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**OPTIMAL INSURANCE DESIGN  
UNDER BACKGROUND RISK**

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## RÉSUMÉ

Nous reconsidérons les travaux de Raviv (1979) portant sur la forme optimale des contrats d'assurance pour intégrer l'existence d'une source de risque non assurable. Tant le cas où ce "bruit de fond" est indépendant du risque assurable que celui où il existe une dépendance positive entre les aléas sont examinés. Nous montrons que, dans le premier cas, les résultats de Raviv se généralisent aisément. De plus, dans le second, la spécification d'une forme particulière de dépendance statistique positive nous permet de rationaliser l'usage de déductibles évanescents. Enfin, l'utilité du concept de prudence pour l'obtention de ces résultats est également démontrée.

Mots clés : contrat d'assurance, "bruit de fond", prudence, déductible évanescent

*Journal of Economic Literature* Classification Numbers : G22.

## ABSTRACT

We extend Raviv's seminal contribution on the design of an optimal insurance policy to include the presence of an uninsurable background risk. Cases where the background risk is independent from the insurable risk and where it exhibits a positive dependence with the insurable risk are both considered. In the first case, we show that Raviv's results carry over. In the second case, a specific characterization of the positive dependence between the two risks offers a reasonable explanation for the current use of disappearing deductibles in some insurance lines. It is also shown how Kimball's prudence concept is much useful in deriving our basic propositions.

Key words : insurance design, background risk, prudence, disappearing deductible

*Journal of Economic Literature* Classification Numbers : G22.

## 1. Introduction

Most models on Pareto optimal insurance policies of a given risk assume that the decision maker's wealth prospect contain only a single source of uncertainty. Such a framework is obviously at odds with a more realistic environment in which insureds must cope with multiple risks, both insurable and uninsurable. In this paper, our objective is to extend Raviv's seminal work on the design of an optimal insurance policy to include the presence of an uninsurable - or background - risk. The case of an independent background risk is first examined. The situation of a positive dependence between the insurable risk and the background risk is then considered. Of course, this setting calls into question the reason for a source of risk not being insurable, which, in fact, represents a type of incompleteness of the market. However, one can argue that many risks involve informational asymmetries (moral hazard and adverse selection) which may preclude their insurability. Several other reasons for the existence of non insurable risks are also given in Mayers and Smith [1983] and Doherty and Schlesinger [1983].

The first issue considered in this paper is: how does an independent background risk affect the Pareto optimal design of an insurance policy. We show that Raviv's results carry over to our more general framework, even though Eeckhoudt and Kimball [1992] have shown that the optimal level of the individual demand for insurance is affected. The second issue shows that Raviv's results are no longer valid as soon as one considers a positive dependence between the insurable risk and the background risk. Indeed, under a specific characterization of the risk dependence, a policy with a disappearing deductible is optimal. This result offers a reasonable explanation for the current use of disappearing deductibles in the homeowners insurance line. It is also shown how Kimball's prudence concept is useful to achieve a better understanding of our results. As soon as the insured exhibits prudence, partial insurance is optimal even though the insurance contract is sold at actuarially fair conditions. A short conclusion summarizes the main findings.

## 2. Assumptions and preliminary results: the case of an independent background risk.

In order to analyze the impact of a background risk on optimal insurance design, we extend the framework of Raviv [1979] in the following way. We suppose that the individual is subject to an insurable loss  $\tilde{x}$  distributed on some interval  $[0, L]$

according to a distribution function  $F(x)$ , and an uninsurable (or background) risk  $\bar{z}$  whose conditional distribution on  $[z, \bar{x}]$  is given by  $G(z|x)$ . Both  $F(x)$  and  $G(z|x)$  are assumed to be continuous<sup>2</sup>, with probability density functions  $f(x)$  and  $g(z|x)$  such that  $f(x) > 0$  and  $g(z|x) > 0$  for all  $x$  and  $z$ .

If both risks were insurable, Raviv [1979] has shown that any optimal insurance policy should be based on the aggregate value of the losses and has the same characteristics as in the basic case where there is only one source of uncertainty. Here, due to the incomplete market hypothesis, any insurance policy can only specify the coverage function of the loss  $x$ , irrespective of the realization of the background risk  $z$ . This implies that the final wealth of the individual is given by:

$$\bar{w}_F = w_0 + \bar{z} - \bar{x} + I(\bar{x}) - P$$

where  $w_0$  represents his initial level of (certain) wealth,  $P$  the insurance premium and  $I(x)$  the coverage function of the loss  $x$ . Moreover, any admissible coverage function must satisfy the following constraint:

$$x \geq I(x) \geq 0$$

for all  $x \in [0, L]$ . That is, any insurance payment must be nonnegative and cannot exceed the size of the insurable loss  $x$ . Now, let  $U(\cdot)$  denotes the individual von Neumann-Morgenstern utility function, with  $U' > 0$  and  $U'' < 0$ , and  $c(I(x))$  represents the transaction cost incurred by the insurer when the insurance payment is equal to  $I(x)$ , with  $c(0) = 0$ ,  $c'(\cdot) \geq 0$  and  $c''(\cdot) \geq 0$ . Then, assuming that the insurer is risk neutral, any Pareto optimal insurance policy  $(P, I(\cdot))$  must solve the following problem:

$$\text{Max}_{I(x), P} \int_0^L \int_0^L U(w_0 + z - x - P + I(x)) dG(z|x) dF(x) \quad (2.1)$$

$$\text{s.t.} : P - \int_0^L (I(x) + c(I(x))) dF(x) \geq 0 \quad (2.2)$$

$$x \geq I(x) \geq 0 \quad (2.3)$$

where the objective function (2.1) is to maximize the insured's expected utility subject to the constraint (2.2) that the insurer's profit is positive.

<sup>2</sup>This is without any loss of generality. Allowing for example  $F(x)$  to have a mass point at  $x = 0$  does not change any result.

This problem can be viewed as an optimal control problem of Hestenes [1966] where  $I(x)$  is the control variable and  $P$  is a control parameter. Moreover, in the case of an independent background risk, we have  $G(z|x) = G(z)$  for all  $(z, x)$ . Under this assumption, the Lagrangian can be written as:

$$L \equiv \psi_0 [V(w_0 - x - P + I(x)) f(x) - \lambda [I(x) + c(I(x)) f(x) + \gamma(x) I(x) + \mu(x)(x - I(x))]$$

where  $\psi_0$  and  $\lambda$  are the multipliers for the objective function (2.1) and constraint (2.2),  $\gamma(x)$  and  $\mu(x)$  are for the constraint (2.3), and  $V(w) \equiv \int_z^L U(w+z) dG(z)$ . Since  $V(w)$  inherits properties of  $U(w)$  (risk aversion, prudence, DARA etc.), the previous problem is identical to Raviv's one for the derived utility function  $V(w)$  in the place of  $U(w)$ . This implies that some well-known results can be easily recovered.

**Theorem 2.1.** Given an independent background risk, any optimal insurance policy has the following properties. There exist  $\bar{x}$ ,  $0 \leq \bar{x} \leq L$ , such that  $I(x) = 0$  for  $x \leq \bar{x}$  and  $0 < I(x) \leq x$  for  $x > \bar{x}$ , with:

$$I'(x) = \frac{A(x)}{A(x) + \frac{c'(x)}{1+x}} \leq 1 \quad (2.4)$$

in that range, where:

$$A(x) = \frac{-\int_z^L U''(w_0 + z - x - P + I(x)) dG(z)}{\int_z^L U'(w_0 + z - x - P + I(x)) dG(z)}$$

Proof: see Raviv's [1979].

Thus, any optimal insurance policy entails a deductible and a coinsurance of losses above the deductible and the properties of the insurance cost function are the driving forces behind the form of the optimal coverage. Hence, assuming for example a proportional insurance cost gives as a by product the classical result of a pure deductible. That is:

**Corollary 2.2.** If  $c(I(x)) = \ell I(x)$ ,  $I(x) = \text{Max}[0, x - \bar{x}]$  for all  $x$ , with  $\bar{x} > 0$  if and only if  $\ell > 0$ .

So, just as in a world without background risk, full coverage of loss  $x$  is optimal when insurance is sold at an actuarially fair price, a result that confirms those of Doherty and Schlesinger [1983] and Eeckhoudt and Kimball [1992]. Finally, a second corollary identical to Raviv's Theorem 3 can also be stated:

**Corollary 2.3.** *A necessary and sufficient condition for the Pareto optimal deductible to be equal to zero under an independent background risk is  $c'(\cdot) = 0$ .*

But, as we will see in the next section, those results are strongly linked to the independence assumption. As soon as we drop this assumption of independence between  $z$  and  $x$ , they cease to be generally true.

### 3. The case of a positive dependence.

Until now, the background risk has been assumed to be independent of the insurable one. However, such an assumption is far from reality. In fact, we can think of many situations characterized by a positive dependence. Several reasons are provided by Doherty and Schlesinger [1983] among others. One can take, for instance, the following example that they give: they consider an individual who is unable to obtain a disability insurance, but can have a medical and hospitalization insurance coverage. As they point out, larger hospitalization claims can be expected to induce longer periods of recovery and of unemployment. Therefore, large hospitalization claims are likely to be accompanied by low income realizations: high losses are more likely to be combined with low values of the background risk.

In order to see how a positive dependence might affect the coverage of an insurable risk, we will retain henceforth the following assumption. From now on, we assume that the conditional distribution of the background risk  $\bar{z}$  deteriorates in the sense of second order stochastic dominance (SOSD) when the insurable loss increases. Formally, this assumption implies that we have the following property:

**Assumption 1:**

$$S_x(z|x) \equiv \int_z^x G_x(s|x) ds \geq 0 \text{ for all } x \text{ and } z \in [z, \bar{z}] \quad (3.1)$$

with a strict inequality for some  $z$ . Finally, we will limit the analysis to the case of a proportional insurance cost:

**Assumption 2:**

$$c(I(x)) = \ell I(x).$$

This cost structure corresponds to the well-known multiplicative loading approach to the pricing of insurance (see Mossin (1968)). Under these assumptions, the Lagrangian of the problem at hand is:

$$\begin{aligned} L \equiv & \psi_0 \int_x^L U'(u_0 + z - x - P + I(x)) dG(z|x) f(x) - \lambda [I(x)(1 + \ell)] f(x) \\ & + \gamma(x) I(x) + \mu(x)(x - I(x)). \end{aligned}$$

Hence, using Hestenes's theorem, the necessary and sufficient conditions for an optimal solution are including:

$$\frac{\partial L}{\partial I(x)} = \psi_0 \int_x^L U''(u_0 + z - x - P + I(x)) dG(z|x) f(x) - \lambda(1 + \ell) f(x) \quad (3.2)$$

$$+\gamma(x) - \mu(x) = 0$$

and the transversality condition:

$$-\lambda + \psi_0 \left[ \int_x^L U'(u_0 + z - x - P + I(x)) dG(z|x) \right] dF(x) = 0. \quad (3.3)$$

with  $\psi_0$ ,  $\lambda$ ,  $\gamma(x)$  and  $\mu(x)$  not vanishing simultaneously on  $x \in [0, L]$ . Suppose now that  $\gamma(x) - \mu(x) = 0$  for all  $x \in [a, b] \subset [0, L]$ . In such a case, implicit differentiation of (3.2) with respect to  $x$  yields the following result:

$$I'(x) = 1 - \frac{\int_x^L U''(u_0 + z - x - P + I(x)) dG_x(z|x)}{\int_x^L U'''(u_0 + z - x - P + I(x)) dG(z|x)}. \quad (3.4)$$

Thus, compared to the situation of an independent background risk, we observe that the slope of the insurance contract is now composed of two terms. The first one, equal to one, is for the optimal coverage of  $x$  absent any background risk. The second one gives the optimal deviation from this rate in order to achieve a partial coverage of the background risk.

To provide a careful analysis of the properties of the optimal insurance policy, it is useful to proceed in two steps. Let us first consider the case where the conditional distribution of the background risk  $z$  deteriorates in the sense of FOSD when the insurable loss  $x$  increases. Since marginal utility decreases with wealth under risk aversion, the following result obtains easily:

Lemma 3.1. If  $G_z(z|x) \geq 0$  for all  $z$ , then:

$$\int_z^{\bar{x}} U''(w_0 + z - x - P + I(x)) dG_z(z|x) \geq 0$$

with a strict inequality if and only if  $G_z(z|x) > 0$  for some  $z$  in  $[z, \bar{x}]$ .

Proof: Integration by parts yields:

$$= - \int_z^{\bar{x}} U''(w_0 + z - x - P + I(x)) G_z(z|x) dz \geq 0$$

since  $G_z(z|x) = G_z(\bar{z}|x) = 0$ , with a strict inequality if and only if  $G_z(z|x) > 0$  for some  $z \in [z, \bar{x}]$ . Q.E.D.

The slope of the insurance contract is thus greater than one under FOSD. Now, let us consider the case where the conditional distribution of  $z$  becomes more risky in the sense of Rothschild and Stiglitz [1971] when the insurable loss increases. Then, risk aversion on the part of the individual is no more sufficient to determine the effect of the background risk on the optimal policy. The reason is easy to understand. Condition (3.2) implies that any deviation in the insurance policy from an optimal coverage function absent any background risk depends on the properties of the derived marginal utility function  $\int U'(w+z) dG(z|x)$ . This entails that the analysis of the effect of the background risk on the insurance policy requires the notion of "prudence" as proposed by Kimball [1990], namely convex marginal utility. Applied here, this notion allows us to establish the following lemma:

Lemma 3.2. If  $S_z(\bar{z}|x) = 0$  and the individual exhibit prudence, then:

$$\int_z^{\bar{x}} U''(w_0 + z - x - P + I(x)) dG_z(z|x) \geq 0$$

with a strict inequality if (and only if)  $S_z(z|x) > 0$  for some  $z$  in  $[z, \bar{x}]$  and  $U''' > 0$ .

Proof: Integrating twice by parts gives:

$$\int_z^{\bar{x}} U''(w_0 + z - x - P + I(x)) dG_z(z|x) = - \int_z^{\bar{x}} U'''(w_0 + z - x - P + I(x)) G_z(z|x) dz$$

$$= \int_z^{\bar{x}} U'''(w_0 + z - x - P + I(x)) S_z(z|x) dz \geq 0$$

since  $S_z(z|x) = S_z(\bar{z}|x) = 0$ , with a strict inequality if (and only if)  $S_z(z|x) > 0$  for some  $z$  and  $U''' > 0$ . Q.E.D.

Again, the slope of the insurance contract is greater than one. Those two lemmas and conditions (3.2) and (3.3) entails the following theorem:

Theorem 3.3. Under the assumption (a) that the conditional distribution of  $z$  deteriorates in the sense of FOSD when the insurable loss increases or (b) that the individual exhibit prudence and the conditional distribution of  $z$  deteriorates in the sense of SOSD when the insurable loss increases, any optimal insurance policy has the following properties. There exist  $\bar{x}$  and  $\hat{x}$ , with  $0 \leq \bar{x} \leq \hat{x} \leq L$ , such that  $I(x) = 0$  for  $x \leq \bar{x}$  and  $0 < I(x) \leq x$  for  $x > \bar{x}$ , with:

- i)  $I'(x) \geq 1 \forall x \in [\bar{x}, \hat{x}]$  and  $I'(x) > 1$  for some  $x$ ;
- ii)  $I(x) = x \forall x > \hat{x}$ .

Proof: see appendix.

What we obtain here is that any optimal insurance policy contains a disapparent deductible. In fact, this result is fairly intuitive. Even if the individual cannot buy market insurance against the total risk he faces, the positive dependence between the two sources of uncertainty implies that any coverage of  $x$  allows him to partially hedge the background risk as well. Since the conditional distribution of  $z$  deteriorates in the sense of a second order stochastic dominance when the insurable loss increases, he can do this more effectively with an increasing coverage function of losses  $x$ .

Our result yields an insurance contract design which is similar to that of Hubbard, Mayers and Smith (1983) in their proposition 3. However, the driving force in their model is not a dependent background risk but the concavity of the cost function. Indeed, a concave cost structure means that economies of scale are to be exploited. It is interesting to observe that disappearing deductibles are quite common on insurance markets. This type of contract is for instance used in homeowners policies (Greene (1980)). As described above, these contracts provide that no liability exists for payments unless the loss from each occurrence exceeds  $\bar{x}$ . If the loss lies between  $\bar{x}$  and  $\hat{x}$ , the insurer pays  $I(x) > 1$  of the loss. For losses above  $\hat{x}$ , no deductible is charged. The so-called homeowners insurance programs are nice candidates for the type of contracts we have just described. They cover a

significant portion of the insured's nonmarketable wealth (dwelling...) while the insured is still exposed to other sources of uncertainty such as unemployment risk. One can indeed conjecture that thefts are more frequent in a period of economic recession, that is in periods of low incomes.

Contracts with a disappearing deductible have however an obvious drawback. Once the damage has occurred, they encourage the insured to inflate the size of the claim. As a result, disappearing deductibles will only be employed when there is no room for ex-post moral hazard. The insurer should for instance have the capacity of monitoring the policyholder to detect abnormal losses.

Finally, it can be easily checked that we have a positive deductible as soon as the proportional insurance cost is positive:

**Corollary 3.4.** *A sufficient condition for the Pareto optimal deductible  $\bar{x}$  to be strictly positive is that  $\ell > 0$ .*

Of course, one may wonder if there are some conditions under which full insurance is optimal. To answer this question, we must consider the case where the insurance is sold at an actuarially fair price. This leads to the following corollary:

**Corollary 3.5.** *If  $\ell = 0$ , the Pareto optimal deductible  $\bar{x}$  is equal to zero if and only if  $S_z(\bar{z} | x) = 0$  and  $U''' = 0$ . Otherwise,  $\bar{x}$  is strictly positive.*

*Proof:* see appendix.

While at first surprising, this result is easy to understand. From lemma 2, we know that any mean preserving spread of the conditional distribution of  $z$  has no effect on the expected marginal utility function of the insured when  $U''' = 0$ . Hence, full insurance is obviously optimal in that case. However, if  $S_z(\bar{z} | x) > 0$  or  $U''' > 0$ , the positive dependence between the two sources of risk implies that any optimal insurance policy would entail a coverage of  $x$  in excess of 100% of the losses if there were no upper limit on the function  $I(x)$ . Since this is not possible under the restriction that any insurance payment cannot exceed the size of the loss  $x$ , a disappearing deductible allows to reduce the insurance premium and therefore to increase the insured's wealth in the bad states of nature. Hence, even if the insurance is sold at a fair price, a partial insurance is optimal. Thus, the next corollary concludes the analysis:

**Corollary 3.6.** *Full insurance is optimal if and only if  $\ell = 0$ ,  $S_z(\bar{z} | x) = 0$  and  $U''' = 0$ .*

#### 4. Conclusion

Several conclusions can be drawn from the previous sections. First, the optimality of Raviv's results is shown to carry over to the case of a setting with an additional independent background risk. Second, most results fall apart when the background risk and the insurable risk are positively dependent. Using some assumptions to describe that dependence, we have been able to characterize the design of the optimal insurance policy. For instance, our model gives some rationale for the existence of disappearing deductibles different from the traditional cost structure explanation.

Further avenues of research are of course possible. The case of a negative dependence between the two risks is an obvious candidate. Further investigation however reveals that this case is less easy to characterize than the ones treated here. Indeed, as shown by Doherty and Schlesinger [1983], the insurance demand depends among other things upon the size and correlations of both the insurable and non insurable losses. A more promising avenue seems to be the introduction of non expected utility preferences, in the spirit of Machina [1982], Chew [1983] or Quiggin (1982).

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## A. Appendix A

In order to establish the theorem 3.3, it is worthwhile noting that we must have  $\psi_0 > 0$  and  $\lambda > 0$  at the optimum. To see this, suppose that  $\psi_0 = 0$  or  $\lambda = 0$ . Then, from complementary slackness, conditions (6) and (7) require that  $\psi_0 = \lambda = \gamma(\bar{x}) = \mu(x) = 0$  on  $x \in [0, \bar{x}]$ , a contradiction. Now, if  $U''' \geq 0$ , and under the assumption that  $S_z(z|x) \geq 0$  for all  $(x, z)$ , we know from lemmas 3.1 and 3.2 that:

$$J^1(z, x) = \int U'(w_0 + z - x - P)dG(z|x)$$

and

$$J^2(z, x) = \int U'(w_0 + z - P)dG(z|x)$$

are both continuous and (weakly) increasing in  $x$ . Thus, assuming that  $\gamma(x)$  is equal to zero for some  $x$  (otherwise there is no insurance), we obtain that if  $I(x) = 0$  for some  $x$ ,  $x \in [0, \bar{x}]$  where  $\bar{x} \geq 0$  solves:

$$\psi_0 \int U'(w_0 + z - \bar{x} - P)dG(z|\bar{x}) - \lambda(1 + \ell) = 0.$$

Furthermore, if  $I(x) = x$  for some  $x$ , the condition 3.2 implies also that  $x \in [\hat{x}, L]$  where  $\hat{x}$  solves:

$$\psi_0 \int U'(w_0 + z - P)dG(z|\hat{x}) - \lambda(1 + \ell) = 0,$$

and  $0 < I(x) < x$  for all  $x \in (\bar{x}, \hat{x})$ , with  $I'(x) \geq 1$  given by equation 3.4. Now, to complete the proof of Theorem 3.3, note that previous results and the combination of 3.2 and 3.3 imply that:

$$\lambda - \int (\gamma(x) - \mu(x))dx = 0.$$

Hence,  $\int \gamma(x)dx \geq 0$ , with a strict inequality when  $\ell > 0$ . Q.E.D.

## B. Appendix B

In order to establish Corollary 3.5, note that  $\lambda > 0$  and the combination of conditions 3.2 and 3.3 yield for  $\ell = 0$ :

$$\int (\gamma(x) - \mu(x))dx = 0. \quad (B.1)$$

Furthermore, we know that implicit differentiation of 3.2 with respect to  $x$  gives:

$$I'(x) = 1 - \frac{\int U'(w_0 + z - x - P + I(x))g_z(z|x)dz}{\int U''(w_0 + z - x - P + I(x))g(z|x)dz} \quad (B.2)$$

for all  $x$  such that  $0 < I(x) \leq x$ . Hence, the sufficiency part of conditions listed in Corollary 3.5 is obvious. For  $S_z(\bar{z}|x) = 0$ , the previous equation reduces to:

$$I'(x) = 1 - \frac{\int U'''(w_0 + z - x - P + I(x))S_z(z|x)dz}{\int U''(w_0 + z - x - P + I(x))g(z|x)dz}.$$

Thus, if  $U''' = 0$ , we have  $I'(x) = 1$ . Now, this last result and equation (B.1) imply that  $\int \gamma(x)dx = \int \mu(x)dx = 0$ . The necessity part can be proved as follows.



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Suppose that  $\bar{x} = 0$ . Then, from complementary slackness, 3.2 and 3.3 imply that we must have:

$$\int \gamma(x) dx = \int \mu(x) dx = 0$$

and:

$$\psi_0 \int U'(u_0 + z - P)g(z|x) dz - \lambda = 0$$

for all  $x$ , that is:

$$-U''(u_0 + z - P)S_z(\bar{z}|x) + \int U''(u_0 + z - P)S_z(z|x) dz = 0.$$

Thus, if  $U''' = 0$ , this last equation reduces to  $-U''(u_0 + z - P)S_z(\bar{z}|x) = 0$ , which - under risk aversion - requires that  $S_z(\bar{z}|x) = 0$ . Hence, one must have  $S_z(\bar{z}|x) = 0$  for  $\bar{x} = 0$ . Moreover, since  $S_z(z|x) \geq 0$  for all  $z$  and  $U''' \geq 0$  by assumption, the second condition of Corollary 3.5 obtains as a by product. Q.E.D.