Département de sciences économiques

2005-14

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Ce cahier a également été publié par le Centre interuniversitaire de recherche en économie quantitative (CIREQ) sous le numéro 18-2005.

This working paper was also published by the Center for Interuniversity Research in Quantitative Economics (CIREQ), under number 18-2005.

ISSN 0709-9231

Choosing Wisely:
The Natural Multi-Bidding Mechanism

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November 2002
(revised June 2005)

Abstract

Pérez-Castrillo and Wettstein (2002) propose a multi-bidding mechanism to determine a winner from a set of possible projects. The winning project is implemented and its surplus is shared among the agents. In the multi-bidding mechanism each agent announces a vector of bids, one for each possible project, that are constrained to sum up to zero. In addition, each agent chooses a favorite a object which is used as a tie-breaker if several projects receive the same highest aggregate bid. Since more desirable projects receive larger bids, it is natural to consider the multi-bidding mechanism without the announcement of favorite projects. We show that the merits of the multi-bidding mechanism appear not to be robust to this natural simplification. Specifically, a Nash equilibrium exists if and only if there are at least two individually optimal projects and all individually optimal projects are efficient.

JEL Classification: D62, D78

Keywords: (natural) multi-bidding mechanism, existence, efficiency.

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I. Introduction

In many situations a group of agents has to choose a winner from a set of possible projects. The winning project is then implemented and the surplus is shared among the agents. Examples include the location of noxious facilities, the selection of a candidate or the siting of a major sports event. In a recent paper, Pérez-Castrillo and Wettstein (2002) propose the *multi-bidding mechanism* to resolve such issues (with possibly conflicting interests). The *multi-bidding mechanism* determines which project will be developed and how the agents will share the surplus from its development. The mechanism is described as follows: each agent announces a vector of bids, one for each possible project, that are constrained to sum up to zero. In addition each agent chooses a favorite project. The project with the highest aggregate bid is chosen as the winner. In case of a tie, the winning project is randomly chosen among those with highest aggregate bid and that are ranked first by at least one agent. Two merits of the *multi-bidding mechanism* are (i) Nash equilibria always exist and (ii) in any Nash equilibrium an efficient project is carried out.

In real life (and in most auctions) usually agents place bids on projects without selecting a favorite project. When a naive or real-world planner is interested in applying the multi-bidding mechanism he will wonder why an agent needs to announce a favorite project in addition to his bids. Since the project with the highest aggregate bid is developed and more desirable projects receive larger bids, it seems natural that an agent's bids reveal his favorite project. In other words isn't the project on which an agent places his maximal bid his favorite project? The real-world planner considers to ask agents to submit bids only and interprets an agent's favorite project to be one with a maximal bid. Does this natural simplification affect the merits of the multi-bidding mechanism?

We demonstrate how crucial it is that agents also indicate separately one of their most favored projects. In the simpler mechanism, in which only bids are submitted, a Nash equilibrium exists if and only if any project, which some individual views optimal, is efficient and there are at least two individually optimal projects. Furthermore, if a Nash equilibrium exists, then all merits of the multi-bidding mechanism carry over to the natural multi-bidding mechanism.

We proceed as follows. In Section II we introduce the environment and the natural multi-bidding mechanism. In Section III we give necessary and sufficient conditions for the existence of Nash equilibria in the natural multi-bidding mechanism. In Section IV we discuss our findings for environments having positive or negative externalities.

II. The Natural Multi-Bidding Mechanism

We consider a set of agents $N = \{1, \ldots, n\}$ and a set of possible projects $K = \{1, \ldots, k\}$. The utility of agent i if project q is carried out is given by $v_q^i \in \mathbb{R}$. The efficient projects are the projects which maximize the sum of the utilities of the agents. We denote by E the set of efficient projects, i.e.

$$E = \{q \in K \mid \sum_{i \in N} v_q^i \geq \sum_{i \in N} v_p^i \text{ for all } p \in K\}.$$

The *multi-bidding mechanism* of Pérez-Castrillo and Wettstein (2002) determines the project which will be developed and how the agents share the surplus from its development. The mechanism is described as follows: each agent i announces k bids, one for each possible project, that are constrained to sum up to zero. In addition i chooses a favorite project. The object with the highest aggregate bid is chosen as the winner. In case there is a tie, the winning project is randomly chosen among those with highest aggregate bid and which are favored by at least one agent. According to Pérez-Castrillo and Wettstein (2002, p. 1578) the multi-bidding mechanism "introduces a measure of relative worth whereby more desirable projects receive larger bids".

This motivates the following natural simplification of the multi-bidding mechanism. Each agent i submits k bids only, which sum up to zero, and does not select a favorite project. The projects for which he submits a highest bid are interpreted to be his favored ones. The *natural multi-bidding mechanism* is described as follows:

Each agent $i \in N$ places on each project q in K a bid $b^i_q \in \mathbb{R}$. A strategy for i is a profile of bids $b^i = (b^i_q)_{q \in K}$ such that $\sum_{q \in K} b^i_q = 0$. A strategy profile $b = (b^i)_{i \in N}$ specifies for each agent a strategy. Given b, let $B_q = \sum_{i \in N} b^i_q$ denote the aggregate bid for project q. Let $\Omega(b) = \{q \in K \mid B_q \geq B_p \text{ for all } p \in K\}$ denote the set of projects with the highest aggregate bid at b. Let $M(b^i) = \{q \in K \mid b^i_q \geq b^i_p \text{ for all } p \in K\}$ denote the projects on which agent i places his maximal bids. Let $M(b) = \cup_{i \in N} M(b_i)$ denote the projects on which some agent places a maximal bid. If $\Omega(b) \cap M(b)$ is non-empty, then the winning project is chosen randomly from the members of $\Omega(b) \cap M(b)$. Otherwise it is randomly chosen from $\Omega(b)$.[1]

If the natural multi-bidding mechanism chooses project q, then q is developed and any agent i receives the payment $\frac{1}{n} B_q - b^i_q$. Agent i's (net) payoff is then $v^i_q + \frac{1}{n} B_q - b^i_q$.[2]

It is easy to check that the necessary conditions for a set of strategies to be a Nash equilibrium (NE) in the multi-bidding mechanism carry over to the natural multi-bidding mechanism.

Lemma 1 (Lemma 5, Pérez-Castrillo and Wettstein, 2002) *If b constitutes a NE of the natural multi-bidding mechanism, then the following three properties hold:*

(a) The aggregate bid for every project is zero, i.e. for all $q \in K$, $B_q = 0$.

(b) An agent's payoff is maximal if an efficient project is chosen and it is the same for all efficient projects, i.e. for all $p, q \in E$, $v^i_p - b^i_p = v^i_q - b^i_q$.

(c) Any project on which some agent puts a maximal bid is efficient, i.e. $M(b) \subseteq E$.

[1] In the *multi-bidding mechanism* each agent is allowed to name exactly one favorite project. When an agent puts the same maximal bid on several projects, in the *natural multi-bidding mechanism* alternatively the planner may choose randomly a favorite project from those. This would not change the analysis.

[2] Any sharing rule other than the equal sharing rule would generate the same results.

III. The Equilibrium Outcomes of the Natural Multi-Bidding Mechanism

It will turn out that the individually optimal projects play the important role for deciding whether a NE of the natural multi-bidding mechanism exists or not. For each agent i we denote by $O_i = \{q \in K \mid v_q^i \geq v_p^i \text{ for all } p \in K\}$ the projects that achieve highest utility for him. These are the projects that agent i views to be *optimal*. Let $O = \cup_{i \in N} O_i$ denote the individually optimal projects.

Lemma 2 *If b is a NE of the natural multi-bidding mechanism, then (i) the set of individually optimal projects coincides with the set of projects on which some agent places a maximal bid, i.e. $O = M(b)$, and (ii) there are at least two individually optimal projects.*

PROOF: (i) First we show that the set of individually optimal projects is included in the set of projects on which some agent places a maximal bid. Suppose not. Then there is some agent i such that $O_i \backslash M(b) \neq \emptyset$. Let $q \in O_i \backslash M(b)$. Then $v_q^i \geq v_p^i$ for all $p \in K$. By (a) of Lemma 1 the winning project is randomly chosen from $M(b)$. Because q is not a project on which agent i puts a maximal bid, there is $p \in M(b)$ such that $b_p^i > b_q^i$. Combining the two inequalities we obtain $v_q^i - b_q^i > v_p^i - b_p^i$. Since by (b) and (c) of Lemma 1, $M(b) \subseteq E$ and agent i's payoff is the same for all projects belonging to $M(b)$, agent i can gain by slightly increasing his bid for q and reducing the bids for all other projects.

Second we show that any project on which some agent places a maximal bid is necessarily individually optimal. Suppose $O \neq M(b)$. We already know $O \subseteq M(b)$. Let $q \in M(b) \backslash O$. Then for all $i \in N$ there is $p(i) \in O$ such that $v_{p(i)} > v_q$. By (b) and (c) of Lemma 1 any agent i's payoff is the same for all projects belonging to $M(b)$. Thus, for any agent i we have $v_{p(i)}^i - b_{p(i)}^i = v_q^i - b_q^i$. Therefore, for all $i \in N$, $b_{p(i)}^i > b_q^i$. In other words there is no agent who puts a maximal bid on q. This is a contradiction to $q \in M(b)$.

(ii) Suppose there is one individually optimal project only. Let $O = \{q\}$. Then by (i), $\{q\} = M(b)$ and for all $i \in N$ we have $b_q^i > b_p^i$ for all $p \neq q$. But then the aggregate bid on q is uniquely maximal, i.e. $B_q > B_p$ for all $p \neq q$. This contradicts (a) of Lemma 1. $\square$

Using the proof of Lemma 5 in Pérez-Castrillo and Wettstein (2002) it is easy to verify that the properties of Lemma 1 and Lemma 2 provide a full characterization of all NE of the natural multi-bidding mechanism.

Lemma 3 *Let b be a set of strategies. Then b constitutes a NE of the natural multi-bidding mechanism if and only if (a), (b) and (c) of Lemma 1 and the following property hold:*

(d) There are at least two individually optimal projects and the set of individually optimal projects coincides with the set of projects with maximal bids, i.e. $|O| \geq 2$ and $M(b) = O$.

Now we are ready to answer the question whether the merits of the multi-bidding mechanism are affected when interpreting an agent's favorite projects to be the ones with his maximal bids. When there are at least two individually optimal projects and any individually optimal project is efficient, then all merits of the multi-bidding mechanism carry over to the natural multi-bidding mechanism. If one of the two conditions is not met, then no equilibrium exists in the natural multi-bidding mechanism.

Theorem 1 *If there are at least two individually optimal projects and any individually optimal project is efficient, then the natural multi-bidding mechanism implements in NE the set of utility vectors*

$$\left\{(u^1, \ldots, u^n) \in \mathbb{R}^N \mid \sum_{i \in N} u^i = \sum_{i \in N} v_q^i \text{ where } q \in E \text{ and } u^i \geq \frac{1}{k} \sum_{q \in K} v_q^i \right\}. \tag{1}$$

Otherwise there exists no NE of the natural multi-bidding mechanism.

PROOF: If $|O| \geq 2$ and $O \subseteq E$, then the proof follows along the lines of Pérez-Castrillo and Wettstein (2002, Appendix): First we show that both the set of Nash equilibria and the set of equilibrium outcomes is convex. Let b and $\bar{b}$ be NE. Then from (b) of Lemma 1 and (d) of Lemma 3 we have for any agent i, $O_i = M(b^i) = M(\bar{b}^i)$. But then for all $\lambda \in [0,1]$, $O_i = M(\lambda b^i + (1-\lambda)\bar{b}^i)$. Since b and $\bar{b}$ are both NE, they satisfy the properties of (a), (b), and (c) of Lemma 1 and (d) of Lemma 3. Now it is straightforward that the strategy profile $(\lambda b^i + (1-\lambda)\bar{b}^i)_{i\in N}$ satisfies the properties of Lemma 1 and Lemma 3. Hence, by Lemma 3, $(\lambda b^i + (1-\lambda)\bar{b}^i)_{i\in N}$ is a NE. Therefore, both the set of Nash equilibria and the set of equilibrium outcomes is convex.

Second we construct for each agent j the "extremal equilibrium point" where any agent i other than j receives as payoff $\frac{1}{k}\sum_{q\in K} v_q^i \equiv \underline{v}^i$. For all $q \in K$, let $b_q^j = \sum_{i\neq j} \underline{v}^i - \sum_{i\neq j} v_q^i$ and $b_q^i = v_q^i - \underline{v}^i$ for all $i \neq j$. The bidding strategies b^i of any agent $i \neq j$ puts his maximal bids on his optimal projects, i.e. $M(b^i) = O_i$. Note that for agent j we have for all $q \in O_j$ and all $p \in K \backslash O_j$, $v_q^j > v_p^j$. Because project q is efficient, it follows that $V^e - \sum_{i\neq j} v_q^i > v_p^j$ (where V^e denotes the maximum total value by developing an efficient project). Then

$$-\sum_{i\neq j} v_q^i > v_p^j - V^e \geq v_p^j - \sum_{i\in N} v_p^i = -\sum_{i\neq j} v_p^i.$$

Therefore, agent j's bid on q is greater than his bid on p, i.e. $b_q^j > b_p^j$. Because any individually optimal project is efficient, the strategies b satisfy properties (c) of Lemma 1 and (d) of Lemma 3. Now it is easy to verify that b also satisfies (a) and (b) of Lemma 1[3] and thus, by Lemma 3, b is a NE. Because Theorem 1 of Pérez-Castrillo and Wettstein (2002) remains unchanged for the natural multi-bidding mechanism, the set of equilibrium outcomes is convex, and any extremal point of the set (1) is supported by a Nash equilibrium, we obtain the first part of Theorem 1.

If there is one individually optimal project only or at least one individually optimal project is not efficient, then (c) of Lemma 1 and (d) of Lemma 3 cannot be satisfied

[3]Note that (b) of Lemma 1 is satisfied because any agent $i \neq j$ receives the payoff $\underline{v}^i$ independently of which project is implemented and agent j receives for any project the difference between the total surplus of it and the previous payoffs. Thus, agent j's payoff is maximal for an efficient project and it is the same for all efficient projects.

simultaneously. Thus, by Lemma 3, no NE exists. □

IV. Discussion

We discuss our findings when each project is associated with one agent.[4] When siting a major sports event the project imposes positive externalities on its associated agent. Then v_i^i is typically a positive number and v_q^i is a negative number if $q \neq i$. Therefore, in environments with positive externalities all projects are individually optimal and by Theorem 1 a NE exists only in the rare case when all projects are efficient. If the real-world planner adapts the natural multi-bidding mechanism, then all merits disappear.

When siting a noxious facility the project imposes negative externalities on its associated agent. Then v_i^i is a negative number and v_q^i is a positive number if $q \neq i$. For example, suppose that a country has decided to build a nuclear plant. Then each region's utility increases with distance (the further the plant is located away, the better) and both the individually optimal regions and the efficient regions are located along the border of the country. If there is no agglomeration around the border, then a NE may exist.

Theorem 1 puts severe restrictions on the existence of NE in the natural multi-bidding mechanism. In particular when one project is unambiguously efficient, then no equilibrium exists. The intuition is that if there are at least two individually optimal projects, then they need to be efficient and one can ensure that both projects receive zero in aggregate while any agent puts his maximal bid on one of the individually optimal projects (counter balanced by negative bids on the other individually optimal project(s)).

Our result also explains why in real life under complete information often no agreement is reached by bids and compensations (even if there is exactly one efficient

[4]This environment is equivalent to the case where one (indivisible) object is sold through an auction to one of the agents and an agent's payoff depends on the identity of the agent who gets the object (see, for instance, Jehiel, Moldovanu, and Stacchetti (1999)).

project).[5] Submitting single or multiple bids does not solve the problem. The crucial feature of the multi-bidding mechanism is that each agent is allowed to name *any* project in addition to his bids, i.e. an agent may name a project on which he placed one of his minimal bids.[6]

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[5]See the discussion by Pérez-Castrillo and Wettstein (2002) on the problem of siting a high-level radioactive waste repository in the US.

[6]However, in other frameworks efficient equilibria may exist under complete information without the need to have bidders name favorite choices (see for instance, Bernheim and Whinston (1986) and Mutuswami, Pérez-Castrillo and Wettstein (2004)).