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On Stackelberg Equilibria with Differentiated Products : The Critical Role of the Strategy Space

by

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ABSTRACT

We show in this paper that in a differentiated product world, the relationship between those products (substitutes or complements) will be an important factor in the determination of the kind of strategic competition (Cournot-Bertrand, Mixed Nash, Stackelberg; through prices or quantities) which we will observe between duopolists. When the goods are substitutes, the competition will tend to be of Cournot type rather than mixed Nash unless one firm can take the leadership and force a Stackelberg type competition. However, if the goods are complements, one should observe the Bertrand type of competition rather than the mixed Nash unless one firm can take a leadership position in which case it can make more profits by doing so. But whether goods are substitutes or complements, the consumers' interest is always best served if the firms are involved in a Bertrand type competition. Therefore, there is no strategic context which meets both the industry and the consumers' interests. Total surplus, and therefore social efficiency is however maximized in a Bertrand framework.

RÉSUMÉ

Nous considérons dans cet article un modèle de duopole avec produits différenciés; nous montrons que le caractère substituts vs compléments de ces produits est un facteur important dans la détermination du mode de concurrence stratégique (Cournot-Bertrand, Nash mixte, Stackelberg; en prix ou quantités) que l'on est susceptible d'observer. Si les produits sont substituts (compléments), la concurrence sera du type Cournot (Bertrand) plutôt que du type Nash mixte à moins qu'une firme puisse affirmer son leadership et forcer une concurrence à la Stackelberg mais quel que soit le rôle tenu par une firme, il sera préférable pour elle que la concurrence s'exprime en quantité (prix). Par ailleurs, la concurrence à la Bertrand est toujours la meilleure du point de vue des consommateurs et du point de vue de l'efficacité sociale, et ce, que les produits soient substituts ou compléments.

SECTION I - INTRODUCTION

It is a well known result in duopoly theory that the equilibrium state of the market (or markets if products are differentiated) depends crucially on the kind of strategy space, prices or quantities, used by the firms. It is a quasi-trivial result if firms sell the same homogeneous product. But homogeneous product is only a case of extreme substituability, and perhaps not the most frequently experienced. However we know very little on this relationship when the goods are differentiated, or when they are complements rather than substitutes. Rather curiously it is only recently that such a problem began to be systematically scrutinized for the Nash equilibria (see for example Deneckere (1983), Singh and Vives (1983)). In Stackelberg equilibria the problem is certainly even more crucial since the profitability of leadership or followership can critically depend upon the kind of strategy, price or quantity, and the sort of cross-relation between the goods. For example, it is quite clear that in case of perfect substituability, if both the leader and the follower are quantity setters, the leader's position will be better than the follower's, and the two duopolists will be fighting for leadership. But if the game is a price game, then it will be better to be the follower. Indeed, the follower (second mover) can always capture all the market by undercutting slightly the price fixed by the leader (first mover) (see Boyer-Moreaux (1983), Ono (1978, 1982) for more on this subject), and in this case the two competitors will fight for the followership. What now if the two goods are not perfect substitutes, or if the two goods are complements? That is the first question that this paper will try to answer. It will be shown that in the Stackelberg duopoly it is always better to struggle in the quantity space when goods are substitutes, and in the price space when goods are complements.

To compare duopoly models in which the strategy spaces are exogeneously given, supposes either that the theory cannot account for the choice of the strategy space itself or that such a choice is preconstrained by something outside the model. But without such an outside constraint a duopoly theory would have to be also a theory of the strategy space choice. We will show that in the Stackelberg setting such a determination can be made endogeneous, contrary to the Nash duopoly theory where such a determination is impossible.

To switch from a Nash equilibrium to a Stackelberg one (with the same strategy space) implies that one of the duopolists will improve his gains and the other one will be made worse off. But the two moves do not necessarily balance. Hence the total welfare and the consumer welfare can change in the process. It will be shown that the two must be carefully distinguished and that their changes can take different directions.

In order to properly isolate the differences resulting only from the asymmetrical leadership or followership positions, it will be assumed that the demands for the two goods are the same, i.e. that the quantity sold of each good by each duopolist is the same function of its own price and that the cross-effects are also the same. Furthermore, we will work with the simplest model allowing the detection of structural changes when passing from a system of substitutes to a system of complements. In other words, the possible changes coming from more or less intense substituability alone, or from more or less intense complementarity alone, will not be investigated (that such alterations could be consequential has been made clear by Deneckere (1983); see also Shubik (1980)). Only substitutability and complementarity in the neighborhood of independence will be studied here.

The paper is organized as follows : the model is presented in section II. The Stackelberg equilibria are studied in section III, and Nash equilibria in section IV. In section V we compare the two sets of equilibria. In the conclusion, we summarize the results.

SECTION II - THE MODEL

We consider an economy with a continuum of consumers with the same utility functions, indexed by θ, $\theta \in [0, 1]$, and distributed as $g(\theta)$ with $\int_0^1 g(\theta)d\theta = 1$. Let the typical utility function be

$$\left.\begin{aligned} U_\theta(q_{1,\theta}, q_{2,\theta}, m_\theta) = \frac{1+\alpha}{1-\alpha^2}(q_{1,\theta} + q_{2,\theta}) \\ - \frac{1}{2(1-\alpha^2)}\left(q_{1,\theta}^2 - 2\alpha\, q_{1,\theta}\, q_{2,\theta} + q_{2,\theta}^2\right) + m_\theta \end{aligned}\right\} \qquad \text{(II.1)}$$

, $-1 < \alpha < 1$, where $q_{i,\theta}$, $i = 1, 2$, is the amount of good i and m_θ the amount of some numeraire good consumed by individual θ. If p_i, $i = 1, 2$, is the price of good i, then consumer θ utility maximization leads to the following system of demand functions :

$$q_i(p_1, p_2, \theta) = \max\{1 - p_i - \alpha p_j, 0\} \qquad j, i = 1, 2 \quad j \neq i \qquad \text{(II.2)}$$

so that the market demand functions :

$$q_i(p_1, p_2) = \int_0^1 q_i(p_1, p_2; \theta) g(\theta) d\theta \qquad i = 1, 2$$

are given by :

$$q_i(p_1, p_2) = \max\{1 - p_i - \alpha p_j, 0\} \qquad i, j = 1, 2 \quad j \neq i \qquad \text{(II.3)}$$

for $p_i \geq 0$, $i = 1, 2$.

For negative, zero or positive α values the non-numeraire goods are respectively substitutes, independent or complements. It will be noticed that in such a model, perfect substituability is excluded, since for whatever value of α, $\alpha \varepsilon (-1, 0)$, the market of good i cannot disappear whatever the quoted price of good $j \neq i$ is. But as explained in the introduction, only α values in the neighborhood of zero will be taken into consideration since we are primarily interested by the qualitative changes when passing from one type of intermarket cross-effect to the other one.

The inverse demand functions are :

$$p_i = \frac{1 - \alpha}{1 - \alpha^2} - \frac{1}{1 - \alpha^2} q_i + \frac{\alpha}{1 - \alpha^2} q_j \qquad j, i = 1, 2 \quad j \neq i \qquad \text{(II.4)}$$

for $1 - \alpha + q_j \geq q_i \geq 0$, $j, i = 1, 2$, $j \neq i$. Any pair (q_1, q_2) which would not satisfy these inequalities imply that the free exchange market rules are violated since some consumers would then buy at least one of the goods in such a quantity that its marginal utility for that good would be negative.

We will assume that each good i is sold by a different firm i, working at zero cost. It would be indifferent to assume that each one is working with some non-zero constant marginal cost lower than one, provided that profits over variable costs cover fixed costs.

In this simple framework total welfare (surplus) is obtained by evaluating the utility function (II.1) at the equilibrium values (q_1^*, q_2^*), and the consumer surplus is obtained by substracting the profits of the two firms from the total surplus.

SECTION III - THE LEADER-FOLLOWER GAMES

Let us first examine the case where the leader, which we will index as #1, selects as a strategy space the price p_1 of good #1. Given the choice of p_1 by the leader, the follower (firm #2)'s profits, π_2, become :

$$\pi_2 = \max\{p_2 - \alpha p_1 p_2 - p_2^2,\ 0\} \qquad p_1, p_2 \geq 0 \qquad \text{(III.1)}$$

so that the follower best reply function $\hat{p}_2(p_1)$ is :

$$\hat{p}_2(p_1) = \max\{\tfrac{1}{2}(1 - \alpha p_1),\ 0\} \qquad p_1 \geq 0 \qquad \text{(III.2)}$$

Taking into account the follower's reaction, the leader maximizes his own profit :

$$\pi_1 = \max\{(1 - p_1 - \alpha \max\{\tfrac{1}{2}(1 - \alpha p_1),\ 0\})p_1,\ 0\} \qquad \text{(III.3)}$$

Direct calculations show that the equilibrium state is defined by the following values of the relevant variables where (LF/p) stands for "in a leader-follower game with prices as the strategy space" :

$$\left.\begin{aligned} p_1^*(LF/p) &= \frac{2-\alpha}{2(2-\alpha^2)}, \quad q_1^*(LF/p) = \frac{2-\alpha}{4}, \quad \pi_1^*(LF/p) = \frac{(2-\alpha)^2}{8(2-\alpha^2)} \\ p_2^*(LF/p) &= \frac{4-2\alpha-\alpha^2}{4(2-\alpha^2)}, \quad q_2^*(LF/p) = \frac{4-2\alpha-\alpha^2}{4(2-\alpha^2)}, \quad \pi_2^*(LF/p) = \frac{4-2\alpha-\alpha^2}{16(2-\alpha^2)^2} \end{aligned}\right\} \text{(III.4)}$$

Before we compare the leader and follower positions, it must be remarked that what matters is the leader's choice of the price as a strategy space, and that the follower's choice of announcing prices rather than quantities is unconsequential. Clearly the follower's profit function could be expressed as :

$$\pi_2 = \max \{(1 - \alpha p_1)q_2 - q_2^2 \text{ , } 0\} \qquad \begin{array}{l} q_2 \geq 0 \\ p_1 \geq 0 \end{array} \qquad \text{(III.5)}$$

so that the quantity best reply function would take the form :

$$\hat{q}_2(p_1) = \max \{\frac{1}{2}(1 - \alpha p_1), 0\} \qquad p_1 \geq 0 \qquad \text{(III.6)}$$

Given the inverse demand function (II.4) of good #1, q_1 can be expressed as a function of q_2 and p_1 :

$$q_1 = 1 - \alpha - \alpha q_2 - (1 - \alpha^2)p_1$$

, so that given the best reply (III.6) of firm #2, the leader's profit becomes :

$$\pi_1 = \max \{(1 - (\alpha/2))p_1 - (1 - (3\alpha^2/2))p_1^2, 0\}, \; p_1 \geq 0 \quad \text{(III.7)}$$

Maximization of π_1 imply $p_1^* = \dfrac{2 - \alpha}{2(2 - \alpha^2)}$, and the same values of p_2^*, q_1^* and q_2^*, hence π_1^* and π_2^*, as in (III.4).

Comparing now the prices, quantities and profits for the leader and the follower, we get

$$\left.\begin{array}{ll} \dfrac{p_1^*(LF/p)}{p_2^*(LF/p)} = \dfrac{K}{K - \alpha^2} & \dfrac{q_1^*(LF/p)}{q_2^*(LF/p)} = \dfrac{K - \alpha^2(1 - \alpha^3)}{K} \\ \multicolumn{2}{c}{\dfrac{\pi_1^*(LF/p)}{\pi_2^*(LF/p)} = \dfrac{K + \alpha^3(4 - 3\alpha)}{K}} \end{array}\right\} \quad \text{(III.8)}$$

where K is a polynomial in α whose value is positive for K in the neighborhood of zero. The value of K is the same in the numerator and denominator of any given ratio, but may be different in different ratios (the same convention on K will be used throughout this paper). Hence :

PROPOSITION III.1 [1] : *In the price leader model :*

1) the price quoted by the leader is always higher than the price quoted by the follower;

2) the quantity sold by the leader is always smaller than the quantity sold by the follower;

3) the leader's profit is smaller (larger) than the follower's when the goods are substitutes (complements).

Suppose now that the leader selects as a strategy space the quantity q_1 of good #1. For a given q_1 the profit function of duopolist #2 is :

$$\pi_2 = \max\left\{\frac{1-\alpha}{1-\alpha^2} + \frac{\alpha}{1-\alpha^2} q_2 q_1 - \frac{1}{1-\alpha^2} q_2^2 \, , 0\right\} \qquad q_1, q_2 \geq 0 \quad \text{(III.9)}$$

so that the quantity best reply function $\hat{q}_2(q_1)$ will be

$$\hat{q}_2(q_1) = \max \{\tfrac{1}{2} (1 - \alpha + \alpha q_1), 0\} \qquad q_1 \geq 0 \qquad \text{(III.10)}$$

Given the follower's reaction, the leader's profit as a function of his own production level q_1 can be written as :

$$\pi_1 = \max \{\tfrac{1}{2}((2+\alpha)(1-\alpha)q_1 - (2-\alpha^2)q_1^2), 0\} \qquad q_1 \geq 0 \quad \text{(III.11)}$$

Maximization of the leader's profit determines the equilibrium state of the markets. Straightforward treatment leads to :

$$\left.\begin{aligned}
&p_1^*(LF/q) = \frac{2+2}{4(1+\alpha)}, \quad q_1^*(LF/q) = \frac{(1-\alpha)(2+\alpha)}{2(2-\alpha^2)}, \\
&\pi_1^*(LF/q) = \frac{(1-\alpha)(2+\alpha)^2}{8(1+\alpha)(2-\alpha^2)}, \quad p_2^*(LF/q) = \frac{4+2\alpha-\alpha^2}{4(1+\alpha)(2-\alpha^2)}, \\
&q_2^*(LF/q) = \frac{(1+\alpha)(4+2a-\alpha^2)}{4(2-\alpha^2)}, \quad \pi_2^*(LF/q) = \frac{(1-\alpha)(4+2\alpha-\alpha^2)^2}{16(1+\alpha)(2-\alpha^2)^2}
\end{aligned}\right\} \text{(III.12)}$$

As in the price leadership model what matters here is the choice by the leader of the quantity as a strategy space, i.e. whatever the choice of a strategy space by the follower, either prices or quantities, the equilibrium values in the markets will remain the same. Given q_1, the profit function of the follower could be rewritten :

$$\pi_2 = \max \{(1 - \alpha + \alpha q_1)p_2 - (1 - \alpha^2)p_2^2 \ , 0\} \quad q_1, p_2 \geq 0 \quad \text{(III.13)}$$

from which one gets the price best reply function :

$$\hat{p}_2(q_1) = \max\left\{\frac{1}{2(1-\alpha^2)} (1 - \alpha + \alpha q_1), \ 0\right\} \qquad q_1 \geq 0 \quad \text{(III.14)}$$

From the demand function of good #1, the price of good #1 can be expressed as a function of its own price p_1 and the quantity sold q_2 of good #2 :

$$p_1 = 1 - \alpha p_2 - q_1$$

Hence given the best reply (III.14) of firm #2, the leader's profit is :

$$\pi_1 = \max\left\{\left(1 - \frac{\alpha(1-\alpha)}{2(1-\alpha^2)}\right)q_1 - \left(1 + \frac{\alpha^2}{2(1-\alpha^2)}\right)q_1^2 \ , 0\right\} \qquad \text{(III.15)}$$

and maximization of π_1 imply that $q_1^* = ((1 - \alpha)(2 + \alpha))/2(2 - \alpha^2)$ giving the same values of p_1^*, q_2^*, p_2^*, π_1^* and π_2^* as in (III.12).

In this quantity leadership model, comparison of the follower's and leader's prices, quantities and profits leads to :

$$\left.\begin{array}{l} \dfrac{p_1^*(LF/q)}{p_2^*(LF/q)} = \dfrac{K - \alpha^2(1 - \alpha)}{K} \qquad \dfrac{q_1^*(LF/q)}{q_2^*(LF/q)} = \dfrac{K}{K - \alpha^2} \\[2ex] \dfrac{\pi_1^*(LF/q)}{\pi_2^*(LF/q)} = \dfrac{K - \alpha^3(4 + 3\alpha)}{K} \end{array}\right\} \qquad \text{(III.16)}$$

hence, the following proposition :

PROPOSITION III.2 : *In the quantity leader model :*

1) the price quoted by the leader is always smaller than the price quoted by the follower;

2) the quantity sold by the leader is always larger than the quantity sold by the follower;

3) the leader's profit is larger (smaller) than the follower's when the goods are substitutes (complements).

Therefore, all the rankings of one model are exactly reversed in the other one!

Yet, it remains to be checked if, in some sense, the choice of the strategy space is more important than the "role" (leader or follower) actually played. Since :

$$\frac{\pi_2^*(LF/q)}{\pi_2^*(LF/p)} = \frac{K}{K + 2\alpha^5} \qquad \text{(III.17)}$$

and given (III.8) and (III.16), profits can be ordered as follows :

$\alpha < 0$ (substitutes) :

$$\pi_1^*(LF/q) > \pi_2^*(LF/q) > \pi_2^*(LF/p) > \pi_1^*(LF/p) \qquad \text{(III.18)}$$

<u>$\alpha > 0$ (complements)</u> :

$$\pi_1^*(LF/p) > \pi_2^*(LF/p) > \pi_2^*(LF/q) > \pi_1^*(LF/q) \qquad \text{(III.19)}$$

so that :

<u>PROPOSITION III.3</u> : *The choice of the strategy space "dominates" the role distribution in the following sense : if the goods are substitutes (complements) profits accruing to any duopolist in the quantity (price) game, whatever his role in this game, will be larger than in the price (quantity) game, whatever his role in this last game.*

It will be noticed that this ranking of profits comes mainly via a price effect in the case of substitutes and a quantity effect in the case of complements. From (III.4) and (III.12), we get :

$$\left.\begin{aligned} &\frac{p_2^*(LF/q)}{p_1^*(LF/p)} = \frac{K}{K - \alpha^2} \qquad \frac{p_1^*(LF/p)}{p_1^*(LF/q)} = \frac{K}{K - \alpha^3} \\ &\frac{p_1^*(LF/q)}{p_2^*(LF/p)} = \frac{K}{K - \alpha^2} \end{aligned}\right\} \qquad \text{(III.20)}$$

$$\left.\begin{aligned} &\frac{q_2^*(LF/p)}{q_1^*(LF/q)} = \frac{K}{K - \alpha^2} \qquad \frac{q_1^*(LF/q)}{q_1^*(LF/p)} = \frac{K}{K + \alpha^3} \\ &\frac{q_1^*(LF/p)}{q_2^*(LF/q)} = \frac{K}{K - \alpha^2} \end{aligned}\right\} \qquad \text{(III.21)}$$

so that taking into account (III.8) and (III.16), the prices and quantities rankings are the following :

<u>Prices</u>

<u>$\alpha < 0$ (substitutes)</u>

$$p_2^*(LF/q) > p_1^*(LF/q) > p_1^*(LF/p) > p_2^*(LF/p)$$

<u>$\alpha > 0$ (complements)</u>

$$p_2^*(LF/q) > p_1^*(LF/p) > p_1^*(LF/q) > p_2^*(LF/p)$$

(III.22)

<u>Quantities</u>

<u>$\alpha < 0$ (substitutes)</u>

$$q_2^*(LF/p) > q_1^*(LF/q) > q_1^*(LF/p) > q_2^*(LF/q)$$

<u>$\alpha > 0$ (complements)</u>

$$q_2^*(LF/p) > q_1^*(LF/p) > q_1^*(LF/q) > q_2^*(LF/q)$$

(III.23)

Hence when the goods are substitutes the prices quoted by any duopolist are always higher in the quantity game than those quoted by any of the duopolists in the price game, but no simple quantity ranking arises. When the goods are complements, the quantity sold by any duopolist are always larger in the price game than those sold by any one in the quantity game, but no simple price ranking appears.

An immediate consequence of the ranking of profits (III.18) - (III.19) is that, as far as the strategy space is not pre-constrained outside the model, the leader will always choose to fight with quantities if the goods are substitutes and with prices if the goods are complements. Hence, since the kind of response, price or quantity, by the follower is unimportant, we get in the leader-follower model an endogeneous determination of the strategy space. As it will be shown in the following section it is not the case with Nash models.

The last point to examine is the consumer and total welfares in the two kinds of competitions. A look at (III.23) shows that in case of substitutes (complements), the price (quantity) game will provide more goods than the quantity (price) game, so that in this symmetrical model consumer and total surpluses would have to be larger in the price (quantity) leadership model than in the quantity (price) leadership model. Simple calculations reveal that consumer and total surpluses are the following where "hot" stands for <u>hi</u>gher <u>o</u>rder <u>t</u>erms, and $X = 32(1+\alpha)(1-\alpha^2)(2-\alpha^2)^2$:

<u>Consumer surpluses</u> (CS)

$$CS(LF/p) = \frac{32 - 32\alpha - 48\alpha^2 + hot}{X}, \quad CS(LF/q) = \frac{32 - 32\alpha - 64\alpha^2 + hot}{X} \qquad (III.24)$$

<u>Total surpluses</u> (TS)

$$TS(LF/p) = \frac{96 - 32\alpha - 192\alpha^2 + hot}{X}, \quad TS(LF/q) = \frac{96 - 32\alpha - 208\alpha^2 + hot}{X} \qquad (III.25)$$

from which we conclude :

<u>PROPOSITION III.4</u> : *Consumer and total surpluses are larger in the price leader-follower competition than in the quantity leader-follower competition in both the substitutes and complements cases.*

SECTION IV - NASH GAMES

In a Nash setting, price competition and quantity competition have already been compared by Singh and Vives (1983). Yet in a Nash game it is also possible to conceive some kind of mixed competition[2], one of the duopolist being a price setter and the other one a quantity setter. This possibility is excluded in the leader-follower models since, as shown in the preceding section, once the strategy space is chosen by the leader, the kind of reply by the follower, in terms of prices or quantities, does not matter. The need to investigate the mixed Nash equilibrium and compare it to Bertrand and Cournot equilibria does not proceed from some kind of completeness-mania. Indeed as we will show later in this section, if the choice of the strategy space itself is not determined outside the model, such an equilibrium cannot be endogeneously excluded.

Starting from the reaction functions (III.2), (III.6), (III.10) and (III.14), the Bertrand, Cournot and mixed equilibria are respectively defined by the three following sets of equilibrium equations :

Bertrand equilibrium

$$p_1 = \frac{1}{2}(1 - \alpha p_2) \quad , \quad p_2 = \frac{1}{2}(1 - \alpha p_1) \qquad \text{(IV.1)}$$

Cournot equilibrium

$$q_1 = \frac{1}{2}(1 - \alpha - \alpha q_2) \quad , \quad q_2 = \frac{1}{2}(1 - \alpha - \alpha q_1) \qquad \text{(IV.2)}$$

Mixed equilibrium

$$p_1 = \frac{1}{2(1-\alpha^2)}(1 - \alpha + \alpha q_2) \quad , \quad q_2 = \frac{1}{2}(1 - \alpha p_1) \qquad \text{(IV.3)}$$

where in this last case, duopolist #1 announces a price p_1 and duopolist #2 announces a quantity q_2.

Solving these equations we get the respective equilibria :

Bertrand equilibrium

$$p^*(B) = \frac{1}{2+\alpha} \, , \; q^*(B) = \frac{1}{2+\alpha} \, , \; \pi^*(B) = \frac{1}{2+\alpha^2} \qquad \text{(IV.4)}$$

Cournot equilibrium

$$p^*(C) = \frac{1}{(1+\alpha)\,(2-\alpha)} \, , \; q^*(C) = \frac{1-\alpha}{2-\alpha} \, , \; \pi^*(C) = \frac{1-\alpha}{(1+\alpha)\,(2-\alpha)^2} \qquad \text{(IV.5)}$$

Mixed equilibrium

$$\left.\begin{aligned} p^*(M/p) &= \frac{2-\alpha}{4-3\alpha^2} \, , \; q^*(M/p) = \frac{(1-\alpha^2)(2-\alpha)}{4-3\alpha^2} \, , \; \pi^*(M/p) = \frac{(1-\alpha^2)(2-\alpha)^2}{(4-3\alpha^2)^2} \\ p^*(M/q) &= \frac{2-\alpha-\alpha^2}{4-3\alpha^2} \, , \; q^*(M/q) = \frac{2-\alpha-\alpha^2}{4-3\alpha^2} \, , \; \pi^*(M/q) = \frac{(2-\alpha-\alpha^2)^2}{(4-3\alpha^2)^2} \end{aligned}\right\} \text{(IV.6)}$$

where $x(M/q)$ and $x(M/p)$ are the equilibrium values of the variable x for the quantity setter and the price setter respectively in the mixed Nash.

Since :

$$\frac{p^*(C)}{p^*(M/p)} = \frac{K}{K+\alpha^3}\ ,\ \frac{p^*(M/p)}{p^*(M/q)} = \frac{K}{K-\alpha^2}\ ,\ \frac{p^*(M/q)}{p^*(B)} = \frac{K-\alpha^3}{K}$$

$$\frac{p^*(C)}{p^*(B)} = \frac{K}{K-\alpha^2} \qquad \text{(IV.7)}$$

$$\frac{q^*(B)}{q^*(M/q)} = \frac{K}{K-\alpha^3}\ ,\ \frac{q^*(C)}{q^*(M/p)} = \frac{K}{K+\alpha^3(1-\alpha^2)}\ ,\ \frac{q^*(M/p)}{q^*(M/q)} = \frac{K-\alpha^2(1-\alpha)}{K}$$

$$\frac{q^*(C)}{q^*(B)} = \frac{K-\alpha^2}{K} \qquad \text{(IV.8)}$$

$$\frac{\pi^*(C)}{\pi^*(M/q)} = \frac{K}{K-\alpha^6(1-\alpha)}\ ,\ \frac{\pi^*(M/p)}{\pi^*(B)} = \frac{K-\alpha^6}{K}$$

$$\frac{\pi^*(M/q)}{\pi^*(B)} = \frac{K-\alpha^3(8-6\alpha^2-\alpha^3)}{K}\ ,\ \frac{\pi^*(C)}{\pi^*(B)} = \frac{K-2\alpha^3}{K} \qquad \text{(IV.9)}$$

$$\frac{\pi^*(M/p)}{\pi^*(M/q)} = \frac{K+2\alpha^3(1-\alpha)}{K}\ ,\ \frac{\pi^*(C)}{\pi^*(M/p)} = \frac{K}{K+\alpha^3(8-6\alpha^2+\alpha^3)}$$

The rankings of prices, quantities and profits are :

Prices

$\alpha < 0$ (substitutes)

$$p^*(C) > p^*(M/p) > p^*(M/q) > p^*(B)$$

$\alpha > 0$ (complements) (IV.10)

$$p^*(M/p) > p^*(C) > p^*(B) > p^*(M/q)$$

Quantities

$\alpha < 0$ (substitutes)

$$q^*(M/q) > q^*(B) > q^*(C) > q^*(M/p) \tag{IV.11}$$

$\alpha > 0$ (complements)

$$q^*(B) > q^*(M/q) > q^*(M/p) > q^*(C)$$

Profits

$\alpha < 0$ (substitutes)

$$\pi^*(C) > \pi^*(M/q) > \pi^*(B) > \pi^*(M/p) \tag{IV.12}$$

$\alpha > 0$ (complements)

$$\pi^*(B) > \pi^*(M/p) > \pi^*(C) > \pi^*(M/q)$$

What appears then is that, whatever the strategy space of the rival, it is always better, at equilibrium, to be a quantity setter than a price setter if goods are substitutes, and to be price setter rather than a quantity setter if goods are complements. And as shown by the prices and quantities rankings, these results come from intricate price and quantity effects, mainly as far as the mixed equilibrium is concerned. The following points are however worthwhile to notice. First, as already pointed out by Singh and Vives (1983), Cournot prices are always higher than Bertrand prices and, of course, the reverse is true concerning the quantities. Yet if higher prices outweight smaller quantities in the case of substitutes, it is no more true in the case of complements. Hence

Cournot profits are larger in the case of substitutes and Bertrand profits are larger in the case of complements. Second, concerning prices in the mixed equilibrium, it is always the price of the good sold by the price setter which is the highest, whatever the kind of relation between the two goods contrary to what could have maybe been guessed from the pure quantity and the pure price competition models; and furthermore, if prices of the mixed equilibrium are between Cournot prices and Bertrand prices in case of substitutes, they switch to the extreme ends of the range in case of complements. Symetrically the quantity sold by the quantity setter is always larger than that sold by the price setter in both cases of substitutes and complements, and these quantities are between the Bertrand and Cournot ones in case of complements, but switch to the extremities of the range in case of substitutes. Lastly, for the profits, to be a quantity setter is a more comfortable position than a price setter if goods are substitutes but to be a price setter is a more comfortable position if goods are complements. However, the best position in the mixed equilibrium is always worse than the best position in pure (quantity if substitutes, prices if complements) competition, and the worst position in that case is always the worst of all. Hence the mixed equilibrium does not appear as some sort of averaging of the Bertrand and Cournot equilibria as far as profits are concerned at the firm level. At the industry level however, the aggregate profits of the two firms are between the Bertrand and Cournot profits in both cases of substitutes and complements.

Consider now a game where each duopolist would not be constrained to move within some strategy space, but rather would be free to choose either some price or some quantity. Then the Nash equilibria of this game would be the four above equilibria (the mixed equilibrium gives rise to two equilibria by permuting the two firms) that comes from the very fact that whatever one duopolist announces, either a price or a quantity, then both the price best response and the quantity best response of the second one give him the same profit so that he is clearly indifferent between them. So mixed equilibria can only be discarded if the choice of the strategy space is pre-constrained outside the model. We have not in the Nash model an endogeneous determination of the strategy spaces.

It remains to assess which one of these equilibria is the most efficient. Since the quantities in the Bertrand equilibrium are always larger than in the Cournot equilibrium, total welfare is always greater in the Bertrand competition than in the Cournot one. But the total welfare in the mixed equilibrium cannot be assessed by a simple look at quantities because in the case of substitutes, the Bertrand and Cournot quantities are between the mixed equilibrium quantities, and in the case of complements, the mixed equilibrium quantities are between the Bertrand and Cournot ones. Also the consumer surpluses cannot be immediately compared since the profits rankings sometimes depart from the quantities rankings. Calculations show that :

Consumer surpluses

$$CS(C) = \frac{128-256\alpha-160\alpha^2+hot}{Y}, \quad CS(B) = \frac{128-256\alpha-32\alpha^2+hot}{Y}$$

$$CS(M) = \frac{128-256\alpha-96\alpha^2 + hot}{Y} \qquad \text{(IV.13)}$$

Total surpluses

$$TS(C) = \frac{384-512\alpha-736\alpha^2+hot}{Y}, \quad TS(B) = \frac{384-512\alpha-608\alpha^2+hot}{Y}$$

$$TS(M) = \frac{384-512\alpha-672\alpha^2+hot}{Y} \qquad \text{(IV.14)}$$

where $Y = 2(1-\alpha^2)\ (2-\alpha)^2(2+\alpha)^2(4-3\alpha^2)^2$. Hence :

$$\forall\alpha : \quad CS(B) > CS(M) > CS(C) \qquad \text{(IV.15)}$$

$$\forall\alpha : \quad TS(B) > TS(M) > TS(C) \qquad \text{(IV.16)}$$

Pure price competition is always better than mixed competition, itself better than quantity competition both for consumer welfare and total welfare, whatever be the intermarket cross-effects.

SECTION V - A COMPARISON OF STACKELBERG AND NASH EQUILIBRIA

In both kinds of models, Stackelberg and Nash, a systematic relation has been made explicit between first the strategy spaces and second the type of intermarket cross-effects. In case of substitutes, quantity competition is more profitable than price competition, and in case of complements, price competition is more profitable than quantity rivalry. It seems a very robust result since when profits for any role in any model are compared, the ranking subsists. Indeed calculations show that :

$$\left.\begin{aligned} &\frac{\pi_1^*(LF/q)}{\pi^*(C)} = \frac{K + \alpha^4}{K} \qquad && \frac{\pi_2^*(LF/q)}{\pi^*(M/q)} = \frac{K + 0\alpha^3 + \text{hot}}{K - 64\alpha^3 + \text{hot}} \\ &\frac{\pi_1^*(LF/p)}{\pi^*(B)} = \frac{K + \alpha^4}{K} \qquad && \frac{\pi_2^*(LF/p)}{\pi^*(M/p)} = \frac{K + 448\alpha^3 + \text{hot}}{K + 512\alpha^3 + \text{hot}} \end{aligned}\right\} \quad (V.1)$$

hence, taking into account (III.18), (III.19), (IV.12), we get :

<u>$\alpha < 0$ (substitutes)</u>

$$\left.\begin{aligned} &\pi_1^*(LF/q) > \pi^*(C) > \pi^*(M/q) > \pi_2^*(LF/q) \\ &\qquad > \pi_2^*(LF/p) > \pi_1(LF/p) > \pi^*(B) > \pi^*(M/p) \end{aligned}\right\} \quad (V.2)$$

<u>$\alpha > 0$ (complements)</u>

$$\left.\begin{aligned} &\pi_1^*(LF/p) > \pi^*(B) > \pi^*(M/p) > \pi_2^*(LF/p) \\ &\qquad > \pi_2^*(LF/q) > \pi_1^*(LF/q) > \pi^*(C) > \pi^*(M/q) \end{aligned}\right\} \quad (V.3)$$

<u>PROPOSITION V.1</u> : *Whatever the role (leader, follower, Nash competitor) it is always more profitable to be a quantity setter if goods are substitutes, and to be a price setter if goods are complements.*

It was also clearly apparent in the two preceding sections that price competition was always better than quantity competition for both total welfare and consumer welfare. For total welfare, more price competition is always better since calculations show that :

$$TS(C) = \frac{V - 104448\alpha^2 + hot}{Y} \qquad TS(B) = \frac{V - 96252\alpha^2 + hot}{Y}$$

$$TS(M) = \frac{V - 100352\alpha^2 + hot}{Y} \qquad TS(LF/q) = \frac{V - 102400\alpha^2 + hot}{Y} \tag{V.4}$$

$$TS(LF/p) = \frac{V - 98304\alpha^2 + hot}{Y}$$

where $V = 24576 - 8192\alpha$

$$Y = 32(1 + \alpha)(1 - \alpha^2)(4 - \alpha^2)^2(2 - \alpha^2)^2(4 - 3\alpha^2)^2,$$

so that $\forall\alpha : TS(B) > TS(LF/p) > TS(M) > TS(LF/q) > TS(C)$ (V.5)

But generally switching from a Nash equilibrium towards a Stackelberg equilibrium (with the same strategy space) implies a loss of total industry profits. And the result is that a Stackelberg equilibrium will be generally better than a Nash equilibrium as far as consumer surplus alone is concerned. Calculations show that :

$$\left.\begin{array}{ll} CS(C) = \dfrac{W - 43008\alpha^2 + hot}{Y} & CS(B) = \dfrac{W - 26624\alpha^2 + hot}{Y} \\ CS(M) = \dfrac{W - 38912\alpha^2 + hot}{Y} & CS(LF/q) = \dfrac{W - 32762\alpha^2 + hot}{Y} \\ \multicolumn{2}{c}{CS(LF/p) = \dfrac{W - 28672\alpha^2 + hot}{Y}} \end{array}\right\} \quad (V.6)$$

and so :

$$\forall\alpha : CS(B) > CS(LF/p) > CS(LF/q) > CS(M) > CS(C) \qquad (V.7)$$

PROPOSITION V.2 : *In both the substitutes and complements cases, the consumer and total surpluses are larger in the Bertrand Equilibrium than in the mixed Nash Equilibrium and larger in the latter than in the Cournot Equilibrium.*

PROPOSITION V.3 : *In both the substitutes and complements cases, the Bertrand Equilibrium dominate the two Stackelberg Equilibria (Leader in price, leader in quantity) which dominate the other two Nash Equilibria (mixed, Cournot). However, when total surplus is considered, then the Bertrand Equilibrium comes on top followed in decreasing order by the Stackelberg-in-price, the mixed Nash, the Stackelberg-in-quantity and the Cournot Equilibria.*

CONCLUSION

We have shown in this paper that in a differentiated product world, the relationship between those products (substitutes or complements) will be an important factor in the determination of the kind of strategic competition which we will observe between duopolists. We also identified which type of competition best serves the consumers' interests and which type maximizes social efficiency. Conflicts arise between the different cases and criteria.

When the goods are substitutes, one should observe that the variable chosen by firms, that is through which the strategic behavior of the firms is transmitted, will be the production programs or quantity. Firms will announce their production programs and let the prices be set by the demand functions. Given that firms will strategically compete in quantity, then the competition will tend to be of Cournot type rather than mixed Nash unless one firm can take the leadership and force a Stackelberg type competition. However, if the goods are complements, then one should observe that firms will compete by strategically chosing prices rather than quantities. Given that the strategies are expressed through prices, one should observe the Bertrand type of competition rather than the mixed Nash unless one firm can take a leadership position in which case it can make more profits by doing so.

But whether goods are substitutes or complements, the consumers' interest is always best served if the firms are involved in a Bertrand type competition through prices. Therefore, there is no strategic context which meets both the industry and the consumers' interests. We showed also that total surplus, and therefore social efficiency, is in fact maximized in a Bertrand framework, that is a context where firms compete in prices without anyone taking a leadership role.

FOOTNOTES

[1]All the propositions must be read as meaning : "In some neighborhood of $\alpha = 0, \ldots$" even if some of them extend over the whole range $\alpha \varepsilon (-1, +1)$.

[2]By mixed equilibrium we do not refer to the usual meaning of the term in game theory (i.e. mixed strategies as probability distributions with pure strategies as support), but only to the fact that the two duopolists do not move in the same strategy space. This type of equilibria have been studied by Salant (1976) and Ulph and Folie (1980) in the context of exhaustible ressource markets.

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