CAHIER 8036

MARKET DETERMINANTS OF MISLEADING ADVERTISING†

by

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Revised September 1980

†Financial support from the Canadian Department of Consumer and Corporate Affairs and the National Science Foundation is gratefully acknowledged. The ideas expressed in this paper are those of the authors and do not necessarily reflect views of these organizations.

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Ce cahier est publié conjointement par le Département de Science Économique et par le Centre de Recherche en Développement Economique de l'Université de Montréal.
Abstract

In this paper, we study the market conditions under which misleading advertising arises and truthful advertising does not. Our aim is to identify the characteristics of markets in which advertising is misleading. We want to focus on factors such as the size of the market, each consumer's demand for the good, the reliability of non-advertising information received by buyers, the costs of advertising, and the amount of the potential quality variation. As we shall see, it will also be necessary to consider the initial beliefs about quality which consumers bring to the market.
Résumé

Nous étudions, dans ce cahier, les conditions de marché favorisant l'émergence de la publicité trompeuse. Nous analysons l'impact de facteurs tels la taille du marché, la demande de chaque consommateur, la fiabilité de l'information non-publicitaire disponible aux consommateurs, les coûts de la publicité et l'importance des variations de qualité potentielle. Comme nous le verrons, il est aussi nécessaire de considérer les probabilités a priori que les consommateurs associent aux différents niveaux de qualité.
In this paper, we study the market conditions under which misleading advertising arises and truthful advertising does not. Our aim is to identify the characteristics of markets in which advertising is misleading. We want to focus on factors such as the size of the market, each consumer's demand for the good, the reliability of non-advertising information received by buyers, the costs of advertising, and the amount of the potential quality variation. As we shall see, it will also be necessary to consider the initial beliefs about quality which consumers bring to the market.

For the most part, the economic theory of advertising has been developed within three different, although interrelated, fields of economics: welfare theory, industrial organization theory and aggregate consumption theory.\(^1\) Welfare economists have studied advertising as a source of product quality information that changes tastes and induces habit formation; see, for example, Nelson [1974], Dixit and Norman [1978], Boyer, Kihlstrom and Laffont [1979], and Kotowitz and Mathewson [1980]. This literature is founded on consumer models of habit formation and information use. Using these models, and the tools of welfare economics, "efficient" levels of advertising information are compared to market-determined levels and measures are proposed for estimating the social costs of inefficient levels of market-determined advertising information.

The industrial organization literature on advertising; see, for example, Doyle [1968], Schmalensee [1972], and Sherer [1980]; is focused on two questions. First, what is the link between market structure and spending on advertising? Second, what are the determinants of the decision to advertise? In this literature, advertising is seen as having two opposite effects. On the one hand, it results in brand loyalties that favor concentration. On the other, it results in well-informed consumers and thereby increases competition.
Contributions to the literature dealing with advertising's effect on aggregate consumption and saving have been made by Schmalensee [1972] and Ashley, Granger and Schmalensee [1980]. The evidence presented there suggest a weak inverse causal relationship between advertising and aggregate consumption.

One paper which exists somewhat apart from the above-mentioned papers is by Arrow and Nerlove [1962]. In that paper, the advertising decision is studied as an investment problem in which the returns to advertising are realized as increases in good will which shifts demand.

Our aim in this paper is quite different from that of the literature just described. As mentioned earlier, we want to derive or identify the market conditions under which misleading advertising is more likely than truthful advertising. The procedure is to construct a model in which we can describe the influence that each of the above factors has on the firm's advertising decision. In the particular model developed below, all of these factors are treated as exogenous. We can then study the differential influence of these exogenous variables when the advertising is truthful and when it is misleading. It is also possible to describe the relationship, implied by the model, between the truthfulness of the advertising and the endogenous variables: price, output, and the amount of advertising expenditures.

The first section of the paper describes the model of the firm and of the consumers who make purchases from the firm. The section concludes with several propositions that characterize the firm's optimal, price, output, and advertising decisions. The second section begins with a formal definition of misleading and truthful advertising. We then proceed to use the characterization of firm decisions obtained at the end of Section 1 to describe the market conditions under which misleading advertising occurs and is more likely than
truthful advertising. For those cases in which both truthful and untruthful advertising are sure to occur, we describe the conditions under which untruthful advertising expenditures are likely to exceed truthful expenditure levels.

1. THE BASIC MODEL

1.1 An Outline

Before describing the formal structure of the model, we outline its basic features. The market under study is assumed to be supplied by a monopolistic firm selling a product whose quality is unknown to buyers. Quality is measured by the level of services provided by a unit of the good produced by the monopolist. To simplify the analysis, we only consider cases in which buyers always buy the same amount of the product. Thus buyers only decide whether to buy or not. They do not decide how much to buy. Simplicity also leads us to consider only two quality levels: high and low.

Buyers may receive quality information of two types from different sources. First, there is advertising information which is provided to some consumers by the firm. Advertising always changes consumer expectations about quality in the same direction. These expectations are formally represented by a subjective probability distribution over the possible quality levels. Advertising is specifically assumed to raise the consumer's subjective probability of high quality. The amount by which this probability is raised is the measure of advertising intensity. More intense advertising is assumed to be more costly.

The second type of information received by consumers is publicly available non-advertising information. We will sometimes interpret this information as government provided, but this interpretation is not essential. Non-advertising information can be viewed as a prediction of quality. The reliability of this
information is measured by the conditional probability of a prediction of high (low) quality given that quality actually is high (low). Since either high or low quality may be predicted, non-advertising information may, unlike advertising information, lower as well as raise the consumer's subjective probability of high quality. The amount by which this probability rises (falls) when high (low) quality is predicted will of course depend on the reliability of the prediction.

Firms have two advertising decisions to make. First, they decide how many consumers will be the objects of advertising. They must then decide how intensively to advertise to each consumer. There is assumed to be constant returns to scale in the first variable; i.e., the cost of advertising is assumed to be the same for all consumers. The costs of increased advertising intensity are assumed to be increasing, however. The first assumption would appear to be satisfied in the case of advertising by mail. In the case of newspaper and magazine advertising, it will hold if advertising rates increase linearly with circulation. For television and radio, it holds when rates increase linearly with audience size.

In addition to the advertising decision, the firm also chooses a price. This decision is related to the advertising decision because at all prices above some critical level, sales will be made only if there is advertising (recall the assumption that consumers make a simple buy-don't buy decision). Because of this, the decision to advertise can be viewed as a decision to charge a price above the critical price. Similarly, the advertising intensity decision is related to the price charged.

Furthermore, when advertising occurs, the only consumers who buy are those who are the objects of advertising. Thus the output and sales decisions are
made implicitly when the firm decides how many consumers to advertise to.

The firm's advertising decision is assumed to be contingent on the prediction made by the non-advertising information. Thus firms are assumed to know this prediction when the price-advertising decisions are made. However, because of the sequence in which advertising and non-advertising information influence consumer expectations, the most natural interpretation is that the firm's advertising decision is taken and carried out before consumers themselves know the non-advertising prediction.

As pointed out earlier, a prediction of high (low) quality will raise (lower) the consumer's subjective probability of high quality. The critical (no-advertising) price is determined by this probability. This probability also influences the firm's pricing-advertising decision. In particular, it determines the conditions under which the firm charges the critical price and decides not to advertise. For these reasons, the firm will, in general, make a different price-advertising choice when high quality is predicted than when low quality if predicted. The nature of this difference is also influenced by the reliability of the non-advertising information. This is so because the extent by which the prediction changes the probability of high quality is determined by the reliability of the prediction. The other exogenous variables mentioned above influence the price-advertising decisions for similar reasons. The characterization of the optimal decisions obtained at the end of this section yield formal expressions which reflect these influences and which are the basis for the analysis of misleading advertising in the following section. The definition of misleading advertising is postponed until the beginning of that section.

1.2 The Formal Model

The model we propose is based on the assumption that the product being sold
provides services. The seller may or may not know how much service a unit of his product will supply. What is essential is that the buyers don't have this information; this makes advertising possible. Formally, we let $a$ represent the level of service provided by each unit of the product being sold. To keep the analysis simple, it is assumed that there are only two possible levels of service that each unit of the product may supply. These are denoted by $a_1$ and $a_2$, and $a_1$ is assumed to exceed $a_2$.

Although the consumers don't know whether $a$ equals $a_1$ or $a_2$, they do have initial expectations about the likely values of $a$. These expectations are assumed to be represented by a subjective probability distribution which specifies the probability of the two outcomes $a_1$ and $a_2$. Thus, for each consumer, $a$ is, in effect, the outcome of a random variable $\tilde{a}$. The subjective probability that $\tilde{a}$ equals $a_1$ will be denoted by $\pi_0$.

In addition to his initial beliefs, each consumer also receives non-advertising information about $a$. This information is assumed to be reliable but less than completely accurate. It may be obtained from previous experience or from the experience of others. It may also be obtained from private or governmental sources which conduct product tests.

Whatever its source, we will represent the information as a random variable $\tilde{x}$ whose distribution depends on $a$. There will be two possible values, $x_1$ and $x_2$, that $\tilde{x}$ may take. These are, in effect, the messages that the information may bring. It will often be convenient to interpret the outcome, $x$, of $\tilde{x}$ as a prediction of the value $a$ that will be taken by $\tilde{a}$. In particular, for each $i$, $x_i$ can be interpreted as a prediction that $\tilde{a}$ will equal $a_i$.

We will assume a particularly simple relationship between $a$ and the
distribution of $\tilde{x}$. Specifically, for each $i$, the probability that $\tilde{x} = x_i$ when $\tilde{a} = a_i$ will be denoted by $\Theta$, where $\Theta \in (\frac{1}{2}, 1)$. This will make it possible to interpret $\Theta$ as the "reliability" of the information provided by the prediction $\tilde{x}$. Note that if $\Theta$ were to equal 1, the prediction $\tilde{x}$ would always be accurate. As mentioned above, completely accurate information is a possibility we wish to rule out. By ruling out perfect accuracy of the information, we are not assuming that the information is unreliable. The information provided by an $\tilde{x}$ observation will be reliable when $\Theta$ exceeds one-half. If, on the other hand, $\Theta$ were to equal one-half, then each prediction $x_1$ and $x_2$ would be equally likely regardless of the value $a$ taken by $\tilde{a}$. In this case the distribution of $\tilde{x}$ would not depend on $a$. As a result, $\tilde{x}$ observations would be unreliable because they provide no useful information about $a$.

In addition to the receiving prediction $x$, some consumers will be subjected to advertising by the firm. We assume that the effect of this advertising is to raise the probability which these consumers attach to the event $a_1$. Thus the advertising causes consumers to believe that the product is more likely to be good; i.e., more likely to provide $a_1$ service units. The level of firm advertising expenditures per consumer is denoted by $\alpha$. For a consumer who is the object of advertising, the probability attached to $a_1$ when the firm has spent $\alpha$ dollars per consumer on advertising is denoted by $\pi(\alpha)$.

It is assumed that

$$\pi(0) = \pi_0, \quad \pi(\infty) = 1,$$

and that $\pi(\alpha)$ increases when $\alpha$ rises; i.e., that $\pi'(\alpha) > 0$. The returns to per person advertising expenditures are also assumed to be diminishing, so that $\pi''(\alpha) < 0$. The assumption $\pi(0) = \pi_0$ simply means that all consumers'
initial beliefs are given by \( \pi_0 \) if there is no advertising. The assumption \( \pi(\infty) = 1 \) means that it requires an infinite advertising expenditure to absolutely convince the consumers that the product will be good (provide \( a_1 \) service units). The function \( \pi(a) \) is shown below.

![Figure 1](image.png)

If the firm spends \( a \) dollars per person on advertising and the consumers receive the prediction \( x_1 \), then the consumers who are the objects of advertising will associate an "a posteriori" probability of

\[
\phi(x_1, \pi(a)) = \frac{\pi(a)\theta}{\pi(a)\theta + [1 - \pi(a)](1 - \theta)}
\]

with the event \( a_1 \). These revised probabilities are computed from "Bayes Law" which specifies how the information obtained from the prediction \( \tilde{x} = x_1 \) causes the consumers to change their beliefs about \( a \) (as measured by the probabilities of \( \tilde{a} = a_1 \), and \( \tilde{a} = a_2 \)). For those consumers who are not the objects of advertising, the a posteriori probability of \( a_1 \) will be given by \( \phi(x_1, \pi_0) \) after \( x_1 \) has been observed. For these consumers, the a posteriori probability of \( a_2 \) will be \( 1 - \phi(x_1, \pi_0) \).

If, on the other hand, a consumer receives the advertising message and
then learns that \( \bar{x} = x_2 \), he will revise the probability of \( a_1 \) to

\[
\phi(x_2, \pi(a)) = \frac{\pi(a)(1 - \theta)}{\pi(a)(1 - \theta) + [1 - \pi(a)] \theta}.
\]

For a consumer who receives no advertising message but does observe \( \bar{x} = x_2 \), the revised probability of \( a_1 \) will be given by \( \phi(x_2, \pi_0) \).

Since

\[
\phi(x_1, \pi(a)) = \frac{1}{1 + \left( \frac{1 - \pi(a)}{\pi(a)} \right) \left( \frac{1 - \theta}{\theta} \right)},
\]

\[
\phi(x_2, \pi(a)) = \frac{1}{1 + \left( \frac{1 - \pi(a)}{\pi(a)} \right) \left( \frac{\theta}{1 - \theta} \right)},
\]

and

\[
\frac{\theta}{1 - \theta} > 1 > \frac{1 - \theta}{\theta},
\]

it is easy to see that, for all \( \pi(a) \) values,

\[
\phi(x_1, \pi(a)) > \phi(x_2, \pi(a)).
\]

Thus consumers assign a higher probability to \( a_1 \) after they receive a prediction \( x_1 \) than they do after receiving a prediction \( x_2 \). Note that this is true even for a consumer who is not the object of advertising; i.e., even if \( \pi(a) = \pi(0) \).

It is also easy to verify from the expressions (3.1) and (3.2) for \( \phi(x_1, \pi(a)) \) and \( \phi(x_2, \pi(a)) \) that, for \( i = 1 \) and 2,

\[
\phi(x_1, \pi(a)) > \phi(x_1, \pi(a'))
\]

if \( a \) exceeds \( a' \). Thus additional advertising expenditures always increase
each consumer's revised estimate of the probability that \( a = a_1 \) where, it will be recalled that, \( a_1 \) is the higher level of services. This is true regardless of the prediction which provides the basis for revising the probability of \( a = a_1 \).

There are assumed to be a continuum of potential consumers. We have already assumed that all consumers begin with the same beliefs, represented by \( \pi_0 \), about \( a \) and that they all obtain the same non-advertising information about \( a \); i.e., they all receive the same prediction. We are now going to assume that these consumers are alike in all other respects as well, with one possible exception. The exception occurs because some consumers are the object of advertising and some are not. But all consumers buy \( z \) units of the good if they buy any at all. They also have the same income \( I \), and they have the same preferences. The per unit price charged by the firm selling the product is \( q \) and this is the price paid by all buyers.

Each consumer's preferences are assumed to be represented by the utility function

\[
(7) \quad u(x, w) = \psi(s) + \beta w
\]

where

\[
s = \text{the level of product services received},
\]

\[
w = \text{money},
\]

and \( \beta \) is a positive constant which measure the marginal utility of money.*

The function \( \psi \) is assumed to be an increasing function of the service level

*Note that, for this utility function, the marginal utility of income is constant. Because this assumption implies that consumer surplus is an accurate measure of welfare losses, this utility function has, implicitly if not explicitly, played an important role in empirical welfare economics.
Furthermore, $\psi$ increases at a decreasing rate; thus $\psi'(s) > 0$ and $\psi''(s) < 0$. If a consumer makes a purchase of $z$ units, his utility will be

$$\psi(za_1) + \beta(I - zq)$$

if $a = a_1$.

If a consumer makes no purchase, his utility will be

$$\psi(0) + \beta I$$

These two situations are illustrated below.

![Diagram showing indifference curves](image)

**Figure 2**

Notice that, in Figure 2, the $(s,w)$ combination $(za_1, I-zq)$ is on a higher indifference curve than $(za_2, I-zq)$. This is true because $za_1$ exceeds $za_2$, and, as a result

$$\psi(za_1) + \beta(I - zq) > \psi(za_2) + \beta(I - zq)$$
When the firm spends \( a \) dollars on advertising and consumers have received the prediction \( x_1 \), each consumer who buys \( z \) units of the product has an expected utility equal to

\[
E(x_1, \pi(a), q) = \left[ \psi(z_{a_1}) + \beta(1 - zq) \right] \phi(x_1, \pi(a)) \\
+ \left[ \psi(z_{a_2}) + \beta(1 - zq) \right] [1 - \phi(x_1, \pi(a))] 
\]

As equation (5) above makes explicit, consumers attach a higher probability to the possibility of receiving \( za_1 \) service units when \( x_1 \) is the prediction than when \( x_2 \) is the prediction. Also, the consumer has a higher utility level when he receives \( za_1 \) service units than when he obtains \( za_2 \) units. Thus the prediction \( x_1 \) attaches a higher probability to an event (obtaining a high quality product) which the consumer prefers. As a result, (5), (10) and (11) imply that each consumer's expected utility is higher when he receives the prediction \( x_1 \) than it is when \( x_2 \) is predicted. This is expressed analytically by

\[
E(x_1, \pi(a), q) > E(x_2, \pi(a), q) 
\]

It should be emphasized that this conclusion holds no matter what price \( q \) the firm charges or what amount \( a \) it spends per person for advertising. It also holds for consumers who do not receive the advertising message; i.e., when \( \pi(a) = \pi_0 \).

Equation (6) also asserts that increases in per person advertising expenditures raise the estimate of the probability of the preferred outcome for consumers who are the object of advertising. This is true regardless of the prediction. Thus (6) implies that, regardless of the prediction \( x \), these consumers perceive themselves to be better off when the firm advertises more.
Formally, the expected utility rises when per person advertising expenditures rise; i.e.,

$$E(x_1, \pi(a), q) > E(x_1, \pi(a'), q)$$

when $a > a'$. This equation makes it clear why advertising can induce consumer purchases in this framework. Specifically, advertising raises the expected utility which the consumer believes he will obtain by buying.

Finally, the consumer is worse off if he pays a higher price for the product. This is true whether the service level is high or low. Thus, for $i = 1$ and 2,

$$\psi(z_{a_1}) + \beta(I - zq) < \psi(z_{a_1}) + \beta(I - zq')$$

if $q > q'$. Since a price rise implies that the consumer is sure to be worse off whether the service supplied is high or low, his expected utility must reflect this by falling when price rises. The decline in expected utility must occur whatever the prediction. Thus, for $i = 1$ and 2,

$$E(x_1, \pi(a), q) < E(x_1, \pi(a), q')$$

when $q$ exceeds $q'$.

The framework described above can now be used to analyze the firm’s pricing and advertising strategies. In particular, we can investigate the influence of the prediction $x$ on the price and advertising decisions. We proceed by first deriving a relationship between advertising costs and the firm's price. That is, for each price $q$ which the firm might possibly charge, and for each prediction $x$, we determine the minimum advertising expenditure required to induce consumers to buy at that price. As we pointed out earlier, advertising
tising induces purchases by raising consumer expectations for the product. If the advertising expenditures \( a \) are large enough to cause

\[
E(x_1, \pi(a), q) \geq \psi(0) + \beta I,
\]

then a consumer who has received the prediction \( x_1 \) will buy because the expected utility of the purchase exceeds the utility of no purchase.

Let us now define \( \alpha_i(q) \) to be the minimum per person advertising expenditure required to raise each consumer's expectations for the product to the point at which he will buy when he obtains prediction \( x_i \); i.e., to the point at which the equality in (6) holds. Formally, \( \alpha_i(q) \) is defined by the equation

\[
E(x_1, \pi(\alpha_i(q)), q) = \psi(0) + \beta I.
\]

It should be clear from Figure 2 that, when \( q \) is low, no advertising is required to induce purchases regardless of the prediction. If, for example, \( q \) is close to zero,

\[
\psi(za_1) + \beta(I - zq) > \psi(0) + \beta I
\]

for both \( i = 1 \) and \( i = 2 \). Thus regardless of how much service the product provides, a purchase is preferred. As a result (16) holds for both \( i = 1 \) and \( i = 2 \) when \( \alpha = 0 \). For such low values of \( q \), \( \alpha_1(q) = \alpha_2(q) = 0 \).

In fact, when \( x_1 \) has been predicted, \( q \) can rise to a level \( q_1 \) at which

\[
E(x_1, \pi_0, q_1) = \psi(0) + \beta I
\]

and still no advertising will be required to make a sale. Thus \( \alpha_1(q) = 0 \).
if \( q \leq q_1 \).

Having thus defined \( a_1(\cdot) \) and \( q_1 \), it will be convenient at this stage to describe them more completely. We first present a formal list of their properties. This list is followed by a series of interpretive remarks and the proofs of these properties.

Properties of \( q_1 \) and of \( a_1(\cdot) \):

P.1 \( q_1 \) satisfies the equation

\[
0 = \phi(x_1, \pi) \{[\psi(a_1 z) - \psi(0)] - \beta z q_1 \} + [1 - \phi(x_1, \pi)] \{[\psi(a_2 z) - \psi(0)] - \beta z q_1 \}.
\]

P.2 \( q_1 \) can also be expressed as

\[
q_1 = \phi(x_1, \pi MRS_1 + [1 - \phi(x_1, \pi MRS_2
\]

where

\[
MRS_1 = \frac{\psi(a_1 z) - \psi(0)}{\beta z}.
\]

P.3 More specific expressions for \( q_1 \) and \( q_2 \) are

\[
q_1 = \left( \frac{\pi_0}{1 - \pi_0} \right) \left( \frac{\theta}{1 - \theta} \right) MRS_1 + MRS_2
\]

(23.1)

and

\[
q_2 = \left( \frac{\pi_0}{1 - \pi_0} \right) \left( \frac{1 - \theta}{\theta} \right) MRS_1 + MRS_2
\]

(23.2)

P.4 \( q_1 \) is greater than \( q_2 \).
\( a_1(q) = 0 \) if \( q < q_1 \) and \( a_1(q) > 0 \) if \( q > q_1 \).

\( a_1(q) \) is an increasing function of \( q \) when \( q > q_1 \).

\( \text{For all } q > q_2 \).

(24) \( a_2(q) > a_1(q) \).

Remarks:

Note that (20) implies that \( q_1 \) must be large enough to cause (18) to fail when \( i = 2 \). But \( q_1 \) cannot be too large because (18) must hold when \( i = 1 \) if (20) is to hold. Thus, when (20) is satisfied, the consumer is better off making a purchase if quality is high but he would prefer not to buy if quality is low. This is the situation represented in Figure 2.

The \( \text{MRS}_1 \) defined in (22) are discrete analogs of the marginal rate of substitution. They represent the value to each consumer of \( a_1z \) service units. This value is measured by the quantity of the other good "money" which the consumer would just be willing to give up to obtain \( a_1z \) service units. Thus (21) asserts that, when \( x_1 \) has been predicted, \( q_1 \) equals the a posteriori expected money value to the consumer of a \( z \) unit purchase which will provide \( a_1z \) service units with a posteriori probability \( \phi(x_1, \pi_0) \) and \( a_2z \) service units with a posteriori probability \( 1 - \phi(x_1, \pi_0) \).

P.4 asserts that the firm will be forced to begin advertising at lower prices if the prediction is unfavorable rather than favorable.

P.5 simply asserts that \( q_1 \) is the highest price at which consumers who have received prediction \( x_1 \) will buy without advertising inducements. At higher prices advertising is necessary if consumers are to buy.

P.6 asserts that more advertising is required to induce purchases at
higher prices.

Inequality (24) states that the per person advertising expenditures required to induce a purchase when the prediction is unfavorable exceeds the expenditure required to induce a purchase when the prediction is favorable.

Proof:

Equation (20) is obtained by simply substituting the expression (11) for $E(x_1, \pi_0, q_1)$ in the equation (17) which defines $q_1$. Equation (21) is derived by solving (20) for $q_1$. To arrive at (23.1) and (23.2), we note that (2.1) and (2.2) respectively imply

\[\phi(x_1, \pi_0) = \frac{\left(\frac{\pi_0}{1 - \pi_0}\right)\left(\frac{\theta}{1 - \theta}\right)}{1 + \left(\frac{\pi_0}{1 - \pi_0}\right)\left(\frac{\theta}{1 - \theta}\right)}\]

and

\[\phi(x_2, \pi_0) = \frac{\left(\frac{\pi_0}{1 - \pi_0}\right)\left(\frac{1 - \theta}{\theta}\right)}{1 + \left(\frac{\pi_0}{1 - \pi_0}\right)\left(\frac{1 - \theta}{\theta}\right)}\]

We then substitute (25.1) and (25.2) in (21) to obtain (23.1) and (23.2).

The proof of P. 4 begins with the observation that because $a_1$ exceeds $a_2$, the money value, $MRS_1$, of $a_1 z$ exceeds $MRS_2$, the money value of $a_2 z$. We also recall inequality (6), which implies, as a special case, that

\[\phi(x_1, \pi_0) > \phi(x_2, \pi_0)\]

Now (21) and (26) imply that the higher money value $MRS_1$ weighted more
heavily in computing \( q_1 \) than in computing \( q_2 \). This implies that \( q_1 \) exceeds \( q_2 \).

We have already noted in the process of defining \( q_1 \) that, when \( q \) is below \( q_1 \), (16) holds if \( \pi(\alpha) \) is replaced by \( \pi_0 \). Thus no advertising is required; i.e., \( x_1(q) = 0 \) when \( q \leq q_1 \). When \( q \) exceeds \( q_1 \), (19) and (15) combine to imply that (16) fails if \( \alpha = 0 \). Because of (13), a rise in \( \alpha \) above zero is therefore required to raise \( E(x_1, \pi(\alpha), q) \) to the point at which (16) holds.

A similar argument is used to prove P. 6. To begin this proof, we note first that, using (11) and (22), (17) can be rewritten to obtain

\[
q = \phi(x_1, \pi(\alpha_1(q)))MRS_1 + [1 - \phi(x_1, \pi(\alpha_1(q)))]MRS_2.
\]

This equation asserts that, for any price \( q \) in excess of \( q_1 \), \( x_1(q) \) must raise \( \phi(x_1, \pi(\alpha_1(q))) \) to the point at which the a posteriori expected money value of a \( z \) unit purchase exactly equals the price \( q \). This equation implies that an increase in \( q \) without a corresponding increase in \( \alpha_1(q) \) would raise the price above the expected value of a purchase. Thus the left side of (27) would exceed the expression for the expected money value on the right side and no purchase would be made. If, however, \( \alpha_1(q) \) were to rise with \( q \), (6) implies that \( \phi(x_1, \pi(\alpha_1(q))) \) would rise, causing the higher money value \( MRS_1 \) to be more heavily weighted in the computation of the expectation on the right side of (27).

The only property which remains to be established is P. 7. When \( q \) lies in the interval \( (q_2, q_1] \), this inequality clearly holds since on this interval \( \alpha_1(q) = 0 < \alpha_2(q) \). When \( q \) exceeds \( q_1 \), the proof is based on the inequality...
\( E(x_2, \pi(a_1(q)), q) < E(x_1, \pi(a_1(q)), q) = \psi(0) + \beta I \),

which is derived from (12) and (17). Inequality (28) simply asserts that the advertising expenditures \( a_1(q) \) which result in purchases when the prediction is favorable are not sufficient to induce purchases when the prediction is unfavorable. In view of (28), inequality (13) implies that \( E(x_2, \pi(a_2(q)), q) \) can equal \( \psi(0) + \beta I \) only if \( a_2(q) \) exceeds \( a_1(q) \).

Having defined and described \( a_1(q) \) and \( q_1 \), these concepts can now be employed in the analysis of the firm's advertising, pricing, and supply decisions. Consider now the case in which the firm knows that the prediction \( x_1 \) has been made. If the firm charges the price \( q > q_1 \), spends \( a_1(q) \) per person on advertising and advertises to \( n \) buyers, it will sell \( nz \) units. Note that if \( q > q_1 \) only those consumers who receive the advertising message will buy. Since, for those consumers who are not the objects of advertising, \( \pi(a) \) will remain at \( \pi_0 \) and (16) will fail to hold because \( q > q_1 \). If, on the other hand, \( q = q_1 \), then all consumers are indifferent between a purchase of \( z \) units and no purchase. In this case, the firm chooses \( n \) and is able to find many buyers. The revenues obtained from these sales are \( nzq \). The firm's profits are then

\[ p_1(n, q) = nzq - na_1(q) - c(nz) \]

where \( na_1(q) \) is the advertising cost and \( c(nz) \) the cost of producing \( nz \) units. We assume that cost is increasing at an increasing rate and that positive profits are possible even when \( x = x_2 \). Formally, the means that \( q_2 > c'(0) > 0 \) and \( c'' > 0 \). In addition, we assume that \( c(0) = 0 \).

The firm chooses \( n \) and decides which price \( q \) to charge by maximizing
\( p_1(n, q) \). It should be noted first that the firm will never find it optimal to charge a price below \( q_1 \). If, for example, \( q \) is below \( q_1 \), then \( a_1(q) = 0 \) and all consumers want to buy. Thus the firm could sell to any number, \( n \), of buyers. Note however, that, for any \( n \), if \( q < q_1 \), then

\[
p_1(n, q) = nzq - c(nz)
\]

For each fixed \( n \), price increases raise revenues, but they do not begin to increase advertising costs until \( q \) has risen above \( q_1 \). Thus, for any \( n \), \( p_1(n, q_1) \), the profits earned by charging \( q_1 \), always exceed \( p_1(n, q) \), the profits obtained by charging a price \( q \) less than \( q_1 \). Furthermore, \( nzq_1 \) is always positive.

The questions which remain are: Will the firm choose \( q_1 \) and refrain from advertising or will it charge a price in excess of \( q_1 \) and advertise to induce sales? What are the factors determining this choice? Finally, how is the firm's supply decision related to its price-advertising decision?

The proposition which we now state characterizes the firm's price, output choice when \( x = x_1 \). This proposition forms the basis for the analysis to follow which provides answers to the questions just posed.

**PROPOSITION 1:**

Assume, in addition to the assumptions made above (recall that these include the cost assumptions: \( c(0) = 0, c'(\cdot) > 0, c''(\cdot) > 0 \) and \( q_2 > c'(0) \)), that \( a_{1''} \) is always positive.

Under these conditions,

a) if

\[
z = a_1'(q_1) > 0
\]
then the profit maximizing price is \( \hat{q}_1 \) at which

\[ z - a'_1(\hat{q}_1) = 0 \ . \]

In this case, the profit maximizing per person advertising expense is positive and equal to \( a_1(\hat{q}_1) \). The firm will advertise to a positive number, \( \hat{n}_1 \), of people where \( \hat{n}_1 \) is such that

\[ z\hat{q}_1 - a_1(\hat{q}_1) - zc(\hat{n}_1z) = 0 \ . \]

b) if, however,

\[ z - a'_1(q_1) < 0 \ , \]

then the profit maximizing price \( \hat{q}_1 = q_1 \) and there is no advertising since \( a_1(q_1) = 0 \). For this case, the firm will again sell to a positive number, \( \hat{n}_1 \), of people where now

\[ q_1 = c'(\hat{n}_1z) \]

defines \( n_1 \).

REMARKS:

Conditions (30) - (33) are simply the first-order necessary conditions for maximization of \( p_1(n,q) \). The proof which follows demonstrates that, in this case, these conditions are sufficient as well as necessary and that the maximum is unique. Normally this involves either a check of the second-order conditions or a demonstration that the maximand is a concave function. The latter approach is preferable since it guarantees a global rather than simply a local maximum. In this case it appears, however, that \( q_1(n,q) \) is not a concave function of \( n \) and \( q \). It is possible, nevertheless, to use the
special properties of $p_i$ to demonstrate by a rather circuitous argument that the first-order conditions yield a unique global maximum.\footnote{That argument constitutes the proof of the proposition given below.}

The interpretation of conditions (29) - (33) is straightforward. Condition (29) asserts that the additional per consumer revenues, $z$, obtained by raising the price one dollar above $q_i$ exceed the additional per buyer advertising expenditures, $a'_i(q_i)$, required to induces purchases at the higher price. In this case, the optimal price will exceed $q_i$; i.e., it will pay advertise; and at the optimal price $q_i$ the revenue gains, $z$, associated with price increases will just be exhausted by the additional advertising costs $a'_i(q_i)$ required at higher prices. This balance of costs and benefits is expressed in (30). If (29) fails; i.e., if (32) holds; then advertising costs are prohibitive. Thus the revenues, $z$, obtained by charging a price above $q_i$ are not sufficient to justify bearing the advertising costs, $a'_i(q_i)$.

Note that conditions (29) and (30) are independent of $n$, and of the cost function, $c$. However, the price $q_i$ does enter equation (32). Thus the firm can be viewed as first deciding what price $q_i$ to charge and simultaneously how much to spend on advertising per person. This choice is characterized by equations (29) and (30) and is independent of the production costs and the number of people who are advertised to. Once the price choice is made, however, it influences, through equation (33), the choice of $n_i$. This feature simplifies the analysis of the price-advertising choice and makes it possible to use (33) alone to analyze the relationship between price and sales. The simplification is possible because of the simple form of the advertising cost function employed here. Specifically, the cost of inducing $n$ people to buy at price $q$ when $x_i$ has been observed is $n a_i(q)$. Thus the marginal adver-
tising cost incurred to reach each additional buyer is constant. This constant marginal cost property follows from the fact that \( \pi(\alpha) \) has been explicitly assumed to be independent of \( n \). In general, one would expect some externalities in advertising. To the extent that these externalities are empirically important, our analysis is biased. Without a complete analysis of the cases not treated here, it is impossible to evaluate the direction or the extent of the bias.

**PROOF:**

Consider first the problem of maximizing

\[ (34) \quad zq - \alpha_1(q) \]

On the set of \( q \)'s such that \( q \geq q_1 \). Note that since the second-order condition \(-c''(\cdot) < 0\) is always satisfied, \( (34) \) is a strictly concave function. Thus \( \bar{q}_1 \) maximizes \( (34) \) if and only if

\[ (35) \quad z - \alpha_1(\bar{q}_1) \leq 0 \text{ with } < 0 \text{ only if } \bar{q}_1 = q_1 \]

Note that, for any positive \( n \), \( \bar{q}_1 \) also maximizes \( p_1(n, q) \) on the set \( \{ q: q \geq q_1 \} \).

Now consider the problem of maximizing \( p_1(n, \bar{q}_1) \) by the choice of a non-negative \( n \). Since the second-order condition \(-c''(\cdot) < 0\) is always satisfied, \( p_1(n, \bar{q}_1) \) is a strictly concave function of \( n \). Thus \( \bar{n}_1 \) maximizes \( p_1(n, \bar{q}_1) \) if and only if

\[ (36) \quad z\bar{q}_1 - \alpha_1(\bar{q}_1) - zc'(\bar{n}_1 z) \leq 0 \text{ with } < 0 \text{ only if } \bar{n}_1 = 0 \]

We can use \( (36) \) and the fact that \( \bar{q}_1 \) maximizes \( (34) \) to obtain two important properties of \( \bar{n}_1 \). The first is that \( \bar{n}_1 \) is positive and the second is
that \( p_1(\tilde{n}_1, q) > 0 \). To prove that \( \tilde{n}_1 > 0 \), observe that since \( \tilde{q}_1 \) maximizes (34) on \( \{q: q \geq q_1\} \), since \( q_1 > q_2 \) and since we are assuming that \( q_2 > c'(0) \), we can conclude that

\[
\tilde{z}q_1 - a_1(\tilde{q}_1) > \tilde{z}q_1 > zq_2 > zc'(0).
\]

Thus (36) can't hold with \( \tilde{n}_1 = 0 \).

The fact that \( p_1(\tilde{n}_1, \tilde{q}_1) \) is positive follows from the strict convexity of \( c \), which is implied by \( c'' > 0 \). To see this, note that when \( c(0) = 0 \), the strict convexity of \( c \), when combined with (36) and the fact that \( \tilde{n}_1 > 0 \), implies that

\[
\tilde{n}_1[\tilde{z}q_1 - a_1(\tilde{q}_1)] = \tilde{n}_1zc'(\tilde{n}_1z) > c(\tilde{n}_1z) ;
\]

i.e., that \( p_1(\tilde{n}_1, \tilde{q}_1) > 0 \).

Graphically,
To summarize the above discussion, we have, for all \( n > 0 \) and all \( q \geq q_1 \), that

\[
\bar{p}_1(\tilde{n}_1, \tilde{q}_1) \geq p_1(n, \tilde{q}_1) \geq p_1(n, q)
\]

(37)

In addition, since \( p_1(\tilde{n}_1, \tilde{q}_1) > 0 \) and \( 0 \geq p_1(0, q) \) for all \( q \geq q_1 \), inequality (37) also holds when \( n = 0 \). Thus \((\tilde{n}_1, \tilde{q}_1)\) maximizes \( p_1(n, q) \) on the set \{(n,q): n > 0, q \geq q_1\}. Furthermore, because (34) is a strictly concave function of \( q \) and \( p_1(n, \tilde{q}_1) \) is a strictly concave function of \( n \), \((\tilde{n}_1, \tilde{q}_1)\) is the unique maximizer of \( p_1(n, q) \) on this set. If we now let \( \hat{q}_1 = \tilde{q}_1 \) and \( \hat{n}_1 = \tilde{n}_1 \), we observe immediately that \( \hat{n}_1 \) is always positive and that (35) and (36) imply (a) and (b). Specifically, (35) implies that \( \hat{q}_1 = q_1 \) if and only if (32) holds. This, of course, means that \( \hat{q}_1 > q_1 \) when (29) holds. In this latter event, (35) asserts that \( \hat{q}_1 \) is characterized by (30). Equations (31) and (33) are the same as (36) for the cases when \( \hat{q}_1 > q_1 \) and \( \hat{q}_1 = q_1 \), respectively.

Since Proposition 1 uses the hypothesis \( a''_1 > 0 \), we must now ask when this condition will be satisfied. For the purpose of answering this question, we use the assumption \( \pi'(\alpha) > 0 \) to invert the function \( \pi \). Let us represent the inverse of \( \pi \) by \( \alpha(\pi) \). The function \( \alpha(\cdot) \) specifies, for each value of \( \pi \) between \( \pi_0 \) and 1, the advertising expenditure \( \alpha \) required to induce consumers to believe (before receiving the prediction \( x \)) that \( \pi \) is the probability that \( \tilde{a} \) equals \( a_1 \), the high quality level. A simple way of guaranteeing that \( a''_1 > 0 \) as required in Proposition 1 is to assume that \( \pi \) is such that its inverse \( \alpha(\pi) \) satisfies

\[
\pi(\alpha) = \xi \left( \frac{1}{1 - \pi} \right)
\]

(38)
where \( \xi(1/(1 - \pi_0)) = 0, \xi' > 0, \xi'' > 0 \) and \( \xi(\pi) = \infty \). It is also easy to check that when the inverse of \( \pi(\cdot) \) has the form (37), then \( \pi'(\xi) > 0, \pi''(\xi) < 0, \pi(0) = \pi_0, \pi(\infty) = 1 \). These statements are proved in the following proposition.

**PROPOSITION 2:**

If \( a(\pi) \) satisfies (28), then \( \pi'(\xi) > 0, \pi''(\xi) < 0, \pi(0) = \pi_0, \pi(\infty) = 1 \), and \( a''(q) > 0 \).

**PROOF:**

First note that (38) implies that

\[
(39) \quad a'(\pi) = \xi'(\frac{1}{1 - \pi})\left(\frac{1}{1 - \pi}\right)^2 > 0
\]

and

\[
(40) \quad \pi'(\xi) = \frac{1}{a'([\pi(\xi)])}.
\]

Together (39) and (40) imply \( \pi'(\xi) > 0 \). Now differentiation of (39) implies that

\[
(41) \quad a''(\pi) = \xi''\left(\frac{1}{1 - \pi}\right)\left(\frac{1}{1 - \pi}\right)^4 + 2\xi'\left(\frac{1}{1 - \pi}\right)\left(\frac{1}{1 - \pi}\right)^3 > 0.
\]

Furthermore, differentiation of (40) yields

\[
(42) \quad \pi''(\xi) = -\frac{1}{[a'([\pi(\xi)])]^2} a''([\pi(\xi)])\pi'(\xi).
\]

Using (39) - (42), \( \pi''(\xi) \) is seen to be negative. The conditions imposed on \( \xi \) in (39) also guarantee that

\[
\pi(0) = \pi\left(\xi\left(\frac{1}{1 - \pi_0}\right)\right) = \pi\left(a(\pi_0)\right) = \pi_0
\]

and
\[ \pi(\omega) = \pi\left(\xi\left(\frac{1}{1 - \theta}\right)\right) = \pi(a(1)) = 1. \]

If we now define \( \pi_1(q) \) and \( \pi_2(q) \) by
\[ \text{def} \quad \pi_1(q) \equiv \pi(a_1(q)) \]

for \( i = 1 \) and \( 2 \), then equations (2.1) and (27) yield
\[ \frac{1}{1 - \pi_1(q)} = 1 + \left(\frac{1 - \theta}{\theta}\right) \left[ q - \frac{\text{MRS}_2}{\text{MRS}_1 - q} \right], \]
and equations (2.2) and (27) imply
\[ \frac{1}{1 - \pi_2(q)} = 1 + \left(\frac{\theta}{1 - \theta}\right) \left[ q - \frac{\text{MRS}_2}{\text{MRS}_1 - q} \right]. \]

First and second differentiation of (44.1) and (44.2) produces
\[ \frac{\partial}{\partial q} \left[ \frac{1}{1 - \pi_1(q)} \right] = \left(\frac{1 - \theta}{\theta}\right) \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{(\text{MRS}_1 - q)^2} \right], \]
\[ \frac{\partial}{\partial q} \left[ \frac{1}{1 - \pi_2(q)} \right] = \left(\frac{\theta}{1 - \theta}\right) \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{(\text{MRS}_1 - q)^2} \right], \]
\[ \frac{\partial^2}{\partial q^2} \left[ \frac{1}{1 - \pi_1(q)} \right] = 2 \left(\frac{1 - \theta}{\theta}\right) \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{(\text{MRS}_1 - q)^3} \right], \]
and
\[ \frac{\partial^2}{\partial q^2} \left[ \frac{1}{1 - \pi_2(q)} \right] = 2 \left(\frac{\theta}{1 - \theta}\right) \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{(\text{MRS}_1 - q)^3} \right]. \]

The first derivatives in (45) and the second derivatives in (46) are all positive since \( a_1 > a_2 \) implies that \( \text{MRS}_1 > \text{MRS}_2 \) and (25) implies \( \text{MRS}_1 > q \).

A consequence of (43) and (38) is that
\( \alpha_i(q) = \alpha(\pi_i(q)) = \xi \left( \frac{1}{1 - \pi_i(q)} \right) \)

for \( i = 1 \) and \( 2 \). (Note that (47) and (44) imply that, as asserted earlier, \( \alpha_2(q) \) exceeds \( \alpha_1(q) \). This follows because the inequality \( \theta > (1 - \theta) \) and equations (44) combine to imply that \( \frac{1}{1 - \pi_1(q)} < \frac{1}{1 - \pi_2(q)} \), and because \( \xi \) is assumed in (38) to be an increasing function.

Using (47), the first and second derivatives of \( \alpha_i(q) \) can be shown to equal, respectively,

\[
\alpha'_i(q) = \xi' \left( \frac{1}{1 - \pi_i(q)} \right) \frac{\partial}{\partial q} \left( \frac{1}{1 - \pi_i(q)} \right)
\]

and

\[
\alpha''_i(q) = \xi'' \left[ \frac{1}{1 - \pi_i(q)} \right] \left( \frac{\partial}{\partial q} \left( \frac{1}{1 - \pi_i(q)} \right) \right)^2
\]

\[ + \xi' \left( \frac{1}{1 - \pi_i(q)} \right) \frac{\partial^2}{\partial q^2} \left( \frac{1}{1 - \pi_i(q)} \right) . \]

Using (38) and (45), (48) is seen to imply that \( \alpha'_1(q) \) is positive. Furthermore, (38), (46) and (49) imply that \( \alpha''_1(q) \) is positive. \( || \)

2. THE CONDITIONS LEADING TO MISLEADING ADVERTISING

We can now use the results of Section 1 to analyze the conditions under which untruthful advertising occurs. It is also possible to describe the conditions under which untruthful advertising is more likely than truthful advertising. The first step is to say exactly what we mean by truthful and untruthful advertising.

Recall that advertising has the effect of raising consumer expectations for the good sold by the firm. Specifically, advertising raises the a priori probability \( \pi(a) \) which the consumers attach to the high quality
state \( a_1 \). This advertising will be said to be truthful if the firm is, in fact, selling a high quality good; i.e., if the product quality actually is \( a_1 \). If, however, the firm advertises when the quality level is low; i.e., when \( \tilde{a} = a_2 \); then the advertising will be called misleading.

In this model, the firm's decision to advertise is contingent only on the nonadvertising information \( \tilde{x} \) received by buyers. Thus the firm's decision is not directly dependent on true quality. In fact, the firm itself may not even know \( a \). The advertising decision is, however, indirectly dependent, in a probabilistic sense, on \( a \). This indirect dependence is introduced by the relationship between the advertising decision and the prediction \( \tilde{x} \) whose probability distribution depends on \( a \). Specifically, the firm will spend \( \hat{a}_1(\hat{q}_1) \) on advertising if \( \tilde{x} \) is observed to equal \( x_1 \).

This implies that the probability that advertising spending equals \( \hat{a}_1(\hat{q}_1) \) is \( \theta \) if \( \tilde{a} = a_1 \) and \((1-\theta)\) if \( \tilde{a} = a_2 \). Similarly, the probability that \( \hat{a}_2(\hat{q}_2) \) is spent for advertising is \((1-\theta)\) if \( \tilde{a} = a_1 \) and \( \theta \) if \( \tilde{a} = a_2 \).

Thus \( \hat{a}_1(\hat{q}_1) \) has a higher probability than \( \hat{a}_2(\hat{q}_2) \) of being the amount spent on advertising when \( \tilde{a} = a_1 \). In this case, the probability of the latter is \((1-\theta) < \theta \). If, however, \( \tilde{a} = a_2 \), then the probability, \( \theta \), that advertising spending equals \( \hat{a}_2(\hat{q}_2) \) exceeds \( 1-\theta \), the probability that \( \hat{a}_1(\hat{q}_1) \) is spent on advertising. Since advertising is misleading when \( \tilde{a} = a_2 \), the level of misleading advertising is more likely to be at the level \( \hat{a}_2(\hat{q}_2) \) than at the level \( \hat{a}_1(\hat{q}_1) \). Similarly, truthful advertising; i.e., that which occurs when \( \tilde{a} = a_1 \); has a higher probability of being at the level \( \hat{a}_1(\hat{q}_1) \) than at the level \( \hat{a}_2(\hat{q}_2) \). Suppose, in particular, that \( \hat{a}_2(\hat{q}_2) = 0 \) while \( \hat{a}_1(\hat{q}_1) > 0 \); i.e., suppose that \( \hat{q}_2 = q_2 \) while \( \hat{q}_1 > q_1 \). In this case, the probability of advertising is \( \theta \) when \( \tilde{a} = a_1 \), and \( 1-\theta \) when \( \tilde{a} = a_2 \). As
a result, the probability of truthful advertising, \( \theta \), exceeds \( (1-\theta) \), the probability of misleading advertising. If, however, \( \hat{a}_2(\hat{q}_2) > 0 \) while \( \hat{a}_1(\hat{q}_1) = 0 \); i.e., if \( \hat{q}_2 > q_2 \) and \( \hat{q}_1 = q_1 \); then the probability of advertising is \( \theta \) if \( \hat{a} = a_2 \) and \( (1-\theta) \) if \( \hat{a} = a_1 \). This is the case in which the probability of misleading advertising, \( \theta \), exceeds \( (1-\theta) \), the probability of truthful advertising.

For a special case, we can now use the preceding analysis to describe the market conditions under which \( \hat{q}_2 > q_2 \) and \( \hat{q}_1 = q_1 \), as well as those under which \( \hat{q}_1 > q_1 \) and \( \hat{q}_2 = q_2 \). As we have just seen, the former case is the one in which the probability of misleading advertising exceeds the probability of truthful advertising. In the latter case, the probability of truthful advertising is higher than the probability of misleading advertising.

We can also use our analysis to determine conditions under which \( \hat{a}_2(\hat{q}_2) \) exceeds \( \hat{a}_1(\hat{q}_1) \) or under which \( \hat{a}_1(\hat{q}_1) \) is above \( \hat{a}_2(\hat{q}_2) \). If \( \hat{a}_2(\hat{q}_2) > (\leq) \hat{a}_1(\hat{q}_1) \), the probability of large advertising expenditures is higher (lower) when the advertising is misleading than when it is truthful.

Before proceeding to the analysis of the special case, we can use Proposition 1 to obtain a result which can be useful in identifying cases of misleading advertising. Specifically, it is a corollary of conditions (31) and (33), and it asserts that sales are likely to be lower when advertising is misleading than when it is truthful. Formally:

**COROLLARY 1:**

Assume that the hypotheses of Proposition 1 hold. Then the firm sells more if the prediction is \( x_1 \) than if the prediction is \( x_2 \); i.e.,

\( \hat{n}_1 > \hat{n}_2 \).

(50)
REMARKS:

1) To interpret this corollary, recall that misleading advertising is advertising which is done when quality is low. If quality is low, sales are more likely to be \( \hat{n}_2 \) than \( \hat{n}_1 \). Thus if there is misleading advertising it is more likely to be at the level \( a_2(\hat{q}_2) \) which is accompanied by sales to \( \hat{n}_2 \) buyers. By similar reasoning, truthful advertising is more likely to be at the level \( a_1(\hat{q}_1) \) and accompanied by sales to \( \hat{n}_1 \) buyers. The corollary therefore asserts that truthful advertisers will supply the large amount, \( \hat{n}_1 \), with a high probability \( \theta \) and the low amount with a low probability \((1 - \theta)\). Conversely, misleading advertisers will supply the low quantity, \( n_2 \), with the high probability \( \theta \).

2) The intuitive basis for this result is the fact that the marginal revenue derived from an additional sale is lower when the prediction is \( x_2 \) than when the prediction is \( x_1 \). Specifically, the marginal revenue is \( z\hat{q}_2 - a_2(\hat{q}_2) \) when \( \hat{x} = x_2 \) but it is \( z\hat{q}_1 - a_1(\hat{q}_1) \) when \( \hat{x} = x_1 \). In either case this marginal revenue is the revenue per sale net of per sale; i.e., per person; advertising costs. It is because per person advertising is more costly at any price when \( \hat{x} = x_2 \) than when \( \hat{x} = x_1 \) (recall inequality (24)) that this marginal revenue is lower when \( x_2 \) has been predicted.

PROOF:

Equation (24) implies that for every \( q \geq q_2 \),

\[
(51) \quad zq - a_2(q) < zq - a_1(q) .
\]

Thus
\[ (52) \quad zq_2 - a_2(q_2) = \max_{q \geq q_2} [zq - a_2(q)] - \max_{q \geq q_1} [zq - a_1(q)] = zq_1 - a_1(q_1) \]

In obtaining (52), we made use of the fact that

\[ \max_{q \geq q_2} [zq - a_1(q)] = \max_{q \geq q_1} [zq - a_1(q)] \]

which follows from the fact that for \( q \) levels between \( q_2 \) and \( q_1 \),

\[ zq - a_1(q) = zq < zq_1 = zq_1 - a_1(q_1) \]

Combining (52) with (31) and (33), we obtain

\[ c'(n_1z) > c'(n_2z) \]

which implies (50) since \( c'' > 0 \).

In order to obtain further results, it is convenient to assume that the function \( \xi \) in (38) is linear. Specifically, we now assume that

\[ (53) \quad \pi(\alpha) = 1 - (1 - \pi_0) \frac{m}{m + \alpha} \]

where \( m \) is a positive constant which we will interpret presently. If the function \( \pi(\alpha) \) in (53) is inverted, the result is

\[ (54) \quad \alpha(\pi) = \left[ \frac{1 - \pi_0}{1 - \pi} - 1 \right]^m \]

This function clearly corresponds to the case in which \( \xi \) is linear and satisfies all restrictions imposed by (38).

The expression (54) provides an interpretation for the parameter \( m \). Notice, in fact, that increases in \( m \) raise the advertising cost required to convince the consumers that the probability of \( a_1 \) is \( \pi \). Thus higher
m's can be associated with less efficient more expensive advertising technologies. When the expressions for \( \frac{1}{1-\pi_1(q)} \) obtained in (44) are substituted in (54), the result in the following special form of (47):

\[
(55.1) \quad \alpha_1(q) = [(1 - \pi_0) \left( \frac{1 - \theta}{\theta} \right) \left( \frac{q - \text{MRS}_2}{\text{MRS}_1 - q} \right) - \pi_0] m,
\]

\[
(55.2) \quad \alpha_2(q) = [(1 - \pi_0) \left( \frac{\theta}{1 - \theta} \right) \left( \frac{q - \text{MRS}_2}{\text{MRS}_1 - q} \right) - \pi_0] m.
\]

In order to further interpret the conditions, derived in Proposition 1, which characterize the firm's price and advertising decisions, it is necessary to obtain explicit expressions for \( \alpha_1'(q) \) and \( \alpha_2'(q) \) in terms of which these conditions are stated. Using (45), (48) and (55), these derivatives are:

\[
(56.1) \quad \alpha_1'(q) = m(1-\pi_0) \frac{1-\theta}{\theta} \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{(\text{MRS}_1 - q)^2} \right]
\]

and

\[
(56.2) \quad \alpha_2'(q) = m(1-\pi_0) \frac{\theta}{1-\theta} \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{(\text{MRS}_1 - q)^2} \right].
\]

To evaluate \( \alpha_1'(q) \) at \( q = q_1 \) and \( \alpha_2'(q) \) at \( q = q_2 \), we use (21) to obtain

\[
(57) \quad \text{MRS}_1 - q_1 = [1 - \phi(x_1, \pi_0)][\text{MRS}_1 - \text{MRS}_2].
\]

Substituting (57) in (56) gives rise to the expressions

\[
(58.1) \quad \alpha_1'(q_1) = m(1-\pi_0) \left( \frac{1-\theta}{\theta} \right) \frac{1}{(\text{MRS}_1 - \text{MRS}_2)[1-\phi(x_1, \pi_0)]^2}
\]

and
(58.2) \[ a'_2(q_2) = m(1-\pi_0) \left[ \frac{\theta}{1-\theta} \right] \left[ \frac{1}{\text{MRS}_1 - \text{MRS}_2} \left( 1 - \phi(x_2, \pi_0) \right)^2 \right] \]

which with the aid of (2) and (22) reduce to

(59.1) \[ a'_1(q_1) = m \left[ \frac{\left( \frac{1}{1-\theta} \right)(1-\pi_0) + \pi_0}{(1-\pi_0)} + \pi_0 \left( \frac{\theta}{1-\theta} \right) \right] \left( \frac{\psi(a_1z) - \psi(a_2z)}{\beta z} \right) \]

(59.2) \[ a'_2(q_2) = m \left[ \frac{\left( \frac{1}{1-\theta} \right)(1-\pi_0) + \pi_0}{(1-\pi_0)} + \pi_0 \left( \frac{1-\theta}{\theta} \right) \right] \left( \frac{\psi(a_1z) - \psi(a_2z)}{\beta z} \right) \]

Substituting these expressions in the appropriate conditions obtained in Proposition 1, we obtain the following proposition which will be interpreted in a series of corollaries and remarks.

PROPOSITION 3:

If \( \pi(a) \) satisfies (53) then there is a higher probability of misleading advertising than of truthful advertising if and only if

(60) \[ \left[ \frac{\theta}{1-\theta} + \left( \frac{\pi_0}{1-\pi_0} \right) \left( 1 - \pi_0 \right) + \pi_0 \left( \frac{1-\theta}{\theta} \right) \right] \leq \frac{1}{m} \left[ \frac{\psi(a_1z) - \psi(a_2z)}{\beta} \right] \]

The probability of truthful advertising will exceed that of misleading advertising if and only if

(61) \[ \left[ \frac{1-\theta}{\theta} + \left( \frac{\pi_0}{1-\pi_0} \right) \left( 1 - \pi_0 \right) + \pi_0 \left( \frac{1-\theta}{\theta} \right) \right] \leq \frac{1}{m} \left[ \frac{\psi(a_1z) - \psi(a_2z)}{\beta} \right] \]
There is sure to be advertising if and only if

\[
(62) \quad \frac{1}{m} \left[ \frac{\psi(a_1z) - \psi(a_2z)}{\beta} \right] > \max \left\{ \left[ \frac{\theta}{(1-\theta)} + \left( \frac{\pi_0}{1-\pi_0} \right) \right] \left[ (1-\pi_0) + \pi_0 \left( \frac{1-\theta}{\theta} \right) \right], \right. \\
\left. \left[ \frac{1-\theta}{\theta} + \left( \frac{\pi_0}{1-\pi_0} \right) \right] \left[ (1-\pi_0) + \pi_0 \left( \frac{1-\theta}{\theta} \right) \right] \right\}.
\]

There is sure to be no advertising if and only if

\[
(63) \quad \frac{1}{m} \left[ \frac{\psi(a_1z) - \psi(a_2z)}{\beta} \right] \leq \min \left\{ \left[ \frac{\theta}{(1-\theta)} + \left( \frac{\pi_0}{1-\pi_0} \right) \right] \left[ (1-\pi_0) + \pi_0 \left( \frac{1-\theta}{\theta} \right) \right], \right. \\
\left. \left[ \frac{1-\theta}{\theta} + \left( \frac{\pi_0}{1-\pi_0} \right) \right] \left[ (1-\pi_0) + \pi_0 \left( \frac{1-\theta}{\theta} \right) \right] \right\}.
\]

Corollaries 2, 3 and 4 which follow describe formally, a series conditions on specific market characteristics which characterize respectively the case in which there is always advertising, the case in which there is no advertising, the case in which misleading advertising is more likely than the truthful variety and finally the case in which truthful advertising occurs with a higher probability than untruthful advertising. After each corollary there is an interpretive remark which provides the intuition of the results stated formally in the corollary.

**COROLLARY 2:**

When \( \pi(a) \) satisfies (53), the probability of misleading advertising exceeds (falls below) that of truthful advertising only if \( \pi_0 \) exceeds (is less than) \( \frac{1}{2} \), i.e., only if consumers initially expect quality to be high (low).
REMARK:

This corollary suggests that optimistic consumers; i.e., those who expect a product to be good are most susceptible to misleading advertising. But since the condition \( \pi_0 > \frac{1}{2} \) is only necessary, it does not guarantee that misleading advertising will be more likely than truthful advertising.

In interpreting this corollary, it should be recalled that there are two cases in which the probabilities of misleading and truthful advertising are equal. When condition (62) holds, both of these probabilities are one; when (63) holds, both probabilities are zero.

PROOF:

Note that (60) can hold only if

\[
0 \leq \left[ \frac{1-\theta}{\theta} + \frac{\pi_0}{1-\pi_0} \right] \left[ (1-\pi_0) + \pi_0 \left( \frac{\theta}{1-\theta} \right) \right] - \left[ \frac{\theta}{1-\theta} + \frac{\pi_0}{1-\pi_0} \right] \left[ (1-\pi_0) + \pi_0 \left( \frac{1-\theta}{\theta} \right) \right].
\]

Also note that the difference on the right hand side of (64) can be rewritten as

\[
\frac{1}{1-\pi_0} \left[ \frac{\theta}{1-\theta} - \frac{1-\theta}{\theta} \right] \left[ \pi_0^2 - (1-\pi_0)^2 \right]
\]

Since \( \theta \) exceeds \( 1-\theta \), (65) is positive only if \( \pi_0 \) exceeds \( 1-\pi_0 \); i.e., only if \( \pi_0 \) exceeds one-half. By a similar argument (61) can hold only if the inequality in (64) is reversed; i.e., only if (65) is negative. But (65) is negative only if \( \pi_0 \) is less than one-half.

COROLLARY 3:

Suppose that \( \pi(a) \) satisfies (53). Also assume that

\[
-\frac{x\psi''(x)}{\psi'(x)} < 1;
\]
i.e., the Arrow-Pratt risk aversion measure is uniformly less than one;
and that

$$\lim_{z \to \infty} [\psi(a_1 z) - \psi(a_2 z)] = \infty.$$  

1) If $\pi_0 > (>) \frac{1}{2}$, then other things equal; i.e., for fixed
values of $\theta, \pi_0, m, \beta, z,$ and $a_2$; there is a range $(a_1, \bar{a}_1)$ of $a_1$
values such that misleading (truthful) advertising is more likely than
truthful (misleading) advertising if $a_1$ lies in this range. If $a_1$ is
at or below $a_1$, there is no advertising and if $a_1$ exceeds $\bar{a}_1$, there is
always truthful and misleading advertising.

The length, $\bar{a}_1 - a_1$, of the interval $(\bar{a}_1, a_1)$ is an increasing
function of $\theta$ and becomes infinite as $\theta$ approaches one. If $\theta > \pi_0$
($(1-\theta) < \pi_0$) then $a_1$ is also an increasing function of $\theta$ and $a_1$ also
approaches infinity as $\theta$ nears one. If $\theta$ or $\pi_0$ are approximately one-
half, then $\bar{a}_1$ is close to $a_1$; i.e., the length of the interval of a
values for which the probability of misleading (truthful) advertising
exceeds that of truthful (misleading) advertising is small.

ii) If $\pi_0 > (>) \frac{1}{2}$, then, other things equal; i.e., for fixed
values of $\theta, \pi_0, m, \beta, z, a_1$ and $a_2$; there is a range of $z$ values $(\bar{z}, z)$
such that misleading (truthful) advertising is more likely than truthful
(misleading) advertising if $z$ lies in this range. If $z$ lies at or below
$\bar{z}$, there is no advertising and if $z$ lies above the range $(\bar{z}, z)$, there is
sure to be both misleading and truthful advertising.
The length $\tilde{z} - z$ of the interval $(z, \tilde{z})$ is an increasing function of $\theta$ and approaches infinity as $\theta$ approaches one. If $\theta > \pi_0(1-\theta) < \pi_0$, then $z$ also increases with $\theta$ and $z$ approaches infinity when $\theta$ tends to one. If $\theta$ or $\pi_0$ are approximately equal to one-half, then the interval $(z, \tilde{z})$ becomes small.

iii) If $\pi_0 > (<) \frac{1}{2}$, then; other things equal; i.e., for fixed values of $\theta$, $\pi_0$, $\beta$, $z$, $a_1$ and $a_2$; there is a range of $m$ values $[m, m]$ such that misleading (truthful) advertising is more likely than truthful (misleading) advertising when $m$ lies in this range. If $m$ lies at or above $\overline{m}$, there is no advertising. If $m$ is less than $\overline{m}$, then there is sure to be misleading and truthful advertising. The length of the interval $\overline{m} - m$ is a decreasing function of $\theta$ and approaches zero as $\theta$ approaches one. If $\theta > \pi_0(1-\theta) < \pi_0$, then $\overline{m}$ is also a decreasing function of $\theta$ and its limit is zero as $\theta$ approaches one.

REMARKS:

The interpretation of this corollary is straightforward if we recall the interpretation of the variables involved. To interpret (1), note that when $a_1$ is large, there is an important difference between high and low quality goods; i.e., between $a_1$ and $a_2$. Thus (1) asserts that if there is little difference between high and low quality then it will not pay to advertise at all. If, however, the difference between high and low quality is great, then there will be truthful and misleading advertising with probability one. The cases in which misleading advertising is more likely than truthful advertising occur when $\pi_0$ exceeds one-half and when the possible quality difference is large but
not too large. If the reliability $\theta$ of the nonadvertising information is improved, then there is an increase in the range of quality differences associated with the case in which misleading advertising is more likely than truthful advertising. As the nonadvertising information nears perfect reliability, this range becomes very large but unless the quality difference is extremely great there will be no advertising. Identical comments apply to describe the conditions under which misleading advertising is less likely than truthful advertising. The only difference is that these cases occur when $\pi_0$ is less than one-half. Unfortunately, $\pi_0$ is an unobservable parameter, thus even if it were known that $a_1$ was between $a_{\overline{1}}$ and $a_{\overline{1}}$, it couldn’t be known whether this will be a case in which misleading advertising is more likely or less likely than truthful advertising.$^6$/(ii) asserts that advertising, truthful or misleading, doesn’t pay if per capita demand $z$ is too low. If, on the other hand, individual demand is high both truthful and misleading advertising are observed with probability one. If consumers are initially optimisitic; i.e., if $\pi_0$ exceeds one-half; then the cases of likely misleading and unlikely truthful advertising occur when individual demand is large but not too large. Intermediate demand levels are also associated with the cases of likely truthful and unlikely misleading advertising. But these cases again occur when consumers are initially pessimistic about quality in the sense that $\pi_0$ is less than one-half. When nonadvertising information is extremely reliable, there is a wide range of individual demand levels consistent with the cases of unlikely truthful and likely misleading advertising or the cases of likely truthful and unlikely misleading
advertising (which of these cases it is again depends on whenever \( \pi_0 \) is greater or less than one-half). But in order to fall in this range demand must be very high. If it is not sufficiently high, then there will not be advertising of any kind.

(iii) implies that the cases in which there is no advertising occur when advertising is expensive in the sense that \( m \) is large. The cases in which advertising is sure to occur arise when \( m \) is low, that is advertising is inexpensive. When advertising is expensive but not too expensive then either misleading advertising is more likely than truthful or truthful is more likely than misleading. Which case occurs depends again on whether \( \pi_0 \) exceeds or falls below one-half. When nonadvertising information is very good there is a small range of \( m \) values which are associated with these two latter cases. Furthermore, when the non-advertising information is good advertising must be very inexpensive to be profitable in any case.

The proof of the corollary is not explicitly given. Instead we simply note that the intervals \( (a_\bar{1}, a_\bar{1}) \), \( (\bar{z}, \bar{z}) \) and \( [m, m] \) can be obtained directly from inequality (60).

**COROLLARY 4:**

If \( \pi(u) \) satisfies (53), then, other things equal, there exists a \( \bar{\theta} \) such that when \( \theta \) is between \( \bar{\theta} \) and one, there is no advertising.

**REMARK:**

This corollary is an immediate consequence of inequality (63). It expresses a point already made in the discussion of Corollary 3. That is, there will be no advertising of any kind if
the nonadvertising information is sufficiently reliable. ||

In those cases where misleading and truthful advertising both occur with probability one, it can be useful to know, for the purpose of identifying when case is likely to have occurred, whether advertising expenditures (per person) when \( \bar{x} = x_2 \) are larger or smaller than advertising expenditures (per person) when \( \bar{x} = x_1 \). For a similar reason, we want to know how the prices and supplies of misleading advertisers relate to those of truthful advertisers. It has already been noted that, in general, misleading advertisers supply less, on average, than truthful advertisers. (Recall Corollary 1 and the corresponding Remark 1.) For the special case currently under investigation it is also possible to show that misleading advertisers charge lower prices, on average. In fact, we can derive an explicit expression for the optimal price when the prediction is \( x_1 \) and when it is \( x_2 \). These expressions can then be used to relate the optimal advertising expenditure (per person) to the market parameters when \( x = x_1 \) and when \( x = x_2 \). This permits a comparison between \( \alpha_1(\hat{q}_1) \) and \( \alpha_2(\hat{q}_2) \) which are respectively the advertising expenditures (per person) most likely to be made by truthful advertisers and untruthful advertisers.

The formal expressions for \( \hat{q}_1, \hat{q}_2, \alpha_1(\hat{q}_1) \) and \( \alpha_2(\hat{q}_2) \) are derived in the following proposition. Interpretive remarks and a corollary follow.

PROPOSITION 3:

If \( \pi(\alpha) \) satisfies (53), and if \( \hat{q}_1 > q_1 \) and \( \hat{q}_2 > q_2 \), then

\[
\hat{q}_1 = \text{MRS}_1 - \left[ m(1 - \pi_0) \left\{ 1 - \frac{q_2}{\text{MRS}_2} \right\} \right]^{\frac{1}{2}}.
\]
\begin{align}
(66.2) \quad \hat{q}_2 &= \text{MRS}_1 - \left\{ \frac{m(1-\pi_0)}{1-\theta} \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{z} \right] \right\}^{1/2}, \\
(67.1) \quad \alpha_1(\hat{q}_1) &= [mz(1-\pi_0)\left( \frac{1-\theta}{\theta} \right)(\text{MRS}_1 - \text{MRS}_2)]^2 - m \left[ (1-\pi_0)\left( \frac{1-\theta}{\theta} \right) - \pi_0 \right] \\
(67.2) \quad \alpha_2(\hat{q}_2) &= [mz(1-\pi_0)\left( \frac{1-\theta}{\theta} \right)(\text{MRS}_1 - \text{MRS}_2)]^2 - m \left[ (1-\pi_0)\left( \frac{1-\theta}{\theta} \right) - \pi_0 \right]. \\
(68) \quad \hat{q}_1 - \hat{q}_2 &= \left\{ \frac{m(1-\pi_0)}{1-\theta} \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{z} \right] \right\}^{1/2} \left[ \left( \frac{1-\theta}{\theta} \right)^{1/2} - \left( \frac{1-\theta}{\theta} \right)^{1/2} \right] \\
(69) \quad \alpha_1(\hat{q}_1) - \alpha_2(\hat{q}_2) &= m(1-\pi_0)\left[ \frac{z(\text{MRS}_1 - \text{MRS}_2)}{m(1-\pi_0)} \right]^{1/2} - \left[ \left( \frac{1-\theta}{\theta} \right)^{1/2} + \left( \frac{\theta}{1-\theta} \right)^{1/2} \right] \left[ \left( \frac{1-\theta}{\theta} \right)^{1/2} - \left( \frac{\theta}{1-\theta} \right)^{1/2} \right].
\end{align}

**PROOF:**

Substituting (56.1) in (38) yields

\begin{align}
(70.1) \quad \text{MRS}_1 - \hat{q}_1 &= \left\{ \frac{m(1-\pi_0)}{1-\theta} \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{z} \right] \right\}^{1/2},
\end{align}

which implies (66.1). Similarly, (56.2) and (30) imply

\begin{align}
(70.2) \quad \text{MRS}_1 - \hat{q}_2 &= \left\{ \frac{m(1-\pi_0)}{1-\theta} \left[ \frac{\text{MRS}_1 - \text{MRS}_2}{z} \right] \right\}^{1/2},
\end{align}

and (66.2).

To obtain (67.1) and (67.2) first note that, because

\begin{align}
(71) \quad (q - \text{MRS}_2) &= (\text{MRS}_1 - \text{MRS}_2) - (\text{MRS}_1 - q),
\end{align}

(55.1) and (55.2) can be rewritten as
(55.1') \[ a_1(q) = \left(1 - \pi_0 \right) \left( \frac{1-\theta}{\theta} \right) \left[ \left( \frac{\text{MRS}_1 - \text{MRS}_2}{\text{MRS}_1 - q} \right) - 1 \right] - \pi_0^m \]

and

(55.2') \[ a_2(q) = \left(1 - \pi_0 \right) \left( \frac{\theta}{1-\theta} \right) \left[ \left( \frac{\text{MRS}_1 - \text{MRS}_2}{\text{MRS}_1 - q} \right) - 1 \right] - \pi_0^m \]

When (70.1) is substituted in (55.1') and (70.2) is substituted in (55.2') we obtain (67.1) and (67.2). Equations (68) and (69) follow immediately from subtraction of (66.2) from (66.1) and of (67.2) from (67.1).

Notice that, because \( \theta \) exceeds one-half, (68) implies that \( \hat{q}_1 \) always exceeds \( \hat{q}_2 \). Thus truthful advertising is more likely to occur at a higher price than untruthful advertising. This is so because truthful advertising is most likely to occur at the price \( \hat{q}_1 \) while untruthful advertising is most likely to occur at the price \( \hat{q}_2 \).

The interpretation of (69) is more complicated since the sign of the expression for \( a_1(\hat{q}_1) - a_2(\hat{q}_2) \) obtained there is ambiguous. The next corollary provides the formal basis for the interpretation of (69).

COROLLARY 5:

Assume that the hypotheses of Corollary 3 are satisfied.

Then:

i) other things equal, there exists an \( a_1 \) level \( a_1^* > a_2 \) such that \( a_1 > a_1^* \) implies that

(72) \[ a_1(q_1) < a_2(q_2) \]

while \( a_1 < a_1^* \) implies that
\( a_1(\hat{q}_1) > a_2(\hat{q}_2) \),

ii) other things equal, there exists a \( z \) level \( z^* \) such that \( z > z^* \) implies (72) while \( z < z^* \) implies (73),

iii) other things equal, there exists an \( m \) level \( m^* \) such that \( m < m^* \) implies (72) while \( m > m^* \) implies (73),

iv) other things equal, there exits a \( \pi_0 \) level \( \pi_0^* \in (0,1) \) such that \( \pi_0 \in (0,\pi_0^*) \) implies (73) while \( \pi_0 \in (\pi_0^*,1) \) implies (72),

(Note: \( \pi_0^* \) may be zero, in which case (72) always holds, and

v) other things equal, there exists a \( \theta^* \in [\frac{1}{2},1) \) such that \( \theta > \theta^* \) implies (73) while \( \theta < \theta^* \) implies (72). (Note that \( \theta^* \) may be equal \( \frac{1}{2} \) in which case (73) always holds.)

PROOF:

Since \( \theta \) exceeds one-half, (69) implies that \( a_1(\hat{q}_1) - a_2(\hat{q}_1) \) has the same sign as

\[
\left( \frac{1-\theta}{\theta} \right)^{\frac{1}{2}} - \left( \frac{\theta}{1-\theta} \right)^{\frac{1}{2}} - \frac{z(\text{MRS}_1 - \text{MRS}_2)}{m(1-\pi_0)} \right)^{\frac{1}{2}}.
\]

(74)

If, for example, \( a_1 \) is large then the difference \( \text{MRS}_1 - \text{MRS}_2 \) will be large (\( a_2 \) being fixed) and (74) will be negative. If however \( a_1 \) is close to \( a_2 \), then the second term in (74) will be approximately zero and (74) will be positive. This proves (i). The proofs of the other results are similar.

REMARK:

Corollary 5 implies that misleading advertising is more likely to exceed truthful advertising if the difference between high and low
quality is large; i.e., if \( a_1 - a_2 \) is large; if per capita demand, \( z \), is high, if advertising costs, \( m \), are low, if consumers are initially optimistic about quality; i.e., \( \pi_0 \) is near one; or if nonadvertising information is unreliable in the sense that \( \theta \) is near one-half.

SUMMARY AND CONCLUSIONS

The conditions under which truthful and misleading information occur are summarized in Proposition 2 of Section 2. Corollaries 2, 3 and 4 and the accompanying remarks summarize these conditions. These results and remarks also describe the market conditions under which truthful advertising is more (less) likely than misleading advertising. Proposition 3 yields expressions for the optimal price and advertising levels. Corollary 5 and the following remark uses this proposition to describe the conditions under which truthful per person advertising expenditures has a higher (lower) probability of being large than misleading per person advertising expenditures. Proposition 3 also implies that truthful advertisers tend to sell at a higher price than misleading advertisers. All of these results depend on the rather special assumptions made about consumer preferences (Equation (7)) and about the advertising technology (Equation (53)). A more general result which does not depend on the special form of the advertising cost function is Corollary 1 which asserts that truthful advertisers tend to sell more than untruthful advertisers.

There is one additional implication of the analysis presented here which has not yet been described. Consider the situation in which the probability of misleading advertising exceeds the probability of truthful advertising. Note that in such a case attempts to improve
the quality of nonadvertising information will, up to a point, only make the situation worse by raising the probability of misleading advertising and lowering the probability of truthful advertising! This happens because the probability of misleading advertising is $\theta$ and the probability of truthful advertising is $(1-\theta)$. But $\theta$ is just the measure of information quality. Note, however, that this only continues to occur until $\theta$ is raised to the point at when it exceeds \( \frac{\theta}{\theta} \) (Corollary 4) and causes the elimination of all advertising.

Although the model developed in this paper does not consider all of the important aspects of the advertising problem; e.g., the dynamics of consumer tastes, the market structure or the accumulated effects of advertising considered as an investment in goodwill capital; we would argue that it captures crucial aspects of the firm's interdependent production, advertising and pricing decisions and that our results do have policy implications. In particular, the model can be viewed as a guide to the identification of markets in which misleading advertising -- as we define it -- is more likely or is likely to be more important than truthful advertising.
FOOTNOTES

1/ Extensive surveys of the econometric literature on advertising can be found in Doyle (1968) and Schmellensee (1972).

2/ A seller who advertises a low quality product without knowing that quality is, in fact, low is not guilty of fraudulent advertising but the advertising is nevertheless misleading to consumers.

3/ Note, in particular, that although \( p_{i,n,n} = -c''(nz)z^2 < 0 \) and \( p_{i,q,q} = -na''(q) > 0 \),

\[
\begin{vmatrix}
   p_{i,q,q} & p_{i,q,n} \\
   p_{i,n,q} & p_{i,n,n}
\end{vmatrix} = [c''(nz)z^2 na''(q)] - [z - a'_i(q)]^2
\]

may be negative, in which case \( p_i \) is not concave.

4/ One property of \( p_i \) that should be observed in this connection is that whenever the first-order condition (30) is satisfied,

\[
\begin{vmatrix}
   p_{i,q,q} & p_{i,q,n} \\
   p_{i,n,q} & p_{i,n,n}
\end{vmatrix} = c''(nz)z^2 na''(q) > 0 .
\]

This fact implies that any interior critical point must yield a local maximum. If corner maxima were impossible, this observation would be sufficient to establish the proposition.

5/ These conditions will hold if, for example, \( \psi(x) = x^\delta \) with \( 0 < \delta < 1 \). They are used only in establishing (ii) of the corollary.

6/ Although \( \pi_0 \) is not directly observable, evidence may be available on the attitude of consumers regarding different products. In some cases, it might, however, be possible to estimate \( \pi_0 \). So although \( \pi_0 \) is not directly observable, proxies for \( \pi_0 \) may be available.
REFERENCES


