Département de sciences économiques

2005-01

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ISSN 0709-9231

FIRST-DEGREE DISCRIMINATION IN A COMPETITIVE SETTING:

PRICING AND QUALITY CHOICE

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December 2004

Abstract

The paper investigates competition in price schedules among vertically differentiated dupolists. First order price discrimination is the unique Nash equilibrium of a sequential game in which firms determine first whether or not to commit to a uniform price, and then simultaneously choose either a single price of a price schedule. Whether the profits earned by both firms are larger or smaller under discrimination than under uniform pricing depends on the quality gap between firms, and on the disparity of consumer preferences. Firms engaged in first degree discrimination choose quality levels that are optimal from a welfare perspective. The paper also reflects on implications of these findings for pricing policies of an incumbent threatened by entry.

1 Introduction

The literature on price discrimination in a competitive setting has looked primarily at discrimination of the second and third degree.[1] First-degree discrimination has received scant attention. It was viewed as unworkable, primarily because information about individual reservation prices remained beyond the reach of sellers. A notable exception arose when firms sold products that were costly to transport across geographical space. Because the distance between buyers and sellers was observable, and because it correlated with transportation cost, one could argue that the maximum price net of transportation that sellers could charge individual buyers, decreased as the distance separating them from these buyers increased. This explains why the spatial economics literature was not reluctant to explore pricing regimes akin to first degree price discrimination (Hurter and Lederer, 1985, Lederer and Hurter, 1986, Thisse and Vives, 1988, Hamilton and Thisse, 1992, Stole, 1995, Ulph and Vulkan, 2000).

Perceptions about the practicality of personalized pricing regimes in non-spatial settings are changing (Shapiro and Varian, 1999, Varian, 2003). The change accompanies a rapid growth of computer-mediated transactions. It is attended by advances in information gathering/processing techniques.[2] This development has spawned a literature that explores how on-line sellers exploit information about consumer preferences via personalization of prices, and product specifications.(Villas-Boas, 1999, Fudenberg and Tirole, 2000, Acquisti and Varian, 2001, Varian, 2003).

The personalization of prices requires that sellers' costs not increase too fast as the number of price categories expands. On-line technologies are valuable in this regard because they make it easier to customize packages in the absence of face-to-face contact with buyers. Also, in some markets - markets of copyrighted material in particular - standard on-line selling methods are used in conjunction with digital rights management techniques to curb resales.

The improved capacity to gather, and process information about consumer preferences, also affects pricing in traditional trading environments. A familiar form of personalized discounting takes place at the check-out counter when promotional offers are particularized. Banks and insurance companies rely increasingly on the history of their clients' activity to mold custom-tailored products.

In markets where know-now is licensed, personalized pricing did not await the emergence of on-line technologies. Royalties have traditionnally depended on licensees' intensity of use of the licensed technology. The reason is that intensity of use correlates with willingness to pay (Farrell and Shapiro, 2004).[3] A related form of discrimination takes place when firms earn high margins in aftermarkets from the sale of complementary products whose wear and tear increase with intensity of use.[4]

This paper examines first degree price discrimination by vertically differentiated duopolists. It addresses several questions. 1)Does rivalry among duopolists produce a Nash equilibrium in discriminatory price schedules? 2) If so, does price discrimination perform better in terms of consumer and overall welfare than uniform pricing? 3)Would firms engage in discriminatory pricing if they had the power to enforce an

[1]Although the literature on price discrimination in competitive settings is not as extensive as the analysis for the monopoly case, it includes a growing number of papers including Katz (1984), Borenstein (1985), Holmes (1989), Corts (1998) and Armstrong and Vickers(2001). A recent survey by Stole (2003) sums up the current state of knowledge about price discrimination and imperfect competition.

[2]In some cases- specifically the on-line distribution of copyrighted works such as music and software- the actual use of products by buyers can also be tracked by means of digital rights management techniques.

[3]Price discrimination in the market for licenses is also facilitated by the fact that licensees cannot legally transfer them to others.

[4]See Emch (2003).

agreement under which each committed to set a uniform price? 4) How do the quality choices made by discriminating duopolists measure up in terms of welfare?

Thisse and Vives (1988) show that first degree discrimination by horizontally differentiated duopolists yields a unique equilibrium in price schedules. They determine that in equilibrium each firm earns less than it would earn if both firms enforced an agreement to price uniformly, and subsequently set prices independently. This paper is akin to Thisse and Vives (1988). However, in contrast to these authors, it shows that a prisoner's dilemma need not arise when differentiation is vertical. Whether it does, depends on the gap in the qualities offered by the two firms, and on the disparity of consumer preferences with respect to quality. A transition from uniform to discriminatory pricing affects profits via two channels: An enhanced capacity to extract surplus from some buyers, and an intensification of competition to obtain the patronage of other buyers. [5]

Section 2 introduces the model. Section 3 characterizes the unique equilibrium that emerges from the simultaneous choice of price schedules. It shows that prices are not monotonic in consumers' willingness to pay, and explains why competition in discriminatory price schedules yields a welfare maximizing market coverage, and a welfare maximizing partition of the market into buyers of high and low quality. In contrast to most of the literature, the paper derives this result - as well as other results - for a general distribution of consumer preferences. Section 4 takes up the question whether discrimination by both firms is an equilibrium strategy when the pricing regime is endogeneous. Section 5 shows that a prior commitment by both firms to uniform pricing does not always enhance their profits. It explains under what conditions it does. Section 6 endogenizes the choice of quality. It establishes that profit maximizing duopolists who engage in discriminatory pricing choose qualities that maximize welfare. Section 7 provides concluding remarks and adresses some policy issues.

2 The model

We consider a market in which two firms serve a continuum of consumers. The size of the market is normalized to one. Each firm produces a single variety of a vertically differentiated product. For convenience we call the varieties "high quality" and "low quality ", and denote them s_H and s_L, where $s_H > s_L > 0$. We call the producers of these qualities the high (H), and the low (L) quality firm.

There is a continuum of consumers with Mussa-Rosen (1978) preferences. Each consumer is identified by a taste parameter θ. The latter is distributed with positive density $f(\theta)$ over the interval $[0, b]$. A consumer buys a single one unit of high or low quality, or nothing at all. The willingness to pay by consumer θ for a unit product of quality s_i is θs_i $(i \in \{H, L\})$. The consumer who pays $p_i(\theta)$ for such unit gets a surplus $\theta s_i - p_i(\theta)$. The consumer who does not purchase has zero surplus. Consumers cannot resell.

Firms observe the $\theta' s$ of all consumers. This fact is common knowledge. The cost of producing q_i units of quality s_i is $C(s_i, q_i) = c(s_i)q_i$ where $c(s_i)$ denotes the unit cost of quality s_i. Firms incur no costs besides production cost. Unit cost $c(s)$ is a differentiable function, strictly increasing and strictly convex in quality. Specifically,

$$c(0) = 0, c^{'}(s) > 0, c''(s) > 0, \forall s > 0 \ (1)$$

Conditions (1) imply that the ranking of cost/quality ratios is

$$0 < \frac{c(s_L)}{s_L} < \frac{c(s_H)}{s_H} < \frac{c(s_H) - c(s_L)}{s_H - s_L}, \ \forall s_H > s_L > 0 \ (2)$$

[5]See Thisse and Vives (1988) and Ulph and Vulkan (2000, 2001). Ulph and Vulkan (2000) find that a switch from uniform to discriminatory pricing boosts profits when transport cost increases rapidly with distance.

Because we focus on the choices between pricing regimes, we want both firms to be active under all the regimes that we consider. Condition (3) below insures such outcome.

$$b > \frac{c(s_H) - c(s_L)}{s_H - s_L}, \ \forall s_H > s_L > 0 \ (3)$$

We define a *price schedule* as a function $p_i(.)$ that specifies the *price* $p_i(\theta)$ at which firm i is willing to sell one unit of quality s_i to consumer θ. We say that a price schedule is *admissible* when all prices that make up the schedule are at least as high as unit cost, and not higher than the reservation price of the consumers they target. The strategy set of firm i is defined as follows:

Definition: The set Σ_i of admissible price schedules for firm i ($i \in \{H, L\}$) is $\Sigma_i = \{p_i(.)/p_i(.)$ is a continuous and bounded real function defined on $[0, b]$ such that, $\forall\theta \in [0, b]$, $c(s_i) \leq p_i(\theta) \leq \theta s_i\}$.

This restriction of the strategy space means that we do not allow a firm to make up a loss incurred in one market segment by a profit earned in another segment.

We consider two admissible price schedules; the uniform schedule and the discriminating schedule. We say that a price schedule is uniform when a single price targets all consumers. We say that a schedule is discriminatory or personalized, when its component prices vary according to the taste parameter of the consumers they target.

We examine four regimes. Under the uniform regime, denoted (U_H, U_L), both firms choose uniform schedules. Under the discriminatory regime, denoted (D_H, D_L), both firms choose discriminatory schedules. The remaining regimes, called mixed are denoted (U_H, D_L) and (D_H, U_L). They occur when one firm sets a uniform price and the other firm discriminates..

3 Discriminatory price schedules: regime (D_H, D_L)

For any pair of admissible schedules $(p_H(.), p_L(.)) \in \Sigma_H \times \Sigma_L$, the *market areas* served by firms H and L are:

$$\Theta_H(p_H(.), p_L(.)) = \{\theta \in [0, b]/\theta s_H - p_H(\theta) \geq Max[0, \theta s_L - p_L(\theta)]\} \ (4)$$

$$\Theta_L(p_H(.), p_L(.)) = \{\theta \in [0, b]/\theta s_L - p_L(\theta) \geq Max[0, \theta s_H - p_H(\theta)]\} \ (5)$$

The profits of firm i ($i = \{H, L\}$) are :

$$\Pi_i(p_H(.), p_L(.)) = \int_{\Theta_i(p_H(.), p_L(.))} [p_i(\theta) - c(s_i)] f(\theta) d\theta \ (6)$$

Proposition 1 below characterizes the Nash equilibrium of the pricing game Γ in which the firms with profit functions defined by (4), (5) and (6) simultaneously choose admissible price schedules within the spaces Σ_H and Σ_L.

To characterize the equilibrium, we use the following terms. We say that firm i has a *monopoly position* with respect to consumer θ if, for any admissible price schedule chosen by its rival j, it can attract that consumer with a price θs_i that leaves zero surplus to that consumer. We say that firm i has a *cost-quality advantage* over it's rival j with respect to consumer θ if there exists an admissible price $p_i(\theta)$ at which it can attract that consumer when the rival firm j targets that consumer with a price equal to its unit cost $c(s_j)$.

Clearly, a firm that holds a monopoly position vis-a-vis a consumer also holds a cost-quality advantage with respect to that consumer. [6]

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Proposition 1 *The unique Nash equilibrium of the pricing game Γ with strategy sets Σ_i and payoffs Π_i $(i \in \{H, L\})$ defined by (6) is the pair of price schedules $(p_H^*(\theta), p_L^*(\theta))$ defined by (7) and (8) below*

$$p_H^*(\theta) = \begin{cases} c(s_L) + \theta(s_H - s_L) & if \quad \frac{c(s_H)-c(s_L)}{s_H - s_L} \leq \theta \leq b \\ c(s_H) & if \quad 0 \leq \theta \leq \frac{c(s_H)-c(s_L)}{s_H - s_L} \end{cases} \tag{7}$$

$$p_L^*(\theta) = \begin{cases} c(s_L) & if \quad \frac{c(s_H)-c(s_L)}{s_H - s_L} \leq \theta \leq b \\ c(s_H) - \theta(s_H - s_L) & if \quad \frac{c(s_H)}{s_H} \leq \theta < \frac{c(s_H)-c(s_L)}{s_H - s_L} \\ \theta s_L & if \quad 0 \leq \theta \leq \frac{c(s_H)}{s_H} \end{cases} \tag{8}$$

Proof:

Existence.

Because the price targeted at one consumer does not constrain the price targeted at another consumer, and because marginal cost does not depend on quantity, competition in price schedules adds up to a collection of Bertrand games for individual consumers. Therefore, it is sufficient to show the following:

i/ For each $\theta \in [0, b]$, $p_H^*(\theta)$ maximizes $\Pi_H(p_H(\theta), p_L^*(\theta))$ over $p_H(\theta) \in \Sigma_H$, given the price $p_L^*(\theta)$;

ii/ for each $\theta \in [0, b]$, $p_L^*(\theta)$ maximizes $\Pi_L(p_H^*(\theta), p_L(\theta))$ over $p_L(\theta) \in \Sigma_L$, given the price $p_H^*(\theta)$.

We examine the equilibrium in four market segments using the simplified notation c_H for $c(s_H)$ and c_L for $c(s_L)$.

1. $\theta \in \left[\frac{c_H - c_L}{s_H - s_L}, b\right]$

When $p_L^*(\theta) = c_L$ consumers derive positive surplus from purchasing low quality. The highest price at which consumers purchase high quality satisfies $\theta s_H - p_H^*(\theta) = \theta s_L - c_L$ which implies (7). This price is admissible because the unit cost function is convex.[7] With respect to consumers in this interval, firm H has a cost quality advantage over firm L, but enjoys no monopoly position .[8] When the H-firm sets $p_H^*(\theta) = c_L + \theta(s_H - s_L)$, consumers purchase low quality only if it is priced lower than unit cost. Because this is not admissible, $p_L^*(\theta) = c_L$ is an optimal response by firm L.

2. $\theta \in [\frac{c_H}{s_H}, \frac{c_H - c_L}{s_H - s_L}[$

When $p_L^*(\theta) = c_H - \theta(s_H - s_L)$ consumers in the interval get a surplus $\theta s_H - c_H$ from low quality. Firm H can attract these consumers only if it prices below unit cost. Because this is not admissible, the best response of firm H is to target these consumers with a price equal to c_H. When $p_H^*(\theta) = c_H$, the highest price at which firm L can attract consumers satisfies the condition $\theta s_L - p_L^*(\theta) = \theta s_H - c_H$ which implies

[6]The converse is not true. Consumers with respect to whom neither firm holds an advantage have $\theta = \frac{c(s_H)-c(s_L)}{s_H - s_L}$. The low quality firm holds a cost-quality advantage or a monopoly position for consumers with a lesser taste for quality; the high quality firm holds a cost-quality advantage for consumers with stronger preference for quality.

[7]For the singleton $\theta = \frac{c_H - c_L}{s_H - s_L}$, there does not exist an admissible price schedule $p_H(\theta)$ that makes the consumer who purchases high quality strictly better off. But, for all other consumers belonging to the interval $]\frac{c_H - c_L}{s_H - s_L}, b]$, there exists an admissible price schedule defined by $p_H(\theta) = p_H^*(\theta) - \epsilon$, where ϵ is a positive and sufficiently small, such that consumers θ are strictly better off by purchasing high quality.

[8]Note that $\theta \in \left[\frac{c_H - c_L}{s_H - s_L}, b\right] \Longrightarrow c_L + \theta(s_H - s_L) < \theta s_H$. Therefore firm H does not hold a monopoly position over such consumer.

(7). Clearly, firm L has a cost-quality advantage but no monopoly position with respect to consumers in the interval.

3. $\theta \in \ [\frac{c_L}{s_L}, \frac{c_H}{s_H}[$.

Consumers would derive negative surplus from high quality even if they purchased such quality at unit cost. Firm L holds a monopoly position over consumers in the interval. It therefore sets $p_L(\theta) = \theta s_L$ capturing their entire consumer surplus. $p_H(\theta) = c_H$ is a best response by firm H because it can sell only by pricing below unit cost.

4. $\theta \in [0, \frac{c_L}{s_L}[$

Within this interval no firm can attract consumers by pricing above unit cost. Therefore both firms set price at unit cost and neither sells any output.

Uniqueness.

We have already indicated that the admissibility restriction, and the assumption that cost is linear in quantity imply that the component parts of the equilibrium price schedules are the solutions Bertrand competition games for individual consumers.[9] Because, the latter are unique, the equilibium price schedules are unique as well.

Figure 1 displays the equilibrium. The lines labeled θs_H and θs_L represent the participation constraints of high and low quality buyers. The line segment KM is the self-selection constraint faced by the high quality firm when its rival sells at unit cost. Similarly, the line segment TL represents the self-selection constraint faced by the low quality firm when its rival offers high quality at unit cost. The high quality firm serves the market segment $[\frac{c_H - c_L}{s_H - s_L}, b]$ setting prices represented by KM. The low quality firm divides its buyers in two segements. With respect to consumers having $\theta \in [\frac{c_L}{s_L}, \frac{c_H}{s_H}]$, it acts as a perfectly discriminating monopolist setting prices at which the participation constraint is binding (VT in Figure 1). With respect to consumers with $\theta \in]\frac{c_H}{s_H}, \frac{c_H - c_L}{s_H - s_L}]$, the low quality firm sets prices at which the consumers' self-selection constraint binds (TL in Figure 1). The firm cannot extract all the surplus from these consumers because they are ready to switch to high quality when the latter is priced at unit cost.

[9]For each $\theta \in [0, b]$, the corresponding θ-game, denoted Γ_θ, is defined as follows. The strategy space of player i ($i \in \{H, L\}$) in Γ_θ is the set of prices p_i^θ belonging to the interval $[c_i, \theta s_i]$. The player i's payoff in Γ_θ is given by

$$\pi_i^\theta(p_i^\theta, p_j^\theta) = \begin{cases} p_i^\theta - c_i & if \quad \theta s_i - p_i^\theta \geq \max(0, \theta s_j - p_j^\theta) \\ 0 & if \quad \theta s_i - p_i^\theta < \max(0, \theta s_j - p_j^\theta) \end{cases}$$

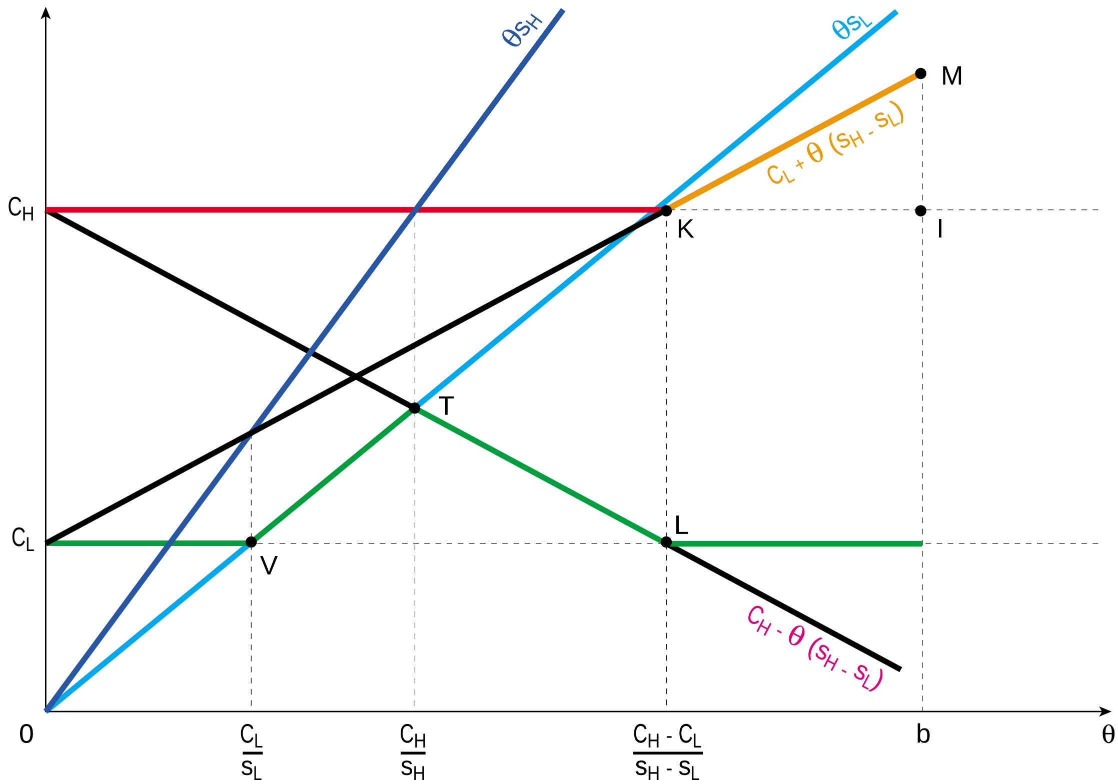

Fig 1 : Regime (D_H, D_L)

Substitution of (7) and (8) into (6) yields the equilibrium profits

$$\Pi_H(D_H, D_L) = \int_{\frac{c_H - c_L}{s_H - s_L}}^{b} [c_L + \theta(s_H - s_L) - c_H] f(\theta) d\theta \; (9)$$

and

$$\Pi_L(D_H, D_L) = \int_{\frac{c_L}{s_L}}^{\frac{c_H}{s_H}} [\theta s_L - c_L] f(\theta) d\theta + \int_{\frac{c_H}{s_H}}^{\frac{c_H - c_L}{s_H - s_L}} [c_H - \theta(s_H - s_L) - c_L] f(\theta) d\theta (10)$$

In Figure 1, the profit earned by the high quality firm from an individual consumer is shown by the vertical distance between the line segment KM and the horizontal line c_H. Because the total profit is a weighted sum of these distances on the segment $\left[\frac{c_H - c_L}{s_H - s_L}, b\right]$ we write $\Pi_H(D_H, D_L) \; = area \; KMI$, keeping in mind that the area is properly defined by the integral (9). Similarly we write $\Pi_L(D_H, D_L) = area \; VTL$

We now note that Proposition 1 implies:

Corollary 2 *Competition in price schedules by quality differentiated duopolists yields a market coverage and a segmentation of consumers into high and low quality buyers that maximize welfare*

Proof:

Welfare is maximized when the following conditions hold: 1) Low quality is sold only to consumers whose willingness to pay is larger than the cost of producing low quality, and whose a surplus from low quality purchased at cost is larger than their surplus from high quality purchased at cost; 2) high quality is sold only to consumers whose surplus from high quality purchased at cost is larger than their surplus from low quality purchased at cost. These conditions are clearly met when the duopolists choose the schedules given by (7) and (8).

4 Selecting a price policy.

We now address the question whether price discrimination is an equilibrium of a game in which each firm can commit to a uniform schedule before the rival producer sets a price or a price schedule. We do so by considering a two-stage game. In the first stage, each firm chooses whether or not to commit to a uniform price. If the two firms commit to a uniform price - regime (U_H, U_L) - both set the price they committed to at the second stage. If no firm commits - regime (D_H, D_L) - each is free to choose an admissible price schedule in the second stage. If one firm commits and the other does not - regimes (U_H, D_L) and (D_H, U_L) - only the firm that did not commit remains free to choose among admissible price schedules at stage 2. [10] Consumers make their purchasing decisions at the second stage.

Committing to a uniform price at stage 1 limits one's freedom at stage 2. Therefore, committing can only be rational if it elicits a pricing response on the part of the rival that is favorable to the firm that makes the commitment. The credibility of a commitment to price uniformly can have different sources. It may derive from sunk investments in a distribution channel that puts intermediaries between manufacturers and consumers, and does not allow the former to ascertain individual preferences. It can arise from a most-favored-customer clause granted by the seller. It may also be based on reputational losses that would ensue from backing down on a pre-announced uniform price.

We assume that firms which commit to a uniform price take measures that lend credibility to that commitment. However, we do not model these measures.

We look first at price equilibria that emerge when one firm commits and the other does not.

4.1 The H-firm commits and the L-firm does not: regime (U_H, D_L)

Because the high quality firm eans zero profits when $p_H = c_H$, we restrict our attention to commitments where $p_H > c_H$. The best response of firm L to such commitment is

[10] We show in appendix 1 that there does not exist a Nash equilibrium of a simultaneous game in pure strategies where the high quality firm commits to a uniform price and the low quality firm price discriminates.

$$p_L(\theta, p_H) = \begin{cases} c_L & for \quad \theta \geq \frac{p_H - c_L}{s_H - s_L} \\ p_H - \theta(s_H - s_L) & for \quad \frac{p_H}{s_H} < \theta < \frac{p_H - c_L}{s_H - s_L} \\ \theta s_L & for \quad \frac{c_L}{s_L} \leq \theta \leq \frac{p_H}{s_H} \end{cases} \quad (11)$$

Profits are

$$\Pi_L(p_H, p_L(.)) = \int_{\frac{c_L}{s_L}}^{\frac{p_H}{s_H}} [\theta s_L - c_L] f(\theta) d\theta + \int_{\frac{p_H}{s_H}}^{\frac{p_H - c_L}{s_H - s_L}} [p_H - \theta(s_H - s_L) - c_L] f(\theta) d\theta \ (12)$$

$$\Pi_H(p_H, p_L(., p_H)) = \int_{\frac{p_H - c_L}{s_H - s_L}}^{b} [p_H - c_H] f(\theta) d\theta \ (13)$$

Referring to Figure 2 and using the same notation as above we write $\Pi_L(p_H, p_L(.)) = area\ VAB$ and $\Pi_H(p_H, p_L(., p_H)) = area\ FGIN$.

Because $\frac{c_L}{s_L} < \frac{c_H}{s_H} < \frac{c_H - c_L}{s_H - s_L} < \frac{p_H - c_L}{s_H - s_L}$, it must be true that $VTL \subset VAB$, or $area\ VTL = \Pi_L(D_H, D_L) < area\ VAB = \Pi_L(p_H, p_L(.))$. Thus, the low quality firm earns higher profits when it does not commit and its rival commits to *any* uniform price $p_H > c_H$, than when no firm commits. Because $\frac{c_H - c_L}{s_H - s_L} < \frac{p_H - c_L}{s_H - s_L} < b$, it must be true that $FGIN \subset KMI$ or $area\ FGIN = \Pi_H(p_H, p_L(., p_H)) < area\ KMI = \Pi_H(D_H, D_L)$.

We therefore conclude that the high quality firm which commits to any uniform price $p_H > c_H$, while its rival does not commit, earns less than when no firm commits.

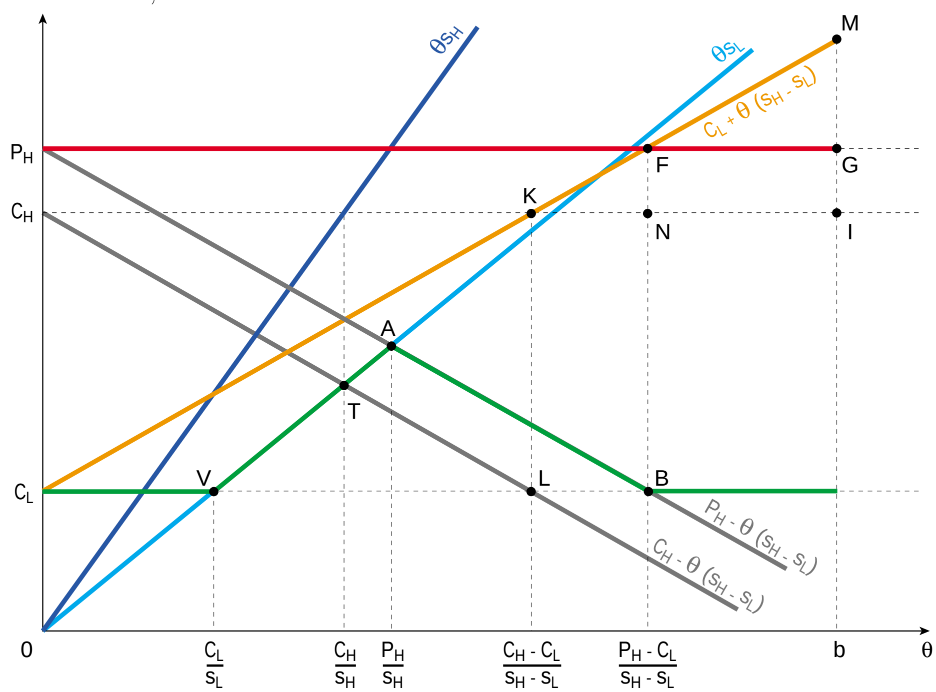

Fig 2 : Regime (U_H, D_L)

Upon defining $\widetilde{p}_H = \arg\max \Pi_H(p_H, p_L(., p_H))$, we can write $\Pi_H(U_H, D_L) \equiv \Pi_H(\widetilde{p}_H, p_L(., \widetilde{p}_H))$ and $\Pi_L(U_H, D_L) \equiv \Pi_L(\widetilde{p}_H, p_L(., \widetilde{p}_H))$ with $p_L(\theta, \widetilde{p}_H)$ given by (11). Because the condition $\Pi_H(p_H, p_L(., p_H)) < \Pi_H(D_H, D_L)$ holds true for any $p_H > c_H$ it holds true for $\widetilde{p}_H$ as well. Therefore:

$$\Pi_H(U_H, D_L) < \Pi_H(D_H, D_L) \ (14)$$

and

$$\Pi_L(U_H, D_L) > \Pi_L(D_H, D_L) \ (15)$$

Figure 2 clarifies the differences between the (U_H, D_L) and (D_H, D_L) regimes: i) Commitment by firm H to a uniform price $p_H > c_H$ shifts the self-selection constraint faced by the low quality firm upward (from TL to AB) and shortens the self-selection constraint faced by the high quality firm (from KM to FM); ii) the market coverage is the same under the two regimes, but under the (U_H, D_L) regime the market segment served by the high quality firm is smaller, and that served by the low quality firm larger than under the (D_H, D_L) regime[11]; iii)the number of low quality buyers that retain no surplus is larger under the (U_H, D_L) regime than under the (D_H, D_L) regime.

4.2 The L-firm commits and the H-firm does not: regime (D_H, U_L)

When the low quality firm commits to a uniform price $p_L \in (c_L, c_H)$ the best response of the high quality producer is

$$p_H(\theta, p_L) = \begin{cases} p_L + \theta(s_H - s_L) & for \quad \theta \in [\frac{c_H - p_L}{s_H - s_L}, b] \\ c_H & for \quad \theta < \frac{c_H - p_L}{s_H - s_L} \end{cases} \quad (16)$$

The profits of the high and low quality firms are

$$\Pi_H(p_H(., p_L), p_L) = \int_{\frac{c_H - p_L}{s_H - s_L}}^{b} [p_L + \theta(s_H - s_L) - c_H] f(\theta) d\theta \ (17)$$

and

$$\Pi_L(p_H(., p_L), p_L) = \int_{\frac{p_L}{s_L}}^{\frac{c_H - p_L}{s_H - s_L}} (p_L - c_L)\, f(\theta)\, d\theta \ (18)$$

Note that (18) assumes $\frac{p_L}{s_L} < \frac{c_H - p_L}{s_H - s_L}$ which is equivalent to $\frac{s_L}{p_L} > \frac{s_H}{c_H}$.This condition must be satisfied if the low quality firm is to remain active.

Figure 3 displays the profits as area $CDBQ$ for $\Pi_L(p_H(., p_L), p_L)$, and area ERI for $\Pi_H(p_H(., p_L), p_L)$. Because $\frac{c_H - p_L}{s_H - s_L} < \frac{c_H - c_L}{s_H - s_L} < b$, we have $KMI \subset ERI$. We conclude that when the high quality firm does not

[11] What brings about this result is that the firm committing to the uniform price has no interest in competing agressively for consumers who are indifferent between the two quelities when each is priced at unit cost. The reason is that the firm would have to accept a lower margin on sales to consumers with a strong preference for its quality.

commit, it earns higher profits when it faces a low quality firm that commits to a uniform price $p_{L,}$ than when it faces a low quality firm that does not commit. Similarly, because $\frac{c_L}{s_L} < \frac{p_L}{s_L} < \frac{c_H}{s_L} < \frac{c_H - p_L}{s_H - s_L} < \frac{c_H - c_L}{s_H - s_L}$, we have $CDBQ \subset \ VTL$. We conclude that when the high quality firm does not commit, the low quality firm earns lower profits by commiting to a uniform price, than by not committing.

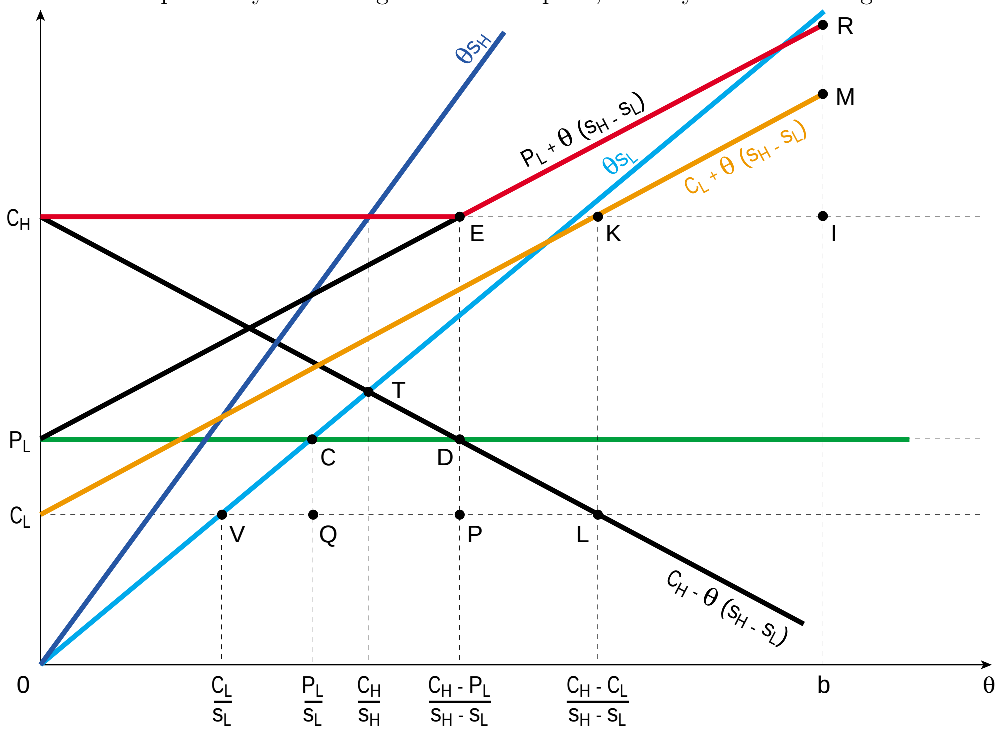

Fig 3 : Regime (D_H, U_L)

Upon defining $\widetilde{p}_L = \arg \max_{c_L < p_L < \frac{s_L}{s_H} c_H} \Pi_L(p_H(., p_L), p_L)$ we can write $\Pi_L(D_H, U_L) \equiv \Pi_L(p_H(., \widetilde{p}_L), \widetilde{p}_L)$ and $\Pi_H(D_H, U_L) \equiv \Pi_H(p_H(., \widetilde{p}_L), \widetilde{p}_L)$, where $p_H(\theta, \widetilde{p}_L)$ is given by (16). Because the inequalities $\Pi_H(p_H(., p_L), p_L) > \Pi_H(D_H, D_L)$ and $\Pi_L(p_H(., p_L), p_L) < \Pi_L(D_H, D_L)$ hold true for all p_L, they hold true for $\widetilde{p}_L \in (\ c_L, \frac{s_L}{s_H} c_H)$ as well. Therefore,

$$\Pi_L(D_H, U_L) < \Pi_L(D_H, D_L)(19)$$

and

$$\Pi_H(D_H, U_L) > \Pi_H(D_H, D_L) \ (20)$$

Figure 3 facilitates a comparison of the outcomes under the (D_H, U_L) and the (D_H, D_L) regimes. Segment ER is the best price response of the high quality firm. It also represents the self-selection constraint confronted by the high quality firm when the low quality firm chooses a uniform price $p_L \in \left(c_L, \frac{s_L}{s_H} c_L\right)$. All consumers with $\theta \in \left[\frac{c_H - p_L}{s_H - s_L}, b\right]$ purchase high quality and keep the same surplus they would retain if they purchased low quality at the price p_L. The segment CD is served by the low quality firm.

The following differences between regimes are apparent: i) Commitment by firm L to a uniform price shifts the self-selection constraint faced by high quality firm upward (KM becomes ER) and it changes the self-selection constraint faced by the low quality firm (VTL becomes CD); ii) total market coverage is smaller under the (D_H,U_L) regime than under the (D_H, D_L) regime; iii) the segment served by the low quality firm is also smaller under the (D_H, U_L) regime (CD instead of VL), whereas the segment served by the high quality producer is larger (EI instead of KI).

Jointly (14) and (19) imply

Proposition 3 *Personalization of prices is a Nash equilibrium of a sequential game in which vertically differentiated duopolists determine whether or not to commit to a specific uniform price before they actually set prices and sell output.*

We now show that when the distribution of consumer preferences is uniform over $[0, b]$, the equilibrium is supported by a dominant strategy for each firm, and is therefore unique.

5 A prisoner's dilemma?

Our finding that price personalization constitutes a Nash equilibrium is the counterpart for vertical differentaition of an earlier result by Thisse and Vives (1998) for the case spatial differentiation. Spatially separated duopolists always face a prisoner's dilemma; both would be better off if they could enforce an agreement to set uniform prices. We show that this is not true is in the case of quality differentiation. We do so for the particular case where the distribution of consumer preferences is uniform.

Table 1 displays the profits of the high and low quality firms for each of the four pricing regimes when $f(.)$ is uniform.[12]

	D_L	U_L
D_H	$\Pi_H = \frac{s_H - s_L}{2b}[b - \frac{c_H - c_L}{s_H - s_L}]^2$ $\Pi_L = \frac{s_L}{2b}[\frac{c_H - c_L}{s_H - s_L} - \frac{c_L}{s_L}][\frac{c_H}{s_H} - \frac{c_L}{s_L}]$	$\Pi_H = \frac{s_H - s_L}{2b}[b - \frac{1}{2}(\frac{c_H - c_L}{s_H - s_L} + \frac{c_H}{s_H})]^2$ $\Pi_L = \frac{s_L}{4b}[\frac{c_H}{s_H} - \frac{c_L}{s_L}][\frac{c_H - c_L}{s_H - s_L} - \frac{c_L}{s_L}]$
U_H	$\Pi_H = \frac{s_H - s_L}{4b}[b - \frac{c_H - c_L}{s_H - s_L}]^2$ $\Pi_L = \frac{1}{2b}\frac{s_L(s_H - s_L)}{s_H}[\frac{1}{2}(b + \frac{c_H - c_L}{s_H - s_L}) - \frac{c_L}{s_L}]^2$	$\Pi_H = \frac{4s_H^2(s_H - s_L)}{(4s_H - s_L)^2 b}[b - \frac{1}{2}(\frac{c_H - c_L}{s_H - s_L} + \frac{c_H}{s_H})]^2$ $\Pi_L = \frac{4s_L s_H(s_H - s_L)}{(4s_H - s_L)^2 b}[\frac{1}{2}(b + \frac{c_H - c_L}{s_H - s_L}) - \frac{c_L}{s_L})]^2$

Table 1: Equilibrium profits for a uniform distribution of preferences over $[0, b]$.

[12] Appendix 4 gives the derivation the profits shown in Table 1.

We note that when one firm discriminates, the profits earned by the rival firm are twice as large when it also discriminates than when it does not. Using $0 < s_L < s_H$, we obtain $\frac{1}{4} < \frac{4s_H^2}{(4s_H - s_L)^2} < \frac{4}{9}$ which entails $\Pi_H(D_H, U_L) > \Pi_H(U_H, U_L)$, and $\Pi_L(U_H, D_L) > \Pi_L(U_H, U_L)$. The latter implies that when the distribution of preferences is uniform, price discrimination is a dominant strategy for both firms.

To show under what circumstances a prisoner's dilemma arises we start with the proof of an intermediate result.

Lemma *There exist two threshold values $b_L(s_H, s_L)$ and $b_H(s_H, s_L)$, each larger than $\frac{c_H - c_L}{s_H - s_L}$, such that*

$$\begin{aligned} sign\left[\Pi_H(U_H, U_L) - \Pi_H(D_H, D_L)\right] &= sign\left[b_H - b\right] \\ sign\left[\Pi_L(U_H, U_L) - \Pi_L(D_H, D_L)\right] &= sign\left[b - b_L\right] \end{aligned}$$

Proof: See appendix 2.

Existence of a prisoner's dilemma requires $b_L < b_H$ and $b \in]b_L, b_H[$. The following proposition states when these conditions are met.

Proposition 4 *Both firms earn higher profits when they simultaneously set uniform prices than when they simultaneously choose discriminatory price schedules, if and only if, $s_H < 3.17s_L$ and $b \in (b_L(s_H, s_L), b_H(s_H, s_L))$ where $b_L(s_H, s_L)$ and $b_H(s_H, s_L)$ are defined by $\Pi_j(D_H, D_L) - \Pi_j(U_H, U_L) = 0$ and $j \in \{L, H\}$.*

Proof: See appendix 3

The intuition behind this result is clear. The duopolists compete for consumers who, loosely speaking, can be divided into two classes. One class is made up of consumers we can call "captive" because they derive far greater surplus from one of the qualities when the both qualities are priced at unit cost. The other class is made up of "non-captive" consumers. Their surplus is not very different across qualities when each quality is priced at unit cost.

For very large values of b, the portion of customers that is captive to the high quality firm is large compared to the portion of consumers that is not captive to that firm. Discrimination allows the high quality firm to extract a large amount of surplus from the class that has a strong preference for its quality. Therefore, the firm earns more when it discriminates than when it does not. For somewhat lower values of b, the high quality firm lacks this opportunity. Also, for lower $b's$ the low quality producer is also better off under uniform pricing because such pricing elicits less agressive pricing by the rival firm. As b decreases further, discrimination becomes the preferred option for the low quality firm. The reason is that discrimination allows greater extraction of surplus from consumers with a low willingness to pay, and entails less of a sacrifice in terms of profits from sales to non-captive consumers. The latter is due to the fact that the optimally chosen uniform prices of both firms fall when b decreases. Therefore, the low quality firm is better off when it discriminates and draws profits (almost) exclusively from its captive consumers.

The proposition also states that an agreement to price uniformly makes the duopolists better off only when the quality gap is below a certain threshold. The reason is that a small quality gap -all else equal-reduces the portion of captive consumers

The following corollary compares the aggregate consumer surplus under the (D_H, D_L) and (U_H, U_L) regime

.

Corollary: *When the conditions insuring the existence of a prisonner's dilemma are met, aggregate consumer surplus is higher when firms engage in personalized pricing than when they price uniformly.*

Proof: The proof follows from the definition of aggregate welfare, and from the earlier result that aggregate welfare is maximized when the two firms choose profit-maximizing discriminatory price schedules.

6 Quality choice by discriminating duopolists.

We now endogenize product specification and address the question how qualities selected by vertically differentiated duopolists compare to welfare maximizing qualities. For this purpose, we add one more stage to the game - an initial stage at which both firms simultaneously choose their quality in a bounded interval $[0, S]$.[13] The subsequent stages are identical to the pricing game studied in the earlier sections.[14].

Because price discrimination is an equilibrium for all values $s_H > s_L > 0$, a Nash equilibrium in qualities must satisfy the first order conditions (21) and (22) below, obtained by differentiating (9) and (10) with respect to s_H and s_L [15]

$$\int_{\frac{c(s_H)-c(s_L)}{s_H-s_L}}^{b} \left[\theta - c'(s_H)\right] f(\theta) d\theta = 0 \quad (21)$$

$$\int_{\frac{c(s_L)}{s_L}}^{\frac{c(s_H)-c(s_L)}{s_H-s_L}} \left[\theta - c'(s_L)\right] f(\theta) d\theta = 0 \quad (22)$$

Conditions (21) and (22) simply state that a pair of qualities constitutes a Nash equilibrium when a small change in either quality affects unit cost by as much as it affects the willingness to pay by the *average* consumer purchasing that quality.[16]We now turn to the welfare analysis.

Proposition 5 *Vertically differentiated single-product duopolists who engage in perfect price discrimination choose socially optimal qualities.*

Proof: The proof is straightforward. One shows that the qualities selected by a welfare maximizing firm that produces two qualities are the same as the equilibrium qualities that satisfy conditions (21) and (22). Total welfare is

$$W(s_H, s_L) = \int_{\frac{c(s_L)}{s_L}}^{\frac{c(s_H)-c(s_L)}{s_H-s_L}} (\theta s_L - c(s_L)) f(\theta)\, d\theta + \int_{\frac{c(s_H)-c(s_L)}{s_H-s_L}}^{b} (\theta s_H - c(s_H)) f(\theta)\, d\theta \quad (23)$$

[13] S is the highest quality allowed by technology.

[14] Because we know that regardless of quality the equilibrium pricing strategy entailss discrimination by both firms, we conclude that the Nash equilibrium in qualities is obtained from maximization of profits under discriminatory pricing.

[15] Convexity of the unit cost function implies that the second order conditions are satisfied.

[16] For a uniform density we have $c'(s_H^*) = \frac{1}{2}[\frac{c(s_H^*)-c(s_L^*)}{s_H^*-s_L^*} + b]$ and $c'(s_L^*) = \frac{1}{2}[\frac{c(s_H^*)-c(s_L^*)}{s_H^*-s_L^*} + \frac{c(s_L^*)}{s_L^*}]$.

Differentiation of (23) with respect to s_H, yields the first order condition

$$\begin{aligned}
&\left[\frac{c(s_H)-c(s_L)}{s_H-s_L}s_L-c(s_L)\right] f\left(\frac{c(s_H)-c(s_L)}{s_H-s_L}\right)\frac{\partial[\frac{c(s_H)-c(s_L)}{s_H-s_L}]}{\partial s_H}\\
&+\int_{\frac{c(s_L)}{s_L}}^{\frac{c(s_H)-c(s_L)}{s_H-s_L}}\left[\theta-c'\left(s_H\right)\right]f(\theta)d\left(\theta\right)\\
&-\left[\frac{c(s_H)-c(s_L)}{s_H-s_L}s_H-c(s_H)\right] f\left(\frac{c(s_H)-c(s_L)}{s_H-s_L}\right)\frac{\partial[\frac{c(s_H)-c(s_L)}{s_H-s_L}]}{\partial s_H}=0
\end{aligned}$$

which simplifies to (21).
Similarly, differentiation of (23) with respect to s_L yields the first order condition

$$\begin{aligned}
&\int_{\frac{c_L}{s_L}}^{\frac{c_H-c_L}{s_H-s_L}}\left[\theta-c'\left(s_L\right)\right]f\left(\theta\right)d\theta\\
&+\left[\frac{c(s_H)-c(s_L)}{s_H-s_L}s_L-c(s_L)\right] f\left(\frac{c(s_H)-c(s_L)}{s_H-s_L}\right)\frac{\partial[\frac{c(s_H)-c(s_L)}{s_H-s_L}]}{\partial s_L}\\
&-\left[\frac{c(s_L)}{s_L}s_L-c(s_L)\right] f\left(\frac{c(s_L)}{s_L}\right)\frac{c(s_L)}{s_L}\frac{\partial[\frac{c(s_L)}{s_L}]}{\partial s_L}\\
&-\left[\frac{c(s_H)-c(s_L)}{s_H-s_L}s_H-c(s_H)\right] f\left(\frac{c(s_H)-c(s_L)}{s_H-s_L}\right)\frac{\partial[\frac{c(s_H)-c(s_L)}{s_H-s_L}]}{\partial s_L}=0
\end{aligned}$$

which simplifies to (22). This completes the proof.

7 Final Remarks

We have shown that first order price discrimination is a Nash equilibrium of a game where quality differentiated duopolists determine first whether or not to commit to a uniform price, and subsequently set prices and sell output. The building blocks of competition in price schedules are Bertrand games for individual consumers. This follows from the assumption that the prices targeted at different consumers can be chosen independently.

Whether specific consumers are better off under discrimination than under uniform pricing depends on the extent to which they are captive to one of the sellers. Discrimination benefits the consumers whose preference for one of the qualities is weak when both qualities are priced at unit cost. With respect to these consumers, the competition effect of discrimination outweighs the enhanced surplus extraction effect. Consumers who have a strong preference for one of the qualities when both are priced at unit cost, are worse off under discrimination.

In contrast to earlier contributions that looked at horizontal differentiation, we find that quality differentiated duopolists are not necessarily better off when they enforce an agreement to price uniformly. Whether they are, depends on the disparity in qualities produced by the rival firm. A necessary condition for both firms to be better off under first degree price discrimination is that the disparity in qualities be sufficiently large. We also establishe that for all possible quality combinations, a unilateral move by one firm from uniform pricing to personalized pricing lowers the rival's profits.

The industrial organization literature has shown that a unilateral grant of a most-favored customer clause can increase the profits of all firms in an industry by softening price competition. Our paper shows that the assumption of uniform pricing is critical to this outcome. In the absence of a commitment to uniform pricing, a unilateral grant of price protection to one's customers is always harmful to the party making such grant.

We have also shown that for any exogenously given quality pair, competition in discriminatory prices schedules yields a welfare maximizing coverage of the market, and a welfare maximizing segmentation into buyers of high and low quality. Furthermore, when firms simultaneously and independently decide on a quality before selecting a price policy, both choose welfare maximizing qualities.

We obtain these results are obtained for a fairly general cost function.The presence of a fixed cost does not change the basic result that personalized pricing is a Nash equilibrium.

One policy implication of the paper is that imposing a minimum quality requirement lowers aggregate welfare when pricing is discriminatory. This is in contrast to earlier results which show that a mildly restrictive quality standard raises welfare when it narrows the quality gap between firms (Ronnen, 1991, Crampes and Hollander, 1995). Under uniform pricing, a minimum quality requirement increases market coverage when the narrowing of the quality gap that ensues, leads to a sufficient intensification of price competition. Such intensification also takes place when the firms engage in price discrimination. The reason is that a narrowing of the quality gap enlarges the range of non-captive consumers. However, when pricing is discriminatory, a minimum quality requirement always restricts market coverage. More importantly, the paper suggests that a narrowing of the quality gap gives firms an incentive to choose distribution channels or contractual arrangements that assure uniform prices. When this happens, the result is a further reduction of market coverage, and a suboptimal segmentation of consumers into high and low quality buyers.

The industrial organization literature has devoted much attention to the question how market structure affects the price level. This paper raises the question how market structure affects the choice of a price policy. Entry for example, may reduce the disparity in qualities offered by different producers. Th paper explains why the latter affects firms' incentives to reach an agreement to price uniformly.

Although the paper looks at competition among active firms, it points to interesting questions in regard to pricing by an incumbent who faces a threat of entry. The standard reply to the question what differentiates an incumbent from an entrant is that the former can credibly commit to a post-entry course of action before the second firm appears on stage. Another distinguishing characteristic between an incumbent and an entrant is that the former is better equipped to engage in differential pricing. Indeed, pre-entry adoption of discriminatory pricing by an incumbent reveals information about buyers' reservation prices that the entrant does not posses. The implication is that the entrant is more likely - at least initially - to set a uniform price, or perhaps divide consumers into fewer classes for pricing purposes than the incumbent.

The paper has shown that a firm which prices uniformly earns lower profits when its rival discriminates that when it does not. This suggests than an incumbent who discriminates is more likely to deter entry than an incumbent who prices uniformly [17].

The assumption that firms posses full information about individual reservation prices, and that the in-

[17] See Aguirre *et al.*, 1998.

formation they do posses is acquired no cost, obviously lacks realism (Varian, 2003). Information about consumer preferences can be obtained, in part at least, from experimentation with various prices. Such experimention carries a cost (Caminal and Matutes, 1990, Shaffer and Zhang, 2000). However, when experimentation takes place prior to entry that cost may be largely sunk when the second firm is ready to enter.

It is clear that discrimination prior to entry does not compel an incumbent to discriminate post entry. However, it makes post-entry discrimination more likely because the incumbent already possesses information about individual reservation prices, and - as shown in the paper- the firm which has costless information about reservation prices, earns higher profits when it discriminates than when it does not, regardless of the policy adopted by the rival producer. This suggests that a firm threatened by entry might decide to incur a non-recoupable cost of acquiring information about individual reservation prices even though it would not do so in the absence of such threat.

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Appendix 1: Non-existence of an equilibrium where D_L and U_H are chosen simultaneously.

Proposition 6 *The one-period game where the high quality firm sets a uniform price and the low quality firm personalizes prices does not have an equilibrium in pure strategies.*

Proof: We restrict the proof to the case where $f(\theta) = \frac{1}{b} \forall \in [0, b]$. We already know that for a uniform price $p_H \geq c_H$ the best response of the low quality firm is given by:

$$p_L(\theta, p_H) = \begin{cases} \theta s_L & for \quad \frac{c_L}{s_L} \leq \theta \leq \frac{p_H}{s_H} \\ p_H - \theta(s_H - s_L) & for \quad \frac{p_H}{s_H} < \theta \leq \frac{p_H - c_L}{s_H - s_L} \\ c_L & for \quad \frac{p_H - c_L}{s_H - s_L} < \theta \leq b \end{cases}$$

The profit of the high quality firm is teherefore

$$\Pi_H(p_H, p_L(., p_H)) = \frac{1}{b}(p_H - c_H)(b - \frac{p_H - c_L}{s_H - s_L})$$

We show first that a pair $(p_H, p_L(\theta, p_H))$ where $p_H > c_H$ cannot be an equilibrium. Assume that $p_H > c_H$ and consider a deviation by firm H lowering its price to $\widetilde{p}_H = p_H - \epsilon$ where $0 < \epsilon < p_H - c_H$. Consumers with $\theta \in]\frac{p_H}{s_H}, \frac{p_H - c_L}{s_H - s_L}]$ switch to the high quality because $\theta s_H - p_H + \epsilon > \theta s_L - p_H + \theta(s_H - s_L)$. Post deviation, the profit of the high quality firm is $\widetilde{\Pi}_H = \frac{1}{b}(\widetilde{p}_H - c_H)(b - \frac{\widetilde{p}_H}{s_H})$. Therefore, $\widetilde{\Pi}_H - \Pi_H = \frac{1}{b}[-\epsilon(b - \frac{\widetilde{p}_H}{s_H}) + (p_H - c_H)(\frac{p_H - c_L}{s_H - s_L} - \frac{\widetilde{p}_H}{s_H})]$. Because the second term on the right hand side can be made larger in absolute value than the first term, the deviation increases the profits of the high quality firm. This proves that there cannot be an equilibrium in pure strategies where $p_H > c_H$.

Consider now the case $p_H = c_H$ which entails $\Pi_H = 0$. The best response of firm L is to sell to consumers $\theta \in \left[\frac{c_H - c_L}{s_H - s_L}, b\right]$ at unit cost c_L. If firm H deviates by choosing $\overline{p}_H = c_H + \frac{1}{2}[b(s_H - s_L) - (c_H - c_L)] > c_H$, the consumer who is indifferent between high quality sold at $\overline{p}_H$ and low quality sold at unit cost must have a preference parameter $\overline{\theta} = \frac{\overline{p}_H - c_L}{s_H - s_L} = \frac{1}{2}[b + \frac{c_H - c_L}{s_H - s_L}]$. Post deviation, the H firm earns $\overline{\Pi}_H = \frac{1}{b}(\overline{p}_H - c_H)(b - \overline{\theta}) > 0$. This completes the proof.

Appendix 2: Proof of lemma 1.

1. Define $A_H(b) \equiv \Pi_H(U_H, U_L) - \Pi_H(D_H, D_L)$ and $h(b) \equiv [1 + \frac{1}{2}\frac{\frac{c_H - c_L}{s_H - s_L} - \frac{c_H}{s_H}}{b - \frac{c_H - c_L}{s_H - s_L}}]^2$. Using Table 1, it is straightforward to show that $sign A_H(b) = sign[h(b) - \frac{(4s_H - s_L)^2}{8s_H^2}]$. Clearly $h(b)$ is positive, continuous and monotonically decreasing in b for $b \geq \frac{c_H - c_L}{s_H - s_L}$. Also, $\lim_{b \to [\frac{c_H - c_L}{s_H - s_L}]^+} h(b) = \infty$ and $\lim_{b \to \infty} h(b) = 1$. Because $\frac{9}{8} < \frac{(4s_H - s_L)^2}{8s_H^2}$ for all $0 < s_L < s_H$ we conclude that there exists a value $b_H > \frac{c_H - c_L}{s_H - s_L}$ such that *sign* $A_H(b) = sign\ [b_H - b]$.

2. Define $A_L(b) \equiv \Pi_H(U_H, U_L) - \Pi_H(D_H, D_L)$ and $g(b) \equiv -A_L(b)\frac{b}{s_L}$. Clearly, $sign\ g(b) = -sign\ A_L(b)$.

Straightforward computation yields $g(b) = \frac{1}{2}[\frac{c_H - c_L}{s_H - s_L} - \frac{c_L}{s_L}][\frac{c_H}{s_H} - \frac{c_L}{s_L}] - \frac{4s_H(s_H - s_L)}{(4s_H - s_L)^2}[\frac{1}{2}(b + \frac{c_H - c_L}{s_H - s_L}) - \frac{c_L}{s_L}]^2$. Clearly, the function $g(b)$, is continuous and monotonically decreasing in b for $b \geq \frac{c_H - c_L}{s_H - s_L}$. Also, $\lim_{b \to \infty} g(b) = -\infty$ and $\lim_{b \to [\frac{c_H - c_L}{s_H - s_L}]^+} g(b) = \frac{1}{2}[\frac{c_H - c_L}{s_H - s_L} - \frac{c_L}{s_L}][\frac{c_H}{s_H} - \frac{c_L}{s_L}] - \frac{4s_H(s_H - s_L)}{(4s_H - s_L)^2}[\frac{c_H - c_L}{s_H - s_L} - \frac{c_L}{s_L}]^2$. Using $[\frac{c_H - c_L}{s_H - s_L} - \frac{c_L}{s_L}] =$

$[\frac{c_H}{s_H}-\frac{c_L}{s_L}]\frac{s_H}{s_H-s_L}$, we obtain $sign\ [g(\frac{c_H-c_L}{s_H-s_L})] = sign\left[1-\frac{4s_H(s_H-s_L)}{(4s_H-s_L)^2}\frac{s_H}{s_H-s_L}\right] = sign\left[1-\frac{4s_H^2}{(4s_H-s_L)^2}\right]$, which is positive because $s_L < s_H$ entails $\frac{4s_H^2}{(4s_H-s_L)^2} < \frac{4}{9}$. Because $g(b)$ is continuous and monotonically decreasing in b, we conclude that there exists a $b_L > \frac{c_H-c_L}{s_H-s_L}$ such that $sign\ A_L(b) = sign\ [b-b_L]$. This completes the proof.

Appendix 3: Proof of proposition 3

Upon defining $z \equiv b_L - \frac{c_H-c_L}{s_H-s_L}$ and $y \equiv b_H - \frac{c_H-c_L}{s_H-s_L}$, we can write $\Pi_L(U_H,U_L) = \frac{4s_Ls_H(s_H-s_L)}{(4s_H-s_L)^2b}\left[\frac{c_H-c_L}{s_H-s_L}-\frac{c_L}{s_L}+\frac{z}{2}\right]^2$, $\Pi_H(D_H,D_L) = \frac{(s_H-s_L)}{2b}y^2$, and $\Pi_H(U_H,U_L) = \frac{4s_H^2(s_H-s_L)}{(4s_H-s_L)^2b}\left[\frac{1}{2}\left(\frac{c_H-c_L}{s_H-s_L}-\frac{c_H}{s_H}\right)+y\right]^2$. Using $[\frac{c_H-c_L}{s_H-s_L}-\frac{c_L}{s_L}] = [\frac{c_H}{s_H}-\frac{c_L}{s_L}]\frac{s_H}{s_H-s_L}$, once again, we write $\Pi_L(D_H,D_L) = \frac{s_L}{2b}\left[\frac{c_H-c_L}{s_H-s_L}-\frac{c_L}{s_L}\right]^2\frac{s_H-s_L}{s_H}$. It is straigthforward to show that $\Pi_L(D_H,D_L) = \Pi_L(D_H,D_L)$ for $z = 2\left[\frac{4s_H-s_L}{2\sqrt{2}s_H}-1\right]\left[\frac{c_H-c_L}{s_H-s_L}-\frac{c_L}{s_L}\right]$ and $\Pi_H(U_H,U_L)$ $=\Pi_H(D_H,D_L)$ for $y = \frac{1}{2}\frac{\left(\frac{c_H-c_L}{s_H-s_L}-\frac{c_H}{s_H}\right)}{\frac{4s_H-s_L}{2\sqrt{2}s_H}-1}$. Clearly $sign\ (b_H-b_L) = sign(y-z)$. Therefore a prisoner's dilemma exists if and only if $y > x$.This condition is satisfied when $4\left[\frac{4s_H-s_L}{2\sqrt{2}s_H}-1\right]^2 < \frac{s_L}{s_H}$ or $1.25s_H^2 - 4.24s_Ls_H + s_L^2 < 0$. The latter condition is met for $s_H < 3.17s_L$. This completes the proof.

Appendix 4: Equilibria under different regimes when the distribution of preferences is uniform

We first show the derivation of profits shown in Table 1. Then we use a numerical example to illustrate when the prisoner's dilemma arises.We denote by $p_i^{jk}(\theta)$ the price targeted at consumer θ by the firm producing quality $i \in \{H,L\}$ when it chooses policy $j \in \{D,U\}$ and its rival chooses $k \in \{D,U\}$.

REGIME (U_H, D_L)

The best response of the L-firm to a commitment by the H-firm is given by (11) in the text The profit of the H-firm is then

$$\Pi_H(U_H,D_L) = \frac{1}{b}[p_H - c_H][b - \frac{p_H-c_L}{s_H-s_L}]$$

It attains a maximimum for

$$p_H^{U_H,D_L} = \frac{1}{2}[c_H + c_L + b(s_H - s_L)]$$

implying

$$\Pi_H(U_H,D_L) = \frac{1}{2b}\left[c_L - c_H + b\left(s_H - s_L\right)\right]\left[b - \frac{1}{2}\left(\frac{c_H-c_L}{s_H-s_L}+b\right)\right] = \frac{s_H-s_L}{4b}[b - \frac{c_H-c_L}{s_H-s_L}]^2$$

Note that $b > \frac{c_H-c_L}{s_H-s_L}$ entails $p_H^{U_H,D_L} > c_H$. Substitution of $p_H^{U_H,D_L}$ into (11) yields

$$p_L^{U_H,D_L}(\theta) = \begin{cases} c_L & for \quad \theta \geq \frac{1}{2}\left[\frac{c_H-c_L}{s_H-s_L}+b\right] \\ \frac{c_H+c_L+(b-2\theta)(s_H-s_L)}{2} & for \quad \frac{1}{2}\frac{c_H+c_L+b(s_H-s_L)}{s_H} < \theta < \frac{1}{2}[\frac{c_H-c_L}{s_H-s_L}+b] \\ \theta s_L & for \quad \frac{c_L}{s_L} \leq \theta \leq \frac{1}{2}\frac{c_H+c_L+b(s_H-s_L)}{s_H} \end{cases}$$

The total number of units sold by the L-firm is $\frac{1}{b}\left(\frac{1}{2}[\frac{c_H-c_L}{s_H-s_L}+b]-\frac{c_L}{s_L}\right)$. The low quality buyer who pays the highest price has $\theta = \frac{1}{2}\frac{c_H+c_L+b(s_H-s_L)}{s_H}$, and pays $\frac{c_H+c_L+b(s_H-s_L)}{2} - \frac{c_H+c_L+b(s_H-s_L)}{2s_H}(s_H-s_L) = \frac{c_H+c_L+b(s_H-s_L)}{2}\frac{s_L}{s_H}$.

The profit earned from that consumer is $\left(\frac{c_H-c_L+2c_L+b(s_H-s_L)}{2s_H}\right)s_L - c_L = \frac{(s_H-s_L)}{2}\frac{s_L}{s_H}\left(b+\frac{c_H-c_L}{s_H-s_L}\right)+\frac{s_L}{s_H}c_L - c_L = \frac{(s_H-s_L)}{2}\frac{s_L}{s_H}\left(b+\frac{c_H-c_L}{s_H-s_L}\right) - \frac{(s_H-s_L)}{s_H}c_L = \frac{s_L(s_H-s_L)}{s_H}\left[\frac{1}{2}\left(b+\frac{c_H-c_L}{s_H-s_L}\right)-\frac{c_L}{s_L}\right]$. Because the distribution of $\theta's$ is uniform, the average profit per unit sold by the L-firm is half that amount. Therefore, $\Pi_L = \frac{1}{2b}\frac{s_L(s_H-s_L)}{s_H}[\frac{1}{2}(b+\frac{c_H-c_L}{s_H-s_L})-\frac{c_L}{s_L}]^2$

REGIME (D_H, U_L)

When the $L-$commits to a uniform price $p_L \in]c_L, c_H[$, the H-firm responds by choosing (16). The profit of the $L-$ firm is

$$\Pi_L(D_H,U_L) = \frac{1}{b}[p_L - c_L][\frac{c_H-p_L}{s_H-s_L} - \frac{p_L}{s_L}]$$

It attains a maximum for

$$p_L^{D_H,U_L} = \frac{1}{2}\left[c_L + \frac{s_L}{s_H}c_H\right] = \frac{s_L}{2}\left[\frac{c_H}{s_H}+\frac{c_L}{s_L}\right]$$

.

implying $p_L^{D_H,U_L} - c_L = \frac{s_L}{2}\left[\frac{c_H}{s_H}-\frac{c_L}{s_L}\right]$. Also, $\frac{c_H-p_L}{s_H-s_L} = \frac{2c_H-c_L-\frac{s_L}{s_H}c_H}{s_H-s_L} = \frac{1}{2}\left(\frac{c_H-c_L}{s_H-s_L}+\frac{c_H}{s_H}\right)$, and $\frac{p_L^{D_H,U_L}}{s_L} = \frac{1}{2}(\frac{c_L}{s_L}+\frac{c_H}{s_H})$. Because $s_L < s_H$, it must be true that $p_L^{D_H,U_L} \in]c_L, c_H[$.Therefore the profit of the L-firm is $\Pi_L = \frac{1}{b}\frac{s_L}{2}\left[\frac{c_H}{s_H}-\frac{c_L}{s_L}\right]\left[\frac{1}{2}\left(\frac{c_H-c_L}{s_H-s_L}+\frac{c_H}{s_H}\right)-\frac{1}{2}(\frac{c_L}{s_L}+\frac{c_H}{s_H})\right] = \frac{s_L}{4b}\left[\frac{c_H}{s_H}-\frac{c_L}{s_L}\right]\left[\frac{c_H-c_L}{s_H-s_L}-\frac{c_L}{s_L}\right]$.The market segment

served by the H-firm is $b - \frac{c_H-p_L}{s_H-s_L} = b - \frac{2c_H--c_L-\frac{s_L}{s_H}c_H}{2(s_H-s_L)} = b - \frac{c_H--c_L+(1-\frac{s_L}{s_H})c_H}{2(s_H-s_L)} = b - \frac{1}{2}\left[\frac{c_H-c_L}{s_H-s_L}+\frac{c_H}{s_H}\right]$. Substitution of $p_L^{D_H,U_L}$into (16) yields :

$$p_H^{D_H,U_L}(\theta) = \begin{cases} \frac{1}{2}\left[c_L+\frac{s_L}{s_H}c_H\right]+\theta(s_H-s_L) & for \quad \frac{1}{2}\left[\frac{c_H-c_L}{s_H-s_L}+\frac{c_H}{s_H}\right] \leq \theta \leq b \\ c_H & for \quad \theta < \frac{1}{2}\left[\frac{c_H-c_L}{s_H-s_L}+\frac{c_H}{s_H}\right] \end{cases}$$

The average profit margin of the H-firm over the segment it serves is $\frac{1}{2}\left(p_H^{D_H,U_L}(b)-c_H\right) = \frac{1}{2}\left[\frac{1}{2}\left(c_L+\frac{s_L}{s_H}c_H\right)+b\,(s_H - \right.$ $\frac{s_H-s_L}{2}\left[b-\frac{c_H-c_L}{2(s_H-s_L)}-\frac{s_H-s_L}{2(s_H-s_L)}\frac{c_H}{s_H}\right] = \frac{s_H-s_L}{2}\left[b-\frac{1}{2}\left(\frac{c_H-c_L}{(s_H-s_L)}+\frac{c_H}{s_H}\right)\right]$. Thus, the profit of the H-firm is $\Pi_H(D_H,U_L) =$

$\frac{s_H-s_L}{2b}\left[b-\frac{1}{2}\left(\frac{c_H-c_L}{(s_H-s_L)}+\frac{c_H}{s_H}\right)\right]^2$

2. When $p_L \geq c_H$. the$H-$firm sells to all consumers with $\theta \in \left[\frac{c_H}{s_H}, b\right]$ for a price $p_H(\theta) = \theta s_H$. No consumer with $\theta \in \left[\frac{c_L}{s_L}, \frac{c_H}{s_H}\right]$ is willing to purchase low quality product at the uniform price $p_L \geq c_H$. More pointedly, when $p_L \geq c_H$, the high quality producer has a monopoly position and the low quality producer has no market at all. Therefore, choosing $p_L \geq c_H$ is never rational on the part of a leader who produces the low quality.

REGIME(U_H, U_L)

This is the standard case examined in the literature[18]. The profits are

$$\Pi_H(p_H, p_L) = \frac{1}{b}(p_H - c_H)(b - \frac{p_H - p_L}{s_H - s_L})$$

$$\Pi_L(p_H, p_L) = \frac{1}{b}(p_L - c_L)(\frac{p_H - p_L}{s_H - s_L} - \frac{p_L}{c_L})$$

Simultaneous choice of prices yields

$$p_H^{U_H,U_L} = \frac{s_H}{4s_H - s_L}[2b(s_H - s_L) + 2c_H + c_L]$$

$$p_L^{U_H,U_L} = \frac{1}{4s_H - s_L}[bs_L(s_H - s_L) + 2c_L s_H + c_H s_L]$$

Substitution into the profit function yields the profits that appear in Table 1.

REGIME (D_H, D_L)

The profits appearing in Table 1 are simply obtained by substitution of (7) and (8) into (6), with $f(\theta) = \frac{1}{b}$ for all $\theta \in [0, b]$

EXISTENCE OF A PRISONER'S DILEMMA: a NUMERICAL EXAMPLE

Take $s_L = 1, c_L = 1, s_H = 2, c_H = 4$ and note that these values satisfy condition (2), and that $\frac{s_H}{s_L} < 3.17$ (see proposition 3). In order to check whether a prisonner's dilemma occurs, we consider two different values of b, specifically $b_1 = 4$ and $b_2 = 8$. We easily check that $b_1 \in [b_L, b_H]$ and $b_2 > b_H$, where b_L and b_H are the thresholds defined in the lemma

When $b_1 = 4$, we find that

1) $p_H^{U_H,D_L} = \frac{9}{2}$and $p_L^{U_H,D_L}(\theta, p_H) = \begin{cases} 1 & for \quad \theta \in \left[\frac{7}{2}, 4\right] \\ \frac{9}{2} - \theta & for \quad \theta \in \left]\frac{13}{4}, \frac{7}{2}\right[\\ \theta & for \quad \theta \in \left[1, \frac{13}{4}\right] \end{cases}$ implying, $\Pi_H(U_H, D_L) = \frac{1}{16}$ and $\Pi_L(U_H, D_L) = \frac{25}{64}$

2) $p_L^{D_H,U_L} = \frac{3}{2}$ and $p_H^{D_H,U_L}(\theta, p_L) = \begin{cases} \frac{3}{2} + \theta & for \quad \theta \in \left[\frac{1}{2}, 4\right] \\ 4 & for \quad \theta < \frac{1}{2} \end{cases}$ implying $\Pi_H(D_H, U_L) = \frac{9}{32}$ and $\Pi_L(D_H, U_L) = \frac{2}{16}$

[18]Gabszewicz and Thisse (1979), Shaked and Sutton (1982), Moorthy (1988, 1991).

3) $p_H^{D_H,D_L}(\theta) = \begin{cases} 1+\theta & for \quad \theta \in [3,4] \\ 4 & for \quad \theta < 3 \end{cases}$ and $p_L^{D_H,D_L}(\theta) = \begin{cases} 1 & for \quad \theta \in [3,4] \\ 4-\theta & for \quad \theta \in]2,3[\\ \theta & for \quad \theta \in [1,2] \end{cases}$ implying $\Pi_H(D_H,D_L) = \frac{1}{8}$ and $\Pi_L(D_H,D_L) = \frac{2}{8}$

4) $p_H^{U_H,U_L} = \frac{34}{7}$and $p_L^{U_H,U_L} = \frac{12}{7}$implying $\Pi_H(U_H,U_L) = \frac{9}{49}$ and $\Pi_L(U_H,U_L) = \frac{25}{98}$,

When $b_2 = 8$, we find that

1) $p_H^{U_H,D_L} = \frac{13}{2}$ and $p_L^{U_H,D_L}(\theta,p_H) = \begin{cases} 1 & for \quad \theta \in \left[\frac{11}{2},8\right] \\ \frac{13}{2}-\theta & for \quad \theta \in \left]\frac{9}{4},\frac{11}{2}\right[\\ \theta & for \quad \theta \in \left[1,\frac{9}{4}\right] \end{cases}$ implying $\Pi_H(U_H,D_L) = \frac{25}{32}$ and $\Pi_L(U_H,D_L) = \frac{81}{256}$

2) $p_L^{D_H,U_L} = \frac{3}{2}$ and $p_H^{D_H,U_L}(\theta,p_L) = \begin{cases} \frac{3}{2}+\theta & for \quad \theta \in \left[\frac{5}{2},8\right] \\ 4 & for \quad \theta < \frac{5}{2} \end{cases}$ implying $\Pi_H(D_H,U_L) = \frac{121}{32}$ and $\Pi_L(D_H,U_L) = \frac{2}{32}$

3) $p_H^{D_H,D_L}(\theta) = \begin{cases} 1+\theta & for \quad \theta \in [3,8] \\ 4 & for \quad \theta < 3 \end{cases}$ and $p_L^{D_H,D_L}(\theta) = \begin{cases} 1 & for \quad \theta \in [3,8] \\ 4-\theta & for \quad \theta \in]2,3[\\ \theta & for \quad \theta \in [1,2] \end{cases}$ implying $\Pi_H(D_H,D_L) = \frac{25}{16}$ and $\Pi_L(D_H,D_L) = \frac{2}{16}$

4) $p_H^{U_H,U_L} = \frac{50}{7}$ and $p_L^{U_H,U_L} = \frac{16}{7}$ implying $\Pi_H(U_H,U_L) = \frac{242}{196}$ and $\Pi_L(U_H,U_L) = \frac{121}{196}$

We note that the regime (D_H,D_L) is an equilibrium in dominant strategies for $b = b_1$ as well as for $b = b_2$. We also observe that this equilibrium is Pareto-dominated by the (U_H,U_L) regime when $b = b_1$and is not dominated when $b = b_2$.