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Estimating preferences with random utility models

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L'abstrait

Ceci est un papier de recherche de maîtrise en science économiques sur le sujet d'estimation des préférences avec les modèles de choix discrets. Le travail du lauréat Nobel Daniel McFadden sur les modèles logit est adapté dans le model d'utilité aléatoire. Ce model permet une perspective intéressante sur la théorie des préférences et de la transitivité, tel que avancé par Amos Tversky, parmi d'autres. La grande partie de ce papier de recherche s'inspire de l'article de Regenwetter et co-auteurs, "Transitivity of preferences" (2011). Les donnes d'un sondage mené en 2011 par Regenwetter et co-auteurs sont utilises ici afin de faire de l'inférence sur le model et estimer des utilités, qui sont ensuite ordonnés de manière de construire des relations de préférence. Une discussion sur les préférences, les modèles logit et ses estimations parait dans ce papier. Ce travail a été fait sous la supervision de professeur William McCausland, Université du Montréal, Quebec.

Abstract

This work is a masters research paper in economics on the topic of estimating preferences with discrete choice models. The work of Nobel laureate Daniel McFadden on logit models was adapted into a model on random utility. This model allows for an interesting perspective on the theory of preferences and transitivity as conjectured by Amos Tversky, among others. The bulk of this research paper draws from the article by Regenwetter and co-authors "Transitivity of preferences" (2011). His data from the 2011 survey was used to run inference on the model and estimate utilities, which then allowed to construct and analyze preference relations. A discussion on preferences, logit models and estimation is featured in this paper. The work was done under supervision of professor William McCausland, Universite du Montreal, Quebec.

INTRODUCTION

This work is a masters research paper on the topic of estimating preferences with discrete choice models. The focus of this work is to motivate and build an empirical model capable of estimating preference relations in a stochastic environment. By using the assumptions of the logit model with respect to estimating discrete choice sets, the model will use the power of the logit model for the purpose of estimating utilities of individuals. The paper will feature discussion on both theory of preferences and inference work.

The paper combines theory of preferences and random utility. The first section will review the theory on preferences and transitivities in dynamic and stochastic environment. Starting with Amos Tversky's seminal paper "Intransitivity of preferences", which has seriously shaken the axioms of rational choice and spurred a lot of research on the subject of choice, rationality and transitivity, this section will discuss the more recent developments in that area, such as the critique of Regenwetter and co-authors on the faults of existing models.

The next sections will discuss choice modelling. The paper will explain the basics of discrete choice models, the logit model and the contribution of Nobel laureate Daniel McFadden to the theory. The paper will then use his econometrics work to build two empirical models; the first is a random utility model and the second is a logit model that supposes a context effect in the utility of individuals. The first model assumes utility is composed strictly of an error term that has a logit distribution. The second model assumes the utility is in part determined by the context, that is what choices the individual is asked to rank.

Furthermore, the paper will present and analyze the results that have been obtained by carrying inference with the aid of the two logit models. All programming was done in R language and all the data was taken from the survey conducted by Regenwetter and co authors in 2011. This model will shed a little light on some conjunctures about cyclicities but unfortunately no definite conclusion can be reached.

Finally, this paper will suggest few ideas for future work. One of such ideas is improving estimation techniques through a better experimental design and surveying methods. One modest experimental design will be proposed but a thorough discussion on the topic is beyond the scope of this paper.

THEORY

This section will briefly cover the theory of transitivity in choice preferences. Transitivity is a crucial axiom in rational choice. Increasingly, researchers challenge the notion of transitivity in preferences. The following section will explain what is transitivity and why transitivity is important, why researchers overestimate cases of transitivity and the difficulties related to uncovering transitivity in general. It will be important to understand the difference between preferences in a stochastic and deterministic environment. This section centers around the theory in Tversky's 1969 paper and the subsequent critique. The following section is a literature review; it introduces the concepts and the notions before dwelling into the somewhat more technical aspects of choice modelling.

Rationality and transitivity

Rational choice is a very vast and complicated idea. Economists think of rational individuals as having a preference relation over choices. Every choice, for example the choice of a car, yields a utility to the chooser, which in turn can be ranked. The choice yielding the highest utility is said to be most preferred, the choice with the second highest utility is second most preferred and so on. Preference relations are crucial as much of the theory of micro economics rests on the idea that rational agents are looking to make the best possible decision. The theory of preference rests on four principal axioms : the preferences are 1) transitive, 2) complete, 3) continuous and 4) independent. The first axiom is also perhaps the most crucial and will be thoroughly discussed in this paper.

Preferences are defined as relations over objects and the objects could be either tangible or decisions like "watching a movie". In a set of three possible objects {A,B,C}, we say an individual n can rank the three in order based on what he prefers most to what he prefers least, for example $(A>_n B>_n C)$, where A is preferred to B, B is preferred to C and A is preferred to C. The $>_n$ (or $<_n$) is a preference operator and denotes an object being preferred to another object according to individual n. When obvious from the context, notation n will be dropped from the operator and preferences are denoted by simply > or < signs.

Transitivity is a fundamental axiom of rational choice. Formally, we define transitivity as: if (A>B) and (B>C) then it must be that (A>C). If choices are not transitive then it is said that they are intransitive. For example, if an individual prefers A over B and B over C but then prefers C over A, we say she violates the transitivity axiom on preferences. This kind of violation contradicts the notion of rationality and it challenges the definition of preferences as we understand it.

Tversky has shown that given particular objects, individuals would, knowingly or not, exhibit preferences that violate this axiom of transitivity. These type of preferences are different from what is hypothesised under classic theory. Termed lexicographic semiorder preferences, they satisfy the definition of transitivity in theory; however in practice it is possible to manipulate the attributes of a choice set such that preferences contain intransitivities. His results spurred a lot of research on rationality, utility, preferences and transitivity. Today this is a very popular research topic in many disciplines and there even exist a number of research centers that study related questions, for example the Center for the Study of Rationality in Jerusalem, Israel and the Center for the Study of Choice in Sydney, Australia. In other fields like biology and neurology, animal brains are tested to determine if rats and birds are capable to calculate expected probability of an event (Glimcher and Rustichni 2004).

Tversky model

The model was originally conceived by psychologist Amos Tversky in 1969. The model was a major contribution to the study of rationality and choice, the understanding of preferences and the discipline of economics in general. The set up will be explained in this section because the rest of this paper will use the same concepts and definitions.

Tversky conducted a survey among 18 Harvard undergraduate students. Each student was asked to answer a series of questions about his preferences. Every question would present the student with two alternatives and ask him to select one that he likes best. The alternatives are objects called a "lottery" because each object has a probability of realization (between 0 and 1) and an outcome (winnings of a lottery) in real U.S. dollars. Intuitively it is possible to calculate an expected value (e.g E(x)=probability of winning * value of prize x) of each lottery but this calculation was not provided nor encouraged. There are a total of 5 lotteries, {A,B,C,D,E} which is a global (or master) choice set. The lotteries will be also called alternatives, objects and items throughout the paper when obvious from the context. A set of alternatives from which an individual was asked to choose is called a "choice set". For the purpose of his survey, the choice set has two items because in each question the individual was tasked to choose one of two choices; effectively, the individual is making a series of pairwise comparisons over all the ten possible ways to construct two-choice object set when the master set contains 5 items. When an alternative has been chosen, it will be referred to as a "choice".

The purpose of the survey is to learn about each individual's preferences. Each question asks an individual to choose between two alternatives. Indifference and non-response are not allowed. Because there are ten possible choice sets with two elements, an individual is asked a total of ten questions and each of the ten questions is repeated twenty times to measure consistency. Plus there are also filler questions asked in the middle of the test to mitigate memory effects, statistical dependencies and otherwise to make sure the real questions are properly administered, but these filler questions are not recorded for the experiment. From observing an individual's answer, it will be possible to discover how she feels about each of the five items {A,B,C,D,E}.

The nature of the test necessitates a stochastic environment so the first thing that was necessary is to introduce a new model to measure behaviour in probabilistic fashion. Formally, object i is said to be weakly stochastically preferred over object j if the probability of choosing i in a set {i,j} is equal to or greater than 1/2, or $Pr(i,j) \geq 1/2$. In turn, transitivity is defined as weak stochastic transitivity where : if $PA(A,B) \geq 1/2$ and $PB(B,C) \geq 1/2$ then $PA(A,C) \geq 1/2$. A higher probability here is tied to the idea of constituency. Individuals that are very consistent in their choice will have a probability approaching 1 (e.g. they behave deterministically).

By conducting the experiment, Tversky has shown that not only do individuals violate the idea of transitivity as we know it but in predictable ways. According to the results of the survey, the majority of individuals violated transitivity in their own preferences. These results were very surprising, including to the respondents themselves who denied at first acting intransitively. This kind of behaviour challenges the notion of preferences as we understand them.

The major contribution of his paper is the introduction of a Lexicographic Semiordering type of preferences which he says individuals occasionally exhibit when faced with particular attributes.

The Lexicographic Semiordering is formally defined as: A > B if and only if (a > b) or (a = b and a' ≥ b'), where (a,a') are respectively the first and second attributes of object A and (b,b') are respectively the first and second attributes of object B. In other words, the first attribute is the most important for an individual's choice. Only when an individual is indifferent or undecided between two alternatives based on the first attribute will she use the second attribute to judge between the objects A,B. An example of a lexicographic semiorder in practice is the system by which a dictionary is composed.

Ranking based on LS preferences does not violate transitivity on its own. Rather, it is the perception of an individual that can be manipulated so to force transitivities in choices. If the difference between the first attribute of two objects (a and b) is minimal, the individual will make the decision based on the 2nd attribute (a',b'). When difference of (a,b) is large again, the individual will make the decision based on the first attribute. This type of decision making can generate intransitivities in preferences.

Regenwetter et co critique

Regenwetter, Dana and Davis-Stober discussed at length the issues with literature on transitivity and the faults with Tversky's model and a number of others (Regenwetter and co 2011). The main component of their paper is that the literature overestimates cases of transitivity due to faulty statistical analysis and bad model specification.

The central remark is how researchers are so far unable to reconcile a dynamic model with a static model. The classic theory on preferences is defined in a static environment whereas empirical testing is set in a stochastic environment. Unfortunately, most dynamic models are not adequate enough to estimate preferences in dynamic environments. Indeed, most of the models have one common shortcoming, namely the inability to distinguish between variability and consistency.

It is impossible to know the difference between variability and consistency in individual's choices. When an individual answers a question 20 times, half of those times she might be in a state of mind where she prefers A and the other half he might be in a state of mind where she prefers B. This kind of variability is an observed phenomenon in real life and thus expected on the part of respondents as well.

By using tricks to mitigate for memory effects, the researchers hope to control for different transitive states but this does not usually work. Therefore, when a respondent is repeated the same question and she answers it differently, she may in fact be expressing variability which is a result of being in a different mental state. This type of behaviour is both natural and anticipated. However, as the researcher cannot observe what transitive state she is, he considers it part of the same preference relation. He assumes that the difference in answers is due to a lack of consistency by the respondent, which is not necessarily true. The models on stochastic preferences is laden with these problems and more. Ignoring it leads to faulty estimation of preferences.

Further criticism by Ragenwetter is how researchers do not focus on all cases of cyclicities. A single respondent has multiple preference relationships and thus multiple ways to break the cycle. Most researchers will assume there is but one preference relationship, so they do not focus on other cases of transitivity (or lack thereof) that might be happening.

Furthermore, there s a problem with Type1 errors. Type1 error is defined as rejecting a hypothesis that should have not been rejected. The error happens when the sample estimate falls outside the confidence interval.

Estimating a preference relation through a statistical test forces the existence of a type1 error. When the researcher estimates multiple preference relations, he runs into multiple type1 errors. A series of pairwise relations extenuates the probability of a type1 error because all it takes is one pairwise relation to be rejected for an individual to be referred to as intransitive. When preferences are "weak" or when PrA(A,B)=½, it is more likely that an estimated cycle is in fact a Type1 error.

In his overview of the literature on transitivities, Regenwetter et co reviewed over 20 papers. Another common mistake researchers make is assume that transitivities are linear, that is they satisfy the triangle inequality. By forcing linearity, researchers force strong assumptions on preferences--linearity of preferences is a lot stronger than transitivity of preferences. By dropping the triangle inequality restriction, they discover that most instances of transitivity are over-reported.

Finally, Regenwetter and co carried out a similar experiment with a few slight modifications. For one, the prizes of lotteries are now adjusted to 2011 price levels and the candidates are not pre-screened. In their experiment, they found that only 4 out of 18 individuals violated transitivity and even that was within the margin of Type1 error.

DISCRETE CHOICE MODEL

It is said the rational individual makes a choice which she thinks will give her the highest utility among all available choices. In a choice set with two alternatives {i,j}, the individual will choose object i over object j if the utility of i is greater than utility of j, $U_i > U_j$. A discrete choice model is often used to describe this type of decision-making because of the discrete nature of the objects. Unlike regressions on continuous variables, a DCM is applicable here because the object set is finite and countable and because the outcomes are discrete (e.g. an individual chooses either object i or object j, she does not choose some fraction of the object).

A discrete choice model describes the relationship between the explained variables (e.g. some attributes) and the outcome. For a model to be effective, the three conditions have to be satisfied: the choice set has to be finite, the alternatives have to be exhaustive (e.g. all possible options have to be presented such that the individual can go through them and choose at least one) and the alternatives have to be mutually exclusive.

Daniel McFadden was one of the main researchers to have made DCM's popular and is credited with much of the development of discrete choice models. In his original work on the San Francisco transit system (McFadden 1974) he used a discrete choice model to analyze individual's decisions to use various modes of transport such as car, train, carpooling and bus as well as the frequency of the transportation. His model would be able to predict the relationship between the observed variables and the outcomes.

The utility accrued to an individual from choosing alternative i, Ui, is composed from an observed part (to the researcher) and an unobserved part. The former is some known attribute, for example the monetary cost of choosing an alternative. The unobserved part is everything else that isn't specified in the utility. Formally, utility is Ui = Vi + Ei where Vi is the observed part and Ei is the error term (unobserved) of object i. The decomposition is fully general in a sense that we say that Ui - Vi = Ei.

The assumptions about the distribution of the error is crucial because it allows the researcher to specify a density function to estimate the otherwise hidden term. In the context of decision-making over discrete variables, the error in the utility, Ei, is assumed to follow an identically, independently, extreme value distribution if the assumptions above are satisfied.

The difference between two error terms that have extreme value distribution, E*ij = Ei - Ej, is said to follow a logistic distribution. This has been proven by McFadden as part of his work on logit models. The assumption about the error term E*, also proven as part of his work, is the motivation for using a logit model such as the one that will be described in the next section. This paper will concentrate on the logit model which, according to Kenneth Train, is the most popular of the discrete choice models today (Train, 2009).

There exist other, more sophisticated, models in the family of logit models, such as nested-logit and the mixed logit models. As well, there are other models such as the multinomial and probit models--they are assumed to have a different error distribution, although the purpose thereof is essentially the same. These models are considered less rigid as they have slightly more relaxed assumptions on the errors, for example the absence of a correlation assumption among errors. Other class of models do not assume distribution of error at all; rather they estimate probability function with monte-carlo simulation. They are usually employed in more sophisticated analysis which is beyond the scope of this paper.

Logit

Originally developed and proved by McFadden in 1974, the logit model is the most popular of the family of discrete choice models. It is simple to understand and to use, has a known closed-form density and was referenced in the literature for decades.

Like all the class of discrete choice models, it deals with discrete outcomes. The purpose of the model is to relate some independent variables X to a discrete outcome Y. In a standard discrete choice model, the X would be the attributes of an object (1: the probability of winning a lottery and 2: the amount of the winning prize in dollars) and the Y would be the individual's choice from a two-choice set {i,j}. The coefficient on X is the likelihood (between 0,1) of impact by X on Y.

The key component of the logit model is the hypothesis on the error distribution, which is said to be distributed logistically. Formally, Y = X + E* where E* is the error term.

The density function of E*ij is : $f(E^{*}ij) = \frac{exp(E^{*}ij)}{1+exp(E^{*}ij)}$

As been mentioned, it is possible to use this error distribution in the model when it assumed that the errors on alternatives (Ei, Ej) are identically, independently, distributed. That is, they are not correlated among themselves, corr(Ei,Ej)=0 for all i≠j, nor are they correlated with the independent variables X.

Some researchers consider this assumption restrictive. After all, it is difficult to presuppose that the alternatives are not correlated. For example, some features about alternative A are also likely to appear in another alternative B.

Kenneth Train explains that the assumption is not as restrictive as it seems at first. In fact, it follows directly from the construction of the discrete choice model itself. Recall the requirements of a discrete choice model is that the choice set is finite, the alternatives of the object set are exhaustive and mutually exclusive. If these conditions hold, then it is straightforward that there can be no correlation among errors. In other words, the errors are nothing but white noise. Thus, rather than a concern, it is actually a characteristic of a well-specified model. This paper will adopt Train's justification.

In a stochastic environment, the probability that object i is chosen in a set {i,j} is $Pi(i,j) = Pr(Vi + Ei > Vj + Ej)$; or,

object i gets chosen if $Pr(Ei > Ej + Vj - Vi)$ = ½.

The probability function relating the probability that i is chosen among all alternatives j is:

$P_i = \frac{exp(Ui)}{\sum_j exp(Uj)}$; where Ui is the utility from alternative i and Uj is the utility from any alternative j in the choice set.

The knowledge over probabilities can be used towards an empirical estimation, which the next section is going to discuss. The probabilities are going to be specified in a likelihood function. Then the negative of the likelihood function is going to be minimized by way of maximum likelihood estimation. Because the MLE is concerned with finding a maximum and because the point at which the maximum occurs does not change if the function is transformed monotonically, the likelihood function can be decomposed logarithmically into a log-likelihood. More of that in the next subsection.

MODEL 1 & 2

Two discrete choice models were produced for the purpose of this paper. Both are based on the logit specification of the error terms. The first model is a true random utility model. The second model is a logit model with a context effect.

The models are constructed based on the theory of decision-making and the theory on probabilities. Then they are adapted so to be used for inference purposes in a stochastic environment. In other words, both Model 1 and Model 2 are empirical models: they are meant to analyze the data collected from the survey by Regenwetter and co-authors and produce estimates for the utilities of objects A,B,C,D, and E. Those estimates can then be used to answer some questions in the research on the theory of preference and transitivity.

The models will examine the pairwise decisions of all the individuals. Recall an individual is asked to choose an alternative from 10 pairwise object sets. Each pairwise object set is asked 20 times, for a total of 200 questions per individual. Using the observations from all the pairwise relations, the empirical models will estimate the utilities for every object for every individual.

The first model is a random utility model. A RUM assumes that the probability that an individual chooses an object i is completely random but has a known distribution. The second model assumes the probability of choosing an object is part random and partly depending on the choice set. The second model will test the hypothesis that an object set is important for individual decision. A discussion on both models will follow.

The models will be used to analyze a number of questions that were postulated in the research about choices and preferences. First, this paper will present some preference relations that were constructed based on the computed utilities. Second, the paper will analyze intransitivities given the constructed preference relations. Third, it will discuss several shortcomings of the models and possible improvements for future work.

As well, there will be a discussion on what can be learned from the models and whether they explain the data very well and the forecasting capacity of the model. A section that deals with results will also talk about the likelihood score and the fit. The second model will answer questions regarding the context of the objects and the importance of the choice set; for example, whether introducing a variable that deals with the context improves the fit of the model.

Model 1

This section will explain the motivation behind Model 1 and how it is applied empirically in order to estimate the utilities from data on individuals.

Model 1 is random utility model. As the name implies, it assumes that utility from selecting an alternative i is random and depends on the distribution of the error term Ei, or Ui = Ei. In other words, an object i is chosen from a set {i,j} when Ei > Ej, or Ui > Uj. Now the probability that Ei > Ej is also an error term, E*ij. If the assumptions on E*ij, as outlined above hold, then it possible to use a logit model for estimation.

The chief purpose behind Model 1 is to test the theory on preferences in a stochastic environment. That purpose is to run inference and to estimate the utility for all the objects in the master set {A,B,C,D,E}. Once the utilities are obtained, it follows that preferences can be constructed. Once preferences are constructed, there can be a discussion on cyclicities.

The simplest and most obvious way is to rank the estimated utilities {UA,UB,UC,UD,UE) ordinally from lowest to highest. This method has its faults, as will be discussed in the section on Preferences, but it is a relevant exercise for the purpose of this paper. It is somewhat more sophisticated than simply looking at all the pairwise comparisons like Tversky did, and given the advances in random utility models, it is only appropriate that somebody finally did it.

True to its name, the utility from the alternatives is assumed to depend on the error term. Some parametric assumptions are imposed on the error but otherwise there are no structural assumptions.

Given the observations on the outcome (e.g. Nij), it is possible to use those observations to calculate what would be the estimated utility to individual possesses.

We have seen that the probability that an object i is chosen in a pairwise comparison {i,j} is $Pi(i,j) = \frac{exp(Ui)}{exp(Ui)+exp(Uj)}$

The probability that object j is chosen in a pairwise comparison {i,j} is

$$Pj(i,j) = \frac{exp(Uj)}{exp(Ui) + exp(Uj)}$$

and Pi(i,j) + Pj(i,j) = 1 because non-choice or indifference is not allowed.

To run inference, it is necessary to combine all the possible pairwise comparisons and specify them in a likelihood function. There exists a total of ten pairwise comparisons, they are :(AB, AC, AD, AE, BC, BD, BE, CD, CE, DE)

Putting them all together in a likelihood function would look like this:

$$PA(A,B)^{Nab} * PB(A,B)^{Nba} * PA(A,C)^{Nac} * PC(A,C)^{Nca} * \ldots\ldots * PD(D,E)^{Nde} * PE(D,E)^{Ned}$$

where $PA(a,b)^{Nab} = (\frac{exp(UA)}{exp(UA)+exp(UB)})^{Nab}$

The exponent on top of each factor (e.g. N_{ab}) denotes the amount of times that particular alternative has been chosen out of 20 times it was asked, and Nab + Nba = 20. The exponents are pulled directly from the results from the survey conducted by Regenwetter.

The goal of this maximization is to find out the unknowns in the likelihood function, which are UA,UB,UC,UD and UE. The unknowns are found by solving for the maximum the likelihood function above. The procedure is known the Maximum likelihood estimation and it is a standard econometric procedure.

The size of the coefficient UA (w.l.o.g) depends on two factors. First, it depends on whether A was chosen the majority of time when compared to the four alternatives B,C,D,E. Second, it depends on how frequently it was chosen in the four possible choice sets (A,B),(A,C),(A,D),(A,E). In other words, a high value of Nab, Nac, Nad, Nae will also imply a high value for coefficient UA, and the size of Nab, Nac, Nad, Nae reflects strong preferences towards alternative A.

However, it is important to remember that the value of the coefficients is not important in itself, rather one has to compare it to other coefficients, including the normaliser which is set at 0. What's important is the difference between the coefficients and the normaliser.

Model 2

The second model is similar to the first in a sense that errors follow a logistic distribution. Its purpose is also to run inference on the data and estimate the coefficients for UA,UB,UC,UD,UE. However, Model 2 has the additional hypothesis that context matters in pairwise decisions so an inference exercise will also measure an additional variable alpha for significance.

Context is said to matter if researchers suspect that the presence of an alternative j in a set {i,j} may influence an individual's decision making. The theory on LS preferences, for example, supposes that adjacent variables mislead an individual into choosing a less preferred alternative because of perception bias.

Model 2 is not a true random utility model as there is another variable added to the utility of an object i, Ui = Ei + $\text{alpha}_{ij}1_X(j)$. The indicator function means that alpha equals some variable when j is in the choice set X={i,j} and 0 otherwise. Unlike Model 1, utility here does not solely depend on the error term, but also the object set X.

The role of object set in decision making is an interesting topic and can be discussed with the additional hypothesis that context plays a role. To test the hypothesis, a variable alphaij is specified in the utility from object i, where alpha is some non-zero variable and j is the object in the choice set that said to matter (e.g. the presence of j is important). The motivation behind the hypothesis is the theory that adjacent objects may lead to perception bias and thus to cyclicities. This goes back to the theory of Tversky that individuals indifferent between the two objects based on the first attribute of the object will make the decision based on the second.

If alphaij is significant then the hypothesis of context may be true. Furthermore, the new model will have a better fit to the data. So by including alphaij to control for the effect of context on decision making, it is possible to produce more powerful results and perhaps test some new ideas about the research.

The first term of the Likelihood function, PA(A,B), now looks like this :

$$PA(A,B)^{Nab} = (\frac{exp(UA + alpha_{ij}1_X(j))}{exp(UA + alpha_{ij}1_X(j)) + exp(UB)})^{Nab}$$

Of course, the variable alpha has to be normalized in terms of another variable for an interpretation to exist. So in the output, there will be information on 4 alphas and the 5th alpha is set to 0 by the researcher. The 4 alphas will be interpreted in terms of the normalized-alpha.

The only caveat with adding more variables to the model is that the likelihood score cannot decrease. In fact, it will increase even when the impact of the new variable is very small. The expanded model can appear like a better fit then the unadjusted model even though it is not necessary the case. In this situation, it is necessary to use caution and perform a fit test such as the Wald test. The difference in estimation between the two models is a good topic for discussion and will be dealt with in the section on results.

Estimation

All values are computed with a program that was coded in R (see appendix D). The log-likelihood function was entered once as seen above, then it was looped over for each of the 18 respondents. Using MLE estimation procedure as composed in one of the R packages, the negative of the LL function was minimized for each respondent. The variables that came in the output are the utilities for A,B,C,D and E. For Model 2, the log likelihood function of the model was modified to allow for an expanded model. The expanded model would test whether the presence of certain alternatives was significant for an individual making his choice. More about that in the next section.

Not least, the important thing about the model is a good fit. A model with a high LL score is said to fit the data well.

ANALYSIS OF THE RESULTS

This section will analyze the results that were obtained from maximizing the log-likelihood functions. The input is the data collected from the survey by Regenwetter and co-authors and the output are the estimated utilities for objects A,B,C,D and E. The results are tabulated in a 18*5 matrix where the rows are respondents and the columns are utilities for individual objects (Appendix A). The first column is the utility from choosing object A, it is normalized at 0 for all respondents. The next four columns are the calculated utility from choosing objects B, C, D, and E respectively. These utilities are interpreted relative to the normaliser which is set to equal 0; so an object with utility 20 is a lot more favourable than an object with utility at 10 and both are more favourable than the normaliser which is 0. An object with a negative estimated utility is said to be less favourable than the normaliser. The last column is the log likelihood score, which is the indication of the fit of the likelihood function.

Recall that the size of the computed utility reflects the choices of individuals from pairwise comparison. Both the choice frequency (e.g Nab) and the fact that A was chosen the majority of time in a set (A,B),(A,C),(A,D),(A,E) impact the coefficient. This section of analysis will not deal with the 4 individuals whose utility was impossible to calculate because they behaved in a deterministic manner, which lead to convergence problems. The interpretation will focus on the 14 individuals that remain. Convergence problems will be discussed in a section about convergence.

Preference relations

A preference relation can be constructed based on some observations of individual decision making. In this example, we have observations on pairwise decisions and the objects as calculated from the maximum likelihood function. The latter are ordinal numbers so it is possible to rank them from highest to lowest. Ranking the master set {A,B,C,D,E} based on estimated utilities {UA,UB,UC,UD,UE} is one way of constructing a preference relation. Theory says it is wrong to aggregate preference draws like that. Further criticism by Regenwetter exposes the faults with assuming all preference draws belong to one preference relationship.

Suppose however a researcher was aggregate preferences and rank the master set from the most preferred to the least. This paper will go ahead and do exactly that and then discuss some possible lessons from constructing such a preference relation, as well as comparing this preference relation to preferences observed from watching individual's choices in a two-object choice-set.

Recall that each object A,B,C,D,E is a lottery with two attributes, 1) the probability of winning the lottery 2) and the prize of the lottery. The first attribute is lowest for object A and increases for B,C,D,E. The second attribute is lowest for object E and increases for objects D,C,B,A. Thus, it is expected that respondents' preferences would mimic the first of the second attribute. For example, an individual who is strictly concerned for the first attribute will probably have a preference relation (A>B>C>D>E) over the master set.

Looking at the results (Appendix B), one can see a wide variation of tastes from respondent to respondent but there are a few patterns that stand out. Indeed, it appears most respondents base their preferences on one of the two attributes. The large majority seem to base their opinion on probability of winning attribute which is highest for object E and then gradually decreases for objects D,C,B and A. Of the 14 respondents that are relevant for the analysis, 11 claim E as their most preferred object. The most common preference relation over the master set is {E>D>C>B>A} but there exist other variations such as {E>C>D>B>A} and {E>D>B>C>A}. Only one respondent based her preferences on the value of prize attribute, which is highest for A and decreasing for B,C,D, and E. This respondent expressed A>B>C>D>E as a preference relation.

Theory, of course, says that this is not an accurate representation of an individual's tastes. Because each draw is random and can belong to any transitive state, it is incorrect to aggregate them as aggregation leads to cycles. The correct way would be to deal with each separately; or better yet, discover a way to unveil an individual's transitive state. Unfortunately, there is no known empirical model that can do that.

Cyclicities

A cyclicity in preferences is said to occur when the axiom of transitivity is violated. Tversky assumed that every cyclicity is proof that transitivity is violated. However, it is possible that cyclicities occur without the axiom of transitivity being violated. This section will discuss why cases of cyclicities are overestimated in literature on preferences and transitivity. It will also point out "divergences" between preferences constructed over a master set and preferences observed in a pairwise comparison. These divergences can sometimes be mistaken for cyclicities but really this is a special case of difference in preferences.

A lot of literature about preferences and transitivity, as reviewed by Regenwetter, finds cases of cyclicities. Many researchers like Tversky assume that observed cases of cyclicity in preferences imply a violation of the axiom of transitivity. Regenwetter argues that it is not necessarily the case. Here are a few remarks on why cases of observed in cyclicity do not necessarily violate the axiom of transitivity.

First, most cases of transitivity are due to the inability of a model to reconcile stochastic transitivity with deterministic transitivity. Recall that the axiom on transitivity is defined in a deterministic context whereas observed cyclicity happens in a stochastic environment. Second, there is a problem with the assumptions that preferences are linear. If a three-way preference relation is not required to satisfy a triangle inequality, the linearity assumption on preferences is forced.

Third, the existence of Type1 errors. As there are multiple draws and thus multiple potential preference relations, accordingly there must be multiple Type1 errors. This increases the probability that a case of false transitivity fails to be rejected following a significance test.

These errors are notorious in the literature on transitivity. The two last remarks are possible to deal with and Regenwetter instructs just how in his paper. The first problem, however, remains so far unresolved.

A "divergence" (for lack of a better word) is said to occur when preferences over a master set do not correspond with one of the preferences expressed under a pairwise comparison. For example, if an individual is assumed to have a preference over a master set as (A>B>C>D>E) but then in a pairwise comparison, she chose B over A, thus B>A, it is said that pairwise preferences diverge from the preferences in the master set. As been pointed out in the previous subsection, the preferences constructed over the master set are not accurate for a host of reasons. Nevertheless, the paper will proceed with this exercise. The results are tabulated in Appendix B.

As seen in the Appendix B, six out of fourteen respondents had the same preferences in the two-choice set as in the five-choice set. There would be a 7th if we counted weakly preferred objects. The remaining 7 out of 14 individuals were found to have divergences between the two-choice set and the five-choice set. The seven had as many as 2-4 "divergences" each in the preferences.

Another possible cause for violating transitivity may be due to bad survey design. While the people responsible for surveying made sure that the questions are separated by filler questions to ensure statistical independency between relevant questions, the order of these questions is not investigated. If the way the questions are posed is not orthogonal, it may create some statistical dependencies or correlation among errors that will bias the estimated results. Furthermore, the order in which the objects are presented matters. Individuals may succumb to perception effects and to cognitive biases when choosing between two alternatives, they will commit errors that otherwise should not happen. These effects may create just enough of a problem to bias results.

As well, some respondents may not choose to reveal their true preferences for any unknown reason. The selections they are making can be in fact some bogus choices that do not reflect their true tastes. Some advanced methods in experimental design can control for or at least mitigate this type of behaviour. Yet the researchers on preferences and choice do not discuss employing any such techniques. More information about experimental design is discussed in the section about "Future work".

Lastly, another potential cause for violation in transitivity is the role of context. It is hypothesised that the choice of the choice-set may be important for an individual's decision because some objects may stand out in the presence of another object. For example, given the presence of j, the respondent will be more likely to choose object i. This kind of preference relationship is not implausible and if it occurs, it could lead to an intransitive choice relation. Perhaps some of the violations of intransitivity are due to exactly this type of effect. To determine if this is indeed possible, a new model will be used to estimate exactly this type of relationship. This model will be called Model 2 and the results are covered in the next subsection.

Model extensions

The random utility model, Model 1, is very flexible and easily adaptable to a number of situations. Due to time limitations, only one adaption was made possible and that is Model 2. However, here is but one idea that could be implemented for future work.

It would be interesting to include an observed variable "expected-value" in the model. On one hand, a pure random utility model has a lot of flexibility in a sense that it can estimate preferences without any structural presupposition on the relations, which is always desirable. On the other hand, it is known by the researcher that the five objects are not equal. If the researcher has information on the objects, which are the two attributes, he should specify this information in the model. Perhaps if this information was embedded under and an observed variable E(i), where E(i) is the expectations operator (e.g. E(i) = expected value of lottery i), the explanatory power of the model would increase and it would reveal more interesting results. By including this kind of variable in the utility, the researcher can control for the observed factors. This kind of tinkering could improve the statistical power of the model and could produce different preference relations. Then it would be possible to compare the new preference relations to current one (as produced under Model 1) and see how the number of cyclicities changes. As the expected values are numeric and hence transitive, it would imply less transitivities.

Results from model 2

The results from model 2 are tabulated in Appendix C. The results are by and large insignificant and irrelevant. In other words, Model 2 does not work. It is not clear why it is the case. The code appears to be okay and the alpha variable is well specified in the likelihood function. However, the results come off as bizarre. For one, there is a problem for each alternative j specified (j=B,C,D,E) and regardless of which default values the parameters are set. Two, the cause for the non-work appears to be a conversion problem. The optimizer is unable to converge for nearly every single respondent. It's bizarre because the only modification to the model is an added variable alpha. It's not like the likelihood function has gone from convex to concave. Therefore, it is not clear why the optimizer is not converging. Maybe multicollinearity is a problem or there exist some other problems not understood by the researcher.

If the model 2 had worked, the output would have been interpreted in the following way. The total utility Ti from the object i equals utility Ui plus the term alphaij when object j is present in the choice set or simply Ui when it is not present. Formally, Ti = Ui + alphaij when j is present or Ti = Ui when it is not present. If alpha is significant and different from zero, it is said that the presence of alternative j matters for a decision-making respondent.

Unfortunately the results here are wholly insignificant so this type of output cannot be used nor interpreted. This paper cannot say very much about the role of context for a decision-making respondent.

Convergence and problems

The only problem with the code is the issue of convergence. Four out of 18 respondents, they are respondent #3,8,11 and 14, exhibited absolute (or almost) consistence in their preference, which is to say they behaved deterministically (e.g. they made the same choice 20 out of 20 times they were presented the choice). Formally, the probability that the individual chooses i in a set of {i,j} equals 1 (or almost). Because of that, the optimizer is unable to converge to a good estimator of the utility. Therefore, these four cases were dropped from the previous portion of the analysis. Also, the log likelihood score is very low for each and one of these four respondents. This confirms that this type of model is unable to adequately measure a case of WST where choices are very consistent.

FUTURE WORK

The following design can be incorporated in the research on preferences. A good design will strongly increase the statistical significance of the model and allow for better interpretation of the results. Unfortunately, applying the design to the model was impossible as it would require to conduct a new survey, which is beyond the scope of this work.

Properties

The following design is embedded with three properties: column balance, adjacency property and maximum row difference. Each property satisfies a particular concern in surveying. A formal definition and explanation of the three properties here follow.

*Column balance***.** The elements A,B,C,D,E are said to be balanced when they each appear an equal number of times on the every column (except for column 5).

This property is desirable because it protects against respondents who choose any column without concern for the element. These respondents only care about completing the survey and not giving any truthful information. For example, there may be respondents who always choose the first column. The column balance property ensures that this type of response does not bias the answers collected from other respondents.

Adjacency property. A is said to be adjacent to B when the two appear together on the same row without space in between them (consider AB as a pair where A is adjacent to B). A row of three elements has 2 adjacent pairs; a row of n elements has n-1 adjacent pairs. The adjacency property ensures that every element is adjacent to another an equal amount of times.

This property addresses the concern that the position of A relative to other elements on the row may be important. For example, if A is a special choice then it might be more visually enticing if it appeared adjacent to B than if it appears adjacent to C (e.g A belongs to ABC is preferred to A belongs to ACB).

In this 26 choice set, there are 49 pairs of elements of which 10 are unique. Thus, my design construction ensures that 9 unique pairs appear five times and 1 unique pair appears four times, which is the optimal construction with respect to adjacency property.

Maximum row difference. Two consecutive rows are said to have maximum difference between them when there is the least number of repeated elements. For example, row AB is said to have the most distance from row CD or row DE because no element is repeated twice. Any two rows of three elements each will have at least one repeat between them. What this property does is ensure there is the least number of repeats possible over the entire 26 row matrix.

This property does not guarantee independence between all rows nor does it maximize distance between rows further apart like in a construction under modulo algebra. However, it does ensure no obvious pattern can be observed as far as the respondent can tell and this should be enough for most purposes.

The reason row independence is desirable is so that we can say that a choice made by the respondent in one row does not influence a choice in another row. This is useful but it removes concerns that a sequence of rows where element A is repeated (e.g. AB, AC, AD) will have a particular effect on a choice due to the repeated presence of A. For example, if A>B>C>D but the three rows AB, AC, AD follow one another then it's more likely that an intransitivity will happen, especially if the difference between the elements are not really large. This type of effect is undesirable and so making the rows different from one another is one way to avoid it.

Design

26-SET ORDERED

CE	0	0	1	0	1
AB	1	1	0	0	0
CD	0	0	1	1	0
EA	1	0	0	0	1
BD	0	1	0	1	0
AC	1	0	1	0	0
DE	0	0	0	1	1
BC	0	1	1	0	0
DA	1	0	0	1	0
EB	0	1	0	0	1
DCA	1	0	1	1	0
EBD	0	1	0	1	1
CAE	1	0	1	0	1
CDB	0	1	1	1	0
BEA	1	1	0	0	1
DEC	0	0	1	1	1
BAD	1	1	0	1	0
ECB	0	1	1	0	1
ADE	1	0	0	1	1
ABC	1	1	1	0	0
DAEB	1	1	0	1	1
EBAC	1	1	1	0	1
ACBD	1	1	1	1	0
BDCE	0	1	1	1	1
CEDA	1	0	1	1	1
CDEAB	1	1	1	1	1

CONCLUSION

This paper is both an overview of the theory on choices and transitivity and an applied exercise in choice modelling. I did my best to cover a bit of everything that relates to the topic in terms of conceptual framework and econometric theory alike, as well as build the model and then run inference on the data. However, as the subject is very expansive, an extensive overview was impossible. More research is required to further develop the original model which would allow to test additional hypothesis about role of context in preferences.

In the section about Future work, I incorporated work on Experimental design that was done independently of the work about Preferences. The two are in fact related but this connection was not sought in this paper. In the future, the work on experimental design could improve statistical methods and increase the explicative power of a survey.

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Appendix A

Results from Model 1 where defaults =0

	U1	U2	U3	U4	U5	minusll
respondent 1	0	0.497227	1.226903	1.13594	2.17501	-113.04
respondent 2	0	-1.28412	-1.49559	-3.00524	-2.88548	-92.51
respondent 3	0	2.991635	5.989139	16.78015	27.85805	-7.99
respondent 4	0	-0.23005	-0.27657	-0.04581	2.064633	-108.59
respondent 5	0	1.693105	3.106339	6.374661	9.371804	-30.7
respondent 6	0	0.448292	0.867551	1.935436	3.22275	-91.52
respondent 7	0	1.16726	2.050719	3.160408	4.829418	-68.55
respondent 8	0	1.793591	4.904423	15.96364	27.30741	-12.59
respondent 9	0	0.293671	0.252259	0.500026	1.055907	-131.36
respondent 10	0	2.469335	4.096593	5.472402	6.554283	-49.22
respondent 11	0	3.107015	5.015965	7.383362	18.27072	-19.42
respondent 12	0	0.440738	0.311293	0.783581	1.570908	-123.46
respondent 13	0	-0.13619	0.349286	1.186854	1.001603	-123.44
respondent 14	0	-2.94422	-14.4266	-25.8081	-28.7525	-7.94
respondent 15	0	0.865325	1.434535	1.861776	2.485558	-106.76
respondent 16	0	0.757396	1.545379	2.353768	3.98601	-81.12
respondent 17	0	0.201585	0.080511	0.161142	-0.24334	-137.1
respondent 18	0	0.576829	1.234083	1.411349	1.593811	-120.5

in bold : convergence problem

Appendix B

Preference relation from model 1

	1st most preferred	2nd preferred	3d preferred	4th preferred	5th preferred	cyclicities
respondent1	E	C	D	B	A	ab,**bc**
respondent2	A	B	C	E	D	de
respondent3	NA					
respondent4	E	A	B	C	D	da,db
respondent5	E	D	C	B	A	none
respondent6	E	D	C	B	A	ab,cb,**cd**
respondent7	E	D	C	B	A	none
respondent8	NA					
respondent9	E	D	B	C	A	none
respondent10	E	D	C	B	A	none
respondent11	NA					
respondent12	E	D	B	C	A	**ab**,cd,de
respondent13	D	E	C	A	B	ac,ce
respondent14	NA					
respondent15	E	D	C	B	A	none
respondent16	E	D	C	B	A	none
respondent17	B	D	C	A	E	ab,**ac**,bd,cd
respondent18	E	D	C	B	A	**ce,de**

in bold: weak cyclicities

Appendix C

Results from Model 2 with alphaij set to 2,3,4

Model with alphaij (j=3) and default values=0

u1	u2	u3	u4	u5	alpha23	alpha43	alpha53	minus ll
0	-12.299675	1.38623	-11.0911	-10.806499	25.74049	24.50266	24.47047	-10.01
0	-13.215245	-1.734595	-13.282807	-13.383981	25.069441	20.305387	23.516747	-8.46
0	-10.24108	13.66828	12.11286	14.20847	34.56924	16.12503	14.20847	0
0	-11.7880719	-0.6169764	-12.4097968	-10.8584259	23.9189634	24.0867578	22.7690304	-12.95
0	-10.001858	2.197183	-8.953580	3.492629	24.106522	22.866268	11.745003	-6.5
0	-11.159763	1.098542	-9.790303	-8.702415	23.932194	22.417021	21.749657	-11.25
0	-10.709997	2.197223	-10.318629	-10.044813	24.904109	25.473991	23.894816	-6.5
0	-11.57530	12.54525	11.64357	13.48811	33.26037	15.03685	13.48811	0
0	-10.5497806	0.6190934	-12.2156831	-10.4817098	23.1035265	25.8922417	23.1070040	-12.95
0	-9.1330412	11.3700930	0.8179845	-9.0907837	31.9752838	24.1997544		0

							33.7390290	
0	-8.533126	11.036553	1.570672	12.141128	30.046337	22.157540	12.141128	0
0	-12.0306243	0.4054924	-12.6669706	-12.0986266	25.9204204	26.5460719	25.7469813	-13.46
0	-11.8228115	-0.2006261	-10.8629975	-11.0544628	23.7053458	22.7230658	24.0754534	-13.76
0	-15.72543	-13.85589	-29.49816	-29.43398	28.40255	16.28202	13.55820	0
0	-10.707084	1.098575	-10.152428	-10.582912	23.677744	23.041651	23.556548	-11.25
0	-3.236855	13.075113	-18.845099	-14.141993	27.222089	48.282894	48.550763	0
0	-1.139719e+01	1.887526e-05	-1.675694e+01	-1.198909e+01	3.223147e+01	3.152129e+01	2.436602e+01	-13.86
0	-11.548390	1.386253	-11.389145	-10.369588	24.389658	24.892349	23.866105	-10.01

<u>model with alphaij (j=3) and default values for alphas = 30</u>

u1	u2	u3	u4	u5	alpha23	alpha43	alpha53	minusll
0	-14.13722	1.38630	-10.99621	-14.94810	30.00006	30.00034	30.04096	-10.01
0	-18.288939	-1.734541	-14.514986	-13.205331	30.000214	30.000048	30.000080	-8.45
0	-12.762429	15.256480	1.259213	1.259208	41.218825	31.259207	31.259178	0
0	-10.884745	-0.618506	-11.129172	-15.817951	30.000000	30.000000	30.000039	-12.95
0	-14.210719566	2.197214332	-14.311197838	-0.002348369				-6.5

	30.000194089 30.470052385 30.000000540	
0	-10.372683 1.098744 -14.282358 -13.580995 30.000001 30.000015 30.000010	-11.25
0	-14.693621 2.197209 -14.583562 -14.500355 30.000348 30.993196 30.812805	-6.5
0	-16.392245 18.035884 3.499634 3.499682 50.752052 33.499546 33.499506	0
0	-12.226076 0.619088 -11.386810 -10.692682 30.000002 30.000001 30.000000	-12.95
0	-11.775998 14.139015 -1.116731 -11.614333 38.674542 30.908477 40.534181	0
0	-13.0281325 15.9023707 0.3113298 1.5726806 43.3008191 32.3913927 31.5726688	0
0	-10.5227853 0.4053481 -10.9034079 -16.1753537 30.0000006 30.0000005 30.0000757	-13.46
0	-16.0433521 -0.2006654 -17.2069636 -17.0252322 30.0001547 30.5579618 30.3313581	-13.76
0	-17.17462 -17.78830 -36.48953 -36.48957 33.27516 30.00001 30.00000	0
0	-13.898017 1.098614 -12.279278 -14.351496 30.000035 30.001904 30.020808	-11.25
0	-3.538448 14.114883 -13.564942 -13.681554 36.264602 42.297078 42.590917	0
0	-1.154697e+01 4.752177e-05 -1.244494e+01 -1.180372e+01 3.000000e+01 3.000000e+01 3.000000e+01	-13.86
0	-13.86636 1.38630 -11.64638 -12.69446 30.00004 30.00106 30.00306	-10.01

Model with alphaij (j=2) and all default values=0

u1	u2	u3	u4	u4	alpha32	alpha42	alpha52	minusll
0	-0.2006768	-12.3545588	-11.4878631	-10.5661190	24.1120907	23.5737003	22.1901692	-13.76
0	-1.734547	-12.982763	-13.699682	-14.333734	25.367850	22.685082	24.571523	-8.45
0	2.944448	-10.170905	6.952230	10.224162	27.113320	10.355430	10.224162	-3.97
0	-2.197151	-14.442191	-13.943415	-12.315449	28.209720	27.932010	25.922309	-6.5
0	2.944329	-9.438684	-9.275966	7.149244	24.506333	25.800443	16.372641	-3.97
0	-0.200675	-11.062810	-9.918823	-1.715243	23.172402	22.521371	15.213558	-13.76
0	2.194198	-9.491577	-10.389325	-9.861736	24.362986	24.480343	24.207493	-6.5
0	1.735157	-5.991030	5.770262	9.758268	20.650922	9.507326	9.758268	-8.45
0	0.6191649	-11.1625175	-12.3678362	-11.4665702	23.9354299	26.0049144	23.2083692	-12.95
0	2.954177	-7.074463	2.760887	-8.980566	21.994452	13.089183	24.399352	-3.97
0	2.944696	-6.507094	4.274082	9.189304	22.824941	11.508597	9.189304	-3.97
0	-3.070476e-05	-1.311004e+01	-1.236514e+01	-1.096421e+01	2.870100e+01	2.613006e+01	2.524358e+01	-13.86
0	1.253683e-05	-1.087681e+01	-1.389536e+01	-1.331490e+01	2.971582e+01	2.615087e+01	2.607002e+01	-13.86

0	-2.944403	-15.220222	-16.716072	-16.657971	12.718997	12.402543	9.536919	-3.97
0	1.098608	-16.709579	-15.245740	-12.276680	32.311125	30.127119	30.106929	-11.25
0	13.7464602	0.6776522	-13.8657996	-13.4135123	28.9202046	42.6402859	42.2208245	0
0	-0.6177744	-11.4963194	-10.8714402	-10.9592872	25.1496083	26.3746230	20.3995736	-12.95
0	0.4054679	-14.0295017	-12.9723529	-13.6655091	29.9895495	29.4485696	29.9388458	-13.46

<u>Model with alphaij (j=4) and all default values = 0</u>

u1	u2	u3	u4	u5	alpha24	alpha34	alpha54	minusll
0	-11.368183	-10.320213	2.188574	-9.985138	25.065024	24.472988	24.710586	-6.5
0	-22.25237	-21.00032	-11.85512	-22.76949	33.60893	32.40999	35.84745	0
0	-11.727045	-0.997094	16.300626	16.177358	26.120137	16.740444	16.177358	0
0	-15.732542	-15.291607	1.386249	-12.915910	31.156325	30.401770	29.012993	-10.01
0	-10.033194	-9.969554	2.948316	-6.431484	21.975624	22.077021	21.512905	-3.97
0	-11.820078	-11.002443	2.944327	-7.919132	24.928928	25.568097	24.272571	-3.97

0	-10.962487 -9.979390 1.734408 -9.502147 23.038214 22.767425 22.916459	-8.46
0	-12.9582132 -0.6536567 15.6169238 15.7697749 27.3033795 17.6604930 15.7697749	0
	-11.2066884 -10.7778116 0.4054029 -11.6573427 23.3262700 22.3657844 24.4759815	-13.46
0	-11.53241928 -0.03047561 12.70252708 -11.40338958 26.95437448 26.31510933 36.86051351	0
0	-12.296382 1.034364 16.368410 16.795217 28.728693 29.100567 16.795217	0
0	-13.953359 -19.633945 1.098592 -13.299253 31.342070 36.168503 28.309578	-11.25
0	-12.170371 -11.886396 1.098529 -11.507440 25.173768 23.446626 23.899186	-11.25
0	-12.90632 -12.90788 -13.30921 -24.97632 23.27467 13.70262 22.38270	0
0	-12.022433 -10.477327 2.197286 -9.834735 25.453040 23.909246 23.960211	-6.5
0	-10.495018 -10.533398 0.847419 -9.926949 22.452117 22.491239 24.060394	-12.22
0	-16.2398094 -11.8746260 0.4054993 -11.7617446 31.2250079 29.0780834 23.5798409	-13.46
0	-10.623554 -12.157278 1.098681 -10.805018 23.453706 26.068141 24.642292	-11.25

Appendix D

R code for Model 1

```
library(bbmle)
llf <- function(u2=0,u3=0,u4=0,u5=0){

  u <- c(0,u2,u3,u4,u5)

  ll <- 0
  for (i in 1:5){
    for (j in 1:5){
      ll <- ll + respondentx[i,j] * (u[i]) - respondentx[i,j] * log(exp(u[i]) + exp(u[j]))
    }
  }
  -ll
}

for (k in 1:18){
  respondentx <- read.csv(paste("respondentx",k,".txt",sep=""), header=TRUE)
  results <- mle2(llf, method = "BFGS")
  print(results)
}
```

Appendix E

R code for Model 2

```
library(bbmle)
llf <- function(u2=0,u3=0,u4=0,u5=0,
               alpha23=0,alpha43=0,alpha53=0){

  alpha = matrix(0, nrow = 5, ncol = 5)

  alpha[, 3] <- c(0, alpha23, 0, alpha43, alpha53)

  u <- c(0,u2,u3,u4,u5)

  ll <- 0
  for (i in 1:5){
    for (j in 1:5){
      ll <- ll + respondentx[i,j] * (u[i]+(alpha[i,3])) - respondentx[i,j] * log(exp(u[i]+
(alpha[i,3])) + exp(u[j]))
    }
  }
  -ll
}

for (k in 1:18){
  respondentx <- read.csv(paste("respondentx",k,".txt",sep=""), header=TRUE)
  results <- mle2(llf, method = "BFGS")
  print(results)
}
```