Université de Montréal

Essais en économie de l'environnement et des ressources naturelles sous incertitude

par
Justin Johnson Kakeu Kengne

Département de sciences économiques
Faculté des arts et des sciences

Thèse présentée à la Faculté des arts et des sciences
en vue de l'obtention du grade de Philosophiæ Doctor (Ph.D.)
en sciences économiques

Septembre, 2010

Université de Montréal
Faculté des arts et des sciences

Cette thèse intitulée:

Essais en économie de l'environnement et des ressources naturelles sous incertitude

présentée par:

Justin Johnson Kakeu Kengne

a été évaluée par un jury composé des personnes suivantes:

Rui Castro,	président-rapporteur
Gérard Gaudet,	directeur de recherche
Benoit Perron,	codirecteur
Marc Henry,	membre du jury
John Livernois,	examinateur externe
Rui Castro,	représentant du doyen de la FES

Thèse acceptée le: 23 novembre 2010

RÉSUMÉ

Cette thèse comprend trois essais en économie de l'environnement et des ressources naturelles sous incertitude. Le premier essai propose un modèle de jeu différentiel qui analyse la pollution globale à travers la quête à l'hégémonie politique entre pays. Le second essai utilise des données boursières pour estimer une version stochastique de la règle de Hotelling et ainsi inférer sur le rôle des ressources naturelles non renouvelables dans la diversification du risque. Le troisième essai montre comment la prise en compte des perspectives futures modifie la règle de Hotelling dans un contexte de diversification du risque.

Mots clés: Hégémonie, Pollution, Ressources non renouvellables, Risque, Capitalisation boursière, Réserves prouvées, Perspectives futures, Utilité différentielle stochastique

ABSTRACT

My thesis is composed of three essays on environmental and natural resource economics under uncertainty. The first essay proposes a differential game analysis of the quest for hegemony among countries as a generator of global pollution. The second essay uses stock market data on market capitalization to estimate a stochastic version of the Hotelling rule of exhaustible resource exploitation and uses it to infer on the riskiness of investment in nonrenewable resources and its effect on the resource price paths. The third essay shows how uncertainty about future prospects modifies the Hotelling rule in a context of risk diversification.

Keywords: Hegemony, Pollution, Exhaustible Natural Resource, Risk, Market Capitalization, Proven Reserves, Future Prospects, Stochastic Differential Utility

TABLE DES MATIÈRES

LISTE DES TABLEAUX

LISTE DES FIGURES

LISTE DES ANNEXES

A ma mère Françoise, mes sœurs et mes frères.

A toutes ces personnes qui, parfois sans le savoir, influent positivement nos vies.

A la femme de mon coeur.

Justin Johnson Kakeu Kengne.

REMERCIEMENTS

Les mots me manquent pour exprimer ma gratitude au Professeur Gérard Gaudet. Durant le cheminement de cette thèse, j'ai largement bénéficié de son expérience à travers ses commentaires et ses encouragements. Je lui en suis infiniment reconnaissant.

Un grand merci à Messieurs Benoit Perron, Sidartha Gordon et Pierre Lasserre pour leurs commentaires et la qualité des cours qu'ils nous ont prodigué.

Merci à l'Atelier d'économie des ressources naturelles et de l'environnement de Montréal. C'est un cadre convivial de réflexion qui m'a permis d'être constamment en éveil aux évolutions de la recherche et de rencontrer de nombreux chercheurs venus d'horizon diverses.

Merci aux membres du Département de sciences économiques, en particulier Jocelyne Demers, Marie-Christine Thirion, Josée Lafontaine, Suzanne Larouche-Sidoti, Arianne Faucher-Gagnon, Micheline Coté, Ruth Charest, Carole Fournier, Pascal Martinolli, Denyse Rompré.

Merci à Eric Momo Nzifack.

Merci aux camarades pour ces nombreux moments de discussion sur des sujets variés.

Merci au Québec et au Canada pour le climat idéal de travail et de réflexion qu'ils offrent.

INTRODUCTION GÉNÉRALE

Cette thèse est composée de trois essais. Le premier essai part de l'hypothèse que les pays, en acquérant plus de puissance économique, augmentent leur probabilité d'atteindre la position hégémonique. La quête à l'hégémonie est modélisée comme un jeu de course où les joueurs sont des pays différenciés par une dotation en capital qui génère un flux de rendement non polluant. Le niveau d'émission d'un pays est supposé relié à sa puissance économique tel que mesurée par le niveau de production. De l'analyse, deux types de pays ressortent : les pays richement dotés, dont le rendement issu de leur dotation est plus grand que le rendement de la récompense en cas de succès dans la course à l'hégémonie, et les pays pauvrement dotés, dont le rendement de la récompense en cas de succès dans cette course est plus grand que celui du flux de rendement issu de leur dotation. Nous montrons que dans un équilibre symétrique constitué de pays pauvrement dotés, le niveau d'équilibre des émissions est plus grand que celui d'un équilibre symétrique constitué de pays richement dotés. Dans un monde asymétrique constitué des deux types de pays, le niveau d'émission d'un pays pauvrement doté est supérieur au niveau d'émission d'un pays richement doté. Les simulations numériques indiquent que si on augmente le nombre de pays richement dotés tout en maintenant fixe le nombre total de pays, le niveau d'équilibre de la pollution globale baisse ; si les dotations des pays pauvrement dotés sont accrues en laissant constante celles des pays richement dotés, la pollution globale diminue ; accroître les dotations des deux types de pays dans les mêmes proportions, et donc accroître la dotation moyenne dans la même proportion, baissera la pollution globale ; redistribuer des pays richement dotés vers les pays pauvrement dotés, tout en maintenant fixe la dotation moyenne, résultera en général en un accroissement du niveau d'équilibre de la pollution globale.

Le second essai est une étude empirique qui utilise les données sur la capitalisation boursière des compagnies minières pour estimer une version stochastique de la règle de Hotelling pour le pétrole, le gaz naturel et le charbon. Un lien formel entre le rendement d'une unité de stock de ressource non renouvelable et la différence entre le taux de croissance de la capitalisation boursière des compagnies minières et celui des réserves prouvées y est formellement établi. La base économique de ce travail est le modèle théorique de Gaudet and Khadr (1991), lequel offre un cadre théorique idéal permettant de connecter la théorie économique des ressources non renouvelables et l'économie financière et par ailleurs d'utiliser les outils de l'économétrie financière pour cette estimation. Les données concernant les capitalisations boursières des entreprises opérant dans le secteur des ressources non renouvelables, et d'autres données comme les rendements des bonds du trésor proviennent de la base de données CRSP qui est l'une des sources de données historiques les plus complètes sur actions échangées sur le New York Stock Exchange, le Nasdaq et l'AMEX. Les données sur les réserves mondiales de pétrole et de charbon proviennent du site web de la compagnie BP. Les données sur la consommation proviennent du site web de la Federal Reserve St Louis. L'approche économétrique est basée sur l'estimation des processus de diffusion en temps continu développé par Nowman (1997). Les résultats montrent qu'investir dans des stocks de ces ressources naturelles peut constituer une assurance contre le risque du marché à long terme.

Dans le troisième essai, je montre comment les perspectives relatives au futur (bonnes ou mauvaises) sont un facteur important pour la formulation de la règle d'Hotelling dans un contexte de diversification du risque. J'utilise les préférences récursives en temps continu de Duffie and Epstein (1992b) dans le modèle d'économie des ressources naturelles de Gaudet et Khadr (1991). Je dérive une formulation générale de la règle de Hotelling qui comprend trois éléments : un élément d'assurance comme dans le modèle de Gaudet et Khadr (1991), un élément relatif aux perspectives sur le futur, et un taux d'actualisation endogène. En comparaison avec la tarification d'un actif reproductible, il ressort que ce taux d'actualisation endogène joue un rôle important dans la tarification des ressources non renouvelables. Il est montré comment cette composante relative aux perspectives futures influe sur le prix du marché de la ressource une fois extraite. Les implications de ces résultats pour les décisions d'investissement dans les stocks de ressources naturelles en terre sont analysées.

Gérard Gaudet est coauteur du premier essai. J'en ai proposé l'idée et la modélisation et j'en ai fait les analyses. Le Professeur Gaudet a contribué à améliorer la structure de l'article et la présentation finale des résultats.

CHAPITRE 1

THE QUEST FOR HEGEMONY AMONG COUNTRIES AND GLOBAL POLLUTION

1.1 Introduction

Concerns about global pollution in general and global warming in particular have led to a considerable body of literature. But an important question which has not yet been formally explored has to do with the relationship between the quest for hegemony and global pollution. Derived from the original Greek word *hegemonia*, which means ”leadership”, hegemony can be seen as an institutionalized practice of special rights and responsibilities conferred on a state with the resources to lead the international system (Clark (2009)). Most historical ages are marked by the presence of a nation capable of dominating the course of international politics. Over the last five centuries, Portugal, the Netherlands, France, Britain, and the United States have played the hegemonic role Modelski (1987)). [1]

The quest for hegemony can be viewed as a status-seeking game among countries which aspire to the hegemonic status and the important benefits that come with it. [2] A basic postulate widely accepted among experts of geopolitics is that relative power differences between states cause them to compete with one another for relative shifts in power and status. For centuries, military force was the main source of primacy in the international system. After the cold war and the advent of nuclear warfare, military force became a costly and risky means of attaining hegemony and economic force gained prominence as the major factor in determining the primacy or the subordination of states. [3]

1. For discussions of the quest for hegemony among countries, see also Kennedy (1987), Black (2007) and Mosher (2002).

2. Weiss and Fershtman (1980) define status as a ranking of individuals (or groups of individuals) based on traits, assets and actions. On the subject of status seeking, see also Moldovanu et al. (2007).

3. To quote the political scientist Kenneth Waltz (Waltz 1993, p. 63 and p. 66) : “Without a considerable economic capability, no state can hope to sustain a world role, as the fate of the Soviet Union has shown.” and “For a country to choose not to become a great power is a structural anomaly. For that reason, the choice is a difficult one to sustain. Sooner or later, usually sooner, the international status of countries

It is safe to say that economic strength has become a necessary condition for attaining hegemony in the international system.

After World War II, the United States, with half the world's gross national product, found itself in a uniquely strong position, much surpassing that of Britain at the height of its power in the nineteenth century, and played the leading role in the international state system. But in the last decades, the United States has been facing new global players, such as China, India and Brazil, who are making their presence felt in international affairs, largely due to the increasing power derived mostly from their expanding economies.[4] Those emerging economies are transforming the hegemony game into a multi-player game (Shenkar 2005, p. 162).

But economic and ecological systems are deeply interlocked, in good part because most of the global pollution released into the atmosphere comes from the combustion of fossil fuels, which is a driving force of the economic system (see Stern (2007), Chombat (1998), Raupach et al. (2007)). Therefore, because economic activity impacts both global pollution and the hegemonic game, the world can be viewed as facing a conflict between the intensity of the hegemony game among countries and the reduction of global pollution. As has been noted by a former Science Advisor to the U.S. President : "No realistic response to climate change can ignore the current geopolitical preoccupation with economic competition among nations" (Marburger 2007, p. 5).

To analyze this issue, this paper builds on the assumption that each country behaves in such a way as to improve, via its economic strength, the probability that it will attain the hegemonic position on the world stage. The quest for hegemony is modeled as a game, with countries being differentiated *only* by the return on some initial endowment

---

has risen in step with their material resources. Countries with great-power economies have become great powers, whether or not reluctantly." Other political scientists, among them Samuel Huntington (p. 72 Huntington 1993), share this view : "Economic activity [. . .] is probably the most important source of power, and in a world in which military conflict between major states is unlikely, economic power will be increasingly important in determining the primacy or the subordination of states." The importance of the economic battle among hegemonic aspirants is also pointed out by the economist Lester Thurow (Thurow 1993, p. 65) : "Those who control the world's largest market get to write the rules. That is as it always has been. When the United States had the world's largest market, it got to write the rules"

4. See Shenkar (2005), Ikenberry (2008) and Elliott (2007). To quote Oded Shenkar (Shenkar 2005, p. 38) : "China's economic aspirations are aligned strongly with its political ambitions, and the regime is aware more than most of the close connection between the two."

which yields a pollution-free flow of income. This return on endowment can be thought of as being related to the country's human capital and economic, social and political institutions. A country's level of pollution is assumed directly related to its economic strength, as measured by its level of production. Two types of countries are distinguished : richly-endowed countries, for whom the return on their endowment is greater than the return they can expect from winning the hegemony race, and poorly-endowed countries, who can expect a greater return from winning the race than from their endowment. As we will see, the latter, having more to gain, are more eager players in the hegemony race and will end up polluting more in equilibrium. The former are more content with the return they get from their endowment and end up polluting less. We may think of emerging economies such as China, India, Brazil and Russia as being in the category of poorly-endowed countries, whereas most North American and Western European countries would fall in the category of richly-endowed countries.

We consider in sequence the equilibria in a world composed of only poorly-endowed countries, a world composed of only richly-endowed countries and a world in which both types of countries coexist. We show that in a symmetric world of poorly-endowed countries the equilibrium level of emissions is larger than in a symmetric world of richly-endowed countries : the former, being less well endowed to begin with, try harder to win the race. In the asymmetric world composed of both types of countries, there can be multiple equilibria. In all of those equilibria, the poorly-endowed countries will be polluting more than the richly-endowed countries. Numerical simulations show that if the number of richly-endowed countries is increased keeping the total number of countries constant, the equilibrium level of global emissions will decrease ; if the lot of the poorly-endowed countries is increased by increasing their initial endowment keeping that of the richly-endowed countries constant, global pollution will decrease ; increasing the endowments of each type of countries in the same proportion, and hence increasing the average endowment in that proportion, will decrease global pollution ; redistributing from the richly-endowed in favor of the poorly-endowed while keeping the average endowment constant will in general result in an increase in the equilibrium level of global pollution.

In the next section, we describe the main features of the model. Section 1.3 presents the hegemony game, which borrows some of its features from the patent-race literature (Reingamun (1982), Lee and Wilde (1980) and Loury (1979)). Section 1.4 analyzes the equilibria under the different scenarios described above and discusses the effect of various ways of modifying the distribution of endowments in the case where poorly-endowed and richly-endowed countries coexist. We conclude with some final remarks in Section 1.5.

1.2 The Model

Consider N countries competing to reach the hegemonic position. The probability for any country of reaching the hegemonic position increases with its output, $Q_i(t)$, a measure of its economic strength. Country i's production gives rise to the emission of pollution at the rate $e_i(t)$. For simplicity it will be assumed that one unit of production gives rise to one unit of emission : $e_i(t) = Q_i(t)$. This pollution is global, in the sense that it will affect each country equally.

To visualize the conditions required to win the hegemony race, we can use Greek foot races or sporting contests as analogies. The first condition to win the game is to get to the finish line. The second condition is to be the first among all players to cross the finish line. The prize for crossing the finish line first is greater than the prize of the losers.

In the present context, the winner gets the hegemon position and gets to enjoy "structural power", which Nye (1990) called the "soft power". This structural power allows the hegemon to occupy a central and prestigious position within the international system and to play a leading role in setting standards (political, cultural, economic) in organizing the world. We borrow from the paper of Moldovanu et al. (2007) the notion of "pure status" prize, which is related to the notion that a contestant is happier when he has other contestants below him. Hence the hegemon enjoys a "pure status" prize A. Any country other than the hegemon gets a "prize" of $B < A$. For simplicity, A and B will be assumed constant.

The time it will take for country i to cross the finish line (i.e., to attain the necessary characteristics that a country must satisfy to get the hegemonic role) is a random variable, τ_i. Uncertainty about the finishing time is determined by the hazard rate $H_i(t)$, which, by definition, is given by : [5]

$$H_i(t) = \frac{P_i(t < \tau_i \leq t + dt)}{P_i(\tau_i > t)}.$$

It represents the propensity to reach the finish line at time t, given that it has not happened before t. The hazard rate is assumed positively related to the country's level of production, $Q_i(t)$, and hence to its rate of pollution emissions, $e_i(t)$. It will be assumed that $H_i(t) = Q_i(t) = e_i(t)$. It follows that the probability of reaching the finish line by time t is the cumulative distribution function $F_i(t)$, which can be expressed, using the hazard rate, as :

$$F_i(t) = 1 - e^{-\int_o^t e_i(u)du}. \tag{1.1}$$

This means that the probability of reaching the finish line by time t increases with country i's cumulative emissions on the interval $[0,t]$, given by the term $\int_o^t e_i(u)du$.

The first country to cross the finish line becomes the winner of this hegemony game and obtains the prize denoted above by A. The time at which one of the countries becomes the hegemon is a random variable and is given by

$$\tau = \min_{i=1,\ldots,N} \tau_i\,.$$

This is the stopping time of the game. It depends stochastically on the vector $(e_1(t),\ldots,e_N(t))$ of emission levels by each country. Given the vector $(e_1(t),\ldots,e_N(t))$ of emission levels, the instantaneous probability that country i will win the hegemony game on the infinitesimal interval $[t, t+dt]$ will be given by :

$$\dot{F}_i(t) \prod_{j \neq i}^{N} \left[1 - F_j(t)\right] dt,$$

5. Cioffi-Revilla (1998) interprets the hazard rate as a force which, although it does not determine the realization of a political event, acts on it by influencing its temporal evolution. In this paper, the hazard rate consists of economic strength, a causal force arising from the country's decisions which influences the hegemony race.

where $\dot{F}_i(t)$ denotes the time derivative of $F_i(t)$.

The instantaneous probability that country i will lose the hegemony race on the infinitesimal interval $[t, t+dt]$ is the probability that one of the $N-1$ other countries becomes the winner over that interval of time. This is given by :

$$\sum_{j=1, j\neq i}^{j=N} \dot{F}_j(t) \left(\prod_{k=1, k\neq j}^{k=N} [1 - F_k(t)] \right) dt.$$

The instantaneous probability that no country wins the hegemony game on the infinitesimal interval of time $[t, t+dt]$ is :

$$\prod_{j=1}^{j=N} [1 - F_j(t)] dt.$$

If we denote by $S(t)$ the stock of pollution at time t, then

$$\dot{S}(t) = e_1(t) + ... + e_N(t) - kS(t),\ S(0) = S_0 > 0 \text{ given}, \tag{1.2}$$

where $0 < k < 1$ is the coefficient of natural purification. Each country is assumed to suffer equally from the global stock of pollution. The damage function is assumed to be a nonlinear increasing and convex function of the stock, more specifically a quadratic :

$$D_i(S(t)) = \frac{b}{2} S(t)^2 \tag{1.3}$$

with b a strictly positive constant.

It will also be assumed that the countries are differentiated solely by the return, π_i, which they get on some initial endowment. This exogenous parameter will capture the idea of disparity between countries and the country's pollution emissions will be assumed independent of this permanent flow of benefits. Among the factors that can affect this return on endowment one can think of human capital and the quality of economic, social and political institutions.

1.3 The hegemony game

The hegemony game bears a lot of similarity to an R&D race, as analyzed in Reingamun (1982), Lee and Wilde (1980) and Loury (1979). The value function of country i, $i = 1, \ldots, N$, in quest of hegemony, is given by :

$$\begin{aligned} V_i(F_1(t), ..., F_N(t), S(t)) &= \max_{e_i(t)} \int_0^{\infty} e^{-rt} \left\{ A\dot{F}_i(t) \prod_{j \neq i} [1 - F_j(t)] \right. \\ &+ \left. B \sum_{j \neq i} \dot{F}_j(t) \prod_{k \neq j} [1 - F_k(t)] + (\pi_i - D_i(S(t))) \prod_{j=1}^{N} [1 - F_j(t)] \right\} dt \end{aligned}$$

where the maximization is subject (1.2) and to $e_i(t) \geq 0$. The stochastic variable τ having been eliminated by formulating the objective functional in terms of expectations, this becomes a deterministic N-player differential game, with control variables $e_1(t), ..., e_N(t)$ and state variables $(F_1(t), ..., F_N(t), S(t))$. The objective functional of country i consists of three terms. The first reflects net benefits if the country succeeds in the quest for the hegemon's position. The second term is the net benefits if the country loses the quest for the hegemon position. The third term represents the pollution damage and the payoff generated by the country's endowment. All three components are weighted by their respective probabilities.

In order to put the emphasis on the characterization of the hegemony game, it will henceforth be assumed that the pollution stock is stationary and hence given by $S(t) = \sum_{i=1}^{N} e_i(t)/k$. We can therefore rewrite the damage function as $D(S(t)) = \beta \left[\sum_{i=1}^{N} e_i(t)\right]^2 /2$, where $\beta = b/k^2$.

Following Dockner et al. (2000), let us now introduce the following state transformation which will help in simplifying the formulation :

$$-\log(1 - F_i(t)) = \int_o^t e_i(u) du \equiv Z_i(t), \tag{1.4}$$

which, upon differentiation with respect to time, gives :

$$\dot{Z}_i(t) = e_i(t). \tag{1.5}$$

The value function of country i can then be rewritten :

$$V_i(Z_1(t),...,Z_N(t)) = \max_{e_i(t)} \int_0^{\infty} e^{-rt} e^{-\sum_{j=1}^{N} Z_i(t)} \left[Ae_i(t) + B\sum_{j\neq i} e_j(t) - \frac{\beta}{2}\left[\sum_{i=1}^{N} e_i(t)\right]^2 + \pi_i \right] dt.$$

where the maximization is with respect to (1.5).

Notice that although each country knows the full state vector $(Z_1(t),...,Z_N(t))$, only a function of it, namely the one-dimensional state variable $X(t) = e^{-\sum_{j=1}^{N} Z_i(t)}$, is payoff relevant.[6] The problem of country i can therefore be transformed into the following one-state variable problem :

$$V_i(X(t)) = \max_{e_i(t)} \int_0^{\infty} e^{-rt} X(t) \left[Ae_i(t) + B\sum_{j\neq i} e_j(t) - \frac{\beta}{2}\left[\sum_{i=1}^{N} e_i(t)\right]^2 + \pi_i \right] dt \qquad (1.6)$$

where the maximization is subject to

$$\dot{X}(t) = -X(t)\left(\sum_{j=1}^{N} e_i(t)\right). \qquad (1.7)$$

The state variable $X(t)$ gives the probability that the game has not yet ended at date t.

The corresponding current value Hamiltonian is given by

$$\mathscr{H}_i(t) = X(t)\left[Ae_i(t) + B\sum_{j\neq i} e_j(t) - \frac{\beta}{2}\left[\sum_{i=1}^{N} e_i(t)\right]^2 + \pi_i \right] - \lambda_i(t)\left[X(t)\left(\sum_{j=1}^{N} e_i(t)\right)\right],$$

where $\lambda_i(t)$ is the shadow value associated to the state variable $X(t)$.

Letting $\theta_i(t) = \sum_{j\neq i} e_j(t)$, that is the sum of the emission rates of country i's rivals, and taking into account that $X(t) > 0$, the following conditions, along with (1.7), become necessary, for $i = 1,\ldots,N$:

$$A - \beta[e_i(t) + \theta_i(t)] - \lambda_i(t) \leq 0, \quad [A - \beta[e_i(t) + \theta_i(t)] - \lambda_i(t)]e_i(t) = 0, \quad e_i(t) \geq 0 \quad (1.8)$$

6. See Dockner et al. (2000), p. 276.

$$\dot{\lambda}_i(t) - r\lambda_i(t) = (e_i(t) + \theta_i(t))\,\lambda_i(t) - \left(Ae_i(t) + B\theta_i(t) - \frac{\beta}{2}\left[e_i(t) + \theta_i(t)\right]^2 + \pi_i\right). \quad (1.9)$$

Differentiating (1.8) with respect to t and substituting into (1.9) we get :

$$\beta\dot{E}(t) = \frac{\beta}{2}E(t)^2 + r\beta E(t) - (A-B)\theta_i(t) - (rA - \pi_i) \quad (1.10)$$

where $E(t) = e_i(t) + \theta_i(t)$. But since by assumption the stock of pollution is in a steady state, so is $E(t)$ and therefore $\dot{E}(t) = 0$. The result is a second degree polynomial in $E(t)$ with roots :

$$E(t) = -r \pm \sqrt{r^2 + \frac{2}{\beta}[(A-B)\theta_i(t) + (rA - \pi_i)]}. \quad (1.11)$$

Notice that we can without loss of generality set $\beta = 1$ and reinterpret A, B and π_i as A/β, B/β and π_i/β. The discriminant is then given by :

$$\delta(\theta_i) = r^2 + 2[(A-B)\theta_i + (rA - \pi_i)].$$

It will be assumed strictly positive to assure distinct real roots. [7] Then, neglecting the negative root and taking into account the nonnegativity constraint on $e_i(t)$, the best response function of country i can, at any given t, be written :

$$e_i(\theta_i) = \max\left\{0, -r - \theta_i + \sqrt{r^2 + 2[(A-B)\theta_i + (rA - \pi_i)]}\right\}. \quad (1.12)$$

This reaction function is not monotone. In fact, in the positive range, the second derivative is strictly negative (since $\delta(\theta_i) > 0$) and hence the best response function is strictly concave in that range and reaches a maximum for $\theta = \left[(A-B)^2 - (r^2 + 2(Ar - \pi_i))\right]/2(A-B)$.

Setting $-r - \theta_i + \sqrt{r^2 + 2[(A-B)\theta_i + (rA - \pi_i)} = 0$ yields the second degree polynomial $\theta_i^2 + 2[(r - [A-B])\theta_i - 2(Ar - \pi_i) = 0$ whose roots are given by

$$\theta_i = -(r - (A-B)) \pm \sqrt{(r - (A-B))^2 + 2(rA - \pi_i)}. \quad (1.13)$$

7. If $\delta(\cdot)$ is negative, then, because of the nonnegativity constraint on e_i, there still exists a solution in real space given by $e_i(t) = 0$ and hence $E(t) = 0$.

The product of those roots is $-2(rA-\pi_i)$. It follows that the roots will be of opposite sign if $rA < \pi_i$ and of the same sign if $rA > \pi_i$. In the latter case, since the sum of the roots is $-2(r-[A-B])$, the two roots are negative if $A-B<r$, in which case the country would choose $e_i(\theta_i)=0$ for all $\theta_i>0$, not participating in the race for hegemony being a dominant strategy. Both roots will be positive if $A-B>r$. We will assume $A-B>r$ so that the quest for hegemony is sufficiently attractive for the country in this situation to participate actively in the game at least for some positive values of θ_i.[8] In the case where $rA<\pi_i$, we retain only the positive root, for obvious reasons.

We will, from this point on, distinguish two types of countries according to their endowments : π_1 and $\pi_2>\pi_1$, with $rA>\pi_1$ and $rA<\pi_2$. Countries of type 1 will be called poorly-endowed countries, in the sense that the interest flow on the hegemon's prize exceeds the return from its endowment ; conversely, countries of type 2 are richly-endowed countries, the return on their endowment being greater than the interest flow on the payoff from winning the quest for hegemony.

By a slight abuse of notation, we will denote by $e_1(t)$ the emissions of the typical poorly-endowed country (type 1) and by $\theta_1(t)$ the sum of the emission of that country's rivals. Similarly for $e_2(t)$ and $\theta_2(t)$ in the case of the richly-endowed countries (type 2).

We may then write the best response to θ_1 of the typical country of type 1 at any time t as :

$$e_1(\theta_1)=\begin{cases} -r-\theta_1+\sqrt{2(A-B)\theta_1+r^2+2(Ar-\pi_1)} & \text{if } \theta_1\in[0,\widetilde{\theta}_1) \\ 0 & \text{if } \theta_1\in[\widetilde{\theta}_1,+\infty] \end{cases} \tag{1.14}$$

where $\widetilde{\theta}_1$ is the positive root of (1.13). This is illustrated in Figure 1.1.

8. Recall that we have earlier set $\beta=1$, so that $A-B$ is to be interpreted as $(A-B)/\beta$, where $\beta=b/k^2$. Hence, written $\beta<(A-B)/r$, the condition can be interpreted as : the additional value of winning the hegemony game rather than losing it, discounted to infinity, exceeds β. The parameter β can be interpreted as the damage cost parameter with respect to the flow of emission in steady-state, whereas b is the damage coat parameter with respect to the stock of pollution.

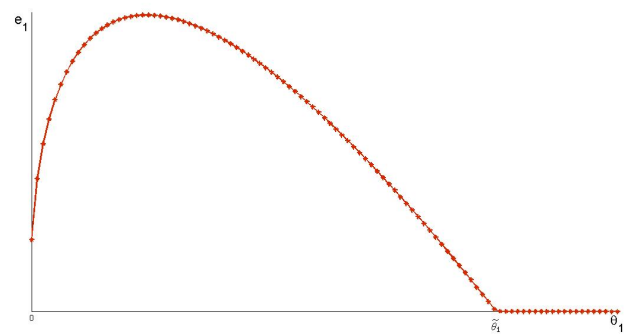

Figure 1.1 – The best response function of a poorly-endowed country

For this type of country, winning the hegemony game is sufficiently rewarding compared to the return it gets from its exogenous endowment that it pays to participate actively in the hegemony game even for low levels of effort by all the other countries, as measured by their emissions ; hence $e_1(\theta_1 = 0) > 0$. As the level of emissions of its rivals increases, its best response is at first to increase its own level of effort and, as a result, its emissions. Beyond some point $\widetilde{\theta}_1$, it becomes optimal to reduce its emissions as the level of emissions of others increases, until it reaches zero and remains there for all greater θ_1s.

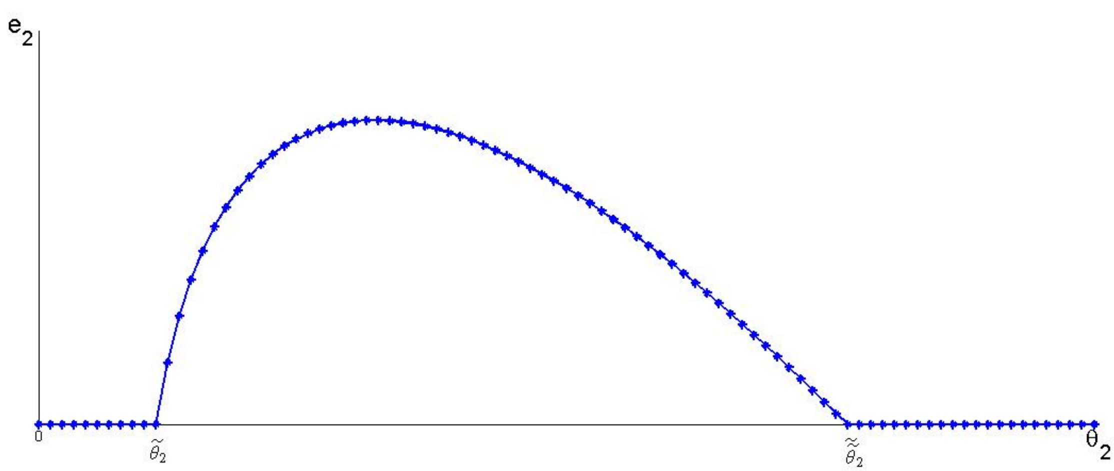

Figure 1.2 – The best response function of a richly-endowed country

In the case of the richly-endowed countries, the best response is similar except for the fact that the high return it gets from its endowment relative to the return from winning the game renders it not optimal to participate actively up to some minimal level of emissions by the other countries. Its best response as a function of θ_2 is therefore :[9]

$$e_2(\theta_2) = \begin{cases} 0 & \text{if } \theta_2 \in [0, \widetilde{\theta}_2] \\ -r - \theta_2 + \sqrt{2(A-B)\theta_2 + r^2 + 2(Ar - \pi_2)} & \text{if } \theta_2 \in (\widetilde{\theta}_2, \widetilde{\widetilde{\theta}}_2) \\ 0 & \text{if } \theta_2 \in [\widetilde{\widetilde{\theta}}_2, +\infty] \end{cases} \tag{1.15}$$

where $\widetilde{\theta}_2 < \widetilde{\widetilde{\theta}}_2$ are the two (positive) roots of (1.13). This is illustrated in Figure 1.2.

As can be seen from (1.13), (1.14) and (1.15), for both types of countries, the greater the gap $(A-B)$ between the winner's prize and the losers' prize, the greater the reaction to any given θ_i, and hence the greater the country's level of emissions. Similarly for the gap $(rA - \pi_i)$ between the interest flow from the hegemon's prize and the return to the country's endowment. But the poorly-endowed countries, whose return on endowment is smaller than the return to be expected from the hegemon's prize, are more eager in the quest for hegemony than are the richly-endowed countries. Each of them therefore reacts in a stronger fashion to any given θ_i than does a richly-endowed country, so that $e_1(x) > e_2(x)$ for any $x < \widetilde{\theta}_1$. The richly-endowed countries, whose return on endowment exceeds the return they can expect on the hegemon's prize, are more content and as a consequence react less strongly to any given level of total effort in the quest for hegemony by their rivals.

1.4 The equilibrium outcomes

In this section we characterize the equilibrium solution to the hegemony game and analyze the consequences for global pollution for, in order, a world of identical poorly-endowed countries, a world of identical richly-endowed countries and a world composed

9. In the limiting case where the discriminant of (1.13) is zero, $\widetilde{\theta}_2 = \widetilde{\widetilde{\theta}}_2 = -(r - [A-B])$, which is positive by assumption. Substituting this value for θ_2 in (1.15), it is easily verified that $e_2(\theta_2) = 0$ for all $\theta_2 > 0$: it is optimal for the typical richly-endowed countries not to participate actively in the hegemony game no matter what the total level of emissions of its rivals. We will assume this uninteresting case away by assuming the discriminant of (1.13) to be strictly positive.

of both poorly-endowed and richly-endowed countries.

1.4.1 Equilibrium in a world of poorly-endowed countries

Consider a world where there are N identical countries of type 1 ($rA > \pi_1$) in quest of hegemony. Then there is a unique symmetric equilibrium, given by

$$e_1^*(N) = \frac{-[(N-1)(A-B)-Nr]+\sqrt{[(N-1)(A-B)-Nr]^2+2N^2(Ar-\pi_1)}}{N^2} \quad (1.16)$$

This is obtained by setting $\theta_1 = (N-1)e_1$ in (1.14), because of symmetry, and keeping only the positive root of the resulting polynomial in e_1.

Figure 1.3 illustrates this equilibrium for $N = 2$, given by the intersection of the reaction function with the 45-degree line, and for $N > 2$, given by its intersection with the lower line $\theta_1/(N-1)$. Since the equilibrium for $N = 2$ will always be in the decreasing part of the best response function, so will the equilibrium for all $N > 2$. It follows that as N increases, $e_1^*(N)$ necessarily decreases. Indeed, from (1.16), it is verified that :

$$\frac{de_1^*(N)}{dN} < 0 \quad \text{and} \quad \lim_{N\to\infty} e_1^*(N) = 0.$$

Hence, if we let N tend to infinity, the contribution of each individual country to global emissions becomes negligible. However the total emissions, $Ne_1^*(N)$, are monotone increasing and, as N tends to infinity, tend to the following positive value :

$$lim_{N\to\infty} Ne_1^*(N) = -(A-B)+r+\sqrt{(A-B)^2+r^2+2(Ar-\pi_1)} > 0.$$

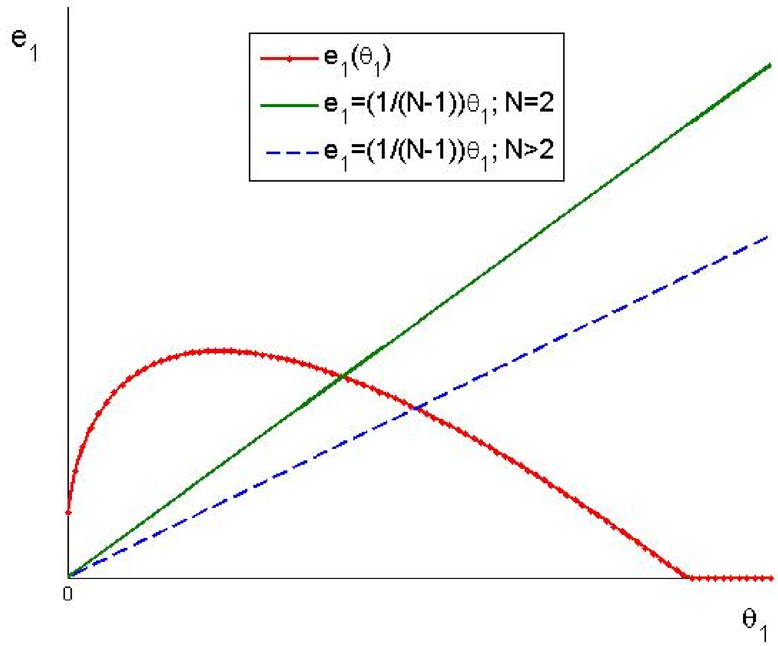

Figure 1.3 – The equilibrium with N poorly-endowed countries

1.4.2 Equilibrium in a world of richly-endowed countries

Consider now a world where all N countries are richly-endowed, hence of type 2 ($rA < \pi_2$). The quest for hegemony in this case can lead to multiple symmetric equilibria.

Setting $\theta_2 = (N-1)e_2$ (by symmetry), there clearly always exists an equilibrium where each country is content with the return from its endowment and hence does not participate actively in the quest for hegemony : $e_2^{0*}(N) = 0$. If the gap between the return from the countries' endowment and the return on the hegemony prize is sufficiently high and the number of countries sufficiently low, this will in fact be the only equilibrium, as illustrated in Figure 1.4 .

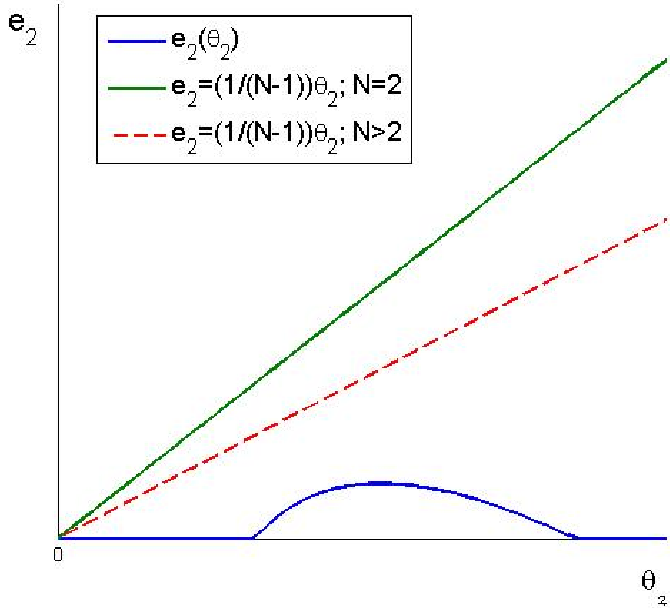

Figure 1.4 – Unique zero-emission equilibrium with N richly-endowed countries

As the gap between the return on endowment and the return on the prize falls (given the number of countries), or as the number of countries increases (given the gap in returns), two positive equilibria will appear in addition to the $e_2^*(N) = 0$ equilibrium. These are given by :

$$e_2^{a*}(N) = \frac{N\left[r-(A-B)\right]+(A-B)-\sqrt{[(N-1)(A-B)-Nr]^2+2N^2(Ar-\pi_2)}}{N^2} \tag{1.17}$$

and

$$e_2^{b*}(N) = \frac{N\left[r-(A-B)\right]+(A-B)+\sqrt{[(N-1)(A-B)-Nr]^2+2N^2(Ar-\pi_2)}}{N^2}, \tag{1.18}$$

obtained from (1.15) with $\theta_2 = (N-1)e_2$. There are then three equilibria, characterized by ($e_2^{0*}(N)$, $e_2^{a*}(N)$, $e_2^{b*}(N)$), with $e_2^{0*}(N)$ and $e_2^{b*}(N)$ being stable and $e_2^{a*}(N)$ unstable, in the sense that any small deviation will lead the system to one of the other two equilibria. [10] The unstable equilibrium $e_2^{a*}(N)$ occurs in the increasing part of the best response function, while the stable equilibrium $e_2^{b*}(N)$ occurs in its decreasing part. This three equilibria situation is illustrated in Figure 1.5.

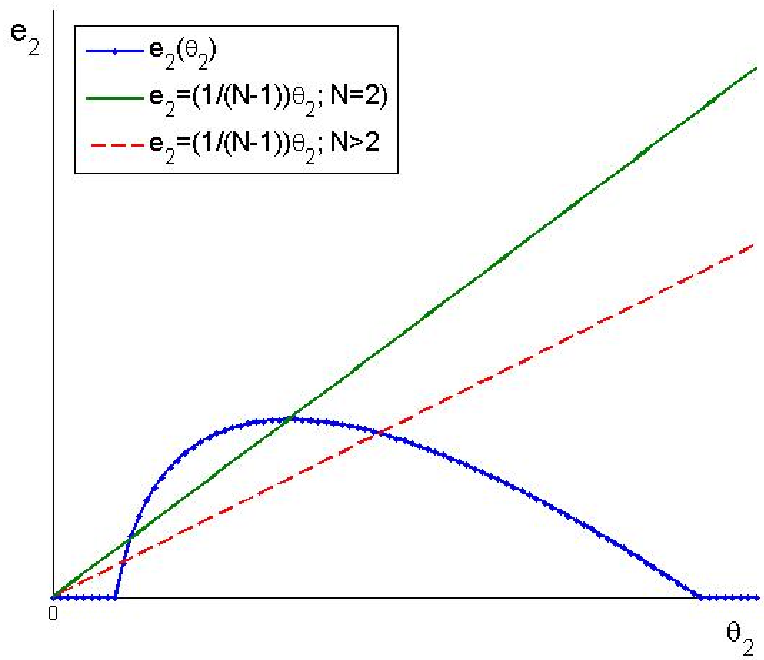

Figure 1.5 – Three equilibria with N richly-endowed countries

Like in the world of poorly-endowed countries of the previous subsection,

$$\lim_{N\to\infty} e_2^{a*}(N) = e_2^{b*}(N) = e_2^{0*}(N) = 0,$$

which simply means that the individual emissions become negligible as the number of countries tends to zero, as can be expected. However, in the unstable equilibrium, we now have

$$lim_{N\to\infty} N e_2^{a*}(N) = 0,$$

10. A two equilibria case can also occur if the discriminant in (1.17) and (1.18) happens to be zero, so that $e_2^{a*}(N)$ and $e_2^{b*}(N)$ coincide. This can be illustrated by the tangency of the line $e_2 = (1/(N-1))\theta_2$ and the best response function. Of the two equilibria, only $e_2^{0*}(N)$ is then stable. We will ignore this possibility in what follows.

so that as the number of countries tends to infinity, both the individual emissions and the total emissions tend to zero. The stable equilibrium $e_2^{b*}(N)$ has the expected property that

$$lim_{N\to\infty} Ne_2^{b*}(N) = -(A-B)+r+\sqrt{(A-B)^2+r^2+2(Ar-\pi_2)} > 0.$$

This expression is the same as in the case of the poorly-endowed countries, except for π_1 being replaced by π_2. Since $\pi_2 > \pi_1$, this positive quantity is therefore smaller than for the poorly-endowed countries.

As can easily be seen by comparing (1.16) to (1.17) and (1.18), the fact that $\pi_2 > \pi_1$ implies that the equilibrium individual emissions will be smaller in a richly-endowed symmetric world than in a poorly-endowed one. This again reflects the fact that more eager poorly-endowed countries will be making greater efforts in the quest for hegemony than more content richly-endowed countries.

1.4.3 Equilibria in a world of both poorly-endowed and richly-endowed countries

A more realistic and more interesting situation is one where both poorly-endowed and richly-endowed countries coexist. Assume now a world composed of N_1 poorly-endowed countries and N_2 richly-endowed countries, with $N_1+N_2 = N$. The configuration of the equilibria will then depend on the distribution of countries between the two types.

From (1.14) and (1.15) we find that three types of equilibria can exist, and they may coexist. These are

$$\left.\begin{array}{l} e_1^*(N_1,N_2) = \frac{2(N_1(1-r-N_1)+\sqrt{[2(N_1(1-r-N_1)]^2+8N_1(Ar-\pi_1)}}{2N_1^2} \\ \\ e_2^*(N_1,N_2) = 0 \end{array}\right\} \tag{1.19}$$

$$\left.\begin{array}{l} e_1^*(N_1,N_2) = \frac{-\left[2(N_1+N_2)(r+\frac{N_2(\pi_1-\pi_2)}{A-B})-2(N_1+N_2-1)(A-B)\right]-\sqrt{\Delta(N_1,N_2,\pi_1,\pi_2,A,B,r)}}{2(N_1+N_2)^2} \\ \\ e_2^*(N_1,N_2) = e_1^*(N_1,N_2)+\frac{\pi_1-\pi_2}{A-B} \end{array}\right\} \tag{1.20}$$

and

$$\left.\begin{aligned} e_1^*(N_1,N_2) &= \frac{-\left[2(N_1+N_2)(r+\frac{N_2(\pi_1-\pi_2)}{A-B})-2(N_1+N_2-1)(A-B)\right]+\sqrt{\Delta(N_1,N_2,\pi_1,\pi_2,A,B,r)}}{2(N_1+N_2)^2} \\ e_2^*(N_1,N_2) &= e_1^*(N_1,N_2)+\tfrac{\pi_1-\pi_2}{A-B} \end{aligned}\right\} \quad (1.21)$$

where the $\Delta(N_1,N_2,\pi_1,\pi_2,A,B,r)$ is given by :

$$\begin{aligned} \Delta(N_1,N_2,\pi_1,\pi_2,A,B,r) = &\left\{2(N_1+N_2)(r+\frac{N_2(\pi_1-\pi_2)}{A-B})-2(N_1+N_2-1)(A-B))\right\}^2 \\ &-4(N_1+N_2)^2(\left[\left(r+\frac{N_2(\pi_1-\pi_2)}{A-B}\right)^2-2N_2(\pi_1-\pi_2)-r^2-2(Ar-\pi_1)\right]. \end{aligned}$$

In the equilibrium described by (1.19), only the poorly-endowed countries participate actively in the quest for hegemony, the richly-endowed countries being content enough with the return from their endowment relative to the return on winning the hegemony race so as not to participate actively. In the other two equilibria, both types of countries participate actively in the race, but the individual emissions of the poorly-endowed countries are always higher than those of the richly-endowed countries. Again, this reflects the greater eagerness of the poorly-endowed countries.

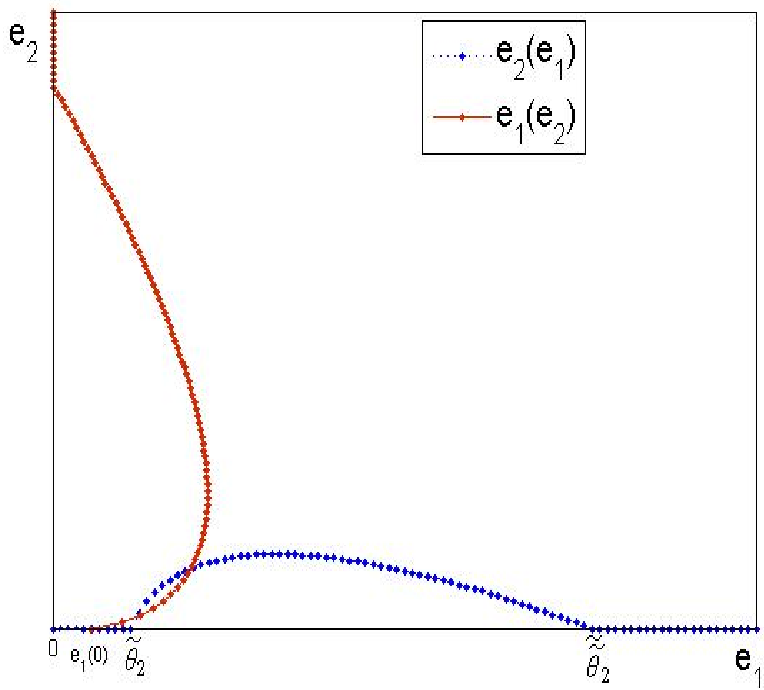

Figure 1.6 – Equilibria in a world composed of poorly-endowed and richly-endowed countries

The expression for $\Delta(N_1, N_2, \pi_1, \pi_2, A, B, r)$ is the discriminant of the second degree polynomial obtained by substituting the best response function of the richly-endowed countries into that of the poorly-endowed countries in order to solve for the equilibrium emissions of the latter. If it is strictly positive, thus eliminating complex roots, the three equilibria can coexist. This is depicted in Figure 1.6 for $N_1 = N_2 = 1$. The equilibrium given by (1.20) occurs in the increasing part of the two reaction functions, with the reaction function of the poorly-endowed country cutting that of the richly-endowed country from above, and is unstable. The other two equilibria are stable. As $\Delta(\cdot)$ is increased there comes a point where $e_1(0) > \widetilde{\theta}_2$. If $\Delta(\cdot)$ is such that $\widetilde{\theta}_2 < e_1(0) < \widetilde{\widetilde{\theta}}_2$, there is then a unique stable equilibrium with $e_1 > 0$ and $e_2 > 0$, characterized by (1.21). If $e_1(0) \geq \widetilde{\widetilde{\theta}}_2$, then the only equilibrium has $e_1 > 0$ and $e_2 = 0$, characterized by (1.19).

If $\Delta(\cdot)$ were negative, then, in the absence of the nonnegativity constraint on the emission rates, $e_1(e_2)$ would lie everywhere above and to the left of $e_2(e_1)$, there would be no intersection between the two best response curves and hence there would be no solution in real space. However, because of the nonnegativity constraint on e_2 the two best response functions intersect along the horizontal axis and there still exists in that case a unique stable equilibrium in real space, characterized by (1.19), with $0 < e_1 < \widetilde{\theta}_2$ and $e_2 = 0$.[11]

The equilibrium level of global emissions is given by $N_1 e_1^*(N_1, N_2) + N_2 e_2^*(N_1, N_2)$. Recall that the date at which the new hegemon is determined and the race to hegemony ends is $\tau = \min\{\tau_1, \ldots, \tau_N\}$. Substituting for $e_1^*(N_1, N_2)$ and $e_2^*(N_1, N_2)$ into (1.1), we find that in equilibrium the probability of reaching the finish line by time t, $P(\tau < t)$, is :

$$F(t) = 1 - e^{-(N_1 e_1^*(N_1,N_2) + N_2 e_2(N_1,N_2))t}.$$

It follows that the expected date at which the new hegemon is determined is given by :

$$E(\tau) = 1/(N_1 e_1^*(N_1, N_2) + N_2 e_2(N_1, N_2)).$$

Hence any change in the configuration of parameters (such as N_1, N_2, π_1 or π_2) which results in a greater equilibrium level of global pollution, will move the expected ending date of the hegemony race closer. This can be interpreted as saying that the more intense the race, the closer the expected date at which the race is won.

A number of sensitivity analyses are of particular interest. The first one consists in simply changing the distribution of countries between the two types, keeping the total number of countries constant. Numerical simulations indicate that increasing the number of richly-endowed countries while keeping N and all the other parameters except N_1 constant results in a monotonic decrease in global pollution in both of the stable equilibria.[12] This makes sense, since, as we get closer to $N_2 = N$, we get closer to the world of

11. If $\Delta(\cdot) = 0$ we have multiple real roots. The equilibria in (1.20) and in (1.21) then coincide at the tangency point of the two curves and the three equilibria reduce to two. The equilibrium given by (1.19) is then the only *stable* equilibrium.

12. All the numerical simulations are done for the interior stable equilibrium characterized by (1.21), in which both types of countries are polluting to begin with. This seems like the most realistic initial situation to consider.

richly-endowed countries described in Subsection 1.4.2. Similarly, if N_1 tends to N under the same conditions, the world converges towards one of poorly-endowed countries only, as described in Subsection 1.4.1, and global emissions increase as N_1 increases.

A second type of sensitivity analysis consist in reducing inequalities by improving the lot of the poorly-endowed countries without changing that of the richly-endowed countries. This might be thought of as measures exogenous to the model that result in improvements in the economic, political and social institutions of the poorly-endowed countries, for instance, or in their human capital. As can be seen from (1.19), (1.20) and (1.21) by letting π_1 tend to π_2 with π_2 fixed, this reduces the equilibrium level of global pollution. Indeed, as π_1 approaches π_2, $e_1^*(N_1,N_2)$ falls and approaches $e_2^*(N_1,N_2)$ and we move towards the equilibrium of a world composed only of richly-endowed countries. At the limit, if $\pi_1 = \pi_2 > rA$, then $e_1^*(N_1,N_2) = e_2^*(N_1,N_2))$ and the level of global pollution will be lower than when the two types of countries coexist with $\pi_1 < rA < \pi_2$, since then $e_1^*(N_1,N_2) > e_2^*(N_1,N_2))$.

Alternatively, we can consider redistributing from the richly-endowed countries towards the poorly-endowed countries by increasing π_1 while keeping constant the mean endowment $n_1\pi_1 + n_2\pi_2$ (where $n_i = N_i/N$) and keeping $\pi_1 < rA < \pi_2$, so that both types of countries continue to coexist. This forcibly means reducing π_2 accordingly. Numerical simulations show that this will result in an increase in the level of emissions of the richly-endowed countries, who become relatively more eager in the hegemony race, and a decrease in the level of emissions of the poorly-endowed countries, who become relatively less eager. But the richly-endowed countries' reaction to the fall in their endowment is stronger than that of the poorly-endowed countries to the increase in their endowment, with the overall result being a monotonic increase in global pollution. [13]

13. The starting point of the simulations is the interior stable equilibrium obtained for parameter values $A = 10$, $B = 3$, $r = 0.027$, $\pi_1 = 0.1$, $\pi_2 = 1$, $N = 100$. The simulations were done for various values of N_1 and N_2, and hence of $n_1\pi_1 + n_2\pi_2$. As long as π_1 is less than $rA = 0.27$, global pollution increases monotonically with π_1. When π_1 exceeds 0.27, all countries become richly-endowed, although unequally so as long as $\pi_1 \neq \pi_2$, and we would have an asymmetric equilibrium in a world of richly-endowed countries. Continuing to redistribute in this way from π_2 towards π_1 beyond this point will continue to increase pollution over some positive interval. But, if we push this redistribution far enough, at some point, if all countries feel sufficiently rich, they will drop out of the race and the world moves to a zero-emissions equilibrium. When this may happen will of course depend, among other things, on the values

Finally, if it were possible to increase π_1, π_2 and the mean endowment in the same proportion, global pollution would decrease monotonically as a function of that proportion : making the whole world better endowed, so that the hegemon's prize does not look as attractive, would result in a reduction in global emissions. The same can be said of a decrease in the hegemon's prize, A.

1.5 Concluding remarks

We have sought to analyze the consequences for global pollution of the quest for hegemony in a world in which economic strength, as measured by the level of economic output, drives this quest by increasing the probability of a country becoming the new hegemon. In doing so, we have differentiated between poorly-endowed and richly-endowed countries. The payoff from winning the hegemony race is more attractive to the poorly-endowed countries than to the richly-endowed countries. As a result they are more aggressive players in the quest for hegemony and end up being bigger polluters in equilibrium. The analysis however suggests ways in which global pollution might be reduced by acting to improve the lot the poorly-endowed countries without impacting directly on the richly-endowed. These would seem to rest on measures designed to improve the major factors that determine the return from their endowment, such as their human capital and their economic, social and political institutions.

In order to emphasize the role of the relative return from initial endowments, we have assumed that it is the only distinguishing factor between countries. In further analysis, one might want to explicitly take into account other distinguishing factors, such as the size of the countries, as measured by their population for instance. There is however no reason to believe that this would change the qualitative results of our analysis.

---

of (N_1, N_2) and of π_2.

CHAPITRE 2

ESTIMATION OF THE HOTELLING RULE FOR OIL AND FOR COAL UNDER STOCHASTIC INVESTMENT OPPORTUNITIES

2.1 Introduction

What role do *in situ* nonrenewable resource stocks play in a context of risk diversification in the long run ? This paper provides an empirical insight on this issue by using market capitalization of companies holding oil or coal in the ground to empirically investigate the optimal rule of extracting exhaustible resource stock derived in the resource economic model of Gaudet and Khadr (1991). Gaudet and Khadr's stochastic Hotelling rule provides a useful theoretical framework to understand the evolution of the return on holding a nonrenewable resource stock in a context of risk diversification. But from an empirical point of view, very few papers have looked at the holding of natural resource stocks from the perspective of risk diversification (Gaudet (2007)).

Slade and Thille (1997) is one of them. They modified the framework of Gaudet and Khadr (1991) and specified an econometric static approach that has the inconvenience of not taking into account the effects of time-varying investment opportunities in resource exploitation. They found a negative beta (β), the parameter measuring the riskiness of holding *in situ* resource stocks of copper, which suggests that copper stocks can be used to hedge against market risk. They proxy the *in situ* value of copper as the difference between price and the marginal extraction costs estimated by Young (1992).

In another study, Young and Ryan (1996) take a different approach for lead, zinc, copper and silver by using a time varying coefficients econometric specification. They proxy the value of a unit of the resource stock by the difference between the Canadian price of the metal and the average operating costs available for the aggregate mining divisions such as copper-gold-silver, nickel-copper-zinc, and silver-lead-zinc.

Also worth mentioning is Livernois et al. (2006) who investigate the riskiness of holding timberland by using the stumpage price bids as a direct measure of scarcity rent.

Finally, Roberts (2001) estimated Euler equations for eight natural resources assets (aluminum, coal, copper, crude oil, lead, natural gas, silver, and zinc) and found that hedging demand can explain why natural assets are held by investors. He proxies the *in situ* value of one unit of the resource stocks by using the gross price.

The proxy used for the scarcity rent in this paper is the difference between the growth rate of market capitalization of mining firms and that of proved reserves. [1] To justify this choice, we derive a formal relationship between the return on a unit of the resource in the ground and the stock market return of a mining company. We then use the market capitalization of mining companies and proved reserves in the empirical investigation of Gaudet and Khadr's stochastic Hotelling rule. The econometric approach used is closely related to that of Nowman (1997) for the estimation of stochastic diffusion processes.

The paper is organized as follows. Section 2 presents Gaudet and Khadr's stochastic Hotelling rule. Section 3 describes the econometric approach. Section 4 relates a firm's market capitalization to the return on its *in situ* stock of the natural resource. Section 4 presents the data, while Section 6 presents the results of the econometric estimation of the model for both oil and coal. The final section offers concluding comments.

2.2 Gaudet and Khadr's stochastic Hotelling rule

In this section we synthesize the main features and results of the Gaudet and Khadr (1991) model, which is the foundation for our empirical work. They assume a two good economy, one of which is a nonrenewable natural resource, whose stock at date t, $S(t)$, is irreversibly reduced by extraction. The other is a reproducible composite good, that can be either consumed or accumulated. If accumulated, it can be either in the form of physical capital, whose accumulated stock is denoted $K(t)$, or of a "bond". The accumulated stock of bonds is assumed to reproduce itself at an exogenously given risk free rate $r(t)$, which represents the force of interest.

Both the production of the composite good and the extraction of the natural resource are assumed to be stochastic. More precisely, if $y(t)$ denotes the flow of production of the

1. Miller and Upton (1985a) is the pioneering paper that relies on the market values of oil reserves to investigate the Hotelling rule. See also Miller and Upton (1985b)

composite good, $x(t)$ denotes the flow of extraction of the resource and $\theta_1(t)$ and $\theta_2(t)$ are two stochastic productivity indices, then the stochastic production and extraction processes are represented respectively by

$$y(t) = F(K_y(t), x(t), \theta_1(t)) \tag{2.1}$$

and

$$x(t) = G(K_x(t), \theta_2(t)), \tag{2.2}$$

where $K_y(t) + K_x(t) = K(t)$. The productivity indices θ_1 and θ_2 are assumed to evolve over time according to Itô processes of the form :

$$d\theta_i = \mu_i dt + \sigma_i \xi_i \sqrt{dt}, \quad i = 1, 2, \tag{2.3}$$

with $\xi_i \sim N(0,1)$, $cov(d\theta_1, d\theta_2) = \sigma_{12}dt + o(dt)$ and $\sigma_{12} = \sigma_1\sigma_2 cov(\xi_1, \xi_2)$. The drift μ_i and the variance σ_i can depend on time and on the state.

The representative consumer derives utility $U(c(t))$ from consuming the flow $c(t)$ of the composite good and is the ultimate owner of the stock of composite good and the stock of the natural resource. He chooses at each date his consumption and the allocation of his wealth between the resource stock and the accumulated composite good so as to maximize his discounted flow of future utility subject to his stochastic wealth constraint. His consumption and portfolio decisions generate consumption and asset demands, through which he sends price signals to the resource firms and the firms producing the composite good. The firms take those demand prices as given in making their extraction and production decisions, which in turn generate the asset returns that the consumer takes as given when establishing his opportunity set, as determined by his wealth constraint. The prices and asset returns are taken to be those that simultaneously equilibrate the markets and are shown to also follow Itô processes, whose drifts and variances are derived from the market equilibrium conditions.

Denote by $\lambda(t)$ the price of a unit of resource *in situ* and let $\mu_S(t) = \frac{1}{\lambda(t)}\frac{1}{dt}E_t(d\lambda(t))$. This is the expected instantaneous rate of return on the resource stock being held in the ground. Gaudet and Khadr show that, in equilibrium,

$$\mu_S(t) = r(t) + A(c(t))\sigma_{Sc}, \tag{2.4}$$

where $A(c) = -U''c/U'$ is the Arrow-Pratt measure of relative risk aversion and $\sigma_{Sc} = cov(d\lambda/\lambda, dc/c)$.

This is a consumption-beta formulation of the Hotelling rule. Notice that if $U'' = 0$, then this intertemporal arbitrage condition simply says that the expected rate of change in the value of a unit of *in situ* resource must equal the rate of interest. If $U'' < 0$, which we will assume, then $A(c(t))$ is positive and the consumer is risk averse. Then, whether the expected rate of change in the price of the *in situ* resource (i.e. the expected rate of return on the resource) is greater or smaller than the rate of interest depends on whether the observed rate of change in the price of reserves, $d\lambda/\lambda$, is positively or negatively correlated to the rate of change of consumption, dc/c. If positively correlated, then the resource is considered a risky asset and its expected return will exceed the rate of interest in equilibrium. If negatively correlated, the resource is considered non risky and the excess return will be negative, since holding the resource then constitutes an insurance against adverse changes in consumption.

If we further denote by $\mu_M(t)$ the expected rate of return on any market portfolio whose observed return has a nonzero covariance with changes in consumption, then, in equilibrium, we will also have

$$\mu_M(t) = r(t) + A(c(t))\sigma_{Mc}. \tag{2.5}$$

From (2.4) and (2.5) it follows that

$$\mu_S(t) - r(t) = \beta(t)[\mu_M(t) - r(t)], \tag{2.6}$$

where $\beta = \sigma_{Sc}/\sigma_{Mc}$ is the so-called “beta coefficient” associated to the resource stock as an asset. Thus, depending on whether $\beta(t)$ is positive or negative, the expected excess return on holding the resource will be positive or negative, meaning that the expected rate of change in the *in situ* price of the resource will be greater or smaller than the rate of interest. This is the formulation of the Hotelling rule which is retained for the empirical investigation that follows.

2.3 An econometric specification of the stochastic Hotelling rule

In what follows we borrow from recent financial econometrics methods to estimate the above stochastic Hotelling rule.

We want to estimate the time varying coefficient $\beta(t)$, which is a measure of the sensitivity of the return on the resource stock to the return on the chosen "market" portfolio. It will be an indicator of that part of the variance on the return on the resource stock that cannot be diversified away by investing in the chosen portfolio of assets.

If the values of $\mu_S(t)$ and $\mu_M(t)$ were known, then, given $r(t)$, the value of $\beta(t)$ could be easily computed by noting that :

$$\beta(t) = \frac{\mu_S(t) - r(t)}{\mu_M(t) - r(t)}. \tag{2.7}$$

Thus estimated values $\mu_S(t)$ and $\mu_M(t)$ will provide us with estimated values of $\beta(t)$.

The rate of return on holding a stock of the resource in the ground is given by the rate of change in its *in situ* price, $d\lambda/\lambda$. It is shown in Gaudet and Khadr (1991) that, given (2.3), it will in equilibrium follow an Itô process of the form :

$$\frac{d\lambda(t)}{\lambda(t)} = \mu_S dt + \sigma_S \xi_S \sqrt{dt}, \tag{2.8}$$

where μ_S, σ_S and $\xi_S \sim N(0,1)$ are equilibrium values which will depend on the parameters, including μ_i, σ_i and ξ_i $(i = 1,2)$ in (2.3). Notice that those equilibrium values will fluctuate randomly over time, even if μ_i, σ_i were not functions of θ_i, since they will depend on ξ_i.

In order to specify an estimable form, it is useful to reformulate (2.8) in terms of the cumulative return on the *in situ* resource stock, $R^S(t) = \int_0^t d\lambda(s)/\lambda(s)$. We can use its infinitesimal variation $dR^S(t) = d\lambda(t)/\lambda(t)$ to rewrite (2.8) as

$$dR^S(t) = \mu_S(t, R^S(t))dt + \sigma_S(t, R^S(t))d\zeta_S(t), \tag{2.9}$$

where the drift and the variance coefficients are now expressed explicitly in terms of time and of $R^S(t)$ and where $d\zeta_S(t) = \xi_S(t)\sqrt{t}$, with $\xi_S \sim N(0,1)$.

Similarly, the stochastic process for the cumulative return on the market portfolio, $R^M(t)$, is given by :

$$dR^M(t) = \mu_M(t, R^M(t))dt + \sigma_M(t, R^M(t))d\zeta_M(t). \tag{2.10}$$

The commonly used strategy to estimate this stochastic process consists first in specifying a functional form for both the drift and the diffusion coefficients and, second, in discretizing the stochastic process in order to estimate the parameters from observed data, which are typically recorded discretely over a certain time interval [0,T].

Consequently, we consider a specific class of continuous time diffusion processes known as the CKLS model (see Chan et al. 1992) and defined as follows :

$$dR(t) = (\alpha_1 + \alpha_2 R(t))dt + \sigma R(t)^{\alpha_3} d\zeta(t). \tag{2.11}$$

This model therefore adopts a linear function of time for the drift and a power function for the volatility. Equation (2.11) allows the conditional mean and variance to depend on the level of $R(t)$, which is the cumulative return on the stock of the resource. The parameters α_1 and α_2 are the unknown drift and mean reversion structural parameters, σ is the volatility parameter and α_3 is the proportional conditional volatility exponent.

Constraining some parameters of the CKLS form to take specific values leads to some well known models, which are summarized in the following table :

	Model	α_1	α_2	α_3	σ
Unconstrained (CKLS)	$dR(t) = (\alpha_1 + \alpha_2 R(t))dt + \sigma R(t)^{\alpha_3} d\zeta(t)$				
Brennan and Schwartz (1980)	$dR(t) = (\alpha_1 + \alpha_2 R(t))dt + \sigma R(t) d\zeta(t)$			1	
Constant Elasticity of Variance	$dR(t) = \alpha_2 R(t)dt + \sigma R(t)^{\alpha_3} d\zeta(t)$	0			
Ornstein-Uhlenbeck process	$dR(t) = \alpha_2 R(t)dt + \sigma d\zeta(t)$	0		0	

The general CKLS specification in (2.11) facilitates the construction of a discrete time version. The discrete approximation of the continuous time diffusion process developed by Nowman (1997) and based on some results found in Bergstrom (1983) is given by :

$$R_t = e^{\alpha_2} R_{t-1} + \frac{\alpha_1}{\alpha_2}(e^{\alpha_2} - 1) + \eta_t, \tag{2.12}$$

where the error term η_t satisfies :

$$E(\eta_s \eta_t) = \begin{cases} 0 & \text{if } s \neq t \\ \dfrac{\sigma^2}{2\alpha_2}(e^{2\alpha_2} - 1) R_{t-1}^{2\alpha_3} & \text{if } s = t. \end{cases}$$

Define $L(\alpha_1, \alpha_2, \alpha_3, \sigma)$ as the logarithm of the likelihood function of the model :

$$L(\alpha_1, \alpha_2, \alpha_3, \sigma) = \sum_{t=1}^{T} \left[\log E(\eta_t^2) + \frac{(R_t - e^{\alpha_2} R_{t-1} - \frac{\alpha_1}{\alpha_2}(e^{\alpha_2} - 1))^2}{E(\eta_t^2)} \right] + constant\, term$$

with $E(\eta_t^2)$ as defined above. Maximum Likelihood Estimation consists in solving for

$$(\widehat{\alpha_1}, \widehat{\alpha_2}, \widehat{\alpha_3}, \widehat{\sigma}) = \underset{\alpha_1, \alpha_2, \alpha_3, \sigma}{\operatorname{argmax}} \ L(\alpha_1, \alpha_2, \alpha_3, \sigma)$$

to get estimates of the unknown parameters in the drift and diffusion coefficients. The method is a sort of quasi-maximum method since (2.12) is not the true discrete model corresponding to equation (2.11), but is merely a conditional Gaussian approximation. In so doing, this method replaces a non-Gaussian process by an approximate Gaussian one.

The estimated drift of the return on the natural resource is then given by

$$\widehat{\mu_S(t)} = \widehat{\alpha_1^S} + \widehat{\alpha_2^S} R^S(t),$$

while that on the return on the market portfolio is given by

$$\widehat{\mu_M(t)} = \widehat{\alpha_1^M} + \widehat{\alpha_2^M} R^M(t).$$

The estimated "short-run" beta can then be computed as :

$$\widehat{\beta(t)} = \frac{\widehat{\mu_S(t)} - r(t)}{\widehat{\mu_M(t)} - r(t)}, \tag{2.13}$$

where $r(t)$ is the risk free rate.

We define a "long-run" beta over the period [0,T] as :

$$\overline{\beta} = \frac{\left[\frac{\sum_{t=1}^{T}(\mu_S(t) - r(t))}{T}\right]}{\left[\frac{\sum_{t=1}^{T}(\mu_M(t) - r(t))}{T}\right]}. \tag{2.14}$$

Notice that if we substitute for $\mu_S(t) - r(t)$ from (2.6), $\overline{\beta}$ can be rewritten in the following way :

$$\bar{\beta} = \frac{\sum_{t=1}^{T} \beta_t \left[\mu_M(t) - r(t)\right]}{\sum_{t=1}^{T} \left[\mu_M(t) - r(t)\right]} \tag{2.15}$$

which is a weighted average of the betas over the interval $[0, T]$, with the weights given by $[\mu_M(t) - r(t)] / \sum_{t=1}^{T} [\mu_M(t) - r(t)]$.

A point estimate of the long-run beta (2.15) is then given by

$$\widehat{\bar{\beta}} = \frac{\sum_{t=1}^{T} \widehat{\beta(t)} \left[\widehat{\mu_M}(t) - r(t)\right]}{\sum_{t=1}^{T} \left[\widehat{\mu_M}(t) - r(t)\right]}. \tag{2.16}$$

In what follows, we adopt an approach similar to that of Lucas (1978); Breeden (1979); Hansen and Singleton (1983); Mankiw and Shapiro (1986), who use the consumption growth risk (or consumption beta) to measure the riskiness of an asset. The betas will be measured relative to consumption growth rather than the market portfolio index. This is analogous to the equations (2.6), (2.13), (2.14) with the expected return on the market portfolio μ_M replaced by the expected rate of growth in consumption μ_C. In this case the beta is the conditional covariance between the return on the resource asset and the consumption growth. A nonrenewable resource asset is more risky if it pays less when consumption is low.

2.4 Market capitalization and the rate of return on the *in-situ* resource stocks

One way for individual investors to invest in nonrenewable reserves (such as oil or coal) is to buy stocks in the companies that hold those reserves. However, at any moment of time, the stock market value of those companies will reflect their holdings not only of *in situ* stocks of the nonrenewable resource, but also of some reproducible assets. Let $K(t)$ represent the stock of capital held at date t in the form of those other assets. If we denote by $\nu(t)$ the value of a unit of that capital, then, if the company owns reserves in the quantity $S(t)$, valued at $\lambda(t)$, the company's market capitalization can be written :

$$\begin{aligned} \Pi(t) &= \lambda(t)S(t) + \nu(t)K(t) && (2.17) \\ &= [\lambda(t) + \gamma(t)\nu(t)]\, S(t), && (2.18) \end{aligned}$$

where $\gamma(t) = K(t)/S(t)$ is the ratio of fixed capital to reserves.

In the absence of reasonable measures of the evolution over time of $\gamma(t)$ and $\nu(t)$ we will assume them to be constant. It follows that :

$$\frac{d\Pi(t)}{\Pi(t)} = \frac{d\lambda(t)}{\lambda(t)}\left[\frac{\lambda(t)}{\lambda(t)+\gamma\nu}\right] + \frac{dS(t)}{S(t)} \tag{2.19}$$

and hence :

$$dR^S(t) = \frac{d\lambda(t)}{\lambda(t)} = \left(1 + \frac{\gamma\nu}{\lambda(t)}\right)\left(\frac{d\Pi(t)}{\Pi(t)} - \frac{dS(t)}{S(t)}\right). \tag{2.20}$$

The return on the resource stock is therefore proportional to the difference between the rate of change in the company's capitalization and the rate of change in its reserves. The factor of proportionality $(1+(\gamma\nu)/\lambda(t))$ is a decreasing function of $\lambda(t)$, but is always positive. In the absence of observations on $\lambda(t)$, it therefore seems reasonable to use

$$\frac{d\Pi(t)}{\Pi(t)} - \frac{dS(t)}{S(t)} \tag{2.21}$$

as a proxy for the return on the resource stock for estimation purposes, since they both must move in the same direction. This is what will done in the estimations that follow.

2.5 The data

2.5.1 Oil

The data used to compute the proxy (2.21) for oil are presented in this subsection. We need data on market capitalization and on reserves. The data on market capitalizations comes from the Center for Research in Security Prices (CRSP) database, on a monthly basis for the period 1959(2)-2006(12). [2] The data on reserves comes from the BP Statistics web site, on a yearly basis for the period 1959-2006. [3]

The proxy (2.21) for the change in the cumulative return of the oil reserves at date t can be computed in discrete time as follows :

$$\Delta R_{\tau}^{oil} = \left[\frac{\sum_{n=1}^{N_{\tau}} X_{n,\tau}^{oil}}{\sum_{n=1}^{N_{\tau-1}} X_{n,\tau-1}^{oil}} - 1\right] - \left[\frac{S_{\tau}}{S_{\tau-1}} - 1\right] \tag{2.22}$$

where $\tau \in \{1959(2),...,2006(12)\}$, $X_{n,\tau}^{oil}$ is the market capitalization of oil company n at date τ and N_{τ} the total number of companies at that date. The number N_{τ} changes over time as the number of companies present on the stock exchange changes. The number of companies in the sample retained for this study varies between 5 and 56 over the period of observation. [4] S_{τ} is the level of world proven oil reserves at date τ. The first term of

2. The CRSP (http ://www.crsp.com) provides a comprehensive database on NYSE, AMEX and NASDAQ stock markets.
3. See www.bp.com
4. A change in the number of companies can be because of the split of an existing company, the entry

this formula reflects the rate of change in the market capitalization of the oil sector and the second term reflects the change in the proven oil reserve.

The 56 oil companies that were at one time or another part of the sample are listed in Table 1 of the Appendix I.1.[5] They include some of the biggest companies owning oil reserves : 39 of them are listed in the top 100 World Oil Companies.[6] The other seventeen are companies in the CRSP database that have an important current market capitalization and are quoted on US stock exchanges for at least 5 years. Figure 1 shows the evolution of the first term of (2.22), which is the monthly growth rate in the index of market capitalization of the oil companies.

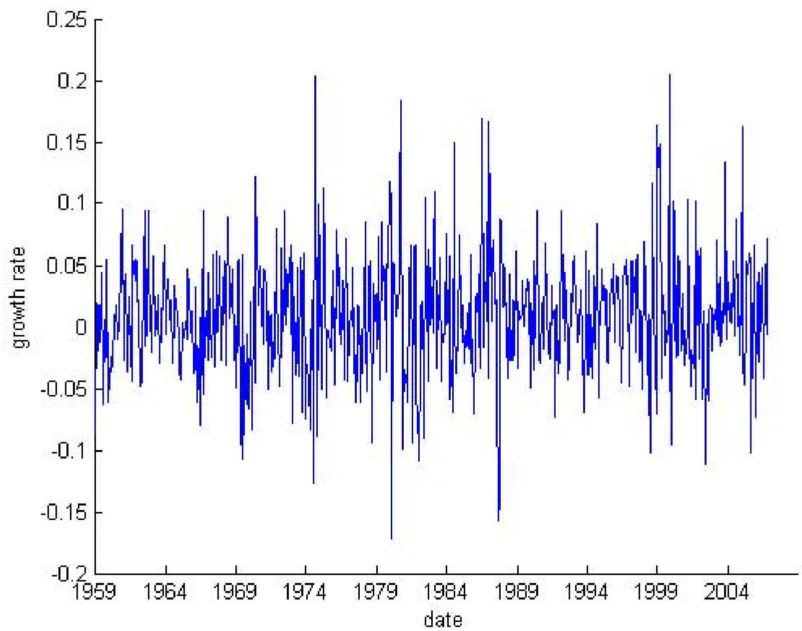

Figure 2.1 – Growth rate of the market capitalization of oil companies

of a new company, the merger of two existing companies or the exit of an existing company. Taking into account all of those changes assures the coherence of the index over time.

5. All of the 56 companies are classified under US Department of Labor SIC number 1311 (Crude petroleum and natural gas).

6. See Energy Intelligence (2006), "The Energy Intelligence Top 100 : Ranking the World's Oil Companies" ($http : //www.energyintel.com/publicationdetail.asp?publication_id = 124$). The 61 other companies in the top 100 were not listed on the US stock exchange, which is a prerequisite for being retained in our sample.

Due to unavailability of data on oil reserves held by the companies in the sample over a sufficiently long period of time, we use data on world's oil proven reserves as its proxy. [7]

The data on world proven oil reserves are from the web site of BP Statistics. Since it is annual in frequency and the market capitalization data is monthly, we need to compute the monthly equivalent of the annual proven reserves. To do this we use an interpolation technique for deriving a monthly series from annual data (see Lisman and Sandee (1964); Boot et al. (1967)). The method generates estimated monthly reserves that are consistent with the annual series on proven reserves. It generates the estimated monthly reserves series by solving the following constrained quadratic minimization problem :

$$\min \sum_{i=2}^{12k} \left(s_i - s_{i-1}\right)^2$$
$$s.t. \sum_{i=12k-11}^{12k} s_i = S_t,\ (t = 1,2,..,k) \tag{2.23}$$

where k is the number of annual observations, S is the annual proven reserve, and s is the estimated monthly proven reserve. The result is shown in Figure 2.

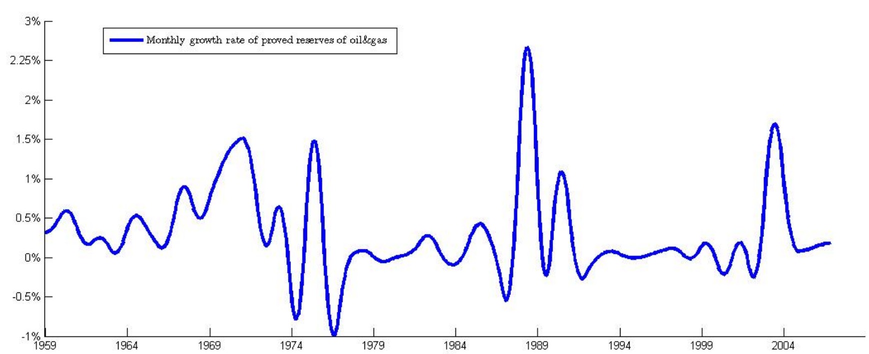

Figure 2.2 – Growth rate of the oil world proved reserves

7. There could be a positive correlation between the evolution of the world's oil proven reserves and the evolution of the total oil proven reserves held by these firms. Thirty nine companies in the sample are listed in the top 100 largest operators that account for more that 85% of world's oil proven reserves.

The proxy for the cumulative return on the reserves can then be calculated as :

$$R_t^{oil} = \sum_{\tau=0}^{t} \Delta R_\tau^{oil} + R_0^{oil} \tag{2.24}$$

where ΔR_τ^{oil} is given by equation (2.22). Figure 3 plots the evolution over time of this proxy for the cumulative return on oil reserves.

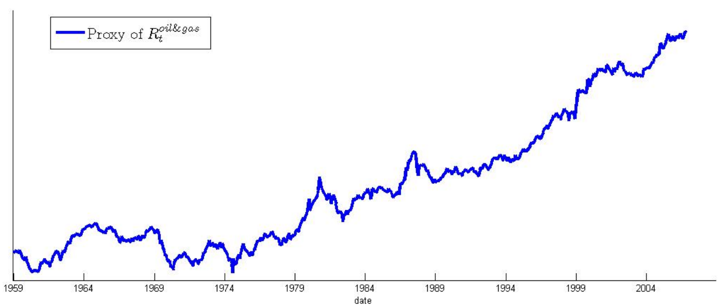

Figure 2.3 – Proxy of the cumulative returns of the oil in the ground (R_t^{oil})

2.5.2 Coal

The CRSP database contains nine coal companies extracting coal and which are listed on the US Stock Exchange. We have constructed the growth rate of market capitalization of mining companies engaged in the coal exploitation and which are in the CRSP data base.

The proxy (2.21) for the change in the cumulative return on *in situ* coal at date t can be computed in discrete time using the following formula :

$$\Delta R_\tau^{coal} = \left[\frac{\sum_{n=1}^{N_\tau} X_{n,\tau}^{Coal}}{\sum_{n=1}^{N_{\tau-1}} X_{n,\tau-1}^{Coal}} - 1 \right] - \left[\frac{S_\tau}{S_{\tau-1}} - 1 \right]$$

where $\tau \in \{1986(2), ..., 2006(12)\}$, where $X_{n,\tau}^{coal}$ denotes market capitalization of the coal mining company n at date τ and N_τ the number of companies changes over time to take into account the number of coal companies present on stock exchange $(1 \leq N_t \leq 9$).

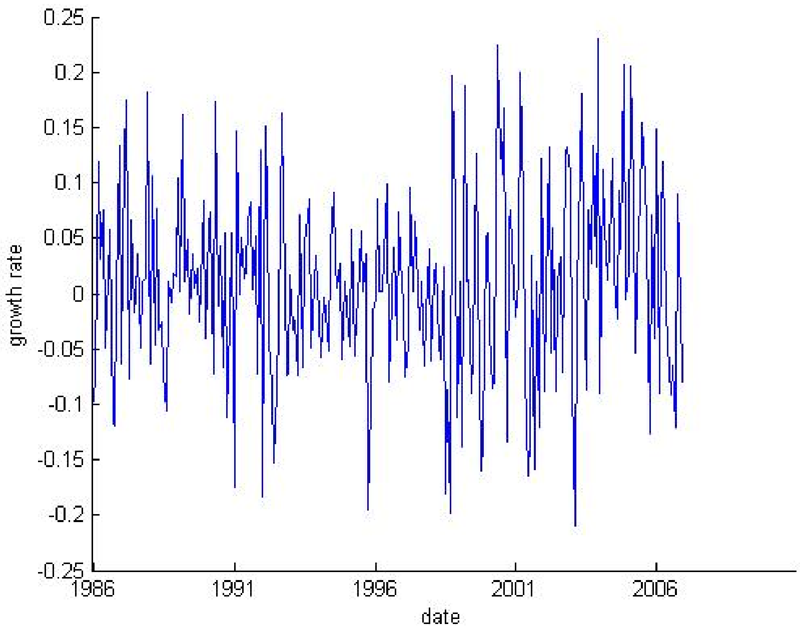

Figure 2.4 – Growth rate of the market capitalization of coal companies

Data on world proved reserves of coal cover the period 1986 to 2006 and are taken from the web site of BP statistics. These are annual which were converted to monthly data, using the same technique as for oil reserves.

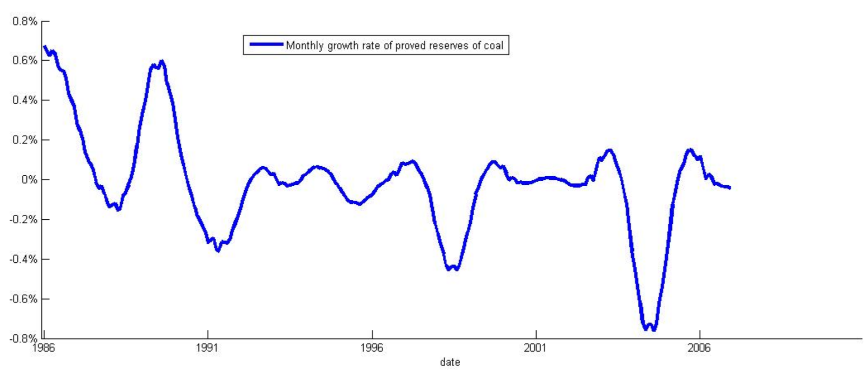

Figure 2.5 – Growth rate of the coal world proved reserves

The cumulative return of the resource in the ground is given by

$$R_t^{coal} = \sum_{\tau=0}^{t} \Delta R_\tau^{coal} + R_0^{coal} \tag{2.25}$$

Using this proxy, the following graphics give an idea of the evolution of the cumulative return of the coal in the ground

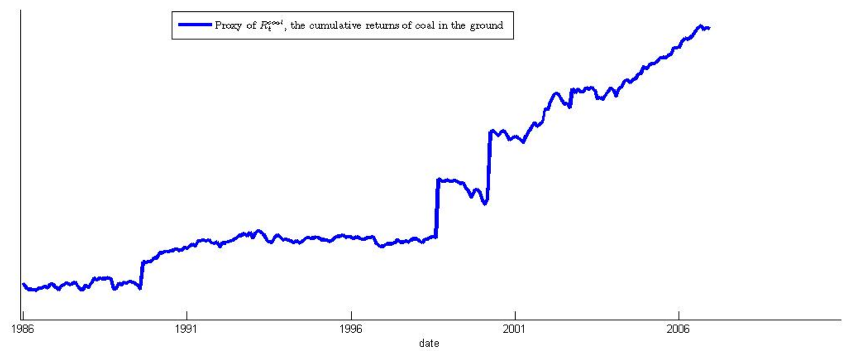

Figure 2.6 – Proxy of the cumulative returns of the coal in the ground (R_t^{coal})

2.5.3 The risk free rate

For the risk-free rate of interest, we use the monthly equivalent of the annual rate on the one-month U.S. Treasury bill, taken from the CRSP database. The data ranges from February 1959 to December 2006.

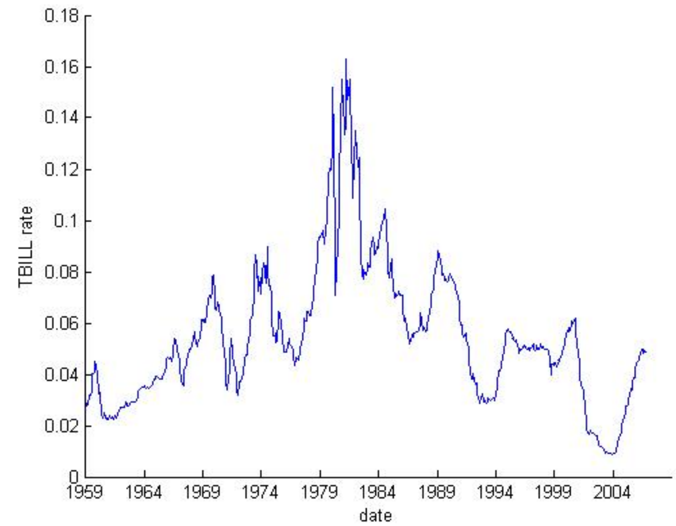

Figure 2.7 – One-month U.S Treasury bill

2.5.3.1 Consumption

Data on consumption are from the St. Louis Federal Reserve Bank and cover the period February 1959 to December 2006. We used monthly per capita consumption of services and non-durable goods.

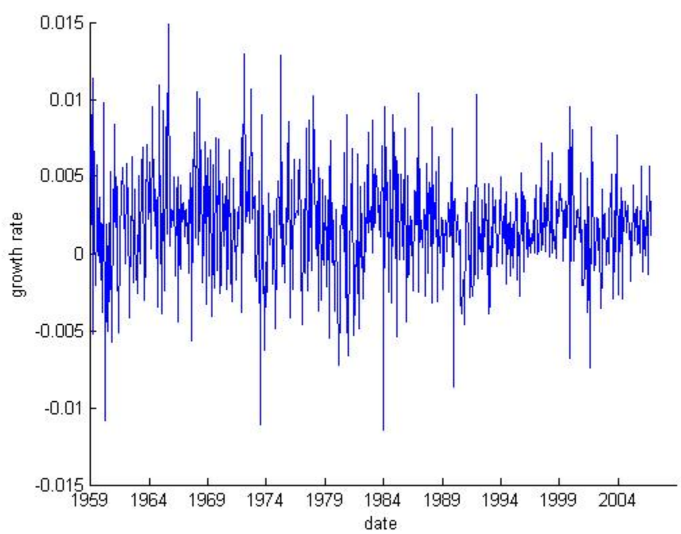

Figure 2.8 – Growth rate of per-capita consumption

2.6 Estimating the stochastic Hotelling rule

2.6.1 The short-run betas and the long-run beta for the *in situ* oil asset

The estimation of the stochastic process for the returns on the *in situ* oil asset was conducted using different specifications of the CKLS models, but only the Brennan and Schwartz (1980) model $\left(\alpha_3 = 1\right)$, given by :

$$dR(t) = (\alpha_1 + \alpha_2 R(t))dt + \sigma R(t) d\zeta(t) \tag{2.26}$$

performed well. [8] The estimations are presented in the following table :

(2.27)

$\widehat{\alpha_1^{oil}}$	$\widehat{\alpha_2^{oil}}$	$\ln(\widehat{\sigma^{oil}})$
0.0530	0.6467	2.4235
(4.7095)	(1.7278)	(10.5996)

All the t-student's coefficients in brackets suggest statistical significance at 10% level.

Suppose that the logarithm of consumption also follows the following stochastic process :

$$d\log(c(t)) = \mu(t, \log(c(t))dt + \sigma(t, \log(c(t)))d\zeta(t), \tag{2.28}$$

where $\mu(.)$ and $\sigma()$ are two scalars measuring respectively the mean and standard deviation of the change in log of consumption.

We estimated the stochastic process for the rate of growth of consumption using different specifications of the CKLS models, but only the Geometric brownian model

$$dR^c(t) = (\alpha_2 R^c(t))dt + \sigma R^c(t)^{\alpha_3} d\zeta(t) \tag{2.29}$$

performed well. The parameters are estimated from 48 years of monthly data (575 observations). The results of the estimation of the diffusion parameters of the rate of growth

8. Among the unrestricted model and the models obtained from imposing the restrictions described at page 6, it is the specification which provides the greatest number of statistically significant coefficients including the second parameter α_2

of consumption are reported in the table below. Estimated t-values are shown in parentheses.

$$\begin{array}{ccc} \widehat{\alpha_2^c} & \widehat{\alpha_3^c} & \ln(\widehat{\sigma^c}) \\ \hline 0.0006 & -1.4738 & -6.2806 \\ \hline (11.7793) & (-5.2675) & (-129.5582) \end{array} \tag{2.30}$$

The estimated beta coefficients are given by :

$$\widehat{\beta}^{oil,C}(t) = \frac{\widehat{\alpha_1^{oil}} + \widehat{\alpha_2^{oil}} R^{oil}(t) - r(t)}{\widehat{\alpha_2^c} R^c(t) - r(t)} \tag{2.31}$$

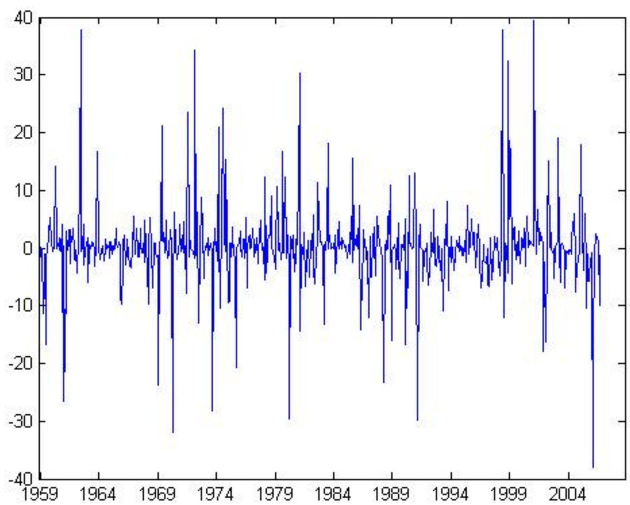

Figure 2.9 – Short-run beta for *in situ* oil over the period 1959(2) to 2006(12)

From the expression (2.16), it follows that the estimated value of the long-run beta is

$$\widehat{\widehat{\beta}}_{oil} = \frac{\sum_{t=1}^{T} \widehat{\beta}^{oil,C}(t) \left[\widehat{\alpha_2^c} R^c(t) - r(t)\right]}{\sum_{t=1}^{T} \left[\widehat{\alpha_2^c} R^c(t) - r(t)\right]} = -3.54 < 0. \tag{2.32}$$

In order to give a confidence interval to the long run beta (2.15) which is a ratio parameter. Building a confidence interval for a parameter ratio is not easy due to the problem of the overdiversification of the model. The denominator can be equal to zero for example, causing the interval to be unbounded. We follow the Bolduc et al. (2007) approach to build the confidence interval for long run beta. They use a procedure which combines the parametric bootstrap, the Fieller method, and the Delta Method. See Appendix I.2 for more details on this procedure. Using this procedure, it follows that the 95% Bootstrap Fieller confidence interval for the long-run beta $\widehat{\overline{\beta}}_{oil}$ is $FC = [-\infty, +\infty]$. It appears the Fieller confidence interval which is unbounded is less informative than the 95% Bootstrap Delta confidence interval for the long-run beta $\widehat{\overline{\beta}}_{oil}$ which is

$$DC = [-4.8; 0.6512].$$

This result suggests that oil in the ground is an asset providing insurance against fluctuations in consumption in the long run. Investing in oil in the ground is a good hedge against the market risk in the long run.

2.6.2 The short-run betas and long-run beta for the *in situ* coal asset

The estimation of the stochastic process for the return on the *in situ* coal asset was conducted using different specifications of the CKLS models but only the Ornstein-Uhlenbeck process $\left(\alpha_1 = 0, \alpha_3 = 0\right)$ given by

$$dR(t) = \alpha_2 R(t)dt + \sigma d\zeta(t) \tag{2.33}$$

performed well.

The parameters are estimated using 22 years of monthly data (253 observations). The results of the estimation of the diffusion parameters of the cumulative return on coal in the ground are reported in following table :

$\widehat{\alpha_2^{Coal}}$	$\ln(\widehat{\sigma^{Coal}})$
0.0040	-1.5366
(2.4661)	(-34.4313)

(2.34)

The estimations of the stochastic process for the consumption growth rate was conducted using different specifications of the CKLS models, but only the Ornstein-Uhlenbeck process given by

$$dR^c(t)) = \alpha_2 R^c(t)dt + \sigma d\zeta(t) \tag{2.35}$$

performed well.[9] The estimations are presented in the following table. T-students's values are in brackets.

$$\begin{array}{cc} \widehat{\alpha_2^c} & \ln(\widehat{\sigma^c}) \\ \hline 0.0005 & -6.7085 \\ \hline (8.7577\ ) & (-149.8206) \end{array} \tag{2.36}$$

The expression of estimated short-run betas are given by :

$$\widehat{\beta}^{coal,C}(t) = \frac{\widehat{\alpha_2^{coal}}\, R^{coal}(t) - r(t)}{\widehat{\alpha_2^c}\, R^c(t) - r(t)}. \tag{2.37}$$

From expression (2.16), it follows that the estimated value of the long-run beta is

$$\widehat{\overline{\beta}}_{coal} = \frac{\sum_{t=1}^T \widehat{\beta}^{coal,C}(t) \left[\widehat{\alpha_2^c} R^c(t) - r(t)\right]}{\sum_{t=1}^T \left[\widehat{\alpha_2^c} R^c(t) - r(t)\right]} = -2.0844 < 0. \tag{2.38}$$

9. The specification that performed best is different than in the case of oil because the period of observation is different.

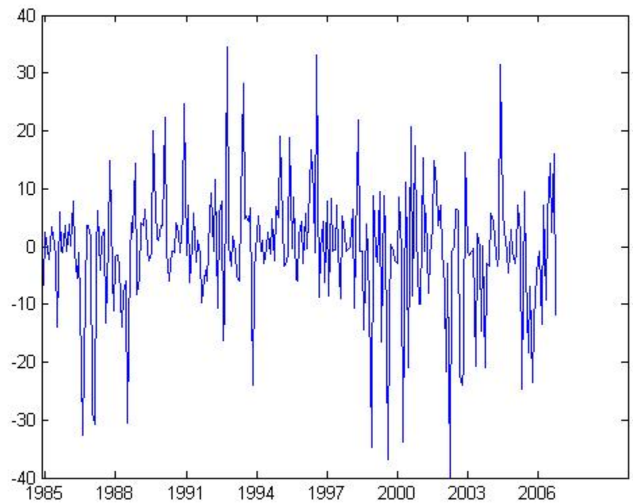

Figure 2.10 – Short-run beta for *in situ* coal over the period 1985(2) to 2006(12)

The 95% Bootstrap Fieller confidence interval for the long-run beta $\widehat{\overline{\beta}}_{coal}$ obtained from the the procedure described in the Appendix I.2 is not informative. But the 95% Bootstrap Delta confidence interval for the long-run beta $\widehat{\overline{\beta}}_{coal}$ is bounded and yields

$$DC = [-4.26; 1.051].$$

As for oil, the point estimation of the long-run beta coefficient is negative, which suggests that, as for oil, coal in the ground as an asset can constitute an insurance against long-run fluctuations in consumption.

From these results, oil in the ground or coal in the ground asset can be used to hedge against risk in the long run. A long-run investor wanting to make a long-term strategy of hedging against risk may include them in his portfolio.

2.7 Concluding comments

Building on the natural resource economic model of Gaudet and Khadr (1991), this paper has investigated empirically the role of nonrenewable natural resource stocks in a context of risk diversification. In doing so, we have formally shown how the market capitalization of mining companies and the world proved reserves can be used in the construction of a proxy for the return on holding reserves of natural resources. We have estimated a stochastic version of the Hotelling rule of exhaustible resource exploitation and used it to infer on the riskiness of investment in oil and coal reserves. The empirical results indicate that investing in oil or coal reserves is a good hedge against risk in the long run. The negative long-run beta coefficients mean that the return on holding oil or coal as assets tends to vary inversely with changes in the market, as captured by the relative change in consumption. In that sense they are both non risky assets relative to the market and constitute a form of insurance against adverse changes changes in consumption.

CHAPITRE 3

FUTURE PROSPECTS AND THE MANAGEMENT OF NATURAL RESOURCE ASSETS

3.1 Introduction

The aim of this paper is to study the evolution of exhaustible natural resource prices using a continuous time setting that merges the stochastic differential utility framework of Duffie and Epstein (1992b) and the natural resource model of Gaudet and Khadr (1991). In doing so, this paper highlights the role played in determining the equilibrium rate of depletion of a nonrenewable natural resource stock by an endogenous risk-adjusted and state dependent discount rate and by another factor related to the growth rate of future utility.

Using a standard linear additive intertemporal utility framework in a model with extraction, production, and consumption under stochastic investments opportunities, Gaudet and Khadr (1991) showed that the pricing of a nonrenewable resource stock must take into account an insurance factor against adverse changes in current consumption. But despite its usefulness in tackling many decisions under uncertainty, the standard additive utility framework does not allow the consumer to care about future prospects. Other well-known weaknesses of the standard additive utility, such as its inability to disentangle risk aversion and intertemporal substitution, and some related consequences in natural resource management have also received attention, including in Shaw and Woodward (2008), Howitt et al. (2005), Knapp and Olson (1996), Peltola and Knapp (2001) and Epaulard and Pommeret (2003).

A continuous time framework allowing consumer preferences to account for future utility is the stochastic differential utility. The future utility index is made a function of the distribution of future consumption ; as noted by Duffie and Epstein (1992a, p.425), a larger future utility index represents an increase in future prospects. The stochastic differential utility framework captures the notion that one's present sense of well-being can depend on one's expected future utility levels in a not necessarily risk-neutral manner. The stochastic differential utility is the continuous time version of the discrete time recursive preferences of Epstein and Zin (1989) and it includes the standard additive utility as a special case.

A convenient way to understand how future utility affects the intertemporal decision making process is to analyze the growth rate of marginal utility (similar to the intertemporal marginal rate of substitution in discrete time). With a standard utility, the growth rate of marginal utility depends only on the current consumption growth. The only source of risk is associated with shocks to current consumption growth. With a stochastic differential utility, the growth rate of marginal utility depends on both the growth rate of consumption and the growth rate of the future utility index. The two sources of risk are related to the current consumption growth and the growth rate of the future utility index.

These findings may have implications in natural resource economics since exhaustible natural resources are used as energy sources and as inputs in the production process of consumption goods and services. Holding nonrenewable resource stock can be risky not only regarding the growth of current consumption but also regarding future prospects as represented by the future utility index.

To investigate these aspects related to future prospects in analyzing the price path of an exhaustible resource stock, this paper merges two ideas : the stochastic differential utility framework by Duffie and Epstein (1992b) and the Gaudet and Khadr (1991) framework, where the arrival of the information on the state of the economy is governed by two stochastic productivity processes. A more general formulation of the Hotelling rule is derived which includes the insurance factor found by Gaudet and Khadr (1991), plus a new factor related to the role of future prospects and an endogenous discount rate. It is shown how future prospects are a determinant of the market price of the resource above ground. It is also shown that, contrary to the case of a reproducible asset, an endogenous discount rate plays a crucial role in determining the pricing of a nonrenewable resource stock.

The paper is structured as follows. Section 2 presents the main features of the stochastic differential utility. Section 3 presents the model. Section 4 derives the general formulation of the Hotelling rule and discusses it. The final section offers concluding comments.

3.2 Stochastic differential utility

The stochastic differential utility is the extension of the notion of recursive utility in a continuous time setting. In this section, I will present its main features.

The stochastic differential utility is identified by a pair of functions (f^*, A^*) called an aggregator, where A^* is local risk aversion and f^* represents the relative preference between immediate consumption and the certainty equivalent of utility derived from future consumption. It determines the degree of intertemporal substitution of consumption and also generates collateral risk attitudes under uncertainty. Given $f^*(\cdot)$, risk attitudes are encapsulated by the function $A^*(\cdot)$ which has no effect on intertemporal substitution. The more negative is $A^*(\cdot)$, the more risk averse is the agent. For a given consumption process, denote by $\mathscr{J}^*(t)$ the continuation utility attributable to future consumption streams given the current information information available at date t. [1] The continuation utility is defined recursively by the following stochastic integral equation :

$$\mathscr{J}^*(t) = E_t\left[\int_t^{\infty}\left(f^*[c(s), \mathscr{J}(s)] + \frac{A^*[\mathscr{J}^*(s)]\,\sigma^2_{\mathscr{J}^*}(s)}{2}\right)ds\right] \tag{3.1}$$

where $\sigma^2_{J^*}(t)$ is the utility volatility process, and E_t denotes the conditional expectation given the information available at date t.

Any stochastic differential utility with aggregator (f^*, A^*) could be represented by an ordinarily equivalent normalized aggregator $(f, 0)$ whose variance multiplier is such that $A = 0$. The normalized aggregator $(f, 0)$ is more tractable for proofs and optimization resolution, while the unnormalized aggregator (f^*, A^*) is convenient for achieving the desired disentangling by changing $A^*()$ with $f^*()$ fixed. With the normalized aggregator,

1. The continuation utility is similar to the prospective utility in the discrete time setting of Koopmans (1960, p.292)

the time t continuation utility becomes

$$\mathscr{J}(t) = E_t \left[\int_t^\infty f\Big(c(s), \mathscr{J}(s)\Big) ds \right] \tag{3.2}$$

Notice that with a stochastic differential utility, the growth rate of marginal utility depends on both the growth rate of consumption and the growth rate of the future utility index. The instantaneous utility is $f(c(t), J(t))$. The growth rate of instantaneous marginal utility can be decomposed into the sum of two terms as follows :

$$\frac{df_c(c(t), \mathscr{J}(t))}{f_c(c(t), \mathscr{J}(t))} = \frac{c(t) f_{cc}}{f_c} \left[\frac{dc(t)}{c(t)} \right] + \frac{\mathscr{J}(t) f_{c\mathscr{J}}}{f_c} \left[\frac{d\mathscr{J}(t)}{\mathscr{J}(t)} \right]. \tag{3.3}$$

The future utility index (or prospective utility) $\mathscr{J}(t)$ is made a function of the distribution of future consumption. The growth rate of the future utility index represents the updates in expectations about the future prospects. [2]

Given the current information available on the state of the economy, a positive growth of the future utility index may be interpreted as brighter future prospects. If the growth of the future utility index is negative, it may be viewed as a worsening future prospects. In this case, it represents an increase in the probability of low levels of consumption in the entire future. With stochastic differential utility, there are two sources of risk : one associated with the current consumption growth and another associated with the growth rate of the future utility index or future growth prospects.

Future prospects may be particularly relevant in the management of natural resources, which by its very nature is a long-term process. During this long-term process, the current information on the state of the economy may lead to revisions in expectations about the future prospects and then affect the resource extraction strategies.

An example of parametric stochastic differential utility for which existence has been

2. In discrete time, an alternative intuitive framework by Koszegi and Rabin (2009, p.4) considers that current instantaneous utility depends on current consumption and a "prospective gain-loss utility" which derives from changes between last period and the current period in beliefs regarding future outcomes.

established by Schroder and Skiadas (1999) is given by the following integral equation :

$$\mathscr{J}^*(t) = E_t \int_t^{\infty} e^{-\rho(s-t)} \left[\frac{c(s)^{\gamma}}{\gamma} + \frac{\alpha \mathscr{J}^*(s)^{-1}}{2} \sigma^2_{\mathscr{J}^*}(s) \right] ds \tag{3.4}$$

where the coefficient ρ denotes time preference and the coefficient $\frac{1}{\gamma}$ denotes the elasticity of intertemporal substitution. Schroder and Skiadas (1999) show that its corresponding normalized aggregator is given by

$$f(c(s), \mathscr{J}(s)) = (1+\alpha) \left[\frac{c(s)^{\gamma}}{\gamma} \mathscr{J}(s)^{\frac{\alpha}{1+\alpha}} - \rho \mathscr{J}(s) \right], \tag{3.5}$$

with the parameters satisfying the following constraints :

$$\alpha > -1 \, , \gamma \neq 0 \, , \gamma < \min \left\{ 1, \frac{1}{1+\alpha} \right\} \tag{3.6}$$

Note that the case where the coefficient $\alpha = 0$ means that the agent pays no attention to the uncertainty of the continuation utility, in which case (3.4) corresponds to the standard additive utility and the aggregator (3.5) becomes linearly dependent upon the future utility. A positive or negative value coefficient α can be viewed as a measure of the risk aversion, when compared to the time additive utility.

When $\gamma > 0$, a negative α penalizes uncertainty about future utility, whereas a positive α rewards uncertainty about future utility. Hence the agent is said to be comparatively more risk-averse (relative to the time-separable utility) if $\gamma > 0$ and $\alpha < 0$, whereas the agent is said to be comparatively less risk-averse (relative to the time-separable utility) if $\gamma > 0$ and $\alpha > 0$. When $\gamma < 0$, the effect of the sign of α on the uncertainty of future utility is reversed. When $\gamma = 0$, the aggregator becomes $f(c(s), \mathscr{J}(s)) = (1+\alpha \mathscr{J}_s) \left[log(c(s)) - \frac{\rho}{\alpha} log(1+\alpha \mathscr{J}(s)) \right]$ and the interpretation of α is the same as in the case of $\gamma > 0$

3.3 The model

Our baseline framework modifies the Gaudet and Khadr (1991) natural resource model, which uses a linear additive utility framework, to allow preferences of the consumer to be represented by a stochastic differential utility. We first describe the Gaudet and Khadr (1991) framework.

3.3.1 The Gaudet-Khadr framework

In the Gaudet and Khadr (1991) economy there are two goods, one of which is a nonrenewable natural resource, whose stock at date t, $S(t)$, is irreversibly reduced by extraction. The other is a reproducible composite good, that can be either consumed or accumulated. If accumulated, it can be either in the form of physical capital, whose accumulated stock is denoted $K(t)$, or of a "bond". The accumulated stock of bonds is assumed to reproduce itself at an exogenously given risk free rate $r(t)$, which represents the force of interest.

Both the production of the composite good and the extraction of the natural resource are assumed to be stochastic. More precisely, if $y(t)$ denotes the flow of production of the composite good, $x(t)$ denotes the flow of extraction of the resource and $\theta_1(t)$ and $\theta_2(t)$ are two stochastic productivity indices, then the stochastic production and extraction processes are represented respectively by

$$y(t) = F(K_y(t), x(t), \theta_1(t)) \tag{3.7}$$

and

$$x(t) = G(K_x(t), \theta_2(t)) = \frac{K_x(t)}{\gamma(\theta_2(t))}, \tag{3.8}$$

where $K_y(t) + K_x(t) = K(t)$.

The production of the composite good is assumed to satisfy $F_K > 0$, $F_x > 0$, $F_{KK} < 0$ and $F_{xx} < 0$, and the Inada conditions with respect to the inputs K_y and x . It also assumed to satisfy $F_1 > 0$, $F_{K1} > 0$ and $F_{x1} > 0$, where the subscript 1 denotes the derivative with respect to θ_1. The function $\gamma(\theta_2)$ represents the number of units of capital required to extract a unit of the natural resource and satisfies $\gamma'(\theta_2) > 0$, $\lim_{\theta_2 \to -\infty} \gamma(\theta_2) = \infty$, $\lim_{\theta_2 \to \infty} \gamma(\theta_2) = 0$.

The productivity indices θ_1 and θ_2 are assumed to evolve over time according to Itô processes of the form :

$$d\theta_i = \mu_i dt + \sigma_i \xi_i \sqrt{dt}, \quad i = 1,2, \tag{3.9}$$

with $\xi_i \sim N(0,1)$, $cov(d\theta_1, d\theta_2) = \sigma_{12} dt + o(dt)$ and $\sigma_{12} = \sigma_1 \sigma_2 cov(\xi_1, \xi_2)$. The drift μ_i and the variance σ_i can depend on time and on the state.

Producers of the composite commodity adjust instantaneously their stock of capital and their use of the nonrenewable resource to maximize profits at each date t. Since the producers take prices as given, the following conditions arise

$$F_K(K_y(t), x(t), \theta_1(t)) = r(t) \tag{3.10}$$

$$F_x(K_y(t), x(t), \theta_1(t)) = p(t) \tag{3.11}$$

The first equation says that in equilibrium the producers ensure that the marginal product of the stock of the composite good is the same in each of its uses. The second equation says that in equilibrium the producers choose the level of the natural resource input such that its marginal product equals its marginal cost which is the price of the resource extracted.

The dynamic programming problem of the resource extraction sector is to choose the extraction path that maximizes the expected present value of the future net benefits :

$$\max_{x(s), s\in[t,\infty]} E_t \int_t^\infty e^{-\rho(s-t)} q(\tau)[p(s) - r\gamma(\theta_2(s))]x(s)ds \tag{3.12}$$

subject to :

$$dS(t) = -x(t)dt \quad \text{and} \quad S(0) = S_0,$$

where ρ is instantaneous time-invariant discount rate, $p(t)$ represents the gross price of a unit of the resource in the ground, expressed in units of the composite commodity, and $q(t)$ denotes the demand price of a unit of the composite commodity, taken as given, as is $\theta_2(t)$. It is shown in Gaudet and Khadr (1991) that this requires :

$$\frac{1}{q(t)\lambda(t)}\frac{1}{dt}E_t d[q(t)\lambda(t)] = \rho \tag{3.13}$$

where $\lambda(t) = p(t) - r\gamma(\theta_2(t))$ is the price of a unit of the resource in the ground. The expression $q(t)\lambda(t)$ is the marginal profit from extracting the resource expressed in term of the composite good. The partial equilibrium rule (3.13) says that the resource producers' optimal extraction rule requires that the expected marginal profit from extraction increase at the constant rate ρ.

The representative consumer derives instantaneous utility $U(c(t))$ from consuming the flow $c(t)$ of the composite good and is the ultimate owner of the stock of composite good and the stock of the natural resource. Given θ_1 and θ_2, he chooses at each date his consumption and the allocation of his wealth between the resource stock and the accumulated composite good so as to maximize his discounted flow of future instantaneous utility, that is

$$\max_{\{c(s),\omega(s)\},s\in[t,\infty]} \int_t^{\infty} e^{-\rho(s-t)}U(c(s))ds$$

subject to his stochastic wealth constraint. The decision variable $\omega(t)$ is the fraction of his wealth held in the form of the composite commodity, the rest being invested in the resource stock. His consumption and portfolio decisions generate consumption and asset demands, through which he sends price signals to the resource firms and the firms producing the composite good. The firms take those demand prices as given in making their extraction and production decisions, which in turn generate the asset returns that the consumer takes as given when establishing his opportunity set, as determined by his wealth constraint. The prices and asset returns are taken to be those that simultaneously equilibrate the markets and are shown to follow Itô processes, whose drifts and variances are derived from the market equilibrium conditions.

The value of a unit of resource in the ground, $\lambda(t) = p(t) - r\gamma(\theta_2(t)$, is the price of the extracted resource minus the marginal cost of extraction. Gaudet and Khadr (1991) show that, given (3.9), the value of a unit of resource in the ground, $\lambda(t)$, will in equilibrium follow an Itô process of the form :

$$\frac{d\lambda(t)}{\lambda(t)} = \mu_S dt + \sigma_S d\zeta(t), \tag{3.14}$$

where $\mu_S(t)$ and $\sigma_S(t)$ are equilibrium values which will depend on the parameters, including μ_i, σ_i $(i = 1,2)$ in (3.9). The drift $\mu_S(t) = \frac{1}{\lambda(t)}\frac{1}{dt}E_t d\lambda(t)$ represents the expected instantaneous rate of return on the resource stock being held in the ground. The volatility of the nonrenewable resource returns, $\sigma_S(t)$, induces time variation in investment opportunities. The returns on both the capital stock $r(t)$ and the resource stock $\frac{d\lambda(t)}{\lambda(t)}$ constitute the signals that the consumer uses in its optimal portfolio decision making.

3.3.2 The investment decision with a stochastic differential utility

Assume now that the consumer's preferences are generated by the stochastic differential utility of Duffie and Epstein (1992b) instead of the linear additive utility assumed in Gaudet and Khadr (1991). The representative consumer's decision problem is then formalized as follows :

$$\max_{c(t),(1-\omega(t))} E_t \left[\int_t^\infty f(c(s), \mathscr{J}(s)) ds \right] \tag{3.15}$$

subject to the wealth constraint :

$$dW(t) = -c(t)dt + W(t)\left[\omega(t)r(t) + (1-\omega(t))\frac{d\lambda(t)}{\lambda(t)}\right], \tag{3.16}$$

where the consumer's total stock of wealth at date t is $W(t) = K(t) + B(t) + \lambda(t)S(t)$, $\lambda(t)$ being the price of the in-ground resource in terms of the composite good. The fraction $\big(1-\omega(t)\big)$ of wealth is invested in the nonrenewable natural resource risky asset at date t, with return $d\lambda(t)/\lambda(t)$, and the rest in the accumulated composite good, with return $r(t)$.

Denote by $J(\theta_1, \theta_2, W(t))$ the maximized utility of the representative consumer in state (θ_1, θ_2) with wealth $W(t)$ at time t. The Hamilton-Jacobi-Bellman equation of the consumption portfolio choice problem described above is then given by :

$$0 = \sup_{c,(1-\omega)} \left[f\left(c(s), J(\theta_1, \theta_2, W(t)\right) + \mathscr{D}J(\theta_1, \theta_2, W(t))\right], \tag{3.17}$$

where $\mathscr{D}J(\cdot) = \frac{1}{dt}E_t dJ(\cdot)$ is the drift coefficient of the Itô process $J(\cdot)$. In other words,

$$\begin{aligned} \mathscr{D}J(\theta_1, \theta_2, W(t)) &= \frac{1}{dt}E_t dJ(\theta_1, \theta_2, W(t) \\ &= J_1\mu_1 + J_2\mu_2 + J_W\Big[W(1-\omega)[\mu_S - r] + Wr - c\Big] + \frac{1}{2}\Big[2J_{11}\sigma_{11}^2 + 2J_{22}\sigma_{22}^2 \\ &+ 2J_{12}\sigma_{12} + 2W(1-\omega)(J_{1W}\sigma_{1S} + J_{2W}\sigma_{2S}) + J_{WW}(1-w)^2W^2\sigma_S^2\Big], \end{aligned} \tag{3.18}$$

where J_i denotes the derivative of J with respect to θ_i and J_{ij} denotes the cross-derivative with respect to θ_i and θ_j.

The first-order condition with respect to consumption c is :

$$J_W = f_c. \tag{3.19}$$

The first-order condition with respect to the portfolio decision $(1-\omega)$ is :

$$J_W W(\mu_S - r) + J_{WW}\sigma_S^2(1-\omega(t))W^2 + W\Big[J_{1W}W\sigma_{1S} + J_{2W}W\sigma_{2S}\Big] = 0. \tag{3.20}$$

Differentiating the maximized Hamilton-Jacobi-Bellman equation (3.17) with respect to W and applying the envelope theorem yields :

$$f_J(c,J)J_W = -\frac{\partial \mathscr{D}J}{\partial W}\,. \tag{3.21}$$

Using the first-order condition (3.20) into the expression of the right-hand side obtained by differentiating (3.18) gives (See Appendix A) :

$$\frac{\mathscr{D}J_W}{J_W} = f_J(c,J) - r(t). \tag{3.22}$$

From the first-order condition (3.19) for optimal consumption we may replace J_W by

$f_c(c,J)$, to obtain :

$$\frac{1}{f_c(c(t),J(t))}\mathscr{D}f_c(c(t),J(t)) = -f_J(c(t),J(t)) - r(t)\,, \tag{3.23}$$

where $\mathscr{D}f_c$ is the drift coefficient of the Itô process $f_c(c,J)$.

The term $-f_J(c,J)$ generalizes the notion of discount rate to the stochastic differential utility. [3] Condition (3.23) states that optimal consumption path is such that the expected rate of change the marginal utility of consumption equals the instantaneous risk free rate $r(t)$ plus the instantaneous stochastic discount rate.

3.4 The Hotelling rule with stochastic differential utility

Substituting for the inverse demand $q(t) = f_c(c(t),J(t))$ into (3.13), the optimal extraction condition of the representative price-taking resource extracting firm, we get :

$$\frac{1}{\lambda(t)f_c(c(t),J(t))}\mathscr{D}\lambda(t)f_c(c(t),J(t)) = \rho, \tag{3.24}$$

where again the notation $\mathscr{D}z = \frac{1}{dt}E_t dz$ is used. Combining (3.24) with condition (3.23), which must be satisfied by the optimal consumption path, and using the fact that $\lambda(t)$, the marginal value of the resource in the ground, must in equilibrium follow the stochastic process (3.14), we obtain the formulation of the Hotelling rule for the stochastic differential utility framework (See Appendix B for the details of the derivation) :

$$\mu_S(t) - r(t) = \rho + f_J(c(t),J(t)) - \frac{c(t)f_{cc}(t)}{f_c(t)}\sigma_{Sc}(t) - \frac{J(t)f_{cJ}(t)}{f_c(t)}\sigma_{SJ}(t) \tag{3.25}$$

where $\mu_S(t) = \frac{1}{\lambda(t)}\frac{1}{dt}E_t d\lambda(t)$ is the expected instantaneous rate of return on the resource stock being held in the ground, $\sigma_{Sc}(t) = cov_t\left(\frac{d\lambda}{\lambda}, \frac{dc}{c}\right)$, and $\sigma_{SJ}(t) = cov_t\left(\frac{d\lambda}{\lambda}, \frac{dJ}{J}\right)$. In addition to accounting for the already discussed endogenous discount rate encountered earlier, it takes the form of a so-called “two-beta asset pricing rule”, where each beta is

3. Note that in the case of the standard additive utility, where the aggregator is $f(c,J) = u(c) - \rho J$, the discount rate becomes $-f_J = \rho$.

associated to a different risk factor. Hence the equilibrium spread between the expected return on holding the stock of resource in the ground and the risk free rate of interest has three components : the endogenous discount rate, the covariance between the return on the resource stock and consumption growth, and the covariance between return on the natural resource stock and the instantaneous change of the utility index.

Notice that in the case where the aggregator f depends linearly on the future utility J, that is $f(c,J) = U(c) - \rho J$, equation (3.25) yields the Hotelling rule derived by Gaudet and Khadr (1991), namely :

$$\mu_S(t) - r(t) = -\frac{c(t)U''(c(t))}{U'(c(t))}\sigma_{Sc}(t) \tag{3.26}$$

This equilibrium condition expresses the pricing equation of the resource stock in the case where the agent ignores future prospects when making economic decisions at the margin (i.e., $(f_{cJ} = 0)$). Furthermore, if the resource investor is assumed to be risk neutral $(U'' = 0)$, the Hotelling rule becomes

$$\mu_S(t) = r(t). \tag{3.27}$$

To understand the role of the discount rate in the management of the resource stock with a stochastic differential utility, it is useful to analyze the first component. The first component $\rho + f_J(c(t), J(t))$ shows how the discount rate is adjusted over time with the resource stock valuation. It may also be thought of as the effect of the time valuation in the resource stock valuation. This effect depends on both the current consumption and the value function that measures the expected future prospects, given the information on the state of the economy available at time t. Note that under certainty $(\sigma_{Sc}(t) = \sigma_{SJ}(t) = 0)$, the Hotelling rule (3.25) reduces to :

$$\frac{\dot{\lambda}}{\lambda} = r(t) + \rho + f_J(c(t), J(t)). \tag{3.28}$$

The equilibrium expected excess return on the resource stock is equal to the rate of time preference adjusted to take into account the effect of the irreversible exhaustion of the resource stock and the level of capital held by the consumer. Hence if $\rho + f_J(c(t), J(t)) \leq 0$ then $\frac{\dot{\lambda}}{\lambda} \leq r(t)$.

The second component is related to the shocks to current consumption growth. It tells us that holding a nonrenewable resource stock is relatively desirable and will, everything else the same, require less of a premium, if its return is negatively correlated with the rate of growth of consumption. The reverse is true if the correlation is positive. Thus if a nonrenewable resource stock is such that its return tends to be high (low) when consumption growth tomorrow is high (low), ($\sigma_{Sc} > 0$), holding such resource stock in one's portfolio makes it difficult to smooth consumption over states of nature. Therefore, risk averse investors require a premium over the riskless return to hold this resource stock, reflecting the investor's aversion to substitution over states of nature. The reverse is true if $\sigma_{Sc} > 0$.

The third component is related to the shocks to future growth prospects. Since the value function utility $J(t)$ is attributable to the entire stream of future consumption stream $\{c_s : s > t\}$, the growth rate of the value function captures revisions in expectations about future prospects.[4] The presence of the covariance between the return on the resource stock and the growth rate of the value function in the pricing equation of nonrenewable resource stocks gives a formal support to Graham-Tomasi et al. (1986, p.244) who pointed out that resource stocks could be held as a hedging strategy against bad future prospects.

If a resource stock is such that its return tends to be high (low) when there are good (bad) news about future prospects, then $\sigma_{SJ} > 0$. Such a resource stock is not attractive for investors who are more risk averse ($f_{cJ} < 0$) than with the time-separable utility and they have a negative hedging demand for this resource stock because it tends to do worse when there is bad news about future prospects. For them, it is an undesirable feature of holding this resource stock which therefore needs to be compensated through a relatively higher risk premium. The risk premium is larger the more positive is the covariance. On the other hand, investors who are less risk averse ($f_{cJ} > 0$) or in other

4. An intuitive connection can be made between this third term and what has been labeled ”long-run risk” in a previous empirical literature, including Bansal and Yaron (2004).

words more aggressive than with the time separable utility have a positive demand for this resource stock since they are willing to trade off a worse performance when news is bad for extra performance when news is good. Holding such an asset smoothes the intertemporal consumption profile. So the more this investor is averse to intertemporal substitution, the more he is willing to hold this resource stock.

If a resource stock is such that the covariance between its return and the change in the value function (capturing how future prospects are expected to be) is negative ($\sigma_{SJ} < 0$), then the future creates more hedging opportunities for investors who are more risk averse than under the time-separable utility ($-f_{cJ} > 0$) and thus makes agents particularly like the resource stock, which contributes to lower its risk premium.

For further interpretation, it is useful to reexpress (3.25) explicitly in terms of the variances and the covariance of the stochastic productivity indices θ_1 and θ_2. To do this, use the Itô product rule to obtain :

$$\mathscr{D}\lambda f_c = f_c\mathscr{D}\lambda + \lambda\mathscr{D}f_c + \frac{\partial f_c}{\partial\theta_1}\frac{\partial\lambda}{\partial\theta_1}\sigma_1^2 + \frac{\partial f_c}{\partial\theta_2}\frac{\partial\lambda}{\partial\theta_2}\sigma_2^2 + \left[\frac{\partial f_c}{\partial\theta_2}\frac{\partial\lambda}{\partial\theta_1} + \frac{\partial f_c}{\partial\theta_1}\frac{\partial\lambda}{\partial\theta_2}\right]\sigma_{12} \tag{3.29}$$

where

$$\begin{aligned}
\frac{\partial f_c}{\partial\theta_1} &= f_{cc}F_1 + f_{cJ}J_1 \\
\frac{\partial f_c}{\partial\theta_2} &= -\gamma'\left(\frac{xF_x f_{cc}}{\gamma}\right) + f_{cJ}J_2 \\
\frac{\partial\lambda}{\partial\theta_1} &= F_{x1} > 0 \\
\frac{\partial\lambda}{\partial\theta_2} &= -\gamma'\left(\frac{xF_{xx}}{\gamma} + r\right).
\end{aligned}$$

Substituting from (3.29) into (3.24) and using (3.23) yields :

$$\begin{aligned}
\frac{1}{\lambda(t)dt}E_t(d\lambda) - r(t) &= \rho + f_J(c(t),J(t)) \\
&- \frac{f_{cc}(t)}{\lambda(t)f_c(t)}\Psi_F(\sigma_1,\sigma_2,\sigma_{12}) - \frac{f_{cJ}(t)}{\lambda(t)f_c(t)}\Psi_J(\sigma_1,\sigma_2,\sigma_{12})
\end{aligned} \tag{3.30}$$

where

$$\Psi_F(\sigma_1, \sigma_2, \sigma_{12}) = \left[F_1\sigma_1^2 - \frac{\gamma'}{\gamma}xF_x\sigma_{12}\right]\frac{\partial\lambda}{\partial\theta_1} + \left[F_1\sigma_{12} - \frac{\gamma'}{\gamma}xF_x\sigma_2^2\right]\frac{\partial\lambda}{\partial\theta_2} \tag{3.31}$$

and, recalling that $J_i = \frac{\partial J}{\partial\theta_i}$, $i = 1, 2$,

$$\Psi_J(\sigma_1, \sigma_2, \sigma_{12}) = \left[J_1\sigma_1^2 + J_2\sigma_{12}\right]\frac{\partial\lambda}{\partial\theta_1} + \left[J_1\sigma_{12} + J_2\sigma_2^2\right]\frac{\partial\lambda}{\partial\theta_2}. \tag{3.32}$$

The term $\frac{f_{cc}}{\lambda f_c}\Psi_F$ expresses the risk related to current production and consumption decisions. An interpretation of this term is provided in Gaudet and Khadr (1991, p. 450).

The term $\frac{f_{cJ}}{\lambda f_c}\Psi_J$ is related to the changes in future prospects. Note that the signs of the coefficients in (3.32) depend crucially, among other things, on the signs of J_1 and J_2. For the sake of argument, assume that $J_1 > 0$ and $J_2 > 0$. This means that improvements in productivity in both the resource extraction sector and the production sector cause future prospects to become brighter.

Assume $\sigma_{12}(t) > 0$. This means that favorable (unfavorable) changes in the productivity in the resource extraction sector are associated with unfavorable (favorable) changes in the productivity in the composite good sector.[5] Assume also $\frac{\partial\lambda}{\partial\theta_2} > 0$, so that productivity improvements in the nonrenewable resource sector decrease the value of the marginal unit of resource held in the ground. If $f_{cJ} < 0$, the consumer may be said averse to risk related to future prospects. Since the uncertainty in both the resource extraction sector and the production sector go in opposite directions, this resource stock will be considered more desirable for such consumers than it would be under the standard utility. Consequently, they require a lower risk premium on the resource stock. Indeed, in this case, the term $\frac{f_{cJ}}{\lambda f_c}\Psi_J$ enters negatively in the risk premium of the resource stock. On the other hand, if $f_{cJ} > 0$ then holding such a resource stock is relatively undesirable and as a consequence a higher risk premium will be required.

In contrast, assume $\sigma_{12}(t) < 0$, which means that favorable (unfavorable) changes in

5. Recall that the specifications of the production functions $F(K_y, x, \theta_1)$ of the composite good sector and $G(K_x, \theta_2)$ of the resource extraction sector are such that $F_1 > 0$ while $G_2 = -\gamma' x/\gamma < 0$, i.e. an increase in θ_2 has a negative effect on the productivity of the resource sector.

the productivity of the resource extraction sector are associated with favorable (unfavorable) changes in the composite good sector. Then, keeping unchanged the other above assumptions, Ψ_J *may* be negative. If this occurs, then the above conclusions are reversed. Clearly, the effect on the risk premium of the risk associated to future prospects is an empirical question which deserves further investigation.

It is interesting to note that, when compared with the risk premium derived by Duffie and Epstein (1992a) for a reproducible asset, an important difference is the presence of the endogenous factor appearing in the discount rate in the case of the nonrenewable resource. This factor derives from condition (3.24), which is specific to the fact that the asset is a fixed stock of a nonrenewable resource and that this non renewability generates an irreversibility in the portfolio decision. Hence, with stochastic differential utility, the discount rate $\rho + f_J(t)$ plays a role in determining the risk premium associated with the decision to invest in a nonrenewable natural resource stock, a role which is absent in the case of a conventional reproducible asset.

Finally, as pointed out by Livernois (2009, p.37) and Krautkraemer (1998, p.2102), changes in expectations regarding future prospects may have an influence on the evolution of resource prices and evaluating its significance is likely to play a greater role in future empirical research. Since $p = \lambda + r\gamma(\theta_2)$, it is a straightforward matter to use the stochastic Hotelling rule (3.25) to derive the equation for the expected evolution of the market flow price of the resource, which is given by :

$$\frac{1}{p}\frac{1}{dt}E_t(dp) = \left(1 - \frac{r\gamma}{p}\right)\left\{r(t) + \rho + f_J(c(t),J(t)) - \frac{c(t)f_{cc}(t)}{f_c(t)}\sigma_{Sc}(t) - \frac{J(t)f_{cv}(t)}{f_c(t)}\sigma_{SJ}(t)\right\} + \left(\frac{r\gamma}{p}\right)\left\{\frac{\gamma'\mu_2 + \frac{\gamma''\sigma_2^2}{2}}{\gamma}\right\} \tag{3.33}$$

Thus, in addition to the effect of the endogenous discount factor and of the two so-called beta coefficients on the expected rate of growth of the resource price, this equation also highlights the effect of the evolution of the cost of extraction, as captured in the last term.

3.5 Concluding remarks

The management of nonrenewable natural resources under risk diversification is a long-term process where future prospects and revisions in expectations about future prospects over time plays a key factor. This paper has extended the natural resource model of Gaudet and Khadr (1991) to the class of stochastic differential utility of Duffie and Epstein (1992b) in order to attempt to take this fact into account. The paper shows how this affects both the rate of return on nonrenewable resource stocks and the gross market price of the extracted resource. One thing that comes out clearly is that the precise direction of the effect of assuming a stochastic differential utility framework depends heavily on a number of assumptions that need to be explored empirically. The formulation presented in this paper could potentially provide a framework for doing so.

CONCLUSION

L'incertitude, qu'elle soit reliée à la quête de l'hégémonie politique, à des chocs sur la consommation, ou aux perspectives futures de l'économie, est un facteur qui influe sur la gestion de l'environnement et des ressources naturelles. Chacun des trois chapitres de cette thèse a traité d'un cas différent de ces trois types d'incertitude. Le premier essai a montré que dans un contexte de course à l'hégémonie politique au niveau international : le niveau d'émission d'un pays pauvrement doté est supérieur au niveau d'émission d'un pays richement doté, si les dotations des pays pauvrement dotés sont accrues, en laissant constante celles des pays richement dotés, alors la pollution globale baissera ; accroître les dotations des deux types de pays dans les mêmes proportions, et donc accroître la dotation moyenne dans la même proportion, baissera la pollution globale ; redistribuer des pays richement dotés vers les pays pauvrement dotés tout en maintenant fixe la dotation moyenne, résultera en général en un accroissement du niveau d'équilibre de la pollution globale. Accroître les dotations des pays pauvres reviendrait à renforcer leurs capacités institutionnelles et à accroître leur capital humain. Le deuxième essai est une étude empirique qui s'appuie sur le cadre théorique de Gaudet and Khadr (1991) et utilise la capitalisation boursière des compagnies minières et les réserves prouvées pour analyser empiriquement le rôle des ressources naturelles dans la diversification du risque. Les résultats obtenus suggèrent que les stocks de ressources non renouvelables, comme le pétrole et le charbon, sont des actifs qui peuvent soutenir une stratégie à long terme d'assurance contre le risque. Le troisième essai montre que la prise en compte de l'incertitude qui prévaut sur les perspectives d'avenir modifie la règle de tarification des ressources en mettant en évidence le rôle du taux d'actualisation endogène et un facteur relatif aux perspectives d'avenir de l'économie. Pour y arriver, le modèle de Gaudet and Khadr (1991) est étendue aux fonctions d'utilité récursive de Duffie and Epstein (1992b) qui prennent en compte l'incertitude liée aux perspectives futures de l'économie dans la prise de décision.

BIBLIOGRAPHIE

Bansal, R., and A. Yaron (2004) 'Risks for the Long Run : A Potential Resolution of Asset Pricing Puzzles.' *The Journal of Finance* 59, 1481–1509

Bergstrom, A. (1983) 'Gaussian estimation of structural parameters in higher-order continuous time dynamic models.' *Econometrica* 51, 117–152

Black, J. (2007) *Great Powers and the Quest for Hegemony The World Order since 1500* (Routledge)

Bolduc, D., L. Khalaf, and C. Yelou (2007) 'Identification robust confidence set methods for inference on parameter ratios with application to discrete choice models.' *Journal of Econometrics, forthcoming*

Boot, J., W. Feibes, and J. Lisman (1967) 'Further methods of derivation of quarterly figures from annual data.' *Applied Statistics* 16, 65–75

Breeden, D. (1979) 'An intertemporal asset pricing model with stochastic consumption and investment opportunities.' *Journal of Financial Economics* 7(3), 265 – 296

Chan, Karolyi, Longstaff, and Sanders (1992) 'An empirical comparison of alternative models of the short-term interest rate.' *Journal of Finance* 47, 1209–1228

Chombat, P. (1998) *L'Empire des énergivores menace la planète* (142 Faubourg Saint-Antoine, 75012 PARIS : Lettres du monde)

Cioffi-Revilla, C. (1998) *Politics and Uncertainty Theory, Models and Applications* (Cambridge university press)

Clark, I. (2009) 'Bringing Hegemony Back In : the United States and International Order.' *International Affairs* 85, 23–36

Dockner, E., S. Jorgensen, N. Long, and G. Sorger (2000) *Differential Games in Economics and Management Science* (Cambridge university Press)

Duffie, J., and L. Epstein (1992a) 'Asset pricing with stochastic differential utility.' *Review of Financial Studies* 5, 411–436

_ (1992b) 'Stochastic differential utility.' *Econometrica* 53, 353–394

Efron, B. (1979) 'Bootstrap methods : Another look at the jackknife.' *Annals of statistics* 7, 1–26

Elliott, M. (2007) 'The chinese century.' *Time magazine*

Epaulard, A., and A. Pommeret (2003) 'Optimally eating a stochastic cake : a recursive utility approach.' *Resource and Energy Economics* 25, 129–139

Epstein, L., and S. Zin (1989) 'Substitution, risk aversion and the temporal behavior of consumption and asset returns : A theoretical framework.' *Econometrica* 57, 937–969

Gaudet, G. (2007) 'Natural Resource Economics under the Rule of Hotelling.' *Canadian Journal of Economics* 40, 1033–1059

Gaudet, G., and A. Khadr (1991) 'The evolution of natural-resource prices under stochastic investment opportunities : An intertemporal asset-pricing approach.' *International Economic Review* 32, 441–455

Graham-Tomasi, Theodore, C. Ford Runge, and William F. Hyde (1986) 'Foresight and expectations in models of natural resource markets.' *Land Economics* 62(3), 234–249

Hansen, L., and K. Singleton (1983) 'Stochastic consumption, risk aversion, and the temporal behavior of asset returns.' *The Journal of Political Economy* 91(2), 249–265

Howitt, R., S. MSangi, A. Reynaud, and K. Knapp (2005) 'Estimating intertemporal preferences for natural resource allocation.' *American Journal of Agricultural Economics* 5, 969–983

Huntington, S. (1993) 'Why International Primacy Matters ?' *International Security* 17, 68–83

Ikenberry, G. (2008) 'The Rise of China and the Future of the West Can the Liberal System Survive ?' *Foreign Affairs.* january/february

Kennedy, P. (1987) *The Rise and Fall of the Great Powers : Economic Change and Military Conflict From 1500 to 2000* (New York : Random House)

Knapp, Keith C., and L. Olson (1996) 'Dynamic resource management : Intertemporal substitution and risk aversion.' *Journal of Agricultural Economics* 78, 1004–1014

Koopmans, T. (1960) 'Stationary ordinal utility and impatience.' *Econometrica* 28(2), 287–309

Koszegi, B., and M. Rabin (2009) 'Reference-dependent consumption plans.' *American Economic Review* 99(3), 909–936

Krautkraemer, J. (1998) 'Nonrenewable resource scarcity.' *Journal of Economic Literature* 36, 2065–2107

Lee, T., and L. Wilde (1980) 'Market Structure and Innovation : A Reformulation.' *Quarterly Journal of Economics* 94, 429–36

Lisman, J., and J. Sandee (1964) 'Derivation of quaterly figures from annual data.' *Applied Statistics* 13, 87–90

Livernois, J. (2009) 'On the Empirical Significance of the Hotelling Rule.' *Review of Environmental Economics and Policy* 3, 22–41

Livernois, J., H. Thille, and X. Zhang (2006) 'A test of the hotelling rule using old-growth timber data.' *Canadian Journal of Economics* 39, 543–559

Loury (1979) ' Market Structure and Innovation .' *Quarterly Journal of Economics* 93, 395–410

Lucas, R. (1978) 'Asset prices in an exchange economy.' *Econometrica* 46(6), 1429–1445

Mankiw, N., and M. Shapiro (1986) 'Risk and return : consumption beta versus market beta.' *The Review of Economics and Statistics* 68, 452–459

Marburger, John (2007) 'Reflections on the Science and Policy of Energy and Climate Change.' *American Geophysical Union Fall Meeting* p. 5. "Union Lecture"

Miller, M., and H. Upton (1985a) 'A test of the Hotelling valuation principle.' *Journal of Political Economy* 93, 1–25

_ (1985b) 'The pricing of oil and gas : some further results.' *Journal of Finance* 40, 1009–1018

Modelski, G. (1987) *Long Cycles in World Politics* (Seattle : University of Washington Press)

Moldovanu, B., A. Sela, and Shi (2007) 'Contests for Status.' *Journal of Political Economy* 115, 338–363

Mosher, S. (2002) *China's Plan to Dominate Asia and the World* (Routledge)

Nowman, K. (1997) 'Gaussian estimation of single-factor continuous time models of the term structure of interest rates.' *The journal of Finance* 52, 1695–1706

Nye, Jr. (1990) *Bound to Lead The Changing Nature of American Power* (New York : Basic Books)

Peltola, J., and K. Knapp (2001) 'Recursive preferences in forest management.' *Forest Science* 47, 455–465

Raupach, M., G. Marland, P. Ciais, C. Le Quéré, J.G. Canadell, G. Klepper, and C.B. Field (2007) 'Global and regional drivers of accelerating CO2 emissions.' *Proceedings of the National Academy of Sciences*

Reingamun, J. (1982) 'A Dynamic Game of R and D : Patent Protection and Competitive Behavior.' *Econometrica* 50, 671–688

Roberts, M. (2001) 'Can portfolio theory explain flat and downward natural resource price trends ?' Paper presented at the Second World Congress of Environmental and Resource Economists, Montery, California, June 24-27, 2002

Schroder, M., and C. Skiadas (1999) 'Optimal consumption and portfolio selection with stochastic differential utility.' *Journal of Economic Theory* 89, 68–126

Shaw, W., and R. Woodward (2008) 'On why environmental and resource economists should care about non-expected utility models.' *Resource and Energy Economics* 30, 66–89

Shenkar, O. (2005) *The Chinese Century : The Rising Chinese Economy and Its Impact on the Global Economy, the Balance of Power, and Your Job* (USA : Wharton School Publisching)

Slade, M., and H. Thille (1997) 'Hotelling Confronts CAPM : A Test of the Theory of Exhaustible Resources.' *Canadian Journal of Economics* 30, 685–708

Stern, N. (2007) *The Economics of Climate Change, The Stern Review* (UK : Cambridge University Press)

Thurow, L. (1993) *Head to Head : The Coming Economic Battle Among Japan, Europe and America* (Warner Books Ed)

Waltz, K. (1993) 'The Emerging Structure of International Politics.' *International Security* 18, 44–79

Weiss, Y., and C. Fershtman (1980) 'Social status and economic performance : A survey.' *European Economic Review* 42, 801–820

Young, D. (1992) 'Cost specification and firm behaviour in a Hotelling model of resource extraction.' *Canadian Journal of Economics* 25, 41–59

Young, D., and D. Ryan (1996) 'Empirical testing of a risk-adjusted Hotelling model.' *Ressources Energy Economics* 18, 265–289

Annexe I

Appendix to Chapter 2

I.1 List of oil and coal companies

Tableau I.I: **Sample of oil Companies**

Companies	**Beginning** [1]	**End**
UNITED STATES STEEL CORP	1959(02)	2006(12)
STANDARD OIL CO CALIFORNIA	1959(02)	2006(12)
PHILLIPS PETROLEUM CO	1959(02)	2006(12)
STANDARD OIL CO N J / EXXON MOBIL CORP	1959(02)	2006(12)
UNOCAL CORP	1959(02)	2005(12)
BRITISH PETROLEUM LTD	1962(07)	2006(12)
KERR MCGEE CORP	1956(03)	2006(12)
CANADA SOUTHERN PETROLEUM LTD	1962(08)	2006(12)
JEFFERSON LAKE PETROCHEMICALS	1962(08)	2006(12)
IMPERIAL OIL LTD	1962(08)	2006(12)
OCCIDENTAL PETROLEUM CORP	1962(08)	2006(12)
BRITALTA PETROLEUMS LTD / WILSHIRE OIL CO TX	1962(08)	2006(12)
MURPHY CORP	1962(02)	2006(12)
APACHE CORP	1963(08)	2006(12)
BARNWELL INDUSTRIES INC	1965(08)	2006(12)
BROWN TOM INC	1972(12)	2004(04)
FOREST OIL CORP	1972(12)	2006(12)
PATRICK PETROLEUM CO	1972(12)	2006(12)
NOBLE AFFILIATES INC	1972(12)	2006(12)

1. In the CRSP, the data on market capitalizations of some companies such as UNITED STATES STEEL CORP begin in 1925, but due to the unavailability of data on monthly consumption at this period, we were constrained to begin in 1959

TIPPERARY LAND & EXPLORATION	1972(12)	2005(09)
WISER OIL CO DE	1972(12)	2004(06)
K R M PETROLEUM CORP	1974(07)	2006(12)
MAYNARD OIL CO	1975(08)	2002(06)
GEORESOURCES INC	1976(04)	2006(12)
PYRAMID OIL CO	1976(08)	2006(12)
CREDO PETROLEUM CORP	1979(03)	2006(12)
HARKEN OIL & GAS INC	1979(12)	2006(12)
DOUBLE EAGLE PETE & MNG CO	1980(01)	2006(12)
BELLWETHER EXPLORATION CO	1980(12)	2005(06)
CENTRAL PACIFIC MINERALS N L	1981(03)	2002(02)
PARALLEL PETROLEUM CORP DE	1981(01)	2006(12)
SANTOS LIMITED	1981(03)	2006(12)
MAGELLAN PETROLEUM CORP	1982(11)	2006(12)
SASOL LTD	1982(05)	2006(12)
ENSERCH EXPLORATION PARTNERS LTD	1985(04)	2002(12)
WALKER ENERGY PARTNERS	1985 (11)	2006(12)
ANADARKO PETROLEUM CORP	1986(10)	2006(12)
NORSK HYDRO A S	1986(07)	2006(12)
PARKER & PARSLEY DEVELOPMENT PAR	1987(12)	2006(12)
REPSOL S A	1989(05)	2006(12)
ENRON OIL & GAS CO	1989(10)	2006(12)
VINTAGE PETROLEUM INC	1990(09)	2005(12)
TOTAL S A	1991(11)	2006(12)
NEWFIELD EXPLORATION CO	1993(12)	2006(12)
SUNCOR INC	1994(01)	2006(12)
CROSS TIMBERS OIL CO	1993(06)	2006(12)
PETRO CANADA	1995(11)	2006(12)
LEVIATHAN GAS PIPELINE PTNERS LP	1998(09)	2004(09)
DEVON ENERGY CORP NEW	1988(10)	2006(12)

PETROLEO BRASILEIRO SA PETROBRAS	2000(09)	2006(12)
PANCANADIAN ENERGY CORP	2001(11)	2006(12)
STATOIL A S A	2001(07)	2006(12)
INTEROIL CORP	2004(10)	2006(12)
BILL BARRETT CORP	2005(01)	2006(12)
ROYAL DUTCH SHELL PLC	2005(07)	2006(12)

Tableau I.II: **Sample of Coal mining Companies**

Companies	**The begin**	**The end**
WESTMORELAND COAL CO	1973(01)	2006(12)
CONSOL ENERGY INC	1999(4)	2006(12)
FORDING INC	2001(10)	2003(01)
SASOL LTD	1982(04)	2006(12)
ALPHA NATURAL RESOURCES INC	2005(02)	2006(12)
ARCH COAL INC	1988(08)	2006(12)
FORDING CANADIAN COAL TRUST	2003(03)	2006(12)
JAMES RIVER COAL CO	2005(01)	2006(12)
NATURAL RESOURCE PARTNERS L P	2002(10)	2006(12)

I.2 The confidence interval for the long-run beta

Assume that the ratio parameter is

$$\bar{\beta} = \frac{\kappa_1}{\kappa_2} \tag{I.1}$$

where $\kappa_1 = \sum_{t=1}^{T} \beta_t \left[\mu_M(t) - r_t\right]$ and $\kappa_2 = \sum_{t=1}^{T} \left[\mu_M(t) - r_t\right]$. A point estimation of the long run beta is given by

$$\widehat{\overline{\beta}} = \frac{\widehat{\kappa_1}}{\widehat{\kappa_2}} \tag{I.2}$$

where $\widehat{\kappa_1} = \sum_{t=1}^{T} \widehat{\beta_t} \left[\widehat{\mu_M}(t) - r_t\right]$ and $\widehat{\kappa_2} = \sum_{t=1}^{T} \left[\widehat{\mu_M}(t) - r_t.\right]$

Let $\widehat{\Sigma_{12}} = \begin{bmatrix} \widehat{v_1} & \widehat{v_{12}} \\ \widehat{v_{12}} & \widehat{v_2} \end{bmatrix}$ denotes the covariance matrix that corresponds to $(\widehat{\kappa}_1, \widehat{\kappa}_2)$.

I.2.1 Bootstrap Fieller confidence interval

The t-test statistic

$$t(\overline{\beta}) = \frac{\widehat{\kappa}_1 - \overline{\beta}\widehat{\kappa}_2}{(\overline{\beta}^2 \widehat{v}_2 - 2\widehat{v_{12}} + \widehat{v}_1)^{\frac{1}{2}}} \rightsquigarrow_{asymp} N(0,1).$$

The Fieller method provides the $(1-\alpha)\%$ confidence interval by inverting the t-test statistic. This corresponds to the values of $\overline{\beta}$ such that $|t(\overline{\beta})| \leq z_{\alpha/2}$.

Let $A = \widehat{\kappa}_2^2 - z_{\frac{\alpha}{2}}^2 \widehat{v}_2$, $B = \widehat{\kappa}_1 \widehat{\kappa}_2 + z_{\frac{\alpha}{2}}^2 \widehat{v_{12}}$, $C = \widehat{\kappa}_1^2 - z_{\frac{\alpha}{2}}^2 \widehat{v}_1$, $\Delta = B^2 - AC$.

The $(1-\alpha)\%$ Fieller confidence interval depends on the sign of $\Delta = B^2 - AC$.

If $\Delta < 0$,

$$FC(\alpha) = [-\infty, +\infty].$$

If $\Delta \geq 0$,

$$FC(\alpha) = \begin{cases} \left[\frac{-B-\sqrt{\Delta}}{A}, \frac{-B+\sqrt{\Delta}}{A}\right] & \text{if } A > 0 \\ \left]-\infty, \frac{-B-\sqrt{\Delta}}{A}\right] \cup \left[\frac{-B+\sqrt{\Delta}}{A}, +\infty\right[& \text{if } A < 0. \end{cases}$$

I.2.2 Bootstrap Delta confidence interval

$$DC(\alpha) = \left[\frac{\widehat{\kappa}_1}{\widehat{\kappa}_2} - z_{\frac{\alpha}{2}} \widehat{L}^{\frac{1}{2}}, \frac{\widehat{\kappa}_1}{\widehat{\kappa}_2} + z_{\frac{\alpha}{2}} \widehat{L}^{\frac{1}{2}}\right]$$

where $\widehat{L}^{\frac{1}{2}} = \begin{bmatrix} \frac{1}{\widehat{\kappa_1}} & -\frac{\widehat{\kappa_1}}{\widehat{\kappa_2}^2} \end{bmatrix} \begin{bmatrix} \widehat{v_1} & \widehat{v_{12}} \\ \widehat{v_{12}} & \widehat{v_2} \end{bmatrix} \begin{bmatrix} \frac{1}{\widehat{\kappa_1}} \\ -\frac{\widehat{\kappa_1}}{\widehat{\kappa_2}^2} \end{bmatrix}$

The parametric bootstrap procedure by Efron (1979) is used to compute the covariance matrix that corresponds to $(\widehat{\kappa_1}, \widehat{\kappa_2})$ as follows.

Let $(\widehat{\alpha_1}^{(0)}, \widehat{\alpha_2}^{(0)}, \widehat{\alpha_3}^{(0)}, \widehat{\sigma}^{(0)})$ an estimate of $(\alpha_1, \alpha_2, \alpha_3, \sigma)$ obtained from the original sample path denoted by $R_t^{(0)}$. The parametric bootstrap procedure consists of the following steps.

Step 1. Generate a bootstrap sample path $R_t^{(b)}$ from

$$R_t^{(b)} = e^{\widehat{\alpha_2}^{(b-1)}} R_{t-1}^{(b)} + \frac{\widehat{\alpha_1}^{(b-1)}}{\widehat{\alpha_2}^{(b-1)}} (e^{\widehat{\alpha_2}^{(b-1)}} - 1) + \eta_t^{(b)} \tag{I.3}$$

where we have simulated the random variable $\widehat{\eta}_t^{(b)}$ such that

$$E(\eta_s^{(b)} \eta_t^{(b)}) = \begin{cases} 0 & \text{if } s \neq t \\ \frac{\widehat{\sigma^2}^{(b-1)}}{2\widehat{\alpha_2}^{(b-1)}} (e^{\widehat{\alpha_2}^{(b-1)}} - 1) R_{t-1}^{2\widehat{\alpha_3}^{(b-1)}} & \text{if } s = t. \end{cases}$$

Step 2. Obtain a new maximum likelihood estimator from the bootstrap sample path by applying the same estimation procedure.

Step 3. Repeat Steps 1 and 2 D number of times and obtain a set of bootstrap estimates.

We can compute $b = 1, ..., D$ bootstraps sample by using the estimated parameters the discrete model and the initial values of the variable.

Step 4. From the original sample, we generate sample $b = 1$ by using with estimates parameters of the original sample in the discrete specification and by simulating random numbers representing the error term $\eta_t^{(b)}$.

Step 5. So we use the parametric bootstrap sample $b-1$ to estimate the new parameters $\widehat{\alpha_1}^{(b-1)}$, $\widehat{\alpha_2}^{(b-1)}$, $\widehat{\alpha_3}^{(b-1)}$, $\widehat{\sigma^2}^{(b-1)}$ and there we plug in the discrete specification and simulate $\eta_t^{(b)}$ to obtain the sample b.

From $R^S(t)$, we use this parametric bootstrap method to obtain the bootstrap replications of the drift $\widehat{\kappa_1^b} = \frac{\sum_{t=1}^{T} (\mu_S^{(b)}(t) - r_t)}{T}$ for the samples $b = 1, ..., D$.

Once we have estimated distribution for $\widehat{\kappa_1}$, we use it to estimate the standard error for $\widehat{\kappa_1}$. This estimate is given by $\widehat{v_1} = \frac{1}{B}\sum_{b=1}^{B}(\kappa_1^b - \overline{\widehat{\kappa_1}})^2$ where $\overline{\widehat{\kappa_1}} = \frac{1}{B}\sum_{b=1}^{B}\widehat{\kappa_1}^b$.

From the consumption $R^C(t)$, we use this parametric bootstrap method to obtain the bootstrap replications of the drift $\widehat{\kappa_2^b} = \frac{\sum_{t=1}^{T}(\mu_M^{(b)}(t)-r_t)}{T}$ for the samples $b = 1, ..., D$.

Using simulated distribution of the estimators $\left(\widehat{\kappa_1^b}, \widehat{\kappa_1^b}\right)_{b=1,...,D}$, we can compute $\widehat{v_{12}}$ which is the estimate of the covariance between $\widehat{\kappa_1}$ and $\widehat{\kappa_1}$.

Annexe II

Appendix to Chapter 3

II.1 The derivation of equation (3.22)

To derive equation (3.22), differentiate the Bellman equation with respect to W to obtain

$$f_J J_W = -\frac{\partial}{\partial W}\Big(\mathscr{D} J\Big),$$

where, from (3.18), we get that :

$$\frac{\partial}{\partial W}\Big(\mathscr{D} J\Big) = \mathscr{D} J_W + J_W r + (1-\omega)\Big[J_W(\mu_S - r) + \Big[J_{1W} W \sigma_{1S} + J_{2W} W \sigma_{2S}\Big] + J_{WW}\sigma_S^2(1-\omega(t))W\Big].$$

From the first-order condition with respect to $(1-\omega)$ (condition (3.20)), the last term vanishes and this equation reduces to :

$$\frac{\partial}{\partial W}\Big(\mathscr{D} J\Big) = \mathscr{D} J_W + J_W r.$$

Using the first-order equation (3.19), equation (3.22) follows immediately.

II.2 The derivation of equation (3.25)

To derive the Hotelling rule (3.25), use the Itô product rule to obtain

$$\frac{d(f_c(c,J)\lambda)}{f_c(c,J)\lambda} = \frac{d\lambda(t)}{\lambda} + \frac{df_c(c,J)}{f_c(c,J)} + \frac{df_c(c,J)d\lambda}{f_c(c,J)\lambda} \tag{II.1}$$

where the quadratic variation is given by

$$\frac{df_c(c,J)d\lambda}{f_c(c,J)\lambda} = \left[\frac{cf_{cc}}{f_c}\sigma_c\sigma_S + \frac{Jf_{cJ}}{f_c}\sigma_J\sigma_S\right]dt + o(dt). \tag{II.2}$$

Now apply the operator $\mathscr{D}z = \frac{1}{dt}E_t dz$ to both sides of equation $(II.1)$ and use equation (3.23) to obtain (3.25).