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David Hume on Probability and the Gambler's Fallacy

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David Hume on Probability and the Gambler's Fallacy

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RÉSUMÉ

Cette présentation examinera le degré de certitude qui peut être atteint dans le domaine scientifique. Le paradigme scientifique est composé de deux extrêmes; causalité et déterminisme d'un côté et probabilité et indéterminisme de l'autre. En faisant appel aux notions de Hume de la ressemblance et la contiguïté, on peut rejeter la causalité ou le hasard objectif comme étant sans fondement et non empirique. Le problème de l'induction et le sophisme du parieur proviennent d'une même source cognitif / heuristique. Hume décrit ces tendances mentales dans ses essais « Of Probability » et « Of the Idea of Necessary Connexion ». Une discussion sur la conception de la probabilité de Hume ainsi que d'autres interprétations de probabilité sera nécessaire. Même si la science glorifie et idéalise la causalité, la probabilité peut être comprise comme étant tout aussi cohérente. Une attitude probabiliste, même si elle est également non empirique, pourrait être plus avantageuse que le vieux paradigme de la causalité.

Mots clés: philosophie, probabilité, David Hume, Pierre Simon de Laplace, incertitude, le sophisme du parieur, induction, hasard, causalité, science.

ABSTRACT

This presentation examines the degree of certainty which can be attained in science. The scientific paradigm is composed of two extremes; causality and determinism on one end and probability and indeterminism on the other. By appealing to Hume's notions of resemblance and contiguity, we can dismiss any claim of objective causality or chance as being ungrounded for lack of an empirical basis. The problem of induction as well as the gambler's fallacy stem from the same cognitive/heuristic source. Hume describes these mental tendencies in his essays 'Of Probability' and 'Of the Idea of Necessary Connexion'. This will necessitate a discussion of Hume's notion of probability, as well as other interpretations of probability. While science has glorified and romanticized causality, probability can be understood as being just as consistent. While a probabilistic stance is as non-empirical as a causal stance, it will be remarked that we may benefit from a paradigmatic switch to probabilism.

Key words: Philosophy, Probability, David Hume, Pierre Simon de Laplace, Uncertainty, Gambler's Fallacy, Induction, Chance, Causality, Science.

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Excerpt from the movie *CASINO* (1995)

ACE and GREEN are seated having coffee and muffins in the Hawaiian lounge by the casino floor.

ACE: Phil, I can understand. You're in the finances, you're upstairs, but you are not on the floor. I got thousands of players. I got five hundred dealers. They're all lookin' to rob me blind, twenty-four hours a day. I have to let them know I'm watching all the details, all the time; that there is not one single thing I will not catch as I am over here.
(Breaks open his blueberry muffin, puts it down and points to Green's.) Look at yours.
GREEN: What?
ACE: Look at that. Look at this. There's nothin' . . . look how many blueberries your muffin has and how many mine has. Yours is falling apart. I have nothing.
GREEN: What are you talking about?
ACE: It's like everything else in this place. You don't do it yourself, it never gets done.
(GREEN follows ACE to the kitchen.)
GREEN: Where you goin'?

ACE, GREEN and the BAKER are gathered around the BAKER's counter surrounded by muffin tins and batter.

ACE: (Handing the BAKER the two muffins) From now on I want you to put an equal amount of blueberries in each muffin. An equal amount of blueberries in each muffin.
BAKER: You know how long that's going to take?
ACE: I don't care how long it takes. Put an equal amount in each muffin.
(ACE leaves, GREEN looks on in amazement as the BAKER holds the muffins. GREEN follows ACE out.)

1 - Introduction

This unanswerable question has fascinated me for some time – is probability objective or subjective? Many great philosophical debates hinge on the answer to this question. Is our universe determinate or indeterminate? Is causality real or a heuristic device found in the mind? Even the question of free-will hinges on the notion of probability. The ontological question of probability is a metaphysical question; any attempt to answer it would be purely conjectural. This question, it appears, is unanswerable because probability, insofar as it is a tool that enables us to quantize uncertainty, is itself subject to the very uncertainty it aims to measure. It will be concluded that uncertainty must be regarded as a permanent feature of the epistemic landscape by virtue of our inability to establish an objective foundation for either causality or probability.

This question, although unanswerable, is extremely insightful. To extract this insight, we can convert this fantastic metaphysical question into an epistemological question. The question then becomes; what degree of certainty can scientific knowledge attain? As an ontological question it is speculative but as an epistemological question, it poses a direct challenge. The former sought to answer what we do not know whereas the latter seeks to question what we think we do know. As we confront this challenge we will need to examine further questions such as; 'what is meaning of 'certainty' in science?', 'what is the relationship between probability and causality?', and 'how are we to interpret probability and understand its place in science?' Our attention consequently moves away from cosmological debates concerning determinism and freewill and turns to practical questions such as these - is it possible to know with precision the electron configuration in a single atom, or the blueberry distribution in a muffin batter?

2 – Causality, Probability, Certainty and Knowledge

On either end of the scientific spectrum, we find causality and probability. These notions of causality and probability can be viewed as antagonistic terms. This is true in a sense; an event is said to be caused insofar as it has necessarily occurred by way of some effect. Exactly the opposite is true for an event which is deemed probabilistic or stochastic; an event occurring by 'chance', without prior determinants. This polarity is especially apparent in the domain of physics where classical mechanics is found on one side of the coin and quantum mechanics on the flipside. Deterministic laws best describe the motions of, and causal interactions between, macroscopic bodies whereas the realm of microscopic phenomena is best understood in probabilistic terms. In this sense, causality and probability both have a rightful yet mutual exclusive place in science. In another sense, causality and probability are complementary concepts, both of which help us map out our knowledge. In this latter sense they work together as brick and mortar in building up and solidifying our epistemic edifice. In either sense, both causality and probability carry with them their respective limits. Beyond these limits, there is only uncertainty. The bulk of this paper will be consecrated to the exploration of these limits.

Epistemologically speaking, one does not start with certainty in hand (unless one purports knowledge through revelation). Certainty is something one must attain. Thus, at the point of departure in his epistemic journey, the scientist is completely lost, confused and perplexed. He has to slowly sift through the dust, piecing together the bits as he painstakingly tries to decipher who he is, where he is and what's up. At the start of the epistemic journey, the scientist is the individual with amnesia - he knows nothing. This tragic flaw may turn out to play to his advantage. He builds up and refines what little he can know and gradually makes his way from

the known into ever further recesses of the unknown. The scientific paradigm, to paint a simple portrait, expands on what is known and extends into that which is not yet known.

Certainty and uncertainty are features of knowledge. Uncertainty is not a property of what is 'unknown' but rather of that which is 'known'. Probability is a tool that enables us to quantitatively gauge our uncertainty and to extract useful information from what would otherwise be ostensible randomness and chaos.

The body of scientific knowledge is analogous to an island amidst a vast ocean representing what is unknown. The highest and most central point in this island should naturally represent what we know with the highest degree of certainty. All talk of causality, being akin to virtual certainty, can be seen as the central and highest point of the island. We should expect physics to occupy this privileged position. This pinnacle would be the ideal point from which to view and map the entire island. As we approach the coastline of this island, the subject matter is intrinsically of a lesser degree of certainty. Upon our descent we may traverse statistical mechanics, followed perhaps by chemistry, biology, and then such fields of inquiry as the statistical or human sciences. Lastly we reach the shoreline itself, the furthest point from the pinnacle, where probability acts as a veritable buffer between what is unknown and what is uncertain. If probability cannot get us any closer to certainty, at the very least, it is apt to deal with, and help us understand, uncertainty. If for no other reason, this alone is sufficient to give it an advantage over the causal paradigm, which has dominated the landscape of the modern scientific paradigm.

Physics is adorned precisely because causal knowledge represents the highest form knowledge. In this schema, all fields of scientific enquiry are subordinate to the laws decreed by

physics. Causality is at the center, it is surrounded by probability which acts as a buffer of uncertainty. On this island of scientific knowledge, probability is understood as a purely epistemic concept. Causality and probability (as epistemic/subjective) go hand in hand. Where a causal nexus denotes the position or 'known' connections in the occurrence of a given phenomenon, epistemic probability denotes the negative or 'secret and hidden' connections in the occurrence of that same phenomenon. Epistemic probability is a gauge of our expectation of the occurrence of a given phenomena based on our ignorance of the true causes.

A scientist who does not understand the role of probability in this epistemic landscape, or views probability as something that ought to be overcome, would like to fancy himself detached, on an island with no shoreline whatsoever. This island floats above the great sea of the unknown as though 'known' and 'unknown' were perfectly distinct from one another without admitting or requiring a buffer between the two. With an outright denial of uncertainty he proceeds to build his castle in the sky. The misunderstanding arises from an unwarranted hasty jump from an epistemic project (empirical observation) to grand ontological claims concerning the underlying state of affairs (the primacy of causation). This unwarranted leap from epistemic inquiry to broader metaphysical claims will be the crux of David Hume's argument against causality.

A scientist who is utterly convinced that everything can be understood, and ultimately explained, via causal connections may view uncertainty as a sort of undesirable static which impedes a pristine perception of the world. Uncertainty, viewed in this light, is something that ought to gradually be eliminated altogether. Like a lingering stain which is observed anew after each consecutive wash, it remains after each attempt by the scientist to reduce its effects on his observations and measurements. This residual property of uncertainty makes it ever more detestable. As for the scientist who does not rigidly adhere to the causal paradigm, uncertainty

may be viewed as a permanent feature of the epistemic landscape. In accepting it as such, this scientist does not look upon it harshly nor will he view himself as any less of a scientist for it. In accepting uncertainty as such, is he forsaking the full potential of the scientific project?

There may be another way of envisaging this body of scientific knowledge altogether, in which probability plays a more pivotal role. One can imagine an island where uncertainty not only acts as a buffer between 'certain' knowledge and the unknown, but where uncertainty forms the very foundation or bedrock of the island itself. Here probability is not merely an epistemic feature but a metaphysical feature of the underlying state of affairs, thus taking on the same metaphysical preeminence of causality in the previous schema. In this schema, all fields of scientific enquiry, not excluding classical physics, are subject to a fundamental and irreducible degree of uncertainty. Regardless of the role that probability is made to play, whether simply epistemic or as a more integral part of the state of affairs, the probabilistic paradigm is as congruent as the causal paradigm in both its description of the world and in its predictive power.

The difficult arises from our ignorance concerning that which is taking place beyond the event horizon of our experience. What does the underlying state of affairs resemble beyond the limits of empiricism (the limits of science)? We do not know what lays behind-the-scenes in this buffer zone named 'uncertainty'. One can ask the rather bizarre question - what is 'uncertainty' made of? Compelled by curiosity, we want to know, and our intellectual appetite is not easily satisfied with the posture that we fundamentally cannot know. We feel uncomfortable with this because we intuit that there must be something, and not merely nothing, taking place beyond the horizon of our experience (lest we should revert back to Bishop Berkeley's *subjective idealism*). Hence, the belief is that if we probe deep enough, we will unearth causes which were previously hidden. We will crack uncertainty open, so to speak, to see what is inside. This is, of course, the

leading view which has dominated the epistemic landscape of science for some time. Alternately, one can proclaim that there is not causality but rather that probability is the metaphysical grounds of ultimate physical reality. Furthermore, the mind is tempted to assert that one of these two alternately must be the state of affairs. The problem is that we have, upon close examination, no empirical basis for either stance. Would the best course of action not be to dismiss both as metaphysical nonsense? Admitting this would be equivalent to stating that one cannot get at the true state of affairs regardless how deep one digs. If we admit this, we are admitting that neither causality nor probability have an objective reality. Uncertainty grows out of the failure of these physical paradigms (the former causal paradigm as well as the newly emerging probabilistic paradigm) to depict the underlying state of affairs in such a manner as to eliminate any room for reasonable doubt. Uncertain is not merely a horizon in our experience but a veritable epistemic 'event-horizon'. Uncertainty does have an objective reality, but one akin to a black hole insofar as it manifests its reality in a purely negative manner. Furthermore, and perhaps surprisingly, this holds even without prescribing a metaphysical primacy to probability.

Ultimately, the task at hand consists in answering this question; can we conceive of an objective uncertainty without any additional metaphysical quality. This is a two-fold task. The first task will be to undermine the notion of causality by appealing to probability. The second task will then be to undermine probability by demonstrate that regardless of the interpretation of probability, that uncertainty is an objective feature of knowledge. I intend to demonstrate that the idea of an attainable absolute certainty, once fantasized by science, is a phantasm. Uncertainty alone is real, it is inevitable and irreducible – it is something that cannot be avoided. We will explore the cognitive mechanisms and fallacies, namely the gambler's fallacy and the problem of induction, which have led people to perceive certainty where there may only be uncertainty.

3 – Rethinking Causality: Hume, *Of Probability*, and the Gambler's Fallacy

It is the business of philosophy to keep alive those unanswerable 'ultimate questions'[1] while keeping the debates that envelope them sound. The first task, then, of philosophy consists of keeping alive our sense of what is possible; not in the sense of fiction or fantasy but in the sense which stems from reason (reasonable doubt) and critical thought. To do this, the philosopher is required to distinguish what we know (with certainty) and what we still do not know from what we cannot know (i.e., classical metaphysics). There have, however, been instances in the history of science where we thought we knew, but on further examination (or re-examination of many centuries of unilateral thought), our knowledge was not as grounded as we had triumphantly presented it to be. Such circumstances of pretentious wisdom put us at a greater disadvantage then if we neither knew nor thought that we knew (remembering Socrates in the *Apology*). It is for this reason that philosophy performs the second critical task of keeping the edifice of human knowledge (i.e., the sciences) in check. This will be my primary focus.

David Hume may be regarded as one such philosopher who excels at both tasks; sparking our sense of wonder and possibility whilst shaking us out of our 'dogmatic slumber'. Essays such as 'Of Probability' and 'Of the Idea of Necessary Connexion' from *An Enquiry Concerning Human Understanding* are a testament to this. His short essay entitled 'Of Probability' will form the base for this presentation. In this essay, Hume will focus on the tendency in the mind to disproportionately favor, and often to take as law-like and necessary, events with a heightened probability of occurrence. This immediately precedes and reinforces Hume's subsequent attacks on induction and causality in 'Of the idea of Necessary Connexion'. Although this famous critique of inductive reasoning and causality follow from his analysis of probability, the latter

[1] See Bertrand Russell '*The Problems of Philosophy*' p.1

has received far less attention[2]. We will explore Hume's idea on probability as well as other philosophical interpretations of probability.

The idea of a world being causally determined is romanticized by science. Certainty, in this romanticized clock-work universe, comes to mean precisely this; foreknowledge of all causes leading to an effect whereby the non-occurrence of that effect is inconceivable. The scientific utopia consists of absolute certainty in which all phenomena can be foreseen and explained via an extremely intricate, yet knowable, causal nexus. It is for this reason that I will briefly suggest that we may benefit from a break from our current Newtonian paradigm to adopt a probabilistic world view. This paradigmatic swap may benefit people such as Sam 'Ace' Rothstein from the film *'Casino'*, who rampantly desire certainty where there may only be uncertainty, in understanding why his blueberry muffin is devoid of blueberries.

3.1 - The Gambler's Fallacy: The fallacies by which we perceive an order out of randomness

It is important to note that Hume does not believe in chance or causality. In the opening passage in 'Of Probability' he says, 'Though there be no such thing as *Chance* in the world; our ignorance of the real causes of any event has the same influence on the understanding, and begets a like species of belief or opinion'[3]. From this passage, one might assume that Hume is advocating determinism in a similar manner to Pierre-Simon Laplace, French astronomer and mathematician. In view of determinism, probability is interpreted as an epistemic notion pertaining to our 'ignorance of the real causes'. In the subsequent essay 'Of the Idea of Necessary Connexion', Hume argues against causality as well. He advocates neither determinism nor indeterminism as he constantly exhibiting a skeptical demeanor.

[2] *Hume on Probability.* Gower (1991), p.2,3 as well as *Hume's Species of Probability.* Hacking (1978) p.22
[3] *An Enquiry Concerning Human Understanding*, p.37

Regardless of this skeptical position, Hume presents ideas in his short essay 'Of Probability' that are simple yet highly informative in regards to how our minds formulate expectations about the outcome of future events on the basis of very restricted past experience. These mechanisms describe the mind a gambler in committing the 'gambler's fallacy'. Such mechanisms do not apply exclusively to the gambler or to games of chance; they may express the mental tendency of anyone seeking to extrapolate patterns from past instances with the wish to make future instances more predictable. In its most extreme form, this fallacy generates ideas of law and necessity – this is precisely the case with the idea of causality or 'necessary connexion'.

To understand was is meant by the Gambler's fallacy, we will use Pierre-Simon Laplace's example of the betting tendencies he observed in the French lottery. He observes,

> Lorsqu'à la loterie de France un numéro n'est pas sorti depuis longtemps, la foule s'empresse de le couvrir de mises. Elle juge que le numéro resté longtemps sans sortir doit, au prochain tirage, sortir de préférence aux autres… Par une illusion contraire aux précédentes, on cherche dans les tirages passés de la loterie de France les numéros le plus souvent sortis, pour en former des combinaisons sur lesquelles on croit placer sa mise avec avantage. Mais vu la manière dont le mélange des numéros se fait à cette loterie, le passé ne doit avoir sur l'avenir aucune influence.[4]

We can define the Gambler's fallacy as the mistaken belief that prior observed instances have a bearing on the outcome of subsequent instances within a system whose outcomes are mutually independent. Laplace illustrates such a system with the use of the national lottery in which, when the manner of 'the mixing of the numbers is considered, the past ought to have no influence upon the future'.

There are two ways in which this 'illusion' takes form. Firstly, it manifests in the belief that a given event is 'due' or 'bound' to occur on the grounds that it has not occurred in recent

[4] *Essai philosophique sur les probabilités*, p. 199, 202

instances. This is also known as the ‘doctrine of the maturity of the chances’ or the ‘Monte-Carlo fallacy’ due to the famous streak of 26 consecutive black numbers at a Monte Carlo roulette table in 1913 which engendered a betting frenzy on red well before the black streak came to an end. Secondly, this fallacy manifests in the belief that an event is more likely to occur on the grounds that it is on a ‘hot streak’ given that it has already occurred, with an inspiring frequency, in recently observed instances. An unwarranted inference results from this skewed line of reasoning.

It is precisely in this second sense that Hume is inviting us to envision the problem of induction – especially pertaining to causality. According to Hume, there exists a strong tendency in the mind to favor highly probable outcomes and, if the probability of their occurrence is high enough, to take their occurrence as ‘necessary’. Simply stated, when the mind has observed two events in constant conjunction, it then tends to form an idea of a ‘necessary connexion’ between the two events. As a result of this tendency, an event with a low order of occurrence is viewed as a discrepancy or an anomaly, as opposed to being accounted for in probabilistic terms. What arises from this is a scientific paradigm founded on determinism and law-like necessity which then rationalizes away, with ever greater vigor, phenomena of ever lower probability.

Why is the gambler’s fallacy significant to scientific justification? It possesses the danger of an unwarranted expectation that the world will present itself in a certain way; as one expects to find it based upon one’s own image of it and not the other way around as empiricism properly entails. When the world does not match this unwarranted expectation, the gambler (or scientist) seeks to explain this non-occurrence by alluding to something having gone wrong. He ought, instead, rightly justify his expectation in the first place – ‘what leads me to believe that it would occur?’ versus ‘why did it not occur?’ The former fits the hypothetical-deductive method whereas the latter does not. Thus, I am not suggesting that one should never seek-out ‘hide and

secret causes', for to do so would be to suggest that chance is real. I am making no such ontological claim. Rather, I am suggesting is that accounting for events which do not readily adhere to a causal nexus by alluding to occult causes carries with it strong metaphysical undertones. Such an insistence is to apply a causal paradigm where it need not necessarily apply.

To illustrate this tendency to disproportionately favor, and often to take as law-like and necessary, an event with at heightened probability of occurrence, Hume asks us to imagine a six-sided die 'marked with one figure or spot on four sides, and with another figure or number of spots on the two remaining sides'. In our minds, we would naturally take these sides as equal and would ascribe that they are 'equiprobable'. Given that there are four sides marked with a figure and only two sides marked otherwise, our minds would be led to expect the former to turn up more often than the latter[5]. Then suppose we have a die with one-thousand sides where 999 sides are similar and only one side is different. How should we expect our minds to formulate an expectancy of the 'more favorable' side to turn up? How would the mind perceive the likelihood of the singular side turning up? The probability assignment to a six-sided die seems more readily apparent to the mind than the probability assessment of a thousand-side die. Herein lays Hume's unique contribution to our understanding, and use of, probability. Hume states clearly,

> I would willingly establish it as a general maxim in the science of human nature, that when any impression becomes present to us, it not only transports the mind to such ideas as are related to it, but likewise communicates to them a share of its force and vivacity.[6]

Hume describes the psychological connection between the observed facts in the world and the state of mind which arises in consequences to these observations. In this case, the connection is seen between chance phenomena and the subjective beliefs formed about their

[5] *Enquiry*, p. 37
[6] *A Treatise of Human Nature*, p. 69

expected frequency of occurrence. In the case of causality, this connection would be seen between an aggregate of 'similar' instances and the subjective tendency of the mind to move from their 'constant conjunction' towards the formation of an idea of cause and effect.

Hume's treatment of probability of 'chances' will gradually take us to his treatment of probability of 'causes' and finally to his treatment of 'necessary connexion'. His treatment of causality can be understood as an extreme case of his treatment of probability. His attack on causality stems directly out of his ideas on probability. For Hume to attack the idea 'necessary connexion', being based on constant conjunction, he requires a theory of less-than-constant conjunction[7]. This is implicit in '*Of the Idea of Necessary Connexion*'. Hacking suggests that Hume did not explicitly spell it out because, as a member of the times, he was not ready to conceptually switch to a world of ontological probabilities[8]. The idea of the necessity of a cause is taken for granted, 'without any proof given or demanded' and is so deeply instilled, in Hume's time as in ours, that it is 'impossible for men in their hearts really to doubt of'[9].

Returning to the six-sided die that Hume has handed us, why should we grant that each side is 'equiprobable'? Before we have accumulated observed experiences to rely upon, we naturally assume a sort of *a priori* probability wherein we take all outcomes as equally likely wherever we have to reason to think otherwise; of one as being more likely than any other. Hume says "a perfect and total indifference is essential to chance, and one total indifference can never in itself be either superior or inferior to another'[10]. This is, in and of itself, in no way sufficient for the evaluation of probability because it is not based on anything except for our initial idea based on equiprobable outcomes. Hume tells us,

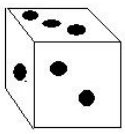
[7] Hacking, 1978, p. 24
[8] *Ibid*
[9] *Treatise*, p.56
[10] *Ibid*, p.125

> When the mind looks forward to discover the event, which may result from the throw of such a die, it considers the turning up of each particular side as alike probable; and this is the very nature of chance, to render all particular events, comprehended in it, entirely equal.[11]

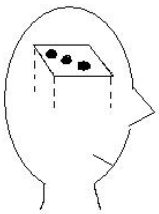
We then need to supplement our knowledge in order to arrive at a more reliable probability evaluation. The most readily available way is to base our knowledge on our experience. This is where observed facts of the world would begin to influence our subjective degrees of belief. Hume demonstrates that experience influences the mind in a very peculiar way.

> It will here be worth our observation, that the past experience, on which all our judgments concerning cause and effect depend, may operate on our mind in such an insensible manner as never to be taken notice of, and may even in some measure be unknown to us.[12]

This interplay between observed facts and degrees of belief will be Hume's unique contribution. The tendency of this mind to amalgamate our perceived experiences, itself goes unperceived in the process and its effect goes unnoticed. This interplay between observed facts and degrees of belief will prove fatal for the notion of causality as well as for any hope of ever ascertaining a truly accurate probability valuation for any event whatsoever.



'A Priori' Probability
Side equal sides
Probability of any side is 1/6



Impression on the imagionation
Heightened propensity to favor the side with three dots.

We assume, upon overlooking our six-side die, that any of its particular sides turning up after a toss is 'alike probable', based on our indifference (principle of indifference). We proceed

[11] *Enquiry,* p.38
[12] *Treatise*, p.72

to toss the die, as part of a game or in hope of uncovering a potential bias in the die itself. The outcome of each toss leaves an impression on our mind.

After a series of tosses of this six-sided die, an on-looking spectator would be content to grant that the turning up of any side is as likely as any other. He would also grant that the next toss will be decided by chance (admitting no law or pattern which would ameliorate his predictive power) unless he is highly prone to commit the gambler's fallacy.

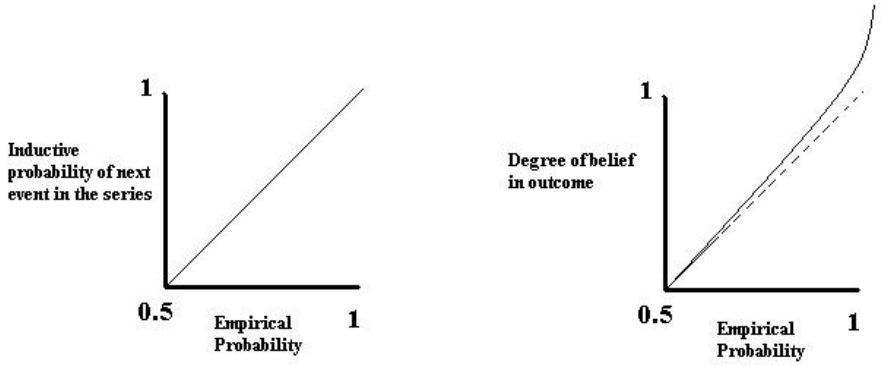
Upon returning to the question concerning the thousand-sided die, will the mind so readily cease the probability of the one lonesome side turning up versus any of the other nine-hundred ninety-nine imposing and opposing sides? Hume describes our psychological propensity to give disproportionate 'preference to that which has been found most usual' in the case of a probabilistic event. Hume describes this process as follows:

> Finding a greater number of sides concur in the one event than in the other, the mind is carried more frequently to that event, and meeting it oftener…begets immediately, by an inexplicable contrivance of nature, the sentiment of belief, and gives that event the advantage over its antagonist, which is supported by a smaller number of views, and recurs less frequently to the mind…The concurrence of these several views or glimpses imprints the idea more strongly on the imagination; gives it superior force and vigor; renders its influences on the passions and affections more sensible; and in a word, begets that reliance and security, which constitutes the nature of belief and opinion.[13]

In Section VI 'Of Probability' from *An Enquiry Concerning Human Understanding*, Hume says "Probability, which arises from a superiority of chances on any side; and according as this superiority increases, and surpasses the opposite chances, the probability receives a

[13] *Enquiry*, p.38

proportional increase, and begets still a higher degree of belief or assent to that side, in which we discover the superiority"[14].



This gradual change to the subjective degree of belief can be graphed as a function of ever greater probabilities (with a marked divergence as probabilities approach 1). A true empiricists ought to adhere to the graph on the left; defining the probability of occurrence of events (i.e., temperature in January, breaking point of a material, etc.) in terms of their ratio of occurrence in sets of past observations (x-axis; empirical probability). We should expect that probability of the next round ought to be what we have experienced thus far. The graph on the right is skewed by the gambler's fallacy and approaches infinity for tangent at 1. As the empirical probability of an event approaches 1, the belief in the occurrence of that event has already increased beyond its empirical value. This trend becomes ever more pronounced with ever greater values. Therefore when the empirical probability readily approaches 1, the mind is already convinced of the impossibility of the non-occurrence of that event; it hastily and falsely assumes a 'necessary connexion'. This is the birth of the idea of law-like necessity in nature.

[14] *Ibid,* p.37 (underline added for emphasis)

A coefficient could be added to the tangent function as a variable describing one's propensity to invoke the gambler's fallacy. When this coefficient is equal to zero, the graph will resemble the one on the left. Beliefs are not biased by incoming information – therein lays the mind of the true empiricist. The greater the coefficient, the more hastily one will evoke the gambler's fallacy and the greater is such a person's propensity to disproportionately prefer the more likely outcomes. This can be expressed by a function of the form $y = \gamma x^3 + x \ , \quad 0 \leq \gamma$ where γ express the propensity to commit the gambler's fallacy. That which seems trivial and even negligible with a two-sided coin (p=1/2) or a six-sided die (p=1/6), becomes philosophically problematic as we approach extreme values of *P*, illustrating how the mind schematizes the world its experiences. If the nuances of this line of thought are appreciated, this underlying problem forces us to reconsider the very probability evaluation of the coin and the die.

Though an event may be highly probably, it should not be thought of as absolutely necessary either. This would mean that we have a hastened tendency to dismiss an event as being impossible while it may still possess a very low possibility of occurrence, or alternately, to assume that a specific effect is necessary though it may only be highly probable. With regards to low orders of probability, this is precisely the tendency in the statistical or human sciences, to discard extreme data (especially when plotting a correlation). This tendency acts as a coat of varnish, making the data appear smoother – one might even say, more 'natural'. Yet the data point is there, it represents an empirical measurement. This discarding of data, I would venture to guess, arises from nothing other than an inferiority complex within the statistical sciences which seek to become more rigorous or 'real' – like all those mathematical sciences, especially physics.

Consequently, as we tend to valorize highly probable events, reciprocally, we have a tendency to rationalize, or utterly ignore, occurrence of a highly improbable nature. Hume says

"When any cause fails of producing its usual effect, philosophers ascribes not this to any irregularity in nature; but suppose that some secret causes, in the particular structure of parts, have prevented the operation"[15]. This is when we must exercise prudence lest we slide into dogmatism as this 'great assurance' that the past will transfer onto the future may eventually 'leave no room for any contrary supposition'[16]. This is especially true for occurrences of a very low order of probability which places them in heighten jeopardy of being rationalized away or dismissed. Thus, Hume warns us that this tendency may lead us to undermine less likely events thereby, jeopardizing the accuracy of our predictions for future events. He says,

> Where different effects have been found to follow from causes, which are to *appearance*, exactly similar, all these various effects must occur to the mind in transferring the past to the future, and enter into consideration, when we determine the probability of an event. Though we give the preference to that which has been found most usual, and believe that this effect will exist, we must not overlook the other effects, but must assign to each of them a particular weight and authority, in proportion as we have found it to be more or less frequent.[17]

Notions like 'necessary connexion' which are supported by notions of 'hidden and secret causes', if unchecked, can lead to all kinds of ungrounded and unempirical ideas. This applies equally to notions like 'chance' which give rise to notions like 'luck' or 'fortune' – all of these notions exude a smell of the occult and of superstition. We revert to these notions just as one would formerly revert to theological notions; for explanative power. When we want science to explain (as the former theological paradigms did) we compromise its true power – the power to account, to measure. Science cannot ultimately explain anything, it can only account, in the Baconian sense. According to Wesley Salmon, this view places me in an exceedingly scarce group of thinkers who believe that "scientific knowledge is descriptive- it tells us what and how. If we seek explanations- if we want to know why- we must go outside of science, perhaps to

[15] *Enquiry*, 38
[16] Ibid.
[17] *Enquiry*, p.38-39

metaphysics or theology"[18]. Why we can continue asking why is because the 'why' question, and that itching inside of us that wants it answered, is never ultimately satisfied – as a curious child may perpetually and systemically ask 'why?' after each of his parents' proposed answers. I, for one, think it is remarkable that this question 'why?' can never be answered. This is why Wittgenstein tells us that "we feel that even when all possible scientific questions have been answered, the problems of life remain completely untouched"[19]. Probability, if it cannot explain fringe phenomena by evoking non-empirical ideas of 'hidden and secret causes', can at least aid us in making more comprehensive measurements.

To illustrate this we can appeal to Salmon's example of the occurrence of leukemia around the hypocenter of a nuclear blast[20]. If we adhere to a strict notion of causality, then 'hidden and secret causes' will be needed to account for illnesses at ever greater distances from the hypocenter of the blast. If we employ a probabilistic conception of causality, we can account for the occurrence of such an illness without needing to appeal to anything else beyond the explosion itself. The definition of cause can be understood as bringing about a change to the probability distribution, or expectation, of a given event. James Woodward describes this as follows - "some intervention on X such that if it were to occur, then Y (or the probability of Y) would change"[21]. This is in stark contrast to the application of an unempirical obsolete conception of causality which relies on occult and obscure notions. In this manner, one can account for fringe occurrences, such as the probability of disease at many kilometers from a hypocenter of a nuclear blast, without evoking 'secret and hidden' causes. In this sense, to account for or describe a given occurrence may be the limit of empirical reasoning – to seek

[18] Salmon, 1978, p.684.
[19] *Tractatus*, 6.52
[20] Salmon, 1978, p.689
[21] Woodward, 2005, p.69

anything beyond this is to try to look beyond the horizon, a task that empirical reasoning is, by virtue of its own design, inapt to accomplish.

This tendency can also be explained with regards to the Texas Sharpshooter fallacy in which data is interpreted in such a way that relevant patterns appear. The idea is to picture a 'sharpshooter' firing some shots into the side of a barn. He then walks over to the barn and paints a target over the largest cluster of hole thereby making himself out to be a 'sharpshooter'. If we leave aside conscious and intention manipulation of data, this fallacy points to the same unconscious cognitive mechanism which gives raise to the gambler's fallacy, namely, the tendency to extrapolate or fabricate patterns out of randomness. This has also been called the 'cluster illusion' in cognitive psychology.[22] Hume's attacks the process of induction, which in turn leads to ideas as causality, are directed to this hasty quality of induction to readily move from an epistemic project of accumulating data to metaphysical proclamation concerning the mechanics of production and propagation of the structure of the world. The data is selected which best suits the model, the model is not selected to best suit the data. A prescribed model of the world is generated within which everything is made to fit. This is where potential implicates for the sciences may arise.

The theory takes precedence over, and is defended at the cost of, observed data. What arises from this are instrumental scientific theories, such as the Ptolemaic model of astronomy. If contradictory evidence is found, the theory can simply accommodate just as the 'sharpshooter' can paint an extra ring around his target. Though such instrumental theories can account for pass observation, they relinquish all predictive power which is the hallmark of science. It is a function of philosophy to insure that science does not regress back to a dogmatic state lest it will lose all

[22] *How we know what isn't so: The fallibility of human reason in everyday life*. Gilovich, Thomas (1991).

its predictive power. This requires the philosophy of science to understand the intrinsic capability of the mind to erroneously observe and/or impose patterns when none exist. It also calls for the need to reexamine the causal paradigm as a potential yet paradoxical hindrance to the predictive power of science, that power which has set science apart from all other paradigms of knowledge.

The thousand-sided die has been a useful aid in understanding the mind's tendency to amplify probability. To illustrate the philosophical problem thoroughly, let us suppose that the structure/ geometric properties of this die were kept hidden from us. If we only ever have access to the outcome of each individual throw of this die, but never to the die itself, how many throws of this die would be needed to generate an idea of its structure? If I had not initially informed you that the object in question had been a die (an object associated with chance), how long would it take (how many throws and subsequent outcomes) to conclude that you were, in fact, dealing with a die (an object of chance)? Alternately stated; being in the world as it is, how could you ever decide whether the world is determinate or indeterminate? Or; how can you ever know if 'chance' in objective (if God plays dice with the cosmos)? You can never know. Our scientist, lost on the island of scientific knowledge, how can he ever determine the ultimate foundation of that island? Furthermore, any assumption or purport to knowledge could rapidly influence the remainder of one's investigation.

The extent to which we can be certain of this die's structure by means of an aggregate of throws is the extent to which we can have certainty in our empirical sciences. This is because all we have is a collection of instances (separate throws of the die). Such a collection, regardless of its magnitude, can never tell us anything absolute about the underlying structure of the object of inquiry, whether that item be a thousand-sided die or the world. We can never be sure of whether we are dealing with a die governed by objective probability or an unwavering law-like generative

principle. It is impossible to know with absolute certainty the structure of this die, or if it is a die, as it is impossible to know that generative principle which governs ultimate physical reality. Hume makes essentially the same argument when he reasons that nothing can be said about the idea of causality or 'necessary connexion' by means of an aggregate of instances.

3.2 - Hume on Probability

Hume formulates probabilities as a difference between 'favorable' versus 'unfavorable' chances. This is opposed to the conventional formulation whereby probabilities are expressed in terms of a fractional number between 0 and 1. Hume claims that 'the mind is determined to the superior only with that force, which remains after subtracting the inferior'[23]. It is the difference between, and not the ratio of, the number of observations favorable to a predicted effect and the number of observations unfavorable to it, which is the measure of probability[24]. For Hume, the 'inferior…destroys the superior, so far as its strength goes' thereby, the probability will be measured by the remaining strength of the 'superior chances'[25].

Hume might advise against yielding probability values between 0 and 1 by dividing the superior chances by the sum total of both the superior and inferior chances ($s/\ s\ +i\ $) Quantifying probabilities as fractional carries an implicit supposition that the total number of observations does not contribute to the probability assessments or degrees of belief.

Let us suppose two people are tallying the relative outcomes of a given phenomenon. In one hundred instances of this phenomenon, one person has observed the outcome *x* on ninety times. The other person, having made just ten observations of the same phenomenon, has

[23] *Treatise,* p.138
[24] *David Hume and the Probability of Miracles.* Gower (1990), p.22
[25] *Treatise*, p.130

tabulated the occurrence of *x* nine times. The conventional formulation of probability will claim that both people yield the same probability, namely, that outcome *x* results from this phenomenon with a probability of occurrence of p=0.9. Contrary to this, Hume suggests that though they have observed the same relative frequency, one person has observed *x* with a greater 'superiority' which will have made a greater 'impression' on that person's beliefs. Gower tells us:

> Hume, as we have seen, wanted to subtract the 'inferior' number of observations from the 'superior' number in order to arrive at a probability, so he could not treat the ten-observation and the one-hundred-observation versions of the example as the same. Nor, it might be said on Hume's behalf, should they be treated the same, for surely the absolute number of observations counts for something when a person 'proportions his belief to the evidence'.[26]

Hume's background as a lawyer may be seen here as influencing his conception of probability[27]. In a case where six witnesses testify against the testimony of two, the jury may be more compelled towards a given verdict than another jury faced with a similar case in which three witnesses testify against the testimony of one. This can be connected to what John Maynard Keynes will later refer to as 'weight', as in weighting the evidence, but what is meant by this ought to be thought of as reliance as in statistical reliability.

Hacking claims this to be the result of Hume skewing levels of probability[28]. This would infer that Hume conflates the idea of probability with confidence or reliability. Whereas the latter increases with an increased number of instances, the former does not – it stays the same. It appears that Hume does not make this distinction. Nevertheless, Hume is pointing to something which is of psychological relevance, and as for as mind is concerned, the latter does have a bearing on its evaluation of the former. A larger number of views, Hume asserts, have a bearing

[26] Gower , 1990, p.22
[27] *Hume on Probability.* Gower (1991), p.8
[28] *Hume's species of probability.* Hacking (1978), p.30

on our degree of belief. This invites discourse for other possible variables which can influence a person's degree of belief concerning outcomes of events. Hume points out that,

> An experiment, that is recent and fresh in the memory, affects us more than one that is in some measure obliterated; and has a superior influence on the judgment, as well as on the passions. A lively impression produces more assurance than a faint one; because it has more original force to communicate to the related idea, which thereby acquires a greater force and vivacity. A recent observation has a like effect; because the custom and transition is there more entire, and preserves better the original force in the communication. [29]

We can venture to imagine that time interval between events, and the magnitude of the event itself, may influence subjective beliefs. Suppose I lose three consecutive hands at blackjack, I may feel more cheated than if I had lost three consecutive plays at roulette betting on color. The odds of one game are exchangeable with that of the other; the only difference may be the time interval between each play. Supposing this time interval to be shorter in blackjack, I may more readily feel these losses versus similar losses at roulette. This would be an example of how time influences our degree of beliefs. Alternately, suppose that opium would not act as a soporific 1 time in 10 and that Mount Vesuvius erupts, on average, once every 10 decades. The disbelief triggered by the volcanic eruption in any given decade would be disproportionately greater than the disbelief generated by witnessing someone having taken opium and not having fallen asleep. If philosophy is to keep a watchful eye over science, these mental tendencies must be well understood as they can easily bestow a subjective flavor to our observations.

3.3 - Hume's attack *Of the Idea of Necessary Connexion* as an extension of *Of Probability*

Hume's work on probability and belief formation extends into his famous critique of induction and causality in his essay 'Of the Idea of Necessary Connexion'. Our concept of

[29] Treatise, p. 98

'necessary connexion' is only a feeling in the mind as it results from the same mental tendency. Hume directly asserts there are metaphysical ideas such as power, force, energy and necessary connexion whose definitions are shrouded in obscurity and uncertainty[30]. To this list, he should have added 'chance' which acts as a sort of antithesis to the idea of 'necessary connexion', being just as obscure and difficult to define.

Hume is an empiricist. If all our ideas are taken to be 'nothing but copies of our impressions', then the cognitive faculties of the mind could not come to define an idea without sensory experience[31]. When long habit is introduced, Hume warns that such impressions, like geometrical definitions, 'compare ideas much wider of each other' such that the mind no longer evaluates individual circles but simply asserts 'Circle!' or ascribes 'circle-ness' on all occasions[32]. This tendency of the mind does not solely apply to properties of simple items or occurrence in our experience, but all facets of our experience including the temporal sequencing of occurrences themselves. The mind seems to have as aptitude for grouping seemingly similar elements of our experience and seeking out or imposing patterns. Hence, having witnessed one event temporally conjoined to another event on multiple occasions, and having witnessed these constantly conjoined, the mind begins to perceive these events as being necessarily conjoined; necessarily entailing one another on all occasions. Thus, the idea of 'necessary connexion' is formed, firstly by seeing *similarity* in separate instances, and then inferring from these 'similar instance' a pattern – in this case, causality.

We cannot, from a single observation, reasonably formulate 'anything which can suggest the idea of power or necessary connexion'. Hume asserts that we cannot derive anything from a

[30] *Enquiry,* p.66
[31] *Ibid.*
[32] *Ibid.*

singular observation of two conjoined events that could suggest the existence of some unseen force which necessarily binds the two events[33]. He goes on to assert that it is equally not in our power to derive from any number of single instances anything which is beyond them. Furthermore, no pattern emerges out of an aggregate of single instances which cannot be derived from a single instance, and no such pattern can be derived from a single case, therefore no pattern can be found. Therefore, says Hume, 'necessary connexion' is inconceivable unless we fathom that in one 'species' of event, or a number of similar events, having always been conjoined with another may allow the mind to conceive of 'causal necessity'. Perhaps 'causal necessity' may develop if we examine an aggregate of similar instances as opposed to any singular instance. Hume denies this flatly:

> It appears, then, that this idea of necessary connexion among events arises from a number of similar instances, which occur, of the constant conjunction of these event…but there is nothing in a number of instances, different from every single instance, which is supposed to be exactly similar; except only, that after a repetition of similar instances, the mind is carried by habit, upon the appearance of one event, to expect its usual attendant, and to believe, that it will exist. This connexion, therefore, which we *feel* in the mind, this customary transition of the imagination from one object to its usual attendant, is the sentiment or impression, from which we form the idea of power or necessary connexion.[34]

We cannot derive 'cause' from a particular instance. Therefore what makes us think we can infer it from an aggregate of similar/uniform instances, as though some occult force or pattern would emerge? There is no objective difference between a singular and the aggregate sum of similar instances. Any difference is in the mind's treatment of these and not anything to be found in, or derived from, the instances themselves. The only notable difference to be found in a collection of uniform instances is in the mind's treatment of them - its own propensity to create an idea or impression of necessity. It is only after regular exposure to similar events which

[33] *Enquiry*, p. 41
[34] *Ibid,* p. 50

we observe in 'constant conjunction' that our mind formulates an 'internal impression' which we call the 'cause *necessitating* effect'. Hume succinctly states these ideas in the *Treatise*, as follows:

> Let men be once fully persuaded of these two principles, that there is nothing in any object, considered in itself, which can afford us a reason for drawing a conclusion beyond it; and, that even after the observation of the frequent or constant conjunction of objects, we have no reason to draw any inference concerning any object beyond those of which we have had experience.[35]

The first principle is that no singular instance can convey anything beyond it. If we then choose to form an aggregate, we must imply a notion of similarity, which is an idealization. Thus, Hume follows up with his second principle which is essentially a pluralized version of his first principle; that there is no good reason to believe that an aggregate can tell us anything beyond itself either. Once an aggregate of 'similar' instances is perceived, the mind is carried to an induction – this is yet another idealization.

Hume presents two more notions which are extremely pertinent to our current discussion. Hume tells us that we can add to our understanding of how observed instances affect the formation of ideas and degrees on belief if we take into consideration 'the effects of contiguity, as well as of resemblance'[36]. Observed similarity in separate instances (resemblance) and temporal proximity and/or sequencing (contiguity) result from *a posteriori* cognitive/heuristic processes. These 'two relations of resemblance and contiguity, are to be considered as associating principles of thought, and as capable of conveying the imagination from one idea to another'[37]. Resemblance and contiguity are two elements which are at the base of pattern formation in the mind. When a heightened degree of regularity is observed, the resemblance and contiguity of these observations combine to form the idea of causality or 'necessary connexion'.

[35] *Treatise*, p. 95
[36] Ibid, p. 70
[37] Ibid, p. 74

Richard von Mises, a proponent of the frequentist interpretation of probability refers to 'resemblance' and 'contiguity' as 'collective' and 'frequency' respectively. When regularity is observed in less-than-constant conjunction, as in the case of stochastic phenomena, resemblance and contiguity combine to form the idea of empirical probability. Causality and its counter-part, empirical probability, are both idealizations just as the notions of their aggregates, 'similarity' and 'collective' respectively, are both idealizations. Causality arises on one end and objective probability arises on the other – both of which arise from the tendencies of the mind in dealing with resemblance and contiguity. The mind is led to conceive of these with additional force and vigor by means of the associating principles; resemblance and contiguity. Resemblance and contiguity represent nothing other than regularity. When the mind is faced with this regularity, it is veritably programmed to grasp at something like causality.

Hume still needs to give an account of why he looks both ways before crossing the street. Having witnessed such events in constant conjunction in the past, he will not admit a 'necessary connexion' but he still feel that these events are conjoined. It is for this reason he requires probability. As Hacking puts it 'Hume needs a theory of probability in which conjunctions, though regular, are less than constant'[38]. Hume says,

> It is the constant conjunction of objects, along with the determination of the mind, which constitutes a physical necessity: and the removal of these is the same thing with chance. As objects must either be conjoined or not, and as the mind must either be determined or not to pass from one object to another, it is impossible to admit of any medium betwixt chance and an absolute necessity.[39]

Just as the mind is determined to ascribe a higher degree of belief to more favorable outcomes, it is also capable of ascribing 'law-like' necessity between objects which seem regularly conjoined. Both of these tendencies arise out of the operation of the same cognitive

[38] Hacking, p. 23
[39] *Treatise*, p.115

mechanism. The belief in physical necessity can be understood as the zenith of the gambler's fallacy. Furthermore, Hume states that either chance or absolute necessity, and nothing in between, constitutes the manner in which events unfold. The problem of induction that Hume lays against causal necessity applies equally to chance.

The error in reasoning in a 'hasty induction' is tantamount to the error made in the gambler's fallacy. The only difference is that in the latter it refers to a 'hasty induction' in a probabilistic system of mutually independent outcomes while the former need not necessarily be dealing with probabilities. There is no doubt that Hume's problem of induction which he lays against 'necessary connexion' stems from his previous treatment of chances. Hume did not formulate it in a probabilistic manner as he could not bring himself to switch to another conceptual mind frame as Hacking suggests. Had he formulated his attack on causality in probabilistic terms, the argument would have essentially remained the same. The problem arises when events that are less-than-constantly conjoined are taken as necessarily conjoined. This is where the propensity to evoke the gambler's fallacy becomes problematic in a practical sense.

Consequently, this notion of law-like necessity which arises out of the mind has the effect of amplifying the very mental mechanism which brought it about to begin with. That is to say, the notion of causality leads one to commit the gambler's fallacy more often than one would without it. One's propensity to commit the gambler's fallacy increases as a result of the idea of causality; itself being a consequence of that fallacy. In other words, we have a sort of iterated function between the application of the gambler's fallacy and the unwarranted ideas it generates.

Hume also points out this iteration 'as belief is almost absolutely requisite to the exciting our passions, so the passions in their turn are very favorable to belief'[40].

Just as causality cannot be inferred from a single observation, neither can the probability value of the occurrence be inferred. Upon making several observation of an event, we gradually form ideas about causality or chance based on the relative results observed. Thus it gradually becomes apparently that both causality and probability are heuristic or epistemic tool which we utilized to sift through our experience to help us arrive at coherent and useful picture of the world. Furthermore, as true empiricists, we must admit that there is nothing in these instances of observation which would suggest that our schematic picture of the world, whether causal or probabilistic, has any reality in its own right beyond its epistemic utility. This unwarranted move from an epistemic schema of the world, to a metaphysical belief of how the world really is; this is what Hume warning of us. The probabilistic paradigm and the causal paradigm are equally coherent in describing our world and equal in epistemic utility, but equally nonsensical in metaphysical terms for lack of any empirical basis. Having done away with causality, our use of the word 'law' in speaking of 'laws of nature' should rightly be interpreted as a figure of speech.

4 - Philosophy of Probability: Seeking Certainty in Uncertainty

The promise of certain knowledge through a causal paradigm has been undermined by exposing the lack of an empirical basis behind such ideas as 'necessary connexion'. Denying an empirical foundation for causality does not, in and of itself, provide any reason to believe that probabilistic paradigm holds true. As we cannot say anything certain in positive or causal terms, an examination of probability is necessitated to exhaust any attempt of saying anything with regards to certainty in negative or probabilistic terms. As a coin is tossed through the air, one

[40] Treatise, p.82

cannot know with certainty which facet will turn up, but can one at least know with certainty the probability of either of these facets turning up? The probability of heads is 50/50. Declarations of this nature will become ever more ambiguous in the course of this examination of probability.

The interpretations of probability presented in this section can generally be categorized into two kinds; the epistemic kind that regards probability as a component of human knowledge and acts as a measure of our subjective ignorance of the 'true causes' of events, and the empirical kind which takes probability to be an inherent ingredient of an indeterminate world.

Probability is, in Hacking's words "Janus-faced. On one side it is statistical, concerning itself with stochastic laws of chance processes. On the other side it is epistemological, dedicated to assessing reason by degrees of belief in propositions quite devoid of statistical background"[41]. The epistemic approach will include the so-called 'classical interpretation' of probability as formulated by Pierre Simon de Laplace as well as the 'subjective interpretation' and 'logical interpretation' of probability. In these interpretations, probability pertains to human knowledge rather than stochastic phenomena, and with the exception of the logical interpretation, probability reflects degrees of belief which are evaluated based on available information. The empirical approach will include the 'frequentist interpretation' of probability as well as Popper's 'propensity interpretation' of probability. The proponents of these interpretations understand stochastic phenomena and probability to be objective.

Let us reconsider this question - Is probability objective or subjective? As the ball spins round and round the roulette wheel, is it possible to predict, if I were capable of processing all the implicated parameters, the final position of this ball on the roulette wheel? In other words, is the final position of this ball causally determined? If this is the case, then probability is simply a

[41] Hacking, 2006, p.12

measure of one's subjective ignorance of, or one's inability to compute in time, the true causes. If this is so, then the final position of the ball is knowable, with total certainty, beforehand (knowable even from start of our universe). Or, as this ball spins around the roulette table, is its final position on the wheel truly unpredictable, with the uncertainty surrounding its final position being the result of 'chance mechanics', chaotic microscopic stochastic phenomena and quantum mechanics? In other words, is the final position of this ball indeterminable? If this is the case, then probability is an objective feature of a random and truly indeterminate universe. If this is so, then the final position of the ball is unknowable beforehand.

The mathematical side of probability, the probability calculus, functions independent of philosophical side which is concerned with the interpretation of probability. For our purposes, it is the philosophical side that will interest us. As the mathematical side is essentially independent of the philosophical side, it will only be mentioned briefly.

4.1 - The Mathematical Side of Probability

The axiomatization for the mathematical properties of probability functions was laid out by Andreï Kolmogorov in 1933 and is regarded as the standard calculus of probabilities. By formulating such axioms, Kolmogorov cleaved the mathematical properties of probability from its application or philosophical interpretation. In doing so, he was able to formulate his axioms in such a way that they would not depend on, and would apply regardless of, the interpretation of probability one uses. This way, he was not burdened with the task of providing a philosophical interpretation of his own. He tells us that,

> The theory of probability, as a mathematical discipline, can and should be developed from axioms in exactly the same way as Geometry and Algebra… independent of the usual concrete meaning of these elements and their

> relationships… Every axiomatic (abstract) theory admits, as is well known, of an unlimited number of concrete interpretations besides those from which it was derived.[42]

In theories of physics, we may use the concept of time, or mass, without need to simultaneously provide a philosophical interpretation of either of these. Just as our intuitive conception of space is divorced from geometry, so too, Kolmogorov insists that the probability calculus must be formulated without needing to provide either a philosophical interpretation of probability or a method for determining prior probabilities. It is important that 'the mathematical features of probability be kept separate from its applications and the foundational and philosophical issues connected with it' as this makes it possible to have 'an autonomous field of enquiry, specifically concerned with the philosophical aspects of probability, involving in the first place its interpretation'[43].

To understand how the mathematical and philosophical sides of probability differ, it is necessary to make a distinction between primitive probabilities and complex probabilities. Primitive probabilities refer to simple or 'atomic' events as is the case with assertions such as 'the next coin flip will turn up heads'. Complex probabilities, as expressed in assertions such as 'the next coin flip will turn up heads and the subsequent flip will turn up tails' express the probability of an aggregate of simple or 'atomic' probabilities. The probability calculus dictates the axioms by which we can calculate the probability of complex events, or complex probabilities, on the basis of the simple events which constitute this complex event. However, the probability calculus is only concerned with complex probabilities and so it does not outline how to ascertain or calculate a primitive probability – the probability of a simple event. This is where the philosophical side of probability comes into play. Ascertaining the probability of

[42] Kolmogorov, 1933, p.1
[43] Galavotti, 2005, p.54

simple events is a philosophical endeavor and the various philosophical interpretations of probability are employed to this end.

It is important to note that the mathematical properties of probability hold true independently of the philosophical interpretation of probability that we attach to it. "The interpretation of probability must be kept separate from its mathematical features…To be sure, an interpretation of probability is adequate only if it satisfies the mathematical properties that form the content of the so-called 'probability calculus'"[44]. Probability may refer to a prediction regarding the occurrence of a singular event or a prediction regarding the behavior of a class of events (also referred to as a collective). Various philosophical interpretations of probability and their methods for calculating initial probabilities are presented with the aim of demonstrating that we cannot, with unmitigated certainty, know the probability of an atomic event. Therein lays the fundamental uncertainty with regards to probability.

4.2 - The Epistemic Interpretations of Probability: The Classical and the Logical Interpretation

In regards to fortuitous events (or seemingly fortuitous), it appears that some interpretations want to keep the Newtonian deterministic worldview alive. The first to propose an interpretation of this kind, now known as the 'classical interpretation', was the ninetieth century mathematician, physicist and astronomer Pierre Simon de Laplace. Laplace was known as 'the Newton of France' and he put a great deal of importance on the English physicist's work. His own work as an astronomer relied heavily on the mechanics developed by Newton and in Newtonian mechanics he saw 'a fully-fledged paradigm of knowledge, strong enough to represent our universe in a true and incontrovertible fashion. A key ingredient of this paradigm is

[44] Galavotti, 2005, p.12

determinism'[45]. From Laplace's perspective, enamored by the elegance of Newton's laws and observing constant, unceasing regularity and order in the celestial bodies as an astronomer, any seeming irregularity or fortuitous event would not conform to this model of the world. Such irregularity must arise from the human intellect alone, which due to its intrinsic limitation would make chance appear where there is only order. Thus, probability is taken as an epistemological feature of human knowledge as opposed to something objective. This idea is presented in his *Essai philosophique sur les probabilities*:

> Tous les événements, ceux mêmes qui par leur petitesse, semblent ne pas tenir aux grandes lois de la nature, en sont une suite aussi nécessaire que les révolutions du Soleil. Dans l'ignorance des liens qui les unissent au système entier de l'univers, on les a fait dépendre des causes finales, ou du hasard, suivant qu'ils arrivaient et se succédaient avec régularité, ou sans ordre apparent ; mais ces causes imaginaires ont été successivement reculées avec les bornes de nos connaissances, et disparaissent entièrement devant la saine philosophie, qui ne voit en elles que l'expression de l'ignorance où nous sommes des véritables causes.[46]

Through the application of a sound philosophy, namely determinism and the clock-work universe paradigm made possible by classical mechanics, humanity gradually sheds itself of its uncertainty to degrees of ever greater certainty. Laplace infers that we gradually approach absolute God-like knowledge which resides only in certainty. This supreme intelligence apprehends the entire causal network through all time:

> Une intelligence qui, pour un instant donné, connaîtrait toutes les forces dont la nature est animée, et la situation respective des êtres qui la composent, si d'ailleurs elle était assez vaste pour soumettre ces données à l'analyse, embrasserait dans la même formule les mouvements des plus grands corps de l'univers et ceux du plus léger atome : rien ne serait incertain pour elle, et l'avenir comme le passé, serait présent à ses yeux.[47]

[45] *Ibid*, p.58
[46] Laplace, 1814, p.2
[47] *Ibid*, p.4

Through human efforts, having been granted a feeble likeness of this intellectual, we gradually approach greater levels of perfection in our knowledge. Laplace tells us that all these efforts in search for truth lead the mind continually towards this supreme intellect, although it will always remain infinitely distant from this intellect. As the bible passage puts it, 'For now we see through a glass, darkly; but then face to face: now I know in part; but then shall I know even as also I am known'[48], the idea being that we can get infinitely close to absolute comprehension of God and still we remain infinitely distant from it.

The important thing to note here is the underlying belief or fantasy that we may reduce our degree of uncertainty to an arbitrarily low level, approaching the limit which is absolute knowledge akin to this supreme intellect. This fantasy, as will be mentioned, is decisively ended by quantum mechanics. It is precisely this fantasy that Laplace wishes to present.

Laplace's interpretation of probability, known as the 'classical interpretation', necessarily reflects this notion of uncertainty as underlying epistemic. Probability becomes a measure of what is known, and is relative to a body of evidence. The probability value (the real number in the interval between 0 and 1 where $0 \leq p \leq 1$and where p=1 denotes an event whose occurrence is certain and where 0 denotes an impossible occurrence [or seemingly impossible based on limited relevant information]), is taken to be the ratio of favorable outcomes to that of all possible outcomes. Laplace spells it out as such:

> La théorie des hasards consiste à réduire tous les évènements du même genre, à un certain nombre de cas également possibles, c'est-à-dire tels que nous soyons également indécis sur leur existence, et à déterminer le nombre de cas favorables à l'évènement dont on cherche la probabilité. Le rapport de ce nombre à celui de tous les cas possibles, est la mesure de cette probabilité qui n'est ainsi qu'une

[48] 1 Corinthians 13:12

> fraction dont le numérateur est le nombre des cas favorables, et dont le dénominateur est le nombre de tous les cas possibles.[49]

Laplace provides a recipe for calculating probabilities in highly symmetric situations in which all outcomes are (thought to be) equal or 'également possibles' such as either face turning up in a coin toss, or of the ball landing on any given number on a roulette wheel. This is precisely the same idea as the 'equiprobability' previously discussed in Hume's ideas of probability. When there is no information by which we can judge or deem one occurrence to be more likely then another, we are to regard their probability of occurrence as equal. This is known as the 'principle of insufficient reason' or 'the principle of indifference' (coined by Keynes).

The underlying theme as we have seen with the 'classical interpretation' is that probability is a measure of our information pertaining to a given hypothesis. This idea is also reflected by John Maynard Keynes, the founder of the logical interpretation of probability, which take probability as a branch of logic dealing with arguments which are not conclusive, that is to say, neither true nor false. Keynes says,

> The terms *certain* and *probable* describes the various degrees of rational belief about a proposition which different amounts of knowledge authorize to entertain. All propositions are true or false, but the knowledge we have of them depends on our circumstances…it is without significance to call a proposition unless we specify the knowledge to which we are relating it.[50]

Between an exposition on the epistemic and empirical interpretations of probability, it makes sense to discuss Rudolf Carnap's interpretation which makes use of both. Carnap sees the two interpretations as dealing 'with two different probability concepts which are both of great importance for science. Therefore, the theories are not incompatible, but rather supplement each

[49] Laplace, 1814, p.7
[50] Keynes, 1921, p.3

other'[51]. The first of these, being the epistemic or logic concept, acts to assess the degree of belief of a hypothesis relative to a given body of evidence. The second refers to 'the relative frequency in the long run of one property with respect to another'[52]. Just as we never know causality, all we know is a series of conjoined phenomena, Carnap says that the probability value is not known, but what is known is the observed relative frequency of one property with respect to another. All we have are separate instances that our mind groups together and out of which we infer what is underlying or metaphysical (namely, causality or cause). What is underlying never shows itself. We never find probabilities, just as we never find causal nexuses; all we have are instances of singular outcomes, which we group together as we deem appropriate.

4.3 - The Advent of Probability in Physics

It was only after Laplace's death that probability became an essential tool in the description of many physical phenomena. It would have been interesting to see how he would have defended his deterministic stance in light of these advents in physics. The first of these advents was the discovery of *Brownian motion* by botanist Robert Brown in 1928, the same year that Laplace died. *Brownian motion* describes the continuous irregular distribution of plant pollen and is now regarded as a general property of all sorts of microscopic particles. Albert Einstein later confirmed the existence of atoms through the application of this general property of small particles. Even where the stochastic element of all these phenomena are arguably dismissed as being fundamentally governed by the same physical laws that predicate all physical phenomena, one can yet turn to quantum mechanics which, on its own suffices to deliver a heavy blow to the deterministic world view. The formulation of quantum theory at the beginning of the 20th century held that probability was an inherent and non-eliminable feature of the constituents

[51] Carnap, *On Inductive Logic*, p.51
[52] *Ibid*

of matter itself. Such phenomena as the kinetic theory of gases and thermodynamics appear to be aleatory on account of their complexity (as Poincaré will describe) making it to difficult, from our limited vantage point, to analyze these phenomena in detail. One who wishes to argue in favor of determinism could state that regardless of their complexity, the underlying laws which govern the phenomena have not changed. This argument does not hold for quantum theory as its dominion is governed by a radically different set of principles. As Werner Heisenberg tells us,

> Quantum theory actually forces us to formulate these laws precisely as statistical laws and to depart radically from determinism.[53]

In quantum mechanics, the physical process of making a measurement has a profound effect which cannot be minimized. This interaction, in turn, decisively ends the possibility of observing things in an entirely passive manner. The process of observing something changes the very thing one is trying to observe; there is a minimum finite disturbance which will be caused to any system through measurement. Observables in quantum mechanics may not be measured simultaneously and this marks a crucial difference with classical mechanics. That is to say, to measure the spin of a given particle will prevent a simultaneous measurement of its velocity, and vice versa, thereby making it an incompatible observable. The fantasy in classical mechanics that the effect of measurement can be infinitely minimized ends with the phenomenon of incompatible observables. This simultaneously dispels the fantasy that we may reduce our degree of uncertainty to an arbitrarily low level as Laplace has envisioned. With the advent of quantum physics, 'uncertainty' becomes objectively insurmountable.

[53] Heisenberg, 1958, p.39

4.4 - How to Conceive of Stochastic Phenomena

It is useful to turn to Henri Poincaré's notions of *Instability* and *Complexity* to understand what constitutes stochastic or aleatory phenomena. When either of these categorizes a phenomenon, the end result is always independent of the initial parameters.

Poincaré's notion of *Instability* can be understood as being synonymous with chaos. In a dynamic system, unpredictable macroscopic changes may emerge out of microscopic changes to the *initial conditions*[54]. This is the case with meteorological phenomena, as discovered Edward Lorenz in 1961 – founding modern chaos theory. The famous example of this is *the butterfly effect* – a butterfly flapping its wings in one hemisphere of the planet (microscopic change) may result in a hurricane sometime later, in the other hemisphere (macroscopic result). In other words, the system (in this case a weather system) is so dynamic and dependent on so many factors that a minute change in one variable will, given enough time, completely change the macroscopic outlook of the event. Poincaré provides the example of a roulette wheel to illustrate this type of aleatory phenomena. He asks us to consider the spin we impute onto a roulette wheel and how any variation in our muscular impulse will be sufficient to change the angular moment, and consequently, the outcome of the spin. Such impulses, he suggests, are so faint that they are beyond the capability of any delicate instrument to measure. Therefore, there are minute variables that affect the outcome of an event which by virtue of their minuteness, completely elude us due to lack of precision in measurement. A slight variation is sufficient to make any attempt at calculation useless.

An aleatory event characterized by *complexity* refers to a system that consists of a vast number of simple events which, due to the sheer number, makes computing an outcome with

[54] *Science and Method*, 1914, p.68

precision unfeasible. An example of this would be the kinetic theory of gases or the batter mixing machine that made Sam Rothstein's blueberry muffin in the opening excerpt from *Casino*. The motion of a singular molecule or a single blueberry may be tracked with ease, but once the quantity of these surpasses a certain number, the task of monitor every single one becomes increasingly difficult and potential unachievable. Therefore we begin to treat them in aggregates. Though they will generally be distributed evenly, a momentary drop in air pressure or a muffin devoid of blueberries may occur, just as the balls on a billiard table may all find themselves in one corner during the course of play. Poincaré's own example is that of a well shuffled deck of cards. There are so many possible permutations that 'the final order which results will no longer be governed by anything by chance'[55].

It is interesting to note that Poincaré takes no stance on the ontological question of probability. Yet, regardless of any metaphysical consideration, the uncertain which result from the *instability* and *complexity* of a given phenomenon is permanent insofar as it exceeds the limits of method (scientific method and measurement apparatuses).

4.5 - The Empirical Interpretations: The Frequentist Interpretation and Propensity Theory

Empirical interpretations view probability not merely as an epistemic tool to describe our subjective ignorance, but as an objective feature within an indeterminate world. These are, namely, the frequentist interpretation and propensity theory of probability. The first to spell out an empirical approach was Robert Leslie Ellis in '*On the Foundations of the Theory of Probabilities*' in 1849. In this essay he lays the foundation for what is now known as the frequentist interpretation of probability. He writes:

[55] Poincaré, 1908, p.74

> For myself, after giving painful degree of attention to the point, I have been unable to sever the judgment that one event is more likely to happen than another, or that it is to be expected in preference to it, from the belief that on the long run it will occur more frequently.[56]

He wanted to address the consistency of the theory of probability with a 'sensational philosophy' and so laid the foundation for an empirical approach to probability. As opposed to examining singular events in isolation, Ellis views these singular events as pertaining to an aggregate, as a member is in long series of similar cases. Thus, on a long series of trails of similar kinds, one can expect every possible outcome to recur in a fixed ratio of frequency. Once this is done, 'fortuitous causes disappear' and what is revealed are the permanent features of these similar cases. On the long series of trails, order is seen in the place of randomness. This is 'an *a priori* truth, supplied by the mind itself, which is ever endeavoring to introduce order and regularity among the objects of its perception'[57]. It is noteworthy to point out Ellis' confidence that something certain can be discovered, namely fixed ratios of frequency or occurrence, amidst randomness and probability. Probability evaluations are generated by the same mental tendency described by Hume; that tendency of the mind which now seeks to introduce order with respect to aleatory events as it has done with the *similar* events upon which it imposed a causal nexus.

In *The Logic of Chance*, John Venn builds upon Ellis' notion by further introducing the notion of a *series* of things or event wherein uniformity starts in large numbers. From this, Venn brings to light the ideas of a 'central limit theorem' and 'the laws of large numbers'. He says:

> As we keep on taking more terms of the series we shall find the proportion still fluctuating a little, but its fluctuations will grow less. The proportion, in fact, will

[56] Ellis, 1849, p.2
[57] *Ibid*, p.3

> gradually approach towards some fixed numerical value, what mathematicians term its *limit*.[58]

Richard von Mises clarified the theoretical presupposition of the frequentist interpretation of probability in *Probability, Statistics and Truth*. His formulation of probability was calculated with reference to mass phenomena or 'collectives', to which large numbers of singular but uniform events or instances belonged. Phenomena of this nature included such things as 'games of chance', 'certain problems relating to mass phenomena', and 'certain mechanical and physical phenomena'[59]. A *collective* is taken to mean 'a sequence of uniform events or processes that differ by certain observable attributes'[60]. Probability refers to the relative frequencies found within classes. As a result, to speak of the probability of a single event or instance is meaningless. He illustrates this via a collective formed by all the throws of a die wherein the resulting dotted side forms a singular event (the dotted surface being the distinctive attribute separating possible events). Another example he provides is the collective formed by all the molecules in a given volume of gas wherein the attribute may be the velocity of a single molecule. Thus a collective must come prior to speaking about the probability of any of its given attributes. Having defined *collective* and *attribute*, he says:

> A collective is a mass phenomenon or a repetitive event, or, simply, a long sequence of observations for which there are sufficient reasons to believe that the relative frequency of the observed attribute would tend to a fixed limit if the observations were indefinitely continued. This limit will be called *the probability of the attribute considered within the given collective*.[61]

If an infinite set of observations were made, our measurements would converge to a fixed limit. This limit would represent a particular attribute to arise in a given set or *collective*. This

[58] Venn, 1866, p.102
[59] Richard von Mises, 1928, p.10
[60] *Ibid*. p.12
[61] *Ibid*. p.15

only, however, is not sufficient to qualify as probabilistic, as what is required in conjunction with this is the feature of 'lawlessness' dispersion of a given attribute over the collective, namely – randomness. On this basis, von Mises defines randomness as *insensitivity to place selection*. The *distribution* - the whole of the probabilities attached to the different attributes of a collective, must be found to be random in the before mentioned sense. Thus, the probability of any given attribute is the frequency with which that attribute has occurred in past observations of a given collective, then the distribution is the hypothesis that that same probability value for that attribute will be found in any new collective of the same kind. We should expect the distribution of any given attribute to be the same as the distribution of that attribute as far as we have observed it in passed instances. Regularity is the expectation that the future will resemble the past.

This formulation of the frequentist interpretation of probability can be applied to physical phenomena. Some areas in physics that von Mises feels can be treated with his probabilistic formulation include the kinetic theory of gases, Brownian motion, radioactivity and Planck's theory of black-body radiation. This is because these fields operate on 'chance mechanisms', having the feature of collectives. However, the frequentist interpretation of probability cannot deal with quantum mechanics which requires an interpretation can address single-case probabilities, as is the case with the famous Schrödinger's cat. Regardless of this, von Mises was an advocate of indeterminism and, like Hume, attacked the notion of causality. He says,

> It now appears inevitable that we must abandon another cherished notion that has its origin in everyday life and pre-scientific thought and has been elevated to the rank of an eternal category of thought by overly zealous philosophers: the naïve concept of causality…the principle of causality is subject to change and it will adjust itself to the requirements of physics.[62]

[62] Richard von Mises, 1928, p.210-211

Lastly, it is worth briefly mentioning Hans Reichenbach who also interpreted probability as a notion of frequency. Reichenbach saw science as a continuous scale of probability and attached a probabilistic meaning to causality which pioneered what is now referred to as 'probabilistic causality'. In this regard he says,

> The characterization of the causal laws of nature as strict laws is justified only for certain schematizations. When all causal factors are known, then an effect can be predicted with certainty; such an idealization would be irrelevant for science, without the addition of further assumptions. It is impossible to know all causal factors; we can only select a limited number of relevant factors and use them to predict future events, but must neglect factors of lesser influence. It is usually assumed that the influence of the less important factors is small, and that we can therefore predict the future within certain limits of exactness. This formulation is inadequate, however, and misses a fundamental point in the epistemological situation. Actually, we can only maintain that it is highly probable that future events will lie within certain limits of exactness.[63]

The frequentist interpretation of probability provides an empirical model for quantifying uncertainty and yielding probability values. However, the underlying suppositions which form its foundation are not exempt from the very uncertainty that they are trying to gauge. Firstly, the probability of a given event is assumed to be static and not dynamic. One could just as easily envision a probabilistic system whose total possible outcomes change dynamically after every singular outcome. Does the impact of the roulette ball on the wheel change the probability of the entire system after one single spin? Has the probability changed after several years of usage in a popular casino? The microscope change affects the outcome of the system in a chaotic way so that one longer simply measures the probability of a given event, but measures that probability in conjunction with its propensity to change over time. This illustrates the difficulty of accurately assessing probability value by empirical means. At best one arrives at an ever more reliable

[63] Reichenbach, 1925, quoted from Galavotti, 2005, p.100

approximation as the number of total observations increased, but ideally an infinite set would be required. This notion of reliability is a statistical notion which is itself based on probability, and therefore it is not devoid of uncertainty which can, in turn, be quantized anew.

The frequentists, in an effort to break from the causal paradigm, slowly start sliding down the very same slippery slope, except they are sliding the opposite slope. Many of Hume's arguments against causality, namely that of moving from a collection of similar instances, which is an idealization in the hope of uncovering a pattern (causality, or in this case – fixed ratios of occurrence) is an idealization as well. This is precisely the tendency of the imagination when it is faced with *resemblance* and *contiguity*. Those who came before them ended up with ungrounded notions of causality. The frequentists end up with the ungrounded notion of objective probability.

All we have are singular instances, the notion of a *series* or of similar cases or kinds are useful abstractions that we employ to aid our understanding but which do not present themselves in nature. Venn admits this. The difficulty of finding a precise value of the empirical probability of a given event is that this relies on the idea of an infinite series. With this arises the possibility that we may have not measured long enough, or deep enough as infinity always eludes us. A counter instance to our measurements may be looming in the upcoming trials. In its crude approach, the empirical interpretation of probability has, at the very least, the humility to admit its own fundamental limitations and thereby regarding uncertainty as absolute. It is this truth which all other interpretations of probability are still willing to deny.

4.6 – A Pragmatic Interpretation of Probability: The Subjective Interpretation

The subjective interpretation of probability also takes probability to be a measure of our beliefs about the occurrence of hypothetical future events. Bruno De Finetti takes a pragmatic

stance, abandoning old metaphysical distinctions of indeterminism and determinism as well as notions such as causality and chance. While De Finetti provides an epistemic view of probability, he does not defend the determinist world view as does Laplace, nor does he argue for the notion of objective probability existing outside of, or independently of, a person's judgment. De Finetti addresses the idea of determinism by stating that either the world is governed by fixed laws – 'necessary and unchangeable laws exist: natural phenomena are determined by their antecedents with absolute accuracy and certainty'. If not, then no such laws exist – 'Real laws do not exist as such; forecasts cannot be certain, but only more or less, and perhaps immensely, likely or probable; the so-called natural laws are no more than the expression of statistical regularities.' He says,

> By rejecting determinism we must accept completely the second of the two propositions that I have stated: then, forecasts will no more be certain, but only more or less probable. One might have a probability so large as to be entitled to name it practical certainty, but this does not change the fact that it is simply a probability. The essential novelty in the scientific method would then be the substitution of logic by probability theory; instead of rationalistic science, where certainty is deduced from certainty; there would be a probabilistic science, where the probable is deduced from the probable. It is not prejudicially necessary to renounce determinism for setting up science on these bases; we may confess not to be able to foresee an event without saying that forecasting is by itself impossible. We may then develop science on this line as 1) acceptable to everybody, either renouncing or not renouncing determinism; 2) independent from the causal principle, which then becomes totally useless. From the acknowledgement of it being useless, to abandon it.[64]

The idea underlying the subjective interpretation of probability is that two people, presented with the same information, may generate different probability evaluations. Thus, the same body of knowledge can constitute different evaluations in different subjects. Furthermore, we can determine the probability evaluation of a given individual by applying the betting method

[64] De Finetti, 1931, p.323-324

– the probability that an individual ascribes to a given event can be seen in the degree of risk he is willing to take for a potential return he deeds worthy. Alternately stated, probability can also be measured by proposing a bet to gauge at what odds the individual is willing to place a wager. A reasonable person would place a bet where the return will be (at least) inversely proportional the risk of losing. A fair bet is where the return on his wager is the inverse of his evaluation of the probability of the event. The essential ideas of the subjective interpretation of probability can be shown via the Monty Hall problem (which is a contemporary variation of Joseph Bertrand's *Box Paradox*).

The Monty Hall problem runs as follows: Suppose you are on a game show and you are given the choice of three doors. Behind one door is a car; behind the others, goats. You pick a door, say, Number 1, and the host, who knows what is behind the doors, opens another door, say Number 3, which has a goat. He then says to you, "Do you want to switch to Door Number 2?" Is it to your advantage to switch your choice?[65]

Intuitively, most people feel that it would be of no advantage to switch as this is not perceived as affecting your chances of winning the car. In actuality, if the car is initially equally likely to be behind each door, a player who picks Door 1 and does not switch has a 1 in 3 chance of winning the car while a player who picks Door 1 and does switch has a 2 in 3 chance. This is the paradox. Contestants who switch double their chances of winning the car. Initially, your chances of selecting the winning door are 1 in 3 as the car is equally likely to be behind each door. Your chances remain the same as they do not change as a result of one door being opened by the host. Therefore, the chance of the remaining door is twice as likely as your door to be the winning door.

[65] This is the *Standard Version* of the problem quote from Krauss and Wang, *Journal of Experimental Psychology*, 2003

For our purposes, we are interested in using this paradox as a means of illustrating the idea of subjective probability rather than providing a proof for the paradox. De Finetti defines subjective probability as meaning 'degree of belief (as actually held by someone, on the ground of his whole knowledge, experience, information) regarding the truth of a *sentence*, or *event E*, whose truth or falsity is, for whatever reason, unknown to the person'[66]. What is known is the winning door in the mind of Monty Hall, the game-show host (where p(car) =1). What is unknown is the winning door in the mind of the contestant whose probability evaluation depends on his knowledge, experience and available relevant information. This is what is meant by subjective probability, that probability is not objective or measurable; it is a pragmatic or operational tool which enables us to assess our own limited knowledge. Probability is nothing more than the odds one would accept when betting on an event - the betting quotient at which he would be ready to bet on its occurrence. In this regard, probability need not necessarily be expressed quantitatively.

If the contestant is familiar with problems such as Bertrand's *Box Paradox* or with the idea of variable change of probability, he might decide to swap doors as this will increase his chances of winning. If the contestant is not familiar with either of these, like the vast majority of us, when he is presented with the two remaining doors, he will evaluate his chances of winning as 1 in 2 regardless of whether or not he swaps doors. Thus far, we have seen three probability evaluations for the occurrence, all of which are subjectively formulated and equally valid; Monty Hall evaluation is p=1 (where he has unquestioned faith that his stage assistants have set up the cars and the goats as planned), the contestant who, ignorant of Bertrand's *box paradox*, evaluates his door as having a 50% chance of winning and the contestant who, having previously

[66] de Finetti, 1968, p.45

encountered Bertrand's *box paradox*, evaluates his door at 33% chance of winning and therefore elects to make the switch.

In this same respect, Émile Borel shows us how the same body of abstract knowledge gives rise to different knowledge in two distinct human minds. He provides an example of two people at sea who are asking themselves what the probability of rain is. Though both are dealing with the same variables (the state of the sky, the wind, and the sea) and the same tools at their disposal such as a barometer, their probability evaluations may differ based on their knowledge, intelligence and past experience.

Past experience, in the form of previously observed frequencies, also influences our subjective evaluation of the probability of a given occurrence. De Finetti allows for a convergence between our degrees of belief and previously observed frequencies if we take observed frequencies to fit Bayes' method, where our probabilities are updated in light of new relevant data. With this, De Finetti provides mathematical arguments that show our inductive habits described by Hume. This shows why we are intuitively inclined to expect that the future frequency of an occurrence will be similar to frequency observed in the past. De Finetti writes:

> No science will permit us say: this fact will come about, it will be thus and so because it follows from a certain law, and that law is absolute truth. Still less will it lead us to conclude skeptically: the absolute truth does not exist, and so that fact might or might not come about. What we can say is this: I foresee that such a fact will come about, and that it will happen in such and such a way, because past experience and its scientific elaboration by human thought make this forecast seem reasonable to me.[67]

This archaic demand for certainty cannot be satisfied. This is essentially the revolutionary approach which Boltzmann adopted in the 19th century. There exists a whole host of natural

[67] de Finetti, 1931, p.170

phenomena which cannot be understood if we demand an account of them in terms of causal interaction. It is appealing to grasp at something like probability in an attempt to understand such phenomena. Natural phenomena exhibit regularity. This regularity can be understood and quantified even when it is characterized by a 'less-than-constant conjunction'. Air pressure is (virtually) always uniformly distributed in a closed environment. A baker is almost always right in thinking that the blueberries have been distributed evenly in the mixing of the muffin batter. The demand for certain, as undisputed knowledge, eventually leads us to degrees of reliability, making these stochastic phenomena knowable in a statistical and probabilistic sense.

A fixed unwavering probability value for a given event is far from indisputable as the various interpretations of probability may all yield different results. As a tool that quantifies our uncertainty, probability is subject to that same uncertainty it is trying to measure. This is especially true for the philosophical interpretation of probability. Any fantasy that we shall uncover something solid amidst stochastic processes which will yield, with law-like precision, the probability of a given event comes to an end. Though we arrive at a probability evaluation, that evaluation is, in turn, submit to a probability evaluation of its own. Just as the domain of science may answer to a question, the resulting answer is in turn subject to the very same line of inquiry. The task is always incomplete. Alas, all epistemic roads seem to converge towards uncertainty.

5 - Concluding Remarks

The world seems to be a two-sided coin with objective probability and indeterminism on one face and causality and determinism on the other. In physics, classical mechanics prevails on one side and quantum mechanics on the other. Both theories are extremely successful. The first enables us to put satellites into orbit and the second enables us to look inside someone's brain

with an MRI without opening their skull. On one side we have classical mechanics that gives rise to the Newtonian clockwork universe paradigm which carries with it four centuries of momentum. On the flipside is quantum mechanics which has not yet elicited a paradigmatic shift in the zeitgeist. If we have no reason to favor one over the other, it is a mistake to be a proponent of one and not the other. As Ramsey says 'We suppose chance to be ultimate if we see no hope of replacing it by law if we knew enough facts…there is no reason to suppose it is not ultimate'[68].

When something known to us fails to bring about its usual effect, we philosophers, says Hume, 'ascribe not this to any irregularity in nature; but suppose that some secret causes, in the particular structure of parts, have prevented the operation'[69]. 'Doubtless when it fails we shall seek out some hidden and secret contrary cause', Hacking reiterates and then goes on to ask, 'but suppose that in reality there is none'[70]? The whole approach of taking certainty to be knowledge of the causal networks between phenomena would be mistaken from the start and much energy would have to be spent on constructing elaborate rationalization to account for anomalies. All phenomena are rationalized to fit the physical view of a clockwork universe. There is, as a result, a rampant reductionism that reduces everything to colliding billiard balls.

We favor the outcomes with greater prevalence and we rationalize, or attempt to account to other outcomes. Hume tells us the higher the probability, the more likely we are to form beliefs of necessity and to ascribe a law-like property to that outcome. Inversely, the lower the chance of an outcome, the greater the likelihood to rationalize or attempt to account for the 'discrepancy' by employing other 'well-established causes' (justified by their higher rate of occurrence, of course). What results is a constant rationalization of data as we have seen with the

[68] Ramsey, *Truth and Probability*, p.209
[69] *Enquiry,* p.38
[70] Hacking 1978, p.26

Texas Sharpshooter fallacy. It is patently ridiculous to modulate your data to fit the picture of the world as generated by an inadequate deterministic paradigm. We are not observing the world as we find it; rather we are observing how we think we ought to find it. When we are served a muffin devoid of blueberries we instantly assume that there is something wrong with the baker or the apprentice as opposed to seeing this muffin as an eventuality of the blueberry muffin making process.

It seems evident, after having examined the role of probability in relation to science, that science itself will, of its own accord, eventually adopt an entirely probabilistic point of view lest it should regress into theology. The idea of prescribed law entered into physics haphazardly from the language in use during the époque in which classical physics was formulated. As in the language of the church and the state, there was the law of God and the law of the land, so to the 'laws' of nature arouse in similar linguistic fashion. Thus, whenever we speak of 'laws of nature' we are simply restating figures of speech which need not apply any longer and offer no advantage as far as making adequate predictions goes. So long as science does not accustom itself entirely to a probabilistic way of seeing things, it will constantly find itself with theological baggage. Probability accounts for what has occurred as it has occurred and never feels the need to go beyond measurement or appeal to anything else – this is empiricism in its purest and simplest form. Probability alone suffices for science.

We know what we know. What we want to know is what we do not know. We want certainty. To this end, we employ science. When investigating natural phenomena, especially physical phenomena, scientists take determinism for granted. Certainty, having taken on a deterministic flavour came to mean something like – absolute knowledge of the causal nexus which brought about this event or phenomenon. So strong becomes the desire for certainty of this

kind that it fashions the world in its own image. It is a shameful mistake for an empiricist to approach the world with unwarranted expectation, wanting to come to know the world as he want it to be and not how it actually is.

When we looked for certainty in the world, in its place all we found was a world of uncertainty, in science and in daily life. As our dream for certainty, along with the promise of science to deliver it, slips through the cracks of our hands like sand, we begin to see that uncertainty is something which cannot be reduced beyond a certain point and can never be eliminated. This is seen in quantum mechanics in the abandoning of the fantasy that the adverse effect of making a measure can be reduce to an infinitely low value. This has aptly been named 'the principle of uncertainty' by Heisenberg. This recognition of uncertainty lays the ground for what Suppes refers to as 'probabilistic empiricism', where 'certainty of knowledge – either in the sense of psychological immediacy, in the sense of logical truth, or in the sense of complete precision of measurement – is unachievable'[71].

The problem, to reiterate the opening question of this presentation, is that the world seemed to be causally determinate, and if it is not, then it would seem as though it were governed by chance. Simply stated, things are either arranged (causality) or they are not (chance). These seem to be the only two logical opinions. It also seems that one of these two opinions for the arrangement of things must be the case. We want to ascribe to one of these a metaphysical pre-eminence. It also seems that unless we raise one of these and proclaiming that this (causality or chance) is indeed the state of affairs, until then we seem to not have a proper grasp on things.

To this question 'What is probability?' we get no definite answer. Probability is like many metaphysical concepts such as 'infinity' in mathematics (debate concerning infinity as

[71] Suppes, *Probabilistic Metaphysics*, p.10

absolute versus potential) or 'causality' in the sciences (the debate whether causality is objective or if it is simply a tendency of the mind to impose patterns). If one has determinism, predictability and causality on one hand and indeterminism, unpredictability and chance in the other hand, there is no way to decide which to choose, and to prefer either is a mistake. Neither determinism nor indeterminism has an empirical foundation, so to choose between them transcends experience[72].

Using all the tools at our disposal to the best of our ability, we map out the world as we map out a thousand-side die, and we are never really sure of what to think of that lone side that turns up from time to time. We have found regularity but certainly never certainty. If you admit that you have discovered the structure of the die, you leap to a hasty conclusion. If you persistently assume that there is, in reality, a definite/absolute structure to this die and that it ought to be known with certainty (as though the benefits of knowing for certain far outweigh the benefits of knowing approximately) then your curiosity exceeds your capacity. As Reichenbach tells us,

> The ideal of an absolute truth is … a phantom, unrealizable; certainty is a privilege pertaining only to tautologies, namely those propositions which do not convey any knowledge.[73]

If on the other hand, we admit after implementing all the faculties of intelligence endowed to us, that we have reached the limit of what is humanly knowable with regards to this die/this world, only then have we found the solution. That which remains is a mystery in the true sense, as there is uncertainty which cannot be traversed. The solution is seen in the vanishing of the problems. That which cannot be spoken of must be passed over in silence as Wittgenstein

[72] Suppes, 1984
[73] Reichenbach, 1937, p.90 quoted from Galavotti, p 93

tells us[74]. In admitting what we cannot know, we know more than when we mistakenly think we do know. On one side of the coin we have causality and on the other we have probability. Both seem logically equivalent, and neither of them makes a difference to our ability to predict the future. How should we decide between them? Or more importantly, why should we decide between them? Probability over causality, at the very least, helps us face uncertainty. It seems that nothing perplexes the mind more than regularity of phenomena. Even Hume acknowledges the idea of an uncaused beginning is a very strange idea to entertain. The mind leaps from one theological notion to the next and creates unobservable nexuses to cope with the strangeness of regularity. The probabilistic paradigm, if it is no better, at least has the courage to look upon regularity as-it-is. Either way, the generative force that transforms what was into what will be, that force of nature which 'loves nothing so much as to change the things which are, and to make new things like them'[75], is completely beyond the capacity of our faculties to comprehend. Uncertainty seems to be a feature of human knowledge which cannot be dispensed with. It forces us to come back to the purest adage of philosophy – The only thing I know is that I don't know, the only thing I can be certain of is uncertainty. To console myself, I remember that 'he is the wisest who, like Socrates, knows that his wisdom is in truth worth nothing.'[76]

This branch of philosophy of science gives rise to many questions. Probability is a notion that extends into philosophy of statistics and the foundations of mathematics. Furthermore, in physics the debate about what place probability should have still remains an interesting question, particularly with respect to interpretations of quantum mechanics. It also challenges the computer sciences to generate algorithms to output random variables. How random can a random number generator really be or how are we to compare pseudo-randomness to 'true' randomness?

[74] *Tractatus*, 7
[75] *Meditations* of Marcus Aurelius
[76] From Plato's *Apology*

My intention has been to juxtapose the causal paradigm to the probabilistic paradigm. The two paradigms are equally useful is describing our experience of the world and yet, both are equally meaningless and unfounded as metaphysical schemas. Hume's texts have proved beneficial in attacking causality and presenting a psychological account of how we treat probability. This, in turn, has help question the idea of justification and certain within the sciences and has led us to conclude the uncertain is a permanent and objective feature of knowledge. Hume's essays mentioned throughout this presentation are of great importance if for no other reason than that they keep alive our sense of what is possible. In this presentation, I have attempted to do the same.

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Excerpt from the movie *CASINO* (1995)

Extreme close-up of a slot-machine. Four reels with sevens across. Suddenly, the power goes out. Lights die down..Machine 'waaaooows' down to a dead stop. Security guards, on their hands and knees, are pulling the plugs on the three giant $15,000 progressive machines, as ACE talks to DON WARD.

ACE: Four reels, sevens across, three fifteen-thousand-dollar jackpots? Do you have any idea what the odds are?
WARD: Shoot, it's gotta be in the millions, maybe more.
ACE: Three fuckin' jackpots in twenty minutes? Why didn't you pull the machines? Why didn't you call me?
WARD: Well, it happened so quick. Three guys won. I didn't have a chance to call you.
ACE: You didn't see the scam? You didn't see what was goin' on?
WARD: Well, there's no way to determine that, Sam.
ACE: Yes, there is. An infallible way! They won!
WARD: Well, it's a casino. People gotta win sometimes.
ACE: Hey . . . Ward, you're pissin' me off. Now, you're insulting my intelligence. What do you think, I'm a fuckin' idiot? You know goddamn well somebody had to get into those machines and set those fuckin' reels. The probability on one-four-reel machine is a million and a half to one. On three machines in a row, it's in the billions. It cannot happen . . . would not happen, you fuckin' momo! What's the matter with you! Didn't you see you were bein' set up on the second win?