

# Legal Institutions, Sectoral Heterogeneity, and Economic Development\*

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## Abstract

Poor countries have lower PPP-adjusted investment rates and face higher relative prices of investment goods. It has been suggested that this happens either because these countries have a relatively lower TFP in industries producing capital goods, or because they are subject to greater investment distortions. This paper provides a micro-foundation for the cross-country dispersion in investment distortions. We first document that firms producing capital goods face a higher level of idiosyncratic risk than their counterparts producing consumption goods. In a model of capital accumulation where the protection of investors' rights is incomplete, this difference in risk induces a wedge between the returns on investment in the two sectors. The wedge is bigger, the poorer the investor protection. In turn, this implies that countries endowed with weaker institutions face higher relative prices of investment goods, invest a lower fraction of their income, and end up being poorer. We find that our mechanism may be quantitatively important.

**Key words.** Macroeconomics, Investment Rate, Overlapping Generations, Relative Prices, Investor Protection, Optimal Contracts.

JEL Codes: E22, F43, G32, G38, O16, O17, O41.

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# 1 Introduction

This paper is about understanding why per-capita income co-varies positively with the PPP-adjusted investment rate and negatively with the relative price of capital goods. Our primary conclusion is that cross-country differences in the quality of legal institutions may be an important factor contributing to such patterns.

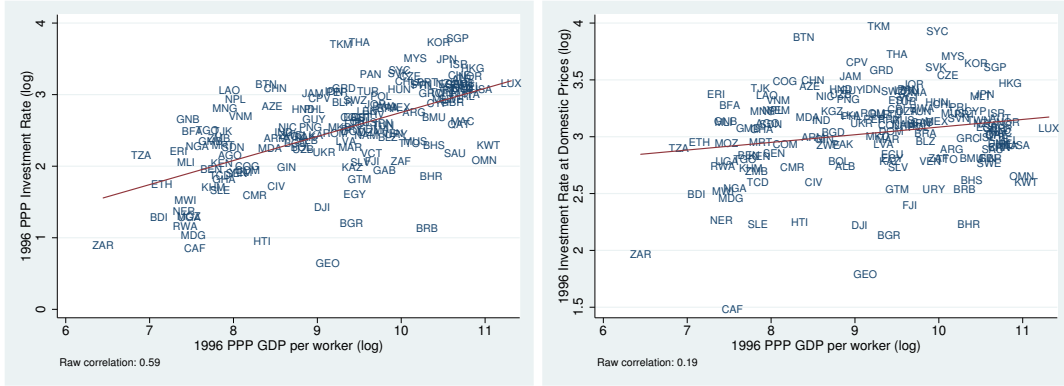


Figure 1: Investment Rates and Income Levels.

Heston and Summers (1988, 1996) first emphasized that the behavior of investment rates in the cross-section of countries is sensitive to the prices used to compute them. When capital goods are valued using international prices, investment rates covary positively with income. But when domestic prices are used, the positive association disappears; see Figure 1, which was constructed using data from Heston, Summers, and Aten’s (2002) Penn World Table, version 6.1. For the two patterns to be compatible, the relative price of investment goods with respect to consumption goods must be negatively correlated with income. This third fact is reported by De Long and Summers (1991), Easterly (1993), and Jones (1994), and documented in Figure 2.<sup>1</sup> These observations suggest that rich and poor countries devote similar fractions of their incomes to investment expenditures, but the former obtain a higher yield in terms of capital goods.

The economic development literature has produced two closely related rationalizations for the just-described evidence. Hsieh and Klenow (2007) argue that poor countries may have lower investment rates simply because they are relatively more efficient in the production of consumption goods. This would make investment goods

<sup>1</sup>The series of relative prices was constructed using the price indexes for consumption and investment goods reported in the Penn World Table 6.1. The methodology followed in constructing these indexes is outlined in Heston and Summers (1991) and in the technical documentation available at <http://pwt.econ.upenn.edu/>

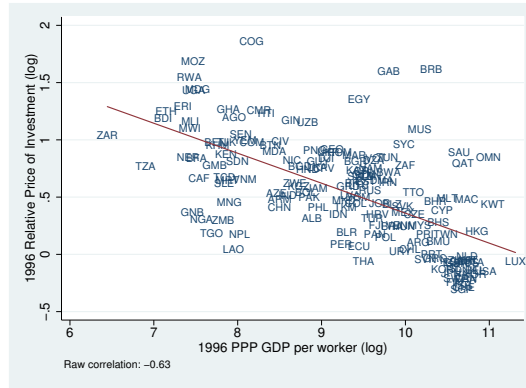


Figure 2: Relative price of Investment Goods and Income Levels.

relatively more expensive, thereby lowering PPP-adjusted investment rates. According to [Chari, Kehoe, and McGrattan \(1996\)](#) and [Restuccia and Urrutia \(2001\)](#), variation in a generic form of investment distortion (wedge) is responsible for the evidence. Both of these mechanisms, when calibrated to match the heterogeneity in prices in the neoclassical growth model, can generate sizeable variation in investment rates. The next challenge is to understand the origin of either form of cross-country heterogeneity (in relative TFP or in wedges). This is crucial if our goal is to identify ways to spur development in poor countries. In this paper, we provide a micro-foundation for the variation in distortions and evaluate its economic significance.

In our model, countries have access to the same technologies. They differ, however, in the extent to which the commercial law and its enforcement protect outside investors, such as bondholders and minority shareholders, from expropriation by company insiders. Several recent papers, e.g., [La Porta, Lopez-de Silanes, Shleifer, and Vishny \(1998\)](#), provide evidence in support of this hypothesis. Our other key assumption is that firms producing investment goods face higher baseline idiosyncratic risk than firms producing consumption goods. Data drawn from COMPUSTAT Files provides strong support for this hypothesis. Even after controlling for a set of observable characteristics and for unobserved heterogeneity, companies producing capital goods display a much higher volatility of sales growth. To our knowledge, this is a novel result.

Ours is a fairly standard two-sector overlapping generation model of capital accumulation. The two industries produce investment goods and consumption goods, respectively. Each individual is born endowed with entrepreneurial talent and decides whether to allocate it to the production of investment or consumption goods. Either way, he will have access to a technology displaying decreasing returns to cap-

ital. Output is assumed stochastic, i.i.d. across technologies, and known only to the technology's owner. The only difference across sectors is that cash flows are more volatile for firms producing investment goods. Young individuals, who we refer to as entrepreneurs, borrow capital from the old through financial intermediaries. The interaction between entrepreneurs and intermediaries takes the familiar form of an optimal contracting problem under asymmetric information. The optimal contract trades off risk-sharing and incentive provision. We model institutions by assuming that entrepreneurs who misreport their outcomes and hide resources face a deadweight loss. The magnitude of this loss reflects the effectiveness of all institutional features that protect outside investors from expropriation by company insiders. The larger the loss, the better the investor protection (the quality of institutions).

The optimal contract dictates that in either sector risk-sharing is increasing in the level of investor protection and decreasing in the volatility of cash flows. Given that risk is higher for firms producing investment goods, this implies a wedge between the returns to investment in the capital and in the consumption good sector, which is only partially compensated by an increase in the price of capital. In turn, this induces a reallocation of resources away from the production of capital goods and towards consumption goods. Since the size of the wedge is larger the poorer the investor protection, better legal institutions yield a lower relative price of capital, higher investment rate, and higher national income.

The quantitative assessment of the model requires taking stands on firm-level volatility in the two sectors, on the quality of legal institutions across countries, and on the degree of international capital mobility. Idiosyncratic risks are calibrated using our own estimates. Whenever possible, a country's investor protection is set in such a way that the model-implied relative price equals its counterpart in the Penn World Table. Finally, barriers to international capital flows are set so that the resulting cross-country dispersion in interest rates is consistent with the data.

Our model accounts for at least 38% and as much as 81% of the variance of log-investment rates implied by the 1996 Penn World Table. Or, alternatively, for at least 74% and as much as 100% of the Gini coefficient of the distribution of investment rates.

Our paper is closely related to recent contributions by [Caselli and Gennaioli \(2003\)](#), [Restuccia and Rogerson \(2007\)](#), [Castro, Clementi, and MacDonald \(2004\)](#), [Restuccia \(2004\)](#), [Erosa and Hidalgo Cabrillana \(2008\)](#), [Burstein and Monge-Naranjo \(2007\)](#), and [Guner, Ventura, and Xu \(2008\)](#). In common with these authors, we investigate the implications of allocative inefficiencies for economic development. Our

paper is closest to [Castro, Clementi, and MacDonald \(2004\)](#) and [Erosa and Hidalgo Cabrillana’s \(2008\)](#). As is the case here, in these papers the allocative inefficiency is the result of information asymmetries in financial markets, and its magnitude depends on the quality of institutions designed to protect investors. Finally, our paper is also part of a recent literature that models investor protection in general equilibrium.<sup>2</sup>

The remainder of this paper is organized as follows. In [Section 2](#) we provide evidence in support of our assumption on the cross-sectoral variation in idiosyncratic risk. We introduce the model in [Section 3](#). In [Sections 4](#) and [5](#) we define and characterize the competitive equilibrium allocation assuming a closed economy. In [Section 6](#) we augment the model to allow for international trade. In [Section 7](#) we describe our calibration procedure. [Section 8](#) is dedicated to comparative statics exercises. The quantitative assessment is conducted in [Section 9](#). In [Section 10](#) we conduct tests of two further restrictions that our theory imposes on the data. Finally, in [Section 11](#) we conclude by discussing a few extensions of our setup.

## 2 Evidence on Firm-Level Volatility

In this section we provide evidence supporting our premise that firms manufacturing investment goods face higher baseline idiosyncratic risk than firms producing consumption goods. For reasons that will become clearer in [Section 3](#), we are interested in assessing the fraction of risk that is not accounted for by factors that would be known to a firm’s financier. Some of these factors are observable by the econometrician, e.g., size and age. Others, such as firm-specific characteristics and sector-specific shocks, are not. Our objective is to test whether the *conditional* standard deviation of sales growth is systematically higher for firms producing investment goods.

Our dataset is an unbalanced panel of 7,070 firms, distributed in 57 3-digit NAICS sectors. It consists of a total of 73,112 firm-year observations, drawn from Standard & Poor’s COMPUSTAT North-America Industrial Annual Database from 1950 to 2005. Our sample selection procedure is detailed in [Appendix A](#). The Bureau of Economic Analysis’ Benchmark Input-Output tables provide information on the contribution of each industry to final demand uses. We classify an industry as in the consumption good sector if the destination of at least 60% of its output is final consumption. We use an analogous rule to assign industries to the investment good category, and we

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<sup>2</sup>[Shleifer and Wolfenzon \(2002\)](#) study the effect of investor protection on the size of the equity market and the number of public firms. [Fabbri \(2007\)](#) extends their analysis to consider the impact of the quality of legal institutions on firm size and aggregate activity. [Albuquerque and Wang \(2008\)](#) look at the asset pricing implications.

discard sectors with very similar, or virtually no contribution to either final use.

Our measure of sales is Compustat item # 12, *net sales*. We first compute the portion of sales growth that is not accounted for by factors, either known or unknown to the econometrician, that are systematically associated with firm growth. We do this by estimating the following equation:

$$\Delta \log(\text{sales})_{ijt} = \alpha_i + \delta_{jt} + \beta_{1j} \log(\text{size})_{ijt} + \beta_{2j} \log(\text{age})_{ijt} + \varepsilon_{ijt}. \quad (1)$$

The dependent variable is the growth rate of real sales for firm  $i$  in sector  $j$ , between years  $t$  and  $t + 1$ . Real sales are net sales over the BEA’s 2–digit sector–specific price deflator for value added. The dummy variable  $\alpha_i$  is a firm–specific fixed effect that accounts for unobserved firm heterogeneity, i.e. for the eventuality that firms have permanently different growth rates for reasons that are unknown to us. We also include a full set of sector–specific year dummies, denoted by  $\delta_{jt}$ . These dummies offer a flexible way to control for changes in sales induced by a variety of industry–wide factors. Among them are events, like weather shocks, changes in the economy’s product mix, and business cycle fluctuations, which tend to have a systematically different impact on different sectors. In particular, investment expenditures are well–known to be much more volatile than consumption expenditures at the business cycle frequency (see for example [Kydland and Prescott \(1990\)](#)). Finally, size and age are included because the empirical Industrial Organization literature has shown that firm growth declines with both of these variables (see [Evans \(1987\)](#) and [Hall \(1987\)](#)). Size is Compustat item # 29, *employees*, whereas age is the time since a firm first appeared in the sample.<sup>3,4</sup>

The objects of our interest are the estimated residuals  $\hat{\varepsilon}_{ijt}$ . We proceed by estimating the following equation:<sup>5</sup>

$$\log \hat{\varepsilon}_{ijt}^2 = \theta_j + u_{ijt}. \quad (2)$$

Letting  $\hat{\theta}_j$  denote the point estimate of the dummy coefficient  $\theta_j$ ,  $\sqrt{\exp(\hat{\theta}_j)}$  is our estimate of the conditional standard deviation of annual sales growth for firms in

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<sup>3</sup>The number of employees is the most common measure of firm size in the empirical IO literature. Its main advantage is that it is relatively immune to measurement problems.

<sup>4</sup>In our theory, firm–level stochastic disturbances are modeled as TFP shocks. In light of this choice, it would be interesting to repeat the exercise of this section using firm–level TFP growth as the dependent variable in regression (1). In principle, this can be accomplished by obtaining a panel of firm–level Solow residuals from our dataset. Unfortunately, Compustat data presents a series of shortcomings that make it ill–suited for this type of analysis. In particular, it severely mismeasures physical capital.

<sup>5</sup>This formulation results from the assumption of a particular functional form for the sectoral variance,  $\sigma_j^2 = \sigma^2 \exp(\theta_j)$ . It is a special case of the multiplicative heteroscedasticity model analyzed by [Harvey \(1976\)](#).

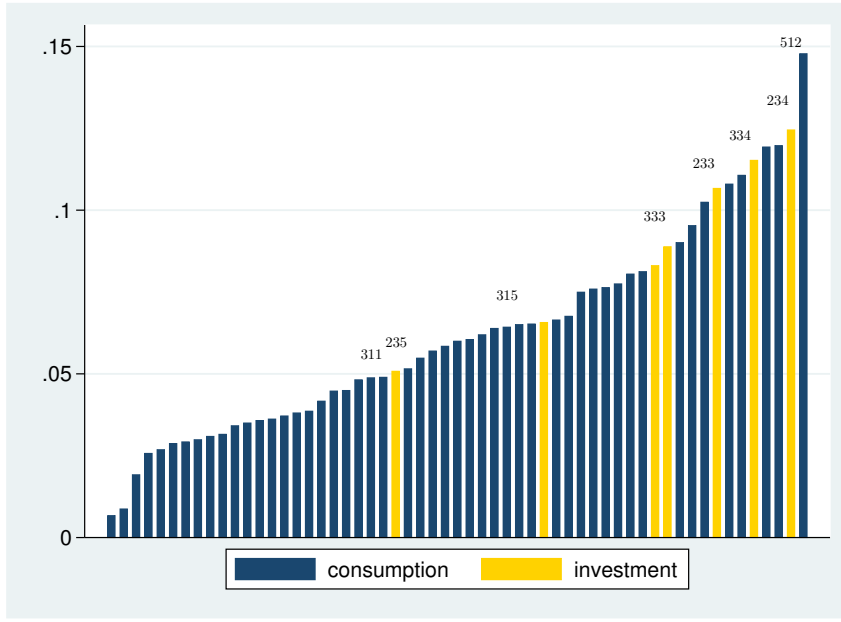


Figure 3: Volatility of sales growth per 3-digit industry.

sector  $j$ . The estimates for all sectors are reported in Table 7 and graphed in Figure 3, sorted in ascending order. The 3-digit figures are the NAICS codes of the largest sectors by value added. Almost all investment good sectors rank among the most volatile in the economy. Among them are Machinery Manufacturing (code 333) and Computer and Electronic Product Manufacturing (334), as well as Construction (233 and 234). Firms in Food Manufacturing (311) and Apparel Manufacturing (315), two of the largest consumption good sectors in most economies, are significantly less volatile.<sup>6,7</sup>

We conducted a series of robustness checks; details are reported in Appendix A. Our results do not change in any appreciable way when we vary the sample selection procedure or the regression equation specification.

Campbell, Lettau, and Malkiel (2001) study the impact of idiosyncratic firm-level shocks on individual stock returns. Since the object and methods of their analysis are different from ours, a direct comparison between our results and theirs is infeasible. Still, we find comfort in the fact that their volatility ranking is broadly consistent with ours. As it is the case here, Campbell, Lettau, and Malkiel (2001) find that firms

<sup>6</sup>We have also tested the null hypothesis that the conditional volatility is the same for firms belonging to consumption and investment good sectors. The Breusch-Pagan test rejects it categorically at the conventional significance level (The Breusch-Pagan test statistic is equal to 804.045, with a p-value lower than 0.0001).

<sup>7</sup>Interestingly, an inspection of the estimates reported in Table 6 reveals that firms producing durable goods tend to be more volatile than those producing non-durables.

producing consumer goods are among the least volatile, while firms manufacturing computers are among the most volatile.

We close this section by briefly addressing three natural questions about this analysis.

Our model assumes that the cross-sectoral heterogeneity in volatility is the same across countries. Would information gathered in other countries deliver estimates similar to those obtained for the US? Lack of detailed micro data for most countries prevents us from answering this question. Notice, however, that our theoretical analysis does not rely on volatilities being the same. Rather, it only needs the rankings of sectors by volatility to be the same.<sup>8</sup>

Firms in Compustat tend to be larger and older than their peers not included in the sample, and therefore are likely to be less volatile. Is selection likely to bias our estimates? We partially addressed this issue by controlling for age and size. However, there may be other, unknown factors, that cause firms' sales growth volatility to be systematically higher or lower than the average US firm's. What is comforting is that as long as these factors have the same impact across sectors, the ranking of volatilities will not be influenced by the selection procedure. Absent data on the whole size distribution, we cannot establish conclusively whether this is the case.<sup>9</sup>

What are the causes of the cross-sectoral variation in conditional sales growth volatility? Since addressing this question is beyond the scope of this paper, in our theory we take a reduced-form approach, assuming that firms are hit by TFP shocks with sector-specific variances. Our conjecture, however, is that in most sectors producing investment and durable consumption goods there is a greater scope for process and product innovation. [Klenow \(1996\)](#)'s findings on the distribution of R&D expenditures across 2- and 3-digit industries seem to be consistent with this hypothesis.<sup>10</sup> We think of firms in these sectors as being arranged on a quality ladder, e.g., [Grossman and Helpman \(1991\)](#) and [Aghion and Howitt \(1992\)](#). The adoption of an innovation ahead of its peers allows a laggard to advance to the frontier and boost its sales,

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<sup>8</sup>To our knowledge, the only attempt at estimating cross-sectoral variation in idiosyncratic risk across countries was carried out by [Michelacci and Schivardi \(2008\)](#). According to their preliminary results, volatilities are somewhat different across countries. However, in most cases, the rankings appear to be the same as in the US.

<sup>9</sup>A separate concern is that cross-sector relative volatilities may have changed over time. Consistent with [Comin and Philippon \(2005\)](#), we find evidence of a positive time trend in volatility for most industries (a phenomenon that, according to [Davis, Haltiwanger, Jarmin, and Miranda \(2006\)](#), did not extend to the whole size distribution). However, the ranking of sectors has essentially stayed the same throughout the sample period. Most importantly for the quantitative exercise of Section 9, the difference between the volatility measures that will be used to calibrate the model has also been roughly constant over time.

<sup>10</sup>See Tables 2 and 3 of that paper.



possibly in a dramatic way. Conversely, the early adoption by a competitor has the potential to severely depress its results.

### 3 Model

We consider a simple extension of the standard two-period, two-sector overlapping generations model. The population is constant and the measure of each cohort is normalized to one. Individuals are risk-averse. Preferences are time-separable and the period utility, denoted by  $u(c_t)$ , displays constant relative risk aversion.<sup>11</sup> Let  $\sigma$  denote the coefficient of relative risk aversion. Agents discount second-period utility at the rate  $\beta$ ;  $\beta > 0$ .

Young individuals are born without any endowment. In order to consume, they engage in the production of either consumption goods or investment goods. Both activities require capital, which is borrowed from the old via financial intermediaries. Old individuals are idle and consume thanks to assets accumulated when young. The technology in sector  $j$ ,  $j = I, C$ , is described by the production function  $y_{jt} = z_{jt}k_{jt}^\alpha$ , where  $\alpha \in (0, 1)$  and  $z_{jt}$  is a random variable i.i.d. across entrepreneurs and over time. In either sector, capital depreciates at the constant rate  $\delta \in (0, 1)$ . The two sectors only differ with respect to the distribution of  $z_{jt}$ . We posit that, for all entrepreneurs in sector  $j$ ,  $j = I, C$ , and for all  $t \geq 0$ ,

$$\log(z_{jt}) = \zeta_j, \quad \zeta_j \sim \mathcal{N}(\mu_j, \eta_j^2).$$

In order to capture the difference in the volatility of growth rates across sectors documented in Section 2, we assume that  $\eta_I > \eta_C$ . We use  $p_t$  to denote the relative price of the investment good in terms of consumption good and  $N_t$  to denote the fraction of entrepreneurs (i.e. the fraction of young agents) engaged in the production of capital goods. Choosing the consumption good as the numeraire means that there is no loss of generality in setting the absolute prices to  $p_{Ct} \equiv 1$  and  $p_{It} \equiv p_t$ , respectively. In spite of this, using absolute prices will sometimes ease the exposition.

The output realization is private information for the entrepreneurs, opening up the option of hiding some of their cash flows from their financiers. Hiding, however, is costly. For every unit of output hidden, an entrepreneur ends up with only  $\xi \in [0, 1]$ .<sup>12</sup> The balance is lost in the hiding process. The parameter  $\xi$  is our measure of the economy-wide level of investor protection. The larger is  $\xi$ , the lower is the protection;

<sup>11</sup>We restrict our attention to the CRRA family, because utility functions in this class display non-increasing absolute risk aversion and imply log-additive indirect utility functions.

<sup>12</sup>Our hiding cost resembles the falsification cost considered by [Lacker and Weinberg \(1989\)](#).

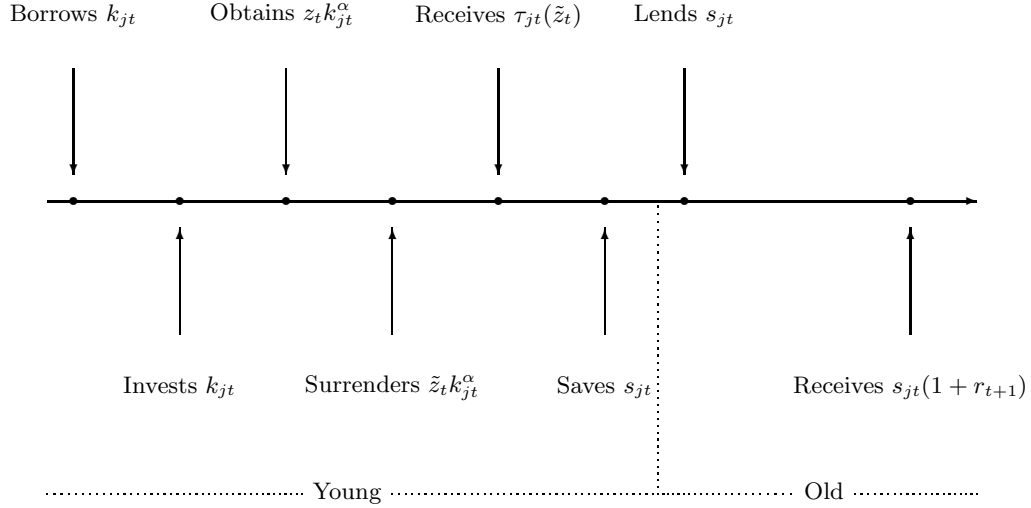


Figure 4: Timing.

the two extreme values correspond to complete absence of protection ( $\xi = 1$ ) and perfect protection ( $\xi = 0$ ). Finally, we assume that the intermediation industry is competitive with free entry.

A financing contract offered to a sector- $j$  entrepreneur consists of a capital advance,  $k_{jt}$ , and of a schedule of contingent transfers,  $\tau_{jt}(z_t)$ . Figure 4 displays the sequence of events during the life of an agent. An individual starts by investing capital  $k_{jt}$  to produce output equal to  $z_t k_{jt}^\alpha$ . Next, he makes a claim about the outcome of his project  $\tilde{z}_t$ , gives the intermediary output consistent with this claim ( $\tilde{z}_t k_{jt}^\alpha$ ), and receives a contingent transfer of consumption good  $\tau_{jt}(\tilde{z}_t)$ . At the end of the first period, entrepreneurs end up with income we denote by  $m_t$ . Having no endowment, an agent is unable to report  $\tilde{z}_t > z_t$ . Truthful reporting yields  $m_t = \tau_{jt}(z)$ . Reporting  $\tilde{z}_t < z_t$  yields  $m_t = \tau_{jt}(\tilde{z}_t) + \xi p_{jt}(z_t - \tilde{z}_t) k_{jt}^\alpha$ . By misrepresenting himself as an agent that received an outcome  $\tilde{z}_t$ , an entrepreneur hit by  $z_t$  will receive a transfer  $\tau_{jt}(\tilde{z}_t)$  and enjoy the fraction  $\xi$  of the hidden output  $(z_t - \tilde{z}_t) p_{jt} k_{jt}^\alpha$ .

At the end of the first stage of their lives, all agents, regardless of their occupations, consume part of their incomes and save the rest. At the beginning of the second stage, they lend their savings to intermediaries at the market rate. Intermediaries channel those funds to the new cohort of young people. Finally, agents receive and consume principal plus interest. Notice that  $r_t$  denotes the return in consumption goods to the investment of one unit of consumption good.

## 4 Competitive Equilibrium

We begin by considering an entrepreneur's consumption–saving decision. This problem is the same for all agents. Let  $v(m_t, r_{t+1})$  denote the indirect utility of an agent born at time  $t$ , conditional on having received an income  $m_t$  and on facing an interest rate  $r_{t+1}$ . Then,

$$v(m_t, r_{t+1}) \equiv u[m_t - s(m_t, r_{t+1})] + \beta u[(1 + r_{t+1})s(m_t, r_{t+1})],$$

where the optimal saving function  $s(m_t, r_{t+1})$  is

$$s(m_t, r_{t+1}) \equiv \arg \max_s \{u(m_t - s) + \beta u[(1 + r_{t+1})s]\}.$$

Under our assumptions on preferences, it follows that

$$s(m_t, r_{t+1}) = \kappa(r_{t+1}) m_t \tag{3}$$

and

$$v(m_t, r_{t+1}) = u(m_t)[u(1 - \kappa(r_{t+1})) + \beta u(\kappa(r_{t+1})(1 + r_{t+1}))], \tag{4}$$

where  $\kappa(r_{t+1}) \equiv [1 + \beta^{-\frac{1}{\sigma}}(1 + r_{t+1})^{\frac{\sigma-1}{\sigma}}]^{-1}$ .

Financing contracts in sector  $j$  consist of a non–negative capital advance  $k_{jt}$  and a function  $\tau_{jt} : \mathfrak{R}^+ \rightarrow \mathfrak{R}^+$  that solve<sup>13</sup>

$$\max_{k_{jt}, \tau_{jt}(z)} \int v[\tau_{jt}(z), r_{t+1}] f_j(z) dz, \tag{P}$$

subject to incentive compatibility, i.e.

$$v[\tau_{jt}(z), r_{t+1}] \geq v[\tau_{jt}(z'), r_{t+1}] + \xi p_{jt}(z - z') k_{jt}^\alpha \quad \forall z, z', z \geq z', \tag{5}$$

and the zero–profit condition for intermediaries:

$$\bar{\tau}_{jt} \equiv \int \tau_{jt}(z) f_j(z) dz = p_{jt} \bar{z}_j k_{jt}^\alpha - (r_t + \delta) p_{It} k_{jt}, \tag{6}$$

with  $\bar{z}_j = \int z f_j(z) dz$ .

We now define a competitive equilibrium.

**Definition 1** *Given an initial aggregate capital stock  $K_0 > 0$ , a competitive equilibrium is a non–negative consumption level of the initial old,  $c_0^o$ , and sequences of young and old agents' non–negative consumption allocations  $\{c_{jt}^y(z)\}_{t=0}^\infty$  and  $\{c_{jt}^o(z)\}_{t=1}^\infty$ , contracts  $\{k_{jt}, \tau_{jt}(z)\}_{t=0}^\infty$ , aggregate capital  $\{K_t\}_{t=1}^\infty$ , measures of entrepreneurs in the investment good sector  $\{N_t\}_{t=0}^\infty$ , relative prices  $\{p_t\}_{t=0}^\infty$ , and interest rates  $\{r_t\}_{t=0}^\infty$ , such that*

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<sup>13</sup>Unless otherwise specified, all integrals are computed on  $[0, +\infty)$ , the natural support of the lognormal distribution.

1.  $c_0^o = p_0 K_0 (1 + r_0)$ ,  
and, at all  $t \geq 0$ :
2.  $c_{jt}^y(z) = \tau_{jt}(z) - s(\tau_{jt}(z), r_{t+1})$  and  $c_{j,t+1}^o(z) = s(\tau_{jt}(z), r_{t+1})(1 + r_{t+1})$ ,  $\forall z$  and for  $j = I, C$ ;
3.  $\{k_{It}, \tau_{It}(z)\}$  solves problem (P) for  $p_{It} = p_t$ ;
4.  $\{k_{Ct}, \tau_{Ct}(z)\}$  solves problem (P) for  $p_{Ct} = 1$ ;
5.  $N_t \in [0, 1]$ ;
6. young individuals are indifferent between the two sectors:

$$\int v(\tau_{It}(z), r_{t+1}) f_I(z) dz = \int v(\tau_{Ct}(z), r_{t+1}) f_C(z) dz; \quad (7)$$

7. aggregate savings are equal to the value of the capital stock:

$$p_t K_{t+1} = N_t \int s(\tau_{It}(z), r_{t+1}) f_I(z) dz + (1 - N_t) \int s(\tau_{Ct}(z), r_{t+1}) f_C(z) dz; \quad (8)$$

8. gross investment equals the production of investment goods:

$$K_{t+1} = (1 - \delta)K_t + N_t \bar{z}_I k_{It}^\alpha; \quad (9)$$

and

9. the market for capital clears:

$$K_t = N_t k_{It} + (1 - N_t) k_{Ct}. \quad (10)$$

## 5 Analysis

### 5.1 Perfect Investor Protection

When investor protection is perfect, our model reduces to the standard two-period, two-sector model of capital accumulation. The necessary conditions for optimality in production are

$$\alpha p_{jt} \bar{z}_j k_{jt}^{\alpha-1} = p_{It} (r_t + \delta), \quad j = I, C. \quad (11)$$

It follows that

$$\tau_{jt}(z) = (1 - \alpha) p_{jt} \bar{z}_j k_{jt}^\alpha. \quad (12)$$

Conditions (11) imply that the relative price of the investment good satisfies

$$p_t \equiv \frac{p_{It}}{p_{Ct}} = \frac{\bar{z}_C}{\bar{z}_I} \left( \frac{k_{Ct}}{k_{It}} \right)^{\alpha-1}. \quad (13)$$

Using (4) and (12), we can rewrite the occupational choice condition (7) as

$$u[(1-\alpha)\bar{z}_C k_{Ct}^\alpha] = u[(1-\alpha)p_t \bar{z}_I k_{It}^\alpha]. \quad (14)$$

Since  $u$  is strictly increasing, conditions (13) and (14) imply that  $k_{Ct} = k_{It}$ . This, along with condition (10), implies that  $k_{Ct} = k_{It} = K_t$ , so that  $p_t = \bar{z}_C/\bar{z}_I$  and  $\tau_{Ct} = \tau_{It}$  for all  $t$ . Finally, by (3), condition (8) leads us to conclude that

$$K_{t+1} = (1-\alpha)\kappa(r_{t+1})\bar{z}_I K_t^\alpha.$$

Aggregation holds: the latter condition, along with (11) for  $j = I$ , can be used to recover the equilibrium sequences for  $K_t$  and  $r_t$ . Then, the sequence for  $N_t$  can be computed using condition (9).

## 5.2 Imperfect Investor Protection

We now turn to the general case of  $\xi \in [0, 1]$ . First, we illustrate the lending contract. Then, we characterize the determinants of the relative price of investment goods. Finally, we describe the general equilibrium.

### 5.2.1 Characterization of the Lending Contract

Our assumptions on preferences imply that Problem (P) is independent of  $r_{t+1}$ . Using condition (4), it can be rewritten as

$$\max_{k_{jt}, \tau_{jt}(z)} \int u[\tau_{jt}(z)] f_j(z) dz,$$

$$\text{subject to } u[\tau_{jt}(z)] \geq u[\tau_{jt}(z') + \xi p_{jt}(z - z') k_{jt}^\alpha] \quad \forall z, z', z \geq z', \quad (15)$$

$$\bar{\tau}_{jt} \equiv \int \tau_{jt}(z) f_j(z) dz = p_{jt} \bar{z}_j k_{jt}^\alpha - (r_t + \delta) p_{It} k_{jt}, \quad (16)$$

$$k_{jt} \geq 0, \tau_{jt}(z) \geq 0 \quad \forall z.$$

Strict concavity of the utility function implies that constraint (15) binds. Then, by strict monotonicity of  $u$ , it follows that, for all  $z, z'$ ,

$$\tau_{jt}(z') = \tau_{jt}(z) + \xi p_{jt}(z' - z) k_{jt}^\alpha. \quad (17)$$

For every  $\xi$ , entrepreneurial income will be an affine and weakly increasing function of  $z$ . Income risk, as measured by its standard deviation, is simply  $std[\tau_{jt}(z)] =$

$\xi p_{jt} k_{jt}^\alpha \times std(z_j)$ . It depends positively on the scale of production and negatively on the level of investor protection (positively on  $\xi$ ). By (17), the contracting problem simplifies further to

$$\max_{k_{jt}, \bar{\tau}_{jt}} \int u[\bar{\tau}_{jt} + \xi p_{jt}(z - \bar{z}_j) k_{jt}^\alpha] f_j(z) dz, \quad (18)$$

$$\text{subject to } \bar{\tau}_{jt} = p_{jt} \bar{z}_j k_{jt}^\alpha - (r_t + \delta) p_{It} k_{jt}, \quad (19)$$

$$k_{jt} \geq 0,$$

$$\bar{\tau}_{jt} + \xi p_{jt}(z - \bar{z}_j) k_{jt}^\alpha \geq 0 \quad \forall z. \quad (20)$$

The following proposition provides a detailed characterization of the optimal transfer schedule.

**Proposition 1** For  $j = I, C$ , (i) There exists a time-invariant function  $g_j : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  such that  $\tau_{jt}(z) = g_j(z) p_{jt} k_{jt}^\alpha$  for all  $t \geq 0$ ; (ii) The function  $g_j$  satisfies the functional equation

$$g_j(z) = \begin{cases} \bar{z}_j(1 - \alpha - \xi) - \alpha \xi \frac{\int u'[g_j(z)](z - \bar{z}_j) f_j(z) dz}{\int u'[g_j(z)] f_j(z) dz} + \xi z & \text{if } \xi \leq \xi_j^*, \\ \xi z & \text{if } \xi \geq \xi_j^*, \end{cases}$$

where  $\xi_j^* \equiv [1 + \frac{\alpha}{1-\alpha} e^{-\sigma \eta_j^2}]^{-1} \in (0, 1)$ .

**Proof.** See Appendix B. ■

Figure 5 shows how  $g_j(z)$  varies with  $\xi$ . The slope of the schedule increases with  $\xi$ . Conditional on the level of capital, the risk imposed on the entrepreneur increases as investor protection worsens. The intercept of the schedule is weakly decreasing in  $\xi$ , strictly for  $\xi < \xi_j^*$ .

How about the level of capital? For  $\xi < \xi_j^*$ , constraint (20) will not bind. The optimal capital is determined by the necessary and sufficient condition for the solution to program (18)–(19):

$$p_{It}(r_t + \delta) = \alpha p_{jt} k_{jt}^{\alpha-1} (\bar{z}_j - \xi \omega_j), \quad (21)$$

where  $\omega_j \equiv -\frac{\int u'[g_j(z)](z - \bar{z}_j) f_j(z) dz}{\int u'[g_j(z)] f_j(z) dz} \geq 0$ . The term  $\xi \omega_j$  is the wedge between the private and social marginal product of capital, which is induced by imperfect risk-sharing. To shed some light on the meaning of (21), rewrite it as

$$[\alpha p_{jt} \bar{z}_j k_{jt}^{\alpha-1} - p_{It}(r_t + \delta)] \int u'[\tau_{jt}(z)] f_I(z) dz = -\alpha p_{jt} k_{jt}^{\alpha-1} \int u'[\tau_{jt}(z)](z - \bar{z}_j) f_j(z) dz. \quad (22)$$

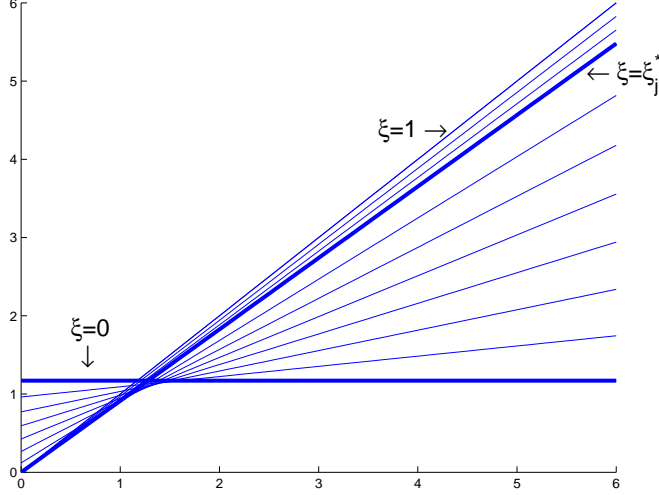


Figure 5: Schedules  $g_j(z)$ .

The term on the left-hand side is the marginal benefit of increasing capital. Raising  $k_{jt}$  affects the resources available for distribution to the entrepreneur in all states of nature. The term on the right-hand side is the marginal cost of increasing capital. Incentive compatibility implies that a larger capital can be accommodated only at the cost of a higher variance in entrepreneurial income.

Consider first the scenario,  $\xi = 0$ , already illustrated in Section 5.1. In that case,  $\omega_j = 0$ . Capital is at its first-best level and the entrepreneur is fully insured against idiosyncratic risk. That is,  $g_j(z) = (1 - \alpha)\bar{z}_j \forall z$ . For all  $\xi > 0$ , capital will be below its unconstrained-efficient level. To see this, let  $\xi = 0$  and think of an infinitesimal increase in  $\xi$ . Since  $\alpha p_{jt} \bar{z}_j k_{jt}^{\alpha-1} = p_{It}(r_t + \delta)$ , such an increase has no first-order effect on the left-hand-side of (22), but induces an increase in its right-hand-side. Holding capital constant would imply an excessive level of income risk. The lender's optimal response is to lower  $k_{jt}$  below its unconstrained-efficient level, therefore reducing risk at the price of a lower average transfer.

For large enough  $\xi$ , the limited liability constraint  $\tau_{jt}(0) \geq 0$  will bind. For  $\xi > \xi_j^*$ , the lender's ability to fine-tune the capital advance  $k_{jt}$  will be limited by that constraint. In that region,  $k_{jt} = \left[ \frac{p_{It}(r_t + \delta)}{(1-\xi)p_{jt}\bar{z}_j} \right]^{-\frac{1}{1-\alpha}}$ . The private marginal benefit of increasing capital will be higher than the marginal cost.

### 5.2.2 The Relative Price of Capital

As long as  $\eta_C > 0$ , we have that  $0 < (1 - \alpha) < \xi_C^* < \xi_I^* < 1$ . This naturally leads to a partitioning of  $\xi$ 's domain into three regions. We consider them in turn.

In Region I,  $[0, \xi_C^*]$ , the choice of capital is unconstrained in both sectors. Let  $Q_t \equiv k_{Ct}/k_{It}$ . Conditions (21) imply that

$$p_t = \frac{\bar{z}_C - \xi\omega_C}{\bar{z}_I - \xi\omega_I} Q_t^{\alpha-1}. \quad (23)$$

Rewriting (7) as

$$\int u[p_t g_I(z) k_{It}^\alpha] f_I(z) dz = \int u[g_C(z) k_{Ct}^\alpha] f_C(z) dz$$

yields the relation

$$p_t = \left[ \frac{E[u(g_C)]}{E[u(g_I)]} \right]^{\frac{1}{1-\sigma}} Q_t^\alpha. \quad (24)$$

By (23) and (24),  $p_t$  is time-invariant and is given by

$$p = \left[ \frac{\bar{z}_C - \xi\omega_C}{\bar{z}_I - \xi\omega_I} \right]^\alpha \left[ \frac{E[u(g_C)]}{E[u(g_I)]} \right]^{\frac{1-\alpha}{1-\sigma}}.$$

The ratio  $Q_t$  is also time-invariant and is given by  $Q = \frac{\bar{z}_C - \xi\omega_C}{\bar{z}_I - \xi\omega_I} \left[ \frac{E[u(g_I)]}{E[u(g_C)]} \right]^{\frac{1}{1-\sigma}}$ .

In Region II,  $[\xi_C^*, \xi_I^*]$ , the choice of capital is unconstrained only in the investment good sector. In the consumption good sector, capital is pinned down by (20), the non-negativity constraint on transfers. Condition (21) is replaced by  $k_{Ct} = \left[ \frac{p_t(r_t + \delta)}{(1-\xi)\bar{z}_C} \right]^{-\frac{1}{1-\alpha}}$ . Along with (21) for  $j = I$  and (24), the latter implies that

$$p = \left[ \frac{(1-\xi)\bar{z}_C}{\alpha(\bar{z}_I - \xi\omega_I)} \right]^\alpha \left[ \frac{E[u(g_C)]}{E[u(g_I)]} \right]^{\frac{1-\alpha}{1-\sigma}}.$$

Finally, in Region III,  $[\xi_I^*, 1]$ , the choice of capital is constrained in both sectors. We have that

$$p = \left( \frac{\bar{z}_C}{\bar{z}_I} \right)^\alpha \left[ \frac{E[u(g_C)]}{E[u(g_I)]} \right]^{\frac{1-\alpha}{1-\sigma}} = e^{(\mu_C - \mu_I) + \frac{1}{2}[(1-\alpha)(1-\sigma) + \alpha](\eta_c^2 - \eta_I^2)}.$$

In Regions I and II, the relative price varies with the parameter  $\xi$ . While analytical results are not easily forthcoming, all numerical experiments we have performed show that in those regions the relative price of capital is higher, the poorer the quality of institutions (the higher is  $\xi$ ). This is not the case in Region III, where  $p$  is invariant with respect to  $\xi$ . In general,  $\xi_I^*$  will imply an upper bound for our model's ability to generate dispersion in relative prices. However, as illustrated in Section 9, for all sensible parameterizations,  $\xi_I^* \approx 1$ . Region III is always very small. For this reason, in the remainder of the paper we will focus exclusively on Regions I and II.



### 5.2.3 The Comparative Statics of the Relative Price: An Example

The purpose of this section is to provide some intuition for the fact that the relative price increases with  $\xi$ . To that end, we consider the case of  $\eta_C = 0$ . In this scenario, the possible values for  $\xi$  are the two regions  $[0, \xi_I^*]$  and  $[\xi_I^*, 1]$ . Here we focus on  $[0, \xi_I^*]$ . Entrepreneurs in the consumption good sector will always be operating at the efficient scale, and their income will simply be  $\tau_{Ct} = (1 - \alpha)\bar{z}_C k_{Ct}^\alpha$ . This means that the pair  $(p, Q)$  is uniquely determined by the conditions

$$p = \frac{\bar{z}_C}{\bar{z}_I - \xi\omega_I} Q^{\alpha-1}, \quad (25)$$

$$p = \frac{(1 - \alpha)\bar{z}_C}{[E[u(g_I)]]^{1/(1-\sigma)}} Q^\alpha. \quad (26)$$

Figure 6 shows the typical comparative statics of  $p$  and  $Q$  with respect to  $\xi$ . The exercise characterizes the effects of reducing investor protection (increasing  $\xi$  from  $\xi_0$  to  $\xi_1$ ). The schedules labeled *FOC* and *IND* depict the relations between  $p$  and  $Q$  implied by equations (25) and (26), respectively. Solid lines refer to the case of high investor protection ( $\xi_0$ ). Dotted lines instead refer to the case of low protection ( $\xi_1$ ).

The *FOC* schedule is simply the locus of the  $(p, Q)$  pairs consistent with the equality of the private marginal products, expressed in consumption goods. These are  $\alpha p k_{It}^{\alpha-1} [\bar{z}_I - \xi\omega_I]$  and  $\alpha \bar{z}_C k_{Ct}^{\alpha-1}$ , respectively. To maintain this equality as  $p$  increases,  $k_C$  must decrease with respect to  $k_I$ . The *IND* schedule is upward sloping because as  $Q = k_{Ct}/k_{It}$  increases, an increase in the relative price is needed to keep agents indifferent between the two occupations.

Since  $\xi\omega_I$  increases with  $\xi$ , the private marginal product of capital in the investment good sector is lower, the poorer the protection. This is why *FOC*<sub>1</sub> lies above *FOC*<sub>0</sub>. Also, while there exist parameter values such that the opposite occurs, in most cases *IND*<sub>1</sub> lies above *IND*<sub>0</sub>, as depicted in Figure 6. We are not able to prove this result with generality because a larger  $\xi$  is accommodated by an increase in both the mean and the variance of the random variable  $g_I(z)$ . Therefore, we cannot sign the impact of such change on the denominator of the right-hand side in (26).

While  $p$  always increases with  $\xi$ , numerical simulations show that the comparative statics of  $Q$  illustrated in Figure 6 are not a robust feature of our environment. There exist parameter values such that  $Q$  is monotone increasing in  $\xi$ .

### 5.2.4 General Equilibrium

We now turn to the full characterization of the equilibrium in Regions I and II. Condition (10) implies that  $k_{It} = \frac{K_t}{N_t + (1 - N_t)Q}$ . Substituting into equations (8), (9),

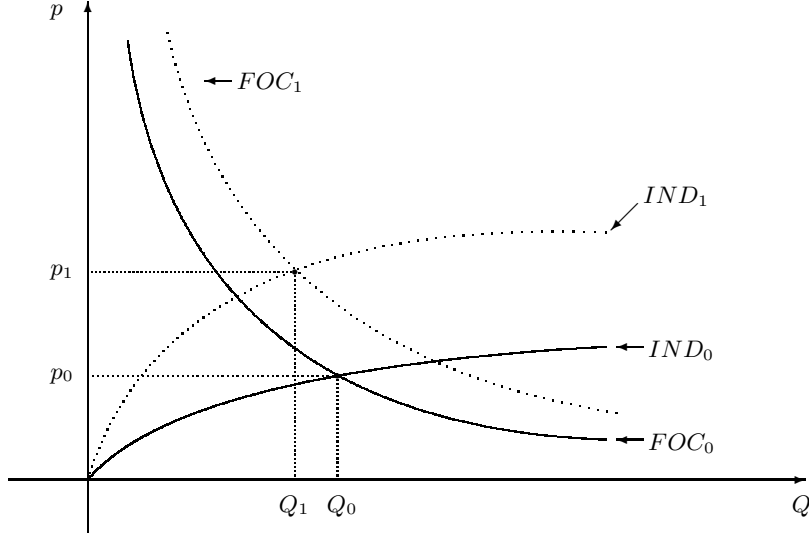


Figure 6: Comparative statics with respect to  $\xi$ .

and (21) yields

$$pK_{t+1} = \kappa(r_{t+1}) \left[ \frac{K_t}{N_t + (1 - N_t)Q} \right]^\alpha [pN_t E(g_I(z)) + (1 - N_t)Q^\alpha E(g_C(z))], \quad (27)$$

$$K_{t+1} = (1 - \delta)K_t + N_t \bar{z}_I \left( \frac{K_t}{N_t + (1 - N_t)Q} \right)^\alpha, \quad (28)$$

$$r_t + \delta = \alpha \left( \frac{K_t}{N_t + (1 - N_t)Q} \right)^{\alpha-1} (\bar{z}_I - \xi\omega_I). \quad (29)$$

Having computed  $p$  and  $Q$ , and given  $K_0$ , the three conditions above are sufficient to characterize the equilibrium paths for  $K_t$ ,  $N_t$ , and  $r_t$ .<sup>14</sup> The sequence of consumption allocations and the other quantities of interest can be easily recovered using the relations outlined earlier in this section.

## 6 International Capital Flows

In principle, there is no reason to expect that the covariance pattern between relative price, investment rate, output and investor protection that obtains under autarky generalizes to a scenario in which international trade is allowed for. For this reason, we now propose an alternative version of our model that allows for trade.

<sup>14</sup>Solving equation (29) for  $r_{t+1}$  and substituting it into (27) yields a bi-dimensional dynamic system in  $K_t$  and  $N_t$ . This implies that, in general, the initial condition  $K_0$  is not enough to determine a solution. In our case, however, it is. This is because, as it turns out, a competitive equilibrium must be a saddle point path. When initialized with pairs  $(K_0, N_0)$  not on the saddle-point path, the system generates sequences that violate one or more equilibrium conditions in finite time. We are not able to prove that the saddle-path solution is unique, but numerical results hint that this is the case. Thus, for given  $K_0$ ,  $N_0$  is pinned down by the requirement that the pair be on the saddle path.

We assume that only capital goods are tradable. All trade is intertemporal. This implies that, consistent with the evidence,<sup>15</sup> the absolute price of capital will be the same across countries and all the variation in relative prices will be due to variation in the price of consumption. Domestic financial intermediaries lend and borrow from foreigners, taking the world interest rate  $r^*$  as given. When referring to the aggregate capital stock, we will now distinguish between capital owned by domestic agents (the *national* capital  $K_t^S$ ), and capital employed by domestic entrepreneurs (the *domestic* capital  $K_t^D$ ). In general, we will have  $K_t^S \neq K_t^D$ .

Trade is not free. We assume that a country engaged in transactions with the rest of the world incurs transaction costs equal to  $\varphi \left(\frac{B_t}{Y_t}\right)^2 Y_t$ , where  $\varphi \geq 0$ ,  $B_t \equiv p_t(K_t^S - K_t^D)$  is the net foreign asset position of the country, and  $Y_t \equiv p_t N_t \bar{z}_I k_{It}^\alpha + (1 - N_t) \bar{z}_C k_{Ct}^\alpha$  is its gross domestic product.

Our specification of the cost schedule is very similar to that adopted by [Backus, Kehoe, and Kydland \(1992\)](#). The simplest way of introducing this friction in our setup is to think of the transactions with the rest of the world as being carried out by the government. When  $K_t^S > K_t^D$ , the country is a net supplier of funds. Domestic banks transfer part of the capital borrowed from residents to the government, who lends it to foreign residents at the rate  $r^*$ . Conversely, when  $K_t^S < K_t^D$ , the government borrows from the rest of the world at the rate  $r^*$  and lends to domestic banks. The domestic interest rate  $r_t$  is pinned down by imposing that the government runs a balanced budget:

$$(r^* - r_t)p_t(K_t^S - K_t^D) = \varphi \left(\frac{B_t}{Y_t}\right)^2 Y_t. \quad (30)$$

The parameter  $\varphi$  governs the interest rate elasticity of foreign net capital supply. When  $\varphi = 0$ , the model becomes that of a small open economy, with infinitely elastic capital supply. With  $\varphi$  arbitrarily large, we obtain the closed economy described above. Since there is no country either fully closed or completely open to foreign capital flows, the empirically relevant case is neither of those, i.e.,  $\varphi$  must be positive and finite. Section 7 develops a procedure to pin down its value. The role played by the elasticity will be explored in Sections 8 and 9.

The definition of equilibrium is the obvious modification of Definition 1 and is omitted for the sake of brevity. Conditions (8) and (10) now become

$$p_t K_{t+1}^S = \kappa(r_{t+1})[N_t \bar{\tau}_{It} + (1 - N_t) \bar{\tau}_{Ct}], \quad (31)$$

$$K_t^D = N_t k_{It} + (1 - N_t) k_{Ct}. \quad (32)$$

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<sup>15</sup>See [Eaton and Kortum \(2001\)](#) and [Hsieh and Klenow \(2007\)](#).

The relative price  $p$  and relative size  $Q$  can be recovered as in Section 4. Finally, given our assumption that consumption goods cannot be either stored or traded, consumption must equal domestic production of the consumption good:

$$\frac{1 - \kappa(r_{t+1})}{\kappa(r_{t+1})} p K_{t+1}^S + (1 + r_t) p K_t^S = (1 - N_t) \bar{z}_C k_{Ct}^\alpha.$$

Given an initial value for the national capital  $K_0^S$ , the latter condition, along with (21), (30), (31), and (32), yields the equilibrium paths for  $K_t^D$ ,  $K_t^S$ ,  $r_t$ , and  $N_t$ .

## 7 Calibration

To make our model more amenable to quantitative analysis, we allow for exogenous TFP growth. This amounts to assuming that, for all  $t \geq 0$ ,  $\log(z_{jt}) = \gamma t + \zeta_{jt}$ , with  $\zeta_{jt} \sim N(\mu_j, \eta_j^2)$ ,  $j = I, C$ . The resulting economy converges to a balanced-growth steady-state path where the interest rate  $r_t$ , the fraction of entrepreneurs in the investment good sector,  $N_t$ , the relative price,  $p_t$ , and  $Q_t$ , are all constant, while the remaining endogenous variables grow at the constant (continuously compounded) rate  $\gamma/(1-\alpha)$ . The equilibrium values of  $p$  and  $Q$  are those of the stationary version. In the remainder of the paper, we confine our attention to the balanced growth path. With a little abuse of notation, from now on all variables will denote detrended values.<sup>16</sup>

The parameter values are listed in Table 1. Individuals are assumed to have a productive life of 60 years. This implies a 30-year model period. The relative risk aversion coefficient  $\sigma$  is set to 1.5, a standard value in quantitative analysis. The parameter  $\alpha$  is 1/3, the value considered by other studies of entrepreneurial behavior such as Burstein and Monge-Naranjo (2007) and Buera (2003). We set  $\delta$  so that the annual depreciation rate is 6%. The world average annual growth rate of real GDP per worker in the Penn World Tables is about 2.3%, equivalent to a 30-year growth rate of output of 97.8%. This implies  $\gamma = (1 - \alpha) \times \log(1.978)$ .

Next, we need to assign values to the variance parameters  $\eta_I$  and  $\eta_C$ . In Section 2 we provided estimates of firm volatility at the 3-digit level. However, the level of aggregation of our model is such that we need summary measures for the entire consumption and investment good sectors. This is a potential problem, as such measures will necessarily depend on the US sectoral composition, while sectoral composition changes with the level of development. In fact, our own model provides a theory of such change. In our benchmark calibration, we will disregard this issue. We have re-run regression (2) imposing only one dummy variable, specifying it to equal 1 if the

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<sup>16</sup>A careful characterization of the model with growth can be found in Appendix B.2.

firm belongs to a consumption good sector, and 0 otherwise. This simple procedure yields volatility estimates for the entire consumption and investment sectors. We set  $\eta_I$  and  $\eta_C$  so that the implied annual standard deviations of growth rates for either sector are equal to such estimates (0.0646 and 0.1042, respectively).<sup>17</sup> In Section 9 we will consider an alternative procedure.

We are left with the difficult task of assigning a value of  $\xi$  to each country. Unfortunately, the available indicators of investor protection, such as those proposed by [La Porta, Lopez-de Silanes, Shleifer, and Vishny \(1998\)](#), are not suitable for this task. A first issue is their ordinal nature. A second is that ex-ante we do not have elements to select one or a particular combination of them as the most informative about  $\xi$ .<sup>18</sup> In the absence of better alternatives, we recover the cross-country distribution of  $\xi$  by imposing that, whenever possible, the model-generated prices equal their empirical counterparts.

Consider the vector of relative prices of capital evinced from the 1996 Penn World Table, sorted in increasing order. Let  $(P_I/P_C)_h$  denote the price for country  $h$ . Country 1, the one with the lowest relative price of capital, is Singapore. Our model predicts that it is also the country with the highest level of investor protection. Accordingly, we set  $\xi_1 \equiv 0$ . Then, we determine every other country  $h$ 's level of investor protection  $\xi_h$  in such a way that the implied relative price, scaled by Singapore's price, equals its empirical counterpart:

$$\frac{p(\xi_h)}{p(0)} = \frac{(P_I/P_C)_h}{(P_I/P_C)_1}. \quad (33)$$

Since we require that  $\xi_h \in [0, 1]$  for all  $h$ , in general there will be countries whose relative prices are so high that a feasible solution to equation (33) does not exist. Let  $M$  be the number of countries such that a solution to equation (33) exists. Then, we set the world price equal to the mean of the price distribution:

$$p_w \equiv \frac{1}{M} \sum_h^M p(\xi_h).$$

The collection of investor protection parameters  $\{\xi_h\}$ , along with  $p_w$ , induces a vector of PPP-adjusted investment rates  $\left\{ \left( \frac{I_h}{Y_h} \right)_{PPP} \right\}$ .

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<sup>17</sup>Strictly speaking, our model does not produce implications for firms' sales growth rate. This is because firms operate for one model's period only. We define the time- $t$  annual detrended average growth rate in sector  $j$  as  $\int \int \frac{1}{30} \log \left( \frac{z' k_{j,t+1}^\alpha}{z k_{j,t}^\alpha} \right) f_j(z) f_j(z') dz dz'$ . In balanced-growth steady state, such statistic is always zero by construction. Therefore the model's annual standard deviation of the sales growth rate is the square root of  $\int \int \frac{1}{30} \left[ \log \left( \frac{z'}{z} \right) \right]^2 f_j(z) f_j(z') dz dz'$ , or  $\frac{\sqrt{2}}{30} \eta_j$ . Our definition of growth rate can be rationalized by assuming that the technologies are infinitely lived and are passed on from generation to generation.

<sup>18</sup>Although not useful for calibration purposes, the available investor protection indicators can still be used to validate our model. We shall do so in Section 10.

It turns out that, no matter the version of our model, the collections of investor protection parameters and investment rates generated by the procedure just described do not change when we change either  $\mu_I$  or  $\mu_C$ . This argument, formalized by Proposition 2 below, implies that the choice of the two parameters is irrelevant.

**Proposition 2** *Along the balanced-growth path, (i) the PPP-adjusted investment rate  $(I/Y)_{ppp}$  is invariant to changes in the parameters  $\mu_I$  and  $\mu_C$  and (ii) for every pair  $(\xi_h, \xi_l)$  the ratio  $p(\xi_h)/p(\xi_l)$  is also invariant.*

**Proof.** For the sake of brevity, we provide a proof only for the case of closed economy, when  $\xi$  lies in Region I. The extension to the other cases is straightforward. The second claim follows immediately from Lemma 3 in Appendix B. To prove the first claim, notice that the PPP-adjusted investment rate is

$$\left(\frac{I}{Y}\right)_{ppp} = \frac{p_w I}{C + p_w I} = \frac{1}{1 + \frac{1-N}{N} \frac{\bar{z}_C}{\bar{z}_I} p_w Q^\alpha}.$$

By definition of  $p_w$ , Corollary 2 implies that  $(\bar{z}_C/\bar{z}_I)p_w$  is invariant to changes in  $\mu_I$  and  $\mu_C$ . Then, the result follows from Lemmas 2 and 4, which show the invariance of  $Q$  and  $N$ , respectively. ■

Notice that, in the parametric choices made so far, nothing hinged upon a particular assumption on the magnitude of  $\varphi$ . We decide to choose the remaining parameters,  $\beta$ ,  $r^*$ , and  $\varphi$ , in such a way that the model generates a mean PPP-adjusted investment rate of 0.147, the statistic generated by the Penn World Table, an average interest rate for the top 5% countries of 4%, and a 4.2% interquartile range for interest rates.

The last two choices deserve some explanation. Since in our model there is no aggregate risk, the return on capital is also the risk-free rate. The model provides essentially no guidance in the choice of the security whose return should be used as a benchmark. It seems sensible to consider the simple mean of the historical average returns on short-term government securities and on the market portfolio, respectively. For the United States and other industrialized countries, that number is roughly 4%, a standard figure in business cycle analysis. Unfortunately, lack of data on equity returns (or lack of equity markets) for developing countries prevents us from obtaining a cross-country distribution of returns consistent with this definition. Needing a target for returns' dispersion, the best available alternative is to use data on sovereign bonds' real returns. This is the case because (i) this data allows for the widest coverage, (ii) the computed measure of dispersion has been essentially constant for the last 25 years, and (iii) there is no reason to expect that dispersion would change dramatically if we considered different securities. Our target for the

interquartile range of interest rates is that implied for the year 1996 by the data on short-term sovereign bonds assembled by [Lustig and Verdelhan \(2007\)](#).

$\beta$	$\sigma$	$\alpha$	$\delta$	$\gamma$	$r^*$	$\varphi$	$\eta_C$	$\eta_I$
0.151	1.5	1/3	0.8437	0.455	2.446	22.785	1.3704	2.2104

Table 1: Parameter Values

## 8 Comparative Statics

The purpose of this section is to develop implications for the co-variation between the quality of legal institutions and other variables of interest for development economics, e.g., GDP and the investment rate measured both in domestic and international prices.

For the parameter values listed in [Table 1](#), [Figure 7](#) depicts the steady-state values implied by all  $\xi \in [0, 1]$ . Besides the economy parameterized in [Section 7](#), which we will refer to as *benchmark*, we consider two scenarios that differ from the latter only in the magnitude of  $\varphi$ . The *autarkic* economy (dotted line) features infinitely high transaction costs. The *free capital flows* economy (dashed line) allows for no transaction cost whatsoever, i.e.  $\varphi = 0$ . Unless otherwise noted, the figures are expressed relative to the outcome under perfect investor protection ( $\xi = 0$ ). In the free-flows economy, an equilibrium does not exist for very large  $\xi$ . Accordingly, we limit our analysis to the subset of  $[0, 1]$  for which existence is guaranteed.<sup>19</sup>

Consider the first two panels on the bottom row. In the autarky case, the investment rate at domestic prices is monotone increasing in  $\xi$ , and the investment rate at world prices follows the same qualitative pattern on most of the domain. At first, this result may be surprising. But it can be rationalized quite intuitively. We already argued that poorer protection distorts the allocation of resources between the capital and the consumption good sector in favor of the latter. This effect goes in the direction of lowering the investment rate. It turns out, however, that raising  $\xi$  also generates an effect on the investment rate of the opposite sign. Here is why. For given aggregate capital stock, poorer investor protection (i.e. a lower demand of capital from entrepreneurs in both sectors) implies a lower interest rate. For given total resources, this translates into higher income for all entrepreneurs. In other words, a higher  $\xi$  prompts a redistribution away from lenders (the old generation) and towards savers (the young generation). If the effect just described is strong enough, current

<sup>19</sup>For the two economies with trade, the first panel on the top row depicts  $K^D$ . The steady-state value of  $K^S$  is also decreasing in  $\xi$ . Notice also that negative interest rates do not signal a pathology. Indeed, in every model in which capital depreciates even when is not utilized, interest rates as low as  $-\delta$  can arise in equilibrium. See for example [Aiyagari \(1994\)](#).

savings and the supply of capital will increase. Given our parameterization, this is exactly what happens in the autarky case.<sup>20</sup> In the other two scenarios, the interest rate adjusts less (benchmark economy) or does not adjust at all (free-flows economy). Therefore both definitions of the investment rate decline with  $\xi$ . The same happens to the capital stock and output. The investment rate at domestic prices does not vary substantially with investor protection because the relative price adjusts for the change in investor protection. The PPP-adjusted rate varies a lot more because investment is valued at the world price. Finally, notice that the comparative statics of  $(I/Y)_{ppp}$  do not depend on the particular value assumed by  $p_w$ . What matters is  $p_w$ 's invariance to the quality of institutions.

Now consider the center panel. When conducting international transactions is costly, but not infinitely so, poorer protection is associated with a lower domestic interest rate, an outflow of capital, i.e.  $K^S > K^D$ , and a trade balance surplus. Financial intermediaries in poor-protection countries invest their clients' savings abroad and use the factor payments to purchase new capital from foreign producers.<sup>21</sup> This is yet another answer to the question raised by Lucas (1990), who wondered why, in contrast to the transitional dynamics of the neoclassical growth model, capital does not flow from rich to poor countries. In the balanced-growth steady-state of our model, capital flows indeed from the poor to the rich in search of the highest return.

Figure 7 also shows that with our benchmark calibration, measured TFP declines with  $\xi$ . Consistent with the empirical literature,<sup>22</sup> the Solow residual is computed as  $Z = Y_{ppp}K^{-\alpha}$ , where  $Y_{ppp} = p_w N \bar{z}_I k_I^\alpha + (1 - N) \bar{z}_C k_C^\alpha$ . This yields:

$$\begin{aligned} Z &= \bar{z}_C \left[ \frac{p_w \bar{z}_I}{\bar{z}_C} N \left( \frac{k_I}{K} \right)^\alpha + (1 - N) \left( \frac{k_C}{K} \right)^\alpha \right] \\ &= \bar{z}_C \left[ \frac{p_w \bar{z}_I}{\bar{z}_C} N \left( \frac{1}{N + (1 - N)Q} \right)^\alpha + (1 - N) \left( \frac{Q}{N + (1 - N)Q} \right)^\alpha \right]. \end{aligned}$$

These expressions help explain why measured TFP varies across countries even in the absence of any heterogeneity in technology. As long as countries differ in investor protection, they will be characterized by different allocations of factors (capital and entrepreneurs) across sectors. That is, in general  $Q$  and  $N$  will vary with  $\xi$ . Over

<sup>20</sup>See Castro, Clementi, and MacDonald (2004) for a careful description of this mechanism in the context of a simpler model.

<sup>21</sup>Without growth, the current account is in equilibrium. The trade balance equals net factor income from abroad. When  $\gamma > 0$ , the current account balance (in absolute value) grows at the same rate as output.

<sup>22</sup>See for example Hall and Jones (1999) and Klenow and Rodriguez-Clare (1997).



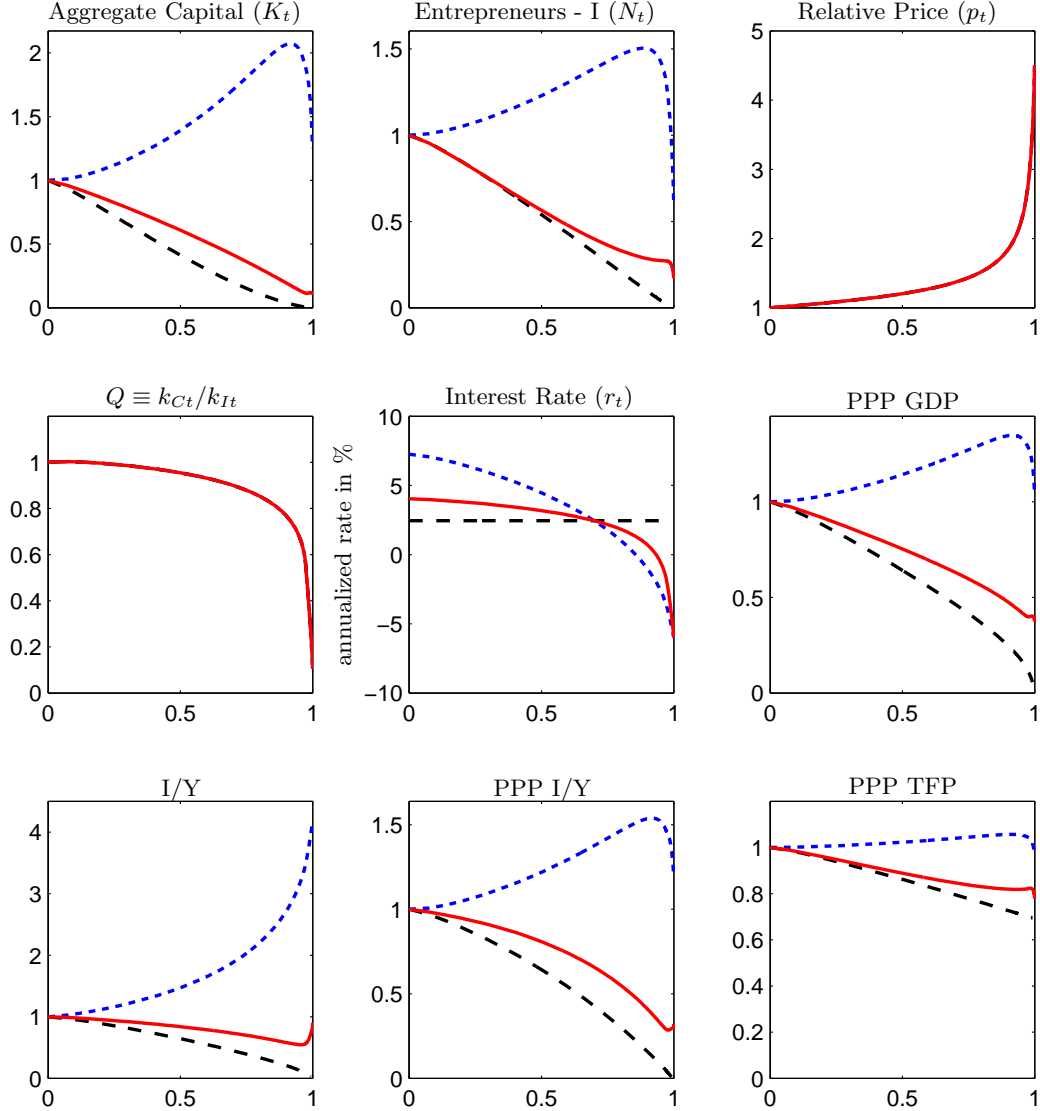


Figure 7: Steady-State Values for  $\xi \in [0, 1]$ .

most of  $\xi$ 's range,  $Q$  declines slowly. The scale of production is about the same across sectors, and so is the productivity of capital. Measured TFP drops mainly because the fraction of entrepreneurs active in the investment good sector decreases with  $\xi$ . Obviously this argument is correct if and only if  $p_w > z_C/\bar{z}_I$ . This condition is always satisfied, since  $\bar{z}_C/\bar{z}_I$ , the domestic price for a country with perfect investor protection, also constitutes a lower bound for the world price. Because of imperfect investor protection, the international price of investment goods is higher than the social marginal rate of transformation. Therefore any factor reallocation away from

the investment sector reduces measured TFP.<sup>23</sup>

The main message of this simple comparative statics exercise is that cross-country differences in legal institutions are able to generate differences in relative prices, investment rates, TFP, and GDP, that are quantitatively significant and in accordance with the empirical evidence.

Chari, Kehoe, and McGrattan (1996), Restuccia and Urrutia (2001), and Hsieh and Klenow (2007) have shown that cross-country differences in the relative price of capital and investment rate can be easily accounted for in a neoclassical framework. What is needed is that the ratio between the net returns to investment in the capital and consumption-good sectors, respectively, varies across countries in the appropriate way. This ratio, or wedge, may reflect cross-sectoral variation in productivity or policy-induced distortions that affect different industries in different ways. So far, however, no one has taken a precise stand on its determinants. We do so in this paper.

Providing a micro-foundation for the wedge goes beyond satisfying a purely intellectual interest. At the very least, it is a necessary condition for formulating policy recommendations. Micro-foundations also provide opportunities for model's falsification. Our exercise highlights why this is the case. Not only must it be that the parameter  $\xi$  belongs to the interval  $[0, 1]$ . Tying the investment wedge to the level of investor protection and to the variation in idiosyncratic risk, our theory generates two further restrictions: (i) the distribution of  $\xi$  must be consistent with the cross-country evidence on the quality of institutions and (ii) the levels of idiosyncratic risk in the consumption and investment good sectors must agree with the data presented in Section 2.<sup>24</sup> The question is whether our mechanism can generate sizeable variation in investment rates and relative prices while satisfying these restrictions. We turn to that next.

## 9 Quantitative Assessment

Refer to Figure 8. The three curves are the loci of the pairs  $\left\{ \frac{p(\xi)}{p(0)}, \left( \frac{I(\xi)}{Y(\xi)} \right)_{PPP} / \left( \frac{I(0)}{Y(0)} \right)_{PPP} \right\}$ , expressed in logarithms, that obtain for  $\xi \in [0, 1]$  in the cases of autarky (dotted line),

<sup>23</sup>Notice that measured TFP is monotone increasing in the world price. This implies that while the ranking of countries with respect to measured TFP is invariant to changes in  $p_w$ , the cross-country variation in this variable is not.

<sup>24</sup>Notice that these restrictions bind. If, contrary to the evidence presented in Section 9, idiosyncratic risk was higher in the consumption good sector, our model would predict that poor countries face lower relative prices of capital!

benchmark (solid line), and free capital flows (dashed line) economies, respectively.<sup>25</sup> The scatter plot identifies the relative price and investment rates for all countries in the 1996 Penn World Table, relative to Singapore’s values.<sup>26</sup>

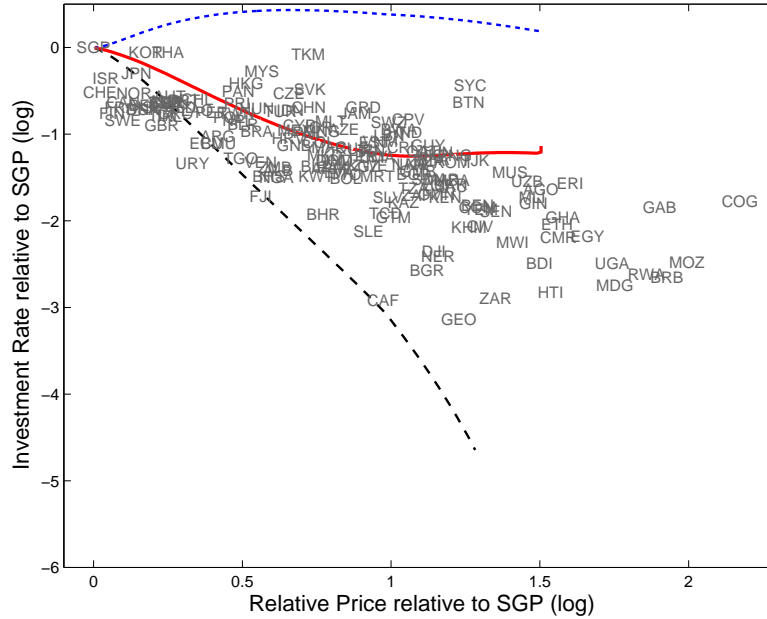


Figure 8: Dispersion in Relative Prices and Investment Rates.

Whenever possible, we follow the simple procedure outlined in Section 7 to assign values of  $\xi$  to countries in our dataset. (To the few whose PWT relative prices are higher than  $p(1)/p(0)$ , our procedure cannot assign a level of investor protection). The investment rates predicted by the model can be read off the loci.

The model-implied cross-country distributions of prices and investment rates yield measures of dispersion that can be compared with the same statistics from the Penn World Table. Refer to Table 2. In the benchmark economy, the variance of (log) prices is 71.5% of the variance in the PWT. The Gini coefficient of the price distribution is 81.4% of its empirical counterpart. For the PPP-adjusted investment rates, these statics are 38.4% and 74.3%, respectively. Consistent with what argued in Section 8, in autarky our model actually produces a positive correlation between price and investment rate, at least for the richer countries. With free capital flows, since capital

<sup>25</sup>Recall that for the free-flows economy, an equilibrium does not exist for very large  $\xi$ . In that case, the locus refers to the subset of  $[0, 1]$  for which this issue does not arise.

<sup>26</sup>We use the 1996 cross-section because of its extensive coverage of benchmark countries. However, in order to maximize the number of countries, we also included some that were not benchmarked.

	Var( $\log p$ )	Gini( $p$ )	Var( $\log I/Y$ )	Gini( $I/Y$ )
Autarky	0.715	0.814	0.024	0.157
Benchmark	0.715	0.814	0.384	0.743
Free Capital Flows	0.593	0.735	3.983	1.875

Table 2: Dispersion of prices and investment rates – Relative to data

supply is infinitely elastic at the international interest rate, the model generates excess variation in investment rates.

The bottom line is that cross-country differences in legal institutions may account for a significant share of the observed dispersion in relative prices and investment rates.

### 9.1 An Alternative Calibration

Our choice of calibrating the variance parameters  $\eta_I$  and  $\eta_C$  using summary estimates of volatility for the US is not problem-free. The reason, again, is that such estimates depend on sectoral composition, which in turn is endogenous to the quality of institutions. An alternative approach, which we follow here, is to select one 3-digit industry as representative of the larger sector to which it belongs. We impose two requirements: that the industry is present in every country, and that it accounts for a sizeable fraction of valued added and employment. Our choice falls on Building, Developing, and General Contracting (NAICS code 233) and Food Manufacturing (311).

The model was entirely re-calibrated to accommodate the new variance parameters. The new loci are displayed in Figure 9. Since the difference between  $\eta_I$  and  $\eta_C$  is now larger, the model is able to account for a larger set of relative prices. Furthermore, the shape of autarky's locus reveals that the effect of larger  $\xi$  on capital supply described in Section 8, is now relatively weaker. For all  $\varphi$ , the reaction of investment rates to changes in  $\xi$  is stronger. In turn, this implies that the model is able to generate greater dispersion in PPP-adjusted investment rates. Refer to Table 3. In the benchmark economy, the Gini coefficient of the price distribution is about 97% of the statistic produced by the PWT. The model only misses Congo's relative price. The Gini coefficient of the investment rates' distribution is essentially the same as in the data.

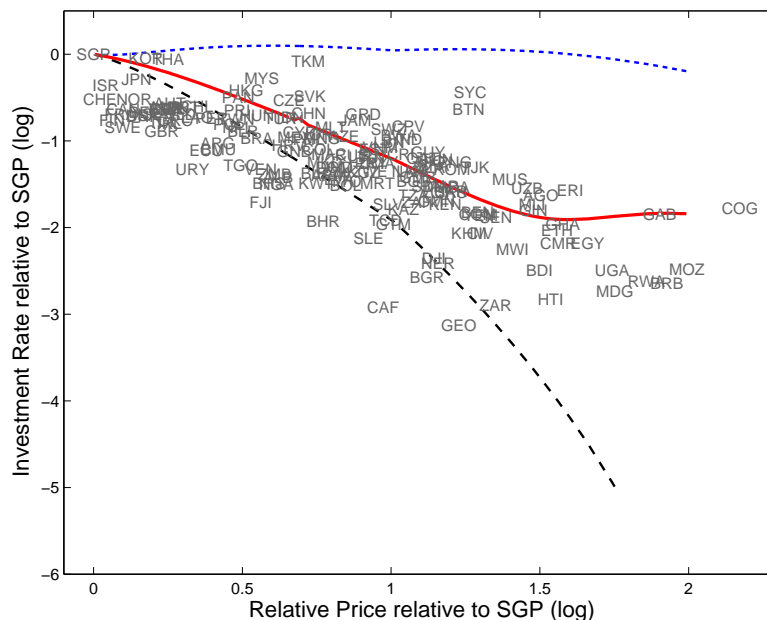


Figure 9: Relative Prices and Investment Rates – Alternative Calibration.

	$\text{Var}(\log p)$	$\text{Gini}(p)$	$\text{Var}(\log I/Y)$	$\text{Gini}(I/Y)$
Autarky	0.956	0.971	0.005	0.066
Benchmark	0.956	0.971	0.810	1.011
Free Capital Flows	0.840	0.895	3.042	1.523

Table 3: Dispersion in prices and investment rates – Alternative Calibration

## 10 Further Tests

In recent years, several scholars have attempted to assess the variation in the quality of institutions across countries directly. These attempts involve assigning scores to countries based on laws’ provisions, their enforcement, and the allocation of powers across institutions. Among the indicators produced by this literature, those provided by [La Porta, Lopez-de Silanes, Shleifer, and Vishny \(1998\)](#) (LLSV from now on) appear to be the most relevant for our analysis. They are geared towards measuring the extent to which the letter of the law protects creditors’ rights (the *CR* indicator) and minority shareholders’ rights (indicators *OV* and *AR*), and the extent to which the law is actually enforced (the *RL* indicator).<sup>27</sup>

In spite of our assessment that the LLSV’s indicators are unsuitable for calibra-

<sup>27</sup>The variable *CR* is higher, the wider the range of creditor rights in firm reorganization and liquidation upon default. The indicator *anti-director rights*, *AR*, and the dummy *one share-one vote*, *OV*, are two indices geared towards assessing the ability of small shareholders to participate in decision-making. Finally, the index *rule of law*, *RL*, proxies for the quality of law enforcement.

tion purposes, it is interesting to compare their cross-country distributions with the model-implied distribution of  $\xi$ . This amounts to regressing relative investment prices on the various indicators. The results, illustrated in Table 4, provide some support

Table 4: Relative Prices and Investor Protection

Dependent Variable:	Relative Price (log)
Rule of law (RL)	-0.09430*** (0.01129)
One share-one vote (OV)	-0.20344** (0.07628)
Antidirector rights (AR)	0.01289 (0.02372)
Creditor rights (CR)	0.04844 (0.02979)
Constant	0.66282*** (0.12289)
Number of countries	46
Adjusted $R^2$	0.538

*Note:* White robust standard errors in parenthesis.

*Sources:* [Heston, Summers, and Aten \(2002\)](#) and [La Porta, Lopez-de Silanes, Shleifer, and Vishny \(1998\)](#).

\*\*\*Significant at 1%; \*\*Significant at 5%.

for our mechanism. The indicators  $RL$  and  $OV$  co-vary negatively with our proxy for  $\xi$ , whereas  $CR$  and  $AR$  are not significantly correlated with them.

Our model also generates implications for the impact of investor protection on the relationship between industry size and firm-level volatility. The relative size of sectors is given by  $(1 - N)k_C/Nk_I$ . In our benchmark calibration this ratio is non-monotone. It increases with  $\xi$  on most of its domain, and decreases with it when investor protection is very poor ( $\xi$  is relatively high). However, when we compute the correlation between the two variables in the cross-country distribution implied by our calibration procedure, we find evidence of a positive association between the two measures. The empirical implication is that on average the positive impact of investor protection on sector size should be greater, the larger firm-level volatility. We test this implication by estimating the following regression:

$$\log(size)_{jh} = \alpha_h + \theta_j + \beta' (vol_j \times IP_h) + u_{jh},$$

where  $size_{jh}$  is our measure of total employment in sector  $j$  in country  $h$ . It is the average between 1985 and 2001 of the sectoral employment variable in the UNIDO dataset.<sup>28</sup> The variables  $IP_c$  are the LLSV's measures of investor protection already

<sup>28</sup>We collected sectoral information at the 3-digit ISIC code (Rev.3) disaggregation level from the United Nations Industrial Development Organization (UNIDO) files. This data is restricted to

used above. The variables  $\alpha_h$  and  $\theta_j$  are country- and sector-specific dummies. Finally,  $vol_j$  is our standard deviation estimate for sector  $j$ .

Our model suggests that  $\beta$  should be a positive-valued vector. The estimates, reported in Table 5, are consistent with this prediction.

Table 5: Sector Size, Volatility, and Investor Protection

Dependent Variable: Log average sector size	
Volatility $\times$ RL	2.20137*** (0.4713515)
Volatility $\times$ AR	0.6593174* (0.8103873)
Volatility $\times$ CR	0.0277885 (0.917312)
Volatility $\times$ OV	0.8447151 (3.13555)
Number of sectors (country average) / countries	14.6 / 37
$R^2$ within/ between/ overall	0.4895 / 0.0078 / 0.1522

*Notes:* Country and sector fixed effects omitted. White standard errors in parenthesis.  
*Sources:* La Porta, Lopez-de Silanes, Shleifer, and Vishny (1998) and UNIDO 1985–2001.

\*\*\*Significant at 1%. \*Significant at 10%.

## 11 Conclusion

This paper contributes to an active line of research that investigates the sources of the sizeable cross-country variation in development experiences. It does so by introducing a theory that links the quality of institutions and cross-sectoral differences in idiosyncratic volatility to macroeconomic outcomes. The quantitative analysis shows that our mechanism can account for a large fraction of the observed variation in relative prices and investment rates.

In order to best illustrate our mechanism, we have abstracted from potentially important features such as alternative sources of heterogeneity and other margins along which economies may adjust in response to changes in institutional quality. Modeling these features may help to understand other facets of economic development.

First, we disregarded ex-ante differences in entrepreneurial ability. The statistical analysis conducted in Section 2 leads us to conclude that residual uncertainty accounts for the largest share of sales growth variance. This is true for nearly all sectors we look at. However, the same analysis has also uncovered a non-negligible amount of heterogeneity in firm fixed-effects, which may signal ex-ante differences in ability.

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manufacturing sectors. We used the correspondence tables from the U.N. Statistics Division to obtain the equivalent NAICS 1997 codes.

Ex-ante heterogeneity could be introduced in our framework in different ways. In the appendix to [Castro, Clementi, and MacDonald \(2007\)](#) we assume that agents are equally successful in the consumption good sector, but differ in their ability to manage technologies in the investment good sector. The allocation of skill across sectors becomes a function of investor protection. A decrease in investor protection can lead to the reallocation of individuals away from the sector where they enjoy a comparative advantage. In turn, this can result in greater effects on investment rates, income, and measured TFP. We would also like to study how our quantitative results would change if we modified our framework to allow a fraction of agents to be financially unconstrained.

We ignored the informal sector, which is thought to be large in most developing countries. Better investor protection, by improving the risk-sharing offered by the banking system, may strengthen entrepreneurs' incentives to leave the underground economy. Indeed, the ability to borrow from banks is often cited among the reasons that lead firms to emerge into the formal sector and start paying taxes.

In this paper we emphasized a particular source of cross-sectoral variation. We realize, however, that considering other types of heterogeneity may lead to similar results. According to [Rajan and Zingales \(1998\)](#), firms engaged in the production of investment goods need to rely on external finance to a larger extent than their counterparts producing consumption goods.<sup>29</sup> This may be the case, we conjecture, because they tend to incur larger initial sunk costs. We think that it is possible to model this alternative form of heterogeneity in such a way that entrepreneurs in investment good sectors are able to finance themselves at a relative lower cost in countries characterized by better investor protection. Besides representing an interesting exercise on its own, building such model would give us the chance to contrast the explanatory power of this form of heterogeneity with that emphasized in this paper.<sup>30</sup>

A version of our model that allowed for multiple tradable goods would generate trade patterns qualitatively consistent with the empirical evidence. In particular, in accordance with what reported by [Eaton and Kortum \(2001\)](#), rich countries would specialize in the production of equipment goods, which are among the riskier ones.<sup>31</sup>

Our last thought is about the possibility that institutions may affect long-term

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<sup>29</sup>This is not exactly the classification adopted by the authors. However, even a cursory look at their data reveals that investment-good producing sectors tend to be classified as industries in which firms require larger access to external finance.

<sup>30</sup>[Erosa and Hidalgo Cabrillana \(2008\)](#) present a model with similar features.

<sup>31</sup>[Cuñat and Melitz \(2007\)](#) conduct an exercise along these lines. They assess cross-sectoral variation in idiosyncratic risk in a way similar to ours, and argue that countries whose labor markets are more flexible tend to specialize in more volatile industries.



growth. Conventional wisdom suggests that entrepreneurial activities leading to technological innovations are also likely to be among the riskier ones. If this is the case, our theory suggests that innovative activities should be concentrated in high-investor protection countries.

## A Data

Our data draws from the COMPUSTAT North–America Industrial Annual Database from 1950 to 2005. After dropping all observations for which either net sales, employment, or the NAICS code are missing, our dataset consists of 265,018 firm–year observations. We then proceed to delete all firms that have less than 3 observations and those belonging to 3–digit NAICS sectors for which the yearly average number of firms in the sample is less than 4. We also eliminate all firm–year observations which are affected by a merger or acquisition occurred the previous year, and those for which COMPUSTAT indicates a potential accounting problem in net sales. Finally, we drop all firms in the Finance and Insurance (3–digit NAICS from 520 to 529), Utilities (220 to 229), and Real Estate (531) industries. We also drop the firms classified by COMPUSTAT in the 3–digit sector 999, which turns out to be a residual category. COMPUSTAT classifies some firms according to the 1997 NAICS system and others according to the 2002 NAICS system. We used the equivalence tables between the two systems published by the BEA to assign every 2002 NAICS code to a corresponding 1997 NAICS code (for example, most firms categorized by COMPUSTAT in sector 236 are included in 233, and all of those in sector 423 are attributed to 421).

Our next task is to assign each of the remaining sectors to either the consumption or the investment category. We rely on the Bureau of Economic Analysis’ 1997 Benchmark Input–Output Use Summary Table (after redefinitions) for the US. The Use Table tells us the fraction of output that flows from each 3–digit sector to any of the other 3–digit industries and to final demand, respectively. We first group final demand uses into two categories, consumption ( $C$ ) and investment ( $I$ ). We do this by aggregating personal, federal, and state consumption expenditures into a single consumption category, and similarly for investment expenditures. Since the Use Table does not provide a breakdown of imports, exports, and changes in inventories into consumption and investment, we choose to ignore these final demand items. We rule out the 3–digit sectors with a contribution to final demand uses of less than 1% of their output. For each remaining 3–digit industry  $j$ , we compute the share of output destined to consumption,  $Y_C(j)/(Y_C(j) + Y_I(j))$ . We assign all industries with a share greater than or equal to 60% to the consumption good sector, and those with a share lower than or equal to 40% to the investment good sector. We discard the remaining industries. An alternative procedure, described in Appendix 2 of [Chari, Kehoe, and McGrattan \(1996\)](#), includes in the consumption output of a given sector

all the intermediate goods whose ultimate destination is final consumption, and similarly for investment. Once sectors that produce only intermediate goods are ruled out, this procedure yields essentially the same assignment as ours.

At the end of this process we are left with an unbalanced panel of 7,070 firms, distributed in 57 sectors, for a total of 73,112 firm–year observations. For each sector, Table 6 reports value added as a fraction of GDP as evinced from the Input–Output Table, and the fraction of output ultimately destined to consumption. Table 7 reports the yearly average number of firms per sector and the results of the estimation of equations (1) and (2).

We carried out a variety of robustness checks. We repeated the analysis by deleting all firm–year observations in which an IPO took place. As expected, the volatility estimates decrease, but they do so across the board, leaving our results on the relative volatility intact. Finally, we also experimented with alternative specifications of the regression equation (1). In particular, we introduced a firm–specific time trend, in order to control for trends in the growth process that are not captured by either age or size. It turns out that adding these factors adds very little to the predictive power of the equation, therefore leaving our results unchanged.

Table 6: Summary Statistics

NAICS	Description	Value Added (%)	Cons. Share (%)
<i>Investment Sectors</i>			
213	Support Activities for Mining	0.13 <sup>32</sup>	12.55
233	Building, Developing, and General Contracting	3.88 <sup>33</sup>	10.49
234	Heavy Construction	3.88	3.42
235	Special Trade Contractors	3.88	3.42
321	Wood Product Manufacturing	0.30	19.89
333	Machinery Manufacturing	1.16	5.28
334	Computer and Electronic Product Manufacturing	1.88	24.24
<i>Consumption Sectors</i>			
111	Crop Production	0.79	100.00
311	Food Manufacturing	1.21	100.00
312	Beverage and Tobacco Product Manufacturing	0.58	100.00
313	Textile Mills	0.18	99.99
314	Textile Product Mills	0.12	86.99
315	Apparel Manufacturing	0.28	100.00
316	Leather and Allied Product Manufacturing	0.04	100.00
322	Paper Manufacturing	0.57	100.00
323	Printing and Related Support Activities	0.50	100.00
324	Petroleum and Coal Products Manufacturing	0.23	100.00
325	Chemical Manufacturing	1.54	98.36
326	Plastics and Rubber Products Manufacturing	0.71	98.40
327	Nonmetallic Mineral Product Manufacturing	0.47	100.00
339	Miscellaneous Manufacturing	0.53	72.24
421	Wholesale Trade, Durable Goods	6.04 <sup>34</sup>	74.70
422	Wholesale Trade, Nondurable Goods	6.04	74.70
441	Motor Vehicle and Parts Dealers	5.31 <sup>35</sup>	94.76
442	Furniture and Home Furnishings Stores	5.31	94.76
443	Electronics and Appliance Stores	5.31	94.76
444	Building Material and Garden Equipment	5.31	94.76
445	Food and Beverage Stores	5.31	94.76
446	Health and Personal Care Stores	5.31	94.76
448	Clothing and Clothing Accessories Stores	5.31	94.76
451	Sporting Goods, Hobby, Book, and Music Stores	5.31	94.76

<sup>32</sup>Data on value added is drawn from the 1997 Use Summary Table.

<sup>33</sup>This figure refers to the aggregate of the I-O Tables' categories "New Residential Construction", "New Nonresidential Construction", and "Maintenance and Repair Construction".

<sup>34</sup>This figure refers to the "Wholesale Trade" category. The I-O Tables do not disaggregate it further.

<sup>35</sup>This figure refers to the "Retail Trade" category. The I-O Tables do not disaggregate it further.

Table 6: (continued)

NAICS	Description	Value Added (%)	Cons. Share (%)
<i>Consumption Sectors</i>			
452	General Merchandise Stores	5.31	94.76
453	Miscellaneous Store Retailers	5.31	94.76
454	Nonstore retailers	5.31	94.76
481	Air Transportation	0.52	94.88
482	Rail Transportation	0.25	77.12
483	Water Transportation	0.07	99.75
484	Truck Transportation	0.98	86.03
486	Pipeline Transportation	0.10	100.00
488	Support Activities for Transportation	0.26 <sup>36</sup>	100.00
492	Couriers and Messengers	0.33	100.00
512	Motion Picture and Sound Recording Industries	0.31	100.00
513	Broadcasting and Telecommunications	2.28	96.56
514	Information and Data Processing Services	0.37	100.00
532	Rental and Leasing Services	0.94	100.00
561	Administrative and Support Services	2.67	100.00
562	Waste Management and Remediation Services	0.26	100.00
611	Educational Services	0.71	100.00
621	Ambulatory Health Care Services	3.12	100.00
622	Hospitals	1.76	100.00
623	Nursing and Residential Care Facilities	0.70	100.00
711	Performing Arts, Spectator Sports, ...	0.35	100.00
713	Amusement, Gambling, and Recreation Industries	0.65	100.00
721	Accommodation	0.63	100.00
722	Food Services and Drinking Places	2.01	100.00
811	Repair and Maintenance	1.63	100.00
812	Personal and Laundry Services	0.69	100.00

<sup>36</sup>This figure refers to the I-O Tables' category "Sightseeing Transportation and Transportation Support".

Table 7: Estimates

NAICS	$\log(size)$	$\log(age)$	volatility (ranking)	avg # firms
<i>Investment Sectors</i>				
213	-.0031302	.0085014	.0657767 (23)	10
233	.0098189	.0137877	.1070622 (7)	14
234	.082332***	-.1942279**	.1245036 (2)	6
235	.0033294	.0130888	.0506282 (33)	6
321	-.0743625***	-.0137304	.0865617 (13)	15
333	-.0220547***	-.111267***	.0888356 (12)	100
334	-.0611239***	-.1653497***	.1152213 (5)	255
<i>Consumption Sectors</i>				
111	-.1939484***	-.3041054***	.0742716 (19)	4
311	-.0250951***	-.0890859***	.0489535 (34)	45
312	-.0025538	-.0378842*	.0515543 (32)	13
313	-.003285	-.0104182	.0548013 (31)	14
314	.0087065	-.0952713**	.0444283 (38)	4
315	-.0745512***	-.106125***	.0650508 (24)	25
316	-.0760973***	-.063782***	.0354913 (45)	11
322	-.0353021***	-.0101161	.0416634 (40)	26
323	.0167837	-.0448362***	.0428692 (39)	14
324	-.0173259	.0125759	.0488046 (35)	22
325	-.0163228	-.114327***	.1197231 (3)	141
326	.0021413	-.0589135***	.0638844 (26)	29
327	.0081283	-.0406237**	.0584274 (30)	21
339	-.0169723	-.1712456***	.0952873 (10)	54
421	-.0069892	-.0977216***	.0812233 (14)	52
422	-.0453721***	-.0646109***	.0763564 (17)	29
441	.0691165	-.1722535***	.0118926 (56)	4
442	-.0933022***	-.0654218*	.0219466 (55)	5
443	-.041673***	-.114911***	.0380338 (41)	6
444	-.002398	-.0944098***	.0299617 (49)	7
445	-.0118256*	-.0275481***	.0308973 (48)	21
446	-.104086***	-.0804354***	.0361896 (44)	9
448	-.0416039***	-.0577344***	.0449255 (37)	22
451	-.0122337	-.0645347***	.0338989 (46)	10
452	-.012334*	-.0380349***	.037139 (42)	15
453	-.0345285	-.2197272***	.0296003 (50)	5
454	.1106638***	-.1841804**	.1192932 (4)	11
481	-.0737513***	-.1017634***	.0604847 (29)	21

Firm fixed effects and sectoral time dummies omitted.

\*\*\*Significant at 1%; \*\*Significant at 5%; \*Significant at 10%.

Table 7: (continued)

NAICS	$\log(size)$	$\log(age)$	volatility (ranking)	avg # firms
<i>Consumption Sectors</i>				
482	−.0611843***	−.0550491***	.0292361 (51)	11
483	−.0014021	−.0636376	.0617809 (27)	8
484	−.018733***	−.0526791***	.0257312 (53)	14
486	−.056379**	−.0527639	.0481128 (36)	7
488	−.0610611**	−.1219306***	.0053382 (57)	5
492	−.0848173***	−.002866	.0314574 (47)	4
512	−.016782	−.1070248*	.1477931 (1)	13
513	−.0973021***	−.1235676***	.064266 (25)	63
514	.0401507	−.4530637***	.1025426 (8)	17
532	−.0538958***	−.0461446	.0804803 (15)	11
561	−.0272675**	−.2128337***	.0900958 (11)	23
562	−.0448732	−.0568337	.1077895 (6)	11
611	.0574297	−.3823941***	.0788587 (20)	7
621	−.057038**	−.2820758***	.1052234 (9)	19
622	−.0193501	−.3326046***	.0225713 (54)	4
623	−.0520427	.0329648	.0284807 (52)	5
711	−.0320651	.0040742	.0680707 (21)	11
713	−.0045382	−.1984318***	.0786522 (22)	9
721	.0134744	−.1671984***	.0775939 (16)	16
722	−.0516385***	−.1395201***	.0599774 (28)	33
811	−.0004886	−.3564935***	.0365723 (43)	4
812	.0758651*	−.1696842**	.0759023 (18)	7

Firm fixed effects and sectoral time dummies omitted.

\*\*\*Significant at 1%; \*\*Significant at 5%; \*Significant at 10%.

## B Proofs and Lemmas

### B.1 Proof of Proposition 1

**Proof.** Refer to the maximization program (18)–(20). Since the schedule  $\tau_{jt}(z)$  is increasing in  $z$ , condition (20) holds if and only if  $\tau_{jt}(0) \geq 0$ , i.e. if and only if  $\bar{\tau}_{jt} \geq \xi p_{jt} \bar{z}_j k_{jt}^\alpha$ . Furthermore, combining (17) and (19) leads us to conclude that

$$\tau_{jt}(z) = p_{jt} \bar{z}_j k_{jt}^\alpha - p_{It}(r_t + \delta) k_{jt} + \xi p_{jt}(z - \bar{z}_j) k_{jt}^\alpha. \quad (34)$$

We can thus rewrite the optimization program as

$$\begin{aligned} \max_{k_{jt}} \int u[p_{jt} \bar{z}_j k_{jt}^\alpha - p_{It}(r_t + \delta) k_{jt} + \xi p_{jt}(z - \bar{z}_j) k_{jt}^\alpha] f_j(z) dz \\ \text{s.t. } k_{jt} \leq \left[ \frac{p_{It}(r_t + \delta)}{(1 - \xi) p_{jt} \bar{z}_j} \right]^{-\frac{1}{1-\alpha}}. \end{aligned} \quad (35)$$

Necessary and sufficient conditions for an optimum are (35), along with

$$\alpha p_{jt} k_{jt}^{\alpha-1} \left[ \bar{z}_j + \xi \frac{\int [(1 - \xi) p_{jt} \bar{z}_j k_{jt}^\alpha - p_{It}(r_t + \delta) k_{jt} + \xi p_{jt} z k_{jt}^\alpha]^{-\sigma} (z - \bar{z}_j) f_j(z) dz}{\int [(1 - \xi) p_{jt} \bar{z}_j k_{jt}^\alpha - p_{It}(r_t + \delta) k_{jt} + \xi p_{jt} z k_{jt}^\alpha]^{-\sigma} f_j(z) dz} \right] \geq p_{It}(r_t + \delta). \quad (36)$$

If (35) holds with strict inequality, then

$$\alpha p_{jt} k_{jt}^{\alpha-1} \left[ \bar{z}_j + \xi \frac{\int u'[\tau_{jt}(z)] (z - \bar{z}_j) f_j(z) dz}{\int u'[\tau_{jt}(z)] f_j(z) dz} \right] = p_{It}(r_t + \delta).$$

Combining the latter with (34) yields

$$\tau_{jt}(z) = \left[ \bar{z}_j(1 - \alpha - \xi) - \alpha \xi \frac{\int u'[\tau_{jt}(z)] (z - \bar{z}_j) f_j(z) dz}{\int u'[\tau_{jt}(z)] f_j(z) dz} + \xi z \right] p_{jt} k_{jt}^\alpha.$$

The transfer function is a fixed point of the functional operator described by the right-hand side of the last condition. In order to prove that  $\tau_{jt}(z) = g_j(z) p_{jt} k_{jt}^\alpha$ , it is enough to show that that operator preserves that property. Given that  $u'[g_j(z) p_{jt} k_{jt}^\alpha] = (g_j(z))^{-\sigma} (p_{jt} k_{jt}^\alpha)^{-\sigma}$ , it is straightforward to show that this is indeed the case and that the function  $g_j$  satisfies the functional equation

$$g_j(z) = \bar{z}_j(1 - \alpha - \xi) - \alpha \xi \frac{\int u'[g_j(z)] (z - \bar{z}_j) f_j(z) dz}{\int u'[g_j(z)] f_j(z) dz} + \xi z. \quad (37)$$

When (35) holds with equality, (34) yields  $\tau_{jt}(z) = \xi p_{jt} z k_{jt}^\alpha$ .

Finally, the value of  $\xi_j^*$  is determined by imposing that both (36) and (35) hold with equality. That is, by imposing  $g_j(z) = \xi z$  in (37). Recalling that  $\int z^x f_j(z) dz = e^{x\mu_j + \frac{1}{2}\eta_j^2 x^2}$  yields  $\xi_j^* = [1 + \frac{\alpha}{1-\alpha} e^{-\sigma\eta_j^2}]^{-1}$ . ■



## B.2 Exogenous TFP Growth

For any variable  $x_t$  that grows at the continuously compounded rate  $\gamma/(1-\alpha)$  along the balanced growth path, define its detrended value as  $\hat{x}_t \equiv e^{-\frac{\gamma}{1-\alpha}t}x_t$ . Let also  $\hat{z}_t \equiv e^{\zeta t}$ . Transfers are given by  $\tau_{jt}(z_t) = g_{jt}(z_t)p_jk_{jt}^\alpha$ , where  $g_{jt}(z_t) \equiv e^{\gamma t}g_j(\hat{z}_t)$  for all  $z_t$ , and  $g_j$  is as characterized in Proposition 1. Detrended transfers are simply  $\hat{\tau}_{jt}(z_t) = g_j(\hat{z}_t)p_j\hat{k}_{It}^\alpha$ .

For given initial capital stock  $\hat{K}_0$ , the competitive equilibrium paths of  $\hat{K}_t$ ,  $\hat{k}_{It}$ ,  $\hat{k}_{Ct}$ ,  $N_t$ , and  $r_t$  are characterized by

$$\begin{aligned} p\hat{K}_{t+1}e^{\frac{\gamma}{1-\alpha}} &= \kappa(r_{t+1})\hat{k}_{It}^\alpha [pN_tE(g_I(\hat{z}_t)) + (1-N_t)Q^\alpha E(g_C(\hat{z}_t))], \\ \hat{K}_{t+1}e^{\frac{\gamma}{1-\alpha}} &= (1-\delta)\hat{K}_t + N_t\bar{z}_I\hat{k}_{It}^\alpha, \\ \hat{K}_t &= N_t\hat{k}_{It} + (1-N_t)\hat{k}_{Ct}, \\ r_t + \delta &= \alpha\hat{k}_{It}^{\alpha-1}[\bar{z}_I - \xi\omega_I], \end{aligned}$$

where the constants  $\bar{z}_I$  and  $\omega_I$  are as defined in Section 5.

## B.3 Lemmas

In the remainder of this Section we state and prove a few Lemmas that are used in the proof of Proposition 2. In order to avoid cluttering the notation further, we use the same notation employed in the description and analysis of the stationary model. For the sake of brevity, we confine our attention to Region I. The same arguments can be used to prove the claims on Regions II and III.

Consider the class of lognormal distributions with the same variance parameter  $\eta$ , but different mean parameter. For all  $\mu \in \mathfrak{R}$ , and with little abuse of notation, let  $f_\mu$  denote the density of the lognormal distribution in such class, that has mean parameter  $\mu$ . For all integrable functions  $g : \mathfrak{R}^+ \rightarrow \mathfrak{R}^+$ , define also the functional operator  $\Gamma_\mu$  as

$$\Gamma_\mu[g(z)] = \bar{z}_\mu(1-\alpha)(1-\xi) - \alpha\xi \frac{\int [g(z)]^{-\sigma} z f_\mu(z) dz}{\int [g(z)]^{-\sigma} f_\mu(z) dz} + \xi z,$$

where  $\bar{z}_\mu \equiv \int z f_\mu(z) dz$ . Finally, denote as  $g_\mu$  the fixed point of the operator  $\Gamma_\mu$ .

**Lemma 1** *For all  $\lambda \in \mathfrak{R}$  and for all  $z \geq 0$ ,  $e^\lambda g_\mu(z) = g_{\mu+\lambda}(e^\lambda z)$ .*

**Proof.** Our strategy is to conjecture that  $e^\lambda g_\mu(z) = g_{\mu+\lambda}(e^\lambda z)$  and to show that  $e^\lambda \Gamma[g_\mu(z)] = \Gamma[g_{\mu+z}(e^\lambda z)]$ . By definition,

$$e^\lambda \Gamma[g_\mu(z)] = e^\lambda \bar{z}_\mu(1-\alpha)(1-\xi) - \alpha\xi \frac{\int [g_\mu(z)]^{-\sigma} e^\lambda z f_\mu(z) dz}{\int [g_\mu(z)]^{-\sigma} f_\mu(z) dz} + \xi e^\lambda z.$$

Now multiply numerator and denominator of the ratio by  $e^{-\lambda\sigma}$ . By our conjecture, it follows that

$$e^{\lambda}\Gamma[g_{\mu}(z)] = e^{\lambda}\bar{z}_{\mu}(1-\alpha)(1-\xi) - \alpha\xi \frac{\int [g_{\mu+\lambda}(e^{\lambda}z)]^{-\sigma} e^{\lambda}z f_{\mu}(z) dz}{\int [g_{\mu+\lambda}(e^{\lambda}z)]^{-\sigma} f_{\mu}(z) dz} + \xi e^{\lambda}z.$$

By introducing the change of variable  $y = e^{\lambda}z$ , one obtains

$$\begin{aligned} \int [g_{\mu+\lambda}(e^{\lambda}z)]^{-\sigma} e^{\lambda}z f_{\mu}(z) dz &= \int [g_{\mu+\lambda}(y)]^{-\sigma} y \frac{1}{\sqrt{2\pi\eta y}} e^{-\frac{(\log -\mu-\lambda)^2}{2\eta^2}} dy \\ &= \int [g_{\mu+\lambda}(y)]^{-\sigma} y f_{\mu+\lambda}(y) dy. \end{aligned}$$

Realizing that  $e^{\lambda}\bar{z}_{\mu} = e^{\mu+\lambda+\frac{1}{2}\eta^2} = \bar{z}_{\mu+\lambda}$  concludes the proof. ■

**Corollary 1** For all  $\sigma \geq 0$ ,  $\sigma \neq 1$ ,  $\int [g_{\mu+\lambda}(z)]^{1-\sigma} f_{\mu+\lambda}(z) dz = e^{\lambda} \int [g_{\mu}(z)]^{1-\sigma} f_{\mu}(z) dz$  and  $\int [g_{\mu+\lambda}(z)]^{1-\sigma} z f_{\mu+\lambda}(z) dz = e^{\lambda} \int [g_{\mu}(z)]^{1-\sigma} z f_{\mu}(z) dz$ .<sup>37</sup>

**Proof.** By Lemma 1,  $g_{\mu+\lambda}(z) = e^{\lambda}g_{\mu}(e^{-\lambda}z)$ . Then, the result follows immediately from the change of variable  $y = e^{-\lambda}z$ . ■

Let  $Q(\mu_C, \mu_I)$  denote the ratio  $Q$ , when the mean parameters of the two distributions are  $\mu_C$  and  $\mu_I$ , respectively. That is:

$$Q(\mu_C, \mu_I) = \frac{\bar{z}_{\mu_C} - \xi\omega_{\mu_C}}{\bar{z}_{\mu_I} - \xi\omega_{\mu_I}} \left[ \frac{E_{\mu_I}[g_{\mu_I}^{1-\sigma}(z)]}{E_{\mu_C}[g_{\mu_C}^{1-\sigma}(z)]} \right]^{\frac{1}{1-\sigma}}.$$

**Lemma 2** The ratio  $Q \equiv \frac{k_C}{k_I}$  is invariant with respect to both  $\mu_C$  and  $\mu_I$ . That is, for all  $\lambda \in \mathfrak{R}$ ,

$$\begin{aligned} Q(\mu_C + \lambda, \mu_I) &= Q(\mu_C, \mu_I), \\ Q(\mu_C, \mu_I + \lambda) &= Q(\mu_C, \mu_I). \end{aligned}$$

**Proof.** We will prove only the first claim. By Corollary 1,

$$\omega_{\mu_C+\lambda} = \bar{z}_{\mu_C+\lambda} - \frac{\int [g_{\mu_C+\lambda}(z)]^{-\sigma} z f_{\mu_C+\lambda}(z) dz}{\int [g_{\mu_C+\lambda}(z)]^{-\sigma} f_{\mu_C+\lambda}(z) dz} = e^{\lambda} \left[ \bar{z}_{\mu_C} - \frac{\int [g_{\mu_C}(z)]^{-\sigma} z f_{\mu_C}(z) dz}{\int [g_{\mu_C}(z)]^{-\sigma} f_{\mu_C}(z) dz} \right] = e^{\lambda}\omega_{\mu_C}$$

and

$$\begin{aligned} E_{\mu_C+\lambda}[g_{\mu_C+\lambda}^{1-\sigma}(z)] &= \frac{1}{1-\sigma} \int [g_{\mu_C+\lambda}(z)]^{1-\sigma} f_{\mu_C+\lambda}(z) dz = \frac{e^{\lambda(1-\sigma)}}{1-\sigma} \int [g_{\mu_C}(z)]^{1-\sigma} f_{\mu_C}(z) dz \\ &= e^{\lambda(1-\sigma)} E_{\mu_C}[g_{\mu_C}^{1-\sigma}(z)]. \end{aligned}$$

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<sup>37</sup>An analogous result can be easily proved in the case of unit elasticity.

This allows us to conclude that

$$\begin{aligned} Q(\mu_C + \lambda, \mu_I) &= \frac{\bar{z}_{\mu_C + \lambda} - \xi\omega_{\mu_C + \lambda}}{\bar{z}_{\mu_I} - \xi\omega_{\mu_I}} \left[ \frac{E_{\mu_I}[g_{\mu_I}^{1-\sigma}(z)]}{E_{\mu_C + \lambda}[g_{\mu_C + \lambda}^{1-\sigma}(z)]} \right]^{\frac{1}{1-\sigma}} \\ &= \frac{e^\lambda \bar{z}_{\mu_C} - e^\lambda \xi\omega_{\mu_C}}{\bar{z}_{\mu_I} - \xi\omega_{\mu_I}} \left[ \frac{E_{\mu_I}[g_{\mu_I}^{1-\sigma}(z)]}{e^{\lambda(1-\sigma)} E_{\mu_C}[g_{\mu_C}^{1-\sigma}(z)]} \right]^{\frac{1}{1-\sigma}} = Q(\mu_C, \mu_I). \end{aligned}$$

■

Let  $p(\mu_C, \mu_I)$  denote the relative price when the mean parameters of the two distributions are  $\mu_C$  and  $\mu_I$ , respectively. That is:

$$p(\mu_C, \mu_I) = \left[ \frac{\bar{z}_{\mu_C} - \xi\omega_{\mu_C}}{\bar{z}_{\mu_I} - \xi\omega_{\mu_I}} \right]^\alpha \left[ \frac{E_{\mu_C}[g_{\mu_C}^{1-\sigma}(z)]}{E_{\mu_I}[g_{\mu_I}^{1-\sigma}(z)]} \right]^{\frac{1-\alpha}{1-\sigma}}.$$

**Lemma 3** For all  $\lambda \in \mathfrak{R}$ ,

$$\begin{aligned} p(\mu_C + \lambda, \mu_I) &= e^\lambda p(\mu_C, \mu_I). \\ p(\mu_C, \mu_I + \lambda) &= e^{-\lambda} p(\mu_C, \mu_I). \end{aligned}$$

**Proof.** It follows the same steps of the proof of Lemma 2. ■

**Corollary 2** Consider the distribution of prices  $\{p_h\}$  generated by any collection of investor protection parameters  $\{\xi_h\}$ . Then, Lemma 3 also applies to the mean and median of that distribution.

Let  $k_j(\mu_C, \mu_I)$  denote the equilibrium capital advancement in sector  $j$  when the mean parameters of the two distributions are  $\mu_C$  and  $\mu_I$ , respectively.

**Lemma 4** The interest rate  $r$  and the fraction of investment sector entrepreneurs  $N$  are invariant with respect to  $\mu_C$  and  $\mu_I$ .

**Proof.** We will prove the claim only in the case of  $\mu_C$ . Our strategy is to: i) conjecture that  $r$  is invariant; ii) prove that, conditional on this conjecture being valid,  $N$  is also invariant; iii) verify our conjecture. By Lemma 3,

$$k_C^{\alpha-1}(\mu_C + \lambda, \mu_I) = \frac{p(\mu_C + \lambda, \mu_I)(r + \delta)}{\alpha[\bar{z}_{\mu_C + \lambda} - \xi\omega_{\mu_C + \lambda}]} = \frac{e^\lambda p(\mu_C, \mu_I)(r + \delta)}{\alpha e^\lambda [\bar{z}_{\mu_C} - \xi\omega_{\mu_C}]} = k_C^{\alpha-1}(\mu_C, \mu_I).$$

On balanced-growth steady-state, (9) and (10) write as

$$\begin{aligned} (e^{\gamma/(1-\alpha)} + \delta - 1)K &= N\bar{z}_I k_I^\alpha, \\ K &= Nk_I + (1 - N)k_C. \end{aligned}$$

Use the two to obtain

$$\frac{\bar{z}_{\mu_I} k_I^{\alpha-1}(\mu_C + \lambda, \mu_I)}{e^{\gamma/(1-\alpha)} + \delta - 1} = 1 + \frac{[1 - N(\mu_C + \lambda, \mu_I)]}{N(\mu_C + \lambda, \mu_I)} Q(\mu_C + \lambda, \mu_I).$$

By Lemma 2,

$$\frac{\bar{z}_{\mu_I} k_I^{\alpha-1}(\mu_C, \mu_I)}{e^{\gamma/(1-\alpha)} + \delta - 1} = 1 + \frac{[1 - N(\mu_C + \lambda, \mu_I)]}{N(\mu_C + \lambda, \mu_I)} Q(\mu_C, \mu_I).$$

It follows that  $N(\mu_C + \lambda, \mu_I) = N(\mu_C, \mu_I)$ . On balanced-growth steady-state, conditions (27) and (28) imply

$$\frac{pN\bar{z}_I}{e^{\gamma/(1-\alpha)} + \delta - 1} = \kappa(r) [pNE[g_I(z)] + (1 - N)Q^\alpha E[g_C(z)]].$$

In turn, this means that

$$\begin{aligned} \frac{p(\mu_C + \lambda, \mu_I)\bar{z}_{\mu_I}}{e^{\gamma/(1-\alpha)} + \delta - 1} &= \\ &= \kappa[r(\mu_C + \lambda, \mu_I)] \left[ p(\mu_C + \lambda, \mu_I)E_{\mu_I}[g_{\mu_I}(z)] + \frac{1 - N(\mu_C + \lambda, \mu_I)}{N(\mu_C + \lambda, \mu_I)} Q^\alpha(\mu_C + \lambda, \mu_I)E_{\mu_C + \lambda}[g_{\mu_C + \lambda}(z)] \right]. \end{aligned}$$

By Corollary 1 and Lemmas 2 and 3, this is equivalent to

$$\frac{p(\mu_C, \mu_I)\bar{z}_{\mu_I}}{e^{\gamma/(1-\alpha)} + \delta - 1} = \kappa[r(\mu_C + \lambda, \mu_I)] \left[ p(\mu_C, \mu_I)E_{\mu_I}[g_{\mu_I}(z)] + \frac{1 - N(\mu_C, \mu_I)}{N(\mu_C, \mu_I)} Q^\alpha(\mu_C, \mu_I)E_{\mu_C}[g_{\mu_C}(z)] \right].$$

In turn, this verifies that  $r(\mu_C + \lambda, \mu_I) = r(\mu_C, \mu_I)$ . ■

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