The Effect of Asymmetric Preferences on the Federal Funds Rate

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Abstract

This paper studies the empirical implications of asymmetric preferences on the nominal interest rate. Under this specification, positive and negative deviations from the inflation objective can be weighted differently by the policymaker. The model nests the usual specification with quadratic preferences. The empirical relevance of the model with asymmetric preferences is evaluated by statistical tests, and results provide evidence in favor of asymmetric inflation preferences. The estimation of the model also reveals idiosyncratic inflation preferences across different FED chairmen during the period 1954:03 1999:01. [1]

[1] All programs and data are available from the author upon request. I want to thank Pr. Francisco J. Ruge-Murcia for academic and financial support during the redaction of this research report, and Pr. Benoît Perron for a careful reading of my paper and useful comments.

1. INTRODUCTION

This paper builds on the paper "Monetary Policy Delegation to an Asymmetric Central Banker" of Ruge-Murcia (1999). Our model uses the work of Ruge-Murcia by developing the implications of his model for the policy reaction function and testing the significance of the conditional variance of inflation parameter in this reaction function. Because of the need to estimate the inflation equation prior to the estimation of the policy reaction function, and since we use a shorter sample than in Ruge-Murcia (1999), we re-estimate the inflation equation for this shorter sample and also test the implications of the model within this equation.

Our paper, as the paper of Ruge-Murcia, builds on the Barro and Gordon (1983) model of monetary policy. These authors shows that in an economic setup where surprise inflation can produce levels of unemployment below the natural rate, a government with preferences over inflation and unemployment is unable to credibly commit to the optimal monetary policy. Within this setting, the (Nash) equilibrium without credible commitment to a policy rule is characterized by an inflation rate larger than the optimal rate. This result arises because at the optimal inflation level, the marginal benefit of reduced unemployment (to the policy maker) is greater than the marginal cost of inflation. A way to attain an optimal solution is to entrust with the conduct of monetary policy a central banker with different preferences. The "conservative" central banker of Rogoff (1985), which has quadratic preferences but attaches a proportionally smaller weight to unemployment in his loss function than the government, is a classic example of this type of welfare improving solution.

As in Ruge-Murcia (1999), we develop and estimate a model similar to the Barro and Gordon model, but where the central banker's preferences are allowed to be asymmetric around the optimal inflation rate. Within this specification, different weights can be assigned to positive and negative deviations from the inflation target. Moreover, this specification nests the quadratic loss function used by Barro and Gordon and common in the literature. While in the quadratic model the associated loss of a deviation from target depends only on its magnitude, under asymmetric preferences both magnitude and sign have an impact. In this model, the potentially asymmetric preferences of the central banker have meaningful implications on the outcome of monetary policy and in particular on the observed relation between the inflation level and its variance. For certain parameter values, the model predicts a deflation bias where inflation is below the optimal rate, while for other parameter value, it predicts an inflation bias where inflation is superior to the optimal rate.

Within this project we will test the empirical implications of the model with US data. It is shown that asymmetric preferences generate testable implications on the time series of inflation and on the exogenous side of the policy reaction function. Specifically, on the one hand inflation follows a GARCH-in-mean process and, on the other hand, the left-hand side of the policy reaction function (the nominal interest rate) has as predetermined regressor, namely the conditional variance of inflation. The parameters that measures the degree of asymmetry in the central banker's loss function corresponds to the coefficients on the conditional variance of inflation in the inflation equation and in the policy reaction function. Provided that the error term of the inflation equation follows a GARCH process, this parameter is identified. Moreover, since our model nests the quadratic model as a special case when the coefficient on conditional variance is zero, it is possible to compare both models by testing the statistical hypothesis that this coefficient equals zero. After doing the appropriate tests, we decide to model inflation, unemployment and the nominal interest rate as I(1) variables.

We do find that inflation preferences are asymmetric while testing both within the inflation equation and within the policy reaction function. We also consider the possibility of different inflation preferences, for each chairmen of the Federal Reserve Board, for the span of the sample. Our results indicate that inflation preferences differ across chairmen.

The paper is organized as follows. Section two presents and solves a dynamic version of the Barro-Gordon model with asymmetric preferences for an inflation equation and a policy reaction function. Section three present different specifications for the conditional variance of the inflation equation. In section four we perform unit root and cointegration tests on the data and present empirical results. We also compare the performance of the Taylor rule with that of our interest rate reaction function. Section five concludes.

2. THE MODEL

The economy consists of a central banker appointed by the government to implement monetary policy and of private individuals, whose expectations are assumed to be rational.

2.1 The Private Sector

An aggregate supply and an aggregate demand equation describe the behavior of the private sector. The aggregate supply is described by the expectations augmented Phillips curve

$$\pi_t = \pi_t^e - \frac{1}{\lambda}\left(u_t - u_t^n\right) + \omega_t \qquad (1a)$$

or

$$u_t = u_t^n - \lambda\left(\pi_t - \pi_t^e\right) + \lambda\omega_t \qquad (1b)$$

where u_t is the unemployment rate, u_t^n is the natural unemployment rate, π_t is the inflation rate, π_t^e is public's forecast of the inflation rate constructed at time t-1, λ is a strictly positive parameter, and ω_t is a supply disturbance that we assume normally distributed with zero mean and potentially time-varying conditional variance. Under the assumption that expectations are formed rationally,

$$\pi_t^e = E_{t-1}[\pi_t] \tag{2}$$

where E_{t-1} stands for expectation conditional upon information available a time t-1. The natural rate of unemployment is assumed to evolve over time as if it where generated by the autoregressive process

$$u_t^n = \psi + \rho(L)u_t^n + \zeta_t \tag{3}$$

where ψ is a constant term, L is the lag operator and $\rho(L)$ is a polynomial in the lag operator

$$\rho(L) = \sum_{i=1}^{q} \rho_i L^i$$

ζ_t is a random shock that is assumed to be white noise, normally distributed and independent of ω_t. No restrictions are imposed on the roots of $\rho(L)$ so that the presence of a unit root in the rate of unemployment is not ruled out.

For realism, we also assume that the econometrician does not observe the natural rate of unemployment. We assume that the public and the central banker observe current and past values of u^n_t. This allows the derivation implications of the effect of the natural rate on observable inflation and unemployment. This hypothesis is based on the fact that households and policymakers have access to more information than the simple time series data used by the econometrician.

Aggregate demand, expressed in terms of unemployment, is a function of the real interest rate:

$$u_t = \mu + \gamma(r_t - \pi_t^e) + \varepsilon_t \tag{4}$$

where μ is an intercept term, r_t is the nominal interest rate, $r_t - \pi_t^e$ is the ex-ante real interest rate, γ is a strictly positive coefficient (the real rate has a negative effect on aggregate demand and thus a positive effect on unemployment), and ε_t is a demand shock that we assume to be white noise, normally distributed and independent of ω_t and ζ_t. Since all that is necessary in the derivations farther in the text is that $\partial u_t/\partial r_t$ be nonzero, the linear nature of (4) is not restrictive and can be relaxed if necessary. Moreover, the inclusion of lagged values of the real interest rate in (4) would not affect the model predictions about the joint inflation-unemployment dynamics. From now on, since equilibrium in the goods market requires that aggregate demand and aggregate supply are equal, no notational distinction will be made between them.

2.2 The Central Banker Preferences

As Ruge-Murcia (1999), we consider the delegation of monetary policy to a central banker with preferences potentially different from the public's ones in terms of optimal inflation rate and employment stabilization. We generalize the policymaker's standard quadratic loss function to allow asymmetric losses from positive and negative deviations of inflation from its objective. Asymmetries in the loss function can arise from the more downward than upward rigidity in prices (that would push the central banker to weight more heavily positive deviations) or the desire to accommodate large supply shocks (that would push the central banker to give more weight to negative deviations). Mathematically, the central banker's preferences are represented by the loss function

$$B(\pi_t, u_t) = \frac{\left(\exp\{a(\pi_t - \pi^*)\} - a(\pi_t - \pi^*) - 1\right)}{a^2} + \frac{\phi}{2}\left(u_t - ku_t^n\right)^2 \qquad (5)$$

where π^* is the socially optimal inflation rate, ku^n_t is the targeted rate of unemployment, and ϕ is a positive constant that measures the relative importance of unemployment stabilization in the objective function. The possibly non-zero value π^* can be interpreted as the one associated with the optimal inflation tax on cash balances. The targeted unemployment rate, ku^n_t, is assumed strictly smaller than the natural rate, u^n_t, $(0 < k < 1)$ because of the assumption that in the presence of distortions in goods and labor markets, such as unemployment compensation and income taxes, the natural rate will tend to exceed the efficient level –that is private agents will choose lower quantities of output and employment.

In (5), the inflation component is described by the LINEX function. For $a > 0$, the LINEX function is asymmetric around $\pi_t = \pi^*$, approximating an exponential function for $\pi_t > \pi^*$ but a linear one for $\pi_t < \pi^*$. Thus, positive deviations from the inflation target are weighted more heavily than negative ones in the central banker's objective function for $a > 0$, while the converse is true for $a < 0$.

The LINEX function nests the usual quadratic function for $a \to 0$.[2] In the case of the quadratic function it is only the magnitude, not the sign of the deviation, that is relevant to the policymaker. Hence, it is possible to test the relevance of our hypothesis that the agent's preferences are asymmetric by the statistical test that the parameter a is different from zero.

2.3 Solution of the Model under Asymmetric Preferences

The central banker disposes as monetary policy tools of: (1) open market operations that changes the monetary base; (2) adjustments to interest rates under his control. However, since the stock of real money balances and the interest rate are inversely

[2] It is easy to show that by taking the limit of the LINEX function as a goes to 0 and by using L'Hôspital rule twice.

related through the money demand function, the central banker cannot simultaneously choose values for both variables. Indeed, when the central bank choose to fix the interest rate, real money balances become endogenous. In this paper, we will assume that the central banker uses the short-term nominal interest rate to control monetary policy.

The determination of inflation and unemployment is characterized by the interaction between the policymaker and the public. This interaction can be described in the following way. The policymaker chooses in period t a value for his policy instrument r_t so as to minimize his loss function described by (5), given the available information I_{t-1}. Simultaneously, the public's form his expectations about π_t. Suppose as in Barro[3] that the public regard the process by which the policymaker choose r_t and π_t as described by the reaction function $h^e(I_{t-1})$. We assume that the public's information set contains all model parameters, including the form of the central bank's loss function and past and current values of u^n_t, u_t, and past values of π_t. Otherwise stated, the public's information set incorporate the knowledge that r_t and π_t will emerge from the policymaker's loss minimization as specified in (5). Thus, $\pi_t^e = E_{t-1}[\pi_t] = h^e(I_{t-1})$. These expectations are based on the same information set that the one available to the policymaker, I_{t-1}. Therefore, as in Barro[4], a solution to the model involves deriving a function $h^e(.)$ such that $\pi_t = h^e(I_{t-1})$ is a solution to the policymaker's loss minimization problem, given that $\pi_t^e = h^e(I_{t-1})$. Since we suppose that expectations are rational, this solution will be as in (2).

Thus, the solution to our model hinges upon a Nash equilibrium where the policymaker chooses the value of the policy variables conditionally on the public's expectations, which are themselves conditional on the policymaker choice function. The policymaker makes his choice knowing the public's expectations internalize his choice function. The public expectations internalize the fact that the policymaker's choice is conditional on his knowledge of their own (rational) expectations building mechanism of the outcome of monetary policy. A Nash equilibrium is this way obtained. As in Barro and Gordon (1983), the (Nash) equilibrium inflation rate is determined by the level of anticipated inflation for which the marginal cost of higher inflation equals the marginal benefit of lower unemployment. Since it implies a higher than socially optimal inflation rate but an equilibrium between the private sector expectations and the solution to the policymaker's problem, it is called a sub-optimal but time-consistent equilibrium.

The problem of the central banker entrusted with the conduct of monetary policy is to choose the sequence of interest rates that minimizes the present discounted value of the loss function (5). Formally,

$$\underset{\{r_t\}_{t=0}^{\infty}}{Min} \quad E_{t-1}\left[\sum_{t=0}^{\infty} \beta^t B(\pi_t, u_t)\right]$$

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[3] Op cit.

[4] Op cit.

where $\beta \in (0,1)$ and where π_t and u_t depend on r_t through the aggregate demand and aggregate supply equations (1) and (4).

Neither the public nor the central banker can observe contemporaneous disturbances. The choice in one particular period of π_t affect unemployment through the aggregate supply, but as our model is built, this effect does not carry forward to affect future unemployment. Indeed, since the process for u^n_t is exogenous, there is no channel for the policy variable to affect future costs, and the objective function therefore reduces to a sequence of static one period problems. The first order condition for the optimization problem is

$$E_{t-1}\left[\frac{\partial B}{\partial \pi_t}\times\frac{\partial \pi_t}{u_t}\times\frac{\partial u_t}{\partial r_t}+\frac{\partial B}{\partial u_t}\times\frac{\partial u_t}{\partial r_t}\right]=0$$

Rearranging and making appropriate substitutions we obtain that the F.O.C. is given by

$$E_{t-1}\left[\frac{\left(\exp\{a(\pi_t-\pi^*)\}-1\right)}{-\lambda a}+\phi\left(u_t-ku_t^n\right)\right]=0 \tag{6}$$

Since the loss function (5) is strictly convex, we obtain a unique minimum. Under the assumptions that expectations are formed rationally and that the supply disturbances are serially uncorrelated, taking conditional expectations in both sides of (1) yield

$$E_{t-1}[u_t]=E_{t-1}\left[u_t^n\right] \tag{7}$$

The assumption that the supply disturbances are normal implies that inflation is also normally distributed. This implies that $\exp\{a(\pi_t-\pi^*)\}$ is distributed log-normal with mean

$$\exp\left\{a\left(E_{t-1}[\pi_t]-\pi^*+a\frac{\sigma_t^2}{2}\right)\right\}$$

where σ^2_t is the conditional variance of inflation. We shall note that because the supply shock can be conditionally heteroskedastic, the conditional variance of inflation could be time varying. Using the last result as well as (7) we can rearrange (6) as

$$\exp\left\{a\left(E_{t-1}[\pi_t]-\pi^*+a\frac{\sigma_t^2}{2}\right)\right\}=1+\lambda a\phi(1-k)E_{t-1}\left[u_t^n\right]$$

As mentioned before, the public's expectation on the inflation rate is conditional upon the policymaker's choice mechanism. Thus, taking natural logs of both sides of the last result and rearranging yields the public's inflation forecast

$$E_{t-1}[\pi_t] = \pi^* - \frac{a}{2}\sigma_t^2 + \frac{1}{a}\ln\left[1 + \lambda a\phi(1-k)E_{t-1}[u_t^n]\right] \quad (8)$$

Note that under the Barro and Gordon framework with quadratic preferences, (8) would be

$$E_{t-1}[\pi_t] = \pi^* + \lambda a\phi(1-k)E_{t-1}[u_t^n]$$

where the second term on the right hand side disappears when k = 1, that is when there is no distortions to the goods and labor markets. In the absence of distortions to the goods and labor market, the marginal benefit of decreased unemployment equals the marginal cost of inflation at $\pi = \pi^*$. However, we can clearly see from (8) that in our model, we have in the absence of distortions

$$E_{t-1}[\pi_t] = \pi^* - \frac{a}{2}\sigma_t^2$$

that is, there remains an inflation (or deflation) bias even in the absence of distortions in the goods and labor markets.

Returning to the derivation of the inflation process, using (3) and (4) we can write

$$u_t^n - u_t = u_t^n - E_{t-1}[u_t] - \varepsilon_t = E_{t-1}[u_t^n] - \varepsilon_t = \varsigma_t - \varepsilon_t$$

Substituting this last result as well as the public's inflation forecast given by equation (8) in the aggregate supply equation (1a), we obtain

$$\pi_t = \pi^* - \frac{a}{2}\sigma_t^2 + \frac{1}{a}\ln\left\{1 + \lambda a\phi(1-k)E_{t-1}[u_t^n]\right\} + \frac{1}{\lambda}(\varsigma_t - \varepsilon_t) + \omega_t \quad (9)$$

Although the econometrician does not observe the natural unemployment rate, the assumption that its current value is included in the public's information set makes it possible to write the unemployment-inflation process in terms of observable variables.

Taking conditional expectations in both sides of the aggregate demand function and using the proceed with (7) we obtain

$$E_{t-1}[u_t^n] = u_t - \varepsilon_t \quad (11)$$

Replacing this expression in (9) yields the stochastic process for inflation :

$$\pi_t = \pi^* - \frac{a}{2}\sigma_t^2 + \frac{1}{a}\ln\{1 + \lambda a\phi(1-k)(u_t - \varepsilon_t)\} + \frac{1}{\lambda}(\varsigma_t - \varepsilon_t) + \omega_t$$

It is useful to consider a first order Taylor series expansion of the logarithmic term above around one. Because the limit a → 0 corresponds to a quadratic loss function, the resulting expression is especially appropriate for small departures of symmetry. While the error involved in the approximation might bias the econometric results against the finding of asymmetries in the policy maker preferences, this approach substantially simplifies the estimation procedure. Using the fact that as a → 0, ln[1 + ax] ≈ ax, we can write

$$\pi_t = \pi^* - \frac{a}{2}\sigma_t^2 + \lambda\phi(1-k)u_t + \frac{1}{\lambda}\varsigma_t - \left(\lambda\phi(1-k) + \frac{1}{\lambda}\right)\varepsilon_t + \omega_t \qquad (12)$$

Note that although inflation and unemployment are non-stationary by assumption, the model implies that the variables should be cointegrated with cointegrating vector

$$[1 \quad -\lambda\phi(1-k)]$$

The error term involves the linear combination of the structural disturbances. If one assumes that ω_t, ζ_t, and ε_t are white noise, a white noise process can represent the reduced form disturbance so that we can write the inflation process implied by the dynamics of our model as

$$\pi_t = \pi^* - \frac{a}{2}\sigma_t^2 + \delta u_t + \upsilon_t \qquad (13)$$

where

$$\delta = \lambda\phi(1-k) \qquad\qquad \upsilon_t = \frac{1}{\lambda}\varsigma_t - \left(\delta + \frac{1}{\lambda}\right)\varepsilon_t + \omega_t$$

We now derive the policy reaction function of the central banker implied by the dynamics of our model. Taking conditional expectations on both sides of the aggregate demand function, using the assumption that expectations are rational and rearranging the proceed we obtain

$$r_t = -\frac{\mu}{\gamma} + \frac{1}{\gamma}E_{t-1}[u_t^n] + E_{t-1}[\pi_t] \qquad (10)$$

3. ARCH PROCESSES

We now have to specify a structural form for the conditional variance of inflation. Since we suspect the variance to be serially correlated, we can model the contemporaneous variance to be dependent on its past values. Engle (1982) proposed in this spirit the class of Autoregressive Conditionally Heteroskedastic, or ARCH, models. These model conditional variance as a distributed lag of past squared residuals :

$$\sigma_t^2 = \omega + \alpha(L)\eta_t^2$$

where $\alpha(L)$ is a polynomial in the lag operator and where to keep the variance positive, we require the coefficients in $\alpha(L)$ to be nonnegative. As a way to model persistent movements in volatility without estimating a large number of coefficients and a high order polynomial $\alpha(L)$, Bollerslev (1986) suggested the Generalized ARCH, or GARCH, model :

$$\sigma_t^2 = \omega + \beta(L)\sigma_t^2 + \alpha(L)\eta_t^2$$

where $\beta(L)$ is a polynomial in the lag operator. A GARCH(p,q) can be thought as an ARMA(p,q) model for the conditional variance where the moving average is on the variance shocks rather than on the regression residuals. This can be easily shown with the help of a simple GARCH(1,1) which can be written as:

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\eta_{t-1}^2$$

$$= \omega + (\alpha + \beta)\sigma_{t-1}^2 + \alpha\left(\eta_{t-1}^2 - \sigma_{t-1}^2\right)$$

The term $(\eta^2_t - \sigma^2_t)$ in the second equality has mean zero and can be thought as a shock (innovation) to the variance process. The coefficient thus measures the extent to which a volatility shock today feeds through into next period's volatility, while α measures the rate at which this effect dies out over time. In this sense, a GARCH(p,q) share similar properties with an ARMA(p,q), although we shall stress that a GARCH(p,q) is not an ARMA(p,q) for the squared residuals, and that a GARCH(p,q) has an heteroskedastic variance while a ARMA(p,q) has an homoskedastic variance. In general, if η_t is described by a GARCH(p,q), then η^2_t follows an ARMA(r,p), where r is the larger of p and q. A GARCH(1,1) is weakly stationary provided that $\alpha + \beta < 1$, as for an ARMA process.

Hentschel (1995) suggests a general Box-Cox transformation of standard deviations that nests various specifications:

$$\frac{\left(\sigma_t^\lambda - 1\right)}{\lambda} = \omega + \alpha\sigma_{t-1}^\lambda\left(\frac{\eta_{t-1}}{\sigma_{t-1}}\right)^v + \beta\frac{\left(\sigma_{t-1}^\lambda - 1\right)}{\lambda}$$

When $\lambda = v = 2$ and $\beta = 0$, this describes a ARCH process, while for $\lambda = v = 2$ it is a GARCH, with $\lambda \to 0$ and $v = 1$ a EGARCH (exponential GARCH : we then consider the natural logarithm of the standard deviations), and finally a IGARCH (integrated ARCH) when $\lambda = v = 2$ and $\alpha + \beta \geq 1$ (that is a non stationary GARCH for which the first difference is stationary).

In the empirical implementation phase, we will thus test the use a Box-Cox transformation of standard deviations *à la* Hentschel against the generally used GARCH(1,1). We also consider some ARCH(p) and GARCH(p,p) specifications by proceeding to a Box and Jenkins like model identification procedure for squared residuals (we limit the GARCH specifications to cases for which the AR and MA polynomials are of the same order because in this case an ARMA(p,p) process for squared residuals is also a GARCH(p,p) for the variance) by looking at autocorrelations and partial correlations.[5]

Since our interest will be in the effect of the conditional variance of inflation on the mean of the inflation process, we will consider a ARCH-in-mean like specification as in Engle et al (1987) where we allow the conditional variance to affect the level of the equation :

$$\sigma_t^2 = \omega + \alpha(L)\eta_t^2$$

$$y_t = \mu + \delta\sigma_t^2 + \eta_t$$

4. EMPIRICAL IMPLEMENTATION

4.1 The Data

The model will be estimated using 179 observations of quarterly, seasonally adjusted US inflation, unemployment and nominal interest rate data between 1954:03 and 1999:01.

The series used as the central bank controlled nominal interest rate is the following: (FEDFUNDS) Federal Funds Rate, average of daily figures, monthly (in annual percentage points) from 1954:09 to 1999:03, released by the Federal Reserve Board of Governors. The value for a quarter is the average of the values for the three months of the quarter. The series of unemployment rates is given by average of the civilian unemployment rate, seasonally adjusted, in the three months of the quarter, from 1954:03 to 1999:01, released by the U.S. Department of Labor, Bureau of Labor Statistics. Finally, the rate of inflation is measured by the percentage change (on an annual basis) in the Consumer Price Index (CPI) for Urban Consumers, from the last month of the previous quarter to the last month of the current quarter. Those last two series are the same that are used by Ruge-Murcia (1999).

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[5] For a presentation of Conditionally Heteroskedastic models, see Hamilton (1994) chap. 21 or Bollerslev et al (1994).

4.2 Unit Root Tests

Before the estimation phase we conduct unit root tests to assess whether the series are stationary or integrated of order one. We will base our decision using three different tests. The first one is an Augmented Dickey Fuller Test (ADF). The null hypothesis is that the series on which we perform the test has a unit root and is thus non-stationary. The idea behind a Dickey-Fuller test is that if a series y_t has a unit root, then the coefficient ρ in the regression equation

$$y_t = \rho y_{t-1} + z_t$$

where z_t is a white noise process, is unity. This last equation is used for a test that the series is stationary in levels (we will call this equation model I), but we can also test that the series is stationary around a constant mean or around a linear trend, as in the next two equations (where the first one allows for a nonzero mean, and we will call it model II, and the second one allows for a linear trend, and we will call it model III) :

$$y_t = \mu + \rho y_{t-1} + z_t$$

$$y_t = \mu + \delta t + \rho y_{t-1} + z_t$$

In the presence of autocorrelation of the white noise process z_t, this test exhibits serious size distortions (the asymptotic distribution of ρ is not invariant to the autocorrelation of the error term). To cope with this problem, Said and Dickey (1984) propose what is known as Augmented Dickey-Fuller unit-root tests. But to implement them, we first have to choose the lag length k in the regression of

$$\Delta y_t = (\rho - 1)y_t + \beta_1 \Delta y_{t-1} + \beta_2 \Delta y_{t-2} + ... + \beta_k \Delta y_{t-k} + \varepsilon_t$$

where under the null hypothesis that the series y_t has a unit root, the coefficient ρ is zero. The order k depends on the autocorrelation structure of the residuals of the regression

$$\Delta y_t = \rho y_t + e_t \qquad (A)$$

Campbell and Perron (1991) suggest to test whether the last included lag in an autoregression of order k (on this residual) is significant (using the standard normal asymptotic distribution), and if not reduce the order of the autoregression by one until the last included lag is significant. In the case of an AR(p), such a procedure will select k greater than or equal to the true order with probability one asymptotically.

We follow the procedure of drawing autocorrelations of those residuals and using the Bartlett's formula standard errors and the 95% confidence level of the asymptotic normal distribution to choose k. We shall note that, as stressed by Campbell and Perron (1991), using too many lags reduces the power of the test while choosing too few can cause size distortions (in the sense of over-rejection of the null hypothesis).

Perron (1988) suggests a testing strategy. First try to reject the null hypothesis of a unit root with the statistic under model III. If you can reject the null, there is no need to go further. But if the true model has a zero mean and no trend, the statistic under model III has very low power compared to the statistics under model I or model II. So if we cannot reject the null with the statistics under model III, we should instead try to do so with the statistics under model II and model I.

For the second test of the null hypothesis of a unit root we use the non-parametric approach of Phillips and Perron (1988). Instead of including lags of the first difference as do Said and Dickey, Phillips and Perron adjust the standard error of their t statistic by using the Newey-West heteroskedasticity and autocorrelation consistent estimate of the variance. This test is interesting because it has more power than the ADF tests to reject the null hypothesis of a unit root (since we do not loose data points by including lags). However, Phillips and Perron (1988) note that in presence of negative autocorrelation of the residual of (A) their statistic suffers from severe size distortions, in the sense that they reject too often the null hypothesis. In this context ADF statistics also suffer similar size distortions but to a lesser extent. The length k used to construct the Newey-West estimate is the same as for ADF tests and the consequences of using too few or too much lags are also the same, that is with too few lags we over-reject the null while with too much lags we lose power.

We use simultaneously both tests because the first one is less sensible to the presence of negative autocorrelation in the residual of (A) while the second has asymptotically more power to reject the null.

The third test is a test of the null hypothesis of stationarity, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) (Kwiatkowski-Phillips-Schmidt-Shin (1992)) test. Like the previous one this test uses the Newey-West consistent variance to allow for serial correlation. The length k used for the Newey-West estimate is the same than for the previous test and the consequences of not using the correct length are also similar. This last test is also sensitive to the presence of negative autocorrelation in the residual of (A), but this time negative autocorrelation causes an under-rejection of the null problem (and since the null is the inverse than for the previous tests it leads to the same false result).

We shall note that in the light of these "instructions" on how to use these tests, mechanically choosing an "optimal" lag length and then making a decision based on the statistic computed with this length is simpler but not necessarily prudent. We instead take our decision on the stationarity of the series considering several lag lengths and the results of different tests. We first report the results of the tests based on an optimal lag length strategy[6] and then report the stances we take on order of integration of the series based on our more careful analysis. Those interested in how we make those decisions can consult appendix II.

Unit Root Tests With Asymptotically Maximum Lag Length

Test		ADF		Phillips-Perron		KPSS	
Series	K	Model II	Model III	Model II	Model III	Levels	Trend
		-3.48 (1%)	-4.01	-3.48	-4.01	0.739	0.216
		-2.88 (5%)	-3.44	-2.88	-3.44	0.463	0.146
		-2.57 (10%)	-3.14	-2.57	-3.14	0.347	0.119
Unemployment	2	-2.95	-2.87	-2.23	-2.21	1.221	0.471
Inflation	3	-2.61	-2.48	-4.66	-4.63	0.805	0.671
Federal Funds	1	-2.75	-2.69	-2.45	-2.34	2.625	1.213

N.B.: Critical Values from Hamilton (1994).

Starting with unemployment we cannot reject the null hypothesis of a unit root under model III but can under model II at 5% using a DF test augmented with two lags and thus we are able to reject the null with an ADF test. However we shall note that the most plausible alternative is model II, considering the pattern of the series (see plot in APPENDIX I). However the Phillips-Perron test is unable to reject the null of a unit root both under model III and II. We shall note that there is no negative autocorrelation in the residual of (A) that would distort the size of the test. The KPSS test rejects the null hypothesis of stationarity in levels as well as around a linear trend in both cases at 1%. There is more evidence for than against the presence of a unit root. Moreover, as a result of a Monte Carlo study, Campbell and Perron (1991) stress that "...near-integrated stationary DGPs are better forecast using integrated forecasting models." Using this last advice, we will thus take the stance of considering the unemployment rate as being I(1). Note that Blanchard and Summers (1987) derive a theoretical model where the wage setting dynamic implies that the unemployment rate follows a random walk.

For the inflation rate series, an ADF test rejects the null of a unit root only at 10% under model II. Again looking at a plot of the series (APPENDIX I) the most plausible alternative is model II. However, a Phillips-Perron test under models III and II rejects the null of a unit root at 1%. Obviously, it is a feature of the Phillips-Perron test that it has more power than an ADF test to reject the null. But the difference is striking and we shall note that there is some clearly significant negative autocorrelation at lag one. As we previously stressed it can cause size distortions leading to over-rejection of the null. And since Phillips-Perron tests are more sensible to this problem than ADF tests, that would explain their clearly different diagnostics. Finally, both KPSS statistics are able to reject the null hypothesis of stationarity at 1%. We thus obtain some rather mixed results (not so mixed because the negative autocorrelation is so important that the result of the PP test cannot be taken seriously). Using the advice of Campbell and Perron (1991) about near integrated process we will, for the remainder of the paper, take the stance of considering the inflation rate series as being I(1).

[6] Note that we choose this lag length ourselves using a plot of the autocorrelations of the residuals in (A). We do not choose the optimal lag length with some information criterion but instead choose the asymptotically maximum lag length. This ensures that our test has correct size if there is no negative autocorrelation in (A) of a lower order than the k that we chose.

Finally, we conduct unit root tests on the federal funds rate series. The ADF test with one lag rejects the null only at 10% under model II, while the Phillips-Perron test is unable to reject the null of a unit root even at 10% (that said, again the most plausible alternative is model II). The KPSS test for his part rejects the null hypothesis of stationarity at 1%. This series is thus clearly I(1).

We report the results of the analysis conducted in APPENDIX II on a broader range of statistics in the next table. The results are basically the same.

Unit Root Tests Diagnostics*

Series	Diagnostic
Unemployment	I(1)
Inflation	I(1)
Federal Funds	I(1)

*For details see appendix II.

4.3 Cointegration Tests

It will be of interest further in the text whether inflation and unemployment on the one hand, and the nominal interest rate and unemployment on the other hand, are cointegrated.

We conduct in this sub-section residual based tests for cointegration from the Phillips-Ouliaris-Hansen procedure as in Hamilton (1994)[7], that is, we estimate by OLS the cointegrating relation, and conduct unit root tests on the residuals from this regression. We employ as before ADF and Phillips-Perron tests with the exception that the critical values are not the same as for unit root tests. We thus use the Phillips-Ouliaris critical values for ADF and Phillips-Perron like tests, provided in table B.9 of Hamilton (1994). Phillips and Ouliaris (1990) suggest that this test might have the same properties as unit root tests. We will thus follow the same procedure as for the case of simple unit root tests.

Suppose that y_t and x_t denote our two variables of interest. Let z_t be the residual of an OLS regression of y_t on x_t

$$z_t = y_t - \beta_{ols} x_t$$

We thus conduct some unit root tests on z_t. Under the hypothesis that y_t and x_t are individually I(1), the null hypothesis that z_t is also I(1) corresponds to a no-cointegration null hypothesis. However, if we can reject the null hypothesis, that is, if there exists a linear combination of two I(1) variables that is I(0), then those two variables are cointegrated. As for the unit root tests, we will test for the hypothesis that z_t is stationary in levels, up to a constant term or around a linear trend, with the usual statistics under model I, II or III, using the asymptotic critical values computed by Phillips and Ouliaris (1990). We report the results in the next table.

[7] Hamilton (1994) p. 599.

Phillips-Ouliaris Residual Based T-Tests of Cointegration With Asymptotically Maximum Lag Length[8]

Underlying Test Relation	K	ADF Model II	ADF Model III	Phillips-Perron Model II	Phillips-Perron Model III
		-3.48 (1%)	-4.01	-3.48	-4.01
		-2.88 (5%)	-3.44	-2.88	-3.44
		-2.57 (10%)	-3.14	-2.57	-3.14
Inflation-Unemployment	3	-3.19	-3.15	-4.93	-4.89
Unemployment-Inflation	3	-3.51	-3.50	-4.83	-4.81
Federal Funds-Unemployment	1	-3.92	-3.98	-3.19	-3.19
Unemployment-Federal Funds	1	-4.17	-4.21	-3.31	-3.30

N.B.: Critical Values From Hamilton (1994)

We can reject the null hypothesis of no-cointegration for the residual of the linear relation inflation-unemployment, using an ADF test with the Phillips-Ouliaris 10% critical values under model III or II. Moreover, we can reject the null with a PP test with the Phillips-Ouliaris critical values at 5% under model III or II. There is negative autocorrelation in the residuals of (A) of the first order and it can influence the results. However a DF test that does not account for autocorrelation but that is not influenced by the negative autocorrelation can reject the null at 1% (see APPENDIX III). However, considering all the information available (including these of APPENDIX III), we conclude that, as implied by our model, inflation and unemployment are cointegrated. As shown in the previous table, these results are robust to normalization.

For the nominal rate-unemployment relation, the ADF test can reject the null of no cointegration at 5% while the PP test can reject the null at 10%. There is no negative autocorrelation of the residual in (A) and thus our statistics are well behaved. Again, as implied by our model, the nominal rate and unemployment are cointegrated.

We will now estimate the inflation process implied by our model. Before estimating this process, we have to choose a specification for the conditional variance. We presented in the section on ARCH models a general specification suggested by Hentschel that nests, as we previously stressed, various more specific specifications.

$$\frac{(\sigma_t^{\lambda}-1)}{\lambda}=\omega+\alpha\sigma_{t-1}^{\lambda}\left(\frac{\eta_{t-1}}{\sigma_{t-1}}\right)^{v}+\beta\frac{(\sigma_{t-1}^{\lambda}-1)}{\lambda}$$

We will first allow the parameters λ and ν to take positive and otherwise unrestricted values and then impose the restrictions λ = ν = 2 (the conditional variance is then represented by a GARCH(1,1)) that we will test with a likelihood ratio test. This LR test that λ and ν are not statistically different from 2 has a p-value of 0.65 so that we cannot reject the restrictions λ = ν = 2. Thus, within the family of specifications nested by Hentschel general specification, the GARCH(1,1) is not rejected by the data.

The next step is to consider various GARCH(p,q) specifications. We first try to identify a specification by looking at autocorrelations and partial correlations *à la*

[8] You can find more details in APPENDIX III.

Box-Jenkins. Then we compare the relative goodness of fit of different specifications with information criteria suggested for ARCH processes by Bollerslev et al (1994). Finally, for each specification, we do a likelihood ratio test of the joint significance of all coefficients, which amounts to a test for the presence of ARCH effects. There is a clear presence of ARCH effects since for each specification considered the LR test is significant at 1%. Within the various specifications considered, we retain an ARCH(1-3-4) specification, that is a model where the conditional variance depends on the first, third and fourth lag of squared residuals.[9]

Finally, we estimate the inflation process alternatively with GARCH(1,1) and ARCH(1-3-4) specifications for the conditional variance. Due to the well known difficulties in obtaining the best maximum of the likelihood function and due to variance explosion problems with GARCH processes, we do a three step estimation using the Maximize procedure of RATS 4.0 : (1) we estimate initial values for the constant and the mean variance as well as the coefficient on the cointegrating relation (we enforce the cointegrating relation coefficient thereafter) by OLS ; (2) we estimate intermediate values for all the parameters by maximum likelihood using the simplex algorithm (this is a grid search method that does not require second derivatives, but we cannot obtain standard errors for the estimates with it) ; (3) we estimate with more precision the parameters by maximum likelihood with the BFGS algorithm and obtain standard errors. We shall note that to make sure that the coefficients of the conditional variance equation are all positive, we consider their absolute value. We could have done a constrained optimization, but our approach is simpler. Here we shall note that we don't know for sure if the standard errors are affected or not by the absolute value function.[10]

The GARCH(1,1) specification provides a better fit: the value of the log-likelihood computed conditionally on the value taken by the vector of estimated coefficients is of –223 for the process with GARCH(1,1) conditional variance and of –231 for the process with ARCH(1-3-4) variance. From now on, we will report results based on GARCH(1,1) errors.[11]

We first estimate the inflation process assuming that the different chairmen[12] of the Federal Reserve Board share the same inflation preferences. We subsequently estimate the inflation process allowing for different preferences. We report the results in the next table. On the first line the parameter a_{global} stands for the common inflation preferences parameter and on the subsequent lines we report particular inflation preferences parameter.

[9] Those interested in the details of this identification procedure can refer to APPENDIX IV.

[10] For details, see APPENDIX V and APPENDIX VI.

[11] We nevertheless compute results based on ARCH(1-3-4) errors. You can find them, as well as more detailed results in APPENDIX V and APPENDIX VI.

[12] These are William McChesney Martin from April 2nd 1951 to January 31st 1970, Arthur Burns from February 1st 1970 to March 7th 1977, William Miller from March 8th 1977 to August 5th 1979, Paul Volcker from August 6th 1979 to August 10th 1987 and Alan Greenspan from August 11th 1987 onwards.

Inflation Preferences Parameters
With GARCH(1,1) Conditional Variance Estimated in the Inflation Process

Parameter	Estimate	Standard Error	T-Statistic	P-value	P-value a = -0.53
a_{global}	-0.53	0.07	-7.66	0.00	-----
a_{Mc}	-0.61	0.24	-2.56	0.01	0.76
a_{Bu}	-1.43	0.34	-4.26	0.00	0.01
a_{Mi}	-1.37	0.59	-2.32	0.02	0.16
a_{Vo}	-0.63	0.19	-3.26	0.00	0.31
a_{Gr}	-1.22	0.28	-4.40	0.00	0.01
Idiosyncrasy	0.59	0.08	7.20	0.00	-----

All coefficients are significant, at least at 5%, and negative. In the context of our model, it implies that positive deviations from the optimal inflation target deliver a smaller loss (to the agent's utility function) than a negative one of the same magnitude. This implies that, on average, the inflation bias is larger than with quadratic preferences. Moreover, it implies that the level and variance of inflation are positively related as suggested by Friedman (1977). In the sixth column we report the p-value of a T test that particular inflation preferences parameters are statistically different from the common parameter. The preferences of chairmen Burns and Greenspan are different, but not those of chairmen McChesney, Miller and Volcker. Finally since the five tests that particular chairman preferences are different from global ones is not a correct proxy for a general test of idiosyncratic preferences because type 1 error adds up for each parameter, we further test for idiosyncratic preferences. We do so by testing the null hypothesis that others chairmen preferences are not different from chairman Greenspan's ones (for details, see APPENDIX V). The results are reported in the last row of the previous table. The null hypothesis that chairmen preferences aren't idiosyncratic is rejected even at 1%.

4.5 Policy Reaction Function

This sub-section derives and estimates the reduced-form version of the policy reaction function under the assumption that the nominal interest rate and unemployment are I(1) and under the assumption that inflation follows the process derived in the previous sub-section. Again, although the natural unemployment rate is not observed by the econometrician, the assumption that its current value forms part of the public's information set makes possible to write the policy reaction function in terms of observable variables.

Substituting (11) in (8) and re-substituting the proceed in (10) yields

$$r_t = -\frac{\mu}{\gamma} + \frac{1}{\gamma}(u_t - \varepsilon_t) + \pi^* - \frac{a}{2}\sigma_t^2 + \frac{1}{a}\ln\{1 + \lambda a \phi (1-k)(u_t - \varepsilon_t)\} \qquad (14)$$

where σ^2_t is the conditional variance of inflation. Considering a Taylor series expansion of the logarithmic term above around one yield

$$r_t = \Im - \frac{a}{2}\sigma_t^2 + bu_t + \tau_t \qquad (15)$$

where

$$\Im = -\frac{\mu}{\gamma} + \pi^* \qquad b = \frac{1}{\gamma} + \lambda\phi(1-k) \qquad b\varepsilon_t = \tau_t$$

Again, if one assumes that ε_t is white noise, τ_t is a linear transformation of ε_t that is also white noise. The model predicts that the nominal interest rate and the unemployment rate are cointegrated with cointegrating vector

$$\left[1 \quad -\frac{1}{\gamma} - \lambda\phi(1-k)\right]$$

As before we estimate the cointegrating relation by OLS and enforce it thereafter. σ^2_t being the conditional variance of inflation, we shall estimate the inflation process first. From our previous estimation of the inflation process, we have saved a conditional variance series[13] that we use here as an exogenous regressor. As before, we estimate the equation in three steps. Here we shall note that to obtain correct standard errors, we should have done a joint estimation of the inflation equation and reaction function. The inflation process that we use as an exogenous regressor in the reaction function is estimated with error and the standard errors that we obtain while estimating the reaction function do not really account for that fact.

We first estimate the policy reaction function, assuming that the different chairman of the Federal Reserve Board shares the same inflation preferences. We subsequently estimate the policy reaction function allowing for different preferences. We report the results in the next table[14].

Inflation Preferences Parameters
With GARCH(1,1) Conditional Variance Estimated in the Policy Reaction Function

Parameter	Estimate	Standard Error	T-Statistic	P-value	P-value a = -0.69
a_{global}	-0.69	0.03	23.58	0.00	-----
a_{Mc}	-0.70	0.12	-5.86	0.00	0.91
a_{Bu}	-1.01	0.13	-7.59	0.00	0.02
a_{Mi}	-1.16	0.30	-3.82	0.00	0.12
a_{Vo}	-0.96	0.05	-19.63	0.00	0.00
a_{Gr}	-1.51	0.03	-45.86	0.00	0.00
Idiosyncrasy	0.66	0.04	18.61	0.00	-----

[13] We report here the results based on the GARCH(1,1) conditional variance series. We also computed a policy reaction function based on the ARCH(1-3-4) conditional variance series. You can find it in Appendix VI.

[14] For details see APPENDIX VI.

All coefficients are significant at 1%, and negative. In the context of our model, it again implies that positive deviations from the optimal inflation target deliver a smaller loss (to the agent's utility function) than a negative one of the same magnitude. The sixth column reports the p-value of a T test that particular inflation preference parameters are statistically different from the common parameter. The preferences of chairmen Burns, Volcker and Greenspan are all different at 5%, but not those of chairmen McChesney and Miller, even at 10%. We finally conduct a general test of the null hypothesis that inflation preferences aren't idiosyncratic across chairmen. We again reject the null at 1% (see the last row of the last table)[15].

4.6 Taylor Rule

We will now compare the performance of our policy reaction function to a policy reaction function *à la* Taylor, as in Taylor (1993), and known as Taylor Rule. In this paper, the author presents a quite straightforward policy reaction function :

$$r = p + \frac{y - y^*}{2} + \frac{p - 2}{2} + 2$$

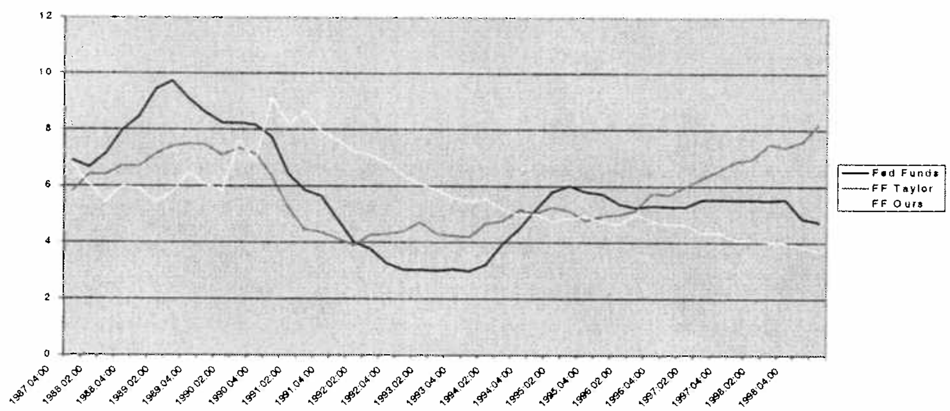
where r is the federal funds rate, p is the rate of inflation over the previous four quarters, y is the log of real output and y^* the log of potential real output. It assumes an equilibrium real interest rate of 2%, a target inflation rate of 2% and equal weighting of the deviation from target of real output and inflation. Taylor shows (graphically) that this rule tracks pretty well the actual outcome for the federal funds rate for the Greenspan's term. As suggested in Taylor (1998), we will fit the equation to data (the Greenspan's term from 1987:04 to 1999:01), by maximum likelihood, assuming no particular value for the real rate, the inflation target nor for the weighting :

$$r_t = \pi_t + rr + \frac{y - y^*}{b} + \frac{\pi_t - \pi^*}{1 - b}$$

where the parameters to be estimated are rr, the equilibrium real rate, π^*, the inflation target and b the weighting. We thus obtain an implied path for the federal funds rate that can be compared to the path implied by our model. The results are plotted in the next figure:

[15] For details, see APPENDIX VI.

Federal Funds Rate vs Taylor's and Ours' Forecast



As we can see, the Taylor function tracks pretty well the actual series of the federal funds rate except for the last two years. This discrepancy can probably be explained by a relative easing of monetary conditions by the FED in reaction to the Asian financial crisis and to the LCTM bailout. On the other hand, our policy reaction function has a hard time tracking the federal funds rate. The Taylor function has a mean squared error of 1.78, compared with a MSE of 4.23 for our function.

4. CONCLUSION

This paper builds on the paper "Monetary Policy Delegation to an Asymmetric Central Banker" by Ruge-Murcia (1999), and test for asymmetry within a policy reaction function. Ruge-Murcia himself builds on the Barro and Gordon (1983) model by allowing the policymaker's preferences to be asymmetric around his inflation objective. His model encompasses the usual model with quadratic preferences and thus allows a natural test of the asymmetry hypothesis. It also provides a theoretical explanation for the positive relation between the level and variance of inflation.

The empirical implication central to this model is that inflation follows a GARCH-in-mean process. It also generates the implication that the policy reaction function contains the conditional variance of inflation as an exogenous regressor. Since the testing of our hypothesis within the policy reaction function requires the prior estimation of the inflation process and since we use a slightly different sample than Ruge-Murcia, we also test the hypothesis within the inflation process equation.

Our estimation procedure slightly differs from the one used by Ruge-Murcia. We use a 3-step estimation procedure where initial values for the parameters are obtained in a first step by OLS, and then refined in a second step by maximum likelihood with the simplex algorithm. The third step involves the final maximization of the likelihood using the BFGS algorithm. We innovate in the utilization of the simplex to obtain improved initial values. It is useful because the variance can explode for certain values of the parameters with GARCH errors.

The model is estimated and tested using US data between 1954:03 and 1999:01 under the assumptions that unemployment, inflation and the nominal interest rate are all I(1) variables. The results we obtain for the inflation preferences when tested within the estimated inflation process as in Ruge-Murcia (1999) or within the policy reaction function are the same as Ruge-Murcia: we reject the nested model with quadratic preferences. Moreover, the asymmetry exacerbates rather than reduces the inflationary bias. The results also show important differences in the preferences of different chairmen of the FED.

We shall finish our paper with three critics. First, we don't know for sure if using the absolute value to constrain the parameters of the conditional variance equation to be positive affects the standard errors. Second, we shall have conducted a joint estimation of the inflation equation and policy reaction function to obtain correct standard errors. Those are two important flaws that would remain to be worked on. Third, whereas our paper provides evidence in favor of asymmetric inflation preferences, our estimated reaction function can't beat a simple Taylor Rule at forecasting the monetary policy stance. Improving the forecasting power of our reaction function, or alternatively integrating the impact of asymmetric preferences in a simple rule *à la* Taylor, would be an interesting new area of research.

APPENDIX I

PLOT OF THE SERIES AND OF THEIR AUTOCORRELATIONS

Figure 1
Unemployment Rate 1954:03-1999:01

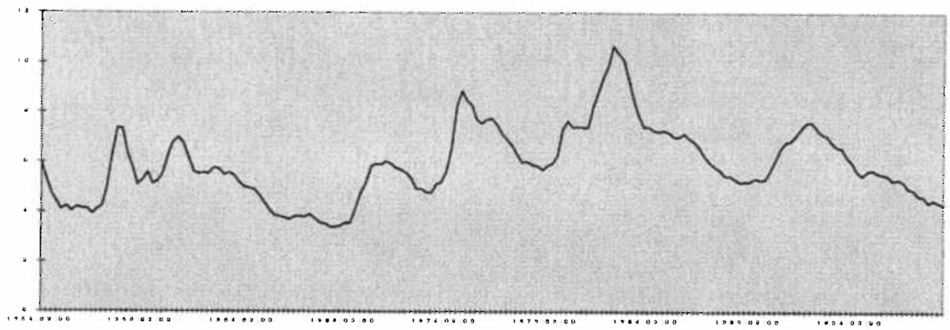


Figure 2
Inflation Rate Quarterly 1954:03-1999:01

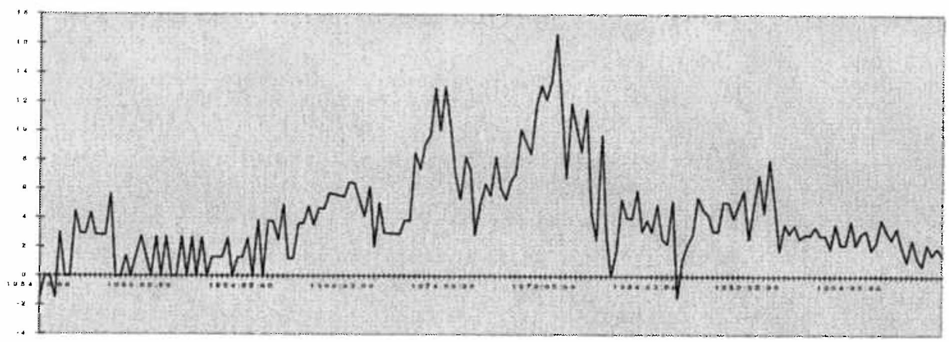


Figure 3
Federal Funds Rate Quarterly 1954:03 1999:01

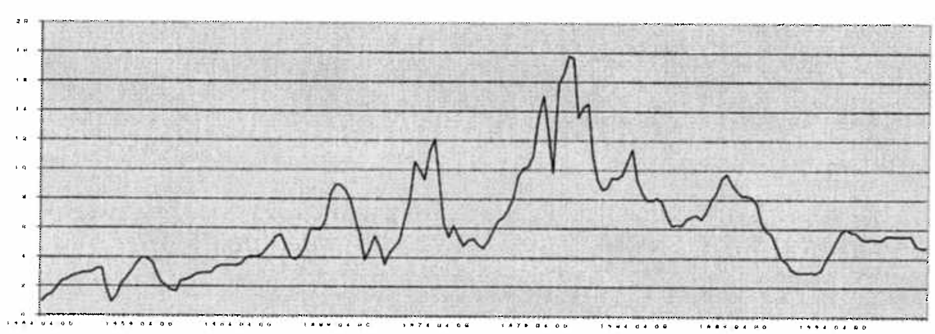


Figure 4
Unemployment Rate Autocorrelations

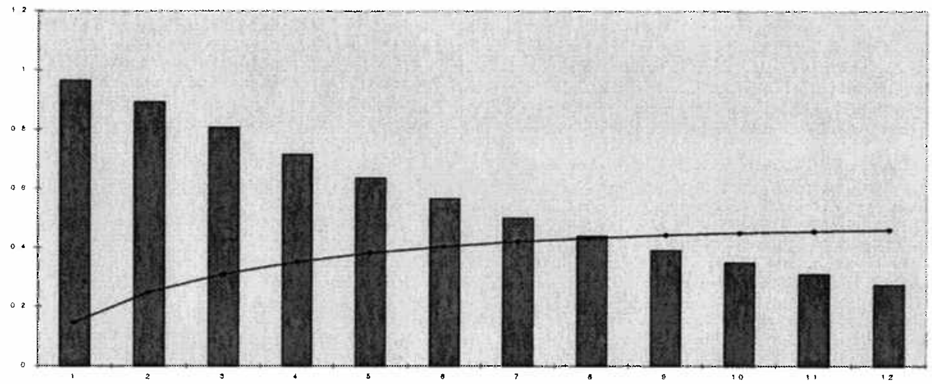


Figure 5
Inflation Rate Autocorrelations

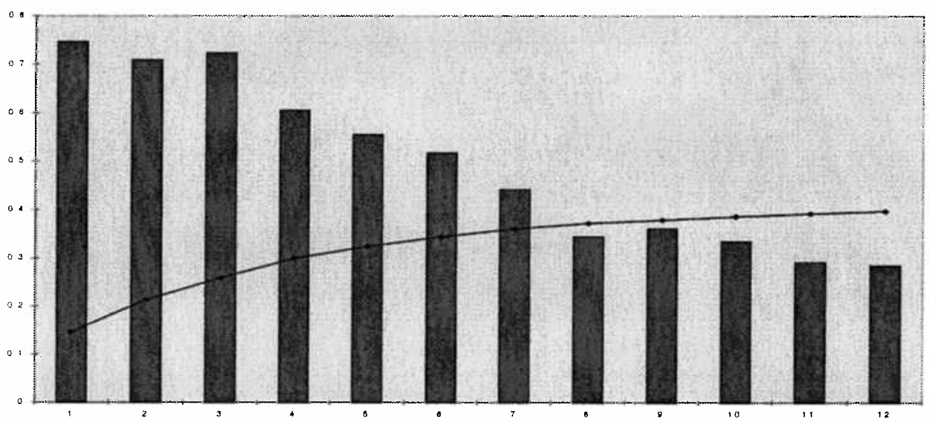
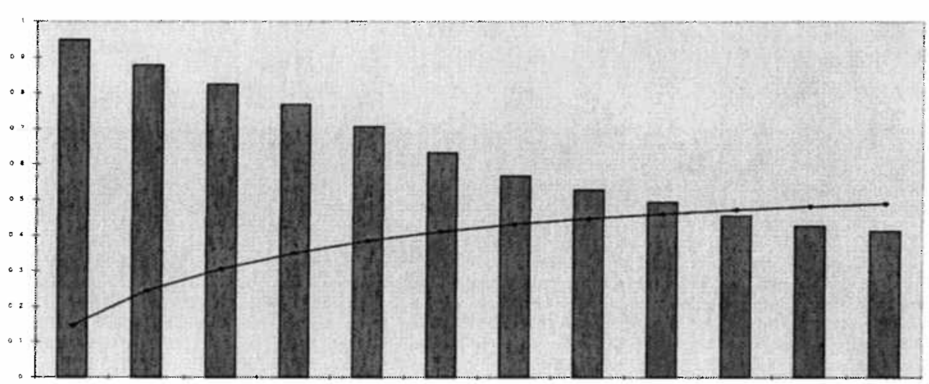


Figure 6
Federal Funds Rate Autocorrelations



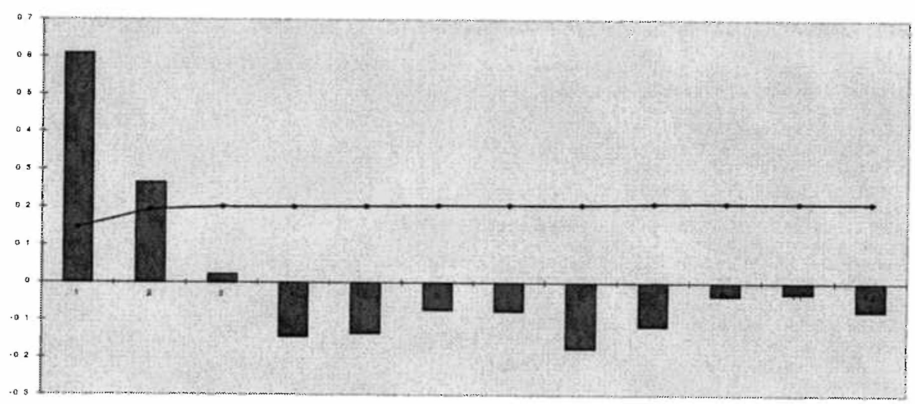
APPENDIX II

UNIT ROOT TESTS

Unemployment

Using the procedure described in the main text, we chose a maximum lag length of two.

Figure 7
Autocorrelations of the Residuals of a DF Test on the Unemployment Rate



Augmented Dickey-Fuller Tests (t-statistics) : Unemployment Rate

Lag		No Constant - No Trend (Model I)	Constant - No Trend (Model II)	Constant Trend (Model III)
Critical Values	1%	-2.59	-3.48	-4.01
	5%	-1.95	-2.88	-3.44
	10%	-1.62	-2.57	-3.14
0		-0.69	-1.49	-1.38
1		-0.82	-3.42	-3.42
2		-0.67	-2.95	-2.87

The ADF test is the nearer to reject the null hypothesis of a unit root under model II. We can reject the null at 5% with one and two lags, but cannot reject the null when no lags are included. We shall note that looking at a plot of the series (APPENDIX I), the most plausible alternative is model II.

Phillips-Perron Unit Root Tests : Unemployment Rate

Lag		Constant - No Trend (Model II)	Constant - Trend (Model III)
Critical Values	1%	-3.48	-4.01
	5%	-2.88	-3.44
	10%	-2.57	-3.14
0		-1.49	-1.38
1		-1.97	-1.93
2		-2.23	-2.21

The PP test never rejects the null of a unit root even at 10%.

KPSS Stationarity Tests : Unemployment Rate

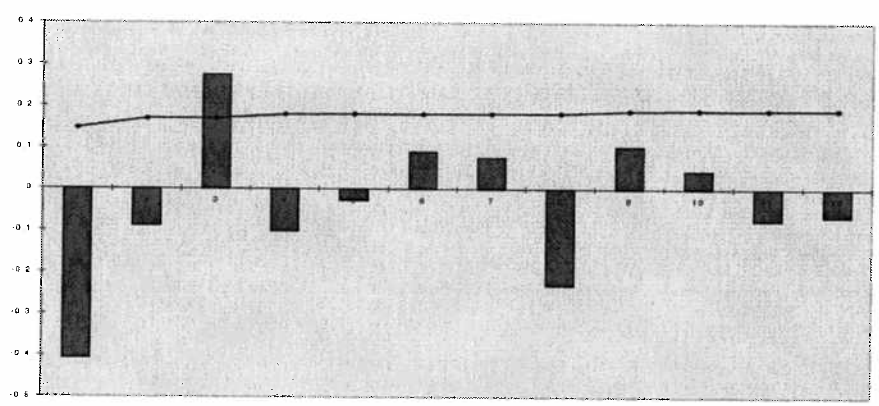
Lag		Level	Trend
Critical Values	1%	0.739	0.216
	5%	0.463	0.146
	10%	0.347	0.119
0		3.521	1.344
1		1.791	0.687
2		1.221	0.471

Finally, the KPSS test always rejects the null of stationarity at 1%. Overall, the tests lead to the conclusion that the unemployment rate series is non-stationary. We shall note that Blanchard and Summers (1987) derive plausible economic conditions under which unemployment would be a random walk.

Inflation Rate

If we don't consider negative autocorrelation, we choose a lag length of at most three.

Figure 8
Autocorrelations of the Residuals of a DF Test on the Inflation Rate



In the next step we conduct ADF tests.

Augmented Dickey-Fuller Tests (t-statistics) : Inflation Rate

Lag		No Constant - No Trend (Model I)	Constant - No Trend (Model II)	Constant Trend (Model III)
Critical Values	1%	-2.59	-3.48	-4.01
	5%	-1.95	-2.88	-3.44
	10%	-1.62	-2.57	-3.14
0		-2.98	-5.09	-5.07
1		-1.87	-3.33	-3.26
2		-1.26	-2.38	-2.25
3		-1.35	-2.61	-2.48

It is under model II that we reject the null the more often: we can reject the null at 1% with no lag, at 5% with one lag and at 10% with three lags. With two lags, we cannot reject the null even at 10%. Here we shall stress that the presence of important negative autocorrelation probably causes the statistic to over-reject the null. This last point, as well as the fact that there was already very few evidence to reject the null lead us to consider that we cannot reject the null hypothesis of a unit root using ADF tests. Again, we shall note that looking at a plot of the series (APPENDIX I), the most plausible alternative is model II.

Phillips-Perron Unit Root Tests : Inflation Rate

Lag		Constant - No Trend (Model II)	Constant - Trend (Model III)
Critical Values	1%	-3.48	-4.01
	5%	-2.88	-3.44
	10%	-2.57	-3.14
0		-5.09	-5.07
1		-4.59	-4.55
2		-4.42	-4.38
3		-4.66	-4.63

The Phillips-Perron test, for his part, draws a very clear conclusion : we can always reject the null hypothesis of a unit root at 1%. The fact that the Phillips-Perron test has more power to reject the null can explain his clear stance in favor of rejection compared to the ADF test but again, we should be careful because it can be caused by negative autocorrelation of the error term of (A). Indeed, there is important negative autocorrelation and the rejection of the null by the PP test is doubtful.

KPSS Stationarity Tests : Inflation Rate

Lag		Level	Trend
Critical Values	1%	0.739	0.216
	5%	0.463	0.146
	10%	0.347	0.119
0		2.569	2.129
1		1.471	1.222
2		1.041	0.866
3		0.805	0.671

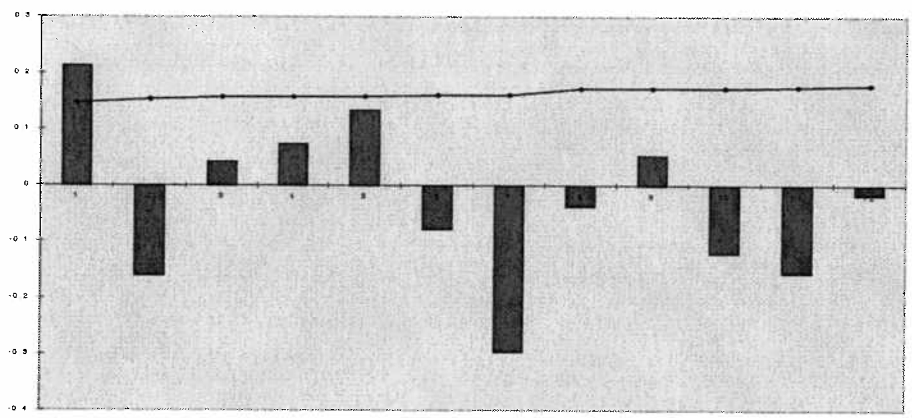
The KPSS test always rejects the null of stationarity at 1%. Again, as stressed by Kwiatkowski et al (1992), the statistic is very sensitive to negative autocorrelation, with a tendency to under-rejection : clearly this is not a problem here. The data does not allow us to make a clear diagnostic on whether the series is stationary or non-stationary. Considering the previous remarks, we would tend to favor the stance of considering the inflation rate as being I(1).

We can add that as a result of a Monte Carlo study, Campbell and Perron (1991) stresses that "...near-integrated stationary DGPs are better forecast using integrated forecasting models." Using this last advice and the previous remarks, we will thus take the stance of considering the inflation rate series as non-stationary.

Federal Funds Rate

Looking at a plot of autocorrelations, we choose a value of at most one if we consider only positive autocorrelations for k. There is no significant negative autocorrelation of low order.

Figure 9
Autocorrelations of the Residuals of a DF Test on the Federal Funds Rate



Considering ADF tests, we can reject the null hypothesis of a unit root only under model II and only at 10%.

Augmented Dickey-Fuller Tests (t-statistics) : Federal Funds Rate

Lag		No Constant - No Trend (Model I)	Constant - No Trend (Model II)	Constant Trend (Model III)
Critical Values	1%	-2.59	-3.48	-4.01
	5%	-1.95	-2.88	-3.44
	10%	-1.62	-2.57	-3.14
0		-0.84	-2.27	-2.10
1		-1.09	-2.75	-2.69

Again, there is significant negative autocorrelation (at lags two and seven) that could cause the statistic to over-reject the null, so that we are hardly able to reject the null with an ADF test. As for the two previously studied series, looking at a plot of the series the most plausible alternative is Model II.

Using the Phillips-Perron test, it is impossible to reject the null even at 1%. The Phillips-Perron statistic is very sensitive to negative autocorrelation and it can cause over-rejection, but again, this is not a problem here since we clearly cannot reject the null.

Phillips-Perron Unit Root Tests : Federal Funds Rate

Lag		Constant - No Trend (Model II)	Constant - Trend (Model III)
Critical Values	1%	-3.48	-4.01
	5%	-2.88	-3.44
	10%	-2.57	-3.14
0		-2.27	-2.10
1		-2.45	-2.34

Finally, the KPSS statistics always reject the null hypothesis of stationarity at the 1% level. Since negative autocorrelation causes under-rejection and we always reject the null, we can have confidence in this result.

KPSS Stationarity Tests : Federal Funds Rate

Lag		Level	Trend
Critical Values	1%	0.739	0.216
	5%	0.463	0.146
	10%	0.347	0.119
0		5.116	2.355
1		2.625	1.213

We thus have here a clear result that the federal funds rate behaves like an integrated series.

APPENDIX III

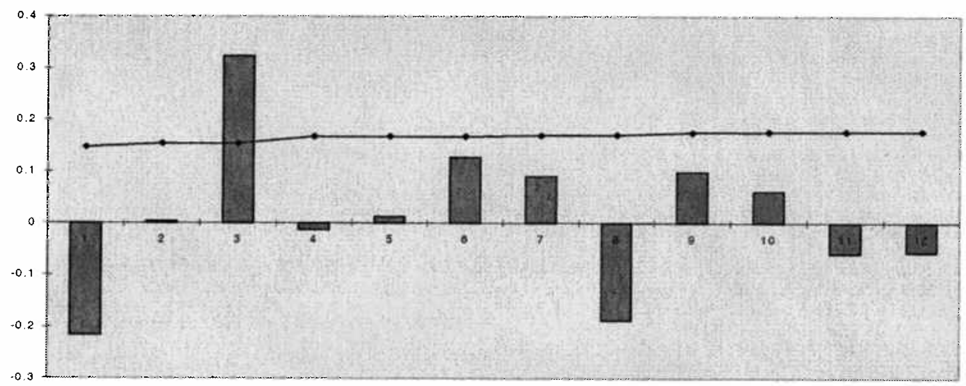
COINTEGRATION TESTS

1) Between The Inflation Rate and The Unemployment Rate

a) Regression of the Inflation Rate on the Unemployment Rate

As for unit root tests, we first have to choose the length k for autocorrelation. In the case of the residual of the cointegrating relation between the inflation rate and the unemployment rate, we choose a k of 3.

Autocorrelations of the Residuals of a DF Test on the Cointegrating Residuals Inflation Rate / Unemployment Rate



As you can see from the previous figure, there is significant autocorrelation of higher order (height), but it is some negative one. There is also some significant negative autocorrelation at lag one. As we know the augmentation is based on the presence of positive autocorrelation and negative autocorrelation can push the statistics to misbehave.

Augmented Dickey-Fuller Tests (t-statistics) : Cointegrating Residual Inflation-Unemployment

Lag		No Constant - No Trend (Model I)	Constant - No Trend (Model II)	Constant Trend (Model III)
Critical Values (Phillips-Ouliaris)	1%	-3.39	-3.96	-3.98
	5%	-2.76	-3.37	-3.42
	10%	-2.45	-3.07	-3.13
0		-5.21	-5.21	-5.18
1		-3.62	-3.63	-3.59
2		-2.74	-2.75	-2.71
3		-3.18	-3.19	-3.15

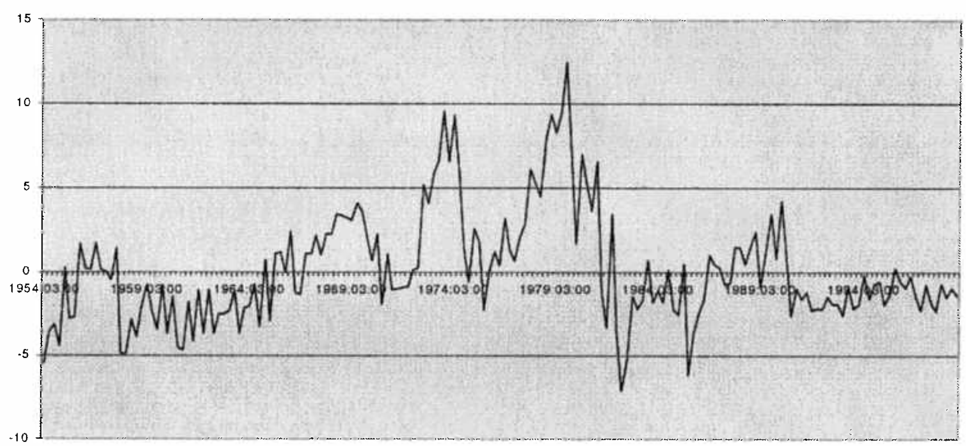
Under model I, considering three lags or less, we can always reject the null hypothesis of no cointegration, sometimes at 1%, sometimes only at 10%. Those results would thus call for a rejection of the null hypothesis but, as before, considering the poor performance in terms of size of the test in presence of negative autocorrelation, we shall take this result with a grain of salt.

Phillips-Perron Unit Root Tests : Cointegrating Residual Inflation-Unemployment

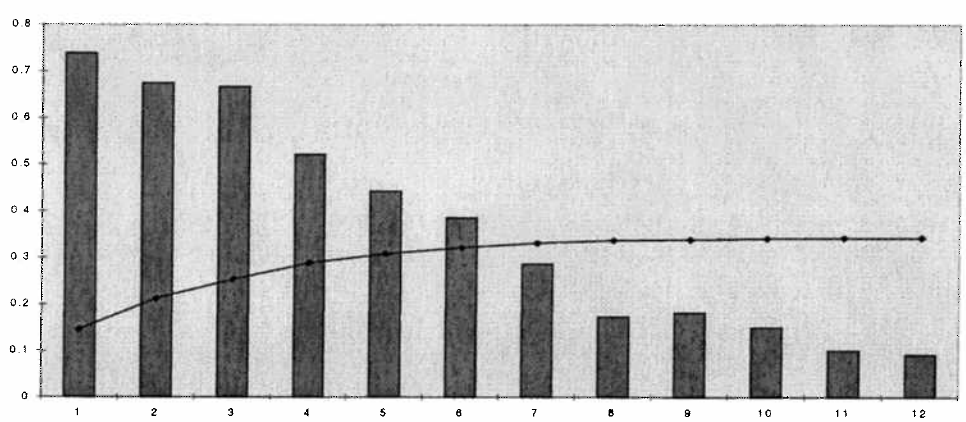
Lag		Constant - No Trend (Model II)	Constant - Trend (Model III)
Critical Values (Phillips-Ouliaris)	1%	-3.96	-3.98
	5%	-3.37	-3.42
	10%	-3.07	-3.13
0		-5.21	-5.18
1		-4.81	-4.77
2		-4.68	-4.64
3		-4.93	-4.89

In the case of the Phillips-Perron test, we always reject the null hypothesis of no cointegration at 1%. Again, the Phillips-Perron statistics are very sensitive to the presence of negative autocorrelation and this conclusion is of limited value.

Cointegrating Residual Inflation Rate / Unemployment Rate



Autocorrelations of Cointegrating Residuals Inflation Rate / Unemployment Rate



If we look at a plot of the residuals of this cointegrating relation it does not look like a stationary series. Negative increments are very often followed by negative increments and positive increments are very often followed by positive increments. As for the autocorrelation, they do not follow a damp exponential pattern common to stationary series, but neither really have the pattern of the autocorrelations of a random walk. If the results of the unit root tests are not corrupted by the negative autocorrelations, and we can really reject the hypothesis of a unit root (of no cointegration), then there is a very weak long-term relationship between the inflation rate and the unemployment rate.

We can reject the null hypothesis of no cointegration using the previous statistics, but we cannot place too much confidence in this result considering the negative autocorrelation of the error term of DF tests.

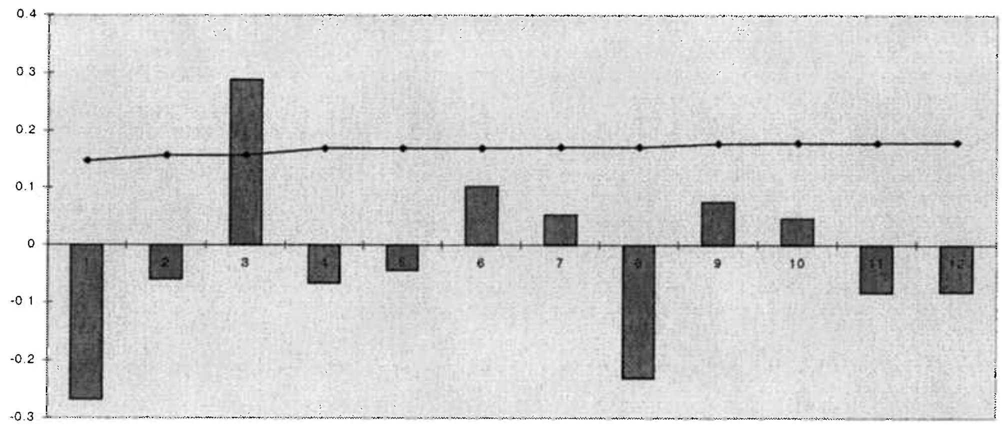
Under the hypothesis that the variables are cointegrated, we estimate by OLS the following cointegrating vector: [1 , 0.6647].[16]

[16] N.B. : Later we will impose a cointegrating relation where the coefficient has been estimated along with a constant. We will nevertheless re-estimate the constant term.

b) Regression of the Unemployment Rate on the Inflation Rate

Looking at the autocorrelations on the next figure, and ignoring significant negative autocorrelation, we choose a value for k of 3.

Autocorrelations of Residuals of a DF Test on Cointegrating Residuals Unemployment Rate / Inflation Rate



Augmented Dickey-Fuller Tests (t-statistics): Cointegrating Residual Unemployment-Inflation

Lag		No Constant - No Trend (Model I)	Constant - No Trend (Model II)	Constant Trend (Model III)
Critical Values (Phillips-Ouliaris)	1%	-3.39	-3.96	-3.98
	5%	-2.76	-3.37	-3.42
	10%	-2.45	-3.07	-3.13
0		-4.13	-5.04	-5.03
1		-2.99	-3.67	-3.66
2		-2.38	-2.91	-2.90
3		-2.81	-3.51	-3.50

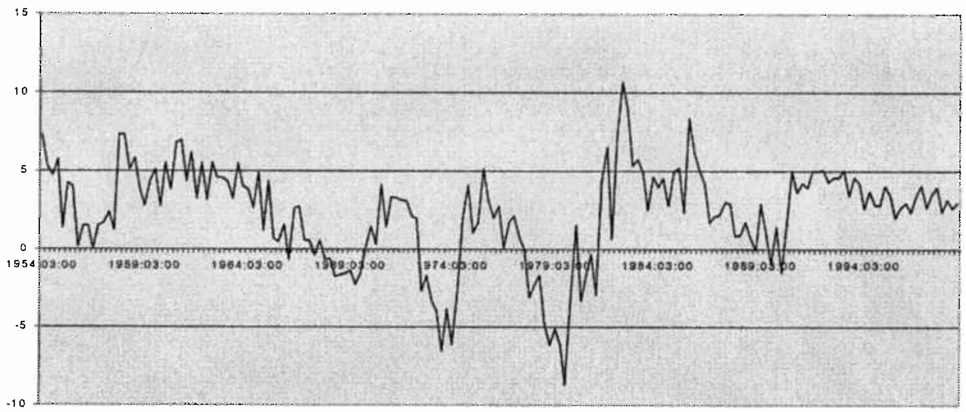
Using statistics under model I, II or III, we can in general reject the null hypothesis of no cointegration at 5%. However, we cannot reject the null under any model when a lag of length 2 is considered, even at 10%. Moreover, the strong presence of negative autocorrelation of the error term can lead to significant size problems. The results are thus more on the side of the rejection of the null, but not unambiguously.

Phillips-Perron Unit Root Tests: Cointegrating Residual Unemployment-Inflation

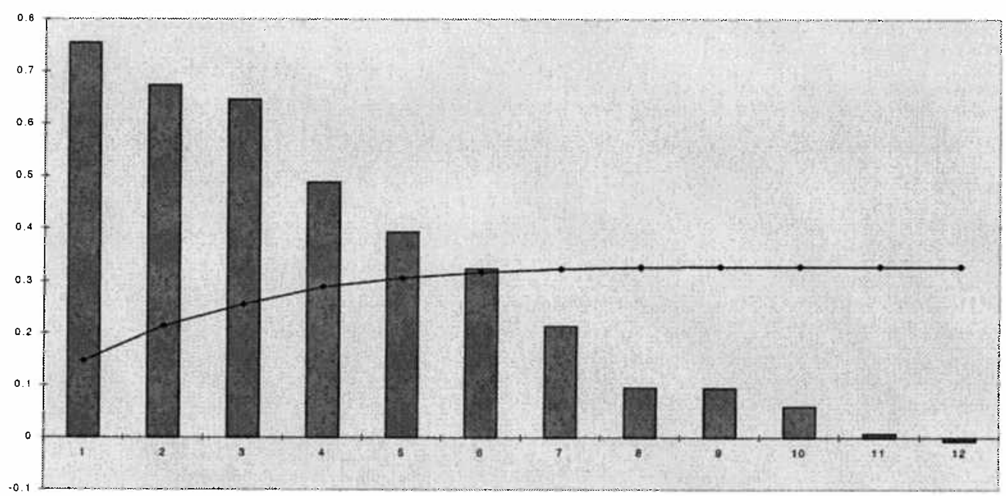
Lag		Constant - No Trend (Model II)	Constant - Trend (Model III)
Critical Values (Phillips-Ouliaris)	1%	-3.96	-3.98
	5%	-3.37	-3.42
	10%	-3.07	-3.13
0		-5.04	-5.03
1		-4.70	-4.69
2		-4.59	-4.57
3		-4.83	-4.81

Using Phillips-Perron statistics, we can reject the null under every models and lag length at 1%. Again, since Phillips-Perron statistics are more vulnerable to negative autocorrelation of the error term than are DF tests, a doubt remains as to the validity of the results.

Cointegrating Residual Unemployment Rate / Inflation Rate



Autocorrelations of Cointegrating Residuals Unemployment Rate / Inflation Rate



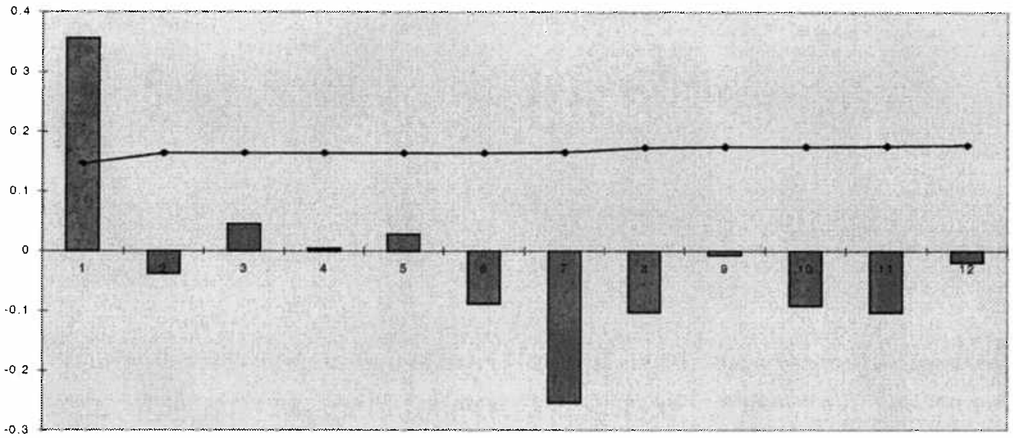
As for the plot of the residuals of the regression of inflation on unemployment, the plot of the residuals of the regression of unemployment on inflation shows much persistence. The autocorrelations also show strong persistence, but in neither case is it really the pattern of a non-stationary series. To sum up, the results of cointegration tests between inflation and unemployment are pretty much the same, whatever the normalization. The lead to the rejection of the no-cointegration null hypothesis. A few doubts however remain.

Between The Federal Funds Rate and The Unemployment Rate

a) Regression of the Federal Funds Rate on the Unemployment Rate

Again, we ignore higher order significant negative autocorrelation to choose a value for k of one.

Autocorrelations of Residuals from a DF Test on the Cointegrating Residual Fed-Funds/Unemployment Rate



There is some negative autocorrelation but no significant one of low order. Under model II and II we can reject the null when we include more than one lag but less than six. Since we have chosen a value of one for k, considering six or seven lags would induce a useless loss of power. Using one lag we can allow for autocorrelation, and indeed there is some significant one. The Dickey-Fuller test on the residuals can always reject the null hypothesis of no cointegration, when at least one lag is included under model I. The Dickey-Fuller like statistics of the Phillips-Ouliaris cointegration test thus reject the null hypothesis of no cointegration.

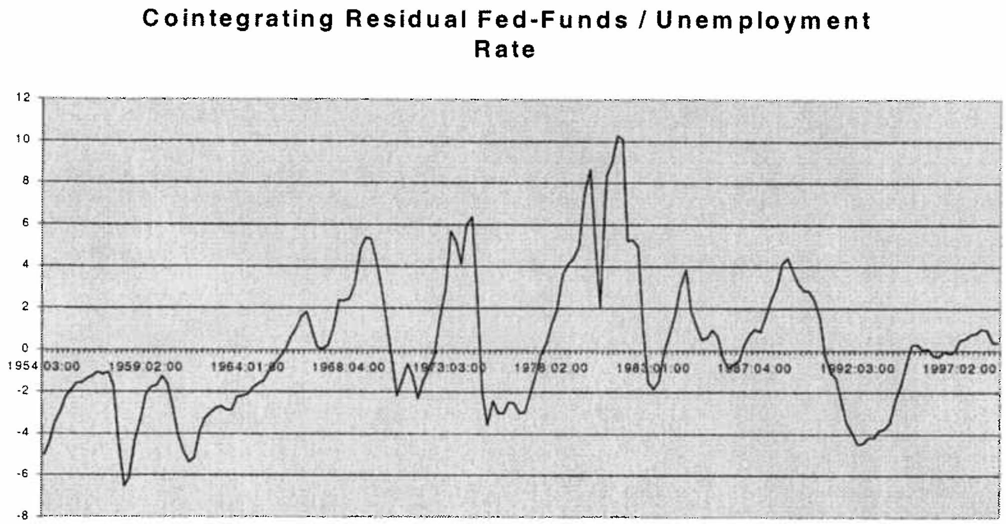
Augmented Dickey-Fuller Tests (t-statistics) : Cointegrating Residual Fed-Funds-Unemployment

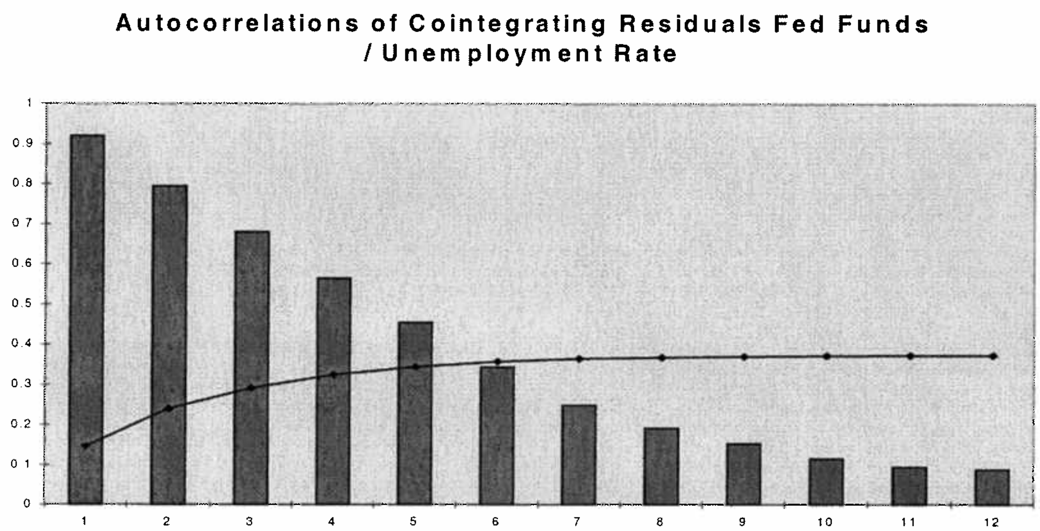
Lag		No Constant – No Trend (Model I)	Constant – No Trend (Model II)	Constant Trend (Model III)
Critical Values (Phillips-Ouliaris)	1%	-3.39	-3.96	-3.98
	5%	-2.76	-3.37	-3.42
	10%	-2.45	-3.07	-3.13
0		-2.83	-2.84	-2.79
1		-3.92	-3.92	-3.98

With the PP test, we can reject the null at 1% when one lag is included but not when any lags are included.

Phillips-Perron Unit Root Tests : Cointegrating Residual Fed-Funds-Unemployment

Lag		Constant – No Trend (Model II)	Constant – Trend (Model III)
Critical Values (Phillips-Ouliaris)	1%	-3.96	-3.98
	5%	-3.37	-3.42
	10%	-3.07	-3.13
0		-2.84	-2.79
1		-3.19	-3.19





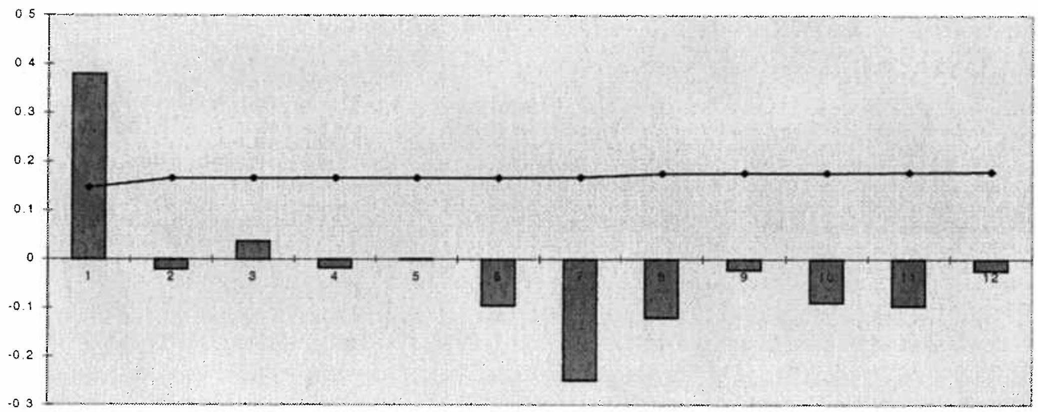
Finally, looking at a plot of the series we have the same pattern than for the previous cointegrating relation. This series is highly persistent but not necessarily non-stationary. The autocorrelations reveal some persistence but not necessarily non-stationarity. Finally, we conclude that the variables are cointegrated.

Under the hypothesis that the variables are cointegrated, we estimate by OLS the following cointegrating vector: [1 , 1.013].

b) Regression of the Federal Funds Rate on the Unemployment Rate

Looking at autocorrelations of the residuals of a DF test on the cointegrating residuals, ignoring significant negative autocorrelation, we choose a lag length of 1 for unit roots (cointegration) tests.

Autocorrelations of Residuals from a DF Test on the Cointegrating Residuals Unemployment Rate/Fed-Funds Rate



Augmented Dickey-Fuller Tests (t-statistics): Cointegrating Residual Unemployment Rate / Fed-Funds

Lag		No Constant – No Trend (Model I)	Constant – No Trend (Model II)	Constant Trend (Model III)
Critical Values (Phillips-Ouliaris)	1%	-3.39	-3.96	-3.98
	5%	-2.76	-3.37	-3.42
	10%	-2.45	-3.07	-3.13
0		-2.78	-2.90	-2.87
1		-3.83	-4.17	-4.21

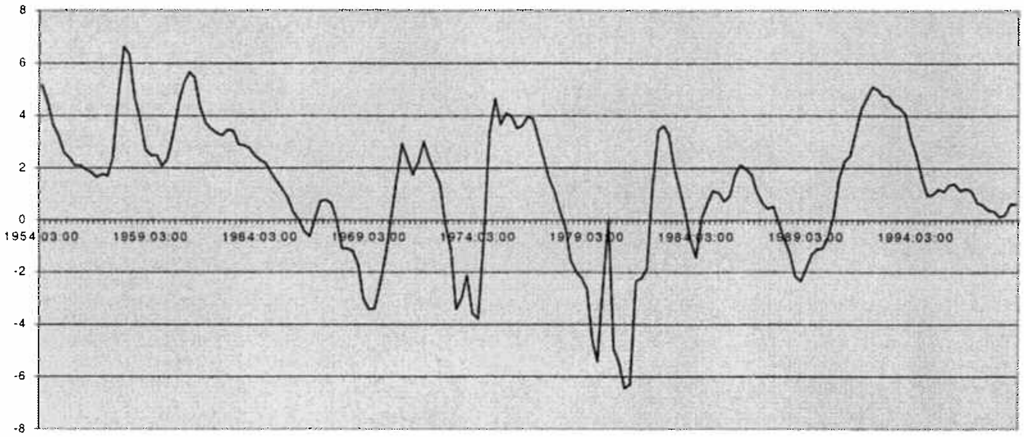
Since there is unambiguously some significant autocorrelation at lag one, we shall consider statistics adjusted for one lag of autocorrelation. Doing that, we can reject the null hypothesis of no cointegration under every model at 1%.

Phillips-Perron Unit Root Tests: Cointegrating Residual Unemployment Rate / Fed-Funds

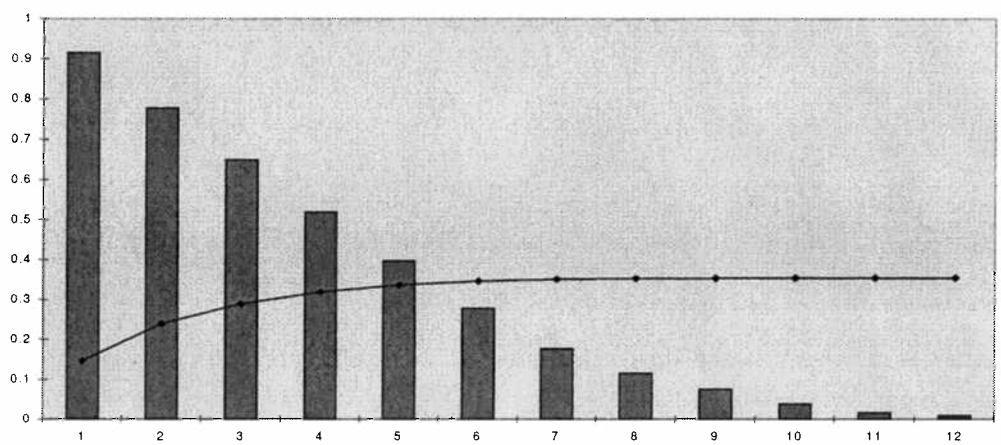
Lag		Constant – No Trend (Model II)	Constant – Trend (Model III)
Critical Values (Phillips-Ouliaris)	1%	-3.96	-3.98
	5%	-3.37	-3.42
	10%	-3.07	-3.13
0		-2.90	-2.87
1		-3.31	-3.30

For their part Phillips-Perron-like tests can reject the null at 10%. Finally, looking at a plot of cointegrating residuals as well as a plot of their autocorrelations, we find this series to be highly persistent, but not necessarily non-stationary. The conclusion regarding the rejection of the no-cointegration hypothesis is the same whichever the normalization.

Cointegrating Residual Unemployment Rate / Fed-Funds



Autocorrelations of Cointegrating Residuals Unemployment Rate / Fed-Funds



APPENDIX IV

CHOICE OF AN ORDER P, Q OF A GARCH(P,Q) FOR THE CONDITIONAL VARIANCE OF INFLATION

We report in the next figure the squared residuals (of the inflation process estimated with constant variance) autocorrelations. Following Box and Jenkins (1994)[17], since the autocorrelation function tails off and the partial correlation function cut off after three lags, we would identify an AR(3) process for the inflation process squared residuals, or an ARCH(3).

Figure 16
Inflation Process Squared Residuals Autocorrelations

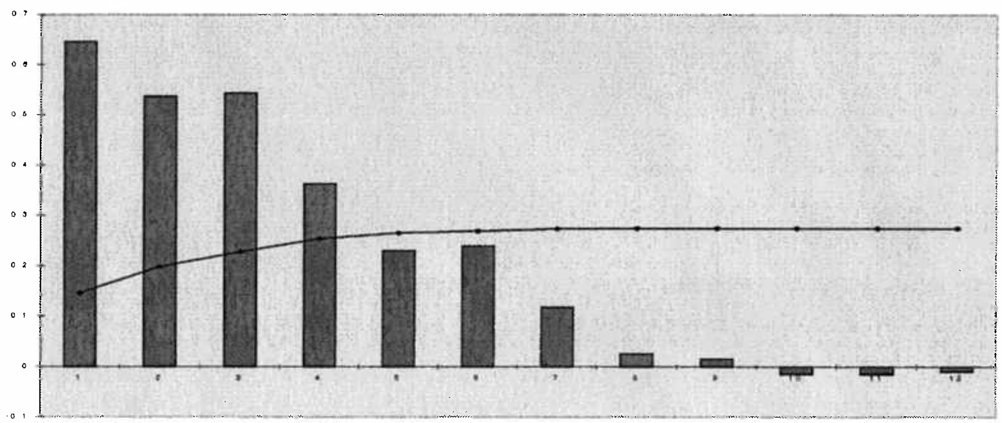
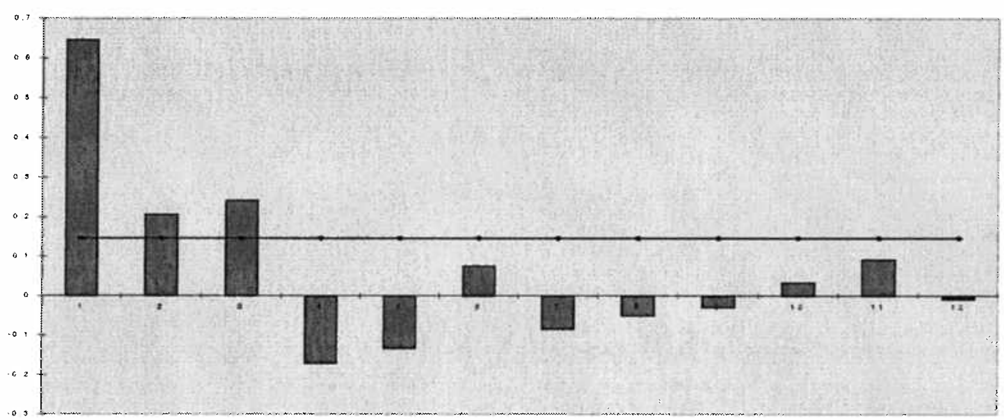


Figure 17
Inflation Process Squared Residuals Partial Correlations



While estimating various ARCH(p) specifications, we note that a fourth lag captures a lot of residual correlation while the coefficient on the second lag is never significant. We thus consider ARCH(1-3) and ARCH(1-3-4) specifications.

[17] Box and Jenkins (1994) p. 186.

We will nevertheless estimate conventional ARMA(p,q) (we impose the restriction p=q to limit the number of cases we have to investigate), AR(p) and MA(q), considering up to four periods lags (one year). To make comparison between specifications possible, we report values for the Akaike and Schwarz information criteria[18] as well as their respective sum of squared residuals. We finally conduct likelihood ratio tests of the joint significance of the coefficients, which amounts to a test for the presence of GARCH in the residuals of the inflation process.

ARMA Specification For The Inflation Process Squared Residuals Under Non-Stationarity

Specification	SSR	Akaike	Schwarz	Likelihood Ratio : p-value
No GARCH	68 741	-1573	-1573	NA
ARMA(1,1)	37 308	-1461	-1467	0.00
ARMA(2,2)	36 584	-1454	-1466	0.00
ARMA(3,3)	33 937	-1438	-1457	0.00
ARMA(4,4)	33 287	-1431	-1457	0.00
AR(1)	39 623	-1469	-1472	0.00
AR(2)	37 882	-1456	-1462	0.00
AR(3)	35 635	-1440	-1450	0.00
AR(4)	34 533	-1430	-1442	0.00
MA(1)	47 918	-1511	-1514	0.00
MA(2)	45 696	-1504	-1510	0.00
MA(3)	38 312	-1475	-1484	0.00
MA(4)	36 493	-1467	-1481	0.00
AR(1-3)	35 840	-1439	-1446	0.00
AR(1-3-4)	34 819	-1429	-1438	0.00

For all estimated specifications we reject the null hypothesis of no GARCH. The specification with a minimal value for the Akaike as well as the Schwarz criteria is an AR(1-3-4), that is an ARCH(1-3-4).

[18] Those are modified versions of the information criteria for use with GARCH specifications. See Bollerslev, Engle and Nelson (1994). To the contrary of the usual criteria, the best specification is the one that maximizes the value of the criteria. Their modified formulas are AIC = 2 X (log-likelihood) - 2 X (nb of regressors) ; SIC = 2 X (log-likelihood) - (nb of regressors) X ln (nb of observations).

APPENDIX V

ESTIMATED INFLATION PROCESS

The inflation process is given by

$$\pi_t = \pi^* - \frac{a}{2}\sigma_t^2 + \delta u_t + \upsilon_t$$

where δ is the already estimated cointegrating relation between the federal funds rate and unemployment. We first estimate δ by the following OLS regression

$$\pi_t = c + \delta u_t + e_t$$

and take its value as given thereafter. The value we obtain by OLS for δ is 0.25590. We then estimate the inflation process with GARCH errors letting c take a new value. As before, σ^2_t denote the conditional variance of inflation that we model as a GARCH (p,q) process

$$\sigma_t^2 = \omega + \beta(L)\sigma_t^2 + \alpha(L)\upsilon_t^2$$

1) With The Same Inflation Preferences for Each Chairman

ARCH(1-3-4)

The ARCH(1-3-4) specification is our previously chosen preferred one. The likelihood function is given by

$$\sigma_t^2 = \omega + |\alpha_1|\upsilon_{t-1}^2 + |\alpha_3|\upsilon_{t-3}^2 + |\alpha_4|\upsilon_{t-4}^2$$

$$\upsilon_t = \pi_t - \pi^* + \frac{a}{2}\sigma_t^2 - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln[\sigma_t^2] - \frac{\upsilon_t^2}{\sigma_t^2}\right)$$

where we assume that the disturbances are white noise. The estimation results are presented in the next table.

Estimation Results For The Inflation Process Parameters With ARCH(1-3-4)

Parameter	Estimate	Standard Error	T-Statistic	P-value
π^*	0.96	0.16	5.90	0.00
ω	1.41	0.26	5.38	0.00
α_1	0.41	0.10	3.96	0.00
α_3	0.31	0.12	2.57	0.01
α_4	0.22	0.10	2.16	0.03
a	-0.21	0.06	-3.72	0.00

GARCH(1.1)

We now estimate a model with GARCH(1,1) errors. The likelihood function is given by

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\upsilon_{t-1}^2$$

$$\upsilon_t = \pi_t - \pi^* + \frac{a}{2}\sigma_t^2 - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln\left[\sigma_t^2\right] - \frac{\upsilon_t^2}{\sigma_t^2} \right)$$

where we assume that the disturbances are white noises.

Estimation Results For The Inflation Process Parameters With GARCH(1,1)

Parameter	Estimate	Standard Error	T-Statistic	P-value
π^*	-0.19	0.16	-1.22	0.22
ω	0.47	0.11	4.21	0.00
α	0.22	0.03	7.93	0.00
β	0.70	0.03	27.78	0.00
a	-0.53	0.07	-7.66	0.00

Note that the GARCH(1,1) conditional variance can be non-stationary. The sum of his coefficients, $\alpha + \beta = 0.22 + 0.70 = 0.92 < 1$, is not for from 1 where it then follows a IGARCH(1,1). The t statistic of a test that $\alpha + \beta = 1$ is -1.93 and the p-value of the test is 0.05 (using the asymptotic normal distribution). Thus we can nearly reject the null hypothesis that the sum of the coefficients is one at 5% and can reject it at 10%.

2) *With Different Inflation Preferences for Each Chairman*

ARCH(1-3-4)

We now estimate the inflation process, allowing the inflation preferences of each chairman to be different, starting with an ARCH(1-3-4). The likelihood function is given by

$$\sigma_t^2 = \omega + |\alpha_1|\upsilon_{t-1}^2 + |\alpha_3|\upsilon_{t-3}^2 + |\alpha_4|\upsilon_{t-4}^2$$

$$\upsilon_t = \pi_t - \pi^* + \frac{a}{2}\sigma_t^2 + \frac{a_1}{2}\sigma_t^2 D_1 + \frac{a_2}{2}\sigma_t^2 D_2 + \frac{a_3}{2}\sigma_t^2 D_3 + \frac{a_4}{2}\sigma_t^2 D_4 - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln\left[\sigma_t^2\right] - \frac{\upsilon_t^2}{\sigma_t^2} \right)$$

where we assume that the disturbances are white noises. D_1 is a dummy variable that takes the value 1 during the term of chairman McChesney-Martin and 0 otherwise, and similarly is D_2 for chairman Burns, D_3 for chairman Miller and D_4 for chairman Volker. Thus, a is the parameter corresponding to the inflation preferences of chairman Greenspan and a_1 to a_4 catches the differences between the respective chairmen preferences and chairman Greenspan ones. In the next table we report estimates for the parameters and in the following one the inflation preferences for each

Inflation Process and ARCH(1-3-4) Conditional Variance of Inflation
Different Inflation Preferences for Each Chairman

Parameter	Estimate	Standard Error	T-Statistic	P-value
π^*	0.70	0.27	2.58	0.01
ω	1.19	0.49	2.43	0.01
α_1	0.41	0.10	4.10	0.00
α_3	0.43	0.15	2.81	0.00
α_4	0.15	0.06	2.38	0.02
a	-0.43	0.17	-2.62	0.01
a_1	0.34	0.15	2.34	0.02
a_2	0.28	0.17	1.60	0.11
a_3	0.05	0.14	0.35	0.73
a_4	0.02	0.12	0.13	0.90

chairman, as well as the p-value[19] of a test that they are not different from the previously estimated common parameter. We compute them by adding the difference parameter to the Greenspan parameter, for example :

$$a_{Mc} = a + a_1$$

[19] Using the asymptotic normal distribution.

where is the inflation preferences parameter of chairman McChesney-Martin and

$$\text{var}(a_{Mc}) = \text{var}(a + a_1) = \text{var}(a) + \text{var}(a_1) + 2\ \text{cov}(a\ ,\ a_1)$$

is his variance.

Inflation Parameters for Each Chairman with ARCH(1-3-4) Conditional Variance Estimated Along With the Inflation Process

Parameter	Estimate	Standard Error	T-Statistic	P-value	P-value a = -0.21
a_{Mc}	-0.09	0.11	-0.86	0.39	0.27
a_{Bu}	-0.16	0.07	-2.11	0.03	0.43
a_{Mi}	-0.38	0.09	-4.05	0.00	0.07
a_{Vo}	-0.42	0.11	-3.80	0.00	0.06
a_{Gr}	-0.43	0.17	-2.62	0.01	0.18

GARCH(1,1)

We now estimate the model allowing for idiosyncratic preferences with a GARCH(1,1) specification for the conditional variance. The likelihood function is given by

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\upsilon_{t-1}^2$$

$$\upsilon_t = \pi_t - \pi^* + \frac{a}{2}\sigma_t^2 + \frac{a_1}{2}\sigma_t^2 D_1 + \frac{a_2}{2}\sigma_t^2 D_2 + \frac{a_3}{2}\sigma_t^2 D_3 + \frac{a_4}{2}\sigma_t^2 D_4 - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln\left[\sigma_t^2\right] - \frac{\upsilon_t^2}{\sigma_t^2}\right)$$

where we assume that the disturbances are white noises. The results are presented in the next two tables.

Inflation Process with GARCH(1,1) Conditional Variance of Inflation Different Inflation Preferences for Each Chairman

Parameter	Estimate	Standard Error	T-Statistic	P-value
π^*	-0.57	0.47	-1.22	0.22
ω	0.56	0.18	3.16	0.00
α	0.20	0.07	2.84	0.00
β	0.69	0.06	11.04	0.00
a	-1.22	0.28	-4.40	0.00
a_1	0.61	0.13	4.80	0.00
a_2	-0.22	0.19	-1.12	0.26
a_3	-0.15	0.46	-0.31	0.75
a_4	0.59	0.15	3.79	0.00

Note that the GARCH(1,1) conditional variance is not necessarily stationary. The sum of his coefficients, $\alpha + \beta = 0.20 + 0.69 = 0.89 < 1$, is inferior to one, but not by much. The t statistic of a test that $\alpha + \beta = 1$ is –0.85 and the p-value of the test is 0.40 (using the asymptotic normal distribution) and thus we cannot reject this hypothesis at the standards confidence levels. Thus, the conditional variance of inflation could be an IGARCH(1,1), which implies that the unconditional variance of inflation is infinite and does not satisfy the definition of a covariance stationary process. However, Nelson (1990) shows that even if a IGARCH(1,1) is generally not covariance stationary, it is nevertheless strictly stationary, and it is thus still possible that υ_t be generated by a strictly stationary process.[20]

Inflation Parameters For Each Chairman
With GARCH(1,1) Conditional Variance Estimated Along With the Inflation Process

Parameter	Estimate	Standard Error	T-Statistic	P-value	P-value a = -0.53
a_{Mc}	-0.61	0.24	-2.56	0.01	0.76
a_{Bu}	-1.43	0.34	-4.26	0.00	0.01
a_{Mi}	-1.37	0.59	-2.32	0.02	0.16
a_{Vo}	-0.63	0.19	-3.26	0.00	0.31
a_{Gr}	-1.22	0.28	-4.40	0.00	0.01

3) General Test for Idiosyncratic Preferences

To test for the presence of idiosyncratic preferences, we will replace the parameters a_1 to a_4 with a parameter a_o and test is significance. This amounts to test the null hypothesis that other chairmen preferences aren't different from chairman Greenspan ones. The likelihood function is given by

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\upsilon_{t-1}^2$$

$$\upsilon_t = \pi_t - \pi^* + \frac{a}{2}\sigma_t^2 + a_o(D_1 + D_2 + D_3 + D_4)\left(\frac{\sigma_t^2}{2}\right) - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln[\sigma_t^2] - \frac{\upsilon_t^2}{\sigma_t^2}\right)$$

The results are presented in the next table.

[20] Campbell et al (1997) p. 484.

Inflation Process with GARCH(1,1) Conditional Variance of Inflation
Different Inflation Preferences Between: 1) Greenspan; 2) The others

Parameter	Estimate	Standard Error	T-Statistic	P-value
π^*	-0.53	0.22	-2.40	0.02
ω	0.41	0.14	2.90	0.00
α	0.20	0.02	11.02	0.00
β	0.72	0.04	19.48	0.00
a	-1.19	0.10	-12.35	0.00
$\mathbf{a_0}$	0.59	0.08	7.20	0.00

The parameter of interest, a_0 is significant at 1%. This leads us to reject the null hypothesis that inflation preferences are not idiosyncratic across chairmen.

APPENDIX VI

ESTIMATED REACTION FUNCTIONS

We follow in this section the same three-step estimation procedure than in the previous section.

The central banker's reaction function is given by,

$$r_t = \Im - \frac{a}{2}\sigma_t^2 + bu_t + \tau_t$$

We shall note that the variable σ^2_t is not the conditional variance of this equation but the conditional variance series of the corresponding estimated inflation process. δ is the coefficient on the cointegrating relation that we first estimate by OLS

$$r_t = q + \delta u_t + e_t$$

We enforce the value of δ in the subsequent estimation by maximum likelihood but re-estimate the constant. We estimate by OLS a numerical value of 0.64152 for δ.

1) With The Same Inflation Preferences for Each Chairman

ARCH(1-3-4)

The system that we want to estimate by maximum likelihood is given by

$$\tau_t = r_t - \Im + \frac{a}{2}\sigma_t^2 - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln\left[\sigma^2\right] - \frac{\upsilon_t^2}{\sigma^2} \right)$$

where we assume that the disturbances are white noises and where the conditional variance of inflation series (σ^2_t) is the one estimated in the previous section with an ARCH(1-3-4) specification.

Reaction Function and ARCH(1-3-4) Conditional Variance of Inflation

Parameter	Estimate	Standard Error	T-Statistic	P-value
$\Im$	0.57	0.19	2.94	0.00
σ^2	6.63	0.57	11.65	0.00
a	-0.34	0.02	-15.94	0.00

GARCH(1,1)

We now estimate a GARCH(1,1). The log-likelihood is given by

$$\tau_t = r_t - \Im + \frac{a}{2}\sigma_t^2 - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln\left[\sigma^2\right] - \frac{\upsilon_t^2}{\sigma^2} \right)$$

where we assume that the disturbances are white noises.

Reaction Function and GARCH(1,1) Conditional Variance of Inflation

Parameter	Estimate	Standard Error	T-Statistic	P-value
$\Im$	-0.59	0.19	-3.12	0.00
σ^2	6.03	0.66	9.20	0.00
a	-0.69	0.03	23.58	0.00

2) *With Different Inflation Preferences for Each Chairman*

We now allow, as before for the inflation process, the coefficient on inflation preferences to be different for each chairman.

ARCH(1-3-4)

We start by our ARCH(1-3-4) preferred specification. The conditional variance of inflation series (σ^2_t) is the one estimated in the previous section with an ARCH(1-3-4) specification. We report the results in the next two tables.

The system that we want to estimate by maximum likelihood is given by

$$\tau_t = r_t - \Im + \frac{a}{2}\sigma_t^2 + \frac{a_1}{2}\sigma_t^2 D_1 + \frac{a_2}{2}\sigma_t^2 D_2 + \frac{a_3}{2}\sigma_t^2 D_3 + \frac{a_4}{2}\sigma_t^2 D_4 - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln\left[\sigma^2\right] - \frac{\upsilon_t^2}{\sigma^2} \right)$$

where we assume that the disturbances are white noises.

Reaction Function With ARCH(1-3-4) Conditional Variance of Inflation
Different Inflation Preferences for Each Chairman

Parameter	Estimate	Standard Error	T-Statistic	P-value
$\Im$	0.47	0.25	1.85	0.06
σ^2	5.91	0.60	9.86	0.00
a	-0.63	0.03	-24.13	0.00
a_1	0.37	0.11	3.38	0.00
a_2	0.45	0.04	12.25	0.00
a_3	0.25	0.09	2.90	0.00
a_4	0.03	0.04	0.78	0.43

Inflation Parameters For Each Chairman
With ARCH(1-3-4) Conditional Variance Estimated in the Reaction Function

Parameter	Estimate	Standard Error	T-Statistic	P-value	P-value a = -0.34
a_{Mc}	-0.26	0.12	-2.06	0.04	0.47
a_{Bu}	-0.18	0.04	-4.48	0.00	0.00
a_{Mi}	-0.38	0.09	-4.38	0.00	0.67
a_{Vo}	-0.60	0.05	-11.72	0.00	0.00
a_{Gr}	-0.63	0.03	-24.13	0.00	0.00

GARCH(1,1)

Now, the conditional variance of inflation series (σ^2_t) is the one estimated in the previous section with a GARCH(1,1) specification.

$$\tau_t = r_t - \Im + \frac{a}{2}\sigma_t^2 + \frac{a_1}{2}\sigma_t^2 D_1 + \frac{a_2}{2}\sigma_t^2 D_2 + \frac{a_3}{2}\sigma_t^2 D_3 + \frac{a_4}{2}\sigma_t^2 D_4 - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln\left[\sigma^2\right] - \frac{\upsilon_t^2}{\sigma^2} \right)$$

where we assume that the disturbances are white noises. We present the results in the next tables.

Reaction Function with GARCH(1,1) Conditional Variance of Inflation
Different Inflation Preferences for Each Chairman

Parameter	Estimate	Standard Error	T-Statistic	P-value
$\Im$	-1.00	0.16	-6.13	0.00
σ^2	5.49	0.63	8.73	0.00
a	-1.52	0.03	-45.86	0.00
a_1	0.81	0.12	7.06	0.00
a_2	0.51	0.13	3.95	0.00
a_3	0.36	0.30	1.18	0.24
a_4	0.56	0.04	15.12	0.00

Inflation Parameters For Each Chairman
With GARCH(1,1) Conditional Variance Estimated in the Reaction Function

Parameter	Estimate	Standard Error	T-Statistic	P-value	P-value a = -0.69
a_{Mc}	-0.70	0.12	-5.86	0.00	0.91
a_{Bu}	-1.01	0.13	-7.59	0.00	0.02
a_{Mi}	-1.16	0.30	-3.82	0.00	0.12
a_{Vo}	-0.96	0.05	-19.63	0.00	0.00
a_{Gr}	-1.51	0.03	-45.86	0.00	0.00

3) General Test for Idiosyncratic Preferences

We again test for idiosyncratic preferences. Using the same method as with the inflation process, the likelihood function is given by:

$$\tau_t = r_t - \Im + \frac{a}{2}\sigma_t^2 + a_o\left(D_1 + D_2 + D_3 + D_4\right)\left(\frac{\sigma_t^2}{2}\right) - \delta u_t$$

$$Log - Likelihood = \sum_{t=1}^{T} -\frac{1}{2}\left(\ln\left[\sigma^2\right] - \frac{\upsilon_t^2}{\sigma^2}\right)$$

where we assume that the disturbances are white noises. The results are presented in the next table.

Reaction Function with GARCH(1,1) Conditional Variance of Inflation
Different Inflation Preferences between 1) Greenspan; 2) others

Parameter	Estimate	Standard Error	T-Statistic	P-value
$\Im$	-1.29	0.21	-6.16	0.00
σ^2	5.62	0.61	9.28	0.00
a	-1.67	0.03	-48.47	0.00
a_o	0.66	0.04	18.61	0.00

Again, a_o the parameter of interest is significant at 1%. We reject the null hypothesis that inflation preferences aren't idiosyncratic across chairmen.

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