where producer i has accepted the contract and the case where it has rejected the contract. In the first case, its marginal cost c_i is equal to $c-\epsilon$ while in the latter case, its marginal cost c_i is simply equal to c. Similarly, c_j will denote producer j's marginal cost with $c_j = c - \epsilon$ whenever j has accepted the contract and $c_j = c$ whenever j has rejected the contract. The profits of producers i and j, denoted respectively by $\Pi_i(p_{ik}, p_{il}, p_{jk}, p_{jl})$ and $\Pi_j(p_{ik}, p_{il}, p_{jk}, p_{jl})$, are thus given as follows.

$$\Pi_i(p_{ik}, p_{il}, p_{jk}, p_{jl}) = (p_{ik} - c_i)x_{ik} + (p_{il} - c_i)x_{il} \tag{9}$$

$$\Pi_j(p_{ik}, p_{il}, p_{jk}, p_{jl}) = (p_{jk} - c_j)x_{jk} + (p_{jl} - c_j)x_{jl} \tag{10}$$

with x_{ik}, x_{il}, x_{jk}, and x_{jl} are given by (3), (5), (4), and (6) respectively. Each producer will choose a price on each marketplace that maximizes its profits. From the first-order conditions of this maximization problem, we can easily obtain the best-response function of each producer in each marketplace since, as can be seen in (9) and (10), the profit of a producer on a marketplace does not depend on the price it chooses to set for its product on the other marketplace.

To begin with, consider the monopoly price, p_i^M, given by $(a + c_i)/2$, i.e. $\arg\max_{p_ik}(p_{ik} - c_i)(a - p_{ik})$. For this price to be a best-response of producer i on marketplace k, the price charged by producer j on this marketplace must be such that its sales x_{jk} are equal to zero whenever producer i's price is equal to the monopoly price. Using the expression of the demand function given in (4), it is easy to verify that the monopoly price is a best-response for producer i to any producer j's price greater than or equal to $[a(2 - \gamma) + \gamma c_i]/2$ since

$$x_{jk}\left(\frac{a + c_i}{2}, p_{jk}\right) = 0 \quad \text{for all} \quad p_{jk} \geq \frac{a(2 - \gamma) + \gamma c_i}{2} \tag{11}$$

We can now turn to producer i's best-response to very small producer j's prices. Consider any producer j's price smaller or equal to $[c_i - a(1 - \gamma)]/\gamma$. From the demand function of producer i in marketplace k given in (3), we obtain that

$$x_{ik}(p_{ik}, p_{jk}) = 0 \quad \text{for all} \quad (p_{ik}, p_{jk}) \text{ such that } p_{ik} \geq c_i \text{ and } p_{jk} \leq \frac{c_i - a(1 - \gamma)}{\gamma} \tag{12}$$

It follows that, for such small producer j's prices, a best-response of producer i is a price equal to its marginal cost c_i. Indeed, if producer i chooses any price above c_i, it will obtain a zero profit on marketplace k since, as noted above, the demand for its product in this marketplace will be equal to zero for such prices. Accordingly, setting a price above marginal cost does not allow producer i to obtain a profit greater than the one it achieves by setting a price equal to its marginal cost[6]. On the other hand, if producer i chooses a price below marginal cost, producer i will obtain either a zero profit or a strictly negative profit and therefore a profit smaller or equal to the profit obtained whenever producer i's price is equal to its marginal cost.

Our next step is to determine what are the best-responses for producer i whenever producer j's price on marketplace k takes intermediate values. Let us immediately define $\tilde{p}_{ik}$ as follows:

$$\begin{aligned}\tilde{p}_{ik} &= \arg\max_{p_{ik}} (p_{ik} - c_i) \left[\frac{a(1-\gamma) - p_{ik} + \gamma p_{jk}}{1-\gamma^2}\right] \\ &= \frac{a(1-\gamma) + c_i + \gamma p_{jk}}{2}. \end{aligned} \tag{13}$$

This means that $\tilde{p}_{ik}$ is the price that maximizes producer i's profit whenever both firms sell a strictly positive quantity on the marketplace k, i.e. whenever the demand for producer i's product in the marketplace k, x_{ik}, is given by $[a(1-\gamma) - p_{ik} + \gamma p_{jk}]/(1-\gamma^2)$. For this to be the case, we must have, from (4), that

$$a(1-\gamma) - p_{jk} + \gamma \tilde{p}_{ik} \geq 0;$$

$$a(1-\gamma) - \tilde{p}_{ik} + \gamma p_{jk} \geq 0.$$

These conditions are verified if

$$\frac{c_i - a(1-\gamma)}{\gamma} \leq p_{jk} \leq \frac{a(1-\gamma)(2+\gamma) + \gamma c_i}{2-\gamma^2}. \tag{14}$$

[6]From this reasoning, it is easy to verify that, for such producer j's prices, any price equal to μc_i, with $\mu \geq 1$, is a best-response for producer i. This multiplicity of best-responses does not lead to any problem since whatever the best-response chosed by producer i, the market outcome will be the same i.e. only producer j has positive sales on the considered marketplace. On the other hand, as we shall see below, we adopt the convention that for such producer j's prices the best-response of producer i is equal to its marginal cost in order to preserve the continuity of producer i's best-response function.

Hence, for any producer j's price verifying (14), producer i's best-response is equal to $\tilde{p}_{ik}$.

There remains a last case to be examined. Indeed, it is easy to see that, whenever γ is strictly positive, there exists some p_{jk} that are strictly greater than $[a(1-\gamma)(2+\gamma)+\gamma c_i]/(2-\gamma^2)$ and that are strictly smaller than $[a(2-\gamma)+\gamma c_i]/2$. This means that, for such p_{jk}, neither $\tilde{p}_{ik}$ nor p_i^M is a best-response since conditions (14) and (11) are not satisfied. To find a best-response of producer i to such p_{jk}, let us define $\bar{p}_{ik}$ as the producer i's largest price such that, for a given price p_{jk}, the demand for product j in marketplace k, x_{jk}, is equal to zero. From (4), we immedaitely obtain that

$$\bar{p}_{ik} = \frac{p_{jk} - a(1-\gamma)}{\gamma}. \tag{15}$$

Now, let us look at the derivative with respect to p_{ik} of producer i's profit when the demand for product j remains positive. If we evaluate this derivative for p_{jk} greater than $[a(1-\gamma)(2+\gamma)+\gamma c_i]/(2-\gamma^2)$ and p_{ik} equal to $\bar{p}_{ik}$, it is easy to see that this derivative is strictly negative. Indeed, we have

$$\begin{aligned}
&\frac{\partial}{\partial p_{ik}}\left[(p_{ik}-c_i)\left(\frac{a(1-\gamma)-p_{ik}+\gamma p_{jk}}{1-\gamma^2}\right)\right]\Bigg|_{p_{ik}=\bar{p}_{ik}} \\
&\qquad = \frac{a(1-\gamma)-2\bar{p}_{ik}+\gamma p_{jk}+c_i}{1-\gamma^2} && (16)\\
&\qquad = \frac{a(1-\gamma)(2+\gamma)+\gamma c_i-(2-\gamma^2)p_{jk}}{1-\gamma^2} && \text{since } \bar{p}_{ik} = \frac{p_{jk}-a(1-\gamma)}{\gamma}\\
&\qquad < 0 && \text{if } p_{jk} > \frac{a(1-\gamma)(2+\gamma)+\gamma c_i}{2-\gamma^2}
\end{aligned}$$

Hence, for such producer j's prices, producer i could increase its profit by choosing a price below $\bar{p}_{ik}$. But for such a price, the demand for product j will be equal to zero so that, for $p_{ik} < \bar{p}_{ik}$ and $p_{jk} \geq [a(1-\gamma)(2+\gamma)+\gamma c_i]/(2-\gamma^2)$, producer i's profit function is actually given by

$(p_{ik} - c_i)(a - p_{ik})$. However, we know that this function is maximized for a price equal to the monopoly price p_i^M but that the demand for product j will be strictly positive if producer i's price is equal to the monopoly price and, as in the case considered here, producer j's price is smaller than $[a(2-\gamma)+\gamma c_i]/2$. Indeed, we have

$$\begin{aligned} \bar{p}_{ik} &= \frac{p_{jk} - a(1-\gamma)}{\gamma} \\ &< \frac{a + c_i}{2} = p_i^M \qquad \text{if } p_{jk} < \frac{a(2-\gamma)+\gamma c_i}{2}. \end{aligned} \tag{17}$$

This immediately implies that

$$\begin{aligned} \frac{\partial}{\partial p_{ik}} \left[(p_{ik} - c_i)(a - p_{ik})\right]\Big|_{p_{ik}=\bar{p}_{ik}} &= a - 2\bar{p}_{ik} + c_i \\ &> 0 \qquad \text{from (17).} \end{aligned} \tag{18}$$

As shown in the following figure, these remarks lead to the conclusion that $\bar{p}_{ik}$ is actually the best-response of producer i to a producer j's price belonging to the interval

$$\left(\frac{a(1-\gamma)(2+\gamma)+\gamma c_i}{2-\gamma^2}, \frac{a(2-\gamma)+\gamma c_i}{2} \right).$$

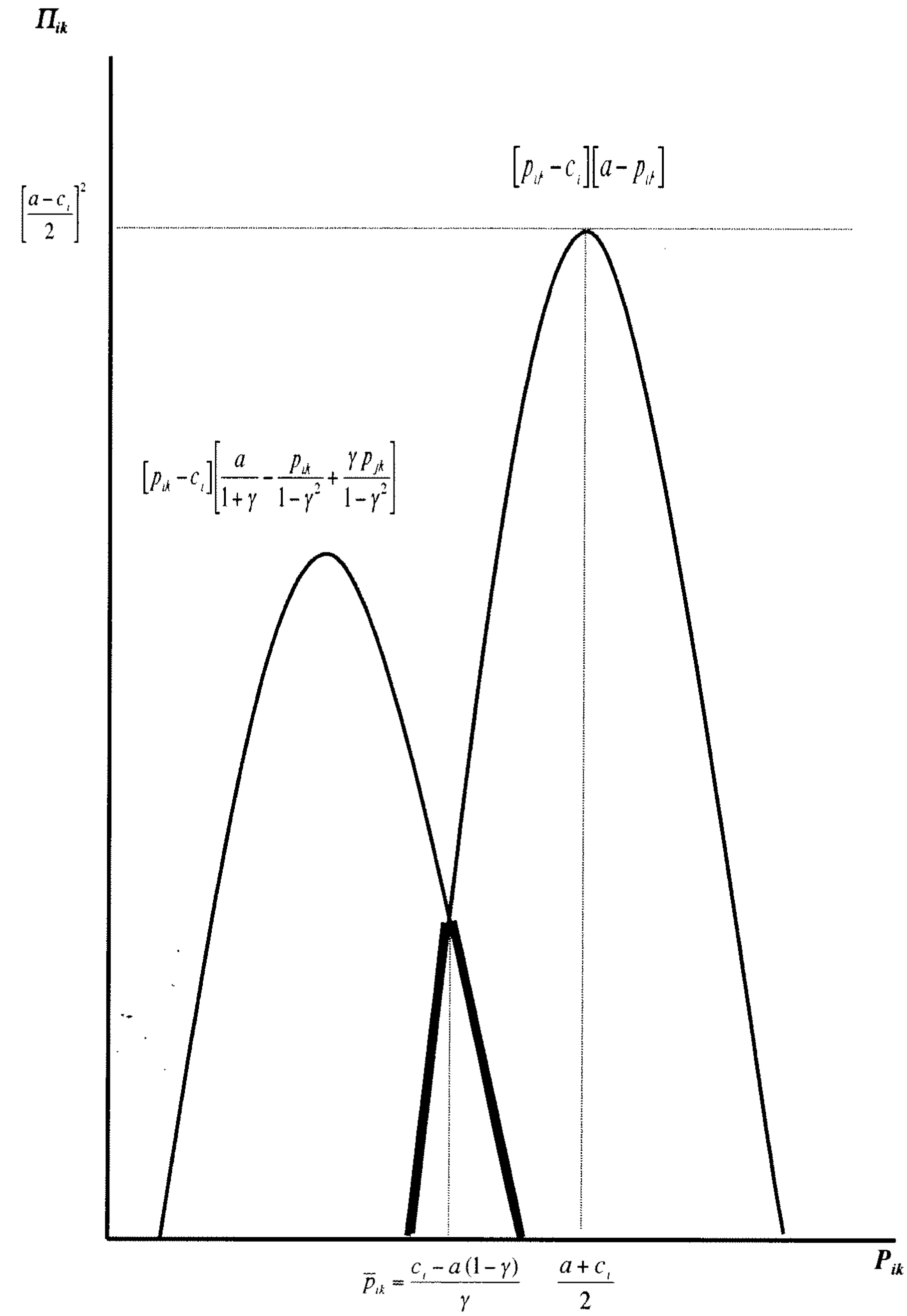

Figure 1: Construction of producer i's best-response function

We can summarize these discussions in the following lemma:

Lemma 1. *Let $s \in \{k, l\}$ be a typical marketplace. Suppose that at least one producer has rejected the proposed licensing contract. Let us denote by c_i, resp. c_j, producer i's, resp. producer j's, marginal production cost with c_i, resp. c_j, being equal to either $c - \epsilon$ whenever producer i, resp. j, has accepted the licensing contract or c if producer i, resp. j, has rejected the proposed licensing contract. Producer i's best-response function on marketplace s, denoted by p_{is}^B, is given as follows:*

$$p_{is}^B = \begin{cases} \dfrac{a + c_i}{2} & \text{if } p_{js} \geq \dfrac{a(2-\gamma) + \gamma c_i}{2}, \\[2ex] \dfrac{p_{js} - a(1-\gamma)}{\gamma} & \text{if } p_{js} \in \left[\dfrac{a(1-\gamma)(2+\gamma) + \gamma c_i}{2-\gamma^2}, \dfrac{a(2-\gamma) + \gamma c_i}{2}\right], \\[2ex] \dfrac{a(1-\gamma) + \gamma p_{js} + c_i}{2} & \text{if } p_{js} \in \left[\dfrac{c_i - a(1-\gamma)}{\gamma}, \dfrac{a(1-\gamma)(2+\gamma) + \gamma c_i}{2-\gamma^2}\right], \\[2ex] c_i & \text{if } p_{js} \leq \dfrac{c_i - a(1-\gamma)}{\gamma}. \end{cases} \tag{19}$$

Similarly, producer j's best-response function on market place s, denoted by p_{js}^B, is given as follows:

$$p_{js}^B = \begin{cases} \dfrac{a + c_j}{2} & \text{if } p_{is} \geq \dfrac{a(2-\gamma) + \gamma c_j}{2}, \\[2ex] \dfrac{p_{is} - a(1-\gamma)}{\gamma} & \text{if } p_{is} \in \left[\dfrac{a(1-\gamma)(2+\gamma) + \gamma c_j}{2-\gamma^2}, \dfrac{a(2-\gamma) + \gamma c_j}{2}\right], \\[2ex] \dfrac{a(1-\gamma) + \gamma p_{is} + c_j}{2} & \text{if } p_{is} \in \left[\dfrac{c_j - a(1-\gamma)}{\gamma}, \dfrac{a(1-\gamma)(2+\gamma) + \gamma c_j}{2-\gamma^2}\right], \\[2ex] c_j & \text{if } p_{is} \leq \dfrac{c_j - a(1-\gamma)}{\gamma}. \end{cases} \tag{20}$$

An illustration of producer i's best-response function is given in the following Figure.

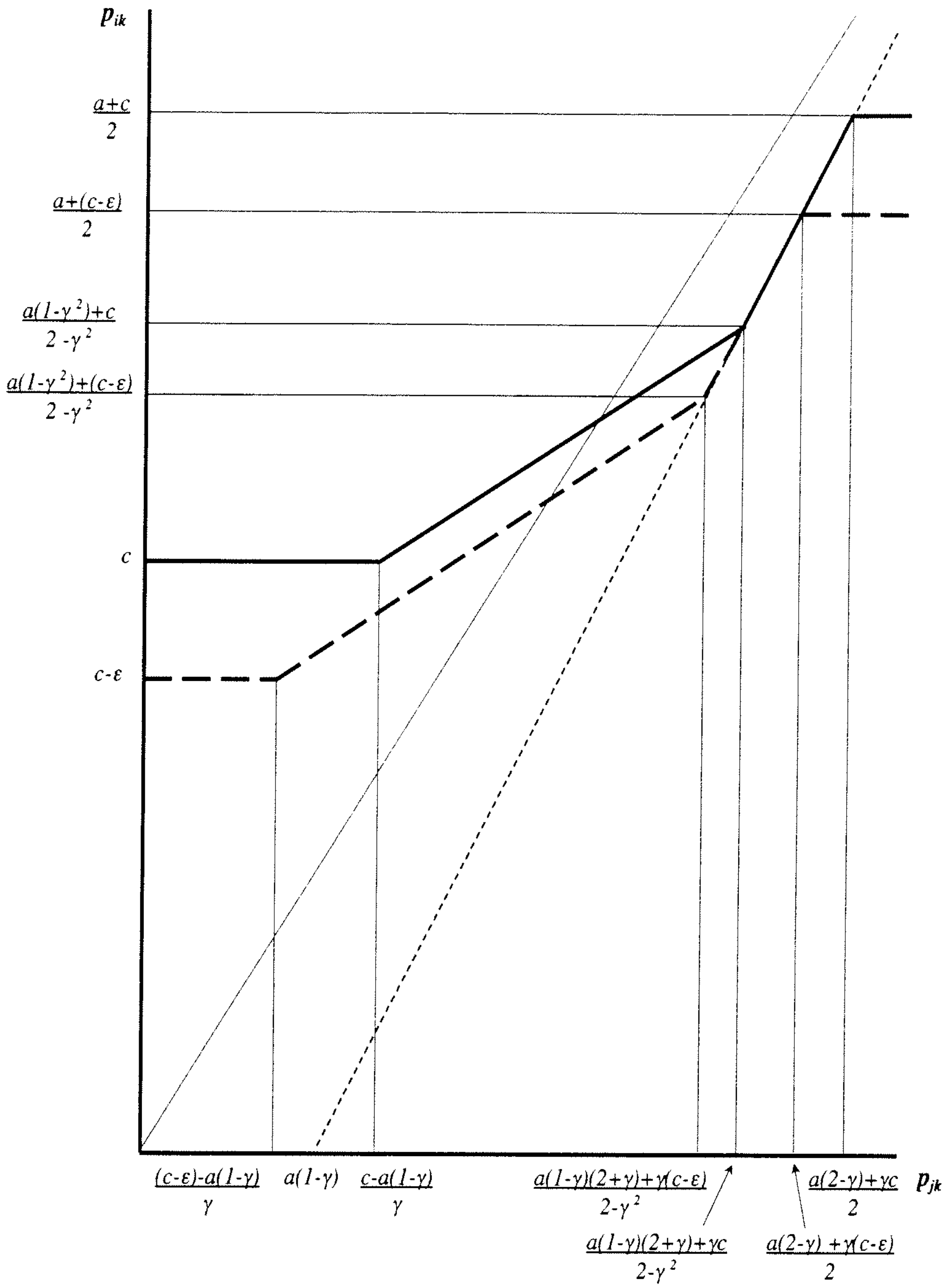

Figure 2: Producer i's best-response function

With these best-response functions in hand, it is quite easy to characterize the equilibrium in a marketplace whenever both producers reject the proposed licensing contract. Indeed, in this case, both firms have a marginal cost equal to c so that, using Lemma 1, we obtain:

Proposition 1. *Suppose both firms reject the proposed licensing contract. Let $p^N(0)$ and $\Pi^N(0)$ be, respectively, the equilibrium price in a marketplace and the equilibrium profits of a producer. We have:*

$$p^N(0) = \frac{[a(1-\gamma)+c]}{(2-\gamma)}, \tag{21}$$

$$\Pi^N(0) = 2\left[\frac{1-\gamma}{1+\gamma}\right]\left[\frac{a-c}{2-\gamma}\right]^2. \tag{22}$$

Notice that, whenever γ tends to 1, i.e. whenever the two commodities tend to be perfect substitutes, the equilibrium price tends to c while the equilibrium profits tends to 0. On the other hand, whenever the two commodities become independent, i.e. whenever γ is equal to 0, the equilibrium price is equal to the monopoly price $(a+c)/2$ and the equilibrium profits are equal to two times the monopoly profits.

Consider now the case where only one producer accepts the proposed licensing contract. We shall denote by subscript A the producer that has accepted the proposed licensing contract while the producer that has rejected the contract will be denoted by subscript R. This means that producer A has a marginal production cost equal to $c-\epsilon$ while producer B's marginal cost is equal to c. In such a case, the equilibrium on the marketplaces will be asymmetric since producers A and R have different marginal costs and the intuition suggests that the larger is this marginal cost asymmetry, i.e. the larger is ϵ, the larger the asymmetry in equilibrium prices will be. In other words, whenever the quality of the innovation, ϵ, is "small", the equilibrium prices are close to those we have found for the case where no producer accepts the proposed licensing contract. As shown by the following result, this is indeed the case.

Proposition 2. *Suppose that only one producer accepts the proposed contract. Let us define ϵ^D as follows:*

$$\epsilon^D = \frac{(a-c)(1-\gamma)(2+\gamma)}{\gamma}. \tag{23}$$

For any $\epsilon \leq \epsilon^D$, the equilibrium prices, denoted by $p_A^N(1)$ and $p_R^N(1)$, and equilibrium profits (gross of the fixed fee), $\Pi_A^N(1)$ and $\Pi_R^N(1)$, are given as follows:

$$p_A^N(1) = \frac{(2+\gamma)\left[a(1-\gamma)+c\right]-2\epsilon}{4-\gamma^2}; \tag{24}$$

$$p_R^N(1) = \frac{(2+\gamma)\left[a(1-\gamma)+c\right]-\gamma\epsilon}{4-\gamma^2}; \tag{25}$$

$$\Pi_A^N(1) = 2\frac{\left[(2+\gamma)(1-\gamma)(a-c)+\epsilon(2-\gamma^2)\right]^2}{(1-\gamma^2)(4-\gamma^2)^2}; \tag{26}$$

$$\Pi_R^N(1) = 2\frac{\left[(2+\gamma)(1-\gamma)(a-c)-\gamma\epsilon\right]^2}{(1-\gamma^2)(4-\gamma^2)^2}. \tag{27}$$

A formal proof of this proposition is given in the Appendix since it only involves tedious computations. We can however obtain some insights by using the following Figure. If ϵ were equal to zero, the equilibrium is the usual symetric one where both producers sell strictly positive quantities on the marketplace. Now, the same kind of equilibrium will arise whenever ϵ is sufficiently close to zero so that, in Figure 3, the two best-responses intersect at a point where the two producers sell strictly positive quantities on the marketplace for such values of ϵ.

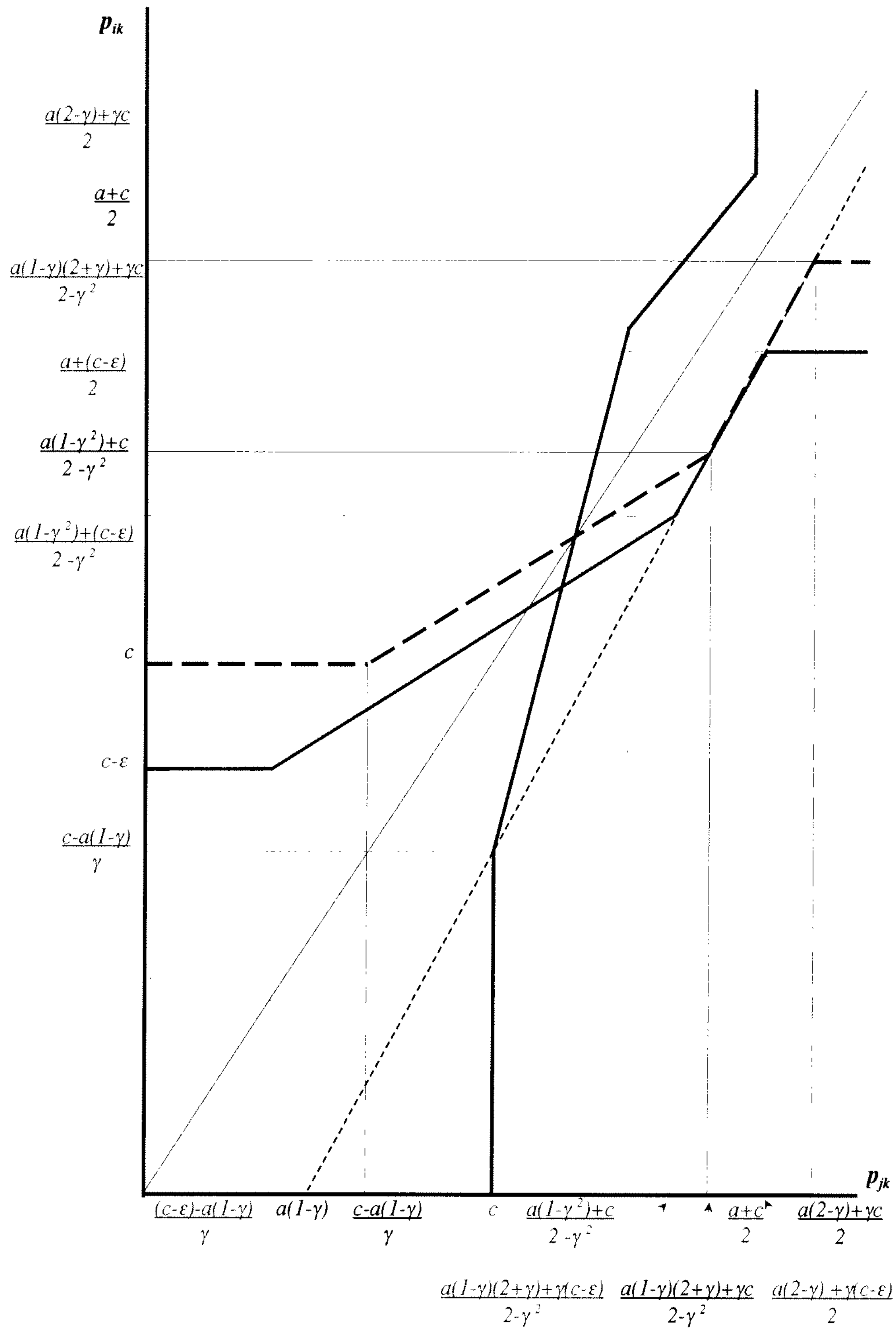

Figure 3: Equilibrium prices for ϵ smaller than ϵ^D.

When ϵ becomes larger, the equilibrium price of both producers will decrease since the best-response of the producer that has accepted the contract, i.e. producer A, is decreasing with respect to its cost and hence ϵ, while the best-response of the producer that has rejected the contract is increasing with respect to the price charged by its competitor. Now, as shown in Figure 4, there exists a critical quality of innovation, denoted by ϵ^D, such that the best-responses intersect at point D where the best-response for producer R is equal to its cost c and where producer R does not sell anything on the marketplace. Indeed, we have that:

$$p^B_{Rs}(p_{As}) = c \qquad \text{whenever } p_{As} \leq \frac{c - a(1-\gamma)}{\gamma}; \tag{28}$$

$$p^B_{As}(c) = \frac{c - a(1-\gamma)}{\gamma} \tag{29}$$

$$\text{whenever } c \in \left[\frac{a(1-\gamma)(2+\gamma) + \gamma(c-\epsilon)}{2-\gamma^2}, \frac{a(2-\gamma) + \gamma(c-\epsilon)}{2}\right]$$

$$\text{i.e. whenever } \epsilon \geq \frac{(a-c)(1-\gamma)(2+\gamma)}{\gamma} \stackrel{\text{def}}{=} \epsilon^D$$

$$\text{and } \epsilon \leq \frac{(a-c)(2-\gamma)}{\gamma} \stackrel{\text{def}}{=} \epsilon^M \tag{30}$$

We can also see on Figure 4 that, even if producer R does not produce anything on the marketplace, producer A cannot charge the monopoly price $[a + (c - \epsilon)]/2$. Indeed, when ϵ is sufficiently close to ϵ^D, producer R's best-response to producer A's monopoly price is greater than its martginal cost and its sales will be positive for such a price.

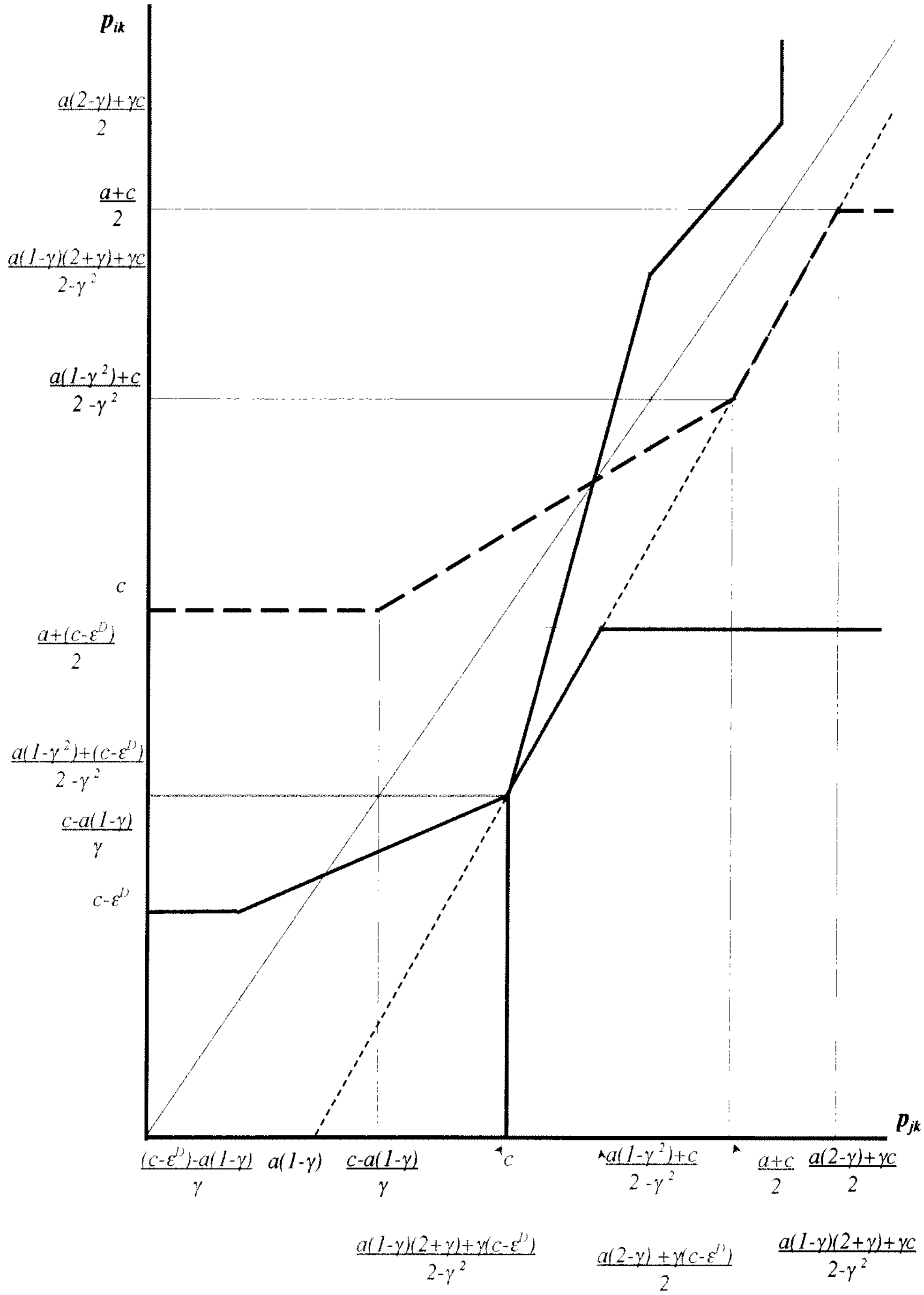

Figure 4: Equilibrium prices for ϵ equal to ϵ^D.

For producer A to be able to charge the monopoly price $[a + (c - \epsilon)]/2$ at equilibrium, ϵ must therefore be greater than or equal to some critical value, namely ϵ^M given in (32). Indeed, we have that

$$p^B_{Rs}(p_{As}) = c \qquad \text{whenever } p_{As} \leq \frac{c - a(1-\gamma)}{\gamma}; \tag{31}$$

$$\frac{a + (c - \epsilon)}{2} \leq \frac{c - a(1-\gamma)}{\gamma} \qquad \text{whenever } \epsilon \geq \frac{(a-c)(2-\gamma)}{\gamma} \overset{\text{def}}{=} \epsilon^M. \tag{32}$$

Such a case is depicted in Figure 5.

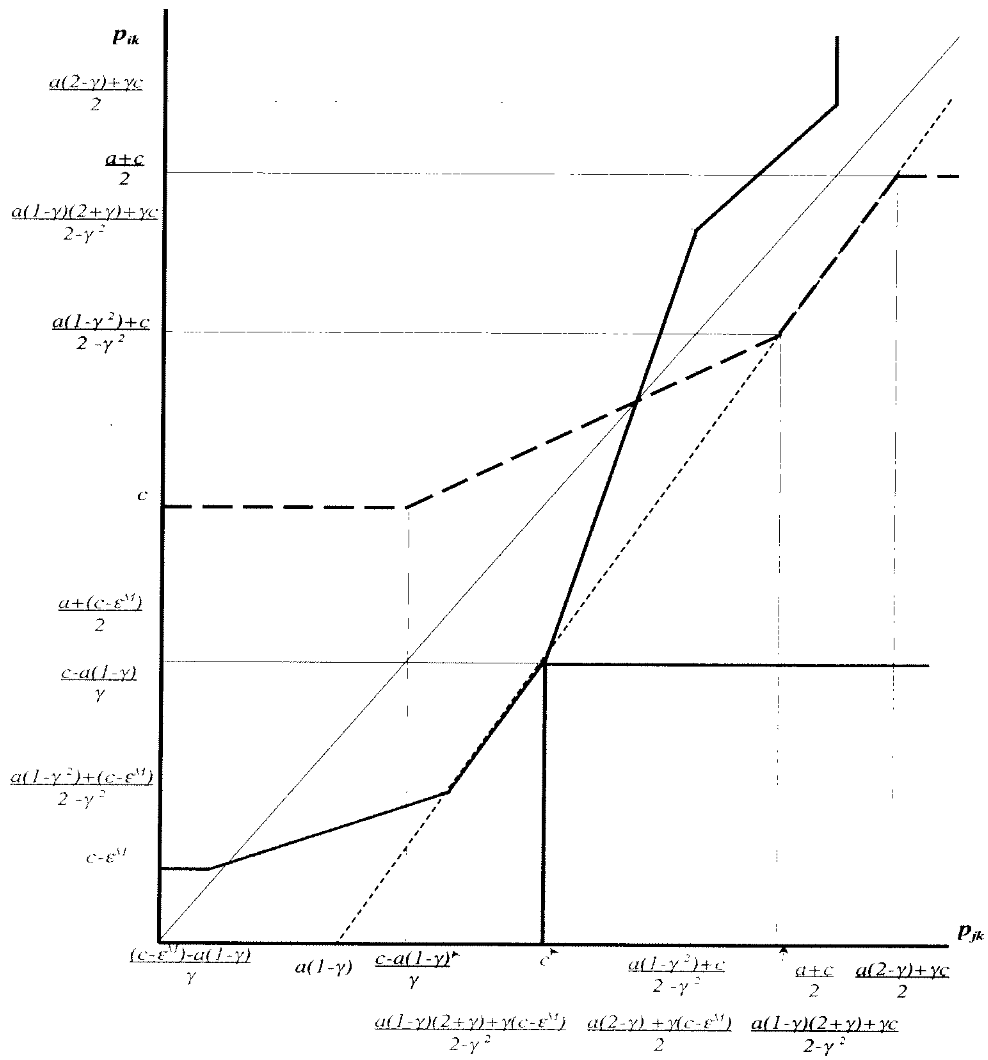

Figure 5: Equilibrium prices for ϵ equal to ϵ^M.

We therefore obtain the following results.

Proposition 3. *Suppose that only one producer accepts the proposed contract. Let ϵ be greater than or equal to ϵ^D. In both marketplaces, the equilibrium price of the producer that has rejected the proposed licensing contract, $p_R^N(1)$, is equal to its marginal cost c while its equilibrium profit, $\Pi_R^N(1)$, is equal to zero. Furthermore, in both marketplaces, the equilibrium price and equilibrium profits (gross of the fixed fee) of the producer that has accepted the proposed contract, $p_A^N(1)$ and $\Pi_A^N(1)$, are given as follows:*

$$p_A^N(1) = \begin{cases} \dfrac{c - a(1-\gamma)}{\gamma} & \text{if } \epsilon \in [\epsilon^D, \epsilon^M], \\[2ex] \dfrac{a + (c - \epsilon)}{2} & \text{if } \epsilon \geq \epsilon^M; \end{cases} \tag{33}$$

$$\Pi_A^N(1) = \begin{cases} 2\left[\dfrac{\gamma\epsilon - (a-c)(1-\gamma)}{\gamma^2}\right][(a-c)(1-\gamma^2)] & \text{if } \epsilon \in [\epsilon^D, \epsilon^M], \\[2ex] 2\left[\dfrac{a-c}{2}\right]^2 & \text{if } \epsilon \geq \epsilon^M. \end{cases} \tag{34}$$

Before we consider the situation where both firms accept the proposed licensing contract, let us mention that the possibility of exclusion of the producer that has rejected the proposed contract depends obviously on the degree of substitution between the two products on the marketplace, γ. We have assumed that ϵ is smaller than or equal to c. Accordingly, if ϵ^D is greater than c, then ϵ is always smaller than ϵ^D and the equilibrium sales and profits of the producer that has rejected the proposed contract are strictly positive. Now, it is easy to verify that:

Lemma 2. *ϵ^D is a decreasing function of the degree of product substitution γ and is equal to zero* *whenever γ is equal to 1. On the other hand, ϵ^D is smaller than the marginal cost c whenever*

$$\gamma > \frac{\sqrt{a^2 + 8(a-c)^2} - a}{2(a-c)}. \tag{35}$$

Moreover, the right-hand side of (35) is smaller than 1 since we have assumed that c is smaller than a. Finally, the right-hand side of (35) as well as ϵ^D tends to 0 whenever c tends to a.

This means that the producer that has rejected the proposed contract cannot be excluded from the marketplace unless products are sufficiently close subsitutes. This is rather intuitive since if products were independent the price charged by one producer cannot affect the sales made by the other. The above result shows also that, whatever the innovation quality, if the equilibrium sales when no producer accepts the licensing contract, i.e. the equilibrium sales before the innovation takes place, are sufficiently small then the producer that rejects the licensing contract will be pushed out of the market by the producer that has accepted the licensing contract. Indeed, as the equilibrium sales when no producer accepts the contract tends to zero, i.e. as c tends to a, ϵ^D tends to zero so that we are in the case of Proposition 3 whatever the value of ϵ.

Now, the same analysis can be made to identify the values of the degree of substitution that allows the producer that has accepted the contract to charge the monopoly price at the equilibrium. We easily obtain the following.

Lemma 3. *ϵ^M is decreasing in γ and is smaller than c whenever*

$$\gamma \geq \frac{2(a-c)}{a}. \tag{36}$$

Furthermore, the right-hand side of (36) as well as ϵ^M tends to 0 whenever c tends to a.

Let us finally turn to the case where both producers accept the proposed licensing contract. In this case, the exclusive territory clause will be in application and, only one product will be available in each marketplace. Each producer will therefore behave as a monopoly so that we obtain:

Proposition 4. *Suppose that both producers accept the proposed licensing contract. In each marketplace* s*, the equilibrium price* $p_s^N(2)$ *is equal to the monopoly price* $[a + (c - \epsilon)]/2$ *and the producer that is active on marketplace* s *obtains equilibrium profits (gross of the fixed fee),* $\Pi_A^N(2)$*, amounting to the monopoly profits* $[a - (c - \epsilon)]^2/4$.

Having described the different equilibrium configurations that can arise in the third stage of the game, we can now examine what happens in the second stage of tha game, i.e. what are producers' decisions regarding the acceptation of the proposed contract along the equilibrium path.

3.2 Decisions relative to the acceptance or rejection of the proposed licensing contract

Roughly speaking, a producer will decide to accept the proposed licensing contract whenever the equilibrium profits it obtains as a licensee less the fixed fee is greater than the equilibrium profits it achieves as a nonlicensee. We therefore easily obtain the following:

Proposition 5. *Suppose that whenever a producer obtains, by accepting the licensing contract, a profit net of the fixed fee equal to the profit it achieves by rejecting the contract, the producer decides to accept the licensing contract. Then, along the equilibrium path, we have that*

1. *no producer accepts the proposed licensing contract whenever* $\Pi_A^N(1) - \alpha < \Pi^N(0)$ *where* $\Pi^N(0)$ *is given by (22) while* $\Pi_A^N(1)$ *is given by (26) if* ϵ *is smaller than or equal to* ϵ^D *and by (34) otherwise;*

2. *only one producer accepts the proposed licensing contract whenever the following condi-*

tions are satisfied

$$\Pi_A^N(1) - \alpha \geq \Pi^N(0)$$

$$\Pi_A^N(2) - \alpha < \Pi_R^N(1)$$

where $\Pi_R^N(1)$ *is either given by (27) if* $\epsilon \leq \epsilon^D$ *or equal to 0 if* $\epsilon \geq \epsilon^D$ *while* $\Pi_A^N(2)$ *is equal to* $[a - (c - \epsilon)]^2/4$*;*

3. *both producers accept the proposed licensing contract whenever* $\Pi_A^N(2) - \alpha \geq \Pi_R^N(1)$.

In other words, if the fixed fee α is greater than $\Pi_A^N(1) - \Pi^N(0)$ then no firm accepts the contract while if α is greater than $\Pi_A^N(2) - \Pi_R^N(1)$ but smaller than or equal to $\Pi_A^N(1) - \Pi^N(0)$ then the contract is accepted by only one producer. Finally, if α is smaller than or equal to $\Pi_A^N(2) - \Pi_R^N(1)$ then both firms accept the contract.

3.3 Existence of an optimal contract

From Definition 2, a strongly optimal contract is such that, at the subgame perfect equilibrium, the innovator's licensing revenue is equal to the sum of profits that a monopoly will achieve on each marketplace when it uses the new technology, i.e., when the innovator's licensing revenue at equilibrium is equal to $2[a - (c - \epsilon)]^2/4$. Obviously, the innovator's revenue is equal to the fixed fee specified in the contract times the number of producers that decide to accept the contract. Consequently, to find an optimal contract, we have to look at the innovator's revenue from two specific contracts. The first contract is the one that specifies a fixed fee equal to $\Pi_A^N(1) - \Pi^N(0)$ so that, as we have seen in Proposition 5, this contract is accepted by only one producer. Notice immediately that, whenever both producers reject the contract, the equilibrium profits of a producer $\Pi^N(0)$, is always strictly positive while the equilibrium profit of the producer that is the only one that has accepted the contract, $\Pi_A^N(1)$, is smaller than or equal to the sum of the profits a monopoly using the new technology will make in each marketplace. Consequently, we obtain

Proposition 6. *If a licensing contract is accepted by only one producer, then this contract is not optimal in the sense of Definition 2.*

The second contract to look at is the one that specifies a fixed fee equal to $\Pi_A^N(2) - \Pi_R^N(1)$ and, following Proposition 5, that is accepted by both producers. Now, as we have seen in Proposition 4, the exclusive territory clause allows both producers to obtain the monopoly profits $[a - (c - \epsilon)]^2/4$. Consequently, for this contract to be optimal, we must have that the equilibrium profit of a producer that rejects the contract is equal to zero whenever the other producer accepts the contract, i.e. $\Pi_R^N(1) = 0$. We have established in Proposition 2 and 3 that a necessary and sufficient condition for this to arise is that the innovation quality ϵ exceeds the threshold ϵ^D given in (23). We therefore have

Proposition 7. *For any $\epsilon \geq \epsilon^D$, the exists a strongly optimal contract that stipulates a fixed fee equal to the monopoly profit $[a - (c - \epsilon)]^2/4$ and an exclusive territory clause. On the other hand, if ϵ is smaller than ϵ^D, then there does not exist an optimal contract that stipulates only a fixed fee together with an exclusive territory clause.*

Having established the condition for the existence of an optimal contract that relies only on a fixed fee together with an exclusive territory clause, we can obtain some comparative static results. First, we obviously have that the larger the quality of the innovation the more plausible is the existence of a fixed fee plus exclusive territory clause optimal contract. However, if we compare the quality of the innovation required for an optimal contract to exist, ϵ^D, with the quality required for an innovation to be drastic in the sense of Definition 1, $(a - c)$, we easily obtain the following.

Proposition 8. *If $\gamma \geq \sqrt{3}-1$, then $\epsilon^D \leq (a-c)$ and there can exist a strongly optimal fixed fee plus exclusive territory clause contract although the innovation is non-drastic. If $\gamma < \sqrt{3}-1$ and an optimal fixed fee plus exclusive territory clause contract exists, then the innovation is drastic.*

Another factor that plays an important role in the existence of an optimal fixed fee plus exclusive territory clause contract is the degree of substitution between the two products in a marketplace, γ. We have indeed seen in Lemma 2 that ϵ^D is a decreasing function of γ and is equal to 0 whenever γ tends to 1, i.e., whenever the two products becomes almost perfect substitutes. An immediate consequence of the properties given in Lemma 2 is the following.

Proposition 9. *For any innovation quality $\epsilon > 0$, any pre-innovation marginal cost c and any demand intercept a, there exists a degree of substitution such that a strongly fixed fee plus exclusive territory clause contract exists. On the other hand, if $\gamma < \left[\sqrt{a^2+8(a-c)^2}-a\right]/[2(a-c)]$ then, for any innovation quality, there does not exist an optimal fixed fee plus exclusive territory clause contract.*

The first result in this proposition comes obviously from the fact that ϵ^D tends to zero whenever γ tends to 1. Accordingly, for any innovation quality ϵ we can find a degree of substitution such that $\epsilon \geq \epsilon^D$. The reason underlying this result is actually quite simple. Whenever the two products are almost perfect substitutes we are in a situation very close to the one of Bertrand competition with homogeneous products so that a quite small advantage in the marginal cost obtained by the licensee allows this producer to prevent the producer that has not accepted the contract to produce. In other words, with the two products being almost perfect substitutes, the equilibrium profits of a non-licensee will be equal to zero whenever its competitor accepts the licensing contract even if the innovation quality is quite small. The non-existence of an optimal contract whenever the two products are not sufficiently substituable can be understood in the same way. If the two products were independent, i.e. $\gamma = 0$, then no optimal contract exists since the equilibrium profit achieved by a non-licensee will always remain positive whatever the cost of its competitor and whatever the innovation quality. It follows that, for the equilibrium profit of a non-licensee to become equal to zero whenever its competitor accepts the licensing contract, the two products must be sufficiently substitutes. Note that the condition given in Proposition 9 is the one given in Lemma 2 for ϵ^D to be smaller than c. Whenever ϵ^D is greater than c, no innovation quality will exceed ϵ^D since ϵ has been assumed smaller than c in order for the licensee's marginal cost, $c - \epsilon$, to remain positive.

The last comparative static result concerns the pre-innovation cost, c. We have seen in Lemma 2 that as c tends to a, i.e., whenever the pre-innovation equilibrium sales tends to 0, ϵ^D tends to 0. Consequently, we have

Proposition 10. *For any innovation quality ϵ and any degree of subsititution γ, there exists a level of pre-innovation cost c such that a strongly optimal fixed fee plus exclusivity clause contract exists.*

This means that it is more plausible for a fixed fee plus exclusive territory clause to be strongly optimal whenever the pre-innovation market is very thin.

4 Concluding remarks

In this work, we consider an innovator who uses licensing contracts that specify a fixed fee and an exclusive territory clause. We have obtained the following results:

- there exists a critical value for the innovation quality such that, if the quality of the innovation is greater than or equal to this critical value, the innovator is able to obtain a revenue equal to the sum of the profits that a monopolist will achieve in each market by using a licensing contract that specifies a fixed-fee with an exclusive territory clause;

- the critical value mentioned above is inversely correlated to the degree of substituability between the products sold by both producers;

- there exists a critical value for the degree of substituability such that, if the degree of substituability is greater than or equal to this critical value, the innovator is able to obtain a revenue equal to the sum of profits that a monopolist will achieve in both markets even if the innovation is non-drastic in the sense of Arrow (1962);

- for any given innovation quality, there exists a degree of substituability such that the licensor is able to obtain a revenue equal to the sum of profits a monopolist will achieve in both markets by licensing his innovation through a fixed fee with an exclusive territory clause contract.

Our analysis points therefore that an innovator can use a licensing contract specifying a fixed fee and an exclusive territory clause to obtain the monopoly profits whenever the pre-innovation equilibrium quantity is small and/or the two products are close substitutes and/or the innovation quality is large. Under such circumstances, there is no need for the innovator to use licensing contracts specifying some royalties scheme as was the case in Erutku and Richelle (2000).

5 References

Anand, B. and T. Khanna (2000) "The Structure of Licensing Contracts," *Journal of Industrial Economics*, **XLVIII**, 103-135.

Arrow, K. (1962) "Economic Welfare and the Allocation of Ressources for Inventions," in *The Rate and Direction of Inventive Activity*, ed. R.R. Nelson. Princeton: Princeton University Press.

Erutku, C. and Y. Richelle (2000) "Optimal Licensing Contract and the Value of a Patent," Cahier de Recherche 2000-7, Département de Sciences Économiques, Université de Montréal.

Gallini, N. and B. Wright (1990) "Technology Transfer under Asymmetric Information," *RAND Journal of Economics*, **21(1)**, 147-160.

Katz, M. and C. Shapiro (1986) "How to License Intangible Property," *Quaterly Journal of Economics*, **101**, 567-589.

Kamien, M. (1992) "Patent Licensing," in *Handbook of Game Theory*, ed. Aumann, R. and S. Hart. Amsterdam: North-Holland/ Elsevier Science Publishers.

Kamien, M. and Y. Tauman (1984) "The Private Value of a Patent: A Game Theoretic Analysis," *Journal of Economics (Supplement)*, **4**, 93-118.

Kamien, M. and Y. Tauman (1986) "Fees versus Royalties and the Private Value of a Patent," *Quaterly Journal of Economics*, **101**, 471-491.

Muto, S. (1993) "On Licensing Policies in Bertrand Competition," *Games and Economic Behavior*, **5**, 257-267.

Rostocker, M. (1983) "PTC Research Report: A Survey of Corporate Licensing," *IDEA - The Journal of Law and Technology*, **24**, 59-92.

6 Appendix

To prove Proposition 2, we begin by noting that

$$p_A^N(1) = \frac{a(1-\gamma) + \gamma p_R^N(1) + (c-\epsilon)}{2} \tag{37}$$

$$p_R^N(1) = \frac{a(1-\gamma) + \gamma p_A^N(1) + c}{2} \tag{38}$$

Accordingly, from the best-response functions given in (24), $p_A^N(1)$ will be a best-response for firm A to $p_R^N(1)$ whenever

$$p_R^N(1) \in \left[\frac{c-\epsilon-a(1-\gamma)}{\gamma}, \frac{a(1-\gamma)(2+\gamma)+\gamma(c-\epsilon)}{2-\gamma^2}\right]. \tag{39}$$

In the same way, $p_R^N(1)$ will be a best response for firm R to $p_A^N(1)$ whenever

$$p_A^N(1) \in \left[\frac{c-a(1-\gamma)}{\gamma}, \frac{a(1-\gamma)(2+\gamma)+\gamma c}{2-\gamma^2}\right]. \tag{40}$$

Let us begin by verifying that (39) is satisfied. First, simple computations show that:

$$\begin{aligned} & \frac{(2+\gamma)[a(1-\gamma)+c]-\gamma\epsilon}{4-\gamma^2} \geq \frac{(c-\epsilon)-a(1-\gamma)}{\gamma} \\ \Leftrightarrow \quad & (1-\gamma)(2+\gamma)(a-c)+\epsilon(2-\gamma^2) \geq 0. \end{aligned}$$

The latter inequality is always satified since γ is smaller than 1 while $a-c$ is positive. Second, we can also verify from simple computations that:

$$\begin{aligned} & \frac{a(1-\gamma)(2+\gamma)+c-\epsilon}{2-\gamma^2} \geq \frac{(2+\gamma)[a(1-\gamma)+c]-\gamma\epsilon}{4-\gamma^2} \\ \Leftrightarrow \quad & 2(1-\gamma)(2+\gamma)a - \gamma c(2+\gamma)(1-\gamma) - \epsilon(4-2\gamma-\gamma^2+\gamma^3) \geq 0 \\ \Leftrightarrow \quad & 2(1-\gamma)(2+\gamma)(a-c) - 2\gamma\epsilon + (c-\epsilon)(4-\gamma^2)(1-\gamma) \geq 0. \end{aligned}$$

This last inequality is verified for any $\epsilon \leq \epsilon^D$ since

$$\epsilon^D = \frac{(2+\gamma)(1-\gamma)(a-c)}{\gamma}$$

Let us then verify that (40) is satisfied. First, easy computations lead to:

$$\frac{(2+\gamma)[a(1-\gamma)+c]-2\epsilon}{4-\gamma^2} \geq \frac{c-a(1-\gamma)}{\gamma}$$

$$\Leftrightarrow \quad (2+\gamma)(1-\gamma)(a-c)-\gamma\epsilon \geq 0.$$

This last inequality is therefore satisfied for any $\epsilon \leq \epsilon^D$. Second, we can also easily obtain that:

$$\frac{a(1-\gamma)(2+\gamma)+c}{2-\gamma^2} \geq \frac{(2+\gamma)[a(1-\gamma)+c]-2\epsilon}{4-\gamma^2}$$

$$\Leftrightarrow \quad 2(1-\gamma)(2+\gamma)(2a-\gamma c)-2(2-\gamma^2)\epsilon \geq 0.$$

This last inequality holds for any value of ϵ. The first part of Proposition 2 is therefore proved. Finally, the profit levels given in this proposition are obtained by substituting the value of p_A and p_B by (24) and (25) respectively.