Université de Montréal

Essais sur la conception de mécanismes et les enchères

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Thèse présentée à la Faculté des études supérieures
en vue de l'obtention du grade de Philosophiæ Doctor (Ph.D.)
en sciences économiques

Novembre, 2009

Université de Montréal
Faculté des études supérieures

Cette thèse intitulée:

Essais sur la conception de mécanismes et les enchères

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RÉSUMÉ

Cette thèse est composée de trois essais liés à la conception de mécanisme et aux enchères.

Dans le premier essai j'étudie la conception de mécanismes bayésiens efficaces dans des environnements où les fonctions d'utilité des agents dépendent de l'alternative choisie même lorsque ceux-ci ne participent pas au mécanisme. En plus d'une règle d'attribution et d'une règle de paiement le planificateur peut proférer des menaces afin d'inciter les agents à participer au mécanisme et de maximiser son propre surplus ; Le planificateur peut présumer du type d'un agent qui ne participe pas. Je prouve que la solution du problème de conception peut être trouvée par un choix max-min des types présumés et des menaces. J'applique ceci à la conception d'une enchère multiple efficace lorsque la possession du bien par un acheteur a des externalités négatives sur les autres acheteurs.

Le deuxième essai considère la règle du juste retour employée par l'agence spatiale européenne (ESA). Elle assure à chaque état membre un retour proportionnel à sa contribution, sous forme de contrats attribués à des sociétés venant de cet état. La règle du juste retour est en conflit avec le principe de la libre concurrence puisque des contrats ne sont pas nécessairement attribués aux sociétés qui font les offres les plus basses. Ceci a soulevé des discussions sur l'utilisation de cette règle : les grands états ayant des programmes spatiaux nationaux forts, voient sa stricte utilisation comme un obstacle à la compétitivité et à la rentabilité. Apriori cette règle semble plus coûteuse à l'agence que les enchères traditionnelles. Nous prouvons au contraire qu'une implémentation appropriée de la règle du juste retour peut la rendre moins coûteuse que des enchères traditionnelles de libre concurrence. Nous considérons le cas de l'information complète où les niveaux de technologie des firmes sont de notoriété publique, et le cas de l'information incomplète où les sociétés observent en privée leurs coûts de production.

Enfin, dans le troisième essai je dérive un mécanisme optimal d'appel d'offre dans un environnement où un acheteur d'articles hétérogènes fait face a de potentiels fournisseurs de différents groupes, et est contraint de choisir une liste de gagnants qui est compatible avec des quotas assignés aux différents groupes. La règle optimale d'attribution consiste à assigner des niveaux de priorité aux fournisseurs sur la base des coûts individuels qu'ils rapportent au décideur. La manière dont ces niveaux de priorité sont déterminés est subjective mais connue de tous avant le déroulement de l'appel d'offre. Les différents coûts rapportés induisent des scores pour chaque liste potentielle de gagnant. Les articles sont alors achetés à la liste ayant les meilleurs scores, s'il n'est pas plus grand que la valeur de l'acheteur. Je montre également qu'en général il n'est pas optimal d'acheter les articles par des enchères séparées.

Mots clés: efficacité, options extérieures endogènes, menaces, coûts virtuels, optimalité, juste retour, libre concurrence, asymétrie, enchère.

ABSTRACT

This thesis is made of three essays related to mechanism design and auctions.

In first essay I study Bayesian efficient mechanism design in environments where agents' utility functions depend on the chosen alternative even if they do not participate to the mechanism. In addition to an allocation rule and a payment rule the designer may choose appropriate threats in order to give agents the incentive to participate and maximize his own expected surplus ; The planner may presume the type of an agent who does not participate. I show that the solution of the design problem can be found by a max - min choice of the presumed types and threats. I apply this to the design of an efficient multi-unit auction when a buyer in possession of the good causes negative externalities on other buyers.

The second essay considers the fair return rule used by the European Space Agency (ESA). It ensures each member state of ESA a return proportional to its contribution, in the form of contracts awarded to firms coming from that state. The fair return rule is in conflict with the principle of free competition since contracts are not necessarily awarded to firms with the lowest bids. This has raised debates on the use of this rule : it is well accepted by small states, but larger states with strong national space programs, see its strict use as an obstacle to competitiveness and cost effectiveness. It is easy to believe that this rule is more costly to the agency than traditional auctions. We show on the contrary that an adequate implementation of the fair return rule may cause it to be less expensive to the agency than the traditional auctions of free competition. We consider the case of complete information where firms' technology levels are common knowledge, and the case of incomplete information where firms observe privately their production costs. In both cases we show that adequate implementation of the fair return rule may help take advantage of asymmetries between countries in order to expect a lower cost than with traditional auctions.

Finally, in the third essay I derive an optimal procurement mechanism in an environment where a buyer of heterogeneous items faces potential suppliers from different groups, and is constrained to choose a winning list that is consistent with some exogenous quotas assigned to the different groups. The optimal allocation rule consists of assigning priority levels to suppliers on the basis of their cost reports. The way these priority levels are determined is subjective but known to all before the auction. The individual reports induce scores for each potential winning list. The items are then purchased from one of the lists with the best score, provided it is not greater than the buyer's valuation for the items. I also find that it is not optimal to purchase the items through separate auctions, unless the buyer's valuation is sufficiently high or low.

Keywords : efficiency, endogenous outside options, threats, Optimal, Virtual costs,fair return, free competition, asymmetry, auction.

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LISTE DES SIGLES

ASE	Agence Spatiale Européenne
CE	Communautés Européenne
EC	European communities
ESA	European Space Agency

REMERCIEMENTS

Je voudrais adresser mes remerciements à Olivier Armantier mon directeur de recherche, à Walter Bossert et Lars Ehlers pour leurs commentaires et encouragements. Merci également à Justin Leroux et Onur Ozgur qui ont bien voulu évaluer cette thèse. Je remercie ma famille et tous mes amis qui m'ont soutenu tout au long de ce travail. Enfin je remercie tout le personnel administratif du département de sciences économiques de l'Université de Montréal.

INTRODUCTION GÉNÉRALE

Prendre une décision quand plusieurs agents font face à des alternatives les affectant différemment est un problème habituellement résolu par l'exécution d'un mécanisme. C'est à dire un ensemble de règles déterminant, sur la base d'informations recueillies auprès des agents (appelées leurs types), quelle alternative sera choisie et éventuellement quels doivent être les paiements à faire ou à recevoir par chaque agent. Des exemples de mécanismes incluent des enchères et des règles de vote. Les agents sont habituellement libres de décider s'ils participeront au mécanisme ou pas. Différentes règles seront employées selon l'objectif visé par le décideur et selon le contexte dans lequel la décision doit être prise. Les deux objectifs les plus communs sont la maximisation du revenu et l'efficacité sociale. Ces deux objectifs sont en général opposés dans un contexte d'information incomplète. Comme discipline, la conception de mécanismes vise à formaliser la création et l'exécution de règles appropriées étant donnés un certain objectif et un environnement particulier. Cette thèse est faite de trois essais liés à la conception de mécanismes avec des applications aux enchères. Les essais diffèrent par l'environnement et par l'objectif du décideur.

Dans la majeure partie de la littérature sur la conception de mécanismes on suppose habituellement que les agents qui ne participent pas au mécanisme ont des options extérieures qui sont indépendantes de l'issue du mécanisme. Cependant dans la pratique les agents qui ne participent pas sont néanmoins concernés par l'alternative finalement choisie par les autres. Par exemple, si un gouvernement veut vendre un permis pour une innovation technologique la société qui obtient le permis exercera des externalités négatives sur les autres sociétés. En fait, qu'une société participe ou pas, sa part du marché est susceptible de diminuer suite au fait que le permis est vendu à un concurrent. *le premier essai* étudie la conception de mécanismes efficaces dans des environnements où les agents ont des informations privées et ont des options extérieures endogènes. C'est à dire que leurs fonctions d'utilité dépendent de l'alternative choisie même si ils ne participent pas au mécanisme. La décision appartient à un planificateur qui, en plus de viser l'efficacité sociale, veut maximiser l'espérance du surplus collecté lors des transfert avec les agents, tout en cherchant à induire une participation honnête de tous les agents. Krishna et Perry (2000) ont généralisé les mécanismes VCG en introduisant le concept de la base. Dans leur théorème 1 ils prouvent qu'un choix approprié de la base permet d'obtenir un mécanisme qui maximise le paiement de chaque agent, et par conséquent le surplus espéré du planificateur, parmi les mécanismes efficaces qui induisent la participation honnête. Leur modèle cependant ne prend pas en considération des environnements où les options extérieures sont endogènes comme c'est le cas dans cet essai. Jehiel et al (1996) ont prouvé que les options extérieures endogènes donnent au planificateur un outil additionnel pour augmenter son surplus espéré et pour induire la participation des agents : le planificateur peut décider comment punir un agent qui décide de ne pas participer. Dans ce contexte un mécanisme implique la description d'une règle d'attribution qui détermine l'alternative qui sera choisie si tous les agents participent, une règle de paiement, mais également "une menace" pour chaque agent au cas où il ne participerait pas.

Je combine les approches de Krishna et Perry (2000) et de Jehiel et autres (1996) pour concevoir un mécanisme qui maximise le surplus espéré (revenu) du concepteur parmi les mécanismes efficaces induisant la participation honnête de tous. Je considère une classe de mécanismes VCG plus large que celle de Krishna et de Perry (2000), et j'utilise des hypothèses moins fortes. Les mécanismes VCG sont désormais caractérisés non seulement par une base mais également par des menaces. Je prouve qu'un choix approprié de type max-min de la base et des menaces maximise le surplus espéré du planificateur parmi les mécanismes efficaces induisant la participation honnête. Je fournis un résultat d'existence pour des fonctions d'utilité extérieures pouvant être décomposées en deux composantes additives : une composante exogène et une composante endogène. J'applique alors ces résultats pour concevoir une enchère multiple efficace dans un environnement où un acheteur en possession du bien cause des externalités négatives sur les autres agents.

Dans la littérature liée à la conception de mécanismes avec des options extérieures endogènes, l'analyse s'est concentrée sur les ventes et sur la maximisation de revenu comme premier objectif. Le mécanisme optimal s'avère souvent inefficace (excepté dans le cas de l'information complète Jehiel et autres (1996)). Jehiel et autres (1996) travaille aussi en information incomplète mais pas avec des options extérieures qui dépendent du type des agents. Figueroa et Skreta (2009) considèrent un modèle avec des options extérieures dépendantes du type des agents ; ils prouvent que quelques fois le mécanisme maximisant le revenu est efficace et d'autres fois il n'est pas efficace. Dans cet essai l'efficacité est plus importante que la maximisation du revenu (le surplus) : l'objectif de revenu est limité aux mécanismes efficaces seulement. Cependant les deux approches se rencontrent chaque fois que le mécanisme maximisant le revenu est efficace. Une intuition qui nous vient de cette littérature est que le planificateur menacerait de minimiser l'utilité d'un agent qui ne participe pas par ce que son option extérieure est une limite potentielle au surplus du décideur ; cependant il peut induire la participation des agents en tendant à réduire leurs paiements s'ils participent au mécanisme. C'est la conjonction de ces deux forces qui mène au choix de type max-min de la base et des menaces dans mon modèle. D'ailleurs dans ce cadre la base peut être interprétée comme types présumés des agents quand ils ne participent pas et donc ne signale pas directement leurs types privés au planificateur.

Dans le deuxième essai nous considérons la règle du juste retour. Les appels d'offre de l'agence spatiale européenne (ASE) sont sujets à cette règle. La règle du juste retour assure à chaque état membre un retour proportionnel à sa contribution, sous forme de contrats attribués aux sociétés venant de cet état. Autrement dit les projets de l'ASE sont divisés en plus petits projets de sorte que les sociétés de tailles différentes et provenant des différents états membres puissent y participer. Un avantage de cette règle est que les sociétés ont l'occasion de partager leurs expériences, leur connaissances scientifiques et leurs technologies. Un autre avantage est d'inciter les états membres à contribuer aux activités de l'ASE. Dans la pratique la règle du juste retour est mise en application de sorte que le rapport entre la part d'un état en valeurs des contrats et sa contribution financière aux projets de l'agence ne soit pas inférieur à un seuil (le taux de retour). Quand ce seuil est 0.98 par exemple, une contribution de 1 euro garanti à un état au moins 0.98 euro sous forme de contrats a attribuer à des sociétés de cet état. Dans le meilleur des cas le taux de retour devrait être égal à 1. À côté de la règle du juste retour, l'ASE cherche également à favoriser la libre concurrence chaque fois que les deux principes ne sont pas en contradiction. La conception traditionnelle de la libre concurrence consiste à assigner des contrats aux sociétés qui font les plus basses offres indépendamment de leurs origines. L'exécution de la règle du juste retour requiert une période relativement longue (5 ans) à l'issue de laquelle chaque état membre devrait avoir un taux de retour dans les normes ; car il est pratiquement impossible d'assurer à chaque état membre un retour égal à sa contribution à tout moment ou à l'issue de chaque enchère. En bref, l'ASE applique la libre concurrence autant que possible et une révision est constamment faite afin d'ajuster les taux de retour si nécessaire. Ceci est fait en appliquant quelques règles particulières d'attribution dans les enchères restantes selon les taux de retour courant. Il y a eu bien des discussions sur l'utilisation de la règle du juste retour. Elle est plutôt acceptée par les petits états mais les états possédant des programmes spatiaux nationaux forts, voient dans son utilisation stricte un obstacle à la compétitivité et à la rentabilité. En outre, l'ASE et le Communauté Européenne travaillent ensemble afin de déterminer une politique spatiale européenne. Dans ce nouveau rapport, la question de la règle du juste retour est également discutée puisque le Communauté Européenne emploie une politique industrielle différente.

L'utilisation du juste retour a généré beaucoup de questions ; ce deuxième essai contribue à répondre à la suivante : les enchères traditionnelles sont-elles moins coûteuses pour l'ASE que la règle du juste retour ? Cette question résulte du fait que la règle du juste retour est (socialement) inefficace, dans le sens où la société ayant la plus basse offre ne gagne pas nécessairement le contrat. Nous faisons un premier pas dans l'analyse de cette question en utilisant un modèle réduit où un acheteur (ASE) cherche à acheter plusieurs articles à des fournisseurs potentiels (sociétés) de diverses origines (états membre de l'ASE). L'agence peut mettre en application la règle du juste retour ou la libre concurrence. Sous la libre concurrence, des contrats pour la fourniture de chaque article sont attribués indépendamment et les gagnants sont les meilleurs soumissionnaires (ceux avec les plus basses offres). Quoique la règle du juste retour soit dynamique dans son exécution, dans cet essai, nous en adoptons une version statique afin de garder les choses simples : selon la règle du juste retour les contrats sont attribués de sorte que tous les états membres soient représentés par les fournisseurs réels. Il est important de noter qu'une version dynamique de la règle du juste retour est faite en réalité d'une suite d'enchères statiques semblables à celle que nous adoptons ici mais différentes aussi par le nombre d'états réellement impliqués dans chaque enchère. Dans la pratique la priorité est accordée à certains états membres selon les taux de retour courants. Lorsque les taux de retour varient la priorité change. Nous considérons le cas de l'information complète où les niveaux de technologie des sociétés sont de notoriété publique, et le cas de l'information incomplète où les sociétés observent en privé leurs coûts de production. De même qu'il y a une multitude d'enchères qui assigneraient les contrats aux meilleurs soumissionnaires (ex. enchères au premier et deuxième prix), il y a plusieurs enchères possibles qui pour satisfaire la règle du juste retour. La plus évidente permettrait d'assigner les contrats à une équipe composée de sociétés venant de tous les états ayant la meilleure offre agrégée. Nous appelons cette enchère FR et la comparons à l'enchère de premier prix dans le cas de l'information complète. Dans le cadre de l'information incomplète nous concevons une enchère dans le genre de l'enchère de deuxième prix et comparons ce format d'enchère (appelé l'enchère SFR) à l'enchère de deuxième prix justement.

En présence d'asymétrie technologique, le conflit entre les petits et grands états peut être illustré comme suit : imaginez une situation où les fournisseurs de l'état l ont des coûts inférieurs pour la production des articles en comparaison avec leurs adversaires de l'état h ; sous l'enchère de premier prix les articles sont uniquement achetés des fournisseurs de l puisqu'ils ont plus de latitude à faire de basses offres. Par conséquent les fournisseurs de l'état h préféreront l'enchère FR à l'enchère de premier prix, contrairement aux fournisseurs de l'état l. D'ailleurs dans un tel contexte il est clair que, si les fournisseurs font les mêmes offres indépendamment de la politique industrielle utilisée, l'agence payerait un prix plus élevé sous l'enchère FR que sous des enchères des premiers prix. Plus généralement, même sans l'hypothèse d'asymétrie, nous pouvons parvenir à cette conclusion. En effet, sous des enchères de premier prix les contrats sont attribués aux plus bas soumissionnaires à un prix égal à ces offres ; mais sous l'enchère FR, les contrats sont attribués à un ensemble de fournisseurs avec la plus basse offre agrégée ; puisque certains fournisseurs de l'équipe gagnante peuvent ne pas avoir la plus basse offre de leur catégorie, le prix payé par l'agence sous l'enchère FR est plus élevé. Ceci étant, une réponse affirmative à la question initiale peut sembler évidente. Cependant, même si l'argument précédent est vrai, il repose sur l'hypothèse que les fournisseurs feraient les mêmes offres peu importe l'enchère en vigueur, ce qui n'est pas forcément le cas. En fait les offres des fournisseurs ne sont que le résultat de leurs différentes stratégies et, celles ci peuvent changer si les règles changent. Nous montrons que la libre concurrence peut parfois être plus coûteuse que la règle du juste retour. En particulier, et contrairement à l'argument précédent et à la première intuition, nous prouvons que ceci se produit souvent quand les fournisseurs d'un état donné sont chacun plus compétitifs que leur adversaire direct de l'autre état. Dans de telles conditions l'un des états possède un avantage technologique par rapport à l'autre dans la production de chaque article. L'intuition : selon la règle du juste retour, puisque les fournisseurs du même état ne peuvent pas gagner simultanément, un fournisseur compétitif est en concurrence non seulement avec le fournisseur du même article mais également avec le fournisseur du même état. Une situation qui force les fournisseurs compétitifs à être plus agressifs dans leurs offres sous la règle du juste retour que sous la libre concurrence, et résulte en un prix plus élevé pour l'agence.

Le dernier essai étudie la conception d'un mécanisme optimal pour un acheteur voulant acheter des articles hétérogènes et faisant face à des fournisseurs potentiels appartenant à différents groupes. Chaque fournisseur peut uniquement fournir un des articles. Les articles sont des compléments et l'acheteur désire les acheter en utilisant un mécanisme d'achat optimal c'est-à-dire, un mécanisme qui maximise son surplus. En outre l'acheteur fait face à la contrainte suivante : à chaque groupe d'agent est assigné des quotas déterminant le nombre maximal et minimal d'articles à acheter des fournisseurs de ce groupe. En parallèle, nous sommes également concernés par le mécanisme optimal quand l'acheteur n'est pas contraint.

Ce modèle pourrait s'appliquer aux deux exemples suivants. Tout d'abord, considérons un organisme gouvernemental désirant réaliser un projet divisé en sous-projets. Des contrats reliés à ces sous-projets sont attribués à des agents par le moyen d'un appel d'offre. Supposons que les participants sont des chercheurs de différentes provinces du pays. Un gouvernement voulant encourager la recherche dans toutes les provinces peut concevoir un mécanisme d'appel d'offre tel que les gagnants proviennent de toutes les provinces. Si l'agence attribue chaque contrat aux chercheurs qui font les plus basses offres, il y a des chances que les gagnants ne viendront pas de toutes les provinces. Quel mécanisme d'attribution l'agence devrait-elle choisir afin d'attribuer les contrats ? En second lieu, supposons qu'une institution internationale a plusieurs postes à remplir et fait face à des candidats venant de différents états. Si l'établissement est financé par ces états, alors l'établissement peut vouloir recruter de tous ces pays dans le but de favoriser l'intégration des différents états. Comment le processus de recrutement devrait-il être conduit ?

Revenant au modèle, on suppose que des fournisseurs ont des informations privées sur leurs coûts d'approvisionnement (coût d'accomplissement de la tâche dans le cas d'un recrutement) et ces coûts sont indépendants les uns des autres. Nous limitons notre attention aux mécanismes directs. Il s'agit de mécanismes dans lesquels chaque fournisseur est tenu de soumettre son information à l'acheteur avant que celui ci ne prenne sa décision. Sous de tels mécanismes, étant donné que les coûts sont connus de façon privée, les fournisseurs peuvent donner une fausse information sur leurs coûts s'il le juge avantageux. Nous nous intéressons aux mécanismes d'incitation c'est-à-dire, des mécanismes dans lesquels c'est un équilibre de rapporter honnêtement l'information privée. En outre nous nous concentrons sur des mécanismes satisfaisant la contrainte de participation : ce sont des mécanismes dans lesquels chaque fournisseur qui participe à l'appel d'offre s'attend à un bénéfice au moins égal à ce qu'il obtient s'il ne participe pas.

Nous généralisons les techniques de Myerson (1981) afin de d'obtenir un mécanisme optimal. Comme dans Myerson (1981) l'acheteur peut refuser d'acheter les articles s'il ne le juge pas avantageux. Branco (1996) caractérise l'enchère multiple optimale dans le cas d'articles homogènes, en revanche nous avons un modèle avec des articles hétérogènes. Des enchères multiples optimales avec des articles hétérogènes ont été largement étudiées. Dans ce type d'environnements chaque soumissionnaire peut habituellement faire concurrence pour plus d'un article. Un thème récurrent dans ces papiers est la question d'acheter les articles par paquets ou par plusieurs enchères séquentielles. Armstrong (2000), Jehiel et moldovanu (2001) ont montré que l'empaquetement est optimal dans le cas de deux articles. Levin (1997) a étudié l'enchère optimale pour des compléments et a prouvé qu'il est avantageux d'empaqueter. Une autre question importante est la dimension des informations disponibles aux soumissionnaires (leur type). La majeure partie de la littérature sur les enchères multiples optimales suppose que les types sont multidimensionnels et discrets (Armstrong 2000, Avery et Hendershott 2000). Nous considérons plutôt des types unidimensionnels et continus (les coûts unitaire des fournisseurs). Tous ces papiers ne considèrent pas des environnement où la liste des gagnants doit respecter des quotas pour chaque groupe de fournisseurs : le sujet de cet essai. Nous ne permettons pas aux fournisseurs d'offrir plus d'un article. Mais le fait que l'acheteur regarde les articles comme des compléments suggère la possibilité de les acheter dans une enchère commune. D'ailleurs la présence d'une contrainte qui assigne différentes quotas aux groupes de fournisseurs semble aussi en faveur d'une enchère commune. Naturellement il est possible d'acheter les articles dans un mécanisme se composant de beaucoup d'enchères séparées : nous appelons ce type de mécanismes "*itemwise*". Nous trouvons une condition nécessaire pour qu'un mécanisme itemwise soit optimal.

CHAPITRE 1

OPTIMAL THREATS AND EFFICIENT MECHANISM DESIGN

Abstract

This paper studies Bayesian efficient mechanism design in environments where agents' utility functions depend on the chosen alternative even if they do not participate to the mechanism. In addition to an allocation rule and a payment rule the designer may choose appropriate threats in order to give agents the incentive to participate and maximize his own expected surplus. The planner may presume the type of an agent who does not participate. I show that the solution of the design problem can be found by a maxmin choice of the presumed types and threats. I apply this to the design of an efficient multi-unit auction when a buyer in possession of the good causes negative externalities on other buyers.

1.1 Introduction

The problem of reaching a decision when a group of agents faces many alternatives affecting them differently is usually solved by the implementation of a mechanism. That is, a set of rules allowing to determine, on the basis of some information gathered from the agents (their types), which alternative will be chosen and what are the payments each agent is to make or receive. Examples of mechanisms include voting schemes and auctions. Agents are free to decide if they will participate to the mechanism. In most of the literature on mechanism design it is usually assumed that agents who do not participate to the mechanism have outside options that are independent of the outcome of the mechanism. [1]In some applications however agents who do not participate are still concerned by the chosen alternative. For example, if a government wants to sell a license for a technological innovation the winning firm will exert negative externality on the other firms. In fact whether a firm participates or not, its share in the market is likely to decrease as a

1. For a review on mechanism design see Jackson (2001) or Myerson (2006).

result of the license being sold to a competitor. The siting of noxious facilities is another example. In this problem agents have to decide who among them will be the host. The agents are concerned by the decision, whether they participate or not to the mechanism, if their utility functions depend on their distance to the site (see Ingberman 1995).

Outside options are said endogenous if the utility functions of agents who do not participate to the mechanism still depend on the chosen alternative. This paper studies efficient mechanism design in environments in which agents are privately informed and have endogenous outside options. The decision belongs to a planner who, beside the objective of efficiency, wants to maximize his own expected surplus and at the same time induce truthful participation of all agents.

Krishna and Perry (2000) generalize the VCG mechanisms by introducing a notion of basis. In their Theorem 1 they show that an appropriate choice of the basis results in a mechanism that maximizes the payment of each agent, and hence the expected surplus of the planner, among efficient mechanisms that induce truthful participation. Their model however does not take into account environments where outside options are endogenous as is the case in this paper. As shown in Jehiel et al (1996) endogenous outside options put in the planner's hand an additional tool to increase his expected surplus and induce the participation of the agents : the planner can decide what to do if a given agent decides to not participate. Thus a mechanism involves the description of an allocation rule which determines the alternative that will be chosen if all agents participate, a payment rule but also a sequence of "threats" to each agent in case he does not participate. I combine the approaches of Krishna and Perry (2000) and Jehiel et al (1996) to solve the announced problem. The class of VCG mechanisms I consider involves not only a basis but also threats.

I show that an appropriate maxmin choice of the basis and threats would maximize the expected surplus of the planner among efficient mechanisms inducing truthful participation. I provide an existence result for outside utility that may be decomposed into two additive components : an exogenous component and an endogenous component. I then apply these results to design an efficient multiunit auction for environments in which a buyer in possession of the good causes negative externalities on other buyers.

I show that a generalization of the Vickrey auction (see Vickrey 1961) maximizes the surplus among efficient mechanisms inducing truthful participation.

Weaker assumptions than those in Krishna and Perry (2000) are made. I do not restrict the study to convex type spaces, nor do I assume that the distributions of the payoff vectors are absolutely continuous. In particular, these assumptions would reduce the set of efficient allocation rules to a "singleton" as is suggested by the argument for theorem 1 in Krishna and Perry (2000) and is explicitly mentioned in Krishna (2002). Instead I directly assume that the "revenue equivalence" property holds and rely on the literature for much weaker sufficient conditions. The results of this paper thus generalize Krishna and Perry (2000).

In the literature of mechanism design with endogenous outside options the analysis has focused on sales and on revenue maximization as a primary objective (See, for instance, Jehiel et al 1996, Jehiel et al 1999, Figueroa and Skreta 2009). The optimal mechanism is often inefficient. Jehiel et al (1996) show that the optimal mechanism is efficient in the case of complete information. They also determine the optimal mechanism under incomplete information and show that it is inefficient. Their model does not allow the outside options of agents to depend on their own type. Figueroa and Skreta (2009) do consider a model with type dependent outside options and show that sometimes the revenue maximizing mechanism is efficient and other times it is not efficient. In the present paper efficiency is more important than revenue (surplus) maximization : the revenue objective is limited to efficient mechanisms only. Clearly, the two approaches meet whenever the revenue maximizing mechanism is efficient. An intuition that comes from this literature is that the planner would threaten to lower the utility of an agent who does not participate because his outside option is a potential limit to the planner's surplus (or revenue). However the planner can induce the participation of the agents by tending to reduce their payments if they participate to the mechanism. It is the conjunction of these two forces that leads to the maxmin choice of the basis and threats in my model. Moreover in this framework the basis may be interpreted as the presumed types of the agents when they do not participate and therefore do not reveal their private types to the planner.

The rest of the paper is organized as follow : the next section describes the model and presents a formal statement of the problem. Section 3 introduces the class of VCG mechanisms and shows that VCG mechanism maximizes the expected surplus of the planner among efficient mechanisms inducing truthful participation of all agents. In section 4, the case of outside utility with endogenous and/or exogenous components is studied. Section 5 presents an application to multi-unit auction design with negative externalities. Section 6 concludes the paper.

1.2 The model

There is a planner who wants to choose among many alternatives. The set of alternatives is denoted A and is finite. There is a set of agents $\mathcal{N} = \{1, 2, ..., n\}$ whose utility funcitons depend on the chosen alternative. Agent i's type is denoted t_i and is a random variable having values in the measurable space $(T_i, \mathcal{T}_i)$ and with a probability image denoted p_i. Denote $T = \underset{i\in\mathcal{N}}{\times} T_i$ and $\mathcal{T} = \underset{i\in\mathcal{N}}{\otimes} \mathcal{T}_i$. The common prior is the product measure $p = \underset{i\in\mathcal{N}}{\times} p_i$. Thus types are independently distributed. Each agent observes privately his own type. I will use the notation $\mathbb{E}Y(t)$ for the expectation of the random variable $Y(t)$ and $\mathbb{E}_{_{-i}}Y(t)$ for the expectation of $Y(t)$ taken over t_{-i}. Agent i's utility when alternative a is chosen is $v_i(a, t_i) - z_i$, where z_i is the payment made by the agent. Assume that $v_i(a, .)$ is integrable for any $a \in A$.

Hereafter we shall denote $\Delta(B)$ the set of probabilities over a finite set B.

An *allocation rule* is a measurable function $q = (q_a)_{a\in A} : T \to \Delta(A)$; $q_a(t)$ is the probability that alternative a is chosen. A *payment rule* is an integrable function $\mu : T \to \mathbb{R}^n$ where $\mu_i(t)$ is the payment made by agent i. A *threat* to agent i is a measurable function $\rho^i : (T_{-i}, \mathcal{T}_{-i}) \to \Delta(A_{-i})$, where $T_{-i} = \underset{j\neq i}{\times} T_j$, $\mathcal{T}_{-i} = \underset{j\neq i}{\otimes} \mathcal{T}_j$ and $A_{-i} \subset A$ denotes the set of available alternatives when agent i does not participate. $\rho^i_a(t_{-i})$ represents the probability that alternative a be chosen when agent i does not participate and the other agents report a type vector t_{-i}. I shall use the notation $\rho = (\rho^i)_{i\in\mathcal{N}}$. I also denote M_i the set of threats to agent i.

By the revelation principle I may focus on direct mechanisms. That is, mechanisms under which agents are asked to report their private information prior to the planner's decision. Formally a *direct mechanism* is a triplet (q,μ,ρ). If several agents do not participate to the mechanism the planner chooses any alternative. However this choice is irrelevant in the subsequent analysis since I am studying mechanisms that induce participation of all agents.

An allocation rule q^e is *(ex post) efficient* if for every type $t \in T$,

$$q^e(t) \in \arg\max_{q \in \Delta(A)} \sum_{i \in \mathcal{N}} \sum_{a \in A} v_i(a,t_i) q_a. \tag{1.1}$$

I shall denote by E the set of efficient allocation rules. An efficient allocation rule chooses the most (socially) valued alternative. A mechanism is *incentive compatible (IC)* if reporting his type honestly maximizes the expected utility of an agent when the other agents are also honest.

Agent i's expected utility when he reports the type r_i and his true type is t_i, is given by :

$$u_i(r_i,t_i;q,\mu) = \mathbb{E}_{-i}\left\{\sum_{a \in A} v_i(a,t_i) q_a(r_i,t_{-i}) - \mu_i(r_i,t_{-i})\right\}. \tag{1.2}$$

Therefore a mechanism is IC if

$$u_i(r_i,t_i;q,\mu) \leq u_i(t_i,t_i;q,\mu) \equiv U_i(t_i;q,\mu). \tag{1.3}$$

Denote $w_i(a,t)$ the utility of agent i if he does not participate to the mechanism and alternative a is chosen. I allow the non participation utility of each agent to depend on his own type and on other agents' types, and to be different from participation payoffs. This assumptions may be justified by the fact that the current mechanism is not the only source of utility (or disutility) for agents. This assumption is a departure from the literature on mechanisms with endogenous outside options in which equality between participation payoffs and outside utility is generally assumed. In the present model this would correspond to the case of "purely endogenous outside options" : $w_i(a,t) = v_i(a,t_i)$.

The expected utility of agent i if he does not participate (or simply his outside option) is given by the formula :

$$IR_i(t_i;\rho^i) = \mathbb{E}_{-i} \sum_{a\in A_{-i}} \rho^i_a(t_{-i}) w_i(a,t). \quad (1.4)$$

When $w_i(a,t)$ is independent of t_i the outside option is type independent : $IR_i = IR_i(\rho^i)$. And if $w_i(a,t)$ is independent of a the outside option is independent of the threat : $IR_i = IR_i(t_i)$. This last case corresponds to the situation where outside options are exogenous.

A mechanism is *individually rational* (IR) if $U_i(t_i;q,\mu) \geq IR_i(t_i;\rho^i)$ for any $i \in \mathscr{N}$ and $t_i \in T_i$. That is, the expected utility if an agent participates truthfully is at least equal to his outside option.

The *surplus of the planner* is the sum of agents' payments. The planner's objective is to design a mechanism that maximizes his expected surplus among efficient, IC and IR mechanisms.

The *revenue equivalence property* is satisfied if for any pair (q,μ,ρ) and (q,μ',ρ) of IC mechanisms, $\mathbb{E}_{-i}\mu_i(t_i,t_{-i}) - \mathbb{E}_{-i}\mu'_i(t_i,t_{-i})$ does not depend on t_i.

Throughout this paper I assume that the revenue equivalence property is satisfied. There is an important literature on sufficient conditions for the revenue equivalence property [2]. The sufficient conditions in Chung and Olszewski (2007) allow for type sets that may not be connected or bounded as well as prior that may not be absolutely continuous. Though they do not consider environments in which outside options are endogenous their result applies to this case provided the outside allocation rules are fixed, as is the case in the previous definition of the revenue equivalence property.

1.3 VCG mechanisms as surplus maximizing mechanism

1.3.1 Definition

Given an efficient allocation rule q, one may define the following functions :

2. See, for instance, Krishna and Maenner (2001), Milgrom and Segal (2002), Heydenreich et al (2007).

$$SW(t;q) = \sum_{i\in\mathcal{N}}\sum_{a\in A} v_i(a,t_i)q_a(t) \tag{1.5}$$

$$SW_i(t;q) = \sum_{a\in A} v_i(a,t_i)q_a(t) \tag{1.6}$$

$$SW_{-i}(t;q) = SW(t;q) - SW_i(t;q) \tag{1.7}$$

$SW(t;q)$ is the social welfare if the planner implements the allocation rule q when the vector of types is t. It is the sum of all the agents utility and the planner's surplus, i.e. the sum of expected payoffs of the agents. $SW_i(t;q)$ represents the payoff of agent i and $SW_{-i}(t;q)$ is the payoff of all agents but i. Note that SW is independent of q (for efficient allocation rules) but this is not true for SW_i or SW_{-i}.

Given an allocation rule and a profile of types $s = (s_i)_{i\in\mathcal{N}}$ one may define the payment rule $m(.;q,\rho|s)$ by :

$$m_i(t;q,\rho|s) = SW(s_i,t_{-i}) - SW_{-i}(t;q) - IR_i(s_i;\rho^i). \tag{1.8}$$

Using the fact that $SW(s_i,t_{-i}) = SW_i(s_i,t_{-i}) + SW_{-i}(s_i,t_{-i})$ we see that, under this payment rule, agent i is asked to pay the externality $(SW_{-i}(s_i,t_{-i}) - SW_{-i}(t;q))$ he causes on the other agents by revealing a type t_i when his true type is s_i, and also his gain $(SW_i(s_i,t_{-i};q) - IR_i(s_i;\rho^i))$ from participating (truthfully) to the mechanism when his type is s_i. It would be helpful to understand s_i as agent i's presumed type if he does not participate to the mechanism (agents' types are private information, thus if they do not participate the planner can only presume their types).

Given an efficient allocation rule q and an outside allocation rule ρ, the mechanism $(q,m(.;q,\rho|s),\rho)$ is called a VCG mechanism with basis s and threats ρ. Hereafter I use the simpler notation (q,s,ρ) to denote the mechanism $(q,m(.;q,\rho|s),\rho)$. The planner has the choice of ρ and s as well as q, and his choice is publicly declared prior to the implementation of the mechanism.

Due to the assumptions in this paper the efficient allocation rules may not be equal almost surely contrary to Krishna and Perry (2000). Thus, there are possibly many VCG mechanisms with the same basis s and threats ρ. Moreover because $SW_{-i}(t;q)$ is sensitive to the allocation rule so is agent i's payment $m_i(t;q,\rho|s)$; two allocation rules, different with positive probability, will give the same agent different expected payments under VCG mechanisms with the same basis and threats.

1.3.2 Properties of VCG mechanisms

By definition a VCG mechanism is efficient. This section shows that VCG mechanisms are incentive compatible and characterizes those that are individually rational.

Lemma 1. *VCG mechanisms are IC.* [3]

Proof. Indeed,

$$\begin{aligned}
& \left\{\sum_{a\in A} v_i(a,t_i)q_a(r_i,t_{-i}) - m_i(r_i,t_{-i};q,\rho|s)\right\} \\
= \; & \sum_{a\in A} v_i(a,t_i)q_a(r_i,t_{-i}) + SW_{-i}(r_i,t_{-i};q) - SW(s_i,t_{-i}) + IR_i(s_i;\rho^i) \\
\leq \; & SW(t_i,t_{-i}) - SW(s_i,t_{-i}) + IR_i(s_i;\rho^i) \\
= \; & \left\{\sum_{a\in A} v_i(a,t_i)q_a(t_i,t_{-i}) - m_i(t_i,t_{-i};q,\rho|s)\right\}
\end{aligned}$$

therefore

$$\mathbb{E}_{-i}\left\{\sum_{a\in A} v_i(a,t_i)q_a(r_i,t_{-i}) - m_i(r_i,t_{-i};q,\rho|s)\right\} \leq \mathbb{E}_{-i}\left\{\sum_{a\in A} v_i(a,t_i)q_a(t_i,t_{-i}) - m_i(t_i,t_{-i};q,\rho|s)\right\}.$$

□

The expected equilibrium utility of agent i is given by :

$$U_i(t_i;s_i,\rho^i) = \mathbb{E}_{-i}\left[SW(t) - SW(s_i,t_{-i}) + IR_i(s_i;\rho^i)\right]. \tag{1.9}$$

3. It is actually dominant strategy incentive compatible.

And in particular,

$$U_i(s_i; s_i, \rho^i) = IR_i(s_i; \rho^i). \tag{1.10}$$

Thus agent i is indifferent between participation and non participation if his type is equal to the basis s_i.

Let us define the function

$$K_i(s_i, \rho^i) = \mathbb{E}_{-i} SW(s_i, t_{-i}) - IR_i(s_i; \rho^i). \tag{1.11}$$

$K_i(s_i, \rho^i)$ represents the difference between the expected social welfare if agent i participates truthfully to the mechanism and his expected utility if he does not participate at all, when his type is s_i and the planner threatens to choose an alternative according to ρ^i. I may rewrite the equilibrium utility as :

$$U_i(t_i; s_i, \rho^i) = \mathbb{E}_{-i} SW(t) - K_i(s_i, \rho^i). \tag{1.12}$$

This expression shows that K_i may be viewed as *the disutility of agent i* if he participates truthfully to a mechanism with basis s_i and threat ρ^i. This disutility is a consequence of the choice of the basis and threat. The planner has a total control on the disutility through s_i and ρ^i. Once an efficient allocation rule has been chosen the planner chooses s and ρ in order to induce truthful participation of the agents while seeking to maximize his own expected surplus. The expression (1.12) implies that K_i is also the expected social welfare, from agent i's perspective, if he is not considered as part of the society. This expected social welfare ($\mathbb{E}_{-i} SW(t) - U_i(t_i; s_i, \rho^i)$) is therefore independent of his true type t_i.

For $i \in \mathcal{N}$, I consider the following set :

$$S_i(\rho^i) \equiv \arg\min_{t_i \in T_i} K_i(t_i, \rho^i). \tag{1.13}$$

$S_i(\rho^i)$ is simply the set of minimizers of the function $K_i(., \rho^i)$. I will say that $S_i(\rho^i)$ is nonempty to mean that $K_i(., \rho^i)$ has a minimum. Let $S(\rho) \equiv \times_{i \in \mathcal{N}} S_i(\rho^i)$; I will say that S

is nonempty valued to mean that $K_i(.,\rho^i)$ has a minimum for every agent i and for every vector of threats ρ.

The following lemma characterizes individual rationality.

Lemma 2. *A VCG mechanism (q,s,ρ) is IR if and only if $s \in S(\rho)$.*

Proof. Using (1.9) and (1.13) one can see that $U_i(t_i;q,\rho^i) \geq IR_i(t_i;\rho^i)$ is equivalent to $\mathbb{E}_{-i}SW(t) - \mathbb{E}_{-i}SW(s_i,t_{-i};q) + IR_i(s_i;\rho^i) \geq IR_i(t_i;\rho^i)$, i.e. to $K_i(t_i,\rho^i) \geq K_i(s_i,\rho^i)$. □

1.3.3 Results

The next theorem shows that an appropriate choice of the basis and threats maximizes the planner's surplus among efficient, IC and IR mechanisms, when S is nonempty valued.

Theorem 1. *Assume that for every $i \in \mathcal{N}$:*

(C_1) S_i is non empty valued,

(C_2) $s_i^* \in S_i(\rho^{i*})$,

(C_3) $\rho^{i*} \in \arg\max_{\rho^i \in M_i} \inf_{s_i \in T_i} K_i(s_i,\rho^i)$.

Let $\rho^* = (\rho^{i*})_{i\in\mathcal{N}}$ and $s^* = (s_i^*)_{i\in\mathcal{N}}$. Then for any $q^* \in E$ the VCG mechanism (q^*,s^*,ρ^*) maximizes the expected surplus among efficient, IC and IR mechanisms. The maximal surplus is equal to :

$$\sum_{i\in\mathcal{N}} K_i(s_i^*,\rho^{i*}) - (n-1)\mathbb{E}SW(t).$$

This shows that it is optimal to choose the basis and threats so that : (C_2) the basis minimizes the disutility from a truthful participation of any agent given the optimal threats and also, (C_3) the minimal disutility of an agent under the optimal threat is at least equal to the minimal disutility under any other threat. (C_2) ensures that the mechanism is IR and (C_3) is meant to maximize the surplus of the planner. The planner wants to maximize the disutility of the agents because it would increases his own surplus ; and he wants to minimize it in order to give agents incentives to participate.

Proof. Let (q,μ,ρ) be efficient, *IC* and *IR*. Take $s \in T$ and let $m = m(.;q,\rho|s)$. By the revenue equivalence property we have :

$$\begin{aligned}
\mathbb{E}_{-i}\mu_i(t_i,t_{-i}) - \mathbb{E}_{-i}m_i(t_i,t_{-i};q,\rho|s) &= \mathbb{E}_{-i}\mu_i(s_i,t_{-i}) - \mathbb{E}_{-i}m_i(s_i,t_{-i};q,\rho|s) \\
&= \mathbb{E}_{-i}u_i(s_i,t_{-i};q,m) - \mathbb{E}_{-i}u_i(s_i,t_{-i};q,\mu) \\
&= IR_i(s_i;\rho^i) - \mathbb{E}_{-i}u_i(s_i,t_{-i};q,\mu) \leq 0.
\end{aligned}$$

Therefore

$$\mathbb{E}\sum_{i\in\mathcal{N}} m_i(t;q,\rho|s) \geq \mathbb{E}\sum_{i\in\mathcal{N}} \mu_i(t).$$

That is : *an efficient, IC and IR mechanism* (q,μ,ρ) *results in an expected surplus weakly lower than any VCG mechanism with the same allocation rule.*

Finally, for $s \in S(\rho)$ we have :

$$\begin{aligned}
\mathbb{E}\sum_{i\in\mathcal{N}} m_i(t;q,\rho|s) &= \mathbb{E}\left\{\sum_{i\in\mathcal{N}}\left[SW(s_i,t_{-i}) - IR_i(s_i;\rho^i)\right] - (n-1)SW(t)\right\} \\
&= \sum_{i\in\mathcal{N}}\left[\mathbb{E}_{-i}SW(s_i,t_{-i}) - IR_i(s_i;\rho^i)\right] - (n-1)\mathbb{E}SW(t) \\
&= \sum_{i\in\mathcal{N}}\min_{s_i'\in T_i}\left[\mathbb{E}_{-i}SW(s_i',t_{-i}) - IR_i(s_i';\rho^i)\right] - (n-1)\mathbb{E}SW(t)\ (s\in S(\rho)) \\
&\leq \sum_{i\in\mathcal{N}}\min_{s_i'\in T_i}\left[\mathbb{E}_{-i}SW(s_i',t_{-i}) - IR_i(s_i';\rho^{i*})\right] - (n-1)\mathbb{E}SW(t)\ (\text{by } (C_3)) \\
&= \sum_{i\in\mathcal{N}}\left[\mathbb{E}_{-i}SW(s_i^*,t_{-i}) - IR_i(s_i^*;\rho^{i*})\right] - (n-1)\mathbb{E}SW(t) \\
&= \mathbb{E}\sum_{i\in\mathcal{N}} m_i(t;q^*,\rho^*|s^*).
\end{aligned}$$

□

It is shown in krishna and Perry (2000) that a VCG mechanism maximizes the expected payment of each agent. Beside the revenue equivalence property, their result also relies on the fact that efficient allocation rules must be equal almost surely. This last condition follows from their assumption and is clearly suggested by their argument for the theorem (see also Krishna 2002, P. 76-77). This result no longer holds in situations where there exist efficient allocations rules that are different with positive probabilities. For instance, this will be the case if type sets are discrete. However the expected surplus may still be maximized provided the revenue equivalence property holds.

A *saddle point* of the function K_i is a couple $(\bar{s}_i, \bar{\rho}^i)$ such that

$$K_i(\bar{s}_i, \rho^i) \leq K_i(\bar{s}_i, \bar{\rho}^i) \leq K_i(s_i, \bar{\rho}^i), \forall (s_i, \rho^i) \in T_i \times M_i \tag{1.14}$$

or equivalently

$$\inf_{s_i \in T_i} \sup_{\rho^i \in M_i} K_i(s_i, \rho^i) = K_i(\bar{s}_i, \bar{\rho}^i) = \sup_{\rho^i \in M_i} \inf_{s_i \in T_i} K_i(s_i, \rho^i). \tag{1.15}$$

As consequence of the theorem, the mechanism design problem may be solved by a search for saddle points of the functions K_i if they exist.

Proposition 1. *Assume that S is non empty valued and K_i possess a saddle point for every agent i. Then for any $q^* \in E$ the sufficient conditions (C_2) and (C_3) of the theorem are satisfied if and only if (s_i^*, ρ^{i*}) is a saddle point of K_i.*

Proof. when S_i is nonempty valued we have $\inf_{s_i \in T_i} K_i(s_i, \rho^i) = \min_{s_i \in T_i} K_i(s_i, \rho^i)$. Since K_i has at least one saddle point then $\inf_{s_i \in T_i} \sup_{\rho^i \in M_i} K_i(s_i, \rho^i) = \sup_{\rho^i \in M_i} \inf_{s_i \in T_i} K_i(s_i, \rho^i)$. If (s_i^*, ρ^{i*}) satisfies (C_2) and (C_3) then $K_i(s_i^*, \rho^{i*}) = \min_{s_i \in T_i} K_i(s_i, \rho^{i*}) \geq \min_{s_i \in T_i} K_i(s_i, \rho^i), \forall \rho^i$. Therefore $K_i(s_i^*, \rho^{i*}) = \max_{\rho^i \in M_i} \min_{s_i \in T_i} K_i(s_i, \rho^i)$ and (s_i^*, ρ^{i*}) is a saddle point. Now assume $(\bar{s}_i, \bar{\rho}^i)$ is a saddle point of K_i and S_i is nonempty valued. then by definition $K_i(\bar{s}_i, \bar{\rho}^i) = \min_{s_i \in T_i} K_i(s_i, \bar{\rho}^i)$ and $K_i(\bar{s}_i, \bar{\rho}^i) = \sup_{\rho^i \in M_i} \inf_{s_i \in T_i} K_i(s_i, \rho^i)$, i.e. (C_2) and (C_3) hold. □

Let us consider the following condition :

(C_4) for a.e $t_{-i} \in T_{-i}$, if $w_i(a, s_i^*, t_{-i}) > \min_{a' \in A_{-i}} w_i(a', s_i^*, t_{-i})$ then $\rho_a^{i*}(t_{-i}) = 0$.

The condition (C_4) means that the planner's threat is to randomly choose an alternative that would generate the worst non participation utility to agent i if his type was s_i^*. It is important to remember that the planner does not observe private types and if agent i does not participate it is not revealed. Under (C_4) the planner acts as though agent i's type was s_i^*. Hence the planner would choose only alternatives that generate the worst utility to agent i with presumed type s_i^*.

Proposition 2. *The conditions* (C_2) *and* (C_4) *characterize the saddle points of* K_i*.*

Proof. Assume that (s_i^*,ρ^{i*}) satisfies (C_2) and (C_4).

For any (s_i,ρ^i),

$$\begin{aligned}
K_i(s_i,\rho^{i*}) &\geq \inf_{t_i\in T_i}\mathbb{E}_{-i}\sum_{a\in A_{-i}}\left[\rho_a^{i*}(t_{-i})\left(SW(t)-w_i(a,t)\right)\right] \\
&= K_i(s_i^*,\rho^{i*}) \text{ (by } (C_2)) \\
&\geq \mathbb{E}_{-i}\sum_{a\in A_{-i}}\left[\rho_a^{i}(t_{-i})\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right)\right] \text{ (by } (C_4)) \\
&= K_i(s_i^*,\rho^{i}).
\end{aligned}$$

Now let (s_i^*,ρ^{i*}) be a saddle point.

By definition $\rho^{i*}\in \arg\max_{\rho^i\in M_i}\inf_{s_i\in T_i}K_i(s_i,\rho^i)$ and $K_i(s_i^*,\rho^{i*})=\inf_{t_i\in T_i}K_i(t_i,\rho^{i*})$ i.e. $s_i^*\in S_i(\rho^{i*})$. I need only show that (C_4) is satisfied. First note that

$$\sum_{a\in A_{-i}}\left[\rho_a^{i*}(t_{-i})\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right)\right]\leq \max_{a\in A_{-i}}\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right).$$

(s_i^*,ρ^{i*}) is a saddle point therefore $K_i(s_i^*,\rho^{i*})\geq K_i(s_i^*,\rho^i)\ \forall\rho^i$. Choosing ρ^i such that

$$\sum_{a\in A_{-i}}\left[\rho_a^{i}(t_{-i})\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right)\right]= \max_{a\in A_{-i}}\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right)\forall t_{-i},$$

I obtain :

$$\mathbb{E}_{-i}\sum_{a\in A_{-i}}\left[\rho_a^{i*}(t_{-i})\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right)\right]= \mathbb{E}_{-i}\max_{a\in A_{-i}}\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right)$$

and

$$\sum_{a\in A_{-i}}\left[\rho_a^{i*}(t_{-i})\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right)\right]=\max_{a\in A_{-i}}\left(SW(s_i^*,t_{-i})-w_i(a,s_i^*,t_{-i})\right)\text{ for a.e }t_{-i}.$$

This implies (C_4). □

Proposition 2 characterizes the saddle points by the conditions (C_2) and (C_4). In general these two conditions are interdependent and constitute an implicit characterization of the saddle points. In some cases however the correspondence S_i may be independent of the threat ρ^i. For example when the non participation utility of an agent is independent of his own type, or if outside options are exogenous (i.e. independent of the chosen alternative) or finally, in case of monotonicity, when type sets are ordered and $S_i(\rho^i)$ is an extremum of the set T_i. In these situations s_i^* is independently determined and the threats are just irrelevant :$s_i^*\in\arg\max_{t_i\in T_i}K_i(t_i)$. This is the optimal choice of the basis in Krishna and Perry (2000).

1.4 The case : $w_i(a,t)\equiv\bar{v}_i(a,t_i)+\hat{v}_i(t)$

In this case each agent's non participation payoffs have an endogenous (own-type dependent) component and an exogenous component that may depend on all agents' types. For example the case of purely endogenous outside options $(w_i(a,t)\equiv v_i(a,t_i))$ corresponds to the situation where the payoffs of an agent who participates to the mechanism equals his utility if he does not participate.

The expected outside option of agent i is :

$$\begin{aligned}IR_i(t_i;\rho^i) &= \mathbb{E}_{-i}\sum_{a\in A_{-i}}\rho_a^i(t_{-i})\bar{v}_i(a,t_i)+\mathbb{E}_{-i}\hat{v}_i(t)\\ &= \sum_{a\in A_{-i}}\bar{v}_i(a,t_i)\mathbb{E}_{-i}\rho_a^i(t_{-i})+\mathbb{E}_{-i}\hat{v}_i(t)\\ &= \sum_{a\in A_{-i}}\bar{v}_i(a,t_i)\bar{\rho}_a^i+\mathbb{E}_{-i}\hat{v}_i(t),\end{aligned}$$

where $\mathbb{E}_{-i}\rho^i=\bar{\rho}^i\in\Delta(A_{-i})$. Therefore $IR_i(t_i;\rho^i)$ depends on ρ^i only through its mean

$\bar{\rho}^i \in \Delta(A_{-i}) : IR_i(t_i;\rho^i) \equiv IR_i(t_i;\bar{\rho}^i)$. Hence the optimal threats can be chosen constant and M_i may be identified with $\Delta(A_{-i})$.

The disutility of agent i for a basis t_i and a threat ρ^i is :

$$K_i(t_i,\rho^i) = \mathbb{E}_{-i}\left[SW(t) - \hat{v}_i(t)\right] - \sum_{a\in A_{-i}} \bar{v}_i(a,t_i)\rho^i_a. \tag{1.16}$$

The following result gives some sufficient conditions for the existence of optimal threats and basis.

Proposition 3. *Assume that for every $i \in \mathcal{N}$:*

(C_5) T_i is a compact convex metric space,

(C_6) $\forall a \in A_{-i}$, $\forall t_{-i} \in T_{-i}$, $SW(.,t_{-i}) - w_i(a,.,t_{-i})$ is lower semicontinuous,

(C_7) for a.e $t_{-i} \in T_{-i}$ $SW(.,t_{-i}) - \max_{a\in A_{-i}} w_i(a,.,t_{-i}) \geq -\theta_i(t_{-i})$, where θ_i is non negative and integrable.

(C_8) $t_i \mapsto \mathbb{E}_{-i}\left[SW(t) - \hat{v}_i(t)\right]$ is quasiconvex and $\forall a \in A_{-i}$ $\bar{v}_i(a,.)$ is concave.

Then K_i has a saddle point (s^*_i,ρ^{i*}) for every $i \in \mathcal{N}$. Let $\rho^* = (\rho^{i*})_{i\in\mathcal{N}}$ and $s^* = (s^*_i)_{i\in\mathcal{N}}$. Then for any $q^* \in E$ the VCG mechanism (q^*,s^*,ρ^*) maximizes the expected surplus among efficient, IC and IR mechanisms.

Proof. $K_i(s_i,.)$ is continuous and linear, and $\Delta(A_{-i})$ is convex and compact in the euclidean topology. (C_8) implies that $K_i(.,\rho^i)$ is quasiconvex $\forall \rho^i \in \Delta(A_{-i})$. By Sion's minimax theorem I need only show that $K_i(.,\rho^i)$ is lower semicontinuous to conclude to the existence of a saddle point. Consider a sequence t^n_i converging to t_i.

$$\begin{aligned}
(C_6) \quad &\Rightarrow \quad \rho^i_a(t_{-i})(SW(t) - w_i(a,t)) \leq \liminf \rho^i_a(t_{-i})(SW(t^n_i,t_{-i}) - w_i(a,t^n_i,t_{-i})), \forall a \in A_{-i}, \forall t_{-i} \in T_{-i}\\
&\Rightarrow \quad \sum_{a\in A_{-i}} \rho^i_a(t_{-i})(SW(t) - w_i(a,t)) \leq \sum_{a\in A_{-i}} \liminf \rho^i_a(t_{-i})(SW(t^n_i,t_{-i}) - w_i(a,t^n_i,t_{-i})), \forall t_{-i} \in T_{-i}\\
&\Rightarrow \quad \sum_{a\in A_{-i}} \rho^i_a(t_{-i})(SW(t) - w_i(a,t)) \leq \liminf \sum_{a\in A_{-i}} \rho^i_a(t_{-i})(SW(t^n_i,t_{-i}) - w_i(a,t^n_i,t_{-i})), \forall t_{-i} \in T_{-i}\\
&\Rightarrow \quad \mathbb{E}_{-i} \sum_{a\in A_{-i}} \rho^i_a(t_{-i})(SW(t) - w_i(a,t)) \leq \mathbb{E}_{-i} \liminf \sum_{a\in A_{-i}} \rho^i_a(t_{-i})(SW(t^n_i,t_{-i}) - w_i(a,t^n_i,t_{-i}))\\
&\Rightarrow \quad \mathbb{E}_{-i} \sum_{a\in A_{-i}} \rho^i_a(t_{-i})(SW(t) - w_i(a,t)) \leq \liminf \mathbb{E}_{-i} \sum_{a\in A_{-i}} \rho^i_a(t_{-i})(SW(t^n_i,t_{-i}) - w_i(a,t^n_i,t_{-i}))
\end{aligned}$$

Where the last implication follows from the extended Fatou's lemma. Therefore $K_i(.,\rho^i)$ is lower semicontinuous. Since T_i is compact I may conclude that $S_i(\rho^i) = \arg\min_{t_i \in T_i} K_i(t_i,\rho^i) \neq \emptyset$. Therefore all the conditions of theorem 1 are satisfied. □

$SW(t) - w_i(a,t)$ is the difference between social welfare if agent i participates and his utility if he does not participate and alternative a is chosen. (C_7) means that this difference has a non positive lower bound that is independent on the chosen alternative. It is satisfied if payoffs and utilities are bounded. Condition (C_6) is satisfied if the functions $w_i(a,.,t_{-i})$ are continuous and the functions $v_i(a,.)$ are lower semicontinuous.

As mentioned earlier the conditions (C_2) and (C_4) may be used to determine the optimal threats and basis. This can be done either analytically or numerically. In particular when K_i is smooth, first and second order conditions for optimization may be used for condition (C_2). Below I propose an algorithm for approximating the threat and the basis.

Algorithm 1. *pick* $\rho^{i0} \in \Delta(A_{-i})$,
$\forall l \geq 0$ *let* $s_i^l \in \arg\min_{t_i \in T_i} K_i(t_i,\rho^{il})$,
$\rho^{il+1} \in \Delta(A_{-i}) : \rho_a^{il+1} = 0$ *if* $\bar{v}_i(a,s_i^l) > \min_{a \in A_{-i}} \bar{v}_i(a',s_i^l)$.

Proposition 4. *Suppose that* $\forall a \in A_{-i}$ $\bar{v}_i(a,.)$ *is continuous. Then under the conditions of proposition 3 the sequence* $(s_i^l,\rho^{il})_{l\geq 0}$ *possess at least one cluster point and every cluster point is a saddle point of* K_i.

Proof. Since $T_i \times \Delta(A_{-i})$ is compact the sequence $(s_i^l,\rho^{il})_{l\geq 0}$ possess a convergent subsequence $(s_i^{l_p},\rho^{il_p})_{p\geq 0}$ with limit $(s_i^*,\rho^{i*}) \in T_i \times \Delta(A_{-i})$. $K_i(s_i,.)$ is continuous and $K_i(.,.)$ is lower semicontinuous. Leininger's maximum theorem implies that $\arg\min_{t_i \in T_i} K_i(t_i,.)$ is upper hemicontinuous and therefore (C_2) is satisfied : $s_i^* \in \arg\min_{t_i \in T_i} K_i(t_i,\rho^{i*})$. If $\bar{v}_i(a,s_i^*) > \bar{v}_i(a',s_i^*)$ then for $n \geq n_0$ we also have $\bar{v}_i(a,s_i^l) > \bar{v}_i(a',s_i^l)$ and $\rho_a^{il+1} = 0$. Taking the limit I obtain $\rho_a^{i*} = 0$: (C_4) is satisfied. the conclusion follows from proposition 2. □

1.5 Application : multi-unit auctions

1.5.1 The market

I consider a seller of m units of an indivisible good who wants to allocate the units efficiently but is also interested in maximizing his expected surplus. In this application the agents are the potential buyers. Buyer i's type is $t_i = (\pi_i, \delta_i)$ where $\pi_i = (\pi_i^l)_{l=1,\dots,m}$ and $\delta_i = (\delta_i^j)_{j\in\mathcal{N}-\{i\}}$. π_i^l is the marginal valuation of the l^{th} unit for buyer i and δ_i^j is the marginal disutility of buyer i when buyer j possesses some units of the good. I assume that marginal valuations are decreasing with the number of units possessed by a buyer. Buyer i's type set is :

$$T_i = \left\{ t_i \in \mathbb{R}_+^{m+n-1} : \pi_i^m \leq \pi_i^{m-1} \leq \dots \leq \pi_i^1 \leq \bar{\pi}_i \text{ and } \delta_i^j \leq \bar{\delta}_i \; \forall j \in \mathcal{N} - \{i\} \right\}.$$

An alternative is a vector $a = (a_i)_{i\in\mathcal{N}}$ where a_i represents the number of units sold to agent i. The set of pure alternatives is $A = \left\{ a \in \mathbb{N} : \sum_{i\in\mathcal{N}} a_i \leq m \right\}$. The payoff functions are

$$v_i(a,t_i) = \sum_{l=1}^{a_i} \pi_i^l - \sum_{j\in\mathcal{N}-\{i\}} \delta_i^j a_j. \tag{1.17}$$

I assume that outside options are purely endogenous so that $w_i(a,t) = v_i(a,t_i)$. The set of available alternatives when buyer i does not participate is $A_{-i} = \{a \in A : a_i = 0\}$.

Let us denote $\delta_+^j = \sum_{i\in\mathcal{N}-\{j\}} \delta_i^j$ the marginal disutility buyer j exerts on all the other buyers when he is allocated the good. I call $\pi_i^l - \delta_+^i$ the relative bid of buyer i for the l^{th} unit it is the difference between his valuation for the l^{th} unit and the marginal disutility he exerts on all the other buyers.

The set T_i is convex and the function $v_i(a,t_i)$ is convex in t_i ; the Hypothesis I in Krishna and Maenner (2001) is satisfied. Therefore the revenue equivalence property is satisfied.

1.5.2 Surplus maximizing auction

The efficient social welfare is given by $SW(t) = \max_{a\in A} \sum_{i\in\mathcal{N}} v_i(a,t_i)$.

$$\sum_{i\in\mathscr{N}} v_i(a,t_i) = \sum_{i\in\mathscr{N}}\sum_{l=1}^{a_i}\pi_i^l - \sum_{j\in\mathscr{N}}\sum_{i\in\mathscr{N}-\{j\}}\delta_i^j a_j = \sum_{i\in\mathscr{N}}\sum_{l=1}^{a_i}\pi_i^l - \sum_{i\in\mathscr{N}}\delta_+^i a_i = \sum_{i\in\mathscr{N}}\sum_{l=1}^{a_i}\left(\pi_i^l - \delta_+^i\right)$$

This last expression shows that *an efficient allocation rule is to allocate an object to the positive relative bids among the m highest.*

I shall denote a^* the efficient allocation rule just described. Formally the following conditions characterize the allocation rule $a^*(t)$.

$$\begin{array}{lc} (E_1) & \forall i\in\mathscr{N}, a_i^*>0 \Rightarrow \pi_i^{a_i^*}-\delta_+^i>0 \\ (E_2) & \forall i\in\mathscr{N}: a_i^*>0, \forall j\in\mathscr{N}: j\neq i, \pi_i^{a_i^*}-\delta_+^i \geq \pi_j^{a_j^*+1}-\delta_+^j \\ (E_3) & \sum_{i\in\mathscr{N}} a_i^* < m \Rightarrow \forall j\in\mathscr{N}, 0\geq \pi_j^{a_j^*+1}-\delta_+^j \end{array}$$

(E_1) and (E_2) means that units are allocated only to the highest positive relative bids. (E_3) means that it is efficient to allocate additional units as long as relative bids are positive.

For every $i\in\mathscr{N}$ and $a\in A$, $v_i(a,.)$ is continuous bounded and affine. Therefore the conditions $(C_5)-(C_8)$ are satisfied. Proposition 3 implies that for every $i\in\mathscr{N}$ there exist an optimal couple of basis and threat (s_i^*,ρ^{i*}).

If he does not participate to the mechanism, buyer i expects a payoff equal to $IR_i(s_i^*;\rho^{i*}) = \min_{a\in A_{-i}} v_i(a,s_i^*)$ (by condition (C_4)).

The payment rule is given by :

$$m_i(t;a^*,\rho^*|s^*) = SW(s_i^*,t_{-i}) - SW_{-i}(t;a^*) - \min_{a\in A_{-i}} v_i(a,s_i^*). \tag{1.18}$$

$SW(s_i^*,t_{-i})$ is the some of the positive relative bids among the m highest if buyer i had type s_i^*. $SW_{-i}(t;a^*)$ is the some of the positive relative bids among the m highest except those of buyer i when he reveals the type t_i.

Hence the efficient mechanism generalizes the Vickrey auction (see Vickrey 1961) which corresponds to the case $s_i^* = 0$.

1.5.3 Absolutely continuous prior

In the rest of this section I assume that hyperplanes in the space of $(\delta_j)_{j\neq i}$ have measure zero with respect to μ_{-i} for every $i \in \mathscr{N}$. This is satisfied if the probabilities μ_i are absolutely continuous with respect to the lebesgue measure. Given that the functions $v_i(a,.)$ are differentiable, I may conclude that the social welfare $SW(t) = \max_{a\in A} \sum_{i\in\mathscr{N}} v_i(a,t_i)$ is differentiable almost surely and $\mathbb{E}_{-i}[SW(t)]$ is differentiable.

Fix $i \in \mathscr{N}$, for $j \neq i$ we have :

$$\begin{aligned}
\frac{\partial}{\partial \delta_i^j}\mathbb{E}_{-i}[SW(t)] &= \mathbb{E}_{-i}\left[\frac{\partial}{\partial \delta_i^j} SW(t)\right] \\
&= \mathbb{E}_{-i}\left[\frac{\partial}{\partial \delta_i^j}\sum_{a\in A}\left(\sum_{k\in\mathscr{N}} v_k(a,t_k)\right).1(a = a^*(t))\right] \\
&= \mathbb{E}_{-i}\left[\sum_{a\in A}\left(\frac{\partial}{\partial \delta_i^j}\sum_{k\in\mathscr{N}} v_k(a,t_k)\right).1(a = a^*(t))\right] \\
&= \mathbb{E}_{-i}\left[\sum_{a\in A}(-a_j).1(a = a^*(t))\right],
\end{aligned}$$

and finally :

$$\frac{\partial}{\partial \delta_i^j}\mathbb{E}_{-i}[SW(t)] = -\sum_{a\in A} a_j.\Pr(a = a^*(t)|t_i). \tag{1.19}$$

The expected disutility of agent i if he participates to the mechanism under the basis t_i and threat ρ^i is given by :

$$K_i(t_i,\rho^i) = \mathbb{E}_{-i}[SW(t)] - \sum_{a\in A_{-i}} \bar{v}_i(a,t_i)\rho_a^i = \mathbb{E}_{-i}[SW(t)] + \sum_{a\in A_{-i}}\sum_{j\in\mathscr{N}-\{i\}} \delta_i^j a_j \rho_a^i \tag{1.20}$$

It is important to keep in mind that this disutility is caused by the choice of the basis and threat by the planner and is different from the disutility caused by other buyers who possess the good.

Equations 1.19 and 1.20 imply that the marginal expected disutility of agent i (when he participates) with respect to δ_i^j, under the basis t_i and threat ρ^i is given by :

$$\frac{\partial}{\partial \delta_i^j} K_i(t_i,\rho^i) = \sum_{a\in A_{-i}} a_j \rho_a^i - \sum_{a\in A} a_j.\Pr(a = a^*(t)|t_i). \tag{1.21}$$

This is just the difference between the expected number of units agent j receives when agent i does not participate and when he participates with type t_i.

Thus, from the perspective of agent i, if he wants to participate, a slight decrease in the (marginal) effect of any other agent is desirable as long as agent i currently expects that agent to obtain more units if agent i decided to not participate.

Note also that the non participation utility of buyer i is $v_i(a,t_i) = -\sum_{j\in\mathcal{N}-\{i\}} \delta_i^j a_j$. It is minimized if all the units are allocated only to buyers causing the highest marginal effect to buyer i, and the minimal value is $-\max_{j\in\mathcal{N}-\{i\}} m\delta_i^j$. I may write the following proposition.

Proposition 5. *Let $\rho^{i*} \in \Delta(A_{-i})$, $\pi_i^* = [0,\bar{\pi}_i]$ and $\delta_i^* \in \left[0,\bar{\delta}_i\right]^{n-1}$. If the prior is absolutely continuous, then $((\pi_i^*,\delta_i^*),\rho^{i*})$ is an optimal choice of the basis and threat if and only if :*

(1) $\pi_i^* = 0$

(2) $\rho_a^{i*} = 0 \; \forall a \in A_{-i} : \sum_{j\in\mathcal{N}-\{i\}} \delta_i^{*j} a_j < \max_{j\in\mathcal{N}-\{i\}} m\delta_i^{*j}$

(3) $\forall j \in \mathcal{N}-\{i\}$,

$$\begin{cases} \sum_{a\in A_{-i}} a_j\rho_a^{i*} = \sum_{a\in A} a_j.\Pr(a=a^*(t)|t_i=(0,\delta_i^*)) & if\ \delta_i^{*j} \in \left(0,\bar{\delta}_i\right) \\ \sum_{a\in A_{-i}} a_j\rho_a^{i*} \geq \sum_{a\in A} a_j.\Pr(a=a^*(t)|t_i=(0,\delta_i^*)) & if\ \delta_i^{*j} = 0 \\ \sum_{a\in A_{-i}} a_j\rho_a^{i*} \leq \sum_{a\in A} a_j.\Pr(a=a^*(t)|t_i=(0,\delta_i^*)) & if\ \delta_i^{*j} = \bar{\delta}_i \end{cases}$$

Proposition 5 shows how the planner may set the basis and threats. (1) He may presume that agents who do not participate do not value the good at all. (2) The object is allocated only to agents who would affect agent i the most if he does not participate and his type is correctly presumed by the planner (i.e. is equal to the basis). (3) If the presumed (marginal) external effect on some agent i is interior then the planner threatens to ensure that, if agent i does not participate, the expected number of units allocated to any other agent equals the expected number of units of that agent if agent i participated and revealed he has no valuation for the units but that the externality of other agents on him correspond to the presumed external effect. If the presumed (marginal) external effect on some agent i is 0 (resp. $\bar{\delta}_i$) then the planner threatens to ensure that, if agent i does not participate, the expected number of units allocated to any other agent is weakly greater (resp. lower) than the expected number of units of that agent if agent i participated and revealed that he has no valuation for the units but that the externality of other agents on him correspond to the presumed external effect.

Proof. We know from propositions 2 and 1 that optimality is equivalent to the conditions (C_2) and (C_4). (2) is simply an expression of the condition (C_4). I need to show that condition (C_2) is satisfied that is :

$$K_i((0,\delta_i^*),\rho^{i*}) \leq K_i((\pi_i,\delta_i),\rho^{i*}), \forall(\pi_i,\delta_i) \in T_i.$$

First observe that

$$K_i((0,\delta_i),\rho^{i*}) \leq K_i((\pi_i,\delta_i),\rho^{i*}), \forall(\pi_i,\delta_i) \in T_i. \tag{1.22}$$

Since the objective is a convex function, the solutions of the optimization program $\arg\min_{\delta_i\in[0,\bar{\delta}_i]^{n-1}} K_i((0,\delta_i),\rho^{i*})$ are characterized by the following condition :

$$\sum_{j\neq i}\frac{\partial}{\partial\delta_i^j}K_i((0,\delta_i),\rho^i).(\delta_i^j-\delta_i^{*j}) \geq 0, \forall\delta_i \in \left[0,\bar{\delta}_i\right]^{n-1}. \tag{1.23}$$

Moreover, using equation (1.21), it is easy to show that the condition (1.23) is equivalent to the conditions (3) in proposition 5. Thus any solution of (3) is optimal :

$$K_i((0,\delta_i^*),\rho^{i*}) \leq K_i((0,\delta_i),\rho^{i*})\ \forall \delta_i \in \left[0,\bar{\delta}_i\right]^{n-1}.$$

Therefore (C_2) holds.

Reciprocally, we want to show that if (C_2) and (C_4) are true then so are (1), (2) and (3). Since (2) and (C_4) are equivalent, we need only show that (1) and (3) are true. In fact we need only show that (1) is true, since (C_2) would then imply that $\delta_i^* \in \arg\min_{\delta_i \in \left[0,\bar{\delta}_i\right]^{n-1}} K_i((0,\delta_i),\rho^{i*})$, which has already been said to be equivalent to (3). The inequality (1.22) together with

$$(C_2) : K_i((\pi_i^*,\delta_i^*),\rho^{i*}) \leq K_i((\pi_i,\delta_i),\rho^{i*}), \forall (\pi_i,\delta_i) \in T_i$$

imply that $K_i((\pi_i^*,\delta_i^*),\rho^{i*}) = K_i((0,\delta_i^*),\rho^{i*})$. This in turn implies that,

for almost every $t_{-i} \in T_{-i}$,

$$SW((\pi_i^*,\delta_i^*),t_{-i}) = SW((0,\delta_i^*),t_{-i}). \tag{1.24}$$

Assume that $\pi_i^* \neq 0$ i.e. $\pi_i^{*1} > 0$. $SW((\pi_i,\delta_i),t_{-i})$ is the sum of the m highest positive relative bids (the top list of relative bids) ; thus $SW((\pi_i^*,\delta_i^*),t_{-i}) = SW((0,\delta_i^*),t_{-i})$ if and only if the highest relative bid of agent i (when $(\pi_i,\delta_i) = (\pi_i^*,\delta_i^*)$) is not positive or is lower than the m^{th} highest relative bid. Indeed when the type of agent i changes from $(0,\delta_i^*)$ to (π_i^*,δ_i^*) the relative bids of all other agents remain unchanged ; with probability 1 the relative bids of agent i are initially negative and thus absent from the top list. The top list of relative bids won't change unless the relative bids of agent i enters the top list. Now consider the set of type vectors

$$H = \left\{t_{-i} \in T_{-i} : \forall j : j \neq i, \pi_i^{*1} - \delta_+^i > \pi_j^1 - \delta_+^j, \pi_i^{*1} - \delta_+^i > 0\right\};$$

H represents the set of types for which agent i has the highest positive relative bid.

It is the non empty interior of a polyhedron with positive lebesgue measure. Elements of H do not satisfy $SW((\pi_i^*,\delta_i^*),t_{-i}) = SW((0,\delta_i^*),t_{-i})$ and yet the probability of H is positive. This is in contradiction with 1.24 and therefore $\pi_i^* = 0$. □

Proposition 5 shows that the planner will always presume that an agent who does not participate does not value the good. However he may presume that there are negative external effects on that agent ($\delta_i^* \neq 0$). When is it optimal to choose $\delta_i^* = 0$? In order to answer the question observe that if $\delta_i^* = 0$ then the condition (2) of proposition 5 is trivially satisfied and all the conditions in proposition 5 reduce to $\pi_i^* = 0$ and to

$$\sum_{a\in A_{-i}} a_j \rho_a^{i*} \geq \sum_{a\in A} a_j . \Pr(a = a^*(0,t_{-i})), \forall j \in \mathcal{N} - \{i\}. \tag{1.25}$$

Therefore the following corollary characterizes the situations under which it is optimal to choose the basis $\delta_i^* = 0$.

Corollary 1. *It is optimal to choose* $t_i^* = 0$ *if and only if the system of inequalities (1.25) has a solution* $\rho^{i*} \in \Delta(A_{-i})$*, which is then the corresponding optimal threat to agent i.*

In other words, *it is optimal to presume that an agent i who does not participate has no valuation for the units and suffers no externality if and only if the planner may threaten to ensure that, if agent i does not participate, each participant will expect at least as much units as he would if agent i revealed he has no valuation for the units and suffers no externality.*

In that case the optimal expected surplus is :

$$\sum_{i\in\mathcal{N}} \mathbb{E}_{-i} SW(0,t_{-i}) - (n-1)\mathbb{E}SW(t).$$

Buyer i pays

$$SW(0,t_{-i}) - SW_{-i}(t),$$

i.e. buyer i pays the difference between the positive presumed relative bids among the m highest and the positive relative bids among the m highest that are not his own. If there are no externalities, relative bids are simply bids and the previous difference is just the a_i highest rejected bids that are not buyer i's if he wins a_i units. This is the payment rule of the Vickrey auction.

I close this section with two examples of situations in which it is optimal to set as a basis $t_i^* = 0$.

Example 1. *A single unit for sale ($m = 1$)*

I denote $[i]$ the alternative in which the good goes to agent i and $[0]$ the alternative in which the seller keeps the good. If an agent does not participate he cannot be given the object. The two terms of the inequality (1.25) are respectively $\sum_{j' \neq i} a_j \rho^{i*}_{[j']} = \rho^{i*}_{[j]}$ and $\sum_{j'} a_j . \Pr([j'] = a^*(0,t_{-i})) = \Pr([j] = a^*(0,t_{-i}))$. Therefore the planner may set the basis $t_i^* = 0$, and sets the threats such that the probability to give the good to an agent if agent i does not participate is at least equal to the probability that the same agent obtains the good when agent i participates even though he does not value the good and is not affected by its possession by others : $\rho^{i*}_{[j]} \geq \Pr([j] = a^*(0,t_{-i})), \forall j \neq i, 0$.

Example 2. *Duopsony ($n = 2$)*

The set of alternatives is $A = \left\{(a_1,a_2) \in \mathbb{N}^2 : a_1 + a_2 \leq m\right\}$. Without loss I shall fix $i = 1$. If agent 1 does not participate then the available alternatives consist of a choice of the number of units to be sold to agent 2 : $A_{-1} = \{0,1,...,m\}$. The expected number of units received by agent 2 if agent 1 does not participate is $\sum_{a_2=0,...,m} a_2 \rho^{i*}_{a_2}$. I can also write $\sum_{a \in A} a_2 . \Pr(a = a^*(0,t_2)) = \sum_{a_2=0,...,m} \sum_{a_1=0,...,a_2-m} a_2 . \Pr(a = a^*(0,t_2)) = \sum_{a_2=0,...,m} a_2 \left(\sum_{a_1=0,...,a_2-m} \Pr(a = a^*(0,t_2)) \right)$. Therefore the inequalities (1.25) are satisfied if $\rho^{i*}_{a_2} = \left(\sum_{a_1=0,...,a_2-m} \Pr(a = a^*(0,t_2)) \right)$. i.e. the probability that agent 2 receives a given number of units if agent 1 does not participate is equal to the probability that he obtains the same number of units if both agents participate though agent 1 does not value the good and is not affected by its possession by agent 2.

1.6 Conclusion

This paper has analyzed the problem of choosing an efficient alternative for a group of privately informed agents with diverse interests. A second objective of the designer was to maximize the surplus collected from the agents among mechanisms that induce truthful participation of all agents. In this environment, agents who do not participate to the decision process might still be affected by the decision. In addition to an allocation rule and a payment rule, the designer must choose appropriate threats in order to give agents the incentive to participate and maximize his own expected surplus. Since an agent who does not participate does not reveal his private information, the planner decides on his own what he considers as that private information (his presumed type) and threatens to choose an alternative that would give the worst utility to this agent. A maxmin choice of the presumed type and the threat is shown to maximize the expected surplus among efficient mechanisms inducing truthful participation of all the agents.

I also provided an existence result for outside utility that may be decomposed into two additive components : an exogenous component and an endogenous component. I then applied the results to design an efficient multiunit auction for environments where a buyer in possession of the good causes negative externalities on other buyers. I showed that a generalization of the Vickrey auction maximizes the surplus among efficient mechanisms inducing truthful participation. Other possible applications include the problem of siting noxious facilities, elections, the siting of sport events, the sale of nuclear weapons etc.

In some situations the planner would seek to implement a mechanism so that there is no additional fund from the planner or any surplus. Such mechanisms are called budget balanced. Formally a mechanism (q,μ,ρ) is budget balanced if : $\sum_{i\in\mathcal{N}} \mu_i(t) = 0$ for every type vector t. As pointed out in Krishna and Perry (2000) the existence of such mechanisms is determined by the sign of the maximal surplus. Though different assumptions are used in this paper, this result remains valid and the constructive proof (which relies mostly on the revenue equivalence) is similar. In fact using the same techniques one may show that it is sufficient that any efficient mechanism inducing truthful participation of all agents results in a positive surplus. This may be useful in applications where a closed form for the surplus maximizing mechanism cannot be found. In such applications it is sufficient to have an approximation of the surplus maximizing mechanism that itself results in a positive surplus and construct from it a budget balanced mechanism.

Finally it would be interesting in further research to relax the assumption that the types of agents are independent and to allow agents to act in a concerted way.

CHAPITRE 2

IS THE FAIR RETURN RULE MORE EXPENSIVE THAN FREE COMPETITION ?

Abstract

We consider the fair return rule used by the European Space Agency (ESA). This rule ensures each member state of ESA a return proportional to its contribution, in the form of contracts awarded to firms coming from that state. The fair return rule is in conflict with the principle of free competition since contracts are not necessarily awarded to firms with the lowest bids. This has raised debates on the use of this rule : it is well accepted by small states, but larger states with strong national space programs, see its strict use as an obstacle to competitiveness and cost effectiveness. It is easy to believe that this rule is more costly to the agency than traditional auctions. We show on the contrary that an adequate implementation of the fair return rule may cause it to be less expensive to the agency than the traditional auctions observing free competition (first price and second price auctions). We consider the case of complete information where firms' technology levels are common knowledge, and the case of incomplete information where firms observe privately their production costs. In both cases we show that by adequately implementing the fair return rule, the agency may even take advantage of asymmetries between countries in order to expect a lower cost than with traditional auctions.

2.1 Introduction

The European space agency (ESA) is an independent organization whose role is to develop space industry in Europe. It is funded by 17 member states and is involved in many activities related to space exploration and technology. Even though many of these member states have developed national space programs, ESA achieves far beyond what is possible within any of these national programs (see Albone et al 2002). ESA's activities are separated into mandatory and optional. Mandatory activities are funded by all member states, each according to its gross national product, and states contribute freely to optional activities. According to ESA's convention (ESA 2003), one objective of the industrial policy is to :

> ensure that all member states participate in an equitable manner, having regard to their financial contribution, in implementing the European space program and in the associated development of space technology ; in particular the agency shall for the execution of the program grant to the fullest extend possible to industry in all member states which shall be given the maximum opportunity to participate in the work of technological interest undertaken for the agency ;

Thus procurements at ESA are globally submitted to the so called fair return rule which ensures each member state a return in the form of contracts (awarded to firms or agents coming from that state) proportional to its contribution. Simply ESA's projects are divided into smaller projects so that firms of different size and from different states may participate. One advantage of this rule is that firms have the opportunity to share experience, scientific knowledge and technology. Another advantage is clearly to give states incentives to contribute to activities. In practice the fair return rule is implemented so that the ratio between the share of a state in the values of contracts and its share in the contribution to the agency's activities (that is the return rate) must not be lower than a given threshold. In the beginning of ESA the threshold was set to 0.8, but it has recently reached 0.98 (ESA 2000). In other words, a contribution of 1 euro from a state guarantees at least 0.98 euro in the form of contracts awarded to firms from the same state. Ideally the return rate should be equal to 1. Note that beside the fair return rule, ESA also seeks to promote free competition whenever the two are not in contradiction. The traditional understanding of free competition is to allocate contracts to firms who place the lowest bids regardless of their origins. The implementation of the fair return rule requires a relatively large period (5 years) at the end of which every member state should have an appropriate return rate (ESA 2003) ; for it is practically impossible to ensure each member state a return equal to its contribution at every moment or at the end of every auction. In short, ESA applies free competition whenever possible and a review is constantly made in order to adjust states' returns when needed. It is done by applying some particular allocation rules in the remaining auctions depending on the current return rates. There have been debates on the use of the fair return rule. It still is well accepted by the small states but larger states, with strong national space programs, see its strict use as an obstacle to competitiveness and cost effectiveness (ESA 2000). In addition, ESA and the European Community have recently been working together in order to write a European space policy. Within this new relationship, the issue of the fair return rule is also discussed since The European Community uses a different industrial policy (EC 2002). In this context a formal analysis of the fair return rule is particularly relevant.

The use of the fair return may generate many questions ; the aim of this paper is to bring a contribution to the following one : are traditional auctions less expensive to ESA than the fair return rule ? This question arises from the fact that the fair return rule is (socially) inefficient, in the sense that a firm with the lowest bid do not necessarily win. To the best of our knowledge, no formal study has been made to address this issue. We make a first step in the analysis of the question using a simple model where a buyer (ESA) is seeking to purchase many items from potential suppliers (firms) of different origins (member state of ESA). The agency may implement the fair return rule or free competition. Under free competition contracts for the provision of each item are awarded independently and the winners are the best bidders (those with lowest bids). Though the fair return rule is a dynamic mechanism, in this paper however, we adopt a static version of it in order to keep things simple : under the fair return rule the contracts are awarded so that all member states are represented by the actual suppliers. It is important to note that a dynamic version of the fair return rule is actually made of a sequence of static auctions similar to the one we adopt here but differing one from another by the number of states involved in each auction. [1] To make things more simple we assume the agency is

1. In practice priority is given to some member states of ESA depending on current return rates. As return rates vary the priority changes.

facing a few states and firms ; the agency is willing to purchase two items (a unit of each) from 4 potential suppliers $1,2,3$ and 4. Where suppliers 1 and 2 come from state l and suppliers 3 and 4 come from state h. We also assume that suppliers 1 and 3 may supply the first item while suppliers 2 and 4 may supply the second item. Let b_i denote supplier i's bid. Under free competition the lowest bidder between supplier 1 and 3 (resp. 2 and 4) wins the contract for the first (resp. second) item. Examples include the traditional first price auction and second price auction. These auctions differ only by their payment rules. Under the first price auction the winner is paid an amount equal to his bid, while under the second price auction the winner is paid an amount equal to the second lowest bid. But the other players receive no payment. Under the fair return rule contracts are awarded to only one of the pairs $\{1,4\}$ and $\{2,3\}$. If we call aggregate bid of a pair of suppliers the sum of their bids, then contracts are awarded to the pair with the lowest aggregate bid. There are many different ways of implementing the static version of the fair return rule. They simply differ by the payment rules. A natural way is to pay the winners an amount equal to their bids and pay nothing to the other suppliers. We shall refer to this auction as the fair return auction (FR). Another way would be to pay each winner his conjugate bid [2] and nothing to the other suppliers ; we call this the second fair return auction (SFR).

In presence of asymmetry the conflict between small and large states may be illustrated as follows : Imagine a situation where suppliers from state l have lower costs for the production of the items than their opponents from state h ; under first price auctions the items are purchased only from suppliers from l since they have more latitude to make lower bids. Therefore suppliers from h will prefer FR auction to first price auctions contrary to suppliers from l. Moreover in such a setting it is clear that, if suppliers make the same bids regardless of the industrial policy used, the agency would pay a higher price under the FR auction than under first price auctions. More generally, without the assumption of asymmetry, we may reach to the same conclusion. Indeed, under first price auctions contracts are awarded to lowest bidders at a price equal to these bids ; but under the FR auction, contracts are awarded to a set of suppliers with the lowest

2. We define this in the next section.

aggregate bid ; Since some suppliers in this winning set may not have the lowest bid of their category, the price paid by the agency under the FR auction is higher. Given this, an affirmative answer to the previous question may seem obvious. But while the previous argument is true, it rests on the assumption that suppliers' bids would be the same under both auctions, and this need not be the case. In fact suppliers' bids are outcomes of their individual strategies and, these strategies may change as the rules of the auction change. We find that free competition may sometimes be more expensive than fair return rule. In particular, and contrary to the previous argument and first intuition, we show that this often happens when suppliers from a given state are more competitive than their direct opponent from the other state. In such conditions one state has a technologic advantage over the other in the production of all the items. The intuition is this : under the fair return rule, since suppliers of the same state cannot win simultaneously, a competitive supplier is in competition not only with the supplier of the same item but also with the supplier from the same state. A situation that forces competitive suppliers to bid more aggressively under the fair return rule than under free competition, and results in a lower cost for the agency.

The rest of the paper is organized as follows. In section 2 we present the model and give the basic definitions. In section 3 we consider the case where suppliers have complete information about the supply costs and find conditions under which the FR auction is less expensive than first price auctions. Then, in section 4, we consider the case of incomplete information where each supplier observes privately his own cost. Due to analytical intractability we do not consider the FR auction, but the SFR auction. We show that this auction induces a lower social cost than the FR auction. We find conditions under which these auctions lead to a lower expected price than second price auctions. Section 5 concludes the paper.

2.2 Preliminaries

An agency is willing to purchase two items from 4 potential suppliers $1,2,3$ and 4. Suppliers 1 and 2 originate from state l and suppliers 3 and 4 from state h. Suppliers 1 and 3 may supply the first item while suppliers 2 and 4 may supply the second item.

The agency may run the following two policies : free competition and the fair return rule. With free competition contracts for the provision of each item are awarded independently and the winners are the best bidders (those with lowest bids). With the fair return rule contracts are awarded such that all the two states are represented by the actual suppliers.

We use the letter i for an arbitrary supplier in $\mathscr{N} = \{1,2,3,4\}$. Given a supplier i the other suppliers (denoted j, i^* and j^*) can be identified by their relationship with i : i and j are supplying the same item while i and i^* are from the same state. As a consequence i and j^* (resp. j and i^*) are from different states and supply different items. Thus under free competition supplier i faces supplier j and supplier i^* faces supplier j^*, while pair $\{i, j^*\}$ faces pair $\{j, i^*\}$ under the fair return rule. In the later case suppliers i and j^* (resp. j and i^*) may be viewed as "partners" since they win or loose the auction together. We assume however that there is no cooperation between suppliers.

If we denote b_i supplier i's bid for all i in $\mathscr{N}$, then we may define *supplier i's conjugate bid* as $\bar{b}_i = b_j + b_{i^*} - b_{j^*}$. This is the algebraic sum of all the other suppliers' bids where his partner bid is counted negatively. Under free competition i wins a contract if $b_i \leq b_j$, and under the fair return rule he wins if $b_i + b_{j^*} \leq b_j + b_{i^*}$ (i.e. if $b_i \leq \bar{b}_i$). Supplier i's conjugate bid somehow summarizes competition under the fair return principle. Under the two policies nothing is paid to suppliers who do not win contracts and ties are solved equiprobably.

Examples of auctions with free competition include the traditional first and second price auctions (they differ only by their payment rules) :

- *Under the second price auction (SP)* the agency compares the bids of suppliers 1 and 3 and buys the first item from the supplier with the lowest bid at a price equal to the second lowest bid (in this case the highest bid). Then it compares the bids of suppliers 2 and 4 and buys the second item from the supplier with the lowest bid at a price equal to the second lowest bid.
- *Under the first price auction (FP)* the agency compares the bids of suppliers 1 and 3 and buys the first item from the supplier with the lowest bid at a price equal to

his bid. Then it compares the bids of suppliers 2 and 4 and buys the second item from the supplier with the lowest bid at a price equal to his bid.

Examples of auctions with the fair return rule include the fair return auction and the second fair return auction. They also differ by their payment rule :

- *Under the fair return auction (FR)* suppliers submit their bids simultaneously and the items are bought from the pair with the smallest aggregate bid between $\{1,\ 4\}$ and $\{2,\ 3\}$,[3] and each winner is paid his bid.
- *Under the second fair return auction (SFR)* suppliers submit their bids simultaneously and the items are bought from the pair with the smallest aggregate bid between $\{1,\ 4\}$ and $\{2,\ 3\}$, and each winner is paid his conjugate bid.

The information held by the suppliers concerns their costs for supplying the items. If supplier i wins it will cost him c_i to provide the item. Otherwise, it will cost him nothing. Denote $\mathbf{c}$ the costs vector $(c_i)_{i\in\mathcal{N}}$. We consider both the cases of complete information (in section 3) and incomplete information (in section 4). Under complete information suppliers know each other cost, and under incomplete information each supplier observes privately his own cost. We will compare the two fair return auctions to the two traditional auctions. Particularly in the next section we compare the FR auction to the FP auction on the basis of the total price of the items.

2.3 Complete information : FR vs FP

We suppose that bids and costs are positive multiples of a given constant $\varepsilon > 0$.[4] We denote fr the total price of the two items under the FR auction. Let fp_j be the price of item $j \in \{1,\ 2\}$ under the FP auction and let $fp = fp_1 + fp_2$. fp_j is also the winning bid for item j. The two auctions induce games of complete information between suppliers. Their strategies are there bids. We suppose that the possible outcomes of these games are Nash equilibria. The sum of the winning bids represents the total price paid by the

3. If seller $i \in \mathcal{N}$ submits a bid b_i then the aggregate bids of pairs $\{1,\ 4\}$ and $\{2,\ 3\}$ are respectively $b_1 + b_4$ and $b_2 + b_3$.

4. ε may be understood as the minimum admissible bid. The main result in this section does not change if bids are positive multiples of a constant and costs are arbitrary positive real numbers.

agency. In all this section the costs vector $c = (c_i)_{i \in \mathcal{N}}$ is fixed.

Proposition 6. *There exists a Nash equilibrium for the FP and FR auctions.*

Actually there may exist multiple Nash equilibria for these two auctions. So we may not always predict the prices of the items, but we may provide bounds for these prices. In the main result of this section (proposition 7) we give an upper bound for the difference between the price under FR and FP auctions ($fr - fp$) as well as sufficient conditions for $fr - fp < 0$.

Lemma 3. *the following assertions are true :*

(i) $fp_1 \geq \max(c_1, c_3) - \varepsilon$ *and* $fp_2 \geq \max(c_2, c_4) - \varepsilon$,

(ii) $fr \leq \max(c_1 + c_4, c_3 + c_2) + 4\varepsilon$.

Under the FP auction fp_j is the winning bid for item j and the winner is the supplier with the smallest cost. Indeed he has more latitude to place a lower bid than his opponent who cannot bid under his cost. Assertion (i) means that the winning bid for item j is at least just below the greatest cost of the potential suppliers of that item.

Under the FR auction the winning pair is the pair with the smallest aggregate cost and fr is the aggregate bid of the winning pair. The winning pair's aggregate bid is close to the greatest aggregate cost but, according to assertion (ii), does not exceed it of more than 4ε.

As a consequence of the lemma, we have an upper bound for $fr - fp$:

$$fr - fp \leq 6\varepsilon + \Delta(\mathbf{c}) \tag{2.1}$$

with

$$\Delta(\mathbf{c}) = \max(c_1 + c_4, c_3 + c_2) - \max(c_1, c_3) - \max(c_2, c_4). \tag{2.2}$$

We are now able to state the main result of this section.

Proposition 7. *the following assertions are true :*

(i) $fr - fp \leq 6\varepsilon$.

(ii) if $c_1 - c_3$ and $c_2 - c_4$ have the same sign and

$\min(|c_1 - c_3|, |c_2 - c_4|) > 6\varepsilon$ *then* $fr - fp < 0$.

Assertion (i) means that even when the FR auction is more expensive than the FP auction, the difference cannot exceed 6ε. In practice the minimum bid is not greater than the smallest monetary unit (a cent for example). Thus in terms of the price of the items the FP auction cannot dominate the FR auction of more than six units.

To understand assertion (ii) recall that suppliers 1 and 3 (resp. 2 and 4) are selling the same item and thus, are in direct competition with one another. Recall also that suppliers 1 and 2 (resp. 3 and 4) are from the same state. If we see suppliers' costs as an index of their technology levels then, $c_1 - c_3$ (resp. $c_2 - c_4$) represents the technology difference between suppliers 1 and 3 (resp. 2 and 4). The fact that $c_1 - c_3$ and $c_2 - c_4$ have the same sign means that suppliers of one state dominate the others with respect to the technology levels. For example if $c_1 - c_3$ and $c_2 - c_4$ are both positive then supplier 3 dominates supplier 1 (his direct opponent) and supplier 4 dominates supplier 2. So state h dominates state l. Now $\min(|c_1 - c_3|, |c_2 - c_4|)$ represents the minimum technology difference between the states. It is therefore an index of the technology gap between the states. Assertion (ii) means that if a state dominates the other and if the technology gap is sufficiently high (greater than 6ε) then the FR auction leads to a lower price than the FP auction. The intuition is this : if suppliers of a state dominate their direct opponents, they have no interest in making aggressive bids under the FP auction. On the contrary under the FR auction they are in direct competition with their fellow state supplier who has a competitive technology (even though he sells another item) and are virtually associated with a less competitive supplier. The suppliers need to be more aggressive in order to win against their fellow state supplier and compensate the weakness of their "virtual partner". This results in a lower price under the FR auction. Note also that the minimum gap required in assertion (ii) (6ε) is small.

Proof of proposition 7. (i) This follows from inequality (2.1) and the fact that $\Delta(\mathbf{c}) \leq 0$. Indeed $c_1 + c_4 \leq \max(c_1, c_3) + \max(c_2, c_4)$ and $c_3 + c_2 \leq \max(c_1, c_3) + \max(c_2, c_4)$.

Now if $c_1 - c_3$ and $c_2 - c_4$ have the same sign then $\Delta(\mathbf{c}) = -\min(|c_1 - c_3|, |c_2 -$

$c_4|)$.[5] By inequality (2.1) a sufficient condition for $fr - fp < 0$ is $6\varepsilon + \Delta(\mathbf{c}) < 0$, i.e. $\min(|c_1 - c_3|, |c_2 - c_4|) > 6\varepsilon$. Hence (ii). □

Note that through the lines of this proof we read the following result :

if $c_1 - c_3$ and $c_2 - c_4$ have the same sign then $6\varepsilon + \min(|c_1 - c_3|, |c_2 - c_4|)$ is a lower bound for the price difference $|fp - fr|$. This lower bound increases as the technology gap $\min(|c_1 - c_3|, |c_2 - c_4|)$ increases. In other words, *if a state dominates the other the price difference under the two auctions tends to increase as the technology gap between states increases.*

In the next section we consider the case where suppliers have private information about their costs. We do not consider the FR auction but the SFR auction which appears to be analytically easier to deal with.

2.4 Incomplete information : SFR vs SP

In this section every supplier observes privately his own cost but not the other suppliers' costs. The common prior for the costs is given by the probability measure μ with support $T = \underset{i \in \mathscr{N}}{\times} T_i$, where $T_i = [\underline{c}_i, \bar{c}_i]$ for all $i \in \mathscr{N}$. We assume that μ is absolutely continuous with respect to the lebesgue measure, and we denote f its probability density function. There is no cooperation between the suppliers.

A strategy for supplier i is a function $\beta_i : T_i \to \mathbb{R}_+$. Under this strategy the value $\beta_i(c_i)$ is supplier i's bid when he observes a private cost c_i. A bid can be any non negative real number.

2.4.1 Nash equilibria

We determine Nash equilibria under the different auctions. *It is well known that under the SP auction it is a weakly dominant strategy to bid his cost (see Krishna 2000).* We show that this remains true under the SFR auction.

5. A proof of this is given in proposition 16, see appendix.

Proposition 8. *It is a weakly dominant strategy for every supplier to bid his cost under the SFR auction.*

Proof. Denote $U_i(c_i, b_i, \mathbf{b}_{-i})$ the payoff of supplier i when he bids b_i rather than his true cost c_i and the other suppliers bid $\mathbf{b}_{-i}$. We will show that $U_i(c_i, c_i, \mathbf{b}_{-i}) \geq U_i(c_i, b_i, \mathbf{b}_{-i})$. Recall that

$$U_i(c_i, b_i, \mathbf{b}_{-i}) = \begin{cases} \bar{b}_i - c_i & \text{if } b_i < \bar{b}_i \\ \frac{1}{2}(\bar{b}_i - c_i) & \text{if } b_i = \bar{b}_i \\ 0 & \text{if } b_i > \bar{b}_i. \end{cases}$$

Assume $b_i < c_i$. If $\bar{b}_i \leq b_i < c_i$ then $U_i(c_i, c_i, \mathbf{b}_{-i}) = 0$ and either $U_i(c_i, b_i, \mathbf{b}_{-i}) = 0$ (when $\bar{b}_i < b_i$) or $U_i(c_i, b_i, \mathbf{b}_{-i}) = \frac{1}{2}(b_i - c_i) < 0$ (when $\bar{b}_i = b_i$).

If $b_i < c_i < \bar{b}_i$ then $U_i(c_i, c_i, \mathbf{b}_{-i}) = \bar{b}_i - c_i = U_i(c_i, b_i, \mathbf{b}_{-i})$. If $b_i < \bar{b}_i \leq c_i$ then $U_i(c_i, c_i, \mathbf{b}_{-i}) = 0 \geq \bar{b}_i - c_i = U_i(c_i, b_i, \mathbf{b}_{-i})$.

Assume $b_i > c_i$. If $b_i > c_i \geq \bar{b}_i$ then $U_i(c_i, c_i, \mathbf{b}_{-i}) = 0 = U_i(c_i, b_i, \mathbf{b}_{-i})$. If $\bar{b}_i > b_i > c_i$ then $U_i(c_i, c_i, \mathbf{b}_{-i}) = \bar{b}_i - c_i = U_i(c_i, b_i, \mathbf{b}_{-i})$. If $\bar{b}_i = b_i > c_i$ then $U_i(c_i, c_i, \mathbf{b}_{-i}) = \bar{b}_i - c_i \geq \frac{1}{2}(\bar{b}_i - c_i) = U_i(c_i, b_i, \mathbf{b}_{-i})$. And finally if $b_i > \bar{b}_i \geq c_i$ then $U_i(c_i, c_i, \mathbf{b}_{-i}) = \bar{b}_i - c_i \geq 0 = U_i(c_i, b_i, \mathbf{b}_{-i})$. □

Thus the two auctions have at least one equilibrium. We now compare the price of the two units assuming that suppliers bid their costs. We first compare the prices of the items expost, i.e. given the actual (though privately observed) costs. Then we compare the expected prices paid by the agency under the two auctions.

2.4.2 Expost Comparison of the prices

Since suppliers bid their costs at equilibriums, the agency pays the greatest cost under the SP auction that is (assuming suppliers costs vector is $\mathbf{c}$) :

$$sp(\mathbf{c}) = \max(c_{1,}c_3) + \max(c_2, c_4) \tag{2.3}$$

In order to determine the price under the SFR auction, note that if $\{1, 4\}$ is the winning pair then because suppliers bid their costs $c_1 + c_4 \leq c_2 + c_3$; and the agency pays the following total amount

$$\bar{c}_1+\bar{c}_4=(c_2+c_3-c_1)+(c_2+c_3-c_4)=2(c_2+c_3)-(c_1+c_4). \tag{2.4}$$

Similarly if $\{2,3\}$ is the winning pair then $c_1+c_4 \geq c_2+c_3$ and the total price of the two units is

$$\bar{c}_2+\bar{c}_3=2(c_1+c_4)-(c_2+c_3). \tag{2.5}$$

Observing that $(\bar{c}_2+\bar{c}_3)-(\bar{c}_1+\bar{c}_4)$ and $(c_2+c_3)-(c_1+c_4)$ have opposite signs we conclude that the price under SFR auction is (assuming suppliers costs vector is $\mathbf{c}$) :

$$sfr(\mathbf{c})=\max(\bar{c}_2+\bar{c}_3,\bar{c}_1+\bar{c}_4). \tag{2.6}$$

Given a costs vector $\mathbf{c}=(c_1,c_2,c_3,c_4)\in T$, The price difference between the two auctions is :

$$\delta(\mathbf{c})=sfr(\mathbf{c})-sp(\mathbf{c})=\max(\bar{c}_2+\bar{c}_3,\bar{c}_1+\bar{c}_4)-\max(c_{1,}c_3)-\max(c_2,c_4). \tag{2.7}$$

Using the equality $\max(a,b)=\frac{a+b+|a-b|}{2}$, we obtain :

$$2\delta(\mathbf{c})=\bar{c}_2+\bar{c}_3+\bar{c}_1+\bar{c}_4+|\bar{c}_2+\bar{c}_3-\bar{c}_1-\bar{c}_4|-(c_1+c_3)-|c_1-c_3|-(c_2+c_4)-|c_2-c_4|.$$

Using (2.4), (2.5) and rearranging :

$$2\delta(\mathbf{c})=3|(c_1-c_3)-(c_2-c_4)|-|c_1-c_3|-|c_2-c_4|. \tag{2.8}$$

It appears that $\delta(\mathbf{c})$ depends solely on the two differences c_1-c_3 and c_2-c_4 which, again, can be understood respectively as the technology gaps between suppliers 1 and 3 and between suppliers 2 and 4. [6] We define the relative technologic gap across the items

6. Remember that suppliers 1 and 3 produce the same good as suppliers 2 and 4 do.

as $\frac{c_1-c_3}{c_2-c_4}$; the sign of this ratio tells us if a state dominates for all items or for only one item ; if it is positive then suppliers of one state have a better technology than their direct opponent for all items ; if it is negative then each state has the advantage for only one item. However the ratio $\frac{c_1-c_3}{c_2-c_4}$ does not give information about which state dominates and for which specific item. Moreover the absolute value of $\frac{c_1-c_3}{c_2-c_4}$ says how great is the advantage on item 1 compared to the advantage on item 2. The advantage on item 1 is greater if $|\frac{c_1-c_3}{c_2-c_4}|$ is greater than 1.

We have the following result :

Proposition 9. *given a realization* $\mathbf{c} \in T$ *of the suppliers' costs vector we have :*
$sfr(\mathbf{c}) - sp(\mathbf{c}) < 0$ *if and only if* $\frac{1}{2} < \frac{c_1-c_3}{c_2-c_4} < 2$.

This proposition gives a necessary and sufficient condition so that, given a realization of the suppliers' costs, the price of the items under the SFR auction is lower than the price under the SP auction. The condition $\frac{1}{2} < \frac{c_1-c_3}{c_2-c_4} < 2$ implies that the relative technologic gap across the items is positive ; thus there is a state with a higher technology for all items. This condition also means that the advantages of the suppliers of the strong state are close enough : the advantage on item 1 (i.e. $|c_1 - c_3|$) is greater than half of the advantage on item 2 (i.e. $|c_2 - c_4|$) but less than twice this advantage.

2.4.3 Comparing the expected prices

In our model the agency does not know the costs of suppliers and therefore cannot predict exactly what would be the price of the items. Instead it formulates beliefs about what these costs could be. We model its beliefs by a probability distribution μ for the costs. We also make the assumption of common beliefs, that is, all the agents (agency and suppliers) have the same beliefs about the costs. To their eyes costs are random variables with probability density function f with support T as mentioned earlier in this section. To compare the two auctions a risk neutral agency would compare the expected

If the costs are such that $c_1 - c_3 < 0$ and $c_2 - c_4 < 0$ then the country l has a higher technology than country h for the production of the two goods.

prices resulting from these auctions. We denote sfr (resp. sp) the expected price of the items under the SFR (resp. SP) auction :

$$sp = E\{\max(c_1, c_3) + \max(c_2, c_4)\}. \tag{2.9}$$

$$sfr = E\{\max(\bar{c}_2 + \bar{c}_3, \bar{c}_1 + \bar{c}_4)\}. \tag{2.10}$$

The difference between the two expected price is :

$$sfr - sp = E(\delta(\mathbf{c})) = \int_T \delta d\mu = \int_T \delta(\mathbf{c}) f(\mathbf{c}) d\mathbf{c}. \tag{2.11}$$

Let

$$N = \{\mathbf{c} \in T : \delta(\mathbf{c}) < 0\} \tag{2.12}$$

and

$$P = T - N = \{\mathbf{c} \in T : \delta(\mathbf{c}) \geq 0\}. \tag{2.13}$$

We may write the difference between the two expected prices as :

$$sfr - sp = \int_T \delta d\mu = \int_P |\delta| d\mu - \int_N |\delta| d\mu. \tag{2.14}$$

This difference depends on the costs' common prior μ. Hence the following proposition.

Proposition 10. *$sfr - sp < 0$ if and only if μ is such that $\int_P |\delta| d\mu < \int_N |\delta| d\mu$.*

Recall that $N = \left\{\mathbf{c} \in T : \frac{1}{2} < \frac{c_1 - c_3}{c_2 - c_4} < 2\right\}$ (proposition 9). Thus, *the expected price of the items under the SFR auction is lower than the expected price of the items under the SP auction if the prior gives "more weight" to the negative side of δ, that is, to situations where it is believed there is a state with a higher technology for all items, and the advantages of suppliers from the strong state are close enough.*

We next give a condition on the support T for the existence of such a prior.

Proposition 11. *The following assertions are equivalent :*

(i) There exists a common prior such that $sfr - sp < 0$

(ii) the lebesgue measure of the set $N = \left\{ \mathbf{c} \in T : \frac{1}{2} < \frac{c_1 - c_3}{c_2 - c_4} < 2 \right\}$ *is positive*

(iii) $\begin{cases} \bar{c}_1 \geq \underline{c}_3 \\ \bar{c}_4 \geq \underline{c}_2 \end{cases}$ *or* $\begin{cases} \bar{c}_3 \geq \underline{c}_1 \\ \bar{c}_2 \geq \underline{c}_4 \end{cases}$

Assertion (iii) is satisfied for example when $\{\bar{c}_i, \underline{c}_i\}$ is the same for all suppliers. So there exists a belief system (not necessarily symmetric) where suppliers' costs have the same support, and for which the expected price under the SFR auction is lower than the expected price under the SP auction. Actually there exist many such belief systems as one reads from the proof of the previous proposition. Such beliefs simply give more weight to situations where there is a state with a higher technology for all items, and where the advantages of suppliers from the strong state are close enough ($\frac{1}{2} < \frac{c_1 - c_3}{c_2 - c_4} < 2$).

Observe that the negation of (iii) can be written as

$$\begin{cases} \bar{c}_1 < \underline{c}_3 \\ \bar{c}_2 < \underline{c}_4 \end{cases} \text{or} \begin{cases} \bar{c}_3 < \underline{c}_1 \\ \bar{c}_4 < \underline{c}_2 \end{cases};$$

So when it is believed that one state *strictly dominates* the other, to the point that the worst costs of each supplier from the dominant state is lower than the lowest costs of its direct opponent, then the expected price under the SFR auction is never lower than the expected price under the SP auction. The agency may expect lower costs under the SFR auction only when this type of dominance is not observed.

2.5 Conclusion

In this paper we've considered the fair return rule : a rule used by the European Space Agency which ensures each member state a return in the form of contracts, awarded to firms coming from that state, globally proportional to its contribution. This rule is somehow in conflict with the principle of free competition since contracts are not necessarily awarded to a firm with the lowest bid.

We showed that an adequate implementation of the fair return rule may cause it to be less expensive than the traditional auctions of free competition (first price and second price auctions). We considered the case of complete information (for the first price auction) where firms' technology levels are common knowledge, and the case of incomplete information (for the second price auction) where firms observe privately their production costs. In both cases we identified an auction under the fair return principle that takes advantage of asymmetries between countries and yields a lower cost than with traditional auctions. The price (resp. expected price) of the items under the fair return rule is lower under the fair return rule in situations where (resp. the agency believes that) : one state has higher technology level for the production of the items than the other, and the advantage of suppliers from the high technology state (over their direct opponents) are close enough.

We have also assumed that the agency does not impose reserve prices. This assumption may be justified by the fact that the buyer's valuation is known to be too high compared to the suppliers' costs. So that the buyer cannot reliably commit to not purchase the items at all. However it would be interesting to compare the two principles in a model that allows for reservation prices.

CHAPITRE 3

OPTIMAL PROCUREMENT WHEN SUPPLIERS' ORIGINS MATTER

Abstract

We derive an optimal procurement mechanism in an environment where a buyer of heterogeneous items faces potential suppliers from different groups, and the buyer is constrained to choose a winning list that is consistent with some exogenous quotas assigned to the different groups. The optimal allocation rule consists of assigning priority levels to suppliers on the basis of their cost reports. The way these priority levels are determined is subjective but known to all before the auction. The individual reports induce scores for each potential winning list. The items are then purchased from one of the lists with the best score, provided it is not greater than the buyer's valuation for the items. Only winning suppliers receive a payment which is at least equal to the highest cost he could have and still win the auction with certainty. We also find that it is not optimal to purchase the items through separate auctions, unless the buyer's valuation is sufficiently high or low.

3.1 Introduction

Consider a government agency desiring to achieve a project. This project is divided into subprojects and contracts related to these subprojects are awarded through a procurement auction. Assume that participants are researchers from different provinces of the country. A government wanting to encourage the research in all the provinces may design a procurement mechanism so that winners come from all the provinces. Any researcher may be awarded a contract, but some subsets of them are "incompatible" : in the sense that they may not win the auction together. For example if one province (or a subset of provinces) has several researchers able to achieve all the subprojects at low cost, they will never appear in the same winning list because it requires all the provinces. In the same time the agency may want to encourage competition in order to minimize the payment. If the agency awards each contract to researchers who make the lowest bids, chances are that the winners will not come from all the provinces. Which mechanism should the agency choose in order to award the contracts ?

Similarly, assume that an international institution has several job positions to be filled and faces job candidates coming from different states. If the institution is funded by these states, then the institution may want to hire from all these countries for the sake of integration of the different states. How should the hiring process be conducted ?

In this paper we study a stylized version of the previous problems. We consider a buyer seeking to purchase some heterogeneous items and facing potential suppliers belonging to different groups. Each supplier may only supply one given item. The items are complements and the buyer is willing to purchase them through an optimal mechanism that is, a mechanism that maximizes the buyer's surplus. As in the above examples, the buyer's environment may be (legally) constrained ; typically groups would be assigned quotas determining the maximal and the minimal number of items purchased from suppliers of a group In parallel, we are also concerned with the optimal mechanism when the buyer's environment is not constrained.

Suppliers are assumed to have private information about their supply costs and these costs are independent. By the revelation principle we may restrict our attention to direct mechanisms. These are mechanisms in which every supplier is required to submit his private information prior to the buyer's decision. Under such mechanisms, given that their costs are private information, suppliers may not be willing to report honestly their costs if it is not judged advantageous. We are concerned with incentive compatible mechanisms that is, mechanisms in which it is an equilibrium to report honestly the private information. In addition we focus on mechanisms satisfying the participation constraint : these are mechanisms in which every supplier who takes part in the procurement expects a profit at least equal to what he gets if he does not participate. In this context of private information the buyer's objective is actually to optimize the *expected value* of the surplus.

This paper is related to the literature on mechanism design. A mechanism is a set of rules that determines which alternative should be chosen among many. Mechanisms may be compared on the basis of their ability to achieve desired goals. These goals would typically depend on the mechanism's designer and on the environment ; by this we mean constraints that may not be modified by the mechanism's designer such as preferences, available information, technology limitations, and institutional constraints.

Part of the literature is concerned with social efficiency. For example, among recent works, Pérez-Castrillo and Wettstein (2002) suggest the use of a multi-bidding auction to efficiently choose an alternative among many. In their mechanism agents make multiple bids (one for every alternative) that must sum up to zero, in addition they each announce one alternative that helps make the decision in case of tie. Ehlers (2008) uses the same environment to show the importance to make the additional announcement. [1]

In contrast, we are rather interested in maximizing the surplus of the buyer in the context of a multi unit procurement. We generalize the techniques in Myerson (1981) in order to derive the optimal mechanism. As in Myerson (1981) the buyer may refuse to purchase the items if it is not judged advantageous. Branco (1996) characterizes optimal multi unit auction in the context of homogeneous items, in contrast our model deals with heterogeneous items. We also make the assumption that the items are complements from the buyer's point of view. Optimal multi unit auctions with heterogeneous items have been widely studied. In these environments each bidder usually may compete for more than one item. A recurrent theme in these papers is the question of whether to purchase the items (or sell in the case of direct auctions) in bundle or in many sequential auctions. Armstrong (2000), Jehiel and moldovanu (2001) show that bundling is optimal in the case of two items. Levin (1997) studied optimal auction of complements and showed that it is advantageous to bundle. Another important issue is the dimension of the information available to bidders (their type). Most of the literature on optimal multi unit auction assumes multidimensional and discrete types (Armstrong 2000, Avery and Hendershott 2000). We rather consider one dimensional and continuous types (the unit costs of the suppliers). All these papers do not study environment where winning lists are constrained to respect exogenous quotas for groups of suppliers : the topic of this

1. For a good review on the subject of socially efficient (and more generally of) mechanism design see Jackson (2001), Serrano (2003), or Myerson (2006).

paper.

We do not allow suppliers to bid for more than one item. But the fact that the buyer views the items as complements suggests the possibility to purchase them at a joint auction. Moreover the presence of a legal constraint that assigns different quotas to groups of suppliers also seems in favor of a joint auction. Of course it is possible to purchase the items in a mechanism consisting of many separate auctions as well : we refer to such mechanisms as *itemwise* mechanisms. In proposition 15 we provide a necessary condition for an itemwise mechanism to be optimal : the buyer's valuation for the items must be too high or too low. Because of the legal constraint the converse is not in general true, yet it is true when the environment is unconstrained.

Armantier and Njiki (2008) consider an environment similar to ours : with four suppliers, two groups and two items. Though not searching for optimal auctions they are interested in some particular auctions. They present two simple constrained mechanisms and show that they yield lower expected prices than first and second price auctions under assumptions implying costs correlation or asymmetry. The assumption of costs independence made in the present paper is the main difference with their environment, beside the fact that we allow for many items, suppliers and groups rather than only two items, four suppliers and two groups as they do.

The rest of the paper is organized as follows. In section 2 we present the model and introduce some useful definitions. In section 3 we clarify which mechanisms are considered feasible and focus on direct mechanisms. In section 4 the optimal mechanism is derived and we discuss about conditions under which an itemwise mechanism may be optimal. Finally an application is considered in section 5 and, Section 7 concludes the paper.

3.2 The model

We consider a buyer seeking to purchase one unit of L heterogeneous items. There are n potential suppliers divided into K groups. Let $\mathcal{N} = \{1,2,...,n\}$ be the set of suppliers. Each supplier supplies exactly one of the heterogeneous items and belongs to exactly one group (or country)[2]. Let I_l be the set of suppliers who may supply item l, and O_k the set of suppliers coming from country k. The set of suppliers can be portioned in two different ways :

$$\bigcup_{l=1}^{L} I_l = \mathscr{N} = \bigcup_{k=1}^{K} O_k \tag{3.1}$$

With

$$I_l \cap I_{l'} = \emptyset \text{ and } O_k \cap O_{k'} = \emptyset, \forall l, l', k, k' \tag{3.2}$$

(3.2) means that every supplier may supply only one item and comes from only one country. Note that this model allows situations in which some items may not be supplied in all countries. If supplier $i \in \mathscr{N}$ wins the procurement it will cost c_i to provide the unit. But if he looses he suffers no cost. Costs are independently distributed and supplier i's cost c_i is distributed according to the probability density function f_i, and the cumulative density function F_i with support $\Gamma_i = [\underline{c}_i, \bar{c}_i]$. Denote $\Gamma = \underset{i \in \mathscr{N}}{\times} \Gamma_i$ the Cartesian product of these supports.[3] The costs' distributions and the countries of the suppliers are common knowledge to the buyer and the suppliers. Every supplier observes privately his own cost but not the other suppliers' costs. There is no cooperation between suppliers. In this set up the winners of the procurement constitute a subset (a list) of L suppliers such that any two of them supply different items. We denote $\mathscr{P}$ the collection of all such lists and call it the set of potential winning lists. The buyer's utility for purchasing the bundle of items is v.

There are many different mechanisms to purchase the bundle. For instance, the buyer may buy the units through L simultaneous first price auctions, one for each item. In this

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2. These groups may be built, for example, according to suppliers' countries or their province of origin or any other characteristic. To be specific, in what follows, we assume that groups are built according to suppliers' countries.

3. In all the paper, scalars and scalar functions are denoted by lowercase letters. Vectors and vector functions are denoted by boldface lowercase letters. $\mathbf{u}_{-i}$ denotes the vector $\mathbf{u}$ without the component of order i : $\mathbf{u} = (u_i, \mathbf{u}_{-i})$. Conditional expectations are denoted by the uppercase of the same letter e.g. : $X(c_i) = E_{\mathbf{c}_{-i}}[x(c_i, \mathbf{c}_{-i})]$.

Finally given a set S, a subset T of S and a vector $\mathbf{u} = (u_i)_{i \in S}$, we denote $(u_i)_{i \in T}$ the vector obtained by removing the component of $\mathbf{u}$ whose orders are not in T.

case, suppliers are asked to bid the price they are willing to accept, and the buyer takes each item from the supplier with the lowest bid among the potential suppliers of the item at a price equal to the lowest bid. He may also buy each item through a second price auction (for each item the potential suppliers submit their bids, and the item is bought from the supplier with the lowest bid at a price equal to the second lowest bid). The buyer can even practice different kind of auctions on each item. In the mechanisms considered so far items are bought separately : the buyer's decision concerning the supplier to whom he purchases an item and its final unit price is independent on his decision concerning the supplier to whom he purchases another item and the related price. In that sense, the procurement of a unit is not related to the procurement of another unit. There exist mechanisms under which the procurements of the units are not separated. For example, consider the following constraint (R1) : "the buyer must buy the items from suppliers coming from all the countries". Let $\mathscr{R}_1$ be the collection of all the potential winning lists that involve all the countries : $\mathscr{R}_1 \subset \mathscr{P}$. The buyer is constrained to purchase only from the lists in $\mathscr{R}_1$.

Example 3. *2 countries, 2 items and 4 suppliers*

Here $n = 4$; *suppliers from the first country are in* $O_1 = \{1,2\}$, *suppliers from the second country are in* $O_2 = \{3,4\}$; *suppliers of the first item are in* $I_1 = \{1,3\}$ *and suppliers of the second item are in* $I_2 = \{2,4\}$;

In this case, since there are two items needed, suppliers win the procurement in pairs : the buyer can only purchase from one of the following pairs $\{1,2\},\{2,3\},\{3,4\}$ *or* $\{4,1\}$: $\mathscr{P} = \{\{1,2\},\{2,3\},\{3,4\},\{4,1\}\}$.[4]

Under constraint (R1) the buyer can buy only from the pairs $\{1,\ 4\}$ *and* $\{2,\ 3\}$: $\mathscr{R}_1 = \{\{1,\ 4\},\{2,\ 3\}\}$.[5] *Thus if he buys the first item from supplier* 1 *he must buy the second item from supplier* 4. *An example of mechanism under (R1) is the following :*

supplier i submits a bid b_i *and the units are bought from the pair* $\{1,\ 4\}$ *if* $b_1 + b_4 <$

4. The pair $\{1,3\}$ is excluded because 1 and 3 sell the same good and the buyer needs only a single unit of it. $\{2,4\}$ is excluded for the same reason.

5. The pair $\{1,2\}$ is excluded because 1 and 2 come from the same country though they sell different goods. $\{3,4\}$ is excluded for the same reason.

b_2+b_3, and from the pair $\{2,\ 3\}$ if $b_1+b_4>b_2+b_3$. In any case the winners are paid a price equal to their bid but the other suppliers receive nothing.

We introduce additional definitions.

In a typical procurement mechanism, the buyer first announces the procurement rules to the suppliers. Suppliers observe privately their own costs and place their bids. Finally the buyer collects the bids and buys the items according to the rules set before.

Let Ω_i be the set of all possible bids for supplier $i \in \mathscr{N}$, and let $\Omega = \prod_{i=1}^{n} \Omega_i$. The set Ω is determined by the type of information that the buyer requires to each supplier during the procurement process. So we may assume that the buyer knows the set Ω of the information he requires. Therefore suppliers cannot bid out of that set.

A *strategy* for supplier i is a function $\beta_i : \Gamma_i \to \Omega_i$ transforming i's cost into a bid.

An *allocation rule* is a function $\mathbf{q} = (q_G)_{G\in\mathscr{P}} : \Omega \to \mathbb{R}_+^{|\mathscr{P}|}$ such that :

$$\text{for any bid vector } \mathbf{b} \in \Omega,\ \sum_{G\in\mathscr{P}} q_G(\mathbf{b}) \leq 1.$$

Where $q_G(\mathbf{b})$ is the probability that list G wins. $\sum_{G\in\mathscr{P}} q_G(\mathbf{b}) = 0$ would simply mean that the buyer refuses to purchase the heterogeneous items when the bid vector is $\mathbf{b}$.

The *individual allocation rule* associated with the allocation rule $\mathbf{q}$ is the function $\mathbf{x} = (x_i)_{i\in\mathscr{N}} : \Omega \to \mathbb{R}_+^n$ such that :

$$\text{for any supplier } i \in \mathscr{N} \text{ and any bid vector } \mathbf{b} \in \Omega, x_i(\mathbf{b}) = \sum_{G\in\mathscr{P}: i\in G} q_G(\mathbf{b}).$$

$x_i(\mathbf{b})$ is supplier i's winning probability when the bid vector is $\mathbf{b}$.[6] Note that this definition implies that $\sum_{i\in I_l} x_i(\mathbf{b}) = \sum_{G\in\mathscr{P}} q_G(\mathbf{b}), \forall l$. i.e. all items have the same probability to be purchased and this probability is the probability to purchase the bundle.

A *payment rule* is a function $\mathbf{t} = (t_i)_{i\in\mathscr{N}} : \Omega \to \mathbb{R}^n$ such that $t_i(\mathbf{b})$ is the amount of money paid to supplier i when the bid vector is $\mathbf{b}$. Note that this payment can be negative, meaning that supplier i will have to make a transfer to the buyer rather than receive from

6. In example 3 , $x_1(\mathbf{b}) = q_{\{1,2\}}(\mathbf{b}) + q_{\{4,1\}}(\mathbf{b})$.

him.

A *procurement mechanism* is defined by a set of bids, an allocation rule and a payment rule. We use the notation $(\Omega, \mathbf{q}, \mathbf{t})$ to refer to a procurement mechanism with a set of bids Ω, an allocation rule $\mathbf{q}$ and a payment rule $\mathbf{t}$. It is important to note that we do not mention the individual allocation rule in the definition of a mechanism since it is uniquely determined by the allocation rule. On the contrary a given individual allocation rule might be associated with many allocation rules. [7]

A *procurement mechanism* $(\Omega, \mathbf{q}, \mathbf{t})$ *is said itemwise* if there exists for every item l two functions $\sigma_l : \underset{i \in I_l}{\times} \Omega_i \rightarrow \mathbb{R}^{|I_l|}$ and $\theta_l : \underset{i \in I_l}{\times} \Omega_i \rightarrow \mathbb{R}^{|I_l|}$ such that for any $\mathbf{b} \in \Omega$,

$$\begin{cases} (x_i(\mathbf{b}))_{i \in I_l} = \sigma_l(\ (b_i)_{i \in I_l}) \\ (t_i(\mathbf{b}))_{i \in I_l} = \theta_l(\ (b_i)_{i \in I_l}) \end{cases},$$

where $\mathbf{x}$ is the individual allocation rule associated with $\mathbf{q}$.

To understand this definition, remember that suppliers in I_l are selling the same item ; the conditions (i.e. allocation and payment) under which one item is purchased depend solely on the message sent by the potential suppliers of that item and not the message sent by the suppliers of other items. Therefore an itemwise mechanism is a mechanism where each item is purchased independently of the others, in a procurement that involves only the potential suppliers of that particular item.

In this paper the buyer may be (legally) constrained to implement allocation rules of a certain type. For instance, in example 3, an allocation rule most satisfy the additional constraint :

$$\text{for any } \mathbf{b} \in \Omega \text{ and } G \in \mathscr{P} \backslash \mathscr{R}_1,\ q_G(\mathbf{b}) = 0.$$

7. Indeed, using the framework of example 3, let $\mathbf{x} = (x_i)_{i \in \mathscr{N}} : \Omega \rightarrow \mathbb{R}^4_+$ be such that $x_1(\mathbf{b}) + x_3(\mathbf{b}) = 1 = x_2(\mathbf{b}) + x_4(\mathbf{b})$. We will build an allocation rule associated with $\mathbf{x}$. Consider a function $\mu : \Omega \rightarrow [0,1]$ such that $\max(0, x_3 + x_4 - 1) \leq \mu \leq \min(x_3, x_4)$; μ exists because $x_3, x_4 \in [0,1] \Rightarrow 0 \leq \max(0, x_3 + x_4 - 1) \leq \min(x_3, x_4) \leq 1$.

Take for example $q_{\{3,4\}} = \mu; q_{\{4,1\}} = x_4 - \mu; q_{\{3,2\}} = x_3 - \mu; q_{\{1,2\}} = 1 - x_3 - x_4 + \mu$;

It is easy to see that $q_G = 1$ and for any $G \in \mathscr{P}, q_G \geq 0$. Thus $\mathbf{q}$ is an allocation rule. Moreover the individual allocation rule associated with $\mathbf{q}$ is precisely x, but the allocation rule $\mathbf{q}$ depends on the selection μ.

More generally let $\mathscr{R}$ be a non empty subset of $\mathscr{P}$.

An *allocation rule* $\mathbf{q}$ *is said* $\mathscr{R}$*-constrained* if the buyer can only purchase from the lists of suppliers in $\mathscr{R}$:

$$\text{for any } \mathbf{b} \in \Omega \text{ and } G \in \mathscr{P} \backslash \mathscr{R},\ q_G(\mathbf{b}) = 0.$$

A *procurement mechanism* $(\Omega, \mathbf{q}, \mathbf{t})$ *is* $\mathscr{R}$*-constrained* if $\mathbf{q}$ *is* $\mathscr{R}$*-constrained.*

If the number of items purchased from suppliers of a group k is constrained to remain between the exogenous parameters α_k and β_k then,

$$\mathscr{R} = \{G \in \mathscr{P} : \alpha_k \leq |O_k \cap G| \leq \beta_k, \forall k \in \{1, 2, ..., K\}\}. \tag{3.3}$$

$|A|$ denotes the number of elements of any set A.

We say that the buyer's environment is *unconstrained* if he is free to implement any allocation rule. Note however that an unconstrained mechanism may be viewed as a $\mathscr{P}$-constrained mechanism.

In the next section we define the set of feasible mechanisms.

3.3 Direct mechanisms

The set of bid vectors Ω can be a complex object, depending on the information the buyer requires from the suppliers. This makes the problem of optimally choosing a mechanism difficult. *Direct mechanisms* are a particular class of mechanisms where each supplier is asked to directly report his cost. Formally a direct mechanism is a mechanism where the set of bid vectors is Γ. When $(\Omega, \mathbf{q}, \mathbf{t})$ is a direct mechanism we shall simply denote it by $(\mathbf{q}, \mathbf{t})$.

3.3.1 The revelation principle

A procurement mechanism induces a game of incomplete information between the suppliers, and the notion of direct mechanism has been defined in the broader context of games with incomplete information. In such games players observe privately an information considered as their types, they send a message and resources are allocated on the basis of the messages sent and predefined rules. The search for an optimal mechanism can be simplified if one can restrict attention to direct mechanisms. The well known revelation principle allows us to make such a restriction without loss of generality. This principle states that : given a game of incomplete information and a Bayesian Nash equilibrium (BNE), [8] there exist a direct mechanism (with the same outcomes as the first game) for which it is a BNE to report honestly the types. For the interested reader we provide (in appendix) a version of the proof of the revelation principle in our framework.

Proposition 12. *(Revelation principle)*

Given a mechanism $(\Omega, \mathbf{q}, \mathbf{t})$ *and a BNE for that mechanism* β*, there exists a direct mechanism* $(\bar{\mathbf{q}}, \bar{\mathbf{t}})$ *in which it is an equilibrium for each supplier to report honestly his cost and the outcomes are the same as in the equilibrium of the first mechanism.*

3.3.2 Incentive compatible and individually rational direct mechanisms

Suppliers need not report their true costs in a direct mechanism since this information is private ; if the buyer cares about truth, he must choose a procurement mechanism that gives them incentives to do so. This condition imposes further restrictions on mechanisms that may be chosen : a procurement mechanism must be *incentive compatible* and *individually rational*. Before we define these two concepts we need to introduce some more notations.

Consider a direct mechanism $(\mathbf{q}, \mathbf{t})$ with an individual allocation rule $\mathbf{x}$.

Let

$$X_i(m_i) =_{\Gamma_{-i}} x_i(m_i, \mathbf{c}_{-i}) f_{-i}(\mathbf{c}_{-i}) d\mathbf{c}_{-i} \tag{3.4}$$

be the (interim) winning probability of supplier i if he reports the value m_i given that the other suppliers report their true costs.

8. A BNE is a profile of strategies such that each player's strategy is optimal against the other players' strategies. We consider interim BNE in which strategies are optimal if they are evaluated when players observe their private types. For more about BNE of games of incomplete information see for example (Fudenberg and Tirole (1991)).

And let

$$T_i(m_i) =_{\Gamma_{-i}} t_i(m_i, \mathbf{c}_{-i}) f_{-i}(\mathbf{c}_{-i}) d\mathbf{c}_{-i} \tag{3.5}$$

be the (interim) expected payment received by supplier i if he reports the value m_i given that the other suppliers report their costs honestly.

The (interim) expected profit of supplier i when he reports m_i (rather than c_i) and the other suppliers report their true costs is :

$$\pi_i(m_i, c_i) = E_{\mathbf{c}_{-i}}[t_i(m_i, \mathbf{c}_{-i}) - x_i(m_i, \mathbf{c}_{-i})c_i] = T_i(m_i) - X_i(m_i)c_i. \tag{3.6}$$

In particular :

$$\pi_i(c_i, c_i) \equiv \ T_i(c_i) - X_i(c_i)c_i. \tag{3.7}$$

$\pi_i(c_i, c_i)$ is the expected profit when supplier i reports his true cost c_i. If honesty (reporting the true cost) is an equilibrium then $\pi_i(c_i, c_i)$ is supplier i's profit at equilibrium. $X_i(c_i)$ and $T_i(c_i)$ are respectively the winning probability and the expected payment of supplier i at equilibrium.

A mechanism is *individually rational* (IR) if :

$$\text{for any } i \in \mathscr{N} \text{ and } c_i \in \Gamma_i,\ \pi_i(c_i, c_i) \geq 0\ . \tag{3.8}$$

This means that even the suppliers with the worst costs will make non negative profits if they participate in the procurement honestly when all the other players do so.

A mechanism is *incentive compatible* (IC) if :

$$\text{for all } i \in \mathscr{N} \text{ and } c_i \in \Gamma_i,\ \ \pi_i(c_i, c_i) = \max_{m_i \in \Gamma_i} \pi_i(m_i, c_i) = \{T_i(m_i) - X_i(m_i)c_i\}. \tag{3.9}$$

This means that reporting honestly his cost give a supplier the highest expected profit when the other suppliers report their true costs. In other words the mechanism $(\mathbf{q}, \mathbf{t})$ is incentive compatible if honesty is an interim BNE of the game induced by $(\mathbf{q}, \mathbf{t})$. The next proposition characterizes an IC mechanism by the winning probabilities and the expected payment functions.

Proposition 13. *A mechanism* $(\mathbf{q},\mathbf{t})$ *is IC if and only if :*

for all $i \in \mathcal{N}$,

$$\textit{the function } X_i \textit{ is decreasing} \tag{3.10}$$

and,

$$\text{for all } c_i \in \Gamma_i,\ T_i(c_i) = T_i(\bar{c}_i) - X_i(\bar{c}_i)\bar{c}_i + X_i(c_i)c_i + \int_{c_i}^{\bar{c}_i} X_i(z)dz. \tag{3.11}$$

Thus, when honesty is a BNE, suppliers with the lowest costs have the highest interim winning probabilities. These winning probabilities determine the expected payments up to a constant. Equation (3.11) is well known in the literature on auction design as the revenue equivalence theorem. Proofs are available in appendix.

In the next section we suppose the buyer has the choice of the procurement mechanism. We look for mechanisms he may choose in order to maximize his expected surplus.

3.4 Surplus maximizing $\mathscr{R}$-constrained mechanism

Assume the buyer uses a direct mechanism $(\mathbf{q},\mathbf{t})$ with an individual allocation rule $\mathbf{x}$. The buyer's expected surplus when suppliers reveal their true cost vector is

$$\pi_0(\mathbf{q},\mathbf{t}) = E\left\{ v(\sum_{G\in\mathscr{P}} q_G(\mathbf{c})) - \sum_{i\in\mathcal{N}} t_i(\mathbf{c}) \right\}.$$

The buyer's goal is to design a direct mechanism maximizing this expected surplus among IC and IR mechanisms.

$$E\sum_{i\in\mathcal{N}} t_i(\mathbf{c}) = \sum_{i\in\mathcal{N}} E[t_i(\mathbf{c})] = \sum_{i\in\mathcal{N}} EE_{\mathbf{c}_{-i}}[t_i(c_i,\mathbf{c}_{-i})] = \sum_{i\in\mathcal{N}} ET_i(c_i)$$

using (3.11),

$$\begin{aligned} E[T_i(c_i)] &= T_i(\bar{c}_i) - X_i(\bar{c}_i)\bar{c}_i + \int_{\underline{c}_i}^{\bar{c}_i} X_i(c_i)c_i f_i(c_i)dc_i \\ &\quad + \int_{\underline{c}_i}^{\bar{c}_i} \left\{ \int_{c_i}^{\bar{c}_i} X_i(z)dz \right\} f_i(c_i)dc_i \end{aligned}$$

Fubini's theorem implies,

$$\begin{aligned} \int_{\underline{c}_i}^{\bar{c}_i} \left\{ \int_{c_i}^{\bar{c}_i} X_i(z)dz \right\} f_i(c_i)dc_i &= \int_{\underline{c}_i}^{\bar{c}_i} \left\{ \int_{\underline{c}_i}^{z} f_i(c_i)dc_i \right\} X_i(z)dz \\ &= \int_{\underline{c}_i}^{\bar{c}_i} F_i(z)X_i(z)dz. \end{aligned}$$

Therefore,

$$\begin{aligned} E[T_i(c_i)] &= T_i(\bar{c}_i) - X_i(\bar{c}_i)\bar{c}_i + \int_{\underline{c}_i}^{\bar{c}_i} [X_i(c_i)\ c_i f_i(c_i) + F_i(c_i)X_i(c_i)]dc_i \\ &= \pi_i(\bar{c}_i, \bar{c}_i) + \int_{\underline{c}_i}^{\bar{c}_i} (c_i + \frac{F_i(c_i)}{f_i(c_i)})X_i(c_i)\ f_i(c_i)dc_i \\ &= \pi_i(\bar{c}_i, \bar{c}_i) + \int_{\Gamma} (c_i + \frac{F_i(c_i)}{f_i(c_i)})x_i(\mathbf{c})\ f(\mathbf{c})d\mathbf{c} \text{ (using (3.4))}. \end{aligned}$$

It follows that,

$$E \sum_{i \in \mathcal{N}} t_i(\mathbf{c}) = \sum_{i \in \mathcal{N}} \pi_i(\bar{c}_i, \bar{c}_i) + \int_{\Gamma} \left\{ \sum_{i \in \mathcal{N}} H_i(c_i)x_i(\mathbf{c}) \right\} f(\mathbf{c})d\mathbf{c} \tag{3.12}$$

where $H_i(m) = m + \frac{F_i(m)}{f_i(m)}$ for all $i \in \mathcal{N}$ and $m \in \Gamma_i$; the function H_i is usually called the *virtual cost* of supplier i.

We may also express the expected price in terms of the allocation rule rather than the individual allocation rule :

$$\begin{aligned}
E\sum_{i\in\mathscr{N}} t_i(\mathbf{c}) &= \sum_{i\in\mathscr{N}} \pi_i(\bar{c}_i,\bar{c}_i) + \int_\Gamma \left\{ \sum_{i\in\mathscr{N}} H_i(c_i) \sum_{G\in\mathscr{P}/i\in G} q_G(\mathbf{c}) \right\} f(\mathbf{c})d\mathbf{c} \\
&= \sum_{i\in\mathscr{N}} \pi_i(\bar{c}_i,\bar{c}_i) + \int_\Gamma \left\{ \sum_{i\in\mathscr{N}} \sum_{G\in\mathscr{P}/i\in G} H_i(c_i) q_G(\mathbf{c}) \right\} f(\mathbf{c})d\mathbf{c} \\
&= \sum_{i\in\mathscr{N}} \pi_i(\bar{c}_i,\bar{c}_i) + \int_\Gamma \left\{ \sum_{G\in\mathscr{P}} \sum_{i\in G} H_i(c_i) q_G(\mathbf{c}) \right\} f(\mathbf{c})d\mathbf{c} \\
&= \sum_{i\in\mathscr{N}} \pi_i(\bar{c}_i,\bar{c}_i) + \int_\Gamma \left\{ \sum_{G\in\mathscr{P}} q_G(\mathbf{c}) \sum_{i\in G} H_i(c_i) \right\} f(\mathbf{c})d\mathbf{c}.
\end{aligned}$$

We define the *aggregate virtual cost* of the list G as $S_G(\mathbf{c}) \equiv \sum_{i\in G} H_i(c_i)$. Using

$$Ev(\sum_{G\in\mathscr{P}} q_G(\mathbf{c})) = E\sum_{G\in\mathscr{P}} q_G(\mathbf{c})v = \int_\Gamma \sum_{G\in\mathscr{P}} q_G(\mathbf{c})vf(\mathbf{c})d\mathbf{c},$$

the expected surplus can be written as :

$$\pi_0(\mathbf{q},\mathbf{t}) = -\sum_{i\in\mathscr{N}} \pi_i(\bar{c}_i,\bar{c}_i) + \int_\Gamma \left\{ \sum_{G\in\mathscr{P}} q_G(\mathbf{c})(v - S_G(\mathbf{c})) \right\} f(\mathbf{c})d\mathbf{c}. \qquad (3.13)$$

For any $i\in\mathscr{N}$, we define the function $K_i : [0,1] \to \mathbb{R}$, as $K_i(z_i) = \int_0^{F_i^{-1}(z_i)} H_i(t)f_i(t)dt$. Note that $K_i'(F_i(c_i)) = H_i(c_i)$.

Let $\hat{K}_i : [0,1] \to \mathbb{R}$ be the convex hull [9] of K_i. $\hat{K}_i$ is differentiable almost surely. Let $\hat{H}_i : \Gamma_i \to \mathbb{R}$ be such that $\hat{H}_i(c_i) = \hat{K}_i'(F_i(c_i))$.

$\hat{H}_i(c_i)$ is called the *ironed out virtual cost* of supplier i and it should be understood as a "priority level" assigned to supplier i when his cost is c_i ; These priority levels are subjective and induce a score for each list of suppliers : $\hat{S}_G(\mathbf{c}) \equiv \sum_{i\in G} \hat{H}_i(c_i)$.

9. i.e. the greatest convex function $g : [0,1] \to \mathbb{R}$ such that $g(z_i) \leq K_i(z_i)$, for any $z_i \in [0,1]$.

Let

$$\hat{\pi}_0(\mathbf{q},\mathbf{t}) = -\sum_{i\in\mathcal{N}} \pi_i(\bar{c}_i,\bar{c}_i) + \int_\Gamma \left\{ \sum_{G\in\mathcal{P}} q_G(\mathbf{c})v \right\} f(\mathbf{c})d\mathbf{c} - \int_\Gamma \left\{ \sum_{i\in\mathcal{N}} \hat{H}_i(c_i)x_i(\mathbf{c}) \right\} f(\mathbf{c})d\mathbf{c}. \tag{3.14}$$

$\hat{\pi}_0$ is obtained from the expression of π_0 by replacing virtual costs by the ironed out virtual costs. Thus we can also write :

$$\hat{\pi}_0(\mathbf{q},\mathbf{t}) = -\sum_{i\in\mathcal{N}} \pi_i(\bar{c}_i,\bar{c}_i) + \int_\Gamma \left\{ \sum_{G\in\mathcal{P}} q_G(\mathbf{c})(v - \hat{S}_G(\mathbf{c})) \right\} f(\mathbf{c})d\mathbf{c}. \tag{3.15}$$

The following three properties of convex hulls are well known :

(a) $\hat{K}_i(0) = K_i(0)$ and $\hat{K}_i(1) = K_i(1)$,

(b) $\hat{K}_i(z_i) \leq K_i(z_i)$ for any $z_i \in [0,1]$,

(c) if $\hat{K}_i(z_i) < K_i(z_i)$ then $\hat{K}_i'$ is constant in some neighborhood of z_i ; hence $\hat{H}_i$ is constant in some neighborhood of $F_i^{-1}(z_i)$.

We are now ready to state our main result. The allocation rule and the payment rule we define just below are optimal in an $\mathcal{R}$-constrained environment ($\emptyset \subsetneq \mathcal{R} \subset \mathcal{P}$).

For any $\mathbf{m} \in \Gamma$, we define the following set :

$$A^{\mathcal{R}}(\mathbf{m}) = \left\{ G \in \mathcal{R} : \hat{S}_G(\mathbf{m}) = \min\left\{ \hat{S}_{G'}(\mathbf{m}) : G' \in \mathcal{R} \right\} \right\}.$$

This set is well defined since $\mathcal{R}$ is finite and it represents the collection of all the potential lists of suppliers having the minimum score among the lists which do not violate the constraint. Thus all the lists in $A^{\mathcal{R}}(\mathbf{m})$ have the same score. Hereafter we refer to elements of $A^{\mathcal{R}}(\mathbf{m})$ as *minimal lists*, and we call $A^{\mathcal{R}}(\mathbf{m})$ the *set of minimal lists* when reported cost is $\mathbf{m}$.

We define the following allocation rule $\mathbf{q}^{\mathscr{R}}$: for any $\mathbf{m} \in \Gamma$ and $G \in \mathscr{P}$,

$$q_G^{\mathscr{R}}(\mathbf{m}) = \begin{cases} 0 & \text{if } G \notin A^{\mathscr{R}}(\mathbf{m}) \\ 0 & \text{if } G \in A^{\mathscr{R}}(\mathbf{m}) \text{ and } \hat{S}_G(\mathbf{m}) > v \\ \frac{1}{|A^{\mathscr{R}}(\mathbf{m})|} & \text{if } G \in A^{\mathscr{R}}(\mathbf{m}) \text{ and } \hat{S}_G(\mathbf{m}) \leq v \end{cases} \tag{3.16}$$

Note that $\mathbf{q}^{\mathscr{R}}$ is $\mathscr{R}$-constrained by definition. Under this allocation rule the buyer purchases from a list of potential suppliers with the lowest score among all the potential lists of suppliers which do not violate the constraint, provided that the lowest score be at most equal to the buyer's valuation of the bundle of items. The items are purchased from any of the minimal lists equiprobably, in case there are many such lists. Observe also that the buyer will not purchase the items when the minimum score exceeds his valuation. Given a bid vector $\mathbf{m} \in \Gamma$ the probability to purchase the items is

$$\sum_{G \in \mathscr{P}} q_G^{\mathscr{R}}(\mathbf{m}) = \begin{cases} 0 & \text{if } v < \min_{G \in \mathscr{R}} \hat{S}_G(\mathbf{m}) \\ 1 & \text{otherwise} \end{cases}. \tag{3.17}$$

Let $\mathbf{x}^{\mathscr{R}}$ be the individual allocation rule associated with $\mathbf{q}^{\mathscr{R}}$; $\mathbf{x}^{\mathscr{R}}$ is such that, for any $\mathbf{m} \in \Gamma$,

$$x_i^{\mathscr{R}}(\mathbf{m}) = \sum_{G \in \mathscr{R} / i \in G} q_G^{\mathscr{R}}(\mathbf{m}). \tag{3.18}$$

We also define the following payment rule :

$$\text{for any } \mathbf{m} \in \Gamma, \ t_i^{\mathscr{R}}(\mathbf{m}) = x_i^{\mathscr{R}}(\mathbf{m}) m_i + \int_{m_i}^{\bar{c}_i} x_i^{\mathscr{R}}(t, \mathbf{m}_{-i}) dt. \tag{3.19}$$

A payment of $x_i^{\mathscr{R}}(\mathbf{m}) m_i$ would mean that supplier i receives an amount equal to his announced cost if he wins the procurement. Beside this, the payment rule $\mathbf{t}$ involves an additional rent $\int_{m_i}^{\bar{c}_i} x_i^{\mathscr{R}}(t, \mathbf{m}_{-i}) dt$ necessary to cause suppliers to reveal their private information honestly.

Remark 1. *In particular $t_i^{\mathscr{R}}(\bar{c}_i, \mathbf{m}_{-i}) = x_i^{\mathscr{R}}(\bar{c}_i, \mathbf{m}_{-i}) \bar{c}_i$ for all $\mathbf{m}_{-i} \in \Gamma_{-i}$. Then taking the expectation we have $T_i^{\mathscr{R}}(\bar{c}_i) = X_i^{\mathscr{R}}(\bar{c}_i) \bar{c}_i$. Thus the expected profit of supplier i with costs*

$\bar{c}_i$ when he bids honestly is $\pi_i^{\mathscr{R}}(\bar{c}_i, \bar{c}_i) = T_i^{\mathscr{R}}(\bar{c}_i) - X_i^{\mathscr{R}}(\bar{c}_i)\bar{c}_i = 0$. A consequence is that the mechanism $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ always yields a worse expected profit than any individually rational mechanism $(\mathbf{q}, \mathbf{t})$ for the supplier i with costs $\bar{c}_i$ when he bids honestly : $\pi_i(\bar{c}_i, \bar{c}_i) \geq 0 = \pi_i^{\mathscr{R}}(\bar{c}_i, \bar{c}_i)$.

The following results shows that the previous mechanism is optimal.

Theorem 2. *The direct mechanism $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ maximizes the expected surplus of the buyer among calR-constrained mechanisms that are IC and IR. The maximum expected surplus is given by $\rho^{\mathscr{R}}(v) = E\left[\max\left[0, v - \min\left\{\hat{S}_G(\mathbf{c}) : G \in \mathscr{R}\right\}\right]\right]$.*

Note that, as expected, $\rho^{\mathscr{R}}(v) \leq \rho^{\mathscr{R}'}(v)$ if $\mathscr{R} \subset \mathscr{R}'$, i.e. the buyer's expected surplus decreases when the environment becomes more constrained. Moreover the allocation rule and the expected surplus increase with v : a buyer with a high valuation for the bundle purchases more often and expect more surplus than a buyer with a lower valuation.

The proof of the theorem is available in appendix and relies on the following results.

Lemma 4. *Let $i \in \mathscr{N}$, $\mathbf{m}_{-i} \in \Gamma_{-i}$ and $\bar{u} \in \Gamma_i$.*

If $i \in G$ and $G \in A^{\mathscr{R}}(\bar{u}, \mathbf{m}_{-i})$ then, for every $\underline{u} \in \Gamma_i : \bar{u} > \underline{u}$, $G \in A^{\mathscr{R}}(\underline{u}, \mathbf{m}_{-i})$ and $A^{\mathscr{R}}(\underline{u}, \mathbf{m}_{-i}) \subset A^{\mathscr{R}}(\bar{u}, \mathbf{m}_{-i})$.

This lemma shows that if a supplier appears in a minimal list with some reported cost, then the list would remain a minimal list if this supplier reported a lower cost while the other suppliers do not change their reports ; Moreover every minimal list would still be minimal if only one supplier changes his report.

The implication $\underline{u} < \bar{u} \Rightarrow A^{\mathscr{R}}(\underline{u}, \mathbf{m}_{-i}) \subset A^{\mathscr{R}}(\bar{u}, \mathbf{m}_{-i})$ is not true in general. The previous lemma provides a sufficient condition for it.

The following lemma is a corollary of lemma 4. It is useful to establish that the mechanism $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ is IC using proposition 13.

Lemma 5. *For any $i \in \mathscr{N}$ and $\mathbf{m}_{-i} \in \Gamma_{-i}$, the functions $q_G^{\mathscr{R}}(., \mathbf{m}_{-i})$ (where $G \in \mathscr{R} : i \in G$), $x_i^{\mathscr{R}}(., \mathbf{m}_{-i})$ and $X_i^{\mathscr{R}}(.)$ are decreasing.*

Finally the following lemma compares the expected surplus $\pi_0(\mathbf{q},\mathbf{t})$ and the modified expected surplus $\hat{\pi}_0(\mathbf{q},\mathbf{t})$ under an IC mechanism.

Lemma 6. *for any IC mechanism* $(\mathbf{q},\mathbf{t})$ *:* $\hat{\pi}_0(\mathbf{q},\mathbf{t}) \geq \pi_0(\mathbf{q},\mathbf{t})$.

3.4.1 Unconstrained environment

Theorem 2 applies in particular in the unconstrained environment, i.e. when $\mathscr{R} = \mathscr{P}$.

If $L = 1$ we fall in the context of a single unit auction. $\mathscr{P}$ is simply the collection of all the singletons of $\mathscr{N}$, and the scores are simply virtual costs :

$$\hat{S}_{\{i\}}(\mathbf{m}) = \hat{H}_i(m_i).$$

The set $A^{\mathscr{P}}(\mathbf{m})$ is the collection of all the potential suppliers who have the minimum virtual cost :

$$A^{\mathscr{P}}(\mathbf{m}) = \left\{i \in \mathscr{N} : \hat{H}_i(m_i) = \min\left\{\hat{H}_j(m_j) : j \in \mathscr{N}\right\}\right\}.$$

The allocation rule and the individual allocation rule are the same (see below). The buyer purchases from a supplier with the lowest virtual cost provided that this virtual cost be at most equal to the buyer's valuation of the single item. When there are many suppliers with the lowest virtual cost he purchases from any one of them with the same probability. The buyer does not purchase the items when the minimum virtual cost exceeds his valuation.

$$q^{\mathscr{P}}_{\{i\}}(\mathbf{m}) = x^{\mathscr{P}}_i(\mathbf{m}) = \begin{cases} 0 & \text{if } i \notin A^{\mathscr{P}}(\mathbf{m}) \\ 0 & \text{if } i \in A^{\mathscr{P}}(\mathbf{m}) \text{ and } \hat{H}_i(m_i) > v \\ \frac{1}{|A^{\mathscr{R}}(\mathbf{m})|} & \text{if } i \in A^{\mathscr{P}}(\mathbf{m}) \text{ and } \hat{H}_i(m_i) \leq v \end{cases}$$

and the payment rule is deduce from this allocation rule. Myerson (1981) designed an optimal mechanism in the context of a single unit auction, our optimal mechanism is simply the procurement version of his mechanism. Our result thus generalizes his.

Below we provide an explicit formula for the individual allocation rule when there is more than one item to buy ($L \geq 2$).

For any item l we define the set of all the suppliers who have the minimum virtual cost among the potential suppliers of that item :

$$A_l(\mathbf{m}) = \left\{ i \in I_l : \hat{H}_i(m_i) = \min \left\{ \hat{H}_j(m_j) : j \in I_l \right\} \right\}. \tag{3.20}$$

The following proposition gives an explicit formula for the individual allocation of the mechanism $(\mathbf{q}^{\mathscr{P}}, \mathbf{t}^{\mathscr{P}})$: the unconstrained surplus maximizing mechanism.

Proposition 14. *The individual allocation rule of the unconstrained surplus maximizing mechanism* $(\mathbf{q}^{\mathscr{P}}, \mathbf{t}^{\mathscr{P}})$ *is given by the equalities :*

$$x_i^{\mathscr{P}}(\mathbf{m}) = \begin{cases} 0 & \text{if } i \notin A_l(\mathbf{m}) \\ 0 & \text{if } i \in A_l(\mathbf{m}) \text{ and } \min_{G \in \mathscr{P}} \hat{S}_G(\mathbf{m}) > v \\ \frac{1}{|A_l(\mathbf{m})|} & \text{if } i \in A_l(\mathbf{m}) \text{ and } \min_{G \in \mathscr{P}} \hat{S}_G(\mathbf{m}) \leq v \end{cases}$$

Remember that under the mechanism $(\mathbf{q}^{\mathscr{P}}, \mathbf{t}^{\mathscr{P}})$ the buyer purchases the items if and only if the minimum score does not exceeds his valuation : $\min_{G \in \mathscr{P}} \hat{S}_G(\mathbf{m}) \leq v$.

The expression of the individual allocation rule shows that when the buyer decides to purchase the items, item l is purchased equiprobably from suppliers who have the minimum virtual cost among potential suppliers of that item. It is true that the set $A_l(\mathbf{m})$ depends only on the bids of suppliers of item l. But in general the minimum score would depend on other suppliers' bids as well. Hence $x_i^{\mathscr{P}}(\mathbf{m})$ would not depend on the bids of suppliers of item l solely. This means that $(\mathbf{q}^{\mathscr{P}}, \mathbf{t}^{\mathscr{P}})$ is not *itemwise* [10] in general. The main reason lies on the fact that the heterogeneous items are complements. The buyer needs them altogether. He has a valuation for the bundle and not for particular items. An itemwise mechanism would have to meet this requirement. This implies more costs. We further discuss this subject when the environment is constrained in the next section.

10. See section 2 for a definition.

3.4.2 Itemwise mechanisms

This section examines the possibility for an itemwise mechanism to be optimal in a constrained environment. We've already shown that the optimal unconstrained mechanism is not itemwise in general. But this alone does not rule out the possibility for some other optimal mechanism to be itemwise. Indeed theorem 2 does not claim the unicity of the optimal solution. We argue below that itemwise mechanisms would not be optimal in general, and we provide a necessary condition for the existence of an optimal mechanism that is also itemwise.

First we must observe that the itemwise property of a mechanism carries to his equivalent direct mechanism. This can be seen through the proof of the revelation principle provided in appendix. So when we are dealing with an itemwise mechanism we may assume without loss that it is a direct mechanism.

Let $(\mathbf{q},\mathbf{t})$ be an itemwise mechanism and let $\mathbf{x}$ be the associated individual allocation rule. By definition of an allocation rule the probability to purchase an item is the same for all items : it is simply the probability to purchase the bundle. Formally :

$$\sum_{i\in I_l} x_i(\mathbf{c}) = \sum_{G\in\mathscr{R}} q_G(\mathbf{c}), \forall c \in \Gamma.$$

Now, because the mechanism is itemwise, the probability to purchase an item l from a supplier i only depends on bids of suppliers of that item $x_i(\mathbf{c}) = x_i((c_i)_{i\in I_l})$. Thus the probability to purchase the bundle itself depends solely on bids by suppliers of item l. Since item l is arbitrary, and since the sets I_l form a partition of the set of suppliers, we conclude that the probability to purchase is constant : $\sum_{G\in\mathscr{R}} q_G(\mathbf{c})$ is constant.

We learn from the proof of theorem 2 that, with an optimal allocation rule, the expected profit of the buyer when costs vector is $\mathbf{c}$ must be :

$$\sum_{G\in\mathscr{R}} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c})) = \max\left\{0, \left((v-\hat{S}_G(\mathbf{c}))\right)_{G\in\mathscr{R}}\right\} \text{ almost surely (a.s).}$$

This implies that the allocation rule eventually gives positive probability only to the maximal elements of the finite sequence $\left\{0, \left((v-\hat{S}_G(\mathbf{c}))\right)_{G\in\mathscr{R}}\right\}$. So,

$$\text{if } G \notin A^{\mathscr{R}}(\mathbf{c}) \text{ then } q_G(\mathbf{c}) = 0.$$

All this implies that

$$\sum_{G\in\mathscr{R}} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c})) = \sum_{G\in A^{\mathscr{R}}(\mathbf{c})} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c})).$$

Given that for any $G \in A^{\mathscr{R}}(\mathbf{c})$ we have $\hat{S}_G(\mathbf{c}) = \min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c})$, we conclude that

$$\sum_{G\in\mathscr{R}} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c})) = (v-\min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c})) \sum_{G\in A^{\mathscr{R}}(\mathbf{c})} q_G(\mathbf{c}) \text{ (a.s).}$$

The same is true for mechanism $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ and therefore :

$$(v-\min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c})) \left[\sum_{G\in A^{\mathscr{R}}(\mathbf{c})} q_G(\mathbf{c}) - \sum_{G\in A^{\mathscr{R}}(\mathbf{c})} q_G^{\mathscr{R}}(\mathbf{c})\right] = 0 \text{ (a.s).}$$

Recall that under $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ the probability to purchase depends on the sign of $v - \min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c})$ (i.e. $\sum_{G\in A^{\mathscr{R}}(\mathbf{c})} q_G^{\mathscr{R}}(\mathbf{c}) = 1(\min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c}) \leq v)$) ; under $(\mathbf{q}, \mathbf{t})$ however the probability to purchase is a constant ρ. We deduce

$$(v-\min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c})) \left[\rho - 1(\min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c}) \leq v)\right] = 0 \text{ (a.s).}$$

This implies that

$$\min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c}) \leq v \text{ (a.s) or } \min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c}) \geq v \text{ (a.s).} \tag{3.21}$$

Hence the buyer's valuation most be too "high" or too "low".

Proposition 15. *There exists an itemwise mechanism that maximizes the buyer's expected surplus only if his valuation (almost) never falls short of the minimum score, or it is (almost) never higher than the minimum score, of the potential lists satisfying the* *constraint. Therefore, when conditions (3.21) are not satisfied, the buyer would prefer to purchase the items in a joint auction rather than through separated auctions.*

Note that in case the buyer's valuation is "high enough", he almost surely purchases the items. In this case maximizing the surplus is simply equivalent to minimizing the payment for purchasing the items. On the contrary if his valuation is not enough high he will not purchase the items.

In general we cannot tell if the converse is true when the environment is constrained. But if the environment is unconstrained the converse is true. Indeed assume that the buyer's valuation is high enough i.e. $\min_{G\in\mathscr{R}} \hat{S}_G(\mathbf{c}) \leq v$ (a.s). Then proposition 14 shows that the individual allocation rule $x_i^{\mathscr{P}}$ is almost surely equal to the individual allocation rule x_i^{∞}, where

$$x_i^{\infty}(\mathbf{m}) = \begin{cases} 0 & \text{if } i \notin A_l(\mathbf{m}) \\ \frac{1}{|A_l(\mathbf{m})|} & \text{if } i \in A_l(\mathbf{m}) \end{cases},$$

and x_i^{∞} is itemwise. Hence the optimal unconstrained mechanism $(\mathbf{q}^{\mathscr{P}}, \mathbf{t}^{\mathscr{P}})$ is almost surely equal to an itemwise mechanism when the buyer's valuation is high enough.

3.5 Example : power distributions

This section is an illustration of our results in the context of the example 3 given in the beginning of the paper.

We suppose $\underline{c}_i = 0$ and $\bar{c}_i = 1$, $F_i(c_i) = (c_i\ )^{a_i}$ and $a_i \geq 1$ for all $i \in \mathscr{N}$ and $c_i \in \Gamma_i$. The probability density functions are given by $f_i(c_i) = a_i\,(c_i)^{a_i - 1}$, and the virtual cost of supplier i is

$$H_i(c_i) = c_i + \frac{F_i(c_i)}{f_i(c_i)} = (1 + \frac{1}{a_i})c_i = \varepsilon_i c_i.$$

Since H_i is increasing we have

$$H_i = \hat{H}_i.$$

An increase in a_i (a decrease in ε_i) results in a decrease of $F_i(c)$ which is the probability that supplier i's cost be lower than c. Thus the parameter a_i somehow describes the technology level of supplier i. The higher a_i the worse i's technology.

We assume that the buyer faces the constraint to purchase from suppliers coming from all the countries. The surplus maximizing mechanism of a buyer with valuation v is given below. The allocation rule $\mathbf{q}^{\mathscr{R}}$ is such that, for any $\mathbf{m} \in \Gamma$:

$$\begin{cases} q^{\mathscr{R}}_{\{1,4\}}(\mathbf{m}) = 1 \text{ if } \varepsilon_1 m_1 + \varepsilon_4 m_4 < \min\left[\varepsilon_2 m_2 + \varepsilon_3 m_3, v\right] \\ q^{\mathscr{R}}_{\{1,4\}}(\mathbf{m}) = q^{\mathscr{R}}_{\{2,3\}}(\mathbf{m}) = \frac{1}{2} \text{ if } \varepsilon_1 m_1 + \varepsilon_4 m_4 = \varepsilon_2 m_2 + \varepsilon_3 m_3 \leq v \\ q^{\mathscr{R}}_{\{2,3\}}(\mathbf{m}) = 1 \text{ if } \varepsilon_2 m_2 + \varepsilon_3 m_3 < \min\left[\varepsilon_1 m_1 + \varepsilon_4 m_4, v\right]. \end{cases}$$

Note also that the individual allocation rule is such that : $x^{\mathscr{R}}_1(\mathbf{m}) = x^{\mathscr{R}}_4(\mathbf{m}) = q^{\mathscr{R}}_{\{1,4\}}(\mathbf{m})$ and $x^{\mathscr{R}}_2(\mathbf{m}) = x^{\mathscr{R}}_3(\mathbf{m}) = q^{\mathscr{R}}_{\{2,3\}}(\mathbf{m})$.

The items are purchased from the list with the smallest weighted sum of reports, if this weighted sum does not exceed the valuation. Reports' weights are subjective and equal ε_i for supplier i. Low technology suppliers are advantaged by the optimal mechanism.

3.6 Conclusion

Most of the literature on procurement auctions does not consider environments where winning lists most respect exogenous quotas for different groups of suppliers. However there are situations in life where such restrictions apply. In this paper we have derived a procurement mechanism maximizing the expected surplus of a buyer in an environment constrained by restrictions such as the "quotas restriction" and where suppliers' information is independent and private. An optimal allocation rule consists of assigning priority levels to suppliers on the basis of their cost report. The way these priority levels are determined is subjective but known to all before the auction. The individual reports induce scores for each potential winning list by summing the priority levels of suppliers in the list. The items are then (equiprobably) purchased from one of the lists with the best score, provided it is not greater than the buyer's valuation for the items. Payments are only made to suppliers who win the auction. Our optimal mechanism generalizes Myerson (1981) concerning both the allocation and the payment rules.

In general it is not optimal to purchase the items through separate auctions. Unless the buyer's valuation for the items is high (so that he cannot reliably commit to not purchase) or too low (when he cannot reliably commit to purchase) it is not optimal to purchase separately. Conversely when the environment is unconstrained, i.e. when one is free to purchase from any list, it is optimal for a high value buyer to purchase the items separately.

In the particular case of power distributions, each supplier is assigned a specific weight : This weight describes the technology level of the supplier : the higher a supplier's weight, the better his technology. The score of a list is simply the weighted sum of the reports by suppliers of that list. Lists with low weighted sum have the priority ; hence the optimal allocation rule advantages low technology suppliers. The buyer cannot reliably commit to not purchase when his valuation is higher than the priority level of the best list, when all suppliers have their worst (highest) possible unit cost.

Future directions of research may include relaxing the assumption of independent costs and allow in the model situations where some suppliers may supply more than one item. We also assumed that supplier's do not act in cooperation ; it would also be interesting to analyze an environment where suppliers may act in a concerted way.

CONCLUSION

Le premier essai a analysé le problème de choisir une alternative pour un groupe d'agents ayant des informations privées et des intérêts divers. L'objectif du concepteur était de maximiser le surplus collecté des agents parmi les mécanismes efficaces qui induisent une participation honnête de tous les agents. Dans cet environnement, les agents qui ne participent pas au processus de décision pourraient néanmoins être affectés par la décision finale. En plus d'une règle d'attribution et d'une règle de paiement, le concepteur peut choisir des menaces appropriées pour chaque agents dans le but de les inciter à participer et de maximiser son propre surplus espéré. Puisqu'un agent qui ne participe pas ne révèle pas son information privée, le planificateur décide de lui même ce qu'il considérera comme tel si un agent venait à ne pas participer, et menace de choisir l'alternative qui donnerait la plus mauvaise utilité à cet agent sous ces conditions. Un choix de type max-min du type présumé et de la menace permettent de maximiser le surplus espéré du planificateur parmi les mécanismes efficaces induisant la participation honnête de tous les agents. Je fournis également un résultat d'existence pour une fonction d'utilité extérieure qui peut être décomposée en deux composantes additives : l'une exogène et l'autre endogène. J'applique ces résultats à la conception d'une enchère multiple efficace dans un environnement où un acheteur en possession du bien cause des externalités négatives sur les autres agents. Je montre qu'une généralisation de l'enchère de Vickrey maximise le surplus parmi mécanismes efficaces induisant une participation honnête. D'autres applications possibles incluent la localisation d'équipements nocifs, les élections, le lieu de déroulement de manifestations sportives, la vente d'armes nucléaires etc.

Dans certaines situations le planificateur aimerait exécuter un mécanisme qui laisserait toujours sa balance en équilibre (surplus nul). De tels mécanismes sont dit de budgets équilibrés. l'existence de tels mécanismes est déterminée par le signe du surplus espéré maximal. Nous argumentons que l'existence des mécanismes de budgets équilibrés par budget est garantie si n'importe quel mécanisme efficace induisant la participation honnête de tous les agents a comme conséquence un surplus espéré positif. Ceci peut être utile dans des applications où une forme analytique pour le surplus maximum ne peut pas être trouvée. Dans de telles applications il est suffisant d'avoir une bonne approximation du mécanisme optimal, laquelle donnerait lieu à un surplus espéré positif. Cette approximation peut alors être utilisée pour construire un mécanisme de budget-équilibré.

Dans le deuxième essai nous avons prouvé qu'une exécution appropriée de la règle du juste retour peut la rendre moins coûteuse que les enchères traditionnelles de libre concurrence (enchères premier prix et deuxième prix). Nous avons considéré le cas de l'information complète (pour l'enchère des premiers prix) où les niveaux de technologie des firmes sont de notoriété publique, et le cas de l'information incomplète (pour enchère de deuxième prix) où les sociétés observent en privé leurs coûts de production. Dans les deux des cas nous avons identifié une enchère sous le principe du juste retour qui tire avantage des asymétries entre les pays et est moins coûteuse que l'enchère traditionnelle. Le prix (resp. prix espéré) des articles sous la règle du juste retour est inférieure au prix sous l'enchère traditionnelle dans les situations où (resp. l'agence croit que) : un état possède un niveau de technologie plus élevé pour la production des articles que l'autre, et de plus les avantages de chacun des fournisseurs de l'état en avance (comparé à ceux de leurs adversaires directs respectivement) sont suffisamment proches l'un de l'autre.

Nous avons également supposé que l'agence n'impose pas des prix de réserve. Cette hypothèse peut être justifiée par le fait que la valeur que l'agence attribue aux articles est reconnue pour être trop haute en comparaison des coûts des fournisseurs. De sorte que l'acheteur ne puisse pas s'engager de façon crédible à ne pas du tout acheter les articles. Toutefois il serait intéressant de comparer les deux principes dans un modèle qui tient compte des prix de réserve.

Le troisième essai élabore un mécanisme d'appel d'offre maximisant le surplus espéré d'un acheteur dans un environnement contraint par des quotas pour chacun des différent groupes d'agents y participant. Une règle optimale d'allocation consiste en l'attribution de niveaux de priorité aux fournisseurs sur la base des coûts unitaires qu'ils rapportent au décideur. Les coûts rapportés induisent des scores pour chaque potentielle listes de gagnant, obtenus en additionnant les niveaux de priorité des fournisseurs dans la liste. Les articles sont alors achetés d'une des listes ayant le meilleur score, pourvu qu'il ne soit pas plus grand que la valeur que l'acheteur attribue aux articles. Des paiements sont seulement effectués aux fournisseurs qui gagnent l'enchère. Notre mécanisme optimal est une généralisation de Myerson (1981).

En général il n'est pas optimal d'acheter les articles par enchères séparées, à moins que la valeur que l'acheteur attribue aux articles ne soit trop haute (de sorte qu'il ne peut pas s'engager de façon crédible à ne pas acheter) soit trop basse (quand il ne peut pas s'engager de façon crédible à l'achat). Réciproquement quand l'environnement n'est pas contraint, c.-à-d. quand on est libre d'acheter de n'importe quelle liste, il est optimal qu'un acheteur de valeurs élevées achète les articles séparément.

Dans le cas particulier où les coûts suivent une distribution "puissance", à chaque fournisseur est assignée un poids : Ce poids décrit le niveau de technologie du fournisseur : le plus haut est le poids meilleur est la technologie. Le score d'une la liste est simplement la somme pondérés des couts rapportés par les fournisseurs de cette liste. Les listes ayant la somme la plus basse ont la priorité ; par conséquent cette règle quoique optimale favorise les fournisseurs les moins avancés technologiquement.

Nous avons supposé tout au long de cette thèse que les agents ou les fournisseurs n'agissent pas en coopération et que leurs types sont indépendants les uns des autres. Il serait également intéressant d'analyser un environnement où ils peuvent agir d'une manière concertée d'une part et d'autre part où leurs types sont plus ou moins corrélés.

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nance, 16 : 8-37.

Annexe I

Appendix to chapter 2

Proof of proposition 6. For the FP auction the argument is this : the supplier with the lowest cost, having more latitude to place low bid than his opponent, maximizes his profit by bidding just above his opponent's cost. If the opponent then bids his own cost, we have an equilibrium provided that the difference between the costs is not small. In this last case an equilibrium is to bid the highest cost for both suppliers.

We prove the existence of a Nash Equilibrium (NE) for the FR auction by assuming without loss of generality that $c_1 + c_4 + \gamma = c_2 + c_3$ with $\gamma \geq 0$. There are only to cases : $\gamma \leq 2\varepsilon$ or $\gamma \geq 3\varepsilon$.

If $\gamma \leq 2\varepsilon$ then $b_1^* = c_1 + \gamma$ and $b_i^* = c_i$ for $i \neq 1$ is a NE. Indeed, we have $b_1^* + b_4^* = b_2^* + b_3^*$ and this implies that no supplier has incentive to increase his bid since this would weaken his pair and cause him to lose certainly. In addition, only supplier 1 may consider to reduce his bid ; if he does reduce his bid of ε then his profit decreases from $\frac{1}{2}\gamma$ to $\gamma - \varepsilon$.

If $\gamma \geq 3\varepsilon$ then $b_1^* = c_1 + \gamma - 2\varepsilon, b_4^* = c_4 + \varepsilon$ and $b_i^* = c_i$ for $i = 2, 3$ is a NE. With such bids we have $b_1^* + b_4^* + \varepsilon = b_2^* + b_3^*$ so that the auction is won by the pair $\{1, 4\}$ and none of suppliers 2 and 3 have incentive to increase their bids ; they cannot reduce their bid since it would lead to a negative profit. Supplier 4, whose profit is ε, makes a worse profit if he increases his bid of more than ε or decreases it. If he increases his bid of ε his profit remains the same ε. Similarly, supplier 1, whose profit is $\gamma - 2\varepsilon$, makes a worse profit if he increases his bid of more than ε or decreases it. If he increases his bid of ε his new profit is $\frac{1}{2}(\gamma - \varepsilon)$; which is less than $\gamma - 2\varepsilon$. So no supplier has incentive to change his bid given the others' bids. □

Proof of lemma 3. (i) The argument is this : supplier with the lowest cost, having more latitude to place low bid than his opponent, maximizes his profit by bidding just above his opponent's cost. Note however that if the difference between the costs is small, it is possible to have an equilibrium where the supplier with the lowest cost submits a bid just equal to his opponent's cost.

(ii) let $(b_i^*)_{i\in\mathcal{N}}$ be a Nash equilibrium. The total price of the items is $fr = \min(b_1^* + b_4^*, b_2^* + b_3^*)$. We will show that $fr \leq 4\varepsilon + \max(c_1+c_4, c_2+c_3)$ when $b_1^*+b_4^* = b_2^*+b_3^*$ and $b_1^*+b_4^* < b_2^*+b_3^*$. The case $b_1^*+b_4^* > b_2^*+b_3^*$ is similar to the last one.

- Suppose $b_1^*+b_4^* = b_2^*+b_3^*$; supplier i's profit is $\frac{1}{2}(b_i^*-c_i)$ and we have $\frac{1}{2}(b_i^*-c_i) \geq (b_i^*-\varepsilon-c_i)$ since he has no interest in changing his bid to $b_i^*-\varepsilon$. This implies successively $b_i^* \leq 2\varepsilon + c_i$ for any i, $b_1^*+b_4^* \leq 4\varepsilon + c_1 + c_4$ and $b_2^*+b_3^* \leq 4\varepsilon + c_2 + c_3$, and finally $fr \leq 4\varepsilon + \max(c_1+c_4, c_2+c_3)$.

- Suppose $b_1^*+b_4^* < b_2^*+b_3^*$; then suppliers 2 and 3 both have zero profit. If supplier 2 changes his bid to $b_2^*-\varepsilon$ then his profit $((b_2^*-\varepsilon-c_2)$ or $\frac{1}{2}(b_2^*-\varepsilon-c_2)$ in case of tie) remains non positive. therefore $b_2^* \leq \varepsilon + c_2$. The same argument leads to $b_3^* \leq \varepsilon + c_3$. So $b_2^*+b_3^* \leq 2\varepsilon + c_2 + c_3 \leq 2\varepsilon + \max(c_1+c_4, c_2+c_3)$ and the result follows. □

Proof of proposition 9. We assume that $c_1+c_4 \leq c_3+c_2$, then $c_1-c_3 \leq c_2-c_4$. The proof is similar in the case $c_1+c_4 \geq c_3+c_2$.

If $c_1-c_3 \leq c_2-c_4 < 0$ then $2\delta(\mathbf{c}) = 3[(c_2-c_4)-(c_1-c_3)]-(c_3-c_1)-(c_4-c_2) = -2(c_1-c_3)+4(c_2-c_4)$ and $\frac{c_1-c_3}{c_2-c_4} \geq 1$; Thus $sfr(\mathbf{c})-sp(\mathbf{c}) < 0$ if and only if $\frac{c_1-c_3}{c_2-c_4} < 2$.

if $0 < c_1-c_3 \leq c_2-c_4$ then $2\delta(\mathbf{c}) = 3[(c_2-c_4)-(c_1-c_3)]-(c_1-c_3)-(c_2-c_4) = -4(c_1-c_3)+2(c_2-c_4)$ and $\frac{c_1-c_3}{c_2-c_4} \leq 1$; in this case $sfr(\mathbf{c})-sp(\mathbf{c}) < 0$ if and only if $\frac{c_1-c_3}{c_2-c_4} > \frac{1}{2}$.

if $c_1-c_3 \leq 0 \leq c_2-c_4$, then $2\delta(\mathbf{c}) = 3[(c_2-c_4)-(c_1-c_3)]+(c_1-c_3)-(c_2-c_4) = -2(c_1-c_3)+2(c_2-c_4) \geq 0$; So that $sfr(\mathbf{c})-sp(\mathbf{c}) \geq 0$.

In all this cases the equivalence $[sfr(\mathbf{c})-sp(\mathbf{c}) < 0 \Leftrightarrow \frac{1}{2} < \frac{c_1-c_3}{c_2-c_4} < 2]$ is true, thus it is always true. □

Proof of proposition 11. We first prove that (i) and (ii) are equivalent.

(i)⇒(ii) Suppose $\lambda(N)=0$; then $\mu(N)=0$ because μ is absolutely continuous with respect to the lebesgue measure λ ; therefore $\int_N |\delta| d\mu = 0$ and, using (2.14), $sfr - sp = \int_P |\delta| d\mu \geq 0$.

(ii)⇒(i) Suppose $\lambda(N) > 0$. We will show that there exist a measure μ with support T such that $\int_T \delta d\mu = \int_P |\delta| d\mu - \int_N |\delta| d\mu < 0$. Consider any measure μ_0 with support T. If $\int_T \delta d\mu_0 < 0$ then simply use $\mu = \mu_0$. Assume now that $\int_T \delta d\mu_0 \geq 0$. Let μ_1 be the uniform distribution on N. It is well defined because $\lambda(N) > 0$; moreover $\int_P |\delta| d\mu_1 = 0$ and $\int_T \delta d\mu_1 = -\int_N |\delta| d\mu_1 < 0$. Define $\mu = (1-\theta)\mu_0 + \theta\mu_1$. We have $\int_T \delta d\mu = (1-\theta)\int_T \delta d\mu_0 + \theta \int_T \delta d\mu_1$ and it is negative for $\theta \in (0,1)$ chosen sufficiently close to 1. Moreover the support of $(1-\theta)\mu_0 + \theta\mu_1$ is necessarily T.

(ii)⇔(iii) let $x = c_1 - c_3$ and $y = c_2 - c_4$ then N has a positive measure if the image set $I = \left\{(x,y) \in D : \frac{1}{2}y < x < 2y\right\}$ has positive measure. Where D is the rectangular $[\underline{x}, \bar{x}] \times [\underline{y}, \bar{y}]$, with $\underline{x}=\underline{c}_1 - \bar{c}_3$, $\bar{x} = \bar{c}_1 - \underline{c}_3$, $\underline{y}=\underline{c}_2 - \bar{c}_4$ and $\bar{y} = \bar{c}_2 - \underline{c}_4$. Consider below the graphic for the set$\left\{(x,y) \in D : \frac{1}{2}y < x < 2y\right\}$:

*itbpF*3.2249*in*2.4249*in*0*inregion.eps*

The bounds $\underline{x}, \bar{x}, \underline{y}$ and $\bar{y}$ can take any value provided that the intersection of the region R and of the rectangular $[\underline{x}, \bar{x}] \times [\underline{y}, \bar{y}]$ have a positive measure. We see that a necessary and sufficient condition for this intersection to have a positive measure is $\begin{cases} \bar{x} \geq 0 \\ \underline{y} \leq 0 \end{cases}$ or $\begin{cases} \underline{x} \leq 0 \\ \bar{y} \geq 0 \end{cases}$. The result follows. □

Proposition 16. *Let* $\Delta(\mathbf{c}) = \max(c_1 + c_4, c_3 + c_2) - \max(c_1, c_3) - \max(c_2, c_4)$.

We have

$$\Delta(\mathbf{c}) = \begin{cases} -\min(|c_1 - c_3|, |c_2 - c_4|) & \textit{if } c_1 - c_3 \textit{ and } c_2 - c_4 \textit{ have the same sign} \\ 0 & \textit{otherwise} \end{cases}$$

Proof. Without loss of generality we may assume that $c_1 + c_4 \leq c_3 + c_2$ (or equivalently $c_1 - c_3 \leq c_2 - c_4$).

If $c_1 - c_3 \leq c_2 - c_4 \leq 0$ then $\Delta(\mathbf{c}) = c_3 + c_2 - c_3 - c_4 = c_2 - c_4 = -|c_2 - c_4|$;

if $0 \leq c_1 - c_3 \leq c_2 - c_4$ then $\Delta(\mathbf{c}) = c_3 + c_2 - c_1 - c_2 = c_3 - c_1 = -|c_1 - c_3|$;

if $c_1 - c_3$ and $c_2 - c_4$ have opposite signs, i.e. if $c_1 - c_3 < 0 < c_2 - c_4$, then $\Delta(\mathbf{c}) = c_3 + c_2 - c_3 - c_2 = 0$. □

Annexe II

Appendix to Chapter 3

Proof of the revelation principle. Consider a mechanism $(\Omega, \mathbf{q}, \mathbf{t})$ and an equilibrium β, define

$$\bar{q}_G(\mathbf{c}) = q_G(\beta(\mathbf{c})) \text{ and } \bar{t}_i(\mathbf{c}) = t_i(\beta(\mathbf{c}));$$

And let $\bar{\mathbf{x}}$ be the individual allocation mechanism associated with $\bar{\mathbf{q}}$.

Suppose that under the direct mechanism $(\bar{\mathbf{q}}, \bar{\mathbf{t}})$ the other suppliers (than i) report their true costs, supplier i's expected profit when he bids $m_i \in \Gamma_i$ rather than the true cost c_i is :

$$E_{\mathbf{c}_{-i}}[\bar{t}_i(m_i, \mathbf{c}_{-i}) - \bar{x}_i(m_i, \mathbf{c}_{-i})c_i];$$

And we have,

$$\begin{aligned}
\bar{t}_i(m_i, \mathbf{c}_{-i}) - \bar{x}_i(m_i, \mathbf{c}_{-i})c_i &= t_i(\beta(m_i, \mathbf{c}_{-i})) - x_i(\beta(m_i, \mathbf{c}_{-i}))c_i \\
&= t_i(\beta_i(m_i), \beta_{-i}(\mathbf{c}_{-i})) - x_i(\beta_i(m_i), \beta_{-i}(\mathbf{c}_{-i}))c_i.
\end{aligned}$$

Observe that $E_{\mathbf{c}_{-i}}[t_i(\beta_i(m_i), \beta_{-i}(\mathbf{c}_{-i})) - x_i(\beta_i(m_i), \beta_{-i}(\mathbf{c}_{-i}))c_i]$ is supplier i's expected profit when he bids $\beta_i(m_i)$, and the other suppliers strategy is β_{-i} under the mechanism $(\Omega, \mathbf{q}, \mathbf{t})$. Since β is an equilibrium,

$$\begin{aligned}
& E_{\mathbf{c}_{-i}}[t_i(\beta_i(m_i), \beta_{-i}(\mathbf{c}_{-i})) - x_i(\beta_i(m_i), \beta_{-i}(\mathbf{c}_{-i}))c_i] \\
\leq\; & E_{\mathbf{c}_{-i}}[t_i(\beta_i(c_i), \beta_{-i}(\mathbf{c}_{-i})) - x_i(\beta_i(c_i), \beta_{-i}(\mathbf{c}_{-i}))c_i] \\
=\; & E_{\mathbf{c}_{-i}}[\bar{t}_i(c_i, \mathbf{c}_{-i}) - \bar{x}_i(c_i, \mathbf{c}_{-i})c_i].
\end{aligned}$$

Thus,

$$E_{\mathbf{c}_{-i}}[\bar{t}_i(m_i, \mathbf{c}_{-i}) - \bar{x}_i(m_i, \mathbf{c}_{-i})c_i] \leq E_{\mathbf{c}_{-i}}[\bar{t}_i(c_i, \mathbf{c}_{-i}) - \bar{x}_i(c_i, \mathbf{c}_{-i})c_i],$$

implying that supplier i's best response is to report his true cost. □

Proof of proposition 13. Denote $\bar{\pi}_i(c_i) \equiv \pi_i(c_i, c_i)$. If $(\mathbf{q}, \mathbf{t})$ is an IC mechanism then the functions $\bar{\pi}_i$ are convex. Indeed let $\lambda \in [0,1]$, and $u, w \in \Gamma_i$:

$$\begin{aligned}
\bar{\pi}_i(\lambda u + (1-\lambda)w) &= \max_{m_i \in \Gamma_i} \{T_i(m_i) - X_i(m_i)(\lambda u + (1-\lambda)w)\} \\
&= \max_{m_i \in \Gamma_i} [\lambda \{T_i(m_i) - X_i(m_i)u\} + (1-\lambda)\{T_i(m_i) - X_i(m_i)w\}] \\
&\leq \lambda \max_{m_i \in \Gamma_i} \{T_i(m_i) - X_i(m_i)u\} + (1-\lambda) \max_{m_i \in \Gamma_i} \{T_i(m_i) - X_i(m_i)w\} \\
&= \lambda \bar{\pi}_i(u) + (1-\lambda)\bar{\pi}_i(w)
\end{aligned}$$

$\bar{\pi}_i$ (being convex) is differentiable almost everywhere in the interior of its domain and the derivative is increasing. The envelope theorem applied to condition (3.9) implies that :

$$\bar{\pi}_i'(c_i) = -X_i(c_i). \tag{II.1}$$

Thus $-X_i$ is increasing and therefore X_i is decreasing.

(II.1) also implies that :

$$\bar{\pi}_i'(c_i) = \bar{\pi}_i(\bar{c}_i) + \int_{c_i}^{\bar{c}_i} X_i(z)dz. \tag{II.2}$$

Replacing $\bar{\pi}_i(c_i)$ by $T_i(c_i) - X_i(c_i)c_i$ leads to (3.11).

Conversely, from (3.11) we can have the expressions of $T_i(c_i)$ and $T_i(m_i)$. Substituting these expressions in what follows, we have :

$$[T_i(c_i) - X_i(c_i)c_i] - [T_i(m_i) - X_i(m_i)c_i] = \int_{c_i}^{m_i} X_i(z)dz - X_i(m_i)(m_i - c_i),$$

or equivalently,

$$\pi_i(c_i, c_i) - [T_i(m_i) - X_i(m_i)c_i] = \int_{c_i}^{m_i} X_i(z)dz - X_i(m_i)(m_i - c_i).$$

This last expression is positive if the function X_i is decreasing. Therefore condition (3.9) holds and the mechanism is IC. □

Proof of lemma 4. We start by showing that if $G \in A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i}) \Rightarrow G \in A^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i})$; indeed if $G \in A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$ then by definition $\hat{S}_G(\bar{u}, \mathbf{c}_{-i}) = \min\left\{\hat{S}_{G'}(\bar{u}, \mathbf{c}_{-i}) : G' \in \mathscr{R}\right\}$. Fix some $G' \in$calR ;

if $i \in G'$ then from $\hat{S}_G(\bar{u}, \mathbf{c}_{-i}) \leq \hat{S}_{G'}(\bar{u}, \mathbf{c}_{-i})$, we deduce successively :

$$\begin{aligned}
\hat{H}_i(\bar{u}) + \sum_{j \in G / j \neq i} \hat{H}_j(c_j) &\leq \hat{H}_i(\bar{u}) + \sum_{j \in G' / j \neq i} \hat{H}_j(c_j), \\
\sum_{j \in G / j \neq i} \hat{H}_j(c_j) &\leq \sum_{j \in G' / j \neq i} \hat{H}_j(c_j), \\
\hat{H}_i(\underline{u}) + \sum_{j \in G / j \neq i} \hat{H}_j(c_j) &\leq \hat{H}_i(\underline{u}) + \sum_{j \in G' / j \neq i} \hat{H}_j(c_j), \\
\text{i.e. } \hat{S}_G(\underline{u}, \mathbf{c}_{-i}) &\leq \hat{S}_{G'}(\underline{u}, \mathbf{c}_{-i}).
\end{aligned}$$

if $i \notin G'$ then, since $\hat{H}_i$ is increasing, we have $\hat{H}_i(\underline{u}) \leq \hat{H}_i(\bar{u})$ and therefore $\hat{S}_G(\underline{u}, \mathbf{c}_{-i}) \leq \hat{S}_G(\bar{u}, \mathbf{c}_{-i})$.

Because $i \notin G'$ we have $\hat{S}_{G'}(., \mathbf{c}_{-i})$ constant ; and from $\hat{S}_G(\bar{u}, \mathbf{c}_{-i}) \leq \hat{S}_{G'}(\bar{u}, \mathbf{c}_{-i})$, we deduce $\hat{S}_G(\bar{u}, \mathbf{c}_{-i}) \leq \hat{S}_{G'}(\underline{u}, \mathbf{c}_{-i})$ and then $\hat{S}_G(\underline{u}, \mathbf{c}_{-i}) \leq \hat{S}_{G'}(\underline{u}, \mathbf{c}_{-i})$.

this shows that $\hat{S}_G(\underline{u}, \mathbf{c}_{-i}) = \min\left\{\hat{S}_{G'}(\underline{u}, \mathbf{c}_{-i}) : G' \in \mathscr{R}\right\}$, i.e. $G \in A^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i})$.

Now we will show that $A^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i}) \subset A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$. Take any $G_0 \in A^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i})$.

- if $i \in G_0$ then, from $\hat{S}_G(\underline{u}, \mathbf{c}_{-i}) = \hat{S}_{G_0}(\underline{u}, \mathbf{c}_{-i})$ we deduce $\hat{S}_G(\bar{u}, \mathbf{c}_{-i}) = \hat{S}_{G_0}(\bar{u}, \mathbf{c}_{-i})$ i.e. $G_0 \in A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$.

- if $i \notin G_0$ then $\hat{S}_{G_0}(., \mathbf{c}_{-i})$ is constant,

$$\begin{aligned}
\hat{S}_{G_0}(\bar{u}, \mathbf{c}_{-i}) &= \hat{S}_{G_0}(\underline{u}, \mathbf{c}_{-i}) \\
&= \hat{S}_G(\underline{u}, \mathbf{c}_{-i}) \text{ (because } G, G_0 \in A^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i}) \text{)} \\
&\leq \hat{S}_G(\bar{u}, \mathbf{c}_{-i}) \text{ (because } \hat{H}_i \text{ is increasing and } i \in G).
\end{aligned}$$

So we also have $G_0 \in A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$. □

Proof of lemma 5. Fix $i \in \mathscr{N}$, $G \in \mathscr{R} : i \in G$, $c_{-i} \in \Gamma_{-i}$ and $\underline{u}, \bar{u} \in \Gamma_i : \underline{u} < \bar{u}$.

We know there are only two cases : either $G \notin A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$ or $G \in A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$.

(i) Suppose $G \notin A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$; then by definition $q_G^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i}) = 0 \leq q_G^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i})$.

(ii) Suppose $G \in A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$ then, by lemma 4, $A^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i}) \subset A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$;

as a consequence $|A^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i})| \leq |A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})|$. Now if $\hat{S}_G(\bar{u}, \mathbf{c}_{-i}) > v$ then $q_G^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i}) = 0 \leq q_G^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i})$. If $\hat{S}_G(\bar{u}, \mathbf{c}_{-i}) \leq v$ then, because $\hat{H}_i$ is increasing and $i \in G$, $\hat{S}_G(\underline{u}, \mathbf{c}_{-i}) \leq v$. Therefore $q_G^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i}) = \frac{1}{|A^{\mathscr{R}}(\underline{u}, \mathbf{c}_{-i})|} \geq \frac{1}{|A^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})|} = q_G^{\mathscr{R}}(\bar{u}, \mathbf{c}_{-i})$.

(i) and (ii) imply that $q_G^{\mathscr{R}}(., \mathbf{c}_{-i})$ is decreasing. Hence $x_i^{\mathscr{R}}(., \mathbf{c}_{-i}) = \sum_{G \in \mathscr{R} / i \in G} q_G^{\mathscr{R}}(., \mathbf{c}_{-i})$ and $X_i^{\mathscr{R}}(.) = E\left[x_i^{\mathscr{R}}(., \mathbf{c}_{-i})\right]$ are decreasing. □

Proof of lemma 6.

$$\begin{aligned}
\pi_0(\mathbf{q},\mathbf{t}) - \hat{\pi}_0(\mathbf{q},\mathbf{t}) &= \int_\Gamma \left\{ \sum_{i \in \mathscr{N}} \hat{H}_i(c_i) x_i(\mathbf{c}) \right\} f(\mathbf{c}) d\mathbf{c} - \int_\Gamma \left\{ \sum_{i \in \mathscr{N}} H_i(c_i) x_i(\mathbf{c}) \right\} f(\mathbf{c}) d\mathbf{c}, \\
&= \sum_{i \in \mathscr{N}} \int_\Gamma \left\{ \hat{H}_i(c_i) - H_i(c_i) \right\} x_i(\mathbf{c}) f(\mathbf{c}) d\mathbf{c}, \\
&= \sum_{i \in \mathscr{N}} \int_{\underline{c}_i}^{\bar{c}_i} \left\{ \hat{H}_i(c_i) - H_i(c_i) \right\} X_i(c_i) f(c_i) dc_i \text{ (using (3.4))}, \\
&= \sum_{i \in \mathscr{N}} \int_{\underline{c}_i}^{\bar{c}_i} \left\{ \hat{K}_i'(F_i(c_i)) - K_i'(F_i(c_i)) \right\} X_i(c_i) f(c_i) dc_i \text{ (by definition of } H_i \text{ and } \hat{H}_i), \\
&= \sum_{i \in \mathscr{N}} \int_{\underline{c}_i}^{\bar{c}_i} \left\{ (\hat{K}_i \circ F_i)'(c_i) - (K_i \circ F_i)'(c_i) \right\} X_i(c_i) dc_i \,.
\end{aligned}$$

Integrating by part,

$$\begin{aligned}
\pi_0(\mathbf{q},\mathbf{t}) - \hat{\pi}_0(\mathbf{q},\mathbf{t}) &= \sum_{i \in \mathscr{N}} \left\{ (\hat{K}_i \circ F_i)(\bar{c}_i) - (K_i \circ F_i)(\bar{c}_i) - (\hat{K}_i \circ F_i)(\underline{c}_i) + (K_i \circ F_i)(\underline{c}_i) \right\} \\
&\quad - \int_{\underline{c}_i}^{\bar{c}_i} \left\{ (\hat{K}_i \circ F_i)(c_i) - (K_i \circ F_i)(c_i) \right\} dX_i(c_i), \\
&= \sum_{i \in \mathscr{N}} \left\{ \hat{K}_i(1) - K_i(1) - \hat{K}_i(0) + K_i(0) \right\} \\
&\quad - \int_{\underline{c}_i}^{\bar{c}_i} \left\{ (\hat{K}_i \circ F_i)(c_i) - (K_i \circ F_i)(c_i) \right\} dX_i(c_i), \qquad \text{(II.3)}
\end{aligned}$$

Finally, using property (a) :

$$\pi_0(\mathbf{q},\mathbf{t}) - \hat{\pi}_0(\mathbf{q},\mathbf{t}) = - \sum_{i \in \mathscr{N}} \int_{\underline{c}_i}^{\bar{c}_i} \left\{ \hat{K}_i(F_i(c_i)) - K_i(F_i(c_i)) \right\} dX_i(c_i) \qquad \text{(II.4)}$$

Since the mechanism is IC, proposition 13 implies that $dX_i(c_i) \leq 0$. Furthermore property (b) implies $\hat{K}_i(F_i(c_i)) - K_i(F_i(c_i)) \leq 0$. It follows that $\pi_0(\mathbf{q},\mathbf{t}) - \hat{\pi}_0(\mathbf{q},\mathbf{t}) \leq 0$. □

proof of Theorem 2. - **First, we prove that $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ is IR.** By definition $t_i^{\mathscr{R}}(\mathbf{m}) \geq x_i^{\mathscr{R}}(\mathbf{m})m_i$ and (integrating over $\mathbf{m}_{-i}$) $T_i^{\mathscr{R}}(m_i) \geq X_i^{\mathscr{R}}(m_i)m_i$, i.e. $\pi_i^{\mathscr{R}}(m_i) \geq 0$. So $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ is IR.

- **Next, we prove that $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ is IC.** Using the definition of $\mathbf{t}^{\mathscr{R}}$: $t_i^{\mathscr{R}}(\mathbf{c}) = x_i^{\mathscr{R}}(\mathbf{c})c_i + \int_{c_i}^{\bar{c}_i} x_i^{\mathscr{R}}(u,\mathbf{c}_{-i})du$,

and integrating this equality over $\mathbf{c}_{-i}$ leads to $T_i^{\mathscr{R}}(c_i) = X_i^{\mathscr{R}}(c_i)c_i + \int_{c_i}^{\bar{c}_i} X_i^{\mathscr{R}}(u,\mathbf{c}_{-i})du$. Since $\pi_i^{\mathscr{R}}(\bar{c}_i,\bar{c}_i) = T_i^{\mathscr{R}}(\bar{c}_i) - X_i^{\mathscr{R}}(\bar{c}_i)\bar{c}_i = 0$, we conclude

$$T_i^{\mathscr{R}}(c_i) = T_i^{\mathscr{R}}(\bar{c}_i) - X_i^{\mathscr{R}}(\bar{c}_i)\bar{c}_i + X_i^{\mathscr{R}}(c_i)c_i + \int_{c_i}^{\bar{c}_i} X_i^{\mathscr{R}}(u,\mathbf{c}_{-i})du.$$

lemma 5 shows $X_i^{\mathscr{R}}$ is decreasing. By proposition 13 we may conclude that $(\mathbf{q}^{\mathscr{R}}, \mathbf{t}^{\mathscr{R}})$ is IC.

- **Finally, we prove that $\pi_0(\mathbf{q},\mathbf{t}) \leq \pi_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}})$** for any IC and IR constrained mechanism $(\mathbf{q},\mathbf{t})$.

Let $(\mathbf{q},\mathbf{t})$ be an $\mathscr{R}$-constrained mechanism IC and IR. Recall that

$$\hat{\pi}_0(\mathbf{q},\mathbf{t}) = -\sum_{i\in\mathscr{N}} \pi_i(\bar{c}_i,\bar{c}_i) + \int_{\Gamma} \left\{ \sum_{G\in\mathscr{P}} q_G(\mathbf{c})(v - \hat{S}_G(\mathbf{c})) \right\} f(\mathbf{c})d\mathbf{c}.$$

Remark 1 shows that $-\sum_{i\in\mathscr{N}} \pi_i(\bar{c}_i,\bar{c}_i) \leq -\sum_{i\in\mathscr{N}} \pi_i^{\mathscr{R}}(\bar{c}_i,\bar{c}_i)$.

Because $(\mathbf{q},\mathbf{t})$ is $\mathscr{R}$-constrained,

$$\sum_{G\in\mathscr{P}} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c})) = \sum_{G\in\mathscr{R}} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c})) = (1-\sum_{G\in\mathscr{R}} q_G(\mathbf{c})).0 + \sum_{G\in\mathscr{R}} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c})). \tag{II.5}$$

This is a weighted mean of the sequence $\left\{0, \left((v-\hat{S}_G(\mathbf{c}))\right)_{G\in\mathscr{R}}\right\}$, and we have :

$$\begin{aligned}\sum_{G\in\mathscr{P}} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c})) &\leq \max\left\{0,\left((v-\hat{S}_G(\mathbf{c}))\right)_{G\in\mathscr{R}}\right\}\\ &= \sum_{G\in\mathscr{R}} q_G^{\mathscr{R}}(\mathbf{c})(v-\hat{S}_G(\mathbf{c})) \text{ (by construction)}\\ &= \sum_{G\in\mathscr{P}} q_G^{\mathscr{R}}(\mathbf{c})(v-\hat{S}_G(\mathbf{c})).\end{aligned}$$

Since $\mathbf{c}$ is arbitrary we conclude that

$$\int_\Gamma \left\{\sum_{G\in\mathscr{P}} q_G(\mathbf{c})(v-\hat{S}_G(\mathbf{c}))\right\} f(\mathbf{c})d\mathbf{c} \leq \int_\Gamma \left\{\sum_{G\in\mathscr{P}} q_G^{\mathscr{R}}(\mathbf{c})(v-\hat{S}_G(\mathbf{c}))\right\} f(\mathbf{c})d\mathbf{c},$$

and $\hat{\pi}_0(\mathbf{q},\mathbf{t}) \leq \hat{\pi}_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}})$. Putting this with lemma 6 gives $\pi_0(\mathbf{q},\mathbf{t}) \leq \hat{\pi}_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}})$. It will be sufficient to show that $\hat{\pi}_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}}) = \pi_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}})$.

Since $(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}})$ is IC, equation (II.4) (in appendix) applies :

$$\hat{\pi}_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}}) - \pi_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}}) = \sum_{i\in\mathscr{N}} \int_{\underline{c}_i}^{\bar{c}_i} \left\{\hat{K}_i(F_i(c_i)) - K_i(F_i(c_i))\right\} dX_i^{\mathscr{R}}(c_i);$$

If $\hat{K}_i(F_i(c_i)) - K_i(F_i(c_i)) \neq 0$ then $\hat{K}_i(F_i(c_i)) - K_i(F_i(c_i)) < 0$ and $\hat{H}_i(c_i)$ is constant in some neighborhood of c_i (property (c)) and so, $X_i^{\mathscr{R}}(c_i)$ is also constant [1] (i.e. $dX_i^{\mathscr{R}}(c_i) = 0$) in some neighborhood of c_i. We then conclude $\int_{\underline{c}_i}^{\bar{c}_i} \left\{\hat{K}_i(F_i(c_i)) - K_i(F_i(c_i))\right\} dX_i^{\mathscr{R}}(c_i) = 0$ and $\hat{\pi}_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}}) - \pi_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}}) = 0$.

- Moreover the expected surplus is :

1.
Recall $X_i^{\mathscr{R}}(c_1) = \int_{\Gamma_{-i}} x_i^{\mathscr{R}}(c_i,c_{-i})f_{-i}(c_{-i})dc_{-i}$ and
$x_i^{\mathscr{R}}(c_i,c_{-i}) = \sum_{G\in\mathscr{P}/i\in G} q_G^{\mathscr{R}}(c_i,\mathbf{c}_{-i})$; this sum depends only on the ironed out virtual costs $\hat{H}_i(c_i)$ and $(\hat{H}_j(c_j))_{j\neq i}$. Because the integral is taken over $\mathbf{c}_{-i}$, we conclude that $X_i^{\mathscr{R}}(c_i)$ depends only on the functions $(\hat{H}_j)_{j\neq i}$ and on $\hat{H}_i(c_i)$; thus it is constant if $\hat{H}_i(c_i)$ is constant.

$$\begin{aligned}\rho^{\mathscr{R}}(v) &= \pi_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}}) = \hat{\pi}_0(\mathbf{q}^{\mathscr{R}},\mathbf{t}^{\mathscr{R}}) = -\sum_{i\in\mathscr{N}} \pi_i^{\mathscr{R}}(\bar{c}_i,\bar{c}_i) + \int_\Gamma \left\{\sum_{G\in\mathscr{R}} q_G^{\mathscr{R}}(\mathbf{c})\hat{S}_G(\mathbf{c})\right\} f(\mathbf{c})d\mathbf{c} \\ &= 0 + \int_\Gamma \max\left\{0, \left((v-\hat{S}_G(\mathbf{c}))\right)_{G\in\mathscr{R}}\right\} f(\mathbf{c})d\mathbf{c} \\ &= E\left[\max\left\{0, \left((v-\hat{S}_G(\mathbf{c}))\right)_{G\in\mathscr{R}}\right\}\right].\end{aligned}$$

□

proof of proposition 14. Fix an item $l \in \{1,2,...,L\}$, a supplier of that item $i \in I_l$ and a report vector $\mathbf{m} \in \Gamma$.

- Suppose $i \notin A_l(\mathbf{m})$; there exists $j \in I_l : \hat{H}_i(m_i) > \hat{H}_j(m_j)$.

Consider some list $G \in \mathscr{P} : i \in G$ and let $G' = G \cup \{j\} - \{i\}$. In the list G' supplier j is awarded the contract for item l instead of i as in G, but suppliers of the other items remain the same.

We have

$$\hat{S}_{G'}(\mathbf{m}) = \hat{S}_G(\mathbf{m}) + \hat{H}_j(m_j) - \hat{H}_i(m_i) < \hat{S}_G(\mathbf{c}).$$

This means that $G \notin A(\mathbf{m})$ and therefore $q_G^{\mathscr{P}}(\mathbf{m}) = 0$. Since G is an arbitrary list in $\mathscr{P}$ such that $i \in G$, we conclude

$$x_i^{\mathscr{P}}(\mathbf{m}) = \sum_{G\in\mathscr{P}/i\in G} q_G^{\mathscr{P}}(\mathbf{m}) = 0.$$

- Now suppose that $i \in A_l(\mathbf{m})$; we now know that $\sum_{j\in A_l(\mathbf{m})} x_j^{\mathscr{P}}(\mathbf{m}) = \sum_{j\in I_l} x_j^{\mathscr{P}}(\mathbf{m})$. We first show that suppliers in $A_l(\mathbf{m})$ have the same probability to win, i.e. $x_i^{\mathscr{P}}(\mathbf{m}) = x_j^{\mathscr{P}}(\mathbf{m})$ for any $j \in A_l(\mathbf{m})$.

Let $j \in A_l(\mathbf{m})$; then $\hat{H}_j(m_j) = \hat{H}_i(m_i)$. The map $\lambda : \{G \in \mathscr{P} : i \in G\} \to \{G \in \mathscr{P} : j \in G\}$, $G \mapsto G \cup \{j\} - \{i\}$ is a bijection. Moreover, for any $G \in \mathscr{P} : i \in G$,

$$\hat{S}_{\lambda(G)}(\mathbf{m}) = \hat{S}_G(\mathbf{m}) + \hat{H}_j(m_j) - \hat{H}_i(m_i) = \hat{S}_G(\mathbf{c}).$$

Thus $G \in A(\mathbf{m})$ if and only if $\lambda(G) \in A(\mathbf{m})$. This means $q_G^{\mathscr{P}}(\mathbf{m}) = q_{\lambda(G)}^{\mathscr{P}}(\mathbf{m})$. It follows that :

$$x_j^{\mathscr{P}}(\mathbf{m}) = \sum_{G' \in \mathscr{P} / j \in G'} q_{G'}^{\mathscr{P}}(\mathbf{m}) = \sum_{G \in \mathscr{P} / i \in G} q_{\lambda(G)}^{\mathscr{P}}(\mathbf{m}) = \sum_{G \in \mathscr{P} / i \in G} q_G^{\mathscr{P}}(\mathbf{m}) = x_i^{\mathscr{P}}(\mathbf{m}).$$

Therefore

$$x_i^{\mathscr{P}}(\mathbf{m}) = \frac{\sum_{j \in I_l} x_j^{\mathscr{P}}(\mathbf{m})}{|A_l(\mathbf{m})|}.$$

The result follows from the fact that the probability to purchase an item is

$$\sum_{j \in I_l} x_j^{\mathscr{P}}(\mathbf{m}) = \sum_{G \in \mathscr{P}} q_G^{\mathscr{P}}(\mathbf{m}) = 1(\min_{G \in \mathscr{P}} \hat{S}_G(\mathbf{m}) \leq v).$$

□