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Ce cahier a également été publié par le Centre interuniversitaire de recherche en économie quantitative (CIREQ) sous le numéro 0398.

This working paper was also published by the Center for Interuniversity Research in Quantitative Economics (CIREQ), under number 0398.

ISSN 0709-9231

CAHIER 9803

UNCOVERING FINANCIAL MARKETS BELIEFS ABOUT INFLATION TARGETS

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March 1998

This project was started while the author was visiting at the IDEI, Université des Sciences Sociales (Toulouse, France) in spring 1997. He would like to thank Serena Ng for providing the code to estimate the density of inflation forecast errors, and Jean Farès at the Bank of Canada for his help in obtaining the data. Financial support from the Social Sciences and Humanities Research Council (SSHRC) of Canada and the Fonds pour la formation de chercheurs et l'aide à la recherche (FCAR) of Québec is gratefully acknowledged.

RÉSUMÉ

Ce papier exploite la structure à terme des taux d'intérêt afin de développer des restrictions économiques testables sur le processus conjoint des taux d'intérêt à long terme et de l'inflation quand cette dernière est soumise à une politique de contrôle à l'intérieur d'une fourchette cible de la part de la Banque centrale. Deux modèles concurrents qui décrivent, en termes économétriques, les inférences des agents sur l'inflation cible sont développés et génèrent des prédictions distinctes sur l'évolution des taux d'intérêt. L'application empirique du modèle au Canada, qui pratique une politique monétaire avec intervalle-cible d'inflation, indique, d'une part, que la fourchette perçue par les agents économiques est significativement plus étroite que celle qui est officiellement annoncée et, d'autre part, que celle-ci est asymétrique par rapport au point médian visé. Ce dernier résultat (i) suggère que les autorités monétaires attribuent des poids différents aux déviations positives et négatives par rapport au point médian, et (ii) remet en question, sur le plan empirique, l'hypothèse fréquemment faite dans la littérature que la fonction de perte du décideur de la politique monétaire est symétrique (habituellement une fonction quadratique) autour du niveau d'inflation désiré.

Mots clés : inflation cible, crédibilité, asymétrie, modèles avec attentes rationnelles non linéaires

ABSTRACT

This paper exploits the term structure of interest rates to develop testable economic restrictions on the joint process of long-term interest rates and inflation when the latter is subject to a targeting policy by the Central Bank. Two competing models that econometrically describe agents' inferences about inflation targets are developed and shown to generate distinct predictions on the behavior of interest rates. In an empirical application to the Canadian inflation target zone, results indicate that agents perceive the band to be substantially narrower than officially announced and asymmetric around the stated mid-point. The latter result (i) suggests that the monetary authority attaches different weights to positive and negative deviations from the central target, and (ii) challenges on empirical grounds the assumption, frequently made in the literature, that the policy maker's loss function is symmetric (usually a quadratic function) around a desired inflation value.

Key words : inflation targets, credibility, asymmetries, nonlinear rational expectations models

1 Introduction and Summary

This paper exploits the information contained in the term-structure of interest rates to draw empirical conclusions about agents' beliefs regarding announced inflation targets. To that end, it is initially shown that the long-term interest rate can be expressed as a function of the agents' inflation forecasts during the bond's holding period. In turn, agents' forecasts depend on their beliefs about the government's statement that monetary policy will be geared to keep inflation within specific upper and minimum levels. Plausible agents inferences about the inflation band are characterized by means of two competing econometric specifications that generate distinct predictions on the time-series properties of the long-term interest rate. First, the band is assumed perfectly credible in the sense that agents genuinely believe that inflation will only take values within the stated upper and lower limits. In econometric terms, this scenario is described by modeling inflation as a two-sided limited-dependent variable. Second, agents might rationally anticipate transitory and/or systematic deviations from the announced bounds. Since in this case the limits would be perceived to be stochastic, rather than fixed, the process of inflation would be akin to a two-sided switching regression.

Since under both specifications inflation follows a nonlinear process, deriving its conditional expectation is nontrivial. For both cases, the analytical representation of inflation forecasts is derived and a stochastic-simulation procedure to calculate their precise numerical value is proposed. It is shown that, as in earlier literature on exchange rate target zones [see, *e.g.,* Krugman (1991)], the expectations of the targeted variable are a nonlinear, S-shaped function of the fundamentals. However, the degree of symmetry and nonlinearity in both models differ [see Pesaran and Ruge-Murcia (1996)], allowing the researcher to statistically identify the model that most faithfully captures the properties of the data.

In an empirical application to the Canadian inflation target zone, results indicate that agents perceive the band to be narrower than officially announced and asymmetrically distributed around the stated mid-value of 2% per year. Specifically, the lower section of the (implicit) band would have a width of 0.77 basis points, while the upper segment would have only 0.47 basis points. This finding suggest that in practice the monetary authority might attach different weights to positive and negative deviations of inflation from the declared mid-value. This outcome would arise theoretically from a policy maker's loss-function that is asymmetric around the desired value rather than the symmetric (usually quadratic) specification frequently used in the literature [see, among others, Barro and Gordon (1983), Rogoff (1985), Walsh (1995), Svensson (1997), and Jensen (1997)]. Specification tests and a formal statistical comparison the two competing representations are also carried out, providing mild support for the stochastic-bound version of the model.

Inflation target zones have been recently adopted by a number of countries [*e.g.,* New Zealand, Canada, Sweden, and the United Kingdom] as a framework for the conduct of monetary policy. Bernanke and Mishkin (1997) and Lafrance (1997) present a detailed description and discussion of this institutional arrangement. Svensson (1997) examines inflation target zones in the commitment-*versus*-discretion setup previously employed by Kydland and Prescott (1977) and Barro and Gordon (1983) and concludes that, due to the inflation bias on the part of the government, realized inflation and inflation expectations would typically exceed the target. In the empirical literature, Svensson (1993) constructs a test of target-zone credibility by calculating the maximum and minimum inflation rates of inflation that are consistent with the targets and substraction them from the nominal yields to maturity to construct upper and lower bounds on the real interest rate. Different versions of the test using *ex-post* real yields, forward rates, and survey data on inflation expectations are presented for a number of countries. Ammer and Freeman (1995) examine the behavior of interest rates and survey data using a Vector Autoregressive (VAR) setup. In contrast to earlier empirical research, this paper addresses the issue of inflation target-zone credibility in the context of a fully-specified framework. That is, the inflation process is econometrically described and then, the agents forecasts consistent with this process and its implications on the long-term nominal interest rate are analytically derived. Finally, the joint inflation/nominal interest rate process is estimated and tested.

The modeling approach in this paper builds on earlier work on limited-dependent rational expectations (LD-RE) models that has examined the behavior of variables subject to announced bounds. For example, Chanda and Maddala (1983), Shonkwiler and Maddala (1985), and Holt and Johnson (1989) study the determination of commodity prices in price-support schemes, and Pesaran and Samiei (1992, 1995) and Pesaran and Ruge-Murcia (1998) analyze exchange rates subject to two-sided limits. However, in contrast to previous LD-RE models, the proposed solution to this model is explicit, numerically exact, and no approximations are required to implement its empirical estimation.

The paper is organized as follows. Section 2 models a perfectly-credible target zone by representing inflation as a two-sided censored variable, derives the appropriate conditional expectations of inflation under this specification, proposes a numerically-exact stochastic-simulation procedure to find the conditional forecasts, and thoroughly discusses the properties of the model solution. Section 3 presents a more general model where agents consider possible transitory and systematic deviations of inflation from the target limits. As in the previous case, the corresponding conditional expectations of inflation and the implication for the interest rate are derived. Section 4 empirically implements both specifications using data for the Canadian inflation target zone and presents estimation and testing results. Finally, Section 5 concludes and discusses some avenues of future research.

2 The Case of Perfect Credibility

The Pure Expectations Hypothesis (PEH) of the term structure of interest rates predicts that the return on a n-period, pure discount bond must equal the average expected return on the sequence of n one-period bonds held over its lifetime.[1] Since, in the presence of uncertainty, both strategies involve different risks, this relationship is usually amended to include a premium that would compensate the bearer of the long-term bond for holding a less-liquid asset. Mathematically,

$$i_t^n = \theta_t^n + (1/n)\left[i_t^1 + E(i_{t+1}^1|I_t) + E(i_{t+2}^1|I_t) + \cdots + E(i_{t+n-1}^1|I_t)\right], \tag{1}$$

for $n \geq 2$, where i_t^n is the nominal return on the n-period bond, I_t is the nondecreasing set of information available to agents at time t, $E(i_{t+s}^1|I_t)$ denotes the conditional expectation of the nominal return on the one-period bond acquired at time $t+s$ for $s = 1, 2, \ldots, n-1$, and θ_t^n is the liquidity premium associated with holding the long-term bond. Following earlier empirical literature [*e.g.,* Shiller (1979) and Hamilton (1988)] and without loss of generality, I assume that *for a given maturity* the liquidity premium is constant. In other words, for a given n, $\theta_t^n = \theta^n$, for all t.

In turn, the relationship between the (short-term) nominal interest rate, the real interest rate, and expected inflation is given by the Fisher equation,

$$i_t^1 = r_t + E(\pi_{t+1}|I_t), \tag{2}$$

where r_t is the *ex-ante* real interest rate and π_{t+1} is the rate of inflation at time $t+1$.[2] Finally, since the *ex-ante* real interest rate is not directly observable, identification restrictions might need to be imposed by the researcher on its stochastic process. In particular, I assume

$$r_t = r + \zeta_t, \tag{3}$$

where r is a constant intercept and ζ_t is a random component that follows an unrestricted ARIMA process. This specification was favored because it renders the model tractable and straightforward to estimate yet, it is general enough to allow nonstationary and conditionally heteroskedastic shocks to the interest rate process,

[1] In the case of coupon-carrying bonds, this hypothesis would equate the long-term return and the *weigthed* average of current and expected future short-term rates where the weights would add up to one [see Shiller (1979)].

[2] Notice that this formulation of the Fisher Equation omits the cross-product term $r_t E(\pi_{t+1}|I_t)$. However for the numerically small values of these variables found in practice, this term is of second order and can be safely ignored. In order to get some idea of the magnitudes involved, assume that the real interest rate and expected inflation are, respectively, 0.01 and 0.05 (that is, 1% and 5% per year). In this case, the cross-product term would be only 0.0005.

if appropriate.[3]

Then, using (2) and (3), (1) can be written as

$$\begin{aligned} i_t^n = \; & \theta^n + r + (1/n)\left[E(\pi_{t+1}|I_t) + E(\pi_{t+2}|I_t) + \cdots E(\pi_{t+n}|I_t)\right] \\ & + (1/(n-1))\left[E(\zeta_{t+1}|I_t) + E(\zeta_{t+2}|I_t) + \cdots + E(\zeta_{t+n}|I_t)\right] \\ & + (1/n)\zeta_t. \end{aligned} \quad (4)$$

Equation (4) explicitly relates the nominal return on a n-period bond to the agents' inflation forecasts during the holding period and will be the focus of this research project. It is easy to show that in the special case when inflation follows a linear stochastic specification, the relation (4) is also linear and, consequently, an innovation in the inflation process yields movements in the long-term interest rate that are symmetric, proportional, and history-independent. Put differently, for linear models, the impulse-response associated with a shock of size 1 (standard deviation) would be the mirror image of the response to a shock of size -1, one-half the response of shock size 2, and independent of the moment the shock is assumed to take place [see Gallant, Rossi, and Tauchen (1993) and Koop, Pesaran, and Potter (1996)].

Instead, this paper models inflation as a nonlinear time series where the nonlinearity arises from a government policy that imposes a publicly-announced target zone on the rate of inflation. This strategy has recently been adopted in New Zealand, Canada, Sweden, Australia, Spain, Israel, and the United Kingdom and entails a commitment by the Central Bank to pursue a monetary policy consistent with keeping inflation between specific lower and upper bounds.[4] Bernanke and Mishkin (1997) and Lafrance (1997) review the basic features of this framework and highlight the emphasis placed by the monetary authority on the timely publication of government statistics and other relevant information that would allow the public to assess the extent to which the Central Bank is close to the desired target.

This section considers the situation when the inflation target zone is perfectly credible in the sense that economic agents genuinely believe that inflation will only take values between the stated lower and upper limits. These limits are denoted by $\underline{\pi}_t$ and $\overline{\pi}_t$, respectively. Notice that the bounds can be time-varying but that, due to the pre-announced and credible nature of the band, their future values are assumed to be included in the agents' set of information at time t.

For this case, I econometrically describe the government policy outlined above by assimilating the inflation process to the one of a two-sided censored variable. Assume that the latent value of inflation (*i.e.*, the one that would take place in the absence of bounds) is generated by the (possibly nonstationary) process,

[3] I also considered modeling r_t as an unobserved component and recasting the model in state-space form. Unfortunately, the estimation of such specification is severely limited by the small sample sizes currently available to empirically examine inflation target zones.

[4] Finland also announced an inflation target of 2% per year in February 1993. However, in contrast to the countries above, its target is not specified as a range but as a sole numerical value.

$$\pi^*_{t+1} = \alpha + \psi(L)\pi_{t+1} + e_{t+1}, \tag{5}$$

where π^*_t is latent inflation, π_t is actual (or realized) inflation, α is an intercept term, L is the lag operator, $\psi(L)$ stands for the polynomial $\sum_{j=1}^{q} \psi_j L^j$, and the disturbance term, e_t, is assumed independently and identically distributed ($i.i.d.$) with mean 0 and variance σ_e^2.[5] In the presence of credible limits on the inflation process, its realized values need not always coincide with the latent ones but the two would be related according to,

$$\pi_{t+1} = \begin{cases} \overline{\pi}_t, & \text{if} \quad \pi^*_{t+1} \geq \overline{\pi}_t, \\ \pi^*_{t+1}, & \text{if} \quad \underline{\pi}_t < \pi^*_{t+1} < \overline{\pi}_t, \\ \underline{\pi}_t, & \text{if} \quad \pi^*_{t+1} \leq \underline{\pi}_t, \end{cases} \tag{6}$$

or, more concisely,

$$\pi_{t+1} = Min\left\{\overline{\pi}_t, Max\left\{\pi^*_{t+1}, \underline{\pi}_t\right\}\right\}.$$

Having proposed a simple econometric characterization for a credible inflation target zone, it only remains to derive the implications of such policy on the long-term interest rate. In order to find the closed-form representation of (4) for this case, it is necessary to find the conditional expectations of future inflation, $E(\pi_{t+s}|I_t)$ for $s = 1, 2, \ldots, n$, during the holding period. The following proposition establishes the appropriate inflation expectations.

Proposition 1. *Assume that inflation follows the two-sided, limited-dependent process (6). Define the composite error term*

$$u_{s,t+s} = e_{t+s} + \sum_{k=1}^{s-1} \psi_{s-k}\eta_{k,t+k}, \tag{7}$$

where $\eta_{k,t+k} = \pi_{t+k} - E(\pi_{t+k}|I_t)$, *with cumulative distribution and density functions denoted by* $F_s(\cdot)$ *and* $f_s(\cdot)$, *respectively. Define the variables*

$$\overline{c}_{t+s} = \overline{\pi}_{t+s} - E(\pi^*_{t+s}|I_t), \tag{8}$$

and,

$$\underline{c}_{t+s} = \underline{\pi}_{t+s} - E(\pi^*_{t+s}|I_t), \tag{9}$$

[5] Lee (1997) examines a LD-RE model with serially correlated shocks. In this case, lagged latent variables are implicitly included among the model regressors. In order to address the difficulties associated with this unobserved explanatory variable, Lee proposes the use of stochastic simulation. Notice that the formulation (5) contains only lagged realizations of inflation (as opposed to its latent values) among the explanatory variables. More general specifications of latent inflation that allow for additional regressors are examined below.

where

$$E(\pi^*_{t+s}|I_t) = \alpha + \sum_{k=1}^{\min\{q,s-1\}} \psi_k E(\pi_{t+s-k}|I_t) + \sum_{j=s}^{q} \psi_j L^j \pi_{t+s}. \tag{10}$$

Then, the conditional expectation of inflation at time $t+s$ is given by,

$$\begin{aligned} E(\pi_{t+s}|I_t) = \ & \overline{\pi}_t \left[1 - F_s(\overline{c}_{t+s})\right] + \underline{\pi}_t F_s(\underline{c}_{t+s}) \\ & + \left\{ \left[E(\pi^*_{t+s}|I_t) + E(u_{s,t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) \right] \right. \\ & \left. \cdot \left[F_s(\overline{c}_{t+s}) - F_s(\underline{c}_{t+s}) \right] \right\}. \end{aligned} \tag{11}$$

Proof. Use the definitions of $u_{s,t+s}$, $\underline{c}_{t+s}$, and $\overline{c}_{t+s}$ to write the process of inflation at time $t+s$ as

$$\pi_{t+s} = \begin{cases} \overline{\pi}_{t+s}, & \text{if} \quad u_{s,t+s} \geq \overline{c}_{t+s}, \\ E(\pi^*_{t+s}|I_t) + u_{s,t+s}, & \text{if} \quad \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}, \\ \underline{\pi}_{t+s}, & \text{if} \quad u_{s,t+s} \leq \underline{c}_{t+s}. \end{cases}$$

Note that (i) $\underline{c}_{t+s}$ and $\overline{c}_{t+s}$ are known at time t because the band is publicly announced and the forecast $E(\pi^*_{t+s}|I_t)$ is known at time t, and (ii) the random term $u_{s,t+s}$ incorporates all uncertainty (including forecast errors) regarding the rate of inflation at time $t+s$. Then, the conditional expectation of inflation, $E(\pi_{t+s}|I_t)$, is given by the probability-weighted average of the predictions conditional on inflation being at the top, bottom, and middle of the band. That is,

$$\begin{aligned} E(\pi_{t+s}|I_t) = \ & E(\pi_{t+s}|I_t, u_{s,t+s} \geq \overline{c}_{t+s}) \Pr(u_{s,t+s} \geq \overline{c}_{t+s}) \\ & + E(\pi_{t+s}|I_t, u_{s,t+s} \leq \underline{c}_{t+s}) \Pr(u_{s,t+s} \leq \underline{c}_{t+s}) \\ & + E(\pi_{t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) \Pr(\underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}). \end{aligned} \tag{12}$$

Using the appropriate expressions for the above conditional expectations [see *e.g.*, Lee (1994)] and the relations $\Pr(u_{s,t+s} \geq \overline{c}_{t+s}) = 1 - F_s(\overline{c}_{t+s})$, $\Pr(u_{s,t+s} \leq \underline{c}_{t+s}) = F_s(\underline{c}_{t+s})$, and $\Pr(\underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) = F_s(\overline{c}_{t+s}) - F_s(\underline{c}_{t+s})$, (12) can be written as,

$$\begin{aligned} E(\pi_{t+s}|I_t) = \ & \overline{\pi}_t \left[1 - F_s(\overline{c}_{t+s})\right] + \underline{\pi}_t F_s(\underline{c}_{t+s}) \\ & + \left\{ \left[E(\pi^*_{t+s}|I_t) + E(u_{s,t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) \right] \right. \\ & \left. \cdot \left[F_s(\overline{c}_{t+s}) - F_s(\underline{c}_{t+s}) \right] \right\}. \| \end{aligned}$$

2.1 Properties of the Solution

A number of important features of the model solution are now examined. *First*, notice that in principle the solution (11) holds regardless of whether the inflation process within the band is stationary or not.[6] Specifically, the solution holds when inflation follows a random walk inside the target zone. In this case the overall inflation process would follow the discrete-time equivalent of a regulated Brownian motion. Restrepo-Londoño (1996) employs precisely this continuous-time specification to describe (tacit) inflation targeting in Colombia during the period 1973 to 1994.

Second, while the solution above was derived for the case when inflation is subject to a two-sided target zone, results for the one-sided case can be easily obtained by letting $\underline{\pi}_t$ decrease without limit. Thus, this model (and its estimation procedure developed below) could be applied, for example, to the case of Spain whose inflation target zone is specified only as an upper ceiling of 3% per year [see Bernanke and Mishkin (1997, p. 99)]. Also, note that in the case when $\overline{\pi}_t \rightarrow \infty$ and $\underline{\pi}_t \rightarrow -\infty$, the solution reduces to the one of the linear model without bounds.

Third, notice that for $s = 1$, $u_{1,t+1} = e_{t+1}$. Thus, the cumulative and density functions of $u_{1,t+1}$ correspond to the ones assumed for e_{t+1}. For example, when e_{t+1} is assumed normally distributed, $E(\pi_{t+1}|I_t)$ can be found analytically using well-known results for censored normal variables [see Maddala (1983, p. 366)],

$$\begin{aligned} E(\pi_{t+1}|I_t) = \; & \overline{\pi}_t \left[1 - F_1(\overline{c}_{t+1})\right] + \underline{\pi}_t F_1(\underline{c}_{t+1}) \\ & + E(\pi^*_{t+1}|I_t)\left[F_1(\overline{c}_{t+1}) - F_1(\underline{c}_{t+1})\right] \\ & + \left[f_1(\underline{c}_{t+1}) - f_1(\overline{c}_{t+1})\right], \end{aligned} \tag{13}$$

where F_1 and f_1 would correspond (respectively) to the cumulative and density function of a normal variable with zero mean and variance σ_e^2. On the other hand, for $s > 1$, $u_{s,t+s}$ includes as components inflation forecast errors, namely $\eta_{k,t+k} = \pi_{t+k} - E(\pi_{t+k}|I_t)$ for $k = 1, 2, \ldots, \min\{q, s-1\}$, that as a result the limited-dependent nature of π_t do not follow a standard distribution.

In order to illustrate this important aspect of the solution, a nonparametric estimate of the density of one-step-ahead forecast errors for a specific numerical example is now presented. Assume that latent inflation is generated by the simple process,

$$\pi^*_{t+1} = 0.1 + 0.95\pi_t + e_{t+1}, \tag{14}$$

where e_t is *i.i.d.* $N(0, 0.5^2)$, and the lower and upper bounds of the inflation target zone are $\underline{\pi}_t = 1$ and $\overline{\pi}_t = 3$, respectively. For this example, the unconditional inflation mean corresponds exactly to the center of the band. A time-series of 10000 observations of inflation was generated iterating on (14) and enforcing numerically the constraint, $\pi_{t+1} = Min\left\{3, Max\left\{\pi^*_{t+1}, 1\right\}\right\}$. The conditional one-step-ahead inflation forecasts, $E(\pi_{t+1}|I_t)$, were obtained using the analytical expression for normal disturbances in (13). Finally, the associated forecasts errors were simply computed

[6] However, the notion that the band is credible would be under question if inflation were to follow a truly explosive process.

as the difference between the two, that is, $\eta_{1,t+1} = \pi_{t+1} - E(\pi_{t+1}|I_t)$. A nonparametric estimate of the density of $\eta_{1,t+1}$ is presented in figure 1. Notice that despite the fact that e_t is normally distributed, $\eta_{1,t+1}$ appears to follows a nonstandard bimodal distribution. Since the actual shape of the distribution might depend on the forecast horizon and model parameters,[7] an analytical description of the composite error $u_{s,t+s}$ appears unfeasible. Moreover, while expectations of $\eta_{k,t+k}$ conditional on I_t are zero, the model solution involves precisely the more complicated conditional expectations of their truncated linear combination defined in (7).

In order to circumvent this problem, this paper proposes the use of stochastic simulation to numerically calculate the values of $F_s(\overline{c}_{t+s})$, $F_s(\underline{c}_{t+s})$, and $E(u_{s,t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s})$ that depend on the distribution of $u_{s,t+s}$. While this strategy might appear computationally demanding, its practical application is facilitated by the observation that all the conditional expectations involved in the term-structure equation (4) can be found recursively by the straightforward application of the law of iterated projections. First, having found (perhaps, analytically) the one-step-ahead forecast $E(\pi_{t+1}|I_t)$, plug its value into the equivalent expressions (8), (9), and (10) for $s = 2$ to find $\overline{c}_{t+2}$ and $\underline{c}_{t+2}$. Then, construct observations of $u_{2,t+2}$ by (*i*) simulating a suitably large number of inflation paths to construct forecast errors $\eta_{1,t+1}$(as in the example above) and (*ii*) combining them with draws from the distribution of innovations e_{t+2} according to (7). The empirical counterparts of $F_2(\overline{c}_{t+2})$ and $F_2(\underline{c}_{t+2})$ are given by the proportion of paths that hit the upper and lower limits while an estimate of $E(u_{2,t+2}|I_t, \underline{c}_{t+2} < u_{2,t+2} < \overline{c}_{t+2})$ can be constructed by taking the arithmetic average of observations of $u_{2,t+2}$ that fall between $\underline{c}_{t+2}$ and $\overline{c}_{t+2}$. Applying the relation (11), delivers $E(\pi_{t+2}|I_t)$. Using $E(\pi_{t+2}|I_t)$, the procedure can then be recursively repeated for $s = 3, 4$, and so forth.

Notice that as presented above, the simulation strategy to numerically find the conditional expectations takes as given the structural parameters. Thus, when estimating the model by the method of Maximum Likelihood (as suggested below), this routine needs to be applied for each of the observations in the sample in each step of the hill-climbing algorithm.

Fourth, in contrast to earlier limited-dependent rational expectations (LD-RE) models, the solution of this model is explicit and numerically exact. For example, in the class of LD-RE models examined by Donald and Maddala (1992), Lee (1994), and Pesaran and Ruge-Murcia (1996, 1998), the RE solution is only implicit and numerical procedures are required to find the fixed point that satisfies the solution equation. Other LD-RE specifications [Holt and Johnson (1989), Pesaran and Samiei (1995), and Pesaran and Ruge-Murcia (1998)] might also require the use of approximations to the solution in order to simplify or to make feasible the estimation of the model. In this case while the solution depends on the number of simulated inflation paths,

[7]This observation is based on similar experiments (not shown) involving different values of s, α, β and σ_e^2.

any discrepancy between the simulated and the true conditional expectation can be made as small as desired by simply increasing the number of simulations.

Fifth, as in previous literature in exchange-rate target zone models in both discrete [*e.g.,* Pesaran and Samiei (1992), Koedijk, Stork, and de Vries (1997)] and continuous time [*e.g.,* Krugman (1991), Flood and Garber (1991)], the solution of the model is nonlinear on the fundamentals as a result of the effect of the band on expectations. In particular, the well-known S-shaped relationship between the targeted variable and its fundamentals obtains.

For the discussion that follows it might useful to illustrate this relationship at different forecast horizons. To that effect, consider the numerical example introduced above where $\pi^*_{t+1} = 0.1 + 0.95\pi_t + e_{t+1}$, e_t is *i.i.d.* $N(0, 0.5^2)$, $\underline{\pi}_t = 1$, $\overline{\pi}_t = 3$, and the band is perfectly credible in the sense the constraint $\pi_{t+1} = Min\left\{3, Max\left\{\pi^*_{t+1}, 1\right\}\right\}$ is perceived to hold at all times. Using the simulation procedure describe in this section, the conditional expectations of inflation were computed for different values of the fundamentals, that in this case consist solely of current inflation. The results of this exercise are plotted in figure 2 for horizons $s = 1, 2, 3$, and 6. Notice that as claimed, the mapping from fundamentals to expectations is nonlinear and S-shaped but becomes noticeably less nonlinear as the horizon increases. This observation can be intuitively explained by the fact that as s increases, the conditional inflation forecast approaches the unconditional mean (that in this case coincides with the mid-point of the inflation target zone) and the effect of the band on expectations decreases.

Up to the extent that the long-term interest rate is a function of the average of inflation forecasts (as expressed in (4)), the properties of the solution derived under the assumption of perfect credibility generate specific implications on the time series properties of i^n_t. In particular, the process of i^n_t is also bounded (up to the idiosyncratic shock $(1/n)\zeta_t$) by the inflation target zone but with its mean shifted by the (possibly) time-varying term

$$\theta^n + r + (1/(n-1))\left[E(\zeta_{t+1}|I_t) + E(\zeta_{t+2}|I_t) + \cdots + E(\zeta_{t+n}|I_t)\right].$$

It also follows that the interest rate and current fundamentals are nonlinearly related. However, since (*i*) the nonlinearity of inflation forecasts decreases with the horizon and (*ii*) the longer the holding period, the smaller the weight received by the shorter-horizon, more nonlinear forecasts, then the effects of the inflation target zone on interest rates should be more evident in bonds of shorter-maturity and might be undetectable in, say, 10-year bonds.

Sixth, this empirical paper abstracts from possible strategic interactions between agents and the Central Bank by assuming that in constructing their inflation forecasts, agents take as given the parametric process of inflation in (5). Bernanke and Mishkin (1997, pp. 101-102) note that Central Banks understand that monetary policy might affect inflation with a lag and, consequently, base their policy decisions on their own forecasts of the likely path of prices and measures of money conditions, commodity prices, capacity utilization, etc. In certain cases, the monetary authority's inflation forecasts might be periodically published.[8] The fact that both agents and policy makers construct expectations about the same variable and know that the other player's actions might affect the final outcome, could result in the well-known forecasting-the-forecast-of-others problem. This issue, that typically arises in rational expectations model with heterogenous agents, has been has been examined by a number of earlier researchers [see, *e.g.,* Townsend (1983)]. Alternatively, one could consider a game-theoretical approach where agents' construct their expectations with knowledge of the Central Bank's loss function and, therefore, understand its incentive to deviate from the announced policy [as Barro and Gordon (1983), Green (1996), Svensson (1997), and Jensen (1997)].

Finally, the model above has postulated and solved an univariate representation of the inflation process solely for the sake of simplicity. It is easy to show that additional explanatory variables (*e.g.,* the rate of growth of money supply) can be easily in introduced to (5). This more general case and its solution for the case of a perfectly credible band are presented in appendix A.

3 The Case of Stochastic Bounds

This section examines a more general version of the model developed above where agents might rationally expect deviations of the inflation rate from the official targets. Conceptually, it will be useful to classify these deviations as either transitory or systematic. The former refer to temporary, short-lived departures of inflation from the announced upper or lower bounds, possibly as a result of shocks outside the control of the monetary authority (*e.g.,* changes in the terms of trade or indirect taxes). That this situation might indeed arise is acknowledged in the current blueprints for inflation target zones. For example, the Policy Target Agreement (PTA) in New Zealand includes specific circumstances where the Reserve Bank would be exempt from meeting the inflation target [see Ammer and Freeman (1995, p. 173)]. The second type of deviations (*i.e.,* systematic) is meant to capture the notion that agents might perceive monetary policy to be inconsistent with the official target zone but compatible with a band that is wider, narrower, and/or less symmetric than the one announced by policy makers. The premise of this paper is that up to the extent that the term structure embodies agents' inflation forecasts, their beliefs regarding the band could be inferred by econometrically examining the joint inflation/interest rate process.

[8] For example, the Bank of England publishes a quarterly *Inflation Report* that contains the Bank's probabilistic forecasts of inflation and a detailed analysis of its monetary policy. Similar publications are produced by the Bank of Canada and the Reserve Bank of New Zealand.

In this case, the inflation process (as perceived by agents) would be characterized by the nonlinear specification,

$$\pi_{t+1} = \begin{cases} \overline{\pi}_{t+1} + \overline{b} + v_{t+1}, & \text{if} \quad \pi^*_{t+1} \geq \overline{\pi}_{t+1} + \overline{b} + v_{t+1}, \\ \pi^*_{t+1}, & \text{if} \quad \underline{\pi}_{t+1} + \underline{b} + v_{t+1} < \pi^*_{t+1} < \overline{\pi}_{t+1} + \overline{b} + v_{t+1}, \\ \underline{\pi}_{t+1} + \underline{b} + v_{t+1}, & \text{if} \quad \pi^*_{t+1} \leq \underline{\pi}_{t+1} + \underline{b} + v_{t+1}, \end{cases} \quad (15)$$

where π_t is realized inflation and π^*_t is latent inflation. As before, π^*_t is assumed to be determined according to the process (5). The notation $\overline{b}$ and $\underline{b}$ denote constant components assumed to be known by agents but not by the econometrician, and v_t is an *i.i.d.* $(0, \sigma^2_v)$ random term contemporaneously uncorrelated with e_t and ζ_t.[9]

In order to develop further the economic intuition about $\overline{b}$, $\underline{b}$, and v_t, consider first the situation when $\overline{b} = \underline{b} = 0$. Then, the process in (15) would differ from the perfectly-credible case (6) only by the random term, v_{t+1}, tagged to the announced upper and lower bounds. For large-enough values of π^*_{t+1} (and positive values of v_{t+1}), it might be possible to observe levels of inflation that are above the upper limit $\overline{\pi}_{t+1}$, but up to the extent that v_{t+1} is *i.i.d.* such deviations would only be temporary.

Now, assume that $\overline{b} > 0$ and $\underline{b} < 0$. In this case (in addition to possible transitory deviations), agents would perceive a *systematically* wider inflation target zone than announced by the monetary authority. The converse would hold when $\overline{b} < 0$ and $\underline{b} > 0$. Moreover, up to the extent that $\overline{b}$ might differ from $\underline{b}$, agents would infer that the band is not symmetric around the official mid-point. The process postulated in (15) captures all these possibilities in a unified framework and includes the case of a perfectly-credible target zone as a special case when $\overline{b} = \underline{b} = 0$ and the distribution of v_t is degenerate and takes value zero at all times.

Notice that in econometric terms, the inflation process in a target zone with stochastic bounds is no longer described by a limited-dependent specification but rather by a two-sided, endogenous switching regression [Pesaran and Ruge-Murcia (1996)]. The appropriate conditional expectations of inflation in this case are given by the following proposition.

Proposition 2. *Assume that inflation follows the two-sided switching-regression process (15). Define the composite error term*

$$u_{s,t+s} = e_{t+s} - v_{t+s} + \sum_{k=1}^{s-1} \psi_{s-k}\eta_{k,t+k},$$

where $\eta_{k,t+k} = \pi_{t+k} - E(\pi_{t+k}|I_t)$, *with cumulative distribution and density functions denoted by* $H_s(\cdot)$ *and* $h_s(\cdot)$, *respectively. Define the variables*

[9] I also considered the possibility of allowing different random shocks to the upper and lower bounds, but was unable to find a tractable solution. The problem arises because non-trivial conditions need to be imposed on the two distributions to rule out the possibility that the upper bound might take a value inferior to the lower bound with the meaningless implication that the width of the target zone would be negative.

$$\overline{c}_{t+s} = \overline{\pi}_{t+s} + \overline{b} - E(\pi^*_{t+s}|I_t),$$

and,

$$\underline{c}_{t+s} = \underline{\pi}_{t+s} + \underline{b} - E(\pi^*_{t+s}|I_t),$$

where

$$E(\pi^*_{t+s}|I_t) = \alpha + \sum_{k=1}^{\min\{q,s-1\}} \psi_k E(\pi_{t+s-k}|I_t) + \sum_{j=s}^{q} \psi_j L^j \pi_{t+s}.$$

Then, the conditional expectation of inflation at time $t+s$ is given by,

$$\begin{aligned} E(\pi_{t+s}|I_t) = \; & (\overline{\pi}_t + \overline{b})\,[1 - H_s(\overline{c}_{t+s})] + (\underline{\pi}_t + \underline{b}) H_s(\underline{c}_{t+s}) \\ & + \{ \left[E(\pi^*_{t+s}|I_t) + E(u_{s,t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) \right] \\ & \cdot \, [H_s(\overline{c}_{t+s}) - H_s(\underline{c}_{t+s})] \}. \end{aligned} \tag{16}$$

Proof. Use the definitions of $u_{s,t+s}$, $\underline{c}_{t+s}$, and $\overline{c}_{t+s}$ to write the process of inflation at time $t+s$ as

$$\pi_{t+s} = \begin{cases} \overline{\pi}_{t+s} + \overline{b} + v_{t+s}, & \text{if} \quad u_{s,t+s} \geq \overline{c}_{t+s}, \\ E(\pi^*_{t+s}|I_t) + e_{t+s} + \sum_{k=1}^{s-1} \psi_{s-k} \eta_{k,t+k} & \text{if} \quad \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}, \\ \underline{\pi}_{t+s} + \underline{b} + v_{t+s}, & \text{if} \quad u_{s,t+s} \leq \underline{c}_{t+s}. \end{cases} \tag{17}$$

Note that (i) $\underline{c}_{t+s}$ and $\overline{c}_{t+s}$ are known at time t because the announced target zone, the forecast $E(\pi^*_{t+s}|I_t)$, and the values of $\underline{b}$ and $\overline{b}$ are known to the agents at time t, and (ii) the random term $u_{s,t+s}$ incorporates all uncertainty (including forecast errors) regarding the rate of inflation at time $t+s$. Then, the conditional expectation of inflation, $E(\pi_{t+s}|I_t)$, is given by the probability-weighted average of the predictions conditional on inflation being at the top, bottom, and middle of the band. That is,

$$\begin{aligned} E(\pi_{t+s}|I_t) = \; & E(\pi_{t+s}|I_t, u_{s,t+s} \geq \overline{c}_{t+s}) \Pr(u_{s,t+s} \geq \overline{c}_{t+s}) \\ & + E(\pi_{t+s}|I_t, u_{s,t+s} \leq \underline{c}_{t+s}) \Pr(u_{s,t+s} \leq \underline{c}_{t+s}) \\ & + E(\pi_{t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) \Pr(\underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}). \end{aligned}$$

Noting that,

$$\begin{aligned} & E(v_{t+s}|I_t, u_{s,t+s} \geq \overline{c}_{t+s})\,[1 - H_s(\overline{c}_{t+s})] \\ & + E(e_{t+s} + \sum_{k=1}^{s-1} \psi_{s-k} \eta_{k,t+k} | I_t, u_{s,t+s} \leq \underline{c}_{t+s})\,[H_s(\overline{c}_{t+s}) - H_s(\underline{c}_{t+s})] \\ & + E(v_{t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) H_s(\underline{c}_{t+s}), \\ & = E(u_{s,t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s})\,[F_s(\overline{c}_{t+s}) - F_s(\underline{c}_{t+s})]\,, \end{aligned}$$

and using the relations $\Pr(u_{s,t+s} \geq \overline{c}_{t+s}) = 1 - H_s(\overline{c}_{t+s})$, $\Pr(u_{s,t+s} \leq \underline{c}_{t+s}) = H_s(\underline{c}_{t+s})$, and $\Pr(\underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) = H_s(\overline{c}_{t+s}) - H_s(\underline{c}_{t+s})$, the conditional expectation can be written as,

$$\begin{aligned} E(\pi_{t+s}|I_t) = \; & (\overline{\pi}_t + \overline{b}) \left[1 - H_s(\overline{c}_{t+s})\right] + (\underline{\pi}_t + \underline{b}) H_s(\underline{c}_{t+s}) \\ & + \left[E(\pi^*_{t+s}|I_t) + E(u_{s,t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s})\right] \\ & \cdot \left[H_s(\overline{c}_{t+s}) - H_s(\underline{c}_{t+s})\right] . \| \end{aligned}$$

3.1 Properties of the Solution

This more general model shares some features with the perfect-credibility version developed above. As before the solution (16) holds regardless of whether the inflation process within the band is stationary or not and, in particular, it holds when inflation follows a random walk inside the target zone. In this case, the overall inflation process follows a renewal process [not unlike Deaton and Laroque (1992)] where the variable follows an integrated process until it reaches (in finite time) either of the time-varying kinks, $\overline{\pi}_t + \overline{b} + v_t$ or $\underline{\pi}_t + \underline{b} + v_t$, where inflation would be generated by the same *i.i.d.* process that generates the stochastic upper or lower bound.

As in the previous case, the one-step-ahead inflation forecast is made tractable by the absence of forecast errors of the endogenous variable in the composite error $u_{1,t}$, but longer horizons require the recursive use of stochastic simulation to numerically find the conditional expectations. The implementation of the procedure for the case of a stochastic band is a simple extension of the routine described in section 2.1.

While the relationship between expectations and fundamentals is also nonlinear in the model with stochastic bounds, its precise form (or shape) crucially depends on the values of $\underline{b}$ and $\overline{b}$, and to a lesser extent on the magnitude of σ^2_v. To illustrate graphically this observation, consider the earlier example where $\pi^*_{t+1} = 0.1 + 0.95\pi_t + e_{t+1}$, and e_t is *i.i.d.* $N(0, 0.5^2)$, but the lower and upper bounds of the inflation target zone are now perceived by the agents to be $\underline{\pi}_t = 0.75 + v_t$ and $\overline{\pi}_t = 3.25 + v_t$, respectively, rather the announced $\underline{\pi}_t = 1$ and $\overline{\pi}_t = 3$ (that is, the perceived band is *wider* than the official one). The random deviation v_t is assumed *i.i.d.* $N(0, 0.5^2)$. Employing the simulation strategy described in section 2.1, the conditional expectations of inflation for different horizons are calculated for different values of the fundamentals (*i.e.,* current inflation) and plotted in figure 3. The reader should notice that, as in the previous case, as the horizon increases the conditional expectation approaches the unconditional inflation mean. More importantly, compared with figure 2, the relation between the inflation forecast and the fundamentals is less nonlinear in the case of stochastic bounds *at all horizons,* reflecting the smaller effect of the less-credible band on expectations. The converse result obtains when $\overline{b} < 0$ and $\underline{b} > 0$, that is the solution is *more* nonlinear than in the perfectly-credible case.

Furthermore, in the situation when $\overline{b} \neq \underline{b}$, this relationship is no longer symmetric. As an illustration, consider the same example above but with $\overline{b} = 0$ and $\underline{b} = -1$.

Then, $\overline{\pi}_t = 3 + v_t$ and $\underline{\pi}_t = 0 + v_t$. The conditional expectations corresponding to this case are plotted as a function of current inflation in figure 4. Notice that only in the upper part of the band does the solution fully reflect the effect of the bounds on expectations.

In an extreme scenario agents might infer that $\overline{b} \rightarrow \infty$ and $\underline{b} \rightarrow -\infty$ (*i.e.*, the actual width of the band is infinite). In this case, from the agents' perspective, the announced band has no bearing for the conduct of monetary policy and the conditional inflation forecast would become a linear function of current inflation. Consequently, the relationship between the long-term interest rate and current inflation would be linear as well.

From these observations, it is clear that the predicted relationship between interest rates and inflation crucially depends on agents' beliefs about the announced target zone. Hence, an econometrician could draw empirical conclusions about these beliefs by constructing a fully-specified model that imposes restrictions of the joint time-series process of inflation and the nominal interest rate. In particular, such restrictions would allow the identification of the model parameters that underlie agents' perceptions about announced inflation targets.

Finally, the model with stochastic bounds can also be extended to allow for additional explanatory variables in the process of latent inflation. This specification and its solution is presented in appendix A.

4 Empirical Implementation

4.1 The Data

The models developed in sections 2 and 3 were estimated using data on inflation and nominal interests rates for the Canadian target zone. These targets were announced in February 1991 and originally envisaged a reduction in the annual rate of inflation to 3% by the end of 1992, 2.5% by mid-1994, and 2% by the end of 1995. In December 1993, the inflation target of 2% per year was extended to the end of 1998. Throughout, the targets are regarded as mid-points in a band of plus or minus one percentage point and apply to a measure of core inflation that excludes the volatile food and energy components and the first-round effect of changes in indirect taxes. However, as pointed out in the original announcement, over time movements in the total Consumer Price Index (CPI) and the CPI excluding food and energy should be broadly similar [see Bank of Canada Review (March 1991, p. 5)]. Figure 4 presents the rate of (core) inflation in Canada between January 1990 and October 1997 and the announced targets.

The nominal interest rate corresponds to the weighted average tender rate of three- and six- month Treasury Bills in the last Tuesday of the month. The average is taken over the successful bids in the weekly auction (that is held each Tuesday).

The yields are calculated on a 365-day true-yield basis and are collected, respectively, in the series B113903 and B113904 of the Bank of Canada. Figure 5 presents these two series for the period January 1990 to October 1997. The fact the behavior of the three- and six- month interest rate is not fundamentally different accounts for the similarities in the results presented below.

The sample consists of monthly observations of core inflation and interest rates between May 1994 to October 1997. It is important to note that the first period of the target zone (that is, between December 1992 and May 1994) *is not* included in the sample. As the reader can see in figure 4, this transitional interval is characterized by a rate of inflation that is for the most part well-below the announced target. Consequently, the models developed above do not generate empirical predictions for the behavior of inflation and interest rates during this period. Finally, while the sample size is severely limited by the recent nature of the target zone policy, the estimated specifications are extremely parsimonious (involving the estimation of only 5 and 7 parameters, respectively) and enough degrees of freedom remain to reliably estimate and test the models.

As discussed above, when the announced bounds are constant, the perfectly-credible and stochastic-bound models both imply that while inflation could be represented by a nonstationary specification *inside* the target zone, the *overall* inflation process should be ergodic. Intuitively, inflation is reset whenever it reaches the fixed or stochastic bound, so that distant-enough observations should be independent. On the other hand, results from Augmented Dickey-Fuller (ADF) tests indicate that the process of the nominal interest rate might contain a unit root.[10] These two observations are reconciled by assuming that the real interest rate follows an integrated process of the form,[11]

$$r_t = r + \zeta_t,$$

where

$$\zeta_t = a + \zeta_{t-1} + w_t,$$

and w_t is *i.i.d.* $(0, \sigma_w^2)$. In other words, shocks to the *ex-ante* real interest rate are assumed to follow a random walk with *i.i.d.* innovations or, equivalently, an ARIMA (0, 1, 0). Some specification tests on the dynamic properties of w_t are presented below. Specializing equation (4) for this case and first-differencing yields,

[10]The ADF tests were performed by running an OLS regression of the interest rate on an intercept term, the lagged interest rate, and three of its lagged differences. The *OLS t* statistics for the three- and six-month rates were respectively -1.335 [-2.93] and -1.305 [-2.93] where the bracketed figures are 95 % critical values of the tabulated distribution. Still, since the ADF test was originally developed under the assumption of linearity, these results should be cautiously interpreted.

[11]Empirical evidence that the *ex-ante* real interest rate might be non-stationary is presented by Rose (1988) for various countries and sample periods. A similar result is reported by Ruge-Murcia (1995) using Israeli data.

$$\Delta i_t^n = \quad a + (1/n) \sum_{s=1}^{n} \Delta E(\pi_{t+s}|I_t) + w_t, \tag{18}$$

where $\Delta i_t^n = i_t^n - i_{t-1}^n$ and $\Delta E(\pi_{t+s}|I_t) = E(\pi_{t+s}|I_t) - E(\pi_{t+s-1}|I_{t-1})$. Under (18), changes in the nominal interest rate primarily reflect the agents' revision about the expected path of inflation during the holding period. In turn, these expectations depend on the agents inferences about the nature of the inflation target zone (*i.e.*, whether perfectly credible or subject to stochastic bounds). Notice that the assumptions about the time-series process of r_t, yield a simple and tractable counterpart to (4), reconcile the theoretical model with empirical evidence, and avoid the need to construct observations of the real interest rate to carry out the estimation procedure.

4.2 Estimation Results

The models developed above can be readily estimated by the numerical maximization of their log likelihood function. The derivation of these functions are presented in appendix B under the assumption that the model disturbances are normally distributed with zero means and constant conditional variances. In the case when the band is perfectly credible, the log likelihood function is akin to the two-sided Tobit model, while in the situation when the bounds are stochastic the inflation density is neither a standard nor a truncated normal but it is instead a mixture of normal distributions. Since, the functions explicitly impose the cross-equation restrictions that arise from the dependence of Δi_t^n (through $E(\pi_{t+s}|I_t)$) on the inflation process, their maximization yields consistent, Full Information Maximum Likelihood (FIML) estimates of the structural parameters.

Prior to the estimation of the model, the number of lags for the inflation process was selected. Processes with lag length 1, 2, and 3 were considered and a set of Likelihood Ratio (LR) tests indicated that the most appropriate specification for inflation was a parsimonious AR(1) process. Then, the maximization the log likelihood functions was performed using the Broyden-Fletcher-Golfarb-Shanno (BFGS) algorithm provided by Gauss 3.2. For both models, the asymptotic variance-covariance matrix was estimated by the inverse of the Hessian of the log likelihood function at the maximum.

A well-known feature of models that involve mixtures of normal distributions is that their log likelihood function might present numerous local maxima. Therefore, for the model with stochastic bounds, the robustness of the global maximum was assessed by considering several randomly chosen starting points for the optimization routine. Furthermore, results indicated that (at the global maximum) the AR coefficient for the process of inflation *inside* the band was quite close to 1 but the numerical inversion of the Hessian proved futile. Thus, for the purpose of calculating standard errors of the estimates, the condition $\psi_1 = 1$ was imposed. Another difficulty associated with this type of models is that the log likelihood function can be unbounded. Such singularities would arise whenever one of the distributions is imputed to have a mean exactly equal to one of the observations with no variance. This issue is further discussed by Maddala (1983, pp. 299-301) and Hamilton (1991). However, Amemiya and Sen (1977) and Kiefer (1978) show that selecting the largest bounded maximum (as done here) yields consistent, asymptotically efficient estimates of the model parameters.

The conditional expectations of inflation that enter into the interest-rate equation were calculated for each observation in the sample in each iteration of the maximization routine, using the stochastic-simulation procedure proposed in section 2.1 with 5000 simulated inflation paths. In order the assess the effect of the number of paths on the parameter estimates, I also employed 100 and 1000 paths with remarkably similar results.[12]

The estimates for the model of a perfect-credible band using the three- and six month nominal interest rate are presented in the second and third columns of table 1. Results are reported in units of 100 basis points. Given the similarities in the two nominal interest rate series documented in figure 5, it is not surprising that the results are quantitatively and qualitatively similar. Under this specification, inflation inside the band would appear to follow a mean-reverting process with an unconditional mean of 1.78 (0.19) basis points with a standard deviation of 0.37 basis points. Notice that the estimated inflation mean is below the announced mid-value of the target zone in this model, but not significantly so.

The model predicts that the coefficient of the weighted inflation forecasts on the interest rate equation equals unity. This restriction was tested by means of a Lagrange Multiplier (LM) test.[13] The calculated LM statistics for the three- and six-month interest rates were respectively, 0.96 [3.841] and 0.12 [3.841], where the bracketed figures indicate the 95% critical value of a χ^2 variate with one degree of freedom. Since these statistics are well-below the critical value, the null hypothesis that the coefficient of $(1/n)\sum_{s=1}^{n}\Delta E(\pi_{t+s}|I_t)$ equals one cannot be rejected at the 5% significance level.

I also examined the possibility of more complicated dynamics in the process of the innovation w_t. First, the hypothesis that this random term is serially uncorrelated was tested against the alternative of higher autocorrelation by means Portmanteau tests. The estimated statistics for first-, second-, and third-order autocorrelation were, respectively, 2.90 [3.841], 3.65 [5.991], and 3.93 [7.815], for the three-month interest rates and 0.85 [3.841], 1.13 [5.991], and 1.70 [7.815] for the six-month interest rate. The numbers in brackets indicate the 95% critical value of a χ^2 variate with 1, 2, and 3 degrees of freedom. Since in all cases, the statistics are well-below the critical

[12] While these results are not shown below, they are available from the author upon request.

[13] Given that the demands involved in the estimation of the alternative model(s), I have opted for the simpler strategy of using LM rather than LR tests to verify the model restrictions. However, the results of both procedures should be asymptotically equivalent.

value, the hypothesis that w_t is serially uncorrelated cannot be rejected at the 5% level. Second, the test for neglected Autoregressive Conditional Heteroskedasticity (ARCH) proposed by Engle (1982) was performed. The test statistic was computed as the product of the number of observations and the uncentered R^2 of the OLS regression of $\hat{w}_t^2$ on a constant and three of its lags, where $\hat{w}_t^2$ denotes the squared residuals of (17). The statistics were 3.34 [7.815] and 0.42 [7.815] for the three- and six-month interest rates, respectively, where the bracketed numbers indicate the 95% critical values of a χ^2 variable with three degrees of freedom. These results would support the conclusion that the innovations to the interest rate process are not conditionally heteroskedastic at the monthly frequency.

The empirical estimates of the model with stochastic bounds are presented in the fourth and fifth columns of table 1. Recall that the condition $\psi_1 = 1$ was imposed for the purpose of calculating standard errors. A central result is that, for both interest rates considered, the estimates of the perceived deviations from the upper and lower bounds, namely $\overline{b}$ and $\underline{b}$, are quantitatively important and statistically significant. Specifically, $\overline{b} = -0.53$ (0.05) and $\underline{b} = 0.23$ (0.05). Thus, the implicit width of the (stochastic) band would be $2 - 0.53 - 0.23 = 1.24$ basis points, where 2 corresponds to the announced width. Furthermore, while the lower section of the band would be 0.77 basis points, the upper segment would be only $1 - 0.53 = 0.47$ basis points. Finally, the transitory deviations from this implicit band appear to be numerically small with a standard deviation of only 0.1 (0.03) basis points, so that for plausible values of the disturbance v_t, the inflation rate would still be contained in the announced target zone.

Since the agents in the model are assumed to be rational and to construct expectations on the basis of the true economic model, uncovering their beliefs provides information about the policy rule that generates inflation. Specifically, the above results suggest that the monetary authority might attach different weights to positive and negative deviations of inflation from the declared mid-value of 2% per year, and contest on empirical grounds the assumption that the policy maker's loss-function is symmetric around a desired value [see, among others, Rogoff (1985), Walsh (1995), and Svensson (1997)]. Moreover, the estimates would appear to contradict the notion that, due to an inflation bias on the part of the government, realized values of inflation should on average exceed the targeted value [see Svensson (1997, p.108)]. In fact, in line with the claim above that positive deviations might be more heavily weighted than negative ones in the social loss function, the available data for the Canadian target zone features 80.5% of the observations below (and only 19.5% above) the target value of 2%. Nonetheless these conclusions should be interpreted with caution given the small sample sizes currently available to empirically examine inflation target zones.

The result that the band might be asymmetric was tested by means of a LR test. The values of the log likelihood function for the model estimated under the restriction that $\bar{b} = -\underline{b}$ were 61.59 and 56.46 for the three- and six-month interest rate, respectively. Comparing these values with the ones reported in Table 1 and constructing the appropriate LR statistic yield 4.84 [3.841] and 4.40 [3.841]. Since the χ^2 statistics are above the 95% critical value, it follows that the hypothesis that band is symmetric cannot be rejected at the standard 5% significance level.[14]

In summary, the findings support the views that the Canadian target zone might be in practice substantially narrower than officially announced, asymmetrically distributed around the stated mid-point, and that the agents' inflation expectations as embodied in the term-structure reflect these beliefs.

A LM test was employed to examine the hypothesis that the coefficient of $(1/n) \sum_{s=1}^{n} \Delta E(\pi_{t+s}|I_t)$ is unity, as predicted by the model. The calculated statistics were 0.35 [3.841] and 0.001 [3.841] for the three- and six-month interest rates, respectively, indicating that indeed this restriction cannot be rejected at the 5% significance level. Tests for neglected ARCH and serial correlation in the process of the interest rate innovation, w_t, were performed as well. The LM statistics for the former hypothesis (computed as described above) were 3.31 [7.815] (0.23 [7.815]) for the three-month (six-month) interest rate. Since, the statistics are well-below the critical value, the hypothesis that the residuals are conditionally homoskedastic cannot be rejected at the 5% significance level.

Regarding the Portmanteau tests for first-, second-, and third-order autocorrelation, the estimated statistics were 3.39 [3.841], 4.09 [5.991], and 4.37 [7.815], respectively, for the three-month interest rate and 0.96 [3.841], 1.30 [5.991], and 1.90 [7.815], for the six-month interest rate. As customary, the figures in brackets denote the 95% critical level. Since the statistics are below the critical value, the hypothesis that w_t is serially uncorrelated cannot be rejected at the 5% level. Hence, these results indicate that no statistical gain would be achieved if the researcher were to postulate a more elaborate, serially correlated and conditional heteroskedastic representation for the interest-rate disturbance term rather than the assumed *i.i.d.* specification.

The two versions of inflation target zones developed and estimated above are based on competing specifications of the agents' beliefs about government policy. Thus, an important part of this project is to statistically compare the two models. Such comparison, however, is complicated by the fact that the two models are nonnested and, consequently, standard LM, LR, or Wald tests are not appropriate. In order to address this issue, I employ the version of the Cox statistic [Cox (1961, 1962)] for nonnested hypothesis proposed by Pesaran and Pesaran (1993). The nonnested test as developed by Cox is a variant of the LR test but the analytical derivation of its statistic can be demanding except in linear or simple nonlinear regression models. Pesaran and Pesaran propose the use of a Monte-Carlo procedure to calculate the

[14]Notice, however, that this hypothesis could be rejected at the more conservative 1% significance level.

statistic in more complicated models and show that its distribution under the null hypothesis is a standard normal.

The test procedure was carried as follows. First, the model with stochastic bounds was postulated as the null hypothesis and the perfectly-credible model as the alternative. The statistic was computed using 200 replications[15] and yielded values of $-0.50[1.96]$ and 0.01 [1.96] for the three- and six-month interest rate, respectively. The terms in brackets denotes the 95% critical value for a two-sided test. This results indicate that the stochastic-bound model cannot be rejected at the 5% significance level. Then, the converse test was performed, that is, the model of a perfectly credible target zone was taken as the maintained null hypothesis and the stochastic-bound model as the alternative specification. In this case, the estimated statistics were -1.22 [1.96] and $-1.03[1.96]$. Thus, while their values are substantially larger than in the previous test, the data still would not reject the perfectly-credible model at the 5% for any of the series considered. It is important to note that since the small-sample distribution of the statistic might differ from the asymptotic one, the above results should only be considered as indicative.

Similarly, it would interesting to compare the preferred, stochastic-bound specification with a linear model that ignores the effect of the band on expectations. As before, the nonnested test was carried in both directions taking first the model with stochastic bounds to be the maintained null hypothesis and the linear model as the alternative. Using 200 replications, the computed statistics were 0.17[1.96] and -0.17 [1.96] for the three- and six-month interest rate, respectively. Thus, the stochastic-bound model cannot be rejected at the 5% significance level. When the converse test was performed, taking the linear model as the null hypothesis and the stochastic-bound model as the alternative, the estimated statistics were -4.14 [1.96] and $-4.02[1.96]$. Hence these results, subject to the caveat above, would lead us to reject the linear model in favor of the stochastic-bound model and would provide statistical evidence on the economic and econometric significance of nonlinearities in inflation and interest rates when the former is subject to a targeting policy by the Central Bank.

5 Concluding Remarks

This paper has examined the joint inflation/nominal interest rate process to construct empirical inferences regarding agents' beliefs about announced inflation target zones. The approach is based on the observation that agents' inflation expectations depend on their beliefs about the policy and that these expectations are encapsulated in the nominal interest rate. Thus, by specifying inflation processes under different

[15] Pesaran and Pesaran (p. 389) report that, for their application, 200 replications "seems to be adequate for a reasonably precise estimate of the Cox statistics.

credibility scenarios and deriving their implication on the rate of nominal interest, the econometrician can establish which one of the competing specifications better describes the data under consideration. Moreover, since agents are rational and construct their forecasts on the basis of the true economic model, this exercise allows one to obtain additional information about the policy that generates inflation.

Two plausible representations of agents' beliefs about the inflation band are characterized. First, the band is assumed perfectly credible in the sense that agents believe that inflation will only take values within the stated upper and lower limits. This scenario is described by modeling inflation as a two-sided limited-dependent variable. A second, more general case, allows agents to anticipate transitory and/or systematic deviations from the announced bounds. Since in this case the limits would be perceived to be stochastic, rather than fixed, the process of inflation would be akin to a two-sided switching regression.

The fact that both specifications of inflation are nonlinear renders the derivation of conditional inflation forecasts nontrivial. To circumvent this problem analytical representations of inflation forecasts are derived and a stochastic-simulation procedure to calculate their precise numerical value is proposed.

The two models are estimated and tested using data for the Canadian inflation target zone. While the small sample currently available does not decisively discriminate between the competing specifications, it does provide some mild support in favor of the stochastic-bound model in that the perceived systematic deviations from the announced band are quantitatively important and statistically significant. In addition, a formal nonnested test provide statistical evidence on the economic and econometric significance of nonlinearities in inflation and interest rates when the former is subject to a targeting policy.

Results also indicate that the target zone might be in practice substantially narrower than officially announced and asymmetrically distributed around the stated mid-point. This observation is consistent with the notion that the monetary authority might weight differently positive and negative deviations from the stated target, possibly as a result of an asymmetric social loss function.

Table 1. Parameter Estimates of Perfectly Credible and Stochastic Bound Models

Parameter	Perfectly Credible		Stochastic Bounds	
	Three-month	Six-month	Three-month	Six-month
α	0.32	0.28	-0.01	-0.02
	(0.16)	(0.06)	(0.02)	(0.05)
ψ_1	0.82	0.84	1.00	1.00
	(0.09)	(0.03)		
a	-0.06	-0.07	-0.06	-0.06
	(0.07)	(0.07)	(0.07)	(0.07)
$\overline{b}$			-0.53	-0.53
			(0.05)	(0.06)
$\underline{b}$			0.23	0.23
			(0.05)	(0.05)
σ_e	0.21	0.21	0.25	0.25
	(0.03)	(0.02)	(0.04)	(0.3)
σ_v			0.10	0.10
			(0.03)	(0.03)
σ_w	0.42	0.47	0.42	0.47
	(0.05)	(0.05)	(0.05)	(0.05)
$L(\hat{\rho})$	59.01	54.04	64.01	58.66

Notes: The figures in parenthesis are asymptotic standard errors. $L(\hat{\rho})$ denotes the maximized value of the log likelihood function.

A Additional Explanatory Variables in the Latent Inflation Process

Consider the more general specification for latent inflation,

$$\pi^*_{t+1} = \alpha + \psi(L)\pi_{t+1} + \beta x_{t+1} + e_{t+1}, \tag{19}$$

where β is a $1 \times m$ vector of parameters, x_t is a $m \times 1$ vector of explanatory variables that are useful in forecasting inflation in addition to the constant α and lagged values of inflation, and the remaining notation was defined above. In turn, the variables in x_{t+1} could be generated by the linear stochastic process,

$$x_{t+1} = Az_t + \delta_{1,t+1}, \tag{20}$$

where A is a $m \times p$ matrix of coefficients, z_t is a $p \times 1$ vector of predetermined variables possibly including past values of x_t, and $\delta_{1,t}$ is $m \times 1$ vector of random disturbances assumed $i.i.d.$ $(0, \Omega^{1/2})$ and uncorrelated with ζ_t, e_t, and v_t. The conditional expectation of inflation in the case of fully credible band is given in the following Proposition.

Proposition 3. *Assume that inflation follows the two-sided, limited-dependent process (6) where latent inflation is determined according to (19). Assume that the set of explanatory variables, x_t, follows the process (20). Define the composite error term*

$$u_{s,t+s} = e_{t+s} + \sum_{k=1}^{s-1} \psi_k \eta_{k-1,t+k-1} + \beta\delta_{s,t+s},$$

where $\eta_{k,t+k} = \pi_{t+k} - E(\pi_{t+k}|I_t)$ and $\delta_{s,t+s} = x_{t+s} - E(x_{t+s}|I_t)$, with cumulative distribution and density functions denoted by $F_s(\cdot)$ and $f_s(\cdot)$, respectively. Define the variables

$$\overline{c}_{t+s} = \overline{\pi}_{t+s} - E(\pi^*_{t+s}|I_t),$$

and,

$$\underline{c}_{t+s} = \underline{\pi}_{t+s} - E(\pi^*_{t+s}|I_t),$$

where

$$E(\pi^*_{t+s}|I_t) = \alpha + \sum_{k=1}^{\min\{q,s-1\}} \psi_k E(\pi_{t+s-k}|I_t) + \sum_{j=s}^{q} \psi_j L^j \pi_{t+s} + \beta E(x_{t+s}|I_t).$$

Then, the conditional expectation of inflation at time $t+s$ is given by,

$$\begin{aligned} E(\pi_{t+s}|I_t) = \; & \overline{\pi}_{t+s}\left[1 - F_s(\overline{c}_{t+s})\right] + \underline{\pi}_{t+s} F_s(\underline{c}_{t+s}) \\ & + \left\{ \left[E(\pi^*_{t+s}|I_t) + E(u_{s,t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s}) \right] \right. \\ & \left. \cdot \left[F_s(\overline{c}_{t+s}) - F_s(\underline{c}_{t+s}) \right] \right\}. \end{aligned}$$

Proof. With suitable redefinition of the variables, the result trivially follows from the proof of Proposition 1.‖

Similarly, the conditional expectations of inflation in the stochastic-bound model when latent inflation follows (19) is given by the following proposition,

Proposition 4. *Assume that inflation follows the two-sided switching regression (15) where latent inflation is determined according to (19). Assume that the set of explanatory variables,* x_t, *follows the process (20). Define the composite error term*

$$u_{s,t+s} = e_{t+s} - v_{t+s} + \sum_{k=1}^{s-1} \psi_{s-k}\eta_{k,t+k} + \beta\delta_{s,t+s},$$

where $\eta_{k,t+k} = \pi_{t+k} - E(\pi_{t+k}|I_t)$ *and* $\delta_{s,t+s} = x_{t+s} - E(x_{t+s}|I_t)$, *with cumulative distribution and density functions denoted by* $H_s(\cdot)$ *and* $h_s(\cdot)$, *respectively. Define the variables*

$$\overline{c}_{t+s} = \overline{\pi}_{t+s} + \overline{b} - E(\pi^*_{t+s}|I_t),$$

and,

$$\underline{c}_{t+s} = \underline{\pi}_{t+s} + \underline{b} - E(\pi^*_{t+s}|I_t),$$

where

$$E(\pi^*_{t+s}|I_t) = \alpha + \sum_{k=1}^{\min\{q,s-1\}} \psi_k E(\pi_{t+s-k}|I_t) + \sum_{j=s}^{q} \psi_j L^j \pi_{t+s} + \beta E(x_{t+s}|I_t).$$

Then, the conditional expectation of inflation at time $t+s$ *is given by,*

$$\begin{aligned} E(\pi_{t+s}|I_t) = \; & (\overline{\pi}_t + \overline{b})\left[1 - H_s(\overline{c}_{t+s})\right] + (\underline{\pi}_t + \underline{b})H_s(\underline{c}_{t+s}) \\ & +\{\left[E(\pi^*_{t+s}|I_t) + E(u_{s,t+s}|I_t, \underline{c}_{t+s} < u_{s,t+s} < \overline{c}_{t+s})\right] \\ & \cdot \left[H_s(\overline{c}_{t+s}) - H_s(\underline{c}_{t+s})\right]\}. \end{aligned}$$

Proof. With suitable redefinition of the variables, the result trivially follows from the proof of Proposition 2.‖

B Formulation of the Log Likelihood Functions

The derivation of the log likelihood functions of the perfectly-credible and stochastic-bounds models is carried out in this section under the assumption that the disturbances w_t, e_t and v_t are normally distributed with zero means and constant conditional variances. While the assumption of normality is not essential and can be relaxed if desired, it does simplify considerably the derivation in the case with stochastic bounds.

Suppose that T data points are available on the variables of the model and use the assumption that w_t is statistically independent of e_t and v_t to write,

$$\Pr(\pi_t, \Delta i_t^n | I_{t-1}) = \Pr(\pi_t | I_{t-1}) \Pr(\Delta i_t^n | I_{t-1}, \pi_t).$$

Under the assumption that w_t is $i.i.d.$ $N(0, \sigma_w^2)$, the density of the $t-th$ observation of the long-term interest rate is simply,

$$\Pr(\Delta i_t^n | I_{t-1}, \pi_t) = \left(2\pi\sigma_w^2\right)^{-1/2} \exp\left(\frac{-\left(\Delta i_t^n - a - (1/n)\sum_{s=1}^{n} \Delta E(\pi_{t+s}|I_t)\right)^2}{2\sigma_w^2}\right),$$

where $\Delta E(\pi_{t+s}|I_t)$ stands for $E(\pi_{t+s}|I_t) - E(\pi_{t+s-1}|I_{t-1})$. The conditional expectations of inflation are given by (11), in the case of a perfectly credible band, or by (16), in the case of a stochastic band.

In the situation when the band is perfectly credible, the inflation density corresponds to the one of the two-sided Tobit model. Write the inflation process (6) as

$$\pi_t = \begin{cases} \overline{\pi}_t, & \text{if} \quad e_t \geq \overline{z}_t, \\ \pi_t^*, & \text{if} \quad \underline{z}_t < e_t < \overline{z}_t, \\ \underline{\pi}_t, & \text{if} \quad e_t \leq \underline{z}_t, \end{cases}$$

where $\overline{z}_t = \overline{\pi}_t - \alpha - \psi(L)\pi_t$ and $\underline{z}_t = \underline{\pi}_t - \alpha - \psi(L)\pi_t$. Then, for an observation at the top of the band,

$$\Pr(\pi_t|I_{t-1}) = 1 - \Phi\left(\overline{z}_t\right),$$

for an observation at the bottom of the band,

$$\Pr(\pi_t|I_{t-1}) = \Phi\left(\underline{z}_t\right),$$

and for an observation in the middle of the band,

$$\Pr(\pi_t|I_{t-1}) = \left(2\pi\sigma_e^2\right)^{-1/2} \exp\left(-\left(\pi_t - \alpha - \psi(L)\pi_t\right)^2 / 2\sigma_e^2\right),$$

where $\Phi(\cdot)$ denotes the cumulative distribution of a normal variable with mean zero and variance σ_e^2.

Collecting the unknown parameters in the set $\rho = \{\alpha, \psi_1, \ldots, \psi_q, a, \sigma_e, \sigma_w\}$ and denoting by Γ_0 the set of observations of π_t at the lower bound, by Γ_1 the set of observations inside the band, and by Γ_2 the set of observations at the upper bound, the log likelihood function for all T observations can be written as

$$\begin{aligned} L(\rho) = & \sum_{t\in\Gamma_0} \log \Phi(\underline{z}_t) - (1/2) \sum_{t\in\Gamma_1} \log(2\pi\sigma_e^2) \\ & -(1/2\sigma_e^2) \sum_{t\in\Gamma_1} (\pi_t - \alpha - \psi(L)\pi_t)^2 + \sum_{t\in\Gamma_2} \log(1 - \Phi(\overline{z}_t)) \\ & -(T/2)\log(2\pi\sigma_w^2) - (1/2\sigma_w^2) \sum_{t=1}^{T} \left(\Delta i_t^n - a - (1/n) \sum_{s=1}^{n} \Delta E(\pi_{t+s}|I_t)\right)^2. \end{aligned}$$

In the situation when the band is stochastic, the inflation density corresponds to the one of an endogenous, switching regression. Write the inflation process (15) as

$$\pi_t = \begin{cases} \overline{\pi}_t + \overline{b} + v_t, & \text{if} \quad \xi_t \geq \overline{z}_t, \\ \pi_t^* & \text{if} \quad \underline{z}_t < \xi_t < \overline{z}_t, \\ \underline{\pi}_t + \underline{b} + v_t, & \text{if} \quad \xi_t \leq \underline{z}_t. \end{cases}$$

where $\xi_t = e_t - v_t$, $\overline{z}_t = \overline{\pi}_t + \overline{b} - \alpha - \psi(L)\pi_t$, and $\underline{z}_t = \underline{\pi}_t + \underline{b} - \alpha - \psi(L)\pi_t$. Then, the density of the $t-th$ observation of inflation is given by,

$$\begin{aligned} \Pr(\pi_t|I_{t-1}) = \; & \Pr(\pi_t|I_{t-1}, \xi_t \geq \overline{z}_t) \Pr(\xi_t \geq \overline{z}_t) \\ & + \Pr(\pi_t|I_{t-1}, \underline{z}_t < \xi_t < \overline{z}_t) \Pr(\underline{z}_t < \xi_t < \overline{z}_t) \\ & + \Pr(\pi_t|I_{t-1}, \underline{z}_t \geq \xi_t) \Pr(\underline{z}_t \geq \xi_t). \end{aligned} \tag{21}$$

In what follows, the appropriate expressions for the conditional probabilities in (21) are derived. Notice that,[16]

$$\Pr(\pi_t|I_{t-1}, \xi_t \geq \overline{z}_t) = \Pr(v_t|I_{t-1}, \xi_t \geq \overline{z}_t), \tag{22}$$

$$\Pr(\pi_t|I_{t-1}, \xi_t \leq \underline{z}_t) = \Pr(v_t|I_{t-1}, \xi_t \leq \underline{z}_t), \tag{23}$$

and

$$\Pr(\pi_t|I_{t-1}, \underline{z}_t < \xi_t < \overline{z}_t) = \Pr(e_t|I_{t-1}, \underline{z}_t < \xi_t < \overline{z}_t), \tag{24}$$

and write the conditional probability in (22) as [see Maddala (1983, p.284)],

$$\Pr(v_t|I_{t-1}, \xi_t \geq \overline{z}_t) = (\Pr(\xi_t \geq \overline{z}_t))^{-1} \Pr(v_t|I_{t-1}) \int_{\overline{z}_t}^{\infty} g(\xi_t|I_{t-1}, v_t) d\xi_t,$$

where $g(\cdot)$ denoted the conditional distribution function of ξ_t, with comparable expressions holding for (23) and (24). Then the relation (21) can be written as,

[16]The Jacobian of these transformation are unity.

$$
\begin{aligned}
\Pr(\pi_t|I_{t-1}) = \quad & \Pr(v_t|I_{t-1}) \cdot \textstyle\int_{\overline{z}_t}^{\infty} g(\xi_t|I_{t-1}, v_t) d\xi_t \\
& + \Pr(e_t|I_{t-1}) \cdot \textstyle\int_{\underline{z}_t}^{\overline{z}_t} g(\xi_t|I_{t-1}, e_t) d\xi_t \\
& + \Pr(v_t|I_{t-1}) \cdot \textstyle\int_{-\infty}^{\underline{z}_t} g(\xi_t|I_{t-1}, v_t) d\xi_t.
\end{aligned}
$$

Using standard results for jointly distributed normal variables [see Mood and Graybill (1963, p. 213)], it possible to derive the following closed-form expressions for the integrals above,

$$
\begin{aligned}
\int_{\overline{z}_t}^{\infty} g(\xi_t|I_{t-1}, v_t) d\xi_t &= 1 - \Phi(D_{1t}), \\
\int_{\underline{z}_t}^{\overline{z}_t} g(\xi_t|I_{t-1}, u_t) d\xi_t &= \Phi(D_{2t}) - \Phi(D_{3t}),
\end{aligned}
$$

and

$$
\int_{-\infty}^{\underline{z}_t} g(\xi_t|I_{t-1}, v_t) d\xi_t = \Phi(D_{1t}),
$$

where

$$
\begin{aligned}
D_{1t} &= (\pi_t - \alpha - \psi(L)\pi_t) / \sigma_e, \\
D_{2t} &= -\left(\pi_t - \overline{\pi}_t - \overline{b}\right) / \sigma_v, \\
D_{3t} &= -\left(\pi_t - \underline{\pi}_t + \underline{b}\right) / \sigma_v,
\end{aligned}
$$

and $\Phi(\cdot)$ denotes the cumulative distribution function of a standardized normal variable. Finally, the density of the $t-th$ inflation observation is given by the mixture of normal distributions,

$$
\begin{aligned}
\Pr(\pi_t|I_{t-1}) = \quad & (2\pi\sigma_v^2)^{-1/2} \exp\left(-(\pi_t - \overline{\pi}_t - \overline{b})^2/2\sigma_v^2\right)(1 - \Phi(D_{1t})) \\
& + (2\pi\sigma_e^2)^{-1/2} \exp\left((\pi_t - \alpha - \psi(L)\pi_t)^2/2\sigma_v^2\right)(\Phi(D_{2t}) - \Phi(D_{3t})) \\
& + (2\pi\sigma_v^2)^{-1/2} \exp\left((\pi_t - \underline{\pi}_t - \underline{b})^2/2\sigma_v^2\right)\Phi(D_{1t}).
\end{aligned}
$$

Finally, the log likelihood function for the T available observations can be concisely written as

$$
L(\rho) = \sum_{t=1}^{T} \log \ \Pr(\pi_t, \Delta i_t^n | I_{t-1}),
$$

where $\rho = \{\alpha, \psi_1, \cdots \psi_q, \underline{b}, \overline{b}, a, \sigma_e, \sigma_w, \sigma_v\}$ denotes the set of unknown parameters the log likelihood function depends upon.

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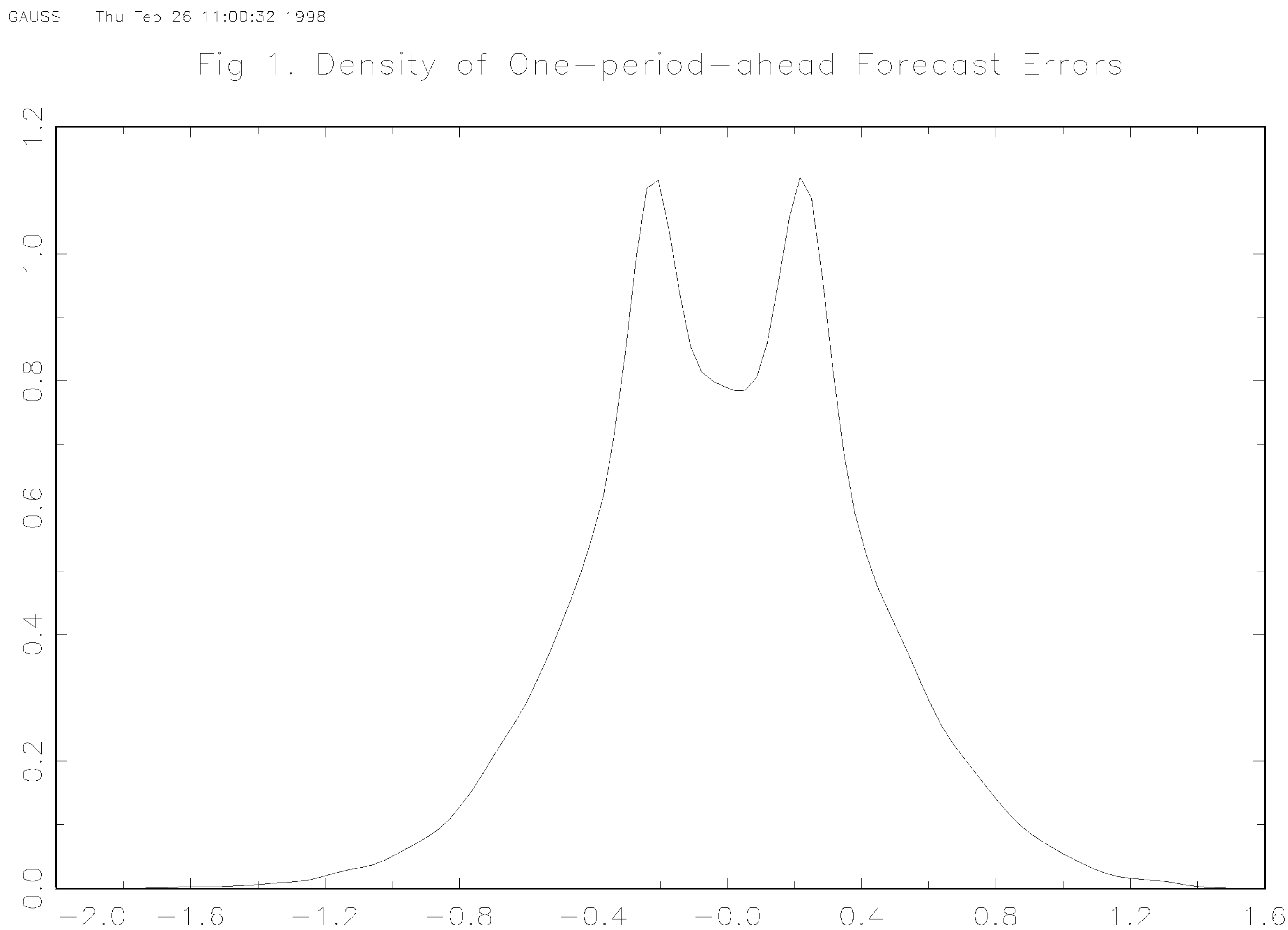
GAUSS Thu Feb 26 11:00:32 1998
Fig 1. Density of One-period-ahead Forecast Errors
0.0
0.2
0.4
0.6
0.8
1.0
1.2
−2.0
−1.6
−1.2
−0.8
−0.4
−0.0
0.4
0.8
1.2
1.6

Fig 2. Inflation Forecasts
Perfectly Credible Band

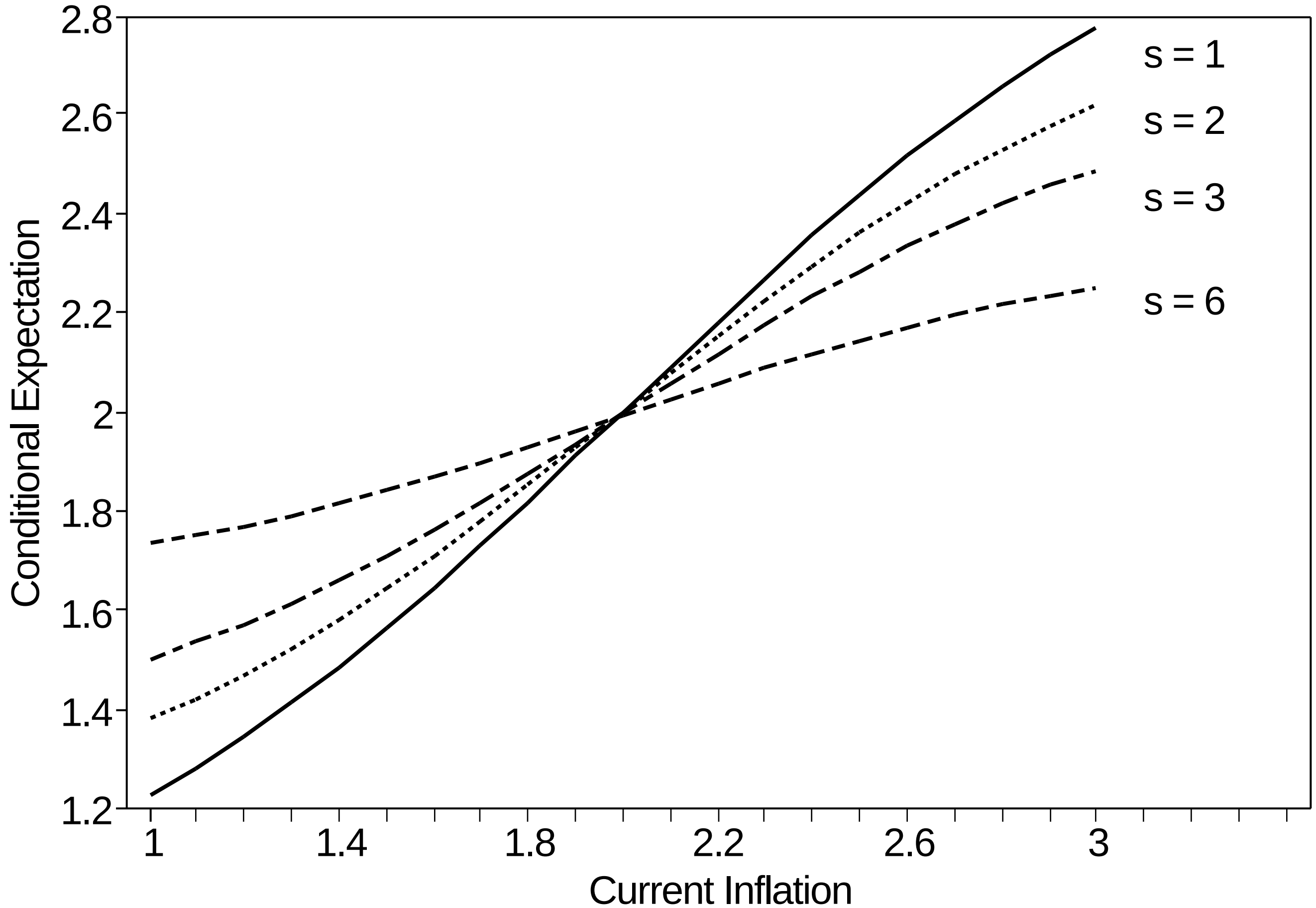

Fig 3. Inflation Forecasts
Symmetric Stochastic Band

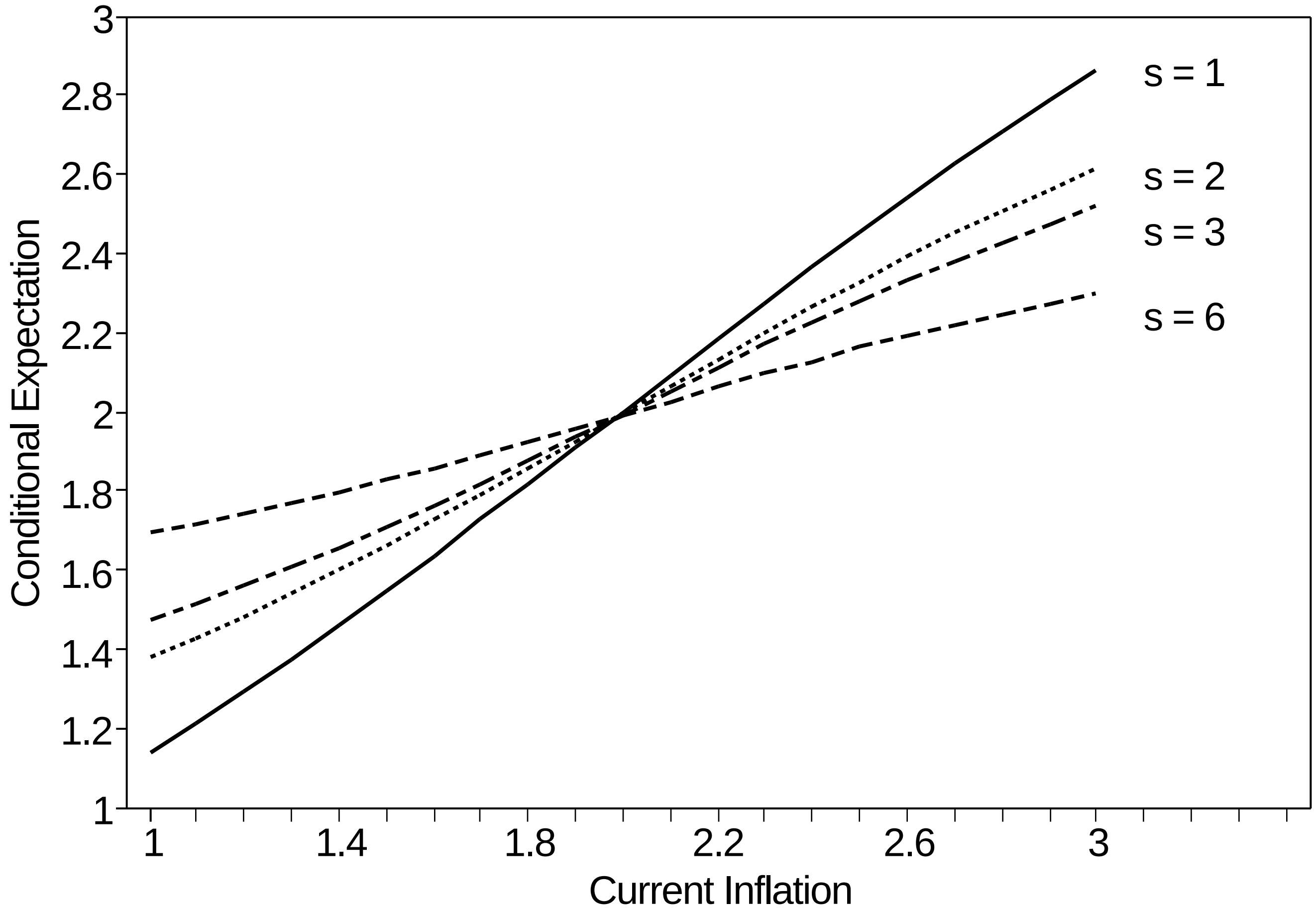

Fig 4. Inflation Forecasts
Asymmetric Stochastic Band

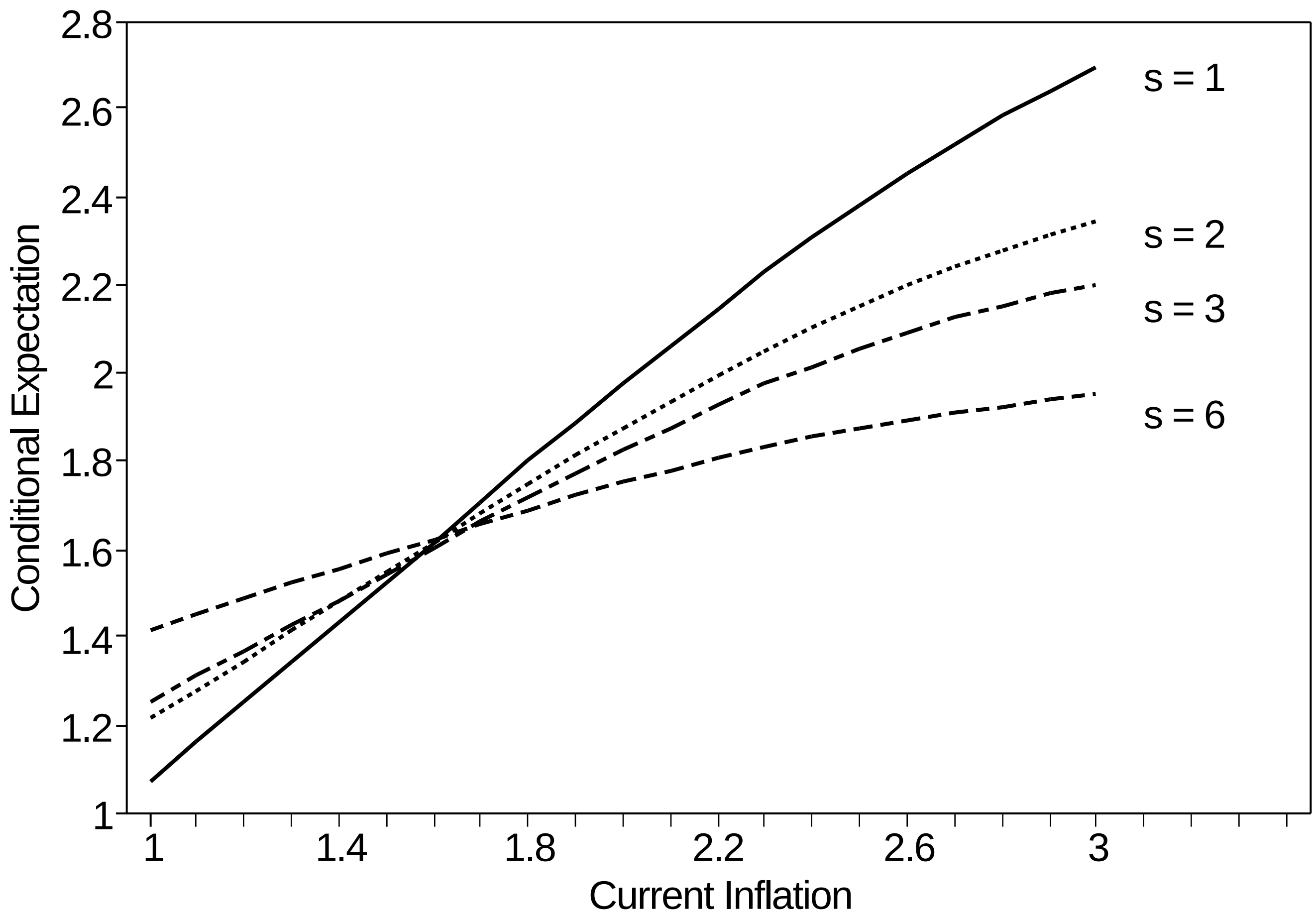

Fig 5. Inflation Targets in Canada

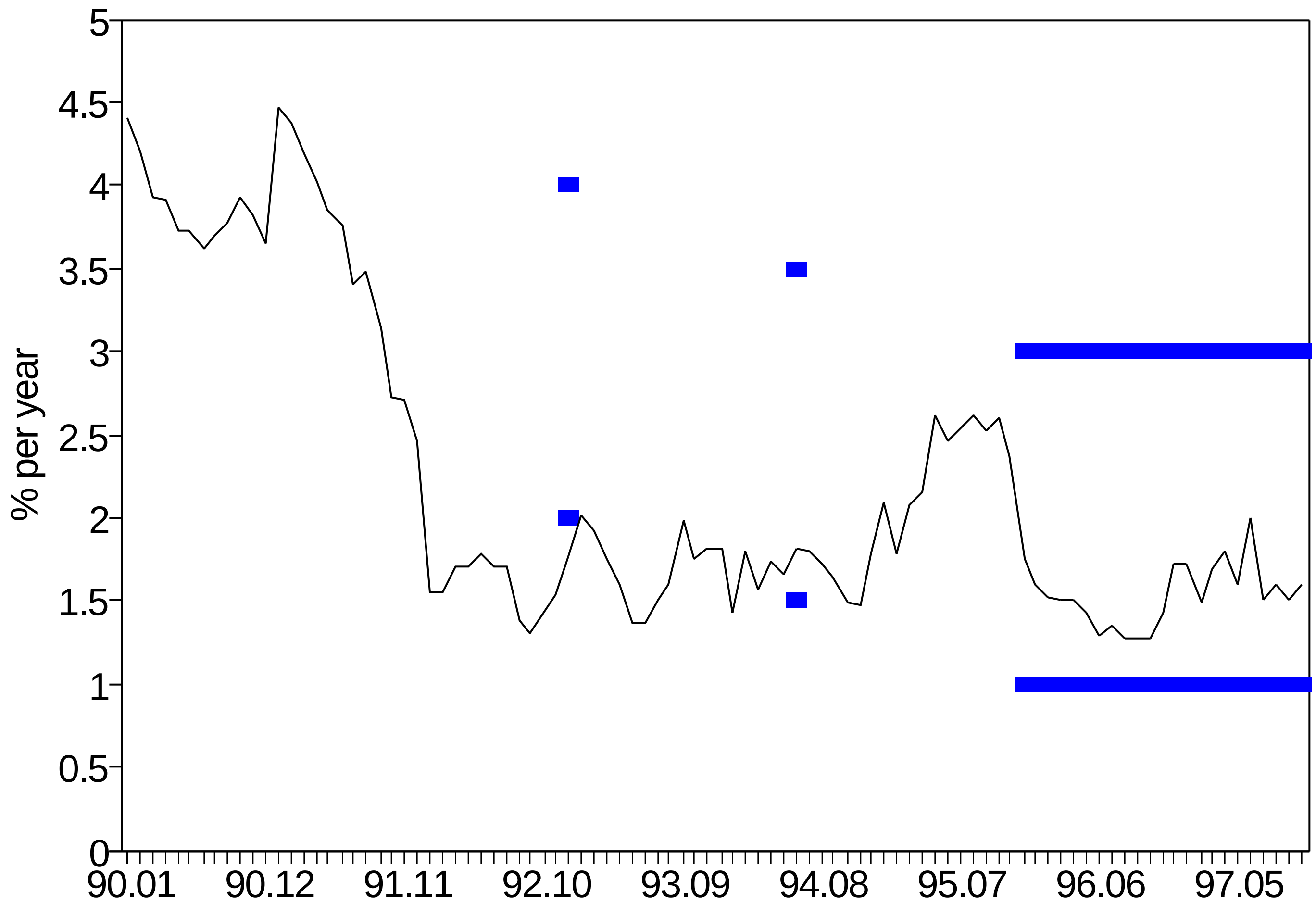

Figure 6. Nominal Interest Rates in Canada