

The Economics of Oil, Biofuel and Food Commodities

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Abstract

We study the effects on the food market of the introduction of biofuels as a substitute for fossil fuel in the energy market. We consider a world economy with an oil cartel and a competitive fringe of farmers producing energy in the form of biofuels. Farmers also produce food and sell it on the world food market. We determine the resulting relationship between prices in the energy and food markets and characterize the cartel's extraction path and the price path of energy. We show that the price of food will be growing as long the oil stock is being depleted, whether population is growing or not, and that it will keep growing after the oil stock is exhausted if population is growing. An analysis of the effects of the productivity of land use in either the food or the biofuel sectors is carried out.

Keywords: Biofuel; Oil depletion; Population growth; Energy price; Food price

Résumé

Nous étudions l'effet sur le marché des aliments de l'introduction sur le marché de l'énergie de biocarburants, comme substitut aux combustibles fossiles. Nous supposons une économie où cohabitent sur le marché de l'énergie un cartel pétrolier et une frange compétitive de cultivateurs qui produit de l'énergie sous forme de biocarburant. Les cultivateurs produisent également des produits agricoles qu'ils vendent sur le marché des aliments. Nous caractérisons la relation qui en résulte entre le prix de l'énergie et le prix des aliments, ainsi que le sentier d'extraction du cartel pétrolier et le sentier de prix de l'énergie. Il est démontré que le prix des aliments va croître aussi longtemps que le stock de pétrole n'est pas épuisé, et cela que la population soit croissante ou non. Il continuera à croître une fois le stock de pétrole épuisé si la population est croissante. Les effets de l'amélioration dans la productivité de la terre dans la production d'aliments ainsi que dans la production de biocarburant sont analysés.

Mots-clés : Biocarburant ; Épuisement du pétrole ; Croissance de la population ; Prix de l'énergie ; Prix des aliments.

1 Introduction

The recent food crisis has become a major concern for world leaders. In June 2008, the World Food Summit organized by the United Nations that took place in Rome raised many questions about the causes of this crisis and what to do about it. Indeed, since the year 2000, major food crop prices have increased for the first time since the 1970s. The prices of corn, rice, wheat as well as other crops reached record highs. According to a recent article by the Economist magazine,¹ food accounts in Botswana and South Africa for a fifth of the consumer price index; in Sri Lanka and Bangladesh it accounts for two-thirds. This might explain the violent clashes that took place in several developing countries (Haiti, Cameroon and Egypt, among others) in the wake of the sharp increase in crop prices that occurred in 2007 and 2008.

Against this backdrop, a number of explanations for this crisis have been proposed. First, a line of argument attributes the increase in major crop prices to the rising world demand for food, which has not been followed by adequate investments in the agricultural sector. The proponents of this view, namely the UN secretary general, declared that global food output must increase by 50% by 2030 in order to maintain ‘food security’. However, such an argument suffers from a drawback. While the lack of investments in agriculture has been a long-term structural problem ever since the end of the ‘first green revolution’ of the 1960s and 70s, it is the case that the recent rise in crop prices has been sharp and dramatic. An alternative view considers that the recent development of the biofuel industry has a lot to do with the food crisis. Advocates of this view include a number of specialized NGOs and renowned international research organizations, like the International Food Policy Research Institute (IFPRI). According to the IFPRI, biofuels account for up to 30% of the increase in the price of agricultural commodities.

From 1999 until the summer of 2008, both global energy demand and fossil fuels prices

¹From The Economist print edition, June 5, 2008, page 70.

have been steadily rising.² This has caused pressure for the development of biofuels as an alternative source of energy.³ This was not the case during the 1990s, when the fossil fuel price was too low to allow for the economic viability of this renewable resource. This increase in the demand for biofuels has generated a ‘crowding-out effect’ in the agricultural sector. Many argue that scarce agricultural resources are being diverted away from food production towards the production of biofuels, which results in a reduction in global crop supplies.

The fact that the prices of oil and food commodities have both tumbled during a period of time following the last quarter of 2008 also suggests that, during the current decade, both prices have become highly positively correlated. In this paper we investigate, within a reasonably tractable model, the mechanisms through which these two markets are linked and how the development of the biofuel industry has affected the correlation between energy and food prices. The model also allows us to look at the possible impacts on food and energy prices of improving land use in either food or biofuel production. As we will show, those impacts are complex and difficult to predict without some careful empirical analyses.

Since the questions arising from the introduction of biofuels are relatively recent, the economic literature on this subject is limited. Moreover, as pointed out by Rajagopal and Zilberman (2007) in a World Bank policy survey, “the environmental literature is dominated by a discussion of net carbon offset and net energy gain, while indicators relating to impact on human health, soil quality, biodiversity, water depletion, etc., have received much less attention”.⁴ Chakravorty, Hubert and Nostbakken (2009) point out that most of the literature focuses on life cycle assessment of biofuels, with the main conclusion being that they are not carbon neutral. There is also a small literature on ‘food versus fuel’ where the price of oil is

²China and India’s staggering growth rates account for a large chunk of that.

³Not to mention environmental lobbying and political pressures that have led to an additional regulation induced demand. For instance, in 2010, the government of Canada imposed a mandatory 5% biofuel content in each liter of gasoline sold in the local market.

⁴See Rajagopal and Zilberman (2007), page 2. They also point out that serious concerns about the carbon benefits of current biofuels can be raised, namely the fact that biofuels consume a significant amount of energy that is derived from fossil fuels. See as well Giampietro, Ulgiati and Pimentel (1997), Lal (2004), Pimentel and Patzek (2005), Farrell *et al.* (2006).

exogenous.⁵ For instance, Hochman, Sexton and Zilberman (2008) study the crowding-out effect of biofuels on the agricultural sector. They propose a two-country general equilibrium trade model with energy as intermediate input. In their model, they consider two sources of energy (fossil and biofuel); both the biofuel and food sectors compete for land and labor.⁶ Their main results suggest that trade liberalization tends to increase the demand for energy, which decreases food production and causes losses in forests and other non-agricultural lands. They also show that neutral technical change in agricultural production, such as biotechnology and second generation biofuel technologies, mitigates this pressure on land. Hubert *et al.* (2008) deal with a related question. They find that backstop technologies will be adopted earlier than expected in response to high increases in food and petroleum prices. They also argue that, as a result, either the demand for energy will decrease or petroleum will be replaced by backstop technologies. Chakravorty *et al.* (2010), for their part, carry out a comprehensive empirical analysis of the long-run effects on food prices of United States and European Union mandatory biofuel mandates, taking into account regional heterogeneity in land quality, consumer preferences and population growth. One of their conclusion is that fears of a large-scale shift from food to biofuel production and its subsequent effect on food prices may be exaggerated. The issue of competition between land and food has also been examined in the agricultural economics literature in the context of an exogenous change in the price of ethanol. Andrade de Sa, Palmer and Engel (2010) study the direct and indirect impacts of ethanol production on land use, deforestation and food production. One of their main results is that land competition between rival uses increases deforestation and decreases food production. Feng and Babcock (2008) examine the effect of the development of ethanol on different types of crops. Closer to our model, Chakravorty *et al.* (2008)

⁵Chakravorty, Hubert and Nostbakken (2009) conclude that "most of them focus on the economics of biofuels supply and in particular address the issue of government policy and how that can affect biofuels production. A smaller sample of the models explicitly considers environmental impacts from biofuels production. A fewer number explicitly consider the role of fossil fuel scarcity and the effect rising prices of energy may have on the supply of biofuels".

⁶In this paper, for simplicity, we consider that only the land resource is shared between food and energy productions. As a matter of fact, many resources are subject to trade-off between these two sectors. See for instance Gaudet, Moreaux and Withagen (2006), where water is shared between oil and agriculture.

propose a centralized Ricardian-Hotelling model with land allocation decisions being decided by a central planner. In their model, as the exhaustible resource becomes scarcer its price increases, thereby making biofuels competitive. As a consequence, land shifts out from food to energy production, which leads to an increase in the price of food. The demand for clean energy is modeled by introducing an exogenous cap on the carbon stock in the atmosphere, which leads to a rise in energy prices and speeds up the adoption of biofuels as a backstop.

In the present paper, unlike Hochman *et al.* (2008) and following Hubert *et al.* (2008) and Chakravorty *et al.* (2008), we study the effects of nonrenewable resource exhaustion over time as the impetus behind the rising global demand for biofuels, which might have a perverse effect on ‘food security’. We propose a decentralized partial equilibrium model with resource dynamics and we consider that the finite land resource is put into two alternative uses by price-taking farmers: food and biofuel production. We abstract from the issue of global atmospheric pollution caused by emissions. We model the effects of oil exhaustion on the supply of biofuels and we study the effects of increased production of biofuel. Finally, our model accounts for population growth and the effect this has on both oil extraction and on ‘food security’. This allows us to distinguish the effect of population growth from that of land use on the price of food. Our main focus is to model the relationship between energy and food prices which follows from the depletion of fossil fuel (oil for short) and the development of biofuels as a substitute. Unlike the papers mentioned earlier, we consider a dynamic framework where the price paths for both energy and food are determined endogenously.

Section 2 presents the model. In Section 3 we solve the farmers’ land allocation problem. Section 4 is devoted to the oil cartel’s optimal depletion and pricing decisions. Section 5 is devoted to an analysis of the effect on food end energy prices of land productivity. We conclude in Section 6.

2 The model

Consider an economy composed of an agricultural sector and an oil sector and of two markets, one for energy and one for food. The energy market is supplied by farmers, in the form of biofuel, and by an oil cartel. The market for food is supplied by price-taking farmers.

2.1 The supply sides

The cartel, acting as a dominant firm, extracts fossil fuel and sells it on the energy market. The finite stock of nonrenewable fossil fuel at date t is $S(t)$. We assume that the stock is homogeneous. We also assume that the size of the stock is known with certainty and we abstract from energy storage issues. This stock is depleted at the rate $E_f(t)$, at zero cost, for simplicity. The evolution of the stock is given by:

$$\dot{S}(t) = -E_f(t) \quad \forall t. \quad (1)$$

The total amount of productive land available is also finite. We assume a representative farmer whose behavior summarizes the production decisions of the mass of all farmers. This representative farmer owns a parcel of arable land of size L . He has to decide how to allocate his land between the production of food and the production of biofuel. The food production of the representative farmer is denoted $Q(t)$ while the amount of biofuel he produces is denoted $E_b(t)$, measured in oil equivalent. At each date t , the fixed amount of arable land is allocated between food and biofuel, so that:

$$L_a(t) + L_b(t) = L, \quad (2)$$

where $L_a(t)$ and $L_b(t)$ stand for the amounts of land allocated respectively to food and biofuel.

We will assume that one unit of oil, or its equivalent in the form of biofuel, generates one

unit of energy.⁷ Therefore, the total supply of energy, $E(t)$, will be:

$$E(t) = E_f(t) + E_b(t). \quad (3)$$

Food output is given by

$$Q(L_a(t)) = AL_a(t), \quad (4)$$

while biofuel output is given by

$$E_b(L_b(t)) = BL_b(t). \quad (5)$$

The constants A and B are conversion parameters related to the technology in use. Parameter B reflects the (linear) conversion efficiency into biofuel of the biomass produced using one unit of land.⁸ We assume that the farmers incur increasing marginal cost of production. Specifically, producing $Q(L_a(t))$ and $E_b(L_b(t))$ will cost respectively $\frac{c_a}{2}(AL_a(t))^2$ and $\frac{c_b}{2}(BL_b(t))^2$, where c_a and c_b are positive cost parameters. This formulation of the costs is equivalent to having decreasing returns to scale.⁹

2.2 The demand sides

The demand for energy at date t is given by the following:

$$E(t) = N(t)(\bar{p}_e - p_e(t)), \quad (6)$$

where $N(t)$ is the population at date t and $p_e(t)$ is the price of energy. The inverse demand is thus given by

$$p_e(t) = \bar{p}_e - \frac{E(t)}{N(t)}. \quad (7)$$

We assume that population grows at a constant rate $\gamma \geq 0$ i.e. $N(t) = N_0e^{\gamma t}$. The world demand for food at date t is given by:

$$Q(t) = N(t)(\bar{p}_a - p_a(t)), \quad (8)$$

⁷Of course it should be understood that the biofuel production represents here the net energetic equivalent of the biomass produced by the farmers. Indeed, in order to produce biofuel, fossil energy is required at various stages (see Rajagopal and Zilberman 2007, p. 34).

⁸For example, in the case of sugarcane one hectare of land yields 4900 liters of ethanol (see Rajagopal and Zilberman 2007, p. 102).

⁹Andrade de Sa, Palmer and Engel(2010) use a Cobb-Douglas production function that exhibits decreasing returns to scale in both land and labor inputs.

where $p_a(t)$ is the price of food. The parameters \bar{p}_e and \bar{p}_a represent the choke prices in the energy and food markets respectively.

3 The farmers' problem

In this section, we solve the land allocation problem faced by the representative farmer. In the energy market, farmers act as a competitive fringe vis-a-vis the oil cartel. The energy price $p_e(t)$ is set by the cartel and this price is taken as given by the representative farmer. The representative farmer also takes as given the price $p_a(t)$ in the food market.

The representative farmer maximizes the sum of his food and biofuel profits subject to the land constraint (2). In other words, at any date t :

$$\max_{L_a(t), L_b(t)} \left[p_e(t)BL_b(t) + p_a(t)AL_a(t) - \frac{c_b}{2}(BL_b(t))^2 - \frac{c_a}{2}(AL_a(t))^2 \right]$$

subject to $L_a(t) + L_b(t) = L$.

Replacing L_b by $L - L_a$, the first-order condition for the determination of L_a can be written, assuming an interior solution:

$$p_a(t)A - c_aA^2L_a(t) = p_e(t)B - c_bB^2[L - L_a(t)]. \quad (9)$$

It says that the allocation of land to food production must be such that it equalizes the marginal net benefit from allocating land to either of its two usages. From (9) we get the solution for land allocation to food production as a function of the two prices:

$$L_a(p_a(t), p_e(t)) = \frac{p_a(t)A - p_e(t)B + c_bB^2L}{c_aA^2 + c_bB^2}. \quad (10)$$

Therefore, recalling (4), food supply is given by

$$Q^S(p_a(t), p_e(t)) = AL_a(p_a(t), p_e(t)). \quad (11)$$

It then follows from (2) that

$$L_b(p_a(t), p_e(t)) = \frac{p_e(t)B - p_a(t)A + c_aA^2L}{c_aA^2 + c_bB^2}, \quad (12)$$

and, from (5), biofuel supply is:

$$E_b^S(p_a(t), p_e(t)) = BL_b(p_a(t), p_e(t)). \quad (13)$$

We will assume that

$$c_b B^2 L > \bar{p}_e B - \bar{p}_a A > -c_a A^2 L. \quad (14)$$

This guarantees that we have an interior solution, so that positive quantities of land will be allocated to both food and biofuel. Indeed, the *full* marginal cost of land allocation to biofuel, given that it can also be used for food production, is $c_b B^2 L_b(t) + p_a(t)A - c_a A^2 [L - L_b(t)]$. When neither food nor biofuel is produced ($L_b = L_a = 0$), this reduces to $\bar{p}_a A - c_a A^2 L$ and assumption (14) guarantees that there exists a positive $L_b(t)$ which equates the *full* marginal cost to $p_b(t)B$, the marginal revenue from land allocation to biofuel. Similarly, the *full* marginal cost of land allocation to agriculture is $c_a A^2 L_a(t) + p_b(t)B - c_b B^2 [L - L_a(t)]$ and, by the same reasoning, assumption (14) guarantees that the solution for $L_a(t)$ is interior. We will assume that the respective *full* marginal costs at $L_b = L_a = 0$ are both nonnegative: $\bar{p}_a A - c_a A^2 L \geq 0$ and $\bar{p}_e B - c_b B^2 L \geq 0$, which of course means $\bar{p}_a - c_a A L \geq 0$ and $\bar{p}_e - c_b B L \geq 0$ since A and B are both positive.

Given the energy price $p_e(t)$ set by the cartel, the market clearing condition, obtained by equating the demand for food (given by (8)) with the supply of food (given by (11)), yields the food price as a function of this given energy price:

$$p_a(p_e(t)) = \frac{\theta N(t) \bar{p}_a - c_b A B^2 L + A B p_e(t)}{A^2 + \theta N(t)}. \quad (15)$$

where

$$\theta = A^2 c_a + B^2 c_b > 0.$$

For any land allocation $L_a = L_b = \ell$, $\theta \ell$ is the sum of the marginal costs of production of food and biofuel.

Thus, because of the competition for the limited amount of land between the production of food and of biofuel, the price of food is linked to the price of energy. As can be seen from

(15), at any date t , *ceteris paribus* the higher the given price of energy, the higher the price of food.

Using (15), the biofuel supply at any date t can now be rewritten as a function of $p_e(t)$ only:

$$E_b^S(p_e(t)) = BL_b(p_a(p_e(t)), p_e(t)). \quad (16)$$

There remains to consider the determination of the energy price by the oil cartel.

4 The oil cartel

Subtracting the farmers' supply of biofuel (16) from the total energy demand (6) gives the residual demand faced by the oil cartel:

$$E_f(p_e(t)) = N(t)(\bar{p}_e - p_e(t)) - BL_b(p_a(p_e(t)), p_e(t)) \quad (17)$$

Applying equations (12) and (15) in (17), one can derive the inverse residual demand which can be written as:

$$P_e(E_f(t)) = \beta(t) - \alpha(t) \frac{E_f(t)}{N(t)}, \quad (18)$$

where

$$\alpha(t) = \frac{A^2 + \theta N(t)}{A^2 + B^2 + \theta N(t)} > 0 \quad (19)$$

$$\beta(t) = \alpha(t) \left\{ \bar{p}_e + \frac{AB(\bar{p}_a - AL[c_a - 1/N(t)])}{A^2 + \theta N(t)} \right\} > 0. \quad (20)$$

Observe that $\beta(t)$ can be viewed as the time-varying effective choke price for the residual demand facing the cartel at each date t . Because of the presence of a fringe of biofuel producers, $\beta(t)$ is smaller than \bar{p}_e , the choke-price of total demand for energy. The cartel has to set a price that is lower than $\beta(t)$ if it wants to sell positive amounts of oil. When $p_e(t) \geq \beta(t)$, the total demand for energy is met exclusively by the biofuel producers. As for $\alpha(t)$, it gives the time-variant slope of the residual inverse demand for oil faced by the cartel.

It can be directly established from (19) that $\alpha(t)$ is increasing over time if population is growing and that $\lim_{t \rightarrow +\infty} \alpha(t) = 1$. As for $\beta(t)$, its time derivative is given by $\dot{\beta}(t) = (\partial\beta/\partial N)\dot{N}(t)$, where

$$\frac{\partial\beta}{\partial N} = \frac{B [(A^4 + A^2B^2 + 2\theta A^2N)L + (B\bar{p}_e - A\bar{p}_a + c_a A^2L)\theta N^2]}{(N(t))^2 [A^2 + B^2 + \theta N(t)]^2}. \quad (21)$$

The right-hand side of (21) is positive, since, from (14), $B\bar{p}_e - A\bar{p}_a + c_a A^2L > 0$. Therefore $\beta(t)$ also increases over time as long as population is growing and $\lim_{t \rightarrow +\infty} \beta(t) = \bar{p}_e$. Note that the residual demand for energy converges asymptotically to the total demand for energy.

We will assume that $N_0 > \tilde{N}$, where \tilde{N} is the positive root of $\beta(0) = 0$, so that $\beta(t) > 0$ for all $t \geq 0$. Since by assumption the marginal cost of oil production is zero, this guarantees that oil production will be positive from the outset.

Given the inverse residual demand, the oil cartel chooses its oil production path and the date of exhaustion of its oil stock so as to maximize its discounted flow of profits:

$$\max_{E_f(t), T} \int_0^T e^{-rt} \left(\beta(t) - \alpha(t) \frac{E_f(t)}{N(t)} \right) E_f(t) dt$$

subject to:

$$\dot{S}(t) = -E_f(t),$$

$$E_f(t) \geq 0,$$

$$S(0) = S_0 \text{ and } S(T) \geq 0.$$

The Hamiltonian of the problem is:

$$H(E_f(t), \lambda(t), t) = e^{-rt} \left(\beta(t) - \alpha(t) \frac{E_f(t)}{N(t)} \right) E_f(t) - \lambda(t) E_f(t)$$

and the following conditions are necessary for optimality:

$$\beta(t) - 2\alpha(t) \frac{E_f(t)}{N(t)} - e^{rt}\lambda(t) \leq 0, \quad \left(\beta(t) - 2\alpha(t) \frac{E_f(t)}{N(t)} - e^{rt}\lambda(t) \right) E_f(t) = 0 \quad (22)$$

$$\dot{\lambda}(t) = 0 \quad (23)$$

$$\dot{S}(t) = -E_f(t) \quad (24)$$

$$\left(\beta(T) - \alpha(T) \frac{E_f(T)}{N(T)} - e^{rT} \lambda(T) \right) E_f(T) = 0 \quad (25)$$

$$\lambda(T) \geq 0 \quad \text{and} \quad \lambda(T) S(T) = 0. \quad (26)$$

The Hamiltonian being concave in the control variable $E_f(t)$, linear in $\lambda(t)$ and independent of the state variable $S(t)$, conditions (22) to (26) are also sufficient for optimality.

Condition (22) says that, if at any date t extraction is positive, the profit derived from the marginal barrel of oil must be equal to its current *in situ* value, $e^{rt} \lambda(t)$.

From (23), we have that $\lambda(t) = \lambda(0) = \bar{\lambda}$ for all $t \in [0, T]$. The current shadow value of *in situ* oil therefore grows at the rate of interest, so that no profitable arbitrage is possible with respect to the stock of oil.

The transversality condition (25) states that the value of marginally delaying the terminal date T , which is given by the Hamiltonian evaluated at T , must be zero. Notice that for any values of $E_f(T) \neq 0$, conditions (22) and (25) cannot both hold at the terminal date T . Therefore the optimal rate of extraction at T must be zero: $E_f(T) = 0$.

The transversality condition (26) states that the value of the remaining stock at the terminal date T must be zero, either because $\lambda(T) = \bar{\lambda} = 0$, or $S(T) = 0$, or both. But $\bar{\lambda} = 0$ would, from (22), contradict the fact that $\beta(T) > 0$. It follows that $\bar{\lambda} > 0$ and $S(T) = 0$: the oil stock will be exhausted. Since the choke price is finite, this will occur in finite time.

Recalling that $N(t) = e^{\gamma t} N_0$, exhaustion of the stock means that:

$$\int_0^T \frac{\beta(t) - \bar{\lambda} e^{rt}}{2\alpha(t)} e^{\gamma t} dt = \frac{S_0}{N_0}. \quad (27)$$

This, along with

$$\bar{\lambda} = e^{-rT} \beta(T), \quad (28)$$

uniquely determines $\bar{\lambda}$ and T as functions of the per-capita initial oil stock, S_0/N_0 . For instance, in the case where population is constant, that is $\gamma = 0$, $N(t) = N_0$, and hence $\beta(t) = \beta$, independent of time, substituting for $\bar{\lambda}$ from (28) into (27), we find that T is given

by:

$$rT + e^{-rT} = \frac{2\alpha r}{\beta} \frac{S_0}{N_0} + 1. \quad (29)$$

The solution for $\bar{\lambda}$ then follows from (28).

The cartel's oil extraction path is therefore given by:

$$E_f(t) = \frac{\beta(t) - \bar{\lambda}e^{rt}}{2\alpha(t)} N(t) \quad \forall t \in [0, T], \quad (30)$$

where $\bar{\lambda}$ and T are the solutions for the shadow price of oil and the date of exhaustion of the stock in terms of S_0/N_0 .¹⁰ Hence, recalling (18), the evolution of the price of energy over time will be given by:

$$p_e(t) = \beta(t) - \alpha(t) \frac{E_f(t)}{N(t)} = \begin{cases} \frac{\beta(t) + \bar{\lambda}e^{rt}}{2} & \forall t \in [0, T] \\ \beta(t) & \forall t \in [T, \infty). \end{cases} \quad (31)$$

Since both $\beta(t)$ and $e^{rt}\bar{\lambda}$ are increasing functions of time, the price of energy rises continuously over time. At date T , $e^{rT}\bar{\lambda} = \beta(T)$, so that $p_e(T) = \beta(T) < \bar{p}_e$ and the stock of oil is exhausted. From date T on, energy demand is supplied exclusively by the biofuel producers, with its market price equal to $\beta(t)$ and tending asymptotically to \bar{p}_e , as long as the population is growing. If the population is constant, then so is $\beta(t)$ and so will be the price of energy for all $t > T$. In all cases however, because the presence of the biofuel fringe lowers the price leader's effective choke price, the oil cartel will choose to exhaust its stock before price reaches \bar{p}_e and hence will exhaust its stock of oil sooner than it would in the absence of the fringe. The switch at T from energy being supplied from both oil and biofuel to biofuel only results in a downward jump in the rate of change of the price of energy at T and hence a kink in its time path. This is illustrated in the top graph of Figure 1.

As for the rate of oil extraction by the cartel, although it must eventually be decreasing to reach zero at T , it cannot be ruled out that it be increasing at the beginning, as illustrated in the bottom graph of Figure 1. Differentiating (31) with respect to time, we find that:

$$\dot{E}_f(t) = \frac{(\dot{\beta}(t) - re^{rt}\bar{\lambda})N(t) + (\beta(t) - e^{rt}\bar{\lambda})\dot{N}(t)}{2\alpha(t)} - \frac{(\beta(t) - e^{rt}\bar{\lambda})N(t)\dot{\alpha}(t)}{2\alpha(t)^2}. \quad (32)$$

¹⁰The result of exhaustion in finite time is robust even to a change in the oil industry structure: the only difference being that the cartel is more conservationist than a fully competitive oil industry.

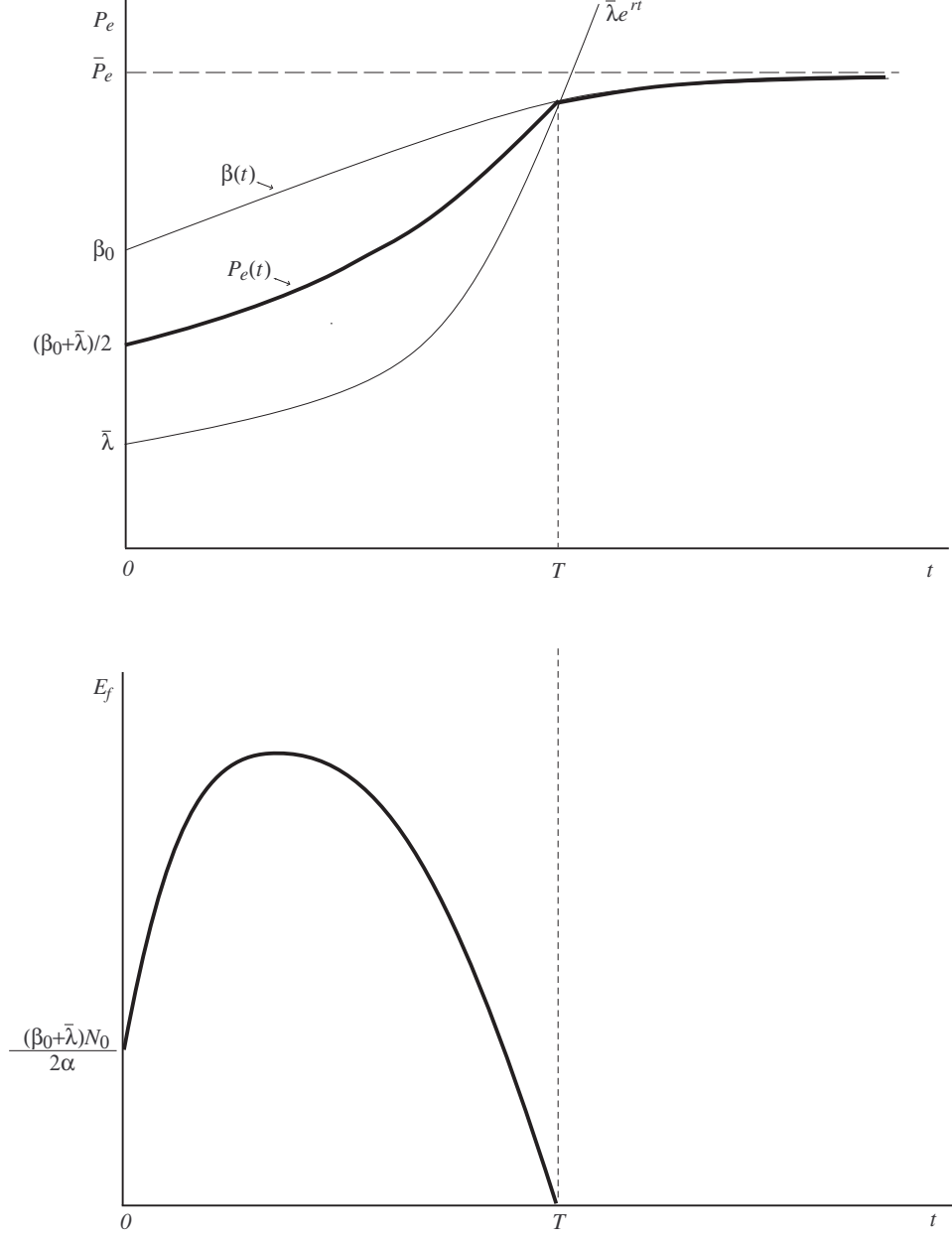


Figure 1: Optimal energy pricing and oil extraction by the cartel

Since the second term is positive, for $E_f(t)$ to be increasing the first term must also be positive. Therefore, in order for oil production to be increasing over some initial interval of time, it is necessary, though not sufficient, that:

$$\left(\dot{\beta}(0) - r\bar{\lambda}\right) N_0 + (\beta(0) - \bar{\lambda}) \gamma N_0 > 0. \quad (33)$$

In the particular case of a constant population ($\gamma = 0$), we have $\dot{\beta}(t) = 0$ for all t and

the necessary condition (33) cannot be satisfied. Therefore, if the population is constant, the production of energy from fossil fuel will be at its maximum at $t = 0$ and will decrease from thereon until it reaches zero at $t = T$. By continuity, the same will be true for some small values of γ .

As for the equilibrium price path of food, substituting for $p_e(t)$ from (31) into (15), it can be written:

$$p_a(t) = \begin{cases} \frac{\theta\bar{p}_aN(t) + (AB/2)(\beta(t) + \bar{\lambda}e^{rt}) - AB^2Lc_b}{A^2 + \theta N(t)} & \forall t \in [0, T] \\ \frac{\theta\bar{p}_aN(t) + (AB/2)\beta(t) - AB^2Lc_b}{A^2 + \theta N(t)} & \forall t \in [T, \infty). \end{cases}$$

Differentiating with respect to time, its evolution over time can be written:

$$\dot{p}_a(t) = \begin{cases} \left[\frac{\partial p_a}{\partial N} + \frac{1}{2} \frac{\partial p_a}{\partial p_e} \frac{\partial \beta}{\partial N} \right] \dot{N}(t) + \frac{1}{2} r \bar{\lambda} e^{rt} \frac{\partial p_a}{\partial p_e} & \forall t \in [0, T] \\ \left[\frac{\partial p_a}{\partial N} + \frac{1}{2} \frac{\partial p_a}{\partial p_e} \frac{\partial \beta}{\partial N} \right] \dot{N}(t) & \forall t \in [T, \infty). \end{cases} \quad (34)$$

As already pointed out, $\partial p_a / \partial p_e$ is positive, from (15). Therefore the second term in the top expression is positive. Also, as established from (21), $\partial \beta / \partial N$ is positive. As for $\partial p_a / \partial N$, it is given by:

$$\begin{aligned} \frac{\partial p_a}{\partial N} &= \frac{\theta[(\theta\{N(t) - 1\})\bar{p}_a + A^2\bar{p}_a - ABp_e(t) + c_bAB^2L]}{[A^2 + \theta N(t)]^2} \\ &\geq \frac{\theta A[A\bar{p}_a - B\bar{p}_e + c_bB^2L]}{[A^2 + \theta N(t)]^2} > 0 \quad (\text{by assumption (14)}). \end{aligned}$$

Therefore the price of food is continuously increasing if the population is growing, as illustrated in Figure 2. It will grow at a faster rate for $t < T$, while the oil stock is being depleted, than for $t > T$, when the only source of supply of energy is biofuel, with a kink in the path occurring at T . If population is constant, it will grow until T and become constant afterwards.

For all $t > T$, the farmers will be the sole suppliers of both the food and the energy market. The equilibrium prices can then be determined using the solution to the farmers'

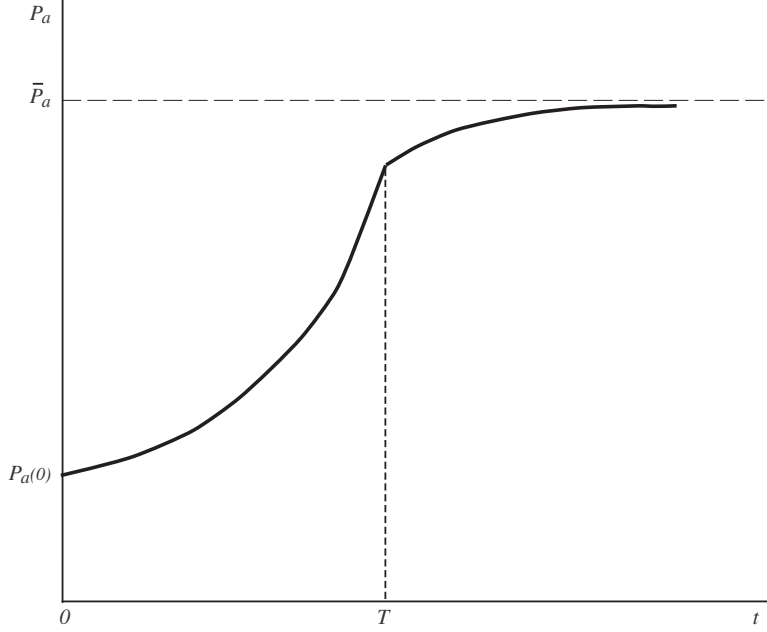


Figure 2: The food price path

land allocation problem of Section 3 and the market clearing conditions. We will have $p_a(t) = p_a(p_e(t))$ given by (15), but with $p_e(t) = \beta(t)$. The price of food will tend asymptotically to \bar{p}_a , while the price of energy tends asymptotically to \bar{p}_e . In the case of a constant population, both of those prices would be constant beyond T , both smaller than their respective choke price.

5 The effects of land productivity on equilibrium prices

It is interesting now to look at the effect on the price paths of energy and food of improving the land use in either biofuel and food production by changing the productivity parameters B and A . As will become clear, the effect will remain ambiguous, for two reasons: because of the fixed stock of oil available and because of the interaction between the two markets due the sharing of the fixed land area available. We will assume for this purpose that population is constant and will normalize by setting $N = 1$. N being constant, so will α and β .

Consider first the effect on the equilibrium price of energy, p_e . From (29) we verify that:

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial (\alpha/\beta)} \frac{\partial (\alpha/\beta)}{\partial x} = \frac{2S_0}{1 - e^{-rT}} \frac{\frac{\alpha}{\beta} [\xi_{\alpha x} - \xi_{\beta x}]}{x}, \quad x = A, B \quad (35)$$

where

$$\xi_{zx} = \frac{\partial z}{\partial x} \frac{x}{z}, \quad z = \alpha, \beta$$

denote the responsiveness (elasticities) of the endogenous choke price and slope of the residual demand curve faced by the oil cartel. Whether a change in the parameters A or B will result in an increase or decrease in the date of exhaustion of the oil stock will depend crucially on those responsiveness, as can be seen from (35).

Differentiating (19) with respect to A and B it is easily established that $\partial\alpha/\partial A > 0$ and $\partial\alpha/\partial B < 0$, so that $\xi_{\alpha A} > 0$ and $\xi_{\alpha B} < 0$. Therefore, if increasing A were to leave the choke price β unchanged or reduce it ($\xi_{\beta A} \leq 0$), thus resulting in a new residual demand for oil which lies everywhere below the old one,¹¹ the effect would be to delay the exhaustion of the oil stock. Similarly, if increasing B were to leave the choke price β unchanged or increase it ($\xi_{\beta B} \geq 0$), thus resulting in new residual demand for oil which lies everywhere above the old one, the effect would be to accelerate the exhaustion of the oil stock. In such cases, the new time path of p_e will necessarily cross the old one, since having the new path either everywhere below or everywhere above the old one would be inconsistent with the given initial stock of oil. In the case of an increase in A with $\xi_{\beta B} \geq 0$, the new path will cut the old one from above at some date $\tau < T$, so that for $t < \tau$ the price of energy will have increased while for $t > \tau$ it will have decreased. In the case of an increase in B with $\xi_{\beta B} \geq 0$, the new path will cut the old one from below at some date $\tau < T$, with the result that p_e will have decreased for all $t < \tau$ and increased thereafter.¹²

As shown in the Appendix, the signs of $\xi_{\beta A}$ and of $\xi_{\beta B}$ are in fact indeterminate. If $\xi_{\beta A} > 0$ and sufficiently large to have $\xi_{\alpha A} - \xi_{\beta A} < 0$, then the effect of an increase in A will be to exhaust the oil stock earlier. The new residual demand faced by the cartel then lies

¹¹The residual demand will then lie everywhere below the old one since both β and β/α are then reduced, β/α being the level of demand of oil that corresponds to $p_e = 0$. Notice that $\xi_{(\beta/\alpha)x} = \xi_{\beta x} - \xi_{\alpha x} = -\xi_{(\alpha/\beta)x}$.

¹²In this last case, measures aimed at improving the productivity of land in biofuel production actually result in an acceleration in the use of oil, a phenomenon which might be viewed as a form the ‘‘Green Paradox’’, a phenomenon first emphasized by Sinn (2008) and subsequently analyzed by Grafton, Kompas and Long (2010), Ploeg and Withagen (2010) and Smulders, Tsur and Zemel (2010), among others. Chakravorty et al. (2010) also note this phenomenon in their analysis of the long-run effect of biofuel mandates on food prices.

everywhere above the old one. Similarly, if $\xi_{\beta B} < 0$ with $\xi_{\alpha B} - \xi_{\beta B} > 0$, then an increase in B will retard the date of exhaustion of the oil stock, the oil cartel's new residual demand lying everywhere below the old one. In both of those cases, the new time path of the price of energy will again necessarily cut the old one, from below in the first case and from above in the second case. Thus, in the first case, an increase in A now results in a lower p_e up to some date τ at which the old and the new price paths cross, and a higher p_e thereafter. In the second case, the reverse is true for an increase in B .

There remains the possibility that $\xi_{\beta A} > 0$ but still $\xi_{\alpha A} - \xi_{\beta A} > 0$, with the result that the new residual demand faced by the oil producer will necessarily cut the old one from above at some positive price level. In this case, and only in this case, it becomes possible for the new price path of energy to be everywhere above the old one subsequent to an increase in A . Similarly, it is possible to have $\xi_{\beta B} < 0$ with $\xi_{\alpha B} - \xi_{\beta B} < 0$, so that the new residual demand curve cuts the old one from below at some positive price level, in which case an increase in B may leave the new price path of energy everywhere below the old one.

That the price paths may cross subsequent to improvements in land use in either food or biofuel production is due to the fact that the available oil stock is fixed. That it is not possible to predict analytically whether the new price path will cut the old one from below or from above, or maybe not at all, is due to the interaction between the market for food and the market for energy, which partly share the fixed availability of agricultural land. Hence there is no definite analytical answer as to the effect on the price of energy. In fact, as shown in the Appendix, the effects of A and B on the equilibrium price of energy at any date t are given by:

$$\frac{\partial p_e}{\partial A} = \left. \frac{\partial p_e}{\partial A} \right|_{E_f=0} - \left(\frac{2A(c_a + 1)}{A^2 + B^2 + \theta} \right) E_f - \left(\frac{AB}{A^2 + B^2 + \theta} \right) \frac{\partial E_f}{\partial A} \quad (36)$$

$$\frac{\partial p_e}{\partial B} = \left. \frac{\partial p_e}{\partial B} \right|_{E_f=0} - \left(\frac{2B}{A^2 + B^2 + \theta} \right) E_f - \left(\frac{A^2 + \theta}{A^2 + B^2 + \theta} \right) \frac{\partial E_f}{\partial B}, \quad (37)$$

where

$$\left. \frac{\partial p_e}{\partial A} \right|_{E_f=0} = \frac{B\bar{p}_a + 2A(c_a + 1)[\bar{p}_e - p_e - BL]}{A^2 + B^2 + \theta} \quad (38)$$

$$\left. \frac{\partial p_e}{\partial B} \right|_{E_f=0} = \frac{A\bar{p}_a + 2B(c_b + 1)[\bar{p}_e - p_e] - 2Bp_e - A^2L(c_a + 1)}{A^2 + B^2 + \theta} \quad (39)$$

are the effects on the equilibrium price of energy when energy supply depends entirely on biofuel production. The price p_e is the equilibrium price (see Appendix) and E_f is given by (30) with $\bar{\lambda}$ given by (28) and T by (29).¹³

For $t > T$ the oil stock is completely depleted and biofuel becomes the only source of energy. Hence the effects of A and B on the price of energy are reduced to (38) and (39). Clearly $\bar{p}_e - p_e > 0$, while $\bar{p}_e - p_e - BL < 0$ since in equilibrium $\bar{p}_e - p_e - BL_b - E_f = 0$, so that $\bar{p}_e - p_e - BL = -BL_a < 0$ when $E_f = 0$, as in the static equilibrium that occurs for $t > T$. Hence, even in a static framework where all energy is obtained from biofuel, the direction of the effects on the price of energy of improving the productivity of land in either food or biofuel production is ambiguous, being crucially dependent on the value of the parameters. This is due to the complex interaction between the food and the energy markets that results from their sharing of the available land. When we add to this ambiguity the fact that for $t < T$ the new and the old price paths of energy can cross, it is to be expected that the directions of those effects cannot be uniquely determined analytically.

Not surprisingly, much the same ambiguity will hold for the price of food as does for the price of energy, and for the similar reasons. The effects of the parameters A and B on the equilibrium price of food at any date t are given (see Appendix) by:

$$\frac{\partial p_a}{\partial A} = \left. \frac{\partial p_a}{\partial A} \right|_{E_f=0} - \left(\frac{B}{A^2 + B^2 + \theta} \right) E_f - \left(\frac{AB}{A^2 + B^2 + \theta} \right) \frac{\partial E_f}{\partial A} \quad (40)$$

$$\frac{\partial p_a}{\partial B} = \left. \frac{\partial p_a}{\partial B} \right|_{E_f=0} - \left(\frac{A}{A^2 + B^2 + \theta} \right) E_f - \left(\frac{AB}{A^2 + B^2 + \theta} \right) \frac{\partial E_f}{\partial B}, \quad (41)$$

¹³Recall that since $N(t)$ is constant and normalized to one, α and β are independent of time.

where

$$\left. \frac{\partial p_a}{\partial A} \right|_{E_f=0} = \frac{B\bar{p}_e + 2A(c_a + 1)(\bar{p}_a - p_a) - 2A\bar{p}_a - B^2(c_b + 1)L}{A^2 + B^2 + \theta} \quad (42)$$

$$\left. \frac{\partial p_a}{\partial B} \right|_{E_f=0} = \frac{A\bar{p}_e + 2B(c_b + 1)[\bar{p} - p_a - AL]}{A^2 + B^2 + \theta} \quad (43)$$

with the price p_a being the equilibrium price (see Appendix) and E_f being given by (30) with $\bar{\lambda}$ given by (28) and T by (29).

Again, even when the supply of energy depends entirely on biofuel production, as is the case for $t > T$, the effects of improving the productivity of land use in either food or biofuel production are ambiguous, because of the common land constraint faced by food and biofuel production which indirectly links those two otherwise independent markets. Indeed, $\bar{p}_a - p_a$ being positive and $\bar{p} - p_a - AL$ being negative, the signs of both (42) and (43) depend on value of the parameters. An analytical determination of the directions of those effects for all t is made all the more difficult by the fact that the price path of energy for $t < T$ can respond in various ways, depending again on the values of the parameters.

6 Concluding remarks

The object of this paper has been the study of the effects on the food sector of the recent development of biofuels as a substitute for fossil fuel in the supply of energy. We have shown how competition for the finite land resource, which takes place between biofuel and food production, explicitly defines a relationship between the energy price and the food price. The rate of depletion of the oil stock may at first increase if population is growing, but it will eventually decrease to zero as the stock gets exhausted. The price of energy will however increase continuously while the stock of oil is being depleted, due to the decline of the remaining *per capita* stock of oil, and this whether population is growing or constant. If population is growing, it will keep increasing after biofuel becomes the only source of energy.

As for the food price, it is also increasing. Two effects account for this growth in the price of food. Firstly, the increase in the energy price raises the opportunity cost of the

use of land for food production, creating an incentive for farmers to reallocate their land in favor of biofuel production. Secondly, population growth increases the demand for food, thus pushing upwards the equilibrium price in the food market.

Although the effect on the price path of food of introducing competition for land between food and biofuel productions is clear, it is not so clear whether investing in productivity enhancing measures in the agricultural food sector, as advocated by the UN secretary general during the 2008 food summit, would alleviate the effect of biofuel production on food prices. What the effect of such productivity measures might be turns out to depend in a complex manner on the various parameters involved in the competition for land between the food and biofuel sectors and in the competition on the energy market between the biofuel and fossil fuel sectors: it may or may not alleviate the pressure on food prices, as it may alleviate it in the short term but not in the long term, or vice-versa. Hence the matter remains an entirely empirical one, but an empirical one which certainly deserves further investigation given its importance for the so-called “food security” issue.

Others might want to emphasize the “energy security” issue and hence focus on improvements in land use in the biofuel sector as a means of generating lower energy prices. Again, for much the same reasons, the effects of such measures on the price of energy, or for that matter on the price of food, are unclear from a purely analytical stand point and would need careful empirical investigation to determine the likely effects of implementing such measures.

Appendix

A The effects of varying A and B on α and β

Assuming N constant and normalized to one, and differentiating (19), we find that:

$$\frac{\partial \alpha}{\partial A} = \frac{2AB^2(c_a + 1)}{A^2 + B^2 + \theta} > 0$$

and

$$\frac{\partial \alpha}{\partial B} = -\frac{2A^2B(c_a + 1)}{A^2 + B^2 + \theta} < 0.$$

Differentiating (20) with respect to A , we get:

$$\begin{aligned} \frac{\partial \beta}{\partial A} &= \frac{\beta}{\alpha} \frac{\partial \alpha}{\partial A} + \alpha \frac{B \overbrace{\{\bar{p}_a - 2AL[c_a - 1]\}}^? - 2A(c_a + 1) \overbrace{\left(\frac{\beta}{\alpha} - \bar{p}_e\right)}^{>0}}{A^2 + \theta} \\ &= \frac{\beta}{\alpha} \frac{\partial \alpha}{\partial A} + \alpha \frac{\partial(\beta/\alpha)}{\partial A}. \end{aligned}$$

The sign of the second term being indeterminate, so is the sign $\partial\beta/\partial A$.

Similarly, differentiating (20) with respect to B , we get:

$$\begin{aligned} \frac{\partial \beta}{\partial B} &= \frac{\beta}{\alpha} \frac{\partial \alpha}{\partial B} + \alpha \frac{A \overbrace{\{\bar{p}_a - AL[c_a - 1]\}}^{>0} - 2Bc_b \overbrace{\left(\frac{\beta}{\alpha} - \bar{p}_e\right)}^{>0}}{A^2 + \theta} \\ &= \frac{\beta}{\alpha} \frac{\partial \alpha}{\partial B} + \alpha \frac{\partial(\beta/\alpha)}{\partial B}. \end{aligned}$$

Again, the sign of the second term is indeterminate and hence so is that of $\partial\beta/\partial B$. The expression $\bar{p}_a - AL[c_a - 1]$ is positive, since $\bar{p}_a - ALc_a$ is positive by assumption (see page 8).

B The effects of varying A and B on the equilibrium prices

Assuming N constant and normalized to one, the equilibrium $(p_a, p_e, L_a, L_b, E_f, T)$ is the solution to the following system of six equations:

$$\text{From (9):} \quad Ap_a - Bp_e - [A^2c_a + B^2c_b]L_a + B^2c_bL = 0 \quad (44)$$

$$\text{From (2):} \quad L_a + L_b = L \quad (45)$$

$$\text{From (4) and (8):} \quad \bar{p}_a - p_a - AL_a = 0 \quad (46)$$

$$\text{From (3), (5) and (6):} \quad \bar{p}_e - p_e - BL_b = \begin{cases} E_f & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases} \quad (47)$$

$$\text{From (30) and (28):} \quad E_f = \begin{cases} \left(\frac{\beta}{\alpha}\right) \left(\frac{1 - e^{-r(T-t)}}{2}\right) & \text{for } t \leq T \\ 0 & \text{for } t > T \end{cases} \quad (48)$$

$$\text{From (29):} \quad rT + e^{-rT} = \left(\frac{\alpha}{\beta}\right) 2rS_0 + 1 \quad (49)$$

Using equations (45) and (46) to eliminate L_a and L_b , we find that (44) and (47) become:

$$(A^2 + \theta)p_a - ABp_e = \theta\bar{p}_a - AB^2c_bL$$

$$Bp_a + Ap_e = B\bar{p}_a + A[\bar{p}_e - BL - E_f].$$

Upon solving for (p_a, p_e) , we get:

$$p_a = \frac{(B^2 + \theta)\bar{p}_a + AB\bar{p}_e - AB^2L(c_a + 1)}{A^2 + B^2 + \theta} - \frac{AB}{A^2 + B^2 + \theta}E_f$$

$$p_e = \frac{(A^2 + \theta)\bar{p}_e + AB\bar{p}_a - A^2BL(c_a + 1)}{A^2 + B^2 + \theta} - \frac{A^2 + \theta}{A^2 + B^2 + \theta}E_f,$$

where E_f and T are to be determined from equations (48) and (49). In both of those equations the first term represents the equilibrium price when all energy is supplied from biofuel, which is the case for $t > T$, the oil stock being then fully depleted. Those first terms can be usefully denoted respectively $p_a|_{E_f=0}$ and $p_e|_{E_f=0}$.

Differentiating the above two equilibrium prices with respect to A and B then yields (36), (37), (40) and (41).

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