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Abstract

Intertemporal social-evaluation rules provide us with social criteria that can be used to assess the relative desirability of utility distributions across generations. The trade-offs between the well-being of different generations implicit in each such rule reflect the underlying ethical position on issues of intergenerational equity or justice.

We employ an axiomatic approach in order to identify ethically attractive social-evaluation procedures. In particular, we explore the possibilities of using welfare information and non-welfare information in a model of intertemporal social evaluation. We focus on the individuals' birth dates and lengths of life as the relevant non-welfare information. As usual, welfare information is given by lifetime utilities. It is assumed that this information is available for each alternative to be ranked.

Various weakenings of the Pareto principle are employed in order to allow birth dates or lengths of life (or both) to matter in social evaluation. In addition, we impose standard properties such as continuity and anonymity and we examine the consequences of an intertemporal independence property. For each of the Pareto conditions employed, we characterize all social-evaluation rules satisfying it and our other axioms. The resulting rules are birth-date dependent or lifetime-dependent versions of generalized utilitarianism. Furthermore, we discuss the ethical and axiomatic foundations of geometric discounting in the context of our model. *Journal of Economic Literature* Classification Number: D63.

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1. Introduction

A social-evaluation functional assigns a social ranking of alternatives to each information profile in its domain. In the classical multi-profile model of social choice, profiles are restricted to welfare information: all non-welfare information is implicitly assumed to be fixed. Because of this, the conventional approach does not allow us to discern the way in which the functional makes use of non-welfare information. For that, multiple non-welfare profiles are needed. Blackorby, Bossert and Donaldson [2005a] analyze a framework in which non-welfare information may vary across information profiles. Each information profile includes a vector of individual utility functions which represent welfare information and a vector of functions which describe social and individual non-welfare information. See also Kelsey [1987] and Roberts [1980] for approaches to social choice where non-welfare information is explicitly modeled.

A social-evaluation functional is welfarist if a single ordering of utility vectors, together with the utility information in a profile, is sufficient to rank all alternatives. The ordering of utility vectors is called a social-evaluation ordering. Welfarism is a consequence of three axioms: unlimited domain, Pareto indifference and binary independence of irrelevant alternatives. Unlimited domain requires that all logically possible profiles are in the domain of the functional. If everyone's well-being is the same in two alternatives for a given profile, then Pareto indifference requires the social ranking generated by that profile to declare the two alternatives equally good. This axiom is implied by a plausible property introduced by Goodin [1991]. If one alternative is declared socially better than another, he suggests it should be better for at least one member of society. Binary independence is a consistency condition across profiles. If welfare and non-welfare information for two alternatives coincide in two profiles, it requires the ranking of the alternatives to be the same for both profiles. If anonymity, which requires individuals to be treated impartially, is added to the three welfarism axioms, anonymous welfarism results. In that case, the social-evaluation ordering is anonymous: any permutation of a utility vector is as good as the utility vector itself.

In this paper, we employ an intertemporal structure for social evaluation. This results in a special case of the above-described model. The non-welfare information that is permitted to vary consists of individual birth dates and lengths of life in each alternative, and all other non-welfare information is assumed to be fixed. To investigate social-evaluation orderings in an intertemporal setting, we employ a period analysis with arbitrary period length. There are multiple information profiles which provide, for each person, lifetime utility, birth date and length of life in each alternative. We assume that no person can live more than a fixed number of periods. The maximal lifetime is finite but can be arbitrarily large.

We employ the standard axioms unlimited domain, binary independence of irrelevant alternatives and anonymity. Unlimited domain allows birth dates and lifetimes to be different in different alternatives. When this axiom is combined with the usual Pareto-indifference condition and binary independence, welfarism results and, as a consequence, dates of birth and lengths of life cannot affect the social ordering. To investigate the influences birth dates or lifetimes may have on social evaluation, therefore, it is necessary to consider weaker axioms. If, in any two alternatives, the same individuals are alive and have the same utility levels, birth dates and lengths of life, conditional Pareto indifference requires them to be ranked as equally good. If a social-evaluation functional satisfies unlimited domain, conditional Pareto indifference, binary independence of irrelevant alternatives and anonymity, there exists an anonymous ordering of compound vectors of individual utilities, birth dates and lifetimes which, together with the information in a profile, can be used to rank the alternatives. Because non-welfare information (in particular, birth dates and lifetimes) can influence the social ordering, not all of these orderings are welfarist.

The model of this paper is a fixed-population version of the variable-population framework analyzed in Blackorby, Bossert and Donaldson [1995, 1997, 1999, 2005b]. However, unlike these earlier papers, we focus on the intertemporal aspect rather than the population aspect of the issue. As a consequence, we can dispense with some assumptions that are required in the variable-population case. This move necessitates some new arguments in our analysis because some of the techniques applied in our earlier work rely on the possibility of varying the population. Moreover, we provide a general result allowing for both birth dates and lifetimes to matter in addition to lifetime well-being.

In our intertemporal setting, a natural independence condition can be imposed. The axiom is called independence of the utilities of the dead and it requires the ranking of any two alternatives to be independent of the utilities of individuals whose lives are over and who had the same lifetime utilities, birth dates and lifetimes in both alternatives. When combined with intertemporal versions of axioms such as continuity, anonymity and the above-mentioned variants of the Pareto conditions, independence of the utilities of the dead has remarkably strong consequences. The axioms imply independence of the utilities of all who are unconcerned, not only of those whose lives are over. In addition, depending on the version of the Pareto axiom employed, several classes of intertemporal generalized-utilitarian orderings are characterized. Intertemporal generalized utilitarianism ranks any two alternatives by comparing their total transformed utilities. Birth-date dependent generalized utilitarianism allows the transformation to depend on birth dates, and lifetime dependent generalized utilitarianism allows it to depend on lengths of life. Finally, birth-date and lifetime dependent generalized-utilitarian orderings are such that the transformation may depend on both birth dates and lifetimes.

In order to assess whether social-evaluation orderings should be sensitive to birth dates or lengths of life, consider a birth-date and lifetime dependent generalized-utilitarian ordering such that the transformation is sensitive to birth dates. In that case, there exist a lifetime-utility level, a lifetime and two different birth dates such that the transformed value of the utility level is different, given the lifetime at the two values for birth date. Now consider an alternative in which a person has the utility level and lifetime just mentioned. We wish to compare two alternatives that differ only in the birth date of the person. Because the conditional transformation is sensitive to birth date at that utility level and lifetime, one birth date will be ranked as better than the other. The transformation is continuous in utility and, therefore, betterness is preserved for some small decrease in utility. Thus, such a ordering approves of changes in birth dates even when, in terms of well-being, no one gains and someone loses. A similar argument applies to sensitivity to lifetimes. This suggests that we should reject the conditional Pareto axioms and, instead, opt in favor of intertemporal strong Pareto, ruling out the effects of non-welfare information. If this is done, intertemporal generalized utilitarianism results: the transformation cannot depend on birth dates or lifetimes.

Special forms of birth-date dependent orderings are employed frequently in economic models. In particular, orderings that are based on geometric discounting are widely used and, for that reason, we investigate them in some detail although we do not endorse them. Using a positive discount factor, the geometric birth-date dependent generalized-utilitarian orderings employ the sum of the discounted transformed utilities. Geometric birth-date dependent generalized utilitarianism is characterized by adding stationarity to the axioms characterizing birth-date dependent generalized utilitarianism. Suppose that the birth date of everyone in each of two alternatives is moved forward in time by a given number of periods. Stationarity requires the ranking of the resulting alternatives to be the same as that of the originals.

The concluding section of the paper addresses two issues. The first is a discussion of our choice of domain, in particular, the possibility of assigning different birth dates to the same person in different alternatives. The second re-examines the practice of discounting and we provide arguments against the discounting of well-being and suggest that concerns for unacceptable suffering of the present generation are better addressed by imposing constraints that prevent this from happening rather than changing the social objective into an ethically undesirable one.

2. Welfare information and non-welfare information

We use $\mathcal{Z}_+$ to denote the set of non-negative integers and $\mathcal{Z}_{++}$ is the set of positive integers. The set of real numbers is denoted by $\mathcal{R}$ and $\mathcal{R}_{++}$ is the set of all positive reals.

For $n \in \mathcal{Z}_{++}$, $\mathbf{1}_n$ is the vector consisting of n ones. Our notation for vector inequalities is $\gg, >, \geq$. For a non-empty set Y and $n \in \mathcal{Z}_{++}$, Y^n is the n-fold Cartesian product of Y.

Suppose there is a universal set of alternatives X with at least three elements. In order to focus on the intertemporal aspect of our investigation, we assume that the population—the set of those who ever live—is fixed and finite but note that, with a few additional assumptions, our model and the results can be reformulated in a variable-population setting; see Blackorby, Bossert and Donaldson [2005b] for a discussion. The population is denoted by $\{1, \ldots, n\}$ where $n \geq 3$. Note that the population is assumed to be finite whereas the universal set of alternatives can be (countably or uncountably) infinite or finite. At least three elements are required in X to obtain a generalization of the welfarism theorem, and the minimal number of individuals is three in order to apply a well-known separability property.

Each individual $i \in \{1, \ldots, n\}$ has a lifetime-utility function $U_i \colon X \to \mathcal{R}$ where, for all $x \in X$, $U_i(x)$ is the lifetime utility of i in alternative x. A utility (or welfare) profile is an n-tuple $U = (U_1, \ldots, U_n)$ and the set of all logically possible utility profiles is $\mathcal{U}$. For $x \in X$, we write $U(x)$ for the vector $(U_1(x), \ldots, U_n(x))$.

Time periods are indexed by non-negative integers and individuals may be born in any period after period zero. There is a finite maximal lifetime $\bar{L} \in \mathcal{Z}_{++}$ but this upper bound on the number of periods in which an individual may be alive can be arbitrarily large.

Because our objective is to examine the intertemporal aspects of social evaluation, we focus on birth dates and lengths of life as the non-welfare information that may be of relevance. For each $i \in \{1, \ldots, n\}$, $S_i \colon X \to \mathcal{Z}_+$ assigns the period just before i is born to each alternative. Analogously, $L_i \colon X \to \{1, \ldots, \bar{L}\}$ is a function that specifies i's lifetime for each alternative. Thus, in alternative $x \in X$, i is alive in periods $S_i(x)+1, \ldots, S_i(x)+L_i(x)$. A period-before-birth-date profile is an n-tuple $S = (S_1, \ldots, S_n)$ and the set of all logically possible period-before-birth-date profiles is $\mathcal{S}$. Analogously, a lifetime profile is an n-tuple $L = (L_1, \ldots, L_n)$ and the set of all logically possible lifetime profiles is $\mathcal{L}$. Furthermore, we define $S(x) = (S_1(x), \ldots, S_n(x))$ and $L(x) = (L_1(x), \ldots, L_n(x))$ for all $x \in X$.

We allow birth dates to vary across alternatives, although it is often argued that birth dates are fixed. Without going into too much detail at this stage, we note that there is some variation because the duration of pregnancy is not fixed. A discussion of possible criticisms to our model and our responses are provided in the concluding section.

An information profile (a profile, for short) collects welfare information and non-welfare information in a vector $\Upsilon = (U, S, L) \in \mathcal{U} \times \mathcal{S} \times \mathcal{L}$. For $x \in X$, we write $\Upsilon(x) = (U(x), S(x), L(x))$. We define $\Omega = \mathcal{R} \times \mathcal{Z}_+ \times \{1, \ldots, \bar{L}\}$. Thus, the set of possible compound vectors (u, s, ℓ) of utility vectors, vectors of periods before birth and vectors of lifetimes is $\Omega^n = \mathcal{R}^n \times \mathcal{Z}_+^n \times \{1, \ldots, \bar{L}\}^n$.

A social-evaluation functional assigns a social ordering of the alternatives in X to each information profile in its domain. Our model is a special case of that studied in Blackorby, Bossert and Donaldson [2005a] where non-welfare information is not restricted to birth dates and lengths of life.

Letting $\mathcal{O}$ denote the set of all orderings on X, a social-evaluation functional is a mapping $F\colon \mathcal{D} \to \mathcal{O}$ where $\emptyset \neq \mathcal{D} \subseteq \mathcal{U} \times \mathcal{S} \times \mathcal{L}$ is the domain of F. We write $R_\Upsilon = F(\Upsilon)$ for all $\Upsilon \in \mathcal{D}$. The asymmetric and symmetric factors of R_Υ are P_Υ and I_Υ. Many of the orderings characterized in this paper are not welfarist—they depend on birth dates or lifetimes in addition to lifetime utilities. Nevertheless, the informational basis required for social evaluation can be simplified in the presence of some mild axioms. We introduce the axioms and state the relevant result but we do not provide a proof because it is analogous to that of the corresponding theorem in Blackorby, Bossert and Donaldson [2005a].

Our first axiom is unlimited domain. It requires the social-evaluation functional to produce a social ordering for all logically possible information profiles.

Unlimited domain: $\mathcal{D} = \mathcal{U} \times \mathcal{S} \times \mathcal{L}$.

The next axiom is a conditional version of the well-known Pareto-indifference condition familiar from traditional social-choice theory. Our version is weaker because it requires the conclusion of the axiom only if not only welfare information but also non-welfare information is the same in two alternatives.

Conditional Pareto indifference: For all $x, y \in X$ and for all $\Upsilon \in \mathcal{D}$, if $\Upsilon(x) = \Upsilon(y)$, then $x I_\Upsilon y$.

Binary independence of irrelevant alternatives is defined as usual. If two profiles and two alternatives are such that the profiles agree on the alternatives, then the ranking of the alternatives must be the same in both profiles.

Binary independence of irrelevant alternatives: For all $x, y \in X$ and for all $\Upsilon, \bar{\Upsilon} \in \mathcal{D}$, if $\Upsilon(x) = \bar{\Upsilon}(x)$ and $\Upsilon(y) = \bar{\Upsilon}(y)$, then

$$x R_\Upsilon y \Leftrightarrow x R_{\bar{\Upsilon}} y.$$

Anonymity requires the social-evaluation functional to be independent of the labels of the individuals—everyone in society is treated equally.

Anonymity: For all $\Upsilon, \bar{\Upsilon} \in \mathcal{D}$, if there exists a bijection $\rho\colon \{1, \ldots, n\} \to \{1 \ldots, n\}$ such that $\Upsilon_i = \bar{\Upsilon}_{\rho(i)}$ for all $i \in \{1, \ldots, n\}$, then $R_\Upsilon = R_{\bar{\Upsilon}}$.

Anonymity is easily defended because it allows non-welfare information to matter. All that is ruled out is the claim that an individual's identity justifies special treatment, no matter what non-welfare information obtains.

An ordering R on Ω^n is anonymous if and only if, for all $(u, s, \ell) \in \Omega^n$ and for all bijections $\rho\colon \{1, \ldots, n\} \to \{1, \ldots, n\}$,

$$\big((u_{\rho^n(1)}, \ldots, u_{\rho^n(n)}), (s_{\rho^n(1)}, \ldots, s_{\rho^n(n)}), (\ell_{\rho^n(1)}, \ldots, \ell_{\rho^n(n)})\big) I (u, s, \ell).$$

The four axioms imply that any two alternatives can be ranked by examining the lifetime utilities, the birth dates and the lifetimes obtained in the two alternatives only—no further information is required. Moreover, anonymity implies that the ranking of the compound vectors of lifetime utilities, birth dates and lengths of life is anonymous—it is insensitive with respect to the labels we give to the individuals. Because the proof is analogous to that of the version of the welfarism theorem stated in Blackorby, Bossert and Donaldson [2005a], we omit it.

Theorem 1: *Suppose F satisfies unlimited domain. F satisfies conditional Pareto indifference, binary independence of irrelevant alternatives and anonymity if and only if there exists an anonymous social-evaluation ordering R on Ω^n such that, for all $x, y \in X$ and for all $\Upsilon \in \mathcal{D}$,*

$$xR_\Upsilon y \Leftrightarrow \big(U(x), S(x), L(x)\big) R \big(U(y), S(y), L(y)\big).$$

The asymmetric and symmetric factors of the social-evaluation ordering R are denoted by P and I.

3. A preliminary result

The proof of our main result makes use of a well-known theorem in atemporal social choice. This theorem characterizes the class of generalized-utilitarian orderings by means of some plausible axioms.

Let $\overset{*}{R}$ be an ordering on $\mathcal{R}^n$ with asymmetric factor $\overset{*}{P}$ and symmetric factor $\overset{*}{I}$. The interpretation of $\overset{*}{R}$ is that of an atemporal social-evaluation ordering—it is a special case of R that depends on utility vectors only. $\overset{*}{R}$ is anonymous if and only if, for all $u \in \mathcal{R}^n$ and for all bijections $\rho\colon \{1, \ldots, n\} \to \{1, \ldots, n\}$, $(u_{\rho(1)}, \ldots, u_{\rho(n)}) \overset{*}{I} u$. $\overset{*}{R}$ is a generalized-utilitarian ordering if and only if there exists a continuous and increasing function $g\colon \mathcal{R} \to \mathcal{R}$ such that, for all $u, v \in \mathcal{R}^n$,

$$u \overset{*}{R} v \Leftrightarrow \sum_{i=1}^{n} g(u_i) \geq \sum_{i=1}^{n} g(v_i).$$

Our first axiom is continuity. As usual, it requires that small changes in utilities do not lead to large changes in the social ranking.

Continuity: For all $u \in \mathcal{R}^n$, the sets $\{v \in \mathcal{R}^n \mid v\overset{*}{R}u\}$ and $\{v \in \mathcal{R}^n \mid u\overset{*}{R}v\}$ are closed in $\mathcal{R}^n$.

Strong Pareto requires unanimity to be respected. If everyone's well-being in a utility vector u is greater than or equal to that person's well-being in v with at least one strict inequality, u is better than v according to $\overset{*}{R}$.

Strong Pareto: For all $u, v \in \mathcal{R}^n$, if $u > v$, then $u\overset{*}{P}v$.

The final axiom of this section is a separability property. It requires $\overset{*}{R}$ to be independent of the utilities of those who are unconcerned. Suppose that a proposed social change affects only the utilities of the members of a population subgroup. Independence of the utilities of the unconcerned requires the social assessment of the change to be independent of the utility levels of people who are not in the subgroup.

Independence of the utilities of the unconcerned: For all $m \in \{1, \ldots, n-1\}$, for all $u, v \in \mathcal{R}^m$ and for all $\bar{u}, \bar{v} \in \mathcal{R}^{n-m}$,

$$(u, \bar{u})\overset{*}{R}(v, \bar{u}) \Leftrightarrow (u, \bar{v})\overset{*}{R}(v, \bar{v}).$$

In this definition, the individuals with utility vectors $\bar{u}$ or $\bar{v}$ are the unconcerned—they are equally well off in $(u, \bar{u})$ and $(v, \bar{u})$ and in $(u, \bar{v})$ and $(v, \bar{v})$. The axiom requires the ranking of pairs such as $(u, \bar{u})$ and $(v, \bar{u})$ to depend on the utilities of the concerned individuals only. If formulated in terms of a real-valued representation, this axiom is referred to as complete strict separability in Blackorby, Primont and Russell [1978]. A corresponding separability axiom for social-evaluation functionals that depend on welfare information only can be found in d'Aspremont and Gevers [1977] where it is called separability with respect to unconcerned individuals. d'Aspremont and Gevers' separability axiom is called elimination of (the influence of) indifferent individuals in Maskin [1978] and Roberts [1980].

In the case of two individuals, the axiom is redundant because it is implied by strong Pareto but, if there are at least three individuals (an assumption we maintain throughout), it has remarkably strong consequences. When combined with continuity and strong Pareto, it characterizes generalized utilitarianism if $\overset{*}{R}$ is assumed to be anonymous. The proof of the following theorem, which employs Debreu's [1959, pp. 56–59] representation theorem and Gorman's [1968] theorem on overlapping separable sets of variables (see also Aczél [1966, p. 312] and Blackorby, Primont, and Russell, [1978, p. 127]), is in Blackorby, Bossert and Donaldson [2002], for instance. Variants of the theorem can be found in Debreu [1960] and Fleming [1952].

Theorem 2: *An anonymous ordering $\overset{*}{R}$ satisfies continuity, strong Pareto and independence of the utilities of the unconcerned if and only if $\overset{*}{R}$ is a generalized-utilitarian ordering.*

4. Intertemporal axioms and orderings

The remaining axioms employed in this paper are formulated for the ordering R. Given the axioms of Theorem 1, this involves no loss of generality. We introduce an intertemporal version of continuity, conditional formulations of the strong-Pareto principle and three variants of an axiom which ensures that the birth date of an individual can be changed to a specific birth date (common for all individuals) without changing the social ranking, provided the individual's lifetime utility is suitably adjusted. These conditions ensure that non-trivial trade-offs are possible.

Continuity can be formulated in an intertemporal model in a way that is analogous to its atemporal version. We require the social ranking to be continuous in lifetime utilities for any fixed pair of birth-date vectors and lifetime vectors.

Intertemporal continuity: For all $(u, s, \ell) \in \Omega^n$, the sets $\{v \in \mathcal{R}^n \mid (v, s, \ell)R(u, s, \ell)\}$ and $\{v \in \mathcal{R}^n \mid (u, s, \ell)R(v, s, \ell)\}$ are closed in $\mathcal{R}^n$.

The intertemporal version of the strong-Pareto principle has two parts. First, if each individual has the same lifetime utilities in two alternatives, they are ranked as equally good by the social ordering. Second, if everyone's utility is greater than or equal in one alternative than in another with at least one strict inequality, the former is better than the latter.

Intertemporal strong Pareto: For all $(u, s, \ell), (v, r, k) \in \Omega^n$,
(i) if $u = v$, then $(u, s, \ell)I(v, r, k)$;
(ii) if $u > v$, then $(u, s, \ell)P(v, r, k)$.

Intertemporal strong Pareto rules out the influence of non-utility information. To allow such information to matter, we introduce the following conditional versions of the axiom. The first of these applies the principle conditionally on birth dates and lifetimes, the second on birth dates only and the final one on lifetimes only.

Conditional strong Pareto: For all $(u, s, \ell), (v, r, k) \in \Omega^n$,
(i) if $s = r$, $\ell = k$ and $u = v$, then $(u, s, \ell)I(v, r, k)$;
(ii) if $s = r$, $\ell = k$ and $u > v$, then $(u, s, \ell)P(v, r, k)$.

Birth-date conditional strong Pareto: For all $(u,s,\ell),(v,r,k) \in \Omega^n$,

(i) if $s = r$ and $u = v$, then $(u,s,\ell)I(v,r,k)$;

(ii) if $s = r$ and $u > v$, then $(u,s,\ell)P(v,r,k)$.

Lifetime conditional strong Pareto: For all $(u,s,\ell),(v,r,k) \in \Omega^n$,

(i) if $\ell = k$ and $u = v$, then $(u,s,\ell)I(v,r,k)$;

(ii) if $\ell = k$ and $u > v$, then $(u,s,\ell)P(v,r,k)$.

Part (i) of conditional strong Pareto is redundant because R is reflexive. It is included in order to have the same structure as in the other strong-Pareto axioms.

The axiom individual intertemporal equivalence and its conditional counterparts ensure that, by suitably changing an individual's lifetime utility, the birth date of the person can be moved to a prespecified period without changing the social ranking. These conditions guarantee the possibility of non-degenerate trade-offs between birth dates or lifetimes and utilities. We require more notation to proceed. Let $i \in \{1,\ldots,n\}$, $(u,s,\ell) \in \Omega^n$ and $(u'_i,s'_i,\ell'_i) \in \Omega$. The vectors $v = (u_{-i},u'_i) \in \mathcal{R}^n$, $r = (s_{-i},s'_i) \in \mathcal{Z}^n_+$ and $k = (\ell_{-i},\ell'_i) \in \{1,\ldots,\bar{L}\}^n$ are defined by

$$v_j = \begin{cases} u_j & \text{if } j \in \{1,\ldots,n\} \setminus \{i\}; \\ u'_j & \text{if } j = i, \end{cases}$$

$$r_j = \begin{cases} s_j & \text{if } j \in \{1,\ldots,n\} \setminus \{i\}; \\ s'_j & \text{if } j = i \end{cases}$$

and

$$k_j = \begin{cases} \ell_j & \text{if } j \in \{1,\ldots,n\} \setminus \{i\}; \\ \ell'_j & \text{if } j = i. \end{cases}$$

Individual intertemporal equivalence: There exists $\lambda_0 \in \{1,\ldots,\bar{L}\}$ such that, for all $(d,\sigma,\lambda) \in \Omega$ and for all $\sigma_0 \in \mathcal{Z}_+$, there exists $\hat{d} \in \mathcal{R}$ such that, for all $(u,s,\ell) \in \Omega^n$ and for all $i \in \{1,\ldots,n\}$,

$$\Big((u_{-i},\hat{d}),(s_{-i},\sigma_0),(\ell_{-i},\lambda_0)\Big)\, I\big((u_{-i},d),(s_{-i},\sigma),(\ell_{-i},\lambda)\big).$$

Birth-date conditional individual intertemporal equivalence: There exists $\lambda_0 \in \{1,\ldots,\bar{L}\}$ such that, for all $(d,\sigma) \in \mathcal{R} \times \mathcal{Z}_+$ and for all $\sigma_0 \in \mathcal{Z}_+$, there exists $\hat{d} \in \mathcal{R}$ such that, for all $(u,s) \in \mathcal{R}^n \times \mathcal{Z}^n_+$ and for all $i \in \{1,\ldots,n\}$,

$$\Big((u_{-i},\hat{d}),(s_{-i},\sigma_0),\lambda_0\mathbf{1}_n\Big)\, I\big((u_{-i},d),(s_{-i},\sigma),\lambda_0\mathbf{1}_n\big).$$

Lifetime conditional individual intertemporal equivalence: There exist $\sigma_0 \in \mathcal{Z}_+$ and $\lambda_0 \in \{1, \ldots, \bar{L}\}$ such that, for all $(d, \lambda) \in \mathcal{R} \times \{1, \ldots, \bar{L}\}$, there exists $\hat{d} \in \mathcal{R}$ such that, for all $(u, \ell) \in \mathcal{R}^n \times \{1, \ldots, \bar{L}\}^n$ and for all $i \in \{1, \ldots, n\}$,

$$\Big((u_{-i}, \hat{d}), \sigma_0 \mathbf{1}_n, (\ell_{-i}, \lambda_0)\Big)\, I \big((u_{-i}, d), \sigma_0 \mathbf{1}_n, (\ell_{-i}, \lambda)\big).$$

The main result of this paper provides characterizations of generalized utilitarianism and related orderings in our intertemporal setting. In each of the definitions of the first three classes of orderings, a condition regarding the possibility of equalizing the values of the requisite transformation for different birth dates or lifetimes is imposed. This condition is required in order to ensure that the relevant individual intertemporal equivalence property is satisfied.

R is a birth-date and lifetime dependent generalized-utilitarian ordering if and only if there exist a function $h\colon \Omega \to \mathcal{R}$, continuous and increasing in its first argument, and $\lambda_0 \in \{1, \ldots, \bar{L}\}$ such that $h(\mathcal{R}, \sigma_0, \lambda_0) \cap h(\mathcal{R}, \sigma, \lambda) \neq \emptyset$ for all $\sigma_0, \sigma \in \mathcal{Z}_+$ and for all $\lambda \in \{1, \ldots, \bar{L}\}$ and, for all $(u, s, \ell), (v, r, k) \in \Omega^n$,

$$(u, s, \ell) R (v, r, k) \Leftrightarrow \sum_{i=1}^{n} h(u_i, s_i, \ell_i) \geq \sum_{i=1}^{n} h(v_i, r_i, k_i).$$

Analogously, R is a birth-date dependent generalized-utilitarian ordering if and only if there exists a function $f\colon \mathcal{R} \times \mathcal{Z}_+ \to \mathcal{R}$, continuous and increasing in its first argument, such that $f(\mathcal{R}, \sigma_0) \cap f(\mathcal{R}, \sigma) \neq \emptyset$ for all $\sigma_0, \sigma \in \mathcal{Z}_+$ and, for all $(u, s, \ell), (v, r, k) \in \Omega^n$,

$$(u, s, \ell) R (v, r, k) \Leftrightarrow \sum_{i=1}^{n} f(u_i, s_i) \geq \sum_{i=1}^{n} f(v_i, r_i). \tag{1}$$

R is a lifetime dependent generalized-utilitarian ordering if and only if there exist a function $e\colon \mathcal{R} \times \{1, \ldots, \bar{L}\} \to \mathcal{R}$, continuous and increasing in its first argument, and $\lambda_0 \in \{1, \ldots, \bar{L}\}$ such that $e(\mathcal{R}, \lambda_0) \cap e(\mathcal{R}, \lambda) \neq \emptyset$ for all $\lambda \in \{1, \ldots, \bar{L}\}$ and, for all $(u, s, \ell), (v, r, k) \in \Omega^n$,

$$(u, s, \ell) R (v, r, k) \Leftrightarrow \sum_{i=1}^{n} e(u_i, \ell_i) \geq \sum_{i=1}^{n} e(v_i, k_i).$$

Finally, R is an intertemporal generalized-utilitarian ordering if and only if there exists a continuous and increasing function $g\colon \mathcal{R} \to \mathcal{R}$ such that, for all $(u, s, \ell), (v, r, k) \in \Omega^n$,

$$(u, s, \ell) R (v, r, k) \Leftrightarrow \sum_{i=1}^{n} g(u_i) \geq \sum_{i=1}^{n} g(v_i).$$

5. Intertemporal independence

When choices are made in a period $t \in \mathcal{Z}_{++}$, all feasible alternatives have a common history but the lifetime utilities, birth dates and lifetimes of some members of society may not be fixed. If person i is alive in period $t-1$, there may be alternatives in which i's life extends to period t and possibly beyond, whereas in other alternatives, i dies at the end of period $t-1$. This suggests that history must matter to some extent if lifetime utilities are to be taken into consideration. On the other hand, some independence property is desirable because decisions should not depend on the utilities of individuals who are long dead, for example. If, in any period, an individual's life is over in two alternatives and he or she had the same lifetime utility, birth date and lifetime in both, a plausible requirement is that the ranking of the two alternatives does not depend on the utility level of that individual. In our setting, this leads to an independence condition whose scope is limited: it applies only if the sets of those whose lives are over are identical in two alternatives and, moreover, everyone in this set had the same lifetime utility, the same birth date and the same lifetime in both.

Independence of the utilities of the dead: For all $m \in \{1, \ldots, n-1\}$, for all $(u, s, \ell), (v, r, k) \in \Omega^m$, for all $(\bar{u}, \bar{s}, \bar{\ell}), (\bar{v}, \bar{r}, \bar{k}) \in \Omega^{n-m}$ and for all $t \in \mathcal{Z}_{++}$, if $\bar{s}_i + \bar{\ell}_i < t$ and $\bar{r}_i + \bar{k}_i < t$ for all $i \in \{1, \ldots, n-m\}$ and $s_i + 1 \geq t$ and $r_i + 1 \geq t$ for all $i \in \{1, \ldots, m\}$, then

$$\big((u, \bar{u}), (s, \bar{s}), (\ell, \bar{\ell})\big) \, R \, \big((v, \bar{u}), (r, \bar{s}), (k, \bar{\ell})\big) \Leftrightarrow \big((u, \bar{v}), (s, \bar{r}), (\ell, \bar{k})\big) \, R \, \big((v, \bar{v}), (r, \bar{r}), (k, \bar{k})\big) \, .$$

Independence of the utilities of the dead is a weak separability condition because it applies to individuals whose lives are over before period t only and not to all unconcerned individuals. Thus, if all generations overlap, it does not impose any restrictions. However, when combined with intertemporal strong Pareto or one of the conditional versions thereof, the axiom becomes more powerful. We now provide characterizations of our intertemporal variants of generalized utilitarianism by combining independence of the utilities of the dead with the various intertemporal versions of strong Pareto and of the intertemporal-equivalence axioms.

Theorem 3:

(i) An anonymous ordering R satisfies intertemporal continuity, conditional strong Pareto, individual intertemporal equivalence and independence of the utilities of the dead if and only if R is birth-date and lifetime dependent generalized-utilitarian.

(ii) An anonymous ordering R satisfies intertemporal continuity, birth-date conditional strong Pareto, birth-date conditional individual intertemporal equivalence and independence of the utilities of the dead if and only if R is birth-date dependent generalized-utilitarian.

(iii) An anonymous ordering R satisfies intertemporal continuity, lifetime conditional strong Pareto, lifetime conditional individual intertemporal equivalence and independence of the utilities of the dead if and only if R is lifetime dependent generalized-utilitarian.

(iv) An anonymous ordering R satisfies intertemporal continuity, intertemporal strong Pareto and independence of the utilities of the dead if and only if R is intertemporal generalized-utilitarian.

Proof. We provide a detailed proof of Part (i). That the birth-date and lifetime dependent generalized-utilitarian orderings satisfy intertemporal continuity, conditional strong Pareto and independence of the utilities of the dead is straightforward to verify. The non-emptiness of $h(\mathcal{R}, \sigma_0, \lambda_0) \cap h(\mathcal{R}, \sigma, \lambda)$ for all $\sigma_0, \sigma \in \mathcal{Z}_+$ and for all $\lambda \in \{1, \ldots, \bar{L}\}$ in the definition of the orderings guarantees that individual intertemporal equivalence is satisfied.

Now suppose R is an anonymous ordering satisfying the axioms of Part (i) of the theorem statement. The proof that R is birth-date and lifetime dependent generalized-utilitarian proceeds as follows. We define an ordering $\overset{*}{R}$ on $\mathcal{R}^n$ (that is, an ordering of utility vectors) as the restriction of R that is obtained by fixing birth dates and lengths of life at specific values. We then show that $\overset{*}{R}$ satisfies the axioms of Theorem 2 and, thus, must be generalized-utilitarian. Finally, we show that all comparisons according to R can be carried out by applying $\overset{*}{R}$ to utilities that depend on birth dates and lifetimes, resulting in birth-date and lifetime dependent generalized utilitarianism.

Let λ_0 be as in the definition of individual intertemporal equivalence. Define the ordering $\overset{*}{R}$ on $\mathcal{R}^n$ by letting, for all $u, v \in \mathcal{R}^n$,

$$u \overset{*}{R} v \Leftrightarrow (u, 0\mathbf{1}_n, \lambda_0 \mathbf{1}_n) \, R \, (v, 0\mathbf{1}_n, \lambda_0 \mathbf{1}_n) \, .$$

Clearly, $\overset{*}{R}$ is an anonymous ordering satisfying continuity and strong Pareto. The last remaining property of $\overset{*}{R}$ to be established is independence of the utilities of the unconcerned. Let $m \in \{1, \ldots, n-1\}$, $u, v \in \mathcal{R}^m$ and $\bar{u}, \bar{v} \in \mathcal{R}^{n-m}$. By repeated application of individual intertemporal equivalence, there exist $\hat{u}, \hat{v} \in \mathcal{R}^m$ such that

$$\big((\hat{u}, \bar{u}), (\bar{L}\mathbf{1}_m, 0\mathbf{1}_{n-m}), \lambda_0 \mathbf{1}_n\big) \, I \, ((u, \bar{u}), 0\mathbf{1}_n, \lambda_0 \mathbf{1}_n) \, , \tag{2}$$

$$\big((\hat{v}, \bar{u}), (\bar{L}\mathbf{1}_m, 0\mathbf{1}_{n-m}), \lambda_0 \mathbf{1}_n\big) \, I \, ((v, \bar{u}), 0\mathbf{1}_n, \lambda_0 \mathbf{1}_n) \, , \tag{3}$$

$$\big((\hat{u}, \bar{v}), (\bar{L}\mathbf{1}_m, 0\mathbf{1}_{n-m}), \lambda_0 \mathbf{1}_n\big) \, I \, ((u, \bar{v}), 0\mathbf{1}_n, \lambda_0 \mathbf{1}_n) \tag{4}$$

and

$$\left((\hat{v},\bar{v}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right) I\left((v,\bar{v}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n\right). \tag{5}$$

(2) and (3) together imply

$$\begin{aligned}&((u,\bar{u}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n)\, R\,((v,\bar{u}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n)\\ \Leftrightarrow\, &\left((\hat{u},\bar{u}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right) R\left((\hat{v},\bar{u}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right).\end{aligned} \tag{6}$$

By independence of the utilities of the dead,

$$\begin{aligned}&\left((\hat{u},\bar{u}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right) R\left((\hat{v},\bar{u}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right)\\ \Leftrightarrow\, &\left((\hat{u},\bar{v}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right) R\left((\hat{v},\bar{v}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right).\end{aligned} \tag{7}$$

(4) and (5) together imply

$$\begin{aligned}&\left((\hat{u},\bar{v}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right) R\left((\hat{v},\bar{v}),(\bar{L}\mathbf{1}_m,0\mathbf{1}_{n-m}),\lambda_0\mathbf{1}_n\right)\\ \Leftrightarrow\, &((u,\bar{v}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n)\, R\,((v,\bar{v}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n).\end{aligned} \tag{8}$$

Combining (6), (7) and (8), we obtain

$$((u,\bar{u}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n)\, R\,((v,\bar{u}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n) \Leftrightarrow ((u,\bar{v}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n)\, R\,((v,\bar{v}),0\mathbf{1}_n,\lambda_0\mathbf{1}_n).$$

By definition of $\overset{*}{R}$, this is equivalent to

$$(u,\bar{u})\overset{*}{R}(v,\bar{u}) \Leftrightarrow (u,\bar{v})\overset{*}{R}(v,\bar{v})$$

which establishes that independence of the utilities of the unconcerned is satisfied.

By Theorem 2, $\overset{*}{R}$ is generalized-utilitarian and, thus, there exists a continuous and increasing function $g\colon \mathcal{R}\to\mathcal{R}$ such that

$$u\overset{*}{R}v \Leftrightarrow \sum_{i=1}^{n} g(u_i) \geq \sum_{i=1}^{m} g(v_i)$$

for all $u,v\in\mathcal{R}^n$. Define the function $\bar{h}\colon\Omega\to\mathcal{R}$ by

$$\bar{h}(d,\sigma,\lambda)=\gamma \Leftrightarrow (d,\sigma,\lambda)I(\gamma,0,\lambda_0)$$

for all $(d,\sigma,\lambda)\in\Omega$ and for all $\gamma\in\mathcal{R}$. This function is well-defined because R satisfies individual intertemporal equivalence.

Consider any $(u,s,\ell),(v,r,k)\in\Omega^n$. By repeated application of individual intertemporal equivalence,

$$\left(\left(\bar{h}(u_i,s_i,\ell_i)\right)_{i=1}^n, 0\mathbf{1}_n,\lambda_0\mathbf{1}_n\right) I(u,s,l)$$

and

$$\left(\left(\bar{h}(v_i,r_i,k_i)\right)_{i=1}^n, 0\mathbf{1}_n,\lambda_0\mathbf{1}_n\right) I(v,r,k).$$

Therefore,

$$\begin{aligned}(u,s,\ell)R(v,r,k) &\Leftrightarrow \left(\left(\bar{h}(u_i,s_i,\ell_i)\right)_{i=1}^{n}, 0\mathbf{1}_n, \lambda_0\mathbf{1}_n\right) R \left(\left(\bar{h}(v_i,r_i,k_i)\right)_{i=1}^{n}, 0\mathbf{1}_n, \lambda_0\mathbf{1}_n\right)\\ &\Leftrightarrow \left(\bar{h}(u_i,s_i,\ell_i)\right)_{i=1}^{n} \overset{*}{R} \left(\bar{h}(v_i,r_i,k_i)\right)_{i=1}^{n}\\ &\Leftrightarrow \sum_{i=1}^{n} g\left(\bar{h}(u_i,s_i,\ell_i)\right) \geq \sum_{i=1}^{n} g\left(\bar{h}(v_i,r_i,k_i)\right).\end{aligned}$$

Letting $h = g \circ \bar{h}$ (where $\circ$ denotes function composition), it follows that

$$(u,s,\ell)R(v,r,k) \Leftrightarrow \sum_{i=1}^{n} h(u_i,s_i,\ell_i) \geq \sum_{i=1}^{n} h(v_i,r_i,k_i).$$

That h satisfies $h(\mathcal{R},\sigma_0,\lambda_0) \cap h(\mathcal{R},\sigma,\lambda) \neq \emptyset$ for all $\sigma_0,\sigma \in \mathcal{Z}_+$ and for all $\lambda \in \{1,\ldots,\bar{L}\}$ follows from the definitions of $\bar{h}$ and h. This completes the proof of Part (i).

The proofs of Parts (ii) through (iv) of the theorem are analogous. Because R is independent of lifetimes in Part (ii) and independent of birth dates in Part (iii), the corresponding weakenings of individual intertemporal equivalence are sufficient for the characterization results. In Part (iv), the equivalence axioms can be dispensed with altogether because, by intertemporal strong Pareto, the ordering cannot depend on either birth dates or lifetimes. ■

6. Geometric discounting

In many intertemporal models, geometric discounting is employed. According to geometric discounting, the transformed lifetime utility of each person $i \in \{1,\ldots,n\}$ is multiplied by δ^{s_i}, where $\delta \in \mathcal{R}_{++}$ is a fixed discount factor. Clearly, higher values of δ are associated with higher relative weights given to future generations. A value of $\delta = 1$ represents the no-discounting case and $\delta > 1$ corresponds to 'upcounting'—putting higher weights on later generations than on earlier generations. The most common case occurs when $\delta < 1$—the later someone is born, the lower the weight attached to this person's lifetime utility.

The ordering R is geometric birth-date dependent generalized-utilitarian if and only if there exist a continuous and increasing function $g\colon \mathcal{R} \to \mathcal{R}$ and $\delta \in \mathcal{R}_{++}$ such that, for all $(u,s,\ell),(v,k,r) \in \Omega^n$,

$$(u,s,\ell)R(v,r,k) \Leftrightarrow \sum_{i=1}^{n} \delta^{s_i} g(u_i) \geq \sum_{i=1}^{n} \delta^{r_i} g(v_i).$$

We do not endorse these orderings (or any other birth-date dependent orderings) because we believe that intertemporal strong Pareto is a compelling axiom and, thus, only lifetime-utility information should matter. However, because of the important status geometric discounting enjoys in intertemporal economic models, we provide a characterization of these orderings to illustrate their properties and the ethical judgments underlying their use. A single axiom is required in addition to the properties employed in Part (ii) of Theorem 3. Stationarity requires that the ranking of any two elements of Ω^n is unchanged if, ceteris paribus, the birth date of everyone is moved into the future by any number of periods in both. This is one of the most commonly used restrictions on multi-period social-evaluation orderings.

Stationarity: For all $(u, s, \ell), (v, k, r) \in \Omega^n$ and for all $\tau \in \mathcal{Z}_{++}$,

$$(u, s + \tau \mathbf{1}_n, \ell) R (v, r + \tau \mathbf{1}_n, k) \Leftrightarrow (u, s, \ell) R (v, r, k).$$

We now obtain a characterization of geometric birth-date dependent generalized utilitarianism by adding stationarity to the axioms characterizing birth-date dependent generalized utilitarianism.

Theorem 4: *An anonymous ordering R satisfies intertemporal continuity, birth-date conditional strong Pareto, birth-date conditional individual intertemporal equivalence, independence of the existence of the dead and stationarity if and only if R is geometric birth-date dependent generalized-utilitarian.*

Proof. That geometric birth-date dependent generalized utilitarianism satisfies the axioms of the theorem statement is straightforward to verify.

Conversely, suppose R satisfies the required axioms. By Part (ii) of Theorem 3, R is birth-date dependent generalized-utilitarian. Stationarity implies that, for all $(u, \ell), (v, k) \in \mathcal{R}^n \times \{1, \ldots, \bar{L}\}^n$ and for all $\sigma, \tau \in \mathcal{Z}_{++}$,

$$(u, (\sigma + \tau)\mathbf{1}_n, \ell) R (v, (\sigma + \tau)\mathbf{1}_n, k) \Leftrightarrow (u, \sigma \mathbf{1}_n, \ell) R (v, \sigma \mathbf{1}_n, k).$$

Letting f be as in the definition of birth-date dependent generalized utilitarianism, this is equivalent to

$$\sum_{i=1}^{n} f(u_i, \sigma + \tau) \geq \sum_{i=1}^{n} f(v_i, \sigma + \tau) \Leftrightarrow \sum_{i=1}^{n} f(u_i, \sigma) \geq \sum_{i=1}^{n} f(v_i, \sigma).$$

Thus, for each $\tau \in \mathcal{Z}_+$, there exists an increasing function $\varphi_\tau \colon \mathcal{R} \to \mathcal{R}$ such that

$$\sum_{i=1}^{n} f(u_i, \sigma + \tau) = \varphi_\tau \left(\sum_{i=1}^{n} f(u_i, \sigma) \right)$$

for all $u \in \mathcal{R}^n$ and for all $\sigma, \tau \in \mathcal{Z}_+$. For each $\sigma \in \mathcal{Z}_+$, define the function $\bar{g}_\sigma: f(\mathcal{R}, \sigma) \to \mathcal{R}$ by

$$\bar{g}_\sigma(\gamma) = z \Leftrightarrow f(z, \sigma) = \gamma$$

for all $\gamma \in f(\mathcal{R}, \sigma)$ and for all $z \in \mathcal{R}$, that is, $\bar{g}_\sigma$ is the inverse of f with respect to its first argument for the fixed value σ of its second argument. Now let $x_i = f(u_i, \sigma)$ for all $i \in \{1, \ldots, n\}$ and $\bar{f}(z, \sigma + \tau) = f(\bar{g}_\sigma(z), \sigma + \tau)$ to obtain the functional equation

$$\sum_{i=1}^{n} \bar{f}(x_i, \sigma + \tau) = \varphi_\tau \left(\sum_{i=1}^{n} x_i \right).$$

This is a Pexider equation in the variables $x_1, \ldots, x_n$ the solution of which satisfies (see Aczél [1966, p. 142])

$$\bar{f}(z, \sigma + \tau) = a(\tau) z + b(\tau)$$

for all $z \in \mathcal{R}$ with functions $a: \mathcal{Z}_+ \to \mathcal{R}$ and $b: \mathcal{Z}_+ \to \mathcal{R}$ which do not depend on σ because φ_τ does not. Substituting back into the definition of $\bar{f}$, we obtain

$$f(z, \sigma + \tau) = a(\tau) f(d, \sigma) + b(\tau) \tag{9}$$

for all $z \in \mathcal{R}$ and for all $\sigma, \tau \in \mathcal{Z}_+$. Setting $\sigma = 0$, it follows that

$$f(z, \tau) = a(\tau) f(z, 0) + b(\tau) \tag{10}$$

for all $z \in \mathcal{R}$ and for all $\tau \in \mathcal{Z}_+$. Therefore,

$$f(z, \sigma + \tau) = a(\sigma + \tau) f(z, 0) + b(\sigma + \tau) \tag{11}$$

for all $z \in \mathcal{R}$ and for all $\sigma, \tau \in \mathcal{Z}_+$. Substituting (10) into (9), we obtain

$$f(z, \sigma + \tau) = a(\tau)[a(\sigma) f(z, 0) + b(\sigma)] + b(\tau) \tag{12}$$

for all $z \in \mathcal{R}$ and for all $\sigma, \tau \in \mathcal{Z}_+$. Combining (11) and (12), it follows that

$$a(\sigma + \tau) f(z, 0) + b(\sigma + \tau) = a(\tau)[a(\sigma) f(z, 0) + b(\sigma)] + b(\tau)$$

or, equivalently,

$$[a(\sigma + \tau) - a(\tau) a(\sigma)] f(z, 0) = a(\tau) b(\sigma) + b(\tau) - b(\sigma + \tau).$$

Because f is increasing in its first argument and the right side of this equation is independent of z, it must be the case that both sides are identically zero which requires

$$a(\sigma + \tau) = a(\tau) a(\sigma) \tag{13}$$

and

$$a(\tau) b(\sigma) + b(\tau) = b(\sigma + \tau) \tag{14}$$

for all $\sigma, \tau \in \mathcal{Z}_+$. Setting $\sigma = 1$ and $\tau = 0$ in (13), it follows that $a(0) = 1$. Thus, defining $\delta = a(1)$, repeated application of (13) implies

$$a(\sigma) = \delta^\sigma \tag{15}$$

for all $\sigma \in \mathcal{Z}_+$. Using (15), (14) implies

$$b(\sigma + \tau) = \delta^\tau b(\sigma) + b(\tau)$$

for all $\sigma, \tau \in \mathcal{Z}_+$. Interchanging the roles of σ and τ in this equation, it follows that

$$b(\sigma + \tau) = \delta^\sigma b(\tau) + b(\sigma)$$

and, therefore, we must have

$$\delta^\tau b(\sigma) + b(\tau) = \delta^\sigma b(\tau) + b(\sigma)$$

for all $\sigma, \tau \in \mathcal{Z}_+$. Setting $\tau = 1$, this implies

$$b(\sigma) = \frac{b(1)}{1-\delta}(1 - \delta^\sigma)$$

for all $\sigma \in \mathcal{Z}_+$. By (10),

$$f(z, \sigma) = a(\sigma) f(z, 0) + b(\sigma) = \delta^\sigma f(z, 0) + \frac{b(1)}{1-\delta}(1 - \delta^\sigma) = \delta^\sigma \left(f(z, 0) - \frac{b(1)}{1-\delta} \right) + \frac{b(1)}{1-\delta}$$

for all $z \in \mathcal{R}$ and for all $\sigma \in \mathcal{Z}_+$. Defining

$$g(z) = f(z, 0) - \frac{b(1)}{1-\delta}$$

for all $z \in \mathcal{R}$, it follows that

$$f(z, \sigma) = \delta^\sigma g(z) + \frac{b(1)}{1-\delta}$$

for all $z \in \mathcal{R}$ and for all $\sigma \in \mathcal{Z}_+$. Substituting into (1), we obtain

$$\begin{aligned}(u, s, \ell) R (v, r, k) &\Leftrightarrow \sum_{i=1}^n \left[\delta^{s_i} g(u_i) + \frac{b(1)}{1-\delta} \right] \geq \sum_{i=1}^n \left[\delta^{r_i} g(v_i) + \frac{b(1)}{1-\delta} \right] \\ &\Leftrightarrow \sum_{i=1}^n \delta^{s_i} g(u_i) \geq \sum_{i=1}^n \delta^{r_i} g(v_i)\end{aligned}$$

for all $(u, s, \ell), (v, k, r) \in \Omega^n$. By birth-date conditional strong Pareto, δ must be positive.
■

7. Concluding remarks

A possible objection to the way we model non-welfare information (in particular, information on birth dates) is the claim that an individual's birth date is fixed and, thus, that our domain which allows us to assign any birth date to an individual is too large. While it is true that a person cannot be born at a completely arbitrary time, his or her birth date may vary over several months because the duration of pregnancy is not fixed. Given the axioms employed in this paper, this possibility is sufficient for our results.

There is another possible criticism, namely, that any change in birth date—even if it is only a matter of a single period—does not allow us to treat the individual born in a period as the same individual as a person born in a later period instead. This position articulates the view that a person's birth date is a characteristic of that person and cannot be changed without changing the person. In a variable-population setting, there is an alternative to the approach that we have chosen that can accommodate this criticism. If each individual is assumed to have a fixed birth date, our axioms intertemporal strong Pareto and anonymity can be replaced with a single axiom that extends the Pareto condition so that it applies anonymously to alternatives with the same population size. If two alternatives have the same population size and the list of utilities in the first is a permutation of the list of utilities in the second, anonymous intertemporal strong Pareto implies that the two alternatives are ranked as equally good. This move is analogous to Suppes's [1966] grading principle. Similar combined axioms correspond to the other Pareto axioms. We think that the combined axioms have strong ethical appeal and, as a consequence, can serve as a convincing defense against the objection.

An argument that is sometimes made in favor of discounting is that, if there is an infinite number of periods and no discounting is employed, the resulting social-evaluation criterion may not be well-defined because it may not yield a finite value for some alternatives. Because the lifetime of the universe is known to be finite, this argument is based on a factual impossibility. But there is another argument that might appear to be more attractive. Very large sacrifices by those presently alive may be justified by larger gains to people who will exist in the distant future only. If these sacrifices are considered too demanding, discounting might be proposed to alleviate the negative effects on the generations that live earlier. However, this argument rests on the false claim that discounting necessarily increases the well-being of the present generation. To see that the claim is not true, consider a three-person society and suppose two alternatives x and y are such that person i is born in period i for all $i \in \{1, 2, 3\}$. In x, utility levels are $u_1 = 28$, $u_2 = 4$ and $u_3 = 44$ and, in y, lifetime utilities are $u_1 = u_2 = u_3 = 24$. If intertemporal generalized utilitarianism with the identity mapping as the transformation is used to evaluate the alternatives, x is better than y and the utility level of person 1, who represents the present generation, is 28. Alternatively suppose that geometric birth-date dependent generalized utilitarianism with the identity mapping and a discount factor of $\delta = 1/2$ is used instead. In that case, the sums of discounted utilities are $28+2+11 = 41$ for x and $24+12+6 = 42$ for y, so y is better and person 1's utility is 24. As a result of discounting, the present generation is worse off.

Our view is that, for the purpose of social evaluation, the well-being of future generations should not be discounted. If maximization of the ethically appropriate objective function requires the present generation to sacrifice most of its consumption for the benefit of others, then such an action can be considered supererogatory: desirable but beyond the call of duty. If these sacrifices are considered to be too demanding, we do not think it is a suitable response to give future generations a smaller weight in the social ordering. Instead, a sufficiently high level of well-being for the present generation can be guaranteed by imposing a floor on their utility as an additional constraint in the choice problem. This is a more natural and ethically attractive way of dealing with problems arising from supererogation than replacing an ethically appropriate social ordering with one that fails to treat generations impartially. See also Blackorby, Bossert and Donaldson [2000] for a details. Cowen [1992] and Cowen and Parfit [1992] present a Paretian argument against discounting, and further discussions can be found in Broome [1992, pp. 92–108, 2004, pp. 126–128].

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