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Factor models, VARMA processes and parameter instability with applications in macroeconomics

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RÉSUMÉ

Avec les avancements de la technologie de l'information, les données temporelles économiques et financières sont de plus en plus disponibles. Par contre, si les techniques standard de l'analyse des séries temporelles sont utilisées, une grande quantité d'information est accompagnée du problème de dimensionnalité. Puisque la majorité des séries d'intérêt sont hautement corrélées, leur dimension peut être réduite en utilisant l'analyse factorielle [1]. Cette technique est de plus en plus populaire en sciences économiques depuis les années 90.

Étant donnée la disponibilité des données et des avancements computationnels, plusieurs nouvelles questions se posent. Quels sont les effets et la transmission des chocs structurels dans un environnement riche en données ? Est-ce que l'information contenue dans un grand ensemble d'indicateurs économiques peut aider à mieux identifier les chocs de politique monétaire, à l'égard des problèmes rencontrés dans les applications utilisant des modèles standards ? Peut-on identifier les chocs financiers et mesurer leurs effets sur l'économie réelle ? Peut-on améliorer la méthode factorielle existante et y incorporer une autre technique de réduction de dimension comme l'analyse VARMA ? Est-ce que cela produit de meilleures prévisions des grands agrégats macroéconomiques et aide au niveau de l'analyse par fonctions de réponse impulsionnelles ? Finalement, est-ce qu'on peut appliquer l'analyse factorielle au niveau des paramètres aléatoires ? Par exemple, est-ce qu'il existe seulement un petit nombre de sources de l'instabilité temporelle des coefficients dans les modèles macroéconomiques empiriques ?

Ma thèse, en utilisant l'analyse factorielle structurelle et la modélisation VARMA, répond à ces questions à travers cinq articles. Les deux premiers chapitres étudient les effets des chocs monétaire et financier dans un environnement riche en données. Le troisième article propose une nouvelle méthode en combinant les modèles à facteurs et VARMA. Cette approche est appliquée dans le quatrième article pour mesurer les effets des chocs de crédit au Canada. La contribution du dernier chapitre est d'imposer

1. Ici, l'analyse factorielle comprend les modèles à facteurs et à composantes principales. D'autres techniques de reduction de dimension basées sur *shrinkage* ne sont pas discutées.

la structure à facteurs sur les paramètres variant dans le temps et de montrer qu'il existe un petit nombre de sources de cette instabilité.

Le premier article analyse la transmission de la politique monétaire au Canada en utilisant le modèle vectoriel autorégressif augmenté par facteurs (FAVAR). Les études antérieures basées sur les modèles VAR ont trouvé plusieurs anomalies empiriques suite à un choc de la politique monétaire. Nous estimons le modèle FAVAR en utilisant un grand nombre de séries macroéconomiques mensuelles et trimestrielles. Nous trouvons que l'information contenue dans les facteurs est importante pour bien identifier la transmission de la politique monétaire et elle aide à corriger les anomalies empiriques standards. Finalement, le cadre d'analyse FAVAR permet d'obtenir les fonctions de réponse impulsionnelles pour tous les indicateurs dans l'ensemble de données, produisant ainsi l'analyse la plus complète à ce jour des effets de la politique monétaire au Canada.

Motivée par la dernière crise économique, la recherche sur le rôle du secteur financier a repris de l'importance. Dans le deuxième article nous examinons les effets et la propagation des chocs de crédit sur l'économie réelle en utilisant un grand ensemble d'indicateurs économiques et financiers dans le cadre d'un modèle à facteurs structurel. Nous trouvons qu'un choc de crédit augmente immédiatement les diffusions de crédit (*credit spreads*), diminue la valeur des bons de Trésor et cause une récession. Ces chocs ont un effet important sur des mesures d'activité réelle, indices de prix, indicateurs avancés et financiers. Contrairement aux autres études, notre procédure d'identification du choc structurel ne requiert pas de restrictions temporelles entre facteurs financiers et macroéconomiques. De plus, elle donne une interprétation des facteurs sans restreindre l'estimation de ceux-ci.

Dans le troisième article nous étudions la relation entre les représentations VARMA et factorielle des processus vectoriels stochastiques, et proposons une nouvelle classe de modèles VARMA augmentés par facteurs (FAVARMA). Notre point de départ est de constater qu'en général les séries multivariées et facteurs associés ne peuvent simultanément suivre un processus VAR d'ordre fini. Nous montrons que le processus dynamique des facteurs, extraits comme combinaison linéaire des variables observées, est en général un VARMA et non pas un VAR comme c'est supposé ailleurs dans la littérature.

Deuxièmement, nous montrons que même si les facteurs suivent un VAR d'ordre fini, cela implique une représentation VARMA pour les séries observées. Alors, nous proposons le cadre d'analyse FAVARMA combinant ces deux méthodes de réduction du nombre de paramètres. Le modèle est appliqué dans deux exercices de prévision en utilisant des données américaines et canadiennes de Boivin, Giannoni et Stevanović (2010, 2009) respectivement. Les résultats montrent que la partie VARMA aide à mieux prévoir les importants agrégats macroéconomiques relativement aux modèles standards. Finalement, nous estimons les effets de choc monétaire en utilisant les données et le schéma d'identification de Bernanke, Boivin et Eliasz (2005). Notre modèle FAVARMA(2,1) avec six facteurs donne les résultats cohérents et précis des effets et de la transmission monétaire aux États-Unis. Contrairement au modèle FAVAR employé dans l'étude ultérieure où 510 coefficients VAR devaient être estimés, nous produisons les résultats semblables avec seulement 84 paramètres du processus dynamique des facteurs.

L'objectif du quatrième article est d'identifier et mesurer les effets des chocs de crédit au Canada dans un environnement riche en données et en utilisant le modèle FAVARMA structurel. Dans le cadre théorique de l'accélérateur financier développé par Bernanke, Gertler et Gilchrist (1999), nous approximons la prime de financement extérieur par les *credit spreads*. D'un côté, nous trouvons qu'une augmentation non-anticipée de la prime de financement extérieur aux États-Unis génère une récession significative et persistante au Canada, accompagnée d'une hausse immédiate des *credit spreads* et taux d'intérêt canadiens. La composante commune semble capturer les dimensions importantes des fluctuations cycliques de l'économie canadienne. L'analyse par décomposition de la variance révèle que ce choc de crédit a un effet important sur différents secteurs d'activité réelle, indices de prix, indicateurs avancés et *credit spreads*. De l'autre côté, une hausse inattendue de la prime canadienne de financement extérieur ne cause pas d'effet significatif au Canada. Nous montrons que les effets des chocs de crédit au Canada sont essentiellement causés par les conditions globales, approximées ici par le marché américain. Finalement, étant donnée la procédure d'identification des chocs structurels, nous trouvons des facteurs interprétables économiquement.

Le comportement des agents et de l'environnement économiques peut varier à travers

le temps (ex. changements de stratégies de la politique monétaire, volatilité de chocs) induisant de l'instabilité des paramètres dans les modèles en forme réduite. Les modèles à paramètres variant dans le temps (TVP) standards supposent traditionnellement les processus stochastiques indépendants pour tous les TVPs. Dans cet article nous montrons que le nombre de sources de variabilité temporelle des coefficients est probablement très petit, et nous produisons la première évidence empirique connue dans les modèles macroéconomiques empiriques. L'approche *Factor-TVP*, proposée dans Stevanovic (2010), est appliquée dans le cadre d'un modèle VAR standard avec coefficients aléatoires (TVP-VAR). Nous trouvons qu'un seul facteur explique la majorité de la variabilité des coefficients VAR, tandis que les paramètres de la volatilité des chocs varient d'une façon indépendante. Le facteur commun est positivement corrélé avec le taux de chômage. La même analyse est faite avec les données incluant la récente crise financière. La procédure suggère maintenant deux facteurs et le comportement des coefficients présente un changement important depuis 2007. Finalement, la méthode est appliquée à un modèle TVP-FAVAR. Nous trouvons que seulement 5 facteurs dynamiques gouvernent l'instabilité temporelle dans presque 700 coefficients.

Mots clés: Analyse factorielle, modèle VARMA, prévision, fonctions de réponse impulsionnelles, analyse structurelle, modèle à paramètres variant dans le temps.

ABSTRACT

As information technology improves, the availability of economic and finance time series grows in terms of both time and cross-section sizes. However, a large amount of information can lead to the curse of dimensionality problem when standard time series tools are used. Since most of these series are highly correlated, at least within some categories, their co-variability pattern and informational content can be approximated by a smaller number of (constructed) variables. A popular way to address this issue is the factor analysis [2]. This framework has received a lot of attention since late 90's and is known today as the large dimensional approximate factor analysis.

Given the availability of data and computational improvements, a number of empirical and theoretical questions arises. What are the effects and transmission of structural shocks in a data-rich environment? Does the information from a large number of economic indicators help in properly identifying the monetary policy shocks with respect to a number of empirical puzzles found using traditional small-scale models? Motivated by the recent financial turmoil, can we identify the financial market shocks and measure their effect on real economy? Can we improve the existing method and incorporate another reduction dimension approach such as the VARMA modeling? Does it help in forecasting macroeconomic aggregates and impulse response analysis? Finally, can we apply the same factor analysis reasoning to the time varying parameters? Is there only a small number of common sources of time instability in the coefficients of empirical macroeconomic models?

This thesis concentrates on the structural factor analysis and VARMA modeling and answers these questions through five articles. The first two articles study the effects of monetary policy and credit shocks in a data-rich environment. The third article proposes a new framework that combines the factor analysis and VARMA modeling, while the fourth article applies this method to measure the effects of credit shocks in Canada. The contribution of the final chapter is to impose the factor structure on the time varying

2. Here, the factor analysis understands both factor and principal components models. Other dimension reduction techniques widely used are shrinkage models, but are not discussed in this work.

parameters in popular macroeconomic models, and show that there are few sources of this time instability.

The first article analyzes the monetary transmission mechanism in Canada using a factor-augmented vector autoregression (FAVAR) model. For small open economies like Canada, uncovering the transmission mechanism of monetary policy using VARs has proven to be an especially challenging task. Such studies on Canadian data have often documented the presence of anomalies such as a price, exchange rate, delayed overshooting and uncovered interest rate parity puzzles. We estimate a FAVAR model using large sets of monthly and quarterly macroeconomic time series. We find that the information summarized by the factors is important to properly identify the monetary transmission mechanism and contributes to mitigate the puzzles mentioned above, suggesting that more information does help. Finally, the FAVAR framework allows us to check impulse responses for all series in the informational data set, and thus provides the most comprehensive picture to date of the effect of Canadian monetary policy.

As the recent financial crisis and the ensuing global economic have illustrated, the financial sector plays an important role in generating and propagating shocks to the real economy. Financial variables thus contain information that can predict future economic conditions. In this paper we examine the dynamic effects and the propagation of credit shocks using a large data set of U.S. economic and financial indicators in a structural factor model. Identified credit shocks, interpreted as unexpected deteriorations of the credit market conditions, immediately increase credit spreads, decrease rates on Treasury securities and cause large and persistent downturns in the activity of many economic sectors. Such shocks are found to have important effects on real activity measures, aggregate prices, leading indicators and credit spreads. In contrast to other recent papers, our structural shock identification procedure does not require any timing restrictions between the financial and macroeconomic factors, and yields an interpretation of the estimated factors without relying on a constructed measure of credit market conditions from a large set of individual bond prices and financial series.

In third article, we study the relationship between VARMA and factor representations of a vector stochastic process, and propose a new class of factor-augmented VARMA

(FAVARMA) models. We start by observing that in general multivariate series and associated factors do not both follow a finite order VAR process. Indeed, we show that when the factors are obtained as linear combinations of observable series, their dynamic process is generally a VARMA and not a finite-order VAR as usually assumed in the literature. Second, we show that even if the factors follow a finite-order VAR process, this implies a VARMA representation for the observable series. As result, we propose the FAVARMA framework that combines two parsimonious methods to represent the dynamic interactions between a large number of time series: factor analysis and VARMA modeling. We apply our approach in two pseudo-out-of-sample forecasting exercises using large U.S. and Canadian monthly panels taken from Boivin, Giannoni and Stevanović (2010, 2009) respectively. The results show that VARMA factors help in predicting several key macroeconomic aggregates relative to standard factor forecasting models. Finally, we estimate the effect of monetary policy using the data and the identification scheme as in Bernanke, Boivin and Eliasz (2005). We find that impulse responses from a parsimonious 6-factor FAVARMA(2,1) model give an accurate and comprehensive picture of the effect and the transmission of monetary policy in U.S.. To get similar responses from a standard FAVAR model, Akaike information criterion estimates the lag order of 14. Hence, only 84 coefficients governing the factors dynamics need to be estimated in the FAVARMA framework, compared to FAVAR model with 510 VAR parameters.

In fourth article we are interested in identifying and measuring the effects of credit shocks in Canada in a data-rich environment. In order to incorporate information from a large number of economic and financial indicators, we use the structural factor-augmented VARMA model. In the theoretical framework of the financial accelerator, we approximate the external finance premium by credit spreads. On one hand, we find that an unanticipated increase in US external finance premium generates a significant and persistent economic slowdown in Canada; the Canadian external finance premium rises immediately while interest rates and credit measures decline. From the variance decomposition analysis, we observe that the credit shock has an important effect on several real activity measures, price indicators, leading indicators, and credit spreads. On the other hand, an unexpected increase in Canadian external finance premium shows no significant effect in Canada. Indeed, our results suggest that the effects of credit shocks in Canada are essentially caused by the unexpected changes in foreign credit market conditions. Finally, given the identification procedure, we find that our structural factors do have an economic interpretation.

The behavior of economic agents and environment may vary over time (monetary policy strategy shifts, stochastic volatility) implying parameters' instability in reduced-form models. Standard time varying parameter (TVP) models usually assume independent stochastic processes for all TVPs. In the final article, I show that the number of underlying sources of parameters' time variation is likely to be small, and provide empirical evidence on factor structure among TVPs of popular macroeconomic models. To test for the presence of, and estimate low dimension sources of time variation in parameters, I apply the factor time varying parameter (Factor-TVP) model, proposed by Stevanovic (2010), to a standard monetary TVP-VAR model. I find that one factor explains most of the variability in VAR coefficients, while the stochastic volatility parameters vary in the idiosyncratic way. The common factor is highly and positively correlated to the unemployment rate. To incorporate the recent financial crisis, the same exercise is conducted with data updated to 2010Q3. The VAR parameters present an important change after 2007, and the procedure suggests two factors. When applied to a large-dimensional structural factor model, I find that four dynamic factors govern the time instability in almost 700 coefficients.

Keywords: Factor analysis, VARMA model, forecasting, impulse responses, structural analysis, time varying parameter model.

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Enfin, à mes collègues du département des sciences économiques de l'UQAM pour leur accueil convivial et leur support au début de ma nouvelle vie professionnelle.

INTRODUCTION GÉNÉRALE

Cette thèse est composée de cinq essais et s'inscrit dans le cadre des modèles à facteurs avec applications en macroéconomie. Les contributions empiriques sont : caractériser la transmission de la politique monétaire au Canada en corrigeant pour la plupart des anomalies répertoriées dans la littérature antérieure, identifier et quantifier, parmi les premiers, les chocs de crédit et leurs effets sur les économies américaine et canadienne, et produire la première évidence empirique sur la structure à facteurs des coefficients aléatoires dans les modèles macroéconomiques. Du point de vue théorique, une nouvelle classe de modèles est proposée et leur importance a été justifiée au niveau de la prévision des agrégats macroéconomique et de l'analyse structurelle utilisant les fonctions de réponse impulsionnelles.

Le premier article analyse la transmission de la politique monétaire au Canada en utilisant le modèle vectoriel autorégressif augmenté par facteurs (FAVAR). Les études précédentes utilisant les modèles VAR structurels ont documenté plusieurs anomalies (*price, exchange rate, delayed overshooting* et *uncovered interest rate parity puzzles*), et les essaies de les corriger n'ont pas connu de grands succès. Puisque l'une des explications des difficultés est le manque d'information dans les petits modèles empiriques, l'approche FAVAR est très attrayante car elle permet d'incorporer un très grand ensemble de données tout en ayant un modèle parcimonieux.

Le modèle est estimé avec un panel non balancé contenant 435 indicateurs économiques et financiers mensuels et trimestriels. Nous trouvons que l'information contenue dans les facteurs est importante pour bien identifier la transmission de la politique monétaire dans les fréquences mensuelle et trimestrielle, et elle aide à corriger les anomalies empiriques présentes dans les modèles VAR. Finalement, le cadre d'analyse FAVAR permet d'obtenir les fonctions de réponse impulsionnelles pour tous les indicateurs dans l'ensemble de données, produisant ainsi l'analyse la plus complète à ce jour des effets de la politique monétaire au Canada.

L'objectif du deuxième article est d'examiner les effets et la propagation des chocs de crédit sur l'économie réelle en utilisant un grand ensemble d'indicateurs économiques et financiers dans le cadre d'un modèle à facteurs structurel. Contrairement aux études précédentes, nous utilisons une procédure d'identification des chocs structurels moins restrictive avec pour but de laisser les réponses des taux d'intérêt et des indicateurs avancés complétement déterminées par les données. Le modèle est estimé en utilisant 187 indicateurs économiques et financiers américains.

Nous trouvons qu'un choc de crédit augmente immédiatement les *credit spreads* et cause une récession significative et persistante accompagnée d'une baisse considérable des niveaux des prix. De plus, les taux d'intérêt baissent significativement à l'impact ainsi que les indicateurs avancés tels que l'indice de marché immobilier, le sentiment des consommateurs, etc. Ces chocs ont un effet important sur des mesures d'activité réelle, indices de prix, indicateurs avancés et financiers. Contrairement aux autres études, notre procédure d'identification du choc structurel ne requiert pas de restrictions temporelles entre facteurs financiers et macroéconomiques. De plus, elle donne une interprétation des facteurs sans restreindre l'estimation de ceux-ci.

Dans le troisième article nous étudions la relation entre les représentations VARMA et factorielle des processus vectoriels stochastiques, et proposons une nouvelle classe de modèles VARMA augmentés par facteurs (FAVARMA). Notre point de départ est de constater qu'en général les séries multivariées et facteurs associés ne peuvent simultanément suivre un processus VAR d'ordre fini. Nous montrons que le processus dynamique des facteurs extraits comme combinaison linéaire des variables observées est en général un VARMA et non pas un VAR comme c'est supposé ailleurs dans la littérature. Deuxièmement, nous montrons que même si les facteurs suivent un VAR d'ordre fini, cela implique une représentation VARMA pour les séries observées. Alors, nous proposons le cadre d'analyse FAVARMA combinant ces deux méthodes de réduction de dimension.

Le modèle est appliquée dans deux exercices de prévision en utilisant des données américaines et canadiennes de Boivin, Giannoni et Stevanović (2010, 2009) respectivement. Les résultats montrent que la partie VARMA aide à mieux prévoir les importants agrégats macroéconomiques relativement aux modèles standards. Finalement, nous estimons les effets de choc monétaire en utilisant les données et le schéma d'identification de Bernanke, Boivin et Eliasz (2005). Notre modèle parcimonieux FAVARMA(2,1) avec six facteurs donne les résultats cohérents et précis des effets et de la transmission monétaire aux États-Unis. Contrairement au modèle FAVAR employé dans l'étude ultérieure où 510 coefficients VAR devaient être estimés, nous produisons les résultats semblables avec seulement 84 paramètres du processus dynamique des facteurs.

L'objectif du quatrième article est d'identifier et mesurer les effets des chocs de crédit au Canada dans un environnement riche en données. Dans le but d'incorporer l'information d'un grand ensemble d'indicateurs économiques et financiers, nous utilisons le modèle FAVARMA structurel. Dans le cadre théorique de l'accélérateur financier développé par Bernanke, Gertler et Gilchrist (1999), nous approximons la prime de financement extérieur par les (*credit spreads*). Le modèle est estimé en utilisant les données mises à jour de Boivin, Giannoni et Stevanovic (2009).

D'un côté, nous trouvons qu'une augmentation non-anticipée de la prime de financement extérieur aux États-Unis génère une récession significative et persistante au Canada, accompagnée d'une hausse immédiate des *credit spreads* canadiens. La composante commune semble capturer les dimensions importantes des fluctuations cycliques de l'économie canadienne. L'analyse par décomposition de la variance révèle que ce choc de crédit a un effet important sur différents secteurs d'activité réelle, indices de prix, indicateurs avancés et *credit spreads*. De l'autre côté, une hausse inattendue de la prime canadienne de financement extérieur ne cause pas d'effet significatif au Canada. Nous montrons que les effets des chocs de crédit au Canada sont essentiellement causés par les conditions globales, approximées ici par le marché américain. Finalement, étant donnée la procédure d'identification des chocs structurels, nous trouvons des facteurs interprétables économiquement.

Finalement, le dernier article innove en imposant une structure à facteurs au niveau des coefficients aléatoires d'un modèle empirique. Il est bien connu que le comportement des agents et de l'environnement économiques peut varier à travers le temps (ex. changements de stratégies de la politique monétaire, volatilité de chocs) induisant de l'instabilité des paramètres dans les modèles en forme réduite. Les modèles à paramètres variant dans le temps (TVP) standards supposent traditionnellement les processus stochastiques indépendants pour tous les TVPs. Dans cet article nous montrons que le nombre de sources de variabilité temporelle des coefficients est probablement très petit, et nous produisons la première évidence empirique connue dans les modèles macroéconomiques empiriques.

L'approche Factor-TVP est appliquée dans le cadre d'un modèle TVP-VAR standard. Nous trouvons qu'un seul facteur explique la majorité de la variabilité des coefficients VAR, tandis que les paramètres de la volatilité des chocs varient d'une façon indépendante. Le facteur commun est positivement corrélé avec le taux de chômage. La même analyse est faite avec les données incluant la récente crise financière. La procédure suggère maintenant deux facteurs et le comportement des coefficients présente un changement important depuis 2007. Finalement, la méthode est appliquée à un modèle TVP-FAVAR. Nous trouvons que seulement 5 facteurs dynamiques gouvernent l'instabilité temporelle dans presque 700 coefficients.

Contribution des coauteurs

Je suis le premier auteur de tous les cinq articles de cette thèse. Cependant, mes coauteurs Nathan Bedock, Jean Boivin, Jean- Marie Dufour, Marc Giannoni et moi-même contribuons à part égale.

Les deux premiers articles sont coécrits avec Jean Boivin et Marc Giannoni. Le troisième article est coécrit avec Jean-Marie Dufour et le quatrième avec Nathan Bedock.

CHAPTER 1

MONETARY TRANSMISSION IN A SMALL OPEN ECONOMY: MORE DATA, FEWER PUZZLES

1.1 Introduction

Conclusions about the role that monetary policy plays in the economy and how it should be conducted in practice depend crucially on the way monetary policy affects the economy. This is why a large empirical literature has attempted to measure the transmission of monetary policy.

A standard approach to uncover the transmission of monetary policy is to use structural vector autoregression (VAR). This method is particularly appealing since it does not require to specify complete model of the economy. It consists of imposing the minimum amount of restrictions needed to identify an exogenous source of variation in monetary policy, in a system of equations capturing the relevant macroeconomic dynamics, that is otherwise left unrestricted. Structural VAR methodology has been largely applied in both assessing the empirical fit of structural models and in policy applications. Some key examples of early and successful implementation on US data are Bernanke and Blinder (1992), Sims (1992), and Bernanke and Mihov (1998). Even if the identification strategy has been a source of disagreement (see Christiano, Eichenbaum and Evans (2000) for a survey), this simple method is still largely used, and delivers some useful information about the effects and the transmission of monetary policy shocks on economy.

However, for small open economies like Canada, uncovering the transmission mechanism of monetary policy through this type of approach has proven to be an especially challenging task. In particular, initial VAR analysis on Canadian data have often documented the presence of anomalies such as price, exchange rate, delayed overshooting and uncovered interest rate parity puzzles.

In Grilli and Roubini (1996) the authors used standard structural VAR model to evaluate the effects of monetary policy shocks in two-country systems (the non-U.S. G-7 countries relative to the U.S.). Their results show strong evidence of several puzzles for most of non-U.S. countries. To solve some of these anomalies the authors replaced the short-term interest rate by the differential between short- and long-term interest rates in order to capture agents' inflation expectations. The same anomalies are reported and resolved in Kim and Roubini (2000) who used a structural VAR setup with non-recursive contemporaneous restrictions where the monetary policy shocks are identified by modeling the monetary authority reaction function and the structure of the economy. Another alternative to simple recursive identification structure is to use some long-run propositions of economic theory. For instance, Fung and Kasumovich (1998) estimate cointegrated VAR models for G-6 countries and then identify monetary shocks by imposing the long-term money neutrality (a permanent change in the nominal stock of money has a proportionate effect on the price level with no long-run effect on real economic activity). In Cushman and Zha (1997) authors argue that puzzles found when estimating the effect of monetary policy shocks in small open countries are due to inappropriate identification schemes of monetary policy in such economies. Using Canada as benchmark case, they estimate a standard VAR model that contains two types of variables, domestic (CAN) and foreign (US), and impose block exogeneity condition on the latter. The monetary policy shock is identified by supposing that monetary authority observe immediately the exchange rate, interest rates, stock of money and world commodity price level. Using this nonrecursive identification they obtain impulse responses that are consistent with standard theory and highlight the exchange rate as a transmission mechanism. Finally, Bhuiyan and Lucas (2007) consider an alternative resolution of these puzzles based on an explicit account of inflation expectations. They first estimate ex-ante real interest rate and inflationary expectations by decomposing the nominal interest rate, and then include these into a fully recursive VAR model to evaluate the effects of monetary policy shocks. Their findings suggest more broadly, that the anomalies reported above might be the result of omitted information from small-scale VARs.

Hence, it is particularly interesting to see if a more systematic use of the relevant information available could yield a more coherent and accurate picture of the effect of monetary policy in a small open economy. In this paper, we use a factor augmented vector autoregression (FAVAR) approach to assess the effect and transmission mechanism of monetary policy shocks on economic activity in Canada [1]. Given that a common potential explanation of all difficulties reported above is the lack of information in small-scale VAR models, the FAVAR approach is appealing in a priori since it incorporates a huge amount of information in a parsimonious way. Moreover, the application to U.S. data by Bernanke, Boivin and Eliasz (2005) was a success story.

In our implementation, we estimate the FAVAR model using an unbalanced data set of 348 monthly and 87 quarterly macroeconomic Canadian time series. We find that the information summarized by the factors is important to properly identify the monetary transmission mechanism in both monthly and quarterly frequencies. Overall, our benchmark FAVAR specification, that includes only the monetary policy instrument as observed factor, leads to broadly plausible estimates of the effects of monetary policy shocks on many macroeconomic variables of interest and contributes to mitigate puzzles mentioned above. Indeed, all price indexes decline after an unexpected increase in short rate while the exchange rates appreciate on impact.

When comparing to standard small open economy VAR model results, we find that adding information through factors into this VAR corrects for price and exchange rate puzzles, and for inconsistent response of industrial production with respect to long-run money neutrality. Also, the maximum response of exchange rates is on impact which corrects for delayed overshooting puzzle. Finally, we find no evidence of the uncovered interest rate parity, meaning that there is no systematic carry trade conditional on a domestic monetary policy shock that rises the domestic interest rate.

Relative to existing literature discussed above, our approach is able to uncover reasonably the monetary policy transmission in a small open economy without searching to include agents' expectations measures or other theoretical concepts proxies, and using the simplest recursive identification scheme. Moreover, the FAVAR framework allows us to check impulse responses for all series in the informational data set, and thus provide, to our knowledge, the most comprehensive picture to date of the effect of Canadian

1. In independent research projects, Mumtaz and Surico (2009), and Forni and Gambetti (2010), obtain similar results for some of the puzzles that we study in this paper.

monetary policy.

The rest of the paper is organized as follows. The FAVAR methodology is detailed in the following section. In Section 3 we explain our application by presenting data and the strategy to identify the monetary policy shocks. The main results are presented and discussed in Section 4, and we conclude in Section 5.

1.2 FAVAR: Motivation, Methodology and Estimation

1.2.1 Motivation

Since Bernanke and Blinder (1992) and Sims (1992), the structural analysis applied macroeconomics employs vector autoregressive (VAR) models to identify and measure the effects of different shocks on macroeconomic variables of interest. Typically, central banks are interested in the behavior of macroeconomic aggregates after a monetary policy shock, and their analysts use widely structural VARs in order to identify the innovation. Several criticisms of the VAR approach are worth of noting. The most important is that it uses only a small number of variables to conserve degrees of freedom. This small number of variables is unlikely to span the information sets used by actual central banks that follow a large number of data series. Then, the lack of information leads to three big potential problems. First, the identification of shock can be contaminated which leads to several "puzzles" observed in the literature. Grilli and Rubini (1996) paper offers a nice overview of puzzles found in several papers:

– *The price puzzle*. When monetary policy shocks are identified with innovations in interest rates, the output and money supply responses are correct as a contractionary increase in interest rate is associated with a fall in the money supply and the level of economic activity. However, the response of the price level is a persistent increase rather than a decrease.
– *The Exchange rate puzzle*. While a positive innovation in interest rates in the US is associated with an impact appreciation of the US dollar relative to the other G-7 countries, such monetary contractions in other G-7 countries are often associated with an impact depreciation of their currency value relative to the US dollar.

– *The forward discount bias puzzle* If uncovered interest parity holds, a positive innovation in domestic interest rates relative to a foreign ones should be associated with a persistent depreciation of the domestic currency after the impact appreciation, as the positive interest rate differential leads to an expected depreciation of the currency. However, the data show that a positive interest differential is associated with a persistent appreciation of the domestic currency for periods up to two years after the initial monetary policy shock.

Further problem implied by lack of information in small-scale VAR models is the omitted variable problem. If important variables are not included in the system (correlated with regressors in the model) this leads to biased estimates of VAR coefficients which is likely to produce biased impulse responses worthless for structural analysis. The typical example in the literature is the omission of commodity prices in structural VAR analysis attempting to measure monetary policy in U.S. (see Sims (1992) for explanation).

The second problem in small-scale VAR model is that the choice of a specific data series to represent a general economic concept is arbitrary. Moreover, measurement errors, aggregation and revisions pose additional problems for linking theoretical concepts to specific data series. Finally, even if the two previous problems do not occur, i.e. a small scale VAR is well defined and the shock is well identified, we can produce impulse responses only for variables included in the VAR.

On the other side, a factor-augmented VAR, which will be discussed deeply in the next section, is a way to introduce additional information and then overcome the previous discussion. It uses a simply dimension reduction with principal components analysis, which permits to resume a big part of information contained in a huge panel, into small number of factors. In the case of the monetary policy studies where the monetary policy instruments is an interest rate, Bernanke, Boivin and Eliasz (2005) show that including only 3 factors correct the price puzzle while keeping a low-dimensional estimated VAR and the easiest identification scheme. Finally, we can compute the impulse response functions for any variable in the informational panel which can be very important if the central bank is interested for example into the behavior of several price indices instead in a total consumer price index only.

1.2.2 Methodology

We apply the Factor Augmented Vector Autoregressive (FAVAR) approach as in Bernanke, Boivin and Eliasz (2005), or BBE for the rest of the paper. Consider a $T \times M$ matrix of observable economic series Y, where T is the time size (number of periods) and M is the cross-sectional size (number of series). In the standard VAR (and structural VAR) models used in monetary literature, Y include several variables assumed to drive the dynamics of the economy and the transmission of monetary policy shocks. The usual candidates are some measures of economic activity (GDP, industrial production, employment, unemployment rate, etc.), an indicator of price level (usually CPI), and a policy instrument (e.g. Federal Funds Rate (FFR) in US, Overnight rate in Canada, Monetary base, etc.). In the traditional (S)VAR approach, Y is modeled alone assuming that all relevant information is contained in several lagged values of Y. However, additional information available in other economic series may be relevant to the dynamic relationships assumed in VAR model, and this lack of information can lead to some unanticipated implications from the estimated model as pointed out in the previous section.

If this additional information can be summarized by a $T \times K$ unobserved factors matrix F, where K is relatively small, we can augment the standard VAR model by adding the factors. As illustrated by an example in BBE, the factors can be seen as proxies for the economic activity, price pressures, credit conditions or other theoretical concepts that are difficult to identify by one or two variables.

Suppose that the joint dynamics of (F_t, Y_t) can be represented by the following equation:

$$\begin{aligned}
\begin{bmatrix} F_t \\ Y_t \end{bmatrix} &= \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \nu_t \\
&= \begin{bmatrix} \phi_{ff}(L) & \phi_{fy}(L) \\ \phi_{yf}(L) & \phi_{yy}(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + e_t
\end{aligned} \tag{1.1}$$

where $\Phi(L)$ is the usual lag polynomial of finite order p, and ν_t is the error term with mean zero and covariance matrix Q. It is easy to see that (1.1) becomes a standard VAR in Y_t if the matrix $\Phi(L)$ is diagonal, i.e. if all terms in $\phi_{fy}(L)$ and $\phi_{yf}(L)$ are zero (implying that there is no direct Granger causal relation between F_t and Y_t). Otherwise, the system (1.1) is defined as a factor augmented vector autoregression (FAVAR).

It is important to notice that since FAVAR nests VAR representation in Y_t, estimating the former allows us to evaluate the marginal contribution of factors by comparing the results with existing VAR analysis. If the best approximation (in reduced form) of the true DGP (data generated process) is a FAVAR, then omitting F_t from (1.1) and estimating the VAR model will lead to biased estimates of the VAR coefficients. Thus, the structural interpretations of the impulse responses are worthless.

If the factors F_t were observed, equation (1.1) would be a standard VAR model and we would use existing structural VAR techniques to estimate the model and identify structural shocks. Unfortunately, F_t are unobservable and we have to learn something about them from the relevant and available economic time series. Suppose that we have a panel of observable and informative economic series contained in a $N \times 1$ vector X_t. The number of series, N, can be arbitrary large relatively to the time series size T, but assumed to be much larger than the number of factors in F_t, K, and observed variables in Y_t, M. Then, we need to assume a relation between our observable series and the factors that we need to estimate. The relation is given in the following observation equation:

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + u_t \tag{1.2}$$

where X_t is an (Nx1) vector of informative time series, Λ^f is an (NxK) matrix of factor loadings, Λ^y is an (NxM) matrix of loadings relating the observable factors in Y_t to X_t, and u_t is the (Nx1) vector of error terms. The errors are of mean zero and can display a small amount of cross-correlation. Note that (1.2) states that both F_t and Y_t explain the dynamics of X_t. Thus, if we condition the statement on Y_t, we can interpret X_t as noisy measures of the underlying unobserved factors F_t.

Hence, defining $\mathbf{F}_t = [F_t' \quad Y_t']'$ and $\Lambda = [\Lambda^f \quad \Lambda^y]$ the FAVAR model can be represented in an approximate static factor model form:

$$X_t = \Lambda F_t + u_t \tag{1.3}$$

$$\mathbf{F}_t = \Phi(L)\mathbf{F}_{t-1} + e_t \tag{1.4}$$

where approximate stands for allowing some weak cross-section and time dependence among idiosyncratic components in e_t, and where F_t contains both observed and unobserved factors. Note that considering a static version, i.e. (1.2) doesn't contain any lagged values of F_t or Y_t, is not a big constraint since dynamic factor model can always be written in a static form.

1.2.3 Estimation

Recall from the previous section that the estimation of the model in (1.1) would be trivial if the factors were observable. Since this is not the case, we have to estimate them from X_t.

The unknown coefficients in (1.3)-(1.4) can be estimated by Gaussian maximum likelihood (or by Quasi ML) using the Kalman filter, see Engle and Watson (1981), Stock and Watson (1989), Sargent (1989). This method is computationally burdensome when N is very large, but also the misspecification becomes very likely. [2]

Instead of the likelihood-based approach, we use the two-step Principal Component

2. However, there are some recent improvements: Kalman filter speedup by Jungbacker and Koopman (2008), using principal components as very good starting values then a single pass of the Kalman filter by Giannone, Reichlin, and Sala (2004), and principal components for starting values then use EM algorithm to convergence by Doz, Giannone, and Reichlin (2006).

Analysis (PCA) estimation method. [3] It is a non-parametric way to uncover the common space spanned by the factors of X_t, denoted by $C(F_t,Y_t)$. In the first step, the equation (1.2) is considered. The space spanned by the factors is estimated by the first K+M principal components of X_t, and is denoted by $\hat{C}(F_t,Y_t)$. One should note that estimating factors in this way is not the most efficient method since we do not exploit the fact that Y_t is observed. However, Stock and Watson (2002a) show that if N is large and the number of principal components is at least as large as the true number of factors, the principal components consistently recover the space spanned by both F_t and Y_t. In that case, we need to identify the part of $\hat{C}(F_t,Y_t)$ that is not spanned by Y_t in order to obtain the estimate of F_t, $\hat{F}_t$. This task depends on identification imposed in the second step where the equation (1.2) is estimated by standard methods since unobserved factors are replaced by $\hat{F}_t$. In the second step, the factors' dynamic process is approximated by standard finite order VAR.

The principal components approach is easy to implement and do not require very strong distributional assumptions. However, since the unobserved factors are estimated and then included as regressors in FAVAR model, the two-step approach suffers from the "generated regressors" problem. In order to get the accurate statistical inference on the impulse response functions, we use a bootstrap procedure proposed by Kilian (1998) that accounts for the uncertainty in the factor estimation.

1.3 Application

The purpose of this paper is to study the dynamic effects of monetary policy shocks on a variety of economic variables in Canada. We previously pointed out some problems with (S)VAR models and we discuss in this section how FAVAR model can deal with some of them.

Since the FAVAR approach consists of adding to a standard VAR K common components from a large number of relevant economic variables, it should deal with the lack of information problem in traditional (S)VAR literature. Moreover, we showed above

3. See Stock and Watson (2002a), and Bai and Ng (2006) for theoretical results concerning the PCA estimator.

that the system (1.1)-(1.2) nests the VAR specification. Then, it is possible to discuss directly if the marginal information brought by estimated factors is relevant or not. Another problem that FAVAR approach can avoid is to assume that theoretical concepts such as real economic activity or price pressure are observed. Also, this approach allows us to study the dynamic responses to monetary policy shock of all variables in X_t, not only in Y_t. Finally, Forni et al. (2009) argues that while non-fundamentalness is generic of small scale model, they cannot arise in a large dimensional dynamic factor models [4]. This is of primary importance since the objective is to identify a relatively new structural shock in empirical macroeconomics.

Let us state now the FAVAR and VAR models that will be used to assess the effect of monetary policy shocks in Canada. The benchmark model is a FAVAR where Y_t contains only one variable, the monetary policy instrument, and F_t contains K unobserved factors. The official monetary policy instrument of the Bank of Canada is the overnight rate. Since this variable is available only from 1975M1, and our application uses data from 1969M1, we take the 3-month Treasury Bill (T-bill) as a proxy [5]. In order to discuss the additional information brought by the factors, we will compare a standard VAR model, where Y contains Industrial production (IP), Consumer price index (CPI), T-Bill and CAN/US Exchange rate (FX-CAN/US), with FAVAR models where Y is augmented by a number of estimated factors.

1.3.1 Data

We estimate the system (1.1)-(1.2) with Canadian data used in Gosselin and Tkacz (2001) and updated with some variables from Galbraith and Tkacz (2007). There are 348 monthly series starting from 1969M1 and ending on 2008M6, and 87 quarterly series covering 1969Q1-2008Q2 time period. These series are initially transformed to induce stationarity. The description of the variables in the data set and their transformation is given in Appendix. To use the two-step approach, we need a balanced panel. Then,

4. If the shocks in the VAR model are fundamental, then the dynamic effects implied by the moving average representation can have a meaningful interpretation, i.e. the structural shocks can be recovered from current and past values of observable series.

5. Since 1975 the correlation coefficient between the two series is 0.97

if we wish to use all available information, we have to mix both monthly and quarterly panels. Hence, we need to replace missing values when transforming the quarterly series to monthly indicators. Moreover, several monthly series contain missing values. To face these irregularities and obtain a balanced data set, we apply the EM algorithm proposed by Stock and Watson (2002b) [6].

Before discussing the estimation results, we need to specify the identification restrictions in the two-step approach, and how the monetary policy instrument is imposed as an observable factor.

1.3.2 Identification in the two-step approach

Different sets of identification restrictions must be imposed before estimating the system (1.1)-(1.2). The first consists of normalization restrictions on the observation equation (1.2) because of the fundamental indeterminacy of this model. Suppose that $\hat{\Lambda}$ and $\hat{F}_t$ are a solution to the estimation problem. However, this solution is not unique since we could define $\tilde{\Lambda} = \hat{\Lambda}H$ and $\tilde{F}_t = H^{-1}\hat{F}_t$, where H is a $K \times K$ nonsingular matrix, which could also satisfy equation (1.2). Then, observing X_t is not enough to distinguish between these two solutions, and a normalization is necessary. We use the standard normalization in the principal components approach, that is, we take $C^{'}C/T = I$, where $C = [C(F_1,Y_1),...,C(F_T,Y_T)]$. Then, $\hat{C} = \sqrt{T}\hat{Z}$, where $\hat{Z}$ are the eigenvectors corresponding to the K largest eigenvalues of $XX^{'}$, sorted in descending order.

The second identification issue is to identify the structural shocks in equation (1.1). As in most VAR model applications in the monetary policy literature, we adopt a recursive structure where the monetary policy instrument is ordered last in Y_t (all the factors

6. The choice of data to include in X_t is not obvious. Theoretically, more data (and that means larger time size, $T \uparrow$, and more series, $N \uparrow$) is better because the estimators in two-step approach are asymptotically consistent and the asymptotic theory here has two dimensions, T and N. But in practice, T is maximized with data availability constraint while augmenting N (and adding relevant information) means more of the same type data (e.g. CPI category has dozens of subcategories). Boivin and Ng (2006) provide examples where adding more data has perverse effects in forecast exercise. The idea is that while the two-step estimators are consistent even in presence of weak cross-correlation between the errors in (1.3), adding many data of the same type in the finite sample context could increase the amount of cross-correlations in the error term and alter the performance of the PCA estimator. However, the pre-screening proposed by Boivin and Ng (2003) is largely *ad hoc*, and the cost from using all series, if any, is marginal in practice.

entering (1.1) respond with a lag to a monetary policy shock). In that case, we don't need to identify the factors separately, but only the space spanned by the latent factors, F_t.

Recall that in the first step, relying on the fact that when N is large, the principal components estimated from X_t, $\hat{C}(F_t, Y_t)$, consistently recover K+M independent, but arbitrary, linear combinations of F_t and Y_t. Since Y_t is not explicitly imposed as a factor in the first step, any of the linear combinations underlying $\hat{C}(F_t, Y_t)$ could involve the monetary policy instrument, which is always ordered last in Y_t. Then, it would not be valid to simply estimate a VAR in estimated factors from entire data set and Y_t, and use the recursive policy shock identification framework. In that case, we need to remove the direct dependance of $\hat{C}(F_t, Y_t)$ on Y_t, where Y_t is T-bill. If linear combinations of F_t and Y_t were known, this would involve subtracting Y_t times the associated coefficient from each of the elements of $\hat{C}(F_t, Y_t)$.

Since these are unknown, to impose Y_t as a factor in the first step we use the iterative principal components approach as in Boivin and Giannoni (2007). Starting from an initial estimate of F_t, F_t^0:

1. Regress X_t on F_t^0 and Y_t, to obtain $\hat{\lambda}_t^0$
2. Compute $\tilde{X}_t^0 = X_t - \hat{\lambda}_t^0 Y_t$
3. Estimate F_t^1 as the first K-1 principal components from $\tilde{X}_t^0$
4. Back to 1.

Contrary to BBE's strategy, it does not rely on any temporal assumption between the observed factors and the informational panel. Hence it can be used for any set of observed factors without imposing any further assumptions. We adopt this approach in our exercise with setting the number of iterations at 15 [7].

1.4 Results

One interesting feature of the FAVAR approach is that we can produce impulse responses for all observable series (in both informational panel and observed factors).

7. In our robustness analysis exercises the convergence is always attained after 10 to 15 iterations.

Hence, we can explore the reaction of the economy to a structural shock on a much broader set of dimensions than in the case of small-scale VAR models. Given our mixed-frequencies approach, we can also conduct the exercise at both monthly and quarterly frequencies.

1.4.1 Effects of a monetary policy shock

Here, we discuss results using mixed-frequencies monthly panel where the benchmark model contains 8 unobserved factors and one observed factor, T-Bill. Figure 1.1 contains impulse response for some economic indicators of interest to a monetary policy shock [8]. We can see that a positive shock on the T-Bill implies a persistent economic slowdown. The production indicators go down progressively, and price indexes present a very persistent decreasing reaction. The leading economic indicators such as housing index, new orders and retail trade, and money aggregates decline significatively. Overall, these results seem to provide a consistent measure of the effect of monetary policy in a small open economy.

The impulse responses of several exchange rates are presented in Figure 1.2. We can see that Canadian dollar appreciates in most of the cases, and especially with respect to the US dollar, meaning that there is no evidence of exchange rate puzzle. Moreover, the maximum response is on the impact, so the delayed overshooting puzzle is corrected too. The impulse responses of interest rates are presented in Figure I.1. They jump initially above the steady state and eventually go down.

Since we have constructed a mixed-frequencies monthly panel, we can produce monthly impulse responses of economic indicators observed only at quarterly frequency. In Figure I.2, in Appendix, we plot impulse responses of some of these constructed monthly indicators. We can see a significative decline in GDP components. Moreover, it is interesting to see if there are some differences in the response to monetary policy shock across different regions in Canada. To do so we grouped some available series of interest in fours regions: Atlantic, Center, Prairie and BC. In Figure I.3 we plot their responses in deviation to the response of corresponding national variable. We can see that

8. In all Figures the 90 percent confidence intervals are obtained using 5000 bootstrap replications.

Atlantic, Center and BC regions present quite similar pattern while the Prairie provinces seem to diverge from the other provinces.

The Table I.II in Appendix presents variance decomposition and R^2 results. The first column reports the contribution of the monetary policy shock to the variance of the forecast error at four year horizon, and the second column contains the R^2 of the common components for 80 variables of interest. As in BBE paper, we find that the monetary policy shock has a small effect on most of the variables, except for interest rates and money supply. Looking at the R^2 results we conclude that the common component explains an important fraction of variability in observable series, meaning that extracted factors do capture important dimensions of the business cycle movements.

1.4.2 Uncovered interest rate parity puzzle

The UIRP puzzle has been a very challenging task in the standard VAR framework. Including the information through factors seems to help in resolving this issue. We construct a measure of the forward discount premium following Scholl and Uhlig (2005). Let i_k and i_k^* be domestic and foreign interest rates impulse responses at horizon k. Define s_0 and s_k as impulse responses of the log of the exchange rate at the impact and at horizon k respectively. The UIRP measure (or the forward discount premium) is calculated as:

$$UIRP = (i_k - i_k^*) + (s_k - s_0).$$

In Figure 1.3, we plot the impulse response function for the UIRP between Canada and US, calculated for 3-month Treasury bills. Surprisingly, conditional on the domestic monetary policy shock, there is no carry trade on the impact. However, the confidence intervals are quite large. After a year, the response is close to zero.

In Figure 1.4, we plot the impulse responses of the same measure but after the US monetary policy shock. It is identified recursively by including the US 3-month T-bill first in the VAR ordering. In that case, the UIRP measure is significatively different from zero on impact. However, we do not interpret this as a violation of the uncovered interest parity hypothesis since the US monetary policy shock is understood as proxy for a global shock, that both monetary authorities respond.

We also tried to identify properly the US monetary policy shock by placing the US interest rate last in the factors' VAR. The idea is that the Canadian central bank does not respond immediately, in a month, to a shock in the US, since the comity meeting do not occur each month. The impulse response of UIRP is plotted in Figure 1.5. We can see that there is no evidence of violation of the uncovered interest parity hypothesis.

Finally, we identified a credit shock following Boivin, Giannoni and Stevanovic (2010). The result is presented in Figure 1.6. We can see that the forward discount premium deviates largely from its steady state value for more than a year, but with very imprecise confidence intervals.

1.4.3 Comparison to SVAR

To see how incorporating more information contained in factors affects standard VAR model results we compare impulse responses from our benchmark model to impulse responses of the VAR model containing $[US-R_t, \quad CPI_t, \quad IP_t, \quad R_t \quad FX-CAN/US_t]$, where $US-R_t$ stands for the US 3-month Treasury bill, and augmented by 1, 3, and 5 factors. The results are presented in Figure 1.6. We can see that in case of standard VAR (VAR + K=0 in the legend), there is an evidence of price, exchange rate, delayed overshooting and UIRP puzzles. The price level stays above its steady state value for more than a year, while the Canadian dollar depreciates on impact and its maximum response arrives several months after the shock. Finally, the last plot in the Figure 1.7 shows that SVAR implies a systematic carry trade. The UIRP responses are created from impulse responses of interest rates and exchange rates [9].

When we start adding factors, some the puzzles are reduced in magnitude. For the response of price index and industrial production adding one factor suffices to produce more reasonable responses. In case of exchange rates, only the benchmark model corrects the puzzles. There is no exchange nor delayed overshooting puzzles, and there is no evidence of the UIRP puzzle.

9. See Scholl and Uhlig (2005)

1.4.4 Monthly estimates of quarterly observed series

An interesting byproducts of our approach are the monthly indicators obtained from quarterly observed series when constructing mixed-frequencies monthly balanced panel. Many important macroeconomic aggregates, such as GDP and its components, are observed only at quarterly frequency and it can be of interest to have an idea about these indicators in monthly domain or to have an estimate of current economic conditions before statistical agencies make them available usually several months later. This problem is also known as nowcasting and there is growing literature that uses several econometric techniques to estimate current economic conditions (see Aruoba, Diebold, and Scotti (2009)).

In our case we construct mixed-frequencies monthly panel by applying the EM algorithm where the number of static factors used when replacing missing values is estimated at each iteration by the second information criteria (in log) in Bai and Ng (2002). In Figure 1.8 we present the standardized monthly estimates of some variables and in Figure 1.9 we plot the monthly estimate of the level of GDP and Consumption. We can see that our simple method gives plausible monthly estimates of quarterly observed variables.

1.5 Conclusion

The objective of this paper was to see if more information can help in assessing the monetary transmission mechanism in a small open economy. To do so, we used a factor augmented vector autoregression (FAVAR) approach to estimate the effects of monetary policy shocks on economic activity in Canada. We found that the information summarized by the factors that have been extracted as principal components from a large data set is important to properly identify the monetary transmission mechanism in both monthly and quarterly frequencies. Overall, our approach gave plausible estimates of the effects of monetary policy shocks on many macroeconomic variables of interest, and, in particular, contributed to mitigate puzzles reported in the literature.

We found that adding information through factors into this VAR corrects for price and exchange rate puzzles, and for inconsistent response of industrial production with respect to long-run money neutrality. Also, the maximum response of exchange rates is on impact which corrects for delayed overshooting puzzle. Finally, our results showed no evidence of the uncovered interest rate parity, meaning that there is no systematic carry trade conditional on a domestic monetary policy shock that rises the domestic interest rate.

Hence, relative to existing literature discussed above, we found that our approach is able to uncover reasonably the monetary policy transmission in a small open economy without searching to include agents' expectations proxies or other theoretical concepts proxies, and still using the simplest recursive identification scheme. Finally, the FAVAR framework allowed us to check impulse responses for all series in the informational data set, and thus provided, to our knowledge, the most comprehensive picture to date of the effect of Canadian monetary policy.

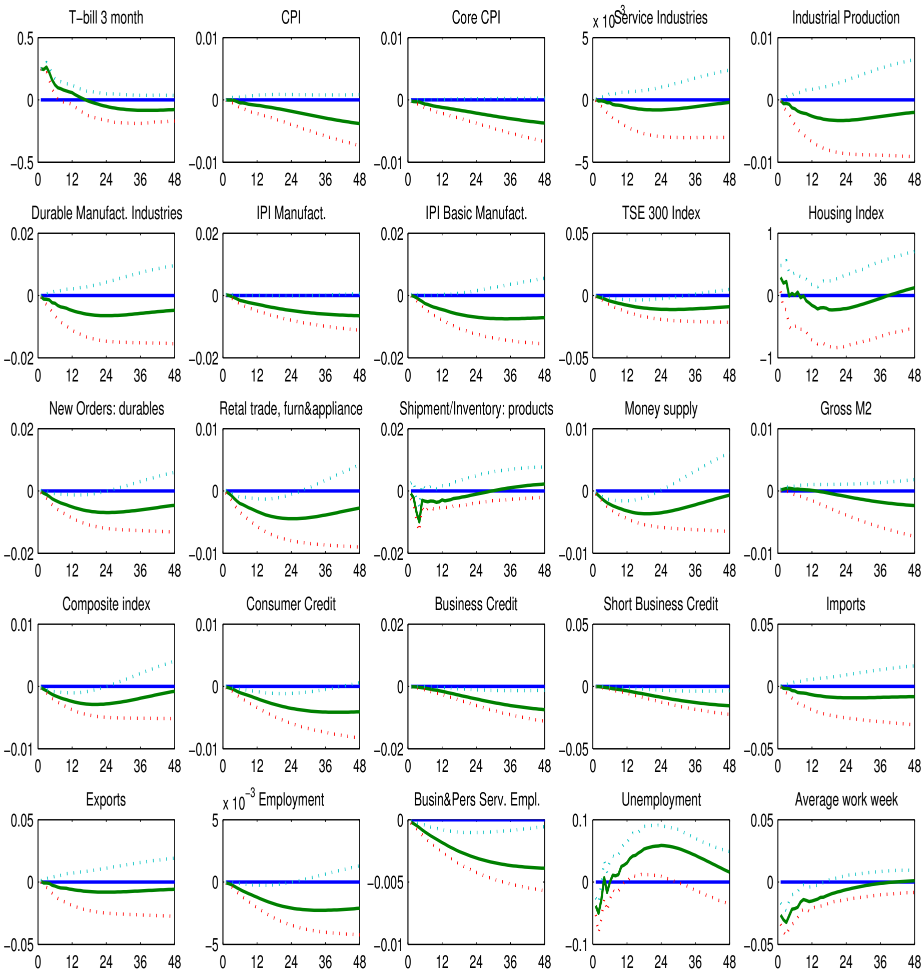

Figure 1.1: Impulse responses of some monthly indicators to national monetary policy shock

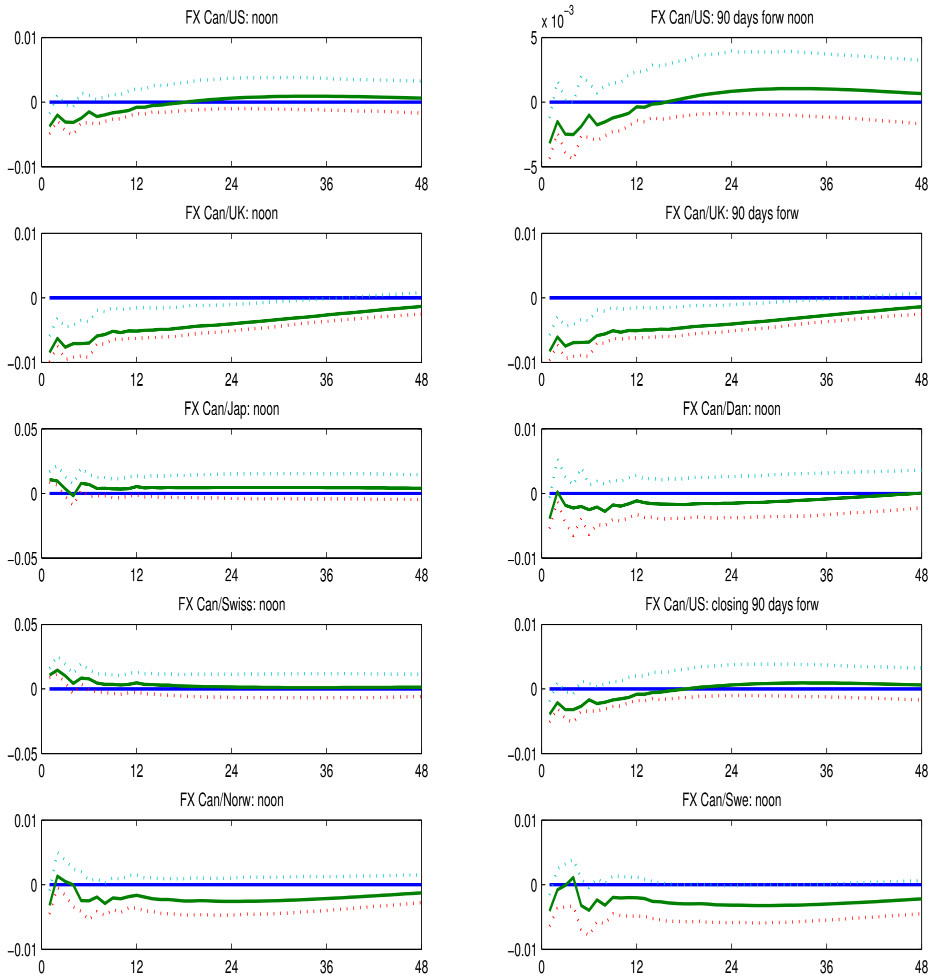

Figure 1.2: Impulse responses of exchange rates to national monetary policy shock

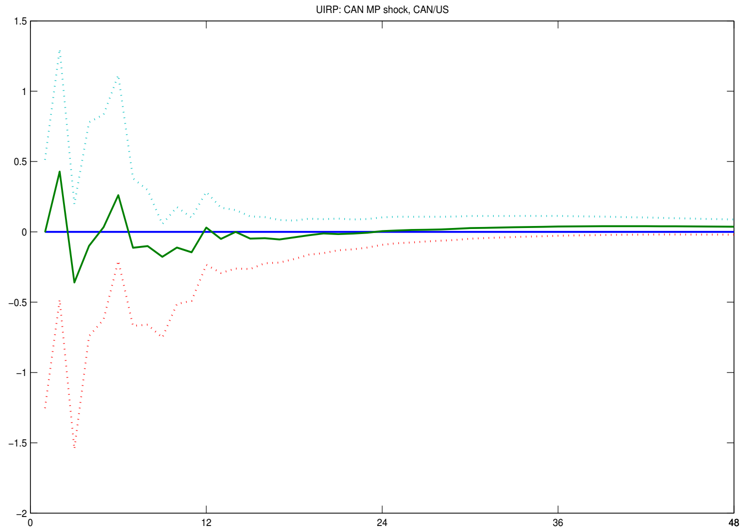

Figure 1.3: UIRP CAN/US, conditional on CA MP shock

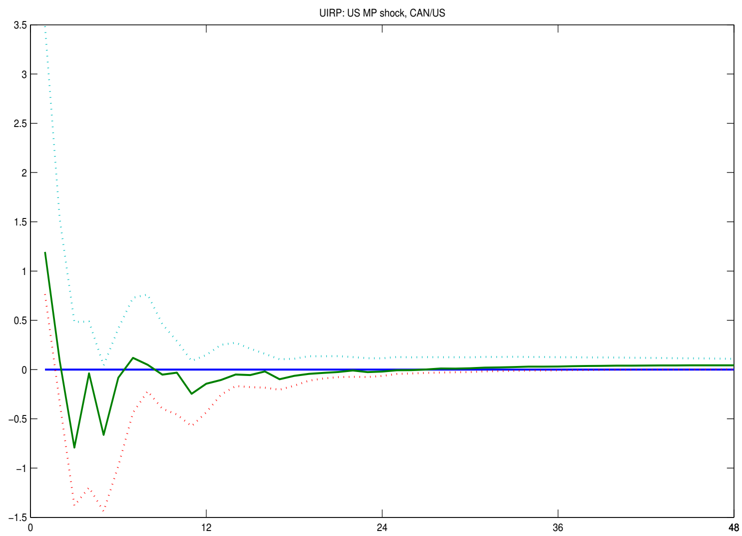

Figure 1.4: UIRP CAN/US, conditional on US Monetary policy shock

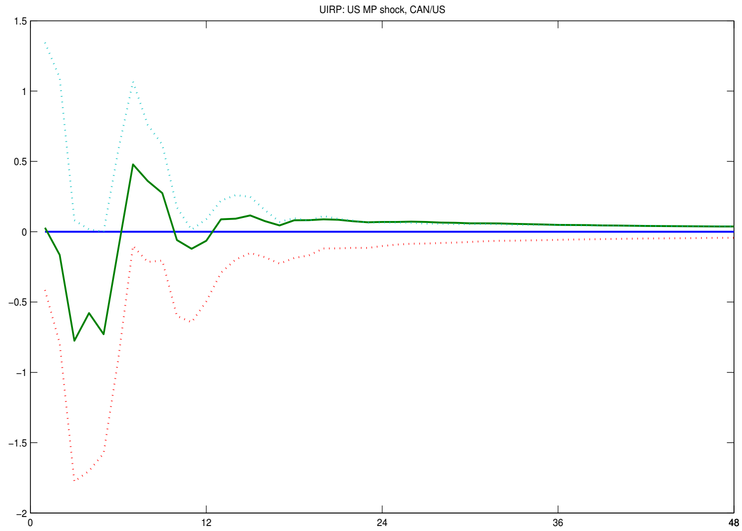

Figure 1.5: UIRP CAN/US, conditional on US MP or a global shock with alternative ordering

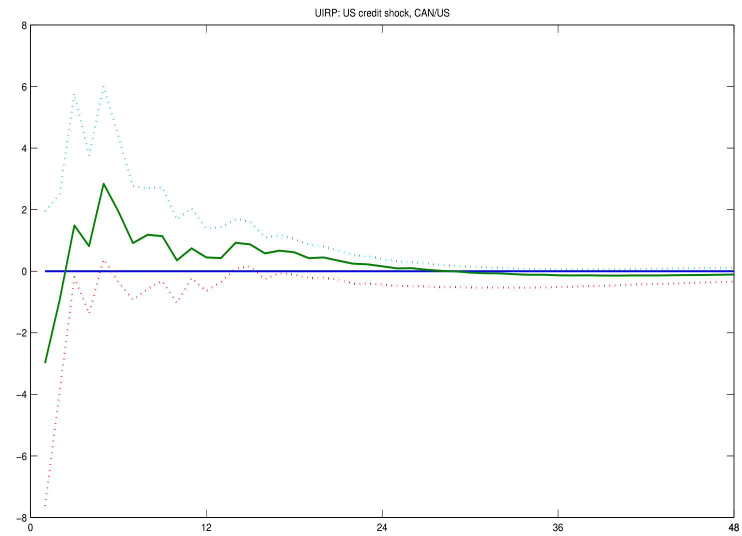

Figure 1.6: UIRP CAN/US, conditional on US (global) credit shock

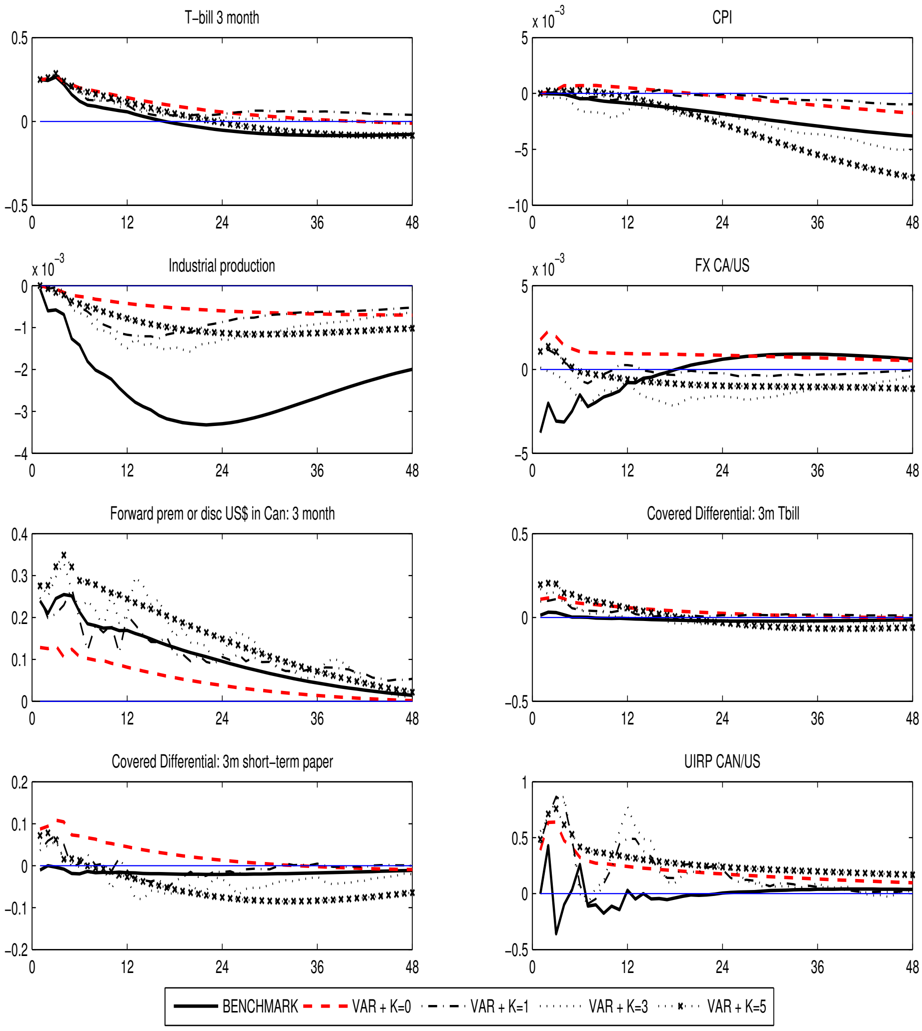

Figure 1.7: FAVAR-VAR comparison. Here, VAR consists of [US Tbill, CPI, IP, CA Tbill, FX CA/US]

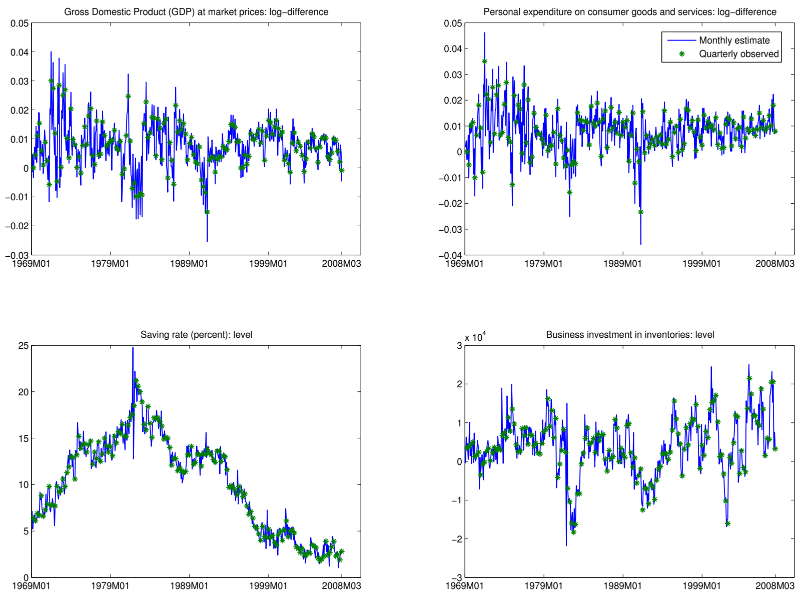

Figure 1.8: Monthly estimates vs quarterly observed series

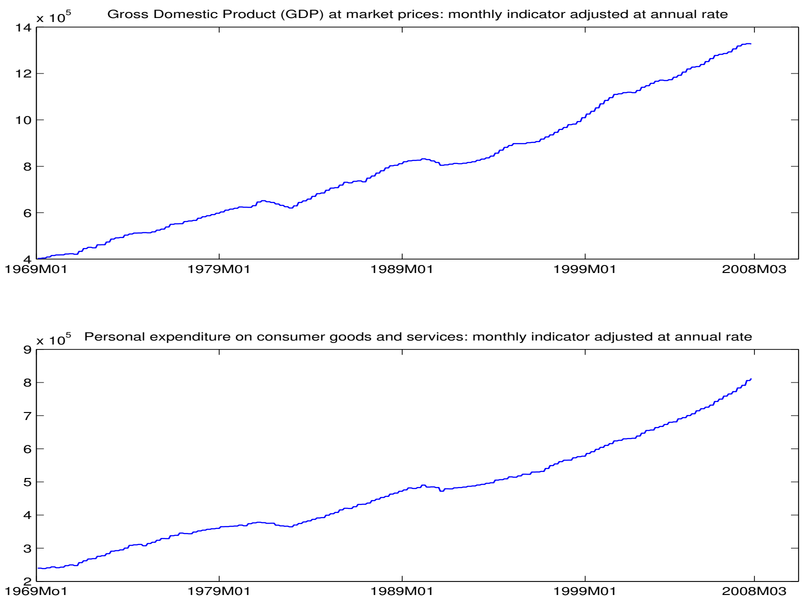

Figure 1.9: Monthly estimates in annualized level

CHAPTER 2

DYNAMIC EFFECT OF CREDIT SHOCKS IN A DATA-RICH ENVIRONMENT

2.1 Introduction

The recent financial crisis caused the most important global economic downturn since the Great Depression. It renewed interest in properly understanding the connection between the real economy and the financial sector. This is important for various reasons. First, by their forward-looking nature, asset prices and credit spreads –i.e., the difference between corporate bond yields and yields on same-maturity Treasury securities –should be useful in predicting fluctuations of economic activity, at least in theory (see Philippon, 2008). Studies, among others, by Stock and Watson (1989), Gertler and Lown (1999), and more recently by Mueller (2007), have found that credit spreads do have significant forecasting power in predicting economic growth. Instead of relying on the usual credit spreads measures, a recent paper by Gilchrist, Yankov and Zakrajšek (2009), henceforth GYZ, re-examines this evidence using a set of new measures of credit market tightness.

Second, understanding the joint dynamics of the real economy and financial sector could lead to more timely – and hopefully more pre-emptive – policy response. The strong tightening in US credit conditions in 2007 and 2008 and the subsequent contraction in economic activity suggests that credit conditions may have significant effects on the economy. This calls for a comprehensive understanding of the quantitative effects of credit shocks on US economic variables and requires an empirical framework that is sufficiently rich to capture the information necessary to account for this joint dynamics.

In this paper, we re-examine the evidence about the propagation mechanism of credit shocks on economic activity and other key macroeconomic variables. We characterize the dynamic effects of credit shocks using a structural factor model estimated with large panels of U.S. monthly and quarterly data. With contrast to structural VAR model, the factor model permits to consider large amount of information potentially observed by agents, is not subject to non fundamentalness and does not pertain to the choice of a specific data series to represent a general economic concept, which may be arbitrary.

Our empirical model is estimated in two steps. First, in order to recover the space spanned by structural shocks (including shocks to credit spreads), we estimate factors as principal components from standardized data panels. Such factors are supposed to capture all aggregate fluctuations in economic and financial series. All economic and financial indicators can respond contemporaneously to movements in those factors, as well as to a series-specific (idiosyncratic) component. Then, a finite-order VAR approximation of the factors dynamics is estimated. The identification of structural shocks is achieved by imposing restrictions on the impact matrix of the structural shocks on a few observable variables, as proposed by Stock and Watson (2005). This allows us to impose as few restrictions as needed in order to identify shocks to credit conditions.

The closest analysis to ours is GYZ. However, in order to determine credit shocks, GYZ impose potentially strong identifying assumptions. In particular, they assume that no macroeconomic variable, including measures of economic activity, prices or interest rates can respond contemporaneously to credit shocks. This assumption may be restrictive, e.g., if changes in credit spreads affect contemporaneously overall financial conditions, including interest rates. It may potentially attribute an overly strong effect of credit spreads on economic variables by preventing a possible contemporaneous drop in the level of interest rate on riskless securities, which might mitigate the effect of a credit tightening. In addition, GYZ assume that the factors summarizing macroeconomic indicators are contemporaneously uncorrelated with the factors summarizing all credit spreads, regardless of the source of disturbances. To the extent that such assumptions are violated, their results might be contaminated. In our identification schemes, these assumptions are relaxed.

Our results show that an unexpected increase in credit spreads causes a significant contemporaneous drop in yields of Treasury securities at various maturities and has a significant effect in the contemporaneous month on other variables such as consumer expectations, commodity prices, capacity utilization, hours worked, housing starts, etc. This unexpected increase in the external finance premium generates also a significant and persistent economic slowdown, in months following the shock, as in GYZ and Mueller (2007). The responses generated by our identifying procedure yield a very realistic picture of the effect of credit shocks on the economy, and provide valuable information about the transmission mechanism of these shocks. In addition, we find that the extracted factors capture an important dimension of the business cycle movements. Furthermore, we find that credit shocks have quantitatively important effects on several indicators of real activity and prices, leading indicators, and credit spreads, as they explain a substantial fraction of the variability of these series. Finally, a further advantage of our identification procedure is that it yields a rotation matrix that can be used to recover structural factors. The latter structural factors maintain the same informational content as the initially extracted factors but they have an interesting economic interpretation.

Our empirical analysis considers a battery of specifications. The findings just described are robust to different data frequencies and identification schemes. The first FAVAR model we consider is estimated using a monthly balanced panel. We impose a recursive assumption to identify structural shocks. The responses of key macroeconomic series to credit shocks are found to be qualitatively similar to those from a small-scale VAR model. However credit shocks are found to generate substantially larger economic fluctuations in the FAVAR model than in the small-scale VAR. This suggests that the VAR may be misspecified and does not properly capture of the source or propagation of key structural shocks. In addition, the factor model gives a more complete and comprehensive picture of the effects of credit shocks since the impulse responses and the variance decomposition of all variables can be obtained. We mentioned above that our approach produces interpretable factors. Indeed, the first structural factor accounts for almost all variation in prices that is explained by the common component. The second factor is important for the unemployment rate, M1, capacity utilization, consumer expectations and credit spreads. The third rotated factor explains well financial indicators and exchange rates, while the fourth factor explains real activity measures, housing starts and new orders.

In a second specification, we consider a mixed-frequencies monthly panel and a recursive identification scheme where we explicitly distinguish between the monetary policy and credit shocks, though the Federal funds rate (the instrument of policy) is allowed to respond on impact to credit shocks. The results are similar to the previous specification. In this specification, interest rates fall significantly on impact, in response to credit shocks. Again, we obtain interpretable factors. The first factor is important for price series, and the second explains well the variability in unemployment rate, money base measures, credit spread and capacity utilization. The third factor seems to be important for consumption series, GDP and investment. The fourth factor contributes mainly in explaining the variations in commodity price index and salaries, and the fifth factor is related to industrial production, employment and new orders.

Lastly, we consider a quarterly balanced panel and identify the structural shocks using sign restrictions. Again, the dynamic effects of the credit shock are quite similar to those observed in previous specifications. The first factor is important for price series, the Federal funds rate and treasury bills yields, and the second explains mostly the real activity measures such as GDP, industrial production, employment, salaries and consumption, and housing starts, new orders and consumer credit. The third and fourth factors seem to be important for monetary measures and exchange rate. Finally, the fifth factor is related to unemployment rate (together with the third factor), capacity utilization rate, and average unemployment duration.

In the rest of the paper, we lay out some theory on the link between credit shocks and economic variables. Section 3 presents the structural factor model and discusses various estimation and identification issues. The main results are presented in Section 4. In Section 5, we compare the results to those obtained from structural VAR anal. Section 6 concludes. The Appendix contains the impulse response results after a monetary policy shock and the description of data sets.

2.2 Some Theory

In this section we briefly discuss various mechanisms that connect financial and economic variables, and channels through which shocks on the credit market could affect economic activity.

Financial frictions are crucial when linking the credit market conditions to economic activity. In their presence, the composition of the borrowers' net worth becomes important due to the incentive problems faced by the lenders (see Bernanke and Gertler (1995), and Bernanke, Gertler and Gilchrist (1999)): a borrower with a low net worth relative to the amount borrowed has a higher incentive to default. Given that agency problem, the lender demands a higher premium to provide external funds, which raises the external finance premium. Therefore, economic downturns and associated declines in asset values tend to produce an increase in the external finance premium for borrowers holding these assets in their portfolio. The higher external finance premium, in turn, leads to cuts in investments, and hence in production, employment, and thus in overall economic activity, which induces asset prices to fall further, and so on. This is essentially the so-called financial accelerator mechanism.

Several other transmission channels, focusing on the credit supply, have also been introduced in the literature. The narrow credit channel focuses on the health of the financial intermediaries and their agency problems in raising funds. The capital channel can transmit credit conditions to the economic activity, if banks' capital is affected. In that case, banks must reduce the supply of loans, resulting in a higher external finance premium. In summary, Bernanke and Gertler (1995) identify two channels through which a shock to the external finance premium can affect the real activity:

1. *Balance sheet channel*, according to which a deterioration of a firm's net worth result in an increase of its external finance premium, and thus causes a reduction in investment, employment, production, and prices. This can be broadly seen as affecting the demand of credit.

2. *Bank lending channel*, according to which a deterioration of the financial intermediaries' external finance premium constrains the supply of loans and hence causes a reduction in economic activity.

More recently, credit risks and their effect on economic conditions have been modeled in general equilibrium frameworks. For instance, Gilchrist, Ortiz and Zakrajšek (2009) augment the medium-size DSGE model of Smets and Wouters (2007) with the financial accelerator mechanism linking conditions on the credit market to the real economy through the external finance premium (i.e., Bernanke, Gertler and Gilchrist (1999)). They also introduce two financial shocks: a financial disturbance shock that affects directly the external finance premium, and a net worth shock affecting the balance sheet of a firm. The first shock is presented as a credit supply shock that Christiano, Motto and Rostagno (2009) interpret as an increase in the agency costs due to a higher variance of idiosyncratic shocks affecting the firms' profitability. The second shock can be viewed as a credit demand shock, whose effect depends on the degree of financial market frictions. After estimating the structural model using US data covering the 1973-2008 period, Gilchrist, Ortiz and Zakrajšek (2009) find that both financial shocks cause an increase in the external finance premium which, through the financial accelerator, implies a persistent slowdown in economic activity and in investment.

2.3 Econometric Framework in Data-Rich Environment

The usual way to attempt to identify structural shocks is through VAR analysis. However, the small-scale VAR model presents several issues. Due to the small amount of information in the model, relative to the information set potentially observed by agents, the VAR can easily suffer from an omitted variable problem that can affect the estimated impulse responses or the variance decomposition. Related to that, Forni et al. (2009) argues that while non-fundamentalness is generic of small scale models, they cannot arise in a large dimensional dynamic factor models [1]. In addition, potential problem pertains to the choice of a specific data series to represent a general economic concept, which may be arbitrary. Moreover, measurement errors, aggregation and revisions present additional problems when linking theoretical concepts to specific data series. Finally, even if the previous problems do not occur, we can produce impulse responses only for the variables included in the VAR.

One way to address all these issues is to take advantage of information contained in

1. If the shocks in the VAR model are fundamental, then the dynamic effects implied by the moving average representation can have a meaningful interpretation, i.e. the structural shocks can be recovered from current and past values of observable series.

large panel data sets using dynamic factor analysis, and in particular a factor-augmented VAR (FAVAR) model. The importance of large data sets and factor analysis is well documented in both forecasting macroeconomic aggregates and structural analysis. Bernanke, Boivin and Eliasz (2005) and Boivin, Giannoni and Stevanović (2009), have shown that incorporating information through a small number of factors corrects for various empirical puzzles when estimating the effects of monetary policy shocks.

The model that we consider takes the form [2]

$$X_t = \Lambda F_t + u_t \tag{2.1}$$

$$F_t = \Phi(L) F_{t-1} + e_t \tag{2.2}$$

where X_t contains N economic and financial indicators, F_t represents K unobserved factors ($N >> K$), Λ is $N \times K$ matrix of factor loadings, u_t are idiosyncratic components of X_t uncorrelated with F_t and the factor innovations e_t. This model is an approximate factor model, as we allow for some limited cross-section correlation among the idiosyncratic components in (2.1). [3]

2.3.1 Estimation

The unknown coefficients in (2.1)–(2.2) could in principle be estimated by Gaussian maximum likelihood using the Kalman filter (or by Quasi ML), as shown in Engle and Watson (1981), Stock and Watson (1989), Sargent (1989). This method is however computationally burdensome and likely leads to misspecification when N is very large. [4]

2. It is worth noting that the static factor model considered here is not very restrictive since an underlying dynamic factor model can be written in static form (see Stock and Watson, 2005).

3. We assume that only a small number of largest eigenvalues of the covariance matrix of common components may diverge when the number of series tends to infinity, while the remaining eigenvalues as well as the eigenvalues of the covariance matrix of specific components are bounded. See Bai and Ng (2008) for an overview of the modern factor analysis literature, and the distinction between exact and approximate factor models.

4. Recently, significant improvements have nonetheless been proposed to this approach. For instance the Kalman filter speedup by Jungbacker and Koopman (2008), using principal components for starting values and then a single pass of the Kalman filter by Giannone, Reichlin, and Sala (2004), and principal components for starting values then use EM algorithm to convergence by Doz, Giannone, and Reichlin (2006).

We adopt instead an alternative estimation approach based on a two-step principal components procedure, where factors are approximated in the first step, and the dynamic process of factors is estimated in the second step. We rely on the result that factors can be approximated by a Principal Components Analysis (PCA) estimator. Stock and Watson (2002a) prove consistency of such an estimator in an approximate factor model when both cross-section and time sizes, N, and T, go to infinity, and without restrictions on N/T. Moreover, they justify using $\hat{F}_t$ as regressor without adjustment. Bai and Ng (2006) improve these results by showing that PCA estimators are $\sqrt{T}$ consistent and asymptotically normal if $\sqrt{T}/N \to 0$. Inference should take into account the effect of generated regressors, except when T/N goes to zero.

The principal components approach is easy to implement and does not require very strong distributional assumptions. Recently, simulation exercises showed that likelihood-based and two-step procedures perform quite similarly in approximating the space spanned by latent factors (see, Doz, Giannone and Reichlin, 2006). Moreover, Bernanke, Boivin and Eliasz (2005) estimated their model using both two-step principal components and single-step Bayesian likelihood methods, and obtained essentially the same results. For these reasons, we consider the PCA approach. However, since the unobserved factors are estimated and then included as regressors in FAVAR model, and given that the number of series in our application is small, relative to the number of time periods, the two-step approach suffers from the "generated regressors" problem. To get the accurate statistical inference on the impulse response functions that accounts for uncertainty associated to factors estimation, we use the bootstrap procedure proposed by Kilian (1998).

2.3.2 Identification of structural shocks

To identify the structural shocks, we employ the contemporaneous timing restrictions procedure proposed in Stock and Watson (2005). After inverting the VAR process of factors in (2.2), assuming stationarity, and plugging it in (2.1), we obtain the moving-average representation of X_t:

$$X_t = B(L)e_t + u_t, \tag{2.3}$$

where $B(L) \equiv \Lambda[I - \Phi(L)L]^{-1}$. We assume that the number of static factors, K, is equal to the number of dynamic factors and that the factor innovations e_t are linear combinations of structural shocks ε_t

$$\varepsilon_t = He_t, \tag{2.4}$$

where H is a nonsingular square matrix and $\mathrm{E}[\varepsilon_t \varepsilon_t'] = I$. Using (2.4) to replace e_t in (2.3) gives the structural moving-average representation of X_t:

$$X_t = B^\star(L)\varepsilon_t + u_t, \tag{2.5}$$

where $B^\star(L) \equiv B(L)H^{-1} = \Lambda[I - \Phi(L)L]^{-1}H^{-1}$. To identify the structural shocks ε_t, we impose contemporaneous timing restrictions on the impact matrix in (2.5). Specifically, we assume that certain structural shocks do not affect the first few indicators in X_t within the period, so that the impact matrix takes the form

$$B_0^\star \equiv B^\star(0) = \begin{bmatrix} x & 0 & \cdots & 0 \\ x & x & \ddots & 0 \\ x & x & \ddots & 0 \\ x & x & \cdots & x \\ \vdots & \vdots & \vdots & \vdots \\ x & x & \cdots & x \end{bmatrix}$$

where x stands for unrestricted elements in the above matrix. It is important to note that our identifying assumptions are imposed on the effects of structural shocks on particular indicators in our data set. They do not require latent factors not to respond contemporaneously to structural shocks.

To estimate the matrix H, we proceed as in Stock and Watson (2005), noting that $B_{0:K}^\star \varepsilon_t = B_{0:K} e_t$ implies $B_{0:K}^* B_{0:K}^{*\prime} = B_{0:K} \Sigma_e B_{0:K}'$, where $B_{0:K}$ contains the first K rows of $B_0 \equiv B(0) = \Lambda$, $B_{0:K}^\star = B_{0:K} H^{-1}$, and Σ_e is the covariance matrix of e_t. Since $B_{0:K}^\star$ is a $K \times K$ lower triangular matrix, then we must have $B_{0:K}^* = \mathrm{Chol}(B_{0:K} \Sigma_e B_{0:K}')$. It follows that $H = \left(B^{*}_{0:K}\right)^{-1} B_{0:K}$, or

$$H = [\mathrm{Chol}(B_{0:K}\Sigma_e B'_{0:K})]^{-1} B_{0:K}. \tag{2.6}$$

To estimate H, we just replace $B_{0:K}$ and Σ_e with their estimates in (2.6).

The impulse responses to structural shocks in ε_t are obtained using (2.5). This identification procedure is similar to the standard recursive identification in VAR models, except that the series-specific term u_t is absent in VARs. By imposing $K(K-1)/2$ restrictions, we just-identify the K structural shocks.

Importantly, the dynamics of the factors is left unconstrained, and identified structural shocks are allowed to have contemporaneous effects on the factors driving our panel of indicators. The identifying restrictions are only imposed on the contemporaneous response of a few of the economic indicators' response within the period. This contrasts with GYZ who assume that credit shocks do not have a contemporaneous effect on any of the economic factors and indicators, including interest rates. Furthermore, contrary to other identification strategies that have been adopted in FAVAR analysis, we do not need to impose any observed factor nor do we rely on the interpretation of a particular latent factor to characterize the responses of economic indicators to structural shocks. [5]

2.3.3 Data and specifications

In our application, we use three different specifications of the FAVAR involving different identifying restrictions and also an increasingly large number of economic and financial indicators. The time span for all panels starts in 1959M01 and ends in 2009M06. All series are initially transformed to induce stationarity. The description of series and their transformation is presented in the Appendix.

Common proxies of the external finance premium of borrowing firms are the credit

5. In Bernanke, Boivin and Eliasz (2005) and Boivin, Giannoni and Stevanović (2009), the authors impose a short-term interest rate as an observed factor, and the monetary policy shock is identified VAR equation pertaining to the interest rate. In contrast, GYZ estimate two sets of factors: those explaining a panel of economic activity indicators and those related to credit spreads, interpreted as "financial factors". The credit shock is identified as a shock on the structural error of the first "financial factor".

spreads for non-financial institutions. Our benchmark measure will be the 10-year B-spread (i.e., the difference between BAA bond yields and Treasury bond yields), although we considered as alternatives the 10-year A-spread and the 1-year B-spread. Table 2.I and Figure 2.1 summarize these measures.

Series description		Time span
FYAAAC	BOND YIELD: MOODY'S AAA CORPORATE	1959M01-2008M12
FYBAAC	BOND YIELD: MOODY'S BAA CORPORATE	1959M01-2008M12
FYGT1	INTEREST RATE: U.S.TREAS CONST MATURITIES,1-Y.	1959M01-2008M12
FYGT10	INTEREST RATE: U.S.TREAS CONST MATURITIES,10-Y.	1959M01-2008M12
FYFF	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE)	1959M01-2008M12
Credit spreads		
10Y B-spread	FYBAAC-FYGT10	1959M01-2008M12
10Y A-spread	FYBAAA-FYGT10	1959M01-2008M12
1Y B-spread	FYBAAC-FYGT1	1959M01-2008M12

Table 2.I: Proxies for the external finance premium

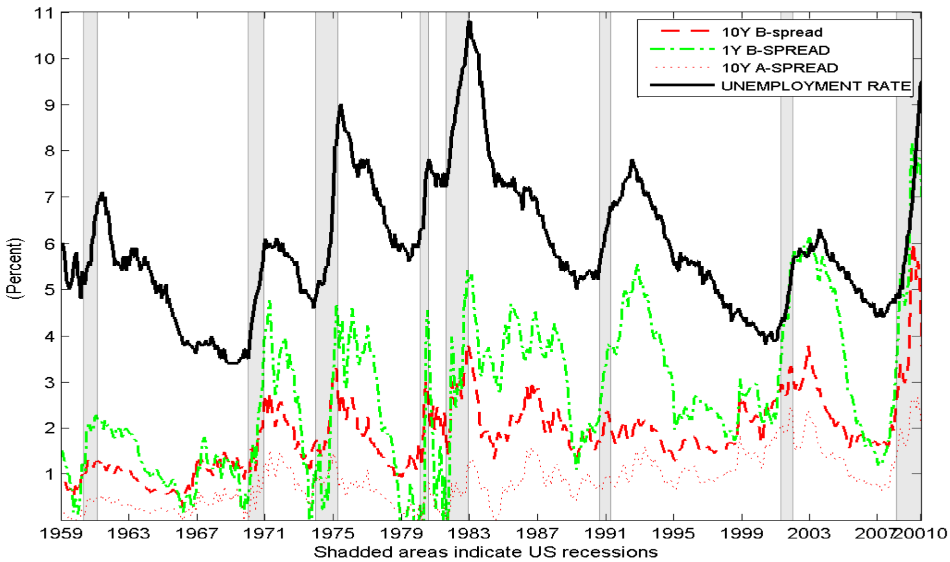

Figure 2.1: Measures of the external finance premium

In our first specification, we consider a monthly balanced panel containing 124 monthly U.S. economic and financial series. This is an updated version of the data set in Bernanke, Boivin and Eliasz (2005). We impose a recursive structure on the following first four economic indicators: [CPI, UR, FFR, B-spread]. This assumption implies that the consumer price index (CPI), the unemployment rate (UR) and the Federal Funds rate (FFR) are the only indicators that do not respond immediately to a surprise increase of the B-spread (we use 10-year B-spread), which is interpreted as the credit shock. This identification scheme is related to the identification strategy in GYZ in sense that the shock is seen as an unexpected increase of the external finance premium. However, it is important to remark that all indicators other than the CPI, UR and FFR may respond contemporaneously to the credit shock. In particular, we do not impose all measures of economic activity, prices and interest rates to respond only with lag to the credit shock. Furthermore, the shock in our approach is a disturbance to the last element of the vector ε_t, and not to the B-spread directly. The impulse response of the B-spread is determined by its factor loading and the corresponding element in the rotation matrix H.

The second specification augments the monthly panel above with 58 important quarterly U.S. macroeconomic series, to yield a mixed-frequencies monthly panel of 182 indicators, over the same period. [6]. The goal is to use the informational content from quarterly indicators to better approximate the space spanned by structural shocks, and to achieve a more reliable identification of these shocks. The recursive structure also differs from the previous specification. We assume a recursive structure in the following indicators [PCE, UR, C, I, FFR], where the credit shock and the monetary policy shock are ordered fourth and fifth in ε_t. This particular identification scheme implies that the Personal Consumption Expenditure Price Index (PCE), the unemployment rate (UR) and real Consumption (C) do not respond immediately to both credit and monetary policy shocks. To identify the credit shock, we impose that Investment (I) can respond immediately to the credit shock, while it does not react to the monetary policy contemporaneously. Finally, we let the Federal Funds Rate (FFR) respond immediately to the credit shock. Note that a measure of the external finance premium is not required to

6. The mixed-frequencies panel is obtained using an EM algorithm as in Stock and Watson (2002b), and Boivin, Giannoni and Stevanović, (2009)

enter in this recursive structure. The impact responses of credit spreads are determined only by their factor loadings and the rotation matrix H.

Finally, we consider a balanced quarterly panel containing 220 quarterly U.S. series for the same period, [7] and identify the credit shock using a sign restrictions strategy. To obtain the initial orthogonalized innovations we start from the recursive structure on the indicators [PCE, GDP, C, I, FFR]

$$X_t \simeq B^\star(L)\varepsilon_t.$$

Then, we generate an orthogonal matrix Q, using a QR decomposition, such that

$$X_t \simeq \tilde{B}^\star(L)\tilde{\varepsilon}_t,$$

where $\tilde{B}^\star(L) \equiv B^\star(L)Q$ and $\tilde{\varepsilon}_t \equiv Q'\varepsilon_t$. The sign restrictions are imposed on the impact matrix $\tilde{B}^\star(0)$:

$$\frac{\partial(PCE_t)}{\partial(\varepsilon_t^{CS})} \leq 0, \quad \frac{\partial(GDP_t)}{\partial(\varepsilon_t^{CS})} \leq 0, \quad \frac{\partial(C_t)}{\partial(\varepsilon_t^{CS})} \leq 0, \quad \frac{\partial(I_t)}{\partial(\varepsilon_t^{CS})} < \frac{\partial(C_t)}{\partial(\varepsilon_t^{CS})}.$$

Hence, we impose that the impact response of PCE inflation, and of the growth rates of real GDP, consumption and investment to a positive credit shock are non positive. The last restriction imposes that investment (nonresidential) responds more negatively than consumption.

2.4 Results

In this section, we present the main empirical results from our three main FAVAR specifications. We could in principle plot the impulse responses of all variables contained in the informational panel X_t but we will focus on a subset of economic and financial indicators of interest. In all cases, the impulse to the component of ε_t corresponding to the credit shock is of size 1. The lag order in VAR dynamics in (2.3) is set to 3. Finally,

7. This last data set is an updated version of the data set used by Boivin and Giannoni (2006).

the 90% confidence intervals are computed using 5000 bootstrap replications.

2.4.1 FAVAR 1 and monthly balanced panel

We estimate the first specification of the FAVAR using the monthly balanced panel and impose the recursive identification scheme is [CPI, UR, FFR, B-spread], implying that we extract four static factors from X_t. Figure 2.2 plots the impulse responses to the credit shock. On impact, the B-spread rises by 19.2 basis points relative to initial value, i.e., its standard deviation. This unexpected increase in the external finance premium generates a significant and very persistent economic downturn through the transmission channels discussed above. For example, economic activity indicators such as industrial production (IP) falls a bit on impact and then by as much as 2% within the first 12 months, employment falls by around 0.7% over the first year, but remains significantly below the initial level for 4 years, and hours worked and capacity utilization also fall. Real personal consumption also falls significantly and persistently along with consumer credit. Aggregate price indices and wages also decline significantly. The price indicators, such CPI, core CPI, and PPI, show a very persistent decline. The labor market indicators, unemployment rate and average unemployment duration, rise significantly for more than 3 years, while employment and wages decline. The leading indicators, such consumer expectations, new orders, housing starts, and commodity prices, react negatively on impact. Finally, the interest rates decline, except for the Federal funds rate which is constrained to remain unchanged on impact, and the monetary aggregates increase progressively.

The impulse responses in Figure 2.2 are broadly similar to the results reported in GYZ, except that in our approach several key economic and financial indicators do respond immediately to an unexpected increase in the external finance premium. Nonetheless, the fact that the overall picture at longer horizons remains similar to that of GYZ suggests that their restrictive identifying assumptions do not distort their findings about medium and long-run responses to credit shocks.

Table 2.II shows the importance of credit shocks in explaining economic fluctuations during our 1959-2009 sample. The first column reports the contribution of the credit

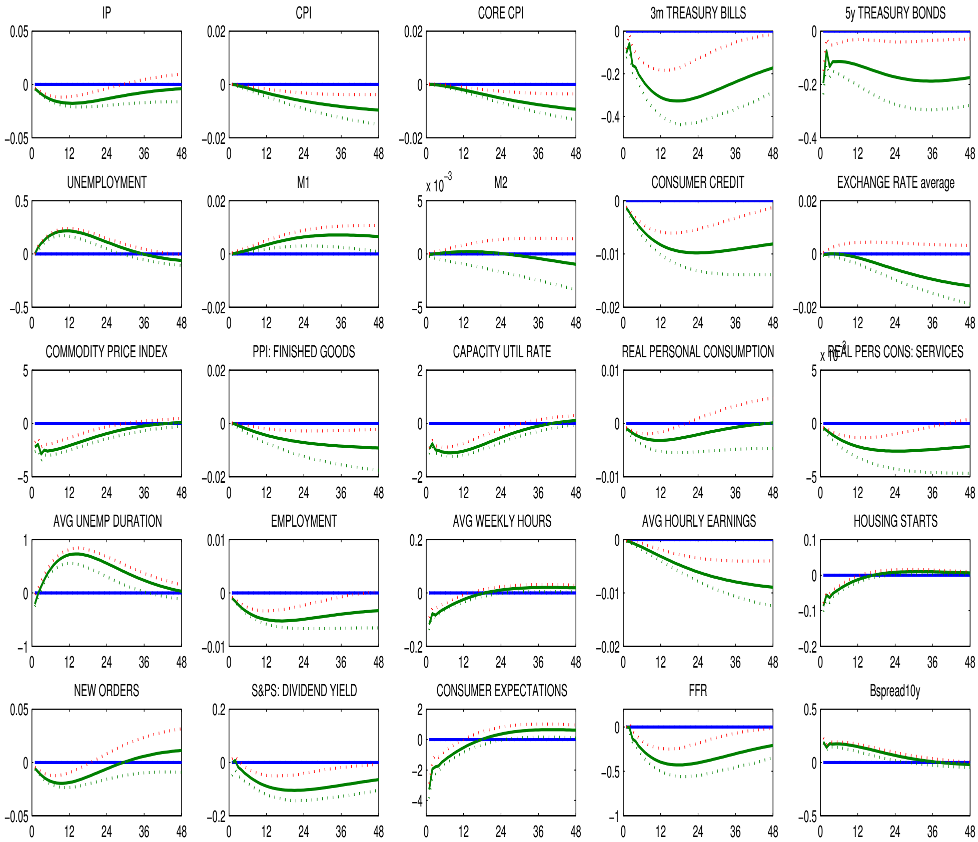

Figure 2.2: Dynamic responses of monthly variables to credit shock

shock to the variance of the forecast error at 48-month horizon, and the second column contains the R^2 of the common component. Surprisingly, the credit shock has important effects on several variables: it explains more than 50% of the forecast error variance of industrial production, consumer credit, capacity utilization rate, labor market series, some leading indicators and credit spreads. Looking at the R^2 statistics, we see that the common component explains a sizeable fraction of the variability in these variables, especially for industrial production, prices, financial indicators, average unemployment duration, capacity utilization and consumer expectations. This means that factors do capture important dimensions of the business cycle movements.

Variables	Variance decomposition	R^2
Industrial production	0.5289	0.7140
CPI: total	0.0591	0.7966
CPI: core	0.1223	0.6123
T-Bill: 3-month	0.1509	0.8839
T-Bond: 5-year	0.1144	0.9132
Unemployment rate	0.2615	0.7089
M1	0.1418	0.0919
M2	0.0308	0.1149
Consumer credit	0.6492	0.1778
Exchange rate: average	0.0326	0.0530
Commodity price index	0.3135	0.5214
PPI: finished goods	0.0424	0.5949
Capacity utilization rate	0.7469	0.7476
Real Pers. Cons.	0.2360	0.1401
Real Pers. Cons.: services	0.2343	0.1283
Avg. unemployment duration	0.4248	0.7597
Employment	0.5946	0.2879
Avg weekly hours	0.4948	0.3819
Avg hourly earnings	0.3949	0.2164
Housing starts	0.6002	0.4676
New orders	0.4452	0.2473
S&P's CCS: dividend yield	0.1605	0.7529
Consumer expectations	0.3188	0.5338
FFR	0.1347	0.8957
B-spread: 10y	0.7727	0.6574

Table 2.II: Variance decomposition and R^2 in FAVAR-1

2.4.1.1 Interpretation of factors

Another interesting feature of our identification approach is that it allows us to obtain the rotation matrix H that can be used to interpret estimated factors. Recall from Section 2.3.2, that structural shocks are linear combination of residuals, $\varepsilon_t = He_t$. Using this hypothesis, we can rewrite the system (2.1)-(2.2) in its structural form

$$X_t = \Lambda^\star F_t^\star + u_t \tag{2.7}$$

$$F_t^\star = \Phi^\star(L) F_{t-1}^\star + \varepsilon_t \tag{2.8}$$

where $F_t^\star = HF_t$, $\Lambda^\star = \Lambda H^{-1}$, and $\Phi^\star(L) = H\Phi(L)H^{-1}$. Hence, given the estimates of F_t and H, we can obtain the estimate of structural factors: $\hat{F}_t^\star = \hat{H}\hat{F}_t$.

In appendix, table II.II presents correlation coefficients between the estimated factors F_t, $F_t^\star$, and the variables used in the recursive identification scheme. The factors are plotted in Figure II.1. Table II.IV reports the marginal contribution of each factor to the total R^2. From columns associated to the elements of F_t in Tables II.II and II.IV, we see that any interpretation in terms of the economic indicators is arbitrary. This is not surprising since the factors are identified up to a rotation, picked by the PCA estimator. The picture changes when we look at columns associated to the elements of the rotated estimated factors. The results, in last four columns of Table II.II, show that $F_{1,t}^\star$ is highly correlated with the CPI growth rate, $F_{2,t}^\star$ with the unemployment rate, $F_{3,t}^\star$ with the Federal funds rate and $F_{4,t}^\star$ with the credit spread. Figure II.1 reveals that the rotation by $\hat{H}$ makes the estimated factors very close to observed indicators used in the recursive identification scheme. However, to have a more reliable idea about the informational content of each rotated factor, we compare its marginal contributions to the total R^2. According to results in Table II.IV, the first rotated factor accounts for almost all variations in prices, that is explained by the common component. The second factor is important for unemployment rate, M1, capacity utilization rate, consumer expectations, and credit spread. The third rotated factor explains well financial indicators and exchange rate, while the fourth factor is related to real activity measures, housing starts, and new orders.

2.4.2 FAVAR 2 and mixed-frequencies panel

While the previous specification used information from a large data set to characterize the effects of credit shocks, our identification assumed in particular that the Federal funds rate would not respond contemporaneously to credit shocks. To assess the robustness of our results, we now present results from our second specification, where we use the mixed-frequencies monthly panel and impose a recursive identification based on the following ordering [PCE, UR, C, I, FFR], where the credit shock and the monetary policy shock are ordered fourth and fifth in ε_t.

The impulse responses are presented in Figure 2.3. The impact response of B-spread is around 0.2. As in the previous specification, this unexpected increase of the external finance premium generates a significative and persistent economic slowdown. The price indexes decline largely and significantly. Industrial production and consumption present a significant downturn for about 18 months after the shock. On the labor market, there are significant positive reactions of unemployment rate and average unemployment duration (and the response of the latter is more persistent), while employment and salaries indicators decline. The leading indicators of economic activity, housing starts, new orders, and consumer expectations, react negatively and significantly on impact. Since the impact response of the Federal funds rate is not restricted in this specification, interest rates respond negatively and significantly on impact.

In Figure 2.4, we present impulse responses of some monthly indicators constructed from the quarterly observed variables. These are GDP components and two price indexes. We can see that GDP and PCE deflators decline in persistent and significative way, while the responses of other variables are quite imprecise. However, we remark that after a positive impact response, most of them decline progressively.

Table 2.III contains variance decomposition and R^2 results, as in Table 2.II. The conclusion is slightly different when compared to the previous specification. According to results in the first column of Table 2.III, the credit shock has a sizeable effect on prices, financial indicators including FFR, capacity utilization rate and consumer credit, but a smaller effect on real economic activity measures than it was the case with the monthly balanced panel. The results from the second column suggest that the common component explains approximately the same amount of variability in data as in the previous specification.

2.4.2.1 Interpretation of factors

As in the previous specification, we can check how the rotation matrix change the correlation structure between the estimated factors and the economic indicators used in the recursive identification scheme. The Tables II.VI and II.VIII, in appendix, contain correlation coefficients and marginal R^2, and Figure II.2 plots the principal components, rotated factors, and the corresponding series. Again, there is no obvious interpretation on correlation structure between principal components and five variables. When we rotate them by matrix $\hat{H}$, we can easily link the first factor to PCE index, the second to unem-

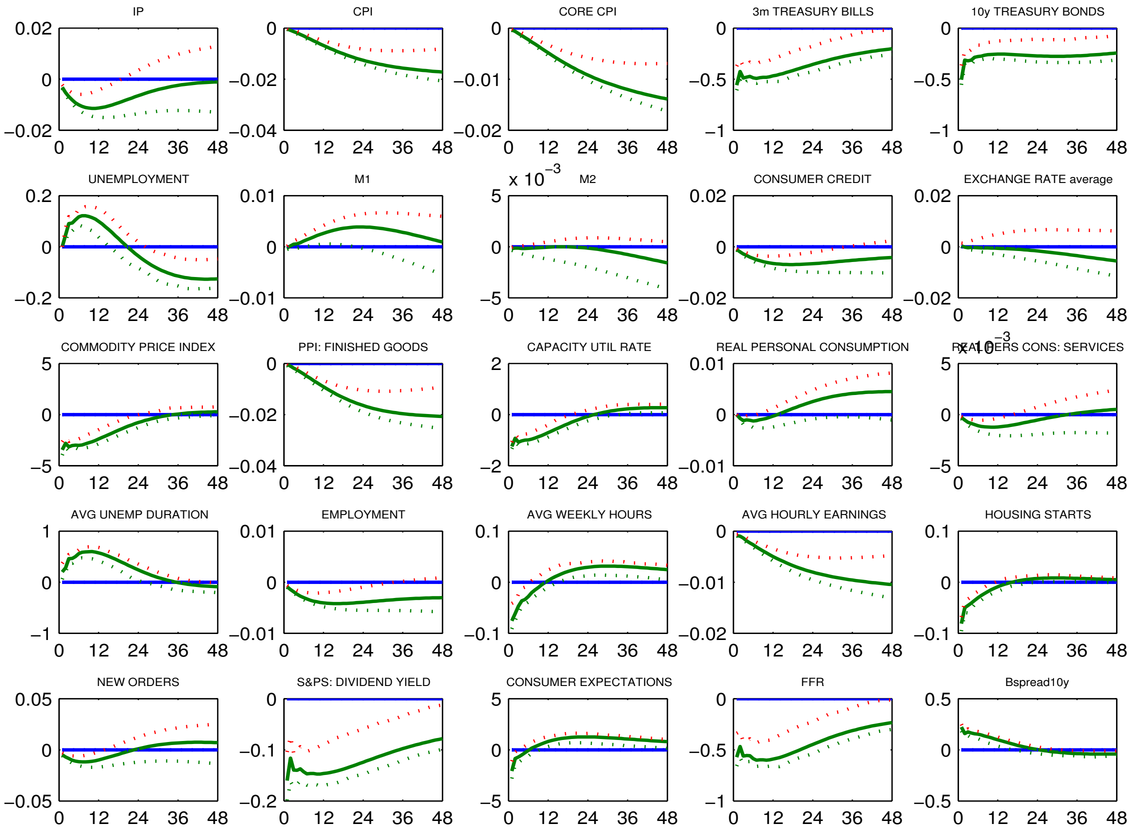

Figure 2.3: Dynamic responses of monthly variables to credit shock

ployment rate and the fifth to short rate. However, the interpretation of third and fourth factors is arbitrary. According to marginal R^2 results in Table II.VIII, the first factor is important for price series, the second for unemployment rate, money base measures, credit spread and capacity utilization. The third factor is related to consumption series, GDP and investment, while the fourth element of F_t^* contributes mainly in explaining variations in commodity price index and salaries. Finally, the fifth factor is important for industrial production, employment and new orders.

2.4.3 FAVAR 3 and quarterly balanced panel

In the final specification, we use a quarterly balanced panel and the sign restrictions framework to identify the credit shock. The results are obtained by simulating 10,000 orthogonal matrices. Among them, 924 have been retained, i.e. they respected the sign

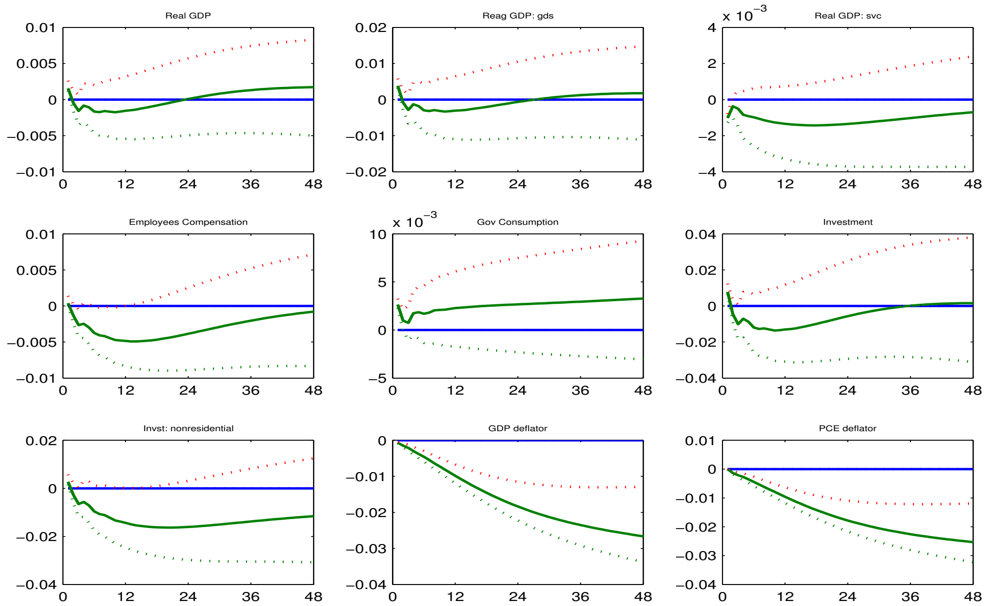

Figure 2.4: Dynamic responses of constructed monthly indicators to credit shock

restrictions. The impulse responses using the median orthogonal matrix are presented in Figure 2.5, and all retained impulse responses are plotted in Figure 2.6. According to results in Figure 2.5, the dynamic effects of the credit shock are similar to what we have observed in previous specifications. There is a sizeable economic downturn: production, employment, consumer credit, and prices decline, while unemployment rate and average unemployment duration rise. The interest rates, housing starts, new orders, and capacity utilization rate react negatively on impact, while credit spreads respond positively as expected. However, compared to previous monthly applications, the effects of credit shock seem to be less persistent. The results in Figure 2.6 show a huge dispersion in impulse responses satisfying sign restrictions. Therefore, the confidence intervals containing all these responses will also contain zero for most of variables and horizons.

In Table 2.IV we present variance decomposition and R^2 results. Contrary to two monthly applications, here the credit shock has a smaller effect on most of the variables. It explains between 20 and 30 percent of forecast error in NAPM production index, FFR, and some leading indicators, but has a small effect on prices and monetary measures. The R^2 results suggest that the extracted factors explain an important fraction of the variability in these series.

2.4.3.1 Interpretation of factors

For the seek of space, we just summarize the findings without reporting the figures and tables. As in previous specifications, there is no obvious interpretation on correlation structure between principal components and five variables. When we rotate them by matrix $\hat{H}$, the first factor becomes linked to PCE index, the second to GDP and the fifth to short rate. However, the interpretation of the third and fourth factors is arbitrary. According to R^2 results, the first factor is important for price series, FFR and treasury bills, and the second explains mostly the real activity measures such as GDP, industrial production, employment, salaries and consumption, and housing starts, new orders and consumer credit. The third and fourth factors seem to be related to monetary measures and exchange rate. Finally, the fifth factor is important for unemployment rate (together with the third factor), capacity utilization rate and average unemployment duration.

2.4.4 Further robustness analysis: Additional FAVAR specifications

In our robustness analysis, we tried two other FAVAR specifications and identification scheme that have been used in the literature: as in Boivin, Bernanke and Eliasz (2005) and in Boivin, Giannoni and Stevanović (2009). A particularity of these is that observable factors are imposed in transition equation along with latent factors:

$$X_t = \Lambda^F F_t + \Lambda^Y Y_t + u_t \tag{2.9}$$

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + e_t \tag{2.10}$$

where F_t contains K latent factors and Y_t has M observable series. Then, when using two-step estimation procedure, the issue is to separate the space spanned by observable and unobservable factors.

In Bernanke, Boivin and Eliasz (2005), the authors split the sample in two parts: series that do not respond immediately to a shock on Federal Funds Rate (FFR, the

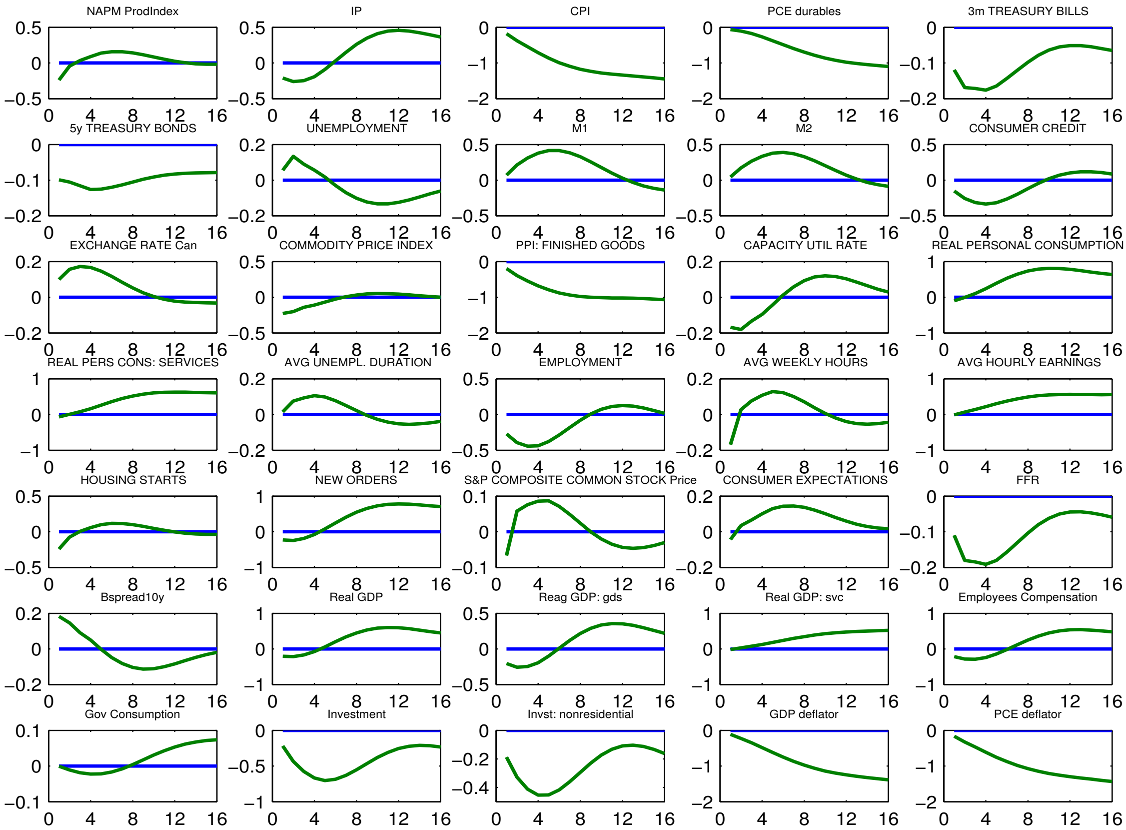

Figure 2.5: Median IRFs of quarterly selected variables to credit shock

observable factor), and the rest of data set that is not restricted. The $K+M$ factors extracted from the entire sample X_t are first regressed on Y_t, and on K factors extracted from the subset of X_t supposed not be contemporaneously linked to unobservable factors. The latter are then obtained as residual of dependent variables and Y_t.

In Boivin, Giannoni and Stevanović (2009), the authors estimate the latent factors through an iterative application of the principal components estimator. Starting from an initial estimate of F_t, F_t^0:

1. Regress X_t on F_t^0 and Y_t, to obtain $\hat{\lambda}_t^0$
2. Compute $\tilde{X}_t^0 = X_t - \hat{\lambda}_t^0 Y_t$
3. Estimate F_t^1 as the first K-1 principal components from $\tilde{X}_t^0$
4. Back to 1.

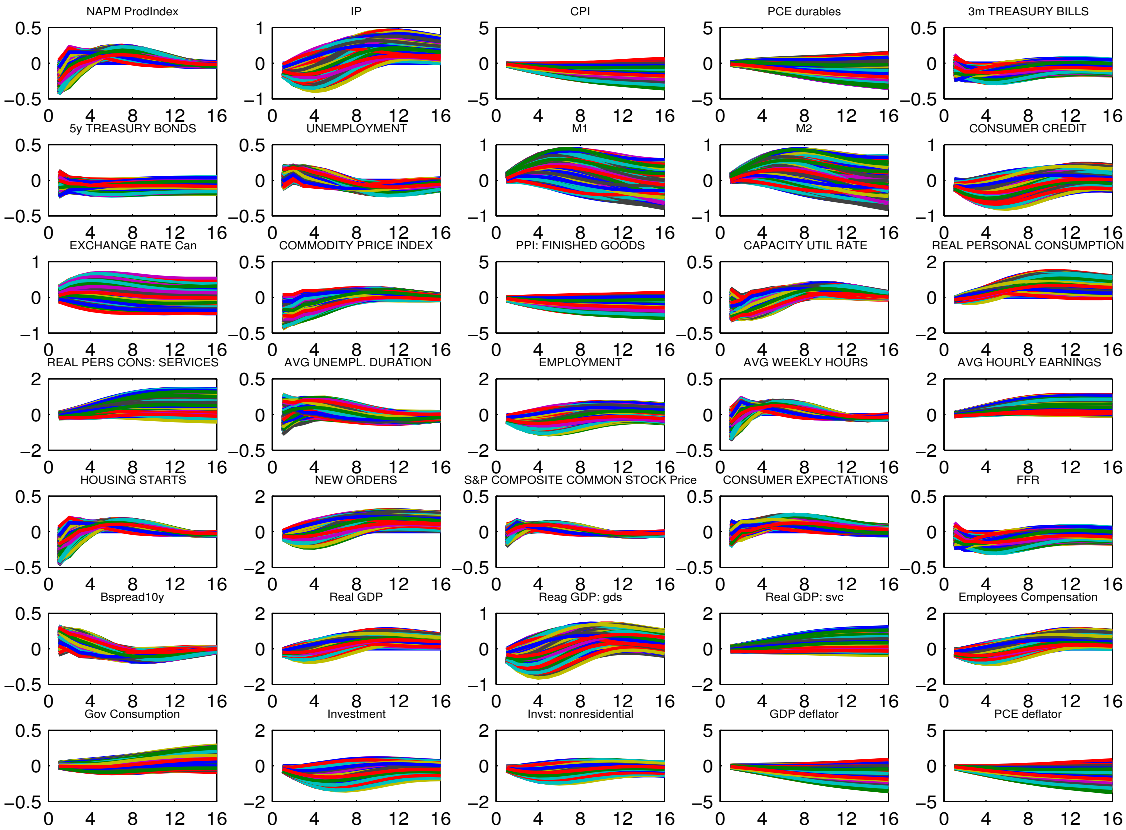

Figure 2.6: All IRFs satisfying sign restrictions

The main advantage of this procedure is that it does not rely on any temporal assumption between the observed factors and the informational panel. Hence it can be used for any set of observed factors without imposing any further assumptions. The identification of structural shocks is achieved by imposing a recursive structure on the VAR residuals in (2.10).

In our context, Y_t contains a proxy of the external finance premium and may contain other observable series. For each estimation procedure, we tried several specifications:

– Y_t contains only one of the credit spreads,
– Y_t contains a credit spread and the FFR (by splitting appropriately the data set in case of BBE estimation) and different orderings in Y_t,
– different numbers of latent factors in F_t.

Overall, the results are very similar to what we have presented here. There is a significative and persistent economic downturn, and depending on the identification procedure, the interest rates and leading indicators respond immediately to a credit shock. Together with results from Gilchrist, Yankov and Zakrajsek (2009), this provides a strong empirical evidence on the real effects of financial disturbances on economic activity.

2.5 Comparison with structural VAR model

In this section we compare our FAVAR results to impulse responses obtained from a standard structural VAR model. Our benchmark VAR model, similar to Mueller (2007), contains the series $[\pi_t, UR_t, R_t, 10yBS_t]$ where π_t is the inflation rate calculated as the first difference in the log of the consumer price index (CPI), UR_t is the unemployment rate, R_t is the Federal funds rate and $10yBS_t$ is the 10-year B-spread. We identify the credit shock by imposing a recursive identification. This implies that inflation, unemployment and the Federal funds rate cannot respond in the same month to an unexpected increase in the credit spread, while the latter is allowed to respond contemporaneously to all other variables included in the VAR.

Figure 2.7 shows the effects of an unexpected 100-basis points increase in the 10-year B-spread. It generates a significant and persistent economic downturn, a fall in the price level, and a persistent reduction in the Fed funds rate.

While the benchmark specification may be restrictive, we check the validity of our results by studying several alternative orderings. For the alternative models we use other credit spreads: $1yBS_t$ (1-year B-spread) and $10yAS_t$ (10-year A-spread). In appendix, Table II.IX lists all models, and Figure II.3 compares the responses implied by alternative models to the benchmark. We can conclude that the results are robust to different empirical measures of the external finance premium and to ordering between monetary policy and credit shocks.

While Figures 2.7–II.3 show the response of the economy to credit shocks, they do not allow us to determine how important credit shocks are in generating economic fluctuations. Table II.X in appendix reports the contribution of credit shocks to the total variance of key macroeconomic series, resulting from a variance decomposition. The

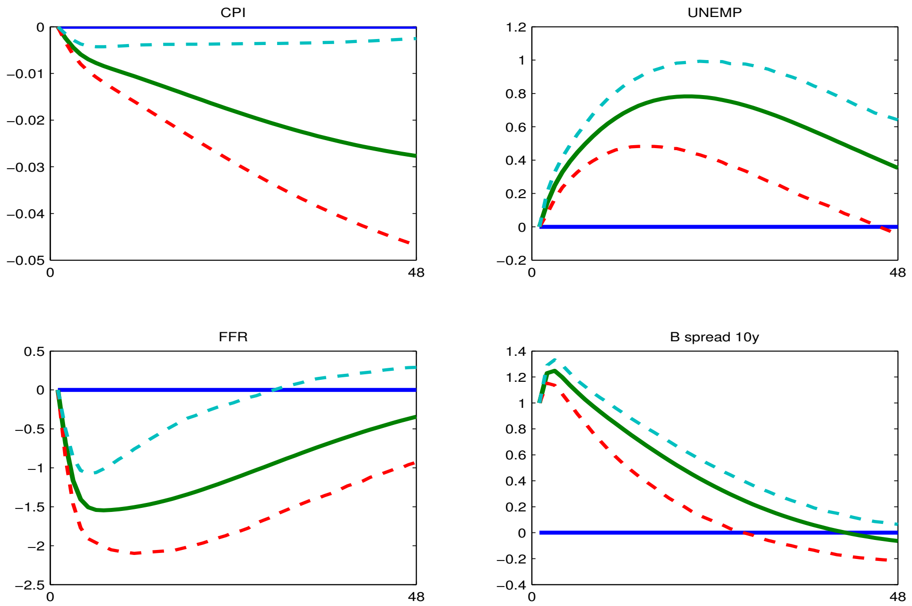

Figure 2.7: Benchmark model, 100 basic points shock to credit spread

credit shocks contribute only little to fluctuations in the CPI (less than 6% at most), a little more to unemployment (around 20% at most) and explain up to 16% of the forecasting error in the Federal funds rate, but are very important for credit spreads.

One interesting finding is that impulse responses to credit shocks from our FAVAR specifications are qualitatively broadly in line with those obtained based on a VAR, at least for the indicators included in the VAR. This suggests that after controlling for past inflation, unemployment and Federal funds rates, shocks to credit market can be well captured by the innovation in the credit spread $10yBS_t$. This contrasts with the findings of Bernanke, Boivin and Eliasz (2005) or Boivin, Giannoni, Stevanovic (2009) who obtain substantial differences between VAR and FAVAR responses of many variables to monetary policy shocks. However, even though the impulse responses to credit shocks appear similar in the VAR and in the FAVAR, obtaining a correct gauge of the quantitative effect of credit shocks in explaining aggregate fluctuations requires also that the transmission mechanism of all shocks, including monetary shocks, be well specified. Given that relevant information is likely omitted in small-scale VARs, at least to explain the response of monetary shocks (see, Bernanke, Boivin and Eliasz, 2005), calculations based on the variance decomposition of variables in the FAVAR are likely to be more reliable. These results suggest that credit shocks are indeed much more important in explaining economic fluctuations than was obtained based on a VAR.

2.6 Conclusion

In this paper, we re-examined the evidence on the propagation mechanism of credit shocks to economic activity. The analysis was done in data-rich environment using a structural factor model. The structural shocks were identified by imposing a minimal number of restrictions on the matrix specifying the impact response of several economic indicators to structural shocks.

The results show that an unexpected increase in the external finance premium generates a significant and persistent economic slowdown. Since we did not impose timing restrictions on forward-looking variables, these leading indicators respond, strongly and significantly on impact. This gives a more realistic picture of the effect of credit shocks on economy, and provides valuable information about the transmission mechanism of these shocks. According to R^2 results, the common components explain an important fraction of variability in observable variables. Hence, the factors capture a sizeable dimension of the business cycle movements.

From the variance decomposition analysis, we observe that credit shocks have important effects on several real activity measures, price indicators, leading indicators, and credit spreads. Moreover, a by-product of our identification approach is a rotation matrix that can be used to recover structural factors that have an interesting economic interpretation. Finally, the results obtained are largely robust to different data frequencies and identification schemes.

Variables	Variance decomposition	R^2
Industrial production	0.2929	0.7313
CPI: total	0.5139	0.6263
CPI: core	0.5656	0.6211
T-Bill: 3-month	0.6723	0.8640
T-Bond: 5-year	0.6611	0.8948
Unemployment rate	0.1915	0.6946
M1	0.1601	0.1090
M2	0.1899	0.0323
Consumer credit	0.4470	0.1893
Exchange rate: average	0.0941	0.0270
Commodity price index	0.7903	0.4731
PPI: finished goods	0.5114	0.3077
Capacity utilization rate	0.7220	0.7405
Real Pers. Cons.	0.0559	0.3819
Real Pers. Cons.: services	0.1930	0.1086
Avg. unemployment duration	0.3727	0.6242
Employment	0.3980	0.3037
Avg weekly hours	0.2261	0.3015
Avg hourly earnings	0.4290	0.3364
Housing starts	0.4582	0.4329
New orders	0.2519	0.2500
S&P's CCS: dividend yield	0.5861	0.6147
Consumer expectations	0.1652	0.5088
FFR	0.6016	0.8802
B-spread: 10y	0.7096	0.6416
Real GDP	0.0737	0.9338
Real GDP: goods	0.0890	0.8860
Real GDP: services	0.0518	0.8769
Employees compensation	0.0641	0.8812
Gov. consumption	0.1032	0.6009
Investment	0.0926	0.8599
Invst.: nonresidential	0.0714	0.9012
GDP deflator	0.1940	0.6547
PCE deflator	0.1302	0.7935

Table 2.III: Variance decomposition and R^2 in FAVAR-2

Variables	Variance decomposition	R^2
NAPM Production index	0.2175	0.7841
Industrial production	0.1611	0.5992
CPI: total	0.0136	0.9387
CPI: core	0.0149	0.8644
T-Bill: 3-month	0.2098	0.8817
T-Bond: 5-year	0.1504	0.8786
Unemployment rate	0.1093	0.6689
M1	0.0699	0.3082
M2	0.0746	0.2859
Consumer credit	0.1182	0.3148
Exchange rate: average	0.1609	0.2084
Commodity price index	0.0395	0.6728
PPI: finished goods	0.0163	0.8151
Capacity utilization rate	0.1402	0.8069
Real Pers. Cons.	0.1514	0.6304
Real Pers. Cons.: services	0.0841	0.5347
Avg. unemployment duration	0.1239	0.5748
Employment	0.1288	0.6847
Avg weekly hours	0.3115	0.4829
Avg hourly earnings	0.0682	0.2523
Housing starts	0.2278	0.5628
New orders	0.2526	0.7960
S&P's CCS: dividend yield	0.3802	0.1922
Consumer expectations	0.0752	0.6804
FFR	0.2270	0.9006
B-spread: 10y	0.1045	0.6476
Real GDP	0.1895	0.6872
Real GDP: goods	0.1782	0.4800
Real GDP: services	0.0514	0.2914
Employees compensation	0.1295	0.7626
Gov. consumption	0.1692	0.0108
Investment	0.0908	0.4821
Invst.: nonresidential	0.0968	0.3160
GDP deflator	0.0152	0.8620
PCE deflator	0.0072	0.9589

Table 2.IV: Variance decomposition and R^2 in FAVAR-3

CHAPTER 3

FACTOR-AUGMENTED VARMA MODELS: IDENTIFICATION, ESTIMATION, FORECASTING AND IMPULSE RESPONSES

3.1 Introduction

As information technology improves, the availability of economic and finance time series grows in terms of both time and cross-section size. However, a large amount of information can lead to the curse of dimensionality problem when standard time series tools are used. Since most of these series are highly correlated, at least within some categories, their co-variability pattern and informational content can be approximated by a smaller number of variables. A popular way to address this issue is to use factor analysis. This framework has received a lot of attention since late 90's and is known today as "the large dimensional approximate factor analysis". [1] It is an extension of the classical factor model that allows for limited cross-section and time correlations among the idiosyncratic components.

While factor models were introduced in finance and macroeconomics by Chamberlain and Rothschild (1983), Sargent and Sims (1977), and Geweke (1977), the literature on the large dimensional factor models started with Stock and Watson (2002a), and Forni et al. (2000). Further theoretical advances were established, among others, by Bai (2003), Bai and Ng (2002), and Forni et al. (2004, 2005). These models were used in forecasting macroeconomic aggregates (Banarjee, Masten and Massimilano (2006), Stock and Watson (2002b), Forni et al. (2005)), in the structural macroeconomic analysis (Bernanke, Boivin and Eliasz (2005), and Favero, Marcellino and Neglia (2005)), in nowcasting or economic monitoring (Aruoba, Diebold and Scotti (2008), and Giannone, Reichlin and Small (2008)), in the weak instrument literature (Bai and Ng (2008), and Kapetanious and Marcellino (2008)), and in the estimation of dynamic stochastic general equilibrium models (Boivin and Giannoni (2006)).

1. The "large dimensional"stands for both time and cross-section size asymptotic.

Vector autoregressive moving average (VARMA) class of models is another way to obtain a parsimonious representation of a vector stochastic process. The VARMA models are very useful in forecasting since they can resume the dynamic relations between the time series while keeping the number of parameters low. Lütkepohl (1987) provides a number of examples where the VARMA model produces the best forecasts, in terms of the mean squared error (MSE), relative to the vector autoregressive (VAR) specification. Moreover, the VARMA structure emerges as the reduced form representation of structural models in macroeconomics. For instance, the linear solution of a standard dynamic stochastic general equilibrium model generally implies a VARMA representation on the observable endogenous variables (see Ravenna (2006)).

In this paper, we study the relationship between VARMA and factor representations of a vector stochastic process, and propose a new class of factor-augmented VARMA models. We start by observing that in general multivariate series and associated factors do not both follow a finite order VAR process. First, we show that when the factors are obtained as linear combinations of observable series, their dynamic process is generally a VARMA and not a finite-order VAR as usually assumed in the literature.[2] Second, we demonstrate that even if the latent factors are generated from a finite order VAR process, this implies a VARMA representation for the observable series. Hence, an important advantage of our approach is to combine two parsimonious ways to summarize the dynamic interactions between a huge number of time series: dynamic factor model and VARMA process.[3] Finally, and contrary to VAR process, VARMA class of models is closed under marginalization. This represents an advantage if the number of factors is

2. The importance of the factor process specification depends on the technique used to estimate the factor model and on the research goal. In the two-step method developed by Stock and Watson (2002a), the factor process does not matter for the approximation of factors. This could be an issue if we use a likelihood-based technique that relies on the completely specified process. Moreover, if we use the factor model to forecast time series, having a reliable and parsimonious approximation of the factor dynamic process is important. Boivin and Ng (2005) compare projection-based models (as in Stock and Watson (2002b)), and those based on the factor structure where the common component is forecasted either using the VAR process or in a nonparametric way (as in Forni et al. (2000)). They conclude that the projection-based method, that uses principal component estimates, generally works best.

3. Chen and Zadrozny (2009) consider weighted-covariance factor decomposition method to reduce the VARMA process of observable series into a smaller VARMA model containing important variables and significant factors.

underestimated. [4]

Once we have argued that the FAVARMA model is a theoretically consistent specification, the objective of this paper is to see if VARMA factors can help in forecasting time series. To do so, we compare the forecasting performance (in terms of MSE relative to benchmark AR(p) model) of four FAVARMA specifications to projection-based models, and those based on the factor structure where the factor dynamics are approximated by a finite order VAR. Moreover, we consider the univariate ARMA model to see how it compares to factor-based models. We perform two pseudo-out-of-sample forecasting exercises using a balanced U.S. monthly panel and a balanced Canadian monthly panel taken from Boivin, Giannoni, and Stevanović (2010, 2009) respectively.

The results show that VARMA factors help in predicting several key U.S. and Canadian macroeconomic aggregates, relative to standard factor models, and across different forecasting horizons. We find important gains, up to 42% of reduction in MSE, when forecasting the growth rate of industrial production and employment, inflation and short-term interest rate. In particular, the VARMA-factor specifications generally outperform the VAR-factor forecasting models, and this is especially the case for two moving average representations. We also perform Monte Carlo simulations in which VARMA factors help a lot especially in small sample cases.

Finally, we perform a structural factor analysis exercise. We estimate the effect of monetary policy shock using the data and the identification scheme as in Bernanke, Boivin and Eliasz (2005). We find that impulse responses from a parsimonious 6-factor FAVARMA(2,1) model give an accurate and comprehensive picture of the effect and the transmission of monetary policy in U.S.. To get similar responses from a standard FAVAR model, Akaike information criterion estimates the lag order of 14. Hence, 84 coefficients governing the factors dynamics need to be estimated in FAVARMA framework, while FAVAR model implies estimating 510 VAR parameters.

In the next section, we recall some important results on linear transformations of vector stochastic processes and present four identified VARMA forms. In Section 3, we

4. If the vector of true factors satisfies a VAR model, the subvectors do not typically satisfy VAR models, but VARMA models.

study the link between VARMA and factor representations. The FAVARMA model is proposed in the Section 4. The estimation of factor models and VARMA processes is discussed in Section 5. Forecasting models that we use in simulations and empirical applications are presented in the Section 6. Monte Carlo simulation is discussed in Section 7, while Sections 8 and 9 contain results on two empirical applications, U.S. and Canadian data respectively. The structural analysis is performed in Section 10. The Appendix contains proofs of theorems, Monte Carlo simulation results, and data description.

3.2 Framework

In this section, we summarize a number of important findings on linear transformations of vector stochastic processes, and then we present four identified VARMA forms that we will use in the forecasting applications.

3.2.1 Linear transformations of vector stochastic processes

Exploring the features of transformed processes is important since the data are obtained by temporal and spatial aggregations, and/or transformed by linear filtering techniques before being used to estimate models and evaluate theories. In macroeconomics, researchers model the dynamic interactions by specifying a multivariate stochastic process on a small number of economic indicators. Hence, they work on the marginalized processes that can be seen as linear transformations of the original process of economic time series. Finally, if we are interested in dimension-reduction methods such as the principal component model, we end up with variables constructed as linear transformations of the observable series. Early contributions on these issues include Zellner and Palm (1974), Rose (1977), Wei (1978), Abraham (1982), and Lütkepohl (1984), among others.

The most important result concerns linear transformations of a zero mean N-dimensional, stationary, nondeterministic stochastic process. Let X_t be such that

$$X_t = \sum_{i=0}^{\infty} \Phi_i u_{t-i} = \Phi(L) u_t, \quad \Phi_0 = I_K, \tag{3.1}$$

where u_t is a white noise with $E(X_t) = 0$, $E(u_t u_t') = \Sigma_u$, $E(X_t X_t') = \Sigma_X$, $E(X_t X_{t+h}') = \Gamma_X(h)$, and $\det(\Phi(z)) \neq 0$ for $|z| < 1$. We consider the following linear transformation of X_t,

$$F_t = CX_t, \tag{3.2}$$

where C is a $(K \times N)$ matrix of rank K that is fixed over time. Then, given the nature of the process of X_t, we have:

1. F_t is also stationary, nondeterministic and has zero mean. Thus, it has an MA representation
$$F_t = \sum_{i=0}^{\infty} \Psi_i v_{t-i} = \Psi(L)v_t, \quad \Psi_0 = I_K, \tag{3.3}$$
where v_t is K-dimensional white noise with $E(v_t v_t') = \Sigma_v$.
2. If Σ_u is nonsingular and C is of full rank M, then $\det(\Psi(z)) \neq 0$ for $|z| < 1$.

This result considers a very general case where X_t is a vector stochastic process with an MA representation. If it is invertible, the cases of finite or infinite VAR processes are covered.

In practice, only a finite number of parameters can be estimated. Suppose an N-dimensional finite order MA(q) process,

$$X_t = u_t + M_1 u_{t-1} + \ldots + M_q u_{t-q} = M(L)u_t \tag{3.4}$$

with $\det(M(z)) \neq 0$ for $|z| < 1$ and nonsingular white noise noise covariance matrix Σ_u. Let C be a $(K \times N)$ matrix of full rank K. Then, it can be shown that $F_t = CX_t$ has an invertible MA(q*) representation

$$F_t = v_t + N_1 v_{t-1} + \ldots + N_q^* v_{t-q^*} = N(L)v_t \tag{3.5}$$

with $\det(N(z)) \neq 0$ for $|z| < 1$ where v_t is a K-dimensional white noise with nonsingular matrix Σ_v, the N_i are $(K \times K)$ coefficient matrices and $q^* \leq q$.

Some conditions in previous results can be relaxed. The nonsingularity of the covariance matrix Σ_u and the full rank of C are not necessary so there may be exact linear dependencies among components of X_t and F_t (see Lütkepohl (1984)). Another remark concerns $q*$. It is easy to construct examples where $q* < q$ by properly choosing C and $M(L)$.

Using previous results it can be shown that the VARMA class of models is closed with respect to linear transformations. Let X_t be an N-dimensional, stable, invertible VARMA(p,q) process

$$\Phi(L)X_t = \Theta(L)u_t, \tag{3.6}$$

and let C be a $(K \times N)$ matrix of rank K<N. Then, as stated in Corollary 11.1.2 in Lütkepohl (2005), $F_t = CX_t$ has a VARMA(p^*,q^*) representation with $p^* \leq (N-K+1)p$, and $q^* \leq (N-K)p+q$. Hence, a linear transformation of a finite order VARMA process still has a finite order VARMA representation but with possibly higher autoregressive and moving average orders.

When modeling economic time series, the most used specification is the finite order VAR. Therefore, it is important to notice that this class of models is not closed with respect to linear transformations reducing the dimension of the original process. It follows from the previous result that such transformation will typically have a VARMA representation.

3.2.2 Identified VARMA processes

The identification problem arises since the VARMA representation of X_t is not unique. There are several ways to identify the process in (3.6). In the following, we state four unique VARMA representations: the well known final equation form and three representations proposed in Dufour and Pelletier (2008).

Definition 1. (Final AR equation form (FAR)) *The VARMA representation in (3.6) is said to be in final AR equation form if* $\Phi(L) = \phi(L)I_N$*, where* $\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$ *is a scalar polynomial with* $\phi_p \neq 0$.

Definition 2. (Final MA equation form (FMA)) *The VARMA representation in (3.6) is said to be in final MA equation form if* $\Theta(L) = \theta(L)I_N$*, where* $\theta(L) = 1 - \theta_1 L - \cdots - \theta_q L^q$ *is a scalar polynomial with* $\theta_q \neq 0$.

Definition 3. (Diagonal MA equation form (DMA)) *The VARMA representation in (3.6) is said to be in diagonal MA equation form if* $\Theta(L) = \mathrm{diag}[\theta_{ii}(L)] = I_N - \Theta_1 L - \cdots - \Theta_q L^q$ *where* $\theta_{ii}(L) = 1 - \theta_{ii,1}L - \cdots - \theta_{ii,q_i}L^{q_i}$, $\theta_{ii,q_i} \neq 0$, *and* $q = \max_{1 \le i \le N}(q_i)$.

Definition 4. (Diagonal AR equation form (DAR)) *The VARMA representation in (3.6) is said to be in diagonal AR equation form if* $\Phi(L) = \mathrm{diag}[\phi_{ii}(L)] = I_N - \Phi_1 L - \cdots - \Phi_p L^p$ *where* $\phi_{ii}(L) = 1 - \phi_{ii,1}L - \cdots - \phi_{ii,p_i}L^{p_i}$, $\phi_{ii,p_i} \neq 0$, *and* $p = \max_{1 \le i \le N}(p_i)$.

An interesting fact from the results on linear aggregations of VARMA processes is that the aggregated process F_t can always have an identified VARMA representation in the final AR equation form. But this representation may not be attractive for several reasons. First, it is quite far from the usual VAR model by excluding lagged values of other variables in each equation. Moreover, AR coefficients are the same in all equations that will require a polynomial of very high order. Second, the interaction between different variables is modeled through the MA part of the model, and may be very complex in structural analysis.

Hence, an interesting representation is the diagonal MA form. It is easy to specify, contrary to the echelon form, since we do not need to deal with rules for the orders of the off-diagonal elements in AR and MA operators. From the point of view of practitioners, it is very appealing since adding lags of u_{it} to the $i^t h$ equation is a natural extension of the VAR model. It also has the advantage of putting the simple structure on MA polynomials, the part which complicates the estimation, rather than the AR part as in the final AR equation form.

3.3 VARMA and factor representations

In this section, we study the link between VARMA and factor representations of a vector stochastic process X_t, and the dynamic process of factors. In Theorem 1, we postulate a factor model for X_t where factors follow a finite-order VAR process. We show that finite-order VAR factors induce a finite-order VARMA process for the observable series.

Theorem 1. *Suppose X_t has the following factor structure:*

$$X_t = \Lambda F_t + u_t \tag{3.7}$$

$$F_t = \Phi(L) F_{t-1} + a_t \tag{3.8}$$

where X_t is an $N{\times}1$ vector of time series, Λ is an $N \times K$ matrix of factor loadings, F_t is a $K{\times}1$ vector of factors, u_t and a_t are uncorrelated white noises with covariance matrices Σ_u and Σ_a, and $\Phi(z) = [\Phi_1 z - \ldots - \Phi_p z^p]$. Then, X_t has a VARMA(p^*,p^*) representation

$$A(L)X_t = B(L)e_t \tag{3.9}$$

where $p^* \leq p$.

This result can be extended to a case where factors have VARMA representation. It is not surprising that the induced process for X_t is again a finite-order VARMA, but with possibly a different MA order that in the previous case. This finding is summarized in Theorem 2.

Theorem 2. *If F_t follows a VARMA(p,q) process*

$$F_t = \Phi(L) F_{t-1} + a_t - \Theta(L) a_{t-1} \tag{3.10}$$

where $\Theta(z) = [\Theta_1 z - \ldots - \Theta_q z^q]$, then X_t has VARMA(p^*,q^*) representation

$$A(L)X_t = B(L)e_t \tag{3.11}$$

where $p^* \leq p$ and $q^* \leq max(p,q)$.

It is worth noting that if the VARMA representation of X_t is invertible, it has a VAR(∞) representation that in practice can be approximated by a finite order VAR.

The next question is what are the implications of the process of X_t on the factors representation? In other words, what are the implications of the underlying structure of

X_t on the representation of latent factors when the latter are calculated as linear transformations of X_t? The results are summarized in Theorem 3.

Theorem 3. *Suppose factors are computed as linear combinations of elements of X_t that has a factor representation as in (3.7). Then the following results hold:*

(i) *if X_t has a VARMA(p,q) representation as in (3.6), then F_t has VARMA(p^*,q^*) representation with $p^* \leq Np$ and $q^* \leq q+(N-1)p$ (or $p^* \leq (N-K+1)p$ and $q^* \leq q+(N-K)p$);*

(ii) *if X_t has a VAR(p) representation, then F_t has VARMA(p^*,q^*) representation with $p^* \leq Np$ and $q^* \leq (N-1)p$;*

(iii) *if X_t has an MA representation as in (3.1) or (3.4), then F_t has an MA or MA(q^*) representation with $q \leq q^*$.*

As in the Section 3.2, the invertibility characteristics of X_t still hold for F_t if Σ_u is nonsingular, and C is of full rank K. The invertibility condition is important in practice. If X_t is invertible (and stable if VARMA or VAR) and if F_t inherits these characteristics, then it has the infinite VAR representation that in practice can be approximated by a finite order VAR.

To resume this section, we recall the arguments in favor of the VARMA modeling of factor process.

(i) Whenever the joint process of series in X_t is VAR or VARMA, if the factors are calculated as principal components they follow a VARMA process. Since most of the series in X_t are usually linearly transformed before estimation (seasonal adjustments, temporal and contemporaneous aggregations), the transformed process from which the factors are extracted is likely to have a VARMA representation. Moreover, we showed in Theorems 1 and 2 that VAR(MA)-factor structure on X_t implies a VARMA representation for the marginal process of X_t.

(ii) VARMA representations are more parsimonious and could produce more relevant statistical inference. As it was found in Dufour and Pelletier (2008) the introduction of the MA operator allows for a reduction of the required AR order so we can get more precise estimates. Moreover, in terms of the forecasting power, VARMA models present theoretical advantages over the VAR representation (see Lütkepohl (1987)).

(iii) Note that imposing a VARMA process on factors can be viewed from two perspectives. First, if one uses the factor analysis as a dimension-reduction method, then the implications of the underlying process of X_t on F_t should be considered. From Theorem 3 we see that VARMA is a natural process for factors. Second, if we suppose that the true representation of the world is a factor representation, i.e. there is a small number of structural shocks that generate observable series, considering a VARMA process on factors instead of a finite order VAR is an interesting generalization motivated by the usual arguments of parsimony, and invertibility and marginalization issues. Moreover, if we underestimate the number of factors, even if F_t has a finite order VAR representation, the subvector of F_t is likely to follow a VARMA process.

We have considered the factor model in its static form without loss of generality, since it is always possible to write the dynamic factor model in the static form. This more general case is studied in the following section where we introduce the dynamic factor model with VARMA process for factors.

3.4 Factor-augmented VARMA models

We showed that the observable VARMA process generally induces a VARMA representation for factors, and not a finite-order VAR process usually assumed in the literature. Following these results, we propose the factor-augmented VARMA (FAVARMA) model. Using the notation as in Stock and Watson (2005), the dynamic factor model (DFM) where factors have a finite order VARMA(p_f,q_f) representation can be written as

$$
\begin{aligned}
X_{it} &= \tilde{\lambda}_i(L) f_t + u_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T & (3.12) \\
u_{it} &= \delta_i(L) u_{i,t-1} + \nu_{it} & (3.13) \\
f_t &= \Gamma(L) f_{t-1} + \Theta(L) \eta_t & (3.14)
\end{aligned}
$$

where $\tilde{\lambda}_i(L)$ is a lag polynomial, $\delta_i(L)$ is a $p_{x,i}$-degree lag polynomial, $\Gamma(L) = [\Gamma_1 L + \ldots + \Gamma_{p_f} L^{p_f}]$, $\Theta(L) = [I - \Theta_1 L - \ldots - \Theta_{q_f} L^{q_f}]$, and ν_{it} is an N-dimensional white noise uncorrelated with q-dimensional white noise process η_t. We see that the standard VAR-factors case is obtained if $\Theta(L) = I$.

The exact (or classical) DFM is obtained if the following assumption is satisfied:

$$E(u_{it} u_{js}) = 0 \quad \forall i, j, t, s, \quad i \neq j.$$

If we allow for some limited cross-section correlations among the idiosyncratic components (such that there exists a small number of largest eigenvalues of the covariance matrix of common components that diverge when the number of series tends to infinity, while the remaining eigenvalues as well as the eigenvalues of the covariance matrix of specific components are bounded), we obtain the approximate DFM. [5]

Subtracting $\delta_i(L) u_{it-1}$ from both sides of (3.12) gives the DFM with serially uncorrelated idiosyncratic errors:

$$X_{it} = \lambda_i(L) f_t + \delta_i(L) X_{it-1} + \nu_{it}, \tag{3.15}$$

where $\lambda_i(L) = (1 - \delta_i(L) L) \tilde{\lambda}_i(L)$.

Then, we can rewrite the DFM in the following form:

5. See Bai and Ng (2008) for an overview of the modern factor analysis literature, and distinction between exact and approximate factor models.

$$X_t = \lambda(L) f_t + D(L) X_{t-1} + \nu_t \tag{3.16}$$
$$f_t = \Gamma(L) f_{t-1} + \Theta(L)\eta_t \tag{3.17}$$

where

$$\lambda(L) = \begin{bmatrix} \lambda_1(L) \\ \vdots \\ \lambda_n(L) \end{bmatrix}, D(L) = \begin{bmatrix} \delta_1(L) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_n(L) \end{bmatrix}, \nu_t = \begin{bmatrix} \nu_{1t} \\ \vdots \\ \nu_{nt} \end{bmatrix}.$$

To obtain the static version of the previous factor model suppose that $\tilde{\lambda}(L)$ has finite degree p-1, and let $F_t = [f_t' \quad f_{t-1}' \cdots f_{t-p+1}']'$. Let the dimension of F_t be K, where $q \leq K \leq qp$. Then,

$$X_t = \Lambda F_t + u_t \tag{3.18}$$
$$u_t = D(L) u_{t-1} + \nu_t \tag{3.19}$$
$$F_t = \Phi(L) F_{t-1} + G\Theta(L)\eta_t \tag{3.20}$$

where Λ is a $N \times K$ matrix where the i^{th} row consists of coefficients of $\tilde{\lambda}_i(L)$, $\Phi(L)$ contains coefficients of $\Gamma(L)$ and zeros, and G is $K \times q$ matrix that loads (structural) shocks η_t to static factors (consists of 1's and 0's).

Again, if $\Theta(L) = I$ we obtain the static factor model that was used to forecast time series (Stock and Watson (2002b, 2008), Boivin and Ng (2005)), and to study the impact of monetary policy shocks in factor-augmented VAR (FAVAR) model (Bernanke, Boivin and Eliasz (2005), Boivin, Giannoni and Stevanović (2009)).

3.5 Estimation

Several estimation methods of factor models and VARMA processes (separately) have been proposed in the literature. One possibility is to estimate the system (3.12)-(3.14) (or in its static form (3.18)-(3.20)) simultaneously after making distributional assumptions on the error terms. This method is already computationally difficult when factors have a simple VAR structure.[6] Hence, adding the MA part to factor process should not help, since estimating the VARMA model is usually not easy.[7]

Instead of the likelihood-based approach, we use the two-step Principal Component Analysis (PCA) estimation method (see Stock and Watson (2002a), and Bai and Ng (2006) for theoretical results concerning the PCA estimator). In the first step, $\hat{F}_t$ are computed as K principal components of X_t. In the second step, we estimate the VARMA representation (3.20) using $\hat{F}_t$.

The number of factors can be estimated using different procedures proposed by Amengual and Watson (2007), Bai and Ng (2002, 2007), Onatski (2009a), and Hallin and Liska (2007). In our forecasting exercises we estimate the number of factors using Bayesian information criterion as in Stock and Watson (2002b), while the number of factors in the structural FAVARMA model is the same as in Bernanke, Boivin and Eliasz (2005).

The standard estimation methods for VARMA models are maximum likelihood and nonlinear least squares. Unfortunately, these methods require nonlinear optimization, that may not be feasible when the number of parameters is relatively large. In this paper, we will use the GLS method proposed in Dufour and Pelletier (2008) that generalize the regression-based estimation method introduced by Hannan and Rissanen (1982). The

6. The unknown coefficients in (3.12)-(3.14) (or in its static form (3.18)-(3.20)) can be estimated by Gaussian maximum likelihood using the Kalman filter (or by Quasi ML), see Engle and Watson (1981), Stock and Watson (1989), Sargent (1989). This method is computationally burdensome when N is very large, but also the misspecification becomes very likely. However, there are some recent improvements: Kalman filter speedup by Jungbacker and Koopman (2008), using principal components as starting values then a single pass of the Kalman filter by Giannone, Reichlin, and Sala (2004), and principal components for starting values then use EM algorithm to convergence by Doz, Giannone, and Reichlin (2006).

7. However, it would be interesting to see if considering VARMA processes can help in the approximation of the true factors, and this is a part of the ongoing research project.

method is described below.

Consider a K-dimensional zero mean process Y_t generated by the VARMA(p,q) model:

$$A(L)Y_t = B(L)U_t \tag{3.21}$$

where $A(L) = I_K - A_1 L - \cdots - A_p L^p$, $B(L) = I_K - B_1 L - \cdots - B_q L^q$, and U_t is a sequence of uncorrelated random variables. Assume $\det[A(z)] \neq 0$ for $|z| \leq 1$ and $\det[B(z)] \neq 0$ for $|z| \leq 1$ so the process Y_t is stable and invertible. Split the whole vector of VARMA parameters, γ, in two parts γ_1 (the parameters for the AR part) and γ_2 (MA part): $\gamma = [\gamma_1 \quad \gamma_2]'$. For VARMA in diagonal MA equation form, we have

$$\gamma_1 = [a_{1\bullet,1}, \ldots, a_{1\bullet,p}, \ldots, a_{K\bullet,1}, \ldots, a_{K\bullet,p}], \tag{3.22}$$

$$\gamma_2 = [b_{11,1}, \ldots, b_{11,q_1}, \ldots, b_{KK,1}, \ldots, b_{KK,q_K}]. \tag{3.23}$$

The estimation method involves three steps.

Step 1. Estimate a VAR(n_T) to approximate the VARMA(p,q) and recuperate the residuals defined as:

$$\hat{U}_t = Y_t - \sum_{l=1}^{n_T} \hat{\Pi}_l^{n_T} Y_{t-l}, \quad T > 2Kn_T. \tag{3.24}$$

Step 2. With the residuals from step 1, compute an estimate of the covariance matrix of U_t, $\hat{\Sigma}_U = \frac{1}{T}\sum_{t=n_t+1}^{T} \hat{U}_t \hat{U}_t'$, and estimate by GLS the following multivariate regression,

$$A(L)Y_t = [B(L) - I_K]\hat{U}_t + e_t,$$

to get estimates $\tilde{A}(L)$ and $\tilde{B}(L)$. The estimator is

$$\hat{\gamma} = [\sum_{t=l}^{T} \hat{Z}_{t-1}' \hat{\Sigma}_U^{-1} \hat{Z}_{t-1}]^{-1} [\sum_{t=l}^{T} \hat{Z}_{t-1}' \hat{\Sigma}_U^{-1} Y_t] \tag{3.25}$$

with $l = n_T + \max(p,q) + 1$. Setting

$$\mathbf{Y}_{t-1}(p) = \left[y_{1,t-1}, \ldots, y_{K,t-1}, \ldots, y_{1,t-p}, \ldots, y_{K,t-p}\right],$$
$$\hat{\mathbf{U}}_{t-1} = \left[\hat{u}_{1,t-1}, \ldots, \hat{u}_{K,t-1}, \ldots, \hat{u}_{1,t-q}, \ldots, \hat{u}_{K,t-q}\right],$$
$$\hat{\mathbf{u}}_{k,t-1} = \left[\hat{u}_{k,t-1}, \ldots, \hat{u}_{k,t-q_k}\right],$$

the matrix $\hat{Z}_{t-1}$ is:

$$\hat{Z}_{t-1} = \begin{bmatrix} \mathbf{Y}_{t-1}(p) & \cdots & 0 & \hat{\mathbf{u}}_{1,t-1} & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{Y}_{t-1}(p) & 0 & \cdots & \hat{\mathbf{u}}_{K,t-1} \end{bmatrix}.$$

Step 3. Using the second step estimates, form new residuals

$$\tilde{U}_t = Y_t - \sum_{i=1}^{p} \tilde{A}_i Y_{t-i} + \sum_{j=1}^{q} \tilde{B}_j \tilde{U}_{t-j}$$

initiating with $\tilde{U}_t = 0$, $t \leq \max(p,q)$, and define

$$X_t = \sum_{j=1}^{q} \tilde{B}_j X_{t-j} + Y_t,$$

$$W_t = \sum_{j=1}^{q} \tilde{B}_j W_{t-j} + \tilde{U}_t,$$

initiating with $X_t = W_t = 0$ for $t \leq \max(p,q)$. Compute a new estimate of Σ_U, $\tilde{\Sigma}_U = \frac{1}{T}\sum_{t=\max(p,q)+1}^{T} \tilde{U}_t \tilde{U}_t'$. Then, regress by GLS $\tilde{U}_t + X_t - W_t$ on $\tilde{V}_{t-1}$ with

$$\tilde{V}_t = \sum_{j=1}^{q} \tilde{B}_j \tilde{V}_{t-j} + \tilde{Z}_t$$

where $\tilde{Z}_t$ is just like $\hat{Z}_t$ from step 2 except it is computed with $\tilde{U}_t$ instead of $\hat{U}_t$ to obtain regression coefficient $\hat{A}_i$ and $\hat{B}_j$:

$$\hat{\gamma} = \left[\sum_{t=\max(p,q)+1}^{T} \tilde{V}_{t-1}' \tilde{\Sigma}_U^{-1} \tilde{V}_{t-1}\right]^{-1} \left[\sum_{t=\max(p,q)+1}^{T} \tilde{V}_{t-1}' \tilde{\Sigma}_U^{-1} \left[\tilde{U}_t + X_t - W_t\right]\right]. \qquad (3.26)$$

The consistency and asymptotic normality of above estimators are derived in DP (2008).

In previous steps the orders of the AR and MA operators were supposed known. In practice they are usually estimated by statistical methods or suggested by theory. Dufour and Pelletier (2008) propose an information criterion to be applied in the second step of estimation procedure above. For all $p_i \leq P$ and $q_i \leq Q$ compute

$$\log(\det\tilde{\Sigma}_U) + \dim(\gamma)\frac{(\log T)^{1+\delta}}{T}, \quad \delta > 0. \tag{3.27}$$

Choose $\hat{p}_i$ and $\hat{q}_i$ as the set which minimizes the information criteria (3.27). The properties of estimators $\hat{p}_i$ and $\hat{q}_i$ are given in the paper.

3.6 Applications in macroeconomics

In macroeconomics, the factor models have been used in several purposes: forecasting (and nowcasting) of macroeconomic aggregates, structural analysis where shocks with meaningful economic interpretation have been identified, testing the implications of DFM structure and estimation of structural macroeconomic models. In this section, we consider a forecasting exercise to see if allowing for VARMA dynamics in estimated factors can help in forecasting some macroeconomic indicators of interest.

3.6.1 Forecasting time series

Consider a simplified version of the static model (3.18)-(3.20)) assuming that F_t is scalar

$$X_{it} = \lambda_i F_t + u_{it} \tag{3.28}$$

$$u_{it} = \delta_i u_{it-1} + \nu_{it} \tag{3.29}$$

$$F_t = \phi F_{t-1} + \eta_t - \theta\eta_{t-1} \tag{3.30}$$

Then, after replacing for two last equations and rearranging, we get the forecast of X_{T+1} based on the informational set at T

$$X_{iT+1|T} = \delta_i X_{iT} + \lambda_i(\phi - \delta_i)F_T - \lambda_i \theta \eta_T. \tag{3.31}$$

We can summarize several implications of (3.31). As discussed in Boivin and Ng (2005), in the standard factor model where idiosyncratic components and factors follow autoregressive processes, if $\lambda_i \neq 0$, i.e. factor structure with respect to variable i exists, and $\phi \neq \delta_i$, meaning that the common and specific components do not have the same dynamics, considering F_t to predict X_{it}, should perform better than the AR forecast in terms of MSE.

Allowing for the MA part in the dynamic process of the common component generalizes this finding. Suppose again $\lambda_i \neq 0$. If the MA coefficients are not zero, $\theta \neq 0$, ignoring the moving average structure will produce higher forecast errors even if $\phi = \delta_i$.

It is important to note that the forecasting performance is affected by the choice of the estimation method to get factors, and by the choice of the forecasting equation. Boivin and Ng (2005) address these issues by considering static and dynamic factors approximations with three types of forecasting equations: unrestricted (where X_{iT+h} is forecasted using X_{iT}, F_T and their lags), direct (where dynamic process of factors is estimated and then used to first forecast F_{T+h} and then to get X_{iT+h}), and nonparametric (no parametric assumptions are made about the dynamics of factors nor their relations to observables). Their simulation results show that the unrestricted forecast equation using static factors generally does best in terms of the relative MSE to the autoregressive alternative. Moreover, it seems that these findings are mainly caused by the choice of the forecasting equation.

In our approach, allowing for the MA structure should help in forecasting X_{it} if the process of factors is well approximated. If coefficients in $[I - \Theta(L)]^{-1}$ vanish slowly, or in more extreme case if the VARMA representation is not invertible, estimating a parsimonious VARMA process should outperform a long VAR approximation. However, in practice, due to estimation error, it will be not surprising to see that simpler method perform better.

3.6.2 Forecasting models

A popular way to evaluate the predicting power of a model is to conduct a pseudo-out-of-sample forecasting exercise. Here, we compare our FAVARMA approach to several standard factor-based forecasting models used in the literature. The forecasting equations are divided into two categories: those that do not consider the dynamic process of factors (called "Unrestricted"in Boivin and Ng (2005) and "Diffusion index", or "DI"and "DI-AR", in Stock and Watson (2002b)), and those that first predict common and specific components separately, from their estimated dynamic processes, and then form the forecast using the estimated observation equation. Moreover, in the latter we distinguish between sequential and direct techniques to obtain forecasts.

In this exercise, we estimate factors as principal components of X_t. Hence, only the second type of forecasting equations can be affected by allowing for VARMA factors. We compare the results for four identified VARMA forms labeled "Diag MA", "Diag AR", "Final MA"and "Final AR".

Before presenting simulation and empirical results, we summarize more formally the factor-based forecasting equations:

– First type:

$$X_{i,T+h|T} = \alpha^h + \sum_{j=1}^{m} \beta_{ij}^h F_{T-j+1} + \sum_{j=1}^{p} \rho_{ij}^h X_{i,T-j+1}$$

- Unrestricted: $m \geq 1, p \geq 0$
- DI: $m = 1, p = 0$
- DI-AR: $m = 1$

– Second type:

$$X_{i,T+h|T} = \lambda_i' F_{T+h|T} + u_{i,T+h|T}$$

where $u_{i,T+h|T}$ is forecasted sequentially or directly using AR(p) process while the factor dynamics is approximated by VAR(p) (giving Sequential and Direct forecasts as in Boivin and Ng (2005)) or by one of four identified VARMA forms in sequential way

- Sequential: $F_{T+h|T} = \hat{\Phi}(L)F_{T+h-1|T}$

- Direct: $F_{T+h|T} = \hat{\Phi}^h(L)F_T$
- VARMA: $F_{T+h|T} = \hat{\Phi}(L)F_{T+h-1|T} + \hat{\Theta}(L)\eta_{T+h-1|T}$ where VARMA contains four different models (Diag MA, Diag AR, Final MA, Final AR) defined by corresponding lag polynomials $\Phi(L)$ and $\Theta(L)$.

The benchmark forecasting model used to compare previous equations is the standard AR(p).[8] However, given the factor structure for observable series, the finite order autoregressive process is only an approximation of the process of X_{it}. After the Theorem 3.1, we have that the marginal process for each element of X_t is possibly an ARMA. Hence, if the MA part for a particular series of interest is very important and close to non-invertibility region, we need a very long autoregressive model to approximate the true process, and this can affect the forecasting performance. For that reason, we also include the ARMA model as an alternative to see if it outperforms the autoregressive specification, and how does it perform relative to factor-based models.

3.7 Monte Carlo simulations

To illustrate the performance of our approach, we run several Monte Carlo simulation exercises where we compare forecasting performance of second type models: FAVARMA (in four identified forms) to FAVAR models. The data are simulated using the static factor model with MA(1) dynamics for factors, and the idiosyncratic component is simulated as in Boivin and Ng (2005) and Onatski (2009b):

$$X_{it} = \lambda_i F_t + u_{it} \tag{3.32}$$

$$F_t = \eta_t - B\eta_{t-1} \tag{3.33}$$

where $i = 1,\ldots,N$, $t = 1,\ldots,T$, $\eta_t \sim N(0,1)$, and the generating process of u_{it} will be specified in each simulation exercise.

We present two different simulation exercises. In the first series of simulations, we vary the time and cross-section dimensions, the nature of the idiosyncratic component,

8. The same benchmark model was used in Stock and Watson (2002b) and Boivin and Ng (2005).

and the importance of the MA component in factor dynamics. In the second exercise, we fix the time and cross-section dimensions to 100 each, simulate the idiosyncratic component as in Boivin and Ng (2005), and vary the number of factors. The main simulation results are presented in the Appendix.

3.7.1 Simulation exercise 1

- Time dimension: $T \in \{50, 100, 600\}$.
- Cross-section dimension: $N = \{50, 100, 130\}$.
- Number of factors: $K \in \{2, 4\}$
- Idiosyncratic component dynamics:

$$\begin{aligned} u_{it} &= \rho_N u_{i-1,t} + \xi_{it} \\ \xi_{it} &= \rho_T \xi_{i,t-1} + \varepsilon_{it} \\ \varepsilon_{it} &\sim N(0,1) \end{aligned}$$

  where the cross-section dependance (CSD) is controlled by parameter $\rho_N \in \{0.1, 0.5, 0.9\}$, and the time dependance (TD) is controlled by parameter $\rho_T \in \{0.1, 0.9\}$.
- VARMA orders: estimated as in Dufour and Pelletier (2008).
- AR order for idiosyncratic component: 1.
  - Case 1:
    - K = 2

$$B = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.3 \end{bmatrix}$$

    - VAR order: 6.
  - Case 2:
    - K = 4

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- VAR order: 4.

The results from this simulation exercise are presented in Tables III.I and III.III (Appendix) for Cases 1 and 2 respectively. The numbers represent MSE of four FAVARMA identified forms over the MSE of FAVAR Direct or Iterative forecasting models.

In Case 1 and for situations where the time dimension is small, i.e. T=50, the FAVARMA models outperform FAVAR Direct model, especially at long horizons. The huge improvement at horizons 24 and 36 is due to the small sample size. When compared to FAVAR Iterative model, the FAVARMA forms still produce better forecasts in terms of MSE, but the improvement is less important relative to the multi-step-ahead forecasting VAR-based model. However, we can see that MA forms outperform FAVAR Iterative model up to 20% at short and long horizons. When the time size increases, T=100 and T=600, the improvement of VARMA-based models is moderate, but they still produce better forecasts, especially at longer horizons when compared to Direct VAR model, and at shorter horizons when compared to Iterative VAR model. The results are similar in Case 2, except that FAVARMA models outperform VAR factor model in most of the specification.

Another interesting result is that FAVARMA models seem to perform better in situations of weaker factor structure, that is in cases where the cross-section size is smaller (N=50 compared to N=100, and for a fixed nature of idiosyncratic component correlation structure). Finally, when time and cross-section sizes are comparable to what we have in data (T=600, N=130), the FAVARMA models perform better in Case 2 than in Case 1, due to very persistent MA part in factor dynamics in Case 2.

3.7.2 Simulation exercise 2

- Time dimension: $T = 100$.
- Cross-section dimension: $N = 100$.
- Number of factors: $K \in \{3,4,6\}$
- Idiosyncratic component dynamics: $u_{it} = \kappa v_{it}$, $v_{it} \sim N(0, \sigma^2_{v_i})$ such that the common component explains a fraction ϑ of the variance of X_t. Following Boivin and Ng (2005), ϑ is set to 0.5 while for the first series in panel X_t, the one that is

forecasted, we have $var(\lambda_1 F_t)/var(X_{1t}) = 0.75$.

– MA coefficients matrices:

  – K = 3

$$B = \begin{bmatrix} 0.2350 & 0 & 0 \\ 0 & 0.2317 & 0 \\ 0 & 0 & 0.5776 \end{bmatrix}$$

  – K = 4

$$B = \begin{bmatrix} 0.3365 & 0 & 0 & 0 \\ 0 & 0.2420 & 0 & 0 \\ 0 & 0 & 0.0610 & 0 \\ 0 & 0 & 0 & 0.4735 \end{bmatrix}$$

  – K = 6

$$B = \begin{bmatrix} 0.1558 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.4827 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.4525 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5320 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.6604 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.2763 \end{bmatrix}$$

– VAR order: 4.

– VARMA orders: estimated as in Dufour and Pelletier (2008).

– AR order for idiosyncratic component: 1.

The results from this simulation exercise are presented in Table III.V. We conclude that FAVARMA models performance improves with the number of factors which is a consequence of parsimony principle. Moreover, in comparison with Direct FAVAR forecasting model, the VARMA based models are better especially at long horizons, except for short horizons in the case of 6-factors model.

3.8 Application to U.S. macroeconomic panel data

We conduct the same out-of-sample forecasting exercise for two different sets of real data. In the first exercise, we use a balanced monthly panel from Boivin, Giannoni and Stevanović (2010) (essentially an upgraded version of the data used in Stock and Watson (2002b)). It contains 128 monthly U.S. economic and financial indicators observed from 1959 to 2008. The second application deals with a Canadian balanced monthly panel from Boivin, Giannoni and Stevanović (2009) containing 332 series observed from 1981 to 2008. The series are initially transformed to induce stationarity.

3.8.1 Main results

The Relative MSE (relative to the benchmark AR(p) model) results are presented in Table 3.I. The pseudo-out-of-sample evaluation period is 1988M01-2008M12. In the forecasting models Unrestricted, DI, and DI-AR, the number of factors, their number of lags and the number of lags of X_{it} are estimated using the Bayesian information criteria, and these can vary over the whole evaluation period. In the case of second-type forecasting models, the number of factors is fixed to 4.

We can see from the Table 3.I that allowing for VARMA factors in the second-type forecasting equations improves the forecasts of some key macroeconomic indicators across several horizons. In the case of Industrial production, diffusion index model performs the best for very short horizon of one month, while diagonal MA and final MA VARMA forms outperform other methods for horizons 2, 4 and 6 months. Finally, the ARMA model produces the smallest RMSE for the long-term forecasts. In the case of Civilian labour force, three identified VARMA forms outperform all other standard factor-based models for short and mid-term horizons while ARMA still produce the smallest RMSE in long-term forecasting, except for 2 and 4 years horizons where Direct and Unrestricted models produce the best results.

On the nominal side, diffusion index model produces the best forecasts of CPI at horizon 1, the model where factors dynamics are modeled in final AR VARMA form works the best for horizons 2, 4 and 6. For the long-term forecasts, VARMA-based

Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
Industrial production: total										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	0.8706	**0.8457**	0.8958	0.9443	0.9443	0.8971	0.9019	0.9132	0.8985	0.9700
2	1.0490	0.9938	1.0106	1.0157	1.0665	**0.9074**	0.9202	0.9112	0.9123	1.0026
4	1.1934	1.0411	1.0527	1.0711	1.2214	**0.8947**	0.9906	0.8970	0.9481	0.9710
6	1.1496	1.0238	1.0245	1.1743	1.3528	0.9248	1.0494	**0.9202**	0.9847	0.9918
12	1.2486	1.0445	1.0389	1.0933	1.3682	1.0008	1.2215	1.0075	1.0371	**0.9713**
18	1.0507	1.0048	1.0207	1.0662	1.2508	1.0511	1.5098	1.0615	1.1206	**0.9910**
24	1.0393	1.0628	1.0748	1.0128	1.0863	0.9858	1.7920	0.9959	1.1061	**0.9604**
36	1.0092	1.0906	1.1437	1.2364	1.0421	0.9855	3.0304	0.9883	1.1795	**0.9826**
48	1.0147	1.1110	1.1212	1.1063	1.0355	0.9921	5.5321	0.9922	1.1681	**0.9856**
Civilian labor force: employed. total										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	0.8264	0.8832	0.8451	0.8202	0.8202	**0.8004**	0.8075	0.8027	0.8008	1.0496
2	0.9407	0.9391	0.9381	0.9477	0.9591	0.8931	**0.8805**	0.8961	0.8852	1.0422
4	0.9766	0.9739	0.9937	1.0204	1.0551	0.9213	0.8997	0.9200	**0.8991**	0.9993
6	1.0776	1.0799	1.0937	1.0714	1.1550	0.9667	0.9526	0.9636	**0.9455**	1.0032
12	1.0741	1.0742	1.0722	1.0137	1.1654	0.9718	0.9912	0.9704	0.9558	**0.9507**
18	1.0471	1.0488	1.0472	0.9735	1.1391	1.0073	1.1386	1.0096	1.0391	**0.9721**
24	1.0237	1.0580	1.0268	**0.9641**	1.1002	1.0154	1.2806	1.0177	1.0856	0.9893
36	0.9573	0.9099	0.9703	0.9507	0.9477	0.9070	1.5452	0.9043	1.0098	**0.8957**
48	**0.9227**	0.9236	0.9250	0.9576	0.9989	0.9652	2.4022	0.9624	1.0482	0.9550
Consumer price index: all items										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	0.8806	**0.8700**	**0.8700**	0.9228	0.9228	0.9144	0.9432	0.8856	0.9072	1.0143
2	0.9866	0.9942	0.9942	0.9612	0.9730	0.9309	0.9427	0.9274	**0.9170**	0.9856
4	1.0656	1.0732	1.0732	1.0398	1.0170	1.0007	1.0665	0.9895	**0.9792**	1.0129
6	1.1343	1.1334	1.1334	1.0349	1.0101	0.9946	1.0752	0.9939	**0.9928**	1.0364
12	1.1173	1.1279	1.1279	1.0821	**0.9513**	0.9572	1.1958	0.9553	1.0408	1.0297
18	1.0311	1.0379	1.0379	1.0430	0.9654	**0.8894**	1.1021	0.8909	0.9673	0.9391
24	0.9644	1.0712	1.0712	0.9510	0.9980	0.8819	1.1851	**0.8791**	0.9713	0.8805
36	0.7645	**0.7627**	**0.7627**	0.9870	0.9470	0.8329	1.4591	0.8385	0.9126	0.8619
48	0.8663	0.8488	0.8488	0.9361	0.9536	**0.8292**	2.2640	0.8335	0.8864	0.8511
Federal funds rate										
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	**0.8375**	7.9025	1.1331	2.5170	2.5170	1.7198	3.1430	1.6070	2.1519	0.9304
2	**0.5932**	2.3977	0.8468	1.2457	1.3039	1.0068	2.0381	0.9462	1.3972	0.8919
4	**0.5156**	1.0252	0.6709	0.7663	0.8053	0.7065	1.5720	0.6562	0.9504	0.9087
6	**0.5576**	0.7488	0.6719	0.7309	0.6908	0.6189	1.4871	0.5742	0.8662	0.9433
12	0.5888	0.5916	0.6790	0.6416	0.6479	0.5757	1.5622	**0.5325**	0.8013	1.0424
18	0.6245	0.6387	0.6407	0.6404	0.6792	0.5591	1.8109	**0.5141**	0.7447	0.8674
24	0.5747	0.6950	0.6169	0.6505	0.6864	0.5449	2.0802	**0.5106**	0.6765	0.7937
36	0.5795	0.5411	**0.3621**	0.7218	0.7389	0.5420	3.0782	0.5312	0.6158	0.7088
48	**0.4743**	0.5066	0.5829	0.8742	0.7607	0.5065	4.6229	0.5088	0.5114	0.6457

Table 3.I: RMSE relative to Direct AR(p) forecasts

models perform the best for horizons 18, 24 and 48 months, while Sequential and Diffusion index models dominate for horizons 12 and 48.

Finally, the Unrestricted model is clearly the best suited to forecast the Federal funds rate at short and mid-term horizons while VARMA-based model in Final MA form performs very well for horizons 12, 18 and 24 months.

We previously showed that the factor structure implies that each observable series has an ARMA representation. Hence, forecasting an observable series, for which factor structure holds, using a factor-based equation or an appropriated ARMA model should give the same forecasts in theory. However, in practice, there are estimation uncertainty and model misspecification that can make theoretically the same models produce completely different forecasts. In Table 3.II we present mean squared errors of all factor-based models predictions relative to ARMA forecasts. The results in bold characters represent cases where ARMA model outperform the factor-based alternative in terms of MSE.

The results in Table 3.II suggest several interesting points. In case of Industrial production, all factor-based models do better than ARMA at short term horizon of one month. For longer horizons, ARMA does much better than Unrestricted and Sequential models, and its performance improve with horizons, while the improvement is more moderate with respect to DI, DI-AR and Direct models. Compared to FAVARMA forecasts, ARMA model does better only after horizon 12. For Employment, the conclusion is quite similar relative to FAVARMA, while the first-type models perform better than ARMA for horizons 1, 2, 4, and 48.

In case of Consumer price index, again all factor-based models perform better at horizon 1 but ARMA seems to be a better choice for the most of horizons relatively to the first-type models. Moreover, we can find a FAVARMA representation that outperforms ARMA model at all horizons. Finally, the picture is quite different in case of Federal funds rate where the Unrestricted forecasts outperform ARMA model completely and the latter beats all other factor-based models at short horizons of one and two months only.

Overall, based on these results we can say that ARMA model is a very good alternative for standard factor-based models in forecasting key macroeconomic indicators, especially for long-term horizons in case of the real activity variables. This is not surprising since ARMA model are very parsimonious. However, it is outperformed in the most of cases by our FAVARMA specifications.

We just discussed the forecasting performance of all alternative models and concluded that the second-type forecasting models with VARMA structure perform generally the best in this particular exercise. But it is also of interest to see more directly how approximating the factor process by a VARMA representation compares to forecasting model where the factor process is assumed to be a finite order VAR. In Table 3.III we present MSE values of VARMA-based forecasting models relative to Direct and Sequential second-type model. The numbers in bold character present cases where VARMA-based model performs better that the VAR alternative.

Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
Industrial production: total									
1	0.8975	0.8719	0.9235	0.9735	0.9735	0.9248	0.9298	0.9414	0.9263
2	**1.0463**	0.9912	**1.0080**	**1.0131**	**1.0637**	0.9050	0.9178	0.9088	0.9099
4	**1.2290**	**1.0722**	**1.0841**	**1.1031**	**1.2579**	0.9214	**1.0202**	0.9238	0.9764
6	**1.1591**	**1.0323**	**1.0330**	**1.1840**	**1.3640**	0.9324	**1.0581**	0.9278	0.9928
12	**1.2855**	**1.0754**	**1.0696**	**1.1256**	**1.4086**	**1.0304**	**1.2576**	**1.0373**	**1.0677**
18	**1.0602**	**1.0139**	**1.0300**	**1.0759**	**1.2622**	**1.0606**	**1.5235**	**1.0711**	**1.1308**
24	**1.0822**	**1.1066**	**1.1191**	**1.0546**	**1.1311**	**1.0264**	**1.8659**	**1.0370**	**1.1517**
36	**1.0271**	**1.1099**	**1.1640**	**1.2583**	**1.0606**	**1.0030**	**3.0841**	**1.0058**	**1.2004**
48	**1.0295**	**1.1272**	**1.1376**	**1.1225**	**1.0506**	**1.0066**	**5.6129**	**1.0067**	**1.1852**
Civilian labor force: employed. total									
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
1	0.7873	0.8415	0.8052	0.7814	0.7814	0.7626	0.7693	0.7648	0.7630
2	0.9026	0.9011	0.9001	0.9093	0.9203	0.8569	0.8448	0.8598	0.8494
4	0.9773	0.9746	0.9944	**1.0211**	**1.0558**	0.9219	0.9003	0.9206	0.8997
6	**1.0742**	**1.0765**	**1.0902**	**1.0680**	**1.1513**	0.9636	0.9496	0.9605	0.9425
12	**1.1298**	**1.1299**	**1.1278**	**1.0663**	**1.2258**	**1.0222**	**1.0426**	**1.0207**	**1.0054**
18	**1.0772**	**1.0789**	**1.0773**	**1.0014**	**1.1718**	**1.0362**	**1.1713**	**1.0386**	**1.0689**
24	**1.0348**	**1.0694**	**1.0379**	0.9745	**1.1121**	**1.0264**	**1.2945**	**1.0287**	**1.0973**
36	**1.0688**	**1.0159**	**1.0833**	**1.0614**	**1.0581**	**1.0126**	**1.7251**	**1.0096**	**1.1274**
48	0.9662	0.9671	0.9686	**1.0027**	**1.0460**	**1.0107**	**2.5154**	**1.0077**	**1.0976**
Consumer price index: all items									
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
1	0.8682	0.8577	0.8577	0.9098	0.9098	0.9015	0.9299	0.8731	0.8944
2	**1.0010**	**1.0087**	**1.0087**	0.9752	0.9872	0.9445	0.9565	0.9409	0.9304
4	**1.0520**	**1.0595**	**1.0595**	**1.0266**	**1.0040**	0.9880	**1.0529**	0.9769	0.9667
6	**1.0945**	**1.0936**	**1.0936**	0.9986	0.9746	0.9597	**1.0374**	0.9590	0.9579
12	**1.0851**	**1.0954**	**1.0954**	**1.0509**	0.9239	0.9296	**1.1613**	0.9277	**1.0108**
18	**1.0980**	**1.1052**	**1.1052**	**1.1106**	**1.0280**	0.9471	**1.1736**	0.9487	**1.0300**
24	**1.0953**	**1.2166**	**1.2166**	**1.0801**	**1.1334**	**1.0016**	**1.3459**	0.9984	**1.1031**
36	0.8870	0.8849	0.8849	**1.1451**	**1.0987**	0.9664	**1.6929**	0.9729	**1.0588**
48	**1.0179**	0.9973	0.9973	**1.0999**	**1.1204**	0.9743	**2.6601**	0.9793	**1.0415**
Federal funds rate									
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
1	0.9002	**8.4937**	**1.2179**	**2.7053**	**2.7053**	**1.8485**	**3.3781**	**1.7272**	**2.3129**
2	0.6651	**2.6883**	0.9494	**1.3967**	**1.4619**	**1.1288**	**2.2851**	**1.0609**	**1.5665**
4	0.5674	**1.1282**	0.7383	0.8433	0.8862	0.7775	**1.7299**	0.7221	**1.0459**
6	0.5911	0.7938	0.7123	0.7748	0.7323	0.6561	**1.5765**	0.6087	0.9183
12	0.5649	0.5675	0.6514	0.6155	0.6215	0.5523	**1.4987**	0.5108	0.7687
18	0.7200	0.7363	0.7386	0.7383	0.7830	0.6446	**2.0877**	0.5927	0.8585
24	0.7241	0.8756	0.7772	0.8196	0.8648	0.6865	**2.6209**	0.6433	0.8523
36	0.8176	0.7634	0.5109	**1.0183**	**1.0425**	0.7647	**4.3428**	0.7494	0.8688
48	0.7346	0.7846	0.9027	**1.3539**	**1.1781**	0.7844	**7.1595**	0.7880	0.7920

Table 3.II: RMSE relative to ARMA(p,q) forecasts

Most of the results in Table 3.III are in bold character meaning that FAVARMA models outperform the standard FAVAR specification for the most of horizons and identified VARMA forms. This is especially the case for Industrial production where all VARMA forms produce smaller MSE than VAR-based forecasts and the improvements seem to be more important with mid-term horizons and relative to Direct model. At best, VARMA-factor model improves the forecasting accuracy for 32% at horizon 12. In the case of Civilian labor force, considering VARMA helps in predicting, but the gain is less important and VAR-based model with multi-step forecasts perform even better for long-term horizons, while the iterative VAR-based model is outperformed by FAVARMA models in MA forms.

On the nominal side, the FAVARMA models in both MA forms seem to perform better than the VAR-based alternatives in predicting CPI, and this improvement rises

	Industrial production: total							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9500**	**0.9551**	**0.9671**	**0.9515**	**0.9500**	**0.9551**	**0.9671**	**0.9515**
2	**0.8934**	**0.9060**	**0.8971**	**0.8982**	**0.8508**	**0.8628**	**0.8544**	**0.8554**
4	**0.8353**	**0.9248**	**0.8375**	**0.8852**	**0.7325**	**0.8110**	**0.7344**	**0.7762**
6	**0.7875**	**0.8936**	**0.7836**	**0.8385**	**0.6836**	**0.7757**	**0.6802**	**0.7279**
12	**0.9154**	1.1173	**0.9215**	**0.9486**	**0.7315**	**0.8928**	**0.7364**	**0.7580**
18	**0.9858**	1.4161	**0.9956**	1.0510	**0.8403**	1.2071	**0.8487**	**0.8959**
24	**0.9733**	1.7694	**0.9833**	1.0921	**0.9075**	1.6496	**0.9168**	1.0182
36	**0.7971**	2.4510	**0.7993**	**0.9540**	**0.9457**	2.9080	**0.9484**	1.1318
48	**0.8968**	5.0005	**0.8969**	1.0559	**0.9581**	5.3424	**0.9582**	1.1281
	Civilian labor force: employed. total							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9759**	**0.9845**	**0.9787**	**0.9763**	**0.9759**	**0.9845**	**0.9787**	**0.9763**
2	**0.9424**	**0.9291**	**0.9456**	**0.9341**	**0.9312**	**0.9180**	**0.9343**	**0.9229**
4	**0.9029**	**0.8817**	**0.9016**	**0.8811**	**0.8732**	**0.8527**	**0.8720**	**0.8521**
6	**0.9023**	**0.8891**	**0.8994**	**0.8825**	**0.8370**	**0.8248**	**0.8343**	**0.8186**
12	**0.9587**	**0.9778**	**0.9573**	**0.9429**	**0.8339**	**0.8505**	**0.8327**	**0.8201**
18	1.0347	1.1696	1.0371	1.0674	**0.8843**	**0.9996**	**0.8863**	**0.9122**
24	1.0532	1.3283	1.0556	1.1260	**0.9229**	1.1640	**0.9250**	**0.9867**
36	**0.9540**	1.6253	**0.9512**	1.0622	**0.9571**	1.6305	**0.9542**	1.0655
48	1.0079	2.5086	1.0050	1.0946	**0.9663**	2.4048	**0.9635**	1.0494
	Consumer price index: all items							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9909**	1.0221	**0.9597**	0.9831	**0.9909**	1.0221	**0.9597**	**0.9831**
2	**0.9685**	**0.9808**	**0.9648**	0.9540	**0.9567**	**0.9689**	**0.9531**	**0.9424**
4	**0.9624**	1.0257	**0.9516**	0.9417	**0.9840**	1.0487	**0.9730**	**0.9628**
6	**0.9611**	1.0389	**0.9604**	0.9593	**0.9847**	1.0644	**0.9840**	**0.9829**
12	**0.8846**	1.1051	**0.8828**	0.9618	1.0062	1.2570	1.0042	1.0941
18	**0.8527**	1.0567	**0.8542**	0.9274	**0.9213**	1.1416	**0.9228**	1.0020
24	**0.9273**	1.2462	**0.9244**	1.0213	**0.8837**	1.1875	**0.8809**	**0.9732**
36	**0.8439**	1.4783	**0.8495**	0.9246	**0.8795**	1.5408	**0.8854**	**0.9637**
48	**0.8858**	2.4185	**0.8904**	0.9469	**0.8695**	2.3742	**0.8741**	**0.9295**
	Federal funds rate							
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.6833**	1.2487	**0.6385**	**0.8549**	**0.6833**	1.2487	**0.6385**	**0.8549**
2	**0.8082**	1.6361	**0.7596**	1.1216	**0.7721**	1.5631	**0.7257**	1.0716
4	**0.9220**	2.0514	**0.8563**	1.2402	**0.8773**	1.9521	**0.8149**	1.1802
6	**0.8468**	2.0346	**0.7856**	1.1851	**0.8959**	2.1527	**0.8312**	1.2539
12	**0.8973**	2.4349	**0.8300**	1.2489	**0.8886**	2.4112	**0.8219**	1.2368
18	**0.8730**	2.8278	**0.8028**	1.1629	**0.8232**	2.6662	**0.7569**	1.0964
24	**0.8377**	3.1978	**0.7849**	1.0400	**0.7939**	3.0306	**0.7439**	**0.9856**
36	**0.7509**	4.2646	**0.7359**	**0.8531**	**0.7335**	4.1659	**0.7189**	**0.8334**
48	**0.5794**	5.2881	**0.5820**	**0.5850**	**0.6658**	6.0772	**0.6689**	**0.6723**

Table 3.III: MSE of VARMA-based models relative to VAR-based forecasting factor model

with forecast horizons, and attains the maximum of 15%. Finally, the same VARMA forms perform quite well relatively to both Direct and Sequential models in case of Federal Funds Rate, and again the gain rises with forecast horizon attaining 42% at horizon 48.

Over all evaluation periods and forecasting horizons the estimated VARMA autoregressive and moving average orders were low: 1 and $[1,1,1,1]$ for DMA form, $[1,2,1,1]$ and 1 for DAR form, 1 and 2 for FMA form, and $[2-4]$ and 1 for FAR form. The estimated VAR order was in most occasions estimated to 2. In our robustness analysis we fixed VAR order to 4, 6 and 12, but the results have not changed substantially.

3.8.2 Number of factors in second-type forecasting models

Note that the previous results are obtained by fixing the number of factors in the second-type forecasting equations at 4 for all evaluation periods. In contrast, the number of factors can vary over time, and is estimated in the first-type forecasting equations. There exists a list of criteria to estimate the number of static factors but their success in practice is mitigated.

Another way to vary the number of factors in forecasting equations of the second type is to set it equal to one of the estimates in the first-type models, but the question remains: to which one? Generally, the estimated number of factors included in "Unrestricted "model is smaller than in "DI "since in the former the information contained in lags of the dependent variable is important enough to force the information criterion to pick a smaller number of factors.

In the robustness analysis we did the same forecasting exercise as in the previous section but where the number of factors in second-type models was the same as in one of the first-type models. The overall forecasting performance results from these exercises produce quite similar picture to the one in Tables 3.I and 3.III. This is not surprising with respect to VARMA-based models since the VARMA representations are closed under the marginalization.

3.9 Application to a small open economy: Canada

In the application with U.S. data, we had a situation where the time size of the informational panel is much bigger than the cross-section size, i.e. 600 time periods versus 128 series. Using Canadian data set from Boivin, Giannoni and Stevanović (2009) we perform the same pseudo-out-of-sample forecasting exercise as in the previous section. In this data set, there are 332 economic indicators measured from 1981 to 2008, which gives a time size of 334. The evaluation period is 1998-2008. All series were initially transformed to induce stationarity.

From the results in Table 3.IV we obtain quite similar conclusions as in the case of US data set, that is, VARMA-based forecasting models perform better for the most of

Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
					Employment					
1	1.0221	1.0165	1.0920	0.9854	0.9854	**0.9410**	0.9854	0.9601	1.0362	1.0151
2	0.9874	0.9751	0.9457	0.9998	0.9920	**0.9059**	0.9920	0.9236	1.0597	1.0092
4	1.0604	1.0865	1.1204	0.9783	0.9399	0.9298	0.9399	**0.9221**	1.0503	1.0060
6	1.1928	1.1408	1.1667	1.1130	0.9760	0.9641	0.9760	**0.9286**	1.0615	1.0011
12	**0.9822**	1.1197	1.2073	1.0402	0.9914	1.0194	0.9914	0.9938	1.0889	1.0760
18	1.2135	1.5923	1.6208	1.3230	**0.9792**	1.0282	1.0740	0.9845	1.1923	1.1054
24	1.3133	1.9476	1.9595	1.1989	**0.9803**	1.0290	1.0022	0.9819	1.1401	1.0937
36	1.7336	2.1289	2.2198	1.5687	0.9201	0.9395	0.9441	**0.9190**	1.0639	1.0442
48	1.7698	1.5115	1.2833	1.7333	0.9788	0.9734	0.9905	**0.9608**	1.0926	1.0829
					Consumer price index: all items					
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	0.8779	**0.8501**	0.8567	0.9146	0.9146	0.8563	0.9130	0.8647	0.9512	0.8811
2	0.9028	**0.8720**	0.8790	0.9946	0.9804	0.8895	0.9804	0.9040	0.9798	0.9226
4	0.9139	0.9082	0.9000	0.9737	0.9328	0.8826	0.9328	**0.8816**	0.9430	0.9069
6	0.8800	0.8701	0.8811	0.9307	0.8853	0.8403	0.8853	**0.8399**	0.8900	0.9062
12	0.9921	1.0585	1.0140	1.0178	0.9845	0.9318	0.9845	**0.9070**	1.0255	1.0207
18	1.0114	1.0143	**1.0083**	1.0362	1.0138	1.0504	1.0847	1.0130	1.0368	1.1184
24	0.9810	1.0563	1.0743	0.9671	0.9460	0.9655	0.9938	**0.9508**	1.0340	1.0804
36	**0.9844**	1.1165	1.1126	1.0140	1.0179	1.0325	1.0309	1.0160	1.1187	1.1287
48	**0.9919**	1.3307	1.3174	1.0908	1.0550	1.0415	1.0318	1.0554	1.1554	1.1832
					Producer price index: all manufacturing					
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR	ARMA
1	1.0079	1.0035	1.0094	1.0097	1.0097	0.9985	1.0070	1.0175	1.0443	**0.9931**
2	1.0088	0.9732	0.9835	1.0317	1.0077	0.9852	1.0077	0.9874	1.0499	**0.9729**
4	0.9841	1.0255	1.0280	1.0115	0.9810	0.9986	0.9810	0.9852	1.0483	**0.9803**
6	0.9759	1.0083	1.0103	0.9885	0.9701	0.9830	0.9701	0.9781	0.9958	**0.9580**
12	1.0246	1.0274	1.0294	1.0183	1.0142	**0.9916**	1.0142	0.9942	0.9973	1.0123
18	**0.9740**	0.9998	1.0026	0.9905	0.9828	0.9789	0.9837	0.9815	0.9894	0.9842
24	**0.9927**	1.0204	1.0230	1.0159	1.0027	0.9956	1.0018	0.9981	0.9984	1.0040
36	1.0363	1.0763	1.0947	0.9850	0.9831	0.9790	0.9814	0.9804	**0.9755**	0.9842
48	**0.9890**	1.0761	1.0632	0.9927	1.0108	1.0032	1.0050	1.0110	0.9969	1.0143

Table 3.IV: RMSE relative to Direct AR(p) forecasts

the horizons. In particular, VARMA factors produce the best forecasts of Employment at all horizons except for 12, 18 and 24 months ahead. It is interesting to note that MA forms perform better than AR forms. In the case of CPI, Diffusion index model does the best at short-term horizons of 1 and 2 months, at 18-month horizon and for the long-term forecasts of 3 and 4 years. FAVARMA in Final MA form outperforms other alternatives at horizons 4, 6, 12 and 24 months. Finally, the ARMA model produces the best predictions of PPI at short horizons of 1, 2, 4 and 6 months, while FAVARMA in Diagonal MA and Final AR forms does the best for horizons 12 and 36 respectively. As in the previous section, we want to see how the ARMA model compares to factor-based models in terms of MSE. The Table 3.V contains mean squared errors of all factor-based models predictions relative to ARMA forecasts. Bold characters represent the cases where ARMA model produces smaller MSE than the alternative. From the first part of Table 3.V, we see that ARMA model outperforms all three first-type factor-based models, except for the 2-month horizon. On the other side, three FAVARMA forms and Sequential VAR factors model do better than ARMA for all horizons, while the univariate model beats the Final AR form. In case of CPI, ARMA model does better at shorter horizons with respect to VARMA factors in both AR forms, and relative to both Direct and Sequential VAR factor specifications. However, it is outperformed by other factor-based models at most of the horizons. Finally, ARMA model seems to perform quite well in case of PPI. It does better than first-type models and Direct VAR-factor model at almost all horizons, while it outperforms other models at short-term horizons.

Finally, in Table 3.VI we redo the same exercise as in Table 3.III, but for Canadian application. The numbers in bold character represent cases where a VARMA-factor form produces smaller MSE than a VAR-factor alternative. As in the case of US data, we find that the FAVARMA models generally outperform the VAR-based factor forecasting models. This is especially the case for two MA forms that seem to be the best choices.

3.10 Structural analysis

In recent empirical macroeconomic literature the structural factor analysis has become very popular and using hundreds of observable economic indicators seems to overcome several problems in standard structural VAR literature. Essentially, bringing more information, and hence, spanning the space spanned by structural shocks, while keeping the model parsimonious, corrects for omitted variables problem and measurement issues.[9] Finally, Forni et al. (2009) shows that non-fundamentalness (case when the space spanned by structural shocks cannot be recovered by current and past observables), which is a likely feature of small-scale models, cannot arise in large dimensional dynamic factor framework.

In this section, we reconsider the empirical application from Bernanke, Boivin and Eliasz (2005). We use exactly the same stationary data, the same method to extract factors (by principal components) and the same way to impose the observed factor (Federal

9. As pointed out in Bernanke, Boivin and Eliasz (2005), the small-scale VAR model presents three issues. Due to small amount of information in the model relative to the information set potentially observed by agents, it easily suffers from omitted variable problem that can alter the impulse response analysis. The second problem in small-scale VAR model is that the choice of a specific data series to represent a general economic concept is arbitrary. Moreover, measurement errors, aggregation and revisions pose additional problems for linking theoretical concepts to specific data series. Even if the two previous problems do not occur, i.e. a small scale VAR is well defined and the shock is well identified, we can produce impulse responses only for variables included in the VAR.

Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
					Employment				
1	**1.0069**	**1.0014**	**1.0758**	0.9707	0.9707	0.9270	0.9707	0.9458	**1.0208**
2	0.9784	0.9662	0.9371	0.9907	0.9830	0.8976	0.9830	0.9152	**1.0500**
4	**1.0541**	**1.0800**	**1.1137**	0.9725	0.9343	0.9243	0.9343	0.9166	**1.0440**
6	**1.1915**	**1.1395**	**1.1654**	**1.1118**	0.9749	0.9630	0.9749	0.9276	**1.0603**
12	0.9128	**1.0406**	**1.1220**	0.9667	0.9214	0.9474	0.9214	0.9236	**1.0120**
18	**1.0978**	**1.4405**	**1.4663**	**1.1969**	0.8858	0.9302	0.9716	0.8906	**1.0786**
24	**1.2008**	**1.7807**	**1.7916**	**1.0962**	0.8963	0.9408	0.9163	0.8978	**1.0424**
36	**1.6602**	**2.0388**	**2.1258**	**1.5023**	0.8812	0.8997	0.9041	0.8801	**1.0189**
48	**1.6343**	**1.3958**	**1.1851**	**1.6006**	0.9039	0.8989	0.9147	0.8872	**1.0090**
					Consumer price index: all items				
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
1	0.9964	0.9648	0.9723	**1.0380**	**1.0380**	0.9719	**1.0362**	0.9814	**1.0796**
2	0.9785	0.9452	0.9527	**1.0780**	**1.0626**	0.9641	**1.0626**	0.9798	**1.0620**
4	**1.0077**	**1.0014**	0.9924	**1.0737**	**1.0286**	0.9732	**1.0286**	0.9721	**1.0398**
6	0.9711	0.9602	0.9723	**1.0270**	0.9769	0.9273	0.9769	0.9268	0.9821
12	0.9720	**1.0370**	0.9934	0.9972	0.9645	0.9129	0.9645	0.8886	**1.0047**
18	0.9043	0.9069	0.9016	0.9265	0.9065	0.9392	0.9699	0.9058	0.9270
24	0.9080	0.9777	0.9944	0.8951	0.8756	0.8937	0.9198	0.8800	0.9571
36	0.8722	0.9892	0.9857	0.8984	0.9018	0.9148	0.9134	0.9002	0.9911
48	0.8383	**1.1247**	**1.1134**	0.9219	0.8916	0.8802	0.8720	0.8920	0.9765
					Producer price index: all manufacturing				
Horizon	Unrestricted	DI	DI AR	Direct	Sequential	Diag MA	Diag AR	Final MA	Final AR
1	**1.0149**	**1.0105**	**1.0164**	**1.0167**	**1.0167**	**1.0054**	**1.0140**	**1.0246**	**1.0516**
2	**1.0369**	**1.0003**	**1.0109**	**1.0604**	**1.0358**	**1.0126**	**1.0358**	**1.0149**	**1.0791**
4	**1.0039**	**1.0461**	**1.0487**	**1.0318**	**1.0007**	**1.0187**	**1.0007**	**1.0050**	**1.0694**
6	**1.0187**	**1.0525**	**1.0546**	**1.0318**	**1.0126**	**1.0261**	**1.0126**	**1.0210**	**1.0395**
12	**1.0122**	**1.0149**	**1.0169**	**1.0059**	**1.0019**	0.9796	**1.0019**	0.9821	0.9852
18	0.9896	**1.0159**	**1.0187**	**1.0064**	0.9986	0.9946	0.9995	0.9973	**1.0053**
24	0.9887	**1.0163**	**1.0189**	**1.0119**	0.9987	0.9916	0.9978	0.9941	0.9944
36	**1.0529**	**1.0936**	**1.1123**	**1.0008**	0.9989	0.9947	0.9972	0.9961	0.9912
48	0.9751	**1.0609**	**1.0482**	0.9787	0.9965	0.9891	0.9908	0.9967	0.9828

Table 3.V: RMSE relative to ARMA(p,q) forecasts

					Employment			
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9500**	**0.9551**	**0.9671**	**0.9515**	**0.9500**	**0.9551**	**0.9671**	**0.9515**
2	**0.8934**	**0.9060**	**0.8971**	**0.8982**	**0.8508**	**0.8628**	**0.8544**	**0.8554**
4	**0.8353**	**0.9248**	**0.8375**	**0.8852**	**0.7325**	**0.8110**	**0.7344**	**0.7762**
6	**0.7875**	**0.8936**	**0.7836**	**0.8385**	**0.6836**	**0.7757**	**0.6802**	**0.7279**
12	**0.9154**	1.1173	**0.9215**	**0.9486**	**0.7315**	**0.8928**	**0.7364**	**0.7580**
18	**0.9858**	1.4161	**0.9956**	1.0510	**0.8403**	1.2071	**0.8487**	**0.8959**
24	**0.9733**	1.7694	**0.9833**	1.0921	**0.9075**	1.6496	**0.9168**	1.0182
36	**0.7971**	2.4510	**0.7993**	**0.9540**	**0.9457**	2.9080	**0.9484**	1.1318
48	**0.8968**	5.0005	**0.8969**	1.0559	**0.9581**	5.3424	**0.9582**	1.1281
					Consumer price index: all items			
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9759**	**0.9845**	**0.9787**	**0.9763**	**0.9759**	**0.9845**	**0.9787**	**0.9763**
2	**0.9424**	**0.9291**	**0.9456**	**0.9341**	**0.9312**	**0.9180**	**0.9343**	**0.9229**
4	**0.9029**	**0.8817**	**0.9016**	**0.8811**	**0.8732**	**0.8527**	**0.8720**	**0.8521**
6	**0.9023**	**0.8891**	**0.8994**	**0.8825**	**0.8370**	**0.8248**	**0.8343**	**0.8186**
12	**0.9587**	**0.9778**	**0.9573**	**0.9429**	**0.8339**	**0.8505**	**0.8327**	**0.8201**
18	1.0347	1.1696	1.0371	1.0674	**0.8843**	**0.9996**	**0.8863**	**0.9122**
24	1.0532	1.3283	1.0556	1.1260	**0.9229**	1.1640	**0.9250**	**0.9867**
36	**0.9540**	1.6253	**0.9512**	1.0622	**0.9571**	1.6305	**0.9542**	1.0655
48	1.0079	2.5086	1.0050	1.0946	**0.9663**	2.4048	**0.9635**	1.0494
					Producer price index: all manufacturing			
	VARMA/Direct				VARMA/Sequential			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	**0.9909**	1.0221	**0.9597**	**0.9831**	**0.9909**	1.0221	**0.9597**	**0.9831**
2	**0.9685**	**0.9808**	**0.9648**	**0.9540**	**0.9567**	**0.9689**	**0.9531**	**0.9424**
4	**0.9624**	1.0257	**0.9516**	**0.9417**	**0.9840**	1.0487	**0.9730**	**0.9628**
6	**0.9611**	1.0389	**0.9604**	**0.9593**	**0.9847**	1.0644	**0.9840**	**0.9829**
12	**0.8846**	1.1051	**0.8828**	**0.9618**	1.0062	1.2570	1.0042	1.0941
18	**0.8527**	1.0567	**0.8542**	**0.9274**	**0.9213**	1.1416	**0.9228**	1.0020
24	**0.9273**	1.2462	**0.9244**	1.0213	**0.8837**	1.1875	**0.8809**	**0.9732**
36	**0.8439**	1.4783	**0.8495**	**0.9246**	**0.8795**	1.5408	**0.8854**	**0.9637**
48	**0.8858**	2.4185	**0.8904**	**0.9469**	**0.8695**	2.3742	**0.8741**	**0.9295**

Table 3.VI: MSE of VARMA-based models relative to VAR-based forecasting factor model

Funds Rate). The difference is that we allow VARMA dynamics on static factors instead of imposing a finite order VAR representation. The monetary policy shock is identified from Cholesky decomposition of the residual covariance matrix in factor transition equation, meaning that the observed factor was ordered last. We considered all four identified VARMA forms, but retained only the diagonal MA representation (see discussion in section 3.2.2). The number of latent factors is five, and we fitted a VARMA (2.1) (these orders were estimated using the information criterion in Dufour and Pelletier (2008)). In Figure 3.1 we present the impulse responses of some economic indicators of interest estimated from model with VAR factors (FAVAR) and model with VARMA factors in DMA form (FAVARMA-DMA). The FAVAR impulse responses were computed for several VAR orders (Akaike information criterion estimate is 14). We can see that, for many series, FAVAR impulse responses tend to FAVARMA impulse responses with increasing the VAR lag order. This result implies that one needs to estimate a very long VAR to get similar response from a more parsimonious VARMA model. Hence, in the case of FAVARMA, only 84 VARMA parameters must be estimated compared to 510 VAR coefficients using FAVAR model with lag order 14.

In Figure 3.2 we present the same FAVARMA-DMA impulse responses but with the 90% confidence intervals computed using 5000 bootstrap replications. As expected from theory, a positive monetary policy shock generates a significant and very persistent economic downturn. A remark on the bootstrap must be made. The approximation of the true factor process could be important when choosing the parametric bootstrap procedure to obtain statistical inference on objects of interest.[10] Following Yamamoto (2009) suggestion, we resample factors transition equation to get the bootstrap factors. These are then used to obtain bootstrap information panel by resampling from the observation equation residuals. Finally, new factors are extracted, their VARMA process is estimated and impulse responses are computed. Hence, having a good approximation of the true factor process can be very important in order to get the right bootstrap procedure. If the finite VAR approximation is far away from the truth, and if the finite VARMA representation does much better, allowing for MA part will provide more reliable inference. Moreover, in practice the number of estimated parameters is important,

10. Another possibility is to use a nonparametric block bootstrap procedure by resampling time-size blocks and keeping the cross-section dimension fixed. Nevertheless, the choice of the size of the time block and the way to induce stationary series seem to be important variables

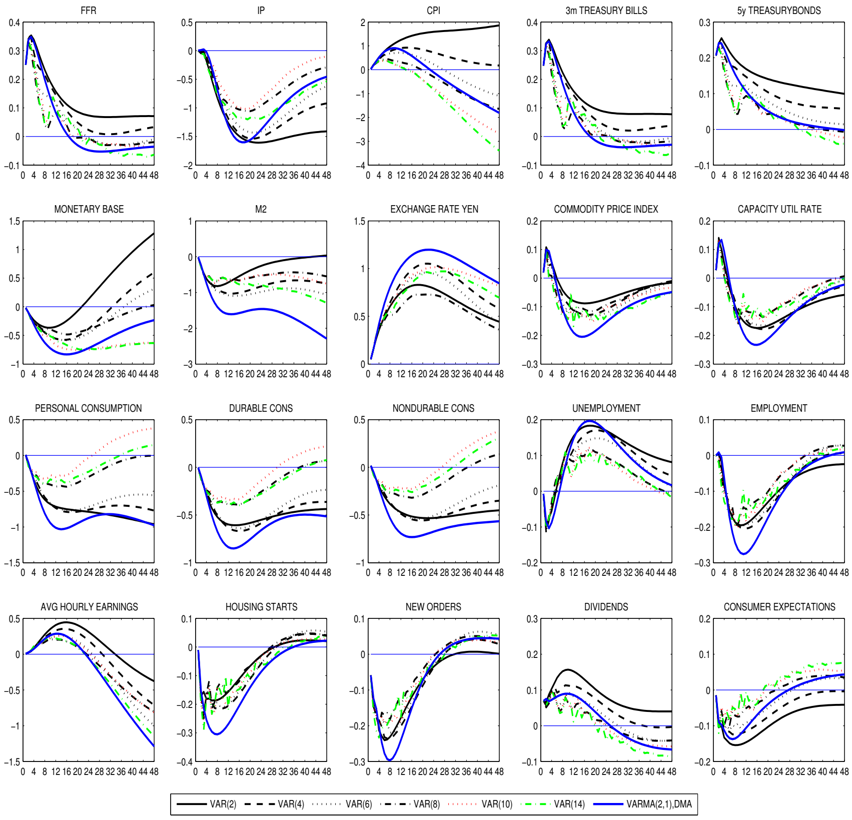

Figure 3.1: Comparison between FAVAR and FAVARMA-DMA impulse responses

and if a long VAR is needed, considering a more parsimonious representation, such as VARMA, should help.

3.11 Conclusion

In this paper, we studied the relationship between VARMA and factor representations of a vector stochastic process and proposed the FAVARMA model. We started by observing that in general multivariate series and associated factors do not both follow a finite order VAR process. When the factors are obtained as linear combinations of observable series, their dynamic process is generally a VARMA and not a finite-order VAR. Also, we showed that even if the factors follow a finite-order VAR process, this

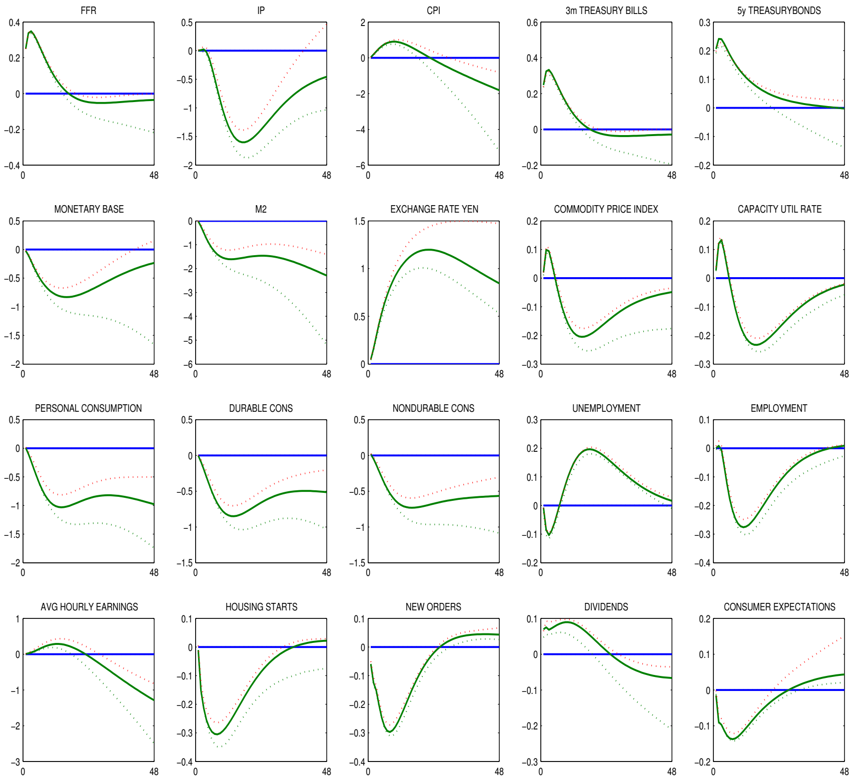

Figure 3.2: FAVARMA-DMA impulse responses to monetary policy shock

implies a VARMA representation for the observable series. As result, we proposed the FAVARMA framework that combines two parsimonious methods to represent the dynamic interactions between a large number of time series: factor analysis and VARMA modeling.

In order to illustrate the performance of our model we performed a series of Monte Carlo simulations and found that VARMA specifications help a lot especially in small sample cases where the best improvement occurred at longer horizons, but also in cases where the sample sizes were comparable to our empirical exercise.

We applied our approach in two pseudo-out-of-sample forecasting exercises using large U.S. and Canadian monthly panels taken from Boivin, Giannoni and Stevanović (2009, 2008) respectively. The results showed that VARMA factors help in predicting several key macroeconomic aggregates relative to standard factor forecasting models. In particular, we found that the FAVARMA models generally outperform the VAR-factor forecasting models, and this is especially the case for two MA VARMA-factor specifications that seem to be the best choices.

Finally, we estimated the effect of monetary policy using the data and the identification scheme as in Bernanke, Boivin and Eliasz (2005). We found that impulse responses from a parsimonious VARMA (2.1) factor model give an accurate and comprehensive picture of the effect and the transmission of monetary policy in U.S.. To get similar responses from a standard FAVAR model, Akaike information criterion estimates the lag order of 14. Hence, only 84 coefficients governing the factors dynamics need to be estimated in the FAVARMA framework, compared to FAVAR model with 510 VAR parameters.

CHAPTER 4

CREDIT SHOCKS TRANSMISSION IN A SMALL OPEN ECONOMY: A FACTOR-AUGMENTED VARMA APPROACH

4.1 Introduction

The current economic downturn suggests that there is information in the financial sector which has not been integrated into our understanding of macroeconomics. Studies, among others, by Stock and Watson (1989), Estrella and Hadrouvelis (1991), Gertler and Lown (1999), Diebold *et al.* (2006), Mueller (2007), and Gilchrist, Yankov, and Zakrajsek (2009) have shown that there is predictive content in financial series. The results in Forni *et al.* (2003) show that financial variables are important when forecasting inflation rates but do not help in predicting industrial production, which is also confirmed in Espinoza *et al.* (2009). Moreover, the non-neoclassical channels of monetary policy transmission mechanisms which are related to credit markets are theoretically and empirically under-documented. We propose to empirically measure the impact of credit shocks in Canada within this theoretical framework.

Due to the complexity of credit markets, we doubt that their informational content can be synthesized in as few variables as a vector autoregressive (VAR) model allows us. In order to incorporate information from a large number of economic and financial indicators, we will use the structural factor analysis approach proposed by Bernanke, Boivin, and Eliasz (2005), Marcellino and Kapetainous (2005), and Stock and Watson (2005), among others. In particular, we will use the factor-augmented VARMA (FAVARMA) model proposed by Dufour and Stevanovic (2010). This is a theoretically coherent model with an approach that combines two dimension reduction techniques: factor analysis and VARMA modeling. The identification of structural shocks is achieved by imposing a recursive structure on the impact matrix of the structural MA representation of observable variables.

Similar studies have been done for the US economy by Boivin, Giannoni, and Stevanovic (2010) (BGS hereafter) and Gilchrist, Yankov, and Zakrajsek (2009). Both studies find that credit shocks have wide effects on the economy that are consistent with a significant economic slowdown. Safei and Cameron (2003) and Atta-Mensah and Dib (2008) have studied the dynamics of the Canadian credit market, the former according to a structural VAR approach, the latter according to a DSGE approach. The conclusions drawn by Safei and Cameron (2003) show a lack of robustness, suggesting that there is information missing in their structural VAR models. As in BGS, the present exercise will correct this problem using a large data set. The results of Atta-Mensah and Dib (2008) are more coherent with dynamic stochastic general equilibrium (DSGE) literature describing credit market models, except that they consider Canada as a closed economy. Our methodology will allow us to include more information about the global financial market and to simulate shocks from outside of Canada, which will be important in our following discussion.

Our results show that an unexpected increase in the external finance premium on global financial markets, approximated by the US credit spread, generates a significant and persistent economic slowdown in Canada. Canadian credit spreads rise immediately, while interest rates and credit measures decline. Contrary to existing work on the Canadian economy, we find that price indexes fall persistently. Since we do not impose timing restrictions on forward-looking variables, these leading indicators respond negatively on impact, as expected. This gives a more realistic picture of the effect of credit shocks on the economy and provides information about the transmission mechanism of these shocks. According to R^2 results, the common component captures an important dimension of the business cycle movements. From the variance decomposition analysis, we observe that the credit shock has an important effect on several real activity measures including price indicators, leading indicators, and credit spreads.

Another piece of important empirical evidence concerns the identification of national financial shocks. Previous studies have treated Canada as a closed economy when identifying a credit shock and have found some real effects. Our results suggest that there is no significant effect of domestic shocks in Canada. Indeed, the effects of credit shocks in Canada are essentially caused by the unexpected changes in foreign credit market conditions.

Finally, a by-product of our identification approach is a rotation matrix that can be used to recover the structural factors. These rotation matrices still have the same informational content, but their interpretation, in terms of the correlation structure, can change. Indeed, we find that the rotated principal components do have an economic interpretation.

In the rest of the paper, we first present the theoretical framework in which credit shocks can occur. Then, we present our econometric framework in a data-rich environment and discuss the estimation and identification issues. The main results are presented in Section 5, followed by a conclusion. The Appendix contains some additional results, the explanation of the bootstrap procedure, and the data description.

4.2 Theoretical framework

In this section we briefly discuss how the financial and economic sides are connected and through which channel(s) shocks on the credit market could affect economic activity. Financial frictions are crucial when linking the credit market conditions to economic activity. In a framework of incomplete information, the Modigliani-Miller theorem does not apply, implying that a firm's value is affected by its capital structure. After aggregation, if credit markets determine capital structure in the economy we should observe informational frictions determining the firm's value. Frictions can arise from both supply and demand.

On the supply side, usually interpreted as the bank lending channel, Bernanke (1993) observes that banks and other financial intermediaries are able to fund projects which are complex to evaluate, using funds from investors that have only partial information about these projects. If banks resolve asymmetric information problems in the credit market, they can be considered credit creators and their health becomes an important macroeconomic parameter. However, because of the democratization of credit in the 1980s, informational frictions on the supply side seem to be less present. Dynan, Elmendorf, and Sichel (2006) provide empirical evidence that households' expenses are less sensitive to their income, encouraging us to look for other kinds of frictions.

On the demand side, which links to the balance sheet channel, BGG introduce the idea of a financial accelerator working through the interaction of two measures. First is the external finance premium, defined as the difference between the external cost of capital and the internal opportunity cost of capital. Second is the net worth of potential borrowers used to measure collateral that firms are able to offer to obtain credit. The idea of the financial accelerator is an inverse relation between these two measures. If the net worth of a firm falls, the collateral value that firms will be able to present to banks will also fall. Similarly, the firm's contribution to capital will also decline. In consequence, the bank will possess relatively more parts of the firm, creating an agency cost to solve the divergence between both parts. This agency cost will raise the external finance premium, i.e. the firm's capital cost. Then the financial accelerator mechanism works as follows: a fall in net worth (due to financial crisis, for example) raises the acquisition capital cost, pushing firms to invest a sub-optimal quantity of capital and creating a persistent effect from the original crisis.

Building on Bernanke, Gertler and Gilchrist (1999) (BGG hereafter), Gilchrist, Ortiz, and Zakrajsek (2009) aim to quantify the role of financial frictions in the business cycle fluctuations. They augment a standard DSGE model with the financial accelerator mechanism that links the conditions on the credit market to the real economy through the external finance premium. Two financial shocks are introduced: financial disturbance shock, which affects external finance premium, and net worth shock affecting the balance sheet of a firm. The first shock is presented as a credit supply shock, which Christiano, Motto and Rostagno (2009) interpret as an increase in the agency costs due to a higher variance of idiosyncratic shocks affecting the firm's profitability. The second shock can be viewed as a credit demand shock. Its effect will depend on the degree of financial market frictions. After estimating the structural model, authors find that both financial shocks cause an increase in external finance premium that, through the financial accelerator, implies a slowdown in economic activity. Finally, Bloom (2009) provides a framework to analyze the impact of uncertainty shocks. He finds that increased volatility generates short, but sharp, recessions an recoveries.

4.3 Econometric framework in data-rich environment

As information technology improves, the availability of economic and finance time series grows in terms of both time and cross-section size. However, a large amount of information can lead to the curse of dimensionality problem when standard time series tools are used. Since most of these series are highly correlated, at least within some categories, their co-variability pattern and informational content can be approximated by a smaller number of variables. A popular way to address this issue is to use factor analysis. The structural factor model approach will here be used to identify a structural shock and its effects on economy.

Previous studies have used standard VAR techniques with recursive identification schemes to identify credit shocks. However, as pointed out in Bernanke, Boivin and Eliasz (2005), the small-scale VAR model presents three issues. First, due to the small amount of information in the model, relative to the information set potentially observed by agents, VAR suffers from an omitted variable problem which can alter the impulse response analysis. The second problem in small-scale VAR model is that the choice of a specific data series to represent a general economic concept is arbitrary. Moreover, measurement errors, aggregations, and revisions present additional problems when linking theoretical concepts to specific data series. Even if the previous problems do not occur, we can only produce impulse responses for the variables included in the VAR. Finally, Forni et al. (2009) argues that while non-fundamentalness is generic of a small scale model, it cannot arise in a large dimensional dynamic factor models [1]. This is of primary importance since the objective is to identify a relatively new structural shock in empirical macroeconomics.

One way to address all of these issues is to take advantage of information contained in large panel data sets using dynamic factor analysis and the factor-augmented VAR (FAVAR) model in particular. The importance of large data sets and factor analysis is well documented in both forecasting macroeconomic aggregates and structural analysis.

1. If the shocks in the VAR model are fundamental, then the dynamic effects implied by the moving average representation can have a meaningful interpretation, i.e. the structural shocks can be recovered from current and past values of observable series.

Boivin, Giannoni and Stevanovic (2009) has recently shown that incorporating information through a small number of factors corrects for several empirical puzzles when estimating the effect of monetary policy shocks in a small open economy. However, Dufour and Stevanovic (2010) argue that in general, multivariate series and associated factors do not both follow a finite order VAR process. Hence, they propose the FAVARMA framework that combines two parsimonious methods to represent the dynamic interactions between a large number of time series: factor analysis and VARMA modeling.

4.3.1 Factor-augmented VARMA model

Using the notation as in Dufour and Stevanovic (2010), the dynamic factor model (DFM) where factors have a finite order VARMA(p_f,q_f) representation can be written as

$$X_{it} = \tilde{\lambda}_i(L)f_t + u_{it}, \quad i = 1,\ldots,N, \quad t = 1,\ldots,T \tag{4.1}$$

$$u_{it} = \delta_i(L)u_{i,t-1} + \nu_{it} \tag{4.2}$$

$$f_t = \Gamma(L)f_{t-1} + \Theta(L)\eta_t \tag{4.3}$$

where $\tilde{\lambda}_i(L)$ is a lag polynomial, $\delta_i(L)$ is a $p_{x,i}$-degree lag polynomial, $\Gamma(L) = [\Gamma_1 L + \ldots + \Gamma_{p_f} L^{p_f}]$, $\Theta(L) = [I - \Theta_1 L - \ldots - \Theta_{q_f} L^{q_f}]$, and ν_{it} is an N-dimensional white noise uncorrelated with q-dimensional white noise process η_t. The equation (4.1) relates observable variable X_{it} to q (latent) factors, f_t, and to its idiosyncratic component, u_{it}. The element $\tilde{\lambda}_i(L)f_t$ is called the common component. We also allow for some limited cross-section correlations among the idiosyncratic components [2].

Subtracting $\delta_i(L)u_{it-1}$ from both sides of (4.1) gives the DFM with serially uncorrelated

2. Such that there exists a small number of largest eigenvalues of the covariance matrix of common components that diverge when the number of series tends to infinity, while the remaining eigenvalues as well as the eigenvalues of the covariance matrix of specific components are bounded. See Bai and Ng (2008) for an overview of the modern factor analysis literature, and distinction between exact and approximate factor models.

idiosyncratic errors:

$$X_{it} = \lambda_i(L) f_t + \delta_i(L) X_{it-1} + \nu_{it}, \tag{4.4}$$

where $\lambda_i(L) = (1 - \delta_i(L)L)\tilde{\lambda}_i(L)$.

Then, we can rewrite the DFM in the following form:

$$\begin{aligned} X_t &= \lambda(L) f_t + D(L) X_{t-1} + \nu_t & (4.5) \\ f_t &= \Gamma(L) f_{t-1} + \Theta(L)\eta_t & (4.6) \end{aligned}$$

where

$$\lambda(L) = \begin{bmatrix} \lambda_1(L) \\ \vdots \\ \lambda_n(L) \end{bmatrix}, D(L) = \begin{bmatrix} \delta_1(L) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \delta_n(L) \end{bmatrix}, \nu_t = \begin{bmatrix} \nu_{1t} \\ \vdots \\ \nu_{nt} \end{bmatrix}.$$

To obtain the static version of the previous factor model suppose that $\tilde{\lambda}(L)$ has finite degree p-1, and let $F_t = [f_t' \quad f_{t-1}' \ldots f_{t-p+1}']'$. Let the dimension of F_t be K, where $q \leq K \leq qp$. Then,

$$\begin{aligned} X_t &= \Lambda F_t + u_t & (4.7) \\ u_t &= D(L) u_{t-1} + \nu_t & (4.8) \\ F_t &= \Phi(L) F_{t-1} + G\Theta(L)\eta_t & (4.9) \end{aligned}$$

where Λ is a $N \times K$ matrix where the i^{th} row consists of coefficients of $\tilde{\lambda}_i(L)$, $\Phi(L)$ contains coefficients of $\Gamma(L)$ and zeros, and G is $K \times q$ matrix that loads (structural) shocks η_t to static factors (consists of 1's and 0's). Note that if $\Theta(L) = I$, we obtain the factor-augmented VAR (FAVAR) model.

Finally, since the VARMA models are not identified in general, we will impose the diagonal moving average representation that is presented in following definition.

Definition 1. (Diagonal MA equation form) *Suppose N-dimensional stochastic process X_t has the following VARMA representation:*

$$\Phi(L)X_t = \Theta(L)u_t$$

This VARMA representation is said to be in diagonal MA equation form if $\Theta(L) = \mathrm{diag}[\theta_{ii}(L)] = I_N - \Theta_1 L - \cdots - \Theta_q L^q$ *where* $\theta_{ii}(L) = 1 - \theta_{ii,1}L - \cdots - \theta_{ii,q_i}L^{q_i}$, $\theta_{ii,q_i} \neq 0$, *and* $q = \max_{1\leq i\leq N}(q_i)$.

From the point of view of practitioners, this form is very appealing since adding lags of u_{it} to the $i^t h$ equation is a natural extension of the VAR model. It also has the advantage of putting the simple structure on MA polynomials, the part which complicates the estimation.

4.3.2 Estimation

We will work with the static version (4.7-4.9). Also, we assume the same number of dynamic and static factors, $G = I$, and no autocorrelations in idiosyncratic component, $D(L) = 0$, which gives the following simplified model:

$$X_t = \Lambda F_t + \nu_t \tag{4.10}$$

$$F_t = \Phi(L)F_{t-1} + \Theta(L)\eta_t \tag{4.11}$$

To estimate this model, we use the two-step Principal Component Analysis (PCA) estimation method (see Stock and Watson (2002a), and Bai and Ng (2006) for theoretical results concerning the PCA estimator). In the first step, $\hat{F}_t$ are computed as K principal components of X_t. In the second step, we estimate the VARMA representation (4.11) using $\hat{F}_t$.

The standard estimation methods for VARMA models are maximum likelihood and nonlinear least squares. Unfortunately, these methods require nonlinear optimization, which may not be feasible when the number of parameters is relatively large. In this paper, we will use the GLS method proposed in Dufour and Pelletier (2008).

Since the unobserved factors are estimated and then included as regressors in the FAVARMA model, the two-step approach suffers from the “generated regressors”problem. To get an accurate statistical inference on the impulse response functions that accounts for uncertainty associated to factors estimation, we use a bootstrap procedure suggested by Yamamoto (2009) and implemented in Dufour and Stevanovic (2010). The details about the bootstrap procedure are presented in the Appendix.

4.3.3 Identification of structural shocks

To identify the structural shocks, we adapt the contemporaneous timing restrictions procedure proposed in Stock and Watson (2005) to the FAVARMA framework. After inverting the VARMA process of factors in (4.11), assuming stationarity, and plugging it in (4.10), we obtain the MA representation of X_t:

$$\begin{aligned} X_t &= \Lambda[I-\Phi(L)L]^{-1}\Theta(L)\eta_t + u_t \\ &= B(L)\eta_t + u_t. \end{aligned} \tag{4.12}$$

We assume that the number of static factors, K, is equal to the number of dynamic factors and that structural shocks ε_t are linear combinations of residuals in (4.11)

$$\varepsilon_t = H\eta_t, \tag{4.13}$$

where H is a nonsingular square matrix and $\mathrm{E}[\varepsilon_t\varepsilon_t'] = I$. Replacing (4.13) in (4.12) gives the structural MA form of X_t:

$$\begin{aligned} X_t &= \Lambda[I-\Phi(L)L]^{-1}\Theta(L)H^{-1}\varepsilon_t + u_t \\ &= B^{\star}(L)\varepsilon_t + u_t. \end{aligned} \tag{4.14}$$

To achieve the identification of shocks in ε_t, the contemporaneous timing restrictions are imposed on the impact matrix in (4.14)

$$B_0^\star \equiv B^\star(0) = \begin{bmatrix} x & 0 & \cdots & 0 \\ x & x & \ddots & 0 \\ x & x & \ddots & 0 \\ x & x & \cdots & x \\ \vdots & \vdots & \vdots & \vdots \\ x & x & \cdots & x \end{bmatrix}.$$

Let $B_{0:K}^\star = B_{0:K}H^{-1}$ be a $K \times K$ lower triangular matrix, where $B_{0:K}$ contains first K rows of B_0. Then, H is obtained as

$$H = [\text{Chol}(B_{0:K}\Sigma_e B_{0:K}')]^{-1}\Lambda_K, \tag{4.15}$$

where Σ_η is covariance matrix of η_t and Λ_K is $K \times K$ matrix of first K rows of Λ. To estimate H, we just plug the estimates of $B_{0:K}$, Σ_e and Λ_K. Hence, the impulse responses to any shock in ε_t are obtained using (4.14). This identification procedure is similar to the standard recursive identification in VARMA models. To just-identify the K structural shocks, we need to impose $K(K-1)/2$ restrictions. Imposing them in a recursive way makes estimation of the rotation matrix H easy. Also, it should be noted that the number of static factors must be equal to the number of series used in recursive identification. Moreover, contrary to other identification strategies in FAVAR literature, we do not need to impose any observed factor or rely on the interpretation of a particular latent factor.

4.4 Data

The majority of our data comes from Dufour and Stevanovic (2010). It contains 332 monthly StatCan series that synthesize real and financial Canadian activity. Also included are variables describing a small open economy: exchange rates and global financial information. The time span is from January 1986 to November 2009.

Credit spreads measuring credit market conditions are also included as additional series. A credit spread is defined by the difference between the actuarial rate of a firm bond and the actuarial rate of a risk-free product (typically a treasury bond). We have built American credit spreads using Moody's bond index as described in BGS. Canadian credit spreads has been built using a Canadian Dex bond index rated AA. Table 4.I synthesizes information about the credit spread for Canada and the US.

Because our results are very similar from one spread to another, we have selected a Canadian 10 Year A Spread and an American 10 Year B spread. The two series are plotted in Figure 4.1.

4.5 Results

The goal of this paper is to measure the dynamic effects of credit shocks on economic activity in Canada. Since we are looking at a small open economy it is important to control for global influence on financial markets when identifying the credit shock effects. In previous studies, authors have considered Canada to be a closed economy, but our empirical evidence suggests this could be misleading. Indeed, our results show that the effect of credit shock is essentially driven by global financial conditions and by US credit markets in particular. Given the fact that the US represents around 80% of foreign trade in Canada, we approximate the world financial conditions with the US proxies. Hence, we use the US 10-year credit spread (USspread10y) in the recursive identification scheme. On the other hand, we take the Canadian 10-year credit spread (CANspread10y) as a proxy to identify the national credit shock. In all specifications the lag order tests suggest a VARMA(2,1) process for extracted factors.

4.5.1 Global credit shock

To identify the global credit shock, we impose the following recursive scheme such that $B^{\star}_{0:K}$ is lower triangular:

$$[USspread10y, \quad CPI, \quad UR, \quad MS, \quad R, \quad FX]$$

Series label	**Description**
SCM2AST(RY)	Bond Yeld: DEX Capital Overall AA Short Term (% per Annum)
SCM2AMT(RY)	Bond Yeld: DEX Capital Overall AA Mid Term (% per Annum)
SCM2ALG(RY)	Bond Yeld: DEX Capital Overall AA Long Term (% per Annum)
v122531	Interest Rateă: T-bills 3 Months (% per Annum)
v122499	Interest Rateă:Gov. of Can.marketable Bond, 1-3 years (% per Annum)
v122501	Interest Rateă:Gov. of Can. marketable Bond, over 10 years (% per Annum)
FYAAAC	Bond Yeld: MoodyŠs AAA Corporate (% per Annum)
FYBAAC	Bond Yeld: MoodyŠs BAA Corporate (% per Annum)
FYGT1.M	Rate: U.S. Treasury Const. Maturities, 1-Year (% Per Annum, NSA)
FYGT10.M	Rate: U.S. Treasury Const. Maturities, 10-Year (% Per Annum, NSA)
	Canadian credit spreads
3 Months A Spread	SCM2AST(RY) - v122531
1 Year A Spread	SCM2AMT(RY) - v122499
10 Year A Spread	SCM2ALT(RY) - v122501
	US credit spreads
10 Year B Spread	FYBAAC - FYGT10.M
10 Year A Spread	FYBAAC - FYGT10.M
1 Year B Spread	FYBAAC - FYGT1.M

Table 4.I: Credit spreads

where CPI is the Consumer Price Index: all items, UR is the Unemployment Rate, MS is the Money Base, R is the 3-month Treasury Bill and FX stands for the Can/US Exchange Rate.

The credit shock is the first element in ε_t. This identification scheme implies that Canadian CPI, UR, MS, R and FX can respond immediately to a credit shock in the US. In other words, the contemporaneous response to a credit shock of all 349 variables is completely unrestricted.

The impulse responses for some variables of interest are presented in Figure 4.2. A one-standard deviation credit shock immediately raises the US credit spread for 0.4 basic point, while the effect is two times smaller on the Canadian spread. This unexpected increase in the global external finance premium generates a significant and persistent economic downturn. We see that economic activity indicators such as production, employment, hours, prices and wages decline significantly. Production measures in particular go down for more than a year. Employment is also negatively affected, especially in the construction sector[3]. All consumer price indexes show approximately the same

3. We have looked at all of the employment series responses and find that the magnitude responses vary across sectors. For sake of space, we will not report the impulse responses on all of the series in our

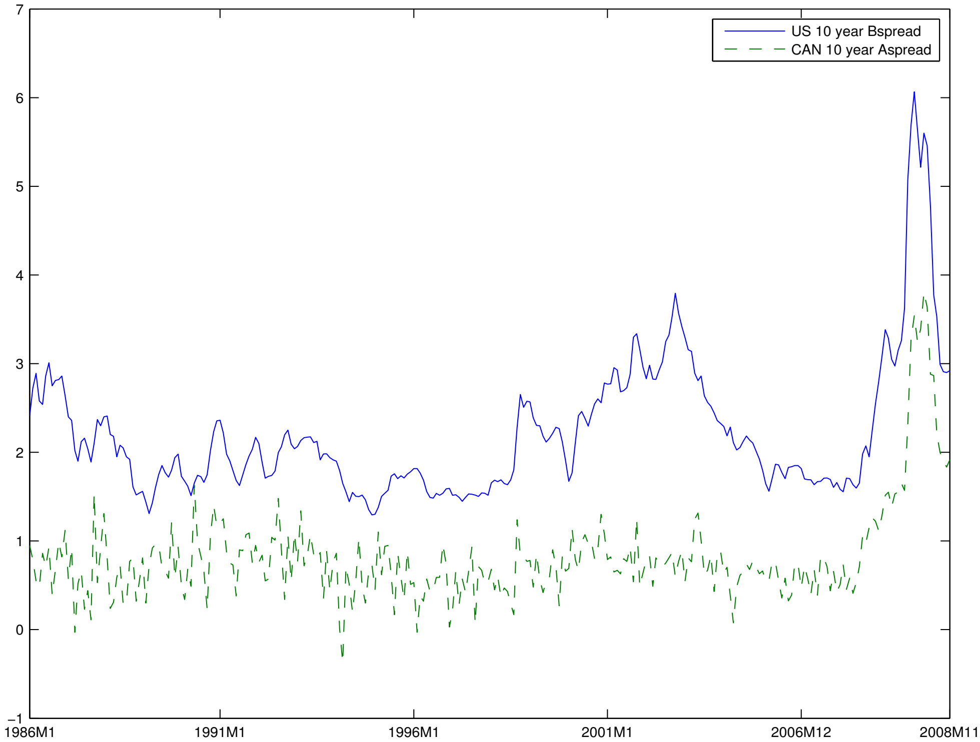

Figure 4.1: Credit spreads used in identification of structural shocks

pattern of a gradual and highly persistent slowdown, but most are non-significative. On the other hand, the industrial and commodities price indexes respond in a statistically significant way and stay a long time under their steady-state value. This result is different from what Atta-Mensah and Dib (2008), and Safaei and Cameron (2003) report where prices rise in response to a credit shock.

The effects on financial markets are even more striking. Treasury bills and government market bonds respond negatively and the effect is significant and persistent. Business and consumer credit measures decline. Leading indicators such as new orders, building permits and housing also start responding negatively on impact.

Our econometric framework allows the possibility of measuring the effects of structural shocks across different economic activity sectors, as well as across geographical regions. This is important in the case of Canada because of its huge territory and small

data set but they are available on demand.

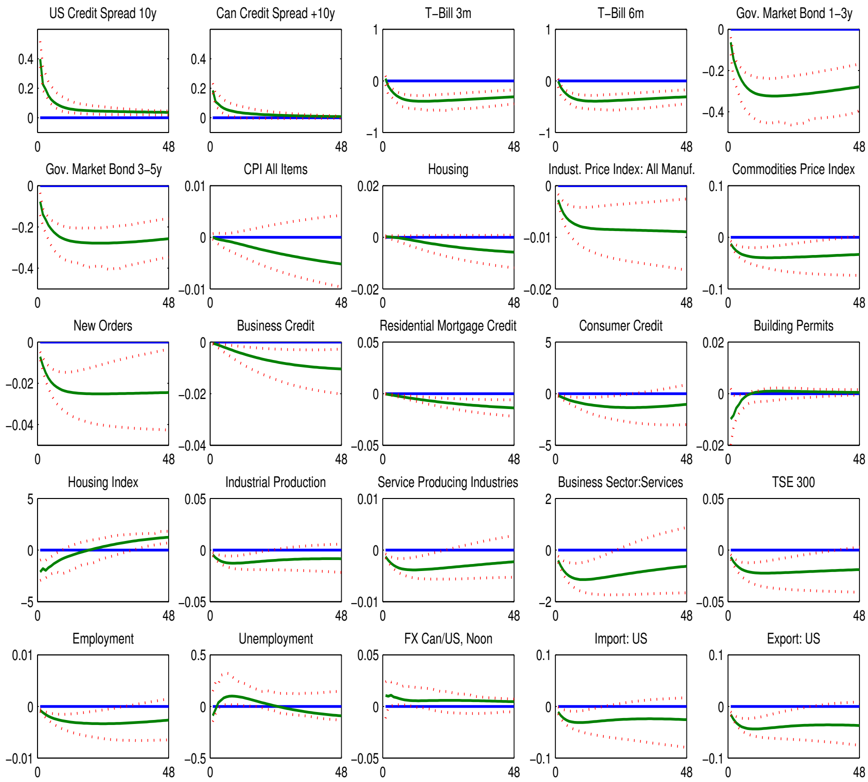

Figure 4.2: Impulse of some variables of interest to one standard deviation global credit shock

overall population density. Thus, it is interesting to see how the credit shocks propagate across different regions. The results are presented in Figure IV.1 in the Appendix. It seems that in general, the Atlantic provinces demonstrate the most inconsistent behavior with respect to the rest of Canada.

It is worth noting that the impulse responses in Figure 4.2 present similar pattern to effects of credit shocks on the US economy reported in BGS and Gilchrist, Yankov and Zakrajsek (2009).

The variance decomposition results are presented in Table 4.II. The second column reports the contribution of the credit shock to the variance of the forecast error at 48-month horizon. According to these results, and contrary to the literature on monetary policy shocks identified in structural VAR framework, the global credit shock has an important effect on several variables: credit spreads, interest rates, industrial price indexes, credit measures, production and employment. This surprising evidence of the importance of credit shocks is also documented in BGS.

Finally, since we are using a factor model, the natural question is how well the extracted factors explain the variability in observable series. Looking at the R^2 results in the third column in Table 4.II, we see that the common component explains a sizeable fraction of the variability in these variables [4]. This means that factors do capture important dimensions of business cycle movements.

4.5.2 Canadian credit shock

In previous section we showed that a global credit shock has significant and meaningful effects on Canadian economy. Now, we will see if a national credit shock, identified using a Canadian external finance premium measure produces any effect. The recursive scheme is the following:

$$[USspread10y, \quad CPI, \quad UR, \quad MS, \quad R, \quad FX, \quad CANspread10y].$$

The credit shock is identified as the last element of ε_t. This identification is similar to what has been done in structural VAR frameworks and in FAVAR frameworks with the US data: activity and price measures do not respond immediately to a credit shock, nor to interest rates or money supply. We also add the exchange rate, considered exogenous to the credit shock [5]. Contrary to other studies we control for the US credit markets by including the US credit spread, but the results do not change if we exclude it.

The impulse responses are presented in Figure 4.3. Overall, the national credit shock doesn't seem to produce any significant effect on economy. In particular, the standard deviation of the credit shock in this identification scheme is more than 8 times smaller than in the case of the global credit shock.

4. One should not forget that only 6 factors were extracted from a data set containing 349 time series presenting different correlation patterns.

5. Other ordering have been tried and results were very similar.

Variables	Variance decomposition	R^2	Marginal contribution to R2 F_t					
			F_1^*	F_2^*	F_3^*	F_4^*	F_5^*	F_6^*
US Credit Spread 10y	0.8813	0.4631	1.0000	0.0000	0.0000	0.0000	0.0000	0.0000
CAN Credit Spread 10y	0.6293	0.5019	0.7730	0.0003	0.0430	0.0209	0.0518	0.1109
T-Bill 3m	0.3947	0.9603	0.3505	0.0281	0.0399	0.5797	0.0016	0.0001
T-Bill 6m	0.4076	0.9685	0.3739	0.0254	0.0396	0.5592	0.0015	0.0005
Gov. Market Bond 1-3y	0.4231	0.9779	0.4052	0.0206	0.0837	0.4841	0.0022	0.0041
Gov. Market Bond 3-5y	0.4088	0.9717	0.4093	0.0183	0.1279	0.4347	0.0026	0.0072
CPI: all items	0.0214	0.9121	0.0313	0.9687	0.0000	0.0000	0.0000	0.0000
Housing price index	0.0520	0.4149	0.0263	0.8049	0.0428	0.0826	0.0066	0.0367
Industrial price index	0.5029	0.4942	0.3727	0.1894	0.0127	0.1834	0.0008	0.2410
Commodity price index	0.5197	0.3525	0.2383	0.2580	0.0523	0.2489	0.0442	0.1583
New orders	0.7074	0.2874	0.5524	0.0012	0.0143	0.2315	0.0696	0.1310
Business credit	0.3425	0.4045	0.4472	0.0000	0.3007	0.0944	0.0302	0.1277
Residential mortgage credit	0.1982	0.6025	0.1181	0.0310	0.1648	0.3405	0.3373	0.0083
Consumer credit	0.4595	0.3332	0.0935	0.0025	0.7411	0.0382	0.0350	0.0896
Building permits	0.1688	0.1184	0.0469	0.0381	0.0053	0.2183	0.2942	0.3971
Housing index	0.1149	0.8045	0.0640	0.0009	0.6939	0.0211	0.2177	0.0024
Indust. Prod.: manufact.	0.5726	0.6352	0.3971	0.0002	0.0451	0.3325	0.0784	0.1467
Indust. Prod.: services	0.6779	0.3501	0.3738	0.1041	0.0278	0.3205	0.0686	0.1052
Business sector: services	0.6749	0.3793	0.3894	0.1336	0.0061	0.3317	0.0516	0.0876
TSE 300	0.6659	0.1972	0.3591	0.0773	0.0210	0.3141	0.2109	0.0176
Employment	0.5691	0.5161	0.3528	0.0081	0.2223	0.1725	0.0013	0.2430
Unemployment rate	0.0840	0.8403	0.0465	0.0049	0.9486	0.0000	0.0000	0.0000
FX Can/US	0.0201	0.7872	0.0092	0.0084	0.0091	0.1638	0.5601	0.2495
Imports: US	0.4857	0.3276	0.3150	0.0142	0.0704	0.2515	0.2310	0.1179
Exports: US	0.7741	0.4445	0.5063	0.0082	0.0284	0.3419	0.1125	0.0028

Table 4.II: Explanatory power of global credit shock and common component

4.5.3 Discussion

The previous results suggest that all effects on Canadian economy are caused by a global (or the US) credit shock. Hence, modeling Canada as a closed economy when identifying and measuring the effects of credit shocks can be misleading in sense that if any effects are found, these are not caused by a national but a global shock.

To understand better this phenomena, we tried another recursive scheme:

$$[CANspread10y, \quad CPI, \quad UR, \quad MS, \quad R, \quad FX].$$

Here, the Canadian credit spread is taken to be exogenous to price, activity, money, interest rate and exchange rate measures. Our *a priori* idea is that the Canadian credit

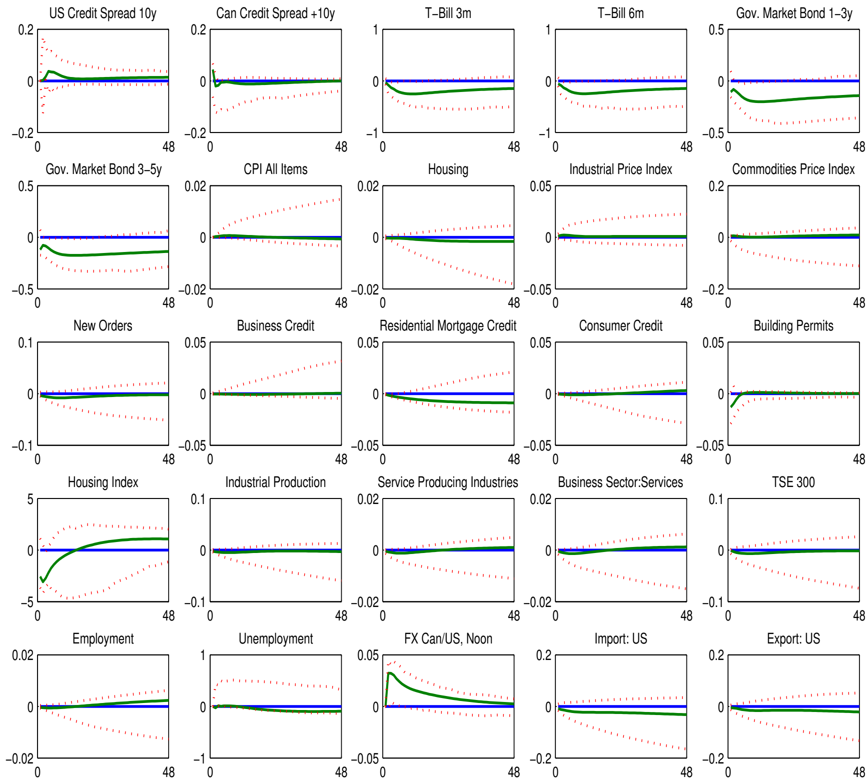

Figure 4.3: Impulse of some variables of interest to one standard deviation Canadian credit shock

spread is Granger caused by the US spread such that this identification scheme would produce similar results to the first one.

In Figure 4.4 we present the results from these two identification schemes. Overall, they are very similar, except that when using the Canadian spread the effects are slightly more important for some variables. This suggests that the same shock can be identified using either Canadian or US external finance premium measures. Moreover, the structural factors from the two models are highly correlated (correlation coefficients are higher than 0.9 in absolute value).

Finally, we tested the Granger causality between the two credit spreads. The results are reported in Table 4.III. According to p-values, we strongly reject the hypothesis that

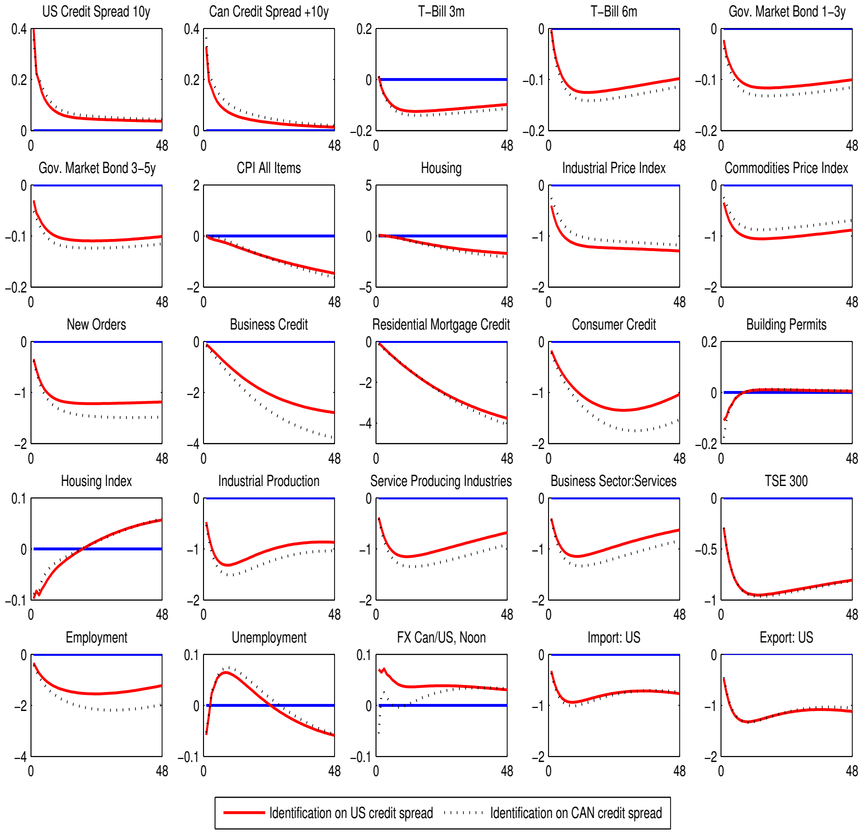

Figure 4.4: Comparison of impulse responses to a credit shock identified by US and Canadian credit spreads

the US credit spread does not cause the Canadian credit spread and posit that there is no evidence to reject the hypothesis that the Canadian credit spread does not Granger cause the US spread. Hence, these results confirm our intuition and suggest that the effects of credit shocks in Canada are essentially caused by the unexpected changes in foreign credit market conditions.

H_0	**F-stat**	**P-value**
US Spread does not Granger cause Can Spread	11.3519	0.0001
Can Spread does not Granger cause US Spread	1.0326	0.3574

Table 4.III: Testing Granger causality between US and Canadian credit spreads

4.5.4 Interpretation of factors

As it was pointed out in BGS, the procedure to identify the structural shocks can produce interpretable factors [6]. Remember that structural shocks are linear combination of residuals, $\varepsilon_t = H\eta_t$. Using this hypothesis, we can rewrite the system (4.10)-(4.11) in its structural form

$$\begin{aligned} X_t &= \Lambda^\star F_t^\star + u_t \\ F_t^\star &= \Phi^\star(L) F_{t-1}^\star + \Theta^\star(L)\varepsilon_t \end{aligned}$$

where $F_t^\star = HF_t$, $\Lambda^\star = \Lambda H^{-1}$, $\Phi^\star(L) = H\Phi(L)H^{-1}$, and $\Theta^\star(L) = H\Theta(L)H^{-1}$. Hence, given the estimates of F_t and H, we can obtain the estimate of structural factors: $\hat{F}_t^\star = \hat{H}\hat{F}_t$. The last six columns in Table 4.II contain the marginal contribution of each structural factor to the total R^2. We can see that the first structural factors explain mostly the two credit spreads. The second is very important for consumer price indexes and housing prices, while the third contributes completely in explaining the unemployment rate. Finally, the fourth factor is important for monetary measures (not reported in the table) and interest rates, while the last two factors do not seem to be interpretable.

4.6 Conclusion

In this paper we measured the impact of a credit shock in Canada in a data-rich environment. To incorporate information from a large number of economic and financial indicators, we used a factor-augmented VARMA (FAVARMA) model proposed by

6. Note however that factors are identified up to a rotation. Hence, any orthogonal rotation matrix will give the same common component even though the interpretation of each factor in terms of correlation can change.

Dufour and Stevanovic (2010). The structural shocks are identified by imposing a recursive structure on the impact matrix of the structural MA representation of observable variables.

We found that an unexpected increase in the external finance premium on global financial markets, approximated by the US credit spread, generates a significant and persistent economic slowdown in Canada. Canadian credit spreads rise immediately, while interest rates and credit measures decline. According to R^2 results, the common component captures an important dimension of business cycle movements. From the variance decomposition analysis, we observed that the credit shock has an important effect on several economic and financial measures.

Another important result is related to the identification of national financial shocks. Previous studies have treated Canada as a closed economy when identifying a credit shock and have found some real effects. Our results suggested however that there is no significant effect of domestic shocks in Canada. Indeed, the effects of credit shocks in Canada are fundamentally caused by the unexpected changes in foreign credit market conditions.

Finally, given the identification approach, we found interpretable structural factors.

CHAPTER 5

COMMON SOURCES OF PARAMETER INSTABILITY IN MACROECONOMIC MODELS: A FACTOR-TVP APPROACH

5.1 Introduction

It is likely that the behavior of economic agents and environment vary over time (e.g. monetary policy authority changes its strategy, shocks hitting economy become more or less volatile) such that some structural relations are not constant any more. This implies time instability in potentially all parameters in reduced-form representations of structural models. For instance, Fernandez-Villaverde and Rubio-Ramirez (2008) find that some structural parameters in dynamic stochastic general equilibrium (DSGE) models are time-varying. Using empirical univariate and bivariate autoregressive models, Stock and Watson (1996) find widespread instability in a large number of U.S. macroeconomic series. In addition, several studies investigated the causes of the Great Moderation: "good luck" (volatility of shocks simply dropped) or "good policy" (monetary authority more effective). [1] Mainly, some structural changes were assumed or tested, such as shifts in monetary policy rule parameters and/or time changes in volatility of shocks (see Boivin (2005), and Stock and Watson (2002, 2003), among others).

On one side, the time-varying parameter VAR models (TVP-VAR hereafter) were used by Boivin and Giannoni (2006), Primiceri (2005) and Sims and Zha (2006) to investigate the time instability of policy functions. Otherwise, a number of studies using DSGE models were conducted by letting some of the structural parameters be time-varying and/or imposing stochastic volatility (see e.g. Justiniano and Primiceri (2008), Fernandez-Villaverde et al. (2010) and Ravenna (2010)). Finally, Inoue and Rossi (2009) used a sequential testing procedure to find which structural parameters in both VAR and DSGE models are time-varying.

All the studies using reduced-form models assumed the same number of sources of

1. The Great Moderation is an empirical finding that volatility of output and prices in most of developed countries has declined since mid '80

time variations as the number of time-varying parameters. Moreover, due to the computational difficulty, independent stochastic processes are imposed for all coefficients. [2] However, it is likely that this time variability presents communalities. The intuition is that only a small number of structural relationships vary over time, inducing instability in all coefficients of reduced-form models, where the correlation structure between TVPs is mainly explained by the common component.

In this paper, I provide new evidence that the number of common sources of parameters' time variation in widely used empirical macroeconomic models is very small. I first show that parameters' instability in the structural model is likely to imply time variation in all (or at least a subset of) coefficients in the reduced-form model. Following Stevanovic (2010), I show how the factor representation of time-varying parameters is obtained, and present the factor time varying parameter model (Factor-TVP) that takes into account the factor structure of the coefficients. The main advantages of the model in the context of this paper are: the correlation structure between the TVPs is unrestricted, and only a small number of states must be filtered. The approach is applied to a standard 3-variable VAR model from Primiceri (2005), and to a factor-augmented VAR (FAVAR) model from Boivin, Giannoni and Stevanovic (2010).

The first objective is to detect the presence of the factor structure, and test its dimensionality in time-varying VAR coefficients, keeping the volatility of shocks constant. Performing the two-step recursive and likelihood-based procedures, I find that a small number of common shocks explain the instability in VAR coefficients. Then, the Factor-TVP model with one latent component is estimated by maximum likelihood. The underlying factor is very persistent, which is in line with the strong collinearity in TVPs found after the first stage estimation in the two-step likelihood procedure (see discussion in Stevanovic (2010)). Moreover, it is highly correlated with the unemployment rate, and moderately related to inflation and interest rate.

When applied to TVP-VAR model with stochastic volatility from Primiceri (2005), the variability in VAR coefficients is mostly explained by only one factor, while the

2. Primiceri (2005) relaxes this hypothesis by letting some VAR time-varying coefficients have correlated error terms.

stochastic volatility part is explained by two additional factors. In particular, when the structural version of the VAR model is obtained by the Choleski decomposition of the residuals time-varying covariance matrix, I find that the contemporaneous relations coefficients are explained by the second factor, while the variances of structural shocks load on the third factor.

These results have an important implication for the standard counterfactual analysis. They suggest that parameters in all VAR equations (even in its structural version), together with the stochastic volatility coefficients, vary with a small number of common factors. Suppose one of these factors represents the change in the monetary policy strategy. Then, doing a counterfactual analysis by replacing only the coefficients in the interest rate VAR equation is inappropriate, since the instability in other VAR parameters, including the volatility of shocks, is also caused by the same factor.

I redo the same exercise for the data updated to 2010Q3. The idea of using the Factor-TVP model is to capture common breaks, and the recent financial crisis is a good example of such an episode. Indeed, the tests suggest one more factor, and their interpretation is changed. The estimated TVPs present an important behavioral change after 2007, indicating the presence of a structural break. The correlation structure between factors and observable variables is such that the first factor is highly positively correlated to the inflation rate, and moderately related to the interest and unemployment rates. The second factor presents a similar pattern, but with correlation coefficients around 0.3. Contrary to pre-crisis data, a majority of TVPs load heavily on factors. In particular, the inflation equation coefficients seem to be time-varying with the two factors, and this is also the case for about half of parameters in other VAR equations.

To complete the empirical exercise, the time-varying FAVAR model is estimated by two-step recursive procedure. I find that four dynamic factors explain most of the covariability between almost 700 TVPs. Again, the stochastic volatility coefficients' instability seems to have different common factors than the regression parameters.

Finally, I simulate the data from a simple New Keynesian model with one time-varying parameter: interest rate target in the standard Taylor rule. The two-step procedures indicate the true factor structure on TVPs of the VAR representation of the simulated data. Simultaneous ML estimation gives the factor that is very correlated with the true common shock, and suggest that most of the time-varying coefficients load on that factor.

In the rest of the paper, I provide examples of common sources of parameters' instability in macroeconomic models. The Section 3 shows how the factor representation of time-varying parameters process is obtained, and the Section 4 presents the Factor-TVP model. The identification and estimation issues are discussed. The main results are presented in Section 5, Section 6 shows that the method can recover the low-dimensional instability in data simulated from a dynamic stochastic general equilibrium (DSGE) model, and Section 7 concludes.

5.2 Examples of reduced-rank parameters instability

In following, I show a simple example where the low number of common sources of parameter instability is a plausible hypothesis. The intuition is that only a small number of structural relationships vary over time, which implies that possibly all the coefficients in reduced-form models are unstable.

Suppose the following simultaneous equation problem. The structural form is

$$q_t = ap_t + b + u_t \tag{5.1}$$

$$s_t = \alpha p_t + \beta x_t + v_t \tag{5.2}$$

with

$$\begin{bmatrix} u_t \\ v_t \end{bmatrix} = N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix} \right). \tag{5.3}$$

where (5.1) and (5.2) are demand and supply equations respectively, and x_t is an exogenous variable that helps in identifying the shifts in supply curve. Imposing the equilibrium condition $q_t = s_t$ and solving for p_t, we obtain the reduced form

$$p_t = \pi_{11} + \pi_{12} x_t + \varepsilon_t \tag{5.4}$$

$$q_t = \pi_{21} + \pi_{22} x_t + \eta_t \tag{5.5}$$

with

$$\begin{bmatrix} \varepsilon_t \\ \eta_t \end{bmatrix} = N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\varepsilon^2 & \sigma_{\varepsilon\eta} \\ \sigma_{\varepsilon\eta} & \sigma_\eta^2 \end{bmatrix} \right). \tag{5.6}$$

where $\varepsilon_t = \frac{v_t - u_t}{a-\alpha}$, $\eta_t = -\frac{a v_t - \alpha u_t}{a-\alpha}$, $\pi_{11} = -\frac{b}{a-\alpha}$, $\pi_{12} = \frac{b}{a-\alpha}$, $\pi_{21} = -\frac{\alpha b}{a-\alpha}$, $\pi_{22} = \frac{a\beta}{a-\alpha}$, $\sigma_\varepsilon^2 = \frac{(\sigma_v^2 - 2\sigma_{uv} + \sigma_u^2)}{(a-\alpha)^2}$, $\sigma_\eta^2 = \frac{(\alpha\sigma_u^2 - 2\alpha a \sigma_{uv} + a\sigma_v^2)}{(a-\alpha)^2}$, and $\sigma_{\varepsilon\eta} = \frac{(\alpha\sigma_u^2 - (\alpha+a)\sigma_{uv} + a\sigma_v^2)}{(a-\alpha)^2}$. Then, if for example α presents stochastic time variation, it will appear in all parameters of the reduced form.

5.3 Factor representation of time-varying parameters' process

Following Stevanovic (2010), I show how to obtain the factor representation of time-varying parameters' process.

5.3.1 Linear approximation

Let α_t be a q-dimensional vector of time varying parameters in the structural model, which dynamic process is characterized by some distribution $\mathfrak{G}$. Let β_t be a k-dimensional vector of (potentially all) time varying *reduced-form* coefficients such that

$$\beta_t = \mathfrak{F}(\alpha_t; \gamma) \tag{5.7}$$

where γ is a vector of m constant coefficients, and $\mathfrak{F}$ is some functional form that relies reduced-form parameters to structural parameters.

In general, β_t is a nonlinear function of structural parameters. If the number of TVPs in the structural model is less than the number of coefficients in β_t, i.e. $q < k$, then $\mathbf{rank}(\Sigma) = q$, $\Sigma = \mathbf{V}(\beta_t)$. Suppose that function $\mathfrak{F}$ in (5.7) is twice continuously differentiable, then by Taylor's theorem

$$\beta_t = \mathfrak{L}(\beta_t) + error \tag{5.8}$$

where $\mathfrak{L}(\beta_t)$ is the linear approximation of β_t around $\bar{\alpha}$:

$$\mathfrak{L}(\beta_t) = \mathfrak{F}(\bar{\alpha};\gamma) + \mathfrak{F}'(\bar{\alpha};\gamma)(\alpha_t - \bar{\alpha}).$$

Define $\mu = \mathfrak{F}(\bar{\alpha};\gamma)$, $\lambda = \mathfrak{F}'(\bar{\alpha};\gamma)$, $f_t = (\alpha_t - \bar{\alpha})$ and e_t as the approximation error, so we have the following factor structure:

$$\beta_t - \mu = \lambda f_t + e_t \tag{5.9}$$

If β_t is demeaned, then $\mu = 0$ and we obtain the usual factor model:

$$\beta_t = \lambda f_t + e_t$$

with $f_t \sim \bar{\mathfrak{G}}$, where $\bar{\mathfrak{G}}$ is the distribution of $(\alpha_t - \bar{\alpha})$, and with dispersion matrices $\mathbf{D}(e_t) = \Psi$ and $\mathbf{D}(f_t) = \Phi$ and

$$\mathbf{D}(\beta_t) = \lambda\Phi\lambda' + \Psi$$

with Ψ diagonal and at least q^2 identifying restrictions.

5.3.2 Dimension reduction

Another way to motivate the factor representation of time-varying parameters is the dimension reduction argument. Since the standard estimation methods are computationally cumbersome when the number of TVPs is large, it may be of practical interest to reduce the dimensionality problem by imposing the factor structure. As in standard applications of principal component analysis, the idea is to approximate a large number of TVPs by few factors.

Suppose we wish to replace the N-dimensional r.v. β_t with its K, $K < N$, linear functions without much loss of information. How are the best K linear function to be chosen? It can be restated as a nonlinear least square problem

$$\beta_{it} = (B_1'\beta_t)\delta_{i1} + \ldots + (B_K'\beta_t)\delta_{iK} + e_{it}, \quad i = 1,\ldots,N, \quad t = 1,\ldots,T.$$

Rao (1973) shows that $B'\beta_t$ should be chosen as first K principal components of β_t. Hence, the last equation in matrix form and defining $A = \delta$, and $F = B'\beta$, we get the factor representation:

$$\beta = AF + e.$$

5.4 Econometric framework

In this section I present the linear factor time varying model (Factor-TVP) proposed by Stevanovic (2010).

5.4.1 Factor-TVP model

The simplest example to illustrate the method is the linear TVP regression:

$$y_t = x_t' \beta_t + w_t \tag{5.10}$$

where y_t is a scalar, x_t is a $k \times 1$ vector of explanatory variables, β_t contains k time varying parameters and w_t is a homoscedastic white noise with $\mathbf{V}(w_t) = R$. A more general case where w_t has a time varying variance is treated in Stevanovic (2010).

There are many ways to specify the time variation of β_t: discrete or stochastic breaks, stochastic continuous processes (random walk and ARIMA). In standard applications of TVP models in macroeconomics, β_t is usually modeled either as a Markov switching process, which includes discrete break as special case, or as a collection of univariate AR(1) processes, which embodies the random walk if the autoregressive coefficient is fixed to unity. Here, we concentrate only on continuous stochastic processes of β_t.[3]

However, as I discussed in previous sections, it is likely that only a small number of common sources has generated the time variability in parameters in (5.10). In that case, the link between β_t and its underlying factors may be approximated as a linear factor model:

3. Nyblom(1989) pointed out that both discrete break and random walk models are martingale processes and are special cases of $\beta_t = \beta_{t-1} + \omega_t$, with $\mathbf{E}(\omega_t) = 0$. Moreover, the continuous TVP model is then an approximation of the discrete break model.

$$\beta_t = \lambda f_t + e_t \quad (5.11)$$

$$f_t = \rho f_{t-1} + v_t \quad (5.12)$$

where f_t is a $(q \times 1)$ vector of latent factors, λ is a $(k \times q)$ matrix of factor loadings, ρ is such that f_t is stationary of martingale, e_t and v_t are white noise processes with $E(e_t e_t') = M$ and $E(v_t v_t') = Q$. Finally, it is assumed that w_t, e_t and v_t are uncorrelated. Note that the error term in (5.11) can be omitted if one is convinced that the time-varying coefficients β_t are exactly linear combinations of f_t. To summarize, the Factor-TVP model is given by the following expressions:

$$y_t = x_t' \beta_t + w_t \quad (5.13)$$

$$\beta_t = \lambda \beta_{t-1} + e_t \quad (5.14)$$

$$f_{t+1} = \rho f_t + v_{t+1} \quad (5.15)$$

Being able to estimate the model without restrictions on the correlation structure between the TVPs is very important in macroeconomic applications. In recent empirical literature many researchers studied the causes of the *Great Moderation* using the counterfactual exercises or TVP-VAR model.[4] If the monetary policy rule has changed over time, such that some parameters in Taylor rule are time-varying, this change is likely to affect the parameters in all equations in the VAR representation of data and cause heteroscedastic errors. Hence, restricting the correlation structure between the time-varying parameters within and across equations, or doing counterfactual exercises by changing parameters values in a particular equation, may be misleading.

4. See among others Stock and Watson (2003), Primiceri (2005), Boivin and Giannoni (2006), Benati and Surico (2009).

5.4.2 Identification

As written above, the Factor-TVP model is not identified. Suppose that $\hat{\lambda}$ and $\hat{f}_t$ are a solution to the estimation problem. However, this solution is not unique since we could define $\hat{\lambda} = \tilde{\lambda} H$ and $\tilde{f}_t = H^{-1}\hat{f}_t$, where H is a $q \times q$ nonsingular matrix, which could also satisfy the model's equations. Then, observing y_t and x_t is not enough to distinguish between these two solutions, and a normalization is necessary.

There are several ways to achieve identification in the factor model. Recall that we have to impose q^2 restrictions. In principal component analysis, this is done by rotating factors such they are orthogonal and with unit variance. This gives $q(q+1)/2$ restrictions. The remaining restrictions are obtained by assuming that $\lambda'\lambda$ is diagonal. Mathematically, factors and loadings are calculated such that

$$\sum f_t f_t'/T = \mathrm{I}_q, \quad \lambda'\lambda/k = \Delta$$

where Δ is a $k \times k$ diagonal matrix.

However, this does not remove the indeterminacy associated to rotation, orthogonal transformation or sign changes of f_t. The main criticism to this normalization is that the factors are not interpretable. If the objective is to estimate the space spanned by factors, and then use this information to improve forecasting of observable series or to help in identifying structural shocks by bringing more information in the model, this normalization is enough. But, as pointed out in Yalcin and Amemiya (2001), if one is interested in causal use of factor analysis, this parametrization is problematic.

Another parametrization of (5.14) is the errors-in-variable representation. The identification is achieved by imposing a $(q \times q)$ identity matrix on q rows of λ:

$$\tilde{\lambda} = \begin{bmatrix} I \\ \lambda \end{bmatrix}.$$

This is equivalent to re-write (5.15) as

$$\begin{aligned}\beta_{1,t} &= f_t + e_{1,t}\\ \beta_{2,t} &= \lambda f_t + e_{2,t}\end{aligned}$$

which implies that f_t is measured with error by q elements of $\beta_{1,t}$.

The main advantage of the errors-in-variable representation is that only q^2 loadings are restricted while the factors distribution is left unrestricted. This can be of interest if the research goal is to study the causal relation between factors and time-varying parameters. For example, if one wants to learn where the instability in VAR parameters is coming from, it is important to not restrict the factors distribution. However, the major problem with this parametrization is that one must arbitrary chose q time-varying parameters that represent factors measured with error.

5.4.3 Estimation

The strength of the Factor-TVP model is that instead of filtering and tracking $k+n(n+1)/2$ states, we only need to filter q states, and q is generally much less than $k+n(n+1)/2$. Moreover, much less covariance coefficients must be estimated. Consider an n-dimensional TVP-VAR(1) model. If no factor structure is imposed one must filter $n+n(n+1)/2$ states, but also estimate the same number of TVP variances. This is the most enthusiastic case in which all coefficients are modeled as random walks with diagonal covariance matrices. If one is interested in more flexible, and probably more realistic case, the number of parameters to estimate will explode.

On the other hand, imposing a factor structure allows for any type of correlation the TVPs, while keeping the dimensionality of the estimation problem relatively low. The number of TVP covariance elements to be estimated is at most $q(q+1)/2$ and the number of factor loadings is at most $(n+n(n+1)/2)q$ in case of orthogonal factors and $(n+n(n+1)/2)q-q^2$ if errors-in-variable representation.

In Stevanovic (2010), three alternative estimation procedures are proposed. The first, and the simplest, is to estimate (5.13)-(5.15) by a recursive (or rolling-window) procedure in two-steps [5]:

Step 1: Estimate TVPs by recursive least squares with fixed or expanding window.

Step 2: Do factor analysis on estimated TVPs.

The second estimation method consists also of two steps. In the first step, the TVPs are estimated within the standard TVP framework without imposing the factor structure, and then the factor model is fitted on estimated TVPs. According to simulation results in Stevanovic (2010), this method provides accurate estimates of the number of common shocks, and of the factors' dynamic process, but is naturally less efficient than the first method since the true restrictions are not imposed.

Finally, if the model is well specified, completely characterizing the system (5.13)-(5.15) in a parametric world, and then use a likelihood-based method to simultaneously estimate the system, is the best option. In addition to elements of β_t, it is possible to estimate the stochastic volatility so the covariance matrix of residuals in (5.13) is heteroscedastic. If no stochastic volatility, and if the number of parameters is not to large, the model can be estimated by ML where the likelihood is calculated using the Kalman filter. Otherwise, Bayesian methods are needed.

5.4.3.1 Note on factor analysis in two-step methods

The factor analysis in two-step methods consists of checking for:

– *Factorability*

A necessary condition for factorability of a data set is the presence of correlation between the variables. An arbitrary criterion is to have several correlation coefficients of at least 0.5 in absolute value.

– *Linearity*

As I discussed previously, the mapping from latent time varying parameters to

5. On one hand, the recursive methods may be inefficient if the true restrictions are not explicitly imposed and the estimates may contain lot of sampling error, but they are more robust to misspecification. Moreover, the results in Stock and Watson (1994) suggest that time varying models have limited success in exploiting this instability to improve upon fixed-parameter or recursive autoregressive forecasts. Finally, Edlund and Søgaards (1993) find that recursive methods are useful to model business cycles in the context of time-varying parameters. Also, they are computationally easy to apply contrary to the simultaneous likelihood methods.

coefficients in estimable models is generally nonlinear. It is then important to check for the linearity between data and factors. Usually, this is done by looking at scatter plots between observed variables, but when the number of series is large, it is practically impossible to keep the track of nonlinearity. Instead, I produce scatter plots of TVPs and the estimated linear factors, and I plot the linear fit.

– *Testing for number of factors*

  The most difficult part in the factor analysis is to test for the number of factors. It is sometimes driven by theory and the type of application (e.g. tests of intelligence), but in exploratory exercises as in this paper, one must use a battery of different tests and some arbitrary criteria to decide on the number of factors. I will use *scree tests*, i.e. plot eigenvalues of covariance matrix of data, *trace test*, see the percentage of variance explained by factors, Bai and Ng (2007) tests for the number of dynamic factors and Horn (1965) parallel tests. [6] As pointed out in Velicer at al. (2000), the scree test is good as adjunct method since this is not a statistical procedure and the trace test is not recommended since it is not robust to presence of irrelevant components. [7] The parallel test, that is in fact a simulation based Monte Carlo test, is one of the most recommended but it is computationally difficult if there is a large number of TVPs. The MAP test (Velicer (1976) and Velicer et al. (2000)) is also a widely used test in psychometric applications of factor analysis, but it tends to overestimate the number of factors (especially in simulations where lot of sampling error is present). Finally, likelihood ratio tests are known to overestimate the number of factors. Note that either of these tests is not robust to nonlinear relationships between measures and factors. [8]

– *Estimation* I will estimate factor model with principal components analysis (PCA) and with maximum likelihood. In former case, the estimator is consistent for

6. Given the sample size and sampling error when TVPs are estimated, I don't report results from Bai and Ng (2002) for number of static factors since these performed quite bad in simulation, see Stevanovic (2010).

7. In simulations, adding irrelevant series to a data set constructed from factor model will decrease arbitrary the trace ratio. This is important here since we do not know a priori how many parameters are time varying due to common shocks.

8. In simulations, when the data are generated from factor model with a polynomial relationship, these test overestimate the number of factors.

large number of periods and variables, and for the presence of a limited amount of cross-correlations between the error elements in observation equation. Using MLE method, I assume an exact factor model on the estimated TVPs.

5.5 Empirical evidence on common sources of parameters instability

Here, I produce new empirical evidence that there exist low-dimension common sources of parameters instability in various reduced-form models widely used in macroeconomics. A 3-variable TVP-VAR model from Primiceri (2005) is estimated using the three methods discussed above. I only consider time variation in VAR coefficients, while the covariances of residuals are assumed constant. The case with stochastic volatility is studied below. According to results from two-step procedures, it seems that the stochastic volatility is governed by trivial additional factors (idiosyncratic variations). In addition, I estimate the factor-augmented VAR (FAVAR) model from Boivin, Giannoni and Stevanovic (2010) with time-varying parameters and stochastic volatilities including factor loadings, factors VAR coefficients and covariances of idiosyncratic and common shocks. Given the large number of TVPs, almost 700, the model is estimated by the recursive procedure only.

5.5.1 VAR model

The VAR model contains the annual growth rate of GDP price index, unemployment rate and 3-month Treasury bill. The data are quarterly and span 1953Q1-2006Q4 period. The lag order is fixed at 2, so the total number of time-varying parameters is 27 (3 constants, 18 VAR coefficients plus 6 covariances). Following notation in Primiceri (2005), the model is

$$y_t = c_t + B_{1,t}y_{t-1} + B_{2,t}y_{t-2} + A_{0,t}^{-1}\Sigma_t\varepsilon_t, \tag{5.16}$$

where $A_{0,t}^{-1}$ is lower triangular with unit diagonal, and Σ_t is diagonal. Let $x_t = I_n \otimes [1, y'_{t-1}, y'_{t-2}]$, $\beta_t = [c'_t \quad vec(B'_{1,t}) \quad vec(B'_{2,t})]'$, and $P_t = A_{0,t}^{-1}\Sigma_t$.[9] Then, the model (5.16)

9. Let d_t be diagonal matrix containing diagonal elements of P_t. Then, $A_{0,t}^{-1} = P_t d_t^{-1}$ and $\Sigma_t = d_t$.

can be written as in (5.10) with $R_t = P_t P_t'$. Let α_t be a vector of lower triangular random elements in $A_{0,t}^{-1}$, and σ_t contains diagonal elements of Σ_t. The Factor-TVP representation is

$$y_t = x_t' \beta_t + P_t \varepsilon_t \tag{5.17}$$

$$\begin{bmatrix} \beta_t \\ \alpha_t \\ \log \sigma_t \end{bmatrix} = \lambda f_t + e_t \tag{5.18}$$

$$f_t = \rho f_{t-1} + v_t \tag{5.19}$$

5.5.1.1 Two-step recursive method

As a starting point, I estimate the time varying parameters and the underlying common sources of instability using the two-step recursive least squares procedure discussed above. Even if this method is not the most efficient, and may suggest time instability in case of constant parameters, the simulation evidence in Stevanovic (2010) shows that it is capable to detect the low-dimension feature of variability in TVPs. Due to sampling errors, it tends to suggest more factors than the actual number, but it gives a good initial sight and provides starting points for the likelihood-based method. In the first step, I estimate TVPs by recursive OLS where the initial window is fixed to 10 years. [10]

A condition for factorability is that θ_t presents enough correlation. In this exercise, more than 58% of unique elements in the correlation matrix of θ_t are higher than 0.5, showing that the correlation structure is strong enough to lead to a factor structure. The scree and trace tests are presented in Figure 5.1. The scree test, suggests there is at least 1 factor, but statistical tests might pick up to 4 factors. Moreover, the Kaiser criterion suggests the number of eigenvalues larger than 1 as an estimate of the number of factors. The cumulative product of eigenvalues is also very informative and indicates a small number of factors. [11] Finally, the trace ratio shows that 2 factors explain almost 80% of

10. Different rolling windows sizes have been tried, and the results do not vary a lot. Here, I present only results from expanding window approach since it appears to be the most robust.

11. If the factor structure is true, than all the eigenvalues after the number of factors should be close to

the variability in data.

Table 5.I presents results from the statistical tests for the number of factors. The MAP tests suggests 4 factors, while the Parallel tests estimate 3 and 4 latent components. [12] The Bai-Ng tests suggest 1 to 5 dynamic factors. Note that I do not report results from Bai and Ng (2002) tests for number of static factors because these need a large cross-section size to be accurate. In this exercise, as long as the economy is well approximated by 3 macroeconomic series in VAR(2), the cross-section size is fixed and is very small compared to number of periods.

The next step is to see if the linear approximation of the factor representation of time varying parameters is reliable. The Figures 5.2 and 5.3 present scatter plots between elements of $\hat{\theta}_t$ and two factors estimated assuming linear relation. Overall, the linear hypothesis seems plausible, but adding a polynomial structure would capture some non-linearities. Finally, Figure 5.4 presents the marginal contribution of each factors to the total R^2. The first seven elements of θ_t correspond to inflation equation, the second seven to unemployment equation, and the final coefficients are from the interest rate equation. According to these results, the coefficients of the interest rate equation are all explained by the first factor. The same factor is important for some parameters in two other VAR equations, together with the second and third common component. The fourth factor has a marginal effect. Remark that θ_t contains reduced-form coefficients, so any structural interpretation is impossible. However, it is interesting that monetary policy rule coefficients are so related to the first factor.

Overall, these results suggest that there are few common sources of parameter instability. However, given the sampling uncertainty involved in recursive procedure, it is recommended to estimate the time varying parameters by an adaptive TVP model and

zero, and hence, the cumulative product should go to zero. Because of the sampling uncertainty, taking products might be more reliable than looking at eigenvalues only.

12. The first specification of MAP test works with squared correlation coefficients and the second with power 4. The first two specifications correspond to PCA & Random data generation and PCA & Raw data permutation, while the last two correspond to PAF/Common factor analysis with random data or raw data permutation. The latter and the MAP test tend to overestimate the number of factors in structures with weaker correlation structure. The PCA specifications of Parallel test consider also the variability of the scores. The last two parallel test specification use adjusted correlation matrices which tends to indicate more factors than warranted (see Buja and Eyuboglu (1992)).

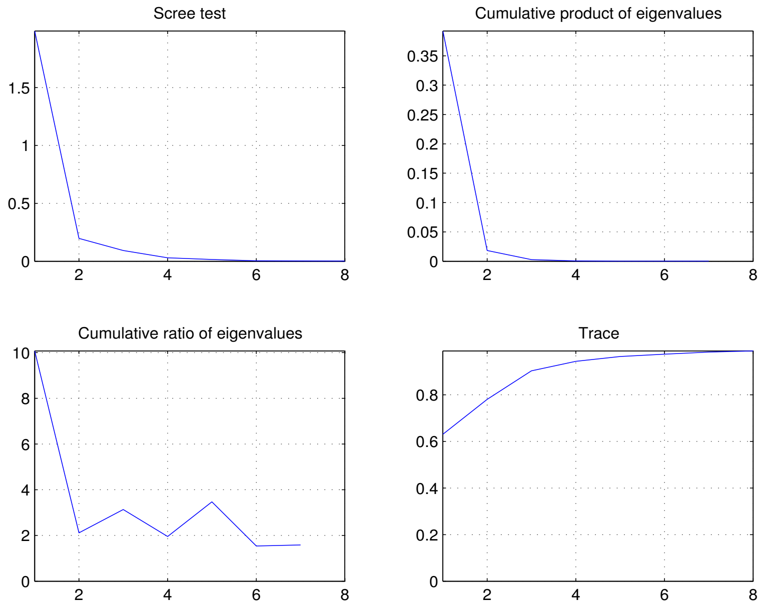

Figure 5.1: Scree and trace tests for VAR model TVPs estimated by recursive OLS

Test	2-step recursive OLS	2-step likelihood
MAP	4 4	8 9
Parallel	3 3 4 4	2 2 4 4
Bai-Ng	1 3 5 2 4 5	1 1 1 1 1 1

Table 5.I: Estimation of the number of factors in VAR time-varying coefficients

then apply the factor analysis to get a more accurate estimation of the number of factors.

5.5.1.2 Two-step likelihood method

In this section, the TVP-VAR model in (5.16) is estimated using Bayesian methods as in Primiceri (2005), and all time-varying parameters are again collected in θ_t. Then, I apply the same factor analysis as in the previous section.

The factorability is much more stronger than in the case of recursive OLS. The Figure 5.5 shows scree and trace tests from which 1 factor seems to explain a great part of variability in data. Also, the percentage of correlation coefficients between elements of $\hat{\theta}_t$ greater than 0.9 is more around 80!. According to simulation results in Stevanovic (2010), such strong correlation structure among TVPs is likely to occur when the under-

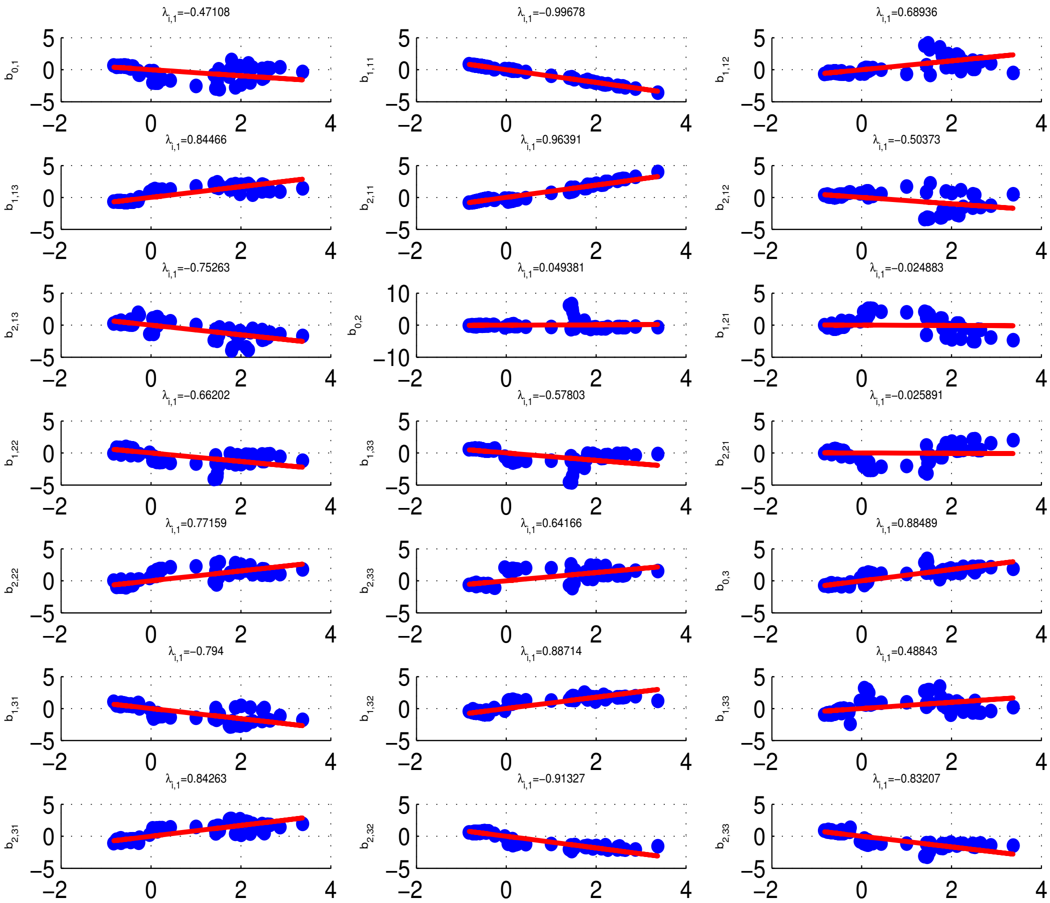

Figure 5.2: Scatter plots for factor 1 and VAR model TVPs estimated by recursive OLS

lying factor structure is strong, and when the factors are very persistent. On testing for number of factors, Parallel and Bai and Ng tests estimate 1 or 2 factors, while the MAP tests surprisingly suggest 11 factors.

On the linearity issue, the scatter plots of θ_t and first factor in Figure 5.6 suggest that linear approximation is very accurate for all time varying parameters. In Figure 5.7, the linear relation between TVPs and the second factor is less evident. In particular, the second factor is very important for $b_{1,11}$ and their relation could be better approximated by a polynomial equation.

Hence, these tests suggest a linear factor model on θ_t with 1 or 2 factors. I estimate the exact 2-factor model on θ_t and Figure 5.8 shows elements of θ_t and fitted values. Given the strong correlation structure among TVPs, it is not surprising that the common

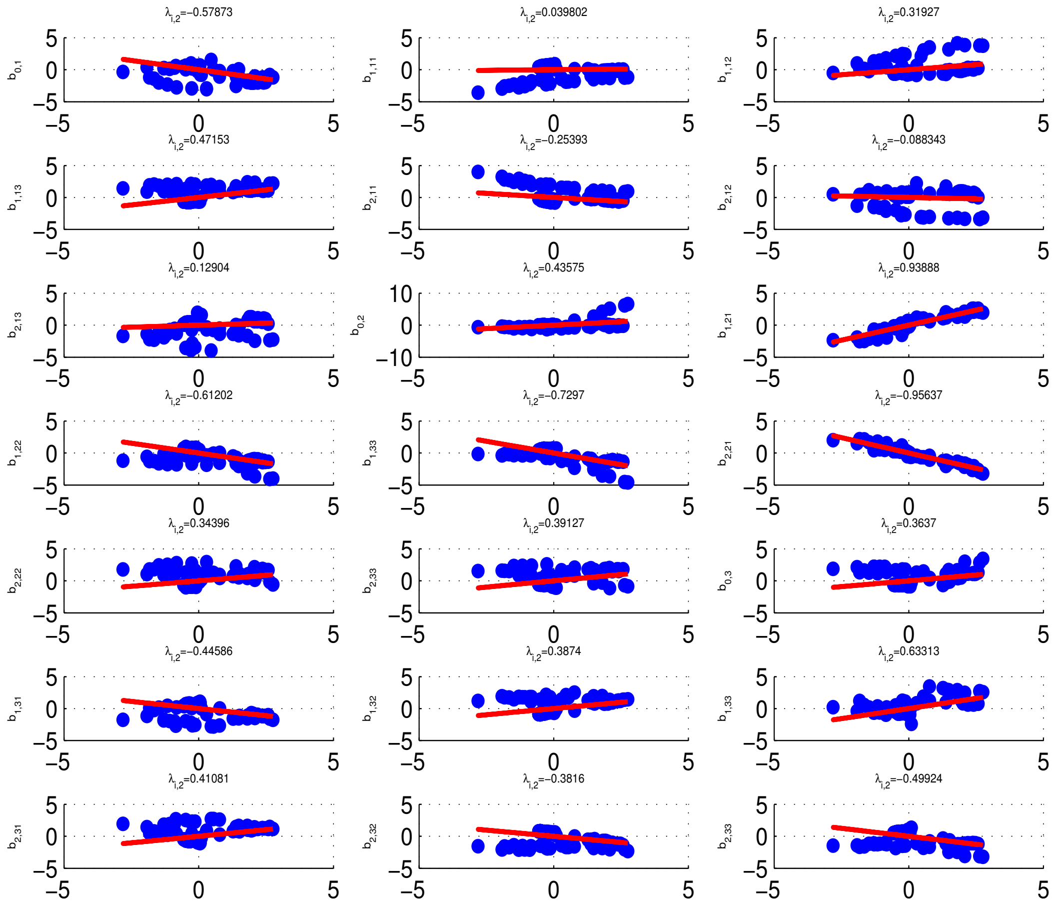

Figure 5.3: Scatter plots for factor 2 and VAR model TVPs estimated by recursive OLS

component reproduce them almost exactly. To get more insight about the factor structure within the time varying VAR coefficients, the Figure 5.9 presents marginal contributions of subsequent factors to the total R^2. First, the 2 factors explain everything in TVPs. Second, contrary to the previous recursive procedure, it is not obvious that the first factor is mostly related to the interest rate equation. The conclusion is quite different in case of stochastic volatility and it is discussed below.

Overall, this exercise shows strong empirical evidence on low-dimension sources of parameters instability in standard monetary VAR model. In particular, it suggests that the VAR coefficients variability is due mostly to one underlying factors. The next step is to take into account this information explicitly and estimate the Factor-TVP model in one step.

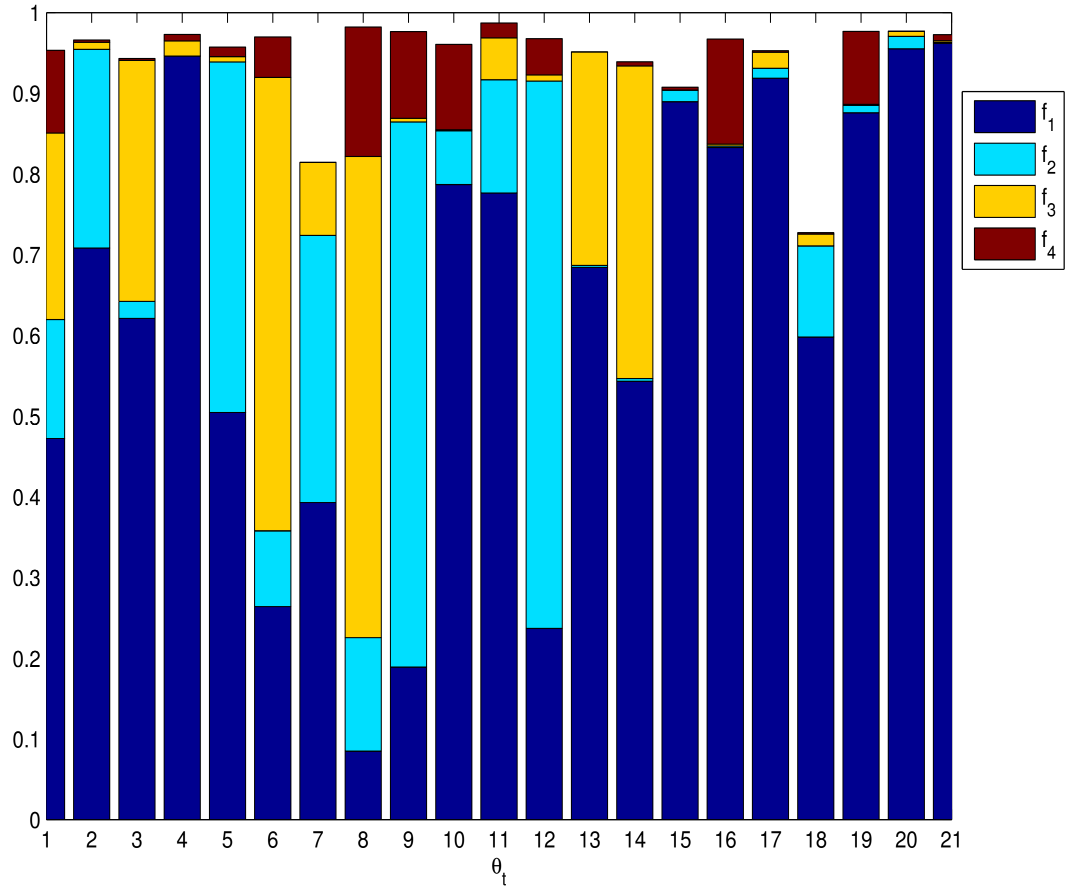

Figure 5.4: Marginal contributions of factors to total R_2 on VAR model TVPs estimated by recursive OLS

5.5.1.3 Simultaneous likelihood method

The maximum likelihood estimation of the Factor-TVP VAR model without stochastic volatility is performed in this section. The Kalman filter is used to calculate the likelihood. I do not assume exact linear relationship between TVPs and factors, so the factor equation (5.18) contains the error term with a diagonal covariance matrix. [13] The estimated θ_t are plotted in Figure 5.10.

13. Even though the maximum likelihood estimation of time-varying parameters models is difficult, the method is still feasible and produces accurate results for small systems. In Stevanovic (2010), this method seems to work well in the context of multivariate linear regression even with 5 dependant and 7 independent variables. However, the next step in this project is to construct the Bayesian estimation procedure that is more suitable given the presence of local maxima and the difficulty of ML to estimate

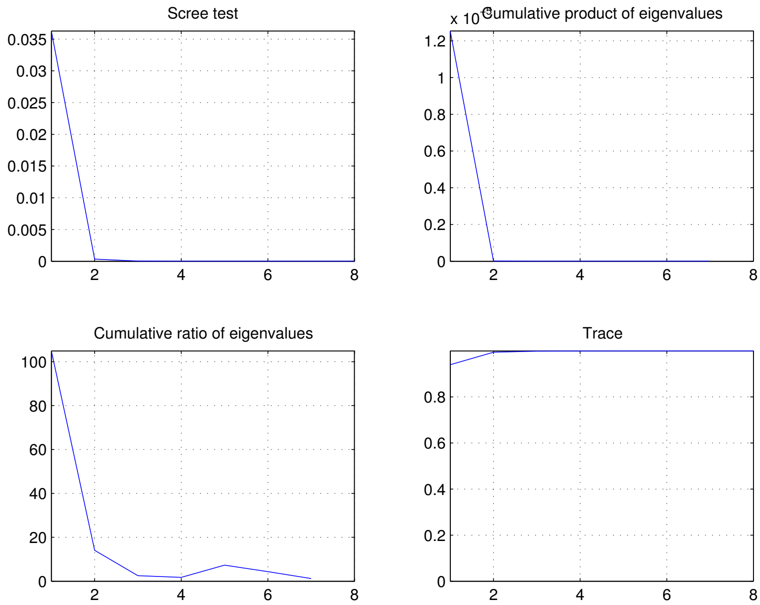

Figure 5.5: Scree and trace tests for TVPs of VAR estimated by two-step likelihood method

The Table 5.II summarizes the results of interest. According to likelihood ratios between 1-, 2- and 3-factor models, not reported here, the 1-factor specification seems to be preferred. Also, the estimation with more that one factor is very instable and the algorithm tends to find different local maxima. The underlying factor is very persistent, $\hat{\rho}$ close to unity, which is in line with the strong collinearity in θ_t found after the first stage estimation in two-step likelihood procedure (see discussion in Stevanovic (2010)).

The underlying component is highly correlated with the unemployment rate, and moderately related to inflation and interest rates. The first part of Figure 5.11 plots the VAR series and the estimated factor. To get a more reliable picture, the series are standardized because the factor is much less volatile then the observable series. Finally, the second part of the Figure 5.11 shows the factor loadings. Note that the errors-in-variable identification is imposed such that first $(q \times q)$ part of λ is identity matrix. Several coefficients across all three VAR equations load on the common factor. However, a majority of TVPs does not have large loading values, so they seem to be either constant or varying in idiosyncratic ways.

the covariances.

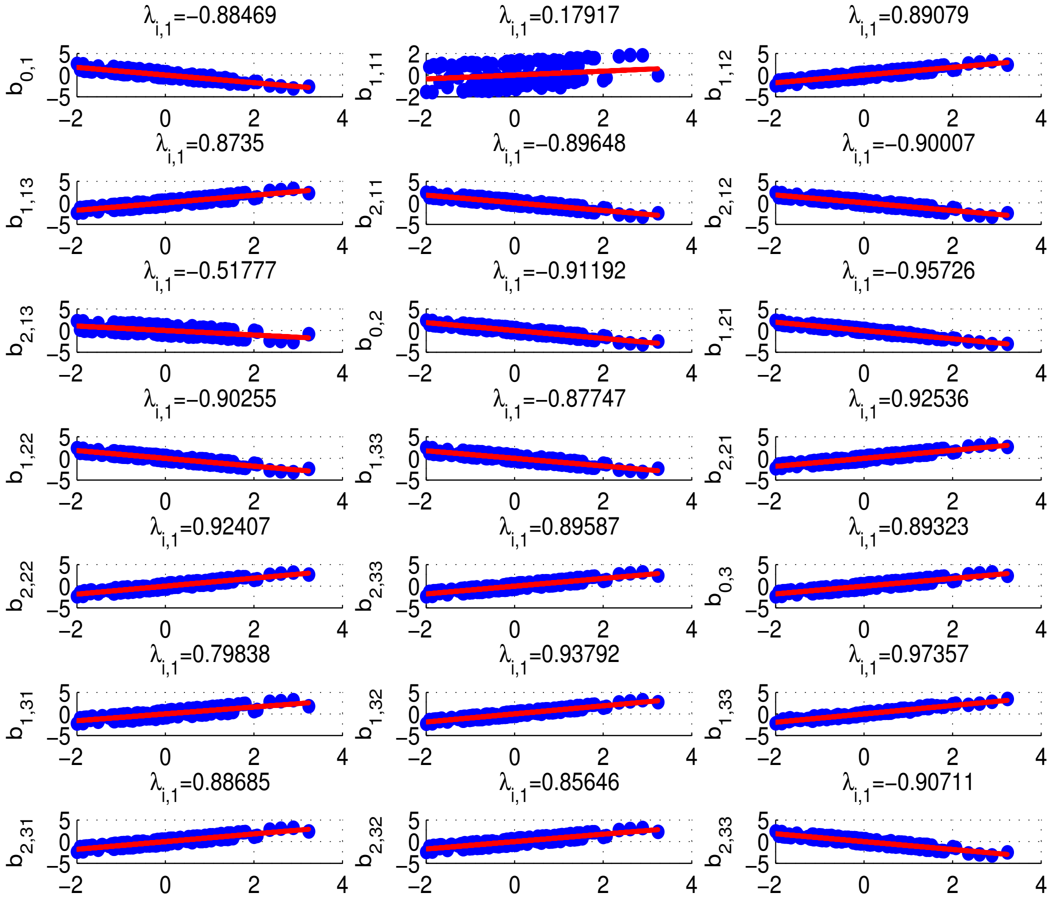

Figure 5.6: Scatter plots for factor 1 and TVPs of VAR estimated by two-step likelihood method

5.5.1.4 TVP-VAR with stochastic volatility

I applied the two-step methods to the TVP-VAR with stochastic volatility as in Primiceri (2005). Compared to the previous case where the volatility was supposed constant, the tests suggest generally the same number of factors. In case of the two-step recursive procedure, the first factor still remains important especially for the interest rate equation coefficients, while the second explains mostly the stochastic volatility part. The picture is clearer after the two-step likelihood estimation. Figure 5.12 present the marginal contribution of factors to the total R^2 of TVPs and stochastic volatility parameters. Elements 22 to 24 corresponds to coefficients in the contemporaneous relations matrix A_0^{-1}, and last three elements are the variances of structural shocks. An interesting result is that

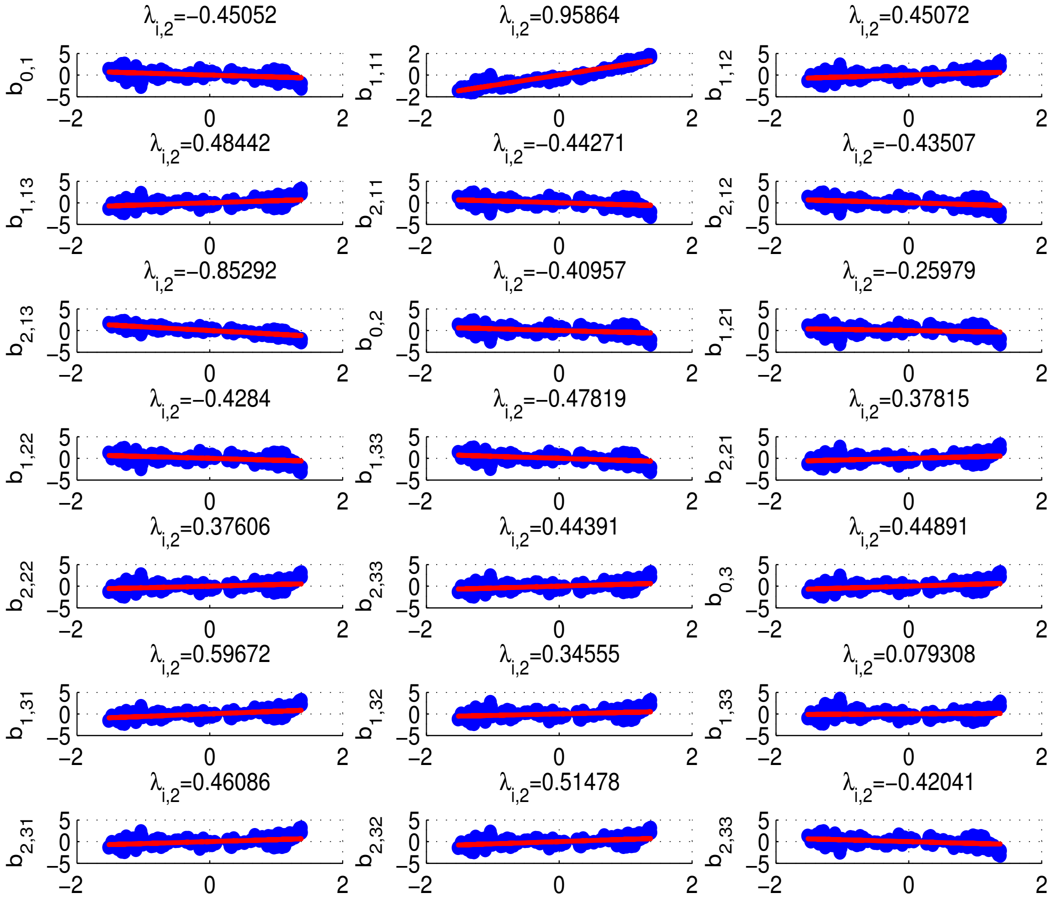

Figure 5.7: Scatter plots for factor 1 and VAR model TVPs estimated by two-step likelihood method

the VAR coefficients load to the first factor, the elements of A_0^{-1} to the second factor, while the structural shocks volatilities are related to the third factor. However, since factor analysis tools suggest a one-factor model, it is likely that the stochastic volatility parameters are either constant or vary in idiosyncratic way.

5.5.1.5 Evidence using post-crisis data

In the previous exercise I used the data up to end of 2006. Remember that the objective of this paper is to establish the empirical evidence on common sources of parameters' instability, following the idea that a small number of structural breaks occur. The recent financial crisis is an example of such a structural change in the economic behav-

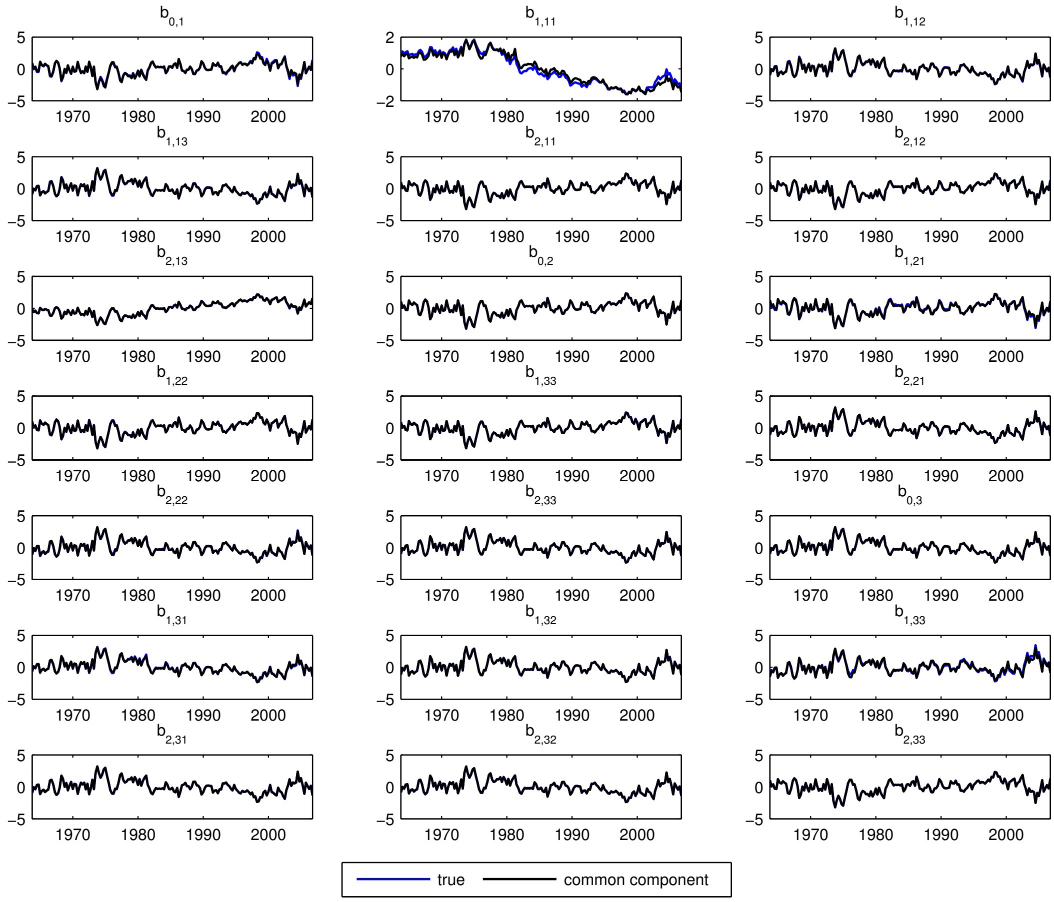

Figure 5.8: Common component and VAR model TVPs estimated by two-step likelihood method

ior, and considering the most recent data may help in identifying the common shocks. In this section, I redo the same exercise but with data updated to 2010Q3.

After applying the two-step recursive procedure, the estimated number of common shocks is generally equal to 2. However, the marginal contribution of factors to the total R^2 has changed, as depicted in Figure 5.13. With the pre-crisis data, the first factor was mostly related to interest rate equation, while using (post-)crisis data produces a picture where the first factor is also important for the unemployment rate equation, and the second factor is mostly related to inflation and unemployment rate equations. The two-step likelihood method produces similar results.

Finally, the simultaneous maximum likelihood method is performed. The suggested

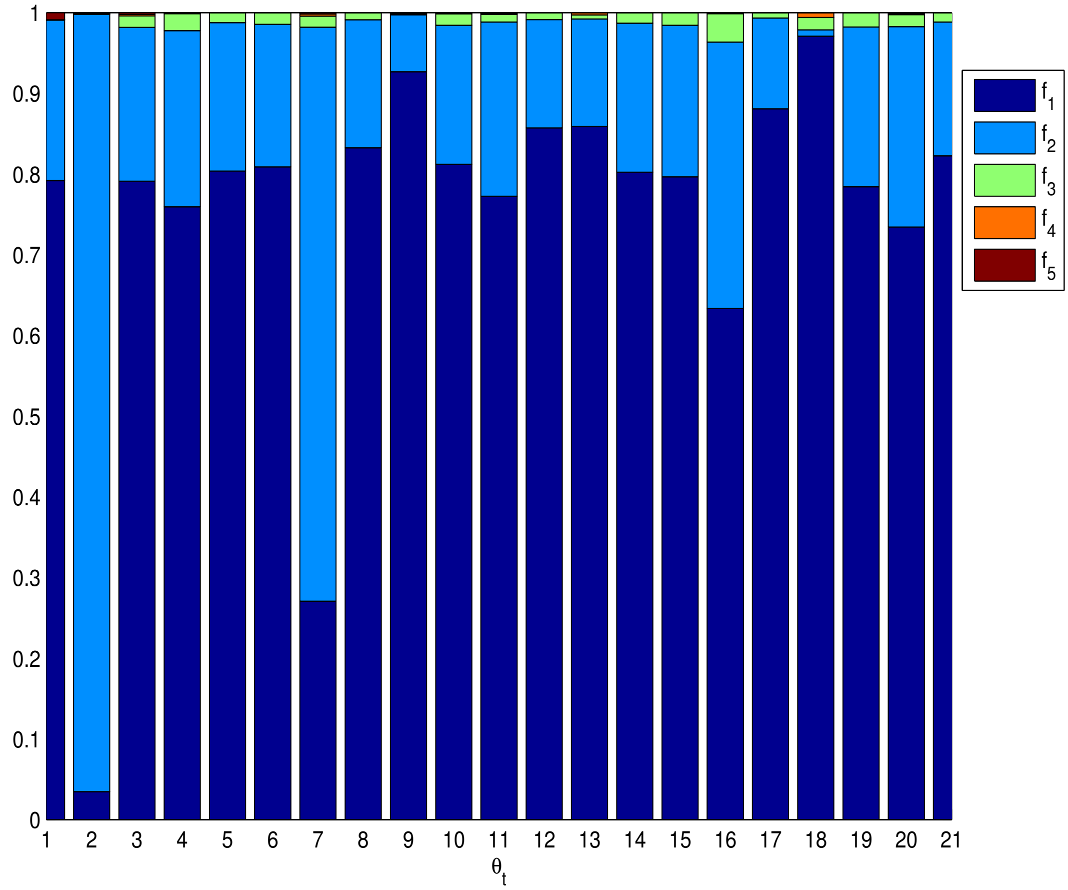

Figure 5.9: Marginal contribution to total R^2 of TVPs of VAR estimated by two-step likelihood method

model contains 2 factors. The time-varying VAR coefficients are plotted in Figure 5.14. For many parameters there is an important change after 2007, indicating the presence of a structural break. The first panel of the Figure 5.15 plots the VAR series and the two factors. The correlation structure is such that the first factor is highly correlated with inflation (0.84), presents a sizeable comovement with interest rate (0.43) and a smaller negative correlation with unemployment rate (-0.27). The second factor shows similar sign pattern but with all correlation being around 0.3. The second part of the Figure 15 presents the factor loadings. Now, contrary to pre-crisis data, a majority of TVPs load heavily on factors. In particular, all inflation equation coefficient seem to be time-varying with the two factors, and this is also the case for about half of the parameters in

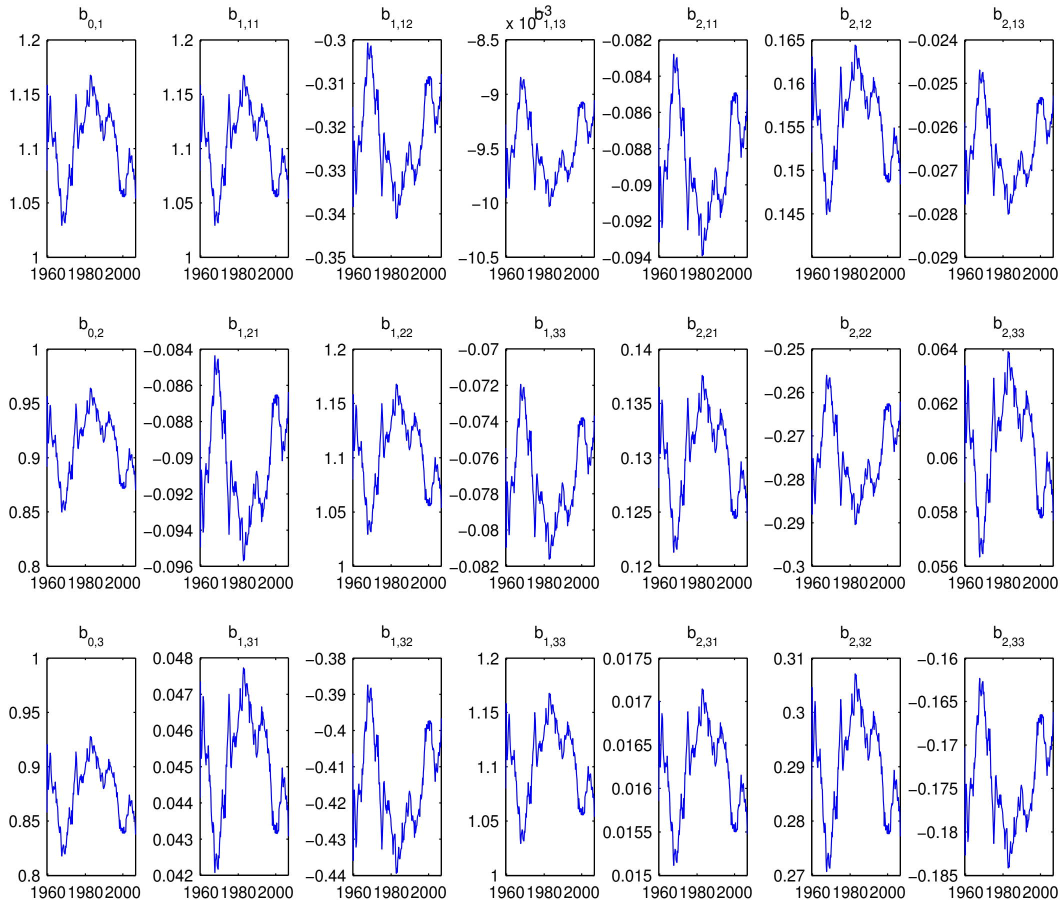

Figure 5.10: Time-varying parameters from ML estimation of Factor-TVP VAR

other VAR equations.

5.5.2 FAVAR model

Since Bernanke, Boivin and Eliasz (2005) there is a growing literature using the large dimensional factor model to measure the effects of structural shocks in economy. This class of models is of a particular interest here because the number of parameters is very large. Stock and Watson (2007) perform a forecasting exercise using a large dimensional dynamic factor model with time varying factor loadings, time varying autoregressive processes for idiosyncratic components and TVP-VAR for latent factors. The results suggest instability in all these parameters. However, given the huge num-

Log-likelihood	-233.52	
$\hat{\rho}$	0.9996	
corr(y,f)	π	0.2430
	ur	0.8419
	r	0.3914

Table 5.II: Summary of ML estimation of Factor-TVP VAR model

ber of TVPs in this empirical application, it is plausible that the underlying variability of these coefficients is of a lower rank. I use the data from Boivin, Giannoni and Stevanovic (2010), and estimate their factor model by recursive procedure. The model is

$$\begin{aligned}
&X_t = \Lambda_t F_t + u_t \\
&F_t = B_t(L) F_{t-1} + e_t \\
&Var(u_t) = \Psi_t, \quad Var(e_t) = \Omega_t
\end{aligned}$$

where X_t is $N \times 1$, F_t is $K \times 1$, $B(L)$ is of order p, Ψ_t is diagonal. The number of TVPs is $N \times K + N + K^2 \times p + 0.5 \times (K \times (K+1))$. In their application, $N = 124$, $K = 4$, $p = 4$, which gives 694 TVPs. These are stacked in vector θ_t:

$$\theta_t = [\mathbf{vec}(\Lambda_t)' \quad \mathbf{diag}(\Psi_t)' \quad \mathbf{vec}(B_t(L))' \quad \mathbf{vec}(\mathbf{tril}(\Omega_t))'],$$

where **vec** operator stacks columns of a matrix, **diag** vectorize the diagonal elements of a matrix, and **tril** operator takes only the lower triangular part of a matrix, including its diagonal elements.

If one is interested to estimate this model by a likelihood method where the likelihood is obtained with Kalman filter, even if latent factors were known, there are still 694 states to track. Moreover, it is hard to find intuition and rationalize the existence of 694 distinct sources of instability.

On the other side, if the factor structure is imposed on θ_t

$$\theta_t = AC_t + \eta_t,$$

with C_t containing $q << 694$ elements, there are only q states to filter. Here, I estimate

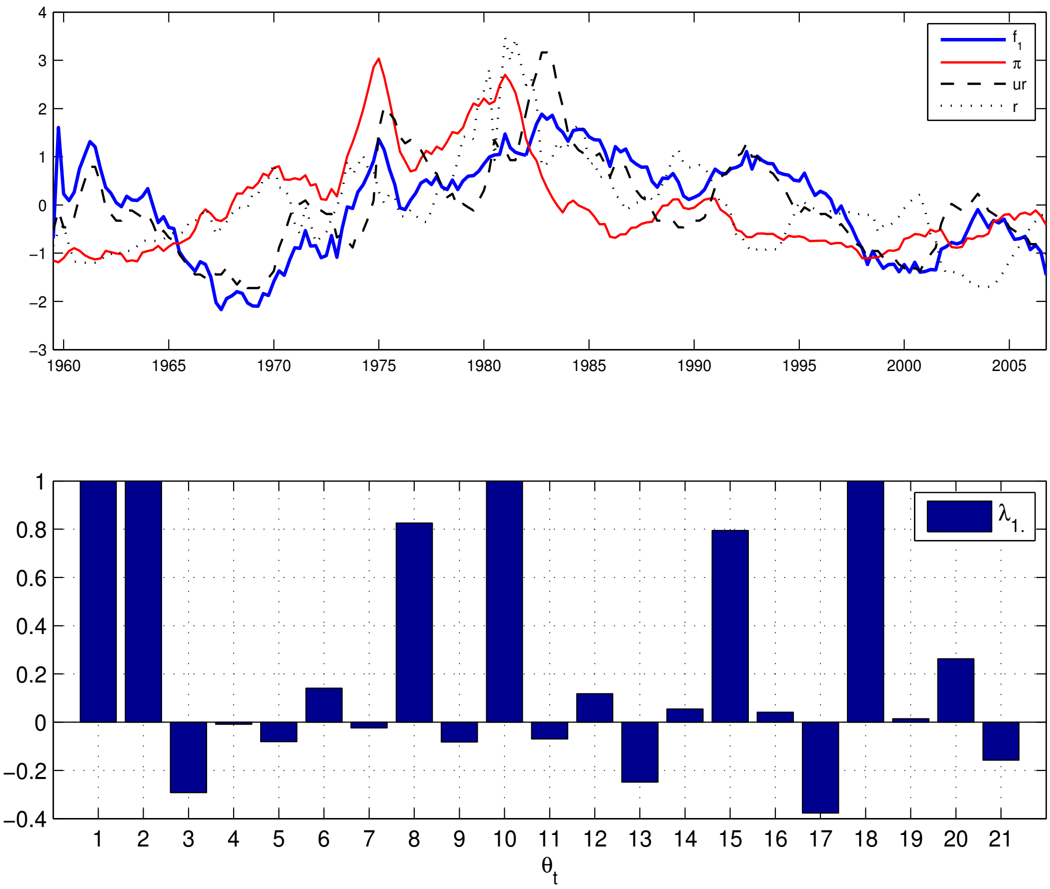

Figure 5.11: Factor loadings from ML estimation of Factor-TVP VAR

the TVP-FAVAR model by the recursive two-step procedure with expanding window: the factors are extracted in step 1, then their VAR process is estimated in the second step. Then, I apply the same factor analysis as in the case of TVP-VAR model to the estimated TVPs in $\hat{\theta}_t$.

For the seek of space, I will just summarize the results without showing figures and tables. The scree and trace tests suggest some weaker factor structure than in the VAR case. The MAP and Parallel tests estimate 5 to 10 factors, and the Bai and Ng tests suggest 2 to 4 common shocks. Hence, 4 factors seems to be satisfactory, but one must keep in mind that $\hat{\theta}_t$ contains 694 elements to approximate. As in the VAR case, I find that few factors explain mostly the variations in factor loadings and VAR coefficients, Λ_t and $\mathbf{vec}(B_t(L))$, but less in stochastic volatilities.

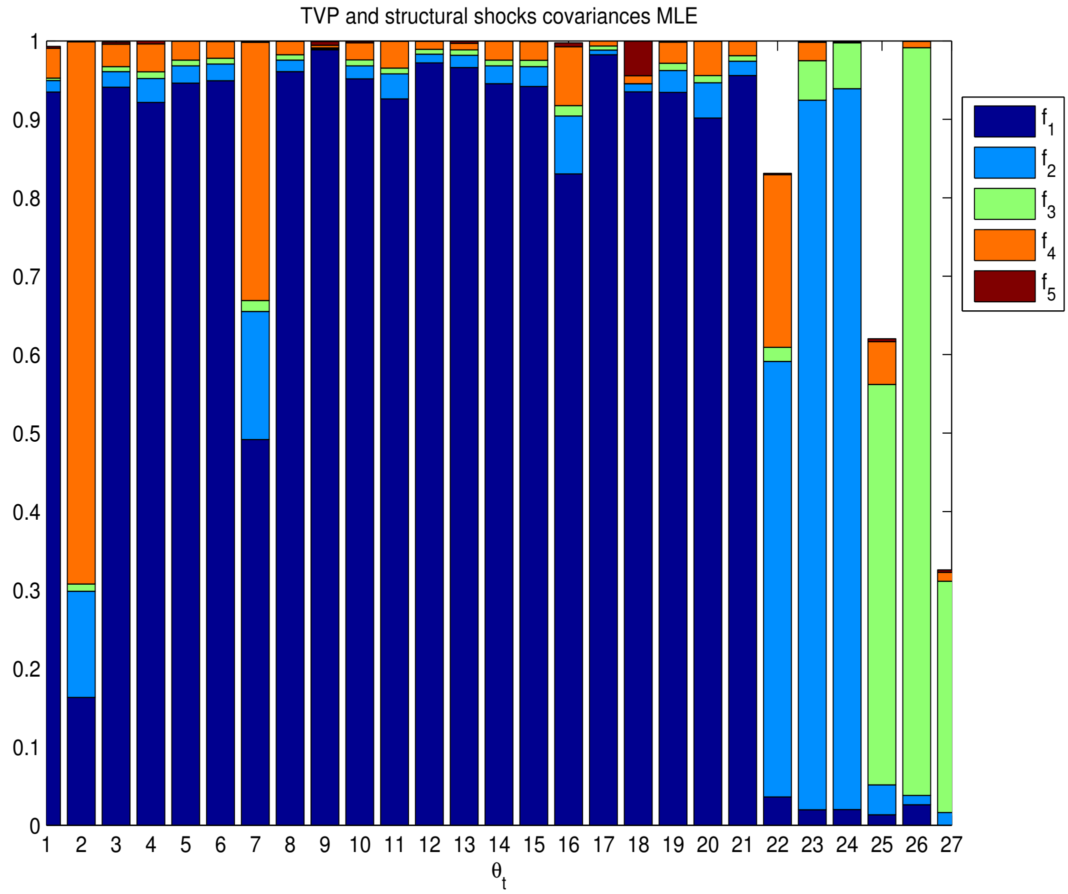

Figure 5.12: Marginal contribution to total R^2 of TVPs and SVs of VAR estimated by two-step likelihood method

When compared to data, the factors have strong predictive power (R^2 between 0.7 and 0.8) for Treasury bonds, especially at long-term maturities, corporate bond yields, some stock market indicators and the personal saving rate. They are also related, with R^2 between 0.4 and 0.7, to many labor market indicators (employment, unemployment rate, hours, earnings) and some price indicators.

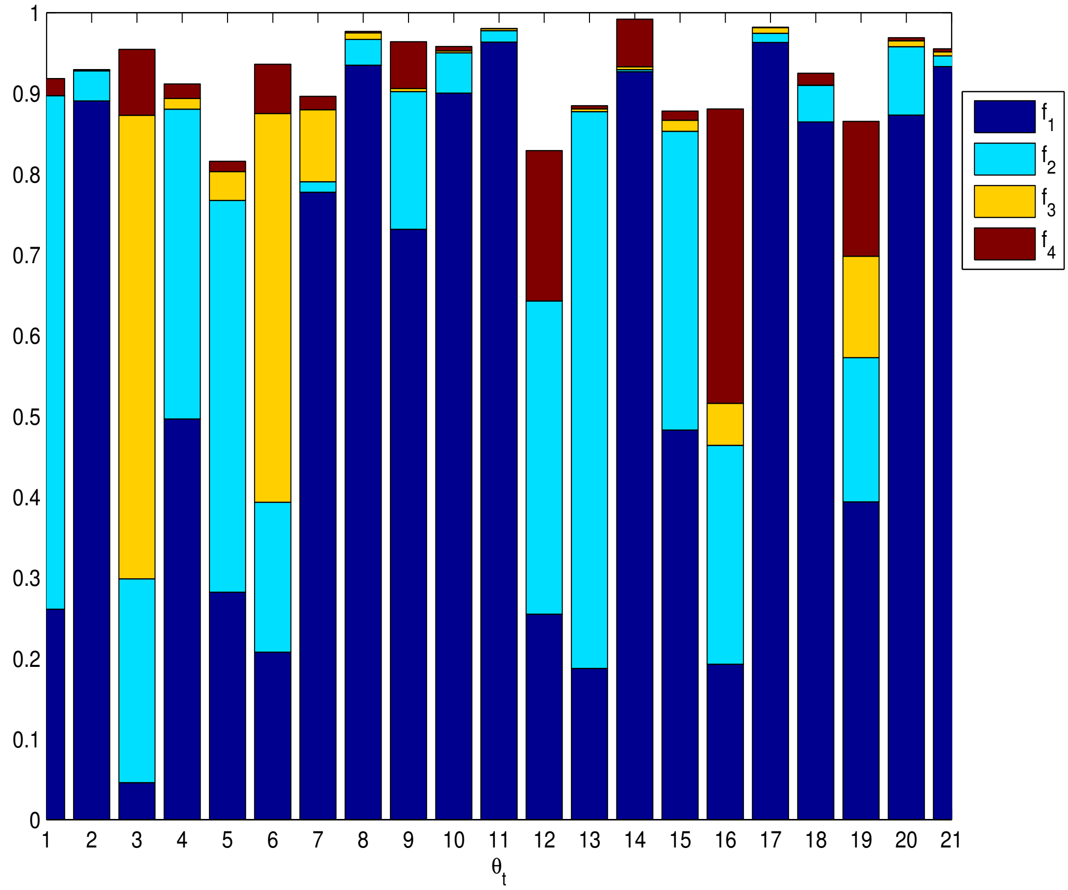

Figure 5.13: Marginal contributions of factors to total R_2 on VAR model TVPs estimated by recursive OLS with post-crisis data

5.6 Evidence from simulated data

In this section, I simulate data from a simple New Keynesian model with log-CRRA preferences. [14] The common source of time instability in parameters of the VAR representation of the model is a shock on the interest rate target in monetary policy Taylor rule:

14. A series of simulation experiments is conducted using the same model with different drifting coefficients and stochastic volatility, and using a simpler 3-variable forward looking model as in Benati and Surico (2009). The conclusions are similar.

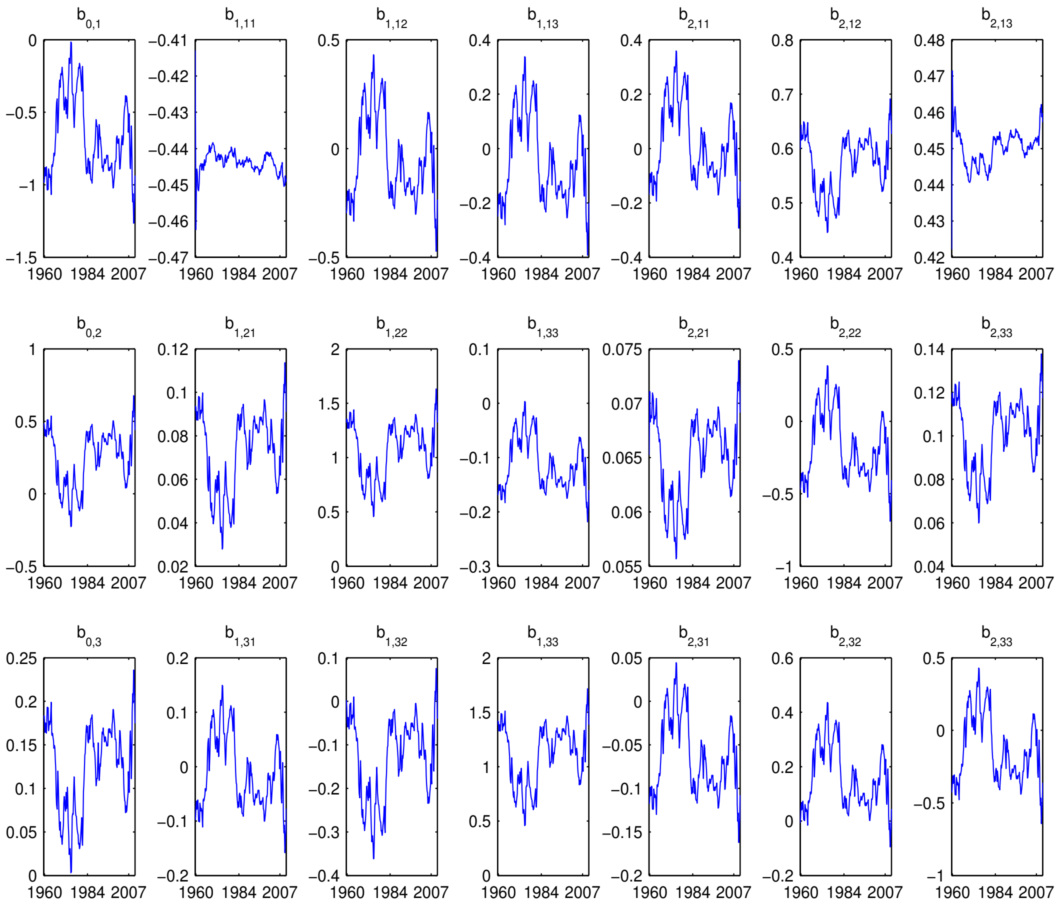

Figure 5.14: Time-varying parameters from ML estimation of Factor-TVP VAR with post-crisis data

$$\frac{R_t}{R_t^\star} = (\frac{\pi_t}{\beta R_t^\star})^\gamma \exp(\sigma e_t) \tag{5.20}$$

$$\log R_t^\star = (1-\rho)0.01 + \rho \log R_{t-1}^\star - \sigma^\star e_t^\star \tag{5.21}$$

Note that the AR(1) process for the time varying target is very persistent, $\rho = 0.99999$, and the variance of its shock is small, $\sigma^\star = 0.01$. Once the data are simulated, I apply the Factor-TVP model to a VAR representation of output gap, inflation and interest rate. [15] The two-step procedure suggest that there exists a factor structure on

15. The VAR representation of all endogenous variables is impossible to estimate since the variables are

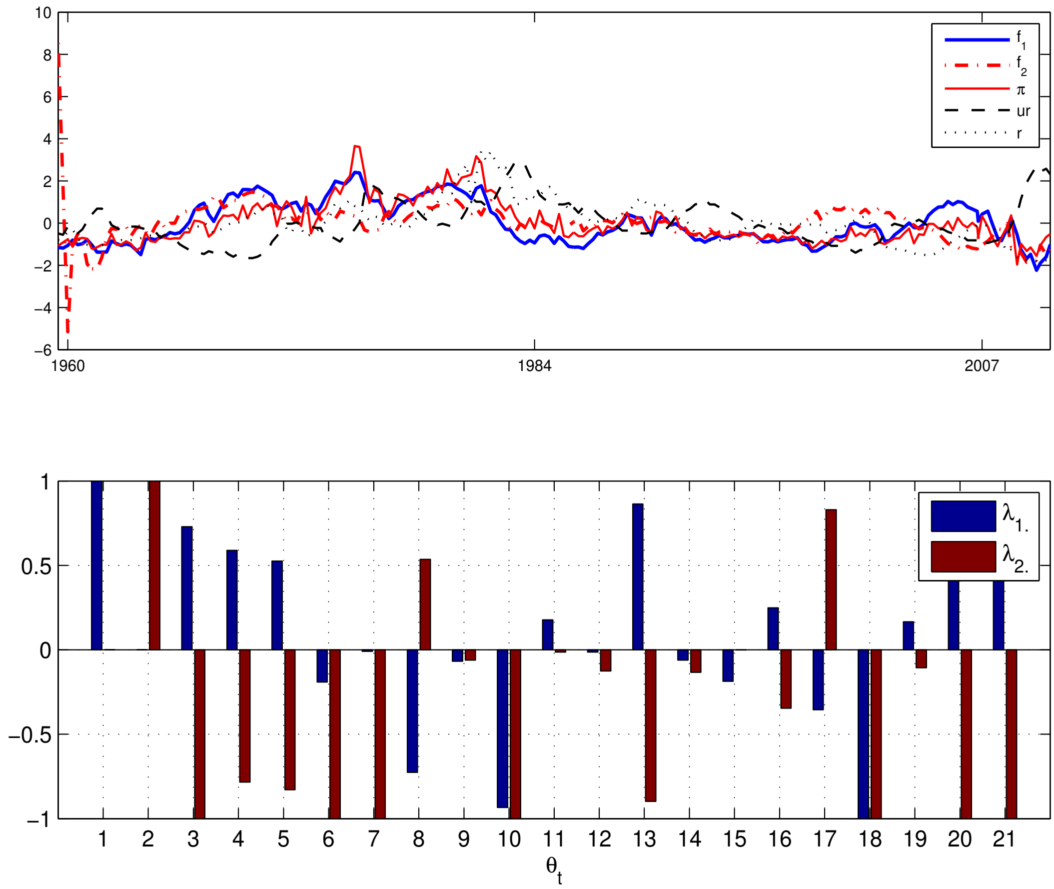

Figure 5.15: Factor loadings from ML estimation of Factor-TVP VAR with post-crisis data

VAR coefficients. The MAP, Parallel and Bai-Ng tests estimate between 1 and 4 factors. The marginal contribution of the first factor is very important for all VAR time varying coefficients, especially for inflation and interest rate equations.

The simultaneous maximum likelihood method suggests one factor model. The estimated common component is highly persistent, actually a random walk, and the majority of factor loadings are close to one. Interesting, the correlation between the estimated factor and the true time-varying inflation target is close to one. Moreover, the estimated factor is highly correlated with simulated output gap and inflation rate.

practically collinear. Another problem is the existence of a VAR representation of a subset of endogenous variables. It is likely that their stochastic representation is actually a VARMA model, but I suppose that it can approximated by a finite order VAR.

5.7 Conclusion

The objective of this paper was to detect the presence of the factor structure, and test its dimensionality, in popular empirical time-varying parameters macroeconomic models. I first showed that structural instability in macroeconomic models is likely to imply time variation in all (or at least a subset of) parameters in reduced-form or estimable models. I discussed how the factor representation of time-varying parameters is obtained, and presented the Factor-TVP that takes into account the factor structure of the parameters. The approach was applied to a standard 3-variable VAR model from Primiceri (2005), and to a factor-augmented VAR (FAVAR) model from Boivin, Giannoni and Stevanovic (2010).

Overall, I found that parameters' instability in these macroeconomic models is caused by a very few number of common factors. In TVP-VAR application, the underlying factor is very persistent and highly correlated with the unemployment rate, and moderately related to inflation and interest rates. When applied to TVP-VAR model with stochastic volatility, I found that time-variability in VAR coefficients is mostly explained by only one factor, while the stochastic volatility part is explained by two additional factors. In particular, when the structural model is obtained by the Choleski decomposition of the residuals time-varying covariance matrix, the contemporaneous relations matrix parameters were mainly explained by the second factor, while the variances of structural shocks were related to the third factor.

To incorporate the recent financial crisis and to see if there have been common structural breaks, the same exercise was conducted with the sample updated to 2010Q3. For two-step procedures, the data suggested one more factor, and their interpretation is changed. The estimated TVPs presented an important change after 2007, indicating the presence of a structural break. Contrary to pre-crisis data, a majority of TVPs load heavily on factors. In particular, all inflation equation coefficients were time-varying with the two factors, and this was also the case for about half of parameters in other VAR equations.

To complete the empirical exercise, the time-varying FAVAR model was estimated by two-step recursive procedure, and 4 dynamic factors were found to govern the dynamics in almost 700 time-varying parameters. Again, the stochastic volatility coefficients instability had different common factors than the regression coefficients.

Finally, I simulated the data from a simple New Keynesian model with time-varying interest rate target in the standard Taylor rule. The two-step procedures found the true factor structure on TVP-VAR representation of the simulated data. Simultaneous ML estimated a factor that is very correlated with the true common shock, and suggested that most of time-varying coefficients load on the factor.

CONCLUSION GÉNÉRALE

Cette thèse consiste de cinq essais ayant pour but d'approfondir la compréhension des mécanismes de transmission des chocs structurels vers l'économie réelle et améliorer la prévision des agrégats macroéconomique. La thèse s'inscrit dans cadre d'analyse factorielle en présence d'un grand nombre de données. Les contributions sont d'ordres empirique et théorique. Premièrement, nous avons caractérisé la transmission de la politique monétaire au Canada en corrigeant pour la plupart des anomalies répertoriées dans la littérature précédente. Deuxièmement, nous avons identifié et quantifié, parmi les premiers, les chocs de crédit et leurs effets sur les économies américaine et canadienne. Enfin, nous avons produit la première évidence empirique sur la structure à facteurs dans les paramètres variant dans le temps dans les modèles macroéconomiques. De point de vue théorique, nous avons proposé une nouvelle classe de modèles combinant l'analyse factorielle et modélisation VARMA et leur importance a été justifiée au niveau de la prévision des agrégats macroéconomique et de l'analyse structurelle utilisant les fonctions de réponse impulsionnelles.

Le premier article avait pour but d'analyser la transmission de la politique monétaire au Canada en utilisant le modèle vectoriel autorégressif augmenté par facteurs (FAVAR). Le modèle FAVAR a été estimé en utilisant un panel non balance de 348 indicateurs économiques mensuels et 87 séries trimestrielles. Nous avons trouvé que l'information contenue dans les facteurs est importante pour bien identifier la transmission de la politique monétaire et qu'elle aide a corriger les anomalies empiriques standard. Enfin, grâce à la possibilité d'obtenir les fonctions de réponse impulsionnelles pour tous les indicateurs dans l'ensemble de données, nous avons produit l'analyse la plus complète à ce jour des effets de la politique monétaire au Canada.

Dans le deuxième article nous avons examiné les effets et la propagation des chocs de crédit sur l'économie réelle dans le cadre d'un modèle à facteurs structurel. Nous avons trouvé qu'un choc de crédit augmente immédiatement les *credit spreads*, diminue les bons de Trésor et cause une récession. Ces chocs ont un effet important sur des mesures d'activité réelle, indices de prix, indicateurs avancés et financiers.

La contribution théorique est apportée dans le troisième article où nous avons étudié la relation entre les représentations VARMA et factorielle des processus vectoriels stochastiques. Comme résultat, nous avons proposé une nouvelle classe de modèles VARMA augmentés par facteurs (FAVARMA). Le modèle a été appliqué dans deux exercices de prévision et les résultats ont montré que la partie VARMA aide à mieux prévoir les importants agrégats macroéconomiques relativement aux modèles standards. Finalement, nous avons estimé les effets de choc monétaire en utilisant les données et le schéma d'identification de Bernanke, Boivin et Eliasz (2005). Notre modèle a donné les résultats cohérents et précis des effets et de la transmission monétaire aux États-Unis.

L'objectif du quatrième article était d'identifier et mesurer les effets des chocs de crédit au Canada dans un environnement riche en données en utilisant le modèle FAVARMA structurel. D'un côté, nous avons trouvé qu'une augmentation inattendue de la prime de financement extérieur aux États-Unis génère une récession significative et persistante au Canada, accompagnée d'une hausse immédiate des *credit spreads* et taux d'intérêt canadiens. De l'autre côté, une hausse inattendue de la prime canadienne de financement extérieur ne cause pas d'effet significatif au Canada. Nous avons montré que les effets des chocs de crédit au Canada sont essentiellement causés par les conditions globales, approximées ici par le marché américain.

Finalement, en utilisant l'approche *Factor-TVP*, dans le cinquième article nous avons montré que le nombre de sources de l'instabilité temporelle des coefficients est très petit. En particulier, nous avons trouvé qu'un seul facteur explique la majorité de la variabilité des coefficients VAR, tandis que les paramètres de la volatilité des chocs varient d'une façon indépendante. La même analyse est faite avec les données incluant la récente crise financière. La procédure suggère maintenant deux facteurs et le comportement des coefficients présente un changement important depuis 2007. Finalement, la méthode est appliquée à un modèle FAVAR avec paramètres instables. Nous avons trouvé que seulement 5 facteurs dynamiques gouvernent l'instabilité temporelle dans presque 700 coefficients.

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ANNEXES

Appendix I

I.1 Appendix to Chapter 1

I.1.1 Additional results with mixed-frequencies monthly data

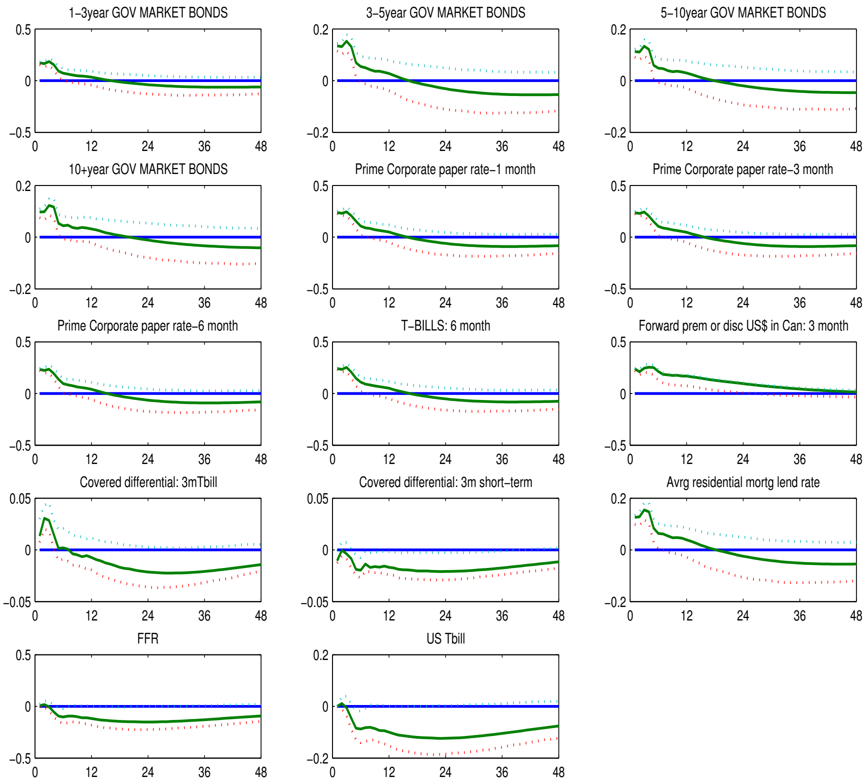

Figure I.1: Interest rates

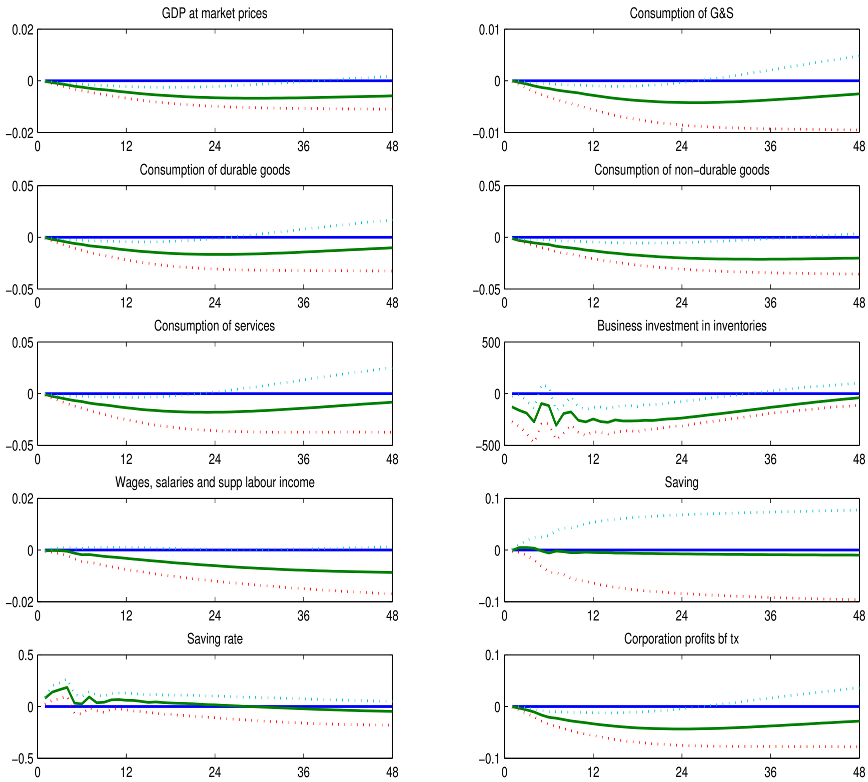

Figure I.2: Quarterly indicators IRFs to CA MP shock

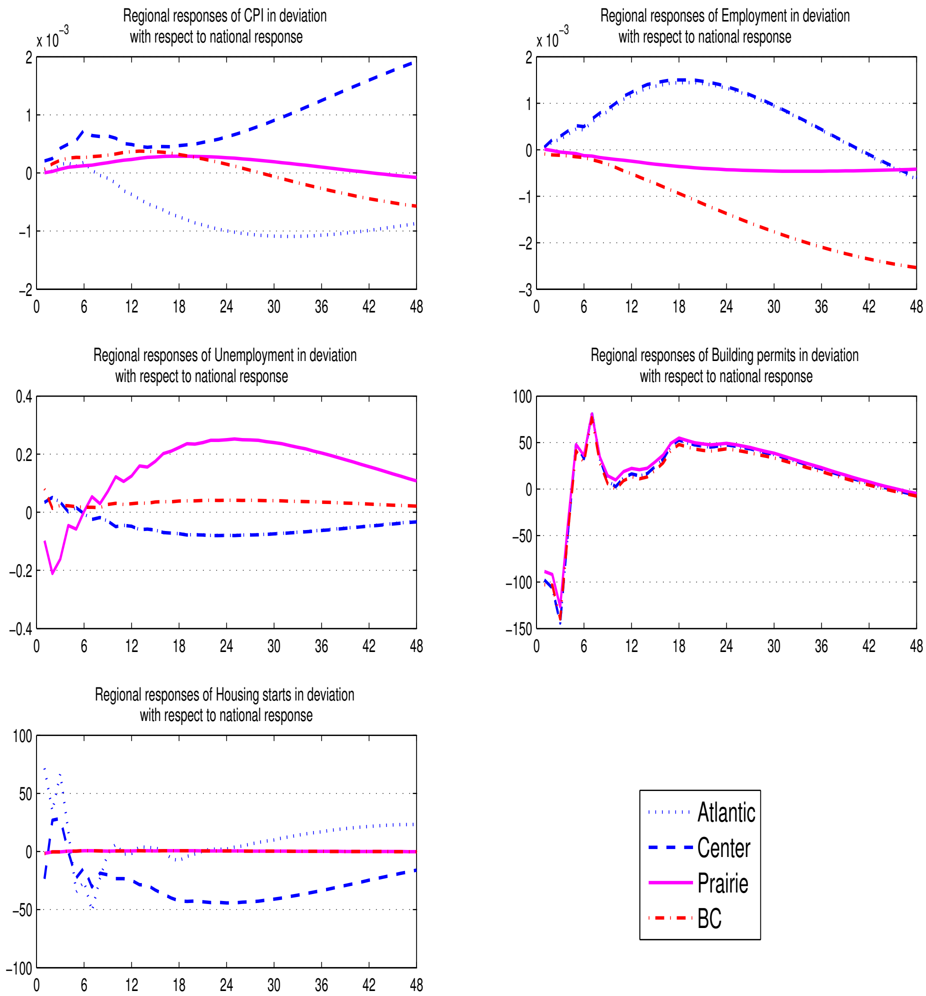

Figure I.3: Comparison of regional economic indicators relative to national impulse responses

I.1.2 Monetary policy shock with mixed-frequencies quarterly data

The frequency in which series are observed can be important in such structural exercise. Since the identification of structural shocks relies on timing restrictions, here contemporaneous ones, these can be more or less realistic across different frequencies. In following, we present impulse responses functions obtained after a positive monetary policy shock in a FAVAR model fitted to mixed-frequencies quarterly data. The benchmark model is composed of eight unobserved factors and one observed factor, T-Bill. The number of lags is set at two and we use the same identification procedure as in the case of monthly panel.

The results for some indicators of interest are presented in Figure I.4. We can see that the responses at quarterly frequency are quite similar to those obtained using monthly panel: slowdown for most of production and price indicators, credit measures and leading indicators. Also, there is no presence of price nor exchange rate puzzles. The impulse response functions of other variables are presented in Appendix. Overall, the effects of monetary policy shock in quarterly frequency are very similar to those in monthly frequency.

According to variance decomposition and R^2 results, not reported here, the monetary policy shock does not have a huge effect on most of the variables, except on interest rates and money supply, but the common component explains an important fraction of variability in observable series.

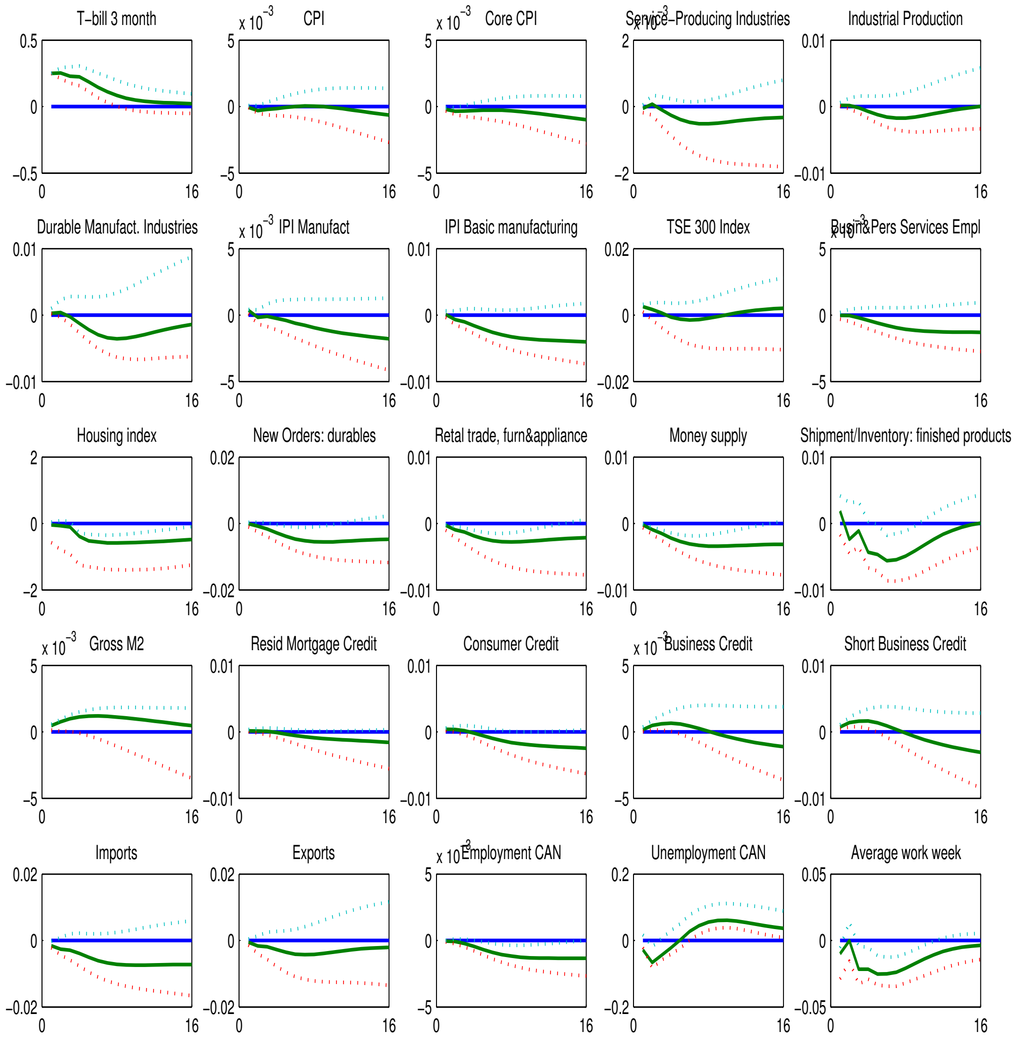

Figure I.4: Impulse responses of some quarterly indicators to identified monetary policy shock

I.1.3 EM Algorithm

When we want to expand the cross-sectional size of the informational panel, i.e. increase N, it is almost sure that we will face some data irregularities causing unbalanced panels. There can be occasionally missing observations, some important data series that start later than the rest of panel, or mixed frequency data. In order to estimate factors by principal component, we need to construct a balanced panel. We present the EM estimation proposed in Stock and Watson (2002b). Consider the least square estimators of Λ and F_t from a generalized factor representation (1) using a balanced panel. The objective function is

$$V(F,\Lambda) = \sum_{i=1}^{N}\sum_{t=1}^{T}(X_{it} - \lambda_i^{'}F_t)^2 \tag{I.1}$$

which can be minimized by the usual eigenvalue calculations. When the panel is unbalanced, least square estimators of F_t can be calculated using an indicator I_{it} equal to 1 if X_{it} is available and 0 otherwise

$$V^*(F,\Lambda) = \sum_{i=1}^{N}\sum_{t=1}^{T} I_{it}(X_{it} - \lambda_i^{'}F_t)^2 \tag{I.2}$$

which requires the following iterative method to be minimized.

Let $\hat{\Lambda}$ and $\hat{F}$ denote estimates of Λ and F from the previous iteration, and let

$$Q(X^*,\hat{F},\hat{\Lambda},F,\Lambda) = E_{\hat{F},\hat{\Lambda}}[V(F,\Lambda)|X^*] \tag{I.3}$$

where X^* denotes the full set of observed data and the RHS of (I.3) is the expected value of the complete data log-likelihood $V(F,\Lambda)$, evaluated using the conditional density of $X|X^*$ evaluated at $\hat{F}$ and $\hat{\Lambda}$. The estimates of F and Λ minimize (I.3).
Developing the equation (I.3) gives

$$Q(X^*,\hat{F},\hat{\Lambda},F,\Lambda) = \sum_i\sum_t E_{\hat{F},\hat{\Lambda}}(X_{it}^2|X^*) + (\lambda_i^{'}F_t)^2 - 2\hat{X}_{it}(\lambda_i^{'}F_t) \tag{I.4}$$

where $\hat{X}_{it} = E_{\hat{F},\hat{\Lambda}}(X_{it}|X^*)$. Since the first term on the RHS of (I.4) does not depend on factors and loadings, we can replace it by $\sum_i\sum_t \hat{X}_{it}^2$, implying that at iteration j, $\hat{F}$ and $\hat{\Lambda}$ minimize $\hat{V}(F,\Lambda) = \sum_{i=1}\sum_{t=1}(\hat{X}_{it} - \lambda_i^{'}F_t)^2$. Then, it reduces to the standard principal component eigenvalue calculation where the missing data are replaced by their expectation conditional on the observed data and using the parameter values from the previous iteration. One way to obtain starting values for $\hat{F}$ and $La\hat{m}bda$ is to estimate them from a subset that constitutes a balanced panel.

The main problem is to calculate $\hat{X}_{it}$ depending on the nature of missing value (occasional missing value, mixed frequency, etc.). Let $\underline{X}_i = (X_{i1},...,X_{iT})^{'}$, and let $\underline{X}_i^*$ be the vector of observations on the ith variable. Suppose that $\underline{X}_i^* = A_i\underline{X}_i$ for some known matrix A_i. Then, $E(\underline{X}_i|X^*) = E(\underline{X}_i|X_i^*) = F\lambda_i + A_i^{'}(A_iA_i^{'})^{-}(\underline{X}_i^* - A_iF\lambda_i)$, where $(A_iA_i^{'})^{-}$ is the generalized inverse. Now, we present five particular cases to calculate $\hat{X}_{it}$.

A. Missing Observations. The easiest and most current case is when some observations on X_{it} are missing. At the iteration j, $\hat{X}_{it} = X_{it}$ if X_{it} observed and $\hat{X}_{it} = \hat{\lambda}_i^{'}\hat{F}_t$ otherwise. The estimate of is then updated by computing the eigenvectors corresponding to the largest r eigenvalues of $N^{-1}\sum_i \hat{\underline{X}}_i\hat{\underline{X}}_i$, where $\hat{\underline{X}}_i = (\hat{X}_{i1},...,\hat{X}_{iT})^{'}$, and $\hat{\Lambda}$ is updated by the OLS regression of $\hat{X}$ onto this updated estimate of F.

B. Mixed Monthly and Quarterly Data - I(0) Stock Variables. If the quarterly observed series is the point-in-time level of a variable at the end of the quarter, stock variable, is integrated of order zero, then it is handled as in case A, i.e. it is treated as a monthly series with missing observations in the first and second months of the quarter.

C. Mixed Monthly and Quarterly Data - I(0) Flow Variable. A quarterly flow variable is the average (sum) of unobserved monthly values. If this series is $I(0)$, the unobserved monthly series, X_{it}, is measured only as the time aggregate $X_{it}^q = (1/3)(X_{i,t-2} + X_{i,t-1} + X_{it})$ for $t = 3,6,9,...$, and X_{it}^q is missing for all other values of t. In this case estimation proceeds as in case A except that $\hat{X}_{it} = \hat{\lambda}_i^{'}\hat{F}_t + \hat{e}_{it}$, where $\hat{e}_{it} = X_{it}^q - \hat{\lambda}_i^{'}(\hat{F}_{\tau-2} + \hat{F}_{\tau-1} + \hat{F}_{\tau})/3$, where $\tau = 3$ when $t = 1,2,3,,$ $\tau = 6$, when $t = 4,5,6,,$ and so forth.

D. Mixed Monthly and Quarterly Data - I(1) Stock Variables. Let X_{it}^1 denote the quarterly first difference stock variable, assumed to be measured in the third month of every quarter, and X_{it} denote the monthly first difference of the variable. Then, $X_{it}^q = (X_{i,t-2} + X_{i,t-1} + X_{it})$ for $t = 3,6,9,...$, and X_{it}^q is missing for all other values of t. In this case estimation proceeds as in case A but with $\hat{X}_{it} = \hat{\lambda}_i^{'}\hat{F}_t + (1/3)\hat{e}_{it}$, where $\hat{e}_{it} = X_{it}^q - \hat{\lambda}_i^{'}(\hat{F}_{\tau-2} + \hat{F}_{\tau-1} + \hat{F}_{\tau})/3$, where $\tau = 3$ when $t = 1,2,3,,$ $\tau = 6$, when $t = 4,5,6,,$ and so forth.

E. Mixed Monthly and Quarterly Data - I(1) Flow Variables. Once again, let X_{it}^q be the quarterly first difference assumed observed at the end of every quarter. The vector of observations is then $\underline{X}_i^* = (X_{i3}^q, X_{i6}^q, ..., X_{i\tau}^q)$, where τ denotes the month of the last quarterly observation. If the underlying quarterly data are averages of monthly series, and if the monthly first differences are denoted by X_{it}, then $X_{it}^q = (1/3)(X_{i,t} + 2X_{i,t-1} + 3X_{i,t-2} + 2X_{i,t-3} + X_{i,t-4})$ for $t = 3, 6, 9$, (which defines implicitly the rows of A_i). Then, the estimate of $\underline{X}_i$ is given by $\hat{\underline{X}}_i = F\lambda_i + A_i'(A_iA_i')^{-1}(\underline{X}_i^* - A_iF\lambda_i)$.

I.1.4 Data Sets

Format contains series number; StatCan number; transformation code; series description and time span. The transformation codes are: 1 - no transformation; 2 - first difference; 4 - logarithm; 5 - first difference of logarithm.

MONTHLY SERIES

Table 326-0020 Consumer Price Index Canada, Provinces

1	v41690973	**5**	All-items (2002=100)	1969-01-01 to 2008-05-01
2	v41690974	**5**	Food (2002=100)	1969-01-01 to 2008-05-01
3	v41690993	**5**	Dairy products (2002=100)	1969-01-01 to 2008-05-01
4	v41691046	**5**	Food purchased from restaurants (2002=100)	1969-01-01 to 2008-05-01
5	v41691051	**5**	Rented accommodation (2002=100)	1969-01-01 to 2008-05-01
6	v41691055	**5**	Owned accommodation (2002=100)	1969-01-01 to 2008-05-01
7	v41691065	**5**	Natural gas (2002=100)	1969-01-01 to 2008-05-01
8	v41691066	**5**	Fuel oil and other fuels (2002=100)	1969-01-01 to 2008-05-01
9	v41691108	**5**	Clothing and footwear (2002=100)	1969-01-01 to 2008-05-01
10	v41691129	**5**	Private transportation (2002=100)	1969-01-01 to 2008-05-01
11	v41691153	**5**	Health and personal care (2002=100)	1969-01-01 to 2008-05-01
12	v41691170	**5**	Recreation, education and reading (2002=100)	1969-01-01 to 2008-05-01
13	v41692942	**5**	All-items excluding eight of the most volatile components (Bank of Canada definition) (2002=100)	1969-01-01 to 2008-05-01
14	v41691232	**5**	All-items excluding food (2002=100)	1969-01-01 to 2008-05-01
15	v41691233	**5**	All-items excluding food and energy (2002=100)	1969-01-01 to 2008-05-01
16	v41691238	**5**	All-items excluding energy (2002=100)	1971-01-01 to 2008-05-01
17	v41691237	**5**	Food and energy (2002=100)	1971-01-01 to 2008-05-01
18	v41691239	**5**	Energy (2002=100)	1969-01-01 to 2008-05-01
19	v41691219	**5**	Housing (1986 definition) (2002=100)	1969-01-01 to 2008-05-01
20	v41691222	**5**	Goods (2002=100)	1969-01-01 to 2008-05-01
21	v41691223	**5**	Durable goods (2002=100)	1969-01-01 to 2008-05-01
22	v41691225	**5**	Non-durable goods (2002=100)	1969-01-01 to 2008-05-01
23	v41691229	**5**	Goods excluding food purchased from stores and energy (2002=100)	1969-01-01 to 2008-05-01
24	v41691230	**5**	Services (2002=100)	1969-01-01 to 2008-05-01
25	v41691231	**5**	Services excluding shelter services (2002=100)	1969-01-01 to 2008-05-01
26	v41691244	**5**	Newfoundland and Labrador; All-items (2002=100)	1978-09-01 to 2008-05-01
27	v41691369	**5**	Newfoundland and Labrador; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
28	v41691363	**5**	Newfoundland and Labrador; Goods (2002=100)	1978-09-01 to 2008-05-01
29	v41691367	**5**	Newfoundland and Labrador; Services (2002=100)	1978-09-01 to 2008-05-01
30	v41691379	**5**	Prince Edward Island; All-items (2002=100)	1978-09-01 to 2008-05-01
31	v41691503	**5**	Prince Edward Island; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
32	v41691497	**5**	Prince Edward Island; Goods (2002=100)	1978-09-01 to 2008-05-01
33	v41691501	**5**	Prince Edward Island; Services (2002=100)	1978-09-01 to 2008-05-01

34	v41691513	**5**	Nova Scotia; All-items (2002=100)	1978-09-01 to 2008-05-01
35	v41691638	**5**	Nova Scotia; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
36	v41691632	**5**	Nova Scotia; Goods (2002=100)	1978-09-01 to 2008-05-01
37	v41691636	**5**	Nova Scotia; Services (2002=100)	1978-09-01 to 2008-05-01
38	v41691648	**5**	New Brunswick; All-items (2002=100)	1978-09-01 to 2008-05-01
39	v41691773	**5**	New Brunswick; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
40	v41691767	**5**	New Brunswick; Goods (2002=100)	1978-09-01 to 2008-05-01
41	v41691771	**5**	New Brunswick; Services (2002=100)	1978-09-01 to 2008-05-01
42	v41691783	**5**	Quebec; All-items (2002=100)	1978-09-01 to 2008-05-01
43	v41691909	**5**	Quebec; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
44	v41691903	**5**	Quebec; Goods (2002=100)	1978-09-01 to 2008-05-01
45	v41691907	**5**	Quebec; Services (2002=100)	1978-09-01 to 2008-05-01
46	v41691919	**5**	Ontario; All-items (2002=100)	1978-09-01 to 2008-05-01
47	v41692045	**5**	Ontario; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
48	v41692039	**5**	Ontario; Goods (2002=100)	1978-09-01 to 2008-05-01
49	v41692043	**5**	Ontario; Services (2002=100)	1978-09-01 to 2008-05-01
50	v41692055	**5**	Manitoba; All-items (2002=100)	1978-09-01 to 2008-05-01
51	v41692181	**5**	Manitoba; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
52	v41692175	**5**	Manitoba; Goods (2002=100)	1978-09-01 to 2008-05-01
53	v41692179	**5**	Manitoba; Services (2002=100)	1978-09-01 to 2008-05-01
54	v41692191	**5**	Saskatchewan; All-items (2002=100)	1978-09-01 to 2008-05-01
55	v41692317	**5**	Saskatchewan; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
56	v41692311	**5**	Saskatchewan; Goods (2002=100)	1978-09-01 to 2008-05-01
57	v41692315	**5**	Saskatchewan; Services (2002=100)	1978-09-01 to 2008-05-01
58	v41692327	**5**	Alberta; All-items (2002=100)	1978-09-01 to 2008-05-01
59	v41692452	**5**	Alberta; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
60	v41692446	**5**	Alberta; Goods (2002=100)	1978-09-01 to 2008-05-01
61	v41692450	**5**	Alberta; Services (2002=100)	1978-09-01 to 2008-05-01
62	v41692462	**5**	British Columbia; All-items (2002=100)	1978-09-01 to 2008-05-01
63	v41692588	**5**	British Columbia; All-items excluding food and energy (2002=100)	1978-09-01 to 2008-05-01
64	v41692582	**5**	British Columbia; Goods (2002=100)	1978-09-01 to 2008-05-01
65	v41692586	**5**	British Columbia; Services (2002=100)	1978-09-01 to 2008-05-01
			Table 026-0001 Building permits, residential values and number of units	
66	v14098	**1**	Canada; Total dwellings (number of units) [D848383]	1969-01-01 to 2008-05-01
67	v41651	**1**	Canada; Total dwellings (dollars - thousands) [D845521]	1969-01-01 to 2008-05-01
68	v13824	**1**	Newfoundland and Labrador; Total dwellings (number of units) [D847651]	1969-01-01 to 2008-05-01
69	v41560	**1**	Newfoundland and Labrador; Total dwellings (dollars - thousands) [D845363]	1969-01-01 to 2008-05-01
70	v13859	**1**	Prince Edward Island; Total dwellings (number of units) [D847658]	1969-01-01 to 2008-05-01
71	v41595	**1**	Prince Edward Island; Total dwellings (dollars - thousands) [D845370]	1969-01-01 to 2008-05-01
72	v13866	**1**	Nova Scotia; Total dwellings (number of units) [D847665]	1969-01-01 to 2008-05-01
73	v41602	**1**	Nova Scotia; Total dwellings (dollars - thousands) [D845377]	1969-01-01 to 2008-05-01
74	v13873	**1**	New Brunswick; Total dwellings (number of units) [D847672]	1969-01-01 to 2008-05-01
75	v41609	**1**	New Brunswick; Total dwellings (dollars - thousands) [D845384]	1969-01-01 to 2008-05-01
76	v13880	**1**	Quebec; Total dwellings (number of units) [D847679]	1969-01-01 to 2008-05-01
77	v41616	**1**	Quebec; Total dwellings (dollars - thousands) [D845391]	1969-01-01 to 2008-05-01
78	v13887	**1**	Ontario; Total dwellings (number of units) [D847686]	1969-01-01 to 2008-05-01
79	v41623	**1**	Ontario; Total dwellings (dollars - thousands) [D845398]	1969-01-01 to 2008-05-01
80	v13894	**1**	Manitoba; Total dwellings (number of units) [D847693]	1969-01-01 to 2008-05-01
81	v41630	**1**	Manitoba; Total dwellings (dollars - thousands) [D845405]	1969-01-01 to 2008-05-01
82	v13901	**1**	Saskatchewan; Total dwellings (number of units) [D847700]	1969-01-01 to 2008-05-01
83	v41637	**1**	Saskatchewan; Total dwellings (dollars - thousands) [D845412]	1969-01-01 to 2008-05-01
84	v13908	**1**	Alberta; Total dwellings (number of units) [D847707]	1969-01-01 to 2008-05-01
85	v41644	**1**	Alberta; Total dwellings (dollars - thousands) [D845419]	1969-01-01 to 2008-05-01
86	v13831	**1**	British Columbia; Total dwellings (number of units) [D847714]	1969-01-01 to 2008-05-01
87	v41567	**1**	British Columbia; Total dwellings (dollars - thousands) [D845426]	1969-01-01 to 2008-05-01
			Table 027-0002 CMHC, housing starts, under constr and completions, SA	
88	v730040	**1**	Canada; Total units (units - thousands) [J9001]	1969-01-01 to 2008-05-01
89	v729972	**1**	Newfoundland and Labrador; Total units (units - thousands) [J7002]	1969-01-01 to 2008-05-01
90	v729973	**1**	Prince Edward Island; Total units (units - thousands) [J7003]	1969-01-01 to 2008-05-01
91	v729974	**1**	Nova Scotia; Total units (units - thousands) [J7004]	1969-01-01 to 2008-05-01
92	v729975	**1**	New Brunswick; Total units (units - thousands) [J7005]	1969-01-01 to 2008-05-01

93	v729976	**1**	Quebec; Total units (units - thousands) [J7006]	1969-01-01 to 2008-05-01
94	v729981	**1**	Ontario; Total units (units - thousands) [J7008]	1969-01-01 to 2008-05-01
95	v729987	**1**	Manitoba; Total units (units - thousands) [J7011]	1969-01-01 to 2008-05-01
96	v729988	**1**	Saskatchewan; Total units (units - thousands) [J7012]	1969-01-01 to 2008-05-01
97	v729989	**1**	Alberta; Total units (units - thousands) [J7013]	1969-01-01 to 2008-05-01
98	v729990	**1**	British Columbia; Total units (units - thousands) [J7014]	1969-01-01 to 2008-05-01
			Table 377-0003 Business leading indicators for Canada	
99	v7677	**1**	Average work week, manufacturing; Smoothed (hours) [D100042]	1969-01-01 to 2008-05-01
100	v7680	**1**	Housing index; Smoothed (index, 1992=100) [D100043]	1969-01-01 to 2008-05-01
101	v7681	**5**	United States composite leading index; Smoothed (index, 1992=100) [D100044]	1969-01-01 to 2008-04-01
102	v7682	**5**	Money supply; Smoothed (dollars, 1992 - millions) [D100045]	1969-01-01 to 2008-05-01
103	v7683	**5**	New orders, durable goods; Smoothed (dollars, 1992 - millions) [D100046]	1969-01-01 to 2008-03-01
104	v7684	**5**	Retail trade, furniture and appliances; Smoothed (dollars, 1992 - millions) [D100047]	1969-01-01 to 2008-03-01
105	v7686	**1**	Shipment to inventory ratio, finished products; Smoothed (ratio) [D100049]	1969-01-01 to 2008-03-01
106	v7678	**5**	Stock price index, TSE 300; Smoothed (index, 1975=1000) [D100050]	1969-01-01 to 2008-05-01
107	v7679	**5**	Business and personal services employment; Smoothed (persons - thousands) [D100051]	1969-01-01 to 2008-05-01
108	v7688	**5**	Composite index of 10 indicators; Smoothed (index, 1992=100) [D100053]	1969-01-01 to 2008-05-01
			Table 379-0027 GDP at basic prices, by NAICS, Canada, SA, 2002 constant prices	
109	v41881478	**5**	All industries [T001] (dollars - millions)	1981-01-01 to 2008-04-01
110	v41881480	**5**	Business sector, goods [T003] (dollars - millions)	1981-01-01 to 2008-04-01
111	v41881481	**5**	Business sector, services [T004] (dollars - millions)	1981-01-01 to 2008-04-01
112	v41881482	**5**	Non-business sector industries [T005] (dollars - millions)	1981-01-01 to 2008-04-01
113	v41881485	**5**	Goods-producing industries [T008] (dollars - millions)	1981-01-01 to 2008-04-01
114	v41881486	**5**	Service-producing industries [T009] (dollars - millions)	1981-01-01 to 2008-04-01
115	v41881487	**5**	Industrial production [T010] (dollars - millions)	1981-01-01 to 2008-04-01
116	v41881488	**5**	Non-durable manufacturing industries [T011] (dollars - millions)	1981-01-01 to 2008-04-01
117	v41881489	**5**	Durable manufacturing industries [T012] (dollars - millions)	1981-01-01 to 2008-04-01
118	v41881494	**5**	Agriculture, forestry, fishing and hunting [11] (dollars - millions)	1981-01-01 to 2008-04-01
119	v41881501	**5**	Mining and oil and gas extraction [21] (dollars - millions)	1981-01-01 to 2008-04-01
120	v41881524	**5**	Residential building construction [230A] (dollars - millions)	1981-01-01 to 2008-04-01
121	v41881525	**5**	Non-residential building construction [230B] (dollars - millions)	1981-01-01 to 2008-04-01
122	v41881527	**5**	Manufacturing [31-33] (dollars - millions)	1981-01-01 to 2008-04-01
123	v41881555	**5**	Wood product manufacturing [321] (dollars - millions)	1981-01-01 to 2008-04-01
124	v41881564	**5**	Paper manufacturing [322] (dollars - millions)	1981-01-01 to 2008-04-01
125	v41881602	**5**	Rubber product manufacturing [3262] (dollars - millions)	1981-01-01 to 2008-04-01
126	v41881606	**5**	Non-metallic mineral product manufacturing [327] (dollars - millions)	1981-01-01 to 2008-04-01
127	v41881637	**5**	Machinery manufacturing [333] (dollars - millions)	1981-01-01 to 2008-04-01
128	v41881654	**5**	Electrical equipment, appliance and component manufacturing [335] (dollars - millions)	1981-01-01 to 2008-04-01
129	v41881662	**5**	Transportation equipment manufacturing [336] (dollars - millions)	1981-01-01 to 2008-04-01
130	v41881663	**5**	Motor vehicle manufacturing [3361] (dollars - millions)	1981-01-01 to 2008-04-01
131	v41881674	**5**	Aerospace product and parts manufacturing [3364] (dollars - millions)	1981-01-01 to 2008-04-01
132	v41881675	**5**	Railroad rolling stock manufacturing [3365] (dollars - millions)	1981-01-01 to 2008-04-01
133	v41881688	**5**	Wholesale trade [41] (dollars - millions)	1981-01-01 to 2008-04-01
134	v41881689	**5**	Retail trade [44-45] (dollars - millions)	1981-01-01 to 2008-04-01
135	v41881690	**5**	Transportation and warehousing [48-49] (dollars - millions)	1981-01-01 to 2008-04-01
136	v41881699	**5**	Pipeline transportation [486] (dollars - millions)	1981-01-01 to 2008-04-01
137	v41881724	**5**	Finance, insurance, realăestate, rental and leasing and management of companies and enterprises [5A] (dollars - millions)	1981-01-01 to 2008-04-01
138	v41881756	**5**	Educational services [61] (dollars - millions)	1981-01-01 to 2008-04-01
139	v41881759	**5**	Health care and social assistance [62] (dollars - millions)	1981-01-01 to 2008-04-01
140	v41881776	**5**	Federal government public administration [911] (dollars - millions)	1981-01-01 to 2008-04-01
141	v41881777	**5**	Defence services [9111] (dollars - millions)	1981-01-01 to 2008-04-01
142	v41881779	**5**	Provincial and territorial public administration [912] (dollars - millions)	1981-01-01 to 2008-04-01
143	v41881780	**5**	Local, municipal and regional public administration [913] (dollars - millions)	1981-01-01 to 2008-04-01
			Tables 329-00(46,38,39) Industrial price indexes, 1997=100	
144	v1575728	**5**	Transformer equipment (index, 1997=100) [P5648]	1969-01-01 to 2008-05-01
145	v1575754	**5**	Electric motors and generators (index, 1997=100) [P5674]	1969-01-01 to 2008-05-01
146	v1575886	**5**	Diesel fuel (index, 1997=100) [P5806]	1969-01-01 to 2008-04-01
147	v1575925	**5**	Light fuel oil (index, 1997=100) [P5845]	1969-01-01 to 2008-04-01
148	v1575903	**5**	Heavy fuel oil (index, 1997=100) [P5823]	1969-01-01 to 2008-04-01
149	v1575934	**5**	Lubricating oils and greases (index, 1997=100) [P5854]	1969-01-01 to 2008-04-01

150	v1575958	**5**	Asphalt mixtures and emulsions (index, 1997=100) [P5878]	1969-01-01 to 2008-04-01
151	v1575457	**5**	Industrial trucks, tractors and parts (index, 1997=100) [P5329]	1971-01-01 to 2008-05-01
152	v1575493	**5**	Parts, air conditioning and refrigeration equipment (index, 1997=100) [P5365]	1969-01-01 to 2008-05-01
153	v1575511	**5**	Food products industrial machinery and equipment (index, 1997=100) [P5383]	1971-01-01 to 2008-05-01
154	v1575557	**5**	Trucks, chassis, tractors, commercial (index, 1997=100) [P5429]	1969-01-01 to 2008-05-01
155	v1575610	**5**	Motor vehicle engine parts (index, 1997=100) [P5482]	1969-01-01 to 2008-05-01
156	v3860051	**5**	Motor vehicle brakes (index, 1997=100) [P5512]	1969-01-01 to 2008-05-01
157	v3822562	**5**	All manufacturing (index, 1997=100) [P6253]	1969-01-01 to 2008-05-01
158	v3825177	**5**	Total excluding food and beverage manufacturing (index, 1997=100) [P6491]	1969-01-01 to 2008-05-01
159	v3825178	**5**	Food and beverage manufacturing [311, 3121] (index, 1997=100) [P6492]	1969-01-01 to 2008-05-01
160	v3825179	**5**	Food and beverage manufacturing excluding alcoholic beverages (index, 1997=100) [P6493]	1969-01-01 to 2008-05-01
161	v3825180	**5**	Non-food (including alcoholic beverages) manufacturing (index, 1997=100) [P6494]	1969-01-01 to 2008-05-01
162	v3825181	**5**	Basic manufacturing industries [321, 322, 327, 331] (index, 1997=100) [P6495]	1978-07-01 to 2008-05-01
163	v3825183	**5**	Primary metal manufacturing excluding precious metals (index, 1997=100) [P6497]	1971-01-01 to 2008-05-01
164	v1574377	**5**	Total, all commodities (index, 1997=100) [P4000]	1969-01-01 to 2008-05-01
			Table 176-0001 Commodity price index, US$ (index, 82-90=100)	
165	v36382	**1**	Total, all commodities (index, 82-90=100) [B3300]	1972-01-01 to 2008-06-01
166	v36383	**1**	Total excluding energy (index, 82-90=100) [B3301]	1972-01-01 to 2008-06-01
167	v36384	**1**	Energy (index, 82-90=100) [B3302]	1972-01-01 to 2008-06-01
168	v36385	**1**	Food (index, 82-90=100) [B3303]	1972-01-01 to 2008-06-01
169	v36386	**1**	Industrial materials (index, 82-90=100) [B3304]	1972-01-01 to 2008-06-01
			Tables 176-00(46,47), 184-0002 Stock market statistics	
170	v37412	**5**	Toronto Stock Exchange, value of shares traded (dollars - millions) [B4213]	1969-01-01 to 2008-03-01
171	v37413	**5**	Toronto Stock Exchange, volume of shares traded (shares - millions) [B4214]	1969-01-01 to 2008-03-01
172	v37414	**5**	United States common stocks, Dow-Jones industrials, high (index) [B4218]	1969-01-01 to 2008-05-01
173	v37415	**5**	United States common stocks, Dow-Jones industrials, low (index) [B4219]	1969-01-01 to 2008-05-01
174	v37416	**5**	United States common stocks, Dow-Jones industrials, close (index) [B4220]	1969-01-01 to 2008-05-01
175	v37419	**5**	New York Stock Exchange, customers' debit balances (dollars - millions) [B4223]	1969-01-01 to 2008-01-01
176	v37420	**5**	New York Stock Exchange, customers' free credit balance (dollars - millions) [B4224]	1969-01-01 to 2008-01-01
177	v122620	**5**	Standard and Poor's/Toronto Stock Exchange Composite Index, close (index, 1975=1000) [B4237]	1969-01-01 to 2008-05-01
178	v122628	**1**	Toronto Stock Exchange, stock dividend yields (composite), closing quotations (percent) [B4245]	1969-01-01 to 2008-05-01
179	v122629	**1**	Toronto Stock Exchange, price earnings ratio, closing quotations (ratio) [B4246]	1969-01-01 to 2008-05-01
180	v6384	**5**	Total volume; Value of shares traded (dollars - millions) [D4560]	1969-01-01 to 2008-06-01
181	v6385	**5**	Industrials; Value of shares traded (dollars - millions) [D4558]	1969-01-01 to 2008-06-01
182	v6386	**5**	Mining and oils; Value of shares traded (dollars - millions) [D4559]	1969-01-01 to 2008-06-01
			Table 176-0064 Foreign exchange rates	
183	v37426	**4**	United States dollar, noon spot rate, average (dollars) [B3400]	1969-01-01 to 2008-06-01
184	v37437	**4**	United States dollar, 90-day forward noon rate (dollars) [B3401]	1969-01-01 to 2008-06-01
185	v37452	**4**	Danish krone, noon spot rate, average (dollars) [B3403]	1969-01-01 to 2008-06-01
186	v37456	**4**	Japanese yen, noon spot rate, average (dollars) [B3407]	1969-01-01 to 2008-06-01
187	v37427	**4**	Norwegian krone, noon spot rate, average (dollars) [B3409]	1969-01-01 to 2008-06-01
188	v37428	**4**	Swedish krona, noon spot rate, average (dollars) [B3410]	1969-01-01 to 2008-06-01
189	v37429	**4**	Swiss franc, noon spot rate, average (dollars) [B3411]	1969-01-01 to 2008-06-01
190	v37430	**4**	United Kingdom pound sterling, noon spot rate, average (dollars) [B3412]	1969-01-01 to 2008-06-01
191	v37431	**4**	United Kingdom pound sterling, 90-day forward noon rate (dollars) [B3413]	1969-01-01 to 2008-06-01
192	v37432	**4**	United States dollar, closing spot rate (dollars) [B3414]	1969-01-01 to 2008-06-01
193	v37433	**4**	United States dollar, highest spot rate (dollars) [B3415]	1969-01-01 to 2008-06-01
194	v37434	**4**	United States dollar, lowest spot rate (dollars) [B3416]	1969-01-01 to 2008-06-01
195	v37435	**4**	United States dollar, 90-day forward closing rate (dollars) [B3417]	1969-01-01 to 2008-06-01
196	v41498903	**4**	Canadian dollar effective exchange rate index (CERI) (1992=100) (dollars)	1981-01-01 to 2008-06-01
			Table 176-0043 Interest rates	
197	v122550	**1**	Bank rate, last Tuesday or last Thursday (percent) [B14079]	1969-01-01 to 2008-06-01
198	v122530	**1**	Bank rate (percent) [B14006]	1969-01-01 to 2008-06-01
199	v122495	**1**	Chartered bank administered interest rates - prime business (percent) [B14020]	1969-01-01 to 2008-06-01
200	v122505	**1**	Forward premium or discount (-), United States dollar in Canada: 3 month (percent) [B14034]	1969-01-01 to 2008-06-01
201	v122509	**1**	Prime corporate paper rate: 1 month (percent) [B14039]	1969-01-01 to 2008-06-01
202	v122556	**1**	Prime corporate paper rate: 2 month (percent) [B14084]	1969-01-01 to 2008-06-01
203	v122491	**1**	Prime corporate paper rate: 3 month (percent) [B14017]	1969-01-01 to 2008-06-01
204	v122504	**1**	Bankers' acceptances: 1 month (percent) [B14033]	1969-01-01 to 2008-06-01
205	v122558	**1**	Government of Canada marketable bonds, average yield: 1-3 year (percent) [B14009]	1969-01-01 to 2008-06-01
206	v122485	**1**	Government of Canada marketable bonds, average yield: 3-5 year (percent) [B14010]	1969-01-01 to 2008-06-01

207	v122486	**1**	Government of Canada marketable bonds, average yield: 5-10 year (percent) [B14011]	1969-01-01 to 2008-06-01
208	v122487	**1**	Government of Canada marketable bonds, average yield: over 10 years (percent) [B14013]	1969-01-01 to 2008-06-01
209	v122515	**1**	Chartered bank - 5 year personal fixed term (percent) [B14045]	1969-01-01 to 2008-06-01
210	v122493	**1**	Chartered bank - non-chequable savings deposits (percent) [B14019]	1969-01-01 to 2008-06-01
211	v122541	**1**	Treasury bill auction - average yields: 3 month (percent) [B14007]	1969-01-01 to 2008-06-01
212	v122484	**1**	Treasury bill auction - average yields: 3 month, average at values (percent) [B14001]	1969-01-01 to 2008-06-01
213	v122552	**1**	Treasury bill auction - average yields: 6 month (percent) [B14008]	1969-01-01 to 2008-06-01
214	v122554	**1**	Treasury bills: 2 month (percent) [B14082]	1969-01-01 to 2008-06-01
215	v122531	**1**	Treasury bills: 3 month (percent) [B14060]	1969-01-01 to 2008-06-01
216	v122499	**1**	Government of Canada marketable bonds, average yield, average of Wednesdays: 1-3 year (percent)	1969-01-01 to 2008-06-01
217	v122500	**1**	Government of Canada marketable bonds, average yield, average of Wednesdays: 3-5 year (percent)	1969-01-01 to 2008-06-01
218	v122502	**1**	Government of Canada marketable bonds, average yield, average of Wednesdays: 5-10 year (percent)	1969-01-01 to 2008-06-01
219	v122501	**1**	Government of Canada marketable bonds, average yield, average of Wednesdays: +10 years (percent)	1969-01-01 to 2008-06-01
220	v122497	**1**	Average residential mortgage lending rate: 5 year (percent) [B14024]	1969-01-01 to 2008-06-01
221	v122506	**1**	Chartered bank - chequable personal savings deposit rate (percent) [B14035]	1969-01-01 to 2008-06-01
222	v122507	**1**	Covered differential: Canada-United States 3 month Treasury bills (percent) [B14036]	1972-10-01 to 2008-06-01
223	v122508	**1**	Covered differential: Canada-United States 3 month short-term paper (percent) [B14038]	1971-04-01 to 2008-06-01
224	v122510	**1**	First coupon of Canada Savings Bonds (percent) [B14040]	1969-01-01 to 2008-06-01
			Table 176-0051 Canada's official international reserves	
225	v122396	**5**	Total, Canada's official international reserves (dollars - millions) [B3800]	1969-01-01 to 2008-06-01
226	v122397	**5**	Convertible foreign currencies, United States dollars (dollars - millions) [B3801]	1969-01-01 to 2008-06-01
227	v122398	**5**	Convertible foreign currencies, other than United States (dollars - millions) [B3802]	1969-01-01 to 2008-06-01
228	v122399	**5**	Gold (dollars - millions) [B3803]	1969-01-01 to 2008-06-01
229	v122401	**5**	Reserve position in the International Monetary Fund (IMF) (dollars - millions) [B3805]	1969-01-01 to 2008-06-01
			Table 176-0032 Credit measures	
230	v36414	**5**	Total business and household credit; Seasonally adjusted (dollars - millions) [B165]	1969-01-01 to 2008-04-01
231	v36415	**5**	Household credit; Seasonally adjusted (dollars - millions) [B166]	1969-01-01 to 2008-04-01
232	v36416	**5**	Residential mortgage credit; Seasonally adjusted (dollars - millions) [B167]	1969-01-01 to 2008-04-01
233	v36417	**5**	Consumer credit; Seasonally adjusted (dollars - millions) [B168]	1969-01-01 to 2008-04-01
234	v36418	**5**	Business credit; Seasonally adjusted (dollars - millions) [B169]	1969-01-01 to 2008-05-01
235	v36419	**5**	Other business credit; Seasonally adjusted (dollars - millions) [B170]	1969-01-01 to 2008-05-01
236	v36420	**5**	Short-term business credit; Seasonally adjusted (dollars - millions) [B171]	1969-01-01 to 2008-05-01
			Table 176-0025 Monetary aggregates	
237	v37148	**5**	Currency outside banks (dollars - millions) [B1604]	1969-01-01 to 2008-05-01
238	v37153	**5**	Canadian dollar assets, total loans (dollars - millions) [B1605]	1969-01-01 to 2008-05-01
239	v37154	**5**	General loans (including grain dealers and installment finance companies) (dollars - millions)	1969-01-01 to 2008-05-01
240	v37107	**5**	Total, major assets (dollars - millions) [B1611]	1969-01-01 to 2008-05-01
241	v37111	**5**	Canadian dollar assets, liquid assets (dollars - millions) [B1615]	1969-01-01 to 2008-05-01
242	v37112	**5**	Canadian dollar assets, less liquid assets (dollars - millions) [B1616]	1969-01-01 to 2008-05-01
243	v37119	**5**	Total personal loans, average of Wednesdays (dollars - millions) [B1622]	1969-01-01 to 2008-05-01
244	v37120	**5**	Business loans, average of Wednesdays (dollars - millions) [B1623]	1969-01-01 to 2008-05-01
245	v41552793	**5**	Currency outside banks and chartered bank deposits, held by general public (including private sector float) (dollars - millions)	1969-01-01 to 2008-05-01
246	v41552795	**5**	M1B (gross) (currency outside banks, chartered bank chequable deposits, less inter-bank chequable deposits) (dollars - millions)	1969-01-01 to 2008-05-01
247	v41552796	**5**	M2 (gross) (currency outside banks, chartered bank demand and notice deposits, chartered bank personal term deposits, adjustments to M2 (gross) (continuity adjustments and inter-bank demand and notice deposits)) (dollars - millions)	1969-01-01 to 2008-05-01
248	v41552797	**5**	Currency outside banks and chartered bank deposits (including private sector float) (dollars - millions)	1969-01-01 to 2008-05-01
249	v37130	**5**	Residential mortgages (dollars - millions) [B1632]	1969-01-01 to 2008-05-01
250	v41552798	**5**	M2+ (gross) (dollars - millions)	1969-01-01 to 2008-04-01
251	v37135	**5**	Chartered bank deposits, personal, term (dollars - millions) [B1637]	1969-01-01 to 2008-05-01
252	v37138	**5**	Total, deposits at trust and mortgage loan companies (dollars - millions) [B1639]	1969-01-01 to 2008-04-01
253	v37139	**5**	Total, deposits at credit unions and caisses populaires (dollars - millions) [B1640]	1969-01-01 to 2008-05-01
254	v37140	**5**	Bankers' acceptances (dollars - millions) [B1641]	1969-01-01 to 2008-05-01
255	v37145	**5**	Monetary base (notes and coin in circulation, chartered bank and other Canadian Payments Association members' deposits with the Bank of Canada) (dollars - millions) [B1646]	1969-01-01 to 2008-05-01
256	v37146	**5**	Monetary base (notes and coin in circulation, chartered bank and other Canadian Payments Association members' deposits with the Bank of Canada) (excluding required reserves) (dollars - millions) [B1647]	1969-01-01 to 2008-05-01

257	v37147	**5**	Canada Savings Bonds and other retail instruments (dollars - millions) [B1648]	1969-01-01 to 2008-06-01
258	v41552801	**5**	M2++ (gross), Canada Savings Bonds, non-money market mutual funds) (dollars - millions)	1969-01-01 to 2008-04-01
259	v37152	**5**	M1++ (gross) (dollars - millions) [B1652]	1969-01-01 to 2008-05-01
			Table 282-0087 LFS, SA, Canada and provinces	
260	v2062811	**5**	Canada; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
261	v2062815	**1**	Canada; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
262	v2063000	**5**	Newfoundland and Labrador; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
263	v2063004	**1**	Newfoundland and Labrador; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
264	v2063189	**5**	Prince Edward Island; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
265	v2063193	**1**	Prince Edward Island; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
266	v2063378	**5**	Nova Scotia; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
267	v2063382	**1**	Nova Scotia; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
268	v2063567	**5**	New Brunswick; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
269	v2063571	**1**	New Brunswick; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
270	v2063756	**5**	Quebec; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
271	v2063760	**1**	Quebec; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
272	v2063945	**5**	Ontario; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
273	v2063949	**1**	Ontario; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
274	v2064134	**5**	Manitoba; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
275	v2064138	**1**	Manitoba; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
276	v2064323	**5**	Saskatchewan; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
277	v2064327	**1**	Saskatchewan; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
278	v2064512	**5**	Alberta; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
279	v2064516	**1**	Alberta; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
280	v2064701	**5**	British Columbia; Employment; Both sexes; 15 years and over; (persons - thousands)	1976-01-01 to 2008-05-01
281	v2064705	**1**	British Columbia; Unemployment rate; Both sexes; 15 years and over; (rate)	1976-01-01 to 2008-05-01
			Table 282-0088 Employment by industry, SA	
282	v2057603	**5**	Total employed, all industries; (persons - thousands)	1976-01-01 to 2008-05-01
283	v2057604	**5**	Goods-producing sector; (persons - thousands)	1976-01-01 to 2008-05-01
284	v2057605	**5**	Agriculture [1100-1129, 1151-1152]; (persons - thousands)	1976-01-01 to 2008-05-01
285	v2057606	**5**	Forestry, fishing, mining, oil and gas [1131-1133, 1141-1142, 1153]; (persons - thousands)	1976-01-01 to 2008-05-01
286	v2057607	**5**	Utilities [2211-2213]; (persons - thousands)	1976-01-01 to 2008-05-01
287	v2057608	**5**	Construction [2361-2389]; (persons - thousands)	1976-01-01 to 2008-05-01
288	v2057609	**5**	Manufacturing [3211-3219, 3271-3279, 3311-3399, 3111-3169, 3221-3262]; (persons - thousands)	1976-01-01 to 2008-05-01
289	v2057610	**5**	Services-producing sector; Seasonally adjusted (persons - thousands)	1976-01-01 to 2008-05-01
290	v2057611	**5**	Trade [4111-4191, 4411-4543]; Seasonally adjusted (persons - thousands)	1976-01-01 to 2008-05-01
291	v2057612	**5**	Transportation and warehousing [4811-4931]; Seasonally adjusted (persons - thousands)	1976-01-01 to 2008-05-01
292	v2057613	**5**	Finance, insurance, real estate and leasing [5211-5269, 5311-5331]; (persons - thousands)	1976-01-01 to 2008-05-01
293	v2057614	**5**	Professional, scientific and technical services [5411-5419]; (persons - thousands)	1976-01-01 to 2008-05-01
294	v2057615	**5**	Business, building and other support services [5511-5629]; (persons - thousands)	1976-01-01 to 2008-05-01
295	v2057616	**5**	Educational services [6111-6117]; (persons - thousands)	1976-01-01 to 2008-05-01
296	v2057617	**5**	Health care and social assistance [6211-6244]; (persons - thousands)	1976-01-01 to 2008-05-01
297	v2057618	**5**	Information, culture and recreation [5111-5191, 7111-7139]; (persons - thousands)	1976-01-01 to 2008-05-01
298	v2057619	**5**	Accommodation and food services [7211-7224]; (persons - thousands)	1976-01-01 to 2008-05-01
299	v2057620	**5**	Other services [8111-8141]; (persons - thousands)	1976-01-01 to 2008-05-01
300	v2057621	**5**	Public administration [9110-9191]; (persons - thousands)	1976-01-01 to 2008-05-01
			Tables 228-00(01,41) Merchandise imports and exports Canada, SA	
301	v183474	**5**	Imports, United States, including Puerto Rico and Virgin Islands (dollars - millions)	1971-01-01 to 2008-04-01
302	v183475	**5**	Imports, United Kingdom (dollars - millions) [D398059]	1971-01-01 to 2008-04-01
303	v183476	**5**	Imports, Other European Economic Community (dollars - millions) [D398060]	1971-01-01 to 2008-04-01
304	v183477	**5**	Imports, Japan (dollars - millions) [D398061]	1971-01-01 to 2008-04-01
305	v191559	**5**	Exports, United States, including Puerto Rico and Virgin Islands (dollars - millions)	1971-01-01 to 2008-04-01
306	v191560	**5**	Exports, United Kingdom (dollars - millions) [D399519]	1971-01-01 to 2008-04-01
307	v191561	**5**	Exports, Other European Economic Community (dollars - millions) [D399520]	1971-01-01 to 2008-04-01
308	v191562	**5**	Exports, Japan (dollars - millions) [D399521]	1971-01-01 to 2008-04-01
309	v21386488	**5**	Imports, total of all merchandise (dollars - millions)	1971-01-01 to 2008-04-01
310	v21386489	**5**	Imports, Sector 1 Agricultural and fishing products (dollars - millions)	1971-01-01 to 2008-04-01
311	v21386492	**5**	Imports, Sector 2 Energy products (dollars - millions)	1971-01-01 to 2008-04-01
312	v21386495	**5**	Imports, Sector 3 Forestry products (dollars - millions)	1971-01-01 to 2008-04-01
313	v21386496	**5**	Imports, Sector 4 Industrial goods and materials (dollars - millions)	1971-01-01 to 2008-04-01
314	v21386500	**5**	Imports, Sector 5 Machinery and equipment (dollars - millions)	1971-01-01 to 2008-04-01

315	v21386505	**5**	Imports, Sector 6 Automotive products (dollars - millions)	1971-01-01 to 2008-04-01
316	v21386509	**5**	Imports, Sector 7 Other consumer goods (dollars - millions)	1971-01-01 to 2008-04-01
317	v21386512	**5**	Imports, Sector 8 Special transactions trade (dollars - millions)	1971-01-01 to 2008-04-01
318	v21386514	**5**	Exports, total of all merchandise (dollars - millions)	1971-01-01 to 2008-04-01
319	v21386515	**5**	Exports, Sector 1 Agricultural and fishing products (dollars - millions)	1971-01-01 to 2008-04-01
320	v21386518	**5**	Exports, Sector 2 Energy products (dollars - millions)	1971-01-01 to 2008-04-01
321	v21386522	**5**	Exports, Sector 3 Forestry products (dollars - millions)	1971-01-01 to 2008-04-01
322	v21386526	**5**	Exports, Sector 4 Industrial goods and materials (dollars - millions)	1971-01-01 to 2008-04-01
323	v21386531	**5**	Exports, Sector 5 Machinery and equipment (dollars - millions)	1971-01-01 to 2008-04-01
324	v21386535	**5**	Exports, Sector 6 Automotive products (dollars - millions)	1971-01-01 to 2008-04-01
325	v21386539	**5**	Exports, Sector 7 Other consumer goods (dollars - millions)	1971-01-01 to 2008-04-01
326	v21386540	**5**	Exports, Sector 8 Special transactions trade (dollars - millions)	1971-01-01 to 2008-04-01
			Regional series	
327		**5**	CPI Atlantic	1978-09-01 to 2008-05-01
328		**5**	CPI Center	1978-09-01 to 2008-05-01
329		**5**	CPI Prairies	1978-09-01 to 2008-05-01
330		**5**	Employment Atlantic	1976-01-01 to 2008-05-01
331		**5**	Employment Center	1976-01-01 to 2008-05-01
332		**5**	Employment Prairies	1976-01-01 to 2008-05-01
333		**1**	Unemployment Atlantic	1976-01-01 to 2008-05-01
334		**1**	Unemployment Center	1976-01-01 to 2008-05-01
335		**1**	Unemployment Prairies	1976-01-01 to 2008-05-01
336		**1**	Building permits Atlantic	1969-01-01 to 2008-05-01
337		**1**	Building permits Center	1969-01-01 to 2008-05-01
338		**1**	Building permits Prairies	1969-01-01 to 2008-05-01
339	v729971	**1**	Housing starts Atlantic	1969-01-01 to 2008-05-01
340		**1**	Housing starts Center	1969-01-01 to 2008-05-01
341	v729986	**1**	Housing starts Prairies	1969-01-01 to 2008-05-01
			Table 026-0008: Building permits, values by activity sector, SA; Canada;	
342	v4667	**5**	Total residential and non-residential (dollars - thousands) [D2677]	1969-01-01 to 2008-05-01
343	v4668	**5**	Residential (dollars - thousands) [D2681]	1969-01-01 to 2008-05-01
344	v4669	**5**	Non-residential (dollars - thousands) [D4898]	1969-01-01 to 2008-05-01
345	v4670	**5**	Industrial (dollars - thousands) [D2678]	1969-01-01 to 2008-05-01
346	v4671	**5**	Commercial (dollars - thousands) [D2679]	1969-01-01 to 2008-05-01
347	v4672	**5**	Institutional and governmental (dollars - thousands) [D2680]	1969-01-01 to 2008-05-01
348		**5**	Nominal Spot oil price: West Texas Intermediate	1969-01-01 to 2008-05-01
			QUARTERLY VARIABLES	
			Table 380-0001: Gross Domestic Product, income-based; Canada; SAAR;	
349	v498077	**5 F**	Corporation profits before taxes (dollars - millions) [D14806]	1969Q1 to 2008Q1
350	v498079	**5 F**	Interest and miscellaneous investment income (dollars - millions) [D14808]	1969Q1 to 2008Q1
351	v498081	**5 F**	Net income of non-farm unincorporated business, including rent (dollars - millions)	1969Q1 to 2008Q1
352	v498082	**1 F**	Inventory valuation adjustment (dollars - millions) [D14811]	1969Q1 to 2008Q1
353	v1992216	**5 F**	Taxes less subsidies, on factors of production (dollars - millions) [D100100]	1969Q1 to 2008Q1
354	v1997473	**5 F**	Taxes less subsidies, on products (dollars - millions) [D100102]	1969Q1 to 2008Q1
			Table 380-0004: Sector accounts, persons and unincorporated businesses; Canada; SAAR;	
355	v498166	**5 F**	Wages, salaries and supplementary labour income (dollars - millions) [D14896]	1969Q1 to 2008Q1
356	v498170	**5 F**	Unincorporated business net income (dollars - millions) [D14897]	1969Q1 to 2008Q1
357	v498171	**5 F**	Interest, dividends and miscellaneous investment receipts (dollars - millions) [D14898]	1969Q1 to 2008Q1
358	v498172	**5 F**	Current transfers from government (dollars - millions) [D14899]	1969Q1 to 2008Q1
359	v498176	**5 F**	Current transfers from corporations (dollars - millions) [D14903]	1969Q1 to 2008Q1
360	v498179	**5 F**	Personal expenditure on goods and services (dollars - millions) [D14906]	1969Q1 to 2008Q1
361	v498180	**5 F**	Current transfers to government (dollars - millions) [D14907]	1969Q1 to 2008Q1
362	v498184	**5 F**	Current transfers to corporations (dollars - millions) [D14911]	1969Q1 to 2008Q1
363	v498185	**5 F**	Current transfers to non-residents (dollars - millions) [D14912]	1969Q1 to 2008Q1
364	v498164	**5 F**	Saving (dollars - millions) [D14913]	1969Q1 to 2008Q1
365	v498186	**5 F**	Disposable income (dollars - millions) [D14914]	1969Q1 to 2008Q1
366	v498187	**1 F**	Saving rate (percent) [D14915]	1969Q1 to 2008Q1
367	v498199	**2 F**	Net financial investment (dollars - millions) [D14939]	1969Q1 to 2008Q1
			Table 380-0002: Gross Domestic Product, expenditure-based; Canada; Chained (2002) dollars; SAAR;	
368	v1992067	**5 F**	Gross Domestic Product (GDP) at market prices (dollars - millions) [D100126]	1969Q1 to 2008Q1

369	v1992044	**5 F**	Personal expenditure on consumer goods and services (dollars - millions) [D100103]	1969Q1 to 2008Q1
370	v1992045	**5 F**	Personal expenditure on durable goods (dollars - millions) [D100104]	1969Q1 to 2008Q1
371	v1992046	**5 F**	Personal expenditure on semi-durable goods (dollars - millions) [D100105]	1969Q1 to 2008Q1
372	v1992047	**5 F**	Personal expenditure on non-durable goods (dollars - millions) [D100106]	1969Q1 to 2008Q1
373	v1992048	**5 F**	Personal expenditure on services (dollars - millions) [D100107]	1969Q1 to 2008Q1
374	v1992049	**5 F**	Government current expenditure on goods and services (dollars - millions) [D100108]	1969Q1 to 2008Q1
375	v1992050	**5 F**	Government gross fixed capital formation (dollars - millions) [D100109]	1969Q1 to 2008Q1
376	v1992052	**5 F**	Business gross fixed capital formation (dollars - millions) [D100111]	1969Q1 to 2008Q1
377	v1992053	**5 F**	Residential structures (dollars - millions) [D100112]	1969Q1 to 2008Q1
378	v1992054	**5 F**	Non-residential structures and equipment (dollars - millions) [D100113]	1969Q1 to 2008Q1
379	v1992055	**5 F**	Non-residential structures (dollars - millions) [D100114]	1969Q1 to 2008Q1
380	v1992056	**5 F**	Machinery and equipment (dollars - millions) [D100115]	1969Q1 to 2008Q1
381	v1992057	**5 F**	Business investment in inventories (dollars - millions) [D100116]	1969Q1 to 2008Q1
382	v1992058	**5 F**	Business investment in non-farm inventories (dollars - millions) [D100117]	1969Q1 to 2008Q1
383	v1992059	**5 F**	Business investment in farm inventories (dollars - millions) [D100118]	1969Q1 to 2008Q1
384	v1992060	**5 F**	Exports of goods and services (dollars - millions) [D100119]	1969Q1 to 2008Q1
385	v1992061	**5 F**	Exports of goods (dollars - millions) [D100120]	1969Q1 to 2008Q1
386	v1992062	**5 F**	Exports of services (dollars - millions) [D100121]	1969Q1 to 2008Q1
387	v1992063	**5 F**	Deduct: imports of goods and services (dollars - millions) [D100122]	1969Q1 to 2008Q1
388	v1992064	**5 F**	Imports of goods (dollars - millions) [D100123]	1969Q1 to 2008Q1
389	v1992065	**5 F**	Imports of services (dollars - millions) [D100124]	1969Q1 to 2008Q1
390	v1992068	**5 F**	Final domestic demand (dollars - millions) [D100127]	1969Q1 to 2008Q1
			Table 380-0003: Gross omestic Product indexes; Canada; Implicit price indexes 2002=100;	
391	v1997756	**5 F**	Gross Domestic Product (GDP) at market prices (2002=100) [D100465]	1969Q1 to 2008Q1
392	v1997738	**5 F**	Personal expenditure on consumer goods and services (2002=100) [D100447]	1969Q1 to 2008Q1
393	v1997739	**5 F**	Personal expenditure on durable goods (2002=100) [D100448]	1969Q1 to 2008Q1
394	v1997740	**5 F**	Personal expenditure on semi-durable goods (2002=100) [D100449]	1969Q1 to 2008Q1
395	v1997741	**5 F**	Personal expenditure on non-durable goods (2002=100) [D100450]	1969Q1 to 2008Q1
396	v1997742	**5 F**	Personal expenditure on services (2002=100) [D100451]	1969Q1 to 2008Q1
397	v1997743	**5 F**	Government current expenditure on goods and services (2002=100) [D100452]	1969Q1 to 2008Q1
398	v1997744	**5 F**	Government gross fixed capital formation (2002=100) [D100453]	1969Q1 to 2008Q1
399	v1997745	**5 F**	Business gross fixed capital formation (2002=100) [D100454]	1969Q1 to 2008Q1
400	v1997746	**5 F**	Residential structures (2002=100) [D100455]	1969Q1 to 2008Q1
401	v1997747	**5 F**	Non-residential structures and equipment (2002=100) [D100456]	1969Q1 to 2008Q1
402	v1997748	**5 F**	Non-residential structures (2002=100) [D100457]	1969Q1 to 2008Q1
403	v1997749	**5 F**	Machinery and equipment (2002=100) [D100458]	1969Q1 to 2008Q1
404	v1997750	**5 F**	Exports of goods and services (2002=100) [D100459]	1969Q1 to 2008Q1
405	v1997751	**5 F**	Exports of goods (2002=100) [D100460]	1969Q1 to 2008Q1
406	v1997752	**5 F**	Exports of services (2002=100) [D100461]	1969Q1 to 2008Q1
407	v1997753	**5 F**	Imports of goods and services (2002=100) [D100462]	1969Q1 to 2008Q1
408	v1997754	**5 F**	Imports of goods (2002=100) [D100463]	1969Q1 to 2008Q1
409	v1997755	**5 F**	Imports of services (2002=100) [D100464]	1969Q1 to 2008Q1
410	v1997757	**5 F**	Final domestic demand (2002=100) [D100466]	1969Q1 to 2008Q1
			Table 380-0031: Saving, investment and net lending; Canada; SAAR;	
411	v498490	**5 F**	Persons and unincorporated businesses; Saving (dollars - millions) [D15234]	1969Q1 to 2008Q1
412	v498495	**5 F**	Persons and unincorporated businesses; Capital consumption allowances (dollars - millions)	1969Q1 to 2008Q1
413	v498499	**2 F**	Persons and unincorporated businesses; Net capital transfers (dollars - millions) [D15243]	1969Q1 to 2008Q1
414	v498504	**5 F**	Persons and unincorporated businesses; Investment in fixed capital and inventories (dollars - millions) [D15248]	1969Q1 to 2008Q1
415	v498508	**5 F**	Persons and unincorporated businesses; Acquisition of existing assets (dollars - millions)	1969Q1 to 2008Q1
416	v498512	**2 F**	Persons and unincorporated businesses; Net lending (dollars - millions) [D15256]	1969Q1 to 2008Q1
417	v498518	**2 F**	Persons and unincorporated businesses; Net financial investment (dollars - millions)	1969Q1 to 2008Q1
418	v498491	**2 F**	Corporations and government business enterprises; Saving (dollars - millions) [D15235]	1969Q1 to 2008Q1
419	v498496	**5 F**	Corporations and government business enterprises; Capital consumption allowances (dollars - millions) [D15240]	1969Q1 to 2008Q1
420	v498500	**5 F**	Corporations and government business enterprises; Net capital transfers (dollars - millions) [D15244]	1969Q1 to 2008Q1
421	v498505	**5 F**	Corporations and government business enterprises; Investment in fixed capital and inventories (dollars - millions) [D15249]	1969Q1 to 2008Q1
422	v498509	**2 F**	Corporations and government business enterprises; Acquisition of existing assets (dollars - millions) [D15253]	1969Q1 to 2008Q1

423	v498513	**2 F**	Corporations and government business enterprises; Net lending (dollars - millions) [D15257]	1969Q1 to 2008Q1
424	v498519	**2 F**	Corporations and government business enterprises; Net financial investment (dollars - millions) [D15263]	1969Q1 to 2008Q1
425	v498492	**2 F**	Government; Saving (dollars - millions) [D15236]	1969Q1 to 2008Q1
426	v498497	**5 F**	Government; Capital consumption allowances (dollars - millions) [D15241]	1969Q1 to 2008Q1
427	v498501	**2 F**	Government; Net capital transfers (dollars - millions) [D15245]	1969Q1 to 2008Q1
428	v498506	**5 F**	Government; Investment in fixed capital and inventories (dollars - millions) [D15250]	1969Q1 to 2008Q1
429	v498510	**2 F**	Government; Acquisition of existing assets (dollars - millions) [D15254]	1969Q1 to 2008Q1
430	v498514	**2 F**	Government; Net lending (dollars - millions) [D15258]	1969Q1 to 2008Q1
431	v498520	**2 F**	Government; Net financial investment (dollars - millions) [D15264]	1969Q1 to 2008Q1
432	v498493	**2 F**	Non-residents; Saving (dollars - millions) [D15237]	1969Q1 to 2008Q1
433	v498502	**2 F**	Non-residents; Net capital transfers (dollars - millions) [D15246]	1969Q1 to 2008Q1
434	v498515	**2 F**	Non-residents; Net lending (dollars - millions) [D15259]	1969Q1 to 2008Q1
435	v498521	**2 F**	Non-residents; Net financial investment (dollars - millions) [D15265]	1969Q1 to 2008Q1

Variables	Variance decomposition	R^2
CPI	0.0074	0.8317
Core CPI	0.0338	0.6732
Service-Producing Industries	0.0137	0.5308
Industrial Production	0.0166	0.8238
Durable Manufact. Industries	0.0195	0.8232
IPI Manufact	0.0318	0.6622
IPI All Commodities	0.0483	0.4418
TSE 300 Index	0.1016	0.1695
Busin&Pers Services Empl	0.1298	0.1309
Housing index	0.0415	0.8438
New Orders: durables	0.0765	0.1977
Retal trade. furn&appliance	0.1060	0.2017
Money supply	0.3347	0.4782
Shipment/Inventory: finished products	0.0614	0.8325
Gross M2	0.0552	0.4896
Resid Mortgage Credit	0.0284	0.7301
Consumer Credit	0.1098	0.5231
Business Credit	0.0481	0.6070
Short Business Credit	0.0570	0.5251
Imports	0.0105	0.5394
Exports	0.0127	0.5509
Employment CAN	0.0632	0.8432
Unemployment CAN	0.1004	0.8923
Average work week	0.1555	0.4915
1-3year GOV MARKET BONDS	0.3461	0.9679
3-5year GOV MARKET BONDS	0.3004	0.9509
5-10year GOV MARKET BONDS	0.2708	0.9373
10+year GOV MARKET BONDS	0.2335	0.9221
Prime Corporate paper rate-1 month	0.3854	0.9823
Prime Corporate paper rate-3 month	0.3843	0.9854
Prime Corporate paper rate-6 month	0.3794	0.9862
Treasury bill: 6 month	0.3882	0.9960
Forward prem or disc US$ in Can: 3m	0.3486	0.3822
Covered differential: Canada-US 3m T-bill	0.1919	0.5533
Covered differential: Canada-US 3m short-term paper	0.0705	0.6333
Avrg residential mortg lend rate	0.3112	0.9253
FX Can/US: noon	0.0891	0.7566
FX Can/US: 90 days forw noon	0.0800	0.7534
FX Can/UK: noon	0.0990	0.4174
FX Can/UK: 90 days forw	0.0905	0.4113
FX Can/Jap: noon	0.0147	0.8502
FX Can/Dan: noon	0.1018	0.5707
FX Can/Swiss: noon	0.0057	0.8529
FX Can/US: closing 90 days forw	0.0910	0.7547
FX Can/Norw: noon	0.0466	0.5015
FX Can/Swe: noon	0.0391	0.5873
All industries	0.0160	0.8537
Business sector: goods	0.0188	0.8347
Business sector: services	0.0152	0.5392
Mining. oil and gas extraction	0.0095	0.1952
Manufacturing	0.0178	0.8758
Finance. insurance. real estate. rental	0.0274	0.1645
Residential build. constr.	0.0405	0.1440
Motor vehicle manuf.	0.0090	0.3591
Building permits CAN	0.0097	0.7936
Housing starts CAN	0.0283	0.7124
CPI Atlantic	0.0059	0.8665
CPI Center	0.0064	0.8331
CPI Prairie	0.0074	0.8048
Employment Atlantic	0.0548	0.2969
Employment Center	0.0616	0.7305
Employment Prairie	0.0925	0.3255
Unemployment Atlantic	0.1131	0.8483
Unemployment Center	0.0887	0.7649
Unemployment Prairie	0.1001	0.8979
Building Permits Atlantic	0.0145	0.6473
Building Permits Center	0.0064	0.7090
Building Permits Prairie	0.0309	0.5858
Housing Starts Atlantic	0.0178	0.5284
Housing Starts Center	0.0217	0.6312
Housing Starts Prairie	0.0754	0.4594
GDP at market prices	0.1229	0.4073
Consumption of G&S	0.0753	0.3978
Consumption of durable goods	0.0929	0.3021
Business gross fixed capital formation	0.0954	0.5971
Residential structures	0.1074	0.3675
Business investment in inventories	0.0444	0.4407
Wages salaries and supp labour inc.	0.0351	0.7553
Saving	0.0156	0.6493
Saving rate	0.0753	0.9023
Corporation profits bf tx;	0.06746	0.4985
Treasury bill 3 month	0.39842	1.0000

Table I.II: Variance decomposition and R^2 with monthly panel

Appendix II

II.1 Appendix to Chapter 2

II.1.1 Results on interpretation of factors

	Correlation[indexF, F_t]				Correlation[indexF, F_t^*]			
	$F_{1,t}$	$F_{2,t}$	$F_{3,t}$	$F_{4,t}$	$F_{1,t}^*$	$F_{2,t}^*$	$F_{3,t}^*$	$F_{4,t}^*$
CPI	0.3267	-0.6760	-0.1618	-0.4547	0.8925	0.2935	0.4822	0.1220
UR	0.5263	0.0171	0.6392	0.1518	-0.0135	0.7906	-0.1070	0.7752
FFR	0.5716	-0.7482	0.0659	0.0700	0.7282	0.6328	0.7091	0.4062
Bspread	0.5499	0.4749	0.3355	-0.1302	-0.1529	0.4996	-0.4542	0.7073

Table II.II: Correlation between factors and variables in recursive identification in FAVAR-1

	F_1	F_2	F_3	F_4	F_1^*	F_2^*	F_3^*	F_4^*
Industrial production	0.5537	0.1393	0.3043	0.0027	0.0136	0.0021	0.0000	0.9843
CPI: total	0.1340	0.5736	0.0329	0.2596	1.0000	0.0000	0.0000	0.0000
CPI: core	0.3179	0.6793	0.0009	0.0019	0.7368	0.1014	0.1616	0.0002
T-Bill: 3-month	0.3261	0.6428	0.0199	0.0113	0.5425	0.2291	0.2237	0.0047
T-Bond: 5-year	0.3626	0.4730	0.1467	0.0177	0.3648	0.4720	0.1459	0.0174
Unemployment rate	0.3907	0.0004	0.5764	0.0325	0.0003	0.9997	0.0000	0.0000
M1	0.2693	0.0420	0.3895	0.2993	0.1275	0.8016	0.0629	0.0080
M2	0.0795	0.0307	0.0756	0.8142	0.0748	0.2914	0.6311	0.0027
Consumer credit	0.6387	0.3348	0.0265	0.0000	0.0136	0.1490	0.0205	0.8169
Exchange rate: average	0.0642	0.0611	0.0711	0.8036	0.0506	0.2640	0.6754	0.0100
Commodity price index	0.0764	0.6462	0.1310	0.1464	0.5902	0.2710	0.0131	0.1257
PPI: finished goods	0.0389	0.3666	0.0654	0.5291	0.8982	0.0396	0.0622	0.0000
Capacity utilization rate	0.3695	0.3580	0.2661	0.0064	0.0803	0.5850	0.1764	0.1583
Real Pers. Cons.	0.4418	0.0015	0.2662	0.2905	0.3390	0.0040	0.0729	0.5841
Real Pers. Cons.: services	0.4361	0.0482	0.0405	0.4752	0.2144	0.0218	0.3180	0.4457
Avg. unemployment duration	0.0001	0.3352	0.5322	0.1326	0.1462	0.2488	0.5861	0.0189
Employment	0.4915	0.3344	0.1669	0.0072	0.0227	0.0233	0.0002	0.9538
Avg weekly hours	0.7537	0.0007	0.0002	0.2454	0.0022	0.3885	0.2424	0.3669
Avg hourly earnings	0.0412	0.8748	0.0153	0.0688	0.4509	0.0052	0.4909	0.0529
Housing starts	0.6263	0.2254	0.0565	0.0918	0.0328	0.1410	0.0319	0.7943
New orders	0.4368	0.0528	0.4952	0.0153	0.0667	0.0181	0.0018	0.9133
S&P's CCS: dividend yield	0.3327	0.4663	0.0134	0.1875	0.2368	0.2710	0.4921	0.0000
Consumer expectations	0.8944	0.0287	0.0011	0.0758	0.3703	0.3290	0.0053	0.2955
FFR	0.3647	0.6250	0.0048	0.0055	0.5921	0.1936	0.2143	0.0000
B-spread: 10y	0.4600	0.3430	0.1712	0.0258	0.0355	0.5156	0.2008	0.2481

Table II.IV: Marginal contribution to R^2 in FAVAR-1

	Correlation[indexF, F_t]					Correlation[indexF, F_t^*]				
	$F_{1,t}$	$F_{2,t}$	$F_{3,t}$	$F_{4,t}$	$F_{5,t}$	$F_{1,t}^*$	$F_{2,t}^*$	$F_{3,t}^*$	$F_{4,t}^*$	$F_{5,t}^*$
PCE	0.1779	0.6894	0.0721	-0.5272	0.0581	0.8908	0.1351	-0.1274	-0.2687	0.3597
UR	-0.2369	0.3634	-0.3249	0.1541	-0.6141	0.0764	0.8319	-0.7171	0.4242	0.7006
C	0.5141	-0.0801	-0.1664	0.2536	-0.1385	-0.1319	0.0245	0.2848	0.0138	-0.0909
I	0.8099	0.2131	-0.3898	-0.0815	-0.0043	0.3431	0.0235	0.3874	0.0303	-0.0247
FFR	-0.1066	0.8734	0.2714	0.1528	-0.0947	0.5801	0.4356	-0.4304	-0.3412	0.7672

Table II.VI: Correlation between factors and variables in recursive identification in FAVAR-2

	F_1	F_2	F_3	F_4	F_5	F_1^*	F_2^*	F_3^*	F_4^*	F_5^*
Industrial production	0.3722	0.0372	0.1777	0.0975	0.3153	0.0240	0.0065	0.2188	0.1865	0.5643
CPI: total	0.0019	0.8011	0.1551	0.0012	0.0407	0.5887	0.0000	0.0070	0.1881	0.2162
CPI: core	0.0137	0.8546	0.1270	0.0047	0.0000	0.4625	0.0628	0.0262	0.2417	0.2069
T-Bill: 3-month	0.0071	0.8433	0.0911	0.0280	0.0304	0.3695	0.1699	0.0041	0.2804	0.1760
T-Bond: 5-year	0.0136	0.7499	0.0329	0.0360	0.1676	0.2731	0.3988	0.0006	0.2403	0.0873
Unemployment rate	0.0808	0.1901	0.1519	0.0342	0.5429	0.0084	0.9916	0.0000	0.0000	0.0000
M1	0.4904	0.0005	0.0042	0.0638	0.4412	0.1032	0.6443	0.2062	0.0343	0.0119
M2	0.1950	0.1857	0.0166	0.1620	0.4407	0.0007	0.6933	0.0465	0.2443	0.0152
Consumer credit	0.6791	0.0094	0.2864	0.0001	0.0251	0.0165	0.1472	0.4326	0.2972	0.1066
Exchange rate: average	0.1637	0.5082	0.1929	0.0611	0.0741	0.5264	0.2092	0.2073	0.0552	0.0019
Commodity price index	0.1450	0.1382	0.5949	0.0053	0.1166	0.2416	0.3128	0.0106	0.4062	0.0289
PPI: finished goods	0.0461	0.6329	0.1662	0.0145	0.1404	0.4157	0.0349	0.0071	0.1940	0.3482
Capacity utilization rate	0.2122	0.0021	0.5721	0.0077	0.2059	0.0396	0.6694	0.0237	0.2672	0.0002
Real Pers. Cons.	0.6921	0.0168	0.0725	0.1684	0.0502	0.0456	0.0053	0.9491	0.0000	0.0000
Real Pers. Cons.: services	0.6349	0.0930	0.0724	0.1551	0.0446	0.0912	0.0437	0.6569	0.1824	0.0257
Avg. unemployment duration	0.0096	0.1435	0.3114	0.0006	0.5350	0.1527	0.4265	0.0208	0.1243	0.2756
Employment	0.2846	0.0028	0.3902	0.1257	0.1967	0.0881	0.0342	0.1034	0.3363	0.4380
Avg weekly hours	0.4182	0.3666	0.1319	0.0833	0.0000	0.0195	0.3945	0.1837	0.0237	0.3786
Avg hourly earnings	0.0824	0.1672	0.6705	0.0768	0.0030	0.0249	0.0003	0.1320	0.7736	0.0691
Housing starts	0.4286	0.0492	0.4255	0.0095	0.0872	0.0025	0.1180	0.2226	0.3908	0.2660
New orders	0.1761	0.0866	0.1372	0.0409	0.5592	0.0019	0.0159	0.1318	0.2023	0.6481
S&P's CCS: dividend yield	0.0224	0.8496	0.0616	0.0398	0.0266	0.3305	0.2061	0.0111	0.2369	0.2154
Consumer expectations	0.2517	0.6571	0.0866	0.0046	0.0000	0.2148	0.3383	0.1360	0.0149	0.2960
FFR	0.0129	0.8667	0.0837	0.0265	0.0102	0.3823	0.1405	0.0103	0.2457	0.2212
B-spread: 10y	0.2776	0.0103	0.5795	0.0045	0.1281	0.0229	0.6147	0.0549	0.3035	0.0041
Real GDP	0.7243	0.0239	0.1817	0.0681	0.0021	0.0107	0.0000	0.8954	0.0467	0.0472
Real GDP: goods	0.7311	0.0512	0.2146	0.0009	0.0023	0.0865	0.0004	0.8019	0.0988	0.0123
Real GDP: services	0.0688	0.0943	0.1299	0.7069	0.0000	0.5760	0.0009	0.0092	0.2823	0.1316
Employees compensation	0.7016	0.0010	0.0412	0.2528	0.0035	0.0141	0.0073	0.8944	0.0012	0.0830
Gov. consumption	0.4116	0.0900	0.2948	0.1792	0.0244	0.3330	0.0075	0.3325	0.3254	0.0015
Investment	0.7627	0.0528	0.1767	0.0077	0.0000	0.1369	0.0010	0.7723	0.0899	0.0000
Invst.: nonresidential	0.7003	0.0234	0.0801	0.1824	0.0137	0.0003	0.0049	0.8734	0.0032	0.1182
GDP deflator	0.0141	0.6406	0.1146	0.2304	0.0002	0.8248	0.0046	0.0982	0.0688	0.0036
PCE deflator	0.0399	0.5990	0.0065	0.3503	0.0043	1.0000	0.0000	0.0000	0.0000	0.0000

Table II.VIII: Marginal contribution to R^2 in FAVAR-2

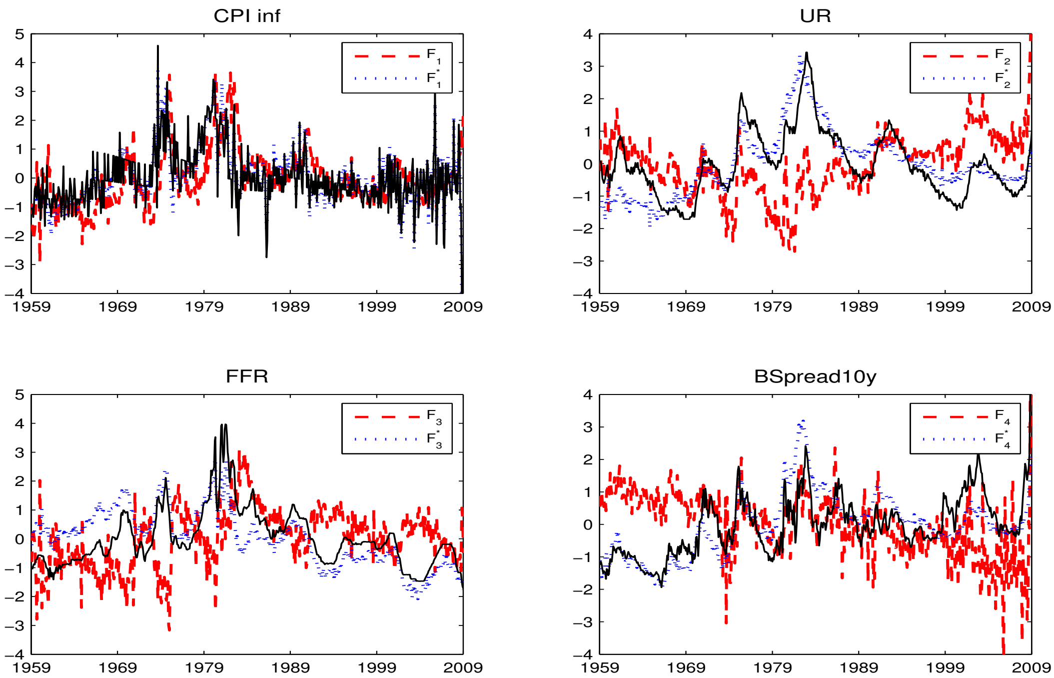

Figure II.1: Principal components, rotated factors and variables used in recursive identification with monthly balanced panel

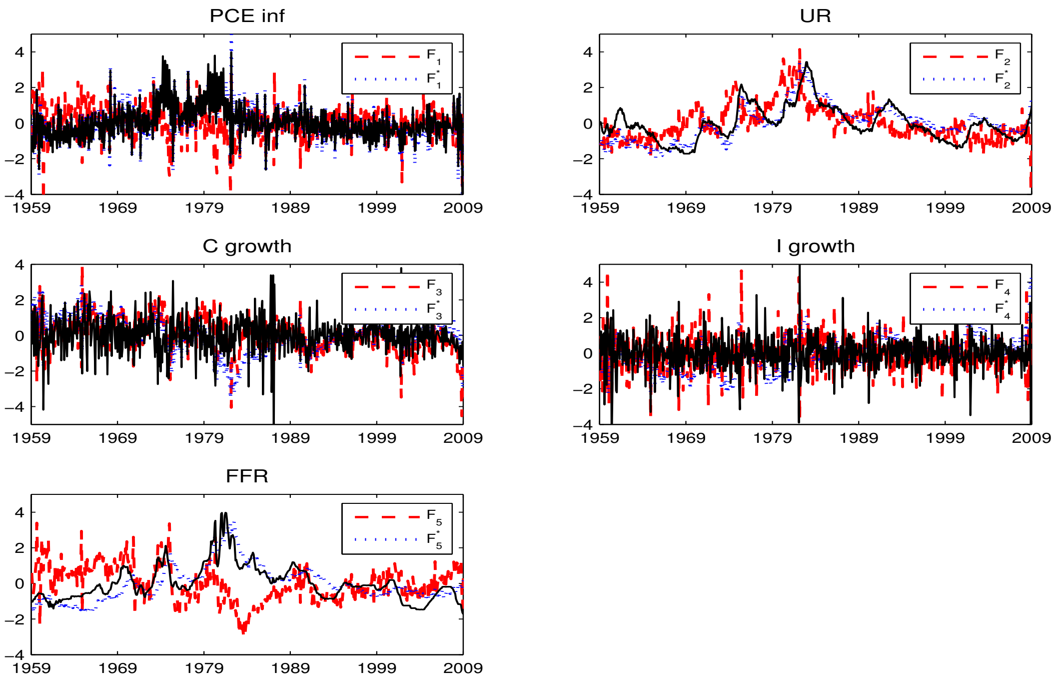

Figure II.2: Principal components, rotated factors and variables used in recursive identification with monthly mixed-frequencies data

II.1.2 Results from structural VAR analysis

Models	Wald causaility ordering
Benchmark	$[\pi_t, UR_t, R_t, 10yBS_t]$
Model 1	$[\pi_t, UR_t, 10yBS_t, R_t]$
Model 2	$[\pi_t, UR_t, R_t, 1yBS_t]$
Model 3	$[\pi_t, UR_t, R_t, 10yAS_t]$

Table II.IX: VAR models used to study effects and identification of financial shock

Variables	Benchmark	Model 1	Model 2	Model 3
CPI	0.0467	0.0569	0.0227	0.0322
Unemployment rate	0.1945	0.1694	0.0477	0.0933
FFR	0.1055	0.1572	0.0882	0.0778
B-spread: 10y	0.9156	0.8968		
B-spread: 1y			0.6069	
A-spread: 10y				0.9437

Table II.X: Variance decomposition: contribution of the credit shock in SVAR models

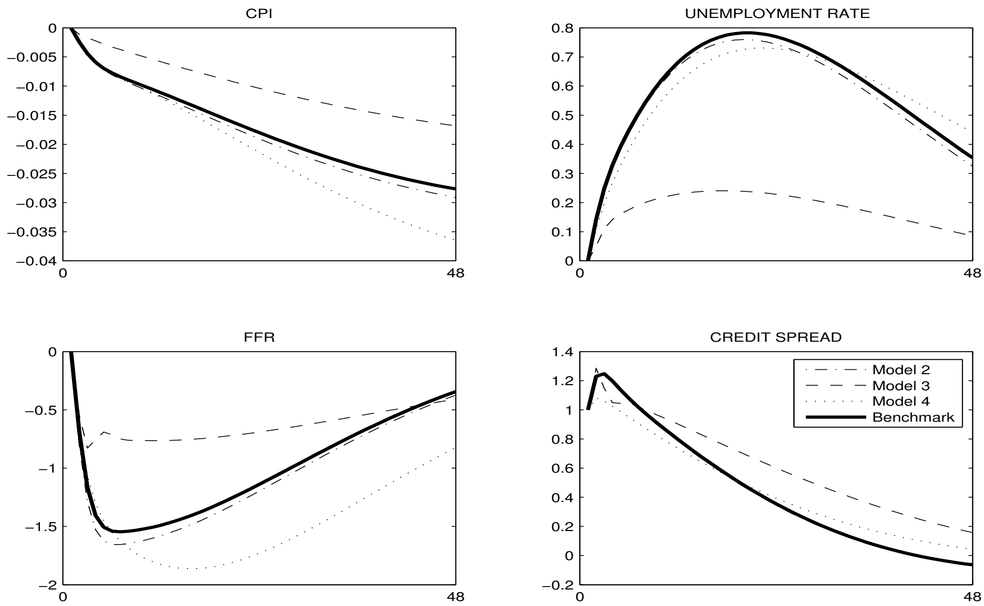

Figure II.3: Benchmark model vs models 1-3, 100 basic points shock to credit spread

II.1.3 Dynamic effects of the monetary policy shock

Here, we present the effects of the monetary policy using the same identification scheme as above, and using the monthly balanced panel and the mixed-frequencies monthly panel. In the first specification the monetary policy shock is ordered third, and in the second specification it is the last element of the vector of identified structural shocks.

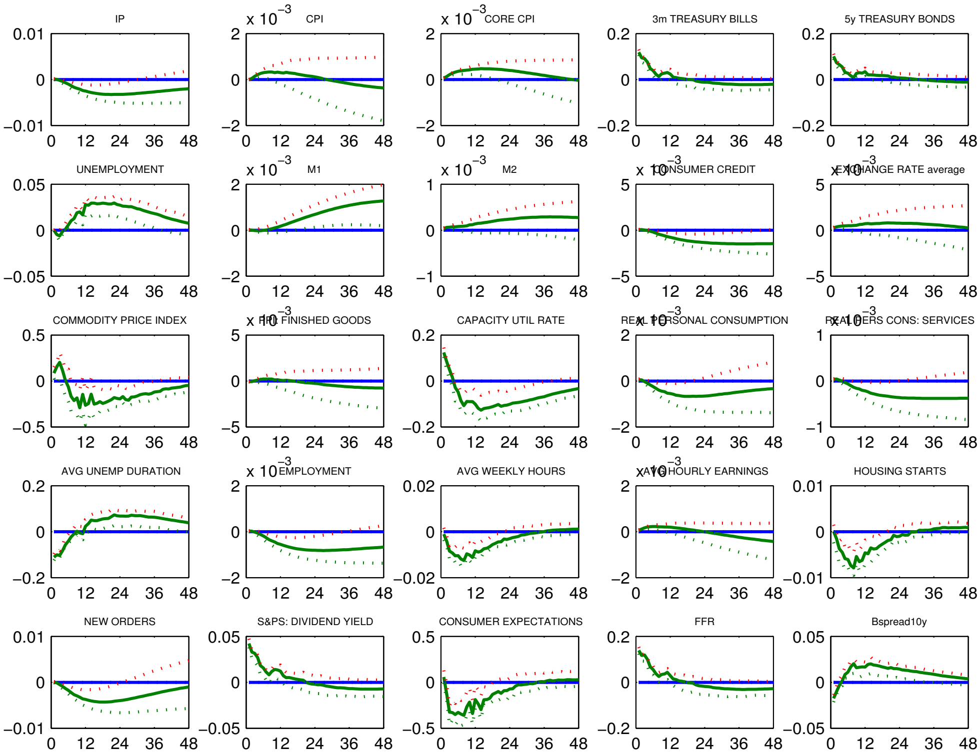

Figure II.4: Dynamic responses of monthly variables to monetary policy shock

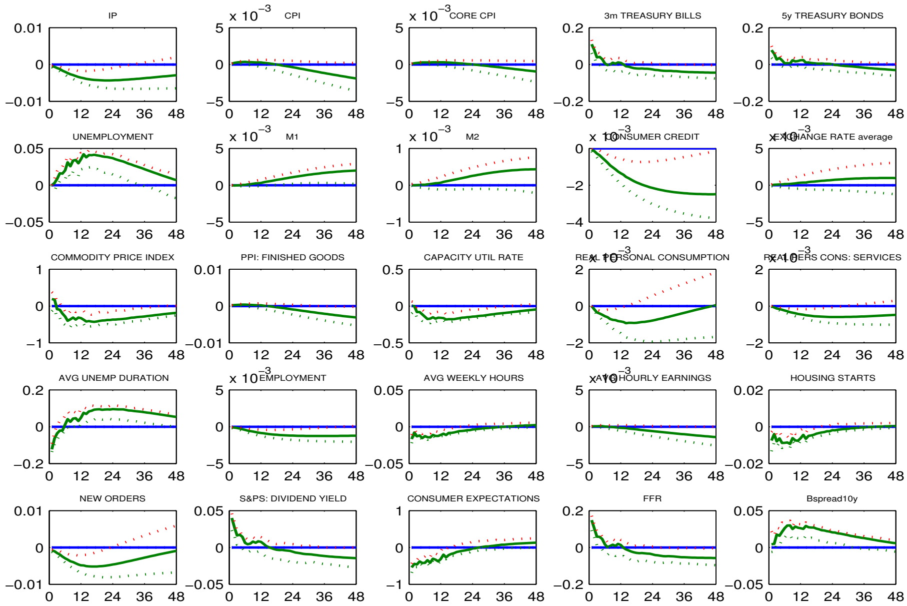

Figure II.5: Dynamic responses of monthly variables to monetary policy shock using mixed-frequencies data

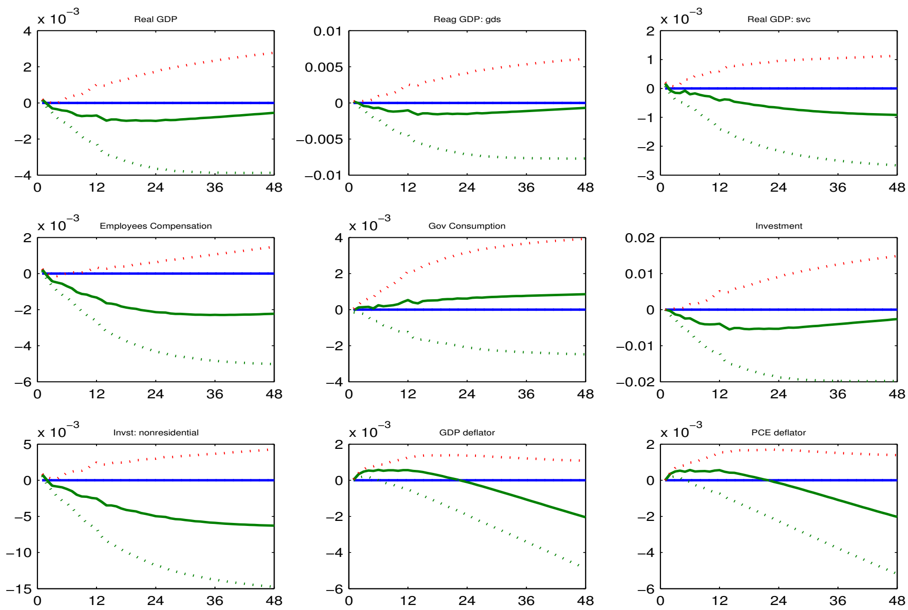

Figure II.6: Dynamic responses of constructed monthly indicators to monetary policy shock using mixed-frequencies data

II.1.4 Data Sets

No.	Series Code	T-Code	Series Description
			Real output and income
1	IPS10	5	INDUSTRIAL PRODUCTION INDEX - TOTAL INDEX
2	IPS11	5	INDUSTRIAL PRODUCTION INDEX - PRODUCTS, TOTAL
3	IPS12	5	INDUSTRIAL PRODUCTION INDEX - CONSUMER GOODS
4	IPS13	5	INDUSTRIAL PRODUCTION INDEX - DURABLE CONSUMER GOODS
5	IPS14	5	INDUSTRIAL PRODUCTION INDEX - AUTOMOTIVE PRODUCTS
6	IPS18	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE CONSUMER GOODS
7	IPS25	5	INDUSTRIAL PRODUCTION INDEX - BUSINESS EQUIPMENT
8	IPS29	5	INDUSTRIAL PRODUCTION INDEX - DEFENSE AND SPACE EQUIPMENT
9	IPS299	5	INDUSTRIAL PRODUCTION INDEX - FINAL PRODUCTS
10	IPS306	5	INDUSTRIAL PRODUCTION INDEX - FUELS
11	IPS32	5	INDUSTRIAL PRODUCTION INDEX - MATERIALS
12	IPS34	5	INDUSTRIAL PRODUCTION INDEX - DURABLE GOODS MATERIALS
13	IPS38	5	INDUSTRIAL PRODUCTION INDEX - NONDURABLE GOODS MATERIALS
14	IPS43	5	INDUSTRIAL PRODUCTION INDEX - MANUFACTURING (SIC)
15	PMP	1	NAPM PRODUCTION INDEX (PERCENT)
16	PMI	1	PURCHASING MANAGERS' INDEX (SA)
17	UTL11	1	CAPACITY UTILIZATION - MANUFACTURING (SIC)
18	YPR	5	PERS INCOME CH 2000 $,SA-US
19	YPDR	5	DISP PERS INCOME,BILLIONS OF CH (2000) $,SAAR-US
20	YP@V00C	5	PERS INCOME LESS TRSF PMT CH 2000 $,SA-US
21	SAVPER	2	PERS SAVING,BILLIONS OF $,SAAR-US
22	SAVPRATE	1	PERS SAVING AS PERCENTAGE OF DISP PERS INCOME,PERCENT,SAAR-US
			Employment and hours
23	LHEL	5	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
24	LHELX	4	EMPLOYMENT: RATIO; HELP-WANTED ADS:NO. UNEMPLOYED CLF
25	LHEM	5	CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
26	LHNAG	5	CIVILIAN LABOR FORCE: EMPLOYED, NONAGRIC.INDUSTRIES (THOUS.,SA)
27	LHTUR	1	UNEMPLOYMENT RATE: (
28	LHU14	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
29	LHU15	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
30	LHU26	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
31	LHU27	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
32	LHU5	1	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
33	LHU680	1	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
34	LHUEM	5	CIVILIAN LABOR FORCE: UNEMPLOYED, TOTAL (THOUS.,SA)
35	AHPCON	5	AVG HR EARNINGS OF PROD WKRS: CONSTRUCTION ($,SA)
36	AHPMF	5	AVG HR EARNINGS OF PROD WKRS: MANUFACTURING ($,SA)
37	PMEMP	1	NAPM EMPLOYMENT INDEX (PERCENT)
38	CES002	5	EMPLOYEES ON NONFARM PAYROLLS - TOTAL PRIVATE
39	CES003	5	EMPLOYEES ON NONFARM PAYROLLS - GOODS-PRODUCING
40	CES004	5	EMPLOYEES ON NONFARM PAYROLLS - NATURAL RESOURCES AND MINING
41	CES011	5	EMPLOYEES ON NONFARM PAYROLLS - CONSTRUCTION
42	CES015	5	EMPLOYEES ON NONFARM PAYROLLS - MANUFACTURING
43	CES017	5	EMPLOYEES ON NONFARM PAYROLLS - DURABLE GOODS
44	CES033	5	EMPLOYEES ON NONFARM PAYROLLS - NONDURABLE GOODS
45	CES046	5	EMPLOYEES ON NONFARM PAYROLLS - SERVICE-PROVIDING
46	CES048	5	EMPLOYEES ON NONFARM PAYROLLS - TRADE, TRANSPORTATION, AND UTILITIES
47	CES049	5	EMPLOYEES ON NONFARM PAYROLLS - WHOLESALE TRADE
48	CES053	5	EMPLOYEES ON NONFARM PAYROLLS - RETAIL TRADE
49	CES088	5	EMPLOYEES ON NONFARM PAYROLLS - FINANCIAL ACTIVITIES
50	CES140	5	EMPLOYEES ON NONFARM PAYROLLS - GOVERNMENT
51	CES151	1	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - GOODS-PRODUCING
52	CES153	1	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - CONSTRUCTION
53	CES154	1	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING

54	CES155	1	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING OVERTIME HOURS
55	CES156	1	AVERAGE WEEKLY HOURS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - DURABLE GOODS
56	CES275	5	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - GOODS-PRODUCING
57	CES277	5	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - CONSTRUCTION
58	CES278	5	AVERAGE HOURLY EARNINGS OF PRODUCTION OR NONSUPERVISORY WORKERS ON PRIVATE NONFARM PAYROLLS - MANUFACTURING
			Real Consumption
59	JQCR	5	REAL PERSONAL CONS EXP QUANTITY INDEX (200=100), SAAR
60	JQCNR	5	REAL PERSONAL CONS EXP-NONDURABLE GOODS QUANTITY INDEX (200=100), SAAR
61	JQCDR	5	REAL PERSONAL CONS EXP-DURABLE GOODS QUANTITY INDEX (200=100), SAAR
62	JQCSVR	5	REAL PERSONAL CONS EXP-SERVICES QUANTITY INDEX (200=100), SAAR
			Real inventories and orders
63	MOCMQ	5	NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1996 DOLLARS (BCI)
64	MSONDQ	5	NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI)
65	PMDEL	1	NAPM VENDOR DELIVERIES INDEX (PERCENT)
66	PMNO	1	NAPM NEW ORDERS INDEX (PERCENT)
67	PMNV	1	NAPM INVENTORIES INDEX (PERCENT)
			Housing starts
68	HUSTSZ	4	HOUSING STARTS: TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)
69	HSFR	4	HOUSING STARTS:NONFARM(1947-58);TOTAL FARM&NONFARM(1959-)(THOUS.,SA
70	HSMW	4	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
71	HSNE	4	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
72	HSSOU	4	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
73	HSWST	4	HOUSING STARTS:WEST (THOUS.U.)S.A.
			Exchange rates
74	EXRCAN	5	FOREIGN EXCHANGE RATE: CANADA (CANADIAN $ PER U.S.$)
75	EXRUK	5	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
76	EXRUS	5	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
			Price indexes
77	PMCP	1	NAPM COMMODITY PRICES INDEX (PERCENT)
78	PW561	5	PRODUCER PRICE INDEX: CRUDE PETROLEUM (82=100,NSA)
79	PWCMSA	5	PRODUCER PRICE INDEX:CRUDE MATERIALS (82=100,SA)
80	PWFCSA	5	PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
81	PWFSA	5	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
82	PWIMSA	5	PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)
83	PUNEW	5	CPI-U: ALL ITEMS (82-84=100,SA)
84	PUS	5	CPI-U: SERVICES (82-84=100,SA)
85	PUXF	5	CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
86	PUXHS	5	CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
87	PUXM	5	CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA)
88	PUXX	5	CPI-U: ALL ITEMS LESS FOOD AND ENERGY (82-84=100,SA)
89	PUC	5	CPI-U: COMMODITIES (82-84=100,SA)
90	PUCD	5	CPI-U: DURABLES (82-84=100,SA)
91	PU83	5	CPI-U: APPAREL & UPKEEP (82-84=100,SA)
92	PU84	5	CPI-U: TRANSPORTATION (82-84=100,SA)
93	PU85	5	CPI-U: MEDICAL CARE (82-84=100,SA)
			Stock prices
94	FSDJ	5	COMMON STOCK PRICES: DOW JONES INDUSTRIAL AVERAGE
95	FSDXP	1	S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD (% PER ANNUM)
96	FSPCOM	5	S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)
97	FSPIN	5	S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS (1941-43=10)
98	FSPXE	1	S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO (%,NSA)
			Money and credit quantity aggregates
99	FM1	5	MONEY STOCK: M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL$,SA)
100	FM2	5	MONEY STOCK:M2(M1+O'NITE RPS,EURO$,G/P&B/D MMMFS&SAV&SM TIME DEP(BIL$,
101	CCINRV	5	CONSUMER CREDIT OUTSTANDING - NONREVOLVING(G19)
			Miscellaneous

102	UOMO83	1	COMPOSITE INDEXES LEADING INDEX COMPONENT INDEX OF CONSUMER EXPECTATIONS
			UNITS: 1966.1=100 NSA, CONFBOARD AND U.MICH.
			Interest rates and bonds
103	FYGM3	1	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,3-MO.(% PER ANN,NSA)
104	FYGM6	1	INTEREST RATE: U.S.TREASURY BILLS,SEC MKT,6-MO.(% PER ANN,NSA)
105	FYGT1	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,1-YR.(% PER ANN,NSA)
106	FYGT10	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)
107	FYGT20	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,20-YR.(% PER ANN,NSA)
108	FYGT3	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,3-YR.(% PER ANN,NSA)
109	FYGT5	1	INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)
110	FYPR	1	PRIME RATE CHG BY BANKS ON SHORT-TERM BUSINESS LOANS(% PER ANN,NSA)
111	FYAAAC	1	BOND YIELD: MOODY'S AAA CORPORATE (% PER ANNUM)
112	FYAAAM	1	BOND YIELD: MOODY'S AAA MUNICIPAL (% PER ANNUM)
113	FYAC	1	BOND YIELD: MOODY'S A CORPORATE (% PER ANNUM,NSA)
114	FYAVG	1	BOND YIELD: MOODY'S AVERAGE CORPORATE (% PER ANNUM)
115	FYBAAC	1	BOND YIELD: MOODY'S BAA CORPORATE (% PER ANNUM)
116	SFYGM3	1	FYGM3-FYFF
117	SFYGM6	1	FYGM6-FYFF
118	SFYGT1	1	FYGT1-FYFF
119	SFYGT5	1	FYGT5-FYFF
120	SFYGT10	1	FYGT10-FYFF
121	SFYAAAC	1	FYAAAC-FYFF
122	SFYBAAC	1	FYBAAC-FYFF
123	FYFF	1	INTEREST RATE: FEDERAL FUNDS (EFFECTIVE) (% PER ANNUM,NSA)
124	Bspread10Y	1	FYBAAC-FYGT10
			Quarterly indicators
125	GDPRC@US.Q	5	NIA REAL GROSS DOMESTIC PRODUCT (CHAINED-2000), SA - U.S.
126	GDPGDR.Q	5	REAL GDP-GDS,BILLIONS OF CH (2000) $,SAAR-US
127	GDPSVR.Q	5	REAL GDP-SVC,BILLIONS OF CH (2000) $,SAAR-US
128	GDPSR.Q	5	REAL GDP-STRUC,BILLIONS OF CH (2000) $,SAAR-US
129	WS@US.Q	5*	NIA NOMINAL TOTAL COMPENSATION OF EMPLOYEES, SA - U.S.
130	CR.Q	5	REAL PCE,BILLIONS OF CH (2000) $,SAAR-US
131	JQCDR.Q	5	REAL PCE-DUR,QTY INDEX (2000=100),SA,SA-US
132	UJQCDMVR.Q	5	REAL PCE-DUR-MV&PARTS,QTY INDEX (2000=100),SA,SA-US
133	JQCDFHER.Q	5	REAL PCE-DUR-FURN&HH EQUIP,QTY INDEX (2000=100),SA,SA-US
134	JQCDOR.Q	5	REAL PCE-DUR-OTH,QTY INDEX (2000=100),SA,SA-US
135	JQCNR.Q	5	REAL PCE-NDUR,QTY INDEX (2000=100),SA,SA-US
136	JQCNFR.Q	5	REAL PCE-NDUR-FOOD,QTY INDEX (2000=100),SA,SA-US
137	JQCNCSR.Q	5	REAL PCE-NDUR-CLO&SHOES,QTY INDEX (2000=100),SA,SA-US
138	JQCNER.Q	5	REAL PCE-NDUR-GASOLINE FUEL OIL&OTH ENERGY GDS,QTY INDEX (2000=100),SA,SA-US
139	JQCNEGAOR.Q	5	REAL PCE-NDUR-GASOLINE FUEL OIL&OTH ENERGY GDS-GASOLINE&OIL,QTY INDEX (2000=100),SAAR-US
140	JQCNEFACR.Q	5	REAL PCE-NDUR-GASOLINE FUEL OIL&OTH ENERGY GDS-FUEL OIL&COAL,QTY INDEX (2000=100),SAAR-US
141	JQCNOR.Q	5	REAL PCE-NDUR-OTH,QTY INDEX (2000=100),SA,SA-US
142	JQCSVR.Q	5	REAL PCE-SVC,QTY INDEX (2000=100),SA,SA-US
143	JQCSVHSR.Q	5	REAL PCE-SVC-HOUSING,QTY INDEX (2000=100),SA,SA-US
144	JQCSVHOPR.Q	5	REAL PCE-SVC-HH OPS,QTY INDEX (2000=100),SA,SA-US
145	JQCSVHOPEAGR.Q	5	REAL PCE-SVC-HH OPS-ELEC&GAS,QTY INDEX (2000=100),SA,SA-US
146	JQCSVHOPOR.Q	5	REAL PCE-SVC-OTH HH OPS,QTY INDEX (2000=100),SA,SA-US
147	JQCSVTSR.Q	5	REAL PCE-SVC-TRNSPRT,QTY INDEX (2000=100),SA,SA-US
148	JQCSVMR.Q	5	REAL PCE-SVC-MEDICAL CARE,QTY INDEX (2000=100),SA,SA-US
149	JQCSVRECR.Q	5	REAL PCE-SVC-RECR,QTY INDEX (2000=100),SA,SA-US
150	JQCSVOR.Q	5	REAL PCE-SVC-OTH,QTY INDEX (2000=100),SA,SA-US
151	JQCENERGYR.Q	5	REAL PCE-ENERGY GDS&SVC,QTY INDEX (2000=100),SAAR-US
152	JQCXFAER.Q	5	REAL PCE EX FOOD&ENERGY,QTY INDEX (2000=100),SAAR-US
153	CGRC@US.Q	5	NIA REAL GOVERNMENT CONSUMPTION EXPENDITURE & GROSS INVESTMENT (CHAINED-2000), SA - U.S.
154	I.Q	5*	GROSS PRIV DOM INVEST,BILLIONS OF $,SAAR-US
155	IF.Q	5*	GROSS PRIV DOM INVEST-FIXED,BILLIONS OF $,SAAR-US
156	IFNRE.Q	5*	GROSS PRIV DOM INVEST-FIXED NONRES,BILLIONS OF $,SAAR-US
157	IFNRES.Q	5*	GROSS PRIV DOM INVEST-FIXED NONRES-STRUC,BILLIONS OF $,SAAR-US
158	IFNRESC.Q	5*	PRIV FIXED INVEST-NONRES-STRUC-COML&HEALTH CARE,BILLIONS OF $,SAAR-US
159	IFNRESMFG.Q	5*	PRIV FIXED INVEST-NONRES-STRUC-MFG,BILLIONS OF $,SAAR-US

160	IFREE.Q	5*	PRIV FIXED INVEST-EQUIP,BILLIONS OF $,SAAR-US
161	IFRESPEMF.Q	5*	PRIV FIXED INVEST-RES-STRUC-MFAM,BILLIONS OF $,SAAR-US
162	IFRESPESF.Q	5*	PRIV FIXED INVEST-RES-STRUC-1 FAM,BILLIONS OF $,SAAR-US
163	IFRESPE.Q	5*	PRIV FIXED INVEST-RES-STRUC-PERMANENT SITE,BILLIONS OF $,SAAR-US
164	IFRES.Q	5*	PRIV FIXED INVEST-RES-STRUC,BILLIONS OF $,SAAR-US
165	IFRE.Q	5*	GROSS PRIV DOM INVEST-FIXED RES,BILLIONS OF $,SAAR-US
166	IFNREEO.Q	5*	GROSS PRIV DOM INVEST-FIXED-NONRES-EQUIP&SW-OTH,BILLIONS OF $,SAAR-US
167	IFNREET.Q	5*	GROSS PRIV DOM INVEST-FIXED-NONRES-EQUIP&SW-TRNSPRT,BILLIONS OF $,SAAR-US
168	IFNREEIND.Q	5*	GROSS PRIV DOM INVEST-FIXED-NONRES-EQUIP&SW-IND,BILLIONS OF $,SAAR-US
169	IFNREEIPO.Q	5*	GROSS PRIV DOM INVEST-FIXED-NONRES-EQUIP&SW-INFO PROC&SW-OTH,BILLIONS OF $,SAAR-US
170	IFNREEIPCS.Q	5*	GROSS PRIV DOM INVEST-FIXED-NONRES-EQUIP&SW-SW,BILLIONS OF $,SAAR-US
171	IFNREEIPCC.Q	5*	GROSS PRIV DOM INVEST-FIXED-NONRES-EQUIP&SW-COMP&PERI,BILLIONS OF $,SAAR-US
172	IFNREEIP.Q	5*	GROSS PRIV DOM INVEST-FIXED-NONRES-EQUIP&SW-INFO PROC,BILLIONS OF $,SAAR-US
173	IFNREE.Q	5*	GROSS PRIV DOM INVEST-FIXED NONRES-EQUIP#&SW,BILLIONS OF $,SAAR-US
174	IFNRESO.Q	5*	PRIV FIXED INVEST-NONRES-OTH STRUC,BILLIONS OF $,SAAR-US
175	IFNRESMI.Q	5*	PRIV FIXED INVEST-NONRES-STRUC-MINING EXPLORATION,SHAFTS,&WELLS,BILLIONS OF $,SAAR-US
176	IFNRESP.Q	5*	PRIV FIXED INVEST-NONRES-STRUC-POWER&COMM,BILLIONS OF $,SAAR-US
177	II.Q	1	GROSS PRIV DOM INVEST-CH IN PRIV INVENT,BILLIONS OF $,SAAR-US
178	IIF.Q	1	GROSS PRIV DOM INVEST-CH IN PRIV INVENT-FARM,BILLIONS OF $,SAAR-US
179	M.Q	5	IMPORTS OF GDS&SVC,BILLIONS OF $,SAAR-US
180	X.Q	5	EXPORTS OF GDS&SVC,BILLIONS OF $,SAAR-US
181	PGDP@US.Q	5	NIA PRICE DEFLATOR - GROSS DOMESTIC PRODUCT, SA - U.S.
182	PCP@US.Q	5	NIA PRICE DEFLATOR - PRIVATE CONSUMPTION EXPENDITURE, SA - U.S.

Appendix III

III.1 Appendix to Chapter 3

III.1.1 Proofs

Proof of Theorem 1 Premultiply (3.7) by $(\Lambda'\Lambda)^{-1}\Lambda'$, assuming that $(\Lambda'\Lambda)$ is nonsingular, and go back one period to get

$$F_{t-1} = (\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} - (\Lambda'\Lambda)^{-1}\Lambda' u_{t-1}.$$

Then, replacing for F_{t-1} in (3.8) yields

$$F_t = \Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} + a_t - \Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1}.$$

Finally, replace for F_t in (3.7) to obtain

$$X_t = \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} + u_t - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1} + \Lambda a_t.$$

Defining $A(L) = [I - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' L]$, $B(L) = [(I - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' L) \quad \Lambda]$ and $e_t = [u_t \quad a_t]'$, we obtain (3.9). This is a VARMA(p,p) process since u_t and a_t are two uncorrelated white noises.

Another way to see that VAR factors induce VARMA structure on observable series is to consider the DFM in VAR form as in Stock and Watson (2005). Consider the model in (3.18)-(3.20), and assume that $\Theta(L) = I$. The VAR($\bar{p}$) on $[X_t \quad F_t]'$ is

$$\begin{bmatrix} X_t \\ F_t \end{bmatrix} = \begin{bmatrix} D(L) & \Lambda\Phi(L) \\ 0 & \Phi(L) \end{bmatrix} \begin{bmatrix} X_{t-1} \\ F_{t-1} \end{bmatrix} + \begin{bmatrix} \nu_t + \Lambda G\eta_t \\ G\eta_t \end{bmatrix}$$

where $\bar{p} = \mathbf{max}(p, \mathbf{max}(p_{x,i})), \quad i = 1, \ldots, N$. Hence, using results from Section 2 it can be shown that the marginal process of X_t is VARMA(p^*,q^*) with $p^* \leq (N-K+1)\bar{p}$,

$q^* \leq (N-K)\bar{p}$.

Proof of Theorem 2 To obtain (3.11), we follow the same steps as in the previous proof except that we use (3.10) instead of (3.8), which yields

$$X_t = \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' X_{t-1} + u_t - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1} + \Lambda a_t - \Lambda\Theta(L)a_{t-1}.$$

Defining $A(L)$ and e_t as above, and $B(L) = [(I - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' L) \quad \Lambda(I - \Theta(L)L)]$ we obtain the result. The VAR part is still of order p, while the MA part is of order max(p,q) since the autocovariances of $v_t \equiv u_t - \Lambda\Phi(L)(\Lambda'\Lambda)^{-1}\Lambda' u_{t-1} + \Lambda a_t - \Lambda\Theta(L)a_{t-1}$ go to zero for $h > max(p,q)$.

Proof of Theorem 3 We have that K-dimensional process F_t is a linear transformation of X_t, i.e. $F_t = CX_t$ where C is $K \times N$. Then we obtain the results using Propositions from Section 2.

For (i) when X_t follows VARMA(p,q) process as in (3.6), then by Corollaries 11.1.1 and 11.1.2 in Lütkepohl (2005), $F_t = CX_t$ has VARMA(p^*,q^*) representation with appropriate bounds for AR and MA orders.

For (ii), if X_t follows VAR(p) process, then by observing this as a special case in Corollaries 11.1.1 and 11.1.2 in Lütkepohl (2005), $F_t = CX_t$ follows VARMA(p^*,q^*) process.

Finally for (iii), if X_t has an MA representation as in (3.1) or (3.4), then by Proposition 4.1 or 4.2 in Lütkepohl (1987) the K-dimensional process $F_t = CX_t$ has an MA or MA(q^*) representation.

III.1.2 Simulation results

	$\rho_T = 0.9, \rho_N = 0.5$								$\rho_T = 0.9, \rho_N = 0.1$							
	RELATIVE MSE (TO VAR(6) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(6) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.0078	1.1405	0.9235	1.3858	1.0061	1.0945	0.9084	1.4722	1.0203	1.0656	0.8897	1.2688	0.9977	1.0303	0.9026	1.3464
2	1.0199	1.0852	0.9483	1.3189	1.0302	1.0762	0.9383	1.3660	0.9689	1.0113	0.8982	1.1708	1.0013	1.0406	0.9038	1.1735
4	0.8872	0.9459	0.8350	1.0746	0.9338	1.0242	0.8745	1.1542	0.9142	0.9508	0.8616	1.0391	0.9032	0.9166	0.8461	1.0029
6	0.8122	0.9181	0.7635	0.9536	0.8514	0.9375	0.7954	1.0010	0.8420	0.8656	0.7851	0.9213	0.8841	0.8798	0.8054	0.9182
12	0.6311	0.8392	0.6072	0.7198	0.6857	0.9278	0.6533	0.8036	0.6401	0.7487	0.6235	0.7038	0.7042	0.8089	0.6850	0.7582
18	0.4913	0.7186	0.4754	0.5339	0.5181	0.8285	0.4955	0.5744	0.5208	0.6774	0.5133	0.5609	0.5469	0.6970	0.5296	0.5742
24	0.3762	0.6192	0.3706	0.4237	0.3846	0.7215	0.3788	0.4291	0.4095	0.5979	0.4124	0.4322	0.4380	0.5724	0.4282	0.4499
36	0.1394	0.2429	0.1369	0.1480	0.1445	0.3006	0.1422	0.1560	0.1417	0.2169	0.1402	0.1447	0.1453	0.2152	0.1424	0.1535
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.0761	1.1170	1.0004	1.6656	1.0130	1.0126	1.0093	1.0070	1.0622	1.0751	0.9990	1.3846	0.9978	0.9980	0.9984	0.9927
2	1.0865	1.1495	1.0193	1.5676	0.9962	0.9956	0.9952	0.9951	1.0578	1.0368	0.9913	1.2818	0.9935	0.9951	0.9935	0.9933
4	1.0537	1.0890	1.0038	1.4432	0.9945	0.9950	0.9947	0.9947	1.0254	1.0088	0.9729	1.2141	0.9890	0.9894	0.9891	0.9891
6	1.0168	1.0392	0.9686	1.3060	0.9945	0.9954	0.9946	0.9946	1.0058	0.9720	0.9477	1.1812	0.9892	0.9892	0.9892	0.9892
12	0.9183	0.9915	0.8960	1.2573	0.9871	0.9883	0.9873	0.9873	0.9480	0.9163	0.8819	1.0303	0.9919	0.9918	0.9919	0.9919
18	0.8886	0.9848	0.8552	1.1123	0.9831	0.9880	0.9832	0.9832	0.9371	0.9068	0.8823	1.0173	0.9784	0.9784	0.9784	0.9784
24	0.8643	0.9706	0.8198	1.1203	0.9831	0.9830	0.9828	0.9828	0.9441	0.8755	0.8626	1.0214	0.9807	0.9807	0.9807	0.9807
36	0.8078	0.9754	0.7956	1.0742	0.9863	0.9846	0.9847	0.9847	0.8591	0.8376	0.8013	0.9264	0.9796	0.9796	0.9796	0.9796
	RELATIVE MSE (TO VAR(6) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(6) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.0078	1.1405	0.9235	1.3858	1.0061	1.0945	0.9084	1.4722	1.0203	1.0656	0.8897	1.2688	0.9977	1.0303	0.9026	1.3464
2	1.0126	1.0774	0.9415	1.3095	1.0234	1.0691	0.9321	1.3570	0.9805	1.0234	0.9090	1.1848	1.0082	1.0477	0.9100	1.1815
4	0.9744	1.0388	0.9170	1.1801	1.0087	1.1063	0.9446	1.2467	0.9902	1.0298	0.9331	1.1254	0.9881	1.0028	0.9257	1.0972
6	0.9771	1.1044	0.9185	1.1472	1.0052	1.1068	0.9390	1.1818	1.0000	1.0281	0.9325	1.0943	1.0380	1.0330	0.9456	1.0781
12	0.9149	1.2165	0.8802	1.0434	0.9656	1.3067	0.9200	1.1317	0.9214	1.0777	0.8975	1.0130	0.9626	1.1057	0.9363	1.0364
18	0.8988	1.3147	0.8699	0.9768	0.9436	1.5088	0.9023	1.0460	0.9115	1.1854	0.8983	0.9816	0.9521	1.2136	0.9221	0.9997
24	0.8625	1.4195	0.8497	0.9714	0.9041	1.6962	0.8905	1.0088	0.8556	1.2493	0.8616	0.9029	0.9393	1.2276	0.9183	0.9649
36	0.8121	1.4144	0.7971	0.8619	0.8697	1.8094	0.8561	0.9391	0.8176	1.2517	0.8088	0.8351	0.8566	1.2680	0.8391	0.9044
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.0761	1.1170	1.0004	1.6656	1.0130	1.0126	1.0093	1.0070	1.0622	1.0751	0.9990	1.3846	0.9978	0.9980	0.9984	0.9927
2	1.0487	1.1095	0.9838	1.5129	0.9959	0.9953	0.9949	0.9949	1.0489	1.0280	0.9829	1.2710	0.9948	0.9965	0.9949	0.9946
4	1.0446	1.0797	0.9952	1.4309	0.9986	0.9990	0.9987	0.9987	1.0364	1.0197	0.9833	1.2272	0.9931	0.9934	0.9932	0.9932
6	1.0440	1.0670	0.9945	1.3409	0.9992	1.0002	0.9993	0.9994	1.0547	1.0192	0.9937	1.2386	0.9981	0.9981	0.9981	0.9981
12	1.0432	1.1264	1.0179	1.4283	1.0002	1.0015	1.0004	1.0004	1.0749	1.0390	0.9999	1.1682	1.0001	1.0001	1.0001	1.0001
18	1.0651	1.1804	1.0251	1.3332	1.0001	1.0051	1.0002	1.0002	1.1030	1.0674	1.0385	1.1973	1.0000	0.9999	1.0000	1.0000
24	1.0808	1.2137	1.0252	1.4010	1.0004	1.0003	1.0000	1.0000	1.1481	1.0647	1.0490	1.2422	1.0000	1.0000	1.0000	1.0000
36	1.0764	1.2997	1.0602	1.4313	1.0016	0.9999	1.0000	1.0000	1.1326	1.1042	1.0564	1.2213	0.9999	0.9999	0.9999	0.9999

Table III.I: Results from simulation exercise 1, case 1

	$\rho_T = 0.1, \rho_N = 0.9$								$\rho_T = 0.1, \rho_N = 0.1$							
	RELATIVE MSE (TO VAR(6) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(6) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.8978	0.9108	0.8924	0.9362	0.9329	0.8880	0.8585	0.9208	0.9439	0.8761	0.7969	0.9618	0.9289	0.8919	0.8155	0.9675
2	0.8522	0.8716	0.8606	0.9168	0.8289	0.8194	0.8228	0.8642	0.8029	0.7863	0.7764	0.8459	0.7888	0.7900	0.7736	0.8569
4	0.8381	0.8420	0.8524	0.8601	0.8195	0.8213	0.8187	0.8238	0.7894	0.7542	0.7533	0.7742	0.7513	0.7533	0.7525	0.7687
6	0.8213	0.8227	0.8225	0.8210	0.7852	0.7806	0.7799	0.7795	0.7580	0.7420	0.7409	0.7438	0.7477	0.7488	0.7446	0.7458
12	0.7923	0.7906	0.7905	0.7907	0.7630	0.7569	0.7567	0.7568	0.6773	0.6751	0.6751	0.6754	0.6575	0.6604	0.6560	0.6613
18	0.6803	0.6770	0.6771	0.6772	0.6582	0.6576	0.6577	0.6577	0.5757	0.5700	0.5701	0.5761	0.5741	0.5753	0.5704	0.5728
24	0.5367	0.5363	0.5364	0.5364	0.4865	0.4864	0.4863	0.4862	0.4106	0.4074	0.4073	0.4084	0.4329	0.4303	0.4304	0.4317
36	0.0946	0.0956	0.0944	0.0944	0.0801	0.0799	0.0799	0.0800	0.0726	0.0721	0.0721	0.0721	0.0719	0.0722	0.0721	0.0721
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9680	0.9676	0.9560	0.9515	0.9931	0.9995	0.9926	0.9921	0.9702	0.9672	0.9290	0.9316	0.9838	0.9874	0.9868	0.9840
2	0.9332	0.9304	0.9306	0.9310	0.9881	0.9929	0.9882	0.9878	0.8998	0.9053	0.8985	0.8993	0.9816	0.9904	0.9811	0.9811
4	0.9338	0.9261	0.9257	0.9257	0.9882	0.9895	0.9893	0.9894	0.9095	0.9003	0.9000	0.8997	0.9891	0.9894	0.9891	0.9891
6	0.9467	0.9350	0.9351	0.9351	0.9831	0.9831	0.9830	0.9830	0.8806	0.8771	0.8767	0.8767	0.9821	0.9822	0.9821	0.9821
12	0.9358	0.9359	0.9359	0.9359	0.9825	0.9825	0.9825	0.9825	0.8855	0.8841	0.8839	0.8839	0.9778	0.9778	0.9778	0.9778
18	0.9297	0.9298	0.9297	0.9297	0.9874	0.9873	0.9873	0.9873	0.8725	0.8704	0.8702	0.8702	0.9852	0.9852	0.9852	0.9852
24	0.9140	0.9142	0.9143	0.9143	0.9887	0.9886	0.9886	0.9886	0.8711	0.8707	0.8709	0.8709	0.9815	0.9815	0.9815	0.9815
36	0.9044	0.9047	0.9043	0.9043	0.9929	0.9930	0.9930	0.9930	0.8183	0.8185	0.8183	0.8183	0.9790	0.9790	0.9790	0.9790
	RELATIVE MSE (TO VAR(6) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(6) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.8978	0.9108	0.8924	0.9362	0.9329	0.8880	0.8585	0.9208	0.9439	0.8761	0.7969	0.9618	0.9289	0.8919	0.8155	0.9675
2	0.8879	0.9082	0.8966	0.9552	0.8644	0.8545	0.8580	0.9012	0.8438	0.8264	0.8159	0.8890	0.8259	0.8271	0.8099	0.8971
4	0.9270	0.9313	0.9427	0.9513	0.9119	0.9139	0.9110	0.9167	0.9067	0.8664	0.8653	0.8893	0.8629	0.8652	0.8643	0.8828
6	0.9552	0.9567	0.9565	0.9547	0.9441	0.9386	0.9378	0.9373	0.9249	0.9055	0.9040	0.9076	0.9174	0.9188	0.9135	0.9151
12	0.9872	0.9850	0.9849	0.9852	0.9840	0.9761	0.9758	0.9759	0.9791	0.9759	0.9758	0.9763	0.9783	0.9826	0.9760	0.9839
18	0.9957	0.9910	0.9910	0.9911	0.9934	0.9925	0.9926	0.9926	0.9937	0.9838	0.9840	0.9944	0.9994	1.0015	0.9928	0.9971
24	0.9973	0.9966	0.9967	0.9968	0.9911	0.9909	0.9908	0.9905	0.9866	0.9788	0.9786	0.9812	1.0005	0.9944	0.9946	0.9977
36	0.9979	1.0082	0.9965	0.9966	0.9925	0.9896	0.9896	0.9908	0.9902	0.9836	0.9835	0.9836	0.9919	0.9958	0.9946	0.9944
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9680	0.9676	0.9560	0.9515	0.9931	0.9995	0.9926	0.9921	0.9702	0.9672	0.9290	0.9316	0.9838	0.9874	0.9868	0.9840
2	0.9508	0.9479	0.9481	0.9485	0.9901	0.9949	0.9903	0.9898	0.9096	0.9152	0.9083	0.9091	0.9847	0.9935	0.9842	0.9842
4	0.9755	0.9675	0.9671	0.9671	0.9936	0.9948	0.9947	0.9948	0.9557	0.9461	0.9458	0.9455	0.9949	0.9953	0.9950	0.9950
6	0.9990	0.9867	0.9867	0.9867	0.9974	0.9974	0.9973	0.9973	0.9717	0.9679	0.9674	0.9674	0.9971	0.9972	0.9971	0.9971
12	0.9987	0.9987	0.9987	0.9987	1.0002	1.0002	1.0002	1.0002	0.9995	0.9979	0.9977	0.9977	1.0000	1.0000	1.0000	1.0000
18	1.0001	1.0002	1.0001	1.0001	1.0001	1.0000	1.0000	1.0000	1.0025	1.0001	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000
24	0.9995	0.9997	0.9998	0.9998	1.0000	0.9999	0.9999	0.9999	0.9998	0.9993	0.9996	0.9996	1.0001	1.0001	1.0001	1.0001
36	1.0003	1.0007	1.0002	1.0002	0.9998	1.0000	1.0000	1.0000	0.9997	0.9999	0.9996	0.9996	1.0000	1.0000	1.0000	1.0000

Table III.II: Results from simulation exercise 1, case 1, cont.

	$\rho_T = 0.9, \rho_N = 0.5$								$\rho_T = 0.9, \rho_N = 0.1$							
	RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.8168	0.8338	0.7830	1.0004	0.8211	0.8364	0.8081	1.0849	0.8024	0.8060	0.7948	0.9410	0.8482	0.8043	0.8569	0.9851
2	0.7715	0.7747	0.7444	0.7682	0.7379	0.7581	0.7481	0.7793	0.7213	0.7259	0.7188	0.7389	0.7198	0.7139	0.7166	0.7150
4	0.6972	0.7636	0.6885	0.7148	0.6981	0.7450	0.6974	0.7262	0.7059	0.7161	0.6937	0.7051	0.6773	0.6853	0.6833	0.6975
6	0.6538	0.7624	0.6533	0.6698	0.6733	0.7685	0.6621	0.6729	0.6507	0.6855	0.6536	0.6589	0.6163	0.6507	0.6318	0.6394
12	0.5453	0.7673	0.5343	0.5335	0.5497	0.7743	0.5494	0.5466	0.5597	0.6293	0.5507	0.5509	0.5290	0.6072	0.5395	0.5395
18	0.4128	0.6560	0.4091	0.4046	0.4379	0.7022	0.4381	0.4331	0.4203	0.5301	0.4205	0.4190	0.4431	0.5177	0.4435	0.4434
24	0.3114	0.5082	0.3001	0.2976	0.3007	0.5244	0.2998	0.2955	0.2956	0.3807	0.3015	0.2995	0.3025	0.3673	0.2975	0.2945
36	0.1427	0.2401	0.1432	0.1412	0.1446	0.2674	0.1454	0.1421	0.1389	0.1853	0.1332	0.1318	0.1403	0.1817	0.1416	0.1407
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9727	0.9732	0.9352	1.1557	0.9965	0.9946	0.9957	0.9979	0.9810	0.9378	0.9130	1.0321	0.9863	0.9899	0.9861	0.9882
2	0.9055	0.9078	0.8919	1.0099	0.9792	0.9800	0.9776	0.9801	0.8919	0.8915	0.8767	0.9066	0.9816	0.9926	0.9822	0.9826
4	0.9020	0.9038	0.8942	0.9336	0.9793	0.9847	0.9796	0.9797	0.8858	0.8770	0.8756	0.8916	0.9769	0.9821	0.9769	0.9770
6	0.8990	0.9078	0.8898	0.9155	0.9782	0.9782	0.9782	0.9782	0.8762	0.8694	0.8697	0.8712	0.9753	0.9755	0.9753	0.9753
12	0.8367	0.9146	0.8277	0.8523	0.9748	0.9749	0.9748	0.9748	0.8448	0.8384	0.8347	0.8370	0.9668	0.9668	0.9668	0.9668
18	0.8115	0.9658	0.7972	0.7983	0.9771	0.9771	0.9771	0.9771	0.8227	0.8171	0.8022	0.8114	0.9738	0.9738	0.9738	0.9738
24	0.7895	1.0204	0.7764	0.7984	0.9791	0.9791	0.9791	0.9791	0.7964	0.7994	0.7781	0.7835	0.9732	0.9732	0.9732	0.9732
36	0.7170	0.9889	0.7026	0.7137	0.9721	0.9721	0.9721	0.9721	0.7362	0.7713	0.7215	0.7249	0.9709	0.9709	0.9709	0.9709
	RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.8168	0.8338	0.7830	1.0004	0.8211	0.8364	0.8081	1.0849	0.8024	0.8060	0.7948	0.9410	0.8482	0.8043	0.8569	0.9851
2	0.8286	0.8320	0.7994	0.8251	0.7797	0.8011	0.7905	0.8235	0.7833	0.7884	0.7807	0.8025	0.7672	0.7610	0.7639	0.7622
4	0.8697	0.9526	0.8589	0.8917	0.8609	0.9187	0.8600	0.8955	0.8698	0.8824	0.8548	0.8688	0.8359	0.8457	0.8432	0.8607
6	0.8943	1.0429	0.8937	0.9163	0.9173	1.0471	0.9021	0.9168	0.8961	0.9440	0.9001	0.9073	0.8810	0.9302	0.9031	0.9141
12	0.9307	1.3097	0.9119	0.9106	0.9372	1.3199	0.9367	0.9318	0.9712	1.0919	0.9556	0.9560	0.9384	1.0770	0.9569	0.9570
18	0.9047	1.4379	0.8967	0.8869	0.9440	1.5138	0.9446	0.9337	0.9320	1.1754	0.9323	0.9292	0.9607	1.1225	0.9614	0.9612
24	0.9289	1.5163	0.8952	0.8877	0.9200	1.6044	0.9174	0.9040	0.9305	1.1987	0.9492	0.9430	0.9554	1.1603	0.9398	0.9302
36	0.8715	1.4667	0.8748	0.8623	0.8821	1.6315	0.8874	0.8671	0.9142	1.2191	0.8767	0.8671	0.9301	1.2043	0.9384	0.9327
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9727	0.9732	0.9352	1.1557	0.9965	0.9946	0.9957	0.9979	0.9810	0.9378	0.9130	1.0321	0.9863	0.9899	0.9861	0.9882
2	0.9165	0.9188	0.9027	1.0221	0.9820	0.9828	0.9804	0.9829	0.9077	0.9074	0.8923	0.9227	0.9830	0.9941	0.9836	0.9840
4	0.9632	0.9651	0.9549	0.9970	0.9884	0.9938	0.9887	0.9888	0.9564	0.9469	0.9454	0.9626	0.9911	0.9964	0.9911	0.9912
6	1.0000	1.0099	0.9898	1.0184	0.9992	0.9993	0.9992	0.9992	0.9935	0.9858	0.9861	0.9878	1.0002	1.0004	1.0002	1.0002
12	1.0057	1.0994	0.9950	1.0245	1.0000	1.0000	1.0000	1.0000	1.0110	1.0033	0.9988	1.0016	1.0001	1.0001	1.0001	1.0001
18	1.0058	1.1971	0.9881	0.9894	1.0000	1.0000	1.0000	1.0000	1.0217	1.0147	0.9962	1.0076	1.0000	1.0000	1.0000	1.0000
24	1.0060	1.3002	0.9893	1.0174	1.0000	1.0000	1.0000	1.0000	1.0175	1.0214	0.9941	1.0010	1.0000	1.0000	1.0000	1.0000
36	1.0081	1.3903	0.9878	1.0034	1.0000	1.0000	1.0000	1.0000	1.0153	1.0636	0.9950	0.9997	1.0000	1.0000	1.0000	1.0000

Table III.III: Results from simulation exercise 1, case 2

	$\rho_T = 0.1, \rho_N = 0.9$								$\rho_T = 0.1, \rho_N = 0.1$							
	RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9207	0.8569	0.9599	0.9455	0.9556	0.8779	0.9330	0.9213	0.9759	0.8416	0.9349	0.9087	0.9368	0.8434	0.9225	0.8822
2	0.6689	0.6830	0.6899	0.6822	0.6670	0.6853	0.6780	0.6627	0.6416	0.6599	0.6531	0.6360	0.6411	0.6687	0.6575	0.6271
4	0.6246	0.6309	0.6261	0.6250	0.6442	0.6652	0.6245	0.6214	0.5737	0.5921	0.5650	0.5602	0.6129	0.6269	0.6215	0.6087
6	0.6142	0.6198	0.6241	0.6240	0.6262	0.6153	0.6106	0.6100	0.5872	0.6063	0.6060	0.6033	0.5694	0.5743	0.5574	0.5562
12	0.5207	0.5402	0.5464	0.5464	0.5372	0.5398	0.5420	0.5418	0.4849	0.4933	0.4871	0.4871	0.5170	0.5173	0.5219	0.5221
18	0.4600	0.4555	0.4419	0.4419	0.4442	0.4465	0.4464	0.4464	0.4073	0.4019	0.3921	0.3921	0.3917	0.3982	0.3957	0.3957
24	0.2904	0.2895	0.2903	0.2903	0.2847	0.2872	0.2817	0.2817	0.2554	0.2597	0.2494	0.2494	0.2696	0.2706	0.2624	0.2624
36	0.1062	0.1074	0.1092	0.1092	0.1021	0.1033	0.0987	0.0987	0.0969	0.0969	0.0967	0.0967	0.0965	0.0994	0.0971	0.0971
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.1330	1.0075	0.9066	1.2783	0.9592	0.9935	0.9641	0.9572	1.5484	0.9748	0.8213	2.1043	0.9237	0.9960	0.9028	0.9907
2	0.8541	0.8709	0.8453	0.8923	0.9719	0.9951	0.9724	0.9727	0.8596	0.8784	0.8165	0.9017	0.9677	1.0016	0.9682	0.9685
4	0.8501	0.8490	0.8428	0.8509	0.9742	0.9914	0.9742	0.9742	0.8287	0.8205	0.8020	0.8184	0.9734	1.0058	0.9733	0.9735
6	0.8452	0.8432	0.8425	0.8425	0.9721	0.9790	0.9721	0.9721	0.8310	0.8218	0.8178	0.8186	0.9699	1.0212	0.9699	0.9696
12	0.8331	0.8326	0.8328	0.8331	0.9761	0.9766	0.9760	0.9760	0.8086	0.7958	0.7957	0.7969	0.9653	0.9949	0.9653	0.9655
18	0.8056	0.8093	0.8052	0.8049	0.9713	0.9829	0.9712	0.9712	0.7650	0.7590	0.7590	0.7579	0.9661	1.0172	0.9661	0.9661
24	0.8044	0.8034	0.8036	0.8038	0.9683	0.9745	0.9682	0.9682	0.7630	0.7624	0.7623	0.7629	0.9721	0.9918	0.9721	0.9718
36	0.7643	0.7610	0.7610	0.7613	0.9680	0.9698	0.9679	0.9679	0.7210	0.7188	0.7188	0.7191	0.9658	0.9918	0.9658	0.9657
	RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS								RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS							
	T=50, N=50				T=50, N=100				T=50, N=50				T=50, N=100			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9207	0.8569	0.9599	0.9455	0.9556	0.8779	0.9330	0.9213	0.9759	0.8416	0.9349	0.9087	0.9368	0.8434	0.9225	0.8822
2	0.7223	0.7375	0.7449	0.7366	0.7213	0.7410	0.7331	0.7167	0.6860	0.7055	0.6983	0.6800	0.6809	0.7102	0.6983	0.6660
4	0.8030	0.8111	0.8050	0.8036	0.8032	0.8294	0.7787	0.7748	0.7679	0.7925	0.7563	0.7498	0.7922	0.8103	0.8032	0.7867
6	0.8952	0.9035	0.9097	0.9096	0.9050	0.8892	0.8825	0.8816	0.8384	0.8657	0.8653	0.8615	0.8590	0.8663	0.8409	0.8391
12	0.9339	0.9690	0.9801	0.9801	0.9716	0.9763	0.9803	0.9799	0.9487	0.9651	0.9529	0.9530	0.9696	0.9702	0.9787	0.9791
18	1.0024	0.9927	0.9629	0.9629	0.9849	0.9900	0.9898	0.9898	0.9950	0.9820	0.9580	0.9579	0.9649	0.9810	0.9748	0.9748
24	0.9944	0.9912	0.9940	0.9940	0.9974	1.0062	0.9868	0.9868	0.9803	0.9967	0.9572	0.9572	0.9890	0.9929	0.9627	0.9627
36	0.9870	0.9976	1.0146	1.0146	0.9876	0.9999	0.9546	0.9546	0.9963	0.9965	0.9948	0.9948	0.9796	1.0089	0.9850	0.9850
	T=100, N=50				T=600, N=130				T=100, N=50				T=600, N=130			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	1.1330	1.0075	0.9066	1.2783	0.9592	0.9935	0.9641	0.9572	1.5484	0.9748	0.8213	2.1043	0.9237	0.9960	0.9028	0.9907
2	0.8822	0.8995	0.8731	0.9216	0.9692	0.9923	0.9697	0.9700	0.8804	0.8997	0.8363	0.9235	0.9610	0.9946	0.9614	0.9618
4	0.9293	0.9281	0.9213	0.9302	0.9814	0.9987	0.9814	0.9814	0.9292	0.9200	0.8992	0.9176	0.9745	1.0069	0.9744	0.9746
6	0.9798	0.9775	0.9767	0.9768	0.9963	1.0034	0.9962	0.9962	0.9759	0.9651	0.9603	0.9613	0.9904	1.0428	0.9904	0.9901
12	0.9978	0.9972	0.9973	0.9977	1.0000	1.0005	0.9999	0.9999	1.0133	0.9972	0.9971	0.9986	0.9996	1.0303	0.9996	0.9998
18	1.0004	1.0049	0.9999	0.9995	1.0001	1.0120	1.0000	1.0000	1.0081	1.0003	1.0003	0.9988	0.9999	1.0528	0.9999	0.9999
24	1.0011	0.9998	1.0001	1.0003	1.0001	1.0065	1.0000	1.0000	1.0012	1.0004	1.0003	1.0011	1.0000	1.0203	1.0000	0.9997
36	1.0040	0.9998	0.9997	1.0002	1.0001	1.0020	1.0000	1.0000	1.0029	0.9998	0.9999	1.0003	1.0000	1.0269	1.0000	0.9998

Table III.IV: Results from simulation exercise 1, case 2, cont.

	RELATIVE MSE (TO VAR(4) DIRECT) RESULTS FOR VARMA-BASED FORECASTING MODELS											
	K=3				K=4				K=6			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9638	0.9643	0.9285	0.9330	0.9194	0.9182	0.8866	0.8927	0.7282	0.6615	0.6905	0.6907
2	0.9085	0.9174	0.9076	0.9133	0.8792	0.8901	0.8805	0.8866	0.8261	0.8615	0.8244	0.8385
4	0.8971	0.8966	0.8965	0.8961	0.8764	0.8775	0.8764	0.8769	0.8030	0.8030	0.8010	0.8072
6	0.9038	0.9037	0.9035	0.9036	0.8548	0.8549	0.8548	0.8549	0.9182	0.9180	0.9182	0.9204
12	0.8808	0.8807	0.8807	0.8807	0.8416	0.8418	0.8418	0.8418	0.7983	0.7997	0.7983	0.7983
18	0.8831	0.8831	0.8831	0.8831	0.8455	0.8454	0.8454	0.8454	0.9393	0.9383	0.9393	0.9393
24	0.8757	0.8756	0.8756	0.8756	0.8425	0.8425	0.8425	0.8425	0.7287	0.7286	0.7287	0.7287
36	0.8344	0.8343	0.8343	0.8343	0.7930	0.7932	0.7932	0.7932	0.5466	0.5466	0.5466	0.5466
	RELATIVE MSE (TO VAR(4) ITERATIVE) RESULTS FOR VARMA-BASED FORECASTING MODELS											
	K=3				K=4				K=6			
Horizon	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR	Diag MA	Diag AR	Final MA	Final AR
1	0.9638	0.9643	0.9285	0.9330	0.9194	0.9182	0.8866	0.8927	0.7282	0.6615	0.6905	0.6907
2	0.9197	0.9288	0.9188	0.9246	0.9092	0.9205	0.9106	0.9168	0.9296	0.9695	0.9277	0.9435
4	0.9685	0.9680	0.9679	0.9675	0.9562	0.9574	0.9562	0.9568	0.9406	0.9406	0.9383	0.9456
6	0.9927	0.9926	0.9925	0.9926	0.9851	0.9852	0.9850	0.9852	0.9467	0.9466	0.9467	0.9490
12	1.0001	1.0000	1.0000	1.0001	1.0002	1.0005	1.0005	1.0005	0.9803	0.9820	0.9803	0.9802
18	0.9997	0.9996	0.9996	0.9996	1.0038	1.0037	1.0037	1.0037	0.9957	0.9947	0.9957	0.9957
24	1.0009	1.0008	1.0008	1.0008	1.0010	1.0009	1.0009	1.0009	0.9978	0.9977	0.9978	0.9978
36	0.9998	0.9997	0.9997	0.9997	0.9993	0.9995	0.9995	0.9995	0.9986	0.9986	0.9986	0.9986

Table III.V: Results from simulation exercise 2

Appendix IV

IV.1 Appendix to Chapter 4

IV.1.1 Additional results

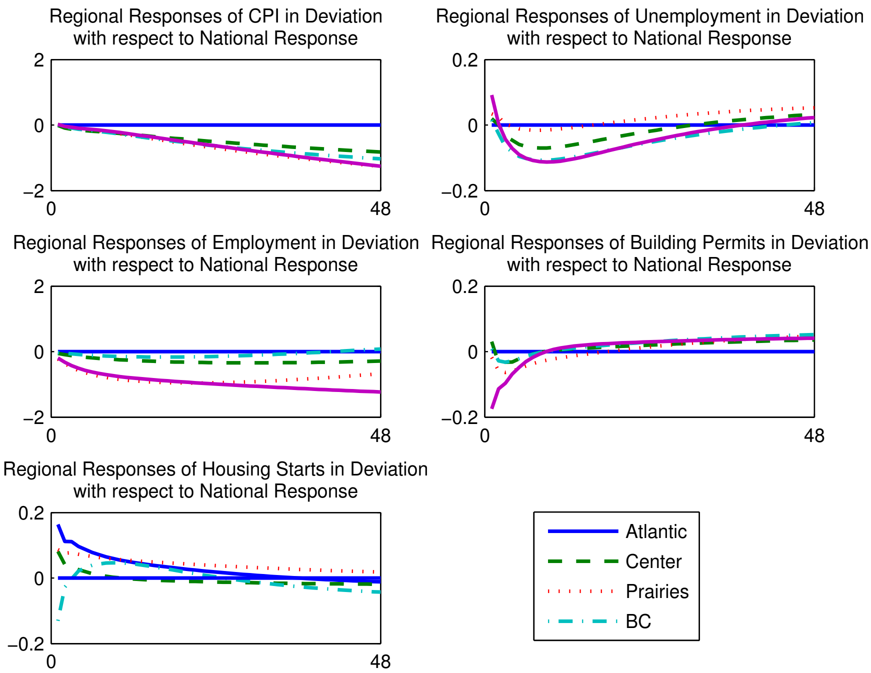

Figure IV.1: Regional impulse responses to a credit shock in deviation with respect to national response

- Atlantic provinces: Newfoundland and Labrador, Prince Edward Island, Nova Scotia and New Brunswick
- Center: Québec and Ontario
- Prairies: Manitoba, Saskatchewan and Alberta
- BC: British Columbia

IV.1.2 Bootstrap procedure

Since there is still no strong theoretical studies that shows the optimal way to produce statistical inference about impulse responses in structural large-dimensional factor models, we explain in details our parametric bootstrap procedure. The goal is to obtain confidence bands for impulse responses to structural shocks in representation (4.10-4.11) with assumption (4.13).

- **Step 1**

  Shuffle time dimension of residuals in (4.11) and resample static factors using estimates of VARMA coefficients:

$$\tilde{F}_t = \hat{\Phi}(L)\tilde{F}_{t-1} + \hat{\Theta}\tilde{\eta}_t$$

- **Step 2**

  Shuffle time dimension of residuals in (4.10), and resample observable series using new factors obtained from the previous step and the estimated loadings:

$$\tilde{X}_t = \hat{\Lambda}\tilde{F}_t + \tilde{u}_t$$

- **Step 3**

  Estimate FAVARMA model on $\tilde{X}_t$, identify structural shock and produce impulse responses.

As it was pointed out in Dufour and Stevanovic (2010), having a good approximation of the true factor process can be very important in order to get the right bootstrap procedure. If the finite VAR approximation is far away from the truth, and if the finite VARMA representation does much better, allowing for MA part should provide a more reliable inference.