The taxation of nonrenewable natural resources[1]

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Abstract

We provide an analytical overview of the distortionary effects of some common forms of taxes faced by the nonrenewable resources sector of the economy. In the category of taxes meant specifically to capture the resource rent, we look at a specific severance tax, an *ad valorem* severance tax, a profit tax and a "lump-sum" tax, with emphasis on their effects on the extraction decisions over time and on the initial reserves to be developed. In the category of taxes meant for all sectors of the economy, we look at the corporate income tax and its special provision for the resource sector in the form of a depletion allowance, with emphasis on the effects on the intra-industry resource extraction decisions and on the inter-industry allocation of investment.

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1 Introduction

A distinctive feature of nonrenewable natural resource markets is that the marginal revenue from the sale of the resource exceeds the marginal cost of extracting it. This difference constitutes a rent, which is due strictly to the nonrenewability of the resource stock. The *in situ* resource being very often under public ownership, taxes can then become instruments used by governments to recuperate this scarcity rent from the private resource extracting firms to which the extraction has been delegated. This use of the tax system is particularly important in the case of nonrenewable natural resources and the taxes used can take different forms. So-called severance taxes are one type of tax that is often used, in the form either of a tax on the quantity extracted (a specific tax) or on the market value of the resource extracted (an *ad valorem* tax). Profit or net revenue taxes, variably defined, are also common. Some form of fixed licence fees, independent of the quantity extracted or of the revenues, may also be used in some cases.

For the tax in question to be socially efficient it should capture the rent without causing distortions in the resource extraction paths: it should be a "pure rent tax". But generally resource taxes, like taxes on any kind of economic activity, will alter behavior, resulting in partial dissipation of the rent. In the case of nonrenewable resource industries the effects are often more complex than for other kind of activities, because the problem faced by the resource extracting firm is inherently dynamic. In the case of the nonrenewable industries, the tax can create an incentive to modify the whole path of extraction, causing the resource stock to be depleted faster or slower over time. In anticipation of the taxes it will face during the extraction phase, the firm may also modify its exploration and development efforts, resulting in a different total resource stock to be exploited. A purpose of this paper is to provide an overview of the impact of those resource taxes on the extraction path and on the capacity of resource owners to capture the resource rent.[1]

[1] When the *in situ* resource is privately owned and the owner licences another party to extract the resource, royalty payments play a role similar to that of the rent taxes in the case of public ownership. Such royalty payments to private citizens are common in the United States, where the *in situ* resource is most often privately owned. To the extent that the payment from the licensee to the licensor can take forms similar to the different types of taxes considered here, the same analysis applies.

Firms exploiting nonrenewable resources are usually subjected also to some form of corporate income taxation. The corporate income tax falls in a different category, since it is not exclusive to the nonrenewable industry; it applies to all incorporated firms, independent of their sector of activity. However nonrenewable resource extraction firms have the particularity that they benefit from a depletion allowance that reduces their corporate income tax base. This depletion allowance is justified by the fact that the firm's reserves constitute an asset that gets depleted over time, as the resource is taken out of the ground. This is very much like the deduction for tax purposes, at a predetermined rate, of the depreciation of the physical capital held by the firms. In the case of nonrenewable resource firms, the two types of assets are present: an *in situ* resource stock and a stock of physical capital used as input in the extraction process. The rate of depletion allowance being an exogenously determined tax parameter, a first issue it raises is: how does it affect the time path of resource extraction and the initial exploration and development effort. In this respect the analysis bears some similarity with that of the taxes specific to the resource sector, aimed at capturing the rent. But since the corporate income tax applies to all incorporated firms, another important issue is raised, which is: to what extent does the depletion allowance create an incentive to transfer investment from the resource industry to other industries, or vice-versa.

Fairly comprehensive early treatments of the taxation of an exogenously given resource stock can be found in Burness (1976), Dasgupta and Heal (1979, chap.12) and Dasgupta, Heal and Stiglitz (1980).[2] Conrad and Hool (1981), Slade (1984), Heaps (1985) and Gaudet and Lasserre (1990) have considered different aspects of resource taxation when the cost of extraction depends on the remaining stock as well as on the flow of the resource, all assuming that the initial resource stock to be exploited is given. For the present purposes we will neglect the stock effect on cost; we will instead assume that initial reserves are endogenously determined, subject to a convex exploration and development cost function. We feel that this at least partially captures the fact that acquiring new reserves becomes increasingly costly as the ultimate resource stocks available in nature get depleted. It also emphasizes

[2] In his seminal paper, Harold Hotelling (1931) also briefly considers the effect of a time-invariant severance tax on the speed of exhaustion of the resource stock.

the importance of taking into account the effect of anticipated taxes on the exploration and development effort.[3] Campbell and Lindner (1983, 1985a,b) are among the few to have considered explicitly the effect of resource taxation on exploration. They assume that risk averse firms use Bayesian updating of their prior beliefs based on information obtained from their exploration efforts in order to establish the value of mineral deposits. They consider the effect that resource rent taxation will have on the selection of deposits to mine and on the government revenues generated by the tax. The resource rent tax they consider is based on the particular form proposed by Garnaut and Ross (1975, 1979) and more recently discussed in Garnaut (2010).[4] The analysis that follows will borrow from many of those papers, as well as from Gaudet and Lasserre (1984, 1986a,b) as concerns the effect of the depletion allowance in the corporate income tax.

The next section looks at taxes meant specifically for the resource industry, with emphasis on their effect on the extraction decisions over time and on the initial reserves to be developed. We consider a specific severance tax, an *ad valorem* severance tax, a "profit" tax and a "lump-sum" tax. Section 3 is devoted to the analysis of the corporate income tax and the role of the depletion allowance, with emphasis on its effects on the intra-industry resource extraction decisions and on the inter-industry allocation of investment. We briefly discuss some remaining issues in Section 4 and offer some concluding remarks in Section 5.

[3]With *given* initial reserves, it is easily verified that when it is optimal to completely exhaust the resource stock, the results obtained in the absence of the stock effect on cost continue to hold when the stock effect is taken into account. With a stock effect, one must take into account the possibility that extraction will, for economic reasons, stop before the initial reserves are exhausted. Taxation will then have an effect on the fraction of the initial reserves that get extracted. However, when the initial reserves are endogenously determined, as we assume here, it will never be optimal to leave developed resources in the ground, as we will show in the next section. Under perfect foresight, this is true whether the costs of extraction depends on the remaining stock or not.

[4]The rent tax proposed by Garnaut and Ross consists essentially in taxing the current net cash flow of the firm (current revenues minus all current expenditures of the mining firm as they occur) when positive and carrying forward negative cash flows with interest, to be deducted from future positive cash flows. This is a form of what is sometimes called the Brown Tax (Brown, 1948). See also Boadway and Keen (2010) for a more detailed discussion of the issues relating to the practical application of this type of rent tax to nonrenewable resource firms.

2 Taxes specific to natural resource extraction

This section is devoted to the study of the equilibrium extraction paths that result from a number of different taxes intended to capture the resource rents (or royalties) and that apply specifically to the nonrenewable resource firms, as opposed to taxes that apply to firms in all sectors of the economy, such as the corporate income tax. The implications of the latter for nonrenewable resource extraction will be the subject of the next section. The taxes under consideration in this section are the lump-sum tax, the specific severance tax, the *ad valorem* severance tax and the profit (or rent) tax. The emphasis is put on comparing the after-tax equilibrium extraction paths with the efficient extraction path that results in the absence of taxes.

The resource industry in question is assumed to be composed of identical price-taking firms, each of which holds at date t a stock $X(t)$ of the resource. The resource is exploited by the firm at the rate $x(t)$, at a cost $C(x(t))$, assumed to satisfy $C(0) = 0$ and $C'(x(t)) > 0$, $C''(x(t)) > 0$ for all $x(t) \geq 0$. Thus the cost function is characterized by strictly decreasing returns, which may be taken to reflect the fact that some quasi-fixed factors are taken as given and their costs sunk.

The firms being assumed identical, we may normalize the number of firms to one and consider the behavior of the representative firm as that of the industry. The inverse market demand for the resource flow may then be written $p(t) = P(x(t))$, with $P'(x(t)) < 0$. We will assume $P(0) = \bar{p}$, $\bar{p}$ being the finite choke-price at which the quantity demanded becomes zero. The conditions that characterize the equilibrium behavior over time of the industry as a whole can be obtained by simply substituting $P(x(t))$ for the price $p(t)$ into the equilibrium conditions obtained for the representative firm.

Most (if not all) previous analyses of resource taxation to be found in the literature assume the initial stock of the resource, X_0, to be exogenously given. In reality exploration and development efforts must be incurred before the initial reserves can be established and rendered exploitable, and the taxes which the firm anticipates having to pay during the extraction phase are likely to have an impact on those efforts. For this reason, we will assume the initial stock of the resource held by the firm to be *endogenously* determined at the outset, at a cost $E(X_0)$. That cost will be assumed to be increasing in X_0 ($E'(X_0) > 0$), and subject to strictly decreasing returns to effort ($E''(X_0) > 0$). This last assumption reflects the fact that acquiring new reserves becomes increasingly costly as the ultimate resource stocks available in nature get depleted. It will also be assumed that $E(0) = 0$ and $E'(0) = 0$, the latter being sufficient (though not necessary) to assure that it is profitable to develop positive reserves to begin with. A situation where this would not be the case is obviously not of interest for our purpose.

At each date t, the firm faces a tax function $G(x(t), p(t), t)$ that takes one of the following forms:

$$\begin{aligned}
G(x(t), p(t), t) &= \beta(t) : \quad \text{a "lump-sum" tax at each date of production;} \\
G(x(t), p(t), t) &= \alpha(t)x(t) : \quad \text{a specific severance tax;} \\
G(x(t), p(t), t) &= \gamma(t)p(t)x(t) : \quad \text{an } \textit{ad valorem} \text{ severance tax;} \\
G(x(t), p(t), t) &= \nu(t)\{p(t)x(t) - C(x(t)) - \mu(t)x(t)\} : \quad \text{a profit (or rent) tax,}
\end{aligned}$$

where $\alpha(t), \gamma(t), \nu(t)$ are positive numbers smaller than one.

The representative firm chooses the extraction rate $x(t)$ at each $t \in [0, T]$, the date T at which extraction ceases, and the initial reserves X_0, so as to maximize the discounted flow of revenues net of extraction costs and of taxes, minus the cost of acquiring the reserves to be exploited.[5] As for the factor prices, the discount rate r (the rate of interest) is assumed time-invariant. The representative firm's objective can therefore be written

$$\max_{\{x(t)\}, T, X_0} \int_0^T e^{-rt}\{p(t)x(t) - C(x(t)) - G(x(t), p(t), t)\}dt - E(X_0)$$

[5]The cost $E(X_0)$ can be viewed indifferently as resulting from exploration and development efforts undertaken by the firm itself, or from expenditures incurred on the market to acquire X_0, discovered and developed by others.

subject to:

$$\dot{X}(t) = -x(t) \tag{1}$$

$$X(0) = X_0, \qquad X(T) \geq 0, \qquad x(t) \geq 0. \tag{2}$$

The present value Hamiltonian associated to this problem is

$$H = e^{-rt}\left\{p(t)x(t) - C(x(t)) - G(x(t), p(t), t)\right\} - \lambda(t)x(t),$$

where $\lambda(t)$ represents the marginal shadow value of reserves at date t, to be determined as part of the firm's optimization problem. The Hamiltonian measures the present value of profit at date t, net of the opportunity cost that represents the present value sacrificed by extracting the quantity $x(t)$ now instead of leaving it in the ground for future use.

The following, along with (1), are necessary conditions for the solution of the representative firm's problem:

$$e^{-rt}\left\{p(t) - C'(x(t)) - \frac{\partial G(x(t), p(t), t)}{\partial x(t)}\right\} = \lambda(t) \tag{3}$$

$$\dot{\lambda}(t) = 0 \tag{4}$$

$$\lambda(0) = E'(X_0) \tag{5}$$

$$\lambda(T)X(T) = 0, \qquad \lambda(T) \geq 0, \qquad X(T) \geq 0 \tag{6}$$

$$e^{-rT}\{p(T)x(T) - C(x(T)) - G(x(T), p(T), T)\} - \lambda(T)x(T) = 0 \tag{7}$$

Condition (3) is necessary for the maximization of the Hamiltonian at each t, assuming an interior solution for $x(t)$. The strict concavity of the cost function guarantees that this condition is also sufficient for a maximum. Condition (4) is an intertemporal arbitrage condition; it says that the present value of a unit of the resource in the ground must remain constant over time, which implies that its current value must grow at the rate of interest. It is the condition that underlies the well known Hotelling rule (Hotelling, 1931) of optimal nonrenewable resource exploitation. The transversality condition (5) says that the value of the marginal unit of the resource in the ground at date $t = 0$ must be equal to the marginal cost of discovering and developing it; it serves in determining X_0. The transversality conditions (6) and (7) serve in determining the terminal stock $X(T)$ and the terminal date T: the first one says that at the terminal date T, it must be the case that either nothing is left unexploited ($X(T) = 0$), or, if some is left, it has no value ($\lambda(T) = 0$). The second one says that at the terminal date T, the value of the Hamiltonian must be zero. This is because extending marginally the interval of time over which extraction takes place adds the value of the Hamiltonian to the maximized present value of the resource stock. The interval of extraction should be increased if this value is positive and decreased if it is negative. Therefore, in order for this interval, and hence T, to be optimal, the value of the Hamiltonian must be zero at T.

If we set $G(x(t), p(t), t) \equiv 0$, the above necessary conditions characterize the tax-free equilibrium. Since by assumption the firms take prices as given, the tax-free equilibrium yields the socially efficient allocation. Where appropriate, we will denote the corresponding necessary conditions and the resulting equilibrium values by an asterisk (*) in what follows.

It seems immediately intuitive that it would not be optimal, either with or without taxes, to develop at $t = 0$ resources that would be left unexploited at date T. This intuition is confirmed by the optimality conditions. Indeed, from (4), we may write $\lambda(t) = \lambda$ for all $T \in [0, T]$ and, from (5),

$$\lambda = E'(X_0) > 0. \tag{8}$$

It then follows from (6) that we must have $X(T) = 0$: the initial stock will be fully depleted.

Taking into account that $X(T) = 0$, substituting the inverse demand function for $p(t)$ and using (8) to eliminate λ from (3) and (7), conditions (3) to (7) can be reduced to the following two conditions:

$$P(x(t)) - C'(x(t)) - \frac{\partial G(x(t), p(t), t)}{\partial x(t)} = e^{rt} E'(X_0) \tag{9}$$

$$P(x(T)) - \frac{C(x(T))}{x(T)} - \frac{G(x(T), p(T), T)}{x(T)} = e^{rT} E'(X_0). \tag{10}$$

Condition (9) must hold for all $t \in [0, T]$, while condition (10) must hold at $t = T$. From condition (9) we see that $e^{rt}E'(X_0)$ is equal to the current marginal scarcity rent (the after-tax marginal profit) accruing to the firm, which must be growing at the rate of interest.

If we now differentiate (9) with respect to t, we get, in the taxed equilibrium,[6]

$$\dot{x} = \frac{dx}{dt} = \frac{r\left[P(x) - C'(x) - \dfrac{\partial G}{\partial x}\right] + \dfrac{\partial^2 G}{\partial x \partial t}}{P'(x) - C''(x) - \dfrac{\partial^2 G}{\partial x^2} - \dfrac{\partial^2 G}{\partial x \partial p} P'(x)}, \tag{11}$$

and in the tax-free equilibrium,

$$\dot{x}^* = \frac{dx^*}{dt} = \frac{r[P(x^*) - C'(x^*)]}{P'(x^*) - C''(x^*)}. \tag{12}$$

Equations (11) and (1) constitute a system of two first-order differential equations which, together with the initial condition $X(0) = X_0$, the terminal extraction rate $x(T)$, and the condition $X(T) = 0$, fully characterize the equilibrium extraction path $\{x(t) | t \in [0, T]\}$. The initial reserves X_0 are obtained from (9) at $t = 0$. Similarly for (12) and (1), and the corresponding initial and terminal conditions in the tax-free case.

The terminal extraction rate $x(T)$ (or $x^*(T^*)$ in the tax-free equilibrium) must satisfy the transversality condition (10) (or (10*)). For that purpose, it is useful to distinguish between the lump-sum tax and the other three taxes. The lump-sum tax has no effect on the marginal cost ($\partial G/\partial x = 0$), but a positive effect on average cost ($G/x(t) = \beta(t) > 0$), whereas the other taxes all have the same positive effect on both marginal and average costs ($\partial G/\partial x = G/x(t) > 0$). In the latter cases, it follows from (9) and (10) that at $t = T$, in both the taxed and tax-free equilibrium,

$$C'(x(T)) = \frac{C(x(T))}{x(T)}. \tag{13}$$

[6] As we do in (11) and (12), we will at times drop the time argument when there is no risk of confusion.

This means that, at the terminal date, the rate of extraction must be such as to minimize the average cost. Given the assumptions on the cost function, which imply that average cost is everywhere increasing, this can only happen at an extraction rate of zero. We therefore must have $x(T) = x^*(T^*) = 0$.

In the case of the lump-sum tax, the effect on marginal and average cost being different, (9) and (10) yield

$$C'(x(T)) = \frac{C(x(T))}{x(T)} + \frac{\beta(T)}{x(T)}. \tag{14}$$

Again, the after-tax average cost must be minimized at $t = T$. But now, since the after-tax average cost at first decreases until some minimum level and then increases, the equality between marginal and average cost will occur at a positive rate of extraction, $\hat{x}(T) = f(\beta(T))$. In the case of the lump-sum tax, we will therefore have $x(T) = \hat{x}(T) > x^*(T^*) = 0$.

The denominators in both (11) and (12) are negative, given the assumptions on the demand, cost, and tax functions. Therefore $x(t)$ and $x^*(t)$ are continuously decreasing functions of t in all cases.[7] In all but the lump-sum tax, the after-tax extraction path decreases to reach zero at T, as for the tax-free path. In the case of the lump-sum tax, it decreases to some positive value at T.

We now take a closer look at the effect of the tax on the equilibrium outcome under each of those forms of taxes. We begin with the lump-sum tax, since, as just seen, it differs from the other three taxes by its differentiated effect on marginal and average costs.

2.1 The lump-sum tax

A tax whose amount at each date is independent of the variables that depend on the firm's decisions at that date, such as output, revenue or profit, is usually viewed as a lump-sum tax. A tax in the amount $\beta(t)$ at each date has those properties. In a static framework, such a tax creates no incentives for the firm to modify its behavior, since it cannot be avoided by doing so. In the inherently dynamic context of nonrenewable resource exploitation, it is

[7] This implicitly assumes that we are never in a situation where $\partial G^2/\partial x \partial t$ is sufficiently negative to render the numerator in (11) positive, in which case we could have an after-tax extraction path that is increasing at first, but eventually decreasing in order to satisfy the terminal condition.

somewhat misleading to qualify such a tax as "lump-sum", as we have for lack of a better term. It is true that the amount of tax paid at any date does not depend on the rate of extraction at that date, but the *total* amount paid in tax can be reduced by depleting the resource stock faster. In fact, as we shall show, such a tax will result in both a smaller initial stock ($X_0 < X_0^*$) and an earlier terminal date ($T < T^*$) than in the tax-free equilibrium.

To see this, first note that since $x(T) = \hat{x}(T) > x^*(T^*) = 0$, as just established, we have $p(T^*) = P(0) = \bar{p}$, $p(T) = P(\hat{x}(T)) < \bar{p}$, and $C'(\hat{x}(T)) > C'(0)$. It follows from (9) and (9*) that

$$E'(X_0^*)e^{rT^*} > E'(X_0)e^{rT}. \tag{15}$$

Now suppose, as a proposition to be contradicted, that $T \geq T^*$. Inequality (15) then implies $E'(X_0^*) > E'(X_0)$, which itself implies $X_0^* > X_0$, since $E''(X_0) > 0$. But if $X_0^* > X_0$ and $T^* \leq T$, it must be the case that the before-tax and after-tax paths cross at least once, for otherwise one or both of the conditions $X(T) = X^*(T^*) = 0$ would be violated. This means that for at least one date τ, we should have $x^*(\tau) = x(\tau)$ and $\dot{x}^*(\tau) \neq \dot{x}(\tau)$. But from (9) and (9*), it then follows that $E'(X_0^*)e^{r\tau} = E'(X_0)e^{r\tau}$ and hence $X_0^* = X_0$, a contradiction. Therefore $T < T^*$: the initial stock will be exhausted faster in the taxed equilibrium than in the tax-free equilibrium.

Not only does the tax create an incentive for the firm to exhaust its stock in less time, but it will create an incentive to develop smaller initial reserves. For suppose, to the contrary, that $X_0 \geq X_0^*$, and hence $E'(X_0) \geq E'(X_0^*)$. Then, since $T < T^*$, there must exist a positive interval of time during which $x(t) > x^*(t)$, for otherwise the condition $X(T) = 0$ would be violated. Let τ be some date in that interval. Since $P'(x) - C''(x) < 0$, in (9) and (9*) we will have $P(x(\tau)) - C'(x(\tau)) < P(x^*(\tau)) - C'(x^*(\tau))$. But this implies $E'(X_0) < E'(X_0^*)$, a contradiction. We may therefore conclude that $X_0 < X_0^*$: the firm will choose to develop and exploit a smaller amount of reserves in the after-tax equilibrium.

We must conclude from this that a lump-sum tax at each date of activity is not a neutral tax, as it would be in a conventional static context. In the inherently dynamic context of nonrenewable resource extraction, for a tax to be truly "lump-sum"it should take the form of an amount levied at the outset, independently not only of the rate of extraction at each date, but of the initial stock which the firms choose to develop and the time they take to exhaust it, and therefore of the present value of the flows of net revenue from the sale of the resource. Provided the tax is not prohibitive, only then will it not result in a deadweight loss because of the distortion it creates.

2.2 The specific, *ad valorem*, and profit taxes

The specific and *ad valorem* severance taxes and the profit tax have in common that their effect on marginal cost is positive and the same as the effect on average cost, except possibly for the profit tax, for which the effect on marginal cost may be made zero by a proper choice of the tax parameters. We will come back to this last point later. Whenever $\partial G/\partial x = G/x > 0$, the tax will create an incentive to develop less reserves initially.

To see this, recall that (13) and (13*) imply $x(T) = x^*(T^*) = 0$. Therefore, from (9) and (9*),

$$E'(X_0^*)e^{rT^*} = E'(X_0)e^{rT} + \frac{\partial G(x(T), p(T), T)}{\partial x} \tag{16}$$

and hence $E'(X_0^*)e^{rT^*} > E'(X_0)e^{rT}$. Now suppose that $X_0 \geq X_0^*$. Since $E''(X_0) > 0$, this would mean $E'(X_0) \geq E'(X_0^*)$. It follows from (16) that we would necessarily have $T \leq T^*$. Since $x(T) = x^*(T^*) = 0$, if $X_0 \geq X_0^*$ and $T \leq T^*$ the before-tax and after-tax extraction paths must then intersect at least once in order for both conditions $X(T) = X^*(T^*) = 0$ to be satisfied. When they do intersect, say at $t = \tau$, we have $x(\tau) = x^*(\tau)$, and hence, from (9) and (9*),

$$E'(X_0^*)e^{r\tau} = E'(X_0)e^{r\tau} + \frac{\partial G(x(\tau), p(\tau), \tau)}{\partial x}.$$

But this implies $E'(X_0) < E'(X_0^*)$, a contradiction. Therefore $X_0 < X_0^*$: as long as $\partial G/\partial x > 0$ the tax will result in a strictly smaller quantity of reserves being developed.

We now look more closely at the other three taxes, beginning with the specific severance tax.

2.2.1 The specific severance tax

Since $\partial G/\partial x = \alpha(t) > 0$ in the case of the specific severance tax, we know from the above discussion that such a tax can never be neutral, since the initial stock will always be lower than in the tax-free equilibrium, no matter the level of the tax or how it evolves over time. How the after-tax time path of extraction compares with the before-tax one at any given time does however depend on the time path of the tax.

In the case of the specific severance tax, equation (11) becomes

$$\dot{x} = \frac{r[P(x) - C'(x)] - [r\alpha - \dot{\alpha}]}{P'(x) - C''(x)}. \tag{17}$$

The case where $\dot{\alpha} = r\alpha$ is of particular interest. The tax rate is then growing at the rate of interest, so that its present value is constant; from (17) and (12), we see that the slopes of the two extraction paths are then the same whenever the rate of extraction is the same. If the initial stock of the resource were given exogenously, and therefore the same before and after tax, as is most often assumed in analyses of nonrenewable resource taxation, then the after-tax and before-tax extraction paths not only would have the same slope, but they would coincide. Such a tax would create no incentives to delay or accelerate the depletion of the exogenously given stock; it would simply capture part of the resource rent without causing any deadweight loss (see for instance Burness (1976), Dasgupta and Heal (1979) or Gaudet and Lasserre (1990)). But when we take into account that the initial reserves are in fact endogenously determined, a specific severance tax will create an incentive to reduce the exploration and development effort, as we have just shown. Since we then have $X_0 < X_0^*$, and must have $x(T) = x^*(T^*) = 0$, it is clear that the two extraction paths cannot coincide. Furthermore, the two paths cannot cross, since whenever the before-tax and after-tax rates of extraction are the same, the slopes must be the same. Therefore the after-tax extraction path will be everywhere below the before-tax path and $T < T^*$.

For there to exist one or more date at which the after-tax and before-tax rates of extraction are the same, it must therefore be the case that $\dot{\alpha} \neq r\alpha$. If at some date t we have

$x(t) = x^*(t)$, we now verify from (17) and (12) that at that date

$$\dot{x}^* - \dot{x} = \frac{r\alpha - \dot{\alpha}}{P'(x) - C''(x)}. \tag{18}$$

The denominator being strictly negative, the right-hand side will be strictly positive if and only if $\dot{\alpha} > r\alpha$ and strictly negative if and only if $\dot{\alpha} < r\alpha$. Since the extraction paths are continuous, it follows that if the after-tax path crosses the before-tax path it will do so only once: from above if $\dot{\alpha} > r\alpha$; from below if $\dot{\alpha} < r\alpha$. Whether they do cross will depend on the level of the tax rate and its evolution over time. If $\dot{\alpha} > r\alpha$, we will necessarily have $T < T^*$, whether the paths cross or not. For suppose $T \geq T^*$. Then, since $X_0 < X_0^*$, the $x(t)$ path must cut the $x^*(t)$ path from below, for otherwise one or both of the conditions $X(T) = X^*(T^*) = 0$ would be violated. But, from (18), this requires $\dot{\alpha} < r\alpha$, a contradiction.

The situation where $\dot{\alpha} < r\alpha$ is probably the most interesting, since it includes the typical case of a constant tax rate, that is $\alpha(t) = \alpha_0$, the tax rate at $t = 0$, and hence $\dot{\alpha} = 0$. There then exists a level of the constant tax rate, say $\hat{\alpha}$, such that $T = T^*$, with $x(t) < x^*(t)$ for $t < T$. For $\alpha_0 < \hat{\alpha}$, we have $T < T^*$ and $x(t) < x^*(t)$ for all $t \leq T$. If $\alpha_0 > \hat{\alpha}$, then $T > T^*$ and the after-tax path necessarily cuts the before-tax path once from below. The reason for this positive relationship between the constant tax rate α_0 and the after-tax terminal date T is that the higher α_0, the lower the initial stock X_0, as can be verified from (9). A smaller X_0 means a smaller surface under the extraction path $\{x(t)\}$, which we already know to be smaller than that under the tax-free path. When the surface under $\{x(t)\}$ gets sufficiently small, the after-tax path must cut the before-tax path once from below in order for both conditions $X(T) = 0$ and $X^*(T^*) = 0$ to be satisfied.

It is a well established fact that in a *static* framework a specific tax is never neutral, since the resulting increase of marginal cost leads the price-taking firm to reduce its production in order to maintain the equality between price and marginal cost. Like in the static framework, the specific severance tax will never be neutral if the initial stock is endogenous, in this case because it leads the firm to produce a smaller *total* quantity (the initial stock) in order to maintain the equality between the marginal benefit of adding to the stock (which is given by the marginal profit at $t = 0$) and the marginal cost of doing so. In such a case, could the specific severance tax be rendered neutral if it was combined with a subsidy to exploration and development? The answer is yes, *provided the severance tax rate is made to grow at the rate of interest.*

Indeed, suppose that along with the specific severance tax we introduce a subsidy of $\alpha_0 X_0$, where α_0 is the tax rate at date $t = 0$. The cost of exploration and development faced by the firm is now $E(X_0) - \alpha_0 X_0$ and condition (9) can be written $P(x(t)) - C'(x(t)) = e^{rt}[E'(X_0) - \alpha_0] + \alpha(t)$. It follows that if and only if $\alpha(t) = \alpha_0 e^{rt}$, the after-tax and before-tax initial stocks will be the same and the after-tax and before-tax extraction paths will coincide. However such a formula is futile, since the expenditures on the subsidy must exactly offset the revenue from the tax, leaving zero net tax revenue. The above discussion also means that the typical time-invariant tax rate ($\alpha(t) = \alpha_0$) cannot be neutral, even with the proposed subsidy on exploration and development effort, let alone with no subsidy.

2.2.2 The *ad valorem* severance tax

The analysis of the *ad valorem* severance tax is qualitatively very similar to that of the specific tax. In particular, since $\partial G/\partial x = \gamma(t)p(t) > 0$, we know that $X_0 < X_0^*$, as for the specific tax. As in the case of the specific tax, the *ad valorem* tax will not be neutral when the initial reserves are endogenously determined.

With an *ad valorem* tax, equation (11) becomes

$$\dot{x} = \frac{r[P(x) - C'(x)] - [r\gamma - \dot{\gamma}]P(x)}{P'(x) - C''(x) - \gamma P'(x)}, \tag{19}$$

which, upon multiplying both sides by the denominator and using the fact that $P'(x)\dot{x} = \dot{p}$, can be rewritten

$$\dot{x} = \frac{r[P(x) - C'(x)] - [r\gamma p - \dot{\gamma}p - \gamma\dot{p}]}{P'(x) - C''(x)}. \tag{20}$$

The difference between the specific and *ad valorem* taxes comes from the fact that, unless demand is perfectly elastic ($P'(x) = 0$), we now have to take into account the effect of the tax on market price. If demand is perfectly elastic, then $\dot{p} = 0$ and it is easily seen, by replacing α by γp in (17) and (18), that the tax becomes equivalent to a specific tax and exactly the same analysis goes through.

But in general demand will not be perfectly elastic. Whenever $x(t) = x^*(t)$, we will have

$$\dot{x}^* - \dot{x} = \frac{r\gamma p - \frac{d\gamma p}{dt}}{P'(x) - C''(x)}. \tag{21}$$

Hence, for $d\gamma p/dt(>)(=)(<)r\gamma p$, the qualitative consequences of the tax for the extraction paths are exactly the same as they were for the specific tax in the cases of $\dot{\alpha}(>)(=)(<)r\alpha$ respectively. In particular, in the typical case of a constant tax rate ($\dot{\gamma} = 0$, $\gamma(t) = \gamma$), the numerator in (21) becomes $(rp - \dot{p})\gamma$. Since marginal cost is positive, the equilibrium price will be growing at less than the rate of interest, which means that $\dot{x}^*(t) < \dot{x}(t)$ whenever $x(t) = x^*(t)$. By the same reasoning as for the constant specific tax rate, we deduce from this that the after-tax extraction path will cut the before-tax path only once, if at all, and from below; in that case $T > T^*$. This happens if the tax rate is not too high. Conversely, if the tax rate is relatively high, initial reserves will be so reduced by the tax that the after-tax extraction path will be everywhere below the before-tax path; in that case $T < T^*$. Also, subsidizing the exploration and development effort in the same fashion described in the specific tax case, this time in an amount $\gamma_0 p(0)$, could not render the constant-rate *ad valorem* tax neutral, since this would require in addition that $\gamma(t)p(t)$ grow at the rate of interest.[8] Hence the typical time-invariant *ad valorem* tax rate is never neutral.[9]

[8] As for the specific tax under similar circumstances, it would generate no net tax revenue.

[9] Gamponia and Mendelsohn (1985) have found, making use of numerical simulations, that a time-invariant *ad valorem* tax yielding the same tax revenue as a time-invariant specific tax generates a smaller loss in efficiency. Hung and Quyen (2009) provide a proof of this. In both cases, the resource stock is assumed exogenously given.

2.2.3 The profit tax

In the case of the profit tax we have $G(x(t), p(t), t) = \nu(t)\{p(t)x(t) - C(x(t)) - \mu(t)x(t)\}$ and therefore $\partial G/\partial x = \nu[P(x) - C'(x) - \mu]$. The tax base is revenue minus extraction cost, minus an adjustment for the fact that reserves are being depleted in the amount $x(t)$ (a depletion allowance). Equation (11) now gives

$$\dot{x} = \frac{[(1-\nu)r + \dot{\nu}][P(x) - C'(x)] + [r\nu\mu - \dot{\nu}\mu - \nu\dot{\mu}]}{(1-\nu)[P'(x) - C''(x)]}. \tag{22}$$

Consider first the case where $\mu \equiv 0$. Then, unless $\dot{\nu} \neq 0$, whenever $x(t) = x^*(t)$ we get that

$$\dot{x}(t) = \dot{x}^*(t) < 0 \quad \text{for all} \quad t \in [0, T]. \tag{23}$$

If initial reserves were given exogenously, and therefore the same before and after tax, such a tax (a constant ν) would be a pure rent tax, leaving the extraction decision at each date unchanged (see Burness (1976), Dasgupta and Heal (1979) or Gaudet and Lasserre (1990)). But with $\mu \equiv 0$ and the initial reserves endogenously determined, we have $\partial G/\partial x = \nu[P(x) - C'(x)] > 0$ and the before-tax and after-tax initial reserves will not be the same: as already established, we will have $X_0 < X_0^*$. Such a profit tax is not neutral anymore. We know that the two extraction paths will not cross, for if they did they would have different slopes at the point of crossing, which violates (23). Therefore, since $X_0 < X_0^*$ and $x(T) = x^*(T) = 0$, the after-tax path must lie everywhere below the before-tax path and $T < T^*$, as in the case of the specific tax with α growing at the rate of interest or the *ad valorem* tax with γp growing at the rate of interest.

Let us now reintroduce the depletion allowance, at rate $\mu(t)$, while keeping the tax rate ν constant, as is typical. Equation (22) then becomes

$$\dot{x} = \frac{(1-\nu)r[P(x) - C'(x)] + \nu[r\mu - \dot{\mu}]}{(1-\nu)[P'(x) - C''(x)]}. \tag{24}$$

We see immediately that if $\mu(t)$ is growing at the rate of interest, we again get (23) whenever

$x(t) = x^*(t)$. If the initial reserves were the same, we could then conclude that the tax is neutral. But having $\mu(t)$ grow at the rate of interest is not sufficient to have $\partial G/\partial x = 0$ and the initial reserves the same. It does however become sufficient if we further specify that $\mu(t)$ at date zero be given by $\mu_0 = E'(X_0)$, so that $\mu(t) = e^{rt}E'(X_0)$. This means that the depletion allowance is set equal to the current marginal scarcity rent at each date. We can then verify, using condition (9), that we will have $\partial G/\partial x = 0$. With $\mu(t) = e^{rt}E'(X_0)$, conditions (9) and (9*) in fact become exactly the same, so that $x(t) = x^*(t)$ for all $t \in [0, T = T^*]$: the after-tax and before-tax extraction paths coincide. The tax revenue will be zero at the margin, but will capture at each date a part of the total surplus (consumer and producer, including resource rent).

If $\mu(t) \neq e^{rt}E'(X_0)$, the after-tax and before-tax initial reserves will differ ($X_0 < X_0^*$), and the after-tax and before-tax extraction paths will have different slopes. Then, at any date at which the after-tax and before-tax paths happen to cross we will have

$$\dot{x}^* - \dot{x} = -\left(\frac{\nu}{1-\nu}\right)\left(\frac{r\mu - \dot{\mu}}{P'(x) - C''(x)}\right). \tag{25}$$

The analysis can be carried out exactly as in the cases of the specific and *ad valorem* taxes. The qualitative results will be the same, except that now, since the depletion allowance constitutes a tax relief rather than a tax (which explains the minus sign on the right-hand side of (25)), the results for $\dot{\mu} < (>) r\mu$ correspond respectively to those obtained above for $\dot{\alpha} > (<) r\alpha$ and $\dot{\gamma} > (<) r\gamma$ for the specific and the *ad valorem* taxes. In particular, if $\mu(t)$ is time-invariant (for instance if $\mu(t) = E'(X_0)$, the marginal rent at $t = 0$), we are in the case of $\dot{\mu} < r\mu$ and will necessarily have $T < T^*$, with paths that may or may not cross, as argued for the *ad valorem* tax in the case of $\dot{\alpha} > r\alpha$.

The cost function we have assumed so far hides the fact that usually some quasi-fixed assets yield a flow of services that enter the resource extraction process. In practice, costs have to be imputed for the services of such factors in order to properly measure profits for tax purposes. A very important example is the physical capital stock held by the firm, for which an implicit rental rate must be taken into account in order to properly determine the tax base. Determining this implicit rental rate in turn requires knowledge of the actual rate of depreciation of such capital. A distortion will result if the depreciation rate used for tax purposes differs from the true rate, a fact that tax authorities sometimes use to encourage investment by allowing depreciation for tax purposes at a rate that exceeds the true rate (accelerated depreciation). For a resource extracting firm, two assets are then involved at any given time: the stock of productive capital, whose flow of services it needs to extract the resource, and the stock of the natural resource itself. In the case of natural resource stocks, the depletion allowance introduced in this section can also be considered a form of depreciation for tax purposes, whose choice of rate will also have important consequences for intra- and inter-industry allocations. We consider those questions in more detail in the next section, which deals with the treatment of nonrenewable resource extraction in the corporate income tax.

3 The corporate income tax and the depletion allowance

Incorporated nonrenewable resource extraction firms (mining firms for short) are generally subject to a "corporate income tax" that applies to all incorporated firms, regardless of their sector of activity. The analysis of the corporate income tax can be relatively complex, if only because of the necessity of imputing a cost for the services of the physical assets owned by the firm and used as input in the production process. The structure of the corporate income tax can involve many parameters designed to deal with particular situations.[10] For our purposes it will suffice to retain two main elements that apply to all incorporated firms, namely the corporate tax rate, denoted υ, and the rate of capital depreciation for tax purposes, denoted θ.[11] The main provision that applies only to the mining firm is the depletion allowance, by

[10]For instance, investment tax credits are often used to encourage new investment in particular regions or in particular sectors of the economy. We will neglect here this aspect of the tax structure. For a treatment of its implications in a context similar to the one modeled here, see Gaudet and Lasserre (1986a).

[11]The tax code usually distinguishes between different classes of assets, each with its own rate of depreciation for tax purposes. For simplicity, we will assume there is only one class of asset. We will also assume declining balance depreciation throughout the life of the asset.

which the firm can deduct a fraction of gross revenues as an allowance for the depletion of its nonrenewable resource stock. We will denote this fraction η. Contrary to what we did in the previous section, we will in this section assume all the tax parameters to be time-invariant, as is typically the case.

We will simplify reality by assuming that the services of capital are the only input in the extraction process. The stock of capital which yields those services at any date t will be denoted $K(t)$ and assumed to depreciate at the rate δ. The rate of extraction is therefore given by $x(t) = F(K(t))$, with $F(0) = 0$, $F'(K(t)) > 0$ and $F''(K(t) < 0$ for all $K(t) \geq 0$.[12] The rate of investment in this stock of capital will be denoted $I(t)$, and, for simplicity, we will assume no adjustment cost.[13] The firm takes as given the market price of investment goods, $q(t)$, as well as that of output, $p(t)$.

We will further simplify reality by assuming that all investments are financed by retained earnings. This means that the discount rate, r, can be taken to be the after-personal-income-tax interest rate, which can be assumed unaffected by the corporate tax parameters and market given. This important simplification avoids having to distinguish between after-tax and before-tax rates of interest when comparing after-corporate-tax and before-corporate-tax situations.

Letting $\widetilde{K}(t)$ denote the value of undepreciated capital for tax purposes, the tax function faced by the mining firm is therefore[14]

$$G(K(t), \widetilde{K}(t), p(t)) = \upsilon[p(t)F(K(t)) - \theta\widetilde{K}(t) - \eta p(t)F(K(t))],$$

[12]For the purpose at hand taking into account variable factors, such as labor, energy and materials, does not affect significantly the analysis. For instance, suppose $L(t)$ represents the flow of some variable factor, $w(t)$ its market price, and $\Phi(K(t), L(t))$ the resource extraction process. If we replace $L(t)$ by its equilibrium quantity $L^*(t) = L(K(t); w(t)/p(t))$ obtained from the optimality condition $\partial\Phi/\partial L = w(t)/p(t)$, then, under proper assumptions on the first and second order derivatives of Φ, we can think of $F(K(t))$ as $F(K(t)) = \Phi(K(t), L(K(t); w(t)/p(t))$.

[13]For the introduction of adjustment cost in the problem of optimal investment in the nonrenewable resource extracting firm, see Gaudet (1983) and Lasserre (1985).

[14]Notice that the tax base is not economic profit; for one thing, it does not account for the rental cost of capital.

and the problem it faces at date $t = 0$ is

$$\max_{\{I(t)\},T,X_0} \int_0^T e^{-rt}\{p(t)F(K(t) - q(t)I(t) - G(K(t), \widetilde{K}(t), p(t))\}dt \\ + e^{-rT}V(K(T), \widetilde{K}(T); q(T)) - E(X_0) \tag{26}$$

where the maximization is subject to

$$\dot{K}(t) = I(t) - \delta K(t), \qquad K(0) = K_0 \tag{27}$$

$$\dot{\widetilde{K}}(t) = qI(t) - \theta\widetilde{K}(t), \qquad \widetilde{K}(0) = \widetilde{K}_0 \tag{28}$$

$$\dot{X}(t) = -F(K(t)), \qquad X(0) = X_0. \tag{29}$$

As before, $E(X_0)$ denotes the cost of discovering and developing the endogenously determined initial reserves X_0, while $V(K(T), \widetilde{K}(T); q(T))$ denotes the after-tax scrap value of the mine at the date of exhaustion T. This function is given by

$$V(K(T), \widetilde{K}(T); q(T)) = q(T)K(T) + \upsilon z[\widetilde{K}(T) - q(T)K(T)], \tag{30}$$

where

$$z = \int_T^\infty e^{-(r+\theta)(t-T)}\theta dt = \frac{\theta}{r+\theta}$$

is the present value of the depreciation of one unit of $\widetilde{K}$ being depreciated indefinitely at rate θ. This formulation of the after-tax scrap value assumes, as is often the case in practice, that once its reserves are exhausted the firm will continue to deduct the depreciation of its remaining stock of capital against other income sources at the same rate θ (or have to "recuperate" at the same rate if the resale value of the remaining stock exceeds the accounting value of undepreciated capital for tax purposes). Thus the after-tax scrap value is the resale value of the remaining stock of capital plus the tax saving from the depreciation of the undepreciated value of the remaining stock of capital (which may be negative if recuperation is required). The assumption of continuity in the tax treatment of depreciation beyond the date of exhaustion T will facilitate the comparison with the analysis of the traditional infinitely-lived firm.[15]

The present value Hamiltonian associated to this firm's problem, at each date t, is

$$H = e^{-rt}\{pF(K) - qI - \upsilon[pF(K) - \theta\widetilde{K} - \eta pF(K)]\} + \psi_K[I - \delta K] + \psi_{\widetilde{K}}[qI - \theta\widetilde{K}] - \lambda F(K).$$

As in the previous section, $\lambda(t)$ represents the present value of a unit of *in situ* resource. As for the shadow values $\psi_K(t)$ and $\psi_{\widetilde{K}}(t)$, they capture, respectively, the present value of a unit of physical capital and the present value of a unit of undepreciated accounting capital.

The Hamiltonian being linear in I, its maximization requires, for all $t \in [0, T]$,

$$I(t) = \begin{cases} +\infty & \text{if} \quad \psi_K(t) > e^{-rt}q(t) - \psi_{\widetilde{K}}(t)q(t) \\ \text{indeterminate} & \text{if} \quad \psi_K(t) = e^{-rt}q(t) - \psi_{\widetilde{K}}(t)q(t) \\ -\infty & \text{if} \quad \psi_K(t) < e^{-rt}q(t) - \psi_{\widetilde{K}}(t)q(t) \end{cases} .$$

This means that the firm's optimal investment program consists in instantaneously adjusting the stock of capital at date $t = 0$ so as to satisfy the so-called singular solution

$$\psi_K(t) = e^{-rt}q(t) - \psi_{\widetilde{K}}(t)q(t), \tag{31}$$

and maintaining it thereafter. The necessary condition (31) thus defines the firm's desired level of capital at any date $t \in [0, T]$.

In addition to (31) and to the constraints (27), (28) and (29), the following intertemporal

[15] If a discontinuity in the tax treatment of depreciation occurs at the date T at which the mine shuts down — for instance if immediate depreciation at T is allowed —, then the present value of the flow of savings resulting from a dollar of depreciable investment would increase for the mining firm as the date T approaches, whereas it is constant for the infinitely-lived traditional firm. For an analysis of such a case, see Gaudet and Lasserre (1984). Notice also that the assumption that capital is perfectly malleable and that investment is perfectly reversible is important here. Otherwise, $K(T)$ could not be valued at the market price $q(T)$. This assumption could be easily relaxed, at the cost of greater complexity in the analysis, but with little gain in insight, at least for our present purposes.

arbitrage conditions must be satisfied

$$\dot{\psi}_K = \delta\psi_K - \left[1 - \upsilon(1-\eta) - e^{rt}\lambda/p\right] e^{-rt} pF'(K) \tag{32}$$

$$\dot{\psi}_{\widetilde{K}} = \left[\psi_{\widetilde{K}} - e^{rt}\upsilon\right]\theta \tag{33}$$

$$\dot{\lambda} = 0 \tag{34}$$

as well as the transversality conditions

$$\lambda(0) = E'(X_0) \tag{35}$$

$$\lambda(T)X(T) = 0, \quad \lambda(T) \geq 0, \quad X(T) \geq 0 \tag{36}$$

$$e^{rT}\psi_K = (1 - \upsilon z)q(T) \tag{37}$$

$$e^{rT}\psi_{\widetilde{K}} = \upsilon z \tag{38}$$

$$e^{rT}H(\cdot)|_T - rV(\cdot) + (1-\upsilon z)\dot{q}(T)K(T) = 0 \tag{39}$$

By an argument similar to that used in the previous section, the initial reserves will be completely depleted at T, so that condition (36) will be satisfied with $X(T) = 0$ and $\lambda(T) > 0$. From conditions (34) and (35), we get that $\lambda(t) = E'(X_0)$: the discounted value of the marginal unit of reserves in the ground must be constant and equal to the cost of acquiring it, also as in the previous section.

Consider next the arbitrage condition (33). It is a first-order differential equation whose general solution is

$$\psi_{\widetilde{K}}(t) = e^{-rt}\upsilon z + \left[e^{r(T-t)}\psi_{\widetilde{K}}(T) - e^{-rt}\upsilon z\right] e^{-(r+\theta)(T-t)},$$

which, using condition (38), has as particular solution

$$\psi_{\widetilde{K}}(t) = \upsilon z e^{-rt}. \tag{40}$$

Substituting this solution into (31), it becomes

$$\psi_K(t) = e^{-rt}(1 - \upsilon z)q(t). \tag{41}$$

If we now differentiate both sides of this equation with respect to t and substitute into the intertemporal arbitrage condition (32) we verify that (41) can be rewritten

$$F'(K) = \left(\frac{1 - \upsilon z}{1 - (1-\eta)\upsilon - e^{rt}E'(X_0)/p}\right)\frac{c}{p}, \qquad \forall t \in [0, T], \tag{42}$$

where

$$c = \left(r + \delta - \frac{\dot{q}}{q}\right) q$$

is the before-tax implicit rental rate of capital of the traditional firm. The right-hand side of (42) is the implicit after-tax rental rate of capital of the mining firm. It differs from that of the traditional non-mining firm in two respects; first by the presence in the denominator of the expression $e^{rt}E'(X_0)$, which represents the current value at date t of a unit of resource in the ground, $E'(X_0)$ being its discounted value; second by the fact that $\eta \equiv 0$ in the case of the non-mining firm.

The value of K that satisfies condition (42), and hence (41), is the firm's desired level of capital. It is the stock of capital that equates the marginal productivity of capital to the implicit after-tax rental rate of capital. Whereas this desired level of capital is constant for the traditional firm if c is constant, it will vary over time in the case of the mining firm because the current *in situ* value of the resource (the resource rent) increases as the reserves are being depleted.

It is instructive to rewrite (42) as

$$p(t) - \frac{c}{F'(K(t))} - \upsilon\left[(1-\eta)p(t) - z\frac{c}{F'(K(t))}\right] = e^{rt}E'(X_0), \tag{43}$$

which is the equivalent of (9) for the corporate income tax. Notice that the corporate income tax affects marginal revenue and marginal cost differently, which would not be the case if it was a pure rent tax. In the absence of taxes, the same condition is

$$p^*(t) - \frac{c}{F'(K^*(t))} = e^{rt} E'(X_0^*). \tag{44}$$

where, as before, the asterisk denotes tax-free equilibrium values.

Recalling that $z = \theta/(r+\theta)$, if we now substitute from (37) and (38) into (39) we find that it yields, after-tax

$$p(T) - \frac{cK(T)}{F(K(T))} - \upsilon \left[(1-\eta)p(T) - z\frac{cK(T)}{F(K(T))} \right] = e^{rT} E'(X_0), \tag{45}$$

which is the equivalent of (10) for the corporate income tax. In the absence of taxes it becomes

$$p^*(T^*) - \frac{cK^*(T^*)}{F(K^*(T^*))} = e^{rT^*} E'(X_0^*). \tag{46}$$

Hence the necessary conditions (31) to (39) reduce to the two conditions (43) and (45) (or (44) and (46) in the absence of the tax).

Evaluating (43) (or (44)) at $t = T$ and substituting into (45) (or (46)), it is easily seen that at the date of exhaustion T it must be the case that, both with and without the tax,

$$F'(K(T)) = \frac{F(K(T))}{K(T)}.$$

This means that the stock of capital at date T must be the same in both the taxed and tax-free equilibrium: it must be such that the marginal product of capital is equal to its average product, which implies that the average product of capital is maximized at that date. This is equivalent to the condition that marginal cost be equal to average cost at T, and hence average cost be minimized at that date. Since, by assumption, $F(0) = 0$ and $F''(K) < 0$ for all $K \geq 0$, this means that $K(T) = K^*(T^*) = 0$, and hence $x(T) = x^*(T^*) = 0$.

Two main issues arise with respect to the corporate income tax at this point. The first has to do with its effect on the extraction path; the second has to do with its effect on the relative incentives to invest in the mining and non-mining industries. We now consider, in order, those two issues. We continue to assume that the firms are identical, so that we can consider the firm as representative of the industry and K as the aggregate industry stock of capital. We can then replace in (43) and (45) the market price by the inverse demand function: $p(t) = P(F(K(t)))$. We will also assume, for simplicity, that c, the before-tax rental cost of capital, is constant.

3.1 The depletion allowance and the resource extraction path

Differentiating both sides of (43) and (44) with respect to t, we find that

$$\dot{K} = \frac{re^{rt}E'(X_0)F'}{D} = \dot{K}^* + \frac{rv[z+\eta-1][P'F'^2 + PF'']}{DD^*}, \tag{47}$$

where

$$\dot{K}^* = \frac{r[PF' - c]}{D^*} \tag{48}$$

and

$$D = [1-(1-\eta)v]P'F'^2 + (1-vz)cF''/F' < 0$$
$$D^* = P'F'^2 + cF''/F' < 0.$$

Given the assumptions on $P(\cdot)$ and $F(\cdot)$, the paths $\{K(t)\}$ and $\{K^*(t)\}$ are both negatively sloped, with

$$\dot{K} - \dot{K}^* \gtreqless 0 \Leftrightarrow \eta \lesseqgtr 1 - z. \tag{49}$$

Since $\dot{x}(t) = F'(K(t))\dot{K}(t)$ and $F'(K(t)) > 0$, those same properties apply to the extraction paths, $\{x(t)\}$ and $\{x^*(t)\}$. The paths being continuous, if they cross they will cross only once, with the after-tax path cutting the before-tax path from below if $\eta < 1 - z$ and from above if $\eta > 1 - z$.

Therefore if and only if $\eta = 1 - z$, the after-tax and before-tax extraction paths have the same slope when $K(t) = K^*(t)$ (i.e. $x(t) = x^*(t)$). If the initial reserves were exogenously given, and therefore the same in the before-tax and after-tax situations, this would mean that the corporate income tax leaves the extraction path unchanged if $\eta = 1 - z$. Indeed, if we substitute this value of η in condition (43), we see that $e^{rt}E'(X_0) = [1 - \upsilon z][P - c/F']$: the after-tax resource rent is proportional to $P - c/F'$, which is the before-tax resource rent. Hence, for an *exogenously given* X_0, the mining firm has no incentive to modify its marginal extraction decision on account of the tax. The tax would then simply capture a part of the resource rent for the benefit of the taxing authority.[16] But when the initial reserves are determined by the exploration and development efforts of the firm, they will not remain unaffected by the tax.

It the initial reserves are endogenously determined, the corporate income tax will in fact never leave the extraction path unchanged. The argument is similar to that of Section 2.2. To see this, subtract (43) from (44) evaluated at $t = T$, recalling that $K^*(T^*) = K(T) = 0$ and that $P(F(0)) = \bar{p}$, to get

$$e^{rT^*}E'(X_0^*) - e^{rT}E'(X_0) = \upsilon\left[(1-\eta)\bar{p} - z\frac{c}{F'(0)}\right]. \tag{50}$$

If the right-hand side of (50) is positive, which will be the case if $\eta \leq 1 - z$, then $e^{rT^*}E'(X_0^*) > e^{rT}E'(X_0)$. Suppose this is the case and suppose $X_0 \geq X_0^*$. This would mean $E'(X_0) \geq E'(X_0^*)$, since $E''(X_0) > 0$, and it follows that we would have $T \leq T^*$. But since $K(T) = K^*(T^*)$ and $X(T) = X^*(T^*) = 0$, if $X_0 \geq X_0^*$ and $T \leq T^*$ the before-tax and after-tax extraction paths must cross at least once, say at $t = \tau$. Then $K(\tau) = K^*(\tau)$ and, from (43) and (44),

$$E'(X_0^*) - E'(X_0) = e^{-r\tau}\upsilon\left[(1-\eta)P(F(K(\tau))) - z\frac{c}{F'(K(\tau))}\right] > 0,$$

which implies $E'(X_0) < E'(X_0^*)$, a contradiction. We can therefore conclude that if either $\eta < 1 - z$ or $\eta = 1 - z$, the corporate income tax will create an incentive to reduce the

[16] With an exogenously given X_0, but $\eta \neq 1 - z$, the tax would create a deadweight loss, dissipating part of the rent. If $\eta > 1 - z$, it will create an incentive to extend the life of the mine ($T > T^*$); if $\eta > 1 - z$, it will create an incentive to deplete the given reserves sooner ($T < T^*$). For a detailed analysis of the case of exogenously given reserves, see (Gaudet and Lasserre, 1986a,b, 1990).

exploration and development effort, resulting in $X_0 < X_0^*$.

As can be seen from (43), the reason is that, with $\eta \leq 1 - z$, the tax results in $E'(X_0) < e^{-rt}[P - c/F']$: it reduces the present value of the marginal rent. If $\eta = 1 - z$, it remains true that the after-tax marginal rent is strictly proportional to the tax-free marginal rent. But since it has a negative effect on the marginal rent, and since the firm is free to determine its exploration and development efforts, it will cause it to reduce the resource stock to be exploited. The after-tax and before-tax extraction paths will have the same slope in that case, but the after-tax path will lie everywhere below the tax-free one, and therefore $T < T^*$. Thus, contrary to a situation where the firm would have to take its initial reserves as given, having $\eta = 1 - z$ does not make the corporate income tax a pure rent tax. If $\eta < 1 - z$, then the after-tax rent is not proportional to the before-tax rent, but it still reduces it. In that case, either the after-tax path lies everywhere below the before-tax one, with $T \leq T^*$, or, if η is sufficiently small, cuts it from below, with then $T > T^*$.

If $\eta > 1 - z$, but such that the right-hand side of (50) remains strictly positive, then the above argument applies and $X_0 < X_0^*$. But the right-hand side of (50) may now be strictly negative if η is sufficiently large, in which case we have $E'(X_0) > e^{-rt}[P - c/F']$: the tax increases the present value of the marginal rent, hence in effect subsidizing investment in initial reserves. We therefore have $X_0 > X_0^*$, with $\dot{K}(t) < \dot{K}^*(t)$ when $K(t) = K^*(t)$, so that if the after-tax extraction path cuts the before-tax path it will do so from above. There then exists a $\hat{\eta}$ such that if $\eta = \hat{\eta}$, we have $T = T^*$. For $\eta > \hat{\eta}$, the after-tax path will be everywhere above the tax-free path and $T > T^*$. For $\eta < \hat{\eta}$, the after-tax path will cut the before-tax from above, and $T < T^*$. It is of course also possible to choose η so that $E'(X_0) = e^{-rt}[P - c/F']$ and therefore $X_0 = X_0^*$. However, it should not be concluded that the extraction path is then not affected by the tax. Since their slopes generally differ, the two paths must necessarily cross if $X_0 = X_0^*$, in order to satisfy the conditions $X(T) = X^*(T^*) = 0$. The after-tax path would cut the before-tax path from above, and $T < T^*$.

3.2 The depletion allowance and the effective tax rates on capital income

We have so far restricted attention to the effect of the corporate income tax on the extraction path of the mining firm. But since the corporate income tax applies to non-mining as well as mining firms, an important issue is how the corporate tax treatment of the mining firm affects the relative attractiveness of investment in the mining and non-mining industries. To analyze this question, it is useful to introduce the notion of the effective tax rate on capital income.[17] The true capital income in this context is gross revenues ($pF(K)$), minus true economic depreciation (δqK), minus the true *in situ* value of the resource extracted in the case of the mining firm ($\pi F(K)$, with π denoting the market price of a unit of the resource in the ground). Let τ_m and τ denote respectively the rate of taxation of true capital income of the mining firm and the non-mining firm. If we replace the corporate income tax by this one-parameter tax structure, the problem facing the mining firm at date $t = 0$ is now

$$\max_{\{I(t)\},T,X_0} \int_0^T e^{-rt}\{pF(K) - qI - \tau_m[pF(K) - \delta qK - \pi F(K)]\}dt + V(K(T); q(T)) - E(X_0)$$

subject to (27) and (29). The Hamiltonian corresponding to this new problem is

$$H = e^{-rt}\{pF(K) - qI - \tau_m[pF(K) - \delta qK - \pi F(K)]\} + \psi[I - \delta K] - \lambda F(K)$$

where ψ and λ keep the same interpretation as above. The equivalent to condition (31), which determines the desired stock of capital, is

$$\psi_K(t) = e^{-rt}q(t), \tag{51}$$

while the equivalent to the arbitrage condition (32) is now

$$\dot{\psi}_K = \delta\psi_K - e^{-rt}\left[1 - \left(1 - \frac{\pi}{p}\right)\tau_m - e^{rt}\frac{\lambda}{p}\right]pF'(K) - e^{-rt}\tau_m\delta q. \tag{52}$$

[17] See for instance Auerbach (1983a,b), where the notion of an effective tax rate is used to study the effect of the corporate income tax on investment decisions of the traditional firm.

The intertemporal arbitrage condition (34) must still hold, as must the transversality conditions (35), (36), and the equivalent to (37) and (39).

Differentiating (51) with respect to t and substituting into (52), after using (34) and (35) to eliminate λ, we find that

$$F'(K) = \frac{c - \tau_m \delta q}{(1-\tau_m)p + \tau_m \pi - e^{rt}E'(X_0)}, \tag{53}$$

which is the equivalent to (42) with this new tax structure. For the non-mining firm, $\lambda \equiv 0$, $\pi \equiv 0$, τ_m becomes τ, and we get

$$F'(K) = \frac{c - \tau \delta q}{(1-\tau)p}. \tag{54}$$

The right-hand sides of (53) and (54) are the real implicit rental rates of capital under such a tax system for, respectively, the mining and non-mining firms.

The effective tax rate on true capital income resulting from the corporate income tax regime for the mining firm is the value of τ_m which equates the implicit rental rate of capital in (42) to that in (53). Solving for this, we get

$$\tau_m = \frac{\left[\dfrac{(1-\upsilon z)(1-e^{rt}E'(X_0)/p)}{1-(1-\eta)\upsilon - e^{rt}E'(X_0)/p} - 1\right] c}{\left[\dfrac{(1-\upsilon z)(1-\pi/p)}{1-(1-\eta)\upsilon - e^{rt}E'(X_0)/p}\right] c - \delta q}. \tag{55}$$

Similarly, the effective tax rate resulting from the corporate income tax for the non-mining firms is

$$\tau = \frac{\left[\left(\dfrac{1-\upsilon z}{1-\upsilon}\right) - 1\right] c}{\left(\dfrac{1-\upsilon z}{1-\upsilon}\right) - \delta q}. \tag{56}$$

Notice that whereas τ is a constant, τ_m is not, because the current marginal resource rent, $e^{rt}E'(X_0)$, is growing at the rate of interest.

When the assets market is in equilibrium, we can expect the current market price of a unit of *in situ* resource, $\pi(t)$, to be equal to the current value of the marginal resource rent, $e^{rt}E'(X_0)$. Substituting for this into (53), we find that

$$\tau_m \gtreqless \tau \quad \Leftrightarrow \quad \frac{1-\upsilon z)(1-e^{rt}E'(X_0)/p)}{1-(1-\eta)\upsilon - e^{rt}E'(X_0)/p} \gtreqless 1,$$

which means

$$\tau_m \gtreqless \tau \quad \Leftrightarrow \quad \eta \lesseqgtr \frac{e^{rt}E'(X_0)}{p(t)}.$$

We can therefore conclude that the corporate income tax will result in equal effective tax rates on capital income for the mining and non-mining firm if and only if the depletion allowance is set equal to the share of the current market value of a unit of *in situ* resource (or current resource rent) in the current market flow price of the resource. It is only then that the corporate income tax would not create an incentive to transfer investment from the non-mining to the mining sector, or vice-versa.

However it is impossible to simultaneously equalize the effective tax rates and not distort the extraction path. Indeed, if we set $\eta = e^{rt}E'(X_0)/p(t)$ at each date t, so as to have $\tau_m = \tau$, and substitute this value of η into (43), we verify that (47) now yields

$$\dot{K} = \dot{K}^* - \frac{r[(1-z)\upsilon][P'F'^2 + PF'']}{DD^*},$$

where now $D = (1-\upsilon)P'F'^2 + (1-\upsilon z)cF''/F' < 0$. Since $z = \theta/(r+\theta) < 1$, we will have $\dot{K}(t) > \dot{K}^*(t)$ and the analysis of the previous subsection for the case $\eta < 1-z$ applies, along with the resulting distortion to the extraction path. In particular, the exploration and development effort will be lower than in the absence of the tax.

4 Some other issues

In this section, we look at a few issues relating to the taxation of natural resources that we believe deserve more attention than they have received so far. We limit attention to three of them: capturing resource rents under asymmetry of information, resource taxation under imperfect competition or other second best situations, and general equilibrium considerations in resource taxation. We could also have added the question of uncertainty, especially in the exploration phase, and the question of fiscal competition between jurisdictions.

4.1 Asymmetry of information

We have so far assumed that the owner of the stock of *in situ* resource, whether it be under government or private ownership, shares the same information about the market price and about the cost of extracting the resource as does the mining firm to which the right to search for and extract the resource has been given. This is usually not the case. The firm that is exploiting the mine will usually be better informed about the true costs of extraction than is the resource owner. Such asymmetry of information creates a situation of adverse selection, which means that the firms would tend to exaggerate their costs in order to benefit from a more attractive royalty contract. This must be taken into account in the determination of the optimal means of capturing the resource rent, whether it be via private royalty payments or a governmental rent tax. In a standard *static* principal-agent problem with adverse selection, it is well known that the optimal contract from the principal's point of view would require that a distortion be introduced, as compared to the symmetric information case, for all but the lowest-cost firms.[18] This is necessary in order to create an incentive for the firms to reveal their true production costs. In Gaudet, Lasserre and Long (1995), the principal-agent framework is used to analyze the inherently dynamic case of resource rent taxation. It is shown that the necessity of taking into account the information constraint will generally result in a modification of the standard Hotelling rule, that holds under symmetric information: it is now marginal profit appropriately corrected for the cost of the information constraint that must grow at the rate of interest. If we assume, for argument's sake, that all firm types are to exhaust in two periods, the modification to the standard Hotelling rule will have all the firms except the most efficient ones extract less in the first period and more in the second, thus tilting the extraction path towards the future as

[18]For useful surveys of mechanism design under incomplete information see, for instance, Besanko and Sappington (1987) and Caillaud, Guesnerie, Rey and Tirole (1988).

compared to the symmetric information case.[19] However, when generalized to an arbitrary number of periods, it is shown that the asymmetry of information will have an ambiguous effect on the optimal number of periods to exhaustion. Furthermore, the optimal royalty contract will generally impose a distortion in the extraction path of all the firms, including the lowest-cost ones, contrary to what would be the case in a purely static context.

Optimal resource rent taxation under asymmetry of information raises important issues of intertemporal commitment on the part of the owner and of correlation over time of the costs of extraction. In Gaudet et al. (1995) it is assumed that the principal, whether a government or a private owner, commits only to the current period's royalty rule. It is also assumed that there is zero temporal correlation of the marginal cost parameter subject to private information. Those are very strong assumptions, which beg to be relaxed in future research.

4.2 Imperfect competition and other second best situations

For some resources, the industry is composed of a small number of players who benefit from considerable market power. In such cases, it could be more appropriate to model the firms as oligopolists, instead of price-takers. Modeling oligopolistic nonrenewable resources markets then becomes a problem in dynamic game theory, a problem that has not been fully resolved to this day. There does exist an important literature framed in open-loop decision making, whereby the players are assumed able to commit to an entire extraction path. But to the extent that the firms can observe the evolution of the resource stocks held by their rivals in the industry, open-loop equilibria suffer from the inconvenience that they are not robust to deviation from the announced equilibrium paths (i.e. they are not subgame perfect), and so may not be suited to analyze many oligopolistic resource extraction games. A closed-loop formulation of the game, whereby firms condition their extraction decisions on the vector of stocks of their rivals (in addition to their own), becomes more appropriate. Solving for the equilibria of closed-loop nonrenewable dynamic games turns out to be quite difficult, which

[19] The analysis in Gaudet et al. (1995) is done in discrete time instead of the continuous-time framework used in the preceding sections.

explains why the literature on the topic is extremely limited. This also probably explains why the literature on oligopolistic resource taxation remains pretty much an open topic, and why the study of resource taxation under imperfect competition has concentrated mainly on the monopoly case, which, as in the case of price-takers, does not involve strategic interactions between industry rivals. Bergstrom, Cross and Porter (1981), Karp and Livernois (1992) and Daubanes (2011) are examples of papers that consider the taxation of a nonrenewable resource monopolist. In order to maximize the present value of its resource stock, the monopolist will want to extract the resource at a rate that assures that its marginal rent (the difference between marginal revenue and marginal cost) grows at the rate of interest. But marginal revenue being smaller than price for a monopolist, this means that price minus marginal cost will not be growing at the rate of interest, an efficiency requirement that is satisfied under perfect competition. The monopolist will therefore not extract the resource efficiently.[20] The issue raised in Bergstrom et al. (1981), Karp and Livernois (1992) and Daubanes (2011) is not one of resource rent collection, as explored here. The question asked is: can the monopolist be induced through taxation to adopt an efficient extraction path. They find that there is a family of time-variant tax-subsidy schemes that would induce a monopoly to replicate the efficient production path of a competitive industry.[21] Whether such a goal can be attained and at the same time generate a positive present value of revenues for the government will depend on the structure of demand for the resource. This highlights the fact that the goal is not rent collection, but rather the correction of a distortion that is already present (the monopoly) by introducing what is essentially another distortion (the tax-subsidy scheme). That leaves the more difficult question of resource rent collection in an oligopolistic context largely unexplored.

More generally, taxes are applied in second best situations. They may alleviate or reduce

[20]The famous special case noted by Stiglitz (1976), where the monopolistic and perfectly competitive extraction paths coincide if demand is isoelastic, depends crucially on the assumption of an exogenously given initial resource stock. If initial reserves are endogenously determined, as we have assumed in the previous sections, then the monopolist will always choose to exploit a smaller stock and hence will never efficiently exploit the resource (see Gaudet and Lasserre (1988)).

[21]In Bergstrom et al. (1981), it is assumed that the government has perfect information about the monopolist's cost function and reserves and is able to precommit to an entire sequence of taxes over time. Karp and Livernois (1992) relax those assumptions.

existing distortions. How should non renewable resources be treated in such circumstances? Daubanes and Lasserre (2012) adapt Ramsey's inverse elasticity rule (Ramsey, 1927) to an economy that includes non-renewable natural resources as well as conventional goods. When reserves are exogenous, they find that non-renewable resources should be taxed at a higher rate than conventional commodities having the same demand elasticity. When reserves are endogenous, they show that the resource acquires some features of a conventional produced commodity so that a variant of Ramsey's inverse elasticity rule applies. Clearly, a lot more remains to be done on this topic.

4.3 General equilibrium considerations

All the resource tax analyses we have described in Sections 2 and 3 have been carried out in a partial equilibrium framework, and have mostly insisted on "neutrality", in the sense of absence of distortion to the extraction paths. But partial equilibrium analysis is limited when it comes to discussing optimal taxation from a social point of view. Tax formulas that are viewed as "neutral" in a partial equilibrium framework may not be "efficient" in a second best world where other taxes exist that already distort consumption and investment decisions in the economy as a whole. We have seen a hint of this in Section 3.2, when we considered the possible effects of the depletion allowance on intersectoral investment decisions. In fact, in one of the rare analyses of resource taxation that uses a general equilibrium framework, although with exogenously given initial reserves, Long and Sinn (1984) have shown that allowing the deduction of the true value of the decrease in the resource stock may not always be desirable, contrary to the usual conclusions drawn from partial equilibrium analysis (see for instance Dasgupta and Heal (1979) or Dasgupta et al. (1980)).

Framing the analysis of resource taxation in a general equilibrium framework is certainly a very important area of future research, both from theoretical and applied stand points.

5 Conclusion

The decision to extract nonrenewable natural resources from the ground is irreversible, since the ultimate stock of resources is given by nature: it cannot be reproduced, contrary to conventional goods. Were it not for this fact, there would be no need for a separate treatment with regard to taxation. Because of this irreversibility, the problem of exploiting the resource stock so as to maximize its present value is inherently dynamic, involving an intertemporal arbitrage between extracting the marginal unit now or leaving it in the ground to satisfy future demand. This in itself distinguishes the analysis of resource taxation: a static framework is not appropriate. This explains the care taken in our presentation in considering the effect of the various tax options on the whole time path of extraction. Another consequence of this irreversibility is that a wedge will appear between market price and marginal cost of extraction: there exists a resource rent. Hence the emphasis in our presentation on the possibility of capturing this resource rent through taxation. Finally, the *in situ* resource stock is an asset, among other assets in the economy. Care must be taken that the tax treatment of the natural resource sector does not distort investment decisions between asset types and between sectors of the economy. This was touched upon when considering the corporate income tax.

Our presentation has relied on assumptions of complete information and perfect competition, in a partial equilibrium framework. A few authors have relaxed to some extent one or the other of those assumptions. More remains to be done in that respect. It seems likely however that the comparative dynamics methodology we have employed will remain a very useful tool in future efforts to extend the analysis of nonrenewable resource taxation.

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