Université de Montréal

Essais sur les logiciels libres : licences doubles, effets de réseau, et concurrence

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Cette thèse intitulée :

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présentée par :

Johan Latulippe

a été évaluée par un jury composé des personnes suivantes :

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Lars Ehlers	président-rapporteur
Sidartha Gordon	membre du jury
Jeffrey Church	examinateur externe

Résumé

Les logiciels libres sont uniques en leur genre : non seulement sont-ils distribués gratuitement, mais on peut aussi les modifier et les copier. Cette thèse étudie l'impact de ces propriétés du logiciel libre sur la compétition et sur les entreprises de logiciel propriétaire. Des modèles propres à l'organisation industrielle sont utilisés.

Le première étude examine l'arrivée d'un logiciel libre sur un marché occupé par un logiciel propriétaire. En utilisant un modèle de différenciation horizontale, le papier considère une firme propriétaire qui investit dans la qualité de son logiciel. L'arrivée d'un logiciel libre cause l'entreprise du logiciel propriétaire à réduire le niveau de son investissement et à augmenter le prix de son produit. Il s'avère alors que l'introduction du logiciel libre sur le marché réduit l'investissement de l'entreprise et engendre même l'augmentation du prix du produit. De plus, l'arrivée du logiciel libre peut réduire le niveau de bien-être des consommateurs. Comme le logiciel libre ne réagit pas aux décisions stratégique de l'entreprise, cette dernière voit son marché réduit peu importe sa stratégie. La firme décide conséquemment de vendre un produit de moindre qualité à un prix plus élevé à une clientèle réduite.

Le deuxième papier propose un modèle qui utilise la différenciation verticale afin d'examiner un monopoleur offrant un produit complémentaire à son logiciel. L'étude compare d'abord les cas d'un logiciel libre et d'un logiciel propriétaire, toujours dans le contexte d'un monopoleur offrant du support professionnel pour son logiciel. Il est établi que le bien-être des consommateurs est plus élevé, et le profit inférieur dans le cas d'un distributeur de logiciel libre. Ensuite, le modèle initial est modifié avec l'ajout d'une seconde entreprise offrant du support professionnel. Dans ce cas, l'offre de support de haut niveau est plus élevée. Finalement, le monopoleur adopte une stratégie de licences doubles. Ce concept permet au monopoleur de proposer la vente d'une licence même si son logiciel est libre. Cette technique génère plus de profits, certaines conditions étant présentes, que si l'entreprise optait pour un logiciel propriétaire.

Un logiciel libre profite des contributions de ses usagers pour améliorer son produit. Le troisième papier examine l'arrivée d'un tel produit sur un marché dominé par un logiciel propriétaire. Le modèle de différenciation verticale utilisé contraste les deux logiciels dans un marché donné et révèle que la contribution des utilisateurs peut diminuer la part de marché du logiciel libre au profit de son conccurrent. De fait, en diminuant ses prix le licenceur du logiciel propriétaire incite le consommateur à délaisser le logiciel libre pour le produit de son concurrent.

Mots-clés : logiciel libre, logiciel propriétaire, différenciation verticale, différenciation horizontale, discrimination par les prix, concurrence

Abstract

This thesis examines the microeconomic consequences of the arrival of open source in the software market. Specifically, it analyzes three features of open source software by using specific models of industrial organization. Open source software is free, and may be modified or duplicated by anyone.

The first paper studies the entry of an open source software in a closed source software market. Using a model of horizontal differentiation, the analysis considers a closed source firm's investment in the quality of its software. The introduction of open source on the market reduces the firm's investment in quality and increases the price of its software. Moreover, the entry of open source software may reduce consumer welfare. Post-entry by an open source software, the reduction in market share lowers the firm's incentive to invest in quality.

The second paper features vertical differentiation to study a monopolist selling supporting product to its software. The study begins by contrasting the supply of support by an open source provider and a closed source vendor. The model shows that in both cases the levels of support offered are the same. In addition, consumer welfare is higher and profit lower under an open source software. Then, the paper considers the competition in the provision of support. Here, the supply of high level support is greater than under a monopolist. Finally, the monopolist adopts a dual licensing strategy to extract more surplus from developers interested in modifying open source software and redistributing the resulting product. This technique, when the developers place high value on the source code, generates more profit if the monopolist chooses to publish as open source rather than closed source.

The last paper studies how a closed source firm is affected by the introduction of an open source benefiting from contributions by users. A vertical differentiation model is used, and reveals that, when contribution of users is present, the closed source vendor may lower its price to a level where it forces the open source out of the market. The firm's lower price not only increases demand for its software, but also induces consumers into switching from open to closed source software therefore reducing the contribution of users.

Keywords : software, open source, closed source, dual license, network effect, price discrimination, location model

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Remarques et abbréviations

- La forme masculine est employée pour désigner à la fois le masculin et le féminin.

- Les abréviations utilisées dans ce texte sont :

ISO	International Organization for Standardization
SQL	Structured Query Language
GPL	General Public License
IP	Internet Protocol
GEDCO	Geophysical Exploration & Development Corporation

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Chapter 1

A Closed Source Firm's Investment in Quality: The Impact of Free Open Source

1.1 Introduction

Software can be introduced on the market as closed source or as open source. Closed source software is distributed as a binary code, which means its source code is not published. Copyrights over the material ensures that it cannot be copied, but not publishing the source code ensures that it cannot be modified. Not publishing the source code provides the licensor with exclusivity over his software, which cannot be modified by an outsider. Specifically, a user cannot change the software's code or hire a programmer to adapt it to his tastes. Microsoft Office, for example, cannot be modified to do tasks other than the office work it is designed to do; only the source code owner, here Microsoft, can adapt the software because the owner is the only one with access to the source code.

Open source software presents an entirely different scenario as it is distributed with its source code. It is copyrighted, but released under licenses that allow free re-distribution. Everyone can study, alter, and improve open source software, so a consumer can change its code or hire a third party to do so. Open source software is widespread. Case in point, the open source HTTP server program Apache which dominates the server market, and the open source graphic tool Gimp which is a reliable alternative to the ubiquitous Photoshop.

This study is interested in the impact of the introdution of an open source software in a closed source market. The open source software's availability raises a series of research questions: How does this entry affect the price and quality of the closed source software? In turn, how are these variables affected by the cost of quality? Finally, how does the presence of the open source software influence consumers' welfare?

The International Organization for Standardization classifies the quality of software as function of the following characteristics: functionality, reliability, usability, efficiency, maintainability and portability [ISO 9126-1]. Each attribute breaks down into subcharacteristics. The performance of software in terms of each component depends on the market targeted by the producer of the software. Consider for example usability and efficiency. The Oracle database ranks high in terms of usability. It is designed to meet the requirements of office workers who need excellent usability. Efficiency is a secondary attribute in their framework. Oracle typically delivers a level of efficiency inferior to a software such as SQL which is designed primarily for efficiency. SQL's target market consists of content providers such as YouTube, Wikipedia, Facebook, and Google which all manage large databases. The aforementioned are willing to forgo the usability of Oracle in order to benefit from the efficiency delivered by SQL. The latter features stability as a primary characteristic while relegating usability as a secondary characteristic.

To simplify this study, I assume that the software products in the model have only two types of characteristic: a primary and a secondary characteristic. Henceforth, I refer to an increase in the performance of a characteristic as an improvement in quality.

The competition between open and closed source products has elicited numerous research papers (see Rossi [2006] and von Krogh and von Hippel [2006] for a literature review of open source software).

Closest to my paper in terms of modeling approach are those of Gaudeul [2009], who uses the Vickrey-Salop model, and Meng and Lee [2005] and Schmidt and Schnitzer [2003], who use a Hotelling model. Gaudeul [2009] focuses on the decisions of consumers to buy proprietary software or contribute to an open source project. She investigates whether the equilibrium number of software products is efficient from the point of view of welfare. Gaudeul is interested in the effect of open source on the market structure (the number of firms producing software), and concludes that in an industry where open and closed source coexist, large open source projects cohabit with specialized closed source projects. Her research findings also demonstrates that an open source software model of production may be more efficient from the point of view of welfare than a closed source model. My model, like hers, assumes spatial differentiation, but I consider a duopoly instead of several firms. My focus is the change in the closed source firm's strategies post-entry.

Meng and Lee [2005] also study the interaction between closed and open source software. However, their concern is to provide insights on how different compatibility strategies result in different profit levels for the closed source firm. Compatibility is the extent to which consumers in one network benefit from the existence of the other network. In their model, the closed source software vendors can promote one of three compatibility options: two-way, inward, or outward compatibility. Briefly summarizing, with two-way compatibility, the software products share the same network externalities because they are compatible with each other. Thus, consumers benefit from both an increase in the number of closed source software users and an increase in the number of open source users. Under inward compatibility, users of the closed source software benefit from network externalities from both products whereas users of the open source benefit only from the software they use because the closed source software is compatible with the open source, but the converse is not true. Reverse the situation and you have outward compatibility, where it is open source users who benefit from the network externalities of both products. Meng and Lee demonstrate that a closed source software producer prefers competing with a closed source rival than with an open source rival. The zero price of open source software makes it a tough competitor. My approach is similar to theirs in that I also rely on an Hotelling model and allow one firm to maximize profit while the open source side is passive. However, the purpose of my paper differs as I do not address compatibility issues, but focus on the effect of open source software's entry on the closed source firm's choice of price and quality.

Other papers in the literature consider the effect of policy. For example, Schmidt and Schnitzer [2003] analyze whether governmental agencies should be obligated to use open source software. Their model concludes that such a policy is detrimental to consumer welfare. Imposing a type of software to part of the market, in this case the part is the government, results in reducing the competition. In my model, the introduction of open source software may be detrimental to some consumers; however, some, if not all, consumers are gaining.

Some studies discuss levels of investment in software. Bitzer and Schröder [2007] study the effect of open source entry on innovation. They compare a monopoly with a duopoly and demonstrate that innovation increases with competition. In their model, firms compete in technology rather than price and quantity which explains why increased competition brings about higher investment in quality. My findings indicate otherwise, in the sense that a competing open source software lowers the quality offered by the closed source vendor. In my model, the presence of the open source software lowers the closed source firm's market size and thus lowers the benefits of investing in quality. Bitzer [2004] and Casadesus-Masanell and Ghemawat [2006] determine that competing with open source discourages traditional closed source firms to invest in research and development. Their research specifically illustrates that a closed source firm may cease to innovate when the price competition from an open source software is too high to allow the firm to recoup its cost.

The two paradigms are also looked at through case studies. Gaudeul [2007] does a case study of the patterns of competition between LaTeX and its closed source alternatives. She observes that the ability of LaTeX to compete with proprietary software is explained by two network externalities: (a) established long before the arrival of closed source software, LaTeX benefits from a larger user base than its rival; (b) ease of use allows for constant quality improvement by users. The models of both Casadesus-Masanell and Ghemawat [2006] and Schmidt and Schnitzer [2003] establish that a government promoting open source software reduces welfare. Promoting open source is taken as being the government forcing its agencies to use open source software. This reduces welfare because those agencies can no longer choose closed source, and this, even when closed source provides higher benefits than the open source solution.

Jaisingh et al. [2008] and Sen [2007] published studies closest to mine. They directly address competition between open and closed source software and explicitly study the impact of open source entry on the strategies of a closed source licensor.

Jaisingh et al. [2008] model a closed source software vendor's investment in quality. They presume that closed source software's demand increases when its quality increases, but decreases when the quality of the open source software increases. Under these assumptions, they conclude that the entry of open source software in the market lowers the closed source software's price and quality. They assume linear demand functions. I derive demand from the consumers' utility functions. Although my modeling approach differs from theirs, the qualitative results are the same. Indeed, in my model, the entry of open source software lowers the closed source firm's investment in quality, and open source software's quality magnifies the impact of entry on investment.

While I am interested in quality effects and consumer welfare, Sen [2007] looks primarily at price competition. He qualifies the level of threat open source software offers to closed source as a variable of the usability of open source and the strength of the network effect.

Unlike both Sen and Jaisingh et al., I do not account for the network effects from which the open source software benefits. My study also differs from theirs in terms of modeling and focus. In this paper, the entry of open source software in the market may compel the firm to increase its price. This result is counter-intuitive since increased competition usually benefits consumers. Here, the competition is between a strategic firm and a non-strategic open source software, so the traditional concept of competition does not apply because the open source software price is always zero. By reducing the market size of the firm, the arrival of open source software lowers the firm's incentive to invest in quality. Consequently, in the presence of open source software, the firm prefers to charge a higher price to fewer consumers because it is too costly for the closed source firm to try to capture consumers with strong preferences for the open source software.

In the next section, I establish the model. Then, I examine how a monopolist, distributing closed source software, sets the price and quality of its software. The monopoly model is fully developed as a benchmark to study the effect of open source software entry. In section 1.4, I introduce competition from the open source software. Finally, I conclude with a discussion on consumer welfare.

1.2 Model setup

There is a continuum of consumers whose preferences are defined over characteristic space. Consumers differ from each other in terms of the value they attach to each characteristic. The consumers who derive most utility from a given software are those whose preferences lean towards that software's main characteristic.

Consumers are distributed uniformly on the unit segment $[0,\ 1]$. The consumer indexed s derives utility $U = \Upsilon - s/\alpha$ from the software whose performance in terms of the secondary characteristic is α. When the software is closed source and is sold at the price P, it is purchased by all consumers whose surplus $S_c(s) = \Upsilon - s/\alpha - P$ is non-negative. The marginal consumer has $s = \tilde{s}_c$ where

$$\tilde{s}_c = \alpha(\Upsilon - P), \tag{1.1}$$

I say that the market is covered when $\tilde{s}_c \geq 1$ and that it is not when $\tilde{s}_c < 1$. The surplus derived by the marginal consumer is zero when $\tilde{s}_c \leq 1$ and positive when $\tilde{s}_c > 1$. Figure 1.1 displays the two possible configurations.

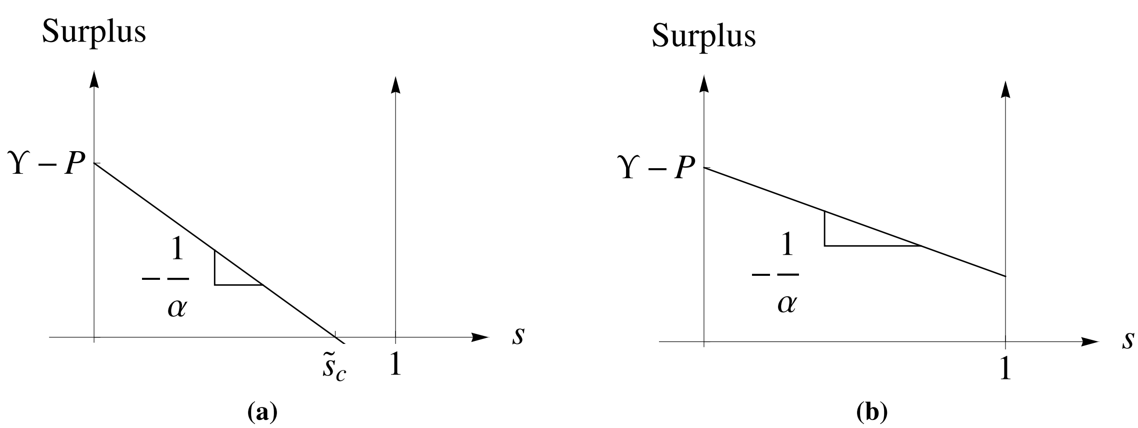

Figure 1.1: Surplus and market coverage

1.3 The closed source product faces no competition

I begin by characterizing a baseline equilibrium in which closed source software is the only product available in the market. The firm which produces that software bears a fixed cost $f(\alpha) = k\alpha^2/2$ when it sets the quality at α. The firm has no other cost.[1]

The firm's profit can then be expressed as

$$\Pi = Pz - k\frac{\alpha^2}{2}, \tag{1.2}$$

where z denotes the quantity sold, and satisfies the condition $z \leq \min\{\tilde{s}_c,\ 1\}$.

Proposition 1. *The profit maximizing price and quality are given by*

$$P_m = \begin{cases} \Upsilon - \sqrt[3]{k} & for\ k \in [0,\ \frac{\Upsilon^3}{8}], \\ \frac{\Upsilon}{2} & for\ k > \frac{\Upsilon^3}{8}, \end{cases} \tag{1.3}$$

and

$$\alpha_m = \begin{cases} \frac{1}{\sqrt[3]{k}} & for\ k \in [0,\ \frac{\Upsilon^3}{8}], \\ \frac{\Upsilon^2}{4k} & for\ k > \frac{\Upsilon^3}{8}. \end{cases} \tag{1.4}$$

Proof. The Lagrange function

$$L(P,\ z,\ \alpha,\ \lambda,\ \mu) = Pz - k\frac{\alpha^2}{2} + \lambda(\tilde{s}_c - z) + \mu(1 - z), \tag{1.5}$$

[1]The cost function is convex because improving the characteristic gets costlier as the characteristic approaches the technological frontier. When the characteristic is not well developed, the cost of increasing α because the programming is not as complex and might be available in technical journals. Furthermore, because the development is done by programmers the firm must spend more to find an additional programmer as the pool of programmers decrease which means that the marginal cost of a programmer increases. This is in tune with Jaisingh et al. [2008].

yields the first-order conditions

$$\frac{\partial L}{\partial P} = \quad z - \lambda\alpha \quad = 0, \tag{1.6}$$
$$\frac{\partial L}{\partial z} = \quad P - \lambda - \mu \quad = 0, \tag{1.7}$$
$$\frac{\partial L}{\partial \alpha} = \quad -k\alpha + \lambda(\Upsilon - P) \quad = 0, \tag{1.8}$$
$$\lambda \geq 0,\ \mu \geq 0,\quad z \leq \tilde{s}_c,\ z \leq 1,\ \lambda(\tilde{s}_c - z) = 0 \quad and\ \mu(1-z) = 0.$$

Clearly the solution cannot have $\lambda = 0$, as it entails $z = 0$ by virtue of (1.6). Thus, there remain two possibilities: a) $\lambda > 0$ and $\mu = 0$, or b) $\lambda > 0$ and $\mu > 0$.

When $\mu = 0$ it follows from (1.6), (1.7), (1.8), and $z = \tilde{s}_c$, that $\alpha_m = \Upsilon^2/4k$ and $P_m = \Upsilon/2$. And, when $\mu > 0$ it follows from (1.6), (1.7), (1.8), $z = \tilde{s}_c$, and $z = 1$, that $\alpha_m = k^{-1/3}$ and $P_m = \Upsilon - k^{1/3}$.

To determine the maximum value of k for which the market is covered, I use $\tilde{s}_c = \alpha(\Upsilon - P) = 1$ and substitute in the equilibrium value $\alpha_m = \Upsilon^2/4k$ and $P_m = \Upsilon/2$. This yields $k = \Upsilon^3/8$. □

The following corollary derives directly from proposition 1.

Corollary 2.

$$z_m = \begin{cases} 1 & for\ k \in [0,\ \frac{\Upsilon^3}{8}], \\ \frac{\Upsilon^3}{8k} & for\ k > \frac{\Upsilon^3}{8}. \end{cases}$$

Figure 2 displays the optimal P, α, and z as a function of k. When k is large and the market is not covered, the optimal price is not affected by a change in k. This is because changes in k induce changes in α, and, in turn, the latter induces an iso-elastic shift in demand. When k is sufficiently small, the market is covered. A smaller k increases α, but now P increases to ensure that the firm captures the marginal consumer's entire surplus.

1.3.1 Consumer welfare

Interestingly, consumer welfare can increase or decrease when k decreases. As long as $k > (\Upsilon/2)^3$ (market not covered), consumer welfare increases when k falls. The reason is that when the market is not covered a lower k brings about an increase in α, but does not

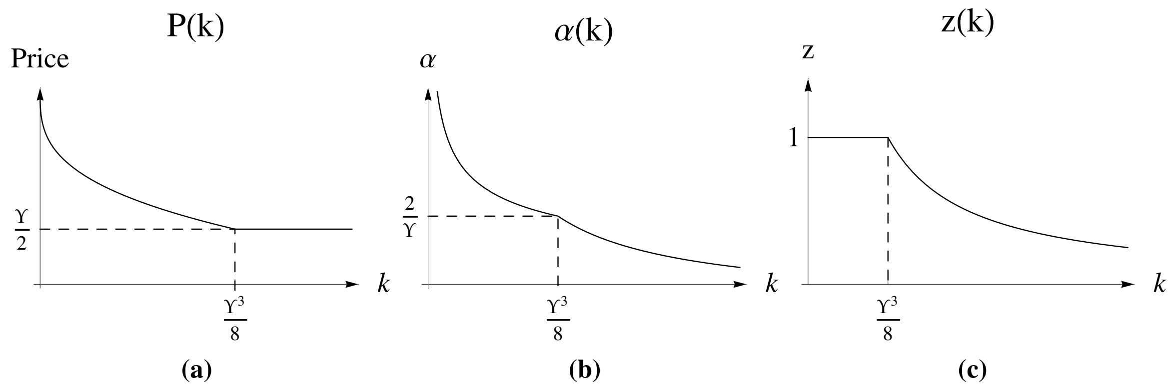

Figure 1.2: Monopoly equilibrium ($\Upsilon = 1$)

affect price. All consumers with $s > 0$ gain, but those with the largest s gain the most. However, as soon as k falls below $(\Upsilon/2)^3$ (market covered), further declines in k lower consumer welfare. The reason is the firms adjusts its price upward in response to the increase in α so as to ensure that the consumer with index $s = 1$ gets zero surplus. But, since that consumer gains more from an increase in α than consumers with indices $s < 1$, it must be true that consumer welfare in general declines as k falls further below $(\Upsilon/2)^3$. The intuition behind the positive relationship between k and consumer welfare when $k < (\Upsilon/2)^3$ can also be understood as follows: by rotating the downward sloping surplus line displayed in figure 1.1(b), α flattens this surplus line. This reduces the variance in the consumers' reservation prices and allows the firms to capture ever larger portion of their surplus.

Consumer welfare may increase or decrease in k, but total welfare, the sum of consumer and producer welfare, always increases as k decreases.

1.4 The firm competes against an open source product[2]

I now consider the question: how the appearance of open source software influences the price of the closed source software and its quality?

I assume that the secondary characteristic of the closed source software is the main characteristic of the open source software, and that the closed source software's main characteristic is the open source software's secondary characteristic.[3] This assumption implies that the consumer indexed $s = 1$ derives the highest utility from the open source software. I also suppose that the quality of the open source software equals β and is given exogenously.[4] Thus, the consumer indexed s derives a surplus $S_{os}(s) = \Upsilon - \frac{1-s}{\beta}$ from the open source software.

The marginal consumer of open source software has $s = \tilde{s}_{os}$ where

$$\tilde{s}_{os} = \begin{cases} 1 - \beta\Upsilon & \textit{when } \beta\Upsilon < 1 \\ 0 & \textit{otherwise} \end{cases} \tag{1.9}$$

In the main body of the paper, I focus on the case where $\beta\Upsilon < 1$. Under this assumption, the consumer indexed $s = \tilde{s}_{os}$ has zero surplus. Those with $s > \tilde{s}_{os}$ enjoy positive surplus whether or not $\beta\Upsilon < 1$ from the open source product.

I consider three a priori possible equilibrium configurations shown in figure 1.3.

Panel (a) shows an equilibrium where $\tilde{s}_c < \tilde{s}_{os}$. The market is not covered, as consumers with $s \in (\tilde{s}_c,\ \tilde{s}_{os})$ acquire neither the closed source nor the open source product.

The condition $S_c(\tilde{s}) = S_{os}(\tilde{s})$ — which describes a consumer who obtains the same surplus from both products — entails

$$\tilde{s} = \frac{\alpha}{\alpha + \beta}(1 - \beta P). \tag{1.10}$$

[2] I do not attempt to explain the reasons behind the presence of the open source software. Its existence is due to the programmer contributions to the source code, a fact which is well documented in the literature [Lerner and Tirole, 2002, Johnson, 2002, Hars, 2002, Haruvy et al., 2005, 2004, Harhoff et al., 2003, Lerner and Tirole, 2005b].

[3] For example, I could say that the primary feature of the closed source software is usability, and its secondary characteristic is efficiency; the open source software, by contrast, has efficiency as its main characteristic and usability as its secondary characteristic.

[4] In reality, β is determined by the inputs of individual contributors to the open source project.

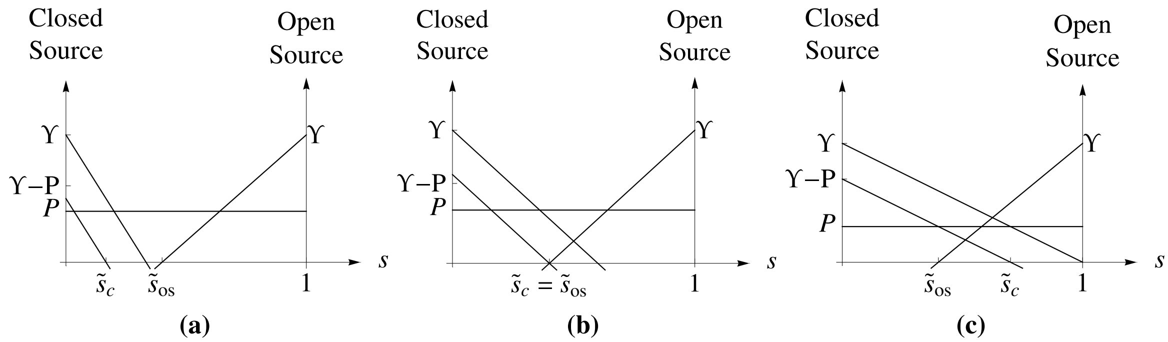

Figure 1.3: Closed and open source surpluses

Panels (b) and (c) display equilibria where the market is covered. When the equilibrium is as shown in panel (c), all consumers get positive surplus, and the consumer indexed $\tilde{s}$ (that consumer is located between $\tilde{s}_{os}$ and $\tilde{s}_c$) is indifferent between the open and the closed source software. Panel (b) displays an equilibrium where consumer indexed $\tilde{s}$ (in this case $\tilde{s}_{os} = \tilde{s}_c$) has zero surplus.

The parameter k determines whether the actual equilibrium is as shown in panel (a), (b), or (c) of figure 1.3. Specifically, two benchmark values $\underline{k}$ and $\overline{k}$ (where $\underline{k} < \overline{k}$) have the following properties:

Case 1: $k \geq \overline{k}$

> The equilibrium is as shown in panel (a). For such large k, the firm chooses a small α and therefore $\tilde{s}_c < \tilde{s}_{os}$. A change in α shifts the demand for the closed source software iso-elastically. The market is not covered and the profit maximizing price does not depend on α. The firm behaves as described in the previous section; it acts like a monopolist.

Case 2: $k \in [\underline{k}, \overline{k})$

> The equilibrium is as shown in panel (b). The firm chooses P and α to ensure that $\tilde{s}_c = \tilde{s}_{os}$. As k falls the firm increases α and P to ensure that $\tilde{s}_c = \tilde{s}_{os}$.

Case 3: $k < \underline{k}$

> The equilibrium is as shown in panel (c). The cost of developing the secondary characteristic is low. Consequently, the firm chooses a large α which

entails $\tilde{s}_c > \tilde{s}_{os}$. Again, the optimal price does not depend on k, because changes in α shift the demand for the closed source software iso-elastically.

Proposition 3 states in precise terms the relationship among P, α, and k.

Proposition 3. *When open source software is available, the profit maximizing price P_c and quality α_c of closed source software are given by*

$$P_c = \begin{cases} \frac{1}{2\beta} & for\ k < \underline{k} \\ \Upsilon - \sqrt[3]{k(1-\beta\Upsilon)} & for\ k \in [\underline{k},\ \bar{k}) \\ \frac{\Upsilon}{2} & for\ k \geq \bar{k} \end{cases}$$

and

$$\alpha_c = \begin{cases} \hat{\alpha} & for\ k < \underline{k} \\ \sqrt[3]{\frac{(1-\beta\Upsilon)^2}{k}} & for\ k \in [\underline{k},\ \bar{k}) \\ \frac{\Upsilon^2}{4k} & for\ k \geq \bar{k}. \end{cases}$$

Where $\hat{\alpha}$ is the solution to

$$\alpha(\alpha+\beta)^2 = \frac{1}{4k}. \tag{1.11}$$

The threshold values are respectively

$$\underline{k} = \left(\frac{2\beta\Upsilon - 1}{2\beta}\right)^3 \frac{1}{(1-\beta\Upsilon)}$$

and

$$\bar{k} = \left(\frac{\Upsilon}{2}\right)^3 \frac{1}{(1-\beta\Upsilon)}$$

Proof. The firm chooses P, α, and z to maximize $Pz - k\alpha^2/2$ subject to the constraints $z \leq \tilde{s}_c$, and $z \leq \tilde{s}$. Upon expressing the Lagrange function as

$$L(P,\ z,\ \alpha,\ \lambda,\ \mu) = Pz - k\frac{\alpha^2}{2} + \lambda(\tilde{s}_c - z) + \mu(\tilde{s} - z). \tag{1.12}$$

the first-order conditions can be expressed as

$$\frac{\partial L}{\partial P} = \quad z - \lambda\alpha - \mu\frac{\alpha\beta}{\alpha+\beta} \quad = 0, \tag{1.13}$$

$$\frac{\partial L}{\partial z} = \quad P - \lambda - \mu \quad = 0, \tag{1.14}$$

$$\frac{\partial L}{\partial \alpha} = \quad -k\alpha + \lambda(\Upsilon - P) + \mu\frac{\beta}{(\alpha+\beta)^2}(1 - \beta P) \quad = 0, \tag{1.15}$$

$$z \leq \tilde{s}_c, \tag{1.16}$$

$$z \leq \tilde{s}, \tag{1.17}$$

$$\lambda \geq 0, \ \mu \geq 0,$$

$$\lambda(\tilde{s}_c - z) = 0 \ and \ \mu(\tilde{s} - z) = 0.$$

I consider the four possible cases.

Case 1: $z < \tilde{s}_c$ and $z < \tilde{s}$

Both constraints cannot be slack in equilibrium since for any given P and α, the firm could increase its revenue by selling to all consumers indexed $s \leq \tilde{s}_c$. As the firm's cost does not depend on z, its profits would also increase.

Case 2: $z = \tilde{s}_c$ and $z < \tilde{s}$ (Panel (a) of figure 1.3)

Jointly $z = \tilde{s}_c$ and $z < \tilde{s}$ entail $\tilde{s} > \tilde{s}_c$. Since the latter inequality implies that the firm sets P and α as if it were alone in the market, I know by virtue of $\tilde{s}$ and $\tilde{s}_c$ (see equations (1.1) and (1.10)) that $\frac{\alpha}{\alpha+\beta}(1 - \beta P) > \alpha(\Upsilon - P)$. Substituting the monopoly equilibrium values, P_m and α_m (see proposition 1), into the latter inequality and rearranging yields $k > \frac{\Upsilon^3}{8(1-\beta\Upsilon)} = \overline{k}$.

Case 3: $z < \tilde{s}_c$ and $z = \tilde{s}$ (Panel (c) of figure 1.3)

Jointly, the conditions (1.13), (1.14), (1.15) and $\tilde{s} = z$ determine the equilibrium values of P, α, z, and μ. Because $\tilde{s} < \tilde{s}_c$, I know by virtue of $\tilde{s}_c$ and $\tilde{s}$ (see equations (1.1) and (1.10)) that $\frac{\alpha}{\alpha+\beta}(1 - \beta P) < \alpha(\Upsilon - P)$. Substituting the equilibrium values of P and α, determined by the first order conditions, into the latter inequality and rearranging yields $k < \left(\frac{2\beta\Upsilon-1}{2\beta}\right)^3 \frac{1}{(1-\beta\Upsilon)} = \underline{k}$.

Case 4: $\tilde{s}_c - z = 0$ and $\tilde{s} - z = 0$ (Panel (b) of figure 1.3)

The conditions (1.13), (1.14), (1.15) along with $\tilde{s}_c - z = 0$ and $\tilde{s} - z = 0$ constitute a system of five equations and five unknowns which yields the equilibrium values of P, α, z, μ, and λ when $k \in [\underline{k},\ \bar{k}]$. □

At equilibrium price, at least one consumer buys the closed source software. The next remark details this result.

Remark 4. The consumer with index $s = 0$ always buys the closed source software. This consumer has surplus $\Upsilon - P$, and I will show that this surplus is strictly positive. For $k > \bar{k}$, the price is $P_c = \Upsilon/2 < \Upsilon$; for $k \in [\underline{k},\ \bar{k})$, the price is $P_c = \Upsilon - \sqrt[3]{1-\beta\Upsilon} < \Upsilon$. For $k < \underline{k}$, the price is $P_c = 1/2\beta < \Upsilon$. The latter inequality is true if and only if $\underline{k} > 0$ (where the definition of $\underline{k}$ is given in 3), and if $\underline{k} \leq 0$ the price is $P_c = \Upsilon - \sqrt[3]{1-\beta\Upsilon} < \Upsilon$.

Corollary 5. *The market size z_c is given by*

$$z_c = \begin{cases} \frac{\hat{\alpha}}{2(\hat{\alpha}+\beta)} & for\ k < \underline{k} \\ 1-\beta\Upsilon & for\ k \in [\underline{k},\ \bar{k}) \\ \frac{\Upsilon^3}{8k} & for\ k \geq \bar{k}. \end{cases}$$

Where $\hat{\alpha}$ is given in proposition 3.

To gain further intuition into the proposition 3, I examine how the firm's revenue and cost evolve as a function of α.

When $\alpha < \frac{2(1-\beta\Upsilon)}{\Upsilon} = \alpha(\bar{k})$ the market is not covered[5] and the marginal revenue from α is $\frac{\partial TR}{\partial \alpha} = (\Upsilon/2)^2$, and is shown as line (a) in figure 1.4. When $\alpha \geq \frac{2(1-\beta\Upsilon)}{\Upsilon} = \alpha(\bar{k})$ the effect on revenue depends on whether or not the price is set to capture consumers who have positive surplus from the open source software. When the firm does not seek to capture these consumers, the firm

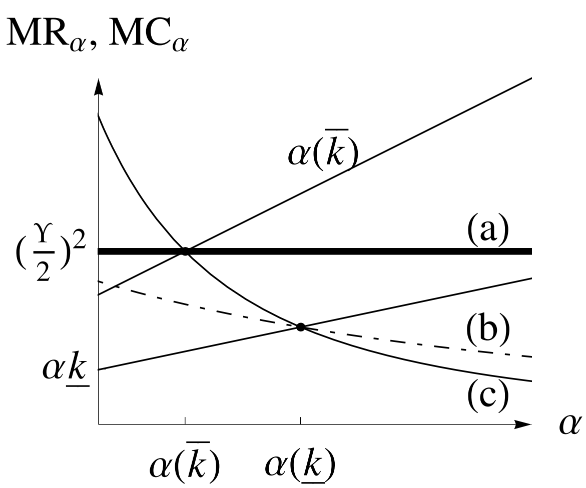

Figure 1.4: Regime change ($\beta = 0.8$ and $\Upsilon = 1$)

[5] There is a one to one mapping between k and $\alpha_c(k)$. Therefore, these two are both equally valid in the specification of the threshold values of k.

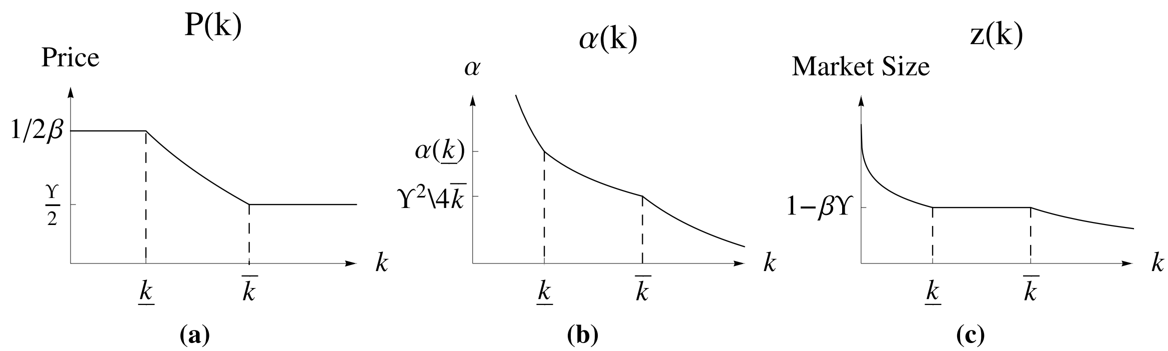

Figure 1.5: Equilibrium with entry ($\beta = 0,8$ and $\Upsilon = 1$)

can set a price which extracts all the surplus from the marginal consumer $\tilde{s}_{os} = 1 - \beta\Upsilon$, the consumer who obtains zero surplus from the open source software.

The marginal revenue from an increase in α determines whether the firm wishes to capture consumers who obtain a positive surplus from the open source software. For $\alpha(\overline{k}) < \alpha < \alpha(\underline{k})$, the marginal revenue in α is higher if the firm does not seek to capture consumers who obtain a positive surplus from the open source software. In that case, the marginal revenue is $\frac{\partial TR}{\partial \alpha} = \frac{(1-\beta\Upsilon)^2}{\alpha^2}$, and is shown as curve (c) in figure 1.4. When the firm seeks to capture consumers who obtain a positive surplus from the open source software, the marginal revenue is $\frac{\partial TR}{\partial \alpha} = \frac{1}{2(\alpha+\beta)}\left(1 - \frac{\alpha}{\alpha+\beta}\right)$ shown as curve (b) in figure 1.4.

The following two propositions clarify how entry by an open source product affects the price and quality of the closed source product.

It follows from proposition 3 and the corollary that P, α, and z evolve as function of k in the way shown in figure 1.5.[6]

Proposition 6. When $\beta\Upsilon > 1/2$ there exists $k^* = \left(\Upsilon - \frac{1}{2\beta}\right)^3 > 0$ such that[7]

- $P_c \geq P_m$ *when* $k \geq k^* = \left(\Upsilon - \frac{1}{2\beta}\right)^3$
- $P_c > P_m$ *when* $k \in (k^*, \overline{k})$
- $P_c < P_m$ *when* $k < k^*$

When $\Upsilon\beta \leq 1/2$*,* $P_m \leq P_c$ *for all k.*

[6] Note that $1/2\beta > \Upsilon/2$ which follows from the assumption $1 - \beta\Upsilon > 0$.
[7] *Note that* $k^* < \underline{k}$ *for* $1 - \beta\Upsilon > 0$ *and* $\beta\Upsilon > 1/2$.

Proof. I consider the three cases:

Case 1: $k \geq \bar{k}$

Recall that for $k > (\frac{\Upsilon}{2})^3$ the market is not covered prior to entry (see Corollary 2) and $P_m = \Upsilon/2$ (see proposition 3). Proposition 3 shows that the firm sells at the very same price post-entry.

Case 2: $k \in [\underline{k}, \, \overline{k})$

By virtue of proposition 3, $P_c = \Upsilon - \sqrt[3]{k(1-\beta\Upsilon)}$. With regard to P_m, I distinguish two cases between the following:

Case 2a: $k > \left(\frac{\Upsilon}{2}\right)^3$

By virtue of proposition 1, $P_m = \frac{\Upsilon}{2}$. Since $k < \overline{k} = \left(\frac{\Upsilon}{2}\right)^3 \frac{1}{1-\beta\Upsilon}$ which rearranges to $\Upsilon - \sqrt[3]{k(1-\beta\Upsilon)} \geq \frac{\Upsilon}{2}$ I conclude that $P_c > P_m$.

Case 2b: $k \leq (\frac{\Upsilon}{2})^3$

By virtue of proposition 1, $P_m = \Upsilon - \sqrt[3]{k}$. Jointly $1-\beta\Upsilon > 0$ and $\beta\Upsilon > 0$ imply $1-\beta\Upsilon \in (0,\ 1)$ because $\beta\Upsilon < 1$. If so, $\Upsilon - \sqrt[3]{k(1-\beta\Upsilon)} > \Upsilon - \sqrt[3]{k}$ showing that $P_c > P_m$.

Case 3: $k < \underline{k}$

I now have $P_c = 1/2\beta$. With regard to P_m I distinguish two cases between the following:

Case 3a: $k > (\frac{\Upsilon}{2})^3$

I now have $P_m = \Upsilon/2$. Jointly $1-\beta\Upsilon > 0$ and $\beta\Upsilon > 0$ imply $1 - \beta\Upsilon \in (0,\ 1)$ because $\beta\Upsilon < 1$. If so, $\frac{1}{2\beta} > \frac{\Upsilon}{2}$ showing that $P_c > P_m$.

Case 3b: $k \leq (\frac{\Upsilon}{2})^3$

I now have $P_m = \Upsilon - \sqrt[3]{k}$. Since P_c is independent of k in this interval, and since P_m is strictly decreasing in k in this interval, the line P_m can cross the curve P_c at most once. Setting $P_m = P_c$ and solving for k yields the threshold $k^* \equiv \left(\Upsilon - \frac{1}{2\beta}\right)^3$. This threshold is positive if and only if $\beta\Upsilon > 1/2$. Finally, note that $\beta\Upsilon \leq 1/2$ entails $\underline{k} \leq 0$. When the latter condition holds, Case 3 cannot arise. Therefore, $P_m > P_c$ when $k < k^*$, and $P_m \leq P_c$ otherwise. The latter proves the last part of the proposition. □

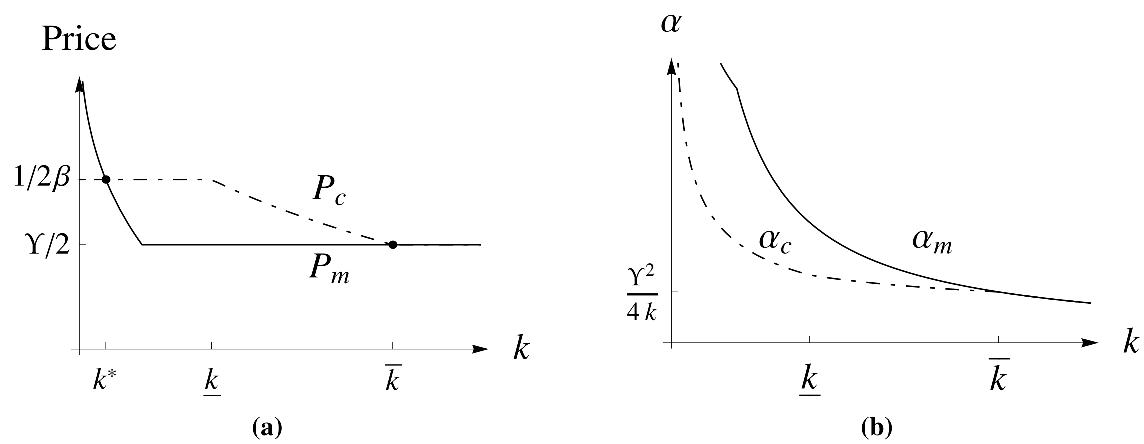

Figure 1.6: The effect of entry on P and α

Figure 1.6 compares the effect of entry on P and α. Panel (a) shows how the price evolves as a function of k. Similarly, Panel (b) shows how the equilibrium value of α evolves as a function of k.[8] The results are dependent on k because this determines the quality the firm sets which, in turn, determines the price the firm can set. With the presence of open source software the closed source firm focuses on a niche market composed of consumers who give high value to the closed source software. The closed source firm sells a lower quality software at a higher price to fewer consumers. This behavior brings higher profit to the firm than trying to compete for consumers who have stong preferences for the open source software. I next report how quality under a monopoly compares with that under competition.

Proposition 7. *$\alpha_c < \alpha_m$ for $k < \overline{k}$ and $\alpha_c = \alpha_m$ for $k \geq \overline{k}$*

Proof. I consider the three cases

Case 1: $k \geq \overline{k}$

[8] Case 3b in the proof of proposition (6) implicitly states that $\underline{k} = k*$ when $\beta\Upsilon = 1/2$. Furthermore, $\underline{k} \to \overline{k}$ and $\overline{k} \to \infty$ as $\beta \to 1/\Upsilon$. In addition, $\underline{k}$ and $\overline{k}$ increase fast while k^* converges slowly to $1/2\beta$ as $\beta \to 1/\Upsilon$.

Recall that for $k > (\frac{\Upsilon}{2})^3$ the market is not covered prior to entry (see Corollary 2) and $\alpha_m = \frac{\Upsilon^2}{4k}$ (see proposition 3). Proposition 3 shows that $\alpha_c = \alpha_m$ in the relevant interval.

Case 2: $k \in [\underline{k}, \overline{k})$

I now have $P_c = \sqrt[3]{(1-\beta\Upsilon)^2/k}$. With regard to P_m I distinguish two cases between the following:

Case 2a: $k > (\frac{\Upsilon}{2})^3$

Prior to entry $\alpha_m = (\frac{\Upsilon}{2})^2\frac{1}{k}$. Since $k < \overline{k} = \frac{\Upsilon^3}{8(1-\beta\Upsilon)}$ which entails $k^{2/3} < \frac{\Upsilon^2}{4(1-\beta\Upsilon)^{2/3}}$. Dividing by k and rearranging yields $\sqrt[3]{\frac{(1-\beta\Upsilon)^2}{k}} < (\frac{\Upsilon}{2})^2\frac{1}{k}$. The latter implies $\alpha_c < \alpha_m$ because its left-hand side is P_c and its right-hand side is P_m (see propositions 1 and 3).

Case 2b: $k \leq (\frac{\Upsilon}{2})^3$

Prior to entry $\alpha_m = \frac{1}{\sqrt[3]{k}}$. The Assumption $1-\beta\Upsilon > 0$ implies $\beta\Upsilon > 0$ imply $1-\beta\Upsilon \in (0,\ 1)$ as $\beta\Upsilon > 0$. If so,

$$\sqrt[3]{\frac{(1-\beta\Upsilon)^2}{k}} = (1-\beta\Upsilon)^{2/3}\frac{1}{\sqrt[3]{k}} \leq \frac{1}{\sqrt[3]{k}}$$

I conclude that $\alpha_c < \alpha_m$ because the left-hand side of the latter inequality is α_c and the right-hand side is α_m (see propositions 1 and 3).

Case 3 $k < \underline{k}$

The optimal α_c is now a solution to $\alpha(\alpha+\beta)^2 = \frac{1}{4k}$ (see proposition 3).

With regard to α_m, I distinguish two cases

Case 3a: $k > (\frac{\Upsilon}{2})^3$

Prior to entry $\alpha_m = \Upsilon^2/4k$. Assume that $\alpha_m \leq \alpha_c$. If so $\alpha_m(\alpha_m + \beta)^2 \leq \alpha_c(\alpha_c+\beta)^2 = \frac{1}{4k}$. Rearranging yields $k \geq \frac{\Upsilon^3}{4(1-\beta\Upsilon)}$. The latter inequality contradicts $k < \overline{k} = \frac{\Upsilon^3}{8(1-\beta\Upsilon)}$ and so the assumption $\alpha_m \leq \alpha_c$ is false.

Case 3b: $k \leq (\frac{\Upsilon}{2})^3$

Prior to entry $\alpha_m = \frac{1}{\sqrt[3]{k}}$. Assume again that $\alpha_m \leq \alpha_c$. If so $\alpha_m(\alpha_m + \beta)^2 \leq \alpha_c(\alpha_c + \beta)^2 = \frac{1}{4k}$. Rearranging yields $\frac{1}{2} + \beta\sqrt[3]{k} < 0$. The latter cannot be true as β and k are positive and so the initial assumption again is false. □

The reason that $\alpha_c < \alpha_m$ for $k < \bar{k}$ can be understood intuitively by referring again to figure 1.3. An increase in α moves $\tilde{s}_c$ to the right. When the market is not covered, the size of the market served by the firm increases by an amount equal to the change in $\tilde{s}_c$. But when the market is covered, the increase in market size is smaller. The reason is that the downward sloping line $\Upsilon - \frac{s}{\alpha} - P$ intersects $\Upsilon - \frac{1-s}{\beta}$ to the left of $\tilde{s}_c$ (as depicted in figure 1.3 (c)). Thus, in the presence of open source software, a unit investment in α increases the market size less than when the closed source firm is a monopolist. This means that the return on investment in α is lower under competition. As a result the firm invests less in α in the presence of open source software.

I now show that the quality is lower when the quality of the open source software is higher.

Proposition 8. $\frac{\partial \alpha_c}{\partial \beta} \leq 0$.

Proof. I consider the three cases:

Case 1: $k \geq \bar{k}$

Recall that for $k > (\frac{\Upsilon}{2})^3$ the market is not covered prior to entry (see Corollary 2) and $\alpha_m = \frac{\Upsilon^2}{4k}$ (see proposition 3). Note that the latter is not a function of β. Proposition 3 shows that in this interval the firm chooses the same α whether or not open source software is available ($\alpha_m = \alpha_c$).

Case 2: $k \in [\underline{k}, \bar{k})$

I now have $\alpha_c = \sqrt[3]{\frac{(1-\beta\Upsilon)^2}{k}}$ which upon differentiation with respect to β gives $\frac{\partial \alpha_c}{\partial \beta} = -\frac{2\Upsilon}{3\sqrt[3]{k(1-\beta\Upsilon)}} < 0$. I conclude that in this interval α_c strictly decreases in β.

Case 3: $k < \underline{k}$

The optimal α_c is now a solution to $\alpha(\alpha+\beta)^2 = \frac{1}{4k}$ (see proposition 3). Taking the total differential of $\alpha(\alpha+\beta)^2 = 1/4k$ and rearranging yields $\frac{\partial\alpha}{\partial\beta} = -\frac{2\alpha(\alpha+\beta)}{(\alpha+\beta)^2+2\alpha(\alpha+\beta)} < 0$. I conclude that in this interval α_c strictly decreases in β. □

An increase in the quality of the open source software reduces the firm's incentive to improve its software. The parameter β increases the surplus of open source software consumers. It is, thus, more costly for the firm to attract a given consumer. Consequently, the presence of the open source software reduces the marginal return of α on revenues. [9]

1.4.1 Consumer welfare

The effect of entry on consumer welfare depends on the value of k. I showed that when $k \geq \overline{k}$ such entry affects neither the P nor the α set by the firm (see proposition 3). I also showed that the closed source market size is unchanged after entry (see corollary 5). Thus, the only consumers whose welfare is affected are those who do not acquire the closed source software before entry and now acquire the open source software. Clearly, these consumers gain; this increases total consumer welfare.

Consider now the case where $k \in [\underline{k},\ \overline{k})$. This case breaks down into the sub-cases $k \leq (\Upsilon/2)^3$ and $k > (\Upsilon/2)^3$. The two sub-cases depend on whether or not the market is covered in the monopoly case (see corollary 2). In those cases, some consumers may lose from the entry of open source software, but total consumer welfare always increases with entry.

When $k \leq (\Upsilon/2)^3$ the market is covered whether or not the open source option exists. The entry of open source software raises the price of the closed source product and lowers quality (see propositions 6 and 7). Thus, the consumers who stay with a closed source software product unambiguously lose. These consumers have index $s < 1-\beta\Upsilon$. Consumers with index $s \geq 1-\beta\Upsilon$ can gain or lose from entry. The group with index $s \geq 1-\beta\Upsilon$ are

[9] Another effect is the reduction in the gap between $\underline{k}$ and $\overline{k}$ that β induces. Remember that when $k \in [\underline{k},\ \overline{k})$ the firm keeps its market size constant and does not fight for consumers who have a positive surplus from the open source software (see figure 1.4). A reduction in the gap means that the firm waits less, in terms of variation in k, to start competing for consumers that have a positive surplus from the open source software.

switchers, that is, consumers who used the closed source software prior to entry but opt for the open source software if it is available. Among these consumers some gain and some lose from entry. The consumer whose surplus is unchanged by entry has $s = \bar{s}$, where $\bar{s} = \frac{1+\beta\left(\sqrt[3]{k}-\Upsilon\right)}{1+\beta\sqrt[3]{k}}$ solves $\Upsilon - \frac{1-\bar{s}}{\beta} = \Upsilon - \frac{\bar{s}}{\alpha_m} - P_m$. Note that $\bar{s} > 1-\beta\Upsilon$. Switchers with $s < \bar{s}$ lose from entry and switchers with $s > \bar{s}$ gain from entry. When $k > (\Upsilon/2)^3$, the analysis proceeds along the same line except that now

$$\bar{s} = \frac{\Upsilon^2(2-\beta\Upsilon)}{2\,(4k\beta+\Upsilon^2)} > 1-\beta\Upsilon.$$

The case $k < \underline{k}$ breaks down into the following sub-cases:

(1) $k > (\Upsilon/2)^3$,

(2) $k^* < k < (\Upsilon/2)^3$,

(3) $k \leq k^* \leq (\Upsilon/2)^3$.

Sub-case (1) yields $\bar{s} = \frac{\Upsilon^2(2-\beta\Upsilon)}{2\left(4k\beta+\Upsilon^2\right)}$ and sub-case (2) yields $\bar{s} = \frac{1+\beta\left(\sqrt[3]{k}-\Upsilon\right)}{1+\beta\sqrt[3]{k}}$. For these two sub-cases, the distribution of gains and losses is the same as above where some consumers gain and some lose from entry.

For the sub-case (3), however, the distribution of gains and losses is different because, contrary to the other cases, the price of the closed source software decreases after entry. Again, some consumers are willing to switch to the open source software while others are willing to continue to purchase the closed source software, but here some consumers with indices sufficiently close to zero gain from entry because of the price reduction (see proposition 6). Although some consumers lose from entry, total consumer welfare is higher after entry. There are now two consumers whose surplus is unaffected by entry.

Remember that α represents the quality of the secondary characteristic for the closed source software. Consumers with index s close to zero benefit from the decrease in price, and are not much affected by the decrease in α. Thus, contrary to the other cases, the consumers with a small s gain from the entry of the open source software. Figure 1.7 illustrates this sub-case. The consumers with indices smaller than $s_L = \frac{\hat{\alpha}(P_c-\hat{\alpha}P_m)}{\hat{\alpha}\alpha_m-1}$ purchase the closed source software and gain from entry ($\hat{\alpha}$ is defined in proposition 3 and, α_m and

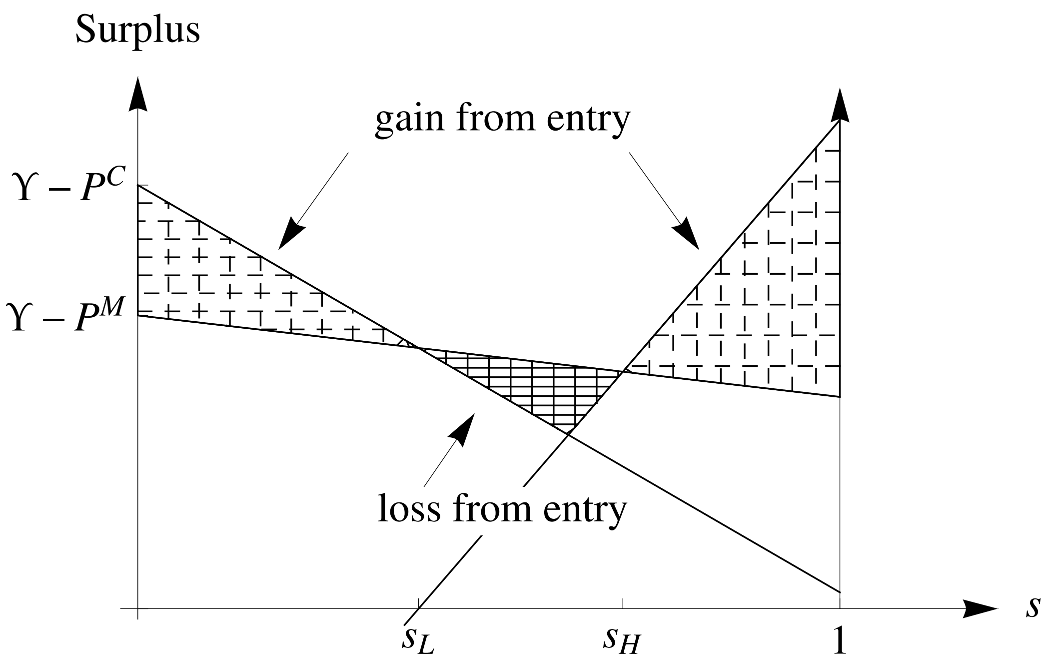

Figure 1.7: Welfare effect of entry when $k < k^*$

P_m are defined in proposition 1). These consumers have gain represented by the leftmost region in figure 1.7. However, consumers farther from zero are strongly affected by the decrease in α. The consumer with indices greater than s_L but smaller than $s_H = \frac{1+\beta\left(\sqrt[3]{k}-\Upsilon\right)}{1+\beta\sqrt[3]{k}}$ lose from entry. These consumers have losses represented by the center region in figure 1.7. Consumers with an index s close to one, those with $s > s_H$, choose the open source software and gain from entry. These consumers are represented by the rightmost region in figure 1.7. Note that for k small enough, all consumers gain from entry. The upper bound strictly increases in k and $\lim_{k\to 0} s_H = (1-\beta\Upsilon)$ while the lower bound decreases in k and $\lim_{k\to 0} s_L = \infty$.

1.5 Conclusion

I consider two main questions in this study: (a) how does a closed source producer adjust price and product quality post-entry; (b) what is the impact on consumer welfare? Answering these questions involves, before all else, establishing the equilibrium of a monopolistic closed source firm as a benchmark. In the model, whether the firm serves all potential consumers determines how the cost of quality affects price and quality at equilibrium. When the market is covered, both the price and the quality chosen increase as the cost of quality falls. Surprisingly, consumer welfare increases as the cost of quality increases. This is caused by declines in the disparity of the consumers' willingness to pay, which allows the firm to capture a higher portion of consumer surplus through a higher price.

When the market is not covered, quality increases as its cost falls, but the price is not influenced by the cost of quality. Here, consumer welfare decreases as the cost of quality increases.

The analysis of the impact of entry by an open source product also depends on whether the market is covered. When it is not, the closed source firm maintains the price and quality level it sets prior to entry. When the market is covered, post-entry, the response depends on whether the firm seeks to sell to consumers who derive a positive surplus from the open source product. When the firm is not concerned with such consumers, it sets a higher price and a lower quality than pre-entry. This is because the marginal consumer dictates pricing, and that consumer is willing to pay more in the duopoly than in the monopoly since the firm market size is smaller under the former.

In contrast, when the firm seeks to sell to consumers who derive positive surplus from the open source product, the price may decrease post-entry. The price is higher in the monopoly case than in the diopoly case if the cost of quality is small. When most of the potential consumers derive a positive surplus from the open source software (when its quality is high), the firm seeks to capture some of these consumers. Therefore, the firm must invest in quality in order to be competitive with open source software. The price set by a monopolist will equal the price the marginal consumer is willing to pay. However, under competition, the firm must set a lower price since the marginal consumer can opt to choose an open source software from which he derives a positive surplus. Under monopoly, as the cost of quality goes to zero, the quality level of the closed source software goes to infinity, in turn driving the variance in consumers' willingness to pay to zero. Thus, the price of the monopolist converges toward the highest willingness to pay as the cost of quality approaches zero. That said, it was found that some consumers have a lower welfare after the introduction of the open source software, but entry unambiguously improves total consumer welfare.

It is important to consider how the model's simplifying assumptions affect the results. First, fixing the location of the software on the Hotelling line simplifies the analysis and ensures that equilibria are easily tractable. This fixity in the level of differentiation explains why the price increases with the entry of the open source software. Furthermore, this result would change if the firm anticipated the entry of the open source, but in my study I compare a static monopoly with a static duopoly.

Second, I assume that the firm could only improve one of its characteristics. This explains why the firm reduces its quality with the entry of the open source software. In reality, software products have a multitude of characteristics and the firm could redirect its effort in a direction where competition is weak or non-existent.

Third, I suppose that the firm only knows the distribution of consumers' preferences and sets a single price. Firms selling applications intended for end-users generally practice uniform pricing, but firms distributing server applications tend to price discriminate. Accordingly, the firm could charge different prices to different consumers to capture more of the consumers' surplus. In the context of my model, price discrimination would enable the firm to price aggressively with consumers further from its location and extract most of its profit from the closest consumers.

Adaptability is a crucial attribute of open source software. In this regard it would be relevant to add competition to the closed source software from a firm adapting the open source software to the consumers' particular needs. Franke and von Hippel [2002] report that the success of Apache, an open source software, is due to its ability to easily adapt to the disparate needs of consumers who are dissatisfied with closed source software.

A series of extensions could be considered. For example, with an asymmetry in consumers' computer literacy, a consumer could decide whether to invest time and effort in improving the open source software to adapt it to his particular needs. Introducing a network effect also changes the scenario as the open source software's quality would increase in the number of users. Furthermore, I have studied a static model; however, in reality, competition takes place over multiple periods. Modeling multiple periods would secure a user base for each software product and generate switching costs. In these conditions, an incumbent firm could accommodate or blockade entry. Lastly, my model features a passive open source firm, but in reality open source projects are often managed by profit maximizing firms, community of users, or individuals. The model could, hence, include a different objective for the open source software, such as maximizing the market size, and act strategically according to that objective.

Chapter 2

Selling Software Complements: The Case of Open Source Software

2.1 Introduction

Open and closed source software are distributed differently. The former is offered with its source code, so that consumers can contribute, and is available for free. In contrast, the closed source firm does not release its source code, and sells its software.

Since open source software is a free product, the distributing firms or individuals are compelled to recoup their investment through means other than the sale of their software. Open source producers invest time and effort to develop and sell complementary products to the source code.

Open source software producers earn profit in two ways: (a) they provide professional support of which various levels exist at different prices.[1] (b) They sell commercial licenses that free the consumers from certain restrictions. Access to open source software may be free but it imposes restrictions unto consumers through licenses. Licenses releasing consumers from many restrictions are typically sold at a higher price than licenses that free consumers from few restrictions [Välimäki, 2003, Riehle, 2009, Campbell-Kelly and Garcia-Swartz, 2010]. For example, a commercial version of MySQL may cost up to 5000$ but the version with the restrictive license is available for free. These practices are known as versioning.[2] Henceforth, any revenue which excludes the sale of the license itself is referred to as support.

Dual licensing is one form of versioning. Consumers can choose a restrictive license, at no cost, or pay for a nonrestrictive license. In order to differentiate the unrestricted from the restricted license, firms typically distribute the free version of their software under a very restrictive license. Dual licensing makes it possible to segment the market in two groups. One group consists of users who download the source code for free, but are forced to contribute any modification they make to the software. The other group, who pays for the license, consists of developers who intend to commercialize their modification of the code. By purchasing the license, they are freed from the obligation to publish their innovations. The Dual licensing model seeks to tap user contributions while extracting rent

[1] In a sample of open source firms gathered by Bonaccorsi et al. [2006], the firms offer an average of 6,4 different services. In their survey the respondents mentioned that their revenues, apart from those emanating from the sale of licenses, came from installation, support, maintenance, consulting, and training.

[2] Versioning is the act of offering information good in different versions designed to appeal to different types of consumers [Shapiro and Varian, 1998].

from consumers through a commercial license.

In this paper, I focus on the following questions: (i) how does support level provided by a monopolistic producer of open source software differ from that offered by a monopolistic closed source firm? (ii) would welfare under a monopolist be lower when the software is distributed as closed source than when it is distributed as open source? (iii) How does competition in the supply of support influences the level of support? (iv) Also, how does the introduction of dual licensing affect the level of support offered when the initial license is restrictive?

My model examines how the aforementioned versioning methods affect the potential to engage in price discrimination, and their impact on profit and welfare. Discrimination is made possible by modeling consumers that differ in their willingness to pay for support and for access to the source code.

In section 2.2, I examine how the provision of support differs among a social planner, a closed source vendor, and an open source distributor. I find that when the monopolistic firm opens its source code, the total welfare is the same as with a closed source software, but consumer welfare is higher and profit lower. Furthermore, under both an open and a closed source regime, the welfare maximizing level of support is offered.

In section 2.3, I extend the model by considering two firms competing for the provision of support. One firm is the licensor and the other is an outsider. I show that the firm provides high quality support to more consumers in the presence of competition than under a monopoly. I assume that the licensor has a cost advantage over the outsider. This assumption generates the surprising result that the firm may increase the level of support it offers even as the cost of support increases. The firm elects to increase its support in order to reduce the competition in price.

In section 2.4, I illustrate that the firm may prefer the dual licensing model over the closed source model even when the benefits of user contribution and of a large user base are absent. I also demonstrate that under the dual licensing model the restriction level of the firm's license increases its profit.

My paper is organized as follows. The first section reviews the relevant literature. In section 2.2, I establish the baseline model considering three distinct allocation regimes: A social planner maximizing welfare, a closed source provider maximizing profit, and an open source firm also maximizing profit. Section 2.3 investigates the effect of competition from an outside firm providing support. In section 2.4, I broaden the analysis to a firm using the dual licensing model. I conclude in section 2.5.

Literature review

The existing literature on this topic is overwhelmingly theoretical. It argues cases of closed source software facing competition by open source software.[3] Some analysis also recognize sales of complementary services by open source firms. Haruvy et al. [2008] postulate that opening the source code benefits the open source firm in two ways. First, a large user base generates increasing demand for software support. Also, the software's value appreciates because users improve the code. They determine when the aforementioned benefits may justify opening the source code. Specifically, they establish the critical level at which the user contributions are sufficiently large to make the open source option the more profitable one.

In Mustonen [2005], a closed source firm invests in a substitute open source product in order to benefit from network externalities. The open source product does not dominate the market because it is an inferior substitute to the closed firm's commercial product.[4] Von Engelhardt and Maurer [2010] examine the motivations of firms to contribute to open source. Using a Cournot model, they derive the welfare implications of such contributions. The model reveals that open source industries generate higher welfare than closed source industries when the competition among firms is weak, and that the converse is true when competition is intense.

Most papers discussing the dual licensing model are found in the business literature (see, for example, Välimäki [2003], Riehle [2009], and Campbell-Kelly and Garcia-Swartz [2010]). Riehle [2009] provides a description of revenue generation strategies in the context of open source. He stresses that dual licensing is attractive for firms because the free license allows for user contributions thereby accelerating the software's development. He mentions that a self-supporting user community is at the core of any successful commercial open

[3]See, for example, Mustonen [2003], Gaudeul [2009] , and Lanzi [2009].

[4]Mustonen assumes that the commercial product valuation is twice as high as the valuation of the open source product.

source project. Campbell-Kelly and Garcia-Swartz [2010] describe, using a series of case studies, the revenue sources of software providers and the strategic interaction among these providers. They compare open source to closed source software, and discuss if there is a convergence happening between these two modes of publication.

Comino and Manenti [2007] show that software vendors prefer the dual licensing model over the closed source model when consumers are sensitive to license restrictions. They assume that the accessible source code implies benefits from user contributions and from a larger user base, and that, for a given investment, the software's quality is greater under a dual licensing model than under a closed source model. Their model describe software development as a two-stage game: A development and a distribution phase. In the first stage, the product's quality increases without any investment by the firm if it releases its source code. The firm benefits because the opened source code generates user contributions, in turn improving the quality of their product. In the second stage, the firm selects a licensing strategy and sets the price. It reveals that the firm distributes its free license under highly restrictive terms in order to induce developers to purchase the nonrestrictive license.

The literature on innovation has also studied open source software. Scotchmer [2010] is interested in how licensing policies affect the rate of innovation. Using a two-period model, she shows that industry profits are higher when the innovators use a restrictive license than when they set in motion a sequence of proprietary license. The closed source license generates more profit for the first innovator, but lowers the profit of subsequent innovators. Henkel [2006] considers what conditions would motivate a firm to share its improvements to an open source.

Lanzi [2009] examines competition between an open and a closed source product that are vertically differentiated. The open source software is supplied for free, but is costly to use because it requires a certain computer expertize. He concludes that the closed source firm may increase or decrease its price after entry of an open source competitor.

2.2 The baseline model

The surplus of consumer indexed θ is $S_\theta = \theta(\underline{s} + s) - p$ where $\underline{s} > 0$ represents some amount of support available for free to all consumers. By paying the price p, consumers obtain extra support in addition to the software. The extra support is denoted s. Thus, the total support enjoyed by consumers who pay p is $\underline{s} + s$. The taste parameter θ is uniformly distributed on $[\underline{\theta},\ \overline{\theta}]$ with $0 < \underline{\theta} < \overline{\theta}$. It captures a consumer's marginal willingness to pay for support.

Software is produced at zero cost.[5] Basic support $\underline{s}$ is provided at zero cost. The total cost of extra support is

$$C(s,\ N) = cs^2N, \tag{2.1}$$

where N is the number of consumers who purchase support s, and $c > 0$ is an efficiency parameter. I assume that the firm offers two levels of extra support denoted l and h, with $s_l < s_h$. I define the marginal cost of extra support[6] per consumer as

$$\frac{\partial^2 C}{\partial N \partial s} = 2cs$$

I examine three ways of providing support. The first way is that of a social planner who maximizes welfare. The second is that of a firm which maximizes profit by selling closed source software and support. The third is that of a firm that maximizes profit by offering an open source product and selling support.

Throughout the paper, I assume that all consumers use software.

Consumers may fall in one of the following groups:

- Group N_b comprises consumers who obtain the base support $\underline{s}$ only.
- Group N_l comprises consumers who obtain the low quality support s_l.
- Group N_h comprises consumers who obtain the high quality support s_h.

[5] I assume that the quality of the software is fixed. For example, the firm could have invested in quality in a previous period and that investment is a sunk cost and, in the present model, does not affect the decisions of the firm.

[6] This is the definition used in the literature.

Consumers belonging to group N_b have $\theta \in [\underline{\theta},\ \theta_o]$; consumers belonging to group N_l have $\theta \in [\theta_o,\ \tilde{\theta}]$; consumers belonging to group N_h have $\theta \in [\tilde{\theta},\ \overline{\theta}]$. The consumer indexed θ_o derives the same surplus whether he belongs to group N_b or N_l. The consumer indexed $\tilde{\theta}$ derives the same surplus whether he belongs to group N_l or N_h.

The social planner maximizing welfare

The allocation of support across consumers maximizes the sum of consumer and vendor surplus. The social planner has three decision variables: price, marginal consumer (quantity), and support. Only two of these are independent because the third one is determined through the demand function. I take the decision variables to be the marginal consumers and support levels.

There are two cases to consider. In the first case, some consumers obtain extra support and some do not. In the second, all consumers obtain extra support.

Case 1.

This case arises when $\underline{\theta} < \overline{\theta}/5$. To see why, I consider the total welfare function:

$$W(s_l,\ s_h) = \int_{\underline{\theta}}^{\theta_0} \theta \underline{s} d\theta + \int_{\theta_o}^{\tilde{\theta}} \left(\theta(\underline{s}+s_l) - cs_l^2\right) d\theta + \int_{\tilde{\theta}}^{\overline{\theta}} \left(\theta(\underline{s}+s_h) - cs_h^2\right) d\theta. \tag{2.2}$$

First-order conditions entail

$$\tilde{\theta} = c(s_l + s_h) \tag{2.3}$$

and

$$\theta_o = \frac{1}{2}(0 + 2cs_l) = cs_l \tag{2.4}$$

Condition (2.3) shows that the marginal utility of support[7] for consumer $\tilde{\theta}$ must equal the average between marginal costs of support s_l and s_h. Similarly, condition (2.4) shows that the marginal utility of support for consumer θ_o must equal the average between marginal costs of support 0 and support s_l.

Having established the optimal location of marginal consumers θ_o and $\tilde{\theta}$, I now turn to

[7]Marginal utility of support's formal definition is $\partial S_\theta / \partial s$.

the choice of optimal support levels. The first-order condition with respect to s_l entails

$$\frac{1}{\tilde{\theta}-\theta_o}\int_{\theta_o}^{\tilde{\theta}} \theta d\theta = 2cs_l \tag{2.5}$$

which shows that the average marginal utility of group N_l equals the marginal cost of support s_l. Likewise for group N_h, the first-order condition with respect to s_h entails

$$\frac{1}{\overline{\theta}-\tilde{\theta}}\int_{\tilde{\theta}}^{\overline{\theta}} \theta d\theta = 2cs_h \tag{2.6}$$

The conditions (2.3), (2.4), (2.5), and (2.6) yield the equilibrium:

$$s_l^w = \frac{\overline{\theta}}{5c},\ s_h^w = \frac{2\overline{\theta}}{5c},\ \theta_o^w = \frac{\overline{\theta}}{5},\ \tilde{\theta}^w = \frac{3\overline{\theta}}{5} \tag{2.7}$$

The condition $\theta_o > \underline{\theta}$, which defines case 1, is met when $\underline{\theta} < \overline{\theta}/5$.

How support is allocated among consumers depends on the θ of the consumer with the highest willingness to pay. As $\overline{\theta}$ increases, the size of groups N_l and N_h increases. However, the proportion of members of these groups to the total number of consumers decreases with $\overline{\theta}$ because $\partial\frac{\overline{\theta}-\theta_o}{\overline{\theta}-\underline{\theta}}/\partial\overline{\theta} < 0$. Finally, note that all the equilibrium values are homogeneous of degree 1 in $\overline{\theta}$.

Case 2. $\underline{\theta} > \overline{\theta}/5$.

In this case, the welfare function is

$$W(s_l,\ s_h) = \int_{\underline{\theta}}^{\tilde{\theta}} \left(\theta(\underline{s}+s_l) - cs_l^2\right) d\theta + \int_{\tilde{\theta}}^{\overline{\theta}} \left(\theta(\underline{s}+s_h) - cs_h^2\right) d\theta. \tag{2.8}$$

The first-order condition of (2.8) with respect to $\tilde{\theta}$ yields $\tilde{\theta} = c(s_l + s_h)$, which is the same condition as (2.3).

Maximizing (2.8) with respect to s_l and s_h yields two conditions: $\frac{1}{\tilde{\theta}-\underline{\theta}}\int_{\underline{\theta}}^{\tilde{\theta}} \theta d\theta = 2cs_l$ and $\frac{1}{\overline{\theta}-\tilde{\theta}}\int_{\underline{\theta}}^{\tilde{\theta}} \theta d\theta = 2cs_h$. These conditions show that the social planner sets support so that the average marginal utility in one group equals the marginal cost of support offered to that group.

The solution is given by

$$s_l^w = \frac{\overline{\theta} + 3\underline{\theta}}{8c},\ s_h^w = \frac{3\overline{\theta} + \underline{\theta}}{8c},\ ,\theta_o^w = \underline{\theta},\ \tilde{\theta}^w = \frac{\overline{\theta} + \underline{\theta}}{2}, \tag{2.9}$$

Both s_l and s_h are higher than what they were in case 1 (the allocation for case 1 is given in (2.7)). This is because the average willingness to pay for support is greater than in case 1.

The profit maximizing firm selling support for a closed source software

As for the social planner, the firm offers two levels of support, but it also charges for the software. As opposed to the social planner, the firm is concerned with the marginal consumer of a group, not the average consumer. The reason is that the marginal consumer dictates the price the firm can charge to consumers in a group.

Again, there are two cases to consider: case 1 where $\underline{\theta} < 3\overline{\theta}/5$ and case 2 where $\underline{\theta} \geq 3\overline{\theta}/5$.

Case 1. $\underline{\theta} < 3\overline{\theta}/5$.

The firm offers three choices: (a) It lets consumers purchase the software without extra support at the price p_b; (b) it lets them purchase support with extra support s_l at the price p_l; and (c) it lets them purchase extra support s_h at the price p_h.

The firm sets the maximum price p_b consistent with the assumption of full market coverage. That is, the price at which the consumer with the lowest willingness to pay for support is left without surplus. That price is $p_b^{cs} = \underline{\theta}\,\underline{s}$. The prices p_l and p_h must relate to the indices θ_o and $\tilde{\theta}$ of the marginal consumers as shown below

$$p_l(\theta_o;\ s_l) = \theta_o s_l + p_b$$

and

$$p_h(\theta_o,\ \tilde{\theta};\ s_l,\ s_h) = \tilde{\theta}(s_h - s_l) + p_l(\theta_o)$$

The firm's profit maximization problem is

$$\max_{\substack{\theta_o,\ \tilde{\theta}\\ s_l,\ s_h}} \pi(s_l,\ s_h) = \int_{\underline{\theta}}^{\theta_o} p_b d\theta + \int_{\theta_o}^{\tilde{\theta}} \left(p_l(\theta_o;\ s_l) - cs_l^2\right) d\theta + \int_{\tilde{\theta}}^{\overline{\theta}} \left(p_h(\theta_o,\ \tilde{\theta};\ s_l,\ s_h) - cs_h^2\right) d\theta \tag{2.10}$$

The first-order condition of (2.10) with respect to s_l entails

$$\theta_o = 2cs_l + (\overline{\theta} - \tilde{\theta}) \tag{2.11}$$

The left-hand side of (2.11) is the marginal willingness to pay of consumer θ_o which is also the increase in price that the firm can charge for support s_l. The right-hand side is the sum of the marginal cost of support s_l and a term which captures the effect of the increase in support s_l on the revenue the firm derives from group N_h. That revenue falls because the firm optimally lowers p_h in response to an increase in s_l.

The first-order condition of (2.10) with respect to s_h yields

$$\tilde{\theta} = 2cs_h$$

The remaining two conditions for optimality are $\theta_o = (cs_l + \overline{\theta})/2$ and $\tilde{\theta} = \left(c(s_l + s_h) + \overline{\theta}\right)/2$. Jointly, the four first-order conditions yield

$$s_l^{cs} = \frac{\overline{\theta}}{5c},\ s_h^{cs} = \frac{2\overline{\theta}}{5c},\ ,\theta_o^{cs} = \frac{3\overline{\theta}}{5},\ \tilde{\theta}^{cs} = \frac{4\overline{\theta}}{5} \tag{2.12}$$

The condition $\theta_o^{cs} > \underline{\theta}$, defining case 1, is met when $\underline{\theta} < 3\overline{\theta}/5$. The equilibrium prices are

$$p_b^{cs} = \underline{s}\,\underline{\theta},\ p_l^{cs} = \frac{3\overline{\theta}^2}{25c} + \underline{s}\,\underline{\theta},\ p_h^{cs} = \frac{7\overline{\theta}^2}{25c} + \underline{s}\,\underline{\theta} \tag{2.13}$$

The prices the firm can charge increase in basic support, $\underline{s}$. In fact, for any given level of extra support, the product is more valuable when there is more basic support. Note, though, that the equilibrium level of extra support does not depend on the amount of basic support.

The profits are

$$\pi^{cs} = \frac{\overline{\theta}^3}{25c} + \underline{\theta}\,\underline{s}(\overline{\theta} - \underline{\theta}) \tag{2.14}$$

Comparing solution (2.12) with the solution (2.7) reveals that all consumers who purchase extra support from the firm would also obtain extra support from the social planner. The converse, though, is not true. Specifically, when $\underline{\theta} \in [0,\ 3\overline{\theta}/5)$, some consumers obtain extra support from a social planner, but do not purchase extra support from a profit maximizing firm. This is because the social planner offers supports at marginal cost, but the firm sets its prices above marginal cost.

A comparison of θ_o^w given by (2.7) with θ_o^{cs} given by (2.12) shows that, in the welfare maximizing case, twice as many consumers enjoy extra support.

Proposition 9. *The firm chooses the welfare maximizing levels of support.*

Proof. A comparison of (2.7) and (2.12) reveals that levels of support are the same. □

The result stated in proposition 9 is due to the linearity of the utility function and the uniform distribution of tastes.[8] To see why, recall first that the monopolist chooses s_h so that $\frac{\partial p_h}{\partial s_h} = \tilde{\theta} = 2cs_h$. The social planner, by contrast, sets s_h so that $\frac{1}{\overline{\theta}-\tilde{\theta}} \int_{\tilde{\theta}}^{\overline{\theta}} \theta d\theta = \frac{\overline{\theta}+\tilde{\theta}}{2} = 2cs_h$ (see (2.2)). Note that $(1/(\overline{\theta} - \tilde{\theta})) \int_{\tilde{\theta}}^{\overline{\theta}} \theta d\theta$ is the average marginal valuation of support. On the whole, the s_h chosen by the firm does not depend on the distribution of θ whereas the s_h set by the social planner does because the social planner equals its marginal cost to the average willingness to pay.

Case 2. $\underline{\theta} \geq 3\overline{\theta}/5$.

In this case, allocation (2.12) is no longer an equilibrium because $\theta_o < \underline{\theta}$. Here, each consumer purchases either s_l or s_h, and the profit maximizing equilibrium is a solution to

$$\max_{\tilde{\theta},\, s_l,\, s_h} \pi(s_l,\ s_h) = \int_{\underline{\theta}}^{\tilde{\theta}} \left(\underline{\theta}(\underline{s} + s_l) - cs_l^2\right) d\theta + \int_{\tilde{\theta}}^{\overline{\theta}} \left(\tilde{\theta}(s_h - s_l) + \underline{\theta}(\underline{s} + s_l) - cs_h^2\right) d\theta$$

[8] Spence [1975] provides a detailed description of how the profit maximizing allocation differs from the welfare maximizing allocation of quality. The difference depends on the demand function and the distribution of consumers.

The first-order conditions entail the following

$$\tilde{\theta} \quad = \quad \frac{c(s_l + s_h) + \overline{\theta}}{2};\ s_l = \frac{\tilde{\theta} - (\overline{\theta} - \underline{\theta})}{2c};\ s_h = \frac{\tilde{\theta}}{2c}$$

Jointly, these optimality conditions yield

$$s_l^{cs} = \frac{3\underline{\theta} - \overline{\theta}}{4c},\ s_h^{cs} = \frac{\underline{\theta} + \overline{\theta}}{4c},\ \tilde{\theta}^{cs} = \frac{\underline{\theta} + \overline{\theta}}{2} \tag{2.15}$$

First note that, in solution (2.15), s_l is positive since $\underline{\theta} \geq 3\overline{\theta}/5$. The social planners solution (2.9) shows that the marginal consumer $\tilde{\theta}$ is at the same location as with the profit maximizer. However, the two extra supports offered by the firm differ from those offered by the social planner. In addition, at $\underline{\theta} = 3\overline{\theta}/5$ the solutions for case 1 given by (2.15) coincides with that for case 2 given by (2.12).

The profit maximizing prices are

$$p_l = \underline{\theta}\frac{3\underline{\theta} - \overline{\theta}}{4c} + \underline{s}\,\underline{\theta},\ p_h = \frac{\overline{\theta}^2 - \overline{\theta}\underline{\theta} + 2\underline{\theta}^2}{4c} + \underline{s}\,\underline{\theta} \tag{2.16}$$

These prices will provide a reference when comparing the closed source with the open source solution.

The profit maximizing firm selling support for open source software

The firm makes more profit with closed source licensing than with open source licensing because under the former the firm sells its license. The purpose here is not to justify the choice of open over closed source software, but to compare the choices of support between a closed and an open source regimes. The question is how the fact that the source code is available for free affects the choice of prices and supports.

By opening the source code, the firm offers a passive substitute to the support it wishes to sell. Thus, the worst a consumer indexed θ can do, when presented the choice of open source software, is to obtain a strictly positive surplus $\theta\underline{s}$. Therefore, with open source software the market is always covered.

Again, there are two cases to consider. The first case has $\underline{\theta} < 3\overline{\theta}/5$ and the second

$\underline{\theta} \geq 3\overline{\theta}/5$.

Case 1. $\underline{\theta} < 3\overline{\theta}/5$.

As with the closed source, consumers have three choices. Consumers use the software without extra support for free or purchase one of the two extra supports the firm offers. Again, the prices p_l and p_h must relate to the indices θ_o and $\tilde{\theta}$ of the marginal consumers. That is, $p_l(\theta_o;\, s_l) = \theta_o s_l$ and $p_h(\theta_o,\, \tilde{\theta};\, s_l,\, s_h) = \tilde{\theta}(s_h - s_l) + p_l(\theta_o,\, s_l)$. These prices equal those in the closed source case minus p_b — where p_b is the price at which the firm sells its software without extra support.

The firm's profit is given by

$$\begin{aligned} \pi(s_l,\, s_h) \quad &= \quad \int_{\theta_o}^{\tilde{\theta}} \left(p_l(\theta_o;\, s_l) - c s_l^2\right) d\theta \\ &\quad + \int_{\tilde{\theta}}^{\overline{\theta}} \left(p_h(\theta_o,\, \tilde{\theta};\, s_l,\, s_h) - c s_h^2\right) d\theta \end{aligned} \tag{2.17}$$

The first-order conditions are

$$s_l(c s_l + \overline{\theta} - 2\theta_o) = 0;\ (s_h - s_l)\left(c(s_l + s_h) + \overline{\theta} - 2\tilde{\theta}\right) = 0$$

and

$$\frac{1}{4} c^2 s_h (s_h - 2 s_l) = 0;\ \frac{1}{4}\left(c^2\left(3 s_h^2 + 2 s_h s_l - s_l^2\right) + \overline{\theta}(\overline{\theta} - 4 c s_h)\right) = 0$$

Jointly, these four conditions yield the solution:

$$s_l^{os} = \frac{\overline{\theta}}{5c},\ s_h^{os} = \frac{2\overline{\theta}}{5c},\ \theta_o^{os} = \frac{3}{5}\overline{\theta},\ \tilde{\theta}^{os} = \frac{4}{5}\overline{\theta} \tag{2.18}$$

Thus, the allocation is the same as with closed source (see (2.12)). Note that the closed source problem reduces to the open source problem when $\underline{s} = 0$. In that sense, the open source problem can be seen as a special case of the closed source problem. Inspection of the closed source solution (2.12) shows that $\underline{s}$ does not enter the expressions defining the equilibrium values. Thus, opening the source code does not affect the extra support offered or the index of the marginal consumers.[9]

The equilibrium prices are

$$p_l = \frac{3\overline{\theta}^2}{25c},\; p_h = \frac{7\overline{\theta}^2}{25c} \tag{2.19}$$

These prices are lower than under closed source licensing (see (2.13)). By not revealing its source code the firm is able to charge $\underline{\theta}\,\underline{s}$ more than by doing so. This price difference corresponds to the value of the software with basic support for the consumer with the lowest willingness to pay. Here, the firm's profit is $\Pi^{os} = \frac{\overline{\theta}^3}{25c}$ which is lower than the closed source profit given by (2.10). These profits differ by the amount $\underline{\theta}\,\underline{s}(\overline{\theta} - \underline{\theta})$ which corresponds to the price the firm charges for its software without extra support multiplied by the total number of consumers.

Case 2. $\underline{\theta} \geq 3\overline{\theta}/5$.

The problem is the same as in case 2 of the preceding subsection. As a result, the allocation is the same as with a closed source software (see (2.15)). The only difference is that, here, the firm must charge a lower price. The prices are

$$p_l = \underline{\theta}\frac{3\underline{\theta} - \overline{\theta}}{4c};\; p_h = \frac{\overline{\theta}^2 - \underline{\theta}\overline{\theta} + 2\underline{\theta}^2}{4c} \tag{2.20}$$

Again, even if all consumers purchase support, the prices under the open source regime are lower than under the closed source regime. (It follows upon comparison of (2.20) with (2.16).) Contrary to expectations, the outside support $\underline{s}$ has no effect for a firm providing its software as open source. Although consumers do benefit from $\underline{s}$, publishing as open source precludes the firm from extracting profits. This is because consumers do benefit from $\underline{s}$ whether they choose the free software alone or purchase support. In contrast, the closed source firm can benefit from the basic support because consumers must purchase the software from the firm in order to enjoy this basic support.

Proposition 10. *Total welfare is the same in the open source case as in the closed source case, but the firm's profit is lower and the consumer's surplus higher.*

[9]This feature of the model supports the conjecture made by Campbell-Kelly and Garcia-Swartz [2010]. Their thesis is that open and closed source strategies, regarding the sale of professional support in particular, are close or getting closer to each other.

Proof. It suffices to note that the choice of support is the same in both cases and that the marginal consumers are the same in both cases. (It follows upon comparison of (2.12) with (2.18), and the fact that (2.15) is also the same in both cases.) Since only the price differs, opening the source code transfers part of the surplus from the firm to the consumers. To see that closed source prices are higher one compares (2.13) with (2.19) and (2.16) with (2.20). □

Having set up the three baselines solutions where there is only one decision-maker, I now consider competition between two providers of support for the open source software — competition between two decision-makers.

2.3 Application 1: Two firms offer support

In this section, an outsider enters the market and competes with the software licensor for the provision of support. This rival exploits the fact that the source code is open. Such a situation is often observed in a market for open source software. Openness enables outside firms to study the licensor's source code and to provide support for it [Campbell-Kelly and Garcia-Swartz, 2010].

Setup and assumptions

Both firms seek to maximize profit. The cost function is as defined in (2.1) except that, here, the outsider is less efficient in the provision of support than the licensor is, viz. $c_l \geq c_h$.[10] The outsider has a higher cost because his knowledge of the source code is not as acute as that of the licensor. To allow for easy comparison with the previous section, I impose that each firm offers a single support. This way two levels of support are available to consumers. In addition, to keep the argument as simple as possible, I assume that consumers choose either the support of the outsider or the support of the licensor. To reduce the number of cases that needs to be treated, I further assume that $s_l < s_h$.[11] Finally, I suppose that $\underline{\theta} > 3\overline{\theta}/5$, so the context is the same as in the monopoly case of section 2.2. The other assumptions of the model are as in the previous section.

The timing of the game is as follows:

Stage 0: Initial costs are revealed to the owner of the source code and to the outside firm: c_h and c_l, respectively.

Stage 1: The owner of the code chooses to offer support, s_h, and the outside firm chooses to offer support, s_l.

[10] The closest to this approach in terms of firm's asymmetry is the paper of Matsubayashi and Yamada [2008]. They model firms that compete not only on price but also on quality. They use a vertical differentiation approach and the firms are dissimilar in that their consumers differ in their loyalty for one or the other firm. They show that when consumers' quality-sensitivity is severe the firm that has a lesser amount of loyal consumers suffers in that quality-sensitive consumers force the firm to lower its price and quality which in turn lowers its profit. To my knowledge, within the framework of vertical differentiation, no paper consider competition between firms that differ in their costs.

[11] Consequently, for any equilibrium in price it must be that $p_l < p_h$ or else no consumer would choose support s_l.

Stage 2: The two firms choose the price associated with the support they offer: p_h and p_l.

Stage 3: Consumers choose the support which maximizes their respective surplus.

I make a distinction between stage 1 (choice of support) and stage 2 (choice of price) because the price can be changed easily, but change in support requires investment in material to provide that support and in time to form employees. Both firms make a decision on price after knowing the support offered by its competitor.[12] Given the chain of events, I look for a subgame perfect equilibrium where the strategies of the firms are price and support.

Equilibrium

The indifferent consumer $\tilde{\theta} = \frac{p_h - p_l}{s_h - s_l}$ determines demand for both firms, that is, $N_h = \overline{\theta} - \tilde{\theta}$ and $N_l = \tilde{\theta} - \underline{\theta}$. Firm i's profit is therefore

$$\pi_i = (p_i - c_i s_i^2) N_i \text{ for } i = l,\ h$$

Each firm maximizes its profit with respect to its price yielding the first-order conditions for a price equilibrium: $(s_h - s_l)N_i = p_i - c_i s_i^2$ for $i = l,\ h.$ The conditions are so that the difference in support times a firm's market size equals the profit per consumer. Thus, the firms' profit per consumer increases with the difference in support. This property show how the relative positions of s_l and s_h dictate the intensity of competition in prices.

Solving these first-order conditions simultaneously yields the equilibrium prices[13] as function of s_l and s_h

$$p_l(s_l,\ s_h) = \frac{c_h s_h^2 + 2c_l s_l^2 + (s_h - s_l)(\overline{\theta} - 2\underline{\theta})}{3};\ p_h(s_l,\ s_h) = \frac{2c_h s_h^2 + c_l s_l^2 + (s_h - s_l)(2\overline{\theta} - \underline{\theta})}{3}$$

[12]Though he is interested in investment in quality, Mustonen [2003] makes a similar distinction in the sequencing of events in his model.

[13]Formally, to call these an equilibrium it must be verified that $p_l < p_h$ and that $p_l/s_l \leq \underline{\theta}$. I assume that these are satisfied and proceed to the equilibrium in support. Then, I verify that the support derived are indeed an equilibrium.

The licensor's profit is then rewritten as a function of s_l and s_h

$$\pi_h(s_l,\ s_h) = \frac{\left(-c_h s_h^2 + c_l s_l^2 + (s_h - s_l)(2\overline{\theta} - \underline{\theta})\right)^2}{9(s_h - s_l)}$$

The outsider's profit is likewise given as

$$\pi_l(s_l,\ s_h) = \frac{\left(c_h s_h^2 - c_l s_l^2 + (s_h - s_l)(\overline{\theta} - 2\underline{\theta})\right)^2}{9(s_h - s_l)}$$

The first-order conditions of these profit functions yield equilibrium support[14] for the licensor

$$s_h^c = \begin{cases} \frac{\left((4c_l - 7c_h)\overline{\theta} + 5c_h\underline{\theta} - 2c_l\underline{\theta} + \sqrt{\Omega}\right)}{12c_h(c_l - c_h)} & \text{for } c_l > c_h \\ \frac{5\overline{\theta} - \underline{\theta}}{8c} & \text{for } c_l = c_h = c \end{cases} \tag{2.21}$$

and equilibrium support for the outsider

$$s_l^c = \begin{cases} \frac{\left((2c_l - 5c_h)\overline{\theta} + (7c_h - 4c_l)\underline{\theta} + \sqrt{\Omega}\right)}{12c_l(c_l - c_h)} & \text{for } c_l > c_h \\ \frac{5\underline{\theta} - \overline{\theta}}{8c} & \text{for } c_l = c_h = c \end{cases} \tag{2.22}$$

where

$$\Omega = 4c_h^2(\overline{\theta} - 2\underline{\theta})^2 + 4c_l^2(2\overline{\theta} - \underline{\theta})^2 - c_h c_l\left(11\overline{\theta}^2 - 14\overline{\theta}\underline{\theta} + 11\underline{\theta}^2\right) \tag{2.23}$$

Note that Ω is positive since $\Omega > \left[2c_h(\overline{\theta} - 2\underline{\theta}) - 2c_l(2\overline{\theta} - \underline{\theta})\right]^2 \geq 0$.

[14]The condition imposed that the support levels are such that $s_l < s_h$ is indeed verified in equilibrium. Also, the equilibrium supports defined in (2.21) and (2.22) for $c_l > c_h$ converge to the solution at $c_l = c_h$. Using l'Hôpital's rule to evaluate $\lim_{c_h \to c_l^-} s_i$, for $i = l,\ h$ shows that support solution for $c_l > c_h$ approaches the solution at $c_l = c_h = c$ as c_h approaches c_l from below.

To ensure that the solution is an equilibrium it is necessary that the assumption that the market is covered is satisfied, that is $p_l^c / s_l^c < \underline{\theta}$. This assumption is satisfied for $\underline{\theta} \geq 5\overline{\theta}/7$. Otherwise the relation between the firms's efficiency parameters must be so that

$$\frac{c_l\left(67\overline{\theta}^2 - 98\overline{\theta}\underline{\theta} + 63\underline{\theta}^2\right)}{32\left(2\overline{\theta}^2 - 7\overline{\theta}\underline{\theta} + 9\underline{\theta}^2\right)} - \frac{3}{32}\frac{c_l(7\underline{\theta} - 3\overline{\theta})}{2\overline{\theta}^2 - 7\overline{\theta}\underline{\theta} + 9\underline{\theta}^2}\sqrt{126\overline{\theta}\underline{\theta} + 9\underline{\theta}^2 - 71\overline{\theta}^2} < c_h$$

In addition, it must be verified that the assumption $s_l < s_h$ holds true. This is indeed satisfied and is discussed in the body of the text.

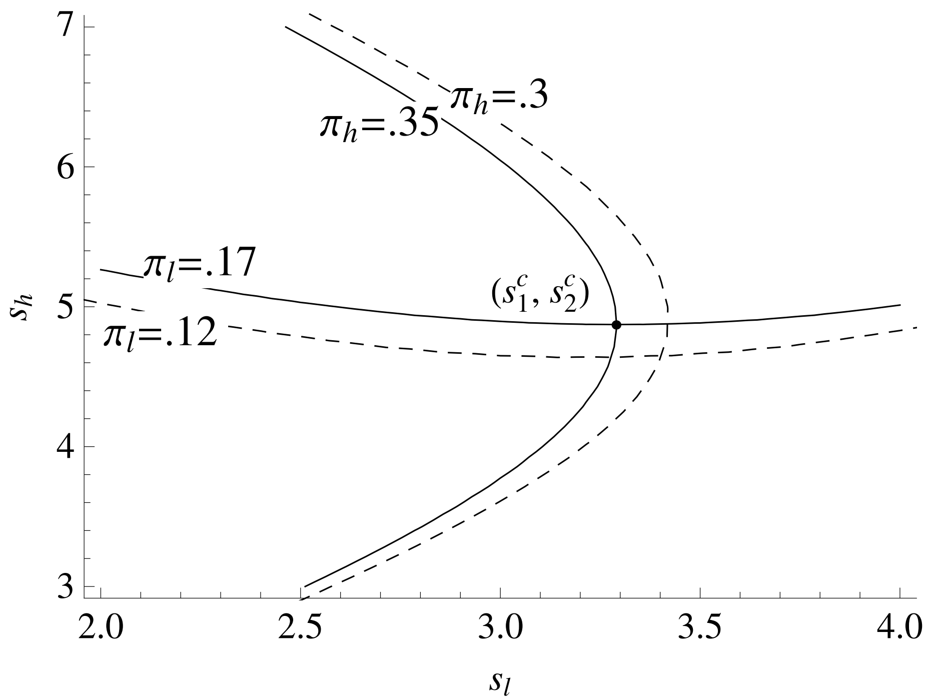

Figure 2.1: Plot of iso-profit contours of the two firms and the associated Nash equilibrium in support: $(s_l^c,\ s_h^c) = (3.29,\ 4.87)$. The values of the parameters are $\overline{\theta} = 3.8$, $\underline{\theta} = 3$, $c_h = 0.4$, and $c_l = 0.41$.

Equilibrium properties

Figure 2.1 shows the equilibrium, and how neither firm has an incentive to deviate from it. In addition, the figure illustrates the property that for a given value of s_h, profit π_h decreases with s_l, but for a given value of s_l, profit π_l increases with s_h. Thus, the firms benefit from support being farther apart. Two of the model's assumptions cause this: vertical differentiation and $s_l < s_h$. Vertical differentiation entails that, unless they offer the same support, one of the firms provides an inferior product. As the outsider's support increases, the price competition increases because $s_l < s_h$; and, for this same reason, increases in the licensor's support dilutes price competition. Thus, as a rival's support varies in one direction, a firm adjust its own support in this same direction.

I can now study how costs affect the level of support offered by each firm. The equilibrium levels of support given by (2.21) and (2.22) show that both decreases in the cost faced by the outsider, viz. $\partial s_i^c / \partial c_l < 0$ for $i = l,\ h$. This is not only because the cost increases, but also because it reduces the competition between the two products. The outside firm reduces its support because its cost of providing it increases. The licensor, in turn, reduces his support because he has lower competitive pressure from the outsider.

For the outsider $\partial s_l / \partial c_h > 0$ because it tries to attract consumers with a high valuation of support when the cost of support of its rival increases. Thus, the level of support of a rival firm has an opposite effect for both firms. From the point of view of the licensor, an increase in support s_l lowers the range from which it can choose its own support. Recall that there is an endogenous upper bound on support due to the convexity of costs and the linearity of tastes. In addition, the licensor does not want to choose a level of support that is too low because doing so intensifies the competition in price. If the support offered by the licensor increases, the price competition is reduced.

How the licensor's support varies in its own cost is more intricate. The level of support s_h^c may either decrease or increase in its cost. The cost, c_h, has both a direct and an indirect effect on s_h^c, and the direction of change depends on the respective magnitude of these effects. The direct effect is simply that as the cost of support increases the firm wishes to offer less of it. The indirect effect follows from the property that $\partial s_l / \partial c_h > 0$ which entails that price competition intensifies as c_h increases. The following proposition summarizes the discussion.

Proposition 11. *The support offered by the owner of the source code increases in c_h for $c_h > c_h^o$ when $\underline{\theta} > 13\overline{\theta}/17$.*

Proof. It must be noted that since $\Omega > 0$, the function $s_h^c(c_h)$ is continuous for $c_h < c_l$. In addition, $s_h^c(c_h) = \frac{5\overline{\theta}-\underline{\theta}}{8c_l}$ for $c_h = c_h^o = c_l \frac{4(7\overline{\theta}-5\underline{\theta})}{3(5\overline{\theta}-\underline{\theta})}$. Then, note that $c_h^o < c_l$ for $\underline{\theta} > 13\overline{\theta}/17$. I now show that the function $s_h^c(c_l,\, c_h)$ is decreasing at point c_h^o when $c_h^o < c_l$. The derivative $\frac{\partial s_h^c}{\partial c_h}$ evaluated at c_h^o yields

$$-\frac{3(17\underline{\theta}-13\overline{\theta})(5\underline{\theta}-\underline{\theta})^2}{64c_l^2(23\overline{\theta}-19\underline{\theta})(2\underline{\theta}-\overline{\theta})}$$

Thus, for $\underline{\theta} > 13\overline{\theta}/17$ the derivative $\frac{\partial s_h(c_l,\, c_h^o)}{\partial c_h} < 0$, and so the function s_h^c is decreasing at point c_h^o. Since the function is continuous for $c_h < c_l$ and $s_h^c \to \frac{5\overline{\theta}-\underline{\theta}}{8c_l}$ as $c_h \to c_l$, I conclude that when $\underline{\theta} > 13\overline{\theta}/17$, the function increases for $c_h \in \left[c_h^o,\, c_l\right]$.

□

Corollary 12. *The support offered by the owner of the code when costs differ may be higher or lower than the support offered when the two firms face the same cost function.*

Proof. It follows directly from proposition 11. □

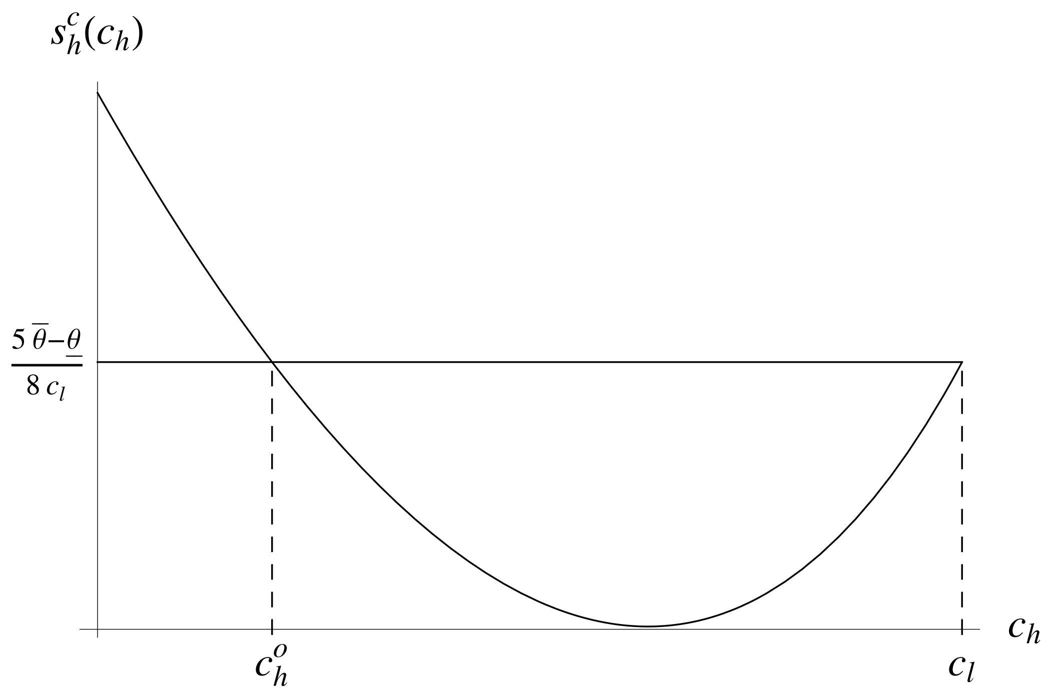

Figure 2.2: Change licensor's optimal support as a function of cost c_h. The range showed is for $c_l \in [.18,\ .2]$, and the parameter values are $\overline{\theta} = 1$, $\underline{\theta} = .2$, $\underline{s} = .1$, and $c_l = .2$.

Figure 2.2 illustrates how the support offered by the owner of the code may be higher or lower than the support offered when both costs are the same. At the same it shows that the support offered by the licensor may be increasing in its own cost.

Comparison with the monopoly

I now compare the competitive case with the monopoly case. I begin with the allocation of support among consumers.

Proposition 13. *The number of consumers purchasing support s_h under competition is greater or equal than under a monopoly.*

Proof. When all consumers purchase support in the monopoly case the number of consumers purchasing s_h is $\overline{\theta} - \frac{\overline{\theta}+\underline{\theta}}{2}$. When two firms offer support that number is $\overline{\theta} - \tilde{\theta}$ where $\tilde{\theta}$ is given by

$$\tilde{\theta} = \frac{c_l(\overline{\theta}+4\underline{\theta}) + c_h(4\overline{\theta}+\underline{\theta}) + \sqrt{\Omega}}{9(c_l - c_h)} \tag{2.24}$$

First note that in the limit case where $c_l = c_h$, the ratio $\tilde{\theta}\left(\frac{\overline{\theta}+\underline{\theta}}{2}\right)^{-1}$ equals one. Thus, for

$c_l = c_h$, the number of consumers purchasing s_h is equal in both the competitive and the monopoly case. When $c_l > c_h$, I show that $\tilde{\theta}\left(\frac{\overline{\theta}+\underline{\theta}}{2}\right)^{-1} < 1$. To simplify the argument, I rearrange the latter inequality as

$$2\sqrt{\Omega} < c_l(7\overline{\theta}+\underline{\theta}) - c_h(\overline{\theta}+7\underline{\theta})$$

The right-hand side of this inequality is linearly decreasing in c_h and the left-hand side is quadratic, convex and decreasing in c_h (see the definition of Ω in (2.23)). To see that Ω is convex it suffices to note that its second derivative is positive $\frac{\partial^2\Omega}{\partial c_h^2} = 8(\overline{\theta}-2\underline{\theta})^2 > 0$. The inequality holds for $c_h = 0$ since $\Omega\Big|_{c_h=0} = 4c_l^2(2\overline{\theta}-\underline{\theta})^2$, and since I assume that $\underline{\theta} > 3\overline{\theta}/5$. Then, this last remark, the fact that the left-hand side is convex, and the fact both sides are equal at $c_l = c_h$ imply that $2\sqrt{\Omega}$ lies below the line $c_l(7\overline{\theta}+\underline{\theta}) - c_h(\overline{\theta}+7\underline{\theta})$ for c_h on the interval $[0,\ c_l)$. □

The result stated in proposition 13 is due to the differences in costs. In fact as $c_l \to c_h$ the market segmentation approaches the segmentation in the welfare maximizing outcome. Otherwise, as cost c_l increases, it forces the outsider to lower the support it offers. This reduces the price competition and allows the licensor to choose a support closer to what the monopolist would choose if it were to offer only one level of support.[15]

I compare the level of support s_l^c offered by the outside firm with the level of support the monopolist offers, $s_l^m = \frac{3\underline{\theta}-\overline{\theta}}{4c}$. The difference between c_l and c_h determine whether the outside firm offers more or less support than the monopolist. When $c_l = c_h = c$, the low support level is higher under competition than under monopoly (this is clear upon comparison of (2.15) with (2.22)). When c_h is low, however, the level of support s_l^c is lower than s_l^m. To see this note that $\lim_{c_h\to 0} s_l^c = \frac{5\underline{\theta}-\overline{\theta}}{12c_l}$ which is lower than $s_l^{os} = \frac{3\underline{\theta}-\overline{\theta}}{4c}$. When the cost of support is low, the monopolist offers a high support in the monopoly case and increases its price accordingly. The outside firm, however, faces competition from a firm facing a low cost for its support s_h. Thus, when the firms face similar costs functions, the consumers benefit from a higher level of support s_l; but when the outside firm faces significantly higher cost, the consumers are served a lower level of support.

[15]Assuming all consumers purchase extra support, a monopolist providing one level of support offers $s = \underline{\theta}/2c$.

I now compare the level of support s_h^c offered by the owner of the code with the level of support the monopolist offers, $s_h^m = \frac{\underline{\theta}+\overline{\theta}}{4c}$. First, the two are equal for $\tilde{c}_h = c_l \frac{-5\overline{\theta}^2+2\overline{\theta}\underline{\theta}+7\overline{\theta}^2}{4(\overline{\theta}-2\underline{\theta})^2}$, and $s_h^{os} > s_h^c$ for $c_h < \tilde{c}_h$ and $s_h^{os} < s_h^c$ for $c_h > \tilde{c}_h$. This threshold means that the support offered by the licensor is higher than in the monopoly case when the firm receives strong competition from the outsider. That is, higher when the cost c_l is not too large relative to the cost faced by the licensor. However, when the outside firm's cost is large, the firm offers a lower support than in the monopoly case. The reason for this is that the firm, since it sells only one level of support, lowers its support in order to attract more consumers when the outside firm faces high costs. It can do so because the prohibitive cost precludes the outside firm from offering a high enough support to serve consumers with a high willingness to pay for support.

2.4 Application 2: The dual licensing model

This section shows that, even without network externalities and user contributions, the firm may prefer the dual licensing model to the closed source model.[16] Software firms typically generate revenues from offering software services, but under dual licensing they may also create revenues both from offering support and selling licenses.

The dual licensing model is especially appropriate in markets where an important proportion of consumers use the code to embed into new software products. Developers use open source codes as an input in the production of software or as part of more complex products.[17] Developers typically want to maintain control over their product, therefore they prefer paying for the source code rather than revealing their technology. Consequently, these consumers not only value the code, but also value a license which allows them to keep their innovation, derived from the source code, private. In other words, they need a license which is unrestricted. To users, the license type is inconsequential because they do not wish to modify its code, license restrictions are concerned with redistribution. [18]

The open and closed source structures introduced in section 2.2 serve as a basis for examining the nature of the equilibrium when two licenses are available. The dual approach borrows from the closed source case in that the firm can sell licenses. The model also borrows from the open source model since the free code enables, under restrictions, consumers to use the software for free. Price discrimination is at the core of this model because the market is segmented into two groups: users downloading the software for free, and developers purchasing an unrestricted license.

In this section, I assume that developers place more value on the open source software than users because the former are interested in a code which they can modify. The restricted software holds less value for developers as it prevents them to publish the work which they will derive from the software in question.

[16] For a discussion on dual licensing see Välimäki [2003], Comino and Manenti [2007], and Campbell-Kelly and Garcia-Swartz [2010].

[17] Open source software is often part of complex products such as mobile phones, cars' on-board computer, or digital media renderers (blue-Ray readers, tablets, picture frames).

[18] Lerner and Tirole [2005b] enumerates the various considerations that figure into the firm's choice of license. Their stylized facts are derived from the study of 40,000 open source projects. They show that the choice of the firm is driven not only by the preferences of the firm itself, but also by the preferences of developers. In particular, they find that software products aimed at users tend to have restrictive licenses, but software products designed to benefit developers are less likely to be published under restrictive licenses.

has lower value for developers because it restrains them in publishing the work which they create from such a software.

Table I shows the utility derived from software where $q > 0$ represents the code's quality and $\rho \in [0,\ 1]$ the level of restriction. A source code with restriction $\rho \in (0,\ 1)$ and quality q has an added value of $q(1-\rho)$ to developers. Users are indexed u, and they comprise a fraction λ of the total number of consumers. Developers are indexed d, and they comprise a fraction $1-\lambda$ of the total number of consumers. The costs are as described by (2.1).

	Consumers	
	Users	Developers
Closed source	$\theta(\underline{s}+s)$	$\theta(\underline{s}+s)$
Open source Restricted	$\theta(\underline{s}+s)$	$\theta(\underline{s}+s)+q(1-\rho)$
Open source Unrestricted	$\theta(\underline{s}+s)$	$\theta(\underline{s}+s)+q$

Table I: Consumers' willingness to pay.

I first assume that the firm can perfectly discern the type of consumers by observing some exogenous characteristic of consumers such as his IP address. I assume that the license has no cost.[19] The firm offers support s_d at price p_d and support s_u at price p_u. In addition, the firm offers an unrestricted license without extra support at price p_b.

No developer chooses the restricted software without support if $p_h < q\rho$. Thus, if the firm wishes to sell to all developers, the profit maximizing price for the unrestricted software without support is $p_h = q\rho$. This is the highest price the firm can charge so that developers are just indifferent between the free restricted license and the unrestricted license without support, of course the firm could charge a price slightly below this value to ensure that developers do indeed opt for the unrestricted license. For simplicity, I assume that developers purchase the unrestricted license when they are indifferent. The firm charges price $p_d(\theta_d,\ s_d) = \theta_d s_d + p_b$ to developers who purchase support s_d, where θ_d is

[19]In reality, the creation of the unrestricted license has a fixed cost. However, once that cost is sunk, there is no variable cost associated with the sale of the license that is if one puts aside the cost of monitoring for copyright infringement.

the marginal developer. The profit the firm earns from selling to developers is

$$\pi_h = (1-\lambda)\left(\int_{\underline{\theta}}^{\theta_d} \rho q d\theta + \int_{\theta_d}^{\overline{\theta}} (p_d(\theta_d,\, s_d) - cs_d^2) d\theta\right) \tag{2.25}$$

The maximum price the firm can charge the marginal user θ_u is $p_u = \theta_u s_u$. The profit from selling to users is

$$\pi_u = \lambda\left(\int_{\theta_u}^{\overline{\theta}} (\theta_u s_u - cs_u^2) d\theta\right) \tag{2.26}$$

I look at a case where the taste parameters are such that some developers do purchase the license alone and where some users use the free software. Regardless of the choices made by the firm, the market is always covered because open source entails the availability of a free version of the software.

The marginal consumers (or equivalently the prices) are obtained from the first-order conditions derived from (2.25) and (2.26). Similarly, equilibrium support levels are determined from the first-order conditions of (2.25) and (2.26) with respect to s_u and s_d. The equilibrium allocation[20] is

$$s_u^h = s_d^h = \frac{\overline{\theta}}{3c},\ \theta_u^h = \theta_d^h = \frac{2}{3}\overline{\theta} \tag{2.27}$$

Thus, the firm offers the same support to developers and to users. The price charged to developers is higher because developers are willing to pay a premium for the unrestricted software. These prices are

$$p_u = \frac{2\overline{\theta}^2}{9c},\ p_b = q\rho,\ p_d = \frac{2\overline{\theta}^2}{9c} + q\rho$$

Recall that I assumed that the firm is able to determine if a consumer is a user or a developer. In fact, the prices and the levels of support chosen by the firm are such that, given the option to select the product they want, developers purchase the unrestricted software and users either use the free restricted software or purchase the restricted software with support. Thus, an equilibrium which maximizes the profit from each group taken separately

[20]This allocation is profit maximizing if $\underline{\theta} < \frac{2}{3}\overline{\theta}$.

induces the consumers to choose the product which the firm would want them to choose. The simple fact that some consumers value the source code and others do not is enough to allow the firm to discriminate.

The firm has profit

$$\pi = \frac{\overline{\theta}^3}{27c} + (1-\lambda)q\rho(\overline{\theta} - \underline{\theta}) \tag{2.28}$$

which is increasing in the level of restriction ρ because it differentiates the restricted software from the unrestricted software. This captures the stylized fact that firms using the dual model tend to choose a restrictive license.[21]

Upon inspection of the dual licensing profit given by (2.28) and the closed source profit given by (2.14), the dual license profit is higher than the single license profit if, and only if,

$$\overline{\theta}^3 \left(\frac{1}{25c} - \frac{1}{27c} \right) < ((1-\lambda)q\rho - \underline{\theta}\,\underline{s})(\overline{\theta} - \underline{\theta}) \tag{2.29}$$

Thus, $(1-\lambda)q\rho > \underline{\theta}\,\underline{s}$ is a necessary condition for dual licensing profit to be higher than closed source profit (otherwise the right-hand side of (2.29) would be negative). This explains why dual licensing is advantageous when the cost of support is sufficiently high. When the valuation of the code is high, that is when q is large, dual licensing provides higher profits than a closed source software. Note that the inequality is most likely to be satisfied when c is large.

This analysis is simple, but it highlights important features of software products that the vertical differentiation model is able to capture. It shows that differentiation alone can justify a dual licensing approach. Thus, the theory that the firm opens its source code to benefit from user contributions and a broader user base is imcomplete. The ability to price discriminate brought forth by the dual licensing is another important factor in choosing open source.

[21]Most software distributed under two licenses use the GPL for the restrictive license. Although, no formal study has been done, it is thought that the GPL is chosen because it encourages contributions. The current model does not account for users contributions, yet the firm prefers a restrictive license.

2.5 Conclusion

In this study, I describe the behavior of a monopolist providing open source software. I model a firm which offers technical support as a complement to its software. The firm offers a level of support that maximizes welfare under both an open and closed source regime. The closed and open source solutions only differ in their pricing of support levels. These prices are lower under an open source regime then under a closed source because consumers have the option of using the free software without support. Thus, consumer welfare is greater and profit lower than under closed source.

When two firms provide support, the number of consumers who purchase superior support is greater than under a monopoly. In addition, the support offered by the licensor may increase in its own cost of support. The increase in this cost triggers two counteracting effects. On one hand, the firm wants to offer less support. On the other hand, the increase intensifies the price competition as the levels of support provided by the firms edge closer to each other.

I also show that dual licensing allows the firm to target those consumers who value the source code, but dislike licenses restrictions. Developers are such consumers and are willing to pay a premium for the unrestricted license. Dual licensing may be more profitable than the closed source option. Two factors dictate whether that is the case: the proportion of developers in the market and their valuation for the source code.

Possible extensions include the following: (a) Allowing free entry of suppliers of support services; (b) Considering the possibility that developers may introduce a competing closed or open source product to the market; (c) Introducing other types of consumers, per example developers interested in the code but not affected by restrictions.

Chapter 3

Competition Between Open Source and Closed Source Software: The Contribution of Users

Two paradigms coexist in the software industry: the open source and the closed source. The former is characterized by a shared code to which anyone can contribute, while the latter has a proprietary code to which only the owner has access. Open source software may be produced through decentralized collaboration. For example, university professors created the statistical software R via contributions from outside the corporate framework.[1] The nature of open source software allows users to contribute their expertize and knowledge directly to the software. Furthermore, these contributions circulate rapidly among users at practically no cost [Demazière et al., 2007].

This paper focuses on user contributions and its impact on the competing environment between closed and open source software. More specifically, I consider three topics: (i) the effect of user contributions to open source software on the industry's market structure, (ii) the impact on welfare from the entry of either closed or open source software into the market, (iii) the consequences of price discrimination on the aforementioned effects.

Early research on this topic has focused on what motivates individuals to freely contribute to open source software; see for example Lakhani and Wolf [2005], Harhoff et al. [2003], and Lerner and Tirole [2005a], or for a review of the literature see von Krogh and von Hippel [2006]. Previous discussions such as Lerner and Tirole [2005b], Economides and Katsamakas [2006], and Scotchmer [2010] explored specific characteristics of open source relative to closed source software. However, direct competition between open and closed source software has received little consideration. Sen [2007], Jullien and Zimmermann [2008], Lin [2008], Lanzi [2009], and Casadesus-Masanell and Ghemawat [2006] study the occurrence, in a closed source software dominant market, of an open source software entrant.

While closed source products are, in general, well advertised and documented, such benefits seldom pertain to open source products. In fact, most consumers may not be aware of the existence of an open source solution to their needs. Taking this into consideration, Sen [2007] assumes that acquiring information regarding open source software is part of its cost. Closed source users, however, have no such cost. He equally suppose that both

[1]Consumers increase a software value by increasing its reliability. Their contribution is twofold: the code contains fewer errors and has increased execution speed. Contributors provide the installation, integration, and maintenance services necessary to operate the open source software efficiently and reliably.

open and closed source software products benefit from a direct network effect. His argument considers the competition between a seller of closed source software and a vendor of services for open source products.[2] He demonstrates that the vendor of services benefits from the open source having a higher cost or lower usability — the ease of use and the conveniences of software — than the closed source software.

Jullien and Zimmermann [2008] also study competition, but allow users to influence the open source software's quality. On one side, the closed source vendor chooses between two levels of quality having different fixed costs. On the other side, an open source provider, given that it invests a minimal amount, benefits from improvement done by users. These contributions determine the quality of the open source software. The authors derive the conditions under which a firm decides to invest in an open source project. Low skilled users motivate the firm to invest little, thus resulting in a lower quality product targeting a relatively price-sensitive market. In the presence of mainly highly skilled users the firm chooses a large and increasing investment. Consequently, it benefits from a high quality product and generates a profit that increases with the skill set of its consumers.

In another duopoly model, Lin [2008] considers a closed source firm competing with an open source distributor, in which consumers differ with respect to skill and experience. She demonstrates that open source may come to dominate the market when its consumers derive significant benefits from this software. However, when the open source software does not provide sufficient benefits to skilled consumers, the mere fact that it is free does not guarantee its success nor its survival.

Using vertical differentiation to study competition, Lanzi [2009] models consumers that face positive switching costs and differ in their ability to use software. Like Meng and Lee [2005], he considers a community of users interested in maximizing, not profit, but the number of open source users. The model is a two-stage game. In the first stage two entities select the quality level of their software, and in the second stage a closed source software licensor sets his price. He concludes that skilled users always choose the open source software. If switching costs are low, then upon entry, the closed source firm lowers its price

[2] Since the source code of open source software is available for free, most firms distributing open source software offer technical support and services to consumers. These services are an important revenue source for such a firm.

below the monopoly level. If switching costs are high, the closed source licensor increases its price relative to its monopoly price, thus compensating for its decreased market share. In fact, Lanzi [2009] finds that a portion of the market is always captured by a high value open source software entrant.

Casadesus-Masanell and Ghemawat [2006] examine competition between a firm owning a closed source software and some entity[3] distributing an open source software. They focus on competition between Linux (an open source software) and Windows (a closed source software). Two assumptions of their model is that some users are constantly entering and exiting the market and that both open and closed source software benefit from a direct network effect. However, the repercussions differ for the two types of firms. Specifically, they find that the closed source software's price (Windows' price) is greater under monopoly than under competition from the open source (Linux). They also claim that the two software types may coexist in equilibrium. Their study suggests that a closed source software monopoly may yield higher welfare than a duopoly. The monopoly outcome may dominate the duopoly because the entry of Linux induces some consumers to switch from Windows to Linux. Fewer Windows' consumers implies lower benefits from network effect for that firm. If this network effect is sufficiently large, the closed source monopoly outcome may, in terms of welfare, outweigh the duopoly outcome.

The framework of this paper considers a closed source software firm exposed to competition from an open source product. While the firm depends on its intrinsic quality to create value, the open source software derives value from user contributions. The open source software may present a lower or higher quality than the closed source software. This difference in quality generates two distinct equilibria, which I treat independently.

First, I suppose that the closed source firm knows only the distribution of tastes but cannot distinguish among buyers; then I relax this assumption by assuming that the firm can perfectly identify each consumer's taste. The latter allows the firm to engage in first-degree price discrimination.

The outcome of the model shows that as contributions to open source software increase, consumers may turn to the closed source rather than to the open source product. This is true whether or not the firm price discriminates. The closed source software may even entirely

[3] The term entity refers to a firm, a community of users, or an individual.

dominate the market because the competition induces the firm to lower its price. In fact, the closed source software licensor subsidizes its own product in order to lower demand for the open source product. Lowering its price not only makes its product more attractive, it reduces consumer contributions to the open source.[4] Consumers are not only impelled to purchase the closed source software they also do not contribute to the open source product.

When the firm engages in price discrimination, one may observe an equilibrium in which it sells to some consumers at a negative price; in effect, the firm is paying certain consumers to use its software. This is in fact an extreme case of the result just discussed: price discrimination magnifies the previous effect.

I also concluded that the entry of a closed source software competing with an open source product may lower welfare, even if the firm price discriminates. However, the entry of an open source software always increases welfare.

The paper is organized as follows. First, as a benchmark, I study the monopoly case for a closed and for an open source software. Second, I analyze how a closed source software firm competes with an open source software when the quality of the former is higher than the quality of the latter. Then, I determine how the equilibrium is affected when the firm price discriminates. Subsequently, I look at the outcome of competition when the ranking of quality is reversed. The final section summarizes the major findings.

[4] If the firm practices uniform pricing it lowers the price for all consumers; but if the firm discriminates, it lowers the price for the consumers with a high valuation of quality.

3.1 Monopoly

The first baseline has a single firm offering a closed source software of quality v_c. In the alternative baseline, open source software of quality v_o is the only product available in the market. Both types of software are produced at zero cost. The utility of consumer indexed σ is $U(\sigma) = \sigma v_i$ where $i = c,\ o$, and the taste parameter σ is uniformly distributed on $[0,\ 1]$. All consumers buy at most one unit. Not consuming the software yields a utility of zero.

3.1.1 A closed source monopoly

I consider two equilibria: one with uniform pricing, the other with price discrimination.

Uniform pricing

When the firm's software is sold at price P, it is purchased by all consumers whose surplus $S_c(\sigma) = \sigma v_c - P$ is non-negative. The marginal consumer has index $\sigma = \sigma_c$ where

$$\sigma_c(P) = \frac{P}{v_c}. \tag{3.1}$$

Demand for the closed source software is $1 - \sigma_c(P)$. Since cost is zero, the profit is $\Pi_m = P(1 - \sigma_c(P))$. The profit maximizing price $P_m^* = v_c/2$ yields an equilibrium market size of $X_c^* = 1/2$.[5]

Discriminatory pricing

When the monopolist sells at personalized price $P(\sigma)$, the consumer indexed σ has surplus $S_c(\sigma) = \sigma v_c - P(\sigma)$. All consumers with a non-negative surplus purchase the software. In fact, the firm sets price $P(\sigma) = \sigma v_c$ to consumer of type σ and, thus, captures the entire surplus.

[5]This result is equivalent to the result of Mussa and Rosen [1978] in their monopoly example.

3.1.2 An open source monopoly

The open source software differs from the closed source software in that its value to any particular user depends on the total number of users. Consumers consider the expected size of the open source software's market when they decide whether or not to use this software. I do not model how consumers' expectations are formed. However, I impose the restriction that consumers' expectations are fulfilled in equilibrium. Each consumer correctly anticipates the total number of consumers that will use the open source software.[6] I denote the expected size of the market for open source software by X_o^e.

I let ψ denote the cost of learning how to use the open source software. The open source software is, then, used by all consumers whose surplus $S_o(\sigma,\, X_o^e) = v_o\sigma - \psi + kX_o^e$ is non-negative,[7] where the parameter $k > 0$ represents the marginal contribution of each user to the benefit that others derive from the software. Following Katz and Shapiro [1985], I refer to $\phi = \psi - kX_o^e$ as the hedonic cost of open source software. This is the cost adjusted for the effect of user contributions. I will henceforth refer to k as contributions.

The following assumption is made throughout the paper:

$$v_o > \psi > k \tag{3.2}$$

This assumption (3.2) implies the following: a) The consumer indexed $\sigma = 0$, has a negative surplus from the open source software, even when he expects all consumers to use it; b) the consumer indexed $\sigma = 1$, has a positive surplus from the open source software, even when he expects no consumer to use it; and c) all consumers face a strictly positive hedonic cost, or more exactly, $\psi - kX_o^e = \phi > 0$.

The consumer indifferent between getting the open source software and choosing the outside option has a taste parameter

$$\sigma_o = \frac{\psi - kX_o^e}{v_o} = \frac{\phi}{v_o}. \tag{3.3}$$

Thus, the demand for open source software is $X_o(X_o^e) = 1 - \sigma_o(X_o^e)$.

[6] In this regard, I follow Katz and Shapiro [1985].

[7] The linear relationship between consumers' expectations of the network size and the surplus is in accordance with research in network economics, for example, Katz and Shapiro [1985] and Shy [2001].

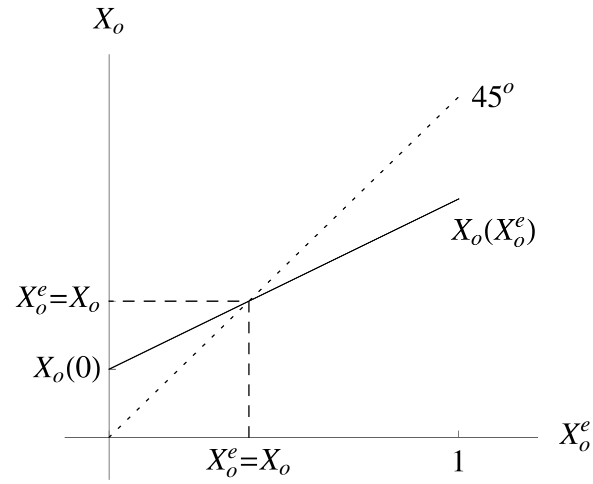

Figure 3.1: Fulfilled Expectations

Figure 3.1 illustrates the fulfilled expectations condition

$$X_o = X_o^e. \tag{3.4}$$

The abscissa represents consumers' expectations, and the ordinate represents their corresponding demand. The line shown as $X_o(X_o^e)$ is the actual demand for the open source product when consumers expect X_o^e. Assumption (3.2) ensures $\frac{\partial X_o}{\partial X_o^e} = \frac{\partial(1-\sigma_o(X_o^e))}{\partial X_o^e} = \frac{k}{v_o} \leq 1$.[8]

Using $X_o^e = X_o(X_o^e)$ and (3.3) yields the equilibrium market size:

$$X_o = \frac{v_o - \psi}{v_o - k} \in (0,\ 1). \tag{3.5}$$

Note the difference with the closed source software equilibrium. The market size is now a function of parameters v_o, ψ, and k. Note in particular that the market size is increasing in k. Whereas the size of the market of an open source monopoly increases in v_o, the size of the market of a closed source monopoly is always equal to $1/2$. The size of the market for open source is larger (smaller) than $1/2$ when $v_o + k > (<) 2\psi$.[9]

In terms of market size, when v_o is low, closed source software performs better than open source software; and when v_o is high and contributions are significant, open source software performs better than closed source software. In the next two sections, I study a situation in which the firm competes against an open source software.

[8]A stability argument can be invoked to justify that the actual demand has slope less than one. To see why, consider the following example. If a consumer underestimates demand (the demand he expects is located to the left of the point depicted as $X_o^e = X_o$ along the horizontal axis), he observes that the actual demand lies on the hard line above his expectation represented by the 45^o line. Realizing that he underestimates demand, the consumer updates his expectation. His new expectation is the point on the 45^o line that corresponds to the actual demand, that is, his updated expectation still lies below the actual demand but is closer than the expectation he made before the update. After doing this an infinity of time, the consumer converges to the fixed point where his expected demand equals actual demand. If the actual demand function is steeper than the 45^o line, the consumer updating his expectation would reach either expected demand of zero or expected demand of infinity.

[9]Remark that assumption (3.2) is sufficient to ensure that the equilibrium market size lies in the set $(0,\ 1)$.

3.2 Duopoly: High-quality closed source software

I now assume that closed and open source software compete in the market place and that $v_c > v_o$. I also assume that consumers form their expectations of the number of open source users on the basis of the price or prices set by the closed source firm. I examine in turn the case of uniform[10] and discriminatory pricing.

3.2.1 Uniform pricing

The consumer indifferent between choosing the open source software and buying the closed source software has a taste parameter $\tilde{\sigma}$ that satisfies $S_c(\tilde{\sigma}) = S_o(\tilde{\sigma},\, X_o^e)$. Thus,

$$\tilde{\sigma}\,(X_o^e,\, P) = \frac{P - \phi}{v_c - v_o}, \tag{3.6}$$

where P denotes the uniform price. In an equilibrium in which both products are consumed, consumers are separated into three groups:

- those with indices $\sigma \in [0,\, \sigma_o)$ do not use any software,
- those with indices $\sigma \in [\sigma_o,\, \tilde{\sigma})$ choose the open source software, and
- those with indices $\sigma \in [\tilde{\sigma},\, 1]$ purchase the closed source software.

Thus, the marginal consumers indexed σ_o and $\tilde{\sigma}$, defined by equations (3.3) and (3.6) respectively, determine demand for open source software which is given by:

$$X_o(X_o^e,\, P) = \tilde{\sigma}(X_o^e,\, P) - \sigma_o(X_o^e). \tag{3.7}$$

Inserting (3.3) and (3.6) into (3.7) yields

$$X_o(X_o^e,\, P) = \frac{P}{v_c - v_o} - \psi \frac{v_c}{v_o(v_c - v_o)} + kX_o^e \frac{v_c}{v_o(v_c - v_o)}. \tag{3.8}$$

[10] If a second closed source software of quality v_c were to enter and act as a Bertrand competitor, it would drive prices down to the marginal cost, zero (from the usual Bertrand argument). Consequently, I do not investigate the matter further.

Differentiating this demand with respect to X_o^e gives $\frac{\partial X_o}{\partial X_o^e} = \frac{kv_c}{v_o(v_c - v_o)}$ which is less than unity if, and only if,

$$v_o(v_c - v_o) - kv_c > 0. \tag{3.9}$$

This condition ensures the stability of the fulfilled expectations condition for the duopoly.

Choosing price

I note first that the firm can choose a price P_a^H which ensures that the demand for open source software is zero. Specifically,

Lemma 14. *For all $P \leq P_a^H = \psi \frac{v_c}{v_o}$, consumers expect a market of size zero for the open source software, that is $X_o^e = 0$.*

Proof. The price P_a^H is the P that solves $X_o(0,\ P_a^H) = 0$. If P_a^H induces a zero market share, so also do all lower prices because $\tilde{\sigma}\,(X_o^e,\ P)$ is decreasing in P and $\sigma_o(X_o^e)$ is independent of P as shown by (3.3) and (3.6). □

Figure 3.1 illustrates lemma 14. When $P = P_a^H$ the intersection of the 45^o line and the actual demand is at the origin. Note that the maximum price which excludes the open source software from the market increases in ψ. This is because the cost of learning about open source software lowers the value of open source software.

To determine the price the firm actually sets under duopoly, I note that the demand for the closed source software is $X_c(P) = 1 - \tilde{\sigma}(X_o^e,\ P)$ for $P > P_a^H$ where $\tilde{\sigma}(X_o^e,\ P)$ satisfies (3.6). It then follows from the fulfilled-expectations condition, (3.4), that

$$X_c(P) = 1 - \frac{P(v_o - k) - v_o\psi}{v_o(v_c - v_o) - kv_c}. \tag{3.10}$$

Assumption (3.9) ensures that the denominator is positive. Thus, the demand for closed source software is decreasing in k for $P > P_a^H$; and since lemma 14 implies that the firm never chooses $P < P_a^H$, it is indeed decreasing for any price that the firm would choose. This is because the demand for closed source software depends on the consumers' expectations of the size of the market for open source software. An increase in k increases the value of the open source software, which in turn lowers demand for the closed source software because more consumers opt for the open source software.

I can now determine the price the firm sets when facing the demand given in (3.10). As cost is zero, the firm's profit is, $\Pi_c^H(P) = PX_c(P)$. The profit is maximum when price[11] is given by (3.11) below[12]

$$P_c^H = \frac{v_c}{2} - \frac{v_o(v_o - \psi)}{2(v_o - k)}. \tag{3.11}$$

The market size X_c^* increases in k and increases in ψ. This follows from inserting the price given in equation (3.11) into the demand function given in equation (3.10) and differentiating the ensuing demand with respect to k which results in $\frac{\partial X_c}{\partial k} = \frac{v_c v_o \psi}{2(v_o(v_c - v_o) - k v_c)^2} > 0$.

I now turn to the question for what values of the parameters $P_c^H > P_a^H$.

Lemma 15. *When $k < \overline{k}(\psi)$, where $\overline{k}(\psi) = v_o\left(1 - \frac{v_o}{v_c}\frac{v_o - \psi}{(v_o - 2\psi)}\right)$, the firm sets price P_c^H.*

Proof. When both software products are in the market, the firm earns a profit:

$$\Pi_c^H = \frac{(v_o(v_c - v_o + \psi) - k v_c)^2}{4(v_o - k)(v_o(v_c - v_o) - k v_c)} \tag{3.12}$$

When $P = P_a^H$ it earns a profit:

$$\Pi_a^H = P_a^H\left(1 - \sigma_c(P_a^H)\right) = \psi \frac{v_c}{v_o}\frac{(v_o - \psi)}{v_o} \tag{3.13}$$

Comparing equation (3.12) and equation (3.13) shows that the profit is larger when $P = P_c^H$ than when $P = P_a^H$ if $k < \overline{k}$. ☐

I can now determine how parameter values, k in particular, determine the number of open source users. As seen, contributions may induce the firm to set a price that reduces the open source market size to zero.

[11] Note that $P_c^H\Big|_{k=\overline{k}} = P_a^H$, implying that price is a continuous function of k.

[12] In an equilibrium where the two software products are in the market, that price is unique since, by assumption (3.9), $\frac{\partial^2 \Pi_c^H}{\partial P^2} = -\frac{2(v_o - k)}{v_o(v_c - v_o) - k v_c} < 0$. The function Π_c^H is twice continuously differentiable with respect to P (over the interval $P \in (P_a^H, \infty)$ which implies that Π_c^H is strictly concave if, and only if, $\frac{\partial^2 \Pi_c^H}{\partial P^2} < 0$. Since P_c^H is such that $\frac{\partial \Pi_c^H}{\partial P}(P_c^H) = 0$, by definition of concavity, Since Π is differentiable, it is strictly concave if and only if for each p_1, p_2 on the relevant interval: $(P_1 - P_2)\frac{\partial \Pi_c^H}{\partial P} > \Pi_c^H(P_1) - \Pi_c^H(P_2)$, $\Pi_c^H(P_c^H) > \Pi_c^H(P)$ for all $P \neq P_c^H$. Furthermore, this price decreases in k and increases in ψ. The effect of k on price becomes stronger for larger k as $\frac{\partial^2 P_c}{\partial k^2} = -\frac{v_o(v_o - \psi)}{(v_o - k)^3} < 0$. On the contrary, the effect of ψ becomes stronger for larger k as $\frac{\partial^2 P}{\partial k \partial \psi} > 0$.

Using the fulfilled-expectations condition in equation (3.7) and differentiating with respect to k yields

$$\frac{\partial X_o\left(P(k)\right)}{\partial k} = v_c \frac{P(k)v_o - v_c\psi}{(v_o(v_c - v_o) - kv_c)^2} + \frac{v_o}{(v_o(v_c - v_o) - kv_c)} \left(\frac{\partial P(k)}{\partial k}\right) \tag{3.14}$$

This shows that the total effect of k is the sum of two components. The first component is the effect of contributions on consumers' expectations of demand. I call this the consumers' expectations effect. The second component is the effect of contributions on price. I refer to the latter as the price effect. The first component is positive and the second component is negative. Whether the market size increases or decreases in k depends on the relative strength of the two effects.

Using (3.4), (3.8) and (3.11) yields the equilibrium number of open source users as

$$X_o^*(k) = \frac{v_o\left(v_o(v_c - v_o + \psi) - kv_c\right) - 2\psi v_c(v_o - k)}{2(v_o - k)\left(v_o(v_c - v_o) - kv_c\right)} \tag{3.15}$$

The effect of k on this demand depends on ψ. To determine how, I define three thresholds. The first threshold is $\hat{\psi}$ below

$$\psi_1 = \{\psi : \frac{\partial X_o^*}{\partial k}\Big|_{k=0}(\psi) = 0\} = \frac{v_o(v_c - v_o)^2}{v_c + (v_c - v_o)^2} \tag{3.16}$$

The threshold ψ_1 is the only value of ψ at which the marginal effect of the first unit of contributions has no impact on the number of open source users. The second threshold is $\psi_2 = \frac{v_o(v_c - v_o)}{2v_c}$. This is the value of ψ for which $\psi = \bar{k}(\psi)$ where $\bar{k}$ is defined in lemma 15. For $\psi < \psi_2$ both products enjoy a positive market share for any value of k satisfying the assumptions of the model. For $\psi > \psi_2$ there exist a sufficiently large k for which the market share of the open source is zero.

The third threshold is $\psi_3 = \frac{v_o(v_c - v_o)}{2v_c - v_o}$. According to lemma 15, the firm always sets price P_a^H when $k > \bar{k}(\psi)$; so its product is alone in the market. Thus, there are two software products in the market only if $\bar{k}(\psi) > 0$ because for $k > \bar{k}(\psi)$ the firm sets price P_a^H. With such a price the size of the market for open source is zero (see lemma 15). Because $\bar{k}(\psi)$

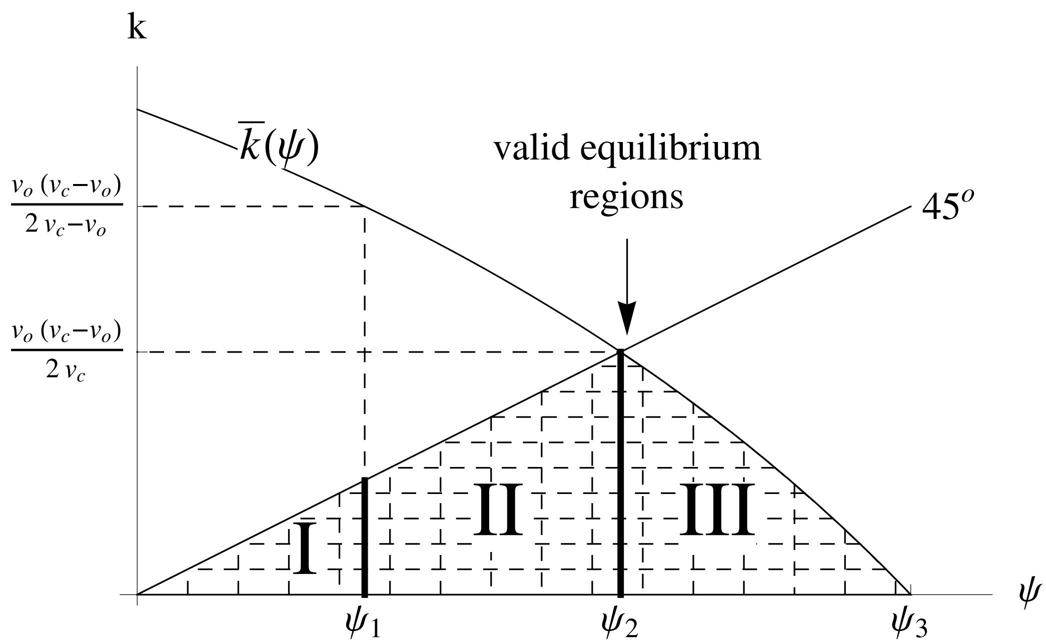

Figure 3.2: Threshold $\bar{k}$ as a function of cost ψ. The three regions represent the parameter range that results in duopolistic equilibria and that is acceptable under the model's assumptions.

is decreasing in ψ on the relevant interval, $\bar{k}(\psi) > 0$ if, and only if, $\psi < \frac{v_o(v_c-v_o)}{2v_c-v_o}$.[13]

Figure 3.2 shows the ranking of thresholds and displays $\bar{k}$ as a function of ψ. Regions I, II, and III shows the combinations of ψ and k for which both software products are in the market.

In regions I and II, the 45^o line lies below $\bar{k}(\psi)$. In these regions, the largest k satisfying assumption (3.2) is smaller than $\bar{k}(\psi)$. Therefore, the firm never sets price P_a^H. In these regions, the cost of learning about open source software is low, and the closed source product cannot be priced so as to keep the open source product out of the market.

In region III, there exists a k large enough so that the equilibrium price is P_a^H, and the open source has a zero market share. It remains to be shown that the number of open source users may increase or decrease in k.

The following lemma yields a condition that I use in later proofs.

Lemma 16. *Jointly $v_c > v_o$ and condition $\psi < \psi_3$ imply $v_o > 2\psi$.*

Proof. Rearanging $\psi < \psi_3$, I obtain $2\psi(v_c - \frac{v_o}{2}) < v_o(v_c - v_o)$. Because $v_c > v_o$, it must

[13]Note that if ψ is larger than ψ_3 then $\bar{k} < 0$ and if so by virtue of lemma 15 the open source market share is zero. In addition, assumption (3.2) ensures that the ranking of the thresholds is always $\psi_1 < \psi_2 < \psi_3$, and it follows from (3.16) that $\psi_1 > 0$.

be true that $v_o > 2\psi$. □

The following proposition characterizes how the equilibrium number of open source users evolves as a function of contributions.

Proposition 17. *a) If $\psi \geq \psi_1$, then $X_o^*(k)$ is concave and strictly decreasing in k. b) If $\psi < \psi_1$, $X_o^*(k)$ always increases in k when contributions are close to zero. c) Furthermore, if $\psi < \psi_1$ and ψ is sufficiently small, then $X_o^*(k)$ increases even when contributions are significant.*

(Proof in Appendix.)

In Region I, the number of open source users may or may not increase in contributions. The consumers' expectations effect is strong because the cost associated with open source software is low. The price effect by contrast is weak because contributions are low. The reason is that price is not sensitive to variations in contributions (recall that $\frac{\partial^2 P}{\partial k^2} < 0$). Although the price decreases in k, the number of open source users can increase in k. As contributions increase, more consumers with a low valuation of quality opt for the open source software.

In region II, the price effect is strong enough to ensure that the size of the open source market decreases monotonically in contributions. Though, it is not strong enough to price the open source software out of the market. Therefore, even at the region's boundary (at $k = \psi$), the market is shared by both products.

In region III, the price effect dominates the consumers' expectation effect. The number of open source users decreases in k. Note that proposition 17 is true only when assumption (3.2) holds true. When it does not the hedonic cost can be negative. In such case, increases in k would eventually lead to capture of the entire market by the open source software. Indeed, if the hedonic cost is negative, the open source software captures consumers with a high valuation of quality.

The welfare effect of open source entry

I can now examine how entry of the open source product into a market initially served by a closed source product affects the number of users and welfare.

Proposition 18. *The price of the closed source product is lower and its market larger under duopoly than under closed source monopoly.*[14]

Proof. There are two parts to the proof. Part 1 concerns the price and part 2 concerns the market size

Part 1. On inspection of the competitive price (3.11), it follows that $P_c^H(0) < P_m$ (P_m is given in subsection 3.1.1). Because P_c^H is strictly decreasing in k, this inequality holds for all k.

Part 2. Upon substitution of P_c^H, given by (3.11), into (3.10), I obtain the number of closed source users in the duopoly as $X_c^* = 1 - \tilde{\sigma} = \frac{1}{2} + \frac{\psi v_o}{2(v_o(v_c - v_o) - kv_c)}$. Because the market size is $1/2$ under a monopoly, it follows from assumption (3.9) that the market size of the closed source software is larger when there also is an open source software in the market. When the firm sets price $P = P_a^H$, its market size is $1 - \frac{\psi}{v_o}$ which is larger than $1/2$ (see lemma 16). I conclude that the market size is always larger under competition than under a closed source monopoly. □

It follows from proposition 18 that entry of the open source product increases the surplus of all consumers who purchased software prior to entry. Clearly, consumers, who do not purchase the closed source product prior to entry, do so post entry and also gain. This group includes users of the open source software and additional users of the closed source product. The addition of this group contributes positively to total welfare.

The welfare effect of closed source entry

I now examine how entry of the closed source product into a market initially served by a open source product affects the number of users and welfare.

[14]In the context of a horizontally differentiated market of Hotelling type, Zacharias [2009] finds that although competition may add to variety, it also leads to increase in prices which may harm the consumer welfare. Here, with vertical differentiation, the price of the closed source software is always decreased with entry.

Proposition 19. *The number of software users is smaller under a duopoly than under an open source monopoly.*[15]

Proof. As $X_o^* + X_c^* = 1 - \sigma_o^*$, I can simply insert X_o^* given by (3.15) into (3.3) to obtain

$$1 - \sigma_o^* = X_o^* + X_c^* = 1 - \frac{k^2 v_c + 2(v_c - v_o)v_o\psi + k(v_o(v_o + \psi) - v_c(v_o + 2\psi))}{2(v_o - k)(v_o(v_c - v_o) - kv_c)} \tag{3.17}$$

This expression gives the size of the market served by both type of software. It follows from (3.17) and (3.5) that the total number of software users is smaller than under the open source monopoly if, and only if,

$$-\frac{k\left((v_o(v_c - v_o) - kv_c) + v_o\psi\right)}{(v_o - k)(v_o(v_c - v_o) - kv_c)} < 0.$$

This expression is indeed negative as assumption (3.9) along with assumption (3.2) entails that both the numerator and the denominator are positive. □

Competition lowers contributions to open source software, which lowers the value of the open source software. Because the open source software serves consumers with a low valuation of quality, some consumers who chose the open source software under the monopoly are no longer interested in using that software. Consequently, the total number of software users declines.

Consider the effect of closed source entry on consumer welfare. One may have expected such entry to increase welfare of all consumers because all consumers retain the option of using open source software. The following numerical example shows that this is not true.

Figure 3.3 shows consumers' surplus as a function of σ under monopoly and duopoly for the particular case where $v_c = 4$, $v_o = 1$, $\psi = 8/25$, and $k = 7/27$. The indices σ_M and σ_D denote the marginal consumer under the monopoly and duopoly. The open source monopoly serves consumers with σ belonging to the interval $[\sigma_M, 1]$ where $\sigma_M = 1/18$. Under duopoly users belong to the interval $[\sigma_D, 1]$ where $\sigma_D = 5/18$. The consumer who has the same surplus under monopoly and duopoly is indexed $\sigma_I = \frac{v_o(v_c - v_o) - kv_c + v_o(2k - \psi)}{2(v_c - v_o)(v_o - k)}$.

[15] This is in contrasts to one of Klemperer's results, in a model in which a competitive market faces entry from another firm which may be more cost efficient then the incumbent, he finds that increased competition always increases the industry output [Klemperer, 1988]. Here, the opposite is observed. Despite the entrant being more efficient, the increased competition has lowered the market size relative to the market size the open source software would enjoy if it were alone in the market.

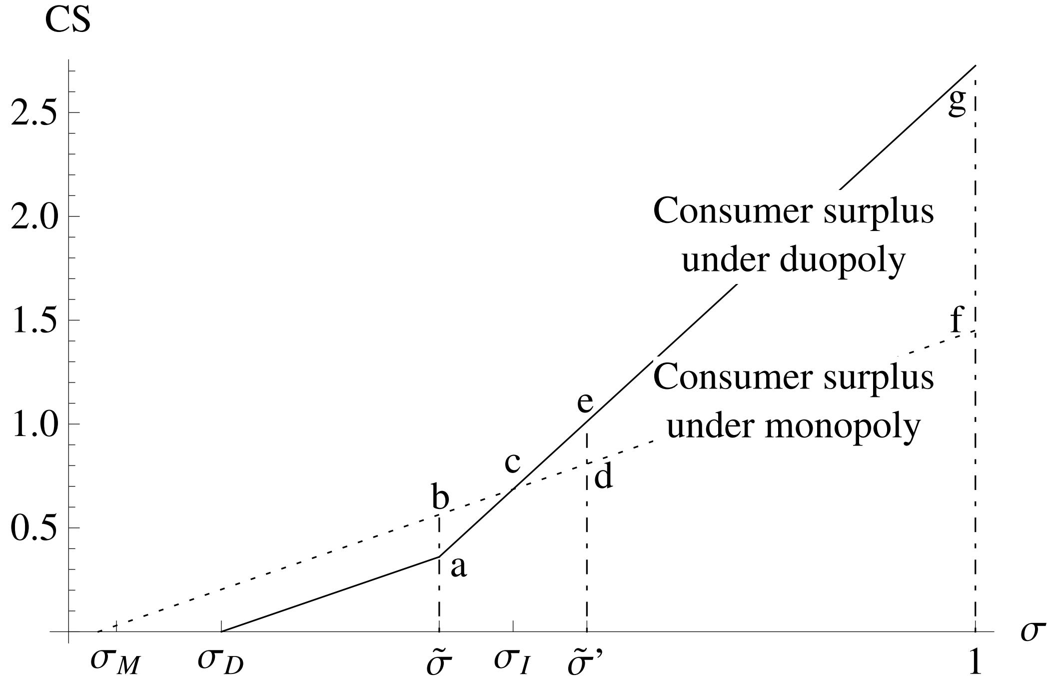

Figure 3.3: Closed source entry: comparison of the consumer's surplus when $v_c = 4$, $v_o = 1$, $\psi = 8/25$, $k = 7/25$

Consumers with index $\sigma < \sigma_I$ lose surplus upon entry. Those with $\sigma \in [\sigma_D,\ \tilde{\sigma}]$ continue to use the open source software, but lose because that software is less valuable to them as a result of the decline in contributions. Those with $\sigma \in [\sigma_M,\ \sigma_D]$ stop using software altogether. Consumers with $\sigma \in [\tilde{\sigma},\ \sigma_I]$ switch to the closed source product but derive less surplus from it than they got from the open source product prior to entry. Finally, those with $\sigma > \sigma_I$ use the closed source product. There surplus is larger than under an open source monopoly because the closed source software is of higher quality than the open source software. Note that $\sigma_I \in (\tilde{\sigma},\ 1/2)$.[16]

The gains of consumers who benefit from entry are larger than the losses of those who are affected adversely by it. To see why, refer again to figure 3.3. Choose a point $\tilde{\sigma}'$ such that the distance between $\tilde{\sigma}$ and σ_I equals the distance between σ_I and $\tilde{\sigma}'$. The area of the triangle abc equals the area of the triangle cde. This means that the aggregate change

[16]The consumer σ_I is always located left of 1/2. To see this, note that $\sigma_I < 1/2$ if and only if $v_o(v_c - v_o) - kv_c + v_o(2k - \psi) < (v_c - v_o)(v_o - k)$ which is true if and only if $-v_o(\psi - k) < 0$. The last inequality holds because of assumption (3.2). Proposition 19 shows that the number of users of the open source software is smaller under a duopoly than under a monopoly. Since its value depends on the size of its market, its value is smaller under a duopoly than under a monopoly. It follows that the user who is indifferent between both software has an index $\tilde{\sigma} < \sigma_I$.

in welfare for consumers on the interval $[\tilde{\sigma},\ \tilde{\sigma}']$ is zero. It remains to be shown that the area degf is larger than the area $\sigma_M$baσ_D. Note that within area degf, the consumer with the smallest gain from entry is consumer indexed $\tilde{\sigma}'$. That consumer's gain equals the loss of the consumer with the largest loss, which is the consumer indexed $\tilde{\sigma}$. There are more gainers than losers as $\sigma_I < 1/2$. Thus, aggregate gains exceed aggregate losses. As a result, consumer welfare increases with entry. Total welfare which includes profits must therefore also increase.[17]

3.2.2 Perfect discrimination

I now assume that the firm knows the preferences of each consumer. To simplify the analysis, I separate the consumers into two groups[18]: a group L composed of individuals with $\sigma \in [0,\ \sigma_o)$, and a group H composed of individuals with $[\sigma_o,\ 1]$ where σ_o is defined by (3.3).

For consumers in group L, the best alternative to the closed source software is the outside option, which gives zero surplus. Hence, the firm sets the price to consumer σ as $P_L(\sigma) = \sigma v_c$. This is the highest price that induces consumers in the group L to buy. Since consumers in group H obtain a positive surplus from the open source software, the firm must leave these consumers with a positive surplus in order to persuade them to buy. The price $P_H(\sigma) = \sigma(v_c - v_o) + \psi$ is the highest price that induces consumers in the group H to buy the closed source software.[19]

Specifically, the discriminatory price schedule is

$$P(\sigma) = \begin{cases} \sigma v_c & for\ \sigma \in [0,\ \psi/v_o] \\ \sigma(v_c - v_o) + \psi & for\ \sigma \in (\psi/v_o,\ 1] \end{cases} \tag{3.18}$$

[17]This is somewhat similar to Klemperer's result that entry of an efficient competitor may be socially detrimental in the sense that some consumers lose [Klemperer, 1988]. Note that Klemperer's model has products that differs in cost, but my model has products that differ in quality. Zacharias [2009] also shows that the entry of a second firm in a Hotelling type market may harm consumers as prices increase and consumers' surplus potentially decreases.

[18]By virtue of assumption (3.2) these two groups always exist.

[19]Note that at these prices, if the fulfilled-expectations condition is satisfied, the consumers expect a market of size zero for the open source software.

Figure 3.4 displays the price schedule (3.18). This price schedule entails zero usage of the open source software. However, the mere existence of the open source software compels the firm to leave positive surplus to consumers in group H.

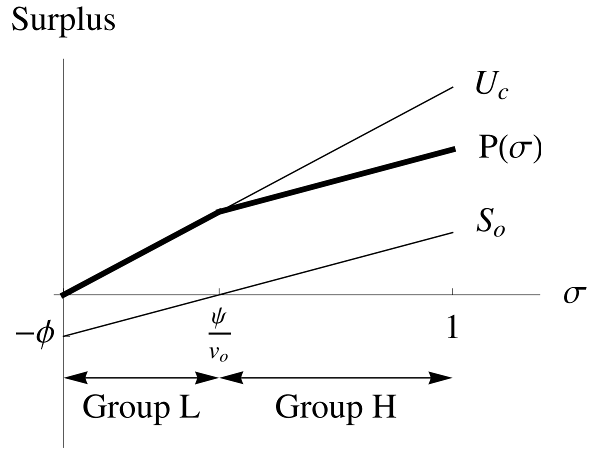

Figure 3.4: High-quality closed source software: utility, surplus, and price.

The welfare effect of open source entry

The entry of an open source software into a market initially served by a closed source monopoly leaves total welfare unchanged, but increases consumer surplus at the expense of profits. The reason is that the presence of the open source software forces the firm to leave a positive surplus to some consumers, those who obtain a positive surplus from an open source software that has an expected market size of zero.

The welfare effect of closed source entry

By contrast, the entry of closed source software into a market served by open source software increases total welfare, but consumers' welfare falls. The reason is that the market is not covered under the open source monopoly, but is fully covered when the closed source firm engages in perfect discrimination.

Recall that the market is never covered under an open source monopoly and that the market is covered when the firm price discriminates. Since $v_c > v_o$, the surplus increases faster in σ when consumers use the closed source software. Since all consumers use the closed source software, the surplus associated with each consumer is higher than the surplus obtained from the opens source software. Clearly, the total surplus is higher after the entry of the closed source software. However, the consumer surplus is reduced. The consumers who did not consume software prior to entry are unaffected by entry. They consume the closed source software, but the firm charges them a price equal to the size of their respective surplus. As a result, these consumers are unaffected by entry. The consumers who used the open source software prior to entry lose from entry. They receive either zero surplus from the closed source software or a surplus equal to the surplus they would receive if the open source software had zero contribution. The latter is clearly lower than the surplus they received under the open source monopoly (since the software did receive contributions). Thus, consumers have a surplus either equal or lower than the surplus they received prior to entry.

3.3 Duopoly: Low-quality closed source software

I now study a market where the closed source software is of lower quality than the open source software. That is $v_c < v_o$. Again, I look at two equilibria: one with uniform pricing, and one with price discrimination.

3.3.1 Uniform pricing

In an equilibrium in which both products are consumed, consumers are separated into three groups:

- those with indices $\sigma < \sigma_c$ do not use software,
- those with indices $\sigma \in [\sigma_c,\ \tilde{\sigma}]$ purchase the closed source software, and
- those with indices $\sigma > \tilde{\sigma}$ choose the open source software.

To address the effect of contributions, I first derive the price chosen by the firm. As in section 3.2.1, the firm can choose a positive price which keeps the open source software out of the market. This price is $P_a^L = v_c + \psi - v_o$.

Unlike the case where the closed source software is of higher quality (see lemma 14), P_a^L may be negative. When the closed source software is of high quality, all consumers prefer the closed to the open source software when the former's price is zero. When the open source software is of high quality, for certain values of the parameter, the only price that can keep the open source out of the market is negative.

The duopoly price is

$$P_c^L = \frac{v_c(\psi - k)}{2(v_o - k)} \tag{3.19}$$

As in case where the closed source software is of higher quality, there is a threshold for parameter k beyond which the firm sets a price sufficiently low to keep the open source software out of the market. When $k > \overline{k}(\psi) = \frac{2v_o(v_c - v_o + \psi) - v_c\psi}{v_c - 2(v_o - \psi)}$, the firm sets the price P_a^L.

I now investigate the effect of consumers' contributions on the number of open source users. The equilibrium number of open source users is given by

$$X_o(P_c^L(k)) = \frac{P_c^L(k) + v_o - v_c - \psi}{v_o - v_c - k} \tag{3.20}$$

which upon substitution for $P_c^L(k)$ from (3.19) yields

$$X_o^*(k) = \frac{2(v_o - k)(v_o - \psi) - v_c(2v_o - \psi - k)}{2(v_o - k)(v_o - v_c - k)}$$

As for the case where the closed source software is of higher quality than the open source software, there is a consumers' expectations effect and a price effect. Since price P_c^L decreases in k and since (3.20) increases in price, the price effect of k is to lower the number of open source users. On the other hand, the direct effect of k captured via the denominator increases the number of open source users.

Before determining the net effect of a change in k, I must ensure that the parameters are such that the open source software enjoys a positive market share. I define three thresholds for ψ.[20] The first threshold is $\psi_1 = v_o - v_c$. The value $v_o - v_c$ is the fixed point of the $\overline{k}(\psi)$ function, viz. $\psi_1 = \overline{k}(\psi_1)$. The second threshold is $\psi_2 = \frac{v_o\left((v_o - v_c)^2 + v_o(v_o - v_c)\right)}{+v_o^2}$. This threshold is the value of ψ at which the marginal effect of the first unit of contributions has no impact on the number of open source users, viz. $\psi_2 = \{\psi : \frac{\partial X_o}{\partial k}\big|_{k=0} = 0\}$.

The third threshold is $\psi_3 = \frac{2v_o(v_o - v_c)}{2v_o - v_c}$. As before, I impose $\overline{k}(\psi) > 0$ which is a necessary condition in order to have an equilibrium where both software products have a positive market share. In equilibrium, there are two software products in the market only if

$$\psi < \frac{2v_o(v_o - v_c)}{2v_o - v_c} = \psi_3 \tag{3.21}$$

These thresholds determine the three regions shown in figure 3.5. In region I, where $\psi < \psi_1$ and $k < \psi$, the cost of learning about open source is small, making the open source software a tough competitor to the closed source software. Consequently, there is no k large enough to allow the firm to price the open source software out of the market.

In region II, where $\psi \in [\psi_1,\ \psi_2]$, the firm prices the open source software out of the market when the contributions to open source software are significant, that is, when k is large. In region III, where $\psi > \psi_2$, the threshold function $\overline{k}(\psi)$ is lower than in region II. The closed source firm prices the open source out of the market at lower values of k.

[20] These thresholds have the same interpretation as in section (3.2.1), although their values differ.

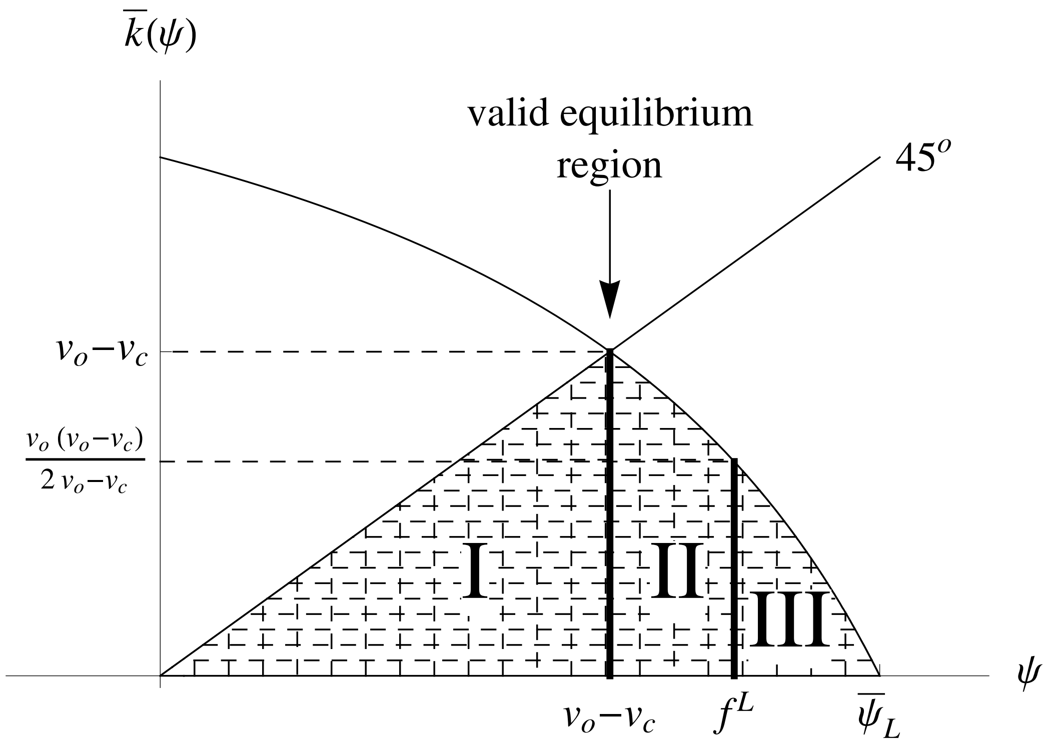

Figure 3.5: Threshold $\overline{k}$ as a function of cost ψ, and equilibrium regions

The following proposition gives some properties of the equilibrium number of open source users.

Proposition 20. *If $\psi \leq \psi_1$, the function X_o^* is convex and increasing in k. If $\psi > \psi_1$, the function X_o^* is decreasing for k sufficiently large. For $\psi > \psi_1$, I do not show whether the function is concave or convex.*

(Proof in appendix.)

In region I, the closed source market size decreases and the open source market size increases in k. The reason is that when k is large the low value of ψ weakens the firm's ability to capture consumers. In that region, the open source software enjoys the benefits from increases in contributions. As the hedonic cost $\phi = kX_o^e - \psi$ goes to zero, even the consumers with a low valuation of quality prefer the open source software. Consequently, the closed source software is squeezed out of the market. The firm's reaction to the increase in k is to lower its price in order to capture consumers with low valuation of quality. However, even for a low price, these consumers prefer the open source software when the hedonic cost goes to zero. This contrasts with the result found in subsection 3.2.1 where at low values of ψ, the firm sells software to more than half the market (see proposition 18).

In region II, the effect of contributions on the number of open source users depends on the magnitude of k. If k is small, the effect on price is weak and the user contributions effect dominates the price effect. Consequently, the number of open source users increases in k. However, if k is sufficiently large, the effect on price becomes more important and user contributions effect is dominated by the price effect. Thus, when k is sufficiently large, the number of open source users decreases in k. In region III, the number of open source users is decreasing in k. In this region, the price effect dominates the consumers' expectations effect for all values of k. Consequently, in this region, the number of open source users always decreases in k.

In summary, the size of the market for closed source software decreases in k in region I but increases in k in regions II and III. Thus, if demand for closed source software increases in k at any level of contribution, then there exists a level of contribution which ensures that the open source software has a zero market share. By contrast, if demand for closed source software decreases in k at any level of contribution, then there exist a level of contribution at which the closed source software has a zero market share.

The welfare effect of open source entry

I now consider the welfare effect of entry by an open source software in a market initially served by a closed source firm.

Consider first consumers who purchased the closed source software prior to entry and continue to do so post-entry. These consumers clearly gain from entry as the post-entry price of the closed source software is lower than the pre-entry price. The consumers who switch from closed source to the open source software also gain because they would have gained if they did not switch. Consumers who did not purchase prior to entry and use either the closed source or the open source product post entry also gain. The remaining consumers do not use any software before and after entry are unaffected. Thus, no consumer loses from entry while some gain.

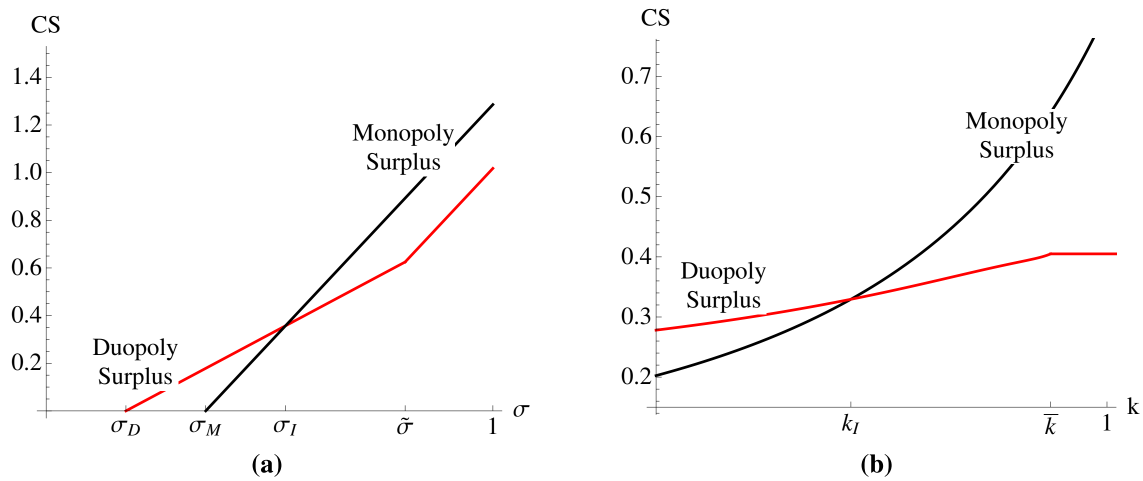

Figure 3.6: Comparison of the consumer's surplus when $v_o = 2$, $v_c = 1$, and $\psi = 1.1$

The welfare effect of closed source entry

The total size of the market is larger under a duopoly than under an open source monopoly. The total number of software users under duopoly is $1-\sigma_c = 1-\frac{1}{2}\frac{\psi-k}{v_o-k}$, whereas the number of open source users in under monopoly is $1-\sigma_o = 1-\frac{\psi-k}{v_o-k}$. Note that in contrast with the result in proposition 19, entry now increases the total number of software users. This is explained by the fact that consumers with a high valuation of quality use software regardless of entry. When the firm is a low-quality entrant, it gets most of its market from consumers who, before entry, did not use software. Furthermore, in the monopoly case as contributions increases the market is eventually covered. However, under a duopoly it may not be covered even when k is large. Note that under the monopoly $X_o \to 1$ as $k \to \psi$, but under the duopoly $X_c + X_o \to 1 - \frac{v_o-\psi}{v_c} < 1$ as $k \to \bar{k} < \psi$ and remains constant for $k \in [\bar{k},\ \psi)$.

I now consider welfare. Figure 3.6(a) contrasts the consumer's surplus before and after entry for the particular case where $v_c = 2$, $v_o = 1$, $\psi = 1.1$, and $k = 0.6$. These values are within region II of figure 3.5. The consumer indexed $\sigma = \sigma_I$ has the same surplus whether or not the closed source software enters the market. Consumers with $\sigma > \sigma_I$ lose from entry, and those with $\sigma < \sigma_I$ gain. The loss incurred by consumers with $\sigma > \sigma_I$ increases in k, whereas the gain of consumers with $\sigma < \sigma_I$ decreases in k. Thus, aggregate consumer surplus falls with entry when the contributions are significant.

Figure 3.6(b) looks at consumers' surplus as a function of k when $v_c = 2$, $v_o = 1$, and

$\psi = 1.1$. The values of k consistent with duopoly belong to the interval $[0,\ 1.1]$. Aggregate consumer surplus falls with entry when $k > k_I$, where k_I is defined as the value of k at which the aggregate consumer surplus under monopoly equals the surplus under duopoly. The aggregate consumer surplus under monopoly and duopoly aggregate surplus increases in k. However, the surplus under duopoly increases less rapidly than under monopoly. The monopoly surplus increases in k because, as k increases, more consumers use the open source software and more surplus accrues to those already using it.

Recall that an increase in k decreases the price of the closed source software. This decrease in price has two effects: 1) It induces consumers with a low valuation of quality to buy the closed source software; 2) it induces consumers who would otherwise choose the open source software to choose the closed source software. The second effect may lower the value of open source software because less consumers are contributing. As a result, open source consumers may not benefit from the increase in contributions.

This result, in terms of welfare, is similar to the result of Casadesus-Masanell and Ghemawat [2006] who found that the monopoly outcome may dominate the duopoly outcome. However, it contrasts with their results in that in their model the entry of the open source Linux induces some consumers to switch from Windows to Linux. In my model, the loss in efficiency is caused by the switch from the open source to the closed source. The reason for the difference is that in Casadesus-Masanell and Ghemawat [2006] the closed source product has more value, while in this section I assumed that the open source software has more value.

3.3.2 Price discrimination

I now assume that the firm knows consumers' willingness to pay. The consumer indifferent between both products has index

$$\overline{\sigma} = \frac{\phi - P(\overline{\sigma})}{v_o - v_c}.$$

I assume that the firm can offer negative prices to some consumers, which is equivalent to sponsoring a consumer to use the software.[21]

The firm's maximization problem is then

$$\begin{aligned}\Pi &= \max_{P(\sigma),\,\overline{\sigma}} \int_0^{\overline{\sigma}(X_o^e)} P(\sigma)d\sigma \\ & \quad subject\ to \\ & \quad (IR)\ \sigma v_c - P(\sigma) \geq 0 for\ all\ \sigma \in [0,\ \overline{\sigma}(X_o^e)] \\ & \quad (IC)\ \sigma v_c - P(\sigma) \geq \sigma v_o - \phi for\ all\ \sigma \in [0,\ \overline{\sigma}(X_o^e)] \\ & \quad X_o^e = 1 - \overline{\sigma}(X_o^e)\end{aligned} \tag{3.22}$$

The last equality in (3.22) is the fulfilled-expectations condition. In group L, defined in subsection 3.2.2, the consumer indexed σ buys the closed source software if, and only if,

$$\sigma v_c \geq P(\sigma)\ \forall\ \sigma \in [0,\ \sigma_o).$$

The firm charges the consumer indexed σ a price $P_L(\sigma) = \sigma v_c$. A consumer in group H buys the closed source software if, and only if, $S_c(\sigma) = \sigma v_c - P(\sigma) \geq \sigma v_o - \phi = S_o(\sigma,\ X_o^e)\ \forall\ \sigma \in [\sigma_o, \overline{\sigma}]$. By collecting the σ terms and rearranging, the latter condition becomes $\phi - \sigma(v_o - v_c) \geq P(\sigma)\ \forall\ \sigma \in [\sigma_o, \overline{\sigma}]$. In group H, the firm charges consumer indexed σ a price $P_H(\sigma) = \phi - \sigma(v_o - v_c)\ \forall\ \sigma \in [\sigma_o, \bar{\sigma}]$. The optimal price function[22] is therefore

$$P(\sigma) = \begin{cases} P_L(\sigma) = \sigma v_c & for\ \sigma \in [0,\ \sigma_o) \\ P_H(\sigma) = \phi - \sigma(v_o - v_c) & for\ \sigma \in [\sigma_o,\ \bar{\sigma}] \\ \infty & otherwise \end{cases} \tag{3.23}$$

[21]Consumer $\overline{\sigma}$ gets paid to use the closed source software. In practice, firms sponsor influential and sophisticated consumers. Firms sponsor universities by providing them with research centers and free software. The negative price found in the model is equivalent to a real-life situation in which a firm sponsors some influential consumer. For example, a software firm may sponsor a university department because it gains from precluding the department from contributing to the quality of an open source software, for example, GEDCO, a Canadian company, helped the Ionian University of Greece with a sponsorship valued at €250 000 for geophysical software.

[22]Note that $P_H'(\sigma) < 0$ and $P_L'(\sigma) > 0$.

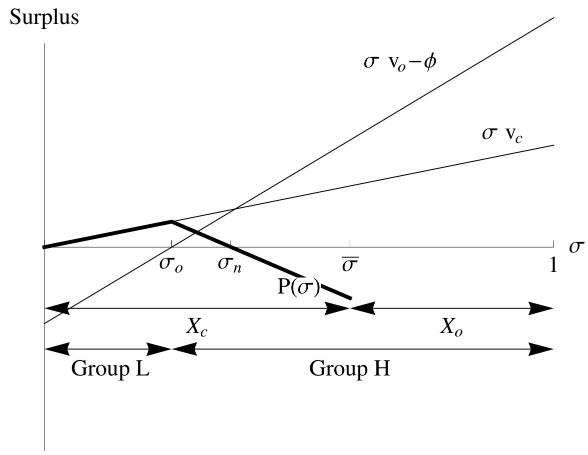

Figure 3.7: Low-quality closed source software and price discrimination: utility, surplus, and price

Because the firm does not sell to consumers with indices $\sigma > \overline{\sigma}$, these consumers choose the open source software. All consumers use software, that is the market is covered. Figure 3.7 illustrates this; the price function is represented by the bold line.[23]

The firm benefits from charging a negative price to some consumers belonging in group H. The firm does so in order to reduce contributions thereby allowing higher prices for the closed source software.

Inserting the price schedule defined in (3.23) into the profit function defined in (3.22) allows me to write the firm's maximization problem as

$$\max_{\overline{\sigma}} \Pi = \int_0^{\sigma_o(X_o^e)} P_L(\sigma)d\sigma + \int_{\sigma_o(X_o^e)}^{\bar{\sigma}(X_o^e)} P_H(\sigma)d\sigma \tag{3.24}$$

subject to

$$X_o^e = 1 - \bar{\sigma}(X_o^e) \;\; Fulfilled\ expectations$$

Profit (3.24) can be rewritten as

$$\Pi = \int_0^{\bar{\sigma}} \sigma v_c d\sigma + \phi(X_o^e) \int_{\sigma_o(X_o^e)}^{\bar{\sigma}} d\sigma - v_o \int_{\sigma_o(X_o^e)}^{\bar{\sigma}} \sigma d\sigma. \tag{3.25}$$

The first term of this expression is the surplus of group L which is entirely appropriated by the firm. The second term is the hedonic cost borne by group H; and the third term is the total utility that group H gets from the open source product. Note that σ_o and ϕ are both functions of X_o^e, which in turn depends on $\overline{\sigma}$. Differentiation of (3.25) with respect to $\overline{\sigma}$

[23]An interesting similarity with the present paper, is in the findings of Thisse and Vives [1988] of a price schedule which decreases in the distance to the firm. In their Hotelling model, the firm lowers its price because the competition is fiercer in remote places. In the present paper, I find that the firm's price schedule may decrease over a certain range. Although my model uses vertical differentiation, the explanation for the decreasing price schedule is somewhat the same. The relation between Hotelling-type models of horizontal differentiation, and models of vertical differentiation is shown in Cremer and Thisse [1991].

and collection of terms yields the first-order condition

$$\frac{d\sigma_o}{d\overline{\sigma}}(v_o\sigma_o - \phi) + k(\bar{\sigma} - \sigma_o) + (\phi - \bar{\sigma}(v_o - v_c)) = 0. \tag{3.26}$$

Since the first term of this expression is zero because it is the surplus of consumer σ_o and by definition of σ_o that surplus is zero. Hence,

$$-k(\bar{\sigma} - \sigma_o) = (\phi - \bar{\sigma}(v_o - v_c)) = P_H(\bar{\sigma}). \tag{3.27}$$

The firm profits from selling to all consumers in group L. Therefore, the firm chooses $\overline{\sigma}$ among the consumers in group H. This means that $\sigma_o > \overline{\sigma}$ which implies that the right-hand side of (3.27) is negative. This shows that in equilibrium the firm always charges a negative price to the consumer indexed $\overline{\sigma}$.[24][25]

The welfare effect of open source entry

In the closed source monopoly the market is covered and all consumers receive zero surplus. For consumers in group L, entry as no effect. The profit associated with these consumers is the same and these consumers still have zero surplus. However, all consumers in group H gain from entry. Either they consume a high quality open source software or they consume the closed source software from which they also obtain a positive surplus. Although the firm's profit is decreased, the gains in consumer surplus outweigh the loss. Total welfare increases.

[24] Bhaskar and To [2004] using a Hotelling-Salop model find that, with a fixed number of firms, perfect price discrimination provides incentives for firms to choose product characteristics in a socially optimal way, but that, with free entry, the number of firms is always excessive. In my model, when the firm sells at a negative price to some consumers, it does not directly create inefficiency as it is simply a transfer of surplus from the firm to the consumers. However, it does create inefficiency by lowering contributions to the open source software.

[25] In an equilibrium where both software products are consumed, the consumer with index $\sigma = \overline{\sigma}$ is indifferent between the open source and the closed source software, and is located at $\overline{\sigma} = \frac{(v_o-k)(\psi-k)}{v_o(v_o-v_c-k)-k(v_o-k)}$. If k is large enough, then the firm sells to all consumers. Using the fact that $X_o = 1-\overline{\sigma}$, it follows that $X_o > 0$ if, and only if, $k < \frac{v_o(v_o-v_c-\psi)}{v_o-\psi}$. Thus, the number of open source users is positive when the level of contributions is low.

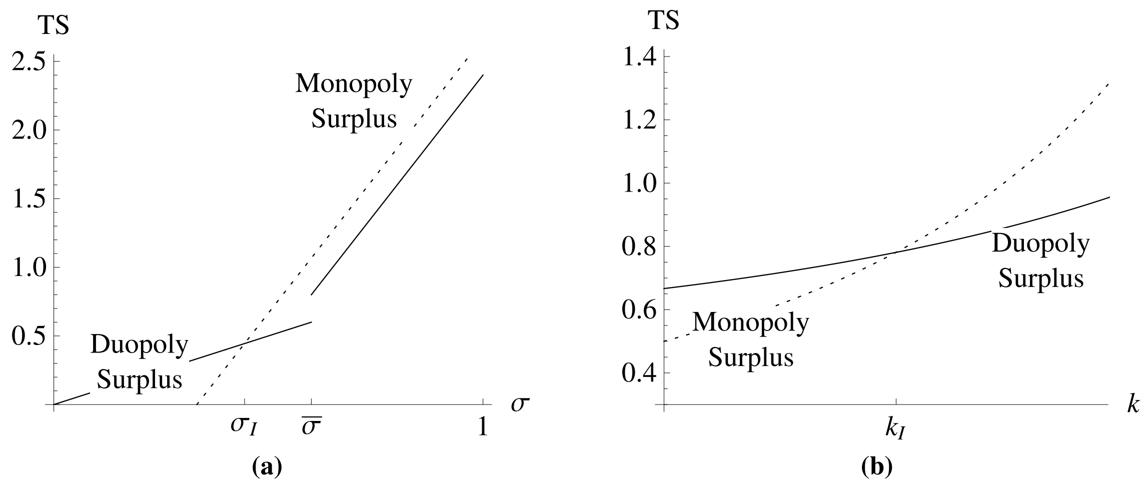

Figure 3.8: Comparison of the total surplus, denoted TS, when $v_o = 4$, $v_c = 1$, and $\psi = 2$

The welfare effect of closed source entry

Consider entry of a closed source software. The total surplus curve shown in panel (a) of figure 3.8 has a discontinuity at $\overline{\sigma}$ because the profit associated with consumer indexed $\sigma = \overline{\sigma}$ is negative. Recall that the firm pays the consumer indexed $\sigma = \overline{\sigma}$ to use the closed source software. Consumers with indices $\sigma > \overline{\sigma}$ use the open source software.[26] Overall the total welfare increases or decreases depending on the relative on the size of k.[27]

In panel (b) of figure 3.8, I compare the total surplus of the monopoly with that of the duopoly when $v_o = 4$, $v_c = 1$, and $\psi = 2$. Panel (a) displays the total surplus associated with each consumer in $[0,\ 1]$ when $k = 1$. The surplus associated with each consumer indexed $\sigma < \sigma_I$ is higher under duopoly than under monopoly. Conversely, the surplus associated with each consumer indexed $\sigma > \sigma_I$ is lower under duopoly than under monopoly. Panel (b) compares the total surplus under duopoly and monopoly as a function of $k \in [0,\ \overline{k}]$. The total surplus is higher under monopoly when $k > k_I$ where k_I is the value of k at which the aggregate surplus under monopoly equals aggregate surplus under duopoly. The intuition is the same as with uniform pricing. When contributions are significant, the loss in value of the open source software is significant and this causes a loss for consumers of the open

[26] Stole [2007] presents an oligopoly game of first-degree price discrimination in which firm differs with respect to their cost functions. He mentions that while the effect on consumer welfare depends on the shape of the particular price function used, the price discrimination always increases total welfare. I will show that in the context of this paper, the entry of a closed source firm practicing first-degree price discrimination may reduce total welfare.

[27] This makes a point in the ongoing argument between Nalebuff [2009] and Elhauge [2009] where the latter claims that one should not suppose that the total welfare effects of price discrimination are positive.

source software.

3.4 Conclusion

This paper addresses three questions: 1) how does the closed source firm pricing strategy affect the industry market structure; 2) how does entry of either closed or open source software affect welfare; and 3) how is the outcome to the aforementioned questions affected by price discrimination.

To address these questions, I establish a baseline by looking at a single closed source software firm first, and then at a single open source software producer. As in Mussa and Rosen [1978], the firm serves half the market in my closed source baseline. My second baseline has a market size that depends on user contributions.

In a competitive environment, the effect of user contributions has important consequences on the closed source firm's strategy. The impact of this effect on the market draws upon the relative quality of the products and the pricing policy of the closed source firm.

The model suggests that user contributions can be detrimental to open source software by indirectly lowering its value. In fact, user contributions may divert consumers towards closed source software; indeed, eradication from the market is particularly a threat when user contributions are significant. If the closed source firm was indifferent to changes in the level of contributions the open source software would actually gain market share as contributions increase, but the firm's aggressive pricing is detrimental the open source.

An increase in user contributions has two effects on the value of open source software. The first effect is direct, and adds value to the open source software through an increase in user contributions. The second is indirect, and counteracts the first effect. The closed source firm reacts by lowering its price thus lessening the open source's value. As more consumers redirect to the closed source product the loss in value for the open source is compounded by the decrease in the number of open source users, therefore contributors.

The relative magnitude of these two effects ultimately determines whether user contributions increases or decreases the value of open source software. Higher levels of user contributions exacerbate the second effect. It amplifies the decrease in price associated with variation in contributions. Consequently, when user contributions are significant, the second effect dominates the first one. The potential gains from increase in contributions are offset by the firm's price decrease. When the closed source firm does react to changes in contributions, its aggressive pricing strategy forces the open source software out of the market. Hence, when the contributions to open source software are significant, the contributions appear as a competitive disadvantage for the open source software.

What do these various scenarios imply for welfare? To answer this question, I look at the case of an open source software targeting consumers who have a high valuation of quality. Such a case results in a lesser value for the open source software given the introduction of a closed source firm targeting consumers who have a low valuation of quality. The lessened value of the open source software engenders a loss for consumers with a high valuation of quality, and this loss is greater when the level of user contributions is high. As indicated, the loss in value associated with a decrease in the size of the market for open source increases with the level of users contributions. The loss is even more important when the firm practices first-degree price discrimination. The firm's motive is as before, it sells at low prices to consumers with a high valuation of quality — it sells at a negative price to some consumers — to stifle the effect of user contributions. Consumers with a low valuation of quality may gain from entry because they may purchase the low quality closed source software. Overall, such entry, regardless of the entrant's pricing technique, may reduce total welfare.

While entry may be detrimental to welfare, it would be natural to assume that entry would at least increase industry output. This paper provides evidence to the contrary. It shows that entry of a closed source software serving consumers with a high valuation of quality lowers the number of software users. As previously noted, the entry reduces the value of the open source software. Therefore, some users with a low valuation of quality abandon the open source even though they were deriving benefit prior to entry.

Economic models of software markets tend to assume that the marginal production cost is zero. I have adopted this approach. However, whether the quality of closed source software is higher or lower than that of open source software could be made endogenous by assuming that the cost of quality is positive. In my model, the fate of the open source software is entirely determined by the decision of the closed source firm. This may well be critical to the conclusion that the open source product may be eradicated from the market. It would be of interest to examine whether such an outcome is possible if the strategy of the open source's managing entity were to maximize its market size. Also, the model assumes that only two software products are competing. One could introduce another competitive closed source firm. There are other potential extensions to the model: a more thorough examination of the indirect network effect (the user contributions) driving the results of the model; the firm could choose its location; a direct network effect can be introduced so that compatibility issues are studied. Another vantage point could consider a dynamic approach where a software benefits from a first-mover advantage or an existing user base.

3.5 Appendix

Proof of proposition 17. I proceed to the analysis of the function, X_o^*, for the three cases (shown in figure 3.2) in which the parameter ψ is in region I, II, and III, respectively.

The derivative of X_o^* evaluated at $k=0$ is $\frac{\partial X_o^*}{\partial k}\Big|_{k=0} = \frac{v_o-\psi}{2v_o^2} - \frac{\psi v_c^2}{2(v_c-v_o)^2 v_o^2}$ which, by definition of f^H (see equation (3.16)), vanishes at $\psi = f^H$. Because the expression is strictly decreasing for $\psi > 0$, it follows that it is positive for $\psi < f^H$ and negative for $\psi > f^H$. Thus, the function X_o^*, when evaluated at $k=0$, is increasing in region I; and decreasing in region II and III.

In region I, the parameter k is bounded by ψ (see assumption (3.2)), and thus, I evaluate the derivative at its boundary $k=\psi$, which yields $\frac{\partial X_o^*}{\partial k}\Big|_{k=\psi} = \frac{v_o^2\left(v_o^2-2v_c(v_o-\psi)\right)}{2(v_o-\psi)\left(v_o^2-v_c(v_o-\psi)\right)^2} + \frac{v_c^2\left((v_o-\psi)^2-\psi(v_o-\psi)\right)}{2(v_o-\psi)\left(v_o^2-v_c(v_o-\psi)\right)^2}$. The denominators are equal and positive, so I focus only on the numerators. The first term (numerator) is negative for $\psi < v_o\left(1-\frac{v_o}{2v_c}\right)$, but ψ is always smaller than this value because I am interested in cases where $\psi < \overline{\psi}$; and $\overline{\psi} < v_o\left(1-\frac{v_o}{2v_c}\right)$. The second term is positive and decreasing in ψ (lemma 16 states that $v_o - 2\psi > 0$). Thus, when the absolute value of the first term is greater than the value of the second term, the expression is negative; and the converse is also true. Because the first term is linear in ψ and the second term quadratic, the line describing the absolute value of the firm term can cross the line described by the second term at most twice. In fact, the two terms crosses at points: $\psi_L = v_o\frac{3v_c-2v_o}{4v_c} - v_o\frac{\sqrt{v_c^2+4v_o(v_c-v_o)}}{4v_c}$ and $\psi_H = v_o\frac{3v_c-2v_o}{4v_c} + v_o\frac{\sqrt{v_c^2+4v_o(v_c-v_o)}}{4v_c}$. I reject ψ_H because $\psi_H > \overline{\psi}$. Thus in the relevant range for ψ, the absolute value of the first term crosses the line described by the second term only once at ψ_L. Now, two properties ensures that the derivative of X_o, when evaluated at $k=\psi$, is increasing in k for $\psi < \psi_L$ and decreasing in k for $\psi > \psi_L$: both terms are decreasing, and the second term is convex in ψ. In region I, $\psi < f^H$, and so $0 < \psi_L < f^H$ entails that, depending on the value of the parameters, the function can be both decreasing and increasing in contributions. The demonstration also shows that X_o^* is decreasing in region II at point $k=\psi$ because $\psi_L < f^H < \psi$.

In region II and III, the demand function is strictly concave in k: I need to show that

$\frac{\partial X_o^{*2}}{\partial k^2} < 0$ where

$$\frac{\partial X_o^{*2}}{\partial k^2} = \frac{v_o}{(v_o-k)^3} - \frac{v_o(v_c-v_o)-kv_c+v_c(v_o-k)}{(v_o-k)^3(v_o(v_c-v_o)-kv_c)^3} \times$$

$$\left(-k\psi v_c \frac{v_o(v_c-v_o)-kv_c+v_ov_c}{(v_o-k)^3(v_o(v_c-v_o)-kv_c)^3} + \psi v_o^2 \frac{(v_c-v_o)^2-v_ov_c}{(v_o-k)^3(v_o(v_c-v_o)-kv_c)^3}\right).$$

To simplify the function, I multiply it by $(v_o-k)^3(v_o(v_c-v_o)-kv_c)^3$; and that does not change the sign of the function because, by assumption (3.9), $v_o(v_c-v_o)-kv_c$ is positive. Then, the sign of $\frac{\partial X_o^{*2}}{\partial k^2}$ is as the sign of

$$v_o\left(v_o(v_c-v_o)-kv_c\right)^3 - \left(v_o(v_c-v_o)-kv_c+v_c(v_o-k)\right)\times$$
$$\left(-kv_c\left(v_o(v_c-v_o)-kv_c+v_cv_o\right)+v_o^2\left((v_c-v_o)^2+v_cv_o\right)\right)\psi.$$

I now show that this expression is decreasing in ψ and use $\psi = f^H$ to find its least upper bound with respect to ψ. The first term does not affect ψ, the second term — before "×" — is positive because of assumption (3.9), and it follows from assumption $v_c > v_o$ and assumption (3.2) that the third term is positive. Evaluating the expression at $\psi = f^H$ yield

$$-v_c^2v_o^2\frac{k^2v_c\left(k(2v_c-v_o)-3v_o(v_c-v_o)\right)+(vc-vo)^2vo^3}{(v_c-v_o)^2+v_c^2}$$

which constitutes a least upper bound for the expression since $\psi > f^H$. Now, the resulting expression is increasing in k. To see this, note that the maximum acceptable k is the fixed point where $\overline{k}(\psi) = \frac{v_o(v_c-v_o)}{2v_c} = \psi$. Now, since $k < \frac{v_o(v_c-v_o)}{2v_c} < \frac{3v_o(v_c-v_o)}{2v_c-v_o}$ it follows that $k(2v_c-v_o)-3v_o(v_c-v_o) < 0$ in the relevant range. Because the expression is increasing in k, evaluating the function at $k = \frac{v_o(v_c-v_o)}{2v_c}$ (the maximum acceptable value for k), again, constitutes a least upper bound. The expression evaluated at $k = \frac{v_o(v_c-v_o)}{2v_c}$ is reduced to $-\frac{v_c^2(v_c-v_o)^2v_o^5}{(2v_c-v_o)^2}$ which is strictly negative. I conclude that X_o^* is strictly concave for $\psi > f^H$.

As for the slope of the function, because X_o is strictly concave, together $\left.\frac{\partial X_o^*}{\partial k}\right|_{k=k_1} < 0$ and $\left.\frac{\partial X_o^*}{\partial k}\right|_{k=k_2} < 0$ imply $\frac{\partial X_o^*}{\partial k} < 0$ $\forall k \in [k_1,\ k_2]$ where $k_1 < k_2$. Thus, it suffices to show that the function is strictly decreasing when evaluated at the boundaries of the relevant parameter space in order to prove that the market size is strictly decreasing.

I already showed that X_o^*, when evaluated at $k = 0$, is decreasing in regions II and III. And I showed that it is decreasing at the boundary $k = \psi$ in region II. It remains to be shown that the function is decreasing at the upper bound of region III.

In region III, $\psi > \overline{k}$ and thus the maximum value that k can take is $k = \overline{k}$ at which point $\frac{\partial X_o}{\partial k}\Big|_{k=\overline{k}} = -\frac{v_c^2(v_o-2\psi)^3}{2v_o^4(v_o-\psi)\psi} < 0$ (by virtue of lemma 16, the numerator is positive). □

The following lemma will be used in the proof of the next proposition.

Lemma 21. *Together assumption 3.2, condition* (3.21)*, and assumption $k < \overline{k}$ imply that $\psi_1 = v_o - v_c > k$.*

The lemma simply states that if both software products are in the market in equilibrium, the difference in quality must be large enough relative to contributions.

Proof of proposition 20. I proceed to the analysis of the function, X_o^*, for the three cases where the parameter ψ is in region I, II, and III, respectively.

First, I show that X_o^* is strictly convex in k when ψ is on the interval $[0,\ v_o - v_c)$, that is in region I. The second-order derivative of X_o^* with respect to k can be written as $\frac{\partial X_o^{*2}}{\partial k^2} = A + B\psi$, where

$$B = -\frac{(2v_o - v_c - 2k)\left(k^2 + v_c^2 + v_c k + v_o(v_o - v_c - k)\right)}{(v_o - k)^3(v_o - v_c - k)^3}.$$

It follows from corollary 21 that B is negative since both the numerator and the denominator are positive. Thus, the second-order derivative is decreasing in ψ. Since $\psi < v_o - v_c$, evaluating the expression at $\psi = v_o - v_c$ constitutes a lower bound. This lower bound $\frac{\partial X_o^{*2}}{\partial k^2}\Big|_{\psi=v_c-v_o} = \frac{v_c}{(v_o-k)^3}$ is positive implying that the second-order derivative is negative for all $\psi \leq v_o - v_c$. I conclude that X_o^* is strictly convex when $\psi < v_o - v_c$.

I now look at the slope of the function. At $k = 0$ the derivative of X_o^* evaluates to

$$\frac{\partial X_o^*}{\partial k}\Big|_{k=0} = \frac{v_o\left((v_o - v_c)^2 + v_o(v_o - v_c)\right)}{2(v_o - v_c)^2 v_o^2} - \frac{\left((v_o - v_c)^2 + v_o^2\right)}{2(v_o - v_c)^2 v_o^2}\psi.$$

The fact that the expression is strictly decreasing in ψ, and the fact that the expression vanishes at $\psi = f^L$ entails that the expression is positive for $\psi < f^L$. Thus, the demand is increasing at $k = 0$ in region I and II, and decreasing at $k = 0$ in region III.

In region I, since I showed that the function is strictly convex and that it is strictly increasing at $k = 0$, it suffices to show that the function is also increasing at $k = \psi$ to prove the first statement of the proposition. The derivative of X_o^* evaluated at $k = \psi$ yields $\frac{\partial X_o^*}{\partial k}\Big|_{k=\psi} \frac{v_o - v_c - \psi + (v_o - \psi)}{2(v_o - \psi)(v_o - v_c - \psi)} > 0$ since $\psi < v_o - v_c$.

In region II and III, I evaluate the function at $\bar{k}$ yielding $\frac{\partial X_o^*}{\partial k}\Big|_{k=\bar{k}} = \frac{(v_o - v_c - \psi + (v_o - \psi))^3}{2v_c^2(v_o - \psi)(v_o - v_c - \psi)}$. The expression has a negative denominator since $\psi > v_o - v_c$. The numerator is positive since $\psi < \overline{\psi}_L < \frac{2v_o - v_c}{2}$, so the expression is negative. □

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