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Exhaustible-Resource Extraction
with capital

by

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RESUME

Cet article porte sur l'introduction du capital et de l'investissement, y compris dans l'hypothèse où celui-ci est soumis à des coûts d'ajustements, dans les modèles théoriques d'extraction des ressources épuisables. Deux résultats classiques font l'objet d'une attention particulière : l'un, la règle d'Hotelling, qui veut que le taux d'extraction diminue avec le temps, s'avère particulièrement robuste; le deuxième, selon lequel une hausse du taux d'escompte accélère l'extraction, ne reste valide que si le capital est relativement moins rare, d'une manière qui peut être définie formellement, que la ressource épuisable.

ABSTRACT

This paper studies alternative ways of introducing capital in theoretical models of resource extraction, including the cost of adjustment model. It focuses on two central results of the mining literature : the first one, known as Hotelling's rule, claims that the extraction rate diminishes over time. It is found to be surprisingly robust. The second one has to do with the effect of a rise in the discount rate on the speed of extraction. Unlike the conventional case, this effect may be negative if capital is relatively scarcer than the resource.

1. INTRODUCTION

Little attention has been devoted in the exhaustible-resource literature to the importance of factors of production, capital in particular, in the extraction process. Given that the approach used to deal with extraction problems has been basically capital theoretic, one may wonder to what extent the presence of one or several other assets or stocks in those problems might affect the traditional results.

In this paper, we investigate the effects of introducing capital on two well-known results of the exhaustible-resource literature: the result that the extraction rate diminishes over time and the result that a rise in the interest rate implies a faster extraction program. Both issues have already received some attention but we hope to throw some more light on the effects of introducing capital, by casting the analysis into a more general framework, by focussing alternatively on the major approaches to investment which have been applied to the conventional firm, and by adapting to the extractive firm some of the analytical methods which are routinely used in standard microeconomics.

In section 2, we propose various ways of introducing capital in extractive models and examine their effects on extraction profiles. In section 3, we use a model with many factors of production but with perfectly malleable capital in order to investigate, following Neher (1981), under what circumstances a rise in the interest rate accelerates extraction. Section 4 provides a brief summary and conclusion.

2. THE EXTRACTION PROFILE

A great deal of efforts have been devoted to characterizing the extraction profile of exhaustible resources. Gray (1914) initiated this long tradition which found another major benchmark with the work of Hotelling (1931), took momentum in the recent period with such research as that by Schultze (1974) or Levhari and Liviatan (1977) and has now reached maturity, that is to say, the status of a full chapter (chapter 6) in the book by Dasgupta and Heal (1979). The major result applies to the industry as a whole in the absence of technical change or demand shifts. It calls for a progressive reduction in the extraction rate as time goes by. Being somewhat unrealistic that result has also generated a search for exceptions, the most meaningful of which arise when the resource is not homogeneous and (or) when capital and investment come into the picture. Two papers, one by Puu (1977) and one by Campbell (1981), fall into the last category and make the connection with an older and less well-known line of research on capital and extraction, one with more practical purposes, which is represented by Massé (1959, 348-53), Billiet (1959) and Ventura (1964).

Before going over the distinctive features of those models with capital, a word on maintained assumptions is warranted. The diminishing extraction rate is the counterpart of the increasing implicit price of the resource, which arises itself from a standard asset management argument: the own interest rate on an otherwise unproductive asset must equal the general rate of interest. It follows that exceptions to the diminishing extraction rate rule may result from two kinds of causes: first, when something breaks or complicates the link between the implicit extraction cost and the extraction rate, as may happen in presence

of capital; second, when the cause itself disappears as may be the case when the resource is heterogeneous. Indeed, when the resource is heterogeneous, its implicit price may decrease and increasing extraction rates may follow (Levhari and Liviatan (1977)). In this section of the paper, since we investigate how the introduction of capital may affect the result that the extraction rate diminishes, we assume that all other conditions for this result are satisfied, in particular, we assume that the resource is homogeneous, that there is no technical change and that the demand, whether elastic or not, does not shift over time (Stiglitz (1976))¹.

Capital raises the issues associated with its durability and its lack of malleability. Those have been addressed in various ways which carry specific assumptions with them: irreversibility, adjustment costs, the putty-clay hypotheses for example (see Lasserre (1981), chapter I). We shall deal with the issue of capital in a somewhat systematic way, by proposing models of the extractive firm which can be classified according to those key-assumptions.

We start with a model of the firm characterized by a concave production function of several factors, some of them having the dimension of stocks and the other ones, called variable factors, having the dimension of flows. There are no adjustment costs to any factors, nor any irreversibility. The sole constraint on factors of production is that they should be non negative. It is also assumed that the industry is competitive or managed by a social planner so that there are no attempts to extract any monopoly rent. The output price may not be constant over time, but is given by a time-autonomous function of output which is such that demand "chokes off" at some finite price. The price trajectory is somehow known to the firm. Factor prices are assumed to be constant. Finally, and

this is the difference between this model and any early neoclassical model of investment (Jorgenson (1963)), the firm extracts ore from a finite homogeneous reserve stock, R . The problem is (time subscripts will be omitted later on when no ambiguity arises):

$$\begin{aligned} \text{Max}_{\{I_t, L_t\}, T2} \quad & \int_{T1}^{T2} \gamma(t) \cdot (p_t \cdot f(x_t, L_t) - w' \cdot L_t - \phi' \cdot I_t) \cdot dt \\ & + \gamma(T2) \cdot \phi' \cdot x_{T2} \end{aligned} \quad (1)$$

Subject to:

$$(a) \dot{x}_t = I_t; (b) x_{T1} = x_1; (c) x_t \geq 0, t \in [T1, T2]; \quad (2)$$

$$L_t \geq 0; \quad (3)$$

$$(a) \dot{R}_t = -q_t = -f(x_t, L_t); (b) R_{T1} = R_1; (c) R_t \geq 0, t \in [T1, T2], \quad (4)$$

where p_t , w , ϕ are respectively the price of the output flow, the rental prices of the variable factors, and the asset prices of the stock factors at time t ; x_t , I_t , L_t are respectively a vector of factor stocks, the vector of adjustments to those stocks, and a vector of variable factors; $\gamma(t) = \exp(-r \cdot (t - T1))$; $T1$ and $T2$ are respectively the initial and final extraction dates. The production function, $f(x_t, L_t)$ is assumed to be concave and twice differentiable and satisfies the Inada conditions. This guarantees that extraction does occur. Since the resource is homogeneous and the price is bounded, the extraction period is finite (Salant et al. (1981)).

Although all conditions for the application of the Maximum Principle are not met, it does yield the solution in this problem. The current-value Hamiltonian, simply called Hamiltonian in the rest of the paper, is:

$$M = (p-\mu) \cdot f(x, L) - w' \cdot L - (\phi' - \lambda') \cdot I, \quad (5)$$

where μ and λ are respectively the costate variable associated with R and the vector of costate variables associated with x . One notes that M is linear in I , so that the optimum decision depends on the sign of each element in the vector of switching functions, $\sigma(t) = (\lambda_t - \phi)$. Consider the case where each switching function is null. Then,

$$\lambda_t = \phi, \text{ and } \dot{\lambda}_t = \dot{\phi} = 0. \quad (7)$$

The first-order conditions governing the costate variables are:

$$(\dot{\lambda} - r\lambda) \cdot \gamma(t) = -\frac{\partial}{\partial x}(\gamma(t) \cdot M) \text{ and } (\dot{\mu} - r\mu) \cdot \gamma(t) = -\frac{\partial}{\partial R}(\gamma(t) \cdot M), \text{ or}$$

$$\dot{\lambda} = r\lambda - (p-\mu) \cdot f_x \text{ and} \quad (8)$$

$$\dot{\mu} = r \cdot \mu. \quad (9)$$

Substituting (7) into (8), we have a conventional user cost relation, except that the output price has been corrected to reflect the implicit value of the resource:

$$(p-\mu) \cdot f_x = r \cdot \phi. \quad (10)$$

Equation (10) defines a singular path for x . That such a path is preferable to any other one results from the fact that the indeterminacy of I in the maximization of the Hamiltonian, when $\sigma=0$, means that no marginal change in x could affect the objective functional. Furthermore, by the concavity of f , each vector x_t^* so characterized is a unique maximum. Hence the singular path cannot be improved upon. Now, if $x_t \neq x_t^*$, the optimal policy is to bring x instantaneously to x^* . That the move is instantaneous results from the bang-bang nature of the problem, and the absence of bounds on I . That the optimal move is toward x^* , not away from it, again results from the concavity of the production function.

Now the Hamiltonian must also be maximized with respect to L . Hence,

$$(p-\mu) \cdot f_L = w. \quad (11)$$

(10) and (11) implicitly define the solutions x^* and L^* as functions of $p-\mu, w, r \cdot \phi$.

(10) and (11) can be interpreted as first-order conditions for the maximization of the modified Hamiltonian, defined below at its maximum:

$$H^*(y, v, w) = y \cdot f(x^*, L^*) - w' \cdot L^* - v' \cdot x^* \quad (12)$$

Where $v = r \cdot \phi$ and $y = p-\mu$.

Since f is concave, H^* can be interpreted as an implicit profit function with the usual properties (Diewert (1974)):

H^* is decreasing in w and v , and increasing in y ; H^* is convex; $\frac{\partial H^*}{\partial w} = -L^*$;

$\frac{\partial H^*}{\partial v} = -x^*$; $\frac{\partial H^*}{\partial y} = f(x^*, L^*) = q^*$.

This last expression, Hotelling's theorem, can be differentiated totally in order to study the behaviour of q^* over time (stars will be omitted in the rest of the argument). One gets:

$$\dot{q} = H_{yy} \cdot \dot{y} \quad (13)$$

Since the market price is a time-autonomous function of output, $p = p(q)$, we have:

$$\dot{y} = p_q \cdot \dot{q} - \dot{\mu}, \text{ so that, substituting into (13),}$$

$$\dot{q} \cdot (1 - H_{yy} \cdot p_q) = -H_{yy} \cdot \dot{\mu}. \quad (14)$$

Since H_{yy} is non negative, p_q non positive, and $\dot{\mu}$ positive, we get the usual result that the extraction rate diminishes over time².

As could be expected from a model which reflects only the durability of capital, and not its lack of malleability, the pattern of diminishing extraction rates is not affected.

We now show that the sign of the change in the extraction rate is not affected either by a fairly common way to model the lack of malleability of capital, the assumption that investment is non negative. Consider the same problem as previously, but with the following two supplementary assumptions:

$$(2) \text{ (d) } I \geq 0, \text{ } t \in (T_1, T_2),$$

and the scrap value of the firm is null, since assumption (2) (d) means that the equipment has no alternative use outside the firm.

The Hamiltonian of the problem is still given by (5), but one must take account of (2) (d) in its maximization. As before, since M is linear in I , three phases must be distinguished for each stock x_j : phase a, where $\phi_j - \lambda_j < 0$, I_j is infinite, and x_j registers a discrete increase; phase b, where $\phi_j = \lambda_j$, and I_j is not determined by the maximization of the Hamiltonian; phase c, where $\phi_j - \lambda_j > 0$, I_j would be infinite and negative if it were not for the non negativity constraint, but the latter is binding and $I_j = 0$. The following remarks are in order: first, since $x_j(t)$ is continuous during phases b and c, its costate variable is continuous during those phases. Consequently, a transition from phase c, when $\phi_j - \lambda_j > 0$, to phase a, when $\phi_j - \lambda_j < 0$, must involve a passage in phase b, when $\phi_j = \lambda_j$; second, since it involves a discrete change in x , any occurrence of phase a must be instantaneous.

In the rest of this section, we shall use the superscripts a, b, c, to denote those stock factors, stock adjustments, stock asset prices or implicit prices which are (or pertain to stock factors which are) in phases a, b, or c, respectively. Consider now a situation where all stock factors are either in phase b or phase c, when I is finite. Since $\lambda^b = \phi^b$, it follows from (8) that:

$$r \cdot \phi^b = y \cdot f_{x^b}(x^b, x^c, L). \quad (15)$$

(11) is unaffected by the new constraint; combined with (15), it implies that the restricted modified Hamiltonian, defined below at its maximum, is being maximized.

$$\tilde{H}(y, v^b, w, x^c) = y \cdot f(x^{b*}, x^c, L^*) - w' \cdot L^* - v^{b'} \cdot x^{b*} \quad (16)$$

\tilde{H} differs from H^* in that it has only a subset of stock-factor user-costs as arguments, those which pertain to stocks currently being adjusted, and it has some stocks, those which are currently fixed, as arguments. Clearly, \tilde{H} can be interpreted as a restricted profit function, with the following properties (Lau (1976)): \tilde{H} is convex in y , v^b , and w ; $\frac{\partial \tilde{H}}{\partial w} = -L$; $\frac{\partial \tilde{H}}{\partial v^b} = -x^b$; $\frac{\partial \tilde{H}}{\partial x^c} = -v^c$; $\frac{\partial \tilde{H}}{\partial y} = q$. Differentiating this last expression with respect to time, one has, following the same argument as in the previous case:

$$\dot{q} \cdot (1 - \tilde{H}_{yy} \cdot p_q) = -\tilde{H}_{yy} \cdot \dot{p}, \text{ since } \dot{x}^c = 0. \quad (17)$$

This expression, which implies that q does not increase over time, is valid whenever all stocks are in phases b or c , although \tilde{H} must be redefined whenever a stock switches from one phase to the other. Given the possibility of a third phase, one may wonder about the generality of this result. However, it is sub-optimal to move from phase b to phase a when, as assumed, prices do not have discontinuities. The reason is intuitively obvious: in phase b , when $\lambda^b = \phi^b$, the Maximum Principle leaves the stock adjustment, I^b , indeterminate precisely because no marginal change in the levels of the corresponding factor stocks can increase the Hamiltonian or, consequently, the objective functional. So, whenever possible, it is optimum for a stock to be in phase b . Since upward stock adjustments are feasible in phase b , λ^b will not be allowed to increase in such a way as to become higher than ϕ^b . It follows that phase a can occur only as an initial instantaneous build-up of capital³, so that (17) holds all the time except, possibly, at T_1 . So even in presence of non negativity constraints, the rate of output is non-increasing.

Are there any interesting special cases? Campbell (1980) presents an elegant treatment of a problem whose optimum involves a constant extraction rate over an

initial period. In his model the short-run marginal extraction cost increases with output and the firm faces a capacity constraint, $q \leq \bar{q}$, with \bar{q} directly proportional to the level of capital, the unique stock factor which may be acquired in any positive amount but has no alternative use. This is clearly a special case of the model just described and we can make use of (17) to infer conditions on the technology which produce Campbell's result. With only one stock factor, K , \tilde{H} reduces to:

$$\tilde{H}(y, w, K) = y \cdot f(K, L^*) - w' \cdot L^*, \text{ when } K \text{ is currently fixed,} \quad (16)a$$

or

$$H^*(y, w, v) = y \cdot f(K^*, L^*) - w \cdot L^* - v \cdot K^*, \text{ when } K \text{ is being adjusted} \quad (16)b$$

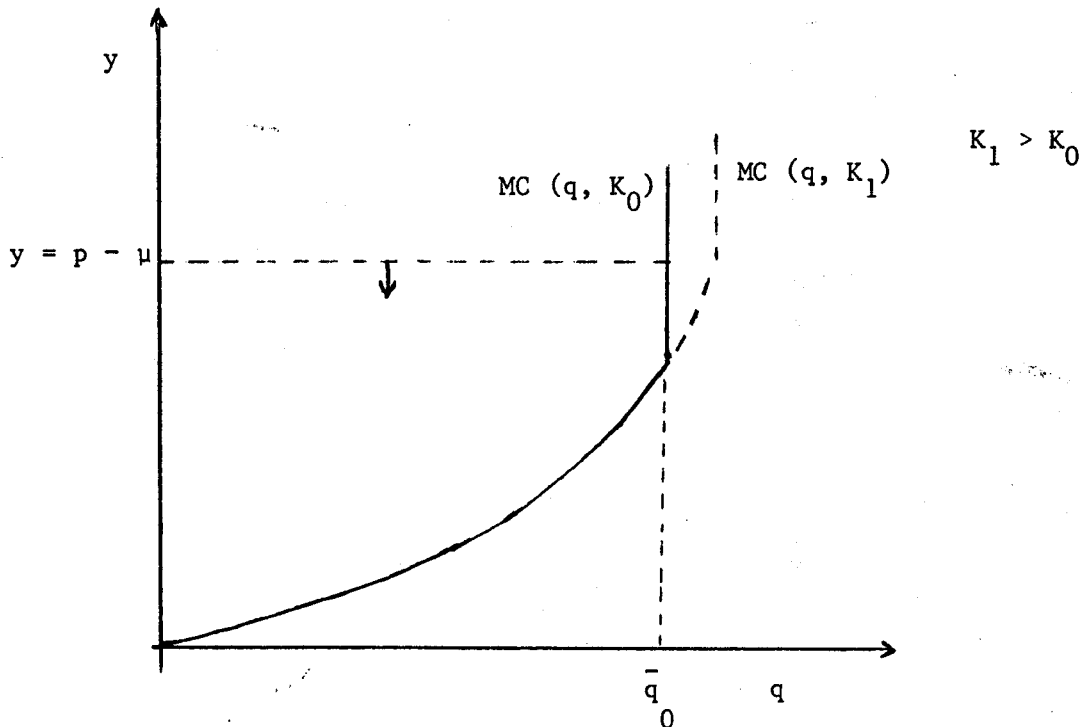
In both cases, (17) holds provided the appropriate function is used. Although this is not crucial to our interpretation, we can also make use of Campbell's result that K is adjusted only at the initial time, which implies that $\tilde{H}(y, w, K)$ is the appropriate function to use. Following Campbell, we also assume that the output price is constant, which implies that:

$$p_q = 0. \text{ Accordingly, (17) reduces to:}$$

$$\dot{q} = -\tilde{H}_{yy}(y, w, K) \cdot \dot{y}.$$

A constant extraction rate, $q' = 0$, implies $\tilde{H}_{yy} = 0$ which requires \tilde{H} to be linear in y . Considering (16)a, this means that L^* is insensitive to y . In other words, the marginal cost curve is vertical at the output rate considered. Following the same argument, a diminishing extraction rate requires the marginal cost curve to be

upward sloping at the levels of output which prevail toward the end of the extraction period, when y is low. Since K is constant, we have just characterized two parts of the same marginal curve: an upward-sloping part at low levels of y ; a vertical part at higher levels of y , as shown in Graph 1.



GRAPH 1: The technology in Campbell's (1980) model.

Graph 1 clearly shows how the reduction in y , which results from the increase in μ over time, may have no effect on extraction as long as the marginal cost curve, MC , is vertical but will cause output to diminish when y becomes low enough to correspond to the upward-sloping part of the curve. It will also be noted that Campbell's assumption that the upward-sloping part of the marginal cost curve is independent of K , as represented in Graph 1, is not crucial to his result.

So far, we have seen that perfectly malleable capital, even with no alternative use outside the firm, does not affect the standard Hotelling's result that the output of a mine or extractive industry does not increase over time. The lack of malleability of capital may also result from internal technological constraints. A primitive way to introduce such constraints is to impose an upper bound on investment. Puu (1977) studies such a model; its solution consists in a phase of increasing output followed by a phase of decreasing output. However he uses a model with variable grade, for which it is known that the standard result does not necessarily hold. Does his result depend on that particular assumption or does it follow from the upper bound which is imposed on investment? Let us use the same model as before, with an additional constraint:

$$(2) \quad (c) \quad I_t \leq \bar{I}.$$

As before, we have a bang-bang problem which involves three possible alternative policies for each stock x_j , according to whether $\lambda_j > \phi_j$ (phase a), $\lambda_j = \phi_j$ (phase b), or $\lambda_j < \phi_j$ (phase c). In the absence of an upper bound on I , phase a was shown to involve an instantaneous upward jump in the stock considered. This could be observed only at T_1 so that, during the extraction period $[T_1, T_2]$, all stocks were in either phases b or c, \dot{q} was given by (17) and was consequently non-positive. This is no longer true in presence of an upper bound on I , and it could be shown that the optimal policy, during phase a, consists in increasing the stock as fast as possible, at a rate $\dot{I}^a = \bar{I}^a$. The optimal extraction policy, derived from (8) and $\lambda^b = \phi^b$, must now satisfy:

$$\dot{q} \cdot (1 - \tilde{H}_{yy} \cdot p_q) = -\tilde{H}_{yy} \cdot \dot{p} + \tilde{H}_{y x^a} \cdot \bar{I}^a. \quad (17)b$$

In the absence of any a priori restriction on the vector $\tilde{H}_{y \ x a}$, the sign of \dot{q} cannot be inferred. This result is not surprising as intuition and Puu's result lead one to conjecture that, if initial factor stocks are low, the firm goes through an initial period during which it builds up its capacity (phase a) and finishes its extractive life with a period where it wished it owned less factor stocks, as equipment has no alternative use, but cannot get rid of those stocks (phase c). Indeed, since we know that the implicit factor stock prices are continuous functions of time in this problem, and since, in the absence of alternative uses for the stocks, they must tend toward zero as t approaches T_2 , it is easily proven that all stocks must be in phase c toward the end of the extraction period. What happens before that is not as clear. One would like to conjecture that the (bliss) phase b when the firm is perfectly happy with the amount of equipment it owns is short-lived to the point of being instantaneous. But if one imposes $\dot{\lambda}^b = 0$ and one differentiates (8) with respect to time, one finds that this is not necessarily the case, unless there is only one factor of production, as in the model of Puu.

So it appears that the lack of malleability of capital, even when it is introduced in a primitive fashion, by imposing upper and lower bounds on investment, affects Hotelling's result about the direction of the change in the extraction rate. One would expect this to be also the case when the lack of malleability takes the more sophisticated form of adjustment costs. In Lasserre ((1981), chapter 2) we studied a model of factor demands and output supply for an extractive firm which faced costs when adjusting its factor stock levels. Unfortunately, we were able to provide only an implicit characterization of the solution, in the form of a system of differential equations. Here, we characterize the optimum programme

explicitely, for a one-factor model with separable adjustment costs. The problem is to choose $I(t)$ so as to maximize:

$$\int_{T1}^{T2} \gamma(t) \cdot (p \cdot f(x) - C(I) - \phi \cdot I) \cdot dt + \gamma(T2) \cdot \phi \cdot x(T2) \quad (18)$$

subject to:

$$(2) (a) \dot{x}=I; (b) x(T1)=x_1, \text{ given}; (c) x \geq 0.$$

$$(4) (a) \dot{R}=-f(x); R(T1)=R_1; (c) R \geq 0,$$

where $C(I)$ is a convex adjustment cost function which reaches a minimum of $C(0)=0$ and tends to positive infinity when $|I|$ tends to infinity. The term $\phi \cdot I$ under the integral may be positive or negative, which implies that the equipment can be resold at no loss, and that the adjustment costs, $C(I)$, are internal to the firm. Consistent with the assumption that the equipment has an alternative use, there is a scrap value to the firm; as formulated it can be realized at no cost, under the assumption that the adjustment costs affect only the operating firm and disappear at $T2^4$. If we ignore those problems which might be associated with using the Maximum Principle in presence of non negativity constraints on the state variables conditions for using it are met. The Hamiltonian is:

$$H = y \cdot f(x) - C(I) - (\phi - \lambda) \cdot I. \quad (19)$$

It is maximized when:

$$C'(I) = -(\phi - \lambda) \quad (20)$$

Other first-order conditions are:

$$\dot{\lambda} = \lambda r - y \cdot f_x, \quad (21)$$

$$\dot{\mu} = r \cdot \mu \quad (22)$$

The transversality conditions are:

$$\mu(T_2) = p[f(x_2)] \quad (23)$$

and

$$\lambda(T_2) = \phi \quad (24)$$

The Legendre second-order conditions are satisfied since $f(x)$ is concave as well as $-(C(I) + \phi \cdot I)$.

Proposition 1: If $\lambda > \phi$, $\dot{\lambda} < 0$

Proof: If $\lambda > \phi$,

We have: $I > 0$, $\dot{q} > 0$, $\dot{p} < 0$, $\dot{y} < 0$

Differentiating (21), we get:

$$\ddot{\lambda} = \dot{\lambda} r - \dot{y} \cdot f_x - y \cdot f_{xx} \cdot I \quad (25)$$

Suppose that $\dot{\lambda} \geq 0$, then by (25), $\ddot{\lambda} > 0$. It follows that λ will keep increasing so that the transversability condition (24) will not be met. Hence $\dot{\lambda}$ must be negative. Q.E.D.

Proposition 2: If $\lambda = \phi$, $\dot{\lambda} < 0$ unless $t = T_2$.

Proof: Using a similar argument as in Proposition 1, one can show that $\dot{\lambda}$ must not be positive unless $t = T_2$. Hence $\dot{\lambda} \leq 0$.
Suppose that $\dot{\lambda} = 0$. Since $\lambda = \phi \Rightarrow \dot{y} < 0$ and $I = 0$, it follows from (25) that $\ddot{\lambda}$ is positive. But if so, $\dot{\lambda}$ will become positive which was just shown not to be feasible unless $t = T_2$. Hence $\dot{\lambda} < 0$ unless $t = T_2$. Q.E.D.

Proposition 3: $\dot{\lambda}(T_2) = r \cdot \phi$ unless $x(T_2) = 0$

Proof: $\lim_{t \rightarrow T_2} \dot{\lambda} = \lim_{t \rightarrow T_2} \lambda \cdot r - \lim_{t \rightarrow T_2} y \cdot f_x$, by (21)

$$= r \cdot \phi - 0 \cdot \lim_{t \rightarrow T_2} f_x \quad \text{using (23) and (24)}$$

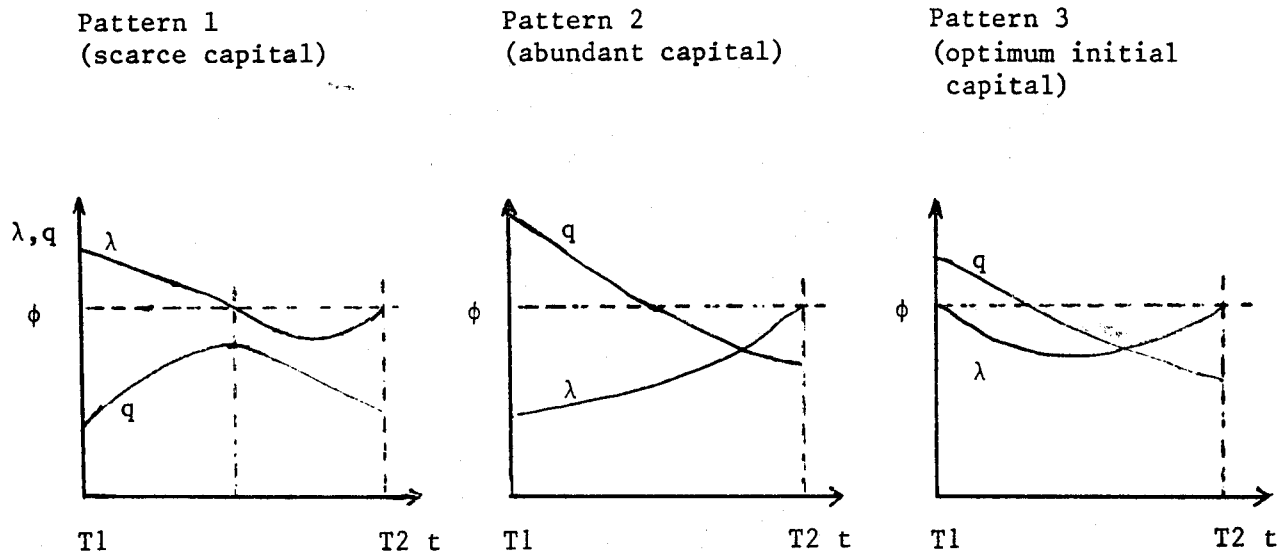
$$= r \cdot \phi, \text{ unless } x \text{ tends toward zero.} \quad \text{Q.E.D.}$$

From propositions 1-3 we deduce the following:

- . If $\lambda > \phi$ at some date during the extraction period, then $\dot{\lambda}(T_1) > \phi$ (from proposition 1);
- . The trajectory of λ can cross the $\lambda = \phi$ line at most once and from above (from proposition 2);
- . If $\lambda > \phi$ at some date during the extraction period, the trajectory of λ must cross the $\lambda = \phi$ line (from proposition 3, since x cannot tend toward zero if λ tends toward ϕ from above as this implies that x is increasing, and since λ must tend toward ϕ from below, if $x(T_2) > 0$).

As a result, referring to Graph 2 below, the optimum program must either follow pattern 1, which we label "scarce capital", or pattern 2, which we label "abundant capital". Pattern 3 is an important limiting case to which we shall refer later.

Graph 2: Optimum programs with costs of adjustment.



As when investment was bounded, a program such as pattern 1, which involves an increasing rate of extraction during an initial period, followed by a period of decreasing output toward the end of the extractive life, may be optimum if the initial capital stock is low. On the contrary, if the initial capital stock is high enough, the standard pattern of decreasing output will obtain. We believe but have not been able to prove that this is also the case for a model with many factors of production.⁵

One weakness of cost of adjustment models of investment is their inability to explain the inherited capital level, x_1 , in a satisfactory manner. One has to believe that the same adjustment cost regime applies during the construction of a firm as during its active existence. A better view of the world, represented by the putty-clay hypothesis and refined by the putty-semi-putty hypothesis (Fuss (1977)), holds that something irreversible happens at the creation time which narrows the ex post choice set of the firm. In Lasserre (1981), this idea is adapted to mean that adjustment costs are negligible ex ante, so that a firm can select any scale of operation and factor proportions to start with, but that adjustment costs become present ex post if the firm wants to modify its initial choice. For problem (18) this amounts to making the initial equipment level a choice variable. The investment program $I(t)$ is then selected accordingly. If we call $V(x_1, R_1, \phi, r)$ the maximized value function obtained by solving problem (18) for a given initial level of x , x_1 , the ex ante choice consists in choosing x_1 so as to maximize:

$$V(x_1, R_1, \phi, r) - \phi \cdot x_1 \quad (26)$$

Since $f(x)$ satisfies the Inada conditions, it can be shown that V is concave and differentiable in x_1 and that an interior solution to (26) exists. That solution must satisfy:

$$\begin{aligned} V_{x_1} &= \phi, & \text{or} \\ \lambda(T_1) &= \phi. \end{aligned} \quad (27)$$

Consequently, when the initial level of capital is a decision variable, the optimum extraction pattern is pattern 3 in graph 1, and it follows that the standard Hotelling's result applies again !

Besides the very sensible extraction pattern implied, this model describes the investment pattern qualitatively. A surprising result implied by proposition 3, which states that λ is increasing when time approaches T_2 , is that investment is becoming less negative when t approaches T_2 , a feature which gets an intuitive explanation from the fact that, in (21), the change in the opportunity cost of capital can be attributed to two sources. The first one, λ_r , reflects the fact that capital is a financial asset which must produce capital gains unless it generates other benefits. Those are productive services, the second source of changes in λ and second term in (21); they tend to be dominated by the first term toward the end of the extraction phase because y , the value of one extracted ore unit net of its resource opportunity cost, becomes very small when the horizon is rear.

A word on the putty-clay hypothesis will complete this review of alternative ways to introduce capital in exhaustible-resource models. Two versions must be considered; in the first one, only stock factors are fixed ex post, while the proportion of variable factors to fixed factors may vary. This is the standard extraction case with increasing marginal costs, which is known to involve a diminishing extraction rate. In the second version, both stock factor levels and factor proportions are fixed ex post. In that case, output is constant. Again, Hotelling's result holds.

3. THE EFFECT OF THE INTEREST RATE

Even a partial survey of the effects of capital and investment on resource extraction could not leave out a discussion on the role of the interest rate. Since Neher (1981) covers the key aspects of this question in an attractive and elegant fashion, we shall only discuss his points briefly, derive them in the more general framework of the model used in the previous section, and provide a few additional results. We shall consider two alternative interest rates, with $r_0 < r_1$. Subscripts "0" and "1", associated with any relevant variable, will refer to the optimum value of that particular variable, when $r=r_0$ or $r=r_1$ respectively. Without loss of generality, the initial extraction date will be $t=0$ and the terminal time will be noted T_i , $i=0,1$, unless the extraction period is infinite.

The traditional view is that extraction is faster, the higher the interest rate, as the opportunity cost of postponing a revenue rises with the interest rate. The notion of a faster extraction is not without ambiguity. When T is finite one could say that extraction is faster when $r=r_1$ than when $r=r_0$ if $T_1 < T_0$. Indeed, Levhari and Liviatan (1977) show that $\partial T / \partial r < 0$. Unfortunately, for many exhaustible resource models, the extraction period is infinite. Such would be the case for our model of last section had we not postulated that demand choked above a certain price. An alternative concept of speed of extraction compares extraction rates: extraction is said to be faster at r_1 than r_0 if the extraction rate is higher at r_1 , other things equal. A privileged date for other things to be equal is $t=0$, as this is when the resource stocks are identical under the two alternative programs. Indeed, whether T is finite or not, $\partial q(0) / \partial r \geq 0$ in all "traditional" extraction models⁶. This last

concept may be deceptive however, for one can conceive of a resource for which $\partial q/\partial r \geq 0$ at some reserve level and $\partial q/\partial r < 0$ at some other level. In fact, this is the essence of Neher's (1978) "Double Cross", an example of an extraction model with capital for which the industry price paths at two alternative interest rates cross twice. Given this possibility one would like to suggest two alternative definitions of "faster extraction". Under the strong definition, extraction is said to be faster at r_1 than r_0 if $\partial q/\partial r \geq 0$ at any reserve level; under the weak definition, extraction is said to be ultimately faster at r_1 than r_0 if there exists a level of reserves below which $\partial q/\partial r \geq 0$. Such a distinction between a strong and a weak definition of "faster extraction" may also usefully be extended to cases where T is finite, for it is easily seen that $\partial q/\partial r \geq 0$ at any reserve level implies $\partial T/\partial r \leq 0$ whatever the initial reserve level (a "strong" concept of faster extraction) and that, if $\partial q/\partial r \geq 0$ when reserves are below a certain level, \bar{R} , then $\partial T/\partial r \leq 0$ if the initial reserve level is not greater than \bar{R} (a "weak" concept of faster extraction).

Before providing a few results on the effect of the interest rate on the extraction speed, we briefly show how the presence of capital may affect the traditional result, as an increase in r increases the user cost of capital and provides an incentive for slow extraction which might offset the increased extractive impatience which underlies the traditional result. Consider the model of extraction with perfectly malleable capital which was studied in the previous section. It is recalled that $q = H^*(y, v, w)$. The interest rate affects q through y , the net-of-resource-rent output price, and through v , the user cost of capital, as:

$y = y(K, r, \phi, w)$, at the optimum, and $v = r \cdot \phi$.

It follows that:

$$\frac{\partial g}{\partial y} = H^*_{yy} \cdot \frac{\partial y}{\partial v} + H^*_{yv} \cdot \frac{\partial v}{\partial r}.$$

$$= H^*_{yy} \cdot (p \cdot \frac{\partial g}{\partial y} - \frac{\partial u}{\partial y}) + H^*_{yv} \cdot \frac{\partial v}{\partial r}, \text{ and}$$

$$(28) \quad \frac{\partial g}{\partial r} \cdot (1-p) \cdot H^*_{yy} = -H^*_{yr} \cdot \frac{\partial y}{\partial r} + H^*_{vr} \cdot \frac{\partial v}{\partial r} \cdot \phi.$$

Since p is non positive and H^* is convex in y , $\partial g / \partial r$ has the sign of the Right Hand Side of (28). For a homogeneous resource, when capital is perfectly malleable,

$\partial u / \partial r \leq 0$, so that the first term on the R.H.S. of (28) is non negative. It follows that, in the absence of capital, when $H^*_{yv} = 0$, $\partial g / \partial r$ is non negative, which is the traditional result. Focussing now on the second term on the R.H.S. of (28), each

term in the vector H^*_{yv} represents the effect on output of an increase in one

particular stock-factor price, ignoring the impact of that price increase on output price y . In general, this effect may be positive or negative so that the combined

effect $H^*_{yv} \cdot \phi$ is unsigned. When there are only stock factors, as in Neher's

one-factor model, this combined effect can be signed because the increase in factor

prices is equivalent to a reduction in the output price, which is known to have a negative impact⁸. When the resource is plentiful and $\partial u / \partial r = 0$, (28) shows that the effect on

the extraction rate of an increase in the interest rate reduces to its effect through

factor prices, which is negative when there are only stock factors. In general, it

appears that a rise in the interest rate may affect the rate of output positively or

negatively, but that the effect is more likely to be positive, the scarcer the

resource relative to capital. This is the essence of Neher's (1981) paper⁹.

This finding satisfies one's intuition and leads to more specific questions. It is clear that when capital is not under the control of the firm, a rise in the interest rate implies a faster extraction program whatever the level of reserves. When capital is variable, however, since the resource becomes relatively scarcer as extraction proceeds, one may wonder whether there is a reserve level below which a rise in the interest rate will imply a faster extraction (the weak definition). It turns out that this is not necessarily the case, but propositions 1 and 2 provide two situations where this is true:

Proposition 1: If the demand does not choke, then there is a reserve level below which a rise in the interest rate implies a faster extraction.

Proposition 2: If the production function satisfies the Inada conditions, then there is a reserve level below which a rise in the interest rate implies a faster extraction.

Proofs:

Both proofs rely on a comparison between \dot{q}_0 and \dot{q}_1 , when t tends toward infinity in the case of proposition 1, and when t tends toward T_0 or T_1 in the case of proposition 2. When the demand does not choke, p tends toward infinity and q tends toward zero as t tends toward infinity. Since $p = \mu + c$, where c represents the marginal extraction cost, and since, with a concave production function, c does not increase as q diminishes, the trajectory of μ tends toward the trajectory of p as q approaches zero. It follows that:

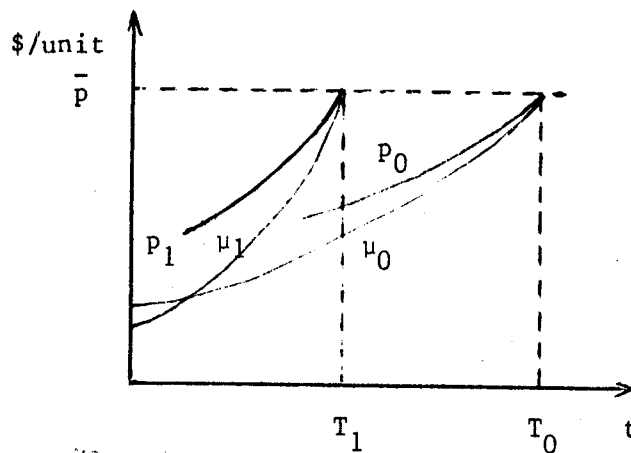
$$\lim_{t \rightarrow \infty} \dot{p}/p = \lim_{t \rightarrow \infty} \dot{\mu}/\mu$$

But $\dot{\mu}_0/\mu_0 = r < \dot{\mu}_1/\mu_1 = r_1$. Consequently, when t tends toward infinity, $\dot{p}_0/p_0 < \dot{p}_1/p_1$, which implies that $\dot{q}_0/q_0 > \dot{q}_1/q_1$. In order that both q_0 and q_1 tend toward zero while q_1 diminishes at a faster rate when t tends toward infinity, it is necessary that:

$$q_1(t) > q_0(t) \text{ when } t \rightarrow \infty$$

This establishes Proposition 1.

When the demand does not choke, Proposition 2 follows from Proposition 1. We now assume that there is an upper limit, \bar{p} , to the output price, which implies that extraction takes place over a finite period. If the production function satisfies the Inada conditions, it can be shown (Lasserre (1981), chapter 2) that $\mu(T) = \bar{p}$, which reflects the fact that $q(t)$ tends toward zero as t approaches T , so that marginal factor products tend toward infinity and the marginal cost tends toward zero. This has two interesting implications: first, since $\mu_0(T) = \mu_1(T)$, $\dot{\mu}_0(t) < \dot{\mu}_1(t)$ when t approaches T ; second, since $p = \mu + c$ and c tends toward zero, $\dot{p} = \dot{\mu}$ when t tends toward T . It follows that $\dot{p}_0(t) < \dot{p}_1(t)$ when t tends toward T_0 (in the case of $\dot{p}_0(t)$) or T_1 (in the case of $\dot{p}_1(t)$). This is illustrated in Graph 3.



Graph 3: Terminal conditions when the production function satisfies the inada conditions and demand chokes at \bar{p} .

In graph 3, the slopes of p_1 and p_0 at T_1 and T_0 respectively are uniquely defined by \bar{p} and the appropriate level of r . But the relative position of T_1 and T_0 depends on the initial level of μ_0 and μ_1 , itself a function of the initial reserve level. It is clear however that the last reserve unit is extracted faster when $r=r_1$ than when $r=r_0$ which implies that there is a level of reserves below which extraction is faster the higher the interest rate. This establishes proposition 2.

Proposition 1 and 2 confirm one's intuition that, although the presence of capital may affect the traditional view on the effect of the interest rate, that view remains valid for a resource which approaches exhaustion. Such intuition must be taken cautiously, however, as can be seen from the following counter-example. Suppose that the technology is such that the minimum marginal cost is $c(r)$, with $c(r_0) < c(r_1)$, as would be the case, for example, if the production function was linear in a unique stock factor. We

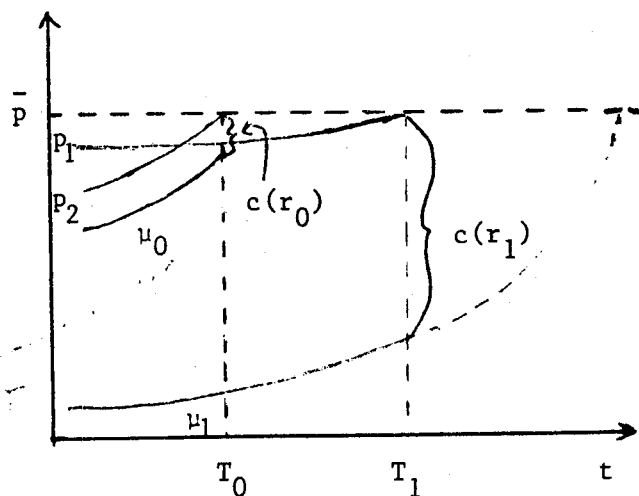
know that:

$$\mu_0(T_0) = \bar{p} - c(r_0) \quad \text{and} \quad \mu_1(T_1) = \bar{p} - c(r_1)$$

it follows that $\mu_1(T_1)$ may be sufficiently lower than $\mu_0(T_0)$ for $\dot{\mu}_1(T_1)$ to be lower than $\dot{\mu}_0(T_0)$ despite the fact that μ_1 grows at a higher rate than μ_0 . If, as drawn in Graph 4, the marginal costs are respectively constant over time, this is sufficient for:

$$\dot{q}_0(T_0) > \dot{q}_1(T_1).$$

In this case, a rise in the interest rate implies a slower extraction at any reserve level. Capital remains scarce relative to the exhaustible resource over the whole extraction program.



Graph 4: Counter-example

4. SUMMARY AND CONCLUSION

In this paper, we have explored some ways and effects of introducing capital in extractive models. The emphasis was put on two well-known results: the proposition that the output of an extractive firm or industry should not increase over time and the proposition that a rise in the interest rate speeds up extraction. When capital is introduced in a somewhat primitive fashion, where only its durability is taken into account, the standard result on the extraction profile remains valid. It is the second key characteristic of capital, its imperfect malleability, which may affect the result, as a firm may find itself in a position where it is optimum progressively to build up its capacity before adopting the standard attitude of reducing output in response to a rising resource opportunity cost and, possibly, to the reduction in the remaining operative life of equipment. The standard result appears to be fairly robust, however, and it is valid again in the case of a more sophisticated one-factor adjustment cost model where the initial factor stock is not left unexplained but results from a rational ex ante decision of the firm.

A rise in the interest rate may slow down extraction in presence of capital but, unlike the previous case, the key characteristic at play is the durability of capital. Neher's (1981) conclusion that this result obtains only if capital is scarce relative to the exhaustible resource was reestablished and explored further. If the demand does not vanish at high prices or if the production function satisfies the inada conditions, there exists a reserve level below which the exhaustible resource is scarce enough

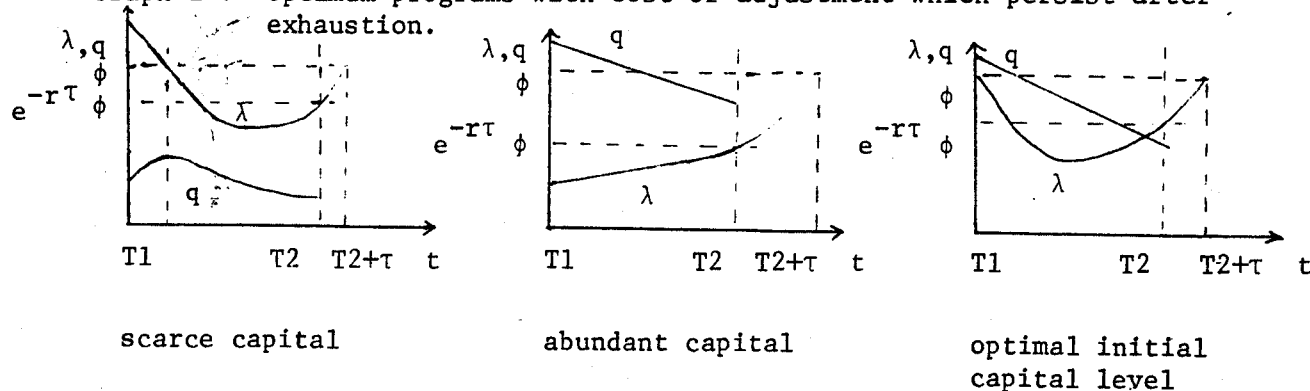
relative to capital for the conventional effect of a rise in the interest rate to obtain. Under other characteristics of the market and the technology, however, the scarcity of the resource will indeed increase over time but may never outweigh that of capital. In such cases, a rise in the interest rate may slow down extraction even if the resource approaches exhaustion.

Throughout the paper, the methods and results of static duality theory were applied to maximized Hamiltonians, allowing the derivation of fairly general results in a simple fashion.

FOOTNOTES

- 1 Aggregation problems may be perverse in presence of exhaustible resources (Blackorby and Schworm (1980)). In fact Lasserre (1982) shows that, even when the conditions outlined below are met, the extraction rates of all individual firms may not decrease over time although industry output conforms to the Hotelling pattern.
- 2 If the aggregation problem is addressed adequately, this result does not follow for all firms (see footnote 1).
- 3 In the foregoing argument, since we are considering upward jumps only, the non-negativity constraint on I can be ignored, so that the argument of Arrow and Kurz ((1970), p. 57) can be used as a formal proof.
- 4 If adjustment costs persists at T_2 , the firm faces ... a standard extraction problem without capital, that of extracting a stock x_2 , at a rate $I = \dot{x}$ using a technology characterized by the cost function $C(I)$, given that the output price will stay at ϕ . For this well-known problem, the imputed price of the resource (capital), λ , grows at the discount rate to reach the output price, ϕ , at the end of the extraction (dismantling) period, with output (negative investment) reaching zero at that time (Lasserre (1981, chap. 2) and also Schulze (1974)). If τ is the length of the dismantling period which starts at T_2 , the imputed value of capital at T_2 is $\lambda(T_2) = e^{-r\tau} \phi$. The whole foregoing treatment of the cost of adjustment model can be adapted by substituting this terminal value to that given by (24). For more comments see footnote 5.
- 5 If adjustment costs persist after T_2 , it is easily checked that proposition 1 holds, proposition 2 holds even at $t = T_2$ and, in proposition 3, ϕ must be replaced by $e^{-r\tau} \phi$, the value of $\lambda(T_2)$ given in footnote 4. The same qualitative results obtain, with Graph 2 being replaced by Graph 2'.

Graph 2': Optimum programs with cost of adjustment which persist after exhaustion.

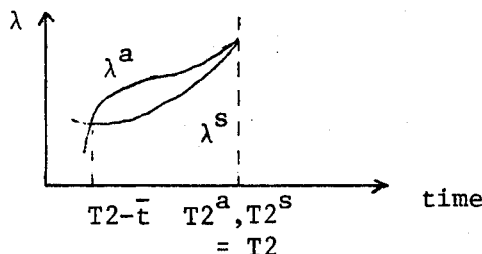


In Graph 2', τ is not lower in the abundant capital case than in the "optimal initial capital" case and τ is not lower in that case than in the "scarce capital" case. To see this, note that $\partial \tau / \partial x_2 > 0$, so that the terminal capital level in the "abundant capital" case would have to be lower than in the "scarce capital" case, if the proposition did not hold in which instance capital trajectories would have to cross. To see

that this is impossible, consider what happens toward exhaustion time. We shall use the indices a and s to refer to the cases of abundant and

scarce capital respectively if $x^a(T2^a) < x^s(T2^s)$, by (21),

$\lambda^a(T2^a - \epsilon) < \lambda^s(T2 - \epsilon)$, since $f_x^a > f_x^s$. But this implies that the trajectory of λ^a tends to the trajectory of λ^s from above if those are represented on such a time scale that $T2^a$ and $T2^s$ are represented by the same point. This in turn implies that $I^a > I^s$ toward $T2$. So if $x^a(T2^a) < x^s(T2^s)$ I^a must have been inferior to I^s , requiring $\lambda^a < \lambda^s$ at some earlier time. The trajectories of λ^a and λ^s must have crossed as indicated in Figure a at, say, $T2 - \bar{t}$. At $T2 - \bar{t}$, $\lambda^a = \lambda^s$, $x^a < x^s$, and $y^a = y^s$. Hence by (21) $\lambda^a < \lambda^s$, which contradicts Figure a. Hence the initial proposition, $x^a(T2^a) < x^s(T2^s)$, cannot hold, hence $x^a(T2^a) \geq x^s(T2^s)$.



- 6 By «traditional» models, we mean positive discount rate models of present-value or additive-utility maximization without capital such as Hotelling (1931), Levhari and Liviatan (1977) or Schultze (1974).

- 7 From resource homogeneity, we have:

$$(a) \quad \mu = J/R, \text{ with } J = \int_0^T \gamma(t) \cdot (f(x, L) - \phi I - w'L) dt + \gamma(T) \cdot \phi' \cdot x(T)$$

where x , L , T are at their optimum. With perfectly malleable capital, J can also be written as:

$$J = \int_0^T \gamma(t) \cdot (f(x, L) - v' \cdot x - w'L) dt + \phi' \cdot x(0)$$

By the concavity of f and since, at the optimum $(p - \mu)f_x = v$ and $(p - \mu)f_L = w$, the expression under the integral is positive at all dates.

Knowing this, suppose that $\partial J / \partial r > 0$, or

- (b) $J(r_1, 1) > J(r_0, 0)$, where i in $J(r_i, i)$ indicates that all relevant variable take the values which are optimal when $r = r_i$. Since the term under the integral is positive at all dates, and $x(T) \geq 0$ $J(r_0, 1) > J(r_1, 1)$. But this means that $J(r_1, 1) < J(r_0, 0)$, since $J(r_0, 0) \geq J(r_0, 1)$ as an optimum. This contradicts (b) and it follows that $\partial J / \partial r < 0$. But by (a), this implies:

$$\partial \mu / \partial r < 0$$

Q.E.D.

- 8 If $\partial v / \partial r = \phi$, $(\partial v / \partial r) / v = 1/r$ which means that all factor prices have been increased by the same percentage $1/r$.
- 9 As Neher's illustration relies on several explicit integrations, it was confined to a one factor model with linear technology under the assumption that demand does not choke. (28) is valid for any concave production function of m variable factors and n perfectly malleable stock factors for which a solution to the mine problem exists, we assume $p_q \leq 0$ but there may be an upper limit to the prices which command a positive demand.

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