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Essais sur la gestion des ressources forestières

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RÉSUMÉ

Cette thèse est composée de trois essais en économie forestière. Les deux premiers s'intéressent à la fixation de la redevance optimale à laquelle fait face le propriétaire d'une ressource forestière dans un contexte d'information asymétrique. Le troisième analyse l'impact à long terme du recyclage sur la surface de terre affectée à la forêt.

La gestion des ressources forestières implique souvent la délégation des droits de coupe par le propriétaire forestier à une entreprise exploitante. Cette délégation prend la forme d'un contrat de concession par lequel le propriétaire forestier octroie les droits d'exploitation aux compagnies forestières, en contrepartie d'une redevance (transfert monétaire). L'octroie des droits d'exploitation s'effectue généralement sous plusieurs modes, dont les plus répandus sont les appels d'offres publics et les contrats de gré à gré, où le propriétaire forestier et la firme exploitante spécifient entre autres la redevance dans les clauses d'exploitation de la forêt. Pour déterminer le mécanisme optimal (choix de la firme, âge de coupe et redevance), le propriétaire forestier a idéalement besoin de connaître les coûts de coupe et de reboisement. Or en réalité, les firmes sont mieux informées sur leurs coûts que le propriétaire forestier. Dans ce contexte d'information asymétrique, le mécanisme optimal doit donc prendre en considération des contraintes informationnelles. Les deux premiers essais caractérisent, sous ces conditions, l'âge de coupe optimal (la rotation optimale) et la redevance optimale.

Le premier essai examine le contrat optimal quand le propriétaire forestier cède les droits de coupes à une firme par un accord de gré à gré ou par une procédure d'appel d'offre public au second prix. L'analyse du problème est menée premièrement dans un contexte statique, dans le sens que les coûts de coupe sont parfaitement corrélés dans le temps, puis dans un contexte dynamique, où les coûts sont indépendants dans le temps. L'examen en statique et en dynamique montre que la rotation optimale va satisfaire une version modifiée de la règle de Faustmann qui prévaudrait en information symétrique. Cette modification est nécessaire afin d'inciter la firme à révéler ses vrais coûts. Dans le cas statique, il en résulte que la rotation optimale est plus élevée en information asymétrique qu'en situation de pleine information. Nous montrons également comment le

seuil maximal de coût de coupe peut être endogénéisé, afin de permettre au propriétaire d'accroître son profit espéré en s'assurant que les forêts non profitables ne seront pas exploitées. Nous comparons ensuite la redevance optimale en information asymétrique et symétrique. Les redevances forestières dans un arrangement de gré à gré étant généralement, en pratique, une fonction linéaire du volume de bois, nous dérivons le contrat optimal en imposant une telle forme de redevance et nous caractérisons la perte en terme de profit espéré qui résulte de l'utilisation de ce type de contrat plutôt que du contrat non linéaire plus général. Finalement, toujours dans le contexte statique, nous montrons à travers un mécanisme optimal d'enchère au second prix qu'en introduisant ainsi la compétition entre les firmes le propriétaire forestier augmente son profit espéré.

Les résultats obtenus dans le contexte dynamique diffèrent pour la plupart de ceux obtenus dans le cas statique. Nous montrons que le contrat optimal prévoit alors que chaque type de firme, incluant celle ayant le coût le plus élevé, obtient une rente strictement positive, laquelle augmente dans le temps. Ceci est nécessaire pour obtenir la révélation à moindre coût à la période courante du véritable type de la firme. Comme implication, la rotation optimale s'accroît aussi dans le temps. Finalement, nous montrons qu'il y a distorsion en asymétrique d'information par rapport à l'optimum de pleine information même pour le coût le plus bas (la réalisation la plus favorable).

La concurrence introduite dans le premier essai sous forme d'enchère au second prix suppose que chaque firme connaît exactement son propre coût de coupe. Dans le deuxième essai nous relâchons cette hypothèse. En réalité, ni le propriétaire forestier ni les firmes ne connaissent avec précision les coûts de coupe. Chaque firme observe de manière privée un signal sur son coût. Par exemple chaque firme est autorisée à visiter un lot pour avoir une estimation (signal) de son coût de coupe. Cependant cette évaluation est approximative. Ainsi, le coût de chaque firme va dépendre des estimations (signaux) d'autres firmes participantes. Nous sommes en présence d'un mécanisme à valeurs interdépendantes. Dans ce contexte, la valeur d'une allocation dépend des signaux de toutes les firmes. Le mécanisme optimal (attribution des droits d'exploitation, redevance et âge de coupe) est exploré. Nous déterminons les conditions sous lesquelles le mécanisme optimal peut être implémenté par une enchère au second prix et dérivons

la rotation optimale et le prix de réserve dans le contexte de ce type d'enchère.

Le troisième essai de la thèse analyse l'impact à long terme du recyclage sur la surface de terre affectée à la forêt. L'un des principaux arguments qui milite en faveur du recours au recyclage est que cela entraînerait une réduction de la coupe de bois, épargnant ainsi des arbres. L'objectif est donc d'aboutir à un nombre d'arbres plus important qu'en l'absence de recyclage. L'idée d'accroître le stock d'arbre tient au fait que les forêts génèrent des externalités : elles créent un flux de services récréatifs, freinent l'érosion des sols et des rives des cours d'eau et absorbent du dioxyde de carbone présent dans l'atmosphère. Étant donné la présence d'externalités, l'équilibre des marchés résulterait en un nombre d'arbre insuffisant, justifiant donc la mise en oeuvre de politiques visant à l'accroître. Le but de ce troisième essai est de voir dans quelle mesure la promotion du recyclage est un instrument approprié pour atteindre un tel objectif. En d'autres mots, comment le recyclage affecte-t-il à long terme la surface de terre en forêt et l'âge de coupe ? Nous étudions cette question en spécifiant un modèle dynamique d'allocation d'un terrain donné, par un propriétaire forestier privé, entre la forêt et une utilisation alternative du terrain, comme l'agriculture. Une fois les arbres coupés, il décide d'une nouvelle allocation du terrain. Il le fait indéfiniment comme dans le cadre du modèle de Faustmann. Le bois coupé est transformé en produit final qui est en partie recyclé comme substitut du bois original. Ainsi, les outputs passés affectent le prix courant. Nous montrons que, paradoxalement, un accroissement du taux de recyclage réduira à long terme la surface forestière et donc diminuera le nombre d'arbres plantés. Par contre l'âge de coupe optimal va s'accroître. L'effet net sur le volume de bois offert sur le marché est ambigu. Le principal message cependant est qu'à long terme le recyclage va résulter en une surface en forêt plus petite et non plus grande. Donc, si le but est d'accroître la surface en forêt, il pourrait être préférable de faire appel à d'autres types d'instruments de politique que celui d'encourager le recyclage.

Mots clés: Règle de Faustmann, Rotation optimale, Asymétrie d'information, Sélection adverse, Redevance, Valeur interdépendante, Enchère, Contrats de gré à gré, Recyclage, Allocation du terrain, Utilisation alternative, Contrats récursifs.

ABSTRACT

This thesis consists of three essays. The first two deal with the design of optimal royalty contracts for forestry exploitation under asymmetric information. The third examines the impact of recycling on the long-run forestry.

The management of forest resources often involves the delegation of the harvesting operation by the forest owner to a harvesting firm. This delegation takes the form of a concession contract in which the forest owner leases logging rights to companies specialized in planting and harvesting, in return for preestablished royalty payments. The royalty (monetary transfers) can be set through different methods. For example, the forest owner can organize an auction among firms. Another way is to negotiate directly with a single firm the terms of the exploitation of the forest and hence the monetary transfers. To set the royalty schedule, the forest owner ideally needs to know the firms' costs, namely the harvesting and planting costs. In practice however firms are better informed about their costs than the forest owner. Under this asymmetry of information, the optimal royalty must therefore take into account informational constraints. The first two essays characterize the optimal royalty and the optimal rotation period under those conditions.

The first essay analyzes the optimal contract under the assumption that the harvesting cost of each firm is perfectly known to itself but not to the forest owner. The problem is examined both in a static context, where the costs are perfectly correlated over time, and in a dynamic context where the costs are intertemporally independent. It is shown that both in the static and in the dynamic cases, the optimal rotation will satisfy a modified version of the Faustmann rule which holds under symmetric information, the modification being necessary in order to induce cost revelation on the part of the harvesting firm. As a result, looking first at the static case, the optimal rotation period will be longer in the asymmetric information case than in the symmetric information case. It is also shown how the cut-off cost can be endogenized, thus increasing the owner's expected profit by making sure that unprofitable forests are not exploited. Finally the comparison is made of the royalty in the symmetric and asymmetric information cases. Because fo-

rest contracts are in practice typically linear in the volume harvested, the optimal royalty is derived under the constraint that it is a linear function of the volume harvested and the loss in expected welfare from using a linear contract instead of the theoretically more general nonlinear contract is characterized. Finally, still in the static context, it is shown that the forest owner could raise its expected profit by allowing competition among firms through public auctions. It is shown in the dynamic context that, unlike in the static case, all firms, including the highest-cost type, get a strictly positive rent. It is also shown that the firm's rent rises over time. This is necessary in order to get revelation at a lower cost in the current period. Therefore, the optimal rotation increases over time as well. The optimal contract under asymmetry of information also has the effect of distorting the lowest-cost firm in this case.

In the second essay, the assumption that the harvesting cost of each firm is perfectly known to itself is dropped. Indeed, in practice neither the forest owner nor the firms know the costs perfectly. Each firm only observes a signal of its cost. For example each firm may be allowed to survey a tract of forest to obtain an estimate (signal) of its cutting cost. Given that its observation is imperfect, a firm's cost will therefore depend on estimations (signals) by the other firms as well, which are private information. This second essay then raises the problem of auction design with firms whose values (costs) are interdependent. In this context, the value of an allocation will depend on the signals of all of the participating firms. The optimal contract is characterized and the conditions under which the optimal mechanism can be implemented by a second price auction are explored. The optimal rotation and the reservation price are derived under this auction mechanism.

The third essay studies the effect of recycling on the land area devoted to forestry in the long run. Interest in recycling of forest products has grown in recent years, one of the goals being to conserve trees or possibly increase their number to compensate for positive externalities generated by the forest and neglected by the market. This paper explores the issue as to whether recycling is an appropriate measure to attain such a goal. We do this by considering the problem of the private owner of an area of land, who, acting as a price taker, decides how to allocate his land over time between forestry

and some other use, and at what age to harvest the forest area chosen. Once the forest is cut, he makes a new land allocation decision and replants. He does so indefinitely, in a Faustmann-like framework. The wood from the harvest is transformed into a final product which is partly recycled into a substitute for the virgin wood, so that past output affects the current price. We show that in such a context, increasing the rate of recycling will result in less area being devoted to forestry. It will also have the effect of increasing the harvest age of the forest, as long as the planting cost is positive. The net effect on the flow of virgin wood being harvested to supply the market will as a result be ambiguous. An important point however is that recycling will result in less trees in the long run, not more. It would therefore be best to resort to other means if the goal is to conserve the area devoted to forestry.

Keywords : Faustmann rule, Optimal forest rotation, Dynamic contracts, Mechanism design, Auction, Adverse selection, Recycling, Land allocation, Alternative uses.

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À la mémoire de mon père, Jacques Poqua.

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INTRODUCTION GÉNÉRALE

L'une des questions traditionnelles dans la gestion des ressources forestière est l'âge de coupe optimal des arbres. L'ingénieur forestier Faustmann (1849) a répondu à cette question en définissant l'âge de coupe optimal comme celui qui maximise la valeur présente d'une séquence infinie de récoltes. Il montre qu'en l'absence d'incertitude et de variation des prix, l'âge de coupe optimal doit être choisi de telle sorte que la valeur de la croissance courante soit égale au coût d'option des arbres debout plus le coût d'option du terrain. Cette règle d'arbitrage inter-temporelle est connue sous le nom de règle de Faustmann.

Dans la détermination de l'âge de coupe optimal, Faustmann suppose implicitement que le propriétaire forestier est l'exploitant de la ressource. En réalité, le propriétaire forestier octroie le plus souvent les droits d'exploitation à des compagnies forestières, en contrepartie d'une redevance. Les deux premiers chapitres de cette thèse déterminent la redevance optimale que doit fixer le propriétaire forestier dans un contexte d'information asymétrique. Le troisième chapitre analyse dans le cadre du modèle de Faustmann l'impact à long terme du recyclage sur le stock d'arbre en forêt.

Dans la plupart des pays, l'octroie des droits d'exploitation s'effectue généralement sous plusieurs modes, dont les plus répandus sont les appels d'offres publics et les contrats de gré à gré, où le propriétaire forestier et la firme exploitante spécifient entre autres la redevance dans les clauses d'exploitation de la forêt. Par exemple, dans le cas du Canada la majeure partie des forêts est du domaine public (approximativement 94% des forêts commerciales sont détenues par les gouvernements provinciaux et le gouvernement fédérale) et est entièrement récoltée par des compagnies forestières privées, dans le cadre de contrats de concession ou par des enchères. Aux USA et en Grande Bretagne, les enchères forestières sont fréquemment utilisées, tandis que certains pays comme la Norvège, la Suède, la Finlande ou l'Allemagne optent plutôt pour des contrats de gré à gré, sous forme de négociation directe entre le propriétaire forestier et la compagnie exploitante.

Dans les deux cas, pour déterminer le mécanisme optimal (choix de la firme, âge de coupe et redevance) le propriétaire forestier a idéalement besoin de connaître les coûts de coupe et de reboisement des firmes. Si les coûts de coupe et de reboisement étaient parfaitement connus à la fois des firmes et du propriétaire forestier et que le prix unitaire par volume de bois était constant et connu de tous, alors, dans un contrat de gré à gré, la redevance optimale sera fixée de telle sorte que la rotation optimale vérifie la règle de Faustmann. Dans le cas d'une concurrence entre les firmes, le propriétaire forestier va céder les droits de coupe à la firme ayant le coût le plus bas et fixera la redevance optimale de manière à ce que l'âge de coupe vérifie aussi la règle de Faustmann. En réalité les firmes sont mieux informées sur leurs coûts que le propriétaire forestier. Dans ce contexte d'information asymétrique, le mécanisme optimal doit donc prendre en considération les contraintes informationnelles. Les deux premiers chapitres caractérisent, sous ces conditions, le mécanisme optimal.

Le premier chapitre examine le contrat optimal principalement dans le cas d'un accord de gré à gré, bien que la compétition soit introduite plus tard sous forme d'un appel d'offre public au second prix. Il s'agit dans ce chapitre de caractériser la rotation optimale et la redevance optimale sous sélection adverse, de les comparer à celles qui prévaudraient en pleine information, et enfin d'analyser leurs évolutions dans le temps. Dans le cas d'un contrat de gré à gré, le problème soulevé s'insère dans un cadre plus général, notamment celui du problème du "principal-agent" avec sélection adverse. La firme (l'agent) est mieux informée sur son coût de coupe que le propriétaire forestier (le principal).

Plusieurs articles et livres modernes ont traité du problème général de sélection adverse (voir par exemple Green et Laffont (1979), Baron et Myerson (1982), et Townsend (1979)), et quelques auteurs ont appliqué la théorie du principal-agent aux ressources naturelles. Par exemple, Gaudet, Lasserre, et Long (1995) ont analysé la redevance optimale dans le cas d'une ressource non renouvelable lorsque l'agent à qui l'exploitation est déléguée à une information privée sur les coûts d'extraction. À ma connaissance la sélection adverse n'a pas encore eu d'application en économie forestière dans le cadre du modèle de Faustmann.

Dans ce chapitre, l'analyse du problème est menée suivant deux grands axes. Dans une première partie, nous supposons que les coûts de coupe sont parfaitement corrélés dans le temps. Ceci fait que le problème peut à toute fin pratique être considéré comme un problème statique, dans la mesure où le propriétaire forestier propose alors le même contrat à chaque rotation. Nous supposons que celui-ci s'engage à appliquer le même contrat indéfiniment pour les récoltes futures, sans possibilité de renégociation. Nous montrons qu'en situation d'information asymétrique le contrat optimal prévoit que les firmes conservent une rente strictement positive, sauf pour la firme ayant le coût le plus élevé. Ceci a pour résultat que la rotation optimale est plus longue en information asymétrique qu'en information symétrique, sauf pour la firme ayant le coût le plus bas, et qu'elle satisfait une version modifiée de la règle de Faustmann. Nous montrons également qu'il peut être possible pour le propriétaire d'accroître son profit espéré en fixant un seuil de participation maximal qui exclut les firmes ayant des coûts trop élevés pour qu'il soit rentable de permettre l'exploitation. Finalement, les redevances étant souvent en pratique exprimées comme une fonction linéaire du volume du bois, nous utilisons les paramètres de la littérature empirique sur les enchères afin d'approximer le gain espéré pour migrer d'un contrat linéaire à un contrat non linéaire beaucoup plus général. La concurrence entre les firmes est introduite à la Section 3 du chapitre, où il est supposé qu'au lieu d'un contrat de gré à gré le propriétaire forestier procède par un appel d'offre au second prix. Dans ce contexte, il est montré que procéder ainsi améliore que le profit espéré du propriétaire forestier.

Dans une deuxième partie de ce chapitre, nous analysons le contrat optimal dans le cadre d'un accord de gré à gré en supposant maintenant une absence de corrélation intertemporelle entre les coûts de coupe. Il s'agit alors d'un problème proprement dynamique à horizon infini qui peut être résolu dans le cadre de l'approche récursive introduite par Green (1987). Certains résultats diffèrent de ceux obtenus dans le cadre statique de la première partie. Nous montrons que le contrat optimal prévoit alors que chaque type de firme, incluant celle ayant le coût le plus élevé, obtient une rente strictement positive. Cette rente augmente dans le temps. Ceci est nécessaire pour obtenir la révélation à moindre coût à la période courante du véritable type de la firme. Il vient alors que la rotation optimale augmente aussi dans le temps. Finalement nous établissons qu'il y a distorsion en situation d'asymétrique d'information par rapport à l'optimum de pleine information même pour le coût le plus bas (la réalisation la plus favorable).

Le deuxième chapitre de la thèse détermine le mécanisme optimal quand plusieurs firmes sont en concurrence pour acquérir les droits d'exploitation de la forêt et que leurs coûts sont interdépendants. Alors que la concurrence introduite au Chapitre 1 sous forme d'enchère au second prix a été faite sous l'hypothèse que chaque firme connaît parfaitement son propre coût de coupe, dans ce deuxième chapitre nous relâchons cette hypothèse. En réalité, ni le propriétaire forestier ni les firmes ne connaissent avec précision les coûts de coupe. Chaque firme observe seulement en privé un signal sur son coût. Par exemple, chaque firme est autorisée à visiter un lot pour avoir une estimation (signal) de son coût de coupe. Cependant, cette évaluation est approximative. Ainsi, le coût de chaque firme va dépendre des estimations (signaux) de ses concurrents. Nous sommes ainsi en présence d'un mécanisme à valeurs interdépendantes. Dans cette situation, la valeur d'une allocation dépend des signaux de toutes les firmes participantes. Ce chapitre a pour but de caractériser le mécanisme optimal dans un tel contexte et de déterminer sous quelles conditions le mécanisme optimal peut être implémenté par un appel d'offre au second prix.

De nombreux travaux ont contribué à développer une littérature abondante liée aux mécanismes à valeurs interdépendantes. McAfee et al. (1989) ont montré que lorsque les signaux sont corrélés, le vendeur peut extraire la totalité du surplus des acheteurs. Toutefois lorsque les signaux sont indépendants, Branco (1996) a montré qu'il n'est pas possible d'extraire entièrement le surplus des acheteurs. Dans le même ordre d'idée que Branco, nous considérons le cas où les signaux sont indépendants. La littérature sur les mécanismes considère en générale une règle d'attribution et un transfert monétaire. Dans notre modèle, les transferts monétaires sont effectués périodiquement à chaque rotation sous forme de redevances. Le propriétaire forestier doit déterminer à la fois les redevances et la rotation. Par conséquent la définition de notre mécanisme se démarque légèrement de la littérature et consiste en une règle d'attribution des droits d'exploitation, une règle de détermination de la rotation et une règle de détermination des redevances. Une autre caractéristique intéressante du mécanisme optimal présenté dans ce chapitre est qu'il nous sert de scénario de référence pour l'implémenter par les enchères traditionnelles. En particulier, dans la Section 3 nous nous intéressons aux conditions sous lesquelles un appel d'offre au second prix avec prix de réserve est optimal. L'appel d'offre au second prix a été choisi pour des raisons de simplifications. Toutefois Riley et Samuelson (1981) ont établi que pour un choix approprié du prix de réserve, les enchères standards sont équivalentes.

Le troisième chapitre de la thèse analyse l'impact à long terme du recyclage sur la surface de terre affectée à la forêt. En effet, l'un des principaux arguments qui milite en faveur du recours au recyclage du papier est que cela entraînera une diminution de la coupe de bois, épargnant ainsi des arbres. Le but visé est donc d'aboutir à une superficie de terre affectée à la forêt plus élevée qu'en l'absence de recyclage. Ceci peut se justifier par le fait que les forêts génèrent des externalités positives. Par exemple, elles créent un flux de services récréatifs, freinent l'érosion des sols et des rives des cours d'eau et absorbent du dioxyde de carbone présent dans l'atmosphère. Étant donné la présence d'externalités, l'équilibre des marchés résulterait en trop peu de terre forestière, justifiant donc la mise en oeuvre de politiques visant à l'accroître. L'objectif de ce troisième chapitre est de voir dans quelle mesure la promotion du recyclage est un instrument approprié pour atteindre un tel objectif.

L'étude de cette question se fera en spécifiant un modèle dynamique simple d'allocation par un propriétaire privé d'un terrain donné entre la forêt et une utilisation alternative du terrain, comme l'agriculture. Le modèle prend en compte le fait que le bois coupé est transformé en produit final qui est en partie recyclé comme substitut du bois original. À chaque date de plantation, le propriétaire forestier choisit d'une part la surface de terrain forestier et d'autre part l'âge de coupe des arbres. On examinera donc l'impact à long terme du recyclage à la fois sur la surface forestière et sur l'âge de coupe optimal des arbres.

On trouve dans la littérature un certain nombre d'articles traitant du problème d'allocation du terrain entre des utilisations compétitives. Par exemple, Ehui, Hertel, et Prectel (1990) utilisent un modèle dynamique à deux secteurs pour étudier, dans un pays en développement, l'allocation optimale du terrain entre l'agriculture et la forêt. Le même modèle a aussi été utilisé par Ehui et Hertel (1989) pour estimer l'état stationnaire optimal du stock de forêt en Côte d'Ivoire. Hartwick, Long et Tian (2001) ont utilisé un modèle dynamique pour analyser la déforestation dans une petite économie avec une large dotation forestière et une faible dotation en agriculture et faisant face aux prix internationaux à la fois pour les produits forestiers et agricoles. Tous ces articles supposent que les produits forestiers sont totalement consommés dans un seul usage et par conséquent excluent le recyclage. De plus, aucun de ces articles n'utilisent la règle de Faustmann dans la détermination de la surface forestière et de l'âge de coupe optimal ; dans certains cas, ils traitent la forêt comme une ressource non renouvelable. La même critique est aussi valable pour Darby (1973), qui, dans une courte note, argumente qu'en réduisant la demande de bois une augmentation du recyclage va diminuer le nombre d'arbres à planter. Dans une certaine mesure, nous formalisons rigoureusement l'argument de Darby en situant notre travail dans le cadre du modèle de Faustmann et en prenant en compte à la fois l'effet dans le temps sur la rotation et sur la surface de terrain allouée à la forêt. Nous montrons que, paradoxalement, un accroissement du taux de recyclage va à long terme réduire la surface forestière et donc diminuer le nombre d'arbre à planter, non l'augmenter. Ceci aura pour effet d'accroître l'âge de coupe optimal, en autant que le coût de reboisement soit strictement positif. Cependant l'effet net sur le volume de bois offert sur le marché demeure ambigu. Le principal message cependant est qu'à long terme le recyclage va résulter en une surface en forêt plus petite et non plus grande. Donc, si le but est d'accroître la surface en forêt, il pourrait être préférable de faire appel à d'autres types d'instruments de politique que celui d'encourager le recyclage.

CHAPITRE 1

OPTIMAL FORESTRY CONTRACTS UNDER ASYMMETRY OF INFORMATION

1.1 Introduction

The management of forest resources often involves the delegation of the harvesting operation by the forest owner to a harvesting firm. This delegation takes the form of a concession contract in which the forest owner leases logging rights to companies specialized in planting and harvesting, in return for preestablished royalty payments. For example, forest land in Canada, which is largely public (approximately 94 percent of the commercial forests are owned by the provincial and federal governments) [1] is harvested entirely by private forest firms, through lease agreements with the provincial or federal governments. [2] The question then arises : What is the optimal royalty contract from the point of view of the forest owner ?

Such a contract should induce the best time to harvest a tree or stand of trees, determining hence the optimal rotation period. If the growth function is known and if price, planting and harvesting costs are constant and known by both the forest owner and the exploiting firm, the answer is straightforward. The Faustmann rule (Faustmann, 1849) dictates that if the forest owner wishes to maximize the value of the forest land (land owner's benefit) from planting and harvesting, the royalty schedule must induce the firm to harvest when the increase in the net value of the standing forest over a unit time interval (rotation period) is equal to the interest on the value of the stand plus the interest on the value of the forest land.

To set such a royalty schedule, the forest owner ideally needs to know the firm's

1. Source : Natural Resources Canada (www.canadaforests.nrcan.gc.ca).

2. The forest owner can also delegate the exploitation of the forest to a harvesting firm through public auctions. This method is frequently use in several forest countries like US, Great Britain... Auction has the advantage that it creates competition among firms competing for a contract. Nevertheless, standing timber auctions are unusual in other countries, such as Norway, Finland, Germany or Sweden : there are negotiations between landowner and forest company (Toivonen, 1997).

costs, namely the harvesting and planting costs. In practice however, the exact costs are known only to the harvesting firm, although the owner may be aware of their distribution. This information asymmetry creates a situation where adverse selection may occur and the optimal royalty must therefore take into account informational constraints. What will be the optimal rotation period and optimal royalty under adverse selection, compared to that which would prevail under full information ? How will it vary over time ? The problem constitutes an application of the well known principal-agent problem with adverse selection.

Adverse selection issues are now well known, and appear in many papers and in modern textbooks (see for example Green and Laffont (1979), Baron and Myerson (1982), and Townsend (1979)).[3] A few authors have applied principal-agent theory to natural resource problems. Gaudet, Lasserre, and Long (1995) study optimal nonrenewable resource royalty contracts when the extracting agent has private information on the costs. In the context of forestry, Bowers (2003) puts into a principal–agent framework the problem a forest owner faces when choosing policy instruments for sustainability in the privately operated forest industry. Many other studies have focused on the impact of various forest taxes on the optimal rotation using the Faustmann or the Hartman model, but with no adverse selection involved.[4] Koskela and Ollikainen (2001) study the impact of harvesting, property and profit taxes on the rotation period in the Hartman model. In a recent paper Alvarez and Koskela (2007) use both single and ongoing rotations to analyze the effect of different forest taxes on the privately optimal rotation under uncertainty. To the best of my knowledge the tree cutting problem under private information on the planting or the harvesting cost and adverse selection has not yet been studied.

To fix ideas the owner of the forest can be thought of throughout as the government. However, it will be obvious that the analysis applies just as well in the case of a private owner who wishes to delegate the management of his forest.

3. Useful surveys of various aspects of mechanism design with incomplete information are contained in Baron and Besanko (winter 1984), Baron (1989), Laffont and Tirole (1988), Besanko and Sappington (1987) and Caillaud, Guesnerie and Tirole (1988).

4. In the Hartman model (see Hartman (1976)), landowners take into account amenity services provided by forest stands as well as net revenues.

In a first part of the paper, I will assume that the harvesting costs of the exploiting firms are perfectly correlated over time. The problem then becomes static, in the sense that the optimal contract is the same for each rotation. In this context, one can explicitly derive the optimal solutions. In a second part of the paper, I will analyze the optimal contract in the absence of this assumption. Then the infinite horizon forest problem coupled with the uncertainty about future costs leads to a truly dynamic problem, which can be solved using a dynamic programming approach introduced by Green (1987). [5]

The paper is organized as follows. Section 1.2 presents the characterization of the optimal contract in the static case. I first derive the optimal royalty and the optimal rotation period in both symmetric and asymmetric information cases. I also discuss the modification to the Faustmann rule under asymmetric information. Thereafter, I consider the case where a cutoff policy that excludes all firms with costs higher than a critical level is optimal. I also characterize the optimal royalty in both symmetric and asymmetric information cases. I end that section by discussing the gain in moving from a suboptimal linear contract to the optimal nonlinear contract. Section 1.3 discusses in the static context the auction mechanism as an alternative to royalty payment. Section 1.4 solves for the optimal contract in the dynamic context. I first present the model. After that, I characterize the incentive compatibility mechanisms and provide a recursive formulation of the problem. I end that section by deriving the solutions in both symmetric and asymmetric information cases and discuss the modification to the Faustmann rule as well as the main features of the optimal contract in both symmetric and asymmetric information cases. I end with some concluding remarks in Section 1.5.

1.2 Static contracts

In this section I model the problem of determining the optimal royalty when the firm's harvesting cost structure is not known by the forest owner and the firms costs are perfectly correlated over time. As a result of this last assumption, the problem can be dealt with as a static problem.

5. There is now an extensive literature dealing with this subject, see, Thomas and Worrall (1990), Atkeson and Lucas (1992), and Spear and Srivastava (1979).

Let $X(T)$ represent the timber growth function, where T represents the age of the trees. It will be assumed strictly concave and twice differentiable, with $X(0) = 0$.[6] Suppose that when it is harvested, a stand of trees of age T yields a net profit in present value given by :

$$(p-\theta)X(T)e^{-rT} - D$$

where p is the given market price of wood, the parameter D represents the total cost of planting a unit of land, r is the discount rate and θ is the unit cost of harvesting.

1.2.1 The model

The forest owner can observe the time when the stand of trees is harvested but it cannot verify the cost incurred by the firm. Therefore it cannot base its royalty schedule on the true harvesting cost of the firm. We must expect that the firm would, if asked for a report, lie about its true cost function whenever it is advantageous to do so.

Denote the firm's cost at period k by θ_k. The firm knows θ_k. The forest owner does not know θ_k, but the cumulative distribution of θ_k, $F(\theta_k)$, defined on $[\theta^L, \theta^H]$, is common knowledge. To this distribution function is associated the density function $f(\theta_k) > 0$, assumed differentiable on $[\theta^L, \theta^H]$. Knowledge of this probability distribution is shared by both the forest owner and the firm. I will assume a monotone hazard rate, which means that

$$h(\theta_k) = \frac{F(\theta_k)}{f(\theta_k)} \text{ is increasing in } \theta_k.$$

Consider a sequence of times $t_1 < t_2 < t_3 ...$ such that at each t_k a tree is cut and a new tree is planted. Then $T_k = t_k - t_{k-1}$ represents the age of the tree at date t_k . Let t_0 be the initial date of planting. The forest owner's problem is to set a royalty schedule $R_k = R(T_k)$ that maximizes expected social welfare. I assume in the remainder of our analysis that royalties are levied at harvesting time.

Using the Faustmann framework (see Faustmann, 1849), we may write the forest

6. Hence $X(T)$ will attain a maximum for some unique value of T, not necessarily finite.

owner's objective function as :

$$W = \sum_{k=1}^{+\infty} R_k e^{-r(t_k - t_0)} + \alpha V$$

where

$$V = \sum_{k=1}^{+\infty} \left\{ [(p - \theta_k) X(t_k - t_{k-1}) - R_k] e^{-r(t_k - t_{k-1})} - D \right\} e^{-r(t_{k-1} - t_0)}$$

is the firm's surplus. The exogenously given price p, the discount rate r and the cost of planting D are assumed to be known to both the government and the firm. I adopt the standard assumption that $0 \leqslant \alpha < 1$: a dollar in the government revenue is valued more highly than a dollar that remains as profits in the hands of the firm. As already stated, for the purpose of this section, I will assume that the firm's costs are perfectly correlated over time ($\theta_k = \theta \; \forall \; k$) and that the government commits itself for the current and future periods.

I model the problem as a direct revelation game. Hence the government chooses an incentive mechanism, in the form of a pair $(R_k(\tilde{\theta}), T_k(\tilde{\theta}))$, that is optimal given the optimal response $\tilde{\theta}$, of the firm, where $\tilde{\theta}$ denotes the value of its cost parameter as reported by the firm. Given that mechanism, the firm then chooses its optimal response, in the form of a $\tilde{\theta}$, the value of which will depend on θ, the true value of its parameter. According to the revelation principle, I can restrict attention to mechanisms in response to which the firm will find it optimal to reveal the true value of its cost parameter : mechanisms such that $\tilde{\theta} = \theta$. Knowing $T_k(\theta)$, and $R_k(\theta)$, I can obtain $R_k(T_k)$, by inverting $\theta = \theta(T_k)$. In order to be feasible, an incentive scheme must also leave the firm with sufficient surplus to cover its opportunity cost. Otherwise it would rationally choose not to participate.

Because costs remain constant over time, the rotation will be the same, so that $T_k = T$, $R_k = R \; \forall k$. Then :

$$V = \frac{e^{-rT}(p - \theta) X(T) - e^{-rT} R(\theta) - D}{1 - e^{-rT}}, \tag{1.1}$$

and

$$W = \frac{R(\theta) e^{-rT}}{1 - e^{-rT}} + \alpha \frac{e^{-rT}(p - \theta) X(T) - e^{-rT} R(\theta) - D}{1 - e^{-rT}}.$$

The social welfare can also be viewed as the present value to the government of all future rotations.

1.2.2 The symmetric information case

Before going on to solve for the optimal royalty scheme under asymmetry of information, I first derive properties of the royalty schedule which would maximize social welfare in the case where the government shares the firm's information about its cost structure. This symmetric information scenario is a useful benchmark, since it yields a first-best solution. The government then wishes to maximize

$$W = \frac{Re^{-rT}}{1-e^{-rT}} + \alpha V$$

subject to $V(\theta) \geq 0$, where $V(\theta)$ is given by (1.1). Clearly, since $0 \leq \alpha < 1$, the solution requires that we set $V = 0$.

In symmetric information, the government will extract the entire producer's surplus. Hence the solution consists in choosing T to maximize

$$\frac{e^{-rT}(p-\theta)X(T) - D}{1-e^{-rT}}$$

and then set $R(\theta)$ so as to collect that maximized value as royalties. This yields the standard Faustman formula :

$$(p-\theta)X'(T_s) = r(p-\theta)X(T_s) + rW_s^*(T_s). \tag{1.2}$$

The subscript s refers to the solution under symmetric information case. A forest stand will be harvested when the rate of change of its value with respect to time is equal to the interest on the value of the stand plus interest on the value of the forest land. Determining T_s in (1.2), the royalty rule can be specified as follows :

$$R(\theta) = (p-\theta)X(T_s) - De^{rT_s}. \tag{1.3}$$

1.2.3 The asymmetric information case

Let us now consider the situation where the true value of θ is known only to the firm. I begin by characterizing the class of incentive compatible mechanisms ; that is mechanisms in response to which the firm will choose to reveal its true cost.

Let $\phi(\tilde{\theta},\theta)$ be the surplus of the firm if it reports $\tilde{\theta}$ when the true cost takes the value θ. The government asks the firm that reports $\tilde{\theta}$ to harvest at $T(\tilde{\theta})$ and to pay the government the total royalty $R(\tilde{\theta})$.

Hence

$$\begin{aligned}\phi(\tilde{\theta},\theta) &= \frac{e^{-rT(\tilde{\theta})}(p-\theta)X(T(\tilde{\theta}))-e^{-rT(\tilde{\theta})}R(\tilde{\theta})-D}{1-e^{-rT(\tilde{\theta})}}\\ &= \frac{(p-\theta)X(T(\tilde{\theta}))-R(\tilde{\theta})-De^{rT(\tilde{\theta})}}{e^{rT(\tilde{\theta})}-1}.\end{aligned}$$

For the firm to respond truthfully, it is necessary that, for all $\theta\in[\theta^L,\theta^H]$,

$$\phi_1(\tilde{\theta},\theta)=0 \quad \text{for } \tilde{\theta}=\theta \tag{1.4}$$

and

$$\phi_{11}(\tilde{\theta},\theta)\leq 0 \quad \text{for } \tilde{\theta}=\theta. \tag{1.5}$$

I will drop the argument and use the notation $T=T(\tilde{\theta})$; $R=R(\tilde{\theta})$ where there is no risk of confusion. Condition (1.4) implies that the incentive scheme must satisfy

$$\begin{aligned}&\left\{(p-\theta)X'(T)\frac{dT}{d\tilde{\theta}}-rDe^{rT}\frac{dT}{d\tilde{\theta}}-\frac{dR}{d\tilde{\theta}}\right\}(e^{rT}-1)\\ &\qquad -re^{rT}\frac{dT}{d\tilde{\theta}}\left\{(p-\theta)X(T)-R-e^{rT}D\right\}=0.\end{aligned} \tag{1.6}$$

In addition, since by condition (1.4) $\phi_1(\theta,\theta)=0$ for all $\theta\in[\theta^L,\theta^H]$, one has

$$\phi_{11}(\theta,\theta)+\phi_{12}(\theta,\theta)=0 \quad \forall\theta\in[\theta^L,\theta^H].$$

It follows that condition (1.5) is equivalent to

$$\begin{aligned}\phi_{12}(\theta,\theta) &= \frac{dT}{d\theta}\left\{-X'(T)(e^{rT}-1)+re^{rT}X(T)\right\} \\ &= \frac{dT}{d\theta}\left\{-X'(T)+\frac{rX(T)}{1-e^{-rT}}\right\}\geq 0. \qquad (1.7)\end{aligned}$$

Conditions (1.6) and (1.7) are local conditions. However, given the linearity of the cost function in θ, they are sufficient for global incentive compatibility to hold (See for example Baron (1989) for the method of proof.).

Note next that if we let $V(\theta)\equiv\phi(\theta,\theta)$, then, by the envelope theorem, I must have

$$\frac{dV}{d\theta}=\phi_2(\theta,\theta)=-\frac{X(T)}{e^{rT}-1}\leq 0. \qquad (1.8)$$

Therefore $V(\theta)$ is a non increasing function of θ : an incentive compatible mechanism must not leave low cost firms with a smaller surplus than high cost firms. Another way of seeing this is to integrate (1.8), which gives :

$$V(\theta)=V(\theta^H)+\int_{\theta}^{\theta^H}\frac{X(T(\tau))}{e^{rT(\tau)}-1}d\tau. \qquad (1.9)$$

As it can be seen, $V(\theta)>V(\theta^H)$ for $\theta<\theta^H$. Conditions (1.8) and (1.9) are local conditions. They must hold in a neighborhood of $\tilde{\theta}=\theta$.

Finally, it is assumed that the firm can decide to opt out in response to the announced incentive scheme. This means that the combined royalty and harvesting time must satisfy a participation constraint, given by :

$$V(\theta)\geq 0 \quad \forall\theta\in[\theta^L,\theta^H]. \qquad (1.10)$$

This set of constraints simply requires that the incentive scheme guarantee each firm a nonnegative surplus. Since $V(\theta)$ is a non increasing function of θ by (1.8), constraints (1.10) can be replaced by the single constraint

$$V(\theta^H)\geq 0. \qquad (1.11)$$

The government's problem can now be stated as choosing $\{(R(\theta), T(\theta)) \mid \theta \in [\theta^L, \theta^H]\}$ to maximize expected welfare, given by :

$$EW = \int_{\theta^L}^{\theta^H} \left[\frac{R(\theta)e^{-rT(\theta)}}{1 - e^{-rT(\theta)}} + \alpha V(\theta) \right] f(\theta) d\theta \tag{1.12}$$

subject to (1.6), (1.7), (1.8), (1.10) or (1.11) and

$$R(\theta) = (p - \theta)X(T(\theta)) - De^{rT(\theta)} - V(\theta)(e^{rT(\theta)} - 1). \tag{1.13}$$

This can be treated as an optimal control problem, with $T(\theta)$ the control variable and $V(\theta)$ the state variable. By substituting (1.13) into (1.12), we obtain :

$$EW = \int_{\theta^L}^{\theta^H} \left[\frac{(p - \theta)X(T(\theta)) - De^{rT(\theta)}}{e^{rT(\theta)} - 1} - (1 - \alpha)V(\theta) \right] f(\theta) d\theta.$$

This gives the government's objective as the expected value of the sum of the royalty receipts and the producers' surplus (the first term), minus that part of producers' surplus which carries no weight in its objective (the second term).

To solve the problem I will ignore the incentive compatibility constraint (1.7), and then verify that the optimal solution satisfies it. The Hamiltonian for this optimal control problem is

$$H(V, T, \mu, \theta) = \left\{ \frac{(p - \theta)X(T(\theta)) - De^{rT(\theta)}}{e^{rT(\theta)} - 1} - (1 - \alpha)V(\theta) \right\} f(\theta) - \mu(\theta) \frac{X(T(\theta))}{e^{rT(\theta)} - 1}, \tag{1.14}$$

where $\mu(\theta)$ is the costate variable corresponding to equation (1.8). Necessary conditions for optimality are given by :

$$\frac{d\mu}{d\theta} = -\frac{\partial H}{\partial V} \tag{1.15}$$

$$\frac{dV}{d\theta} = \frac{\partial H}{\partial \mu} \tag{1.16}$$

$$\frac{\partial H}{\partial T} = 0 \text{ (for an interior solution).} \tag{1.17}$$

The terminal conditions require that :

$$\mu(\theta^L) = 0 \tag{1.18}$$

$$V(\theta^H)\mu(\theta^H) = 0. \tag{1.19}$$

Equation (1.15) yields

$$\frac{d\mu(\theta)}{d\theta} = (1-\alpha)f(\theta).$$

Hence the costate variable $\mu(\theta)$ satisfies $\mu(\theta) = (1-\alpha)F(\theta) + cte$. The terminal condition (1.18), implies that $cte = 0$ and then

$$\mu(\theta) = (1-\alpha)F(\theta).$$

The terminal condition (1.19) requires that $V(\theta^H)\mu(\theta^H) = 0$. Since $\mu(\theta^H) = 1-\alpha > 0$, this implies that $V(\theta^H) = 0$.

Substituting for $\mu(\theta)$ into (1.14) and differentiating the Hamiltonian with respect to the control variable, the solution of equation (1.17) must satisfy :

$$\begin{aligned} [p-\theta-(1-\alpha)h(\theta)]X'(T_a) &= r[p-\theta-(1-\alpha)h(\theta)]X(T_a) \\ &+ r\frac{[p-\theta-(1-\alpha)h(\theta)]X(T_a)e^{-rT_a}-D}{1-e^{-rT_a}} \end{aligned} \tag{1.20}$$

where the subscript a refers to the solution under asymmetry of information. The term $(1-\alpha)h(\theta)$ may be interpreted as the marginal cost of dealing with the informational asymmetry resulting from the firm's private information (the marginal information rents) and $p-\theta-(1-\alpha)h(\theta)$ represents the net price of a cubic meter of wood in asymmetric information. I will assume that $p-\theta-(1-\alpha)h(\theta) > 0$ for all θ.

To verify that (1.20) satisfies the incentive compatibility constraint (1.7), rewrite it as :

$$[p-\theta-(1-\alpha)h(\theta)][-X'(T_a)+r\frac{X(T_a)}{1-e^{-rT_a}}] = \frac{rD}{1-e^{-rT_a}}. \tag{1.21}$$

The right hand side of (1.21) being positive and $p-\theta-(1-\alpha)h(\theta)>0$, therefore :

$$-X'(T_a)+r\frac{X(T_a)}{1-e^{-rT_a}}>0. \tag{1.22}$$

To determine the sign of $\frac{dT_a}{d\theta}$, rewrite the first-order condition (1.20) as :

$$[X'(T_a)-rX(T_a)]=r\frac{[p-\theta-(1-\alpha)h(\theta)]X(T_a)e^{-rT_a}-D}{[p-\theta-(1-\alpha)h(\theta)](1-e^{-rT_a})}.$$

Totally differentiating this expression yields :

$$[X''(T_a)-rX'(T_a)]dT_a=-r\frac{D(1+(1-\alpha)h'(\theta))}{[p-\theta-(1-\alpha)h(\theta)]^2(1-e^{-rT_a})}d\theta. \tag{1.23}$$

Because $X'(T_a)$ must be positive, the left-hand side of (1.23) is negative. So is the right-hand side, since $h'(\theta)\geq 0$. Therefore $\frac{dT_a}{d\theta}\geq 0$ and T_a satisfies the incentive compatibility constraint (1.7).

The maximized Hamiltonian being linear in the state variable V, it follows from Arrow's sufficiency theorem (Kamien and Schwartz, 1981) that the solution to the necessary conditions (1.15) to (1.19) is a solution to the optimization problem (1.12).

1.2.4 The modified Faustmann rule

We can rewrite (1.20) as :

$$[p-\theta-(1-\alpha)h(\theta)]X'(T_a)=r[p-\theta-(1-\alpha)h(\theta)]X(T_a)+rZ(\theta), \tag{1.24}$$

where $H^*(\theta)=H(V(\theta),T_a(\theta),\mu(\theta),\theta)$ is the maximized hamiltonian and

$$\begin{aligned} Z(\theta) &= \frac{H^*(\theta)}{f(\theta)}+(1-\alpha)V(\theta) \\ &= \frac{[p-\theta-(1-\alpha)h(\theta)]X(T_a)-De^{rT_a}}{e^{rT_a}-1} \end{aligned} \tag{1.25}$$

represents the value of the forest land to the forest owner adjusted for the information constraints. In auctions terms, this if often called the *virtual surplus* (Myerson 1981).

Equation (1.24) defines the optimal rotation period under asymmetric information. This is the usual Faustmann Rule, modified in order to take into account the information constraints. Hence a forest stand will be harvested when the rate of change of its value with respect to age is equal to the interest on the value of the stand plus interest on the value of the forest land for all future rotation. It should be noted that the net price and the value of the forest land are properly corrected for the cost of the informational constraints. Since $h(\theta^L) = 0$, the usual Faustmann rule is unmodified for the lowest-cost firm.

The value of the expected welfare at the optimum is given by

$$EW^* = \int_{\theta^L}^{\theta^H} \left[\frac{H^*(\theta)}{f(\theta)} + (1-\alpha)V(\theta) \right] f(\theta) d\theta. \tag{1.26}$$

Let us now analyze the impact of asymmetric information on the optimal rotation period.

Proposition 1. *The optimal rotation period in the asymmetric information case is longer than that in the symmetric information case, except for the lowest cost firm, for which the rotation period remains unchanged. In other words,* $T_a(\theta) > T_s(\theta)$ *for all* $\theta \in (\theta^L, \ \theta^H]$ *and* $T_a(\theta^L) = T_s(\theta^L)$.

Proof. Let

$$g(T) = (1 - e^{-rT})[-X'(T) + r\frac{X(T)}{1 - e^{-rT}}] = -X'(T)(1 - e^{-rT}) + rX(T) > 0. \tag{1.27}$$

Then

$$g'(T) = (1 - e^{-rT})[-X''(T) + rX'(T)] > 0.$$

Hence $g(T)$ is strictly increasing in T. I can rewrite (1.21) and (1.2) as :

$$(p-\theta)g(T_s) = rD$$
$$(p-\theta-(1-\alpha)h(\theta))g(T_a) = rD.$$

This implies that

$$(p-\theta)g(T_a) = (p-\theta)g(T_s) + (1-\alpha)h(\theta)g(T_a).$$

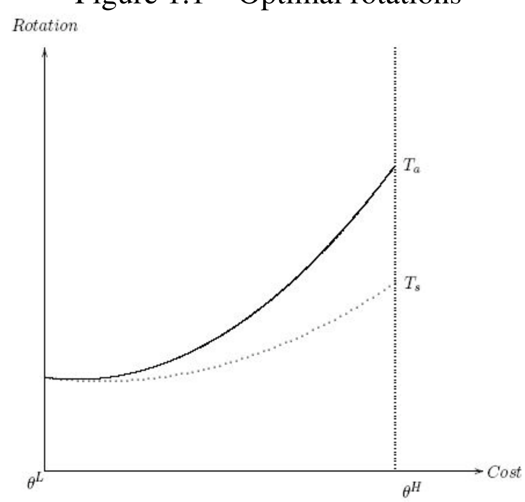
Notice that since $h(\theta^L) = 0$, $T_a(\theta^L) = T_s(\theta^L)$. For $\theta \in (\theta^L,\ \theta^H]$, $h(\theta) > 0$ and $g(T_a) > 0$, hence $(1-\alpha)h(\theta)g(T_a) > 0$. Therefore :

$$(p-\theta)g(T_a) > (p-\theta)g(T_s) \Rightarrow g(T_a) > g(T_s).$$

Since $g(T)$ is strictly increasing in T, then $T_a > T_s$. ∎

This situation is illustrated in Figure 1.1. The intuition behind this proposition is

Figure 1.1 – Optimal rotations



as follows : under asymmetric information, the optimal contract must give sufficiently high rent to low contractors. The principal induces some distortion in efficiency in an attempt to improve its share of the remaining rent. This distortion takes the form of longer rotations, except for the must efficient type.

1.2.5 The optimal cutoff cost

In the previous subsection we have seen that under asymmetric information the net price and the value of the forest land need to be corrected for the informational constraints. The information constraints are increasing in θ, the firm's harvesting cost parameter, and a high value of the harvesting cost can yield a negative virtual surplus to the forest owner. In order to avoid a negative virtual surplus, the forest owner will want to determine a critical value of the cost, which we may call the "cutoff cost", such that all types above this value are excluded from exploiting the forest.[7] I show in the next section that the cutoff cost is the analogous of the optimal reserve price in auctions.

To make sure that unprofitable forests are not exploited, the government should therefore endogenize the highest cost of the firm. Hence the government wants to find an optimal value $\bar{\theta} \in (\theta^L, \theta^H]$ that maximizes the expected social welfare. We can then replace θ^H by $\bar{\theta}$ in the optimal control problem (1.12) and determine its optimal value by solving the following problem :

$$\max_{T(\theta)\ ,V(\theta),\ \bar{\theta}} EW = \int_{\theta^L}^{\bar{\theta}} \{ \frac{(p-\theta)X(T(\theta)) - De^{rT(\theta)}}{e^{rT(\theta)} - 1} - (1-\alpha)V(\theta)\} f(\theta) d\theta \qquad (1.28)$$

subject to (1.6), (1.7), (1.8), and

$$\begin{aligned} V(\bar{\theta}) &\geq 0 && (1.29) \\ \bar{\theta} &\leq \theta^H && (1.30) \\ V(\theta^L) &\ \text{free}\ . && (1.31) \end{aligned}$$

A similar argument to that above establishes that the condition that determines the optimal value of $T(\theta)$ is the same as in the preceding problem, given by (1.20). The

7. The concept of excluding undesirable types of agent is in itself not new. Baron and Myerson (1982) allow the principal to opt out an inefficient monopolist. Moorthy (1984) showed that the supplier can increase it profit by excluding a certain types in designing product line.

terminal value $V(\bar{\theta})$ satisfies $V(\bar{\theta}) = 0$. The optimal value of $\bar{\theta}$ is determined by the following transversality conditions :

$$(\bar{\theta} - \theta^H) H(V(\bar{\theta}), T(\bar{\theta}), \mu(\bar{\theta}), \bar{\theta}) = 0 \quad (1.32)$$

$$\bar{\theta} \leq \theta^H. \quad (1.33)$$

If $\bar{\theta} < \theta^H$ then $H(V(\bar{\theta}), T(\bar{\theta}), \mu(\bar{\theta}), \bar{\theta}) = 0$ and

$$(p - \bar{\theta} - (1-\alpha)h(\bar{\theta}))X(T(\bar{\theta})) - De^{rT(\bar{\theta})} = 0. \quad (1.34)$$

Hence we can state the following proposition, which gives necessary and sufficient conditions for existence and uniqueness of an optimal interior value of $\bar{\theta}$, denoted by θ_a^*.

Proposition 2. *There exists a unique $\theta_a^* \in (\theta^L, \theta^H)$ such that $H^*(\theta_a^*) = 0$ if and only if $H^*(\theta^H) < 0$.*

Proof. Suppose that $H^*(\theta^H) < 0$. Differentiating the virtual surplus $Z(\theta)$, given by ((1.25)), and using the envelope theorem, we get :

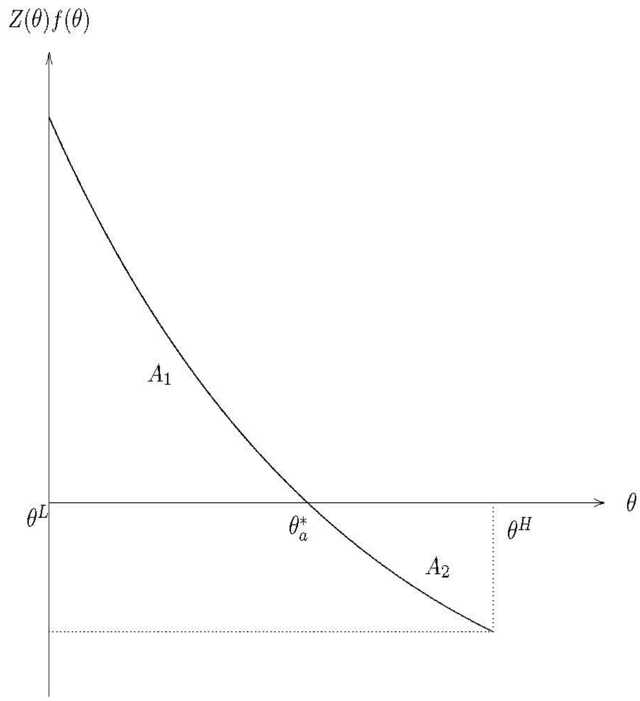
$$\frac{dZ(\theta)}{d\theta} = -\frac{(1 + (1-\alpha)h'(\theta))X(T_a)}{e^{rT_a} - 1}.$$

Hence $Z(\theta)$ is strictly decreasing in θ. In addition $Z(\theta^L) > 0$ and $Z(\theta^H) = H^*(\theta^H) < 0$. By the intermediate value theorem, there exist $\theta_a^* \in (\theta^L, \theta^H)$ such that $Z(\theta_a^*) = 0$. Given that $V(\theta_a^*) = 0$, then $H^*(\theta_a^*) = 0$. Because $Z(\theta)$ is strictly decreasing in θ, θ_a^* is unique.

Let us now assume that there exists a $\theta_a^* \in (\theta^L, \theta^H)$ such that $H^*(\theta_a^*) = 0$. It follows that $Z(\theta_a^*) = 0$ and $H^*(\theta_a^*) = 0$. Since $Z(\theta)$ is strictly decreasing in θ, $Z(\theta^H) = H^*(\theta^H) < 0$. ∎

Figure 1.2 summarizes the problem that the government is facing. The welfare when the cut-off cost θ^H is imposed is given by $EW^* = A_1 - A_2$. The government can increase this welfare by excluding types that belong to the interval $(\theta_a^*,\ \theta^H]$ and the welfare becomes $EW^* = A_1$. The gain in welfare is A_2, as compared to imposing θ^H, so that no type is excluded.

Figure 1.2 – Optimal cutoff cost



We can interpret θ_a^* as the limiting quality of commercially viable forest in asymmetric information. Let $T_a^* = T_a(\theta_a^*)$ and substitute $(p - \theta_a^* - (1-\alpha)h(\bar{\theta}^*))X(T_a^*)$ into (1.20). We then see that the optimal rotation period T_a^* satisfies :

$$-X'(T_a^*) + rX(T_a^*) = 0. \tag{1.35}$$

The optimal rotation period T_a^* for the firm with cost parameter θ_a^* is independent of θ_a^* and satisfies the Wicksell rule for a single rotation : $T_a^* = T_w$ where T_w is the Wicksell rotation. [8] Having determined T_a^* and substituting into (1.34), I can implicitly find θ_a^* by

8. The Wicksell rule is an arbitrage condition that determines the best time to harvest a piece of land when its has no alternative uses. It states that a forest stand shall be cut when its relative value growth equals the rate of interest. The Wicksell rule is a particular case of the Fautsmann rule when the value of land is zero.

solving the equation :

$$(p-\theta_a^*-(1-\alpha)h(\theta_a^*))X(T_w)-De^{rT_w}=0. \tag{1.36}$$

Let us denote the equivalent of θ_a^* in the symmetric information case by θ_s^*. We can find θ_s^* by setting $W_s^*(\theta_s^*)=0$. The resulting rotation period would then the satisfies Wicksell rule. Hence we have :

$$\theta_s^*=p-\frac{De^{rT_w}}{X(T_w)}. \tag{1.37}$$

It follows that $\theta_s^*-\theta_a^*=(1-\alpha)h(\theta_a^*)>0$. The difference between θ_s^* and θ_a^* is equal to the marginal cost of information at θ_a^*. In the remaining of our analysis, I will assume, for purposes of discussion, that θ_s^*=θ^H.

1.2.6 Interpreting the optimal cutoff cost

I will focus here on the interesting case where the optimal cutoff cost satisfies, $\theta_a^*<\theta^H$, indicating that the interval $[\theta^L,\ \theta^H]$ can be divided into two regions : a region $[\theta^L,\ \theta_a^*]$ in which the firm produces (a commercially viable forest) and a region $(\theta_a^*,\ \theta^H]$ in which the firm does not participate and earns zero profit. This means that $V(\theta)=0$ for $\theta>\theta_a^*$. Since $V(\theta_a^*)=0$, and $V(\theta)$ is strictly decreasing in θ, then $V(\theta)>0$ for $\theta^L<\theta<\theta_a^*$.

By endogenizing the highest cost the government also discourages the firm from falsely reporting a high cost. Indeed, if a firm with $\theta<\theta_a^*$ was to report $\tilde{\theta}>\theta_a^*$, it would not participate and would have zero profit instead of $V(\theta)>0$. In doing this, the government raises its expected profits and induces the firm to report truthfully its cost parameter.

Using (1.34), comparative statics results on the limited quality of commercial forest

θ_a^*, are :

$$\frac{\partial \theta_a^*}{\partial D} < 0 \tag{1.38}$$
$$\frac{\partial \theta_a^*}{\partial \alpha} > 0 \tag{1.39}$$
$$\frac{\partial \theta_a^*}{\partial r} < 0. \tag{1.40}$$

Expression (1.38) indicates that as the cost of planting decreases, the commercially viable forest interval increases. Expression (1.39) indicates that the greater is the weight the government places on the firm's profits, the greater is the commercially viable forest interval. In the limit, when α is equal to one, the government will find it optimal to let all the firms participate, because by doing this the firms will use the available information to maximize their profits and this will yield the same solution as in the symmetric information case. Finally, expression (1.40) indicates that when the interest rate increases, the commercially viable forest interval decreases.

1.2.7 Impact of the endogenous cutoff cost

To determine the effect of the optimal cutoff cost on the profit of the firm, let $V^*(\theta)$ denote the profit of the firm when $\theta_a^* < \theta^H$ occurs. If $\theta_a^* < \theta^H$, the optimal profit of the cutoff firm is lower than in the situation where the cutoff cost is set exogenously to θ^H. This profit is given by :

$$V^*(\theta) = V(\theta) - \int_{\theta_a^*}^{\theta^H} \frac{X(T_a(\tau))}{e^{rT_a(\tau)} - 1} d\tau.$$

The difference $V^*(\theta) - V(\theta)$ in the profit of the firm is independent of θ. Since the profit of the firm equals the rent it earns on its private information, setting the cutoff cost endogenously reduces this rent. The optimal royalty satisfies :

$$R^*(\theta) = R(\theta) + (V^*(\theta) - V(\theta))(e^{rT_a} - 1).$$

As we can see, it is higher than when θ^H is imposed as the cutoff cost. The difference $R^*(\theta) - R(\theta)$ in the royalty of the government is strictly increasing in θ.

1.2.8 Implementing the optimal royalty

The determination of the royalty rule is given by the expression

$$R(\theta) = (p-\theta)X(T_a(\theta)) - De^{rT_a(\theta)} - (e^{rT_a(\theta)} - 1)\int_{\theta}^{\theta_a^*} \frac{X(T_a(\tau))}{e^{rT_a(\tau)} - 1} d\tau. \tag{1.41}$$

We can also express the royalty as a function of the optimal rotation period by inverting $\theta = \theta(T)$ in (1.20) and substituting into (1.41). The optimal royalty under asymmetric information can be expected to be nondecreasing in the optimal rotation period, while the royalty in symmetric information can be expected to be nonincreasing. The intuition is the following : since the information rent $V(\theta)$ depends on the royalty, the government can reduce this rent by increasing the optimal rotation period, so as to remove the incentive to the firm to exaggerate its cost.

Now let us compare royalties in symmetric information with those in asymmetric information.

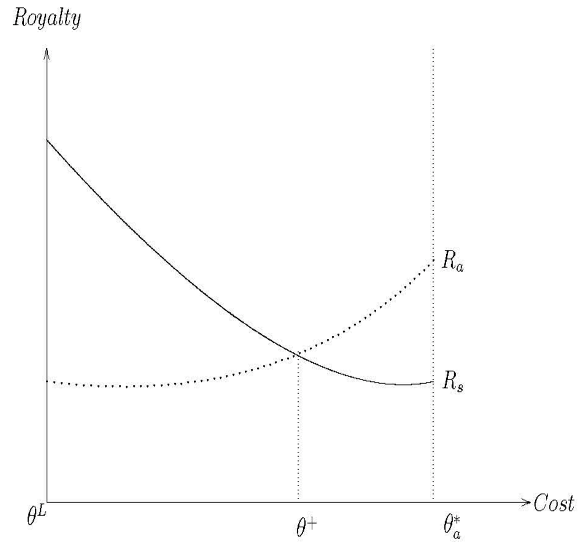
Proposition 3. *There exist $\theta^+ \in (\theta^L, \theta_a^*)$ such that $R_s(\theta^+) = R_a(\theta^+)$ and $R_s(\theta) > R_a(\theta)$ for any $\theta^L \leq \theta < \theta^+$.*

Proof. See the Appendix. ∎

Figure 1.3 depicts the optimal royalties in both the symmetric (solid line) and asymmetric information (dotted line) cases and illustrates the result of Proposition 3. It should be noted that although a firm with cost parameter θ^+ pays the same royalty under both asymmetric and symmetric information, it earns a positive profit in the asymmetric information case, but a zero profit in the symmetric information case. This is due to the fact that the optimal rotation period is higher in the asymmetric information case than in the symmetric information case.

In practice it is often more practical to express royalties as a function of the volume of wood. Let $x = X(T(\theta))$ be the volume of timber when the forest is cut at time $T(\theta)$.

Figure 1.3 – Optimal royalties



The royalties can be expressed as a function of this volume by inverting $\theta(x)$. Hence,

$$R_a(x) \quad \text{is defined for} \quad x \in [x^L, x_a] \quad x = X(T_a(\theta)), \quad \theta \in [\theta^L, \theta_a^*]$$
$$R_s(x) \quad \text{is defined for} \quad x \in [x^L, x_s] \quad x = X(T_s(\theta)), \quad \theta \in [\theta^L, \theta_a^*]$$

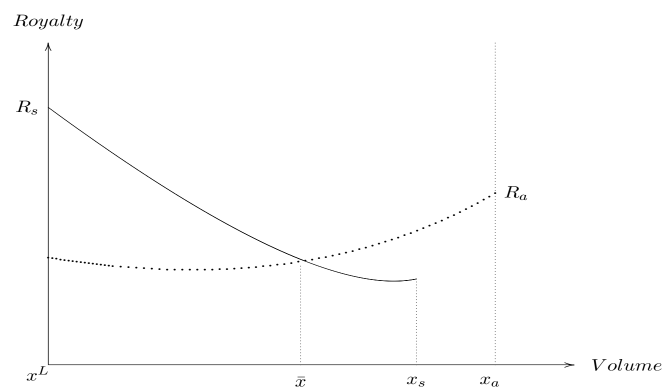
where $x^L = X(T_s(\theta^L)) = X(T_a(\theta^L))$, $x_s = X(T_s(\theta_a^*))$, $x_a = X(T_a(\theta_a^*))$. Since $T_a(\theta_a^*) > T_s(\theta_a^*)$ and $X(\cdot)$ is strictly increasing, then $x_a > x_s$. If x is known, the cost can be found by inverting $X(T)$. It follows that $\theta_s = T_s^{-1}(X^{-1}(x))$ and $\theta_a = T_a^{-1}(X^{-1}(x))$, which implies that $T_s(\theta_s) = T_a(\theta_a)$. Thus, I can state the following proposition.

Proposition 4. *There exist $\bar{x} \in (x^L, x_s)$ such that $R_s(\bar{x}) = R_a(\bar{x})$ and $R_s(x) > R_a(x)$ for any $x^L \leq x < \bar{x}$.*

Proof. See the Appendix. ∎

This proposition is illustrated in Figure 1.4. As can be seen, the royalty function in the symmetric information case is not defined beyond x_s, since it is not profitable for the firm nor the government to harvest in excess of the volume x_s. In the asymmetric information case, however, the firm will have a higher profit and will therefore want to wait longer to harvest. This explains why $x_a > x_s$. Figures 1.3 and 1.4 also show that in order to reduce the informational rent left to the exploiting firm in the case of adverse selection, the forest owner sets a higher royalty than in the situation of full information for a high cost (beyond θ^+) or a high volume of wood harvested (beyond $\bar{x}$).

Figure 1.4 – Optimal royalties as a function of the volume of wood



1.2.9 Uniform contracts

Typically forest contracts are in practice set as a linear function of the volume harvested, independently of the cutting cost. [9] In this section I analyze the optimal policy with incomplete information when the forest owner constrains itself to choose such a uniform contract. This kind of contract is suboptimal, in the sense that it cannot do better than a more general optimal nonlinear contract derived above. I will characterize the loss to a

9. Various aspects of linear pricing in forestry are contained in Paarsh (1993), and Størdal (2004).

forest owner who uses a uniform contract instead of the nonlinear one and illustrate it using simulation results.

The problem of determining the loss resulting from the application of a second-best policy is not new. In a related procurement environment, Bower (1993) showed, using simulation results, that a simple single contract often can contain a large fraction of the surplus secured by a full menu. Rogerson (2003) proposed a very simple FPCR contract (Fixed Price Cost Reimbursement) and showed that it can capture at least 75 percent of the gains that contains the optimal complex menu of the standard principal-agent model. Sheriff (2009) used empirical results to calibrate a second-best land conservation mechanism and evaluate its cost relative to other alternatives. I will follow Sheriff and use some parameter values from published empirical studies to perform my simulations.

For a uniform contract, the forest owner sets a royalty of b per unit of volume of timber harvested, independent of types. The firm chooses a rotation period T and pays the total royalty $R(T) = bX(T)$ to the forest owner. One can then rewrite the firm's profit and the welfare function respectively as :

$$V(T,b,\theta) = \frac{e^{-rT}(p-\theta-b)X(T)-D}{1-e^{-rT}}$$

and

$$W(T,b,\theta) = \frac{e^{-rT}bX(T)}{1-e^{-rT}} + \alpha V. \tag{1.42}$$

Thus the firm's objective function is independent the distribution of the harvesting cost and is of the same form as in the absence of private information, except for the fact that net price now accounts for the royalty rate, in addition to the harvesting cost. Given that objective function, the firm's optimal rotation will be given by the following version of the Faustmann rule :

$$(p-\theta-b)X'(T) = r(p-\theta-b)X(T) + rV(T(b,\theta),b,\theta).$$

I will assume that the cutoff cost, denoted by $\bar{\theta}$ is endogenous. Hence the government knows that the optimal rotation period at $\bar{\theta}$ would yield zero profit and determines a royalty b that maximizes expected social welfare given those constraints. Given that b is the same regardless of types, the welfare maximization problem then consists of choosing b and $\tilde{\theta}$ to maximize :

$$\int_{\theta^L}^{\bar{\theta}} \left[\frac{bX(T(b,\theta))e^{-rT(b,\theta)}}{1-e^{-rT(b,\theta)}} + \alpha V(T(b,\theta),b,\theta) \right] f(\theta)d\theta \tag{1.43}$$

subject to

$$\begin{aligned} (p-\theta-b)X'(T) &= r(p-\theta-b)X(T)+rV(T(b,\theta),b,\theta) && (1.44)\\ V(T(b,\bar{\theta}),b,\bar{\theta}) &\geq 0 \text{ with equality if } \bar{\theta} < \theta^H && (1.45)\\ \bar{\theta} &\leq \theta^H. && (1.46) \end{aligned}$$

The characterization of this suboptimal mechanism is given in the Appendix.

To illustrate the gains from moving from the linear to the nonlinear contract I borrow parameters from the empirical study on timber auction presented by Paarsh (1997). He assumed that the harvesting cost follows a Weibull distribution and estimated the parameters of that distribution to be $\delta_1 = 0.3686$ and $\delta_2 = 3.323$. I also use that distribution, with the same parameters. The market price he uses is $p = 46.89$, the average price for timber sales from 1984 to 1987. The planting cost is $D = 3891.87$. I borrow the growth function from Payandeh (1973), who estimated it to be $X(T) = 5680.7 - 26660T^{-0.5}$ for pine. I truncate the distribution to $[\theta^L, \theta^H]$ to ensure that they are well defined. I use two ranges of harvesting cost : $[\theta^L, \theta^H] = [7.18, 23.68]$, and $[\theta^L, \theta^H] = [12.13, 18.73]$. The left side of the table is performed with r=0.03 and the right side is performed with $r = 0.02$. The results are given in the following table, where the optimal cutoff cost under a uniform contract is denoted by θ_u^*.

Tableau 1.I – **Gain in moving from the uniform contract to the nonlinear contract**

	$[7.18, 23.68]$	$[12.13, 18.73]$	$[7.18, 23.68]$	$[12.13, 18.73]$
Gain (%)	*0.20*	*0.87*	*0.42*	1.73
θ_a^*	20.51	18.73	21.77	18.73
θ_u^*	20.59	18.73	20.60	18.73

The table shows that the gain in moving from a uniform contract to a nonlinear contract decreases with the length of the interval $[\theta^L, \theta^H]$ and with the interest rate. As can seen, the uniform contract captures between 98.27 and 99.8 percent of rent captured by the nonlinear contract. It would appear that the gain in moving from a uniform contract to a nonlinear one is small.

1.3 Forest auctions

The previous sections consider a bilateral contract between a forest owner and a harvesting firm. In this section, I introduce a competition between firms and analyse the optimal contract using a standard auction, namely the second price auction. I determine how a forest owner can lease the right of a forest to N competing firms. Let $\theta^i, i = 1,2...,N$ be the cost of firm i. Suppose that each θ^i is drawn independently from the same distribution with the cumulative distribution function $F(\cdot)$ and the density function $f(\cdot)$ on the interval $[\theta^L, \theta^H]$. Moreover $F(\cdot)$ is common knowledge. I will focus on symmetric equilibria with increasing bids.

A second price auction is defined by a rotation period $T(b)$ and a reserve price $\underline{R} \geqslant 0$ where $b = (b^i)_{i=1,...,N}$ is a bid vector. Let us assume that the payment will be made every time the trees are cut and this time is determined by the rotation period $T(b)$. The bid of firm i depends on its cost θ^i according to a decreasing function γ, where γ is the symmetric equilibrium strategy so that $b^i = \gamma(\theta^i)$. Without loss of generality, let us assume that the firm considering an alternative bidding strategy is firm 1. Suppose that firm 1 bids $b^1 = \gamma(z)$ and view firm 1 as choosing z when its type is θ^1. Firm 1 will win if and only if it submits the highest bid which is greater than the reserve price $\underline{R}$. This means if and only if $\forall i = 2,...,N$, $b^1 \geqslant \max\{\gamma(\theta^i), \underline{R}\}$, which implies that $z < \min_{j \neq 1} \theta^j = y$. It must pay the second highest bid $\gamma(y)$ or the reserve price $\underline{R}$ if $\gamma(y) < \underline{R}$. The revenue of firm 1 when it wins is

$$w(\theta^1, z) = \frac{(p - \theta^1)X(T(\theta^1, z))e^{-rT(\theta^1, z)} - D}{1 - e^{-rT(\theta^1, z)}}.$$

Let $\underline{R} = \gamma(\delta)$; if $z \leqslant \delta$, the expected payment of firm 1 is given by

$$E[\gamma(y)\mathbf{1}(z < \delta)|z] = \int_z^\delta \gamma(y) f_y(y) dy + \gamma(\delta)[1 - F_y(\delta)] \tag{1.47}$$

where $F_y(z) = 1 - (1 - F(z))^{N-1}$. The expected profit of firm 1 when it chooses to enter the auction can be expressed as

$$U(\theta^1, z) = w(\theta^1, z)(1 - F_y(z)) - \int_z^\delta \gamma(y) f_y(y) dy - \gamma(\delta)[1 - F_y(\delta)]. \tag{1.48}$$

$\gamma(\cdot)$ is the equilibrium bidding strategy only if firm 1 chooses $z = \theta^1$ and bids $\gamma(\theta^1)$. Therefore, the following first-order condition must be satisfied

$$\left.\frac{\partial U(\theta^1, z)}{\partial z}\right|_{z=\theta^1} = 0 \tag{1.49}$$

which means that,

$$w_2(\theta^1, z)(1 - F_y(z)) - w(\theta^1, z) f_y(z) + \gamma(z) f_y(z) = 0 \text{ at } z = \theta^1$$

where $w_2(\cdot) = \frac{\partial w(\cdot)}{\partial z}$. The equilibrium bidding strategy is then obtained as

$$\gamma(\theta^1) = w(\theta^1, \theta^1) - \frac{1 - F_y(\theta^1)}{f_y(\theta^1)} w_2(\theta^1, \theta^1).$$

As shown in the Appendix, the forest owner's optimization can be reduced to

$$\begin{aligned} &\max_{T^1(\theta^1), V_{sa}^1(\delta), \delta} \quad EW_{sa} \qquad (1.50) \\ &= N \int_{\theta^L}^{\delta} [w(\theta^1, T^1(\theta^1)) - (1 - \alpha) \frac{X(T^1)}{e^{rT^1 - 1}} h(\theta^1)](1 - F_y(\theta^1)) f(\theta^1) d\theta^1 \\ &- N(1 - \alpha) V_{sa}^1(\delta) F(\delta) \end{aligned}$$

subject to $V_{sa}^1(\delta) \geqslant 0$, $\delta \leqslant \theta^H$ and $\gamma(\cdot)$ is nonincreasing in θ^1, where the subscript *sa* refers to a second price auction. Clearly $V_{sa}^1(\delta) = 0$. The optimal rotation period function

$T^1_{sa}(\theta^1)$ satisfies

$$[p-\theta^1-(1-\alpha)h(\theta^1)]X'(T^1_{sa}) = \qquad (1.51)$$
$$r[p-\theta^1-(1-\alpha)h(\theta^1)]X(T^1_{sa})+r\frac{[p-\theta^1-(1-\alpha)h(\theta^1)]X(T^1_{sa})e^{-rT^1_{sa}}-D}{1-e^{-rT^1_{sa}}}.$$

It follows from (1.51) that $T^1_{sa}=T_a$, and hence $T^i_{sa}=T_a$ for all $i=1,...,N$. Recall that T_a is the optimal rotation period under bilateral contract. Finally, by assuming that $\delta<\theta^H$, I obtain that the optimal reserve price satisfies $(p-\delta-(1-\alpha)h(\delta))X(T_w(\delta))-De^{rT_w(\delta)}=0$ which is the same equation that determined θ^*_a, hence $\delta=\theta^*_a$. In short, the solution to the optimization problem shows that the optimal rotation period function is the same as in the bilateral contract case, and the optimal reserve price is equal to the cutoff cost.

Now let us compute the gain or loss in expected welfare from having an auction. Denote the expected welfare in a second price auction by EW_{sa} and in a bilateral contract by EW. Then :

$$EW_{sa}-EW = N\int_{\theta^L}^{\delta}(1-F_y(\theta))Z(\theta)f(\theta)d\theta - EW = \int_{\theta^L}^{\delta}Z(\theta)f(\theta)d\theta \qquad (1.52)$$

where $Z(\cdot)$ follows from (1.25). Recalling that $F_y(\theta)=1-(1-F(\theta))^{N-1}$ and integrating (1.52) by part, one obtains that

$$EW_{sa}-EW = \int_{\theta^L}^{\delta}\frac{1+(1-\alpha)h'(\theta)}{e^{rT_a}-1}[1-(1-F(\theta))^{N-1}]F(\theta)d\theta > 0. \qquad (1.53)$$

Equation (1.53) shows that the second price auction yields a higher expected welfare compared to the case of a bilateral contract. This result can be summarized by the following proposition

Proposition 5. *With N competing firms whose types are drawn independently from the same distribution,* ***(i)*** *an optimal second price auction awards the contract to the firm with the lowest cost.* ***(ii)****The optimal rotation period function of the winning firm is the same as in the bilateral contract case as well as the reservation price.* ***(iii)*** *The optimal* *auction yields a higher expected welfare than in the case of a bilateral contract.*

Proposition 5 deserves some comments. The fact that the optimal rotation function under the second price auction is the same as that obtained under the bilateral contract means that in the case of an auction the optimal rotation period is independent of the number of firms. The optimal auction can be viewed as a two step process. The first step consists of selecting the winning firm, which is the lowest cost firm ; the second step is to propose a bilateral contract. In this case, the winning firm knows that if its cost is above the cutoff cost it will not participate, which means that the cutoff cost is equal to the reserve price. Finally, when the number of firms increases, the profit of the winner decreases, leading to a higher value for the forest owner :

$$V_{sa}(\theta) - V^*(\theta) = \int_{\theta}^{\theta_a^*} \frac{X(T_a(\tau))}{e^{rT_a(\tau)} - 1} F_y(\tau) d\tau < 0.$$

1.4 Dynamic contracts

In the previous sections I have assumed that the costs are perfectly correlated over time and that the government is committed to present and future harvests. Under the assumption of perfect correlation cost, the principal cannot credibly commit to the contract. Indeed, after one rotation the principal knows the cost and has an incentive to switch to the symmetric information in order to capture all of the firm's rents. In this section I will depart from this assumption.

Let us assume that there is no intertemporal correlation between harvesting cost : the $\theta' s_k$, $k = 1, 2, ..., \infty$, are not correlated over time. Recall that under asymmetric information the firm observes the realization of θ_k at the planting date t_{k-1} or period $k-1$ and the principal does not. Let $\theta^k = (\theta_1, \theta_2, ..., \theta_k) \in \Theta^k$ where $\Theta = [\theta^L, \theta^H]$ denote a k period history of costs. Because the costs θ_k are not correlated over time, each firm is assumed to privately observe its history of costs. But the forest owner's only sources of information about this history are the reports provided by the firm itself. Define a reporting strategy h to be a sequence of functions $\{h_k\}_{k=1}^{\infty}$ with $h_k : \Theta^k \to \Theta$, and refer to H as the set of all reporting strategies. The truthful reporting strategy is denoted by $h^* = \{h_k^*\}_{k=1}^{\infty}$ where for all $t,\ \theta^k,\ h_k^*(\theta^k) = \theta_k$. Recall that the harvesting time $t_k : \Theta^k \to \mathbb{R}_+$, the age of the tree at date t_k (rotation period) $T_k : \Theta^k \to \mathbb{R}_+$ and the forest owner's royalty $R_k : \Theta^k \to \mathbb{R}$ are random variables.

The timing of events is as follows. Period $k-1$ begins after history $\theta^{k-1} \in \Theta^{k-1}$ has taken place and the sequence of reports $h^{k-1}(\theta^{k-1})$ has been submitted to the forest owner. The firm observes its period $k-1$ harvesting cost θ_k and reports its realization to the forest owner according to the strategy $h_k(\theta^k)$. The forest owner allows the firm to cut the tree at age $T_k(h^k)$ and in return the firm pays a royalty $R_k(h^k)$. Given a sequence of strategies $T = \{T_k(h^k)\}_{k=1}^{\infty}$ and $R = \{R_k(h^k)\}_{k=1}^{\infty}$, the expected profit to the firm at time t_{k-1} given a history h^{k-1} is given by :

$$\begin{aligned} & V_{k-1}(T,R,h;h^{k-1}) \\ & = E\sum_{s=k-1}^{\infty} \pi_s(T,R;h^s)e^{-r(t_s(h^s)-t_{k-1}(h^{k-1}))} \\ & = \int_{\Theta} [\pi_{k-1}(T,R;h^{k-1}) + e^{-rT_k(h^k)}V_k(T,R,h;h^k)]f(\theta_k)d\theta_k \end{aligned} \tag{1.54}$$

and the expected welfare of the principal is given by :

$$\begin{aligned} & W_{k-1}(T,R,h;h^{k-1}) \\ & = E\sum_{s=k-1}^{\infty} [R_{s+1}(h^{s+1})e^{-rT_{s+1}(h^{s+1})} + \alpha\pi_s(T,R;h^s)]e^{-r(t_s(h^s)-t_{k-1}(h^{k-1}))} \\ & = \int_{\Theta} \left[R_k(h^k)e^{-rT_k(h^k)} + \alpha\pi_{k-1}(T,R;h^{k-1}) + e^{-rT_k(h^k)}W_k(T,R,h;h^k)\right] f(\theta_k)d\theta_k \end{aligned} \tag{1.55}$$

where $\pi_{k-1}(T,R;h^{k-1}) = (p-\theta_k)X(T_k(h^k))e^{-rT_k(h^k)} - D - R_k(h^k)e^{-rT_k(h^k)}$ is the current profit of the firm at time t_{k-1}, $V_{k-1}(T,R,h;h^{k-1})$ and $W_{k-1}(T,R,h;h^{k-1})$ represent respectively the continuation profit of the firm and the continuation expected welfare of the principal at date t_k after history h^k occurred. Let $V(T,R,h)$ and $W(T,R,h)$ be respectively the expected profit of the firm and the principal's expected welfare at the initial date t_0.

By the revelation principle I can, without loss of generality, restrict the forest owner's choice of mechanism to those that induce the firms to be truthful. Thus I require the allocation (T,R) to satisfy the incentive-compatibility condition :

$$V(T,R,h^{*}) \geq V(T,R,h) \quad \forall h \in H. \tag{1.56}$$

In other words, truth-telling weakly dominates any other reporting strategy. The allocation (T,R) is temporarily incentive-compatible if $\forall\, k, h^{k}, \theta$and θ' :

$$\begin{aligned} &\pi_{k-1}(T,R;(h^{k-1},\theta)) + e^{-rT_k(h^{k-1},\theta)}V_k(T,R,h;(h^{k-1},\theta)) \\ \geqslant\ &\pi_{k-1}(T,R;(h^{k-1},\theta')) + e^{-rT_k(h^{k-1},\theta')}V_k(T,R,h;(h^{k-1},\theta')). \end{aligned} \tag{1.57}$$

The preceding constraints imply that after each history of harvesting costs, a firm is better off truthfully reporting its costs, rather than lying and being truthful thereafter. If the allocation (T,R) is incentive-compatible then it is temporarily incentive-compatible. According to Green (1987) an allocation which is temporarily incentive-compatible and that satisfies the following transversality condition (1.58) is incentive-compatible

$$\lim_{k\to\infty} \sup_{\theta^k,h} e^{-r\left(t_k\left(\theta^k\right)-t_0\right)} V_k\left(T,R,h \mid \theta^k\right) = 0 \tag{1.58}$$

Condition (1.58) simply means that starting the problem in the remote future will have negligible consequences for current decisions. I assume that the transversality condition (1.58) is fulfilled.

Finally I require a contract (T,R) to be self-enforcing, which implies that the continuation profits of firms must satisfy the following participation constraints :

$$V_{k-1}(T,R,h;h^{k-1}) \geqslant 0 \quad \forall k, h^{k-1}. \tag{1.59}$$

This definition assumes that the firm does not commit to the contract : at each period $k-1$ it could abandon the contract if it is advantageous to do so.

An optimal contract is an allocation (T,R) such that (T,R) maximizes $W(T,R,h)$ subject to the participation constraints (1.59) and the temporary incentive compatibility conditions (1.57)

1.4.1 The recursive formulation

This subsection provides a recursive formulation for the problem described above. Let $V(h^{k-1}) = V_{k-1}(T,R,h;h^{k-1})$ and $W(h^{k-1}) = W_{k-1}(T,R,h;h^{k-1})$. Let $\mathscr{V} = \{V(h^{k-1})\}$ and $\mathscr{S} = \{W(h^{k-1})\}$. Where these values are computed at the optimal contract. For each $v \in \mathscr{V}$, consider solving the following problem : maximize the forest owner's time t_0 (period 0) expected profit subject to the firm getting a minimum profit v ; let $S(v)$ denote the expected profit to the forest owner in the solution to this problem ; let θ be the cost of the firm at time t_0 ; let $T(v,\theta)$ and $R(v,\theta)$ be respectively the optimal rotation and the optimal royalty at time t_0 ; let $U(v,\theta)$ be the profit promised to the firm at time t_1 (period 1). This means that if $v = V(h^{k-1})$, then $S(v) = W(h^{k-1})$, $T(v,\theta) = T(h^{k-1},\theta)$, $R(v,\theta) = R(h^{k-1},\theta)$, and $U(v,\theta) = V(h^{k-1},\theta) = V_k(T,R,h;(h^{k-1},\theta))$. At period 1, having received the report θ, the forest owner has to give the firm profit $U(v,\theta)$. Then $T(U(v,\theta),\theta')$, $R(U(v,\theta),\theta')$, and $U(U(v,\theta),\theta')$ are respectively the optimal rotation, the optimal royalty and promised future profit in period 1, where θ' is the firm's cost in that period. This argument is repeated indefinitely.

The argument above implies that the forest owner's problem is to find four functions :

$$U : \mathscr{V} \times \Theta \to \mathscr{V} \qquad S : \mathscr{V} \to \mathscr{S} \qquad T : \mathscr{V} \times \Theta \to \mathbb{R}+ \qquad R : \mathscr{V} \times \Theta \to \mathbb{R}$$

so as to maximize

$$\begin{aligned} S(v) = \int_{\theta^L}^{\theta^H} & \{R(v,\theta)e^{-rT(v,\theta)} + \alpha[(p-\theta)X(T(v,\theta))e^{-rT(v,\theta)} \\ & -D - R(v,\theta)e^{-rT(v,\theta)}] + e^{-rT(v,\theta)}S(U(v,\theta))\}f(\theta)d\theta \end{aligned} \tag{1.60}$$

subject to

$$\begin{aligned} \int_{\theta^L}^{\theta^H} & [(p-\theta)X(T(v,\theta))e^{-rT(v,\theta)} \\ & -D - R(v,\theta)e^{-rT(v,\theta)} + e^{-rT(v,\theta)}U(v,\theta)]f(\theta)d\theta = v \end{aligned} \tag{1.61}$$

$$(p-\theta)X(T(v,\theta))e^{-rT(v,\theta)} - D - R(v,\theta)e^{-rT(v,\theta)} + e^{-rT(v,\theta)}U(v,\theta)$$
$$\geqslant (p-\theta)X(T(v,\theta'))e^{-rT(v,\theta')} - D - R(v,\theta')e^{-rT(v,\theta')}$$
$$+e^{-rT(v,\theta')}U(v,\theta'), \quad \forall v,\theta,\theta' \tag{1.62}$$

$$U(v,\theta) \geqslant 0. \tag{1.63}$$

Equations (1.60), (1.61), (1.62), and (1.63) follow respectively which (1.55), (1.54), (1.57), and (1.59). Equation (1.61) states that v is the expected profit of the firm, (1.62) says that (T,R) is incentive compatible, and (1.63) is the participation constraints of the firm, which says that the firm will accept the contract if and only if the next period promised profit is non-negative.

Finally, $S(v)$ must be nonincreasing, for otherwise the forest owner could be better off by offering more to the firm. As shown by Spear and Srivastava (1987), the value function $S(v)$ is concave. Notice that, given the participation constraints, the lower level of $\mathscr{V}$ is 0. The upper level of $\mathscr{V}$ denoted by $\bar{v}$ can be taken to be the point where the firm pays zero royalty and gets the same profit at each period. Hence $\bar{v}$ is given by

$$\bar{v} = \max_{T} \frac{(p-E[\theta])X(T)e^{-rT} - D}{1-e^{-rT}}$$

hence $\mathscr{V} = [0,\bar{v}]$.

1.4.2 Characterization of the optimal contract

In this subsection, I analyze the properties of a differentiable solution to the preceding problem. Assume that the forest is profitable, which means that the forest owner can achieve a positive benefit by promising a zero profit to the firm. This is equivalent to $S(0) > 0$.

1.4.2.1 The symmetric information case

Let us first analyze the results when there is no private information. From (1.61) I have that

$$\int_{\theta^L}^{\theta^H} R(v,\theta)e^{-rT(v,\theta)}f(\theta)d\theta$$
$$= \int_{\theta^L}^{\theta^H} [(p-\theta)X(T(v,\theta))e^{-rT(v,\theta)} - D + e^{-rT(v,\theta)}U(v,\theta) - v]f(\theta)d\theta.$$

Substituting this into (1.60), the problem of the forest owner under symmetric information is to maximize

$$S(v) = \int_{\theta^L}^{\theta^H} \{(p-\theta)X(T(v,\theta))e^{-rT(v,\theta)} - D + (1-\alpha)e^{-rT(v,\theta)}U(v,\theta)$$
$$-(1-\alpha)v + e^{-rT(v,\theta)}S(U(v,\theta))\}f(\theta)d\theta \qquad (1.64)$$

subject to the participation constraints (1.63).

If we attach Lagrange multipliers $\mu(v,\theta)e^{-rT(v,\theta)}f(\theta)$ to expression (1.63), we then get the following first-order conditions for T, U and μ :

$$(p-\theta)[X'(T) - rX(T)] = r[(1-\alpha) + \mu(v,\theta)]U(v,\theta) + rS(U(v,\theta)) \qquad (1.65)$$

$$1-\alpha + \mu(v,\theta) + S'(U(v,\theta)) = 0 \qquad (1.66)$$

$$\mu(v,\theta)U(v,\theta) = 0;\ \mu(v,\theta) \geqslant 0;\ U(v,\theta) \geqslant 0. \qquad (1.67)$$

By the envelope theorem,

$$S'(v) = -(1-\alpha). \qquad (1.68)$$

Equation (1.68) shows that the value function is linear in v, which means that, $S(v) = -(1-\alpha)v + S(0)$. Recall that $S(0) > 0$. Substitute for this into (1.66) and into (1.65). It follows that $\mu(v,\theta) = 0$, so that (1.67) is satisfied for all $U(v,\theta) \geqslant 0$ and the equation determining the optimal rotation period becomes : [10]

$$(p-\theta)X'(T_s) = r(p-\theta)X(T_s) + rS(0). \qquad (1.69)$$

Equation (1.69) simply states that the optimal rotation period satisfies the Faustmann rule. The term $S(0)$ represents the expected value of the forest land or the value of bare land under symmetric information. Since $S(\cdot)$ is non increasing, this is the highest value of bare land that the forest owner can get for subsequent rotations. Equation (1.69) also tells us that the optimal rotation period depends only on the cost parameter. Since the optimal rotation does not depend on v, it is constant over time. Finally the net transfer $(R_s - U_s)(v,\theta)$ is given by :

$$\int_{\theta^L}^{\theta^H} (R_s - U_s)(v,\theta)e^{-rT_s(\theta)} f(\theta)d\theta$$
$$= \int_{\theta^L}^{\theta^H} [(p-\theta)X(T_s(\theta))e^{-rT_s(\theta)} - D]f(\theta)d\theta - v. \qquad (1.70)$$

Differentiating (1.70) with respect to v, I obtain, $\int_{\theta^L}^{\theta^H} \frac{\partial (R_s-U_s)(v,\theta)}{\partial v} e^{-rT_s(\theta)} f(\theta)d\theta = -1$ which implies that $(R_s - U_s)(v,\theta) = -ve^{rT_s(\theta)} + K$, where K is a constant. Substituting this into (1.70), gives

$$K = S(0)\frac{1-E[e^{-rT_s}]}{E[e^{-rT_s}]}$$

thus

$$R_s(v,\theta) - U_s(v,\theta) = -ve^{rT_s(\theta)} + S(0)\frac{1-E[e^{-rT_s}]}{E[e^{-rT_s}]}. \qquad (1.71)$$

The forest owner can promise any future profit $U(v,\theta) \in [0,\bar{v}]$ to the firm. By doing so, he imposes a royalty payment in order to get the highest expected profit. This royalty satisfies (1.71). From this equation it can be seen that the optimal royalty varies over time. The preceding results prove the following proposition :

Proposition 6. *Under symmetric information , the optimal rotation period is invariant over time and satisfies the Faustmann rule. The forest owner can achieve the same goal*

10. Recall that the subscript s refers to the solution under symmetric information.

by promising any future profit $U(v,\theta) \in [0,\bar{v}]$ to the firm. Finally, the optimal royalty varies over time and is given by (1.71).

The intuition behind those results is as follows. For simplicity let assume that the objective of the forest owner is revenue maximization ($\alpha = 0$). For a given v, consider an optimal contract $\{T,R,U\}$. It is possible for the forest owner to increase U and R by the same amount d, leaving the net transfer unchanged so that the contract $\{T,(R+d),(U+d)\}$ satisfies the promise-keeping condition (1.61) and the participation constraints. Since the value function is linear ($S(U+d) = -(U+d)+S(0) = S(U)-d$) this implies that the contract $\{T,(R+d),(U+d)\}$ is also optimal. [11]

Differentiating (1.70) yields

$$\frac{dT_s}{d\theta} = \frac{X'(T_s) - rX(T_s)}{X''(T_s) - rX'(T_s)} = \frac{rS(0)}{(p-\theta)[X''(T_s) - rX'(T_s)]} < 0.$$

It follows from (1.71) that the net transfer $R_s - U_s$ is decreasing in v and increasing in θ. [12] I can therefore state the following proposition :

Proposition 7. *Unlike in the static case, in the dynamic context, the optimal rotation period decreases with the firm's cost parameter. The net transfer also decreases with the firm's cost parameter, but its evolution over time is ambiguous.*

1.4.2.2 The asymmetric information case

In this subsection I analyze the optimal contract when the true value of the harvesting cost is known only to the firm. I will first characterize the class of incentive compatible mechanism, that is mechanism which induce the firm to reveal its cost truthfully. I show

11. Notice that $S(v) = \int_{\theta^L}^{\theta^H} [Re^{-rT} + S(U)e^{-rT}]f(\theta)d\theta = \int_{\theta^L}^{\theta^H} [(R+d)e^{-rT} + (S(U)-d)e^{-rT}]f(\theta)d\theta$.
12. The evolution of R over time will be ambiguous, because v and U cannot be compared.

in the Appendix how this class of mechanisms can be characterized by :

$$(p-\theta)[X'(T(v,\theta))-rX(T(v,\theta))]\frac{\partial T(v,\theta)}{\partial \theta} = \frac{\partial (R-U)(v,\theta)}{\partial \theta} - r(R-U)(v,\theta) \quad (1.72)$$

$$\frac{\partial T(v,\theta)}{\partial \theta}[X'(T(v,\theta))-rX(T(v,\theta))] \leqslant 0 \quad (1.73)$$

$$\frac{\partial \varphi(v,\theta)}{\partial \theta} = -X(T(v,\theta))e^{-rT(v,\theta)}. \quad (1.74)$$

Having characterized the incentive compatible mechanisms, the forest owner's problem can now be stated as choosing $\{(T(v,\theta),U(v,\theta),R(v,\theta))|(v,\theta)\in\mathscr{V}\times[\theta^L,\theta^H]\}$ to maximize (1.60) subject to (1.61),(1.72), (1.73), (1.74), and (1.63). A simple way to solve this problem is to transform the objective function into functions of T and U, by direct substitution. From (1.74), and using integration by part I get

$$\int_{\theta^L}^{\theta^H}\varphi(v,\theta)f(\theta)d\theta = \varphi(v,\theta^H)+\int_{\theta^L}^{\theta^H}X(T(v,\theta))e^{-rT(v,\theta)}F(\theta)d\theta.$$

Substituting this into (1.60) and (1.61) the objective function and the promise-keeping condition become respectively

$$\begin{aligned} S(v) \;=\; & \int_{\theta^L}^{\theta^H}\{(p-\theta-(1-\alpha)h(\theta))X(T(v,\theta))e^{-rT(v,\theta)}-D \quad (1.75)\\ & +(1-\alpha)e^{-rT(v,\theta)}U(v,\theta)+e^{-rT(v,\theta)}S(U(v,\theta))\}f(\theta)d\theta-(1-\alpha)\varphi(v,\theta^H) \end{aligned}$$

and

$$\varphi(v,\theta^H)+\int_{\theta^L}^{\theta^H}X(T(v,\theta))e^{-rT(v,\theta)}f(\theta)d\theta = v. \quad (1.76)$$

The forest owner's problem can then be rewritten as that of choosing $\varphi(v,\theta^H)$ and $\{(T(v,\theta),U(v,\theta))|(v,\theta)\in\mathscr{V}\times[\theta^L,\theta^H]\}$ to maximize (1.75) subject to the promise-keeping condition (1.76), the incentive compatibility constraint (1.73) and the participation constraints (1.63). I will first ignore the incentive compatibility condition (1.73) and I will show later that the optimal contract satisfies it.

Let $\lambda(v)$ be the Lagrange multiplier associated to (1.76) and $\mu(v,\theta)e^{-rT(v,\theta)}f(\theta)$ the Lagrange multiplier associated to (1.63). For simplicity I will use $T=T(v,\theta)$ and

$U = U(v,\theta)$ where there is no risk of confusion. The Lagrangian can be written as :

$$\begin{aligned}\mathscr{L}(T,U,\lambda,\mu) \;=\; & \int_{\theta^L}^{\theta^H} \{[p-\theta+(\lambda(v)-(1-\alpha))h(\theta)]X(T)e^{-rT} \\ & -D+(1-\alpha+\mu(v,\theta))e^{-rT}U \\ & +e^{-rT}S(U)\}f(\theta)d\theta+(\lambda(v)-(1-\alpha))\varphi(v,\theta^H)-\lambda(v)v.\end{aligned}$$

Differentiating $\mathscr{L}$ with respect to T, U, μ gives the following necessary conditions :

$$[p-\theta+(\lambda(v)-(1-\alpha))h(\theta)][X'(T)-rX(T)] = r(1-\alpha)U+rS(U) \qquad (1.77)$$

$$1-\alpha+\mu(v,\theta)+S'(U) = 0 \qquad (1.78)$$

$$\mu(v,\theta)U = 0;\; \mu(v,\theta) \geqslant 0;\; U \geqslant 0. \qquad (1.79)$$

By the envelope condition,

$$S'(v) = -\lambda(v). \qquad (1.80)$$

In order to determine the optimal contract, let us first show that

$$0 \leqslant \lambda(v) < 1-\alpha.$$

Since $S(\cdot)$ is non increasing in v, it follows from (1.80) that $\lambda(v) \geqslant 0$. Now, if $\lambda > 1-\alpha$, equation (1.76) implies $\varphi(v,\theta^H) \leqslant v$. Given that $\mathscr{L}$ is linear in $\varphi(v,\theta^H)$ and $\lambda(v)-(1-\alpha) > 0$, optimality requires that we set $\varphi(v,\theta^H) = v$. Considering that $\varphi(v,\theta)$ is decreasing in θ, this condition implies that $\forall\theta \in [\theta^L,\theta^H]$ $\varphi(v,\theta) > \varphi(v,\theta^H) = v$. Integrating this, it follows that $\int_{\theta^L}^{\theta^H} \varphi(v,\theta)f(\theta)d\theta > v$ which is a contradiction. Finally if $\lambda(v) = 1-\alpha$. The preceding system of equations gives the same solution as under symmetric information. So the mechanism is not incentive compatible in the sense that the incentive compatibility condition (1.72) is not satisfied. Hence I have shown that $\lambda(v) < 1-\alpha$.

Since $\lambda(v) < 1-\alpha$ and $\mu(v,\theta) \geqslant 0$, it follows from (1.78) and (1.80) that $S'(v) >$

$S'(U)$. This condition, combined with the assumption that $S(\cdot)$ and $U(\cdot)$ are differentiable, implies that $S(\cdot)$ is strictly concave in $\mathscr{V}$, or as worst, $S(\cdot)$ is strictly concave in an interval. For simplicity, let us assume that $S(\cdot)$ is strictly concave in $\mathscr{V}$. With $S'(v) > S'(U)$, I obtain by the concavity of $S(\cdot)$ that $v < U(v,\theta)$. Given that $v \geqslant 0$, then $U(v,\theta) > 0$ and it follows that

$$0 < v < U(v,\theta). \tag{1.81}$$

This is an important result. First it states that an efficient contract must require the promised future profit to rise ($v < U(v,\theta)$). The economic intuition behind the result seems to be that the forest owner can obtain truthful revelation at a lower cost by promising a higher profit to the firm at the next rotation. Secondly the fact that $v > 0, U(v,\theta) > 0$ simply means that at each period all firm types earn a strictly positive rent. This result contrast with that obtained in the static case, where the highest cost firm earned zero profit. Notice that in the static case the firm incurs the same cost forever. However, in a dynamic context, a higher-cost type will be of a lower-cost type with positive probability in some future period and will take advantage of it. The foregoing discussion implies the following proposition.

Proposition 8. *Under asymmetric information, the firm's promised expected profit rises. Unlike in the static setup, all firm types including the highest cost one enjoy strictly positive rent.*

I can now determine the optimal rotation period. It follows from (1.81) that $\mu(v,\theta) = 0$. Substituting this into (1.78), it follows that $S(U(v,\theta)) = -(1-\alpha)U(v,\theta) + \mathbf{A}$ where $\mathbf{A}$ is a constant. Plugging this into (1.77), the equation determining the optimal rotation period becomes : [13]

$$[p-\theta-(S'(v)+(1-\alpha))h(\theta)]X'(T_a) = r[p-\theta-(S'(v)+(1-\alpha))h(\theta)]X(T_a)+r\mathbf{A} \tag{1.82}$$

To verify that the incentive compatibility condition (1.73) is satisfied, differentiate (1.82)

13. Recall that the subscript a refers to the solution under asymmetric information.

and use (1.78) to get :

$$[X'(T_a) - rX(T_a)]\frac{\partial T_a(v,\theta)}{\partial \theta} = \frac{(S'(v) + (1-\alpha))h'(\theta)[X'(T_a) - rX(T_a)]^2}{[X''(T_a) - rX'(T_a)]} \leqslant 0.$$

Thus the condition (1.73) indeed holds.

1.4.3 The modified Faustmann rule

Equation (1.82) defines the optimal rotation period under asymmetric information. This equation deserves some comments. First, the expression $(S'(v) + (1-\alpha))h(\theta)$ represents the marginal information rent in the dynamic context. Unlike in the static setup, the marginal information rent includes a new term $S'(v)h(\theta)$. By promising an additional unit of profit to the firm, the forest owner can lower the cost of the informational constraints in the current period by $S'(v)h(\theta)$. The constant **A** is positive and it represents the future expected profit of the forest owner (expected welfare) or the value of bare land under asymmetric information. Notice that if **A** is negative, the forest owner would get out of forestry and devote the land to its "highest use".

Equation (1.82) is the Faustmann rule modified in order to take into account the cost of information. It is different from the standard Faustmann rule under symmetric information given in equation (1.69) in two aspects. The first is the net price which is corrected for the cost of informational constraints and the second is that **A**, the value of bare land under asymmetric information, is lower than $S(0)$, the value of bare land under symmetric information. This is because, under asymmetric information, the forest owner has to pay in order to obtain revelation. This implies that unlike in the static case, the lowest cost firm chooses a different rotation under asymmetric information than under symmetric information. The reason is that a firm of type θ^L in the current period has a positive probability of being a higher cost type in future periods.

Differentiating (1.82), it follows that :

$$\frac{\partial T_a(v,\theta)}{\partial \theta} = \frac{(S'(v)+(1-\alpha))h'(\theta)[X'(T_a)-rX(T_a)]}{[X''(T_a)-rX'(T_a)]} < 0 \qquad (1.83)$$

$$\frac{\partial T_a(v,\theta)}{\partial v} = \frac{S''(v)h(\theta)[X'(T_a)-rX(T_a)]}{[X''(T_a)-rX'(T_a)]} \geqslant 0. \qquad (1.84)$$

Equation (1.83) shows that at each period k, the optimal rotation decreases with the firm's cost parameter. It follows from (1.84) that $\frac{\partial T_a(v,\theta)}{\partial v} > 0$ when $\theta \in (\theta^L, \theta^H]$. Since $U(v,\theta) > v$, this implies that $T(U(v,\theta),\theta) > T(v,\theta)$, which means that for a given value of the cost, the rotation at period $k+1$ is longer than the rotation at period k. Hence :

Proposition 9. *Under asymmetric information, (i) for any given period, the optimal rotation period decreases with the cost parameter ; (ii) the optimal rotation period strictly increases over time, except for the lowest cost firm, for which the rotation is constant.*

Now let us compare optimal rotations in the symmetric information and asymmetric information cases. It follows immediately from (1.69) and (1.82) that for all v, $T_s(\theta^L) < T_a(v,\theta^L)$. Since T_s and T_a are continuous functions of θ, this implies that for all v, $T_s(\theta^L) < T_a(v,\theta^L)$ in the neighborhood of θ^L. In other words, for any given period k, the optimal rotation in the asymmetric information case is longer near the lower bound of the cost parameter.

In order to provide a complete comparison of optimal rotations and optimal royalties in $[\theta^L, \theta^H]$, we have to compute the value functions under symmetric information as well as under asymmetric information. This can only be done by using numerical methods.

1.5 Conclusion

I have characterized the optimal royalty contracts in the forestry when the harvesting firm has private information on its harvesting cost. Given the informational constraints that arise from this situation, it has been shown that both in the static and in the dynamic contexts the optimal rotation must satisfy a modified version of the usual Faustmann rule which holds under symmetric information. This modification is necessary in order to induce cost revelation on the part of the harvesting firm. As a result, in the static context, the optimal rotation period will be longer in the asymmetric information case than in the symmetric information case. It was also shown how the cutoff cost can be endogenized, thus increasing the owner's expected profit by making sure that unprofitable forests are not exploited. Also in the static context, it was shown that the forest owner could increase its expected profit by allowing competition among firms through public auctions.

In the dynamic context, I find that unlike in the static case all firms, including the highest-cost type, get a rent associated with the continuation part of the rotation choice under asymmetry information. It was also shown that the firm's rent rises over time. This is necessary in order to get revelation at a lower cost in the current period. Therefore the optimal rotation increases over time as well. The optimal contract under asymmetry of information also has the effect of imposing a distortion on the rotation of the lowest-cost firm. This contrasts with the result in the static case, where the lowest-cost firm remains efficient under asymmetric information.

It would be desirable to characterize the optimal contract within a dynamic context when the forest owner uses auctions as alternative royalty payment and to compare it with the mechanism derived above. This raises an inherent difficulty due to the fact that the value functions can only be obtained through numerical methods. It is left for future work. It could also be interesting to consider the effect of asymmetry of information on other parameters, such as the replanting cost.

CHAPITRE 2

OPTIMAL FORESTRY CONTRACTS WITH INTERDEPENDENT VALUES

2.1 Introduction

In many countries, governments and private landowners delegate the exploitation of their forests to firms specialized in planting and harvesting operations. The decisions regarding the firm that will exploit the land and the monetary transfers that occur between the firms and the owner can be reached through many different means. The owner can organize an auction among competing firms for example. The owner may also select one of the firms randomly or through subjective criteria and negotiate directly with the selected firm the terms of the exploitation. The following question then arises : What is the optimal allocation mechanism from the point of view of the forest owner ?

The answer to this question should induce the best time to harvest the forest, determining hence the optimal rotation period. If the growth function is known and if the price, the planting and harvesting costs are constant and known by the forest owner and the firms together, then the answer is straightforward. The forest owner will sell the right to harvest the forest to the low cost contractor and the optimal rotation period will satisfy the well known Faustmann rule (Faustmann, 1849). This means that if the forest owner wishes to maximize the present value of the forest through planting and harvesting operations, the optimal contract must induce the selected firm to cut down the forest when the increase in the net value of the standing forest over a unit time interval (rotation period) is equal to the interest on the value of the stand plus the interest on the value of the forest land.

In practice however neither the forest owner nor the firms fully know the costs. Each firm only observes a signal of its cost. For example each firm may be allowed to cruise a tract of forest and obtain an estimate (signal) of its cost. Given that its observation is incomplete, a firm's cost will therefore depend on estimations (signals) by the other firms as well which are private information. We are then facing the problem of auction design with firms that have interdependent values. In this setting, the value of an allocation will depend on the private signals of the other firms. The purpose of this paper is to characterize the optimal mechanism in this context and determine conditions under which the optimal mechanism can be implemented by a second price auction. The optimal mechanism presented in this paper induces a rule for the optimal rotation period that is a modified version of the Faustmann rule. Indeed we find that the selected firm will cut down the forest when the increase in the net virtual value of the standing forest over a unit rotation period is equal to the interest on the net virtual value of the stand plus the interest on the virtual value of the forest land. The virtual values of the stand and of the land are corrections of the same values in order to account for the asymmetry of information between the owner and the firms.

Optimal mechanism design has received a lot of attention since the seminal paper of Myerson (1981) who designed an optimal selling mechanism for one unit of an indivisible good. Since then many authors have studied optimal mechanisms in a more general context of interdependent values, with one or many units of an indivisible good. In this context bidders receive private signals which may be correlated (see Crémer and McLean (1985, 1988), and McAfee et al. (1989)) or independent (see Branco, (1996)), and which jointly determine the valuations of all bidders. In the case of correlated signals McAfee et al. (1989) showed that the seller can extract almost all of the full surplus of the buyers, in the sense that the surplus left to the buyers can be as low as desired. In the case of independent signals however, Branco (1996) showed that it is not possible to fully extract the surplus of buyers.

Following Branco (1996) we consider the case of independent signals. Hence our optimal mechanism is inefficient. Moreover the literature considers mechanisms that consists of an assignment rule and a payment rule. In our set up the payments have to be made periodically in the form of royalties, at each rotation period. The owner of the land must determine both the royalties and the rotation period. Thus our definition of a mechanism design slightly departs from the literature and consists of an assignment rule, a rule for the determination of the rotation period and a rule for the determination of the royalties at each rotation.

An interesting feature of the optimal mechanism is that it provides a benchmark for the comparison of standard auctions. In particular in Section 3 we are interested in situations under which a second price auction with reservation price is optimal. The choice of this auction format is guided by analytical tractability. Moreover Riley and Samuelson (1981) indicate that the standard auctions format are equivalent for appropriate reservation prices.

This is not the first paper to characterize the optimal contract in the presence of asymmetry of information the using Faustmann framework. Indeed Tatoutchoup (2010) analyzed the optimal contract in forestry in the case where the firm has private information about its cost. But it is assumed in that paper that each firm knows perfectly its harvesting cost. In the present paper, the firms only observe signals about these costs. In this setting, the value of an allocation will also depend on the private signals of the other firms ; hence costs are interdependent. Finally, to the best of our knowledge, in the literature on resource economics, standing timber auctions are most often studied in the case independent private value (see Elyakime et al., (1994) or Li and Perrigne, (2003)), contrary to what is assumed in this paper.

We will assume that the forest contracts are signed into perpetuity. Hence we will analyze the case where the harvesting costs are perfectly correlated over time, and we will assume that the owner cannot renegotiate the contract.

The paper is organized as follows. In Section 2.2, we first describe the problem and the model and then solve it by determining an optimal contract. We finally discuss the resulting modified Faustmann rule. Section 2.3 discusses the conditions under which the optimal mechanism can be implemented by a second price sealed-bid auction with reservation price. Some concluding remarks are provided in Section 2.4.

2.2 Characterization of the optimal mechanism

In this section we model the problem of determining how a forest owner will allocate the right to harvest the forest among N competing harvesting firms. The firms cannot perfectly observe their costs and the harvesting cost structure of the firms are unknown to the forest owner. Let $X(T)$ be the timber growth function, where T represents the age of a tree, and assume that it is twice differentiable, strictly increasing and concave. Suppose that when it is cut down, a stand of trees of age T yields a net profit in present value given by :

$$(p-C)X(T)e^{-rT}-K.$$

Where p is the given market price of wood, the parameter K represents the total cost of planting a unit of land, r is the discount rate and C denotes the cutting cost per unit of wood.

2.2.1 The model

Each firm $i \in \{1,...,N\}$ privately observes a signal (or type) θ_i that is drawn from $[\underline{\theta}_i, \overline{\theta}_i]$ according to a continuous density function $f_i(\theta_i) > 0$, independently of the other firms' signals. We will assume a monotone hazard rate, that is :

$$h_i(\theta_i) = \frac{F_i(\theta_i)}{f_i(\theta_i)} \text{ is increasing in } \theta_i,$$

where $F_i(\theta_i)$ is the cumulative distribution function associated to the density function $f_i(\theta_i)$. The knowledge of the probability distribution is shared by all the agents (forest owner and firms).

The harvesting cost $C_i(\theta)$ of firm i depends on the signals received by all the firms ($\theta = (\theta_1,...,\theta_N)$). Let us assume that $\partial_i C_i(\theta) = \frac{\partial C_i(\theta)}{\partial \theta_i} > 0$ and $\partial_j C_i(\theta) = \frac{\partial C_i(\theta)}{\partial \theta_j} \geq 0$, $j \neq i$. This means that the costs functions are differentiable and non decreasing with respect to any firm's signal. In particular the cost of a firm is increasing in its own signal.

We will also assume that the cost functions satisfy the single crossing condition, that is :

$$\text{for all } i \text{ and } j \neq i \text{ and for all } \theta, \qquad \frac{\partial C_i(\theta)}{\partial \theta_i} \geqslant \frac{\partial C_j(\theta)}{\partial \theta_i}. \tag{2.1}$$

Condition (2.1) states that, keeping all others signals fixed, the cost of firm i is increasing at least as quick as the cost of any other firm, when firm i's own signal is increasing. So the two costs cross at most once.

We will further assume that the costs incurred by a firm are perfectly correlated over time. So at each rotation k, the cost of the firm i is the same, $C_i(\theta)$.

By the revelation principle we may restrict our attention to revelation mechanisms. In such mechanisms, the owner requires each firm to reveal its private signal and, on the basis of the information collected, he chooses the winning firm, the rotation periods (the cutting ages of the trees) and the payments of each firm at each rotation period. Formally a revelation mechanism is a triplet of functions (q,T,R) defined on $\times_{i=1}^{N}[\underline{\theta}_i,\overline{\theta}_i] \equiv \Theta$, where $q(\theta) = (q_i(\theta))_{i=1}^{N}$, $T(\theta) = (T_i(\theta))_{i=1}^{N}$ and $R(\theta) = (R_i(\theta))_{i=1}^{N}$.
For any i and every vector θ, $T_i(\theta) \in \mathbb{R}^+$ denotes the cutting age of the trees, $q_i(\theta) \in [0,1]$ denotes the probability that firm i receives the right to harvest the forest land and $R_i(\theta) \in \mathbb{R}$ denotes the payment that firm i must make to the forest owner when it harvests the tree. [1] Consequently, the sum of the probabilities, $\sum_{i=1}^{N} q_i(\theta)$, must never exceed unity.

Throughout the paper, θ_{-i} denotes the vector $(\theta_1,...,\theta_{-i},\theta_{i+1},...,\theta_N)$. Firm $i's$ expected profit over infinite cycles of rotations $T_i(\theta)$ given that it reports its type to be $\widehat{\theta}_i$ and that all firms $j \neq i$ truthfully reveal their types is given by :

$$\varphi_i(\widehat{\theta}_i,\theta_i) = E_{\theta_{-i}}[\pi_i(\widehat{\theta}_i,\theta_i,\theta_{-i})],$$

where

$$\pi_i(\widehat{\theta}_i,\theta_i,\theta_{-i}) = \omega_i(\widehat{\theta}_i,\theta_i,\theta_{-i})q_i(\widehat{\theta}_i,\theta_{-i}) - \frac{R_i(\widehat{\theta}_i,\theta_{-i})e^{-rT_i(\widehat{\theta}_i,\theta_{-i})}}{1-e^{-rT_i(\widehat{\theta}_i,\theta_{-i})}}$$

and

$$\omega_i(\widehat{\theta}_i,\theta_i,\theta_{-i}) = \frac{(p-C_i(\theta))X(T_i(\widehat{\theta}_i,\theta_{-i}))e^{-rT_i(\widehat{\theta}_i,\theta_{-i})} - K}{1-e^{-rT_i(\widehat{\theta}_i,\theta_{-i})}}.$$

1. A priori we allow this payment to be negative. This corresponds to a situation where a firm is subsidized in order to harvest the land for its own profit.

2.2.2 Characterization of the incentive compatibility constraints

Since the signals are observed privately the firms may have the incentive to misreport their types in order to reduce their payments. To avoid this possibility the forest owner may want to design a mechanism that induces the truthful participation of the firms, i.e. an incentive compatible mechanism. A mechanism (q,T,R) is (Bayesian) incentive compatible if the expected profit of any firm is maximized when it reports its true signal, given that the other firms do the same. Formally :

$$\text{for all i=1,..., N, } \varphi_i(\theta_i,\theta_i) \geq \varphi_i(\widehat{\theta}_i,\theta_i)\ \forall \widehat{\theta}_i,\theta_i \in [\underline{\theta}_i,\overline{\theta}_i]. \tag{2.2}$$

We are now going to break this condition into two. This will prove useful in the determination of the optimal mechanism. Equation (2.2) implies that

$$\begin{aligned}
\varphi_i(\theta_i,\theta_i) &\geq \varphi_i(\widehat{\theta}_i,\theta_i) \quad \forall \widehat{\theta}_i,\theta_i \\
\varphi_i(\theta_i,\theta_i) &\geq \varphi_i(\widehat{\theta}_i,\widehat{\theta}_i)+\varphi_i(\widehat{\theta}_i,\theta_i)-\varphi_i(\widehat{\theta}_i,\widehat{\theta}_i) \quad \forall \widehat{\theta}_i,\theta_i \\
\varphi_i(\theta_i,\theta_i) &\geq \varphi_i(\widehat{\theta}_i,\widehat{\theta}_i)+E_{\theta_{-i}}[(\omega_i(\widehat{\theta}_i,\theta_i,\theta_{-i})-\omega_i(\widehat{\theta}_i,\widehat{\theta}_i,\theta_{-i}))q_i(\widehat{\theta}_i,\theta_{-i})]. \qquad (2.3)
\end{aligned}$$

It is convenient to define $V_i(\theta_i)=\varphi_i(\theta_i,\theta_i)$, $\partial_2\varphi_i=\frac{\partial\varphi_i(\widehat{\theta}_i,\theta_i)}{\partial\theta_i}$, and $\partial_2\omega_i=\frac{\partial\omega_i(\widehat{\theta}_i,\theta_i,\theta_{-i})}{\partial\theta_i}$. Then by the envelope theorem, we must have

$$\frac{dV_i}{d\theta_i}=\partial_2\varphi_i(\theta_i,\theta_i)=E_{\theta_{-i}}[\partial_2\omega_i(\theta_i,\theta_i,\theta_{-i})q_i(\theta_i,\theta_{-i})]. \tag{2.4}$$

Since

$$\partial_2\omega_i(\theta_i,\theta_i,\theta_{-i})=-\frac{\partial_i C_i(\theta)X(T_i(\theta))}{e^{rT_i(\theta)}-1}<0,$$

we may conclude that $\frac{dV_i}{d\theta_i}<0$. Therefore $V_i(\theta_i)$ is a non increasing function of θ_i.

We can rewrite $V_i(\theta_i)$ as :

$$V_i(\theta_i)=V_i(\overline{\theta}_i)-\int_{\theta_i}^{\overline{\theta}_i}E_{\theta_{-i}}[\partial_2\omega_i(\tau,\tau,\theta_{-i})q_i(\tau,\theta_{-i})]d\tau. \tag{2.5}$$

Using this last expression of the envelope theorem (i.e. equation 2.5), incentive compatibility (2.2) implies that :

$$\begin{aligned} & E_{\theta_{-i}} \int_{\theta_i}^{\widehat{\theta}_i} -\partial_2 \omega_i(\tau,\tau,\theta_{-i}) q_i(\tau,\theta_{-i}) d\tau \\ \geq \ & E_{\theta_{-i}}[(\omega_i(\widehat{\theta}_i,\theta_i,\theta_{-i}) - \omega_i(\widehat{\theta}_i,\widehat{\theta}_i,\theta_{-i}))q_i(\widehat{\theta}_i,\theta_{-i})]. \end{aligned} \tag{2.6}$$

Reciprocally, equation (2.6) and the envelope theorem (2.5) imply equation (2.3), which is equivalent to the incentive compatibility constraints (2.2). Therefore incentive compatible mechanisms can be characterized by the following lemma :

Lemma 10. *(q,T,R) is incentive compatible if and only if conditions (2.5) and (2.6) are satisfied.*

Finally we assume that firms' participation is voluntary. They can decide to opt out in response to the announced incentive scheme. We will normalize the outside options of the firms to zero. This means that the revelation mechanism (q,T,R) must satisfy the individual rationality constraints, given by :

$$\text{for all i=1,..., N, } V_i(\theta_i) \geq 0 \quad \forall \theta_i \in [\underline{\theta}_i, \overline{\theta}_i]. \tag{2.7}$$

Since $V_i(\theta_i)$ is a non increasing function of θ_i, by the envelope theorem (2.5), the constraints (2.7) can be replaced by the constraints

$$V_i(\overline{\theta}_i) \geq 0, \quad \text{for all i=1,..., N.} \tag{2.8}$$

The forest owner's objective function is the expected social welfare given by :

$$EW = E\sum_{i=1}^{N}[\frac{R_i(\theta)e^{-rT_i(\theta)}}{1-e^{-rT_i(\theta)}} + \alpha_i \pi_i(\theta_i,\theta_i,\theta_{-i})]. \tag{2.9}$$

The social welfare $W = \sum_{i=1}^{N} W_i$ where $W_i = \frac{R_i(\theta)e^{-rT_i(\theta)}}{1-e^{-rT_i(\theta)}} + \alpha_i \pi_i(\theta_i,\theta_i,\theta_{-i})$ is the weighted sum of the owner's revenue and firms' surplus. [2] We adopt the standard assumption

2. To simplify the problem, we assume that the country is a price taker in the world market so that domestic production of resource good does not give rise to consumer's surplus.

that $0 \leqslant \alpha_i < 1$: a dollar in the owner revenue is valued more highly than a dollar that remains as profits in the hands of a firm.[3] The social welfare can also be viewed as the present value to the owner of all future rotations. From the definition of $\pi_i(\theta_i, \theta_i, \theta_{-i})$, it follows that $\frac{R_i(\theta)e^{-rT_i(\theta)}}{1-e^{-rT_i(\theta)}} = \omega_i(\theta_i, \theta_i, \theta_{-i})q_i(\theta_i, \theta_i, \theta_{-i}) - \pi_i(\theta_i, \theta_i, \theta_{-i})$. Substituting this into (2.9), we may therefore rewrite EW as :

$$E\sum_{i=1}^{N} \omega_i(\theta_i, \theta_i, \theta_{-i})q_i(\theta_i, \theta_i, \theta_{-i}) - E\sum_{i=1}^{N}(1-\alpha_i)\pi_i(\theta_i, \theta_i, \theta_{-i}). \tag{2.10}$$

Using the law of iterated expectations we can write

$$E\sum_{i=1}^{N}(1-\alpha_i)\pi_i(\theta_i, \theta_i, \theta_{-i}) = \sum_{i=1}^{N}\int_{\underline{\theta}_i}^{\overline{\theta}_i}(1-\alpha_i)V_i(\theta_i)f_i(\theta_i)d\theta_i.$$

Now substituting for $V_i(\theta_i)$ from (2.5) and using integration by parts, we verify that :

$$\int_{\underline{\theta}_i}^{\overline{\theta}_i}(1-\alpha_i)V_i(\theta_i)f_i(\theta_i)d\theta_i = V_i(\overline{\theta}_i) - \int_{\underline{\theta}_i}^{\overline{\theta}_i} E_{\theta_{-i}}[\partial_2\omega_i(\theta_i, \theta_i, \theta_{-i})q_i(\theta_i, \theta_{-i})]F_i(\theta_i)d\theta_i.$$

Substituting this into (2.10) and rearranging we can therefore rewrite the owner's objective function as :

$$EW = E\sum_{i=1}^{N} g_i(\theta, T_i(\theta))q_i(\theta) - \sum_{i=1}^{N}(1-\alpha_i)V_i(\overline{\theta}_i), \tag{2.11}$$

where

$$g_i(\theta, t_i) = \frac{[p - C_i(\theta) - (1-\alpha_i)\partial_i C_i(\theta)h_i(\theta_i)]X(t_i)e^{-rt_i} - K}{1-e^{-rt_i}}.$$

The owner's problem is to choose the functions $\{T_i(\theta), q_i(\theta), V_i(\overline{\theta}_i)\}_{i=1}^{N}$ in order to maximize the expected welfare given by

$$EW = \int_{\underline{\theta}_1}^{\overline{\theta}_1} \ldots \int_{\underline{\theta}_N}^{\overline{\theta}_N} \left[\sum_{i=1}^{N} g_i(\theta, T_i(\theta))q_i(\theta)\right]\left[\prod_{i=1}^{N} f_i(\theta_i)\right] d\theta_N \ldots d\theta_1 - \sum_{i=1}^{N}(1-\alpha_i)V_i(\overline{\theta}_i) \tag{2.12}$$

subject to (2.5), (2.6), and (2.8).

3. This is the standard interpretation of α_i, as presented by Baron (1989).

We will now construct a direct mechanism (q,T,R) that maximizes the expected welfare (2.12) subject to the constraints (2.5),(2.6) and (2.8). Consequently this mechanism will be optimal for the forest owner. To construct this mechanism, let us define :

$$g_i^*(\theta) = \max_{t_i>0} g_i(\theta,t_i) \quad \forall i,\ \forall \theta.$$

We show in the Appendix that $g_i^*(\cdot)$ satisfies the following condition :

$$g_i^*(\theta) = g_j^*(\theta) \Rightarrow \frac{\partial g_i^*(\theta)}{\partial \theta_i} < \frac{\partial g_j^*(\theta)}{\partial \theta_i} \quad i \neq j. \tag{2.13}$$

Let us define the optimal rotation period as follows :

$$T_i^*(\theta) \in arg\max_{t_i>0} g_i(\theta,t_i). \tag{2.14}$$

To complete the mechanism, we define the probability assignment and the royalty respectively as :

$$q_i^*(\theta) = \begin{cases} 1 & \text{if} \quad g_i^*(\theta) > \max\{0, \max_{j\neq i} g_j^*(\theta)\} \\ 0 & \text{if} \quad g_i^*(\theta) < \max\{0, \max_{j\neq i} g_j^*(\theta)\} \end{cases} \tag{2.15}$$

and

$$\frac{R_i^*(\theta)e^{-rT_i^*(\theta)}}{1-e^{-rT_i^*(\theta)}} = \omega_i^*(\theta_i,\theta_i,\theta_{-i})q_i^*(\theta) - \int_{\theta_i}^{\overline{\theta}_i} \partial_2\omega_i^*(\tau,\tau,\theta_{-i})q_i^*(\tau,\theta_{-i})d\tau. \tag{2.16}$$

This implies that the profit is given by

$$\pi_i^*(\theta_i,\theta_i,\theta_{-i}) = \int_{\theta_i}^{\overline{\theta}_i} \partial_2\omega_i^*(\tau,\tau,\theta_{-i})q_i^*(\tau,\theta_{-i})d\tau$$

.

The following Proposition shows that the mechanism (q^*,T^*,R^*) defined previously is optimal.

Proposition 11. *The mechanism (q^*,T^*,R^*) maximizes the expected welfare among in-*

centive compatible and individually rational mechanisms.

To show this we will need the results of Lemma.

Lemma 12.
(i) $T_i^*(.,\theta_{-i})$ *is increasing* $\forall i$, $\forall \theta_{-i}$.
(ii) $g_i^*(\theta_i^h,\theta_{-i}) > \max\{0, \max_{j\neq i} g_j^*(\theta_i^h,\theta_{-i})\}$
$\Rightarrow \forall \theta_i < \theta_i^h, \quad g_i^*(\theta_i,\theta_{-i}) > \max\{0, \max_{j\neq i} g_j^*(\theta_i,\theta_{-i})\}$.
(iii) $-\partial_2\omega_i^*(.,\theta_i,\theta_{-i})q_i^*(.,\theta_{-i})$ *is non increasing in*
$\triangle_i(\theta_{-i}) = \{\theta_i : g_i^*(\theta) \neq \max\{0, \max_{i\neq j} g_j^*(\theta)\}\}$.

The proof of the Lemma is provided in the Appendix. Part (i) means that the cutting age of trees is increasing with a firm's own signal. Part (ii) is rather technical and is used to prove part (iii). Part (iii) means that the expected marginal revenue of a firm with respect to its own signal is non increasing (almost surely). It is used to prove that the mechanism (q^*,T^*,R^*) is incentive compatible.

Proof. (**of Proposition 11**)
First we need to show that the mechanism (q^*,T^*,R^*) is incentive compatible and individually rational. In other words we will show that it satisfies the equations (2.5), (2.6) and (2.8).

By construction $V_i(\overline{\theta}_i) = \varphi(\overline{\theta}_i,\overline{\theta}_i) = 0$ so individual rationality constraint (2.8) is satisfied. The envelope condition (2.5) is also satisfied by construction. The incentive compatibility constraint (2.6) follows from part (iii) of Lemma (12). Indeed if $\widehat{\theta}_i > \theta_i$

then

$$\begin{aligned}
& E_{\theta_{-i}} \int_{\theta_i}^{\widehat{\theta}_i} [-\partial_2 \omega_i^*(\tau,\tau,\theta_{-i}) q_i^*(\tau,\theta_{-i})] d\tau \\
= \;& E_{\theta_{-i}} \int_{\theta_i}^{\widehat{\theta}_i} [-\partial_2 \omega_i^*(\tau,\tau,\theta_{-i}) q_i^*(\tau,\theta_{-i}) 1_{\triangle_i(\theta_i)}(\tau)] d\tau \\
\geq \;& E_{\theta_{-i}} \int_{\theta_i}^{\widehat{\theta}_i} -\partial_2 \omega_i^*(\widehat{\theta}_i,\tau,\theta_{-i}) q_i^*(\widehat{\theta}_i,\theta_{-i}) 1_{\triangle_i(\theta_i)}(\tau) d\tau \\
= \;& E_{\theta_{-i}} \int_{\theta_i}^{\widehat{\theta}_i} -\partial_2 \omega_i^*(\widehat{\theta}_i,\tau,\theta_{-i}) q_i^*(\widehat{\theta}_i,\theta_{-i}) d\tau \\
= \;& E_{\theta_{-i}} [(\omega_i^*(\widehat{\theta}_i,\theta_i,\theta_{-i}) - \omega_i^*(\widehat{\theta}_i,\widehat{\theta}_i,\theta_{-i})) q_i^*(\widehat{\theta}_i,\theta_{-i})].
\end{aligned}$$

Finally we need to show that the mechanism (q^*, T^*, R^*) is optimal, i.e. it maximizes the expected welfare (2.12) among all incentive compatible and individually rational mechanisms. We have $V_i(\overline{\theta}_i) = 0$ for individually rational mechanisms that are incentive compatible. Moreover by equation (2.14),

$$\sum_{i=1}^{N} g_i(\theta, T_i^*(\theta)) q_i^*(\theta) \geq \sum_{i=1}^{N} g_i(\theta, T_i(\theta)) q_i(\theta), \forall q, T.$$

Therefore $W(q^*, T^*, R^*) \geq W(q, T, R)$ for any mechanism (q, T, R) that is incentive compatible and individual rational. ∎

2.2.3 Discussion and Interpretation

The value $g_i^*(\theta, t_i)$ can be thought of as the (virtual) surplus of the forest owner when firm i is allowed to harvest the forest under an infinite cycle of rotation t_i. In the equation (2.14) the optimal rotation is chosen so as to maximize the virtual surplus. Thus the value $g_i^*(\theta_i)$ represents the optimal (virtual) surplus (hereafter virtual surplus or simply surplus) of the forest owner when he allows the firm i to harvest the forest. We can also interpret $g_i^*(\theta_i)$ as the value of the forest land, under imperfect information, when the forest is exploited by the firm i.

Equation (2.15) implies that the right to harvest the forest is eventually allocated to the firm with the highest virtual value provided its virtual value is positive. Observe that the forest owner can deny all firms the right to harvest the forest, and thus leave it unexploited, if the value of the forest land (the virtual surplus) is negative regardless of the firm that would exploit the land. Efficiency would require that it be exploited and that it be exploited by the agent that values the land the most (i.e. has the lowest harvesting cost). This means that in such a case the optimal mechanism is inefficient, first because the forest is not exploited and second because it is not exploited by the lowest cost firm.

Equation (2.22) implies that the payment of a firm that is not allowed to exploit the land is zero (i.e. $q_i^*(\theta) = 0$ implies that $R_i^*(\theta) = 0$). Moreover the profit of firms whose cost are lower than the market price are positive. So the owner is unable to extract the full surplus of the firms due to the asymmetry of information. Instead he must pay the firms an informational rent to provide them with incentives to reveal their private information. We can show that

$$g_i^*(\theta) = W_i(\theta) - (1-\alpha_i)\partial_i C_i(\theta) h_i(\theta_i) \frac{X(T_i)}{1-e^{-rT_i}}.$$

So the gap between W_i and g_i^* (i.e. $(1-\alpha_i)\partial_i C_i(\theta) h_i(\theta_i) \frac{X(T_i)}{1-e^{-rT_i}}$) captures the amount of the informational rent. These remarks on equations (2.15) and (2.22) are in tune with the literature on mechanism design (see Myerson (1981) and Branco (1996)).

It is also important to discuss the relationship between our result and the case of perfect information. In the case of perfect information, where the forest owner perfectly observes the cost of each firm, the firm with the lowest cost will receive the right to harvest the forest. The forest owner will set the payment in order to extract the entire surplus of the selected firm. Therefore the optimal rotation period of the firm will satisfy the Faustmann rule given by :

$$(p - C_i(\theta))X'(T_i) = r(p - C_i(\theta))X(T_i) + rW_i(\theta). \tag{2.17}$$

Our result allows us to derive a similar relationship in the case where the signals are private. Indeed, $g_i(\theta, t_i)$ may be rewritten as $\frac{\psi_i(\theta)X(t_i) - Ke^{rt_i}}{e^{rt_i}-1}$, where $\psi_i(\theta) = p - C_i(\theta) -$

$(1-\alpha_i)\partial_i C_i(\theta)h_i(\theta_i)$. For interior solutions, it can be shown that $T_i^*(\theta)$ satisfies the equation :

$$\psi_i(\theta)\lambda(T_i^*(\theta)) - rK = 0, \tag{2.18}$$

where $\lambda(t_i) = -X'(t_i)(1-e^{-rt_i}) + rX(t_i)$. The proof is provided in the proof of Lemma 12 (see equation (II.2)).

Let us rewrite equation (2.18) as

$$[p-\hat{C}_i(\theta)]X'(T_i) = r[p-\hat{C}_i(\theta)]X(T_i) + rg_i^*(\theta), \tag{2.19}$$

where

$$\hat{C}_i(\theta) = C_i(\theta) + (1-\alpha_i)\partial_i C_i(\theta)h_i(\theta_i).$$

Equation (2.19) defines implicitly the optimal rotation period of the selected firm under imperfect information. $\hat{C}_i(\theta)$ is often called the virtual costs of firm i. It is the cost of firm i augmented by the informational rent per unit of wood. Put simply, under the optimal mechanism the owner views it as firm i's cost and he views g_i^* as the value of the land if it is exploited by firm i. In that perspective equation (2.19) is the analogous of the Faustmann rule described in equation (2.17). The difference between the two rules lies on the informational rent. In equation (2.19) we use virtual costs instead of true cost as is the case in equation (2.17). Note that for the lowest type, the usual Faustmann rule is unmodified. The term $p-\hat{C}_i(\theta)$ represents the net price of the winning firm corrected by the informational rent per unit of wood.

It is important to note that, when the owner allocates the right to harvest to one firm the net price is necessarily positive. Otherwise this will imply that the highest virtual surplus is negative and then the forest owner will deny all firms the right to harvest the forest. The modified Faustmann rule can be stated as follows : the selected firm is induced to cut down the forest when the increase in the net virtual value of the standing forest over a unit time interval is equal to the interest on the net virtual value of the stand plus the interest on the virtual value of the forest land.

2.3 Implementation as a second price auction

This section studies the implementation of the optimal auction in the form of a standard auction, namely the second price auction. We give sufficient conditions for the optimality of a second price auction.

In standard auctions, participants are usually treated equally in terms of the payment and allocation rules. We will thus consider second price auctions in the context of a symmetric model. Therefore we make the following assumptions :

(i) The cost functions are symmetric :

$\forall i, \quad \forall \theta, \quad C_i(\theta) = C(\theta_i, \theta_{-i})$, where the function $C's$ last $(n-1)$ arguments are permutable.

(ii) The firms' signals are identically distributed ($f_i = f, \quad \forall i$) and all signals θ_i are drawn from the same interval $[\underline{\theta}, \overline{\theta}]$. Moreover, $\alpha_i = \alpha$.

Note that the symmetry of the cost functions implies the symmetry of functions g_i^* ; $g_i^* = G(\theta_i, \theta_{-i})$, G is permutable in the last arguments. We define $\theta_i^*(\theta_{-i})$, the maximum winning signal of firm i given the other firms' signals, as :

$$\theta_i^*(\theta_{-i}) = \min[\underline{\theta}_i^*(\theta_{-i}), \theta_{ij}^*(\theta_{-i})],$$

where

$$\begin{aligned}
\underline{\theta}_i^*(\theta_{-i}) &= \sup\{\theta_i \in [\underline{\theta}, \overline{\theta}] : G(\theta_i, \theta_{-i}) > 0\}, \\
\theta_{ij}^*(\theta_{-i}) &= \sup\{\theta_i \in [\underline{\theta}, \overline{\theta}] : G(\theta_i, \theta_{-i}) > G(\theta_j, \theta_{-j})\}.
\end{aligned}$$

Hence the optimal mechanism can be rewritten as

$$T_i^*(\theta) \in arg \max_{t_i > 0} g_i(\theta, t_i) \tag{2.20}$$

$$q_i^*(\theta) = \begin{cases} 1 & \text{if} \quad \theta_i < \theta_i^*(\theta_{-i}) \\ 0 & \text{otherwise} \end{cases} \tag{2.21}$$

$$\mathfrak{R}_i^* = \omega_i^*(\theta_i,\theta_i,\theta_{-i})q_i^*(\theta) - \int_{\theta_i}^{\theta_i^*(\theta_{-i})} \partial_2\omega_i^*(\tau,\tau,\theta_{-i})d\tau, \qquad (2.22)$$

where $\mathfrak{R}_i^* = \frac{R_i^*(\theta)e^{-rT_i^*(\theta)}}{1-e^{-rT_i^*(\theta)}}$ is the actualized payment.

Equation (2.22) follows from equation (2.13) and from the fact that G is decreasing in its first argument. This new expression of the optimal mechanism allows us to write the expected welfare as :

$$EW^* = E\sum_{i=1}^{N}[w_i(\theta_i,\theta_i,\theta_{-i}) + (1-\alpha)h(\theta_i)\partial_2 w_i(\theta_i,\theta_i,\theta_{-i})]\mathbf{1}_{(\theta_i<\theta_i^*(\theta_{-i}))}.$$

A second price auction is defined by a rotation function $T : \mathbb{R}^N \to \mathbb{R}_+$ and a reservation price function $r : \mathbb{R}^{N-1} \to \mathbb{R}_+$.

Given a bid vector $b = (b_i)_{i=1,\ldots,N}$, firm i wins if its bid exceeds the other firm's bid as well as the reservation price, i.e. if $b_i > b_j, \forall j \neq i$ and $b_i > r(b_{-i})$. The winning firm pays the minimum between the second lowest bid and the reservation price $r(b_{-i})$.

Observe that we consider a general class of second price auctions where the reservation price depends on the other participants' bids. The payment will have to be made every time the trees are cut and this period is determined by the rotation $T(b)$. To proceed we introduce the following functions :

$$\begin{aligned} v(z,\tau,y) &= E[w_i(z,\theta_i,\theta_{-i})\mathbf{1}(z<\theta_i^*(\theta_{-i})|\theta_i^*(\theta_{-i})=y,\theta_i=\tau)] \\ \gamma(\theta_i) &= v(\theta_i,\theta_i,\theta_i) - \frac{1}{f_{\theta_i^*(\theta_{-i})}(\theta_i)}\int_{\theta_i}^{\overline{\theta}} \frac{\partial v}{\partial z}(\theta_i,\theta_i,y)f_{\theta_i^*(\theta_{-i})}(y)dy. \end{aligned}$$

The following proposition shows how to determine a second price auction that is optimal.

Proposition 13. *If γ is decreasing and if $\theta_i^*(\theta_{-i}) \leq \min_{j\neq i}\theta_i \quad \forall i$, then the second price auction defined by $T(b) = T^*(\gamma^{-1}(b))$, $\gamma^{-1}(r(b)) = \theta_i^*(\gamma^{-1}(b_{-i}))$, is optimal.*

Proof. **Step 1** : γ is a symmetric equilibrium under the second price auction.
Assume firm i bids $\gamma(z)$ when its type is θ_i and all other firms bid $\gamma(\theta_j),\ j\neq i$. Firm i's

expected utility is

$$U(\theta_i, z) = E\{[w_i(z, \theta_i, \theta_{-i}) - \gamma(\delta(\theta_{-i}))]\mathbf{1}(z < \delta(\theta_{-i}) \mid \theta_i\},$$

. where $\delta(\theta_{-i}) = \min(\theta_i^*(\theta_{-i}), \min_{j\neq i}\theta_i) = \theta_i^*(\theta_{-i})$, by assumption. Therefore

$$U(\theta_i, z) = E\{[w_i(z, \theta_i, \theta_{-i}) - \gamma(\theta_i^*(\theta_{-i}))]\mathbf{1}(z < \theta_i^*(\theta_{-i})) \mid \theta_i\}.$$

Using the previous definition of v we obtain

$$\begin{aligned} U(\theta_i, z) &= E\{[v(z, \theta_i, \theta_i^*(\theta_{-i})) - \gamma(\theta_i^*(\theta_{-i}))]\mathbf{1}(z < \theta_i^*(\theta_{-i})) \mid \theta_i\} \\ &= \int_z^{\overline{\theta}} (v(z, \theta_i, y) - \gamma(y)) f_{\theta_i^*(\theta_{-i})}(y)dy. \end{aligned}$$

and

$$\frac{dU(\theta_i, z)}{dz} = \int_z^{\overline{\theta}} \frac{\partial v}{\partial z}(z, \theta_i, y) f_{\theta_i^*(\theta_{-i})}(y)dy - [v(z, \theta_i, z) - \gamma(z)] f_{\theta_i^*(\theta_{-i})}(z). \tag{2.23}$$

Setting (2.23) equal to zero, the first-order condition implies that :

$$\gamma(\theta_i) = v(\theta_i, \theta_i, \theta_i) - \frac{1}{f_{\theta_i^*(\theta_{-i})}(\theta_i)} \int_{\theta_i}^{\overline{\theta}} \frac{\partial v}{\partial z}(\theta_i, \theta_i, y) f_{\theta_i^*(\theta_{-i})}(y)dy.$$

Step 2 : The welfare under the second price auction is equivalent to the optimal welfare. We have $EW = \sum EW_i$, where $EW_i = \alpha EU(\theta_i, \theta_i) + E\mathfrak{R}_i^*(\theta)$ and

$$EU(\theta_i, \theta_i) = E\{w_i(\theta_i, \theta_i, \theta_{-i})\mathbf{1}(\theta_i < \theta_i^*(\theta_{-i}))\} - E\mathfrak{R}_i^*(\theta).$$

Therefore $EW_i = (\alpha - 1)EU(\theta_i, \theta_i) + E\{w_i(\theta_i, \theta_i, \theta_{-i})\mathbf{1}(\theta_i < \theta_i^*(\theta_{-i})) \mid \theta_i\}$. Using integration by part, we obtain :

$$\begin{aligned} EW_i = &- (1-\alpha)U_i(\overline{\theta}, \overline{\theta}) + E\{[w_i(\theta_i, \theta_i, \theta_{-i}) \\ &+ (1-\alpha)\int_{\theta_i}^{\overline{\theta}} \partial_2 w_i(y, y, \theta_{-i})F(y)dy]\mathbf{1}(\theta_i < \theta_i^*(\theta_{-i})) \mid \theta_i\}. \end{aligned}$$

By definition,

$$U_i(\overline{\theta},\overline{\theta}) = E\{[w_i(\overline{\theta},\theta_i,\theta_{-i}) - \gamma(\theta_i^*(\theta_{-i}))]\mathbf{1}(\theta_i < \theta_i^*(\theta_{-i})) \mid \theta_i = \overline{\theta}\}.$$

But $\mathbf{1}(\overline{\theta} < \theta_i^*(\theta_{-i})) = 0$ and hence $U_i(\overline{\theta},\overline{\theta}) = 0$. Therefore

$$\begin{aligned}
EW_i &= E\{[w_i(\theta_i,\theta_i,\theta_{-i}) + (1-\alpha)\int_{\theta_i}^{\overline{\theta}} \partial_2 w_i(y,y,\theta_{-i})F(y)dy]\mathbf{1}(\theta_i < \theta_i^*(\theta_{-i})) \mid \theta_i\} \\
&= \int_{\theta_i}^{\overline{\theta}} f(\theta_i)d\theta_i\{\int_{\theta_{-i}} w_i(\theta_i,\theta_i,\theta_{-i})\mathbf{1}(\theta_i < \theta_i^*(\theta_{-i}))f_{\theta_{-i}}d\theta_{-i} \\
&+ (1-\alpha)h(\theta_i)E[\partial_2 w_i(\theta_i,\theta_i,\theta_{-i})\mathbf{1}(\theta_i < \theta_i^*(\theta_{-i})) \mid \theta_i] \\
&= E\{[w_i(\theta_i,\theta_i,\theta_{-i}) + (1-\alpha)h(\theta_i)\partial_2 w_i(\theta_i,\theta_i,\theta_{-i})]\mathbf{1}(\theta_i < \theta_i^*(\theta_{-i}))\} \\
&= EW_i^*
\end{aligned}$$

and $EW = \sum EW_i^* = EW^*$. ∎

2.4 Conclusion

We have determined an optimal forestry contract when firms' harvesting costs are unknown to the forest owner and to the firms themselves, in a context where each firm observes privately a signal about its harvesting cost. We shown that the forest owner will allocate the right to harvest the forest to the firm with the highest (virtual) value of the forest land when it is nonnegative. Given the informational constraint, we have shown that the optimal rotation of the winning firm must satisfy a modified version of the usual Faustmann rule which holds under full information. This modification is necessary in order to induce the revelation of their true cost signals by all participating firms. We also determined conditions under which the optimal mechanism can be implemented by a second price auction.

These results have been obtained under the assumptions that the harvesting costs of firms are perfectly correlated over time and that the owner cannot renegotiate the contract. The optimal harvesting age is then the same for each rotation, a fact which considerably simplifies the problem. Those are not totally unrealistic assumptions in many situations. However characterizing the solution for the optimal contract in the absence of those assumptions remains a desirable goal. The problem is then complicated by the necessity of considering variable rotation length. It is left for future work to study this more complex case.

CHAPITRE 3

THE IMPACT OF RECYCLING ON THE LONG-RUN FORESTRY

3.1 Introduction

In recent years, it has become fashionable to promote recycling of forest products, in particular paper. The main argument in favor of encouraging recycling is that it saves trees, the implicit objective therefore being to end up with more trees than in the absence of recycling. The reason for wanting to do this is that the forest generates externalities : it procures direct amenities, it protects against soil erosion and it serves as a carbon sink. To the extent that positive externalities are involved, the market equilibrium will result in an insufficient area being devoted to foresty, which may justify policies meant to increase it. The purpose of this paper is to consider to what extent the promotion of recycling is an appropriate means of attaining such a goal.

To do this, we specify a simple dynamic model of land allocation by a private owner between forestry activities and alternative uses, such as agriculture. The model takes into account that the product of the forest can be partly recycled and it allows for two decision variables on the part of the land owner, namely the area of land allocated to forestry at any time and the age at which the forest is cut and replanted. This enables us to examine how recycling affects both the long-run equilibrium quantity of forest land and the long-run cutting age of the forest.

The question of the allocation of land between competing uses is of course, in itself, not new. For instance, in the recent literature involving use of land for forestry, Barbier and Burgess (1997) propose an intertemporal model to analyze the optimal conversion of land from timber to agriculture and use it to estimate the demand relationship for converted land. McConnell (1989) and Lopez, Shah and Altobello (1994) examine the optimal allocation of land between agricultural land, park and public land and urban land in the United States, using a static model. Ehui, Hertel, and Preckel (1990) use a two-sector dynamic model to study the optimal allocation of land between agriculture and forestry in a developing country. This model was also used by Ehui and Hertel (1989) to estimate the optimal steady-state forest stock in Ivory Coast. Hartwick, Long and Tian (2001) use a two-sector dynamic model to analyze land clearing in a small open economy with a large endowment in forestry and a small endowment in agriculture, and facing given world prices for both agricultural and forest products. All of those papers treat the output of the forest as being totally consumed in a single usage and consequently exclude recycling.

Furthermore, whereas the original Faustmann rule (Faustmann, 1849), which is the basic intertemporal arbitrage rule for determining the optimal forest rotation, takes the land area planted in forest as given, all of those papers ignore the optimal rotation question and consider only the land allocation decision, in some cases treating the forest as a nonrenewable resource. None of them makes use of the Faustmann rule to determine the optimal harvest age and replanting decisions. The same is true of Darby (1973), who, in a short note, makes a stylized argument to the effect that recycling paper, by reducing the demand for wood, will result in less trees being planted. In a sense, this paper formalizes Darby's argument, by explicitly setting it into the optimal forest rotation model *à la* Faustmann and taking into account the effect on both the rotation over time and the area devoted to forestry.

We show that increasing the rate of recycling reduces the equilibrium area of land allocated to forestry in steady state and hence results in less trees being planted. On the other hand, as long as the planting cost is positive, it leads to an increase in the harvest age of those trees. As a consequence, the effect of increasing the recycling rate on the equilibrium volume of virgin wood being supplied to the market is ambiguous, being more likely to be negative the smaller is the planting cost.

The next section serves to describe the model. In Section 3.3 we solve it and derive the comparative static results of varying the recycling rate on the steady-state equilibrium. We end with a few concluding remarks in Section 3.4.

3.2 The model

Consider a piece of land of fixed area A, to be allocated by its owner between forestry and some alternative use, say agriculture. Once the forest is cut, a new allocation is determined and the area devoted to forest is immediately planted. This process is repeated indefinitely. Let i denote the ith such rotation. Then the forest cut at date t_i will have been planted at date t_{i-1}, which is the harvest date of the previous rotation. The cutting age for rotation i will therefore be $T_i = t_i - t_{i-1}$.

Let $X(T_i)$ denote the volume of wood per unit of area devoted to forestry obtained at age T_i. The growth function $X(T_i)$ is assumed to be an increasing and strictly concave function of T_i. If areas f_{i-1} and a_{i-1} were assigned respectively to forestry and to agriculture at planting date t_{i-1}, then the total volume of wood harvested at date t_i will be $h_i = f_{i-1}X(T_i)$. Since the total land area will be devoted either to forestry or agriculture, we will have :

$$f_{i-1} + a_{i-1} = A, \quad i = 1, ..., \infty. \tag{3.1}$$

Any use of the forest other than for the production of wood is neglected by this representative land owner. Once harvested, the wood is transformed into some recyclable final product, say paper. The final product can be equally well produced from virgin wood or from the recycled product.[1] We will assume that this is the only use for the wood being harvested.

Now let $S(t_i)$ denote the total quantity of input available for transformation into final product at date t. If a fraction δ of the stock of input available at date t_{i-1} is recycled, we will have :

$$S(t_i) = f_{i-1}X(T_i) + \delta S(t_{i-1}), \quad i = 1, ..., \infty \tag{3.2}$$

with $S(t_0) = S_0$, the given stock available at the initial planting date t_0. For simplicity, it will be assumed that one unit of this input can be transformed into one unit of the final product. If we let p_{t_i} denote the price of this input, then the inverse demand can be

1. That virgin wood and the recycled product are perfect substitutes is a simplifying assumption. Assuming otherwise does not yield any additional insight towards the issue addressed in this paper.

written :

$$p_{t_i} = P(S_{t_i}), \quad \text{with} \quad P(S_{t_i}) \geq 0 \quad P'(S_{t_i}) < 0 \quad \text{and} \quad \lim_{S_{t_i} \to \infty} P(S_{t_i}) = 0. \tag{3.3}$$

Note that since virgin wood and the recycled product are perfect substitutes, p_{t_i} is also the price of wood.

The factors determining the fraction that is recycled are exogenous to the land owner. One element that will obviously affect the rate of recycling is the cost faced by the recycling industry. But the recycling cost will only affect the allocation of land by the land owner through its effect on the rate of recycling. To the extent that the fraction recycled can be assumed a decreasing function of the cost of recycling, the effect of a decrease (increase) in that cost will go in the same direction as an increase (decrease) in the recycling rate δ. Thus the only parameter that in the end matters for the market allocation of land to forestry is the rate of recycling. We will concentrate our sensitivity analysis in the next section to this parameter, but the reader may want to keep in mind that it will itself be affected by a number of factors, including recycling cost and possibly public policies favoring recycling.

Let $c \geq 0$ denote the cutting cost per unit of wood and $k \geq 0$ denote the planting cost per unit of area planted. It will be assumed that $P(0) > c$, so that it is profitable to exploit the forest to begin with. The present value at t_{i-1} of the net benefits from rotation i if an area f_{i-1} is planted and it is cut at age T_i will be :

$$\Pi_f(T_i, f_{i-1}; S(t_i)) = [P(S(t_i)) - c] f_{i-1} X(T_i) e^{-rT_i} - k f_{i-1},$$

where r is the discount rate. The semi-colon (;) in front of $S(t_i)$ is meant to reflect the fact that the representative land owner will, as a price taker, neglect the effect of his individual decisions on $S(t_i)$.

If an area f_{i-1} is devoted to forestry, then an area $a_{i-1} = A - f_{i-1}$ is devoted to agriculture. Let $g(a_{i-1})$ represent the instantaneous net benefit function from agriculture, with $g'(a_{i-1}) > 0$ and $g''(a_{i-1}) < 0$. The present value at t_{i-1} of the net benefits from agriculture over the same interval of time T_i will be :

$$\Pi_a(T_i, a_{i-1})) = \int_{t_{i-1}}^{t_i} g(a_{i-1}) e^{-r(\tau - t_{i-1})} d\tau \ = \ \frac{g(a_{i-1})}{r}(1 - e^{-rT_i}).$$

The value at t_{i-1} of the net discounted benefits from total land use over the interval T_i is therefore :

$$\pi_i = \Pi(T_i, f_{i-1}, a_{i-1}; S(t_i)) = \Pi_f(T_i, f_{i-1}; S(t_i)) + \Pi_a(T_i, a_{i-1}). \tag{3.4}$$

3.3 The equilibrium land allocation and harvesting age

The representative land owner's decision problem at t_0, acting as a price taker, is to choose the sequence $\{T_i, f_{i-1}, a_{i-1}\}_{i=1}^{\infty}$ so as to maximize :

$$V(S_0) = \sum_{i=1}^{\infty} \Pi(T_i, f_{i-1}, a_{i-1}; S(t_i)) e^{-r(t_{i-1} - t_0)} \tag{3.5}$$

subject to (3.1) and to $f_{i-1} \geq 0, \quad a_{i-1} \geq 0, \quad i = 1, ..., \infty$. We will hereafter consider only interior solutions for f_{i-1} and a_{i-1}, so that the nonnegativity constraints can be ignored. Substituting for a_{i-1} from (3.1) into (3.5), the problem can then be reformulated as choosing the sequence $\{T_i, f_{i-1}\}_{i=1}^{\infty}$ to maximize :

$$\begin{aligned} V(S_0) &= \sum_{i=1}^{\infty} \Pi(T_i, f_{i-1}, A - f_{i-1}; S(t_i)) e^{-r(t_{i-1} - t_0)} \\ &= \Pi(T_1, f_0, A - f_0; S(t_1)) + \sum_{i=2}^{\infty} \Pi(T_i, f_{i-1}, A - f_{i-1}; S(t_i)) e^{-r\sum_{j=1}^{i-1} T_j} \end{aligned} \tag{3.6}$$

where we have used the fact that $t_{i-1} - t_0 = \sum_{j=1}^{i-1} T_j$ for $i > 1$.

The first-order necessary conditions for interior solutions will be given by :

$$\frac{\partial V(S_0)}{\partial f_0} = \frac{\partial \pi_1}{\partial f_0} - \frac{\partial \pi_1}{\partial a_0} = 0 \tag{3.7}$$

$$\frac{\partial V(S_0)}{\partial f_{i-1}} = e^{-r\sum_{j=1}^{i-1} T_j} \left\{ \frac{\partial \pi_i}{\partial f_{i-1}} - \frac{\partial \pi_i}{\partial a_{i-1}} \right\} = 0, \quad i = 2, \ldots, \infty \tag{3.8}$$

$$\begin{aligned} \frac{\partial V(S_0)}{\partial T_1} &= \frac{\partial \pi_1}{\partial T_1} - r \sum_{k=2}^{\infty} \pi_k e^{-r\sum_{j=1}^{k-1} T_j} \\ &= \frac{\partial \pi_1}{\partial T_1} - re^{-rT_1} \left[\pi_2 + \sum_{j=2}^{\infty} \pi_{i+j} e^{-r\sum_{k=1}^{j-1} T_{i+k}} \right] \\ &= \frac{\partial \pi_1}{\partial T_1} - re^{-rT_1} V(S(t_1)) = 0 \end{aligned} \tag{3.9}$$

$$\begin{aligned} \frac{\partial V(S_0)}{\partial T_i} &= \frac{\partial \pi_i}{\partial T_i} e^{-r\sum_{j=1}^{i-1} T_j} - r \sum_{k=i+1}^{\infty} \pi_k e^{-r\sum_{j=1}^{k-1} T_j} \\ &= e^{-r\sum_{j=1}^{i-1} T_j} \left\{ \frac{\partial \pi_i}{\partial T_i} - re^{-rT_i} \left[\pi_{i+1} + \sum_{j=2}^{\infty} \pi_{i+j} e^{-r\sum_{k=1}^{j-1} T_{i+k}} \right] \right\} \\ &= e^{-r\sum_{j=1}^{i-1} T_j} \left\{ \frac{\partial \pi_i}{\partial T_i} - re^{-rT_i} V(S(t_i)) \right\} = 0, \quad i = 2, \ldots, \infty, \end{aligned} \tag{3.10}$$

where the partial derivatives of π_i are obtained from (3.4). After substituting for these, we find that conditions (3.7) to (3.10) will be satisfied if and only if, for all $i = 1, \ldots, \infty$:

$$[P(S(t_i)) - c]X(T_i)e^{-rT_i} - k = \frac{g'(A - f_{i-1})}{r}(1 - e^{-rT_i}) \tag{3.11}$$

$$[P(S(t_i)) - c]f_{i-1}X'(T_i) + g(A - f_{i-1}) = r\{[P(S(t_i)) - c]f_{i-1}X(T_i)\} + rV(S(t_i)). \tag{3.12}$$

The left-hand side of (3.11) is the net marginal benefit of allocating land to forestry, while the right-hand side is the net marginal benefit of allocating land to agriculture. The condition simply says that the net benefit from the marginal unit of land must be the same in the two allocations.

Condition (3.12) says that the net benefit from delaying the cutting age marginally, which is given by the increment in the volume of wood resulting from forest growth valued at its net price plus the net benefit from agriculture obtained on area a_{i-1} during that marginal delay in cutting the forest, must be equal to the interest on the net benefit foregone from not harvesting the forest immediately, plus the interest foregone from delaying all future rotations.

Consider now a steady state, such that $T_i = T_{i-1} = T$, $f_i = f_{i-1} = f$ and $S(t_i) = S(t_{i-1}) = S$. We then find that, for all i :

$$V(S(t_i)) = \frac{[P(S)-c]fX(t)e^{-rT} - kf}{1-e^{-rT}} + \frac{g(A-f)}{r},$$

from which it follows that (3.11) and (3.12) become :

$$F(f,T,S) = [P(S)-c]X(T)e^{-rT} - k - \frac{g'(A-f)}{r}(1-e^{-rT}) = 0 \tag{3.13}$$

$$G(f,T,S) = [P(S)-c]X'(T) - r\frac{(P(S)-c)X(T)-k}{1-e^{-rT}} = 0. \tag{3.14}$$

The steady-state stock of input S available for transformation into the final product is given by :

$$S = fX(T)/(1-\delta). \tag{3.15}$$

Condition (3.13) says that, given the harvesting age, the area devoted to forestry must be such as to equate the net marginal benefit between the two possible uses of the land. Condition (3.14) says that, given the area devoted to forestry, the harvesting age must be chosen so as to satisfy the Faustmann rule (Faustmann, 1849). Conditions (3.13) and (3.14), together with (3.15), determine the steady-state equilibrium area devoted to forestry, f, and harvesting age, T.

Notice that an interior solution for f is possible only if $P(S)-c > 0$. For if $P(S)-c \leq 0$, then $F(f,T,S) < 0$ and no land would be devoted to forestry ($f = 0$). Notice also that, if $P(S)-c > 0$, it follows from (3.13) and (3.14) that $X'(T) > 0$. It also follows from (3.14) that

$$X'(T) - \frac{rX(T)}{1-e^{-rT}} = \frac{-rk}{(1-e^{-rT})(P(S)-c)} \leq 0.$$

We show in the Appendix that a steady state defined by (3.13) and (3.14) will be locally stable if and only if

$$\triangle = F_f G_T + \frac{1}{1-\delta}\left[G_T F_S X(T) + F_f G_S f X'(T)\right] > 0, \tag{3.16}$$

where [2]

$$F_f = \frac{g''(A-f)}{r}\left(1-e^{-rT}\right) < 0 \tag{3.17}$$

$$G_T = [P(S)-c][X''(T) - rX'(T)] < 0 \tag{3.18}$$

$$F_S = P'(S)X(T)e^{-rT} < 0 \tag{3.19}$$

$$G_S = P'(S)\left[X'(T) - \frac{rX(T)}{1-e^{-rT}}\right] = \frac{-rkP'(S)}{(1-e^{-rT})(P(S)-c)} \geq 0. \tag{3.20}$$

To see the impact of the rate of recycling on the long-run equilibrium land allocation and harvesting age, differentiate totally (3.13) and (3.14) taking into account (3.15), to get :

$$\frac{df}{d\delta} = -\frac{1}{\triangle}\left[\frac{fX(T)}{(1-\delta)^2}F_S G_T\right] < 0$$

$$\frac{dT}{d\delta} = -\frac{1}{\triangle}\left[\frac{fX(T)}{(1-\delta)^2}F_f G_S\right] \geq 0.$$

Therefore, increasing the rate of recycling (from any admissible level, including $\delta = 0$) results unambiguously in less land being allocated to forestry in the long run and hence less trees. At the same time, the long run harvest age of those trees will either increase or stay the same. This means that the overall effect on $h = fX(T)$, the volume of virgin wood being supplied to the market, will be ambiguous. This effect is given by

$$\begin{aligned}\frac{dh}{d\delta} &= fX'(T)\frac{dT}{d\delta} + X(T)\frac{df}{d\delta} \\ &= -\frac{1}{\triangle}\frac{fX(T)}{(1-\delta)^2}\left[G_T F_S X(T) + F_f G_S f X'(T)\right].\end{aligned}$$

2. The second-order conditions for a maximum require $F_f \leq 0$, $G_T \leq 0$ and $F_f G_T \geq 0$, which are all satisfied given (3.17) to (3.20). Notice that $G_f \equiv 0$ and $F_T = [P(S)-c][X'(T) - rX(T)]e^{-rT} - g'(A-f)e^{-rT} = 0$ when (3.13) and (3.14) are satisfied.

The sign of this expression is indeterminate. It will depend on whether the longer growth allowed before harvesting compensates for the smaller area devoted to the forest. One of the important parameters is k, the planting cost. Since G_S is increasing in k, so is the marginal effect of δ on the harvest age T. The reason is that letting the trees grow is a way for the owner to delay the replanting cost. In particular, if the planting cost is zero, changing the recycling rate has no effect on the harvest age and the net effect on the volume of virgin wood supplied to the market is negative. The same holds for small planting costs. But if the planting cost is sufficiently large, increasing the recycling rate can result in a greater volume of virgin wood being produced.

3.4 Concluding remarks

One result that clearly comes out of our analysis is that increasing the rate of recycling will result in less, not more, land being allocated to forestry in the long run. If the only goal is to end up with more trees in order to compensate for external benefits that are neglected by the market, it would seem that to encourage recycling is not an appropriate measure. Measures aimed directly at the land allocation decision are more appropriate, whether they be incentive mechanisms, such as taxes or subsidies, or regulation aimed at maintaining the forest area or increasing it. There may of course be other reasons to pursue a recycling policy, but from the strict standpoint of protecting the forest area it is likely to have the reverse effect in the long run.

CONCLUSION

Cette thèse a utilisé le cadre d'analyse du modèle inter-temporelle de Faustmann pour proposer des solutions optimales à quelques problèmes liés à la gestion des ressources forestières. Les deux premiers chapitres de la thèse ont établi et caractérisé les mécanismes optimaux d'octroie des droits d'exploitation d'une ressource forestière aux potentielles firmes exploitantes quand celles-ci sont mieux informées de leurs coûts de coupe que le propriétaire forestier. Le troisième chapitre a montré qu'une augmentation du taux de recyclage va réduire à long terme la surface forestière.

Le premier chapitre a analysé la redevance optimale et la rotation optimale quand le propriétaire forestier cède les droits de coupe à une firme qui l'exploitera par un accord de gré à gré. Il a été montré à la fois dans un cadre statique (les coûts de coupe sont parfaitement corrélés dans le temps) et un cadre dynamique (indépendance inter-temporelle des coûts de coupe) que la rotation optimale va satisfaire une version modifiée de la règle de Faustmann qui prévaudrait en pleine information. Cette modification est nécessaire afin d'inciter la firme exploitante à révéler ses vrais coûts. Dans le contexte statique, il en résulte que la rotation optimale est plus longue en information asymétrique qu'en situation de pleine information. Nous avons montré également comment le seuil maximal de coût de coupe peut être endogénéisé, afin de permettre au propriétaire forestier d'accroître son profit espéré en s'assurant que les forêts non profitables ne seront pas exploitées. Nous avons déterminé et comparé les redevances optimales à la fois en information asymétrique et symétrique. Les redevances forestières étant généralement, en pratique, une fonction linéaire du volume de bois, le contrat (sous optimal) est dérivé en imposant une telle forme de redevance. Nous avons caractérise le gain en terme de profit espéré qui résulterait de l'utilisation du contrat non linéaire plus général plutôt que ce type de contrat. Finalement toujours dans le contexte statique, nous avons établi à travers un mécanisme optimal d'enchère au second prix qu'en créant ainsi la concurrence entre les firmes, le propriétaire forestier augmente son profit espéré. Dans le cas dynamique, certains des résultats obtenus contrastent avec ceux obtenus dans le contexte statique. Nous avons montré que chaque type de firme, incluant celle ayant le coût le plus élevé, conserve alors une rente strictement positive. Ceci s'explique par le fait qu'étant donné l'absence de corrélation inter-temporelle entre les coûts, un exploitant dont le coût est élevé à la période courante peut se retrouver, avec une probabilité strictement positive, avec des coûts bas lors des rotations futures. Nous avons aussi montré que cette rente augmente dans le temps. Ceci est nécessaire afin d'obtenir à moindre coût à la période courante la révélation du véritable coût de la firme. Il vient alors que la rotation optimale s'accroît aussi dans le temps. Finalement nous avons montré qu'il y a distorsion en asymétrique d'information par rapport à l'optimum de pleine information pour le coût le plus bas.

Dans le deuxième chapitre de la thèse, nous avons déterminé le contrat optimal lorsque les coûts de coupe ne sont connus ni par le propriétaire forestier, ni par les firmes elles même. Cependant chaque firme observe un signal privé sur son coût. Nous avons établi que le propriétaire forestier attribuera les droits d'exploitation de la forêt à la firme dont la valeur virtuelle du terrain (valeur du terrain corrigé pour la rente informationnelle) est la plus élevée, en autant que celle-ci demeure positive. Nous avons montré que la rotation optimale de la firme gagnante satisfait, comme précédemment, une version modifiée de la règle de Faustmann. Cette modification a pour but d'inciter toutes les firmes participantes à révéler la vérité sur leurs coûts. Enfin, nous avons déterminé sous quelles conditions le contrat optimal peut être mis en application par une enchère optimale au second prix. Les résultats de ce chapitre ont été obtenus sous l'hypothèse que le contrat est signé à perpétuité et que les coûts de coupe sont parfaitement corrélés dans le temps. Bien que cela ne soit pas totalement irréaliste dans beaucoup de situation, il n'en demeure pas moins que la détermination du contrat optimal en l'absence de ces hypothèses est un objectif souhaitable. Ce problème est rendu très complexe, de par la nécessité de considérer des rotations variables ainsi que la décision de désigner à chaque rotation la firme gagnante. Ceci est une recherche en cours.

Le troisième chapitre de la thèse s'est penché sur l'impact à long terme du recyclage sur la surface de terre en forêt. Il ressort clairement de notre analyse qu'une augmentation du taux de recyclage va entraîner à long terme une diminution de la surface forestière et non une augmentation comme on s'y attend généralement. Si le but ultime du recyclage est d'accroître le stock d'arbre afin de compenser les externalités négligées par l'équilibre des marchés, il semble qu'encourager le recyclage ne soit pas l'instrument approprié pour l'atteindre. Des mesures axées directement sur l'allocation du terrain seront beaucoup plus appropriées, ou encore des mesures incitatives telles que les taxes, les subventions et d'autres mesures de régulation visant à maintenir la surface forestière ou l'accroître. Il se peut qu'il existe d'autres raisons pour poursuivre une politique de recyclage, mais du seul point de vue de protéger la surface forestière, ceci semble avoir l'effet inverse.

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Annexe I

Appendix to Chapter 1

I.1 Proof of Proposition 3

Proof. Let

$$\begin{aligned}\overline{R}_s(\theta) &= R_s(\theta)e^{-rTs(\theta)} = (p-\theta)X(T_s(\theta))e^{-rTs(\theta)} - D,\\ \overline{R}_a(\theta) &= R_a(\theta)e^{-rTa(\theta)} = (p-\theta)X(T_a(\theta))e^{-rT_a(\theta)} - D - (1-e^{-rT_a(\theta)})V_a(\theta)\end{aligned}$$

and let $m(T) = X(T)e^{-rT}$, where T satisfies the first-order condition (1.2) or (1.20) for the optimal rotation in the symmetric or the asymmetric information cases. It follows that $m'(T) = [X'(T) - rX(T)]e^{-rT} > 0$. Hence $m(T)$ is strictly increasing in T. Since $T_a(\theta^L) = T_s(\theta^L)$, it follows that $\overline{R}_s(\theta^L) - \overline{R}_a(\theta^L) = (1-e^{-rT_a(\theta^L)})V_a(\theta^L) > 0$. Furthermore, we have $\overline{R}_s(\theta_a^*) - \overline{R}_a(\theta_a^*) = (p-\theta_a^*)[m(T_s(\theta_a^*)) - m(T_a(\theta_a^*))]$. Therefore, since $T_s(\theta_a^*) < T_a(\theta_a^*)$ and $m(T)$ is strictly increasing in T, it is the case that $\overline{R}_s(\theta_a^*) - \overline{R}_a(\theta_a^*) < 0$.

Using the intermediate value theorem, we conclude that there exists a $\hat{\theta} \in (\theta^L, \theta_a^*)$ such that $\overline{R}_s(\hat{\theta}) = \overline{R}_a(\hat{\theta})$ and hence $R_s(\hat{\theta})e^{-rTs(\hat{\theta})} = R_a(\hat{\theta})e^{-rTa(\hat{\theta})}$. Therefore :

$$R_s(\hat{\theta}) - R_a(\hat{\theta}) = \overline{R}_s(\hat{\theta})e^{-rTs(\hat{\theta})}\left[1 - e^{r(Ta(\hat{\theta}) - Ts(\hat{\theta}))}\right] < 0$$

and

$$R_s(\theta^L) - R_a(\theta^L) = (\overline{R}_s(\theta^L) - \overline{R}_a(\theta^L))e^{rTs(\theta^L)} > 0.$$

Using again the intermediate value theorem, we conclude that there exists $\theta^+ \in (\theta^L, \hat{\theta})$ such that $R_s(\theta^+) = R_a(\theta^+)$ and hence $R_s(\theta) > R_a(\theta)$ for any $\theta^L \leq \theta < \theta^+$. ∎

I.2 Proof of Proposition 4

Proof. For simplicity, denote : $\theta_a = \theta_a(x)$ and $\theta_s = \theta_s(x)$. Then

$$\begin{aligned} R_a(x) &= (p-\theta_a)x - De^{rT_a(\theta_a)} - (e^{rT_a(\theta_a)}-1)V(\theta_a) \\ R_s(x) &= (p-\theta_s)x - De^{rT_s(\theta_s)}. \end{aligned}$$

Since $T_s(\theta_s) = T_a(\theta_a)$, I can write :

$$R_s(x) - R_a(x) = (\theta_a - \theta_s)x + (e^{rT_a(\theta_a)}-1)V(\theta_a). \tag{I.1}$$

For $x = x^L$, $\theta_a(x^L) = \theta_s(x^L) = \theta^L$, and from (I.1) I have :

$$R_s(x^L) - R_a(x^L) = (e^{rT_s(\theta^L)}-1)V(\theta^L) > 0.$$

For $x = x_s$, $\theta_s(x_s) = \theta_a^*$ and $\theta_a(x_s) = \bar{\theta}_a$, where $\bar{\theta}_a < \theta_a^*$. It follows from (I.1) that :

$$R_s(x_s) - R_a(x_s) = (\bar{\theta}_a - \theta_a^*)x_s + (e^{rT_a(\bar{\theta}_a)}-1)V(\bar{\theta}_a),$$

where

$$V(\bar{\theta}_a) = \int_{\bar{\theta}_a}^{\theta_a^*} \frac{X(T_a(\tau))}{e^{rT_a(\tau)}-1} d\tau.$$

By (1.8), $\frac{dV}{d\theta_a} = -\frac{X(T_a)}{e^{rT_a}-1} \leq 0$, and therefore :

$$\frac{d^2V}{d\theta^2} = -\frac{X'(T_a)(e^{rT_a}-1) - re^{rT_a}X(T_a)}{(e^{rT_a}-1)^2}\frac{dT_a}{d\theta_a} > 0.$$

Hence $\frac{X(T_a)}{e^{rT_a}-1}$ is a strictly decreasing function of θ. This implies that

$$V(\bar{\theta}_a) < (\theta_a^* - \bar{\theta}_a)\frac{X(T_a(\bar{\theta}_a))}{e^{rT_a(\bar{\theta}_a)}-1} = (\theta_a^* - \bar{\theta}_a)\frac{x_s}{e^{rT_a(\bar{\theta}_a)}-1}.$$

Therefore $R_s(x_s) - R_a(x_s) < 0$. Using the intermediate value theorem, we can conclude that there exists a $\bar{x} \in (x^L, \bar{x}_s^*)$ such that $R_s(\bar{x}) = R_a(\bar{x})$ and $R_s(x) > R_a(x)$ for any $x^L \leq$

$x < \bar{x}$. ∎

I.3 Characterization of the optimal linear contract

Proposition 14. *Let the subscript u refer to the solution under a uniform contract and denote by θ_u^* the optimal value of $\bar{\theta}$. The optimal policy under a uniform contract can then be summarized by :*

Case 1 : $\theta_u^* = \theta^H$

$$\begin{aligned} &\textit{if} \quad V_u(\theta^H) = 0 \quad \textit{then, } b^* = p - \theta^H - \frac{De^{T_w}}{X(T_w)} \\ &\textit{if} \quad V_u(\theta^H) > 0 \quad \textit{then, } b^* \textit{ solves} \end{aligned}$$

$$\int_{\theta^L}^{\theta^H} \left[\frac{\partial T}{\partial b} \frac{e^{-rT_u} g(T_u)}{(1-e^{-rT_u})^2}\right] (b^* - (1-\alpha)h(\theta)) f(\theta) d\theta - (1-\alpha) \frac{X(T_u^H) e^{-rT_u^H}}{1-e^{-rT_u^H}} = 0 \quad \text{(I.2)}$$

and T_u solves

$$(p - b^* - \theta) g(T_u) = rD \quad \text{(I.3)}$$

where $g(T)$ is as defined in expression (1.27) and T_w is the Wicksell rotation.

Case 2 : $\theta_u^* < \theta^H$

θ_u^* solves

$$\frac{(p - \theta_u^* - (1-\alpha)h(\theta_u^*)) X(T_w) e^{-rT_w} - D}{1 - e^{-rT_w}} f(\theta_u^*) + \int_{\theta^L}^{\theta_u^*} \left[\frac{\partial T}{\partial b} \frac{e^{-rT_u} g(T_u)}{(1-e^{-rT_u})^2}\right] (b^* - (1-\alpha)h(\theta)) f(\theta) d\theta = 0 \quad \text{(I.4)}$$

and b^* satisfies

$$b^* = p - \theta_u^* - \frac{De^{T_w}}{X(T_w)}. \quad \text{(I.5)}$$

Proof. The solution to the problem (1.43)-(1.46) has as a necessary condition for the determination of b :

$$\frac{bX(T(\bar{\theta}))e^{-rT(\bar{\theta})}}{1-e^{-rT(\bar{\theta})}}f(\bar{\theta})\frac{d\bar{\theta}}{db}+\int_{\theta^L}^{\bar{\theta}}\frac{dW}{db}f(\theta)d\theta=0. \tag{I.6}$$

Differentiating (1.42) and using (1.44) to eliminate superfluous terms we find that :

$$\frac{dW}{db}=-b\frac{e^{-rT}g(T)}{(1-e^{-rT})^2}\frac{\partial T}{\partial b}+(1-\alpha)\frac{X(T)e^{-rT}}{1-e^{-rT}}.$$

Integration by part gives

$$\int_{\theta^L}^{\bar{\theta}}\frac{X(T)e^{-rT}}{1-e^{-rT}}f(\theta)d\theta=\frac{X(T(\bar{\theta}))e^{-rT(\bar{\theta})}}{1-e^{-rT(\bar{\theta})}}F(\bar{\theta})+\int_{\theta^L}^{\bar{\theta}}\frac{e^{-rT}g(T)}{(1-e^{-rT})^2}\frac{\partial T}{\partial\theta}F(\theta)d\theta.$$

Equation (1.44) implies that $\frac{\partial T}{\partial\theta}=\frac{\partial T}{\partial b}$. We also have that

$$\frac{\partial T}{\partial b}=\frac{g(T)}{(p-\theta-b)g'(T)}$$

and

$$\frac{bX(T(\bar{\theta}))e^{-rT(\bar{\theta})}}{1-e^{-rT(\bar{\theta})}}=\frac{(p-\bar{\theta})X(T(\bar{\theta}))e^{-rT(\bar{\theta})}-D-V(T(\bar{\theta}),b,\bar{\theta})}{1-e^{-rT(\bar{\theta})}}.$$

Therefore condition (I.6) becomes :

$$\frac{(p-\bar{\theta})X(T(\bar{\theta}))e^{-rT(\bar{\theta})}-D-V(T(\bar{\theta}),b,\bar{\theta})}{1-e^{-rT(\bar{\theta})}}f(\bar{\theta})\frac{d\bar{\theta}}{db}-(1-\alpha)\frac{X(T(\bar{\theta}))e^{-rT(\bar{\theta})}}{1-e^{-rT(\bar{\theta})}}F(\bar{\theta})$$
$$+\int_{\theta^L}^{\bar{\theta}}\left[\frac{\partial T}{\partial b}\frac{e^{-rT}g(T)}{(1-e^{-rT})^2}\right](b-(1-\alpha)h(\theta))f(\theta)d\theta=0. \tag{I.7}$$

If $\theta_u^*=\theta^H$, then $\frac{d\bar{\theta}}{db}=0$, $T(\bar{\theta})=T_w$ and we have the first part of the proposition. If $\theta_u^*<\theta^H$, then $V(T(\bar{\theta}),b,\bar{\theta})=0$, $T(\bar{\theta})=T_w$ and the second part of the proposition follows. ∎

I.4 Characterization of the expected welfare

The equilibrium being symmetric, it is sufficient to consider the expected payment of firm 1, which, from (1.47), is given by

$$R_{sa}(\theta^1) = \int_{\theta^1}^{\delta} \gamma(y) f_y(y) dy + \gamma(\delta)[1 - F_y(\delta)].$$

Using equation (1.48), I can therefore write the expected profit of firm 1 as

$$V_{sa}(\theta^1) = U(\theta^1, \theta^1) = w(\theta^1, \theta^1)(1 - F_y(\theta^1)) - R_s a(\theta^1). \tag{I.8}$$

The preceding equation implies that

$$\frac{dV_{sa}^1(\theta^1)}{d\theta^1} = \frac{\partial w^1(\theta^1, \theta^1)}{\partial \theta^1}(1 - F_y(\theta^1)) = -(1 - F_y(\theta^1))\frac{X(T^1(\theta))e^{-rT^1(\theta)}}{1 - e^{-rT^1(\theta)}}.$$

Hence V_{sa}^1 is nonincreasing in θ^1, so that firm 1's individual rationality constraint is satisfied for all θ^1 if it is satisfied at $\theta^1 = \delta$. The contribution of firm 1 to the expected welfare is $W^1 = R_{sa}^1 + \alpha V_{sa}^1$. Substituting R_{sa}^1 from (I.8), I get $W^1 = w(T^1(\theta), \theta^1)(1 - F_y(\theta^1)) - (1 - \alpha)V_{sa}^1(\theta)$. Finally, the forest owner's optimization problem is, for all $i = 1, ..., N$:

$$\max_{T^i(.), V_{sa}^i(.), \delta} E_\theta \sum_{i=1}^{N} [w(\theta^i, T^i(\theta))(1 - F_y(\theta^i)) - (1 - \alpha)V_{sa}^i(\theta)]$$

subject to

$$\frac{dV_{sa}^i(\theta^i)}{d\theta^i} = -(1 - F_y(\theta^i))\frac{X(T^i(\theta))e^{-rT^i(\theta)}}{1 - e^{-rT^i(\theta)}},$$

(a) $\gamma(.)$ is nonincreasing in θ^i, (b) $V_{sa}^i(\delta) \geqslant 0$, (c) $\delta \leqslant \theta^H$.

As proved in Laffont and Tirole (1987), at the optimum, $T^i(\theta)$ is a function of θ^i only, $i = 1, ..., N$. The optimization can then be simplified to :

$$\begin{aligned} \max_{T^1(\theta^1), V_{sa}^1(\delta), \delta} EW_{sa} \quad = \quad & N \int_{\theta^L}^{\delta} \left[w(\theta^1, T^1(\theta^1)) - (1 - \alpha)\frac{X(T^1)}{e^{rT^1 - 1}} h(\theta^1) \right] (1 - F_y(\theta^1)) f(\theta^1) d\theta^1 \\ & - N(1 - \alpha)V_{sa}^1(\delta)F(\delta) \end{aligned}$$

subject to $V^1_{sa}(\delta) \geqslant 0$, $\delta \leqslant \theta^H$ and $\gamma(\cdot)$ is nonincreasing in θ^1.

I.5 Characterization of incentive compatible mechanisms.

In this appendix I characterize for the dynamic case the class of incentive compatible mechanisms when functions are differentiable. This class of incentive compatible mechanism satisfies conditions (1.62). Let

$$\varphi(v,\theta,\theta') = (p-\theta)X(T(v,\theta'))e^{-rT(v,\theta')} - D - R(v,\theta')e^{-rT(v,\theta')} + e^{-rT(v,\theta')}U(v,\theta')$$

be the profit of the firm as expected today if it reports θ' when the true cost parameter is θ. The firm will respond truthfully if and only if

$$\theta = \arg\max_{\theta'} \varphi(v,\theta,\theta')$$

for which the folowing conditions are necessary :

$$\text{(i)} \quad \left.\frac{\partial \varphi}{\partial \theta'}\right|_{\theta'=\theta} = 0 \qquad \text{and} \qquad \text{(ii)} \quad \left.\frac{\partial^2 \varphi}{\partial \theta'^2}\right|_{\theta'=\theta} \leqslant 0.$$

Differentiating totally (i) gives $\left.\frac{\partial^2 \varphi}{\partial \theta' \partial \theta}\right|_{\theta'=\theta} d\theta + \left.\frac{\partial^2 \varphi}{\partial \theta'^2}\right|_{\theta'=\theta} d\theta' = 0$ and it follows from (ii) that : (iii) $\left.\frac{\partial^2 \varphi}{\partial \theta' \partial \theta}\right|_{\theta'=\theta} \geqslant 0$.

Conditions (i), (iii) and the envelope theorem together characterize the class of incentive compatible mechanism. This can be summarized by :

$$(p-\theta)\left[X'(T(v,\theta)) - rX(T(v,\theta))\right]\frac{\partial T(v,\theta)}{\partial \theta} = \frac{\partial (R-U)(v,\theta)}{\partial \theta} - r(R-U)(v,\theta)$$

$$\frac{\partial T(v,\theta)}{\partial \theta}\left[X'(T(v,\theta)) - rX(T(v,\theta))\right] \leqslant 0$$

$$\frac{\partial \varphi(v,\theta)}{\partial \theta} = -X(T(v,\theta))e^{-rT(v,\theta)}$$

where

$$\begin{aligned}\varphi(v,\theta) &= (p-\theta)X(T(v,\theta))e^{-rT(v,\theta)} - D - R(v,\theta)e^{-rT(v,\theta)} + e^{-rT(v,\theta)}U(v,\theta) \\ &= \max_{\theta'} \varphi(v,\theta,\theta').\end{aligned}$$

Annexe II

Appendix to Chapter 2

II.1 Proof of condition (2.13)

We have that $g_i^*(\theta) = g_i(\theta, t_i^*) = \frac{\psi_i(\theta)X(t_i^*) - Ke^{rt_i^*}}{e^{rt_i^*} - 1}$ where $\psi_i(\theta) = p - C_i(\theta) - (1 - \alpha_i)\partial_i C_i(\theta) h_i(\theta_i)$ and t_i^* satisfies

$$\psi_i(\theta)\lambda(t_i^*) - rK = 0. \tag{II.1}$$

Recall that $\lambda(t_i) = -X'(t_i)(1 - e^{-rt_i}) + rX(t_i)$. Using (II.1) we can rewrite $g_i^*(\theta)$ as

$$g_i^*(\theta) = K\frac{X'(t_i^*) - rX(t_i^*)}{\lambda(t_i^*)} = S(t_i^*)$$

where $S(t) = K\frac{X'(t) - rX(t)}{\lambda(t)}$.
Hence

$$S'(t) = \frac{[X''(t) - rX'(t)]\lambda(t) - \lambda'(t)[X'(t) - rX(t)]}{\lambda(t)^2}.$$

For all t satisfying (II.1), $S'(t) < 0$. Then $g_i^*(\theta) = g_j^*(\theta)$ is equivalent to $S(t_i^*) = S(t_j^*)$. Since $s(\cdot)$ is strictly decreasing it follows that $t_i^* = t_j^*$. By the envelope theorem we get :

$$\begin{aligned}
\frac{\partial g_i^*(\theta)}{\partial \theta_i} &= -\partial_i C_i(\theta)\frac{X(t_i^*)}{e^{rt_i^*} - 1} - \{(1-\alpha)h_i'(\theta_i)\partial_i C_i(\theta) + (1-\alpha)\partial_{ii}C_i(\theta)h_i(\theta_i)\}\frac{X(t_i^*)}{e^{rt_i^*} - 1} \\
\frac{\partial g_j^*(\theta)}{\partial \theta_i} &= -\partial_i C_j(\theta)\frac{X(t_j^*)}{e^{rt_j^*} - 1}.
\end{aligned}$$

If $g_i^*(\theta) = g_j^*(\theta)$ then $t_i^* = t_j^*$ and, given that $\partial_i C_i(\theta) > \partial_i C_j(\theta)$, it follows from the preceding equations that

$$\frac{\partial g_i^*(\theta)}{\partial \theta_i} < \frac{\partial g_j^*(\theta)}{\partial \theta_i}.$$

II.2 Proof of Lemma 12

Part (i) : We can rewrite $g_i(\theta,t_i)$ as $\frac{\psi_i(\theta)X(t_i)-Ke^{rt_i}}{e^{rt_i}-1}$, where $\psi_i(\theta) = p - C_i(\theta) - (1-\alpha_i)\partial_i C_i(\theta)h_i(\theta_i)$. For interior solutions, $T_i^*(\theta)$ satisfies the equation

$$\psi_i(\theta)\lambda(T_i^*(\theta)) - rK = 0, \tag{II.2}$$

where $\lambda(t_i) = -X'(t_i)(1-e^{-rt_i}) + rX(t_i)$. [1] Assume $\theta_i^h > \theta_i^l$. Then

$$\psi_i(\theta_i^h,\theta_{-i})\lambda(T_i^*(\theta_i^h,\theta_{-i})) = \psi_i(\theta_i^l,\theta_{-i})\lambda(T_i^*(\theta_i^l,\theta_{-i})) = rK > 0$$

and

$$\frac{\partial\psi_i(\theta)}{\partial\theta_i} = -(1+(1-\alpha_i)h_i'(\theta_i))\partial_i C_i(\theta) - (1-\alpha_i)h_i(\theta_i)\partial_{ii}C_i(\theta) < 0.$$

Hence $\psi_i(.,\theta_{-i})$ is decreasing in θ_i, which implies that $\psi_i(\theta_i^h,\theta_{-i}) < \psi_i(\theta_i^l,\theta_{-i})$ and therefore $\lambda(T_i^*(\theta_i^h,\theta_{-i})) > \lambda(T_i^*(\theta_i^l,\theta_{-i}))$. Observe that $\lambda'(t_i) = (1-e^{-rt_i})[-X''(t_i) + rX'(t_i)] > 0$ because X is increasing and strictly concave. Thus λ is increasing and we may conclude that $T_i^*(\theta_i^h,\theta_{-i}) > T_i^*(\theta_i^l,\theta_{-i})$.

Part (ii) : The proof follows from the application of the following Lemma 15 to the function $g_i^*(.,\theta_{-i}) - g_j^*(.,\theta_{-i})$.

Lemma 15. *If v is differentiable in $[\underline{\theta},\overline{\theta}]$ and satisfies $v(\theta)=0 \Rightarrow v'(\theta)<0$, then $V(\theta^h)>0 \Rightarrow v(\theta)>0\ \forall\theta<\theta^h$.*

The proof of Lemma 15 is provided in Appendix II.3.

Indeed $g_i^*(\theta_i^h,\theta_{-i}) > \max\{0,\max_{j\neq i} g_j^*(\theta_i^h,\theta_{-i})\}$ means that $g_i^*(\theta_i^h,\theta_{-i}) > 0$ and $g_i^*(\theta_i^h,\theta_{-i}) - g_j^*(\theta_i^h,\theta_{-i}) > 0$ for $i \neq j$. Using condition (2.13) and Lemma 15, we deduce that $g_i^*(\theta_i,\theta_{-i}) - g_j^*(\theta_i,\theta_{-i}) > 0 \quad \forall\theta_i < \theta_i^h$, and since $\psi_i(.,\theta_{-i})$ is decreasing in θ_i we also deduce that $g_i(.,t_i)$ is decreasing in θ_i. Therefore $g_i^*(.,\theta_{-i})$ is non increasing as the maximum of decreasing functions, thus $g_i^*(\theta_i^l,\theta_{-i}) > 0$.

1. $\frac{\partial g_i(\theta,t_i)}{\partial t_i} = \frac{\psi_i(\theta)\lambda(t_i)-rK}{(e^{rt_i}-1)^2}, \frac{\partial g_i(\theta,t_i)}{\partial t_i} = 0$ implies that $\psi_i(\theta)\lambda(t_i) - rK = 0$.

Part (iii) : Fix $\theta_i^h > \theta_i^l \in \triangle_i(\theta_{-i})$. If $g_i^*(\theta_i^h, \theta_{-i}) < \max\{0, \max_{j \neq i} g_j^*(\theta_i^h, \theta_{-i})\}$ then $q_i^*(\theta_i^h, \theta_{-i}) = 0$ and $-\partial_2 \omega_i^*(\theta_i^h, \theta_i, \theta_{-i}) q_i^*(\theta_i^h, \theta_{-i}) = 0 \leq -\partial_2 \omega_i^*(\theta_i^l, \theta_i, \theta_{-i}) q_i^*(\theta_i^l, \theta_{-i})$. Now suppose that $g_i^*(\theta_i^h, \theta_{-i}) > \max\{0, \max_{j \neq i} g_j^*(\theta_i^h, \theta_{-i})\}$. Then Part (ii) implies that $g_i^*(\theta_i^h, \theta_{-i}) > \max\{0, \max_{j \neq i} g_j^*(\theta_i^h, \theta_{-i})\}, \quad \forall \widehat{\theta}_i \leq \theta_i^h$. Furthermore :

$$
\begin{aligned}
g_i^*(\widehat{\theta}_i, \theta_{-i}) > 0 \quad &\Rightarrow \quad \psi_i(\widehat{\theta}_i, \theta_{-i}) X(T_i^*(\widehat{\theta}_i, \theta_{-i})) > K e^{r T_i^*(\widehat{\theta}_i, \theta_{-i})} \\
&\Rightarrow \quad \psi_i(\widehat{\theta}_i, \theta_{-i}) > 0.
\end{aligned}
$$

It follows from (II.2) that : $\lambda(T_i^*(\widehat{\theta}_i, \theta_{-i})) > 0$. Using the mean value theorem we may write the following equalities :

$$
\begin{aligned}
&\partial_2 \omega_i^*(\theta_i^h, \theta_i, \theta_{-i}) q_i^*(\theta_i^h, \theta_{-i}) - \partial_2 \omega_i^*(\theta_i^l, \theta_i, \theta_{-i}) q_i^*(\theta_i^l, \theta_{-i}) \\
&= \partial_2 \omega_i^*(\theta_i^h, \theta_i, \theta_{-i}) - \partial_2 \omega_i^*(\theta_i^l, \theta_i, \theta_{-i}) \\
&= \frac{\partial}{\partial \widehat{\theta}_i} [\partial_2 \omega_i^*(\widehat{\theta}_i, \theta_i, \theta_{-i})] \,|_{\widehat{\theta}_i = \theta_i^o} (\theta_i^h - \theta_i^l), \theta_i^o \in (\theta_i^l, \theta_i^h) \\
&= \partial_i T_i^*(\theta_i^o, \theta_{-i}) \partial_i C_i(\theta) \frac{\lambda(T_i^*(\theta_i^o, \theta_{-i})) e^{r T_i^*(\theta_i^o, \theta_{-i})}}{(e^{r T_i^*(\theta_i^o, \theta_{-i})} - 1)^2} > 0.
\end{aligned}
$$

II.3 Proof of Lemma 15

Assume that $\underline{\theta} < \theta^o < \theta^h < \bar{\theta}$, $v(\theta^o) \leq 0$ and $v(\theta^h) > 0$. We can assume without loss that $v(\theta^o) = 0$. Indeed, if $v(\theta^o) < 0$ then, since $v(\theta^h) > 0$, the intermediate value theorem implies that there exists $\hat{\theta^o} \in (\theta^o, \theta^h)$ such that $v(\hat{\theta^o}) = 0$. Let $\theta^1 = Sup\{\theta \in [\theta^o, \theta^h] : v(\theta) = 0\}$. Because v is continuous we have $v(\theta^1) = 0$ and therefore $v'(\theta^1) < 0$. This means that v is locally decreasing around θ^1 : there exists $\theta^2 \in (\theta^1, \theta^h) : v(\theta^2) < 0 = v(\theta^1)$. Again the intermediate value theorem implies that $v(\theta^3) = 0$ for some $\theta^3 \in (\theta^2, \theta^h)$. This contradicts the fact that θ^1 is the supremum since $\theta^3 > \theta^1$.

Annexe III

Appendix to Chapter 3

III.1 Existence and stability of a steady state

Conditions (3.11) and (3.12) yield $f_{i-1}(S(t_i))$ and $T_i(S(t_i))$, the solution for the forest owner's decisions for f_{i-1} and T_i as a function of $S(t_i)$, from which we may write $h_i(S(t_i)) = f_{i-1}(S(t_i))X(T_i(S(t_i)))$, the solution for the total volume of wood harvested as a function of $S(t_i)$. Equation (3.2) therefore becomes :

$$S(t_i) = h_i(S(t_i)) + \delta S(t_{i-1}).$$

In steady state, $S(t_i) = S(t_{i-1}) = S$ and hence :

$$h(S) = (1-\delta)S.$$

The function $h(S)$ takes a positive value at $S = 0$. This follows directly from (3.13), considering that $P(0) > c$. It also goes to zero as S goes to infinity. This last property holds because, in view of (3.3), as S increases it will eventually reach a value $\bar{S}$ such that $P(\bar{S}) = c$ and beyond which $P(S) - c < 0$, with the result that $F(f, T, S) < 0$ in (3.13). The value of f could then not be interior and must be zero : no land would be allocated to forestry by the land owner and hence $h = fX(T) = 0$. The function $h(S)$ being continuous and the right-hand side of the equation being a monotone increasing function of S that goes through the origin, it follows that there exists at least one steady state. Such a steady state will be locally stable if and only if, in its neighborhood,

$$h'(S) = -\frac{G_T F_S X(T) + fF_f G_S X'(T)}{F_f G_T} < 1-\delta,$$

or, since $F_f G_T > 0$ (see (3.17) and (3.18)),

$$\triangle = F_f G_T + \frac{1}{1-\delta}\left[G_T F_S X(T) + F_f G_S f X'(T)\right] > 0.$$

Note that $h'(S)$ was obtained by applying Cramer's rule to the system (3.13)-(3.14).