# The Nature and Origin of Spatial and Temporal Variations in the Gravity Fields of Telica and Masaya Volcanoes, Nicaragua 

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Ce mémoire intitulé

# Gravity and Microgravity Experiments at Telica and 

 Masaya Volcanoes, NicaraguaPrésenté par:
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## Frontispiece



Telica


Masaya


#### Abstract

Knowledge of the internal structures and dynamics of volcanoes is an important element in understanding and being able to forecast volcanic activity. developments in the field of gravity studies during the last decade have permitted better definition of the principal structures in volcanic environments and the processes governing the internal dynamics of volcanoes. This study is a presentation of different applications of gravity at two Nicaraguan volcanoes, Telica and Masaya. A static gravity survey was carried out at Telica to identify Bouguer gravity anomalies. The presence of a positive anomaly centered on the active crater defines a large, shallow intrusion in the Telica complex. It is oriented north-northwest-southsoutheast, with dimensions of $2 \mathrm{~km} \times 2 \mathrm{~km} \times 6 \mathrm{~km}$, and located at an approximate depthto top of 1 km . North-trending faults and the alignment of cones in the complex have a similar orientation to this intrusion. Microgravity measurements were made at Masaya. Modelling of the gravity changes at Masaya showed that the gravity changes are produced by fluctuations of the density, caused in turn by variations in the degree of vesiculation in the magma. A possible correlation of these fluctuations with solar and lunar tides was observed. A model for density changes in magma in terms of their exsolved and dissolved volatile contents was developed. This model shows that observable changes in gravity at the surface are produced by density variations caused by fluctuations in dissolved or exsolved volatile contents for shallow to relatively deep magma chambers ( $1-6 \mathrm{~km}$ ). The volatile element that produces the largest density changes is $\mathrm{H}_{2} \mathrm{O}$.


## Resumen

Un conocimiento de la estructura y la dinámica de los volcanes constituye un aspecto importante para poder comprender y pronosticar su comportamiento geológico. Los desarrollos científicos dentro del campo de estudios de la gravedad durante la última década nos han permitido una mejor definición de las estructuras principales que caracterizan el ambiente volcánico, y también de los procesos que controlan la dinámica interna de los volcanes. Este artículo presenta las varias aplicaciones técnicas de la gravedad en el estudio de dos volcanes nicaraguenses: Telica y Masaya. Una investigación de la gravedad estática fué realizada en Telica con el objeto de identificar anomalías de gravedad Bouguer. La preséncia de una anomalía positiva con su centro colocado sobre el cráter activo indica la existencia de un gran cuerpo intrusivo de poca profundidad dentro del complejo de Telica. La intrusión tiene una orientación de norte noroeste (NNW) hacia el sur sudeste (SSE), y sus dimensiones son 2 km por 2 km por 6 km . La superficie del cuerpo se encuentra a una profundidad de aproximadamente 1 km . Fallas con orientación norte - sur, y el arreglo linear de los conos volcanicos del complejo demuestran una orientación paralela a esta intrusión. Medidas microgravimétricas fueron coleccionadas en Masaya. Un modelo de los cambios de la gravedad en Masaya fue desarrollado, y este muestra que las variaciones en el campo de gravedad son debidas a la fluctuación de la densidad, debido, a su vez, a variaciones en el nivel de vesicularisación dentro del magma. Fué notada una posible relación entre estas fluctuaciones y las mareas lunares. Además, el modelo explica bien las variaciones de densidad dentro del magma como resultado del contenido de componentes volátiles disueltos. Este
modelo nos enseña que las variaciones de la gravedad notadas en la superficie son debidas a las variaciones en la densidad producidas por fluctuaciones en el contenido de elementos volatiles disueltos o exdisueltos en cámaras magmáticas a profundidades entre 1 y $6 \mathrm{~km} . \mathrm{H}_{2} \mathrm{O}$ es el componente volátíl que produce la mayor variación en densidad.

## Résumé

Dans le cadre de ce mémoire, deux volcans nicaraguayens ont été étudiés. Tous deux sont situés dans la chaîne volcanique d'Amérique Centrale, dans la partie ouest du Nicaragua, proche de l'Océan Pacifique. Telica est un stratovolcan situé à $12.603^{\circ} \mathrm{N}$ and $86.845^{\circ} \mathrm{W}$ dans le sud-ouest du Nicaragua. Il fait partie d'un complexe volcanique composé de plusieurs édifices (Santa Clara, Cerro Aguero et San Jacinto) situés dans la chaîne des Marabios. Le cône volcanique est pentu et contient un cratère de 700 m de diamètre et d'environs 120 m de profondeur. Les roches du complexe volcanique de Telica consistent en un chevauchement de coulées de lave, de tephras, de dépôts alluvionnaires et de lahars. L'activité volcanique à Telica depuis la conquête espagnole consiste en des périodes allongées d'émission de soufre et de nombreuses petites éruptions stromboliennes et phréatiques. Une augmentation de l'activité sismique est présentement en cours depuis 1996, le nombre d'événements étant passé de 100/jour à 220/jour entre le mois de décembre 1996 et le mois de juin 1997. Le dégazage du volcan reste très faible pendant cette période.

Le volcan Masaya est situé à $11.984^{\circ} \mathrm{N}$ et $86.161^{\circ} \mathrm{W}, 25 \mathrm{~km}$ au sud-ouest de Managua, capital du Nicaragua. Il fait partie de la caldeira de Masaya qui a des dimensions de 11.5 km par 6 km allongée selon une direction nord-ouest et sud-est, parallèlement à la chaîne volcanique. Dans la caldeira, une série d'évents en forme semi-circulaire se sont développés après la formation de la caldeira; ce sont les cônes de Masaya, de Nindirí, de Comalito, de Cerro Montosa et d'Arenal. Des cratères d'effondrement se sont formés dans les deux cônes principaux (Masaya et Nindirí): Santiago, Masaya, Nindirí et San Pedro. Santiago est présentement en phase de
dégazage intense depuis 1993, il rejette dans l'atmosphère plusieurs centaines à quelques milliers de tonnes de $\mathrm{SO}_{2}$ par jour. Durant les 150 dernières années, Masaya a connu plusieurs épisodes de dégazage semblable de façon cyclique. Deux coulées de lave se sont produites dans la caldeira: en 1670, d'un débordement du lac de lave de Nindirí au nord, et en 1772, d'une fissure sur le flanc nord-est du cône de Masaya.

Les objectifs principaux de ce mémoire sont d'expérimenter deux façons d'utiliser les méthodes gravimétriques sur les volcans et de démontrer leur utilité en ce qui a trait à l'accroissement de la connaissance sur la structure et le dynamisme interne des volcans. Premièrement, une carte des anomalies de Bouguer du volcan Telica est construite et des modélisations sont effectuées sur ces anomalies. Deuxièmement, des études microgravimétriques temporelles à différentes échelles de temps (annuelle, mensuelle, hebdomadaire et quotidienne) sont effectuées à Masaya. Troisièmement, un modèle théorique sur la variation de la densité des magmas en relation avec leur contenu en volatiles est échafaudé. Et quatrièmement, le modèle précédent est utilisé pour discuter des causes des variations temporelles de gravité à Masaya. Afin de combler ces objectifs, deux saisons de terrain furent conduites pendant lesquelles un total de 245 mesures de gravité et de GPS ont été prises dans la région du complexe volcanique de Telica et plusieurs mesures de microgravité ont été prises de façon répétitive à des échelles de temps différentes à Masaya.

L'étude de gravité statique à Telica a permis d'ébaucher une carte des anomalies gravimétriques du complexe volcanique. Des modélisations faites à partir de profils sur la carte des anomalies ont démontré la présence d'une intrusion à faible profondeur sous le complexe. Ce corps aurait une taille approximative de $2 \times 2 \times 6$
km , allongé selon une direction nord-nord-ouest et sud-sud-est, à une profondeur de 1 km . Le contraste de densité entre ce corps et les roches encaissantes serait entre 400$600 \mathrm{~kg} \mathrm{~m}^{-3}$. Les structures régionales concorderaient avec la présence d'une telle intrusion. Cette intrusion pourrait vraisemblablement être le réservoir magmatique ayant nourri les volcans du complexe de Telica.

Les études temporelles de microgravité à Masaya ont permis d'observer un lien entre les marées lunaire et solaire et les variations gravimétriques. Effectivement, il est démontré que les variations de gravité observées à Masaya sont causées par des changements de densité du magma sous le cratère de Santiago. Ces changements de densité seraient eux-mêmes causés par des fluctuations dans la quantité de bulles dans le magma. Un lien mécanique entre ce processus et les marées est une possibilité. Le dégazage important de Masaya montre peut-être aussi une relation avec les variations de gravité, mais il est difficile de trouver une relation directe puisque les mesures de $\mathrm{SO}_{2}$ sont plus éparses que celles de gravité. De toute façon, si les variations de gravité sont causées par des changements dans la quantité de gaz dans le magma, il est sûr qu'il y a un lien entre le dégazage et les variations de gravité.

La modélisation théorique des changements de densité des magmas à des pressions différentes avec les volatiles montre qu'il est possible d'observer des changements de gravité pour des chambres magmatiques, des dômes ou des conduits magmatiques d'une certaine taille. L'exsolution de volatiles dans un magma provoque un changement de densité important facile à repérer pour des corps magmatiques importants et de faible profondeur ( $1-6 \mathrm{~km}$ ). Des variations du contenu en volatile en milieu sous-saturé sont aussi possible à observer, même si elles
produisent des changements de densité de moindres importances. Il est aussi démontré qu'à de faible profondeur, le volatile le plus efficace pour faire varier la densité est $\mathrm{H}_{2} \mathrm{O}$, le $\mathrm{CO}_{2}$ étant à des concentrations trop faibles à ces profondeurs.

Ce travail à permis de démontrer la présence d'un réservoir magmatique à Telica. La surveillance de l'activité volcanique peut être focalisée au-dessus de ce corps afin de mieux cerner tout renouvellement de magma ou d'augmentation de pression à l'aide de microgravité ou de méthodes sismiques. Une meilleure connaissance du système volcanique interne de Masaya peut aider à mieux prévenir les futures périodes de dégazage et de possibles éruptions.

L'utilisation des techniques gravimétriques en milieu volcanique augmente les connaissances des processus magmatiques et de la structure interne sous les volcans. En intégrant ces données avec d'autres méthodes géologiques, géochimiques et géophysiques, notre connaissance des systèmes volcaniques s'accroît. Cela permet de mieux prévoir les comportements volcaniques qui peuvent poser des dangers pour les populations vivant aux alentours des volcans actifs.

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## Table Of Contents

Abstract ..... ii
Resumen ..... iii
Résumé ..... $-v$
Acknowledgments ..... ix
List of Figures ..... xii
List of Tables ..... xvii
General Introduction ..... 1
Introduction ..... 2
Objectives ..... 3
Locations ..... 4
Masaya Volcano ..... 4
Telica Volcano ..... 8
Geological Setting ..... 8
Regional Geology ..... 8
Geology of Masaya Volcano ..... 16
Geology of Telica Volcano ..... 22
Volcanic Activity ..... 25
Volcanic Activity of Masaya ..... 25
Volcanic Activity of Telica ..... 28
References ..... 29
CHAPTER I - The nature and origin of gravity anomalies at Telica volcano, Nicaragua ..... 32
Abstract ..... 33
Introduction ..... 33
Methodology ..... 37
Results ..... 47
Discussion ..... 53
Conclusion ..... 55
Acknowledgements ..... 56
References ..... 57
CHAPTER II - Temporal variations of microgravity at Masaya volcano, Nicaragua ..... 59
Abstract ..... 60
Introduction ..... 61
Methodology ..... 67
Results ..... 70
Daily Gravity Variations ..... 70
Weekly Gravity Variations in 1997 ..... 79
Weekly Gravity Variations in 1998 ..... 82
Annual Gravity Variation, 1997-1998 ..... 85
Discussion ..... 85
Conclusions ..... 94
Acknowledgements ..... 95
References ..... 96
CHAPTER III - Gravity changes induced by variations of the volatile content in magmas ..... 98
Abstract ..... 99
Introduction ..... 100
Methodology ..... 101
Shape Used for the Models ..... 101
Density Variation of Magmas ..... 104
Volatile Content and Solubility ..... 104
Density Calculation of a Crystal-and Volatile-BearingMagma ..... 105
Density of Modelling of Magma with a Pure $\mathrm{H}_{2} \mathrm{O}$ Dissolved Gas Fraction ..... 108
Density Modelling of Basaltic Magma with a Pure $\mathrm{CO}_{2}$ Dissolved Gas Fraction110
Density Modelling of Magmas with a Mixed $\mathrm{CO}_{2}-\mathrm{H}_{2} \mathrm{O}$ Dissolved-Gas Fraction ..... 112
Density Changes as a Function of Volatile Content for Undersaturated Magma112
Modelling of Gravity as a Function of Density Variation ..... 115
Spherical Model ..... 115
Vertical Cylinder Model ..... 120
Cubic Shape Chamber ..... 123
Discussion ..... 126
Conclusion ..... 130
Acknowledgements ..... 131
References ..... 132
Conclusions ..... 135
General Conclusions ..... 136
Recommendations for Future Work ..... 137
Appendix A ..... A1
Appendix B ..... B1
Appendix C ..... C1

## List of Figures

Figure I-1 Location of Telica and Masaya volcanoes in the Central American Front. Shaded strips are proposed segment boundaries of Soitber and Carr(1973)

Figure I-2 Locations of volcanic centers in the Masaya caldera (From Maciejewski, 1995)

Figure I-3 Aerial photo of Masaya caldera showing the different pit crater (From McClelland et al., 1989)

Figure I-4 Volcanoes of the Marabios Range. Volcano El Viejo and San Jacinto are now called San Cristobal and Santa Clara, respectively (From Lefebure, 1986)

Figure I-5 A view of Telica crater from the east side
Figure I-6 Plate tectonic setting of the Middle American island arc (From Lefebure, 1986)

Figure I-7 Map of the Quaternary volcanoes of Central America (From Weyl, 1980) 12

Figure I-8 Geologic map of Nicaragua (From Weyl, 1980) 14

Figure I-9 Generalized geological section through southwestern Nicaragua (From Weyl, 1980) 15

Figure I-10 Masaya caldera and the 1670 and 1772 lava flows (From Kieffer and Creusot-Eon, 1992) 19

Figure I-11 A view of the active vent at the bottom of Santiago crater $\qquad$ 20

Figure I-12 Geologic map of Masaya caldera showing the different geologic units (From Walker et al., 1993)

Figure I-13 Map of the Telica volcanic complex and surrounding towns (From Lefebure, 1986)23

Figure 1.1 Map of the Quaternary volcanoes of Central America showing the location of Telica volcano (From Weyl, 1980) $\qquad$ 34

Figure 1.2 Geological map of the Marabios Range, Nicaragua (Form Weyl, 1980)

Figure 1.3 Topographic map of the Telica complex showing location of the base station, the two gravity profiles and a sketch of the intrusion 38

Figure 1.4 Location of station Basel near the house at the foot of Telica volcano. Photo of the house

Figure 1.5 Nettleton correction for a gravity profile on Telica volcano using density variation of $1800 \mathrm{~kg} \mathrm{~m}^{-3}$ to $2900 \mathrm{~kg} \mathrm{~m}^{-3}$. (a) The gravity profile for the different densities. (b) The corresponding elevation profile on Telica

Figure 1.6 Contour map of the gravity anomaly showing the two profiles for density reductions of (a) $2400 \mathrm{~kg} \mathrm{~m}^{-3}$, (b) $2300 \mathrm{~kg} \mathrm{~m}^{-3}$ and (c) 2500 kg $\mathrm{m}^{-3}$. Contour interval is in mGal. Two profiles are shown which are used for GRAVMAG interpretation

Figure 1.7 Stratigraphic profiles of the Telica complex which correspond approximately to the gravity profile: (a) the north-south profile; (b) the east-west profile 46

Figure 1.8 Modelling of the east-west gravity profile using GRAVMAG. (a) An isolated shallow body. (b) A deeper and larger body. (c) The stratigraphic units without an intrusion. (d) The stratigraphic units with a shallow intrusive body. (e) Best fit with a density contrast of 600 kg $\mathrm{m}^{-3}$ (intrusion). (f) Best fit with dikes. (g) Best fit with a density contrast of $450 \mathrm{~kg} \mathrm{~m}^{-3}$ 49

Figure 1.9 Modelling of the north-south gravity profile using GRAVMAG 51

Figure 1.10 Modelling of the east-west gravity profile with a simple body of 3000 m half-strike for density reductions of (a) $2300 \mathrm{~kg} \mathrm{~m}^{-3}$, (b) $2400 \mathrm{~kg} \mathrm{~m}^{-3}$ and $2500 \mathrm{~kg} \mathrm{~m}^{-3}$ 52

Figure 2.1 The Central American volcanic chain showing the segments of Carr (1984) and Masaya volcano (From Metaxian, 1994) 62

Figure 2.2 Map of Masaya caldera showing the different cones and craters (From Viramonte et al., 1997)63

Figure 2.3 Gravity changes at stations within Masaya caldera; station locations are shown in Figure 2.4 (From Rymer et al., 1998)

Figure 2.4 Location of the gravity stations in Masaya caldera. Stations B1A and B2 are proximal and are represented by B (From Rymer et al., 1998)

Figure 2.5 (a) Scintrex and LCR daily gravity changes at Masaya volcano at station A7, March 12 1997. (b) Temporal variation of $\mathrm{SO}_{2}$ vs temporal variation of microgravity for the LCR meter at Santiago crater, 12 March 1997

Figure 2.6 Daily gravity variation at station A7 for (a) 25/02/98, (b) 06/03/98 and (c) $13 / 03 / 98$. (d) Maximum gravity changes vs. maximum tidal amplitude for daily variations in 1997 and 1998. (e) Maximum gravity changes vs. maximum tidal amplitude for daily variations in 1998 only. (f) Fluctuation of $\mathrm{SO}_{2}$ vs. gravity changes on 13 March 1998 at station A7

Figure 2.7 Daily gravity changes vs. pressure fluctuations at station A7 for (a) $06 / 03 / 98$ and (b) 13/03/98. (c) Pressure variation rate vs. gravity variation rate for daily variations in 1998 at station A7 77

Figure 2.8 (a) Weekly gravity changes for the LCR in 1997. (b) Gravity changes vs. diurnal tidal variation, 4-17 March 1997 at stations A3, A7, B1A and B280

Figure 2.9 (a) Gravity changes at Masaya between 27/01/98 and 14/03/98. (b) Gravity changes at Masaya between 24/02/98 and 09/03/98. (c) Gravity changes vs. diurnal tidal variation from 27 January to 14 March 199884

Figure 2.10 Annual gravity changes at Masaya for station A7, 1993-1998
Figure 2.11 Gravity changes of a body of magma under Santiago crater 120 m thick with an initial density contrast of (a) $-120 \mathrm{~kg} \mathrm{~m}^{-3}$ and than increasing to (b) $-70 \mathrm{~kg} \mathrm{~m}^{-3}$, (c) $-50 \mathrm{~kg} \mathrm{~m}^{-3}$ and (d) $-25 \mathrm{~kg} \mathrm{~m}^{-3}$

Figure 2.12 (a) Fluctuation of $\mathrm{SO}_{2}$ flux on March 12, 1997. (b) Fluctuation of $\mathrm{SO}_{2}$ flux on 13 March 1998

Figure 3.1 Gravity effect of a sphere, the horizontal axis $x / z$ is the horizontal distance from the center of the sphere, $x$, and $z$ the depth of the center; the vertical axis is the gravity of a point at a certain horizontal distance from the center of the sphere versus maximum gravity (From Telford et al., 1990). 102

Figure 3.2 Density changes with variation in the water content at different pressures for (a) rhyolitic magma and (b) basaltic magma $\qquad$ 107

Figure 3.3 Density changes with variation in the water content at different crystal fractions at 100 MPa for (a) rhyolitic magma and (b) basaltic magma. Density changes with variation in the crystal content at 100 MPa and 4 wt $\% \mathrm{H}_{2} \mathrm{O}$ for (c) rhyolitic magma and (d) basaltic magma $\qquad$ 109

Figure 3.4 (a) Density changes of a basaltic magma at the onset of gas exsolution with variation in the $\mathrm{CO}_{2}$ content at different pressures. (b) Density variation of a basaltic magma with variation in the $\mathrm{CO}_{2}$ content with different crystal fractions at 100 MPa . (c) Density variation of a basaltic magma with variation in the crystal content at 100 MPa and at different $\mathrm{CO}_{2}$ contents

Figure 3.5 Solubility of $\mathrm{H}_{2} \mathrm{O}+\mathrm{CO}_{2}$ as a function of pressure and fluid composition for (a) tholeiitic basalt at $1200^{\circ} \mathrm{C}$ and (b) rhyolite at 850 ${ }^{\circ} \mathrm{C}$ (From Holloway and Blank, 1994) 113

Figure 3.6 Density changes of ascending non-degassed magma and descending degassed magma as function of depth. Arrows schematically show a path for convective transport of magma caused by density changes. Numbers show water content in wt. \% (From Kazahaya et al., 1994)

Figure 3.7 Gravity effect of density variation of a $0.1 \mathrm{~km}^{3}$ spherical magma chamber at depths of (a) 1900 m , (b) 3800 m and (c) 7600 m $\qquad$ 117

Figure 3.8 Gravity effect of density variation of a $1 \mathrm{~km}^{3}$ spherical magma chamber at depths of (a) 1900 m , (b) 3800 m and (c) 7600 m $\qquad$ 118

Figure 3.9 Gravity effect of density variation of a $10 \mathrm{~km}^{3}$ spherical magma chamber at depths of (a) 1900 m , (b) 3800 m and (c) 7600 m $\qquad$ 119

Figure 3.10 (a) Gravity change caused by lowering of the level of magma in a vertical cylinder of 50 m radius and 100 m length at a depth of 100 m with a density contrast of $300 \mathrm{~kg} \mathrm{~m}^{-3}$. (b) Gravity change of a vertical cylinder of 50 m radius, 100 m deep and 100 m long caused by variation in density of the magma. (c) Gravity change caused by lowering the level of magma for a vertical cylinder of 220 m radius and 100 m length at a depth of 300 m with a density contrast of 300 kg $\mathrm{m}^{-3}$. (d) Gravity change of a vertical cylinder of 220 m radius, 300 m deep and 100 m length caused by variation in density of the magma 122

Figure 3.11 Gravity change produced in the top layer ( 100 m thick) of a magma chamber at a depth of 2000 m by a density decrease of (a) $70 \mathrm{~kg} \mathrm{~m}^{-3}$ and (b) $140 \mathrm{~kg} \mathrm{~m}^{-3}$ 124

Figure 3.12 Gravity change produced in the top layer ( 100 m thick) of a magma chamber at a depth of 6000 m by a density decrease of (a) $30 \mathrm{~kg} \mathrm{~m}^{-3}$ and (b) $55 \mathrm{~kg} \mathrm{~m}^{-3}$ $\qquad$ 125

Figure 3.13 Gravity change produced by a foam accumulation at the top of a 2000 m deep basaltic magma chamber 127

## List of Tables

Table I-1 Stratigraphic correlation chart of southwestern Nicaragua (From Viramonte et al., 1997)

Table I-2 Formations of the Telica volcanic complex (From Lefebure, 1986) _24
Table I-3 Description of the volcanic units of the Telica complex (From Lefebure,1986)

Table 2.1 Maximum gravity and tidal variations and their ratio in 1997 and 1998

## General introduction

$\qquad$

## Introduction

Gravity surveys of volcanic areas provide an excellent means to define and understand the subsurface structures of volcanoes. With the technological improvements in the past few years in the field of the Global Positioning System (GPS), new approaches in the field of gravity are now possible. These techniques give us a way to study mass movement and accumulation or loss of gas beneath volcanoes, which in turn help us understand their activity and forecast eruptions. Gravity variations observed in the past have been difficult to interpret because of a lack of good elevation control (Eggers et al., 1976; Eggers and Chavez, 1979; Eggers, 1983; Vieira et al., 1986). Microgravity measurements with precision to better than $10 \mu \mathrm{Gal}$ and levelling with precision to better than 2 cm can now be acquired rapidly on volcanoes (Rymer, 1989; Brown et al., 1991). With accumulation of data and experience, interpretation of time-varying gravity features on volcanoes becomes more reliable and meaningful (Rymer, 1994; Rymer and Locke, 1995).

In the present study, two volcanoes were surveyed using two different methods of gravity monitoring. At Telica volcano, Nicaragua, a conventional static gravity survey was conducted to make a map of the Bouguer anomalies of the crater area. At Masaya volcano, microgravity monitoring was conducted to observe temporal variations at different scales (yearly, monthly and daily variations).

Telica is an active volcano in northwestern Nicaragua that is part of the chain of Quaternary volcanoes along the western margin of Central America. The Santa Clara, Cerro de Aguero and San Jacinto edifices coalesce with Telica to form a group of volcanic centers that is part of the Marrabios range. Historic activity at Telica for the last 500 years consists of extended periods of solfataric activity and numerous
small explosive eruptions (Lefebure, 1986). Masaya volcano is part of Masaya caldera, situated in the same Quaternary volcanic chain as Telica. Historic activity at Masaya includes lava lake formation, pit crater formation and two lava flows which were erupted in 1670 from an overflow of the lava lake of Nindirí to the north and in 1772 from a fissure on the northeast flank of the Masaya cone (Rymer et al., 1998). Geological evidence also shows episodes of pyroclastic cone-building eruptions. Plinian airfalls are also known to have occurred from Masaya caldera (Williams, 1983). During the past 150 years, episodes of strong degassing have occurred, showing a cyclic interval of about 25 years. These degassing crises, five in total since 1852, represent the degassing of approximately $10 \mathrm{~km}^{3}$ of basaltic magma (Stoiber et al., 1986). Rymer et al. (1998) monitored the volcano from 1993 to 1997 using microgravity techniques, levelling instruments, GPS and COSPEC measurements. Masaya caldera also has been investigated using other geophysical techniques (Bouguer gravity, seismology and magnetotellurics) (Metaxian, 1994; Metaxian and Lesage, 1997). Presently, Masaya is monitored on a continuous basis with one seismic station by INETER (Instituto Nicaragüense de Estudios Territoriales).

## Objectives

The main goals of this thesis are to (1) map and model the gravity anomalies of Telica volcano, (2) monitor microgravity variations on different timescales (yearly, monthly, weekly and daily) at Masaya volcano, (3) build a model for gravity changes related to variations in the dissolved and exsolved volatile content in the magma under volcanic edifices, and (4) discuss possible causes of gravity variations with
time at Masaya and, using the previous theoretical model, explain the changes observed. To achieve these goals, two seasons of fieldwork were conducted where gravity and microgravity data were acquired at Telica and Masaya volcanoes. At Telica, 245 gravity and GPS measurements were taken around the crater area. Microgravity measurements at Masaya volcano were taken repeatedly each day and week during the two field seasons to obtain adequate data with which to observe temporal variations at different timescales. Leica GPS 200 dual-frequency differential receivers were used for positioning of the gravity stations at Telica and for monitoring altitude variations at Masaya. LaCoste and Romberg meter G-513 and Scintrex meter were use for the gravity work.

## Locations

## Masaya Volcano

Masaya volcano is a large basaltic shield volcano situated at $11.984^{\circ} \mathrm{N}$ and $86.161^{\circ} \mathrm{W}, 25 \mathrm{~km}$ southeast of Managua, which is the capital of Nicaragua (Fig. I-1). The summit of Masaya volcano is situated at 624 m above sea level. Masaya caldera is 11.5 km by 6 km , elongated in the northwest-southeast direction, and parallel to the volcanic chain. An $8 \mathrm{~km}^{2}$ lake at 135 m altitude is situated in the southeastern part of the caldera. Inside the caldera, a semi-circular set of vents have developed from postcaldera eruptions; they include Masaya, Nindirí, Comalito, Cerro Montosa and Arenal cones (Fig. I-2). Four pit craters have been formed in the two main cones (Masaya and Nindirí), including the Masaya, Santiago, Nindirí and San Pedro pit craters (Fig. I-3) (Rymer et al., 1998). Santiago is the active crater at the present time.

## FIGURE I-1

Location of Telica and Masaya volcanoes in the Central American Front. Shaded strips are proposed segment boundaries of Soitber and Carr (1973).


FIGURE I-2
Locations of volcanic centers in the Masaya caldera (From Maciejewski, 1995).


## FIGURE I-3

Aerial photo of Masaya caldera showing the different pit crater (From McClelland et al., 1989).


## Telica Volcano

Telica volcano is a composite volcano located at $12.603^{\circ} \mathrm{N}$ and $86.845^{\circ} \mathrm{W}$, 19 km north of León, Nicaragua's second largest city, at the northwestern edge of a large volcanic complex (Fig. I-1). The summit of Telica is 1040 m above sea level. The Telica volcanic complex is situated in the central part of the Marabios Range (Fig. I-4). Telica has a very steep sided cone with a double crater measuring 700 m in diameter (Fig. I-5). The southern crater, which is presently active, is at least 120 m in depth. None of the other cones in the volcanic complex which comprise the large ridge of El Liston are active.

## Geological Setting

## Regional Geology

Telica and Masaya volcanoes are situated in the Nicaraguan Quaternary volcanic chain located near the southern end of the active Central American volcanic front. This front is part of the Meridional Structural Domain which includes Costa Rica, Panama and southwestern Nicaragua. The volcanic front is the result of plate convergence between the Cocos and Caribbean plates (Malfait and Dinkelman, 1972; Carr, 1984) (Fig. I-6). The Central American volcanic chain, consisting of 38 active volcanoes, has been divided into 7 linear segments along the Pacific margin of Central America (Stoiber and Carr, 1973) (Fig. I-7). In Nicaragua, two tectonic segments divide the volcanic chain. Masaya caldera is located in the eastern Nicaraguan segment, about 25 km southeast of the boundary between the two segments, and Telica is found in the western Nicaraguan segment (Fig. I-1). The

## FIGURE I-4

Volcanoes of the Marabios Range. Volcano El Viejo and San Jacinto are now called San Cristobal and Santa Clara, respectively (From Lefebure, 1986).


## FIGURE I-5

A view of Telica crater from the east side.


Plate tectonic setting of the Middle American island arc (From Lefebure, 1986).


FIGURE I-7
Map of the Quaternary volcanoes of Central America (From Weyl, 1980).

northern part of this volcanic chain where the Telica complex is found, the Marabios chain, is mainly dominated by volcanic complexes, individual stratocones, cinder and spatter cones. In the central part of the volcanic chain lie the main ignimbrite centers: the Malpaisillo caldera at the north and the Managua-Las Sierras-Masaya Complex at the south (Metaxian, 1994; Viramonte et al., 1997; Lefebure, 1986).

Central America is divided by Weyl (1980) into two units: the northern part which is underlain by a basement of continental crust (Paleozoic) and the southern part which consists of oceanic crust overlain by younger sediments and volcanics (Tertiary). The contact between the two units is thought to underlie the Managua Graben in the area of the Nejapa Alignment in Managua (Bice, 1990, from Maciejewski, 1995). Nine structural provinces, corresponding to the distribution of formations of different age, were identified in Nicaragua (Garayar, 1977, from Weyl, 1980). This scheme is reflected in the general geological map of the country (Fig. I8). McBirney and Williams (1965) proposed the following as the most important physiographic units in Nicaragua: the Atlantic Coastal Plain, the Interior Highland, the Nicaraguan Depression and the Pacific Coastal Plain. This structural scheme is shown in a northeast-southwest geological cross-section (Fig. I-9).

The basement rocks of Central America extend from Guatemala southward to northern Nicaragua. They form a sequence of schists, phyllites, marbles and quartzites called the Nueva Segovia Formation of Paleozoic age that was intruded by Cretaceous plutonic rocks (Lefebure, 1986). The Bocay basin, in the northwestern frontier region of Nicaragua near Honduras, is oriented northeast-southwest and is mainly composed of terrestrial and marine sediments (Bracchi and Giudice, 1958;

FIGURE I-8

Geologic map of Nicaragua (From Weyl, 1980).


## FIGURE I-9

Generalized geological section through southwestern Nicaragua (From Weyl, 1980).


Rivera, 1962, from Weyl, 1980). The Mosquitia basin, which is the northeastern continuation of the Bocay Basin, is one of the largest Mesozoic-Tertiary sedimentary basins in Central America (Karim et al., 1966; Mills et al., 1967; Mills and Hugh, 1974; Arden, 1969 and 1975, from Weyl, 1980). The Interior Highlands are mainly composed of Tertiary volcanic sequences that are divided into the Matagalpa, Lower and Upper Coyol Groups. They are composed of accumulations of pyroclastic flows, lava flows, tuffs and epiclastic breccias with minor associated sediments (Lefebure, 1986). The Nicaraguan Depression, a broad shallow graben, is the dominant structural feature in Nicaragua and is probably late Miocene in age (McBirney and Williams, 1965). It is filled in part with pyroclastic material coming form the Las Sierras group into the central part of the depression (Metaxian, 1994). The sedimentary formations in Nicaragua consist of the Rivas, Brito, Masachapa, El Fraile and El Salto Formations (Lefebure, 1986) (Table I-1).

## Geology of Masaya Volcano

The origin of Masaya caldera is controversial. McBirney (1956) believed that the caldera was formed after a series of collapses due to a migration of the magma chamber. For Williams (1983) and Bice (1985), the formation of the caldera results from a series of collapses due to strong magmatic eruptions, all of basaltic composition, including plinian and ignimbrite eruptions. Kieffer and Creusot-Eon (1982) suggested that the caldera was the result of an enormous phreatomagmatic eruption which produced a huge depression of "maar" type. Masaya caldera is itself situated within another caldera, the Las Sierras caldera formed $20000-30000$ years ago, which resulted in the formation of the Masaya Lapilli Bed (van Wyk de Vries,

Table I-1
Statigraphic correlation chart of southwestern Nicaragua (From Viramonte et al., 1997)

1993). Masaya caldera was formed during the eruption that deposited the Masaya Tuff between between 6000 and 2000 years ago (Walker et al., 1993).

Masaya caldera is different from the other volcanoes in the volcanic chain. Not only does it have large-volume explosive basaltic activity, the pre-caldera activity resulted in the construction of a low shield volcano with a form that does not match the composite volcanoes which dominate the Nicaraguan volcanic front (Walker et al., 1993, Metaxian, 1994). Poorly vegetated lavas cover the floor of the caldera. Since the sixteenth century, only two lava flows have been known to erupt. The first lava flow was the result of an overflow in 1670 from the Nindiri pit which contained a 1 km wide lava lake at the time (Rymer et al., 1998). The second lava flow was erupted from a fissure on the flank of the Masaya cone in 1772 (Fig. I-10). The only other historic lava is found in Santiago crater which is still active at present, and in Nindirí crater in 1852 (Rymer et al., 1998).

Santiago crater is the main site of activity since it formed in 1858-1859. A lava lake covered the floor of Santiago in 1948 and 1965; the solidified lava is now broken by concentric faults (McBirney, 1956). The active vent of Santiago is now situated in an inner crater 150 m deeper than the main crater which is itself 150 m deep (Fig. I-11). Incandescence is often visible at the bottom of the crater. The volcanic rocks from Masaya are all basalts or basaltic andesites showing very little compositional variation compared to other Central American volcanoes. They also have a low content in $\mathrm{Al}_{2} \mathrm{O}_{3}$ and a high concentration in FeO , showing a tholeiitic differentiation trend. These compositional variations and the evolution of the caldera is the result of open-system magmatic differentiation in a large, shallow magma chamber (Walker et al., 1993). A gravity anomaly was first reported by Connor and Williams (1990) and

FIGURE I-10
Masaya caldera and the 1670 and 1772 lava flows (From Kieffer and Creusot-Eon, 1992).


FIGURE I-11

A view of the active vent at the bottom of Santiago crater.


## FIGURE I-12

Geologic map of Masaya caldera showing the different geologic units (From Walker et al., 1993).

refined by Metaxian (1994). From a Bouguer survey of the caldera area, Metaxian (1994) postulated the presence of a dense body $500 \mathrm{~kg} \mathrm{~m}^{-3}$ higher than the surrounding rocks and centered at 6 km depth, of 6 km thickness, with the roof at a depth of 2-3 km. There is also a magnetic anomaly which has been modlled in terms of the same structure (Metaxian, 1994).

The basement of the caldera comprises the Las Sierras Formation (Williams, 1983). In the west, the caldera walls are made up only of basaltic ignimbrites, a 73 m thick sequence that formed the Fontana Lapilli, which is a major regional stratigraphic marker (Williams, 1983). Pyroclastic eruptions have produced extensive Strombolian deposits on the caldera floor and on the slopes of the volcano (Williams, 1983). Voluminous and widespread surge deposits are found around the caldera, and overlain by pyroclastic flow and fallout deposits which are believed by Williams (1983) to be associated with caldera collapse. Detailed mapping of the geologic units in the vicinity of the caldera was done by Williams (1983) (Fig I-12).

## Geology of Telica Volcano

The Telica Volcanic Complex covers approximately $80 \mathrm{~km}^{2}$ and extends from the town of Telica in the south to Las Marias in the north. Also near the complex, the city of San Jacinto lies to the east and Colonia Agricola Cristo Rey to the west (Fig. I13). Telica volcano is a stratovolcano of basaltic to andesitic composition. The volume of the Telica complex is estimated at $30 \mathrm{~km}^{3}$ of volcanic material (Stoiber and Carr, 1973). The following description of the Telica complex comes mainly from Lefebure (1986).

The Telica complex can be divided geomorphologically into elevated areas which are Cerro Los Portillo and San Jacinto, a lava "apron" and the volcanic cones

FIGURE I-13

Map of the Telica volcanic complex and surrounding towns (From Lefebure, 1896).


## Table 1-2

Formations of the Telica volcanic complex (Adapted from Lefebure, 1986)

| Period | Epoch | Formation | Lithology | Unit |
| :---: | :---: | :---: | :---: | :---: |
| Quaternary | Recent |  | alluvium and Colluivium | 9 |
|  |  | Telica | basalt lava and tephra | 8 |
|  | Recent or Pleistocene |  | andesite lava | 7 |
|  |  | San Jacinto | altered basalt lapillistone and tuff | 6 |
|  |  | Santa Clara | basalt lava and tephra | 5 |
|  |  | Cerro de Aguero | basalt lava and tephra | 4 |
|  |  | El Najo | basalt lava | 3 |
|  |  | La Ceiba | andesite lava | 2 |
| Tertiary or Quaternary | Pliocene or Pleistocene | Cerros Los Portillos | basalt lava | 1 |

of Telica, Santa Clara and Cerro de Aguero. The Telica complex consist of lavas, tephra, alluvium, lahars and intrusive rocks (Table I-2). The most abundant rock types are lava flows and tephra of basaltic and andesitic composition. Epiclastic rocks can be found in valleys on the flanks of the volcanic centers and in the surrounding plains. Outcrops of hypabyssal intrusive rocks are visible on the older volcanic centers. Telica, the only active cone in the complex at present, is also the youngest cone and rests on the other volcanic centers of the complex. McBirney and Williams (1965) believed the volcanoes of the Marrabios Range, including the Telica Complex, to be of Quaternary age. Lefebure (1986) separated the Telica Complex into eight different volcanic units (Table I-3) and identified 14 lava flows. Fumarolic activity occurs in the crater of Telica volcano, which is most intense during and immediately after the rainy season from April to July. North-south-oriented faults are common on the volcano (van Wyk de Vries, 1993). Hot springs are also present at 8 localities in the central, eastern and northern parts of the volcanic complex. The shape of the Telica edifice represents the construction and destruction of at least three successive edifices (Lefebure, 1986).

## Volcanic Activity

## Volcanic Activity of Masaya

Masaya volcano has been persistently active since the beginning of the sixteenth century. The activity is characterised by episodes of lava lake formation associated with strong gas emissions since the formation of Santiago crater in 18581859 (Stoiber et al., 1986). Plinian and ignimbrite eruptions also occurred in the

## Table I-3

Description of the volcanic units of the Telica complex (From Lefebure, 1986)

| Formation | Unit | Name | Phenocrysts | Groundmass |
| :---: | :---: | :---: | :---: | :---: |
| Telica | 8c | Black pyroxene basalt porphyry tephra deposits | plag ( $\mathrm{An}_{70-65}$ ) <br> augite <br> olivine ( $\mathrm{Fo}_{75}$ ) | labradorice and clinopyroxene microlites, opaques and sideromelane |
|  | 8b | Gray pyroxene basalt porphyry flows | plag ( $\mathrm{An}_{50-65}$ ) augite olivine ( $\mathrm{Fo}_{85-80}$ ) orthopyroxene opaques | labradorite, clinopyroxene and olivine microlices. opaques, +/sideromelane |
|  | 8a | Gray pyroxene basalt porphyry lapilli and ash | plag ( $\mathrm{An}_{80-70}$ ) <br> augite <br> opaques | labradorite microlites and sideromelane or tachylite glass |
|  | 7 | Gray andesite porphyry flows | $\begin{aligned} & \text { plag }\left(\mathrm{An}_{85-60}\right) \\ & +/ \text { - augite } \\ & \text { opaques } \end{aligned}$ | andesine and clinopyroxene microlites, opaques and and intersertal glass |
| San Jacinco | 6* | Brown or orangebrown altered basalt lapilli-stone and tuff | Plag Augite olivine | reddish-brown to black glass |
| Santa Clara | 5 | Gray olivine basalt porphyry flows and tephra | plag ( $\mathrm{An}_{75 \text {-70 }}$ ) <br> opaques <br> olivine ( $\mathrm{Fo}_{\mathrm{s}_{0}}$ ) | labradorite and augite microlites, opaques and minor glass |
| Cerro de Aguero | 4 | Gray olivine andesite porphyry flows and tephra | plag ( $\mathrm{An}_{80-70}$ ) opaques olivine ( $\mathrm{FO}_{75}$ ) | labradorite and augite microlites, olivine, opaques and glass |
| El Najo | 3 | Gray olivine pyroxene basalt porphyry flows | plag ( $\mathrm{An}_{80.75}$ ) <br> augite <br> olivine | labradorite and clinopyroxene microlites, opaques and intersertal glass |
| La Ceiba | 2 | Grey and red pyroxene andesite flows | plag $\left(\mathrm{An}_{85-65}\right.$ <br> ) bronzite <br> opaques <br> +/- olivine | plagioclase and clinopyroxenes microlites in glass |
| Cerro Los Portillos | 1 | Gray olivine basalt porphyry flows | plag $\left(\mathrm{An}_{55.60}\right)$ <br> olivine ( $\mathrm{FO}_{95.85}$ ) <br> $+/$ - augite | labradorite and clinopyroxene microlites, opaques, minor glass |

history of Masaya (Williams, 1983). Caldera-forming eruptions have been frequent in Masaya's volcanic history; at least four are known between 2700 and 30000 years BP (Williams, 1983; van Wyk de Vries, 1991, from Rymer et al., 1998). Stoiber et al. (1986) estimated that approximately $33 \times 10^{6} \mathrm{~m}^{3}$ of basaltic lavas were erupted since the Spanish Conquest (1524). This gives an average rate of $0.07 \times 10^{6} \mathrm{~m}^{3} \mathrm{yr}^{-1}$, which is significantly lower compared to what Williams (1983) calculated for the average prehistoric rate of $1.9-5.5 \times 10^{6} \mathrm{~m}^{3} \mathrm{yr}^{-1}$.

One unusual aspect of Masaya is the degassing crises that have occurred since 1852 (Stoiber et al., 1986). Five of these crises have occurred in the past, and one is ongoing since 1993 (Rymer et al., 1998). The gas is coming out of the active Santiago crater. During the episode from 1977 to 1985, Stoiber et al. (1986) measured $\mathrm{SO}_{2}$ fluxes in the gas plume by COSPEC and estimated an average flux of 1275 metric tonnes/day. This degassing rate implies that $10 \mathrm{~km}^{3}$ of magma have been degassed over the last century. This indicates a discordance between the gas emission rate and the lava emission rate. The ratio of erupted solid material to the volume of intrusive degassed magma ( $0.1 \mathrm{~km}^{3} \mathrm{yr}^{-1}$ ) is only 0.0007 (Metaxian, 1994). An active lava lake was sometimes visible in Santiago from 1965 to 1979. Cooling of this lava lake formed the platform which is visible today at a depth of 150 m in Santiago. After partial collapse of this platform in 1989, a new lava lake was visible 150 m deeper than the platform (SEAN Bulletin, 1989). This state lasted for $11 / 2$ months and was terminated by further collapse of the southern part of the crater (Metaxian, 1994). From 1990 to 1993, activity was restricted to very small gas emissions (less than 25 metric tonnes/day $\mathrm{SO}_{2}$ ) (Bulletin of the Global Volcanism Network, 1992). In 1993, the volcano entered a new degassing phase, and a new lava
lake was visible (Rymer et al., 1998). This renewal in activity was accompanied by an increase in the permanent tremor amplitude by a factor of 4 as recorded at the volcano (Metaxian, 1994). The source of tremor is located beneath the active crater of Santiago (Metaxian and Lesage, 1997).

## Volcanic Activity of Telica

During historic time, two volcanoes in the Telica Complex have been active: Santa Clara and Telica. Historical volcanic activity at Santa Clara was restricted to solfateric activity during the sixteenth century (Lefebure, 1986). Telica has had extended periods of solfateric activity and numerous small explosive eruptions during the last 500 years (Lefebure, 1986). Since the beginning of the twentieth century, the activity has increased, ejecting ash during at least 22 different eruptive periods (Lefebure, 1986). A lava lake was observed in 1971 at the bottom of Telica's deep circular crater. Historic eruptions at Telica have resulted in vertical ejections of basaltic ash and gases over periods of days or weeks with associated seismic activity. During stronger eruptions, lapilli, bombs and blocks were sometimes ejected onto the crater rim and flanks. The eruptions resemble many of the features of Stromboliantype volcanoes (Lefebure, 1986).

## References

Bice, D.C., 1985. Quaternary volcanic stratigraphy of Managua, Nicaragua: correlation and source assignment for multiple overlapping plinian deposits. Geol. Soc. Am. Bull., 96: 553-566.

Brown, G.C., Everett, S.P., Rymer, H., McGarvie, D.W., and Foster, I., 1991. New light on caldera evolution - Askja, Iceland. Geology, 19: 352-355.

Bulletin of the Global Volcanism Network, 1994. Masaya volcano. Smithsonian Institution 18: 7, 11.

Carr, M.J., 1984. Symmetrical and segmented variations of physical and geochemical characteristics of the Central American volcanic front. J. Volcanol. Geotherm. Res., 20: 231-252.

Connor, C.B., and Williams, S.N., 1989. Interpretation of gravity anomalies, Masaya caldera complex, Nicaragua. Transactions of the $12^{\text {th }}$ Caribbean Geological Conference.

Eggers, A., Krausse, J., Rush, H., and Ward, J., 1976. Gravity changes accompanying volcanic activity at Pacaya volcano, Guatemala. J. Volcanol. Geotherm. Res., 1: 229-236.

Eggers, A.A., and Chavez, D., 1979. Temporal gravity variations at Pacaya volcano, Guatemala. J. Volcanol. Geotherm. Res., 6: 391-402.

Eggers A.A., 1983. Temporal gravity and elevation changes at Pacaya volcano, Guatemala. J. Volcanol. Geotherm. Res., 19: 223-237.

Kieffer, G., and Creusot-Eon, A., 1992. La caldeira de Masaya (Nicaragua): une dépressiom polyphasée de type « maar». C.R. Acad. Sci. Paris, 315: 14031409.

Lefebure, D.V., 1986. The mina El Limon area and the Telica complex: two examples of Cenozoic volcanism in northwestern Nicaragua, Central America. Unpublished Ph.D. Thesis, Department of Geology, Carleton University, Ottawa, Canada, 269 pp.

Maciejewski, A.J.H., 1995. Evolution and present-day activity of the Masaya volcanic complex. Unpublished manuscript, The Open University, 37 pp .

Malfait, B.T., and Dinkelman, M.G., 1972. Circum-Caribbean tectonic and igneous activity and the evolution of the Caribbean plate. Geol. Soc. Am. Bull., 83: 251-272.

McBirney, A.R., 1956. The Nicaraguan volcano Masaya and its caldera. EOS Trans. AGU, 37: 83-96.

McBirney, A.R., and Williams, H., 1965. Volcanic history of Nicaragua. University of California, Publications in Geological Sciences, 55: 1-73.

McClelland, L., Simkin, T., Summers, M., Nielsen, E., and Stein, T. C. 1989. Global Volcanism 1975-1985. The first decade of reports from the Smithsonian Institution's Scientific Event Alert Network (SEAN). Prentice Hall, Englewood Cliffs, New Jersey, AGU, 655 pp.

Metaxian, J.-P., 1994. Etude sismologique et gravimétrique d'un volcan actif: Dynamisme interne et structure de la Caldeira Masaya, Nicaragua. Unpublished Ph.D. Thesis, Université de Savoie, Savoie, France, 319 pp.

Metaxian, J.-P., and Lesage, P., 1997. Permanent tremor of Masaya volcano, Nicaragua: Wave field analysis and source location. J. Geophys. Res., 102: 22 529-22 545.

Rymer, H., 1989. A contribution to precision microgravity data analysis using Lacoste and Romberg gravity meters. Geophys. J., 97: 311-322.

Rymer, H., 1994. Microgravity change as a precursor to volcanic activity. J. Volcanol. Geotherm. Res., 61: 311-318.

Rymer, H., and Locke, C.A., 1995. Microgravity and ground deformation precursors to eruption: a review. Cahiers du Centre Européen de Géodynamique et de Séismologie, 8: 21-39.

Rymer, H., van Wyk de Vries, B., Stix, J., and Williams-Jones G., 1998. Pit crater structure and processes governing persistent activity at Masaya volcano, Nicaragua. Bull. Volcanol., 59: 345-355.

Stoiber, R.E., and Carr, M.J., 1973. Quaternary volcanic and tectonic segmentation of Central America. Bull. Volcanol., 37: 304-325.

Stoiber, R.E., Williams, S.N., and Huebert, B.J., 1986. Sulfur and halogen gases at Masaya caldera complex, Nicaragua: total flux and variations with time. J. Geophys. Res., 91: 12 215-12 231.

Van Wyk de Vries, B., 1993. Tectonics and magma evolution of Nicaraguan volcanic systems. Unpublished Ph.D. Thesis, Department of Earth Sciences, Open University, Milton Keynes, UK, 328 pp.

Vieira, R., Toro, C., and Araña, V., 1986. Microgravity survey in the caldera of Teide, Tenerife, Canary Islands. Tectonophys., 130: 249-257.

Viramonte, J.G., Navarro Collado, M., and Malavasi Rojas, E., 1997. NicaraguaCosta Rica Quaternary Volcanic Chain. IAVCEI 1997 General Assembly, Puerto Vallarta, Mexico, Guidebook for Field Trip 17, 59 pp.

Walker, J.A., Williams, S.N., Kalamarides, R.I., and Feigenson, M.D., 1993. Shallow open-system evolution of basaltic magma beneath a sudbuction zone volcano: the Masaya caldera complex, Nicaragua. J. Volcanol. Geotherm. Res., 56: 379-400.

Weyl, R., 1980. Geology of Central America. Gebruder Borntraeger, Berlin, Germany, 371 pp .

Williams, S.N., 1983. Plinian airfall deposits of basaltic composition. Geology, 11: 211-214.

## CHAPTER I

# The nature and origin of gravity anomalies at Telica volcano, Nicaragua 

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#### Abstract

The Telica complex, Nicaragua, has been very active in historic time and is a potentially dangerous volcano. Telica has undergone extended periods of fumerolic activity and numerous small explosive eruptions. An interpretation of gravity data collected in 1997 and 1998 was made to better delineate the geologic structure of the Telica Complex. During the static gravity survey, a total of 245 stations were occupied in the vinicity of the volcano. Compilation of the gravity data revealed a large anomaly oriented north-south. This body has a positive density contrast of around $400-600 \mathrm{~kg} \mathrm{~m}^{-3}$. This anomaly of dimensions $2 \mathrm{~km} \times 2 \mathrm{~km} \times 6 \mathrm{~km}$ at a depth of about 1 km is probably an intrusion or magma reservoir. It is situated in the middle of the Telica complex and may feed all the volcanoes situated in the complex. The regional structure is concordant with this intrusion, with three north-trending tensional faults and the alignment of the cones being oriented approximately parallel to the gravity anomaly. The ongoing seismic activity may indicate magma replenishment into this reservoir or the occurence of a pressure buildup, since there is practically no visible degassing at the surface of the volcano.


## Introduction

Telica is an active volcano in northwestern Nicaragua forming part of the chain of Quaternary volcanoes along the western margin of Central America (Fig. 1.1). Telica and its adjacent volcanoes, including Santa Clara, Cerro Aguero and San Jacinto, are part of the Telica complex situated in the Marrabios Range on the southwestern margin of the Nicaraguan Depression (Fig. 1.2). Telica rises to a

## FIGURE 1.1

Map of the Quaternary volcanoes of Central America showing the location of Telica volcano (From Weyl, 1980).


FIGURE 1.2
Geological map of the Marabios Range, Nicaragua (Form Weyl, 1980).

maximum elevation of 1060 m above sea level; elevation at the base of the volcano is around 100 m . The volcano has a very steep-sided cone with an active crater of 700 m diameter. The Telica Complex consists of overlapping lava flows, tephra, alluvium, lahars and intrusive rocks. One of Nicaragua's most active volcanoes, Telica has erupted intermittently since the time of the Spanish conquest. Historical activity at Telica consists of extended periods of fumerolic emissions and numerous small explosive eruptions (Lefebure, 1986). Eruptions in the sixteenth century have been reported at Santa Clara (Lefebure, 1986), but its eroded and breached crater has been covered by forests throughout historical time. Telica is currently monitored with a telemetered seismic station by INETER (Instituto Nicaragüense de Estudios Territoriales). From December 1996 to June 1997, the number of volcanic/seismic and seismic events increased from $\sim 100$ /day to $\sim 220 /$ day (GVNB 22:03, 22:05 and 22:06). Solfateric activity is very low at Telica, with the $\mathrm{SO}_{2}$ flux measured by COSPEC on 17 March 1996 averaging $40 \pm 20 \mathrm{t} / \mathrm{d}$ and nearly zero in March 1997, based on nine measurements (GVNB 21:04, 22:03). The volcano is also monitored with microgravity; measurements since 1993 have shown no significant changes (H. Rymer, personal communication, 1998).

Knowledge of the internal structure and dynamics of volcanoes improves our means to understand volcanic activity. Knowing the location of a large dike or a shallow intrusion makes it possible to focus monitoring on that particular point. This knowledge also may be used for economic purposes; e.g., geothermal energy which could provide electricity. Investigation of volcanoes by geophysical means enhances our understanding of their internal structure when integrated with geological, petrological and geochemical methods. Gravity surveys are well suited to the study
of internal structures of volcanoes because the density contrasts encountered in a volcanic environnement can be substantial. A gravity survey permits the detection of bodies which have a different density than the surrounding rocks, for example, deep or shallow magma chambers, magma dikes or pipes and low-density fragmental material which may fill a caldera or a crater depression. The utility of gravity and microgravity data on volcanoes has been amply demonstrated (Rymer and Brown, 1986; Eggers, 1987; Metaxian, 1994; Rymer, 1994). To better define the subsurface and internal structure and to delineate possible intrusive magma volumes, a gravity survey of Telica volcano and the surrounding area was made in the winters of 1997 and 1998. From this survey, a series of Bouguer anomaly maps were made.

## Methodology

A total of 245 stations were occupied in the vinicity of Telica volcano during the months of February and March in 1997 and 1998. Gravity measurements were made with Lacoste and Romberg meter G-513 (see Table A1 in Appendix A). Leica GPS 200 dual-frequency differential receivers provided positioning and elevation control of the stations (see Table A2 in Appendix A). In the course of the first gravity survey in 1997, measurements were made only in the vinicity of the crater with a dense array. In 1998, the gravity grid was extended farther from the volcano to enlarge the gravity anomaly map. Four base stations (Base1, Base2, Base3 and Basefinal) were used to connect all the stations together. The stations are not connected to an absolute gravity station, so the gravity values are calculated relative to Base3. The locations of these base stations are shown in Figure 1.3. Base2, Base3 and Basefinal

FIGURE 1.3
Topographic map of the Telica complex showing location of the base station, the two gravity profiles and a sketch of the intrusion.

are temporary stations located on the road to Telica and are difficult to relocate. Basel is a good station to use for a new survey or to continue this survey. The station is located about 5 m west of a house (Fig. 1.4) at the foot of the volcano on its northeast side. The latitude and longitude is: $12^{\circ} 36^{\prime} 33.2394^{\prime \prime} \mathrm{N}, 86^{\circ} 49^{\prime} 56.9752^{\prime \prime}$ W , at an elevation of 734 m . The station is situated on a round dark gray rock 50 cm in diameter and rising up about 15 cm from the soil. Alejandro Acosta, a driver for INETER, is a good contact for more information on the exact location of this station.

All gravity measurements were tide-corrected in the field using GRAVPAC, a solar and lunar tide calculator provided by Lacoste \& Romgerg Inc.. Meter drift was not accounted for on a daily basis, since it was generally only of the order of 15-25 $\mu \mathrm{Gal}$ (average $17 \mu \mathrm{Gal}$ ) after 6-7 hours at the end of the day. Meter drift on a weekly and yearly basis was corrected, since there was an overall drift of $131 \mu \mathrm{Gal}$ between the first and the last day of measurements at Basel in 1997. The drift of the meter between 20/03/97 and 1 1/02/98 was 3.690 mGal .

Gravity corrections were made for all stations; a summary of these corrections and measurements is presented in Table A2 in Appendix A. Because of the equatorial bulge and the rotation of the Earth, there is an increase of gravity with latitude (Telford et al., 1990). Correction for latitude $\Delta \mathrm{G}_{1}$ is calculated as follows:

$$
\Delta \mathrm{Gl}=0.811 \sin 2 \phi \Delta \mathrm{~s} \quad \mathrm{mGal}
$$

where $\Delta \mathrm{s}$ is the north-south horizontal distance in kilometers of a station from Base3 and $\phi$ the latitude in degrees ( $12.6^{\circ} \mathrm{N}$ in our case). This correction is positive as we

## FIGURE 1.4

Location of station Base1 near the house at the foot of Telica volcano. Photo of the house.


House

move toward the Equator. Because gravity varies inversely with the square of distance, a correction for elevation differences between stations is required. The freeair correction $\left(\Delta \mathrm{G}_{\mathrm{FA}}\right)$ is calculated as follows:

$$
\Delta \mathrm{G}_{\mathrm{fa}}=0.3086 \Delta \mathrm{z} \mathrm{mGal}
$$

where $\Delta z$ is the elevation difference in meters between a station and Base3. Then, a Bouguer correction is made to account for the material between the stations and the reference station that the free-air correction ignores (Base3 in our case). This correction $\left(\Delta G_{B}\right)$ is calculated from:

$$
\Delta \mathrm{G}_{\mathrm{B}}=0.00004192 \rho \Delta \mathrm{z} \mathrm{mGal}
$$

where $\rho$ is the density in $\mathrm{kg} \mathrm{m}^{-3}$ of the material determined by the Nettleton method (1939) and $\Delta z$ is the elevation difference in meters between a station and Base3. For accurate corrections (latitude, free-air and Bouguer), the station position and elevation have to be known with accuracy. For an accuracy of $10 \mu \mathrm{Gal}$, the position of each station has to be known within 10 m and elevation to 3 cm .

Position and elevation of each station were acquired with Leica GPS 200 dualfrequency differential receivers. The precision obtained is about 1 cm horizontally and $1-2 \mathrm{~cm}$ vertically. This precision is relative between stations when they are connected to each other. For the absolute position of the gravity network, no benchmarks were available. Thus, during the survey of 1997 , the reference GPS was always positioned at the same point and measured several time for 6-7 hours each day. This is not a gravity station but a relative reference for the rover GPS. A single-
point computation was made at this point and gave an absolute position to about 10 m of precision. The corresponding error for latitude, Free-air and Bouguer corrections is $3.5 \times 10^{-6} \mathrm{mGal}, 6 \times 10^{-3} \mathrm{mGal}$ and $2 \times 10^{-3} \mathrm{mGal}$, respectively. The error for latitude correction is negligible and will not be considered further.

For the terrain correction, only a topographic map of 1:50 000 scale was available. For terrain corrections « $\mathrm{A} »$ through « $\mathrm{D} »$ ( $0-170 \mathrm{~m}$ radius), the method of Sandberg (1958) was used. It involves approximating the slope near the station, divided into quadrants. The Sandberg tables were used to calculate the correction (Sandberg, 1958). For terrain corrections « $\mathrm{E} »$ through « $\mathrm{K} »$ (170-9900 m radius), the Hammer (1939) method was used on the topographic map available. Unfortunately, because of the lack of other maps of larger scale, corrections «M » and « $\mathrm{L} »(9900-21950 \mathrm{~m})$ were not made. The terrain correction in the tables of Hammer (1939) and Sanberg (1958) are calculated for a density of $2000 \mathrm{~kg} \mathrm{~m}^{-3}$. A conversion has to be made to account for the average density evaluated with the Nettleton (1939) method. For the terrain correction, the error is comparatively high. It is normally about $10 \%$ of the average terrain corrections (Barrows, 1996), which is equal here to $0.62 \mathrm{mGal}(6.2 \mathrm{mGal}$ being the average terrain correction).

The density used in the modelling was evaluated from the Nettleton profile made on the volcano (Fig. 1.5a, b). The Nettleton method is a mean to estimate the nearsurface density, using a gravity profile over topography, that is not related to density variations. Field readings are reduced using different densities for the Bouguer and terrain corrections. The profile that least reflects the topography is the profile with the best estimated density. An average density of $2400 \mathrm{~kg} \mathrm{~m}^{-3}$ was chosen from the

## FIGURE 1.5

Nettleton correction for a gravity profile on Telica volcano using density variation of $1800 \mathrm{~kg} \mathrm{~m}^{-3}$ to $2900 \mathrm{~kg} \mathrm{~m}^{-3}$. (a) The gravity profile for the different densities. (b) The corresponding elevation profile on Telica.



Nettleton correction. For matters of comparison, gravity also was corrected using values of $2300 \mathrm{~kg} \mathrm{~m}^{-3}$ and $2500 \mathrm{~kg} \mathrm{~m}^{-3}$. Using these densities, three Bouguer anomaly maps were made. The density was used in the Bouguer and terrain corrections. After being corrected, the gravity data were interpolated using a linear method to a $50 \mathrm{~m} \times 50 \mathrm{~m}$ grid and contoured using MATLAB.

The error due to gravity measurements is about 0.015 mGal , while the error from elevation control depending on the density estimation (Free-air and Bouguer) should not exceed 0.01 mGal . The estimated error from the terrain correction is 0.62 mGal. Thus, we can say that the error for terrain corrections should not exceed 1.0 mGal . The error on the density estimate, which should not exceed $200 \mathrm{~kg} / \mathrm{m}^{3}$, corresponds to an error of 0.1 mGal . The total error (E) is equal to the following:

$$
E=\left[E_{G}^{2}+E_{T}^{2}+E_{A}^{2}+E_{\rho}^{2}\right]_{2}^{1}
$$

where $\mathrm{E}_{\mathrm{G}}$ is the error for the Lacoste and Romberg measurement, $\mathrm{E}_{\mathrm{T}}$ is the error from the terrain corrections, $\mathrm{E}_{\mathrm{A}}$ the error from elevation control and $\mathrm{E}_{\mathrm{p}}$ the error from the density evaluation. Thus, the error on the contour map is about 0.63 mGal , mainly due to the terrain correction.

Three Bouguer gravity anomaly maps were made for densities of $2300 \mathrm{~kg} \mathrm{~m}^{-3}$, $2400 \mathrm{~kg} \mathrm{~m}^{-3}$ and $2500 \mathrm{~kg} \mathrm{~m}^{-3}$. There are only small differences among the three maps in terms of the anomalies, but the magnitude of the anomalies is different for each map. The gravity anomalies are calculated using Base3 as a zero gravity reference value. The anomaly map for modelling used a density of $2400 \mathrm{~kg} \mathrm{~m}^{-3}$ (Fig. 1.6a), and the two other maps are shown for comparison (Fig 1.6b, c). Two profiles

## FIGURE 1.6

Contour map of the gravity anomaly showing the two profiles for density reductions of (a) $2400 \mathrm{~kg} \mathrm{~m}^{-3}$, (b) $2300 \mathrm{~kg} \mathrm{~m}^{-3}$ and (c) $2500 \mathrm{~kg} \mathrm{~m}^{-3}$. Contour interval is in mGal . Two profiles are shown which are used for GRAVMAG interpretation.
A

B
(
C


FIGURE 1.7
Stratigraphic profiles of the Telica complex which correspond approximately to the gravity profile: (a) the north-south profile; (b) the east-west profile.

were made on the anomaly map, one nearly east-west and the other approximately north-south (Fig.1.6a). The east-west profile has not been place orthogonally to the structural direction but is offset of about $15^{\circ}$. This position was choosen because it better represent the observed anomaly, e.i., it is directly situated on the station array. For the modelling of the anomalies, an interactive 2.5 D gravity program, GRAVMAG, was used (Pedley et al., 1993). In conjunction with the modelling, stratigraphic descriptions of the volcano made by Lefebure (1986) were used for the construction of two generalized stratigraphic profiles (Fig. 1.7a, b). Unfortunately, the thicknesses of the various formations are unknown to the author, so the modelling first considers the Lefebure (1986) estimates, and then the thickness of each formation was varied to fit the observed anomalies. For comparison, the east-west profile also was made using reduction densities of $2300 \mathrm{~kg} \mathrm{~m}^{-3}$ and $2500 \mathrm{~kg} \mathrm{~m}^{-3}$ (Fig $1.6 \mathrm{~b}, \mathrm{c})$.

## Results

Examination of the Bouguer anomaly map revealed a general positive anomaly oriented north-northwest-south-southeast (Fig 1.6a). The interval contour are in mGal. Three distinct positive anomalies are visible: (1) at 517400 and 1393 700 , (2) at 518000 and 1396000 and (3) the highest anomaly at 518500 and 1393 500. These positions represent UTM coordinates based on the WGS 84 datum. Other anomalous zones may not be real, since they are extrapolations of the data. Modelling of the profiles was done using a step-by-step analysis. First, simple models and shapes were used to explore the limits of the possible solutions for the
anomalies. From these results, the best approximation was used to create a more detailed model. The east-west profile was modelled first. Since the two profiles intersect in the anomalous zone, interpretations made with the east-west profile were then used in the modelling of the north-south profile.

Simple approximations of the anomaly in the crater area (profile east-west in Figure 1.6a) are shown in Figure 1.8a and 1.8b. The modelling demonstrates that the anomaly is created by a relatively small, shallow body rather than a larger, deeper body. A deeper body needs to be very large to match the amplitude of the doublypeaked positive anomaly; in so doing, the overall anomaly is too large (Fig 1.8b). A smaller and shallower body gives a better fit to anomaly (Fig. 1.8a). The density contrast used for the two bodies is the same $\left(+650 \mathrm{~kg} \mathrm{~m}^{-3}\right)$, but the half-strike (extension along strike) is 1000 m for the shallow body and 2000 m for the deep body. Modelling using only the volcano stratigraphy along the same profile shows that basalt lava flows (density contrast of $+350 \mathrm{~kg} \mathrm{~m}^{-3}$ ) are responsible, in part, for the overall anomaly in the eastern part of the profile (Fig. 1.8c). This observation, in conjunction with the two previous models, shows that a shallow body ( $+600 \mathrm{~kg} \mathrm{~m}-3$ and half-strike of 1000 m ) better fits the high-amplitude doubly-peaked positive anomaly (Fig 1.8d).

Once it was determined that the anomaly is caused by a shallow body of positive density contrast $\left(+600 \mathrm{~kg} \mathrm{~m}^{-3}\right)$, more detailed modelling was done to match the observed anomaly (Fig. 1.8e). For modelling of the doubly-peaked anomaly, shallow less dense material ( $-800 \mathrm{~kg} \mathrm{~m}^{-3}$ ) was used to fit the trough of the anomaly (Fig. 1.8e, Unit 7). Modifications of the shallow subsurface body alone did not match the anomaly, nor did introduction of two smaller and shallower bodies of positive

## FIGURE 1.8

Modelling of the east-west gravity profile using GRAVMAG. (a) An isolated shallow body. (b) A deeper and larger body. (c) The stratigraphic units without an intrusion. (d) The stratigraphic units with a shallow intrusive body. (e) Best fit with a density contrast of $600 \mathrm{~kg} \mathrm{~m}^{-3}$ (intrusion). (f) Best fit with dikes. (g) Best fit with a density contrast of $450 \mathrm{~kg} \mathrm{~m}^{-3}$.



D



F


density contrast, such as dikes, on each side of the trough (Fig 1.8f). The density contrast of these two dikes needs to be large and the size of the dikes enormous; moreover, low-density material at shallow depth was still needed to fit the anomaly. The density contrast of the large body in Figure 1.8 e (Unit 4) is $600 \mathrm{~kg} \mathrm{~m}^{-3}$, and its half-strike is increased to 3000 m . This increase was made to match the anomaly modelled in the north-south profile (Fig 1.6a, 1.9). Modelling of the two profiles together (east-west and north-south, Fig 1.6a, 1.8e and 1.9) was used to create a realistic scenario. In Figure 1.8 g , the body (Unit 4) has a density contrast of 450 kg $\mathrm{m}^{-3}$ and a half strike of 3000 m . The thickness and density contrast of the different formations were then modified slightly to fit the anomaly. The half-strike used in the modelling of the north-south profile (Fig.1.6a, 1.9) corresponds to half the width of the body ( 1000 m ) modelled in the east-west profile (Fig. 1.8 g ), since the bodies in the two profiles intersect each other perpendicularly near the anomaly. The density contrast needed to match the anomaly in the north-south profile is $500 \mathrm{~kg} \mathrm{~m}^{-3}$, which is $50 \mathrm{~kg} \mathrm{~m}^{-3}$ higher than the body for the east-west profile (Fig 1.9). For this northsouth profile, thicknesses of formations again were adjusted to fit the anomalies, but not the densities which are the same as for the east-west profile. A shallow intrusion (Unit 4) had to be introduced into Unit 2 to match the positive peak (Fig. 1.9).

Finally, to compare the anomaly using Bouguer maps made with different densities $\left(2300 \mathrm{~kg} / \mathrm{m}^{3}, 2400 \mathrm{~kg} / \mathrm{m}^{3}\right.$ and $\left.2500 \mathrm{~kg} / \mathrm{m}^{3}\right)$, a body of similar size and halfstrike was modelled in each east-west profile (Fig. 1.10a, b, c). The density contrasts needed to match the anomaly in each profile are: (1) $470 \mathrm{~kg} \mathrm{~m}^{-3}$ for the $2300 \mathrm{~kg} \mathrm{~m}^{-3}$ map, (2) $420 \mathrm{~kg} \mathrm{~m}^{-3}$ for the $2400 \mathrm{~kg} \mathrm{~m}^{-3}$ map and (3) $360 \mathrm{~kg} \mathrm{~m}^{-3}$ for the $2500 \mathrm{~kg} \mathrm{~m}^{-3}$ map.

## FIGURE 1.9

Modelling of the north-south gravity profile using GRAVMAG.


## FIGURE 1.10

Modelling of the east-west gravity profile with a simple body of 3000 m half-strike for density reductions of (a) $2300 \mathrm{~kg} \mathrm{~m}^{-3}$, (b) $2400 \mathrm{~kg} \mathrm{~m}^{-3}$ and $2500 \mathrm{~kg} \mathrm{~m}^{-3}$.


## Discussion

Modelling of the anomalies defined on the contour maps is not meant to be definitive. The modelling shows possible configurations of structure under a volcano which best fit the observed anomalies and which are geologically plausible. Most importantly, the modelling shows that the anomaly does not seem to be produced by a deep structure but instead by a shallow body with positive density contrast around $400-600 \mathrm{~kg} \mathrm{~m}^{-3}$. A deeper body gives a broader anomaly than is observed. The best model, made in conjunction with the stratigraphy of the volcano and the two profiles (Fig. 1.8 g and 1.9 ), gives a shallow body 1 km deep with dimensions of 2000 m by 2000 m by 6000 m . This body strikes north-northwest-south-southeast with a density contrast of around $400-600 \mathrm{~kg} / \mathrm{m}^{3}$. The body is probably an intrusion at shallow depth representing some sort of shallow magma chamber or reservoir.

The doubly-peaked anomaly in the east-west profile and the trough in the north-south profile are due to variations in thickness at the surface or very shallow depth low-density material such as tephra. Although the possibility of a dike system is not ruled out, this effect alone could not reproduce the doubly-peaked anomaly observed on the east-west profile. Comparison among the Bouguer maps made with different densities shows that there is not much difference between the density contrast of the bodies. The sum of the density contrasts and the density reduction of the anomaly maps are $2770 \mathrm{~kg} \mathrm{~m}^{-3}, 2820 \mathrm{~kg} \mathrm{~m}^{-3}$ and $2860 \mathrm{~kg} \mathrm{~m}^{-3}$, respectively, for the $2300 \mathrm{~kg} \mathrm{~m}^{-3}, 2400 \mathrm{~kg} \mathrm{~m}^{-3}$ and $2500 \mathrm{~kg} \mathrm{~m}^{-3}$ Bouguer maps. Thus, use of $2400 \mathrm{~kg} \mathrm{~m}^{-3}$ density is considered to be justified, since there is not much difference among the
other maps. This is confirmed by the anomalies on the three maps (Fig. 1.6a, b, c) which are not very different except for their amplitudes.

A structural study of Central American volcanoes by Carr (1976) shows that north-south-trending faults are tensional. Three north-northwest-south-southeast trending faults have been identified in the Telica complex (Lefebure, 1986). One is situated in the fumarole field of San Jacinto, and the two others to the east (Fig. 1.2). Arcuate or linear scarps, up to 40 m high and 2 km long, are found on the northeastern flanks of the San Jacinto hills and strike in a north-northwest direction. These probably also represent fault traces. Telica erupts predominantly basalts and andesite, with low pressure fractionation and mixing trends (van Wyk de Vries, 1993). Magma intrusion occurs rapidly to high levels due to crustal extension in the area. A central magma chamber located at shallow depth is likely to be formed (van Wyk de Vries, 1993). This idea also is postulated by Lefebure (1986), who suggested that lava flows can erupt laterally from the volcano and tap both the top and sides of a magma reservoir. Effusion of lava at different levels is an indication of the shallow nature of the magma chamber (Lefebure, 1986). Isotope analyses made by Lefebure (1986) showed uniform ${ }^{87} \mathrm{Sr} /{ }^{86} \mathrm{Sr}$ ratios for the volcanic rocks of the Telica complex, indicating a common parent magma or a relatively uniform source.

This gravity survey of Telica has revealed the presence of a large positive anomaly at shallow depth. This anomaly is probably an intrusion of magma which represents the magma chamber that feeds the volcanoes of the Telica complex. It is oriented north-northwest-south-southeast, similar to the faults in the region. Moreover, Santa Clara, Telica and Cerro Aguerro are nearly aligned (northwestsoutheast) with this anomaly. A sketch of the anomaly is presented in Figure 1.3.

## Conclusions

A large shallow intrusion of dimensions $2 \mathrm{~km} \times 2 \mathrm{~km} \times 6 \mathrm{~km}$ with a depth to top of about 1 km and a density contrast of $400-600 \mathrm{~kg} \mathrm{~m}^{-3}$ is defined in the Telica complex. Telica has been active in its historic past and is currently still very active, showing seismic activity since December 1996 increasing from $\sim 100$ events/day to ~220 events/day in June 1997. Practically no visible degassing is occurring, which may reflect a pressure buildup in the system. The seismic activity may also indicate magma movement under the volcano. The intrusion defined with this gravity survey probably represents the magma chamber that fed past eruptions of the volcanoes in the Telica complex. The present seismic activity could indicate that some sort of intrusive or convective activity is occuring presently in this chamber, such as magma replenishment or a buildup of pressure. This gravity survey proved useful in delineating the anomaly, but further investigations are needed to better define the body. Longer profiles should be used to better model the anomaly's depth and size, which was not possible to model with confidence using the available profiles from the contour map.

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## References

Barrows, L., 1996. Microgravity surveying: new developments, new capabilities. The Professional Geologist, November 1996, p. 6-17.

Bulletin of the Volcanism Network, 1996. Telica volcano, 21: 4.
Bulletin of the Volcanism Network, 1997. Telica volcano, 22: 3.
Bulletin of the Volcanism Network, 1997. Telica volcano, 21: 5.
Bulletin of the Volcanism Network, 1997. Telica volcano, 21:6.
Carr, M. J., 1976. Underthrusting and Quaternary faulting in northern Central America. Geological Society of America Bulletin, 87: 825-829.

Eggers, A.A., 1987. Residual gravity changes and eruption magnitudes, J. Volcanol. Geotherm. Res., 33: 210-216.

Hammer, S., 1939. Terrain corrections for gravity stations. Geophysics, 4: 184-194.
Lefebure, D.V., 1986. The mina El Limon area and the Telica Complex: two examples of Cenozoic volcanism in northwestern Nicaragua, Central America. Unpublished Ph.D. thesis, Department of Geology, Carleton University, Ottawa, Canada, 255 pp .

Metaxian, J.-P., 1994. Etudes sismologique et gravimétrique d'un volcan actif: Dynamisme interne et structure de la caldeira de Masaya, Nicaragua. Unpublished Ph.D. thesis, Université de Savoie, France, 309 pp.

Nettleton, L. C., 1939. Determination of density of reduction of gravimeter observation. Geophysics, 4: 176-183.

Rymer, H., and Brown G.C., 1986. Gravity fields and the interpretation of volcanic structures: geological discrimination and temporal evolution. J. Volcanol. Geotherm. Res., 27: 229-54.

Rymer, H., 1994. Microgravity change as a precursor to volcanic activity. J. Volcanol. Geotherm. Res., 61: 311-328.

Sandberg, C.H., 1958. Terrain correction for an inclined plane in gravity computations. Geophysics, 23, 4: 701-711.

Telford, W. M., Geldart, L. P., Sheriff, R. E., 1990. Applied Geophysics: $2^{\text {nd }}$ edition. Cambridge: Cambridge Univerity Press, 770 pp.

Van Wyk de Vries, B., 1993. Tectonics and magma evolution of Nicaraguan volcanic systems. Unpublished Ph.D. thesis, Department of Earth Sciences, Open University, Milton Keynes, UK, 328 pp.

Weyl, R., 1980. Geology of Central America. Berlin: Gebruder Borntraeger, 371 pp .

## CHAPTER II

# Temporal variations of microgravity at <br> Masaya volcano, Nicaragua 

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#### Abstract

Temporal variations of microgravity were observed at Masaya caldera, Nicaragua, from 4 March to 17 March 1997, and from 27 January to 14 March 1998. During the survey of 19 days in 1997, conducted with Scintrex and Lacoste \& Romberg gravity meters, a decrease of $55-80 \mu \mathrm{Gal}$ was observed near the active crater, Santiago. Some differences between the two instruments were observed during the survey, but in general, they show the same trend of temporal variation, except at a single station (B1A). In 1998, the observed microgravity variations were of the same amplitude ( $55-63 \mu \mathrm{Gal}$ ) but were also seen on a shorter timescale and were increases instead of decreases. A detailed experiment was conducted at one station (A7) during one day in 1997 and during three days in 1998 during which microgravity was measured continuously. Two instruments were used simultaneously for approximately 7 hours in 1997 and one instrument for about 13-14 hours for each day in 1998. COSPEC measurements of $\mathrm{SO}_{2}$ flux were made to examine degassing trends of the active vent during the days of continuous gravity measurements. In 1998, during the continuous gravity monitoring, pressure and seismicity were monitored at the same time. In 1997, both instruments showed an increase in microgravity, $45 \mu \mathrm{Gal}$ for the Lacoste \& Romberg and $20 \mu \mathrm{Gal}$ for the Scintrex. Microgravity variations with amplitudes of 40,20 and $35 \mu \mathrm{Gal}$ were observed on 25 February, 6 March and 13 March 1998, respectively. A tentative correlation is made between microgravity changes and diurnal tides. No direct correlations were observed with the $\mathrm{SO}_{2}$ flux in both cases. Gravity variation induced by atmospheric pressure changes is unlikely, since no direct links are observed. The


cause of these gravity variations is linked to density variation in the magma which in turn is an effect of fluctuation in the vesicularity of the magma in the order of 3-5 \% that could be related mechanically to the diurnal and fortnightly tidal variations.

## Introduction

Masaya Caldera is situated approximately 25 km south-southeast of Managua, the capital of Nicaragua. The caldera is oriented northwest-southeast, parallel to the alignment of the volcanic chain (Fig. 2.1). The length of the longest axis of Masaya caldera is 11.5 km and its width is 6 km . The caldera consists of a lake (Laguna de Masaya) and four pit craters: Masaya, Santiago (presently active), Nindirí and San Pedro (Fig. 2.2). Historical activity at Masaya consists of lava flows in 1670 and 1772, episodic lava lake formation associated with pit crater formation, small strombolian eruptions and periodic degassing crises. There is also geological evidence of pyroclastic and plinian eruptions from Masaya in the more distant geologic past (Williams, 1983). Recurrent lava lake episodes with significant degassing activity have occurred several times in recorded history from Santiago crater. Stoiber et al. (1986) estimated that $10 \mathrm{~km}^{3}$ of basaltic magma have been degassed in five degassing crises since 1852. This output of gas is devastating to the coffee crop surrounding the affected area; damaged and dead vegetation is most conspicuous on the slopes leading up to the Llano Pacaya rim (Johnson and Parnell, 1986). Significant degassing was occuring from Santiago crater during our microgravity surveys in March 1997 and February-March 1998. In 1997, the $\mathrm{SO}_{2}$ fluxes observed were between $300-500 \mathrm{t} / \mathrm{d}$. In 1998 , the $\mathrm{SO}_{2}$ fluxes were

FIGURE 2.1
The Central American volcanic chain showing the segments of Carr (1984) and Masaya volcano (From Metaxian, 1994).


## FIGURE 2.2

Map of Masaya caldera showing the different cones and craters (From Viramonte et al., 1997).

significantly higher, ranging from 700 to $4000 \mathrm{t} / \mathrm{d}$.
The present study was conducted in two field trips (March-April 1997 and February-March 1998) during which microgravity and GPS measurements were taken at five stations in 1997 and seven stations in 1998. The stations used to monitor the volcano were part of an existing microgravity survey line at Masaya caldera. This line of seventeen stations was established by French researchers (Metaxian, 1994) and continued by Rymer et al. (1998). The microgravity was monitored using two gravimeters, a Scintrex CG-3 \#9101184 (loan by Scintrex Ltd) and a Lacoste and Romberg \#G-5 13 (LCR) (loan by Hazel Rymer) in 1997, and in 1998 with only the LCR \#G-513. Altitudes were monitored using Leica GPS 200 dual-frequency differential receivers to assess possible changes in altitude on the microgravity results. During the same period, COSPEC measurements of $\mathrm{SO}_{2}$ flux were made to examine degassing trends of the active vent. Rymer et al. (1998) made microgravity measurements once a year for the past five years (1993-1997) at Masaya; between 1993 and 1994 they observed decreases on the order of $90 \mu \mathrm{Gal}$ at stations near Santiago crater (Fig 2.3). They correlated this decrease to an increase in the gas flux, indicating a change in the shallow plumbing system. Between 1994 and 1997, they recorded gradual increases in gravity of about $56 \mu \mathrm{Gal}$ in conjunction with minor pit crater collapse and a decline in degassing. They related the gravity variations to a convective overturn at shallow depth and not to a major intrusion.

The main goal of the current experiment was to obtain microgravity measurements at some of the stations used by Rymer et al. (1998) in order to obtain measurements over a shorter period of time. This would allow us to observe

## FIGURE 2.3

Gravity changes at stations within Masaya caldera; station locations are shown in Figure 2.4 (From Rymer et al., 1998).


## FIGURE 2.4

Location of the gravity stations in Masaya caldera. Stations BIA and B2 are proximal and are represented by B (From Rymer et al., 1998).

microgravity variations on timescales of years, weeks, and days. During the survey, $\mathrm{SO}_{2}$ flux, atmospheric pressure and seismicity measurements were also made to better understand the gravity changes. Comparisons with tidal variations was also made.

## Methodology

The stations that were used from the existing line are easy to locate and access. The stations are MUSEO, A1, A3, A5, A7, B2 and B1A, A1 being the base station; MUSEO and A5 are the two stations that were added in 1998 (Fig. 2.4). A secondary base station, REGIS, was established in 1998 outside of the caldera, at the Hotel Regis in Masaya City, about 7 kilometers east of Santiago, to examine gravity variations other than in the crater area (station A7, B2 and B1A). Microgravity measurements (tide-corrected) and GPS measurements for the altitude control were taken repeatedly at each station over a period of about three weeks in 1997 and 1998. In order to obtain gravity variations, a microgravity survey line was established, in this case represented by stations MUSEO, $\mathrm{A} 1, \mathrm{~A} 3, \mathrm{~A} 5, \mathrm{~A} 7, \mathrm{~B} 2$ and B 1 A , with A 1 as base station. Then a relative difference in microgravity was calculated between the base station and the other stations (Rymer, 1989). The differences obtained were then monitored with time. Changes in excess of $25 \mu \mathrm{Gal}$ are considered significant at the $95 \%$ confidence level (H. Rymer, personal communication, 1997). Since the base station (AI) is several kilometers from the active vent in the caldera in a more stable environment, it is considered not to vary during the survey. Measurements taken at REGIS verify this: the standard deviation of the differences between REGIS and MUSEO and REGIS and A1 are $14 \mu \mathrm{Gal}$ and $12 \mu \mathrm{Gal}$, respectively. Variations of
the REGIS station, measured at the beginning and the end of each day, are about 20 $\mu \mathrm{Gal}$ and always in the same direction which probably represent a drift. This slight drift of the meter occurred since it was slightly unstable after a power failure on 14 March 1998, which caused a decrease in the meter temperature. The second set of REGIS measurements were usually made late in the day, so there was a long lapse of time when the meter was stored in a car and not used. This may account for the drift.

The same procedure was followed for microgravity measurements at each station to minimize sources of error due to manipulations and readings (Rymer, 1989). Two different ways of taking measurements were used in the surveys of 1997 and 1998. In 1997, a measurement was taken at the base station and then at all the other stations sequentially and finally at the base station again. With a larger number of stations, repeating certain stations along the line is recommended for better precision and to locate possible tares. In the case of the present survey, only the base station was repeated because of the small number of stations (5 in 1997). In 1998 measurements were made in order to obtain three gravity differences between two individual stations. For example, a first measurement was taken at A1, a second at A3, a third at A1 and a fourth at A3. The line was continued in this manner for each pair of stations until the last station was connected to the base station. The differences obtained were then averaged and used as reference values to which the results of subsequent days are compared. The number of measurements is higher in the second technique, but the precision is better and the tares easily located. Both techniques give good results when done with care. Continuous daily-variation measurements were made only at station A7, once in 1997 and three times in 1998. During the daily

FIGURE 2.5
(a) Scintrex and LCR daily gravity changes at Masaya volcano at station A7, March 12 1997. (b) Temporal variation of $\mathrm{SO}_{2}$ vs temporal variation of microgravity for the LCR meter at Santiago crater, 12 March 1997.

survey in 1998, the atmospheric pressure was monitored constantly at the Hotel Regis in Masaya City using a Vaisala PTB 100 pressure meter. Pressure measurements were taken every five minutes. Precision of the pressure meter is around 0.03 mbar but because of the analog-to digital converter which digitizes the raw data, precision is of the order of 0.5 mbar . There was also a portable seismometer installed 10 m from station A 7 to record the volcanic tremor which is present in the crater area.

## Results

## Daily Gravity Variations

An initial experiment was conducted in 1997 at a single station near Santiago crater to look for gravity variations during the course of a day. This was conducted at station A7 for half a day on 12 March with the LCR and the Scintrex instruments. The two gravity meters were set at station A7, with 2 m distance between them. The Scintrex was set to measure during 120 seconds with a cycle time of 5 minutes; for the LCR, we took measurements every 30 minutes (Appendix B, Table B1). The LCR shows an increase of about $45 \mu \mathrm{Gal}$ over 7 hours (Fig. 2.5a). Although the Scintrex also shows an increase (Fig. 2.5a), it is of lower amplitude (about $20 \mu \mathrm{Gal}$ ). These results were calculated from linear regression of the data. In Figure 2.5a, the earth tide variation is plotted with our observed gravity variations of both meters. In general, the observed gravity changes increase in magnitude sympathetically with the tidal variations. The gravity changes may be shifted with respect to the tides by 3-4 hours, i.e., the observed gravity minimum and maximum may occur later than those for tides. However, this relation is not clear due to a lack of data. During this
experiment, a second team monitored the amount of $\mathrm{SO}_{2}$ degassing from Santiago by use of a COSPEC (Fig. 2.5b), since our microgravity measurements were conducted next to the crater. There does not appear to be a direct correlation between the increase in microgravity and the $\mathrm{SO}_{2}$ degassing. The difference of $25 \mu \mathrm{Gal}$ between the two instruments is difficult to interpret. Since the Scintrex was in cycling mode, it was not touched during the day except for checking the levelling once. Thus, there should not have been any tare or variation due to movement of the instrument. As for the LCR, it was not moved during the experiment. It was only clamped and unclamped before and after each measurement. Thus, there should not be much tare for this instrument either. There is no error from reading the Scintrex since it is digital. The LCR may have some error due to the reading dial, but it should not exceed $10 \mu \mathrm{Gal}$ if the correct procedure is followed (Rymer, 1989). The measurements for the LCR were done by two readers, with a switch made at 14:06 local time. This may account for an error of about $10 \mu \mathrm{Gal}$ considering the different way to interpret the nulling point on the meter by different readers. Another difference between the two instruments may be the tide corrections. The Scintrex has a tide correction program, and there may be a slight difference between this program and the one used (GRAVPAC) for correcting the tide for the LCR. But it is certainly not of the order of 15 to $20 \mu \mathrm{Gal}$. There is also the possibility of drift in the case of the Scintrex; even with the correction that is made by the Scintrex, there may be additional drift that may affect the data over an extended period of time. A third possibility is that the Scintrex gives better results in unstable environments than the LCR, which may be affected by vibration and seismicity. To achieve this, the

Scintrex must average a number of measurements and then reject values that are too far from the average. This rejection could induce additional errors. In summary, the difference between the two gravimeters is difficult to explain, and a repeat of this experiment would be necessary to better explain the cause of this difference (Appendix C). But because it is not of large amplitude and is of the same order and in the same direction, it is possible to say that the two instruments' response to the microgravity variation is approximately the same.

The experiment was continued in 1998, measurements being taken continuously every 10-15 minutes with the LCR instrument during the course of three days (25 February, 6 March and 13 March 1998) (Appendix B, Table B2). During two days, atmospheric pressure was monitored (pressure measurements are lacking for the first day because of equipment failure), and seismicity was monitored in the vinicity of station A 7 for the three days. COSPEC measurements of $\mathrm{SO}_{2}$ flux were acquired in the afternoon of 13 March. Compared to 1997 when gravity measurements lasted 7 hours, the time in 1998 was extended to about 13 -14 hours each day in order to better characterize the microgravity changes. The first day of measurements (25 February) started at 10:01 local time (16:01 GMT). All the measurements are tide-corrected using tidal values given by the GRAVPAC software. There is an increase in gravity of about $32 \mu \mathrm{Gal}$ at a rate of $6.8 \mu \mathrm{Gal} /$ hour from 10:01 to $15: 22$, then the gravity decreases about $40 \mu \mathrm{Gal}$ at a rate of $-5.8 \mu \mathrm{Gal} / \mathrm{hour}$, and finally, gravity seems to increase again, but there are not sufficient data to define a slope (Fig. 2.6a). On the second day (6 March), results are different. A gravity increase of $22 \mu \mathrm{Gal}$ at a rate of $5.6 \mu \mathrm{Gal} /$ hour is observed in the morning starting at

09:58 and ending at 13:21 (Fig. 2.6b). Subsequently, there are no significant gravity variations. The rate of gravity variation during this period is about $-0.7 \mu \mathrm{Gal} / \mathrm{hour}$, which is trivial. During the third day of continuous monitoring (13 March), an increase in gravity is observed in the morning, followed by a decrease (Fig. 2.6c). From $10: 50$ to about $15: 32$, gravity increases about $15 \mu \mathrm{Gal}$ at a rate of 2.8 $\mu \mathrm{Gal} /$ hour; from $15: 32$ to $22: 55$, gravity decrease about $35 \mu \mathrm{Gal}$ at a rate of $3.8 \mu \mathrm{Gal} /$ hour. Similar to 25 February, the gravity then appears to increase, but there are not sufficient data to confirm this.

A certain correlation is observed between the daily earth tides and the observed gravity variations, particularly on 25 February and 13 March. On these two days, the maxima of the gravity variation seem to be offset by approximately three to four hours after the first maximum of the tidal variation (Fig. 2.6a and 2.6c). The minima of gravity are also offset by approximately four hours after the tidal minimum. Moreover, the observed gravity variation is higher when the amplitude of the tidal variation is larger. The gravity variation on the first day is of the order of $40 \mu \mathrm{Gal}$, while the amplitude of the tides is $263 \mu \mathrm{Gal}$ from the first maximum to the first minimum. On the second day, the gravity variation, which is not so clearly linked to the tides as for the other two days, is on the order of $20 \mu \mathrm{Gal}$, while the amplitude of the tides from the first minimum to the maximum is of $135 \mu \mathrm{Gal}$. For the third day, the gravity variation is about $36 \mu \mathrm{Gal}$ and the tidal amplitude is $233 \mu \mathrm{Ga}$. Results obtained in 1997 for the continuous measurements are not sufficiently long to obtain such maxima, so they are extrapolated. This extrapolated maximum gravity variation is about $45 \mu \mathrm{Gal}$ and the maximum tidal amplitude is

FIGURE 2.6
Daily gravity variation at station A7 for (a) 25/02/98, (b) 06/03/98 and (c) 13/03/98. (d) Maximum gravity changes vs. maximum tidal amplitude for daily variations in 1997 and 1998. (e) Maximum gravity changes vs. maximum tidal amplitude for daily variations in 1998 only. (f) Fluctuation of $\mathrm{SO}_{2}$ vs. gravity changes on 13 March 1998 at station A7.


$222 \mu \mathrm{Gal}$. When plotted, a clear positive correlation appears between the maximum tidal amplitude and the maximum gravity variation (Fig. 2.6d). The ratios of maximum gravity variations to maximum tidal amplitudes are similar for each day, except for the 1997 data, so a direct correlation of the gravity variations to tidal variations is tempting (Table 2.1). The Scintrex ratio of maximum gravity to maximum tidal amplitude for 1997 do not match with the ratio of 1998 so they are not plotted in Figure 2.6d. Plotted alone, the positive correlation for the 1998 ratios of maximum gravity variations to maximum tidal amplitudes is very good $\left(\mathrm{r}^{2}=0.999\right)$ (Fig 2.6e). More data are necessary to verify this relationship. When comparing the gravity changes to the $\mathrm{SO}_{2}$ flux variations for the same period of time for the 13 March 1998, a positive correlation is tempting (Fig 2.6f). However, the period of time is not very representative since the $\mathrm{SO}_{2}$ fluxes were measured only for 2.5 hours beginning at noon.

Atmospheric pressure measurements were taken at the Hotel Regis in Masaya City during the course of the second and the third days of continuous gravity monitoring. A first look at figure 2.7 a and 2.7 b gives the impression that gravity variations are related inversely to pressure variations. But on closer inspection, the rates of the pressure and gravity variations show that there is no clear relation between pressure and gravity. For 6 March (Fig 2.7a), the gravity increase of 5.6 $\mu \mathrm{Gal} /$ hour between 09:58 and 13:32 corresponds to decreasing pressure at a rate of $0.8 \mathrm{mbar} /$ hour. After $13: 32$, the gravity does not vary significantly ( $-0.7 \mu \mathrm{Gal} /$ hour $)$, while the pressure increases at a rate of $0.5 \mathrm{mbar} / \mathrm{hour}$. For 13 March , the increase in gravity at rate of $2.8 \mu \mathrm{Gal} /$ hour from $10: 50$ to $15: 32$ occurs at a pressure decrease of

Table 2.1: Maximum gravity and tides variation and their ratio in 1997 and 1998

|  | Max Grav Max tide | Ratio G/T |  |
| :--- | ---: | ---: | ---: |
| LCR-98 | 40 | 263 | 0.1520913 |
| LCR-98 | 20 | 135 | 0.1481481 |
| LCR-98 | 36 | 233 | 0.1545064 |
| LCR-97 | 45 | 222 | 0.2027027 |
|  | Scintrex | 20 | 222 |

FIGURE 2.7
Daily gravity changes vs. pressure fluctuations at station A7 for (a) 06/03/98 and (b) 13/03/98. (c) Pressure variation rate vs. gravity variation rate for daily variations in 1998 at station A7.

$-0.7 \mathrm{mbar} /$ hour (Fig. 2.7b). After $15: 32$, the gravity decrease of $-3.8 \mu \mathrm{Gal} /$ hour corresponds to a pressure increase of $0.7 \mathrm{mbar} / \mathrm{hour}$. On a plot of gravity variation rate vs. pressure variation rate, it appears that there could be a relation between the two parameters (Fig. 2.7c). However, a closer look at the plot shows that for pressure variation rates of $0.5 \mathrm{mbar} / \mathrm{hour}$ and $0.7 \mathrm{mbar} / \mathrm{hour}$, the corresponding gravity rates are $-0.7 \mu \mathrm{Gal} /$ hour and $-3.8 \mu \mathrm{Gal} /$ hour, which correspond to ratios of -1.4 $\mu \mathrm{Gal} / \mathrm{mbar}$ and $-5.4 \mu \mathrm{Gal} / \mathrm{mbar}$, respectively. The lack of a linear trend on this diagram and the difference in the ratios strongly imply that the gravity variations are not caused by variations in atmospheric pressure. Even if the gravity variation were affected by a pressure leak in the LCR meter, there is no clear relation between pressure and gravity variations observed at Masaya volcano on a daily basis. For 6 March, the maximum pressure change was on the order of 5 mbar , which corresponds to approximately $-2 \mu \mathrm{Gal}$. For 13 March , the maximum pressure change was about 7 mbar, corresponding to about $-3 \mu \mathrm{Gal}$. These gravity changes are much smaller than those observed on a daily basis, indicating that atmospheric pressure does not play an important role.

The response of the meter to the temperature variation over the durations of the experiements also does not appear to be significant. For the case of 25 February, the ambient temperature first increased, then decreased during the course of the day. It might be possible to correlate these temperature changes with the initial increase and subsequent decrease of gravity for 25 February (Fig. 2.6a). However, gravity begins to increase a second time at about 22:00 local time. This increase cannot be
caused by temperature, since the region was cooling during the nightime. Similar observations can be made for 6 March and 13 March 1998.

Continuously occurring volcanic tremors is recorded beneath Santigo (Metaxian and Lesage, 1997). It is possible that variations in the intensity of tremors may reflect changes in the magmatic activity, which in turn may also cause changes in microgravity. During the gravity measurements, seismicity has been monitored constantly. Investigation of the seismic data show that there is a continuous background tremor of 10 digital units of amplitude. No significant variations in amplitude were observed for the maxima and minima of observed gravity on 25 February, 6 March and 13 March 1998. Some isolated seismic events from an unknown source were also detected. No correlation was found between the seismicity and gravity changes.

## Weekly Gravity Variations in 1997

Results presented in this section are from the microgravity line consisting of stations $\mathrm{A} 1, \mathrm{~A} 3, \mathrm{~A} 7, \mathrm{~B} 2$ and B 1 A . The relative differences are calculated with Al as the base (Appendix B, Table B3); they represent variations between 4-17 March 1997. Measurements on 4 March were made by Hazel Rymer. The LCR instrument shows a decrease of approximately $30-40 \mu \mathrm{Gal}$ between $4-10$ March for the three stations near the crater $(\mathrm{A} 7=36 \mu \mathrm{Gal}, \mathrm{B} 1 \mathrm{~A}$ and $\mathrm{B} 2=31 \mu \mathrm{Gal})$ (Fig. 2.8a). Between 10-13 March, gravity was mostly stable except for station A7 where there was an increase of $19 \mu \mathrm{Gal}$. Between 13-17 March, a decrease was observed at every station near the crater. This decrease was of variable amplitude depending on the station (62 $\mu \mathrm{Gal}$ for $\mathrm{A} 7,38 \mu \mathrm{Gal}$ for B 2 and $24 \mu \mathrm{Gal}$ for BIA ). In general, the temporal

FIGURE 2.8
(a) Weekly gravity changes for the LCR in 1997. (b) Gravity changes vs diurnal tidal variation, 4-17 March 1997 at station A3, A7, B1A and B2.


variation of microgravity during this period is similar among the three stations. For station A3, located in the caldera three kilometers north of the active crater (Fig. 2.4), there was no significant variation observed. The gravity changes observed between 4-13 March consist of a decrease of $15 \mu \mathrm{Gal}$, while between 13-17 March an increase of $11 \mu \mathrm{Gal}$ was observed. However, these variations are not significant at the $95 \%$ confidence level, so as stated above, there are no significant variations at A3. Therefore, the gravity changes are confined to the vinicity of the crater area; this conclusion is also made by Rymer et al. (1998).

The microgravity changes occurred at stations near the crater due to its higher level of activity. During the period February 1993 to March 1994, precision levelling within the caldera revealed an uplift of $2-3 \mathrm{~cm}$ at the summit relative to a station 5 km east (J.B. Murray personal communication, 1997; Global Volcanism Network Bulletin, 1994). GPS data also indicate that there have been no vertical movements in excess of 2 cm or horizontal movements in excess of 1 cm between 1994 and 1997. This altitude variation corresponds to $6 \mu \mathrm{Gal}$ and is small compared to the microgravity variations observed (Rymer et al. 1998). It is therefore clear that the microgravity variations observed are not due simply to altitude variations. As the tidal amplitude decreased from 9 March to 16 March, so did the gravity during the same period (Fig. 2.8b). However, as the gravity decreased between 4 March to 10 March, the fortnightly tidal amplitude increased, which is in the opposite sense. Thus, no clear relation between the gravity changes observed from 4-17 March and the tidal variations is observed.

## Weekly Gravity Variations in 1998

Gravity variations observed in 1998 extend from 27 January to 14 March. In total, 10 days of microgravity measurements were taken over a duration of 47 days, starting on 27 January. Measurements taken on 27 January were made by Hazel Rymer, and those taken on the 18 and 27 Febuary, and 14 March were made by Glyn Williams-Jones. From 27 January to 18 February, there are no consistent gravity variations except for a large decrease of $55 \mu \mathrm{Gal}$ at Museo, which is due to an error from the misplacement of the station between 27 January and 18 February (Fig. 2.9a). For certain days (18 and 27 February and 14 March 1998), variations of less $30 \mu \mathrm{Gal}$ are considered not to be significant (G. Williams-Jones, personal communication, 1998). Because of a problem with the electrical wire connecting the LCR meter to the battery, there was a power failure on 14 February which caused the meter temperature to fall by $3-4^{\circ} \mathrm{C}$. Because of this problem, the meter was unstable for about a week; the associated error is higher than usual, depending on the technique used to acquire data. Between 18-24 March, there is no variation in the crater area (A7, B2 and B1A), but an increase of about $27 \mu \mathrm{Gal}$ is observed at stations A 3 and A5. This is surprising, since there are no variations at the summit area and at MUSEO. This variation is near the error limit of $25 \mu \mathrm{Gal}$, thus not too much weight should be placed on this difference. Between 24 February and 1 March, a peak is observed at all stations except MUSEO. This peak is more prominent for stations A7, B 2 and BlA , where there are increases of 62,63 and $55 \mu \mathrm{Gal}$, respectively, from 24 to 27 February. From 27 February to 1 March, there are decreases of $53 \mu \mathrm{Gal}$ for A7, $46 \mu \mathrm{Gal}$ for B 2 and $41 \mu \mathrm{Gal}$ for B 1 A . The gravity variation at A5 for the 24

February-1 March peak corresponds to an increase of $29 \mu \mathrm{Gal}$ and a subsequent decrease of $33 \mu \mathrm{Gal}$. For A 3 , the peak is represented by an increase of $10 \mu \mathrm{Gal}$ and then a decrease of $12 \mu \mathrm{Gal}$. It seems that either an event occurred between 24 February and 1 March which increased the magma density in the crater area, or alternatively there is a problem with the data of 27 March. After this peak, gravity variations are generally less than $25 \mu \mathrm{Gal}$, except for stations B2 and B1A which show increases of $32 \mu \mathrm{Gal}$ and $24 \mu \mathrm{Gal}$, respectively, between 5-9 March. Interestingly, A7 does not follow this trend, and actually shows a small decrease instead, increasing afterward while B2 and B1A decrease between 5-14 March. In general, except for the peak between 24 February and 1 March, there are no consistent variations, considering the level of precision for most of the variations (25$30 \mu \mathrm{Gal})$.

If the gravity variations measured by the second data acquisition technique (discussed above) are considered alone (24 February and 1, 2, 3, 5 and 9 March 1998), the results are the same but the precision is higher. The precision with this technique is around $12 \mu \mathrm{Gal}$ (twice the standard deviation), so variations larger than $15 \mu \mathrm{Gal}$ should be considered significant at the $95 \%$ confidence level. The only gravity variations observed between 24 February and 1 March are at stations B2 and B1A where there are increases of $33 \mu \mathrm{Gal}$ and $40 \mu \mathrm{Gal}$, respectively (Fig. 2.9b). All the gravity variations at other stations are generally within the level of error. Again, station A7 does not follow the trend of stations $B 2$ and $B 1 A$; it shows no real variation during the entire survey. GPS measurements made in 1998 do not show any vertical variation greater than $1-2 \mathrm{~cm}$ (G. Williams-Jones, personal communication,

FIGURE 2.9
(a) Gravity changes at Masaya between 27/01/98 and 14/03/98. (b) Gravity changes at Masaya between 24/02/98 and 09/03/98. (c) Gravity changes vs. diurnal tidal variation from 27 January to 14 March 1998.


C

1998), which corresponds to about 3-6 $\mu \mathrm{Gal}$ in gravity. No direct correlation was found with the fortnightly and diurnal tidal variations during this these weekly measurements (Fig. 2.9c).

## Annual Gravity Variation, 1997-1998

Variations on a yearly basis are not very significant. If we compare values acquired in 1997 to those of 1998, we see that there are no consistent variations (Fig. 2.3). It is difficult to interpret the gravity on a year-to-year basis, since variations on weekly and even daily basis are sometimes quite large. For example, gravity variations observed at station A7 since 1993 show a clear trend initially because there is only one measurement per year (Fig. 2.10). When one looks at gravity variations in 1997 and 1998, however, the major observation is that gravity varies significantly. If only one measurement was made in 1997 and 1998, the trend of the gravity variation would be variable depending on the day the measurement was taken.

## Discussion

Microgravity monitoring of Masaya volcano was initiated by Hazel Rymer in 1993 after the renewal of degassing activity at the active Santiago crater. The goal of the survey was to constrain the shallow structure of the magma system and its geophysical signature in a way to forecast future changes. From 1993 to 1994, Rymer et al. (1998) observed a gravity change at all stations near the active crater and little or no change at stations away from the crater in the caldera (Fig. 2.4). Subsequently, gravity increased slightly each year at stations near the crater. These data indicate that the anomaly causing the gravity changes is centered at the crater

FIGURE 2.10

Annual gravity changes at Masaya for station A7, 1993-1998.

and at shallow depth. The model that best fits the observed gravity variation is a cylindrical body of reduced density of 440 m diameter and 100 m thick (Rymer et al., 1998). They related this decrease of density by vesiculation of the shallow magma beneath Santiago crater as a result of convective overturn of the magma remaining in the plumbing system from the previous episode of activity in the 1980's.

We now analyze the short-time scale microgravity variation observed at Masaya during 1997-98 and compare it to the above model. Results obtained in 1997 over a period of two weeks showed overall decreases of gravity of about $55-80 \mu \mathrm{Gal}$ at the three stations near the active crater (A7, B2 and B1A) (Fig. 2.8a). In 1998, there were variations of the order of $55-63 \mu \mathrm{Gal}$ over very short periods (3 days) (Fig. 2.9a). One-day experiments in 1997 and 1998 showed gravity changes varying from 20 to $45 \mu \mathrm{Gal}$ at Station A7 near Santiago (Fig. 2.5a, 2.6a,b and c). These variations may be linked to changes in the density of the magma, which in turn probably depend on the bubble content. The dissolved gas content in the magma beneath Santiago does not cause the density to vary much. At low pressure, the variation in the quantity of volatiles dissolved in the magma does not change the magma density significantly. At 50 MPa , a variation of $1 \mathrm{wt} \% \mathrm{H}_{2} \mathrm{O}$ dissolved in the magma is required to produce a density variation of $30-40 \mathrm{~kg} / \mathrm{m}^{3}$ (Lange and Carmichael, 1990; Lange, 1994). This would not be expected at Masaya considering the amount of volatiles in the magma (about 1 wt \%) (K. St-Amand, personal communication, 1998) and the lithostatic pressure of 5 MPa at the base of the magma body at Masaya. The corresponding quantity of $\mathrm{H}_{2} \mathrm{O}$ at this pressure for a saturated magma is about 0.95 wt $\%$. Therefore, variation in the dissolved content of $\mathrm{H}_{2} \mathrm{O}$ in the magma should not be
high enough to produce the density variation needed to account for the gravity changes observed at the surface. An easier way to produce the density variations in a magma is to vary the amount of exsolved volatiles in the magma. At a pressure of 5 MPa , very small fluctuations in the $\mathrm{H}_{2} \mathrm{O}$ content ( $0.01 \mathrm{wt} \%$ ) can produce density variations of $50-100 \mathrm{~kg} / \mathrm{m}^{3}$. This corresponds to $3-5 \%$ vesicularity.

Short-term variations were observed at Poás volcano in Costa Rica during a period of 43 days where ten sets of gravity measurements were made at different stations around and in the crater area by Rymer and Brown (1987). They concluded that the most probable cause of these gravity fluctuations was changes in the density of the magma. They stated that a variation in the density of the magma of about 30 $\mathrm{kg} / \mathrm{m}^{3}$ was needed to produce the observed gravity variation at the crater station of $120-140 \mu \mathrm{Gal}$. They also stated that a vesicularity variation of $1 \%$ would be sufficient to produce the inferred change in the density of the magma. At Masaya, the observed gravity variations are of the same order ( $40-90 \mu \mathrm{Gal})$. Because of a lack of rain during our field season, the gravity variations observed on a daily basis are clearly not produced by variations in the water table level. Another possible cause to explain the gravity variation observed near the crater is changes in the level of the magma. The problem with this model is that the magma level would have to vary by tens of meters in a very short time to produce the daily gravity changes observed near Santiago at station A7. We can model this using the finite vertical cylinder model to calculate the maximum gravity variation:

$$
\mathrm{G}_{\max }=2 \pi \gamma \rho\left[\mathrm{~L}+\left(\mathrm{z}^{2}+\mathrm{R}^{2}\right)^{1 / 2}-\left[(\mathrm{z}+\mathrm{L})^{2}+\mathrm{R}^{2}\right]^{1 / 2}\right]
$$

where $\gamma$ is the universal gravity constant, $\rho$ the density contrast in $\mathrm{kg} / \mathrm{m}^{3}, \mathrm{~L}$ the length of the cylinder in $\mathrm{m}, \mathrm{z}$ the depth to the roof of the cylinder and R the radius. Using a large body of magma at a depth of 360 m , having a length of 200 m and a radius equal to the Santiago crater radius of 300 m and a depth variation of 10 m for a magma density contrast of $300 \mathrm{~kg} / \mathrm{m}^{3}$, the corresponding gravity variation is $15 \mu \mathrm{Gal}$. This correspond to the maximum gravity variation that could be observed at the center of the anomaly for a 10 m level variation.

The four continuous sets of measurements made near the crater area during 7 13 hours in 1997 and 1998 showed variations in gravity of the order of $40 \mu \mathrm{Gal}$. These changes do not seem related to any inherent problems of the meter in response to environmental variations such as atmospheric pressure, temperature and tares, since the meter was not touched for the duration of the experiments. Instrumental drift is not considered for this particular meter (G-513) because it is known to be fairly stable. In fact, the same meter was used to make a Bouguer survey at Telica volcano, and there it proved to be very stable. For example, the average difference between the starting and ending measurements made each day at the reference station during the Bouguer survey was about $17 \mu \mathrm{Gal}$. This is quite low, considering that fewer precautions were taken for the precision during this survey compared with a microgravity survey. The changes observed with this meter in such a short time period must be the result of changes beneath Santiago crater. As stated above, the most realistic probability over a short time is a fluctuation in the density of the magma, which is more likely in an upper, less dense layer of magma where there are many gas bubbles and degassing activity. Modelling of the anomaly under Santiago
showed that the variations observed are caused by a local body of shallow depth and of a size similar to the diameter of Santiago crater. To account for the observed gravity variations, the most plausible body of magma would be about $100-200 \mathrm{~kg} / \mathrm{m}^{3}$ less dense than the surrounding rocks. Density variations of the order of $25-75 \mathrm{~kg} / \mathrm{m}^{3}$ within the magma body would be necessary to produce the observed gravity changes (Fig. 2.11a,b,c and d). These density variations would be more efficient if distributed in a larger body instead of a very thin layer of vesiculated magma like a foam. If the density variations are produced only in a thin layer of about 10-20 meters, they must be very large $\left(300-600 \mathrm{~kg} / \mathrm{m}^{3}\right)$ to produce the observed gravity changes.

A possible response of the magma to diurnal tides was observed for the oneday experiments. there appears to be a time-lag of about 4 hours between the maximum tidal amplitude and the maximum gravity variations, with the maximum gravity variation occuring after the maximum tidal amplitude. The same phenomenon is observed for the tidal and gravity minima. Moreover, the amplitude of the gravity variation appears to be linked to the amplitude of the diurnal tides. It is not yet clear in what way the physical or mechanical effect of the tides modify the magma beneath Santiago. If the daily gravity variations observed at Masaya volcano are considered to be real, then the weekly gravity variations are probably not representative of the processes occuring at Masaya. The same conclusions may apply to the annual variation to a certain extent. Yet one may consider that such short-time fluctuations are residual and should not affect the general trend of the gravity variations on a yearly basis.

The processes causing fluctuations in the density and vesicularity of the magma are not fully understood. Permanent tremor is ongoing at Masaya; its source

## FIGURE 2.11

Gravity changes of a body of magma under Santiago crater 120 m thick with an initial density contrast of (a) $-120 \mathrm{~kg} \mathrm{~m}^{-3}$ and than increasing to (b) $-70 \mathrm{~kg} \mathrm{~m}^{-3}$, (c) $-50 \mathrm{~kg} \mathrm{~m}^{-}$ ${ }^{3}$ and (d) $-25 \mathrm{~kg} \mathrm{~m}^{-3}$.



FIGURE 2.12
(a) Fluctuation of $\mathrm{SO}_{2}$ fluxon March 12 1997. (b) Fluctuation of $\mathrm{SO}_{2}$ flux on 13 March 1998.

is located under the active crater of Santiago. Metaxian and Lesage (1997) observed that the permanent tremor at Santiago is probably generated by the continuous degassing. There may be convective processes in the magma that bring gas-rich magma from deep within the body to shallow levels where it vesiculates to lower the density of the magma. Fluctuations of the $\mathrm{SO}_{2}$ flux were observed over a period of a day at Masaya (Fig. 2.12a and b). This may indicate that the quantity of gas may vary at the top of the magma over a very short time. Pulses of gas-rich magma from rapid convection could be the cause of these $\mathrm{SO}_{2}$ fluctuations. These pulses may be driven thermally by convection or by injection of new magma. Another possibility is that batches of gas bubbles could be rising from a deeper part of the magma body under Santiago. Vergniolle (1996) has show that the ascent rate for 1 mm-diameter bubbles is about $1.5 \times 10^{-4} \mathrm{~m} / \mathrm{s}$. This corresponds to a rise time of about one month for the 440 meter-thick magma body under Santiago. This timescale is two orders of magnitude higher than that of 4 hours observed between the tidal and gravity variations. Thus, the bubble rise mechanism is not possible for the daily gravity variations which are linked to the diurnal tides. The possibility of a new magma injection into the chamber is unlikely, since it would have left a larger and more widespread gravity signature than the one observed at Masaya by Rymer et al. (1998). Nevertheless, it is clear that the cause of the gravity changes is linked to variations in the density of the magma, which in turn is related to fluctuations in the vesicularity. However, the cyclicity observed in these changes could be driven by several possible mechanisms.

## Conclusions

Gravity variations observed at Masaya volcano are linked to fluctuations in vesiculation of the magma beneath Santiago crater. A possible link of earth tides to daily gravity variations was observed at one station (A7) near the active crater. The tides appear to affect the vesiculation of the magma. All the gravity changes observed at different timescales are consider to be the result of the same process, which is changes in density of the magma. These changes are linked to vesiculation fluctuations in the magma. Varying the vesicularity of the magma is an efficient way to produce the gravity variations observed at Santiago. This could occur in many different ways, such as different configurations of bubbles in the magma body; gas pockets heterogeneously distributed in the magma; an upper vesiculated layer; or simply small bubbles scattered uniformly in the magma. No direct links were found between the variation in the $\mathrm{SO}_{2}$ flux and the gravity changes in 1997, but a possible positive correlation was observed on 13 March 1998. If the cause of the gravity variation is a fluctuation in density due to variation in the vesicularity of the magma, there is probably a direct or indirect link to the $\mathrm{SO}_{2}$ flux.

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## References

Carr, M.J., 1984. Symmetrical and segmented variations of physical and geochemical characteristics of the Central American volcanic front. J. Volcanol. Geotherm. Res., 20: 231-252.

Johnson, N., and Parnell, R.A., 1986. Composition, distribution and neutralisation of "acid rain" derived from Masaya volcano, Nicaragua. Tellus, 38b, 106-117.

Lange, R. A., and Carmichael, I. S. E., 1990. Thermodynamic properties of silicate liquids with emphasis on density, thermal expansion and compressibility. In: Nicholls, J., and Russell, J. K., eds., Modern Methods of Igneous Petrology: Mineralogical Society of America Reviews in Mineralogy, 24: 44-54.

Lange, R. A., 1994. The effect of $\mathrm{H}_{2} \mathrm{O}, \mathrm{CO}_{2}$ and F on the density and viscosity of silicate melts. In Carroll, M., and Holloway, J. R., eds., Volatiles in Magmas: Mineralogical Society of America Reviews in Mineralogy, 30: 331-369.

Metaxian, J.-P., 1994. Etude sismologique et gravimétrique d'un volcan actif: Dynamisme interne et structure de la Caldeira Masaya, Nicaragua. Unpublished Ph.D. Thesis, , Université de Savoie, Savoie, France, 319 pp.

Metaxian, J.-P., and Lesage, P., 1997. Permanent tremor of Masaya volcano, Nicaragua: Wave field analysis and source location. J. Geophys. Res., 102: 22 529-22 545.

Rymer, H., and Brown, G.C., 1987. Causes of microgravity changes at Poás volcano, Costa Rica: an active but non-eruptive system. Bull. Volcanol., 49: 389-398.

Rymer, H., 1989. A contribution to precision microgravity data analysis using Lacoste and Romberg gravity meters. Geophys. J., 97: 311-322.

Rymer, H., van Wyk de Vries, B., Stix, J., and Williams-Jones G., 1998. Pit crater structure and processes governing persistent activity at Masaya Volcano, Nicaragua. Bull. Volcanol., 59: 345-355.

Stoiber, R.E., Williams, S.N., and Huebert, B.J., 1986. Sulfur and halogen gases at Masaya caldera complex, Nicaragua: total flux and variations with time. J. Geophys. Res., 91: 12 215-12 231.

Vergniolle, S., 1996. Bubble size distribution in magma chambers and dynamics of basaltic eruptions. Earth Planet. Sci. Lett., 140: 269-279.

Viramonte, J.G., Navarro Collado, M., and Malavasi Rojas, E., 1997. NicaraguaCosta Rica Quaternary Volcanic Chain. IAVCEI 1997 General Assembly, Puerto Vallarta, Mexico, guidebook for Field Trip 17, 59 pp.

Williams, S.N., 1983. Plinian airfall deposits of basaltic composition. Geology, 11: 211-214.

Bulletin of the Global Volcanism Network, 1994. Masaya Volcano, 18: 7, 11.

## CHAPTER III

# Gravity changes induced by variations of the 

## volatile content in magmas

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#### Abstract

Microgravity surveys have been carried out frequently on volcanoes and calderas to monitor signs of activity, re-activation and potential eruptions. This enables us to better define our concept of the subsurface of volcanoes and the processes occurring magmatic systems. As demonstrated in this study, volatile oversaturation, mainly as a free $\mathrm{H}_{2} \mathrm{O}$ phase, is an important factor in triggering eruptions due to overpressure in large silicic magma chambers. It is also possible to monitor an increase in the dissolved volatile content. $\mathrm{CO}_{2}$ concentrations in magma at the depths modelled in this study $(2-8 \mathrm{~km})$ are not sufficiently large to produce significant gravity changes under saturated or undersaturated conditions. Modelling shows that even at depths of 3800 m , it is possible to observe overpressure due to exsolution of gas bubbles induced by crystallisation. These overpressures could produce gravity variations at the surface on the order of $50-100 \mu \mathrm{Gal}$ for a $10 \mathrm{~km}^{3}$ magma chamber. This is a long process which may take hundreds of years; it is thus difficult to see year-to-year changes for very deep systems. Yet this may prove useful for a long-term survey. For a shallower magma body, the density changes caused by gas exsolution are easier to observe with gravity. The density decrease, they could be as high as $40 \mathrm{~kg} \mathrm{~m}^{-3}$ for an increase of $0.3 \mathrm{wt} \% \mathrm{H}_{2} \mathrm{O}$ under saturated conditions; such a decrease in density in a $1 \mathrm{~km}^{3}$ magma body 1900 m deep would produce a gravity change of $50 \mu \mathrm{Gal}$. In conjunction with other means of monitoring, microgravity can be used to understand subsurface processes in volcanic systems.


## Introduction

The importance of volatiles for causing overpressure in a magma chamber and volcanic eruptions is well known. Volatiles such as carbon dioxide, water, and sulfur dioxide can accumulate in a magma chamber under certain conditions. First, a certain amount of $\mathrm{H}_{2} \mathrm{O}$ or $\mathrm{CO}_{2}$ is needed in the magma to produce a free gas phase. Under high-pressure conditions, the magma can accumulate volatile components which remain dissolved in the melt. At some point, however, the magma reaches a solubility limit and becomes saturated in volatiles. After this point, when the solubility of volatiles in the magma has been exceeded, there is free-vapor production and small bubbles forms in the magma. Many factors are involved in the amount of gas that can be exsolved; the solubility of $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{CO}_{2}$ depends on pressure, temperature, composition of the magma and the presence of other volatiles.

Several processes in a magma chamber can affect the production or the quantity of volatiles. Crystallisation may occur, which would cause the volatile content to increase. There also could be an injection of new magma which is richer in volatiles. Convection could occur to bring magma richer in gas to shallow levels. A trap is also needed for these volatiles to be retained in the magma chamber. In other words, the system needs to be closed; if not, the bubbles in the magma will escape. If the magma chamber is closed, these bubbles can accumulate at the top of the magma chamber or they can be distributed throughout the chamber. This would inevitably cause the magma density to be lowered by a certain amount. If this density variation is large enough, it could eventually lead to an observable gravity variation at the surface. If the system is open, there can still be density variations due to changes in
the bubble content in the magma. In shallow environments, such as beneath Masaya volcano, Nicaragua, convection may transport gas-rich magma to shallow levels where it will degas; this process could be repetitive and produce variations in gravity (Rymer et al., 1998; Beaulieu et al., 1998). Any volcanic system where there is a large gas emission could potentially be monitored for gravity variations. Fluctuation in the gas emission is linked to variations in the amount of gas bubbles in the magma, so this could possibly create observable density variations. In a volatileundersaturated magma, variation in the dissolved volatile content also could lead to a density variation that could be observed by gravity methods. This is a useful tool for a volcanologist to help forecast volcanic eruptions linked to gas accumulations and overpressure in a volcano or lava dome.

The main objective of this study was to show, using a theoretical model, that density changes produced by volatiles in magma are possible to observe at the surface with gravity methods. Calculations of the density of magma were made using different volatile content $\left(\mathrm{H}_{2} \mathrm{O}, \mathrm{CO}_{2}\right.$, or both) at different depths, e.i., different pressures. Examples demonstrating that gravity changes can be observed at the surface are then modeled using plausible density changes within magma bodiesof various dimensions.

## Methodology

## Magma chamber shape:

In order to define models that are simple and representative of a magmatic chamber and system, spherical, cubic and cylindrical magma chambers and conduits

FIGURE 3.1
Gravity effect of a sphere, the horizontal axis $x / z$ is the horizontal distance from the center of the sphere, $x$, and $z$ the depth of the center; the vertical axis is the gravity of a point at a certain horizontal distance from the center of the sphere versus maximum gravity (From Telford et al., 1990).

will be used to model the gravity variation from density fluctuations caused by changes in the dissolved and exsolved volatile content. Different sizes and emplacement depths will also be used. For the spherical model, the magma chamber sizes are $0.1 \mathrm{~km}^{3}, 1 \mathrm{~km}^{3}$ and $10 \mathrm{~km}^{3}$ at three different depths of $1900 \mathrm{~m}, 3800 \mathrm{~m}$ and 7600 m , each depth representing 50,100 and 200 MPa , respectively, calculated from:

$$
\begin{equation*}
\mathrm{P}(\mathrm{z})=\rho \mathrm{gz} \tag{1}
\end{equation*}
$$

where $P(z)$ is the lithostatic pressure, $z$ the depth, $g$ the acceleration due to gravity $\left(9.81 \mathrm{~m} \mathrm{~s}^{-2)}\right.$ and $\rho$ the density of the country rocks ( $2700 \mathrm{~kg} \mathrm{~m}^{-3}$ ). In the spherical model, the bubbles in the magma chamber are assumed to be uniformly distributed in a closed system. The gravity effect in this model is calculated from:

$$
\begin{equation*}
\mathrm{g}_{\mathrm{x}}=2.79 \times 10^{-2} \Delta \rho \mathrm{zr}^{3} /\left(\mathrm{x}^{2}+\mathrm{z}^{2}\right)^{3 / 2} \tag{2}
\end{equation*}
$$

where $\mathrm{g}_{\mathrm{z}}$ is the gravity in mGal, $\Delta \rho$ is the density contrast between the magma chamber and the surrounding rocks, $z$ the depth at the center of the sphere, $x$ the horizontal distance from the center of the sphere, and $r$ the radius of the sphere (Fig. 1) (Telford et al., 1990). The cubic magma chamber is used to define a model where the bubbles are not distributed uniformly in the magma chamber but instead are clustered in a thin foam layer at the top of the chamber or as layers of different thickness in the lower or the middle part of the magma chamber. For the calculation of the gravity effect, GRAVMAG is used; this is a 2.5 -dimensional gravity modelling program (Pedley et al., 1993). Finally, to represent variations at smaller scales and closer to the surface, a vertical cylinder will be used to model a magmatic conduit. The maximum gravity effect of a cylinder is calculated as follows:

$$
\begin{equation*}
\mathrm{G}_{\max }=4.19 \times 10^{-2} \Delta \rho\left[\mathrm{~L}+\left(\mathrm{z}^{2}+\mathrm{R}^{2}\right)^{1 / 2}-\left\{(\mathrm{z}+\mathrm{L})^{2}+\mathrm{R}^{2}\right\}^{1 / 2}\right] \tag{2}
\end{equation*}
$$

where $\mathrm{G}_{\text {max }}$ is the maximum gravity in $\mathrm{mGal}, \Delta \rho$ the density contrast between the magma chamber and the surrounding rocks, z the depth at the top of the cylinder, L the length of the cylinder and $R$ the radius of the cylinder.

## Density Variation of Magmas

## Volatile Content and Solubility

In order to reach overpressure in a magma chamber or in a volcanic complex, a certain amount of $\mathrm{H}_{2} \mathrm{O}$ and/or $\mathrm{CO}_{2}$ needs to be initially present in the magma. Depending on the solubilities of these components, they will be in the form of free gas or volatiles dissolved in the structure of the magma itself. In the quenched glass inclusions of explosive silicic and andesitic explosions, high water concentrations (4$6 \mathrm{wt} \%$ ) are commonly found (Stix and Layne, 1996). Infrared spectroscopic measurements of glass inclusions within quartz phenocrysts from a plinian fallout of the Pine Grove, southwestern Utah, USA, show very high concentration of $\mathrm{H}_{2} \mathrm{O}(6-8$ \%) (Lowenstern, 1994). For more mafic magma, there is less dissolved water in general. At Kilauea, the parental melt contains approximately $0.30 \mathrm{wt} \% \mathrm{H}_{2} \mathrm{O}$ and $0.65 \mathrm{wt} \% \mathrm{CO}_{2}$ (Gerlach, 1986). However, high water concentrations are sometimes observed in arc-related basaltic and basaltic andesite magmas; glass inclusions from the 1974 Fuego eruption in Guatemala had values up to $6 \mathrm{wt} \% \mathrm{H}_{2} \mathrm{O}$ (Harris and Anderson, 1984). Basaltic andesite glass inclusions from Goosenest Volcano, Oregon, contain up to $3.3 \mathrm{wt} \% \mathrm{H}_{2} \mathrm{O}$ (Sisson and Layne, 1993). At Mt Pinatubo, the initial dacitic melt had concentration of dissolved $\mathrm{CO}_{2}$ up to $0.4 \mathrm{wt} \%$ (Wallace and Gerlach, 1994).

The parameter that will determine the fraction of a volatile component as a free gas phase is the solubility of that component in the magma. For rhyolitic melt, the calculation of the solubility of water is assumed to be proportional to the square root of pressure (Stolper, 1992), with a solubility constant of:

$$
\begin{equation*}
\mathrm{S}_{\mathrm{H}_{2} \mathrm{O}}=4.186 \times 10^{-6} \mathrm{P}^{0.5} \tag{4}
\end{equation*}
$$

where P is the pressure in Pascals. For basaltic magma, the solubility law for $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ used in the modelling is (Stolper and Holloway, 1988):

$$
\begin{equation*}
\mathrm{S}_{\mathrm{CO}_{2}}=4.4 \times 10^{-12} \mathrm{P}^{1.0} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{S}_{\mathrm{H}_{2} \mathrm{O}}=6.8 \times 10^{-8} \mathrm{P}^{0.7} \tag{6}
\end{equation*}
$$

## Density Calculation of a Crystal- and Volatiles-Bearing Magma

The density of a volatile-bearing magma, $\rho$, may be defined in terms of the partial volumes occupied by the exsolved volatiles of density $\rho_{g}$, the magmatic liquid of density $\rho_{m}$, and the crystals of density $\rho_{c}$ (Bowers and Woods, 1997). If the mass fraction of volatiles in the mixture is $n$ and the mass fraction of crystals is x , then

$$
\begin{gather*}
\frac{1}{\rho}=\frac{n}{\rho_{g}}+\frac{1-n}{\rho_{l}}  \tag{7}\\
\rho_{1}=(1-\mathrm{x}) \rho_{m}+\mathrm{x} \rho_{c}
\end{gather*}
$$

where $\rho_{l}$ is the density of a cristal-bearing magma and $\rho_{\mathrm{y}}$ is the density of the gas phase assuming that the exsolved volatiles obey the ideal gas law (Tait et al., 1989):

$$
\begin{equation*}
\rho_{\mathrm{g}}=\frac{\mathrm{P}}{\mathrm{RT}} \tag{9}
\end{equation*}
$$

where R is the universal gas constant divided by the molar mass of water or carbon dioxide ( $\mathrm{R}=461.5 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ for $\mathrm{H}_{2} \mathrm{O}$ and $189.0 \mathrm{~J} \mathrm{~kg}^{-1} \mathrm{~K}^{-1}$ for $\mathrm{CO}_{2}$ ) and T is the temperature in Kelvin ( 1173 K for rhyolite and 1473 for basalt). The density of the magmatic liquids ( $\rho_{\mathrm{m}}$ ) is calculated with the method of Bottinga et al. (1982). For rhyolitic and basaltic melts at different pressures and $\mathrm{H}_{2} \mathrm{O}$ undersaturated conditions, compositions used are from Shaw (1963) (sample DC-1), and from Lange and Carmichael (1990) (Kilauea Tholeiite). Density calculated for the anhydrous magmas with no effect of the pressure is $2333 \mathrm{~kg} \mathrm{~m}^{-3}$ for the rhyolite and $2650 \mathrm{~kg} \mathrm{~m}^{-3}$ for the basalt.

At equilibrium the mass fraction of dissolved volatiles in the melt $n_{\mathrm{s}}$ at pressure $P$ is given by Henry's Law. It is calculated by

$$
\begin{equation*}
n_{\mathrm{s}}=\mathrm{S}(\mathrm{P}) \tag{10}
\end{equation*}
$$

where S is the solubility constant and P the pressure in Pa . If the total initial mass fraction of volatiles in the magma is $n_{0}$ and assuming the crystals to be anhydrous (Huppert et al., 1982), the exsolved mass fraction of volatiles $n(P)$ is:

$$
\begin{equation*}
n(\mathrm{P})=n_{0}-\mathrm{SP}^{\mathrm{n}}\left((1-\mathrm{x}) \rho_{\mathrm{m}} \rho_{\mathrm{l}}\right) \tag{11}
\end{equation*}
$$

For the compressibility of magma, it is assumed that the volume V of a given mass of liquid magma and crystals changes with pressure $P$ according to the following relation (Blake, 1981; Druitt and Sparks, 1984):

$$
\begin{equation*}
\frac{d \mathrm{~V}}{d \mathrm{P}}=-\frac{\mathrm{V}}{\beta} \tag{12}
\end{equation*}
$$

where $\beta$ is the elastic bulk modulus of the liquid mixture. Typical values of $\beta$ for silicate liquids range from $10000-40000 \mathrm{MPa}$ (Tait et al., 1989). The value used in

## FIGURE 3.2

Density changes with variation in the water content at different pressures for (a) rhyolitic magma and (b) basaltic magma.

this calculation is 10000 MPa . Integration of (12) gives an expression for the meltcrystal density $\rho_{1}$ as a function of pressure:

$$
\begin{equation*}
\rho_{l}(\mathrm{P})=\rho_{i_{0}} \exp \left(\frac{\mathrm{P}-\mathrm{P}_{\mathrm{a}}}{\beta}\right) \tag{13}
\end{equation*}
$$

where $\rho_{l o}=\rho_{l}\left(\mathrm{P}_{\mathrm{a}}\right)$ is the density of the mixture of melt and crystals at atmospheric pressure $\mathrm{P}_{\mathrm{a}}=0.1 \mathrm{MPa}$ (Bowers and Woods, 1997). The density of a magma-volatiles mixture varying with pressure $P$ can be obtained by combining (7) and (13):

$$
\begin{equation*}
\frac{1}{\rho}=\frac{n(\mathrm{P}) R T}{\mathrm{P}}+\frac{(1-n(\mathrm{P}))}{\rho_{l_{0}}} \exp \left(-\frac{\mathrm{P}-\mathrm{P}_{a}}{\beta}\right) \tag{14}
\end{equation*}
$$

## Density Modelling of Magma with a Pure $\mathrm{H}_{2} \mathrm{O}$ Dissolved-Gas Fraction

In the examples below, modelling was done with a pure $\mathrm{H}_{2} \mathrm{O}$ phase varying from 0-7 wt $\%$ in a closed system at constant temperature ( 1173 K for rhyolite and 1473 K for the basalt) for rhyolitic and basaltic magmas. For simplicity, the pressure is assumed to remain constant everywhere in the magma chamber. Figure 2 a and 2 b shows the density changes for rhyolitic and basaltic magmas with variations in the water content at different pressures $(50,100$ and 200 MPa$)$. For both the rhyolite and the basalt, once the exsolution of volatiles begin, the density is lowered drastically at lower pressure ( 50 MPa and 100 MPa ). At 200 MPa , the effect of volatile exsolution is nearly insignificant for rhyolite and low for basalt. In the basaltic magma, the density begins to decrease at lower water content than the rhyolite, indicating that the solubility of water in basalt is lower than in rhyolite. Density is also decreasing faster in the basalt probably because of the greater density contrast between basaltic melt and the $\mathrm{H}_{2} \mathrm{O}$ gas phase. For the crystal fraction in the magma (Fig. 3a and 3b), density changes with water content are shown for three different crystal contents. As

FIGURE 3.3
Density changes with variation in the water content at different crystal fractions at 100 MPa for (a) rhyolitic magma and (b) basaltic magma. Density changes with variation in the crystal content at 100 MPa and $4 \mathrm{wt} \% \mathrm{H}_{2} \mathrm{O}$ for (c) rhyolitic magma and (d) basaltic magma.

the crystal fraction is increased under volatile-saturated conditions, the density decreases as the crystal fraction is increased. The magnitude of density decrease for the rhyolite $\left(47 \mathrm{~kg} \mathrm{~m}^{-3}\right)$ is nearly twice that for the basalt $\left(24 \mathrm{~kg} \mathrm{~m}^{-3}\right)$ when the crystal fraction increases from 0 to $10 \%$. The relation between density and crystal fraction is demonstrated differently in Figure 3c and 3d, where the density varies with the crystal fraction at a pressure of 100 MPa and initial water contents of $2.5 \mathrm{wt} \%$ and 4 wt \% for basalt and rhyolite, respectively. As the crystal fraction increases, the residual melt becomes more enriched in volatiles, and a larger mass of these volatiles are exsolved as vapor for a given pressure. First, the density increases and then decreases as exsolution of volatiles begins. As cited above, the density decrease is larger for the basalt because of the larger density contrast between basaltic melt and the $\mathrm{H}_{2} \mathrm{O}$ gas phase. The crystal densities used in the modelling are $2700 \mathrm{~kg} \mathrm{~m}^{-3}$ (plagioclase) in the rhyolite and $3300 \mathrm{~kg} \mathrm{~m}^{-3}$ (olivine) for the basalt.

## Density Modelling of Basaltic Magma with a Pure $\mathrm{CO}_{2}$ Dissolved-Gas Fraction

In the following examples, the effect of $\mathrm{CO}_{2}$ gas exsolution on the density of a basaltic magma is shown; calculation of density for $\mathrm{CO}_{2}$ undersaturated magma is not included in modelling. Because of the very low solubility of $\mathrm{CO}_{2}$ in the magma (Holloway and Blank, 1994; Dixon and Stolper, 1995; Papale, 1997), the onset of exsolution of magma starts at a very low $\mathrm{CO}_{2}$ content compared to $\mathrm{H}_{2} \mathrm{O}$, even at high pressure (Fig. 3.4a). The density change is of much lower amplitude than for $\mathrm{H}_{2} \mathrm{O}$ because of the low $\mathrm{CO}_{2}$ content in the magma. Similar to $\mathrm{H}_{2} \mathrm{O}$, however, the density decrease is proportionately larger at low $\mathrm{CO}_{2}$ contents and low pressures (Fig. 3.4a). For density variations with crystal content, the density increases instead of decreasing (Fig. 3.4 b and 3.4 c ). In this example, the density of the crystals used is $3300 \mathrm{~kg} \mathrm{~m}^{-3}$

FIGURE 3.4
(a) Density changes of a basaltic magma at the onset of gas excolution with variation in the $\mathrm{CO}_{2}$ content at different pressures. (b) Density variation of a basaltic magma with variation in the $\mathrm{CO}_{2}$ content with different crystal fractions at 100 MPa . (c) Density variation of a basaltic magma with variation in the crystal content at 100 MPa and at different $\mathrm{CO}_{2}$ contents.


B


(olivine); the effect of the increasing crystal fraction is larger than the effect of $\mathrm{CO}_{2}$ bubbles on the density of the magma.

## Density Modelling of Magmas with a Mixed $\mathrm{CO}_{2}-\mathrm{H}_{2} \mathrm{O}$ Dissolved-Gas Fraction

The effect of combined volatiles on their solubility is well demonstrated by Dixon and Stolper (1995). They show that for different ratios of $\mathrm{CO}_{2}: \mathrm{H}_{2} \mathrm{O}$, the solubilities of $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ decrease correspondingly. For a melt containing 1.0 wt $\% \mathrm{H}_{2} \mathrm{O}$ and 46 ppm CO 2 , the saturation pressure is 20 MPa with a vapor phase having a water fraction of 0.5 ( Fig 3.5 a ). If there was only $\mathrm{H}_{2} \mathrm{O}$ present at this pressure, the amount necessary to saturate the melt would be $1.4 \mathrm{wt} \%$. Similarly, for a melt with $\mathrm{CO}_{2}$ as the only volatile, the amount for saturation at this pressure would be 95 ppm . This effect is very important since natural magmas often contain multiple volatile species. The density of a basaltic magma in a closed system with $2.5 \mathrm{wt} \% \mathrm{H}_{2} \mathrm{O}$ and no $\mathrm{CO}_{2}$ at 1473 K and 50 MPa is about $2315 \mathrm{~kg} \mathrm{~m}^{-3}$, and the solubility of water at this pressure is about $2.24 \mathrm{wt} \%$ (Fig. 3.5a). For the same magma, if $100 \mathrm{ppm} \mathrm{CO} \mathrm{CO}_{2}$ is added, the solubility of water is lowered to 1.9 wt \%. Since the lowered $\mathrm{H}_{2} \mathrm{O}$ solubility means that more water is exsolved, the density decreases a further 220 kg $\mathrm{m}^{-3}$; the density for this magma is thus decreased to $2095 \mathrm{~kg} \mathrm{~m}^{-3}$. The same kind of relation is demonstrated in Figure 3.5b for a rhyolite at 1173 K . Thus, the effect of $\mathrm{CO}_{2}$ on the solubility of $\mathrm{H}_{2} \mathrm{O}$ and on the density of magma is considerable.

## Density Changes as a Function of Volatile Content for Unsaturated Magma

Variation in the volatile content of ansaturated magma also causes the magma density to change. A good example is shown by Kazahaya et al. (1994). They demonstrated the density changes of basalt for ascending non-degassed magma and descending degassed magma as a function of depth (Fig. 3.6). The effect of the

FIGURE 3.5
Solubility of $\mathrm{H}_{2} \mathrm{O}+\mathrm{CO}_{2}$ as a function of pressure and fluid composition for (a) tholeiitic basalt at $1200^{\circ} \mathrm{C}$ and (b) rhyolite at $850^{\circ} \mathrm{C}$ (From Holloway and Blank, 1994).


FIGURE 3.6
Density changes of ascending non-degassed magma and descending degassed magma as function of depth. Arrows schematically show a path for convective transport of magma caused by density changes. Numbers show water content in wt. \% (From Kazahaya et al., 1994).

dissolved volatiles, mainly $\mathrm{H}_{2} \mathrm{O}$ at low pressure is to lower the melt density as modelled in Figure 3.2a and 3.2b. In both models (Kazahaya et al., 1994 and this work), the effect of pressure variation on $\mathrm{H}_{2} \mathrm{O}$ undersaturated magma is not very important. For a variation of 150 MPa (from 200 MPa to 50 MPa ), the corresponding density decreases of a volatile-undersaturated basaltic melt is $50 \mathrm{~kg} \mathrm{~m}^{-3}$ according to Kazahaya et al. (1994) and $60 \mathrm{~kg} \mathrm{~m}^{-3}$ for the present modelling.

The effect of $\mathrm{CO}_{2}$ is not considered since the amount of $\mathrm{CO}_{2}$ dissolved in the melt at the pressures used in this model $(50 \mathrm{MPa}, 100 \mathrm{MPa}$ and 200 MPa ) is less than 900 ppm (corresponding to $0.09 \mathrm{wt} \%$ ). Moreover, Lange (1994) demonstrated that for an amount of $3 \mathrm{wt} \% \mathrm{CO}_{2}$ in an alkali olivine basalt, the density decrease by 3.3 $\%$, and that for the same amount of $\mathrm{H}_{2} \mathrm{O}$, the corresponding density decrease is $4.8 \%$. The effect of $\mathrm{CO}_{2}$ on the density is thus lower than that of $\mathrm{H}_{2} \mathrm{O}$; this is due to the difference in the molar volume between $\mathrm{H}_{2} \mathrm{O}$ and $\mathrm{CO}_{2}$. Thus, only the effect of $\mathrm{H}_{2} \mathrm{O}$ on the melt density is considered in the modelling.

## Modelling of Gravity as a Function of Density Variation

## Spherical Model

Modelling here represents the changing density of magma on gravity observed at the surface for different sizes and depths of magma chambers of spherical shape. The density variations used for the modelling range from $0-100 \mathrm{~kg} \mathrm{~m}^{-3}$ and represent possible density changes in a magma body due to various mechanisms such as crystallisation producing increases in the volatile content or convection in the magma
chamber bringing deep, gas-rich magma to lower lithostatic pressures where exsolutions begins, thereby decreasing the density of the magma.

In the following models, the residual gravity changes from density variation of $0-100 \mathrm{~kg} \mathrm{~m}^{-3}$ in a spherical magma chamber are presented as 20000 m -wide horizontal profiles. The maximum gravity changes are produced above the center of the sphere, and they decrease rapidly as the horizontal distance increases. This effect is stronger for shallower magma bodies. The gravity effect of a $0.1 \mathrm{~km}^{3}$ sphere at $1900 \mathrm{~m}, 3800 \mathrm{~m}$ and 7600 m is shown in Figure 3.7. For the magma body at a depth of 1900 m , a large density variation is required $\left(>100 \mathrm{~kg} \mathrm{~m}^{-3}\right)$ to observe a gravity variation at the surface. A density change of $100 \mathrm{~kg} \mathrm{~m}^{-3}$ produces a maximum gravity change of $18 \mu \mathrm{Gal}$ (Fig. 3.7a). At depths of 3800 m and 7600 m , very large density variations would be needed to produce observable gravity changes (Fig. 3.7b, c). For example, a density variation of $100 \mathrm{~kg} / \mathrm{m}^{3}$ at a depth of 3800 m produces a gravity change of the same amplitude as that produced at 1900 m for a $25 \mathrm{~kg} \mathrm{~m}^{-3}$ density variation $(4.6 \mu \mathrm{Gal})$. At 7600 m depth, there is no significant gravity variations observed for a density change of $100 \mathrm{~kg} \mathrm{~m}^{-3}(1.1 \mu \mathrm{Gal})$ : at this depth, even large density variations up to $500 \mathrm{~kg} \mathrm{~m}^{-3}$ are insufficient to obtain an observable gravity change at the surface ( $>20-30 \mu \mathrm{Gal}$ ). For larger magma bodies, the gravity effect is accentuated. Thus, the maximum gravity effect for a $1 \mathrm{~km}^{3}$ chamber at a depth of 1900 m is of the order of $45 \mu \mathrm{Gal}$ for a density change of $25 \mathrm{~kg} \mathrm{~m}^{-3}$ (Fig. 3.8a). The density variation needed to obtain an observable gravity variation is less for a large magma body than for a smaller body; this is true only if the density variation occurs throughout the magma body. For a deeper magma body of the same size $\left(1 \mathrm{~km}^{3}\right)$, a

## FIGURE 3.7

Gravity effect of density variation of a $0.1 \mathrm{~km}^{3}$ spherical magma chamber at depths of (a) 1900 m , (b) 3800 m and (c) 7600 m .

## A



Figure 3.8

FIGURE 3.8
Gravity effect of density variation of a $1 \mathrm{~km}^{3}$ spherical magma chamber at depths of (a) 1900 m , (b) 3800 m and (c) 7600 m .


FIGURE 3.9
Gravity effect of density variation of a $10 \mathrm{~km}^{3}$ spherical magma chamber at depths of (a) 1900 m , (b) 3800 m and (c) 7600 m .


density variation of $25 \mathrm{~kg} \mathrm{~m}^{-3}$ is still visible, altough very small, in the surface gravity ( $12 \mu \mathrm{Gal}$ ) for a 3800 m deep body but is invisible for a 7600 m deep body ( $3 \mu \mathrm{Gal}$ ) (Fig. $3.8 \mathrm{~b}, \mathrm{c}$ ). For a density variation of $25 \mathrm{~kg} \mathrm{~m}^{-3}$ in a $10 \mathrm{~km}^{3}$ magma body, the maximum corresponding gravity changes at $1900 \mathrm{~m}, 3800 \mathrm{~m}$ and 7600 m are 460 $\mu \mathrm{Gal}, 115 \mu \mathrm{Gal}$ and $30 \mu \mathrm{Gal}$ (Fig 3.9a, b,c). Thus, it is clear that for a larger magma body, changes in the density can be readily observed at the surface.

## Vertical Cylinder Model

The vertical cylinder model presented here is used to represent either a magma pipe or a large body of magma at shallow depth. For simplicity, pressure, temperature and density are consider to be constant throughout the cylinder. Many variations are possible in a magma pipe or large cylindrical body; there can be variations in the level of magma, in the radius of the pipe or body, in the density, or any combination of these parameters. Depending on the gravity anomaly observed at the surface, certain of these processes better reproduce the anomaly. For example, Rymer and Brown $(1984,1987)$ modelled gravity variations at Poás volcano, Costa Rica, and concluded that the best model was a density variation of magma in a pipe. They were also able to model gravity variations they observed at the crater stations by variation in the level of magma. However, this model did not fit the observed gravity variations on the flanks of the volcano. They also examined variations in the radius of the pipe but encounted the same problem. There are several possible ways to model gravity variation close to the surface, but the most efficient way is by varying the density. Gravity variations can be large and rapid as seen at Masaya volcano, Nicaragua (Chapter 2).

Figure 3.10a shows gravity changes produced by variation of magma level in a cylinder of 50 m radius, 100 m initial depth and 100 long with a density contrast of $300 \mathrm{~kg} \mathrm{~m}^{-3}$. At this depth, the gravity changes are small even for large changes in the level (a 10 m change produces a gravity variation of less than $10 \mu \mathrm{Gal}$ ). For a deeper body of the same size, only very large variations in the level of magma would produce gravity changes. It is clear that for smaller bodies such as a thin magma pipe of less than 10 m radius, gravity changes would be difficult if not impossible to detect. For the same size of cylinder, variations in density are also not easy to observe since a density variation of $50 \mathrm{~kg} \mathrm{~m}^{-3}$ produces only a $13 \mu \mathrm{Gal}$ gravity change (Fig. 3.10b). Modelling of the Masaya volcano system with the cylinder model gives interesting results (Fig $3.10 \mathrm{c}, \mathrm{d}$ ). The values used for the model of size, emplacement and density contrast are from Rymer et al. (1998). Gravity changes caused by magma level variations can be observed at the surface if they are large (more than 20 m ) (Fig. 3.10c). Gravity changes at Masaya occur through the course of one day (Chapter II) and changes in the level of magma would have to be rapid and large to produce the observed temporal gravity anomaly. Seismic monitoring during the daily gravity surveys at Masaya did not show any changes in the seismicity and only rare isolated events (Chapter II). On the other hand, density variation can better explain the gravity variation observed at Masaya (Fig. 3.10d). Rymer et al. (1998) demonstrated that for a density change of $100 \mathrm{~kg} \mathrm{~m}^{-3}$, only a $5 \%$ increase in the degree of vesiculation would be necessary.

FIGURE 3.10
(a) Gravity change caused by lowering of the level of magma in a vertical cylinder of 50 m radius and 100 m length at a depth of 100 m with a density contrast of 300 kg $\mathrm{m}^{-3}$. (b) Gravity change of a vertical cylinder of 50 m radius, 100 m deep and 100 m long caused by variation in density of the magma. (c) Gravity change caused by lowering the level of magma for a vertical cylinder of 220 m radius and 100 m length at a depth of 300 m with a density contrast of $300 \mathrm{~kg} \mathrm{~m}^{-3}$. (d) Gravity change of a vertical cylinder of 220 m radius, 300 m deep and 100 m length caused by variation in density of the magma.


## Cubic Shape Chamber

This model is used to show variations of gravity caused by fractionation in a magma chamber where there is creation of a thin low-density layer ( $10-100 \mathrm{~m}$ thick) at the top of the magma chamber or to show accumulation of gas bubbles at the top of a magma chamber such as at Kilauea (Verginolle and Jaupart, 1990). In the case of gravity changes from crystallization in a magma chamber, the model of Blake (1984) is used. Two examples are presented based on his model. First, a magma chamber 4 km thick and 4 km wide at a depth of 2 km is shown (Fig. 3.11a). A thin layer of about 100 m , which represent $2.5 \%$ of the thickness of the chamber, is formed by accumulation of gas at the top of the magma chamber. If the initial $\mathrm{H}_{2} \mathrm{O}$ content is 3 wt $\%$, which is near saturation at the top of the magma chamber, a maximum gravity decrease of $105 \mu \mathrm{Gal}$ would be observed after only $6 \%$ crystallisation. For a larger amount of crystallisation (i.e., $12 \%$ ), the maximum corresponding gravity change would be about $210 \mu \mathrm{Gal}$ (Fig. 3.11b). These gravity changes result from density changes of $70 \mathrm{~kg} \mathrm{~m}^{-3}$ and $140 \mathrm{~kg} \mathrm{~m}^{-3}$, corresponding to overpressure of 5 MPa and 10 MPa , respectively. The crystallization required for the overpressure were inferred from Tait et al. (1989). The modelling was also done for a deeper magma chamber (depth of 6 km ). At this depth, less crystallization is required to produce the $5-10$ MPa overpressure. Thus the density changes are smaller because less gas is produced, and it is more dense at higher pressure $(\sim 200 \mathrm{MPa})$. At this depth $(6000$ m ), the gravity variations are very small and probably cannot be detected at the surface since crystallization can be a long process. Examples of density decreases of $30 \mathrm{~kg} \mathrm{~m}^{-3}$ and $55 \mathrm{~kg} \mathrm{~m}^{-3}$ are shown in Figure 3.12a and 3.12b; the gravity changes

## FIGURE 3.11

Gravity change produced in the top layer ( 100 m thick) of a magma chamber at a depth of 2000 m by a density decrease of (a) $70 \mathrm{~kg} \mathrm{~m}^{-3}$ and (b) $140 \mathrm{~kg} \mathrm{~m}^{-3}$.


FIGURE 3.12
Gravity change produced in the top layer ( 100 m thick) of a magma chamber at a depth of 6000 m by a density decrease of (a) $30 \mathrm{~kg} \mathrm{~m}^{-3}$ and (b) $55 \mathrm{~kg} \mathrm{~m}^{-3}$.

caused by these density changes are 8 and $15 \mu \mathrm{Gal}$, respectively. The density changes at this depth correspond to crystallization of 17.5 and $31 \%$ for a near $\mathrm{H}_{2} \mathrm{O}$ saturated rhyolitic magma, which are much higher than that needed to attain overpressure suggested by Tait et al. (1989) (around 3-6\%). For the third example, modelling by Vergniolle and Jaupart (1990) and Vergniolle (1996) on foam accumulations at Kilauea is used. The effect of accumulation of a 1 m thick foam with $70 \%$ bubbles at the top of the subchamber of Kilauea is shown in Figure 3.13. The gravity variation produced by this thin slab of density contrast equal to about $1800 \mathrm{~kg} \mathrm{~m}^{-3}$ is around $17 \mu \mathrm{Gal}$.

These examples show that gravity variation induced by gas in a magma chamber can be observed at the surface under certain conditions. By comparison, microgravity surveys were conducted in the Kuparuk River oil field (Alaska, United States) to observe gravity changes caused by gas movement and gas displacing oil. At a depth of 1848 m , a horizontal slab of 15.2 m thick, with horizontal dimensions of 4900 m by 4900 m , created a negative anomaly of $26 \mu \mathrm{Gal}$, and inferred density variations were on the order of $47 \mathrm{~kg} \mathrm{~m}^{-3}$ (Brady et al, 1996).

## Discussion

The modelling examples presented above do not fully reproduce real systems, but they can be used to study a particular aspect of dynamic volcanic systems. The simplicity of the models permit manipulations of different models and situations. The first goal was to model gravity variation caused by accumulation of gas in a magma chamber. Swelling of volcanic edifices is often observed, which is evidence for

FIGURE 3.13
Gravity change produced by a foam accumulation at the top of a 2000 m deep basaltic magma chamber.

overpressure in a magmatic system or upward movement of magma. The gravity change produced by a fractional volume change of 0.001 is less than $10 \mu \mathrm{Gal}$ for a 10 $\mathrm{km}^{3}$ magma chamber at a depth of 1900 m . In comparison, to account for this volume change, an overpressure of $5-10 \mathrm{MPa}$ is necessary, which corresponds crystallization of 5-10 \%. The gravity change produced over the same magma chamber would be $1200 \mu \mathrm{Gal}$. The overpressure is generated by the presence of free volatiles at the top of the magma chamber (Blake, 1984; Druitt and Sparks, 1984; Tait et al, 1989; Bower and Woods, 1997). This eventually lead to a decrease in the density of the magma at the top. However, there is a certain point beyond which the gas cannot accumulate, since the overpressure exceeds the fracture criterion (Tait et al, 1989). Thus, the density of the gas-melt mixture at the top of the chamber reaches a certain limit depending on the depth of the chamber. Rymer (1994) has demonstrated the possibility of monitoring rhyolitic volcanoes and calderas in order to anticipate a major buildup of pressure. The examples above prove that it is possible to observe gravity variations for shallow to relatively deep systems (2-4 km). For the Kilauea example, the small gravity variations ( $17 \mu \mathrm{Gal}$ ) are more difficult to monitor because other processes are also occurring, such as fracture opening and magma chamber replenishment. Unless frequent or even continuous measurements are made, it is difficult to recognise eruption precursors using microgravity (Rymer, 1989; Rymer, 1994).

For gravity variations above a magma chamber, crystallization is one process that could produce an overall increase in the dissolved volatile content in the magma. Density changes produced in volatile undersaturated conditions could possibly lead to
gravity changes large enough to be observed at the surface. On the other hand, it is unlikely for deep systems because the density contrasts are not large enough to produce any appreciable gravity changes. As examples, an overall increase of 0.5 wt $\% \mathrm{H}_{2} \mathrm{O}$ in a rhyolitic magma chamber of $10 \mathrm{~km}^{3}$ at a depth of 7600 m would cause a gravity decrease of $90 \mu \mathrm{Gal}$, and an increase of $0.2 \mathrm{wt} \% \mathrm{H}_{2} \mathrm{O}$ in a rhyolitic magma chamber of $1 \mathrm{~km}^{3}$ at a depth of 1900 m would produce a $19 \mu \mathrm{Gal}$ gravity decrease. It is clear that for very large magma chambers $\left(10-100 \mathrm{~km}^{3}\right)$, variation in the dissolved volatile content can be observed at the surface. Thus, the quantity of dissolved volatiles needed to produce a large gravity variation is reduced. Increase in dissolve coponents can occur by crystallization, convection or an input of new magma. It is not clear, however, how long it would take for such a dissolved volatile variation to occur. Tait et al. (1989) calculated that the rates of respose times, hence typical times for fractional crystallization, for dacitic to rhyolitic melt are around 400-850 years.

Many possible mechanisms may serve to change the exsolved volatile content in a magmatic system. New magma injected into a chamber may be rich in gas than the existing magma. Convection in a magma system can move saturated magma upward to depthswhere it becomes oversaturated. This is what appears to have happened at Masaya between 1993 and 1994 (Rymer et al., 1998). Rapid crystallization and volatile saturation and exsolution could occur within a magma layer (Pyle and Pyle, 1995). At shallow levels in open-systems, gravity variations also could be produced in magma pipes where there is constant degassing and variation in the bubble content in the magma, such as at Masaya.

## Conclusions

Gravity variations induced by a variation in the volume fraction of free gas in magma are observable at the surface for a shallow magma chamber, for a subchamber or even in large magma pipes of volcanic edifices. In many cases, volatiles are responsible for overpressure and eruptions. Although density variations produced by exsolution of volatiles are most important, even variations in the dissolved volatile content can be observe at the surface. Although many volatiles are present in magma, $\mathrm{H}_{2} \mathrm{O}$ is the most important for varying the density and hence, the gravity. The possibility of monitoring the exsolution of volatiles in a magma chamber, beneath a dome or in the conduit under a volcanic edifice, could enhance our means of forecasting eruptions. Volatiles are not the only cause of gravity variations in volcanic systems; there also could be intrusion of new magma, hydrothermal activity, gas loss and fracture opening and closing. Yet, volatiles are an important aspect of volcano instability, and it is important to understand and monitor their evolution. Microgravity techniques have improved significantly in the past few years. Changes in gravity linked to vesiculation in the magma have already been observed by Rymer and Brown (1984) at Poás volcano and by Beaulieu et al. $(1997,1998)$ at Masaya volcano. Microgravity monitoring is also being used at oil fields to observe the movement of gas pockets at depths of 1500-3000 m (Brady et al., 1996).

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## References

Beaulieu, A., Stix, J., St-Amand, K., Rymer, H., Gaonac'h, H., and Lovejoy, S., 1997. Gravity studies at Telica and Masaya volcanoes, Nicaragua. AGU Fall Meeting, San Francisco, p. 797.

Beaulieu, A., Williams-Jones, G., St-Amand, K., Stix, J., and Rymer, H, 1998. Gravity studies at Telica and Masaya volcanoes, Nicaragua. AGU Fall Meeting, San Francisco, in press.

Blake, S., 1981. Volcanism and the dynamics of open magma chambers. Nature, 289: 783-785.

Blake, S., 1984. Volatile oversaturation during the evolution of silicic magma chamber as an eruption trigger. J. Geophys. Res., 89: 8237-8244.

Bottinga, Y., Weill, D., and Richet, P., 1982. Density calculations for silicate liquids. I. Revised method for aluminosilicate compositions. Geochim. Cosmochim. Acta, 46: 909-919.

Bower, S. M. and Woods, A. W., 1997. Control of magma volatile content and chamber depth on the mass erupted during explosive volcanic eruptions. J. Geophys. Res., 102: 10273-10290.

Brady, J. L., Wolcott, D. S., and Aiken, C. L. V., 1996. Gravity methods: useful techniques for reservoir surveillance. The Log Analyst July-August 1996: 4556.

Dixon, J. E., Stolper, E. M., and Holloway, J. R., 1995. An experimental study of water and carbon dioxide solubilities in Mid-Ocean Ridge basaltic liquids. J. Petrol. 36: 1607-1631.

Dixon, J. E., and Stolper, E. M., 1995. An experimental study of water and carbon dioxide solubilities in Mid-Ocean Ridge basaltic liquids. Part II: application to degassing. J. Petrol. 36: 1633-1646.

Druitt, T. H., and Sparks, R. S. J., 1984. On the formation of calderas during ignimbrite eruptions. Nature, 310: 679-681.

Gerlach, T. M., 1986. Exsolution of $\mathrm{H}_{2} \mathrm{O}, \mathrm{CO}_{2}$, and S during eruptive episodes at Kilauea volcano, Hawaii. J. Geophys. Res., 91: 12177-12185.

Harris, D. M., and Anderson, A. T., 1984. Volatiles $\mathrm{H}_{2} \mathrm{O}, \mathrm{CO}_{2}, \mathrm{Cl}$ in a subduction related basalt. Contrib. Mineral. Petrol., 87: 120-128.

Holloway, J. R., and Blank, J. G., 1994. Application of experimental results to C-O-H species in natural melts. In Carroll, M., and Holloway, J. R., eds., Volatiles
in Magmas: Mineralogical Society of America Reviews in Mineralogy, 30: 187-230.

Kazahaya, K., Shinohara, H., and Saito, G., 1994. Excessive degassing of IzuOshima volcano: magma convection in a conduit. Bull. Volcanol., 56: 207-216.

Lange, R. A., and Carmichael, I. S. E., 1990. Thermodynamic properties of silicate liquids with emphasis on density, thermal expansion and compressibility. In: Nicholls, J., and Russell, J. K., eds., Modern Methods of Igneous Petrology: Mineralogical Society of America Reviews in Mineralogy, 24: 44-54.

Lange, R. A., 1994. The effect of $\mathrm{H}_{2} \mathrm{O}, \mathrm{CO}_{2}$ and F on the density and viscosity of silicate melts. In Carroll, M., and Holloway, J. R., eds., Volatiles in Magmas: Mineralogical Society of America Reviews in Mineralogy, 30: 331-369.

Lowenstern, J. B., 1994. Dissolved volatile concentrations in an ore-forming magma. Geology, 22: 893-896.

Papale, P., 1997. Modeling of the solubility of a one-component $\mathrm{H}_{2} \mathrm{O}$ or $\mathrm{CO}_{2}$ fluid in silicate liquids. Contrib. Mineral. Petrol., 126: 237-251.

Pedley, R. C., Busdy, J. P., and Dabek, Z. K., 1993. GRAVMAG user manual: Interactive 2.5 D gravity and magnetic modelling. British Geological Survey, Natural Environment Research Council, Technical Report WK/93/26/R, 73 pp.

Pyle, D. M., and Pyle, D. L., 1995. Bubble migration and the initiation of volcanic eruptions. J. Volcanol. Geotherm. Res., 67: 227-232.

Rymer H., and Brown, G. C., 1984. Periodic gravity changes at Poás volcano, Costa Rica. Nature, 311: 243-245.

Rymer H., and Brown, G. C., 1987. Causes of microgravity changes at Poás volcano, Costa Rica: an active but non-eruptive system. Nature, 311: 243-245.

Rymer, H., 1994. Microgravity changes as a precursor to volcanic activity. J. Volcanol. Geotherm. Res., 61: 311-328.

Rymer, H., 1989. A contribution to precision microgravity data analysis using Lacoste and Romberg gravity meters. Geophys. J., 97: 311-322.

Rymer, H., van Wyk de Vries, B., Stix, J., and Williams-Jones, G., 1998. Pit crater structure and processes governing persistent activity at Masaya volcano, Nicaragua. Bull. Volcanol., 59: 345-355.

Shaw, H. R., 1963. Obsidian- $\mathrm{H}_{2} \mathrm{O}$ viscosities at 1000 and 2000 bars in the temperature range 700 to $900^{\circ} \mathrm{C}$. J. Geophys. Res., 68: 6337-6343.

Sisson, T. W., and Layne, G. D., 1993. $\mathrm{H}_{2} \mathrm{O}$ in basalt and basaltic andesite glass inclusions from four subduction-related volcanoes. Earth Planet. Sci. Lett., 117: 619-635.

Stix, J., and Layne, G. D., 1996. Gas saturation and evolution of volatile and light lithophile elements in the Bandelier magma chamber between two calderaforming eruptions. J. Geophys. Res., 101: 25 181-25 196.

Stolper, E., 1992. Water in silicate glasses: an infrared spectroscopic study. Contrib. Mineral. Petrol., 81: 1-17.

Stolper, E., and Holloway, J. R., 1988. Experimental determination of the solubility of carbon dioxide in molten basalt at low pressure. Earth Planet. Sci. Lett., 87: 397-408.

Tait, S., Jaupart, C., and Vergniolle, S., 1989. Pressure, gas content and eruption periodicity of a shallow, crystallising magma chamber. Earth Planet. Sci. Lett., 92: 107-123.

Telford, W. M., Geldart, L. P., Sheriff, R. E., 1990. Applied Geophysics: Second edition. Cambridge, Cambridge Univerity Press. 770 pp.

Vergniolle, S., and Jaupart, C., 1990. Dynamic degassing at Kilauea volcano, Hawaii. J. Geophys. Res., 95: 2793-2809.

Vergniolle, S., 1996. Bubble size distribution in magma chambers and dynamics of basaltic eruptions. Earth Planet. Sci. Lett., 140: 269-279.

Wallace, P. J., and Gerlach, M., 1994. Magmatic vapor source for sulfur dioxide released during volcanic eruptions: evidence from Mount Pinatubo. Science, 265: 497-499.

## Conclusions

## General conclusions

Gravity is a useful tool to better define and understand the internal structure of volcanoes and to monitor their activity. Noteworthy conclusions of this work are the following:

1. The gravity survey at Telica volcano revealed the presence of a large shallow intrusion of dimensions $2 \mathrm{~km} \times 2 \mathrm{~km} \times 6 \mathrm{~km}$ at a depth of about 1 km , with a density contrast of $400-600 \mathrm{~kg} \mathrm{~m}^{-3}$ and trending nearly north-south.
2. Regional structures and the geomorphology of Telica, such as the north-northwest-south-southeast trending faults and the north-west crater alignment, are parallele to the shallow intrusion.
3. Gravity variations observed at Masaya volcano are linked to fluctuations in the quantity of bubbles in the magma beneath Santiago crater.
4. A possible link of earth tides to short-time gravity variations was observed at one station (A7) near the active crater at Masaya; tides appear to affect the ability of the magma to release gas by vesiculation.
5. If gravity varies with density due to fluctuation in the vesicularity of the magma, there is probably a direct or indirect link with $\mathrm{SO}_{2}$ flux at Masaya. This effect was not observed in 1997, but a possible positive correlation was observed on 13 March 1998.
6. Rymer et al.'s (1998) model of convective overturn of magma driven by density changes for renewal of degassing activity at Masaya is confirmed by this work.
7. Theoretical modelling shows that gravity chamges induced by a variation in the volume fraction of free gas in magma are observable at the surface for a shallow magma chamber.
8. Density variations produced by exsolution of volatiles from a magma are larger than density changes due to variations in the quantity of dissolved volatiles in a magma chamber. Nevertheless, changes in the amounts of dissolved species are also able to be seen gravimetrically on the surface if the changes are sufficiently shallow.
9. Although many volatiles are present in the magma, $\mathrm{H}_{2} \mathrm{O}$ is the most important for causing variations in the density and hence the gravity.

## Recommendations for Future Work

For Telica volcano, the gravity maps produced by this study do not cover a sufficiently large area to define the anomaly with confidence. A longer profile (50 km ) would be useful to verify the depth and size of the anomaly. Some stations should be added where there is a lack of points, such as in the southwestern, northeastern and northwestern parts of the volcano. The southeastern part of the volcano was not covered much because of the steepness of the cone and the presence of vegetation. With knowledge of the location of a shallow intrusion at Telica, microgravity stations could be better situated to monitor temporal changes.

At Masaya volcano, the daily gravity monitoring should be done for longer periods of time to better define the relation between the tides and the changes in density in the magma chamber. If this relation holds, monitoring on longer
timescales (e.g., years) would be difficult to interpret. The continuous gravity monitoring should be done at other stations in the caldera and beyond. Monitoring of $\mathrm{SO}_{2}$ flux and gravity should be done simultaneously to better define a correlation, if one exists.

For the modelling of the magma density with dissolved and exsolved volatiles, a better way to calculate the density of the melt under undersaturated condition should be used, such as the MELTS program. The solubility laws for $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$ in rhyolite and basalt should be verified, since new experimental data are being produced on a fairly regular basis.

## Appendix A

Telica gravity and GPS data

## Table A1

1997 and 1998 raw gravity data at Telica volcano

| Station | $G_{\text {raw }}{ }^{*}$ | Hour (GMT) | date |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| 1997 data |  |  |  |
| base1 | 1700.094 | $16: 29$ | $20-03-97$ |
| tel01 | 1693.636 | $17: 59$ | $20-03-97$ |
| tel02 | 1658.969 | $18: 30$ | $20-03-97$ |
| tel03 | 1664.052 | $19: 25$ | $20-03-97$ |
| tel04 | 1664.122 | $20: 20$ | $20-03-97$ |
| tel05 | 1661.416 | $20: 42$ | $20-03-97$ |
| tel06 | 1654.692 | $21: 09$ | $20-03-97$ |
| basel(fin) | 1700.308 | $22: 00$ | $20-03-97$ |
|  |  |  |  |
| basel | 1700.078 | $15: 37$ | $21-03-97$ |
| tel07 | 1697.412 | $16: 02$ | $21-03-97$ |
| tel08 | 1693.700 | $16: 16$ | $21-03-97$ |
| tel09 | 1688.324 | $16: 35$ | $21-03-97$ |
| tel10 | 1680.789 | $16: 58$ | $21-03-97$ |
| tel11 | 1672.407 | $17: 26$ | $21-03-97$ |
| tel12 | 1662.583 | $18: 30$ | $21-03-97$ |
| tel13 | 1666.138 | $18: 51$ | $21-03-97$ |
| tel14 | 1669.735 | $19: 14$ | $21-03-97$ |
| tel15 | 1671.756 | $19: 41$ | $21-03-97$ |
| tel16 | 1672.109 | $20: 06$ | $21-03-97$ |
| tel17 | 1673.509 | $20: 25$ | $21-03-97$ |
| tel18 | 1673.137 | $20: 47$ | $21-03-97$ |
| tel19 | 1672.285 | $21: 08$ | $21-03-97$ |
| tel20 | 1671.368 | $21: 30$ | $21-03-97$ |
| basel(fin) | 1700.288 | $22: 17$ | $21-03-97$ |
|  |  |  |  |
| base1 | 1700.042 | $16: 44$ | $22-03-97$ |
| tel21 | 1692.442 | $17: 00$ | $22-03-97$ |
| tel22 | 1693.522 | $17: 23$ | $22-03-97$ |
| tel23 | 1696.877 | $17: 47$ | $22-03-97$ |
| tel24 | 1696.742 | $18: 33$ | $22-03-97$ |
| tel25 | 1697.269 | $18: 56$ | $22-03-97$ |
| tel26 | 1690.455 | $19: 27$ | $22-03-97$ |
| tel27 | 1688.690 | $20: 01$ | $22-03-97$ |
| tel28 | 1687.531 | $20: 17$ | $22-03-97$ |
|  |  |  |  |


| tel29 | 1685.885 | $20: 32$ | $22-03-97$ |
| :---: | :---: | :---: | :---: |
| tel30 | 1685.207 | $20: 52$ | $22-03-97$ |
| tel31 | 1683.101 | $21: 06$ | $22-03-97$ |
| tel32 | 1679.529 | $21: 17$ | $22-03-97$ |
| basel(fin) | 1700.251 | $22: 04$ | $22-03-97$ |
|  |  |  |  |
| basel | 1700.127 | $15: 46$ | $01-04-97$ |
| tel47 | 1656.755 | $17: 03$ | $01-04-97$ |
| tel48 | 1658.137 | $17: 23$ | $01-04-97$ |
| tel49 | 1660.449 | $17: 42$ | $01-04-97$ |
| tel50 | 1660.299 | $18: 10$ | $01-04-97$ |
| tel51 | 1662.304 | $18: 28$ | $01-04-97$ |
| tel52 | 1663.577 | $18: 46$ | $01-04-97$ |
| tel53 | 1664.708 | $19: 08$ | $01-04-97$ |
| tel54 | 1665.748 | $19: 26$ | $01-04-97$ |
| tel55 | 1665.096 | $19: 44$ | $01-04-97$ |
| tel56 | 1667.845 | $20: 08$ | $01-04-97$ |
| tel57 | 1668.277 | $20: 33$ | $01-04-97$ |
| tel58 | 1669.270 | $20: 49$ | $01-04-97$ |
| tel59 | 1666.050 | $21: 08$ | $01-04-97$ |
| basel(fin) | 1700.131 | $22: 07$ | $01-04-97$ |
|  |  |  |  |
| basel | 1700.082 | $15: 29$ | $02-04-97$ |
| tel60 | 1667.917 | $16: 14$ | $02-04-97$ |
| tel61 | 1669.587 | $16: 33$ | $02-04-97$ |
| tel62 | 1669.128 | $16: 55$ | $02-04-97$ |
| tel63 | 1668.224 | $17: 13$ | $02-04-97$ |
| tel64 | 1666.003 | $17: 35$ | $02-04-97$ |
| tel65 | 1665.764 | $17: 53$ | $02-04-97$ |
| tel66 | 1668.088 | $18: 28$ | $02-04-97$ |
| tel67 | 1668.429 | $18: 44$ | $02-04-97$ |
| tel68 | 1672.821 | $19: 18$ | $02-04-97$ |
| tel69 | 1672.242 | $19: 27$ | $02-04-97$ |
| tel70 | 1673.639 | $19: 43$ | $02-04-97$ |
| tel71 | 1673.866 | $20: 03$ | $02-04-97$ |
| tel72 | 1673.609 | $20: 24$ | $02-04-97$ |
| tel73 | 1671.361 | $20: 39$ | $02-04-97$ |
| tel74 | 1671.196 | $20: 55$ | $02-04-97$ |
| tel75 | 1674.510 | $21: 15$ | $02-04-97$ |
| tel76 | 1675.439 | $21: 35$ | $02-04-97$ |
| basel(fin) | 1700.169 | $22: 15$ | $02-04-97$ |
|  |  |  |  |
| basel | 1699.967 | $15: 29$ | $05-04-97$ |


| tel110 | 1696.105 | $16: 05$ | $05-04-97$ |
| :---: | :---: | :---: | :---: |
| tel111 | 1692.218 | $16: 20$ | $05-04-97$ |
| tell12 | 1688.316 | $16: 35$ | $05-04-97$ |
| tel113 | 1682.795 | $16: 52$ | $05-04-97$ |
| tel114 | 1669.909 | $17: 15$ | $05-04-97$ |
| tel115 | 1670.037 | $17: 37$ | $05-04-97$ |
| tel116 | 1669.687 | $18: 16$ | $05-04-97$ |
| tell17 | 1670.869 | $18: 30$ | $05-04-97$ |
| tel118 | 1684.221 | $18: 51$ | $05-04-97$ |
| tel119 | 1683.147 | $19: 07$ | $05-04-97$ |
| tel120 | 1678.173 | $19: 26$ | $05-04-97$ |
| tel121 | 1689.142 | $19: 45$ | $05-04-97$ |
| tel122 | 1697.574 | $20: 14$ | $05-04-97$ |
| basel(fin) | 1700.167 | $21: 13$ | $05-04-97$ |
|  |  |  |  |
| basel | 1699.973 | $15: 29$ | $06-04-97$ |
| tel123 | 1660.680 | $16: 42$ | $06-04-97$ |
| tel124 | 1661.779 | $16: 59$ | $06-04-97$ |
| tell25 | 1662.928 | $17: 20$ | $06-04-97$ |
| tel125 | 1662.881 | $17: 45$ | $06-04-97$ |
| tel126 | 1662.906 | $18: 14$ | $06-04-97$ |
| tell27 | 1663.783 | $18: 28$ | $06-04-97$ |
| tell28 | 1665.388 | $18: 47$ | $06-04-97$ |
| tell29 | 1668.675 | $19: 06$ | $06-04-97$ |
| tel130 | 1671.925 | $19: 28$ | $06-04-97$ |
| tel131 | 1674.792 | $19: 47$ | $06-04-97$ |
| tel132 | 1672.632 | $20: 08$ | $06-04-97$ |
| tel133 | 1670.800 | $20: 26$ | $06-04-97$ |
| tel134 | 1671.243 | $20: 39$ | $06-04-97$ |
| basel(fin) | 1700.140 | $22: 06$ | $06-04-97$ |
|  |  |  |  |
| basel | 1700.024 | $15: 46$ | $08-04-97$ |
| tel151 | 1676.958 | $17: 05$ | $08-04-97$ |
| tel152 | 1680.138 | $17: 15$ | $08-04-97$ |
| tel153 | 1682.481 | $17: 33$ | $08-04-97$ |
| tel154 | 1681.667 | $17: 48$ | $08-04-97$ |
| tel155 | 1681.667 | $17: 48$ | $08-04-97$ |
| tel156 | 1681.844 | $18: 00$ | $08-04-97$ |
| tel157 | 1682.380 | $18: 33$ | $08-04-97$ |
| tel158 | 1675.879 | $18: 48$ | $08-04-97$ |
| tel159 | 1678.578 | $19: 06$ | $08-04-97$ |
| tel160 | 1670.312 | $19: 55$ | $08-04-97$ |
| tel161 | 1673.510 | $20: 11$ | $08-04-97$ |


| tel162 | 1674.018 | $20: 25$ | $08-04-97$ |
| :---: | :---: | :---: | :---: |
| basel(fin) | 1700.009 | $21: 35$ | $08-04-97$ |
|  |  |  |  |
| BASE2 |  |  |  |
| station |  |  |  |
|  |  |  | $23-03-97$ |
| base2 | 1716.159 | $19: 43$ | $23-03-97$ |
| tel34 | 1718.594 | $19: 58$ | $23-03-97$ |
| tel35 | 1720.559 | $20: 21$ | $23-03-97$ |
| tel36 | 1722.902 | $20: 39$ | $23-03-97$ |
| tel37 | 1724.786 | $21: 01$ | $23-03-97$ |
| tel38 | 1726.533 | $21: 25$ | $23-03-97$ |
| base2(fin) | 1716.341 | $21: 57$ |  |
|  |  |  |  |
| base2 | 1716.252 | $15: 35$ | $27-03-97$ |
| tel39 | 1713.842 | $15: 48$ | $27-03-97$ |
| tel40 | 1710.191 | $16: 06$ | $27-03-97$ |
| tel41 | 1706.452 | $16: 25$ | $27-03-97$ |
| tel42 | 1702.976 | $16: 42$ | $27-03-97$ |
| tel43 | 1696.187 | $17: 23$ | $27-03-97$ |
| tel44 | 1692.820 | $17: 49$ | $27-03-97$ |
| tel45 | 1688.274 | $18: 07$ | $27-03-97$ |
| tel46 | 1681.271 | $18: 26$ | $27-03-97$ |
| base2(fin) | 1716.091 | $18: 59$ | $27-03-97$ |
|  |  |  |  |
| base2 | 1716.096 | $15: 10$ | $03-04-97$ |
| tel77 | 1715.914 | $15: 37$ | $03-04-97$ |
| tel78 | 1714.403 | $15: 55$ | $03-04-97$ |
| tel79 | 1711.448 | $16: 11$ | $03-04-97$ |
| tel80 | 1708.363 | $16: 30$ | $03-04-97$ |
| tel81 | 1705.131 | $16: 51$ | $03-04-97$ |
| tel82 | 1700.659 | $17: 07$ | $03-04-97$ |
| tel83 | 1694.353 | $17: 24$ | $03-04-97$ |
| tel84 | 1685.111 | $17: 42$ | $03-04-97$ |
| tel85 | 1682.338 | $18: 17$ | $03-04-97$ |
| tel86 | 1690.102 | $18: 35$ | $03-04-97$ |
| tel87 | 1695.727 | $18: 55$ | $03-04-97$ |
| tel88 | 1701.455 | $19: 10$ | $03-04-97$ |
| tel89 | 1706.767 | $19: 32$ | $03-04-97$ |
| tel90 | 1710.032 | $19: 49$ | $03-04-97$ |
| tel91 | 1713.308 | $20: 10$ | $03-04-97$ |
| tel92 | 1715.729 | $20: 25$ | $03-04-97$ |
| tel93 | 1718.518 | $20: 40$ | $03-04-97$ |


| base2(fin) | 1716.280 | $21: 00$ | $03-04-97$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
| base2 | 1716.060 | $15: 13$ | $04-04-97$ |
| tel94 | 1714.117 | $15: 40$ | $04-04-97$ |
| tel95 | 1712.004 | $15: 59$ | $04-04-97$ |
| tel96 | 1706.902 | $16: 16$ | $04-04-97$ |
| tel97 | 1702.417 | $16: 47$ | $04-04-97$ |
| tel98 | 1696.730 | $17: 06$ | $04-04-97$ |
| tel99 | 1689.649 | $17: 26$ | $04-04-97$ |
| tel100 | 1682.663 | $17: 46$ | $04-04-97$ |
| tel101 | 1673.097 | $18: 36$ | $04-04-97$ |
| tel102 | 1671.659 | $18: 53$ | $04-04-97$ |
| tel103 | 1672.836 | $19: 14$ | $04-04-97$ |
| tel104 | 1669.733 | $19: 41$ | $04-04-97$ |
| tel105 | 1681.371 | $20: 04$ | $04-04-97$ |
| tel106 | 1688.202 | $20: 22$ | $04-04-97$ |
| tel107 | 1692.689 | $20: 39$ | $04-04-97$ |
| tel108 | 1696.652 | $21: 02$ | $04-04-97$ |
| tel109 | 1700.724 | $21: 20$ | $04-04-97$ |
| base2(fin) | 1716.277 | $21: 51$ | $04-04-97$ |
|  |  |  |  |
| base2 | 1716.052 | $15: 13$ | $07-04-97$ |
| tel135 | 1719.764 | $15: 35$ | $07-04-97$ |
| tel136 | 1717.574 | $15: 50$ | $07-04-97$ |
| tel137 | 1713.165 | $16: 03$ | $07-04-97$ |
| tel138 | 1709.058 | $16: 22$ | $07-04-97$ |
| tel139 | 1705.036 | $16: 39$ | $07-04-97$ |
| tel140 | 1700.439 | $16: 58$ | $07-04-97$ |
| tel141 | 1695.518 | $17: 12$ | $07-04-97$ |
| tel142 | 1688.738 | $17: 25$ | $07-04-97$ |
| tell43 | 1679.268 | $17: 45$ | $07-04-97$ |
| tel144 | 1678.489 | $18: 17$ | $07-04-97$ |
| tel145 | 1688.682 | $18: 33$ | $07-04-97$ |
| tel146 | 1695.051 | $18: 49$ | $07-04-97$ |
| tel147 | 1701.040 | $19: 07$ | $07-04-97$ |
| tel148 | 1706.634 | $19: 25$ | $07-04-97$ |
| tel149 | 1709.993 | $19: 43$ | $07-04-97$ |
| tel150 | 1718.535 | $20: 12$ | $07-04-97$ |
| base2(fin) | 1716.070 | $20: 35$ | $07-04-97$ |
|  |  |  |  |
| basefinal | 1739.562 | $15: 17$ | $10-04-97$ |
| basel | 1700.015 | $16: 25$ | $10-04-97$ |
| base2 | 1716.032 | $17: 17$ | $10-04-97$ |
|  |  |  |  |


| basefinal | 1739.427 | 18:19 | 10-04-97 |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  |  |  |  |
| 1998 data |  |  |  |
| base1 | 1696.234 | 16:45 | 02-02-98 |
| tel175 | 1697.632 | 17:26 | 02-02-98 |
| tel176 | 1697.874 | 17:47 | 02-02-98 |
| tel177 | 1703.121 | 18:08 | 02-02-98 |
| base1 | 1696.175 | 18:48 | 02-02-98 |
|  |  |  |  |
| base1 | 1696.226 | 15:02 | 03-02-98 |
| tel178 | 1708.796 | 15:39 | 03-02-98 |
| tel179 | 1712.193 | 15:58 | 03-02-98 |
| tel180 | 1714.848 | 16:25 | 03-02-98 |
| tel181 | 1730.580 | 16:50 | 03-02-98 |
| tel182 | 1734.869 | 17:15 | 03-02-98 |
| tel183 | 1743.189 | 17:40 | 03-02-98 |
| base1 | 1696.138 | 22:35 | 03-02-98 |
|  |  |  |  |
| base3 | 1790.854 | 14:05 | 04-02-98 |
| tel191 | 1789.234 | 14:27 | 04-02-98 |
| tel192 | 1786.615 | 14:45 | 04-02-98 |
| tel193 | 1785.628 | 15:05 | 04-02-98 |
| tel194 | 1783.536 | 15:25 | 04-02-98 |
| tel195 | 1780.116 | 15:42 | 04-02-98 |
| tel196 | 1775.516 | 16:07 | 04-02-98 |
| tel197 | 1770.582 | 16:25 | 04-02-98 |
| tel198 | 1768.676 | 16:43 | 04-02-98 |
| tel199 | 1763.323 | 17:05 | 04-02-98 |
| tel200 | 1758.571 | 17:31 | 04-02-98 |
| tel201 | 1749.420 | 18:22 | 04-02-98 |
| tel202 | 1742.476 | 18:42 | 04-02-98 |
| tel203 | 1739.040 | 19:02 | 04-02-98 |
| tel204 | 1740.312 | 19:22 | 04-02-98 |
| tel205 | 1739.106 | 19:47 | 04-02-98 |
| tel206 | 1732.659 | 20:26 | 04-02-98 |
| tel207 | 1725.054 | 20:43 | 04-02-98 |
| tel208 | 1720.396 | 21:01 | 04-02-98 |
| base2 | 1712:332 | 21:35 | 04-02-98 |
| bafinal | 1735.727 | 21:58 | 04-02-98 |
| base3 | 1790.828 | 22:35 | 04-02-98 |
|  |  |  |  |
| base1 | 1696.274 | 15:03 | 05-02-98 |


| tel209 | 1697.476 | $15: 46$ | $05-02-98$ |
| :---: | :---: | :---: | :---: |
| tel210 | 1702.905 | $16: 07$ | $05-02-98$ |
| tel211 | 1705.256 | $16: 32$ | $05-02-98$ |
| tel212 | 1704.810 | $16: 55$ | $05-02-98$ |
| tel213 | 1704.203 | $17: 15$ | $05-02-98$ |
| tel214 | 1702.480 | $17: 35$ | $05-02-98$ |
| tel215 | 1699.727 | $18: 35$ | $05-02-98$ |
| tel216 | 1692.611 | $19: 24$ | $05-02-98$ |
| tel217 | 1687.701 | $19: 46$ | $05-02-98$ |
| tel218 | 1687.254 | $20: 10$ | $05-02-98$ |
| tel219 | 1691.294 | $20: 35$ | $05-02-98$ |
| base1 | 1696.325 | $21: 50$ | $05-02-98$ |
|  |  |  |  |
| base2 | 1712.381 | $15: 06$ | $06-02-98$ |
| tel220 | 1720.468 | $15: 33$ | $06-02-98$ |
| tel221 | 1725.569 | $15: 56$ | $06-02-98$ |
| tel222 | 1726.158 | $16: 27$ | $06-02-98$ |
| tel223 | 1742.789 | $16: 54$ | $06-02-98$ |
| tel224 | 1750.223 | $17: 23$ | $06-02-98$ |
| tel225 | 1762.819 | $17: 55$ | $06-02-98$ |
| tel226 | 1763.116 | $18: 43$ | $06-02-98$ |
| tel227 | 1764.740 | $19: 07$ | $06-02-98$ |
| tel228 | 1770.301 | $19: 26$ | $06-02-98$ |
| tel229 | 1771.672 | $19: 44$ | $06-02-98$ |
| base2 | 1712.454 | $21: 55$ | $06-02-98$ |
|  |  |  |  |
| tel216 | 1692.573 | $14: 57$ | $07-02-98$ |
| tel230 | 1706.187 | $15: 36$ | $07-02-98$ |
| tel231 | 1716.644 | $16: 01$ | $07-02-98$ |
| tel232 | 1729.839 | $16: 24$ | $07-02-98$ |
| tel233 | 1741.050 | $16: 55$ | $07-02-98$ |
| tel234 | 1755.301 | $17: 21$ | $07-02-98$ |
| tel235 | 1759.926 | $18: 40$ | $07-02-98$ |
| tel236 | 1761.907 | $19: 07$ | $07-02-98$ |
| tel216 | 1692.703 | $21: 45$ | $07-02-98$ |
|  |  |  |  |
| bafinal | 1735.946 | $14: 40$ | $10-02-98$ |
| tel246 | 1715.989 | $16: 02$ | $10-02-98$ |
| tel247 | 1699.865 | $16: 33$ | $10-02-98$ |
| tel248 | 1691.321 | $17: 03$ | $10-02-98$ |
| tel249 | 1684.897 | $18: 20$ | $10-02-98$ |
| tel250 | 1700.448 | $19: 00$ | $10-02-98$ |
| tel251 | 1707.243 | $19: 35$ | $10-02-98$ |


| tel252 | 1714.336 | $20: 10$ | $10-02-98$ |
| :---: | :---: | :---: | :---: |
| tel253 | 1724.802 | $20: 35$ | $10-02-98$ |
| tel254 | 1731.585 | $20: 57$ | $10-02-98$ |
| bafinal | 1736.024 | $21: 35$ | $10-02-98$ |
|  |  |  |  |
| base1 | 1696.497 | $15: 12$ | $11-02-98$ |
| tel255 | 1677.139 | $16: 13$ | $11-02-98$ |
| tel256 | 1679.016 | $16: 41$ | $11-02-98$ |
| tel257 | 1659.611 | $17: 27$ | $11-02-98$ |
| tel258 | 1656.562 | $18: 25$ | $11-02-98$ |
| tel259 | 1667.429 | $18: 53$ | $11-02-98$ |
| tel260 | 1669.959 | $19: 23$ | $11-02-98$ |
| tel261 | 1673.672 | $19: 25$ | $11-02-98$ |
| base1 | 1696.509 | $21: 12$ | $11-02-98$ |

* After correction factor of Lacoste \& Romberg meter G-513
Table A2
Summary of the gravity corrections and the resulting gravity differences relative to Base3

$$
\mathrm{G}_{\text {obs }} \text { is the gravity measurements which are tides and drift corrected }
$$

$\Delta G_{\text {obs }}$ is the difference between the gravity measurements at a station and Base3
$\Delta G_{1}$ is the latitude correction
$\Delta \mathrm{G}_{\mathrm{FA}}$ is the altitude correction
$\Delta G_{g}$ is the Bouguer correction
ter (a-d) is the terrain correction for Sanberg sections A through D ( $0-171 \mathrm{~m}$ ) (a-d)* $\rho$ is the terrain correction for Sanberg sections A through D corrected for a chosen density ( $2.3-2.5 \mathrm{~g} / \mathrm{cm}^{3}$ ) ter (e-k) is the terrain correction for Hammer sections E through K (171-9900 m)
$(e-k)^{*} \rho$ is the terrain correction for Hammer sections $E$ through $K$ corrected for a chosen density ( $2.3-2.5 \mathrm{~g} / \mathrm{cm}^{3}$ )
$\Delta G$ is the sum of $\Delta G_{\text {obs }}$ and all the corrections reported to zero relative to Base3

| Station | $\mathrm{G}_{\text {obs }}$ | $\Delta \mathrm{G}_{\text {obs }}$ | $\triangle \mathrm{G}_{1}$ | $\Delta \mathrm{G}_{\mathrm{FA}}$ | $\Delta \mathrm{G}_{\mathrm{B}}$ | $\operatorname{ter}(\mathrm{a}-\mathrm{d}){ }^{(\mathrm{a}-\mathrm{d})^{\star} \mathrm{P}}$ |  | ter (e-k) | (e-k)* ${ }^{\text {a }}$ | $\Delta \mathrm{G}$ | Coordinates in UTM (WGS 84 datum) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | North (m) |  |  | East (m) | Alt (m) |
| Base3 | $179+821$ | 0 | 0.000 | 0.000 | 0.000 | 0.005 | 0.006 |  | 0.313 | 0.375 | 0.000 | 1392895 | 510694 | 222.9 |
| Basefina! | 1739.701 | -55.12 | -0.660 | 92.762 | -30.242 | 0.000 | 0.000 | 1.902 | 2.283 | 8.641 | 1394807 | 515700 | 523.5 |
| Base! | 1700.209 | -94.612 | -0.357 | 157.740 | -51.425 | 0.074 | 0.089 | 3.756 | 4.507 | 15.560 | 1393929 | 518193 | 734.0 |
| Base2 | 1716.273 | -78.548 | -0.651 | 132.933 | -43.338 | 0.070 | 0.084 | 2.827 | 3.392 | 13.490 | 1394780 | 517431 | 653.6 |
| tel01 | 1658.981 | -135.84 | -0.247 | 203.660 | -66.396 | 0.759 | 0.911 | 10.114 | 12.136 | 13.843 | 1393610 | 517965 | 882.8 |
| tcl02 | 1659.035 | -135.786 | -0.262 | 203.423 | -66.319 | 0.434 | 0.521 | 9.557 | 11.469 | 12.663 | 1393655 | 517885 | 882.1 |
| tel03 | 1664.077 | -130.744 | -0.278 | 198.120 | -64.590 | 0.248 | 0.298 | 7.830 | 9.395 | 11.820 | 1393700 | 517760 | 864.9 |
| tel04 | 1664.105 | -130.716 | -0.279 | 197.712 | -64.457 | 0.103 | 0.124 | 7.742 | 9.291 | 11.293 | 1393703 | 517666 | 863.6 |


| (ek) ${ }^{\text {a }}$ | 1661.385 | -133.436 | -0.279 | 2000018 | -6. 209 | 0.134 | 0.161 | 10.152 | 12.182 | 13.056 | 1393703 | 517583 | 871.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| tel106 | 1654.646 | -140.175 | -0.272 | 206.200 | -67.224 | 1.723 | 2.068 | 12.271 | 14.725 | 14.940 | 1393684 | 517485 | 891.1 |
| tel07 | 1697.554 | -97.267 | -0.326 | 161.382 | -52.613 | 0.017 | 0.020 | 3.892 | 4.670 | 15.485 | 1393839 | 518228 | 745.8 |
| tcl08 | 1693.846 | -100.975 | -0.296 | 165.927 | -54.095 | 0.167 | 0.200 | 4.185 | 5.023 | 15.403 | 1393752 | 518221 | 760.6 |
| tel09 | 1688.473 | -106.348 | -0.288 | 172.023 | -56.082 | 0.467 | 0.560 | 4.556 | 5.467 | 14.950 | 1393730 | 518173 | 780.3 |
| tel10 | 1680.939 | -113.882 | -0.280 | 180.512 | -58.849 | 0.693 | 0.832 | 5.455 | 6.547 | 14.497 | 1393705 | 518129 | 807.8 |
| tell1 | 1672.553 | -122.268 | -0.261 | 189.926 | -61.919 | 0.440 | 0.528 | 7.045 | 8.454 | 14.079 | 1393651 | 518087 | 838.3 |
| tel 12 | 1662.698 | -132.123 | -0.234 | 200.196 | -65.267 | 0.813 | 0.976 | 9.005 | 10.806 | 13.972 | 1393573 | 518032 | 871.6 |
| tel13 | 1666.237 | -128.584 | -0.217 | 196.721 | -64.134 | 0.541 | 0.649 | 8.085 | 9.702 | 13.756 | 1393525 | 518084 | 860.4 |
| tel 14 | 1669.817 | -125.004 | -0.191 | 193.009 | -62.924 | 0.486 | 0.583 | 7.260 | 8.712 | 13.805 | 1393448 | 518136 | 848.3 |
| tel15 | 1671.814 | -123.007 | -0.162 | 190.646 | -62.153 | 0.334 | 0.401 | 6.798 | 8.157 | 13.500 | 1393364 | 518164 | 840.7 |
| tel16 | 1672.144 | -122.677 | -0.132 | 189.734 | -61.856 | 0.559 | 0.671 | 6.481 | 7.778 | 13.136 | 1393276 | 518153 | 837.7 |
| tcl17 | 1673.503 | -121.318 | -0.104 | 187.762 | -61.213 | 0.299 | 0.359 | 6.150 | 7.380 | 12.485 | 1393196 | 518136 | 831.3 |
| tel18 | 1673.137 | -121.684 | -0.079 | 187.103 | -60.998 | 0.559 | 0.671 | 6.103 | 7.323 | 11.955 | 1393123 | 518093 | 829.2 |
| tel19 | 1672.266 | -122.555 | -0.055 | 186.678 | -60.860 | 0.374 | 0.449 | 6.048 | 7.258 | 10.534 | 1393053 | 518023 | 827.8 |
| tcl20 | 1671.338 | -123.483 | -0.043 | 186.692 | -60.864 | 0.582 | 0.698 | 6.750 | 8.100 | 10.719 | 1393018 | 517953 | 827.9 |
| tel21 | 1692.612 | -102.209 | -0.272 | 167.690 | -54.669 | 0.358 | 0.430 | 4.270 | 5.124 | 15.712 | 1393682 | 518275 | 766.3 |
| tel22 | 1693.693 | -101.128 | -0.24t | 166.529 | -54.291 | 0.567 | 0.680 | 4.166 | 4.999 | 16.164 | 1393601 | 518304 | 762.5 |
| tel23 | 1697.045 | -97.776 | -0.219 | 162.742 | -53.056 | 0.570 | 0.684 | 3.898 | 4.678 | 16.672 | 1393529 | 518375 | 750.2 |
| tcl24 | 1696.768 | -98.053 | -0.195 | 163.160 | -53.192 | 0.699 | 0.839 | 3.910 | 4.692 | 16.868 | 1393460 | 518392 | 751.6 |
| tel25 | 1697.401 | -97.42 | -0.172 | 162.616 | -53.015 | 1.006 | 1.207 | 3.895 | 4.674 | 17.509 | 1393392 | 518429 | 749.8 |
| tel26 | 1690.562 | -104.259 | -0.147 | 170.775 | -55.675 | 0.331 | 0.397 | 4.313 | 5.176 | 15.885 | 1393321 | 518399 | 776.3 |
| tel27 | 1688.766 | -106.055 | -0.173 | 172.802 | -56.336 | 0.331 | 0.397 | 4.462 | 5.355 | 15.609 | 1393395 | 518344 | 782.8 |
| tel28 | 1687.592 | -107.229 | -0.201 | 173.992 | -56.724 | 0.635 | 0.762 | 4.570 | 5.483 | 15.702 | 1393477 | 518303 | 786.7 |
| tci29 | 1685.932 | -108.889 | -0.227 | 175.670 | -57.271 | 0.693 | 0.832 | 4.784 | 5.741 | 15.475 | 1393553 | 518261 | 792.1 |
| tel30 | 1685.234 | -109.587 | -0.253 | $176.23+$ | -57.455 | 0.693 | 0.832 | 4.944 | 5.933 | 15.322 | 1393629 | 518222 | 794.0 |
| tel31 | 1683.115 | -111.706 | -0.274 | 178.278 | -58.121 | 0.693 | 0.832 | 5.173 | 6.208 | 14.835 | 1393688 | 518162 | 800.6 |


| tel 32 | 1679.533 | -115.288 | -0.247 | 182.471 | -59.488 | 0.693 | 0.832 | 5.730 | 6.876 | 14.774 | 1393609 | 518182 | 814.2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| tel34 | 1718.695 | -76.126 | -0.671 | 129.596 | -42.250 | 0.039 | 0.047 | 2.789 | 3.347 | 13.561 | 1394839 | 517372 | 642.8 |
| ${ }_{\text {Lel } 135}$ | 1720.632 | -74.189 | -0.695 | 126.958 | -41.390 | 0.039 | 0.047 | 2.725 | 3.270 | 13.619 | 1394909 | 517337 | 634.3 |
| tel36 | 1722.962 | -71.859 | -0.725 | 124.088 | -40.454 | 0.010 | 0.012 | 2.654 | 3.185 | 13.865 | 1394995 | 517337 | 625.0 |
| tel37 | 1724.824 | -69.997 | -0.750 | 121.407 | -39.580 | 0.028 | 0.034 | 2.741 | 3.289 | 14.020 | 1395068 | 517303 | 616.3 |
| tel38 | 1726.478 | -68.343 | -0.770 | 119.021 | -38.802 | 0.039 | 0.047 | 2.652 | 3.182 | 13.953 | 1395126 | 517244 | 608.6 |
| tel39 | 1713.872 | -80.949 | -0.624 | 135.444 | -44.157 | 0.027 | 0.032 | 2.934 | 3.520 | 12.886 | 1394701 | 517408 | 661.8 |
| (tel40 | 1710.238 | -84.583 | -0.589 | 140.089 | -45.671 | 0.066 | 0.079 | 4.120 | 4.944 | 13.887 | 1394602 | 517360 | 676.8 |
| tel41 | 1706.515 | -88.306 | -0.564 | 144.614 | -47.146 | 0.060 | 0.072 | 4.331 | 5.197 | 13.485 | 1394529 | 517333 | 691.5 |
| tel42 | 1703.055 | -91.766 | -0.539 | 148.587 | -48.441 | 0.078 | 0.094 | 5.463 | 6.555 | 14.108 | 1394456 | 517315 | 704.4 |
| tel43 | 1696.302 | -98.519 | -0.512 | 156.476 | -51.013 | 0.128 | 0.154 | 4.351 | 5.221 | 11.425 | 1394376 | 517286 | 729.9 |
| tel44 | 1692.955 | -101.866 | -0.486 | 160.438 | -52.305 | 0.178 | 0.214 | 4.737 | 5.684 | 11.297 | 1394303 | 517262 | 742.8 |
| tel45 | 1688.421 | -106.4 | -0.461 | 165.618 | -53.994 | 0.426 | 0.511 | 4.985 | 5.982 | 10.874 | 1394231 | 517244 | 759.6 |
| tel46 | 1681.429 | -113.392 | -0.434 | 173.432 | -56.541 | 0.898 | 1.078 | 5.937 | 7.124 | 10.885 | 1394152 | 517210 | 784.9 |
| tcl47 | 1656.806 | -138.015 | -0.295 | 203.761 | -66.429 | 2.534 | 3.041 | 11.238 | 13.485 | 15.166 | 1393750 | 517437 | 883.2 |
| tcl48 | 1658.182 | -136.639 | -0.318 | 202.832 | -66.126 | 2.223 | 2.668 | 10.426 | 12.511 | 14.546 | 1393816 | 517354 | 880.2 |
| tel49 | 1660.488 | -134.333 | -0.332 | 200.405 | -65.335 | 0.528 | 0.634 | 9.557 | 11.468 | 12.125 | 1393858 | 517278 | 872.3 |
| tel50 | 1660.331 | -134.49 | -0.342 | 199.956 | -65.188 | 0.862 | 1.034 | 9.431 | 11.317 | 11.905 | 1393886 | 517189 | 870.8 |
| tel51 | 1662.332 | -132.489 | -0.357 | 196.756 | -64.145 | 0.599 | 0.719 | 11.183 | 13.420 | 13.522 | 1393928 | 517101 | 860.5 |
| tel52 | 1663.604 | -131.217 | -0.367 | $19+314$ | -63.349 | 0.615 | 0.738 | 10.235 | 12.282 | 12.019 | 1393958 | 517018 | 852.5 |
| tel53 | 1664.733 | -130.088 | -0.369 | 191.668 | -62.487 | 0.618 | 0.742 | 9.953 | 11.944 | 11.029 | 1393964 | 516926 | 844.0 |
| tel54 | 1665.774 | -129.047 | -0.366 | 188.925 | -61.592 | 0.879 | 1.055 | 9.034 | 10.841 | 9.434 | 1393955 | 516842 | 835.1 |
| tel55 | 1665.124 | -129.697 | -0.350 | 188.010 | -61.294 | 1.071 | 1.285 | 8.117 | 9.740 | 7.313 | 1393909 | 516763 | 832.1 |
| tel56 | 1667.877 | -126.944 | -0.397 | 186.627 | -60.843 | 0.620 | 0.744 | 8.514 | 10.217 | 9.022 | 1394045 | 516912 | 827.6 |
| tel57 | 1668.316 | -126.505 | -0.404 | 187.611 | -61.164 | 0.593 | 0.712 | 8.410 | 10.092 | 9.961 | 1394066 | 517042 | 830.8 |
| tel58 | 1669.315 | -125.506 | -0.407 | 186.985 | -60.960 | 0.320 | 0.384 | 8.274 | 9.929 | 10.043 | 1394074 | 517121 | 828.8 |
| tel59 | 1666.102 | -128.719 | -0.388 | 191.921 | -62.569 | 0.341 | 0.409 | 9.579 | 11.495 | 11.768 | 1394018 | 517215 | 844.8 |


| $\left\|\begin{array}{c} 7 \\ 0 \\ 0 \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} m \\ \vdots \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\substack{4 \\ \infty}}{ } \end{array}\right\|$ | $\left\|\begin{array}{l} n \\ \vdots \\ \infty \\ \infty \end{array}\right\|$ | $\underset{\substack{+\underset{\infty}{2} \\ \hline}}{ }$ | $\stackrel{n}{\infty}$ | $\left\|\begin{array}{c} \hat{e} \\ \dot{子} \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ \dot{0} \\ \infty \end{array}\right\|$ | $\underset{\infty}{\infty}$ | $\left\|\begin{array}{c} \stackrel{\rightharpoonup}{\infty} \\ \stackrel{\infty}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{2}{\infty} \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{l} \infty \\ \infty \\ \infty \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} 7 \\ \dot{\omega} \\ \underset{\infty}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \infty \\ \infty \\ \infty \end{array}\right\|$ | $\mid$ | $\begin{array}{\|c\|} \hline \\ 0 \\ \dot{\infty} \\ \hline \end{array}$ | $\underset{\infty}{\sim}$ | $\stackrel{-}{3}$ |  | $\begin{aligned} & 0 \\ & \text { í } \\ & \hline 1 \end{aligned}$ | $\stackrel{\sim}{\underset{0}{0}}$ | $0$ | $\stackrel{m}{n}$ | $\left\lvert\, \begin{gathered} \infty \\ \stackrel{i}{\Sigma} \end{gathered}\right.$ | $\mathfrak{c}$ | $\stackrel{7}{\sim}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|\begin{array}{c} 9 \\ \underset{y}{c} \\ \hat{n} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ 0 \\ i \\ i n \\ i \end{array}\right\|$ | $\left\|\begin{array}{c} 9 \\ 0 \\ 0 \\ i n \end{array}\right\|$ | $\left\|\begin{array}{c} \mathbf{O} \\ \stackrel{0}{c} \\ \stackrel{i}{n} \end{array}\right\|$ | $\stackrel{2}{2}$ | $\begin{array}{\|c} \infty \\ \stackrel{\infty}{n} \\ \stackrel{n}{n} \end{array}$ | $\underbrace{2}_{0}$ | $\frac{\text { A }}{\hat{E}}$ | $\frac{0}{n}$ | $\left\|\begin{array}{c} \bar{n} \\ \stackrel{i}{n} \end{array}\right\|$ | $\left\|\begin{array}{l} \circ \\ \stackrel{\circ}{7} \\ \stackrel{n}{2} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} n \\ \infty \\ \end{gathered}\right.$ | $\left\|\begin{array}{l} \infty \\ \frac{\infty}{x} \\ \frac{1}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \mathbf{d} \\ \mathbf{D} \\ \end{array}\right\|$ | $\left\|\begin{array}{c} 2 \\ \vdots \\ i \\ i n \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ 0 \\ \stackrel{n}{n} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \hat{n} \\ \dot{\circ} \\ \dot{n} \\ \hline \end{gathered}\right.$ | $\begin{aligned} & \circ \\ & \stackrel{0}{2} \\ & i \end{aligned}$ | $\tilde{n}$ | 合 | $\begin{gathered} \vec{E} \\ \underset{n}{n} \end{gathered}$ | 寺 | 苦 |  | $\stackrel{2}{2}=\frac{7}{5}$ | 菏 |
| $\left[\begin{array}{c} n \\ c \\ 2 \\ m \\ m \end{array}\right]$ | $\begin{gathered} त \\ \tilde{n} \\ \underset{\sim}{n} \end{gathered}$ | $\left\|\begin{array}{c} \infty \\ \underset{\sim}{2} \\ \underset{\sim}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{2} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ \underset{\sim}{2} \\ \underset{\sim}{2} \end{array}\right\|$ |  | $2 \begin{gathered} i n \\ \underset{n}{n} \\ \underset{n}{n} \end{gathered}$ | $\stackrel{c}{2}$ | $\left\lvert\, \begin{gathered} \stackrel{\sim}{\mathrm{N}} \\ \stackrel{\sim}{2} \end{gathered}\right.$ | $\left.\begin{gathered} \overline{0} \\ \underset{\sim}{2} \\ \underset{\sim}{2} \end{gathered} \right\rvert\,$ | $\left\lvert\, \begin{gathered} o \\ c \\ \underset{\sim}{2} \\ \hline \end{gathered}\right.$ | $\begin{gathered} \underset{\sim}{\underset{\sim}{e}} \\ \stackrel{\rightharpoonup}{*} \end{gathered}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{2} \\ & \stackrel{\rightharpoonup}{2} \\ & \stackrel{e}{2} \end{aligned}$ | $\left\lvert\, \begin{gathered} \underset{O}{2} \\ \tilde{2} \\ \underset{\sim}{2} \end{gathered}\right.$ | $\left\|\begin{array}{c} \underset{\sim}{2} \\ \underset{\sim}{2} \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{c} \stackrel{0}{2} \\ \stackrel{\rightharpoonup}{2} \\ \hline \end{array}\right\|$ | $\begin{aligned} & \underset{\sim}{w} \\ & \text { N } \\ & \text { in } \end{aligned}$ | $\begin{aligned} & \frac{0}{0} \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ | ion | $\begin{aligned} & \text { 윰 } \\ & \text { in } \end{aligned}$ | $\begin{gathered} \underset{m}{n} \\ \vdots \\ \vdots \\ \hline \end{gathered}$ | 암 | $\begin{gathered} \stackrel{\rightharpoonup}{\tilde{j}} \\ \hline \end{gathered}$ | $\begin{gathered} \stackrel{c}{0} \\ \stackrel{y}{c} \\ \stackrel{y}{c} \end{gathered}$ |  | 年 |
| $\begin{gathered} \underset{\sim}{n} \\ \underset{\sim}{2} \end{gathered}$ | $\stackrel{ \pm}{=}$ | $\left\|\begin{array}{l} z \\ \underset{\sim}{0} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} 0 \\ 0 \\ 0 \end{gathered}\right.$ | $\stackrel{9}{7}$ | $\begin{gathered} \pm \\ n \\ \vdots \\ \sigma \end{gathered}$ | $\underset{\sim}{2}$ | $=\begin{gathered} 9 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ |  | $\stackrel{\substack{\infty \\=\\=}}{ }$ | $\stackrel{\infty}{9}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{2} \\ & \underset{\sim}{3} \end{aligned}$ | $\left\|\begin{array}{c} 0 \\ \\ \underset{y}{c} \end{array}\right\|$ | $\begin{aligned} & \pm \\ & \stackrel{5}{6} \\ & = \end{aligned}$ | $\begin{aligned} & \stackrel{\infty}{c} \\ & = \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline \infty \\ \stackrel{\infty}{\infty} \\ \underset{y}{2} \end{array}$ | $\left\lvert\, \begin{aligned} & \text { is } \\ & \text { and } \end{aligned}\right.$ | $\begin{array}{\|c\|} \hline \left.\begin{array}{c} \infty \\ \substack{4 \\ ~} \end{array} \right\rvert\, \end{array}$ | $\begin{gathered} \infty \\ \infty \\ \underset{\sim}{\infty} \\ \hline \end{gathered}$ | $\stackrel{i n}{n} \underset{\sim}{n} \mid$ | $\begin{aligned} & \infty \\ & \stackrel{n}{\infty} \\ & \underset{y}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{n} \\ & \underset{\sim}{n} \end{aligned}$ | 딩 | $\stackrel{\substack{2 \\=\\ \hline}}{ }$ | $\begin{aligned} & n \\ & \stackrel{e}{n} \\ & 0 \end{aligned}$ | －さ |
| $\left\|\begin{array}{c} 9 \\ \infty \\ \infty \end{array}\right\|$ | $\stackrel{\overrightarrow{7}}{\sim}$ | $\left\lvert\, \begin{gathered} \sim \\ \sim \\ \sim \end{gathered}\right.$ | $\left\|\begin{array}{l} 0 \\ n \\ n \end{array}\right\|$ | $\left\|\frac{\infty}{n}\right\|$ | $0$ | E\|c| | $\stackrel{\otimes}{\otimes}$ | $\left\lvert\, \begin{gathered} \frac{m}{a} \\ \frac{0}{2} \end{gathered}\right.$ | $\begin{aligned} & 0 \\ & 0 \\ & \end{aligned}$ | $\begin{array}{\|l\|l} 8 \\ 0 \\ 0 \\ \hline \end{array}$ | $\stackrel{\substack{\infty \\ \infty \\ \infty}}{ }$ | $\underset{8}{\infty}$ | $\stackrel{\substack{7 \\ \sim}}{ }$ | $\stackrel{n}{n}$ | $\left\|\begin{array}{c} \mathbf{t} \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} \tilde{N} \\ \underset{0}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} 2 \\ m \\ m \end{array}\right\|$ | $\dot{m}$ | $\underset{i}{i}$ | $\begin{aligned} & N \\ & \tilde{C} \\ & 子 \end{aligned}$ | $\stackrel{\rightharpoonup}{7}$ | $\begin{gathered} \infty \\ \stackrel{8}{6} \\ - \end{gathered}$ | $\left.\begin{gathered} 0 \\ 0 \\ \vdots \\ i \end{gathered} \right\rvert\,$ | $\begin{array}{c\|c\|} \substack{0 \\ n \\ 0 \\ 0 \\ \hline \\ \hline \\ \hline \\ \hline} \\ \hline \end{array}$ | ¢ |
| $\left\|\begin{array}{c} \circ \\ \hat{心} \\ \dot{心} \end{array}\right\|$ | $1$ | $\left\|\frac{\Delta}{6}\right\|$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right\|$ | $0$ |  | $\begin{array}{l\|l\|} \hline 8 \\ 0 \\ 0 \\ 0 & 0 \\ \hline \end{array}$ | B | $\left\|\begin{array}{l} \vec{b} \\ i n \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{l} \tilde{\infty} \\ \infty \\ x \end{array}\right\|$ | $0$ | $\begin{aligned} & 8 \\ & \stackrel{8}{6} \\ & i \end{aligned}$ | $\begin{aligned} & \hat{2} \\ & \stackrel{n}{2} \\ & i \end{aligned}$ | $\frac{2}{9}$ | $\frac{9}{6}$ | $\left\|\begin{array}{c} n \\ n \\ n \end{array}\right\|$ | $\left\|\begin{array}{c} \overrightarrow{6} \\ 0 \\ i n \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \infty \\ \infty \\ \underset{i}{2} \\ \underset{y}{2} \end{gathered}\right.$ | $\left.\begin{gathered} x \\ 0 \\ \vdots \\ i \end{gathered} \right\rvert\,$ | $\frac{\tilde{m}}{m}$ | $\underset{\sim}{\underset{\sim}{n}} \underset{\sim}{n}$ | $\begin{gathered} 2 \\ c \\ m \\ m \end{gathered}$ | $\begin{gathered} \hat{\infty} \\ \underset{\infty}{n} \\ \hline \end{gathered}$ | $\begin{aligned} & 8 \\ & \stackrel{8}{2} \\ & 7 \end{aligned}$ | $\begin{gathered} \infty \\ \stackrel{\infty}{4} \\ \dot{n} \end{gathered}$ | ¢ |
| $\left\|\begin{array}{c} \infty \\ \underset{y}{0} \\ \hline \end{array}\right\|$ | $\begin{aligned} & \text { rex } \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 7 \\ & 0 \\ & 0 \end{aligned}$ | $\left[\begin{array}{c} a \\ 0 \\ 0 \end{array}\right]$ | $\begin{gathered} 2 \\ \hline \end{gathered}$ | $\begin{array}{l\|l\|} \hline 9 & 2 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{array}{\|c\|c} 7 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{array}{l\|l\|} \hline & \hat{6} \\ 0 & 0 \\ 0 \end{array}$ | $\frac{\mathbf{m}}{0}$ | $\begin{gathered} 9 \\ 0 \\ 0 \end{gathered}$ | $\begin{gathered} \infty \\ \hline 0 \\ \hline \end{gathered}$ | $\begin{aligned} & \text { od } \\ & 0 \\ & 0 \end{aligned}$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\|\begin{array}{c} 1 \\ 3 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} 9 \\ 0 \\ 0 \end{array}\right\|$ | 敛 | $\left\|\begin{array}{c} \hat{O} \\ \hline 0 \end{array}\right\|$ | $\stackrel{\hat{0}}{0}$ | $\begin{gathered} \mathbf{T} \\ \hline \\ 0 \end{gathered}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\frac{ \pm}{0}$ | $\frac{2}{6}$ | $\begin{aligned} & \stackrel{\infty}{\infty} \\ & \underset{8}{2} \end{aligned}$ | $\overrightarrow{\vec{n}} \underset{0}{2}$ | $\cdots{ }_{3} \stackrel{0}{3}$ |
| $\left\|\begin{array}{c} \text { ㅈ̃ } \\ \text { a } \end{array}\right\|$ | $\begin{aligned} & \circ \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{gathered} \tilde{N} \\ 0 \\ 0 \end{gathered}$ | $0$ | $\begin{array}{l\|l\|} \hline 0 \\ \hline & 0 \\ 0 \\ 0 \\ \hline \end{array}$ |  | $\begin{array}{l\|l\|} \hline & \mathbf{J} \\ \hline 0 \\ 0 \\ \hline \end{array}$ | $\frac{ \pm}{0}$ | $\frac{0}{0}$ | $\frac{\pi}{2}$ | $\begin{aligned} & \hat{0} \\ & 0 \end{aligned}$ | $\left\|\begin{array}{c} \mathbf{t} \\ \vdots \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} 3 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\frac{\mathrm{N}}{3}$ | $\underset{0}{0}$ | 등 | $\left\|\begin{array}{c} \frac{2}{3} \\ 0 \end{array}\right\|$ | $\begin{aligned} & 2 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \hat{6} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\stackrel{\rightharpoonup}{\mathbf{0}}$ | $\stackrel{\pi}{m}$ | $$ | － |
|  | $\begin{gathered} 9 \\ \underset{\sim}{7} \\ \underset{\sim}{4} \end{gathered}$ | $\begin{array}{\|c} \ddot{9} \\ \underset{i}{i} \\ \hline \end{array}$ | $\begin{array}{\|c} \hat{6} \\ \text { ì } \\ \hline \end{array}$ | $6 \begin{gathered} c \\ \substack{3 \\ 子 \\ \vdots \\ \vdots \\ \hline} \end{gathered}$ |  | $\begin{array}{c\|c} \hat{n} \\ 0 \\ 0 \\ \substack{0 \\ \hline} \\ \hline \end{array}$ | $\begin{array}{c\|c} 0 \\ \hline \\ \hline \\ \hline \end{array}$ | $\left\|\begin{array}{c} n \\ \vdots \\ \vdots \\ \vdots \end{array}\right\|$ |  | $\begin{gathered} 0 \\ \underset{\sim}{6} \end{gathered}$ |  | $\begin{array}{\|c} \overrightarrow{9} \\ \vec{~} \end{array}$ | $\left\|\begin{array}{c} 0 \\ 0 \\ \vdots \\ \vdots \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \infty \\ \underset{c}{\infty} \\ \vdots \end{gathered}\right.$ | $\begin{array}{\|} \vec{n} \\ \hat{0} \\ \vdots \end{array}$ | $\left\|\begin{array}{c} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{array}{\|l\|} \hline \infty \\ 0 \\ 0 \\ \underset{子}{4} \\ \hline \end{array}$ | $\begin{gathered} \underset{\sim}{4} \\ \text { 寸 } \end{gathered}$ | $\begin{array}{\|c} 9 \\ 4 \\ \text { a } \\ \text { 子 } \end{array}$ | $\begin{array}{\|r\|} 0 \\ \vdots \\ \vdots \\ 0 \\ \hline \end{array}$ | $\begin{gathered} 0 \\ 0 \\ \vdots \\ 子 \\ \hline \end{gathered}$ | $\begin{aligned} & \stackrel{a}{2} \\ & \stackrel{y}{寸} \end{aligned}$ | $\begin{aligned} & \bar{\infty} \\ & \stackrel{y}{\infty} \\ & \stackrel{n}{n} \end{aligned}$ |  | （1） |
| $\left.\begin{array}{\|l\|} \hat{6} \\ 0 \\ 0 \\ 0 \end{array} \right\rvert\,$ | $\begin{aligned} & \frac{N}{y} \\ & \frac{1}{\mathrm{~S}} \end{aligned}$ |  | $\begin{aligned} & 9 \\ & \underset{y}{9} \\ & \underset{a}{i} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \infty \\ & \dot{2} \\ & \dot{a} \\ & \hline \end{aligned}\right.$ | $\begin{array}{\|c\|c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{array}{l\|l\|} \hline & \bar{a} \\ \dot{a} & \bar{i} \\ \hline & 2 \\ \hline \end{array}$ |  | $\left\|\begin{array}{c} n \\ \infty \\ \infty \\ \infty \\ \infty \end{array}\right\|$ | $\begin{aligned} & \stackrel{0}{n} \\ & i n \\ & \infty \end{aligned}$ | $\begin{gathered} \infty \\ \infty \\ \infty \\ \underset{\sim}{\infty} \end{gathered}$ | $\begin{aligned} & \stackrel{N}{n} \\ & \infty \\ & \infty \end{aligned}$ | $\begin{array}{\|l\|l} \hline 8 \\ \vdots \\ \vdots \\ \\ \hline \end{array}$ | $\begin{aligned} & \text { ã } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \text { 가 } \\ 0 \\ 0 \end{gathered}$ | $\begin{gathered} 2 \\ \tilde{0} \\ \text { en } \\ \hline \end{gathered}$ | $\begin{array}{\|l} \mathbf{7} \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline 0 \\ 0 \\ \vdots \\ \hline \\ \hline \end{array}$ | $\begin{aligned} & \text { in } \\ & \underset{\sim}{c} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\infty} \\ & \infty \\ & \underset{\sim}{4} \\ & \hline \end{aligned}$ |  | $\begin{array}{\|c} 1 \\ \mathbf{3} \\ \mathbf{3} \\ \hline \end{array}$ | $\begin{aligned} & n \\ & \stackrel{n}{n} \\ & \stackrel{n}{2} \end{aligned}$ | $\begin{aligned} & \underset{c}{c} \\ & \substack{0 \\ i} \end{aligned}$ |  | 管 |
| $\left\|\begin{array}{c} 9 \\ \underset{\sim}{4} \\ i \end{array}\right\|$ | $\begin{array}{\|c} \frac{\infty}{n} \\ \vdots \\ \vdots \end{array}$ | $: \begin{gathered} \infty \\ \hline \\ \hline \end{gathered}$ | $\frac{n}{i}$ | $\left.5 \begin{gathered} i \\ -1 \\ i \end{gathered} \right\rvert\,$ | $\begin{array}{l\|l\|} 2 & \overrightarrow{0} \\ i & 0 \\ \hline \end{array}$ | $\begin{array}{l\|l} 5 & 0 \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $$ | $\begin{array}{\|c} 9 \\ 9 \\ \hline \end{array}$ | $\frac{8}{8}$ | $\begin{gathered} \overrightarrow{4} \\ \stackrel{y}{4} \\ \hline \end{gathered}$ | $$ | $\begin{array}{\|l\|} \hline 9 \\ \hline \end{array}$ | $\frac{2}{6}$ | $\begin{gathered} 0 \\ 0 \\ \vdots \\ \hline \end{gathered}$ | $\begin{gathered} \mathbf{~} \\ \hline- \\ \hline \end{gathered}$ | $\begin{gathered} \pi \\ i \\ i \end{gathered}$ | $\begin{array}{\|c\|} \hline 0 \\ \hline 0 \\ \vdots \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline \stackrel{\rightharpoonup}{6} \\ \stackrel{1}{3} \\ \hline \end{array}$ | $\begin{aligned} & \hline 2 \\ & 2 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{array}{\|c} \hat{6} \\ 0 \\ i \\ i \end{array}$ | $\begin{array}{\|c\|} \hline 9 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \stackrel{\pi}{n} \\ i \\ \hline \end{array}$ |  | $\stackrel{\rightharpoonup}{i}$ |  |
| $\left\|\begin{array}{l} 2 \\ \vdots \\ 9 \\ 9 \end{array}\right\|$ | $\begin{gathered} \frac{9}{2} \\ \hdashline \\ \hdashline \end{gathered}$ | $\begin{array}{\|l\|} \hline 8 \\ 0 \\ 0 \\ \hdashline \end{array}$ | $\begin{aligned} & \vec{\sim} \\ & \stackrel{1}{0} \end{aligned}$ |  |  | $\begin{array}{c\|c} 0 \\ 0.0 \\ \vdots \\ \vdots & 0 \\ & \underset{1}{1} \\ \hline \end{array}$ | $\begin{array}{l\|l} 0 \\ 0 & 0 \\ 0 \\ 0 & 0 \\ & 0 \\ \end{array}$ | $\begin{aligned} & 2 \\ & \vdots \\ & \square \end{aligned}$ | $\begin{array}{\|l\|} \vec{i} \\ \stackrel{y}{a} \\ \text { an } \end{array}$ | $\begin{array}{\|l\|} \hline \frac{\rightharpoonup}{\infty} \\ \underset{\sim}{7} \\ \hline \end{array}$ | $\begin{array}{\|l\|} \hline \infty \\ \text { N } \\ \text { तो } \end{array}$ | $\stackrel{8}{8}$ |  | $\begin{gathered} \text { ה } \\ 0 \\ \text { ल्ju } \end{gathered}$ | $\begin{aligned} & \hat{0} \\ & \stackrel{y}{n} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{gathered} \frac{n}{n} \\ \\ \hline \end{gathered}$ | $\left\lvert\, \begin{gathered} \underset{\sim}{\infty} \\ \infty \\ \end{gathered}\right.$ |  |  | $\left\|\begin{array}{c} \stackrel{\rightharpoonup}{\mathrm{N}} \\ \stackrel{\rightharpoonup}{\mathrm{P}} \end{array}\right\|$ | $\left[\begin{array}{l} \mathbf{n}_{2}^{2} \\ 0 \\ x_{1} \end{array}\right]$ | $\left\|\begin{array}{c} \tilde{y} \\ \dot{0} \\ \dot{1} \end{array}\right\|$ | $\begin{aligned} & \overline{7} \\ & \underset{8}{6} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & \vdots \\ & \vdots \\ & \\ & \hline \end{aligned}$ | $\begin{aligned} & \dot{6} \\ & \dot{d} \end{aligned}$ |
| $\begin{array}{\|c\|} \hline 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ |  | $\begin{array}{\|c} \stackrel{n}{7} \\ 0 \\ -0 \\ \hline \end{array}$ | $\begin{gathered} n \\ 0 \\ 0 \\ 0 \end{gathered}$ | $0 \left\lvert\, \begin{gathered} \hat{0} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}\right.$ | $\begin{array}{l\|l\|} \hline 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \hline \end{array}$ | $\begin{array}{l\|l} 0 & 0 \\ \infty & 9 \\ \hline 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ |  | $1 \begin{gathered} \infty \\ \infty \\ \text { in } \\ \end{gathered}$ |  | $\begin{aligned} & \overrightarrow{0} \\ & \stackrel{0}{0} \\ & \underline{0} \end{aligned}$ | $\begin{array}{\|l} \hline \tilde{0} \\ \infty \\ \text { N } \\ \hline 0 \end{array}$ | $\hat{S}_{6}^{6}$ |  | $\begin{aligned} & \dot{g} \\ & \underset{i}{2} \\ & \underline{s} \end{aligned}$ |  | $\left[\begin{array}{c} \infty \\ \vdots \\ \vdots \\ \vdots \\ \hline \end{array}\right.$ | $\begin{array}{\|l\|} \hline \vec{a} \\ \stackrel{\rightharpoonup}{\mathbf{n}} \\ \hline \end{array}$ | $\begin{aligned} & \mathrm{N} \\ & \mathrm{n} \\ & \stackrel{f}{\mathrm{I}} \\ & \hline \end{aligned}$ | $$ | $\begin{aligned} & \dot{r} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{\|c\|} \vec{a} \\ \underset{y}{n} \\ \end{array}$ | 2 $\stackrel{2}{2}$ $\stackrel{0}{0}$ -1 |  | $\underset{~}{\text { }}$ | － |
| 析 | － | T | ¢ | 岩 | 进\|: | $$ |  | $\begin{gathered} \infty \\ \stackrel{6}{2} \\ \hline \end{gathered}$ | $\begin{array}{\|c\|c\|} \underline{0} \\ \hline \end{array}$ | $\frac{?}{9}$ | 들 | $\underset{3}{5}$ | $\frac{ल}{2}$ | ＋ | 录 | $0$ | 즐 | $\cdots$ | 2 | $\begin{array}{\|l} \hline \infty \\ \hline \end{array}$ | ¢ | $\frac{\sim}{e}$ | $\stackrel{\text { \％}}{\sim}$ | 寺 | \％ |


| $\underset{\sim}{c} \mid$ | $\|\stackrel{\ddot{\theta}}{\dot{\theta}}\|$ | $\begin{array}{\|c\|} \hat{8} \\ \hat{0} \end{array}$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ \infty \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \substack { 8 \\ \begin{subarray}{c}{6{ 8 \\ \begin{subarray} { c } { 6 } } \\ {\hline} \end{gathered}\right.$ | 佥\| | 尓 | $\left\lvert\, \begin{gathered} \dot{a} \\ \dot{8} \\ \hline \end{gathered}\right.$ | $\begin{gathered} m \\ 0 \\ 0 \\ \end{gathered}$ | $\stackrel{\infty}{\stackrel{\infty}{\wedge}}$ | $\stackrel{i}{n}$ | $\left\|\begin{array}{c} 0 \\ i \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ \infty \end{array}\right\|$ | $\left\lvert\, \begin{array}{\|c\|} \substack{\text { In }} \end{array}\right.$ | $\left\|\begin{array}{c} \vec{\infty} \\ \infty \\ \infty \end{array}\right\|$ | $\begin{aligned} & \underset{\infty}{\sim} \\ & \underset{\infty}{2} \end{aligned}$ |  | $\begin{aligned} & \infty \\ & \infty \\ & \infty \\ & \infty \end{aligned}$ | $\begin{gathered} a \\ \stackrel{0}{2} \end{gathered}$ | $\left\|\begin{array}{c} 0 \\ \stackrel{0}{d} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \underset{\sim}{c} \\ \hline \end{gathered}\right.$ | $\underset{\sim}{\underset{\sim}{n}}$ | $\left\|\begin{array}{c} 0 \\ \vdots \\ \vdots \end{array}\right\|$ | $\begin{gathered} \text { N } \\ \stackrel{\rightharpoonup}{\mathrm{N}} \end{gathered}$ | $\left.\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned} \right\rvert\,$ | $\stackrel{\sim}{\sim}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} c \\ \substack{0 \\ \vdots \\ i n} \end{gathered}$ | $\left\|\begin{array}{c} \frac{4}{d} \\ \stackrel{y}{n} \\ 心 \end{array}\right\|$ | $\left\|\begin{array}{l} \vec{n} \\ \frac{n}{n} \end{array}\right\|$ | $\left\|\begin{array}{l} \frac{4}{n} \\ \frac{i n}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \circ \\ i \\ i n \\ n \end{array}\right\|$ | $\left\|\begin{array}{c} \stackrel{\sim}{6} \\ \stackrel{\rightharpoonup}{n} \\ \hline \end{array}\right\|$ | $\frac{7}{2}$ | $8$ | $\frac{4}{0}$ | $\left\lvert\, \begin{gathered} \text { 古 } \\ 0 \\ \text { in } \end{gathered}\right.$ | $\begin{array}{\|c} 2 \\ i \\ i n \\ \hline \end{array}$ | 会 | $\frac{0}{n}$ | $\frac{2}{2}$ | $\frac{2}{9}$ | $\left\|\begin{array}{c} c \\ \underset{c}{c} \\ n \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \underset{\sim}{n} \\ \underset{i}{2} \end{gathered}\right.$ | $\begin{aligned} & \frac{\infty}{x} \\ & \frac{p}{2} \end{aligned}$ |  | $\stackrel{n}{n}$ |  | $\begin{aligned} & \hat{0} \\ & \hat{0} \\ & \stackrel{n}{n} \end{aligned}$ | $\left\|\begin{array}{l} n \\ e \\ \stackrel{n}{n} \end{array}\right\|$ | $\stackrel{\underset{N}{N}}{\stackrel{N}{n}}$ | $\begin{aligned} & 0 \\ & \infty \\ & \stackrel{\infty}{n} \end{aligned}$ | $\begin{array}{\|c} \substack{c \\ \stackrel{n}{n} \\ \hline \\ \hline} \end{array}$ | $\left.\begin{aligned} & 0 \\ & \stackrel{y}{7} \\ & i \end{aligned} \right\rvert\,$ |  |
| $\left\|\begin{array}{c} \infty \\ \stackrel{\infty}{2} \\ \underset{\sim}{c} \\ \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{n} \\ \stackrel{y}{c} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\right\|$ | $\left\|\begin{array}{c} \underset{2}{2} \\ \stackrel{3}{6} \\ \mathbf{c} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ \vdots \\ \end{array}\right\|$ | $\left\|\begin{array}{c} i \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{array}\right\|$ |  | $\left\|\begin{array}{c} 9 \\ 0 \\ 0 \\ \vdots \\ 0 \end{array}\right\|$ |  | $\begin{gathered} \stackrel{\rightharpoonup}{\infty} \\ \vdots \\ \vdots \\ \stackrel{\rightharpoonup}{2} \end{gathered}$ | $\left\|\begin{array}{c} \underset{2}{2} \\ \underset{\sim}{2} \\ \underset{\sim}{2} \end{array}\right\|$ |  | $\frac{\underset{y}{2}}{\underset{\sim}{2}}$ | $\begin{gathered} n \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ |  |  | $\begin{gathered} n \\ \substack{n \\ \vdots \\ \\ \hline} \end{gathered}$ | $\begin{aligned} & 2 \\ & 0 \\ & \\ & \end{aligned}$ |  | $\begin{gathered} \stackrel{\circ}{\circ} \\ \stackrel{\omega}{c} \end{gathered}$ | $\left\|\begin{array}{c} \infty \\ \tilde{m} \\ \vdots \\ \dot{m} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{m}{m} \\ \underset{\sim}{7} \\ \mid \end{array}\right\|$ | $\left\|\begin{array}{c} \hat{2} \\ \frac{2}{f} \\ \underset{\sim}{2} \end{array}\right\|$ | $\begin{aligned} & \mathscr{O} \\ & \underset{\sim}{\mathscr{0}} \\ & \stackrel{y}{2} \end{aligned}$ | $\begin{gathered} \infty \\ \stackrel{\infty}{0} \\ \vdots \\ \stackrel{y}{c} \end{gathered}$ | $\begin{gathered} \underset{C}{2} \\ \vdots \\ \vdots \\ \end{gathered}$ |  | － |
| $\left\|\begin{array}{c} \underset{\sim}{\tilde{q}} \\ = \end{array}\right\|$ | $\left\|\begin{array}{c} \vec{\circ} \\ = \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & 6 \\ & \underset{\sim}{d} \\ & \underset{\sim}{2} \end{aligned}\right.$ | $\left\|\begin{array}{c} \stackrel{\infty}{\infty} \\ \underset{j}{j} \end{array}\right\|$ | $\left\|\begin{array}{c} \circ \\ \stackrel{\circ}{\dot{n}} \end{array}\right\|$ | $\left\|\begin{array}{c} \bar{\infty} \\ \underset{寸}{f} \\ \underset{寸}{ } \end{array}\right\|$ | $\left.\begin{gathered} 0 \\ 0 \\ \underset{\sim}{\mathbf{j}} \end{gathered} \right\rvert\,$ | $\left\|\begin{array}{c} 0 \\ 0 \\ \mathbf{~} \\ \underset{\sim}{2} \end{array}\right\|$ |  | $\begin{array}{\|l} \stackrel{a}{\tilde{m}} \\ \underset{\sim}{2} \end{array}$ | $8$ |  |  | $0$ | $\underset{\sigma}{\sim}$ | $\begin{aligned} & 9 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 9 \\ & \stackrel{0}{0} \end{aligned}$ | $\begin{aligned} & \vec{\infty} \\ & 0 \\ & 0 \end{aligned}$ |  | $\begin{aligned} & \stackrel{\infty}{\infty} \\ & \stackrel{0}{0} \end{aligned}$ | $\left\|\begin{array}{c} \infty \\ \stackrel{\infty}{=} \\ = \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \text { ? } \\ = \\ = \end{gathered}\right.$ | $\begin{gathered} \mathscr{\infty} \\ \stackrel{\infty}{=} \end{gathered}$ | $\underset{\underset{\sim}{7}}{\underset{\sim}{7}}$ | $\begin{gathered} \underset{\sim}{c} \\ \underset{\sim}{m} \end{gathered}$ | $\stackrel{\stackrel{O}{\infty}}{\stackrel{\sim}{\mathfrak{d}}}$ | $\begin{gathered} \underset{0}{0} \\ \underset{\sim}{i} \end{gathered}$ | 喜 |
| $\begin{array}{\|c\|} \hline 0 \\ 0 \\ 子 \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \underset{子}{子} \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline 0 \\ 0 \\ \hline \end{array}$ | $\left.\begin{array}{\|c\|} \vec{\circ} \\ \underset{n}{n} \end{array} \right\rvert\,$ | $\left\|\begin{array}{l} \overrightarrow{\mathbf{3}} \\ \underset{\sim}{n} \end{array}\right\|$ |  | $\left\lvert\, \begin{gathered} \underset{\underset{c}{c}}{\underset{m}{2}} \end{gathered}\right.$ | $$ |  | $\begin{array}{\|c\|} \hline 0 \\ 0 \\ \dot{子} \end{array}$ | $\begin{array}{\|c\|} \hline 0 \\ \\ \rightarrow \end{array}$ |  | $\underset{\sim}{m}$ | $\left\lvert\, \begin{gathered} e_{0}^{\infty} \\ \end{gathered}\right.$ | $\underset{\sim}{c}$ | $\left.\mathfrak{c} \left\lvert\, \begin{array}{l} \infty \\ \vdots \\ \dot{子} \end{array}\right.\right\}$ | $\frac{\sqrt{2}}{\infty}$ | $\underset{\sim}{2} \underset{\infty}{2}$ |  | $\underset{\substack{0 \\ \hline \\ \hline}}{ }$ | $\left.\begin{array}{\|c\|} \hline 0 \\ 0 \\ 0 \\ c \end{array} \right\rvert\,$ | $\frac{0}{2}$ | $\begin{gathered} \hat{Q} \\ \underset{\sim}{0} \end{gathered}$ | $\begin{gathered} \infty \\ \underset{子}{\infty} \end{gathered}$ | $\begin{gathered} \underset{\sim}{2} \\ \underset{子}{2} \end{gathered}$ | $\begin{gathered} \text { ron } \\ \text { 和 } \end{gathered}$ | $\begin{gathered} \infty \\ \underset{\sim}{\infty} \\ \underset{n}{2} \end{gathered}$ | 鿖 |
| $\begin{array}{\|c\|} \hline \frac{m}{f} \\ \hline \end{array}$ | $\left\|\begin{array}{l} \hat{N} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ m \\ m \end{array}\right\|$ | $\stackrel{\bar{n}}{\stackrel{\rightharpoonup}{n}} \mid$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ \text { in } \end{array}\right\|$ | $\begin{aligned} & \hat{e} \\ & \underset{\sim}{n} \end{aligned}$ | $\stackrel{\substack{\infty \\ \infty \\ \underset{\sim}{\infty} \\ \hline}}{ }$ | $\left\|\begin{array}{c} \dot{\infty} \\ \underset{\sim}{c} \end{array}\right\|$ |  | $\left\|\begin{array}{c} 1 \\ \underset{m}{n} \\ \end{array}\right\|$ | $\stackrel{y}{t}$ |  | $\mathfrak{c}$ | $\underset{\substack{a \\ \underset{\sim}{n} \\ \text { n }}}{ }$ |  | $\stackrel{\infty}{\infty}$ | $\begin{gathered} 4 \\ 0 \\ 0 \end{gathered}$ |  | $\underset{\dot{b}}{\vec{j}}$ | ત্ત̃ | $\left\|\begin{array}{l} \overrightarrow{6} \\ \underset{\sim}{2} \end{array}\right\|$ | $\begin{gathered} \stackrel{\rightharpoonup}{4} \\ \underset{\sim}{2} \end{gathered}$ | $\underset{\substack{+\infty \\ \infty}}{ }$ | $\stackrel{0}{0}$ | $\stackrel{\underset{\infty}{\circ}}{\substack{0}}$ | $\frac{\hat{y}}{7}$ | $\stackrel{\vec{a}}{\vec{寸}}$ | $\pm$ |
| 商 | $\frac{9}{5}$ | $\begin{aligned} & \text { in } \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 1 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{\|c\|c\|} \hline 0 \\ 0 \\ 0 \end{array}$ | $\begin{aligned} & \text { Y } \\ & \stackrel{\rightharpoonup}{0} \\ & \hline \end{aligned}$ | $\begin{aligned} & 2 \hat{\circ} \\ & 0 \\ & 0 \end{aligned}$ | $\|\overrightarrow{0}\|$ | $\begin{array}{l\|l\|} \hline \\ 0 & 0 \\ 0 \end{array}$ | $\left\|\begin{array}{l} \mathbf{0} \\ \mathbf{0} \\ 0 \end{array}\right\|$ | $\begin{array}{l\|l\|} \hline \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \frac{\infty}{\circ} \\ \hline 0 \end{array}$ | $\begin{gathered} \frac{c}{m} \\ \substack{2} \\ \hline \end{gathered}$ | $\begin{array}{\|c\|} \hline \\ \\ \hline \end{array}$ | $\begin{gathered} \substack{t \\ n \\ 0} \\ \hline \end{gathered}$ | $\underbrace{\mathfrak{e}}_{0}$ | $\stackrel{\widetilde{\otimes}}{\stackrel{\otimes}{\otimes}}$ |  | of | $\begin{array}{\|c\|} \hline \stackrel{\rightharpoonup}{m} \\ \dot{n} \end{array}$ | $\left\|\begin{array}{l} \mathbf{6} \\ 0 \end{array}\right\|$ | $\begin{aligned} & 0 \\ & \cdots \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0.0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \vdots \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\stackrel{y}{2}$ | $\overline{\bar{o}}$ |  |
| $\left\|\begin{array}{c} \text { ה } \\ \text { ה } \end{array}\right\|$ | $\frac{\infty}{5}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}\right.$ | $\begin{aligned} & \text { 층 } \\ & 0 \end{aligned}$ | $\hat{0}$ | $\begin{aligned} & \stackrel{6}{6} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 5 \\ & 0 \\ & 0 \end{aligned}$ | $$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{array}{l\|l\|} \hline 0 \\ 0 \\ 0 \\ 0 \end{array}$ | S | $\underset{\substack{0 \\ \\ \\ \hline \\ \hline}}{ }$ | $8$ | $\underbrace{\infty}_{0}$ |  | 苜 |  | $\infty$ | $\left\|\begin{array}{c} 2 \\ \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{c} \mathbf{4} \\ \hline 0 \end{array}\right\|$ | $\frac{i}{i}$ | 苟 | 웅 | $\begin{aligned} & \pi \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} 8 \\ 8 \\ \hline 0 \end{gathered}$ | $\underset{\sim}{2}$ | － |
| $\left\|\begin{array}{l} r_{i} \\ \frac{1}{v_{i}} \end{array}\right\|$ | $\begin{array}{\|c} 0 \\ 0 \\ 9 \\ \text { 号 } \end{array}$ | $\begin{aligned} & 6 \\ & 8 \\ & \underset{f}{9} \end{aligned}$ | $\begin{aligned} & \stackrel{a}{c} \\ & \stackrel{0}{c} \end{aligned}$ | $\begin{gathered} 0 \\ \infty \\ \substack{f \\ \hline \\ \hline} \end{gathered}$ | $\begin{gathered} \infty \\ \stackrel{\infty}{\infty} \\ \underset{寸}{2} \end{gathered}$ | $\left\|\begin{array}{\|c} \hat{e} \\ \infty \\ \underset{i}{f} \end{array}\right\|$ | 化守 | $\underset{f}{\underset{\sim}{c}} \underset{\sim}{\infty}$ |  |  | $\begin{array}{c\|c} \hat{0} \\ 0 & 0 \\ 0 \\ \hline & 0 \\ 0 & 0 \end{array}$ |  |  | $\dot{S}$ | $\begin{gathered} 0 \\ 5 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{gathered} \bar{a} \\ \stackrel{\rightharpoonup}{d} \end{gathered}$ |  | $\begin{aligned} & 0 \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & 9 \\ & 0 \\ & 0 \\ & i \\ & i \end{aligned}$ |  | $\begin{gathered} \mathscr{\infty} \\ \underset{\sim}{\infty} \end{gathered}$ | $\frac{\stackrel{r}{m}}{\stackrel{m}{r}}$ |  | $\begin{aligned} & \tilde{\sim} \\ & \underset{\sim}{i} \end{aligned}$ |  |  | 析 |
| $\left[\begin{array}{c} \tilde{m} \\ \underset{\sim}{n} \end{array}\right]$ | $\begin{array}{\|c} \substack{7 \\ \vdots \\ 5 \\ \hline} \end{array}$ | $\begin{gathered} 0 \\ \vdots \\ \vdots \\ \hline \end{gathered}$ | $\begin{array}{\|l\|} \hline 0 \\ 0 \\ 0 \\ 9 \end{array}$ | $\begin{gathered} \infty \\ \vdots \\ \vdots \\ \\ \hline \end{gathered}$ | $\begin{gathered} \stackrel{\rightharpoonup}{0} \\ \stackrel{1}{2} \\ \stackrel{\rightharpoonup}{2} \\ \hline \end{gathered}$ | $\underset{\sim}{n}$ | $2 \begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |  | $\begin{array}{ll} 9 \\ 9 \\ 9 \end{array}$ | $\dot{c}$ | $0$ | An |  |  | $\begin{aligned} & \underset{\infty}{\infty} \\ & \underset{\infty}{\infty} \end{aligned}$ |  | $\begin{aligned} & \hat{6} \\ & 0 \\ & \infty \\ & \infty \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & \pm \\ & \pm \end{aligned}$ | $\left.\begin{array}{\|c\|} \hline 0 \\ \mathbf{0} \\ \vdots \\ \vdots \end{array} \right\rvert\,$ |  | $\begin{aligned} & \vec{a} \\ & \stackrel{\rightharpoonup}{a} \\ & \stackrel{y}{c} \end{aligned}$ |  | $\circ$ 2 2 2 2 | $\mathrm{C}_{6}^{\mathrm{C}}$ |  | － |
| $\left.\begin{array}{\|c} \bar{x} \\ \vdots \\ \vdots \end{array} \right\rvert\,$ | $\begin{gathered} = \\ n_{1} \end{gathered}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{1} \\ & \substack{1 \\ \hline} \end{aligned}$ | $\begin{gathered} 7 \\ 6 \\ \vdots \\ i \end{gathered}$ | $\begin{aligned} & \overrightarrow{5} \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{array}{\|c} \hline 0 \\ \vdots \\ \hline \end{array}$ | $0 \begin{aligned} & \mathrm{E} \\ & \hline \end{aligned}$ | $0$ | $$ | $0$ |  |  | $\begin{aligned} & 7 \\ & i \end{aligned}$ | $\begin{aligned} & \text { to } \\ & \vdots \\ & \vdots \\ & i \end{aligned}$ |  | $\begin{aligned} & \hat{\infty} \\ & \underset{o}{0} \end{aligned}$ | $\begin{aligned} & \bar{子} \\ & \dot{̣} \\ & \hline \end{aligned}$ |  |  | $\left.\begin{array}{\|c\|} \hline 8 \\ \stackrel{0}{6} \\ i \end{array} \right\rvert\,$ | $$ | $\begin{aligned} & 9 \\ & 9 \\ & \hline \end{aligned}$ | $\begin{array}{\|c} \hline 8 \\ 9 \\ \vdots \\ \hline \end{array}$ | $\begin{aligned} & 0 \\ & \hline \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \stackrel{0}{4} \\ & i \end{aligned}$ | $\begin{gathered} 9 \\ \hline \end{gathered}$ | $\begin{gathered} i \\ i \\ i \end{gathered}$ | 志 |
| $\left\|\begin{array}{l} 9 \\ ⿳ 亠 口 子 丸 \\ \vdots \\ 8 \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \tilde{\sim} \\ \tilde{2} \end{array}\right\|$ | $\begin{array}{\|c\|c\|c\|c\|} \substack{\infty \\ \infty \\ \infty} \end{array}$ | $\begin{aligned} & \hat{N} \\ & \underset{D_{2}}{ } \end{aligned}$ | $\left\lvert\, \frac{\vec{n}}{\frac{1}{x}}\right.$ | $\left\|\begin{array}{c} \hat{a} \\ \mathbf{C} \\ \vdots \\ i \end{array}\right\|$ | $\begin{array}{\|c\|c\|c} \overrightarrow{0} \\ 0 \\ 0 & 0 \end{array}$ | $\begin{gathered} 0 \\ \hline \end{gathered}$ |  | $i_{i}^{\infty}$ | $\begin{array}{c\|c} 0 \\ 0 \\ \vdots \\ \\ \hline \end{array}$ |  | $\begin{array}{l\|l} n \\ 0 & \stackrel{n}{6} \\ \vdots \\ \vdots \end{array}$ | $\underset{\sim}{n}$ |  |  | $\begin{aligned} & Q_{\infty}^{\infty} \\ & \underset{\sim}{7} \end{aligned}$ |  | $\begin{gathered} \text { I } \\ \end{gathered}$ | $\begin{aligned} & \stackrel{r}{⿱ ㇒} \\ & \vdots \\ & \stackrel{n}{7} \end{aligned}$ | $\begin{array}{\|l\|} \hline \vec{a} \\ 0 \\ 0 \\ \vdots \end{array}$ |  | $\underset{\substack{\underset{\sim}{\infty}\\}}{ }$ | ${ }_{c}^{\infty}$ |  | 志 |  | 筞 |
| $\begin{array}{\|l\|} \hline \stackrel{\rightharpoonup}{\hat{y}} \\ \stackrel{y}{\hat{G}} \\ \hline \end{array}$ | $\begin{aligned} & 9 \\ & \stackrel{n}{c} \\ & \stackrel{B}{8} \end{aligned}$ | $$ | $\begin{array}{\|l\|l} \hline ⿱ 士 口 犬 ~ \\ 0 \\ \vdots \\ \end{array}$ | $\begin{aligned} & \stackrel{n}{e} \\ & \underset{\sim}{n} \\ & \hline \end{aligned}$ | $$ | $2 \begin{gathered} 8 \\ x \\ x \\ x \end{gathered}$ |  | $$ |  |  | $\begin{array}{l\|l\|} \hline & 7 \\ i & 0 \\ i \\ 0 \\ 0 \\ 0 \end{array}$ |  | $0 \begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ |  |  |  |  | $\begin{aligned} & \overline{0} \\ & 0 \\ & 0 \\ & \hline-1 \end{aligned}$ |  | $\begin{aligned} & \underset{y}{m} \\ & \infty \\ & 0 \\ & \hline 0 \end{aligned}$ | $\begin{aligned} & 3 \\ & \vdots \\ & \vdots \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{gathered} 8 \\ i \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ |  | 禺 |  |  | （1） |
| $\frac{\times}{0}$ | $\stackrel{\infty}{\times}$ | $\frac{2}{3}$ | $\frac{2}{2}$ | $\stackrel{\vec{a}}{\stackrel{\rightharpoonup}{2}}$ |  | $\frac{2}{2}$ | $\begin{array}{\|c} \mathbf{t} \\ \hline \mathbf{0} \\ \hline \end{array}$ | $\begin{array}{c\|c} \frac{0}{2} \\ \hline 9 \\ \hline \end{array}$ | $\frac{2}{2}$ | $\begin{array}{l\|l\|} \hline 2 \\ 0 & \frac{0}{2} \\ \hline \end{array}$ | $\begin{array}{\|c\|c\|} \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline \end{array}$ | $\frac{2}{2}$ | $5$ | $8$ | $\stackrel{5}{4}$ | $\stackrel{\text { O}}{0}$ |  | 志 | $\begin{aligned} & 6 \\ & \stackrel{3}{3} \\ & \hline \end{aligned}$ | $\frac{0}{2}$ | $\frac{-0}{9}$ | $\stackrel{\infty}{\circ}$ |  | $\begin{aligned} & \text { 을 } \\ & \stackrel{3}{3} \end{aligned}$ | 으르를 | Eie | $\stackrel{\mathrm{m}}{\overline{\mathrm{E}}}$ |


| $\left\|\begin{array}{c} \underset{\infty}{\infty} \\ \underset{\infty}{2} \end{array}\right\|$ | $\left\|\begin{array}{l} \infty \\ \infty \\ \infty \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \underset{\infty}{ \pm} \\ \hline \end{gathered}\right.$ | $\left\|\begin{array}{c} 0 \\ \infty \\ \infty \\ \infty \end{array}\right\|$ | $\begin{aligned} & 7 \\ & 0 \\ & 2 \end{aligned}$ | $\frac{9}{2}$ | $\begin{gathered} \infty \\ \infty \\ \infty \\ \infty \end{gathered}$ | 范\| | $\begin{aligned} & 9 \\ & \vdots \\ & \hdashline \end{aligned}$ | $\left\|\begin{array}{c} 0 \\ \infty \\ 0 \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ \infty \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{l} \vec{~} \\ \dot{0} \\ \dot{\infty} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \stackrel{0}{\infty} \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} m \\ \stackrel{m}{\infty} \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{\tilde{\infty}} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \substack{0 \\ \\ \hline \\ \hline} \end{gathered}\right.$ | $\overrightarrow{\substack{\infty}} \mid$ | $\left.\begin{aligned} & \hat{e} \\ & \underset{\infty}{\infty} \end{aligned} \right\rvert\,$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\stackrel{\substack{- \\ \hline \\ \hline}}{ }$ | $\left\lvert\, \begin{aligned} & \hat{\sim} \\ & \stackrel{\infty}{\infty} \end{aligned}\right.$ | $\begin{aligned} & \overrightarrow{0} \\ & \stackrel{0}{0} \end{aligned}$ | $\left\lvert\, \begin{aligned} & \text { n } \\ & \text { 岩 } \end{aligned}\right.$ | $\left\|\begin{array}{c} n \\ \text { N } \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{c} a \\ 0 \\ b \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \underset{\sim}{\hat{6}} \\ \stackrel{0}{2} \end{gathered}\right.$ | $\stackrel{\text { 읏 }}{ }$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|\begin{array}{c} \frac{\pi}{2} \\ \hat{n} \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{c} 7 \\ 0 \\ \vdots \\ \vdots \end{array}\right\|$ | $\left\|\begin{array}{c} 7 \\ \infty \\ \stackrel{\infty}{n} \\ \stackrel{1}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} 2 \\ \frac{2}{2} \\ i n \end{array}\right\|$ | $\frac{2}{2}$ | $\begin{aligned} & \infty \\ & \infty \\ & \stackrel{\infty}{n} \\ & \hline \end{aligned}$ |  | $\left\|\begin{array}{c} \infty \\ \stackrel{\infty}{n} \\ \stackrel{n}{n} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \infty \\ \underset{\sim}{\infty} \\ \stackrel{0}{2} \\ \hline \end{gathered}\right.$ | $\frac{9}{2}$ | 首 | $\left\|\begin{array}{c} 2 \\ \stackrel{a}{2} \\ \stackrel{n}{i} \end{array}\right\|$ | $\left.\begin{gathered} n \\ 0 \\ 0 \\ n \end{gathered} \right\rvert\,$ | $\stackrel{y}{4}$ | $\left\|\begin{array}{c} 0 \\ \text { in } \\ i \end{array}\right\|$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\mathrm{A}} \\ & \hline \end{aligned}$ | $\frac{\pi}{\hat{n}}$ | $\begin{array}{\|c} 2 \\ \stackrel{2}{n} \end{array}$ | $\left\|\begin{array}{l} \mathrm{t} \\ \stackrel{\rightharpoonup}{n} \end{array}\right\|$ | $\begin{gathered} \frac{\pi}{0} \\ \stackrel{n}{n} \\ \hline \end{gathered}$ | $\left\|\begin{array}{c} \underset{n}{n} \\ \stackrel{n}{n} \end{array}\right\|$ | $\begin{gathered} \stackrel{\circ}{N} \\ \stackrel{N}{n} \end{gathered}$ | $\left\|\begin{array}{c} \dot{\alpha} \\ \underset{N}{n} \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ \hat{n} \\ i \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \underset{\sim}{n} \\ i \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{N}{N} \\ i n \end{array}\right\|$ | $\stackrel{3}{5}$ |
| $\begin{gathered} \underset{\sim}{2} \\ \tilde{y} \\ \underset{\sim}{2} \end{gathered}$ | $\left\|\begin{array}{c} 0 \\ \underset{\sim}{e} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} n \\ 0 \\ 0 \\ \end{array}\right\|$ | $\begin{gathered} \frac{0}{2} \\ \substack{2 \\ 9 \\ 9} \end{gathered}$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ \\ \end{gathered}$ | $\begin{gathered} \infty \\ \stackrel{\infty}{\infty} \\ \underset{\sim}{m} \end{gathered}$ |  | Con | $\mathfrak{c}$ | $\begin{gathered} \underset{\sim}{A} \\ \text { n } \\ \end{gathered}$ | $\mathfrak{c}$ | $\left\|\begin{array}{c} \vec{~} \\ \underset{\sim}{\sim} \end{array}\right\|$ | $\begin{aligned} & 0 \\ & \underset{\sim}{0} \\ & \underset{m}{2} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & 0 \\ & \underset{\sim}{2} \end{aligned}$ | $\left\|\begin{array}{c} \underset{\substack{2}}{ } \\ \underset{\sim}{\mathbf{0}} \end{array}\right\|$ | $\begin{aligned} & \text { 等 } \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{array}{\|c} \underset{\sim}{\circ} \\ \underset{\sim}{2} \end{array}$ | $\left\lvert\, \begin{gathered} \underset{0}{2} \\ \underset{\substack{2}}{ } \\ \hline \end{gathered}\right.$ | $\left\|\begin{array}{c} 0 \\ \mathbf{e} \\ \mathbf{e} \\ \mathbf{c} \end{array}\right\|$ |  | $\left\|\begin{array}{c} \infty \\ 2 \\ 0 \\ 0 \\ 0 \\ \end{array}\right\|$ |  | $\begin{gathered} \infty \\ \infty \\ \stackrel{\infty}{4} \\ 0 \end{gathered}$ | $\left\|\begin{array}{c} \underset{0}{0} \\ \vdots \\ \dot{d} \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{l} 9 \\ \stackrel{0}{0} \\ \vdots \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \stackrel{0}{2} \\ \underset{\sim}{3} \end{array}\right\|$ | 諦 |
| $\left\lvert\, \begin{gathered} \stackrel{\infty}{9} \\ = \\ = \end{gathered}\right.$ | $\begin{gathered} \frac{y}{\infty} \\ = \end{gathered}$ | $\stackrel{C}{c}$ | $\begin{aligned} & 8 \\ & n \\ & n \\ & \hdashline \end{aligned}$ | $\left\lvert\, \begin{gathered} \underset{\sim}{c} \\ \underset{m}{2} \end{gathered}\right.$ | $\hat{c}$ |  | $\hat{i} \mid$ | $\frac{7}{4}$ | $\stackrel{0}{\infty}$ | $\dot{c}$ | $\stackrel{\sim}{\sim}$ | $\begin{gathered} n \\ \underset{\sim}{7} \end{gathered}$ | $\left\|\begin{array}{c} \hat{5} \\ 0 \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \hat{G} \\ \stackrel{\rightharpoonup}{a} \end{array}\right\|$ | $\left\lvert\, \begin{array}{\|c} \stackrel{\rightharpoonup}{0} \\ \hline \end{array}\right.$ | $\left\|\begin{array}{l} n \\ \underset{子}{f} \end{array}\right\|$ | $\left\|\begin{array}{c} o \\ \vdots \\ \infty \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \vec{~} \\ \stackrel{a}{\infty} \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \hat{2} \\ \infty \\ \infty \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \stackrel{r}{\underset{\sim}{2}} \\ \underset{\sim}{2} \end{gathered}\right.$ | $\left\|\begin{array}{l} \vec{m} \\ \underset{m}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{n} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ \underset{1}{0} \end{array}\right\|$ |  | $\left\|\begin{array}{c} n \\ \tilde{y} \\ \underset{\theta}{0} \end{array}\right\|$ |  |
| $\left\|\frac{9}{\infty}\right\|$ | $\left\|\begin{array}{c} \infty \\ 0 \\ \infty \\ \infty \end{array}\right\|$ | $\begin{gathered} \infty \\ \stackrel{i}{2} \\ \sim \end{gathered}$ | $\stackrel{\rightharpoonup}{\mathbf{a}} \underset{\sim}{c}$ | $\left\|\begin{array}{c} \circ \\ \stackrel{0}{\infty} \\ \dot{n} \end{array}\right\|$ | $\begin{array}{l\|l\|} 0 \\ 0 \\ i & 0 \\ i n \\ i n \end{array}$ | $6\left\|\begin{array}{l} 2 \\ 2 \\ 0 \end{array}\right\|$ |  | $\mathfrak{c}$ |  | $\dot{\alpha}$ | $\begin{array}{\|c\|} \hline \frac{g}{9} \\ \hline \end{array}$ | $\begin{array}{\|c} \hline \stackrel{0}{2} \\ \underset{n}{2} \end{array}$ | $\begin{gathered} \stackrel{\rightharpoonup}{m} \\ \underset{\sim}{2} \end{gathered}$ | $\underset{\sim}{n}$ | $\left\|\begin{array}{c} \underset{\sim}{\omega} \\ \infty \\ \infty \end{array}\right\|$ | $\begin{aligned} & m \\ & \\ & \end{aligned}$ | $\left\|\begin{array}{c} \infty \\ o \\ o \\ \vdots \end{array}\right\|$ | $\left\|\begin{array}{c} \vec{t} \\ \stackrel{\rightharpoonup}{0} \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c}  \pm \\ \stackrel{\rightharpoonup}{\infty} \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \vec{\infty} \\ \substack{\infty \\ \infty} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{c} \\ \underset{m}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} n \\ \underset{m}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} 2 \\ \underset{\sim}{n} \\ \underset{m}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} n \\ \infty \\ \infty \\ n \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{\sim} \\ \underset{\sim}{2} \end{array}\right\|$ | $\stackrel{4}{4}$ |
| $\left\|\begin{array}{l} \infty \\ \substack{\infty \\ 0 \\ 0} \end{array}\right\|$ | $\left\|\begin{array}{l} \hat{0} \\ 0 \\ 0 \end{array}\right\|$ | $\stackrel{\substack{0}}{ }$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{gathered} \frac{7}{a} \\ 9 \end{gathered}$ | $\begin{array}{c\|c} t \\ \stackrel{\rightharpoonup}{0} \\ \underset{\sim}{2} \end{array}$ |  |  | $\left.\begin{array}{r} \bar{r} \\ \bar{m} \end{array} \right\rvert\,$ | $\hat{i}$ | $\underset{\sim}{\infty}$ | $\begin{array}{\|c} \vec{a} \\ \underset{\sim}{2} \end{array}$ | $\stackrel{\underset{c}{\underset{~}{2}}}{\substack{n}}$ | $\stackrel{\rightharpoonup}{\infty}$ | $\left\|\begin{array}{c} 0 \\ \stackrel{0}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \mathfrak{2} \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} \hat{y} \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} ⿱ 士 口 寸 \\ \text { in } \end{array}\right\|$ | $\left\|\begin{array}{l} x \\ \underset{1}{n} \\ \end{array}\right\|$ | $\begin{array}{\|c\|} \hline \stackrel{\sim}{n} \\ \underset{\sim}{2} \end{array}$ | $\stackrel{\bar{n}}{\stackrel{n}{n}}$ | $\left\|\begin{array}{c} \underset{\underset{i}{2}}{ } \end{array}\right\|$ | $\left.\begin{array}{\|c\|} \hline \underset{o}{\infty} \\ \underset{i}{ } \end{array} \right\rvert\,$ | $\left\|\right\|$ | $\stackrel{\substack{\mathrm{c} \\ \underset{\sim}{n} \\ \hline}}{ }$ | $\left\|\begin{array}{c} t \\ \stackrel{t}{0} \\ \mathrm{c} \end{array}\right\|$ | $\stackrel{\square}{2}$ |
| $\left\|\begin{array}{c} \hat{O} \\ \stackrel{c}{0} \\ 0 \end{array}\right\|$ | $\begin{aligned} & \text { c } \\ & \text { co } \end{aligned}$ | $\stackrel{\substack{e \\ 0}}{ }$ | $\hat{0}$ | $0 \begin{gathered} \infty \\ \infty \\ 0 \\ 0 \\ \hline \end{gathered}$ |  | $\begin{array}{c\|c\|c} \substack{2 \\ 0 \\ 0 \\ 0} \\ 0 \end{array}$ | $\begin{array}{l\|l\|} \hline & \stackrel{a}{n} \\ 0 & \tilde{0} \\ \hline \end{array}$ | $5 \frac{4}{0}$ | E | $a_{0}^{0}$ | $\begin{gathered} \infty \\ \stackrel{8}{子} \\ 0 \end{gathered}$ | $\left\|\begin{array}{l} \bar{\infty} \\ n \\ \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \infty \\ 0 \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \tilde{\sim} \\ \text { in } \\ \hline \end{gathered}\right.$ | $\left\|\begin{array}{c} \tilde{q} \\ \underset{寸}{子} \end{array}\right\|$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\left\lvert\, \begin{aligned} & 0 \\ & \infty \\ & 0 \\ & 0 \end{aligned}\right.$ | $\begin{array}{\|c} \stackrel{\rightharpoonup}{6} \\ 0 \\ 0 \end{array}$ | $\left\lvert\, \begin{gathered} \underset{\sim}{\underset{~}{2}} \end{gathered}\right.$ | I | $\left\|\begin{array}{l} 8 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} 2 \\ \frac{2}{5} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\text { cin }}{0} \end{array}\right\|$ | $\stackrel{\infty}{9}$ | $\begin{array}{\|c\|} \hline \frac{m}{0} \\ \hline \end{array}$ | － |
| $\left\|\begin{array}{c} \text { ren } \\ 0 \\ 0 \end{array}\right\|$ | $0$ | $0$ | $0$ | $\begin{aligned} & 0 \\ & \vdots \\ & \vdots \end{aligned}$ | $\begin{array}{l\|l\|} 0 \\ 0 & 0 \\ \hline & 0 \\ 0 \end{array}$ | $\begin{array}{l\|l\|} \substack{0 \\ \hline \\ 0 \\ \hline \\ \hline \\ \hline \\ \hline} & \end{array}$ | $\begin{array}{l\|l\|} \hline & 0 \\ 0 & \\ 0 & 0 \end{array}$ | $\overbrace{5}^{9} \underset{0}{\infty}$ | $\theta_{0}$ | $\hat{0}_{0}^{\infty} \underset{\sim}{\infty}$ | $\begin{aligned} & i n \\ & 0 \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{aligned} & \mathbf{士} \\ & \substack{d \\ \vdots} \end{aligned}\right.$ | $\left\|\begin{array}{c} \underset{\sim}{m} \\ \underset{\sim}{2} \end{array}\right\|$ | 寺 | $\begin{array}{\|c} 7 \\ 7 \\ 0 \end{array}$ | $\left\lvert\, \begin{gathered} \left.\begin{array}{c} \mathrm{n} \\ \hat{n} \\ 0 \end{array} \right\rvert\, \end{gathered}\right.$ | $\left\lvert\, \begin{gathered} \underset{8}{\infty} \\ \underset{8}{2} \end{gathered}\right.$ | $\stackrel{\pi}{8}$ | $\left\|\begin{array}{c} 0 \\ 0 \\ -1 \end{array}\right\|$ | $\begin{aligned} & N \\ & \infty \\ & \infty \end{aligned}$ | $\left\|\begin{array}{c} \infty \\ 0 \\ 0 \\ \hline \end{array}\right\|$ | $\left\|\frac{9}{9}\right\|$ | $\left\lvert\, \begin{gathered} \infty \\ \stackrel{\infty}{0} \\ \hline \end{gathered}\right.$ | $\stackrel{n}{0}$ | $\stackrel{0}{0}$ | $\stackrel{\text { 2 }}{\sim}$ |
| $\left.\begin{aligned} & \hat{\lambda} \\ & \vec{~} \end{aligned} \right\rvert\,$ | $\begin{gathered} 7 \\ 0 \\ 0 \end{gathered}$ | $\left\|\begin{array}{c} \underset{0}{c} \\ \underset{i}{c} \end{array}\right\|$ | $\begin{gathered} 0 \\ \vdots \\ \vdots \end{gathered}$ | $\stackrel{o}{6}$ | $\begin{array}{c\|c} 0 \\ 0 \\ 0 \\ n \\ i \\ i \end{array}$ |  |  | $\stackrel{?}{i}$ | $\begin{gathered} i \\ i \end{gathered} \left\lvert\, \begin{gathered} 8 \\ \vdots \\ \hline \end{gathered}\right.$ |  | $\begin{gathered} \text { N} \\ \underset{\substack{j}}{ } \end{gathered}$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} \hat{0} \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{gathered} \text { c } \\ \text { ç } \\ \text { ¢ } \end{gathered}$ | $\begin{array}{\|c} 2 \\ \vdots \\ \vdots \\ \hline \end{array}$ | $\begin{array}{r} 0 \\ 0 \\ 0 \\ 0 \end{array}$ |  | $\left\|\begin{array}{c} m \\ 0 \\ 0 \\ \dot{c} \end{array}\right\|$ | $\left\|\begin{array}{l} a \\ \underset{\sim}{2} \\ \vdots \\ 0 \end{array}\right\|$ | $\left(\begin{array}{c} \infty \\ \infty \\ \infty \\ \infty \\ \infty \end{array}\right)$ | $\begin{array}{\|c\|} \hline 8 \\ \frac{n}{7} \\ \hline \end{array}$ | $\begin{aligned} & \frac{\infty}{7} \\ & \stackrel{y}{f} \end{aligned}$ | $\begin{gathered} \stackrel{\rightharpoonup}{a} \\ \stackrel{寸}{寸} \end{gathered}$ | $\begin{aligned} & \vec{\infty} \\ & \stackrel{\infty}{\infty} \\ & \underset{寸}{\prime} \end{aligned}$ | $\begin{aligned} & \hline \stackrel{\rightharpoonup}{0} \\ & \stackrel{\sim}{寸} \end{aligned}$ | － |
| $\begin{array}{\|l\|l} \hat{0} \\ \tilde{o} \\ \underset{\sim}{2} \end{array}$ |  | $\begin{array}{\|l\|} \hat{0} \\ 0 \\ \dot{0} \\ \hline \end{array}$ |  |  |  |  | $\begin{array}{l\|l\|} \hline 0 \\ 0 & 0 \\ 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ | $\begin{aligned} & 6 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $6$ | $\dot{c}$ | $\begin{aligned} & \tilde{\sim} \\ & \underset{\sim}{2} \\ & \text { ת } \end{aligned}$ | $$ | $\left.\begin{array}{\|l\|} \hline \hat{0} \\ 0 \\ 9 \end{array} \right\rvert\,$ | $\overrightarrow{\vec{\sigma}}$ | $\begin{array}{\|c} \hat{0} \\ 2 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \tilde{2} \\ \underset{\sim}{2} \\ \hline \end{gathered}$ | $\begin{gathered} \underline{a} \\ 0 \\ \underset{\infty}{\dot{x}} \\ \hline \end{gathered}$ | $\begin{gathered} n \\ n \\ \underline{\infty} \\ \underline{n} \\ \hline \end{gathered}$ | $\begin{aligned} & 0 \\ & \vdots \\ & \vdots \\ & 0 \end{aligned}$ | $\begin{aligned} & \bar{n} \\ & \stackrel{\rightharpoonup}{\lambda} \end{aligned}$ | $\begin{aligned} & \exists \\ & \vdots \\ & 0 \\ & \hline \end{aligned}$ | $\begin{array}{l\|} \hline 6 \\ \dot{6} \\ \dot{n} \end{array}$ | $$ | $\begin{aligned} & \underset{\sim}{\mathbf{N}} \\ & \underset{\sim}{c} \end{aligned}$ | － |
| $\begin{array}{\|c} 9 \\ 9 \\ 9 \end{array}$ | $\begin{aligned} & 1 \\ & \hline \\ & \\ & \hline \end{aligned}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \\ & \hline 1 \end{aligned}$ | $\begin{gathered} \text { Bn } \\ \hline \end{gathered}$ | $\underset{i}{c}$ |  |  |  | $\begin{gathered} t \\ \hline \end{gathered}$ | $\stackrel{c}{n}$ | $\begin{gathered} 2 \\ \vdots \\ i \end{gathered}$ | $\left[\begin{array}{l} \infty \\ \stackrel{0}{0} \\ 0 \end{array}\right.$ | $$ | $6$ | $\begin{array}{\|l\|l} \hline 0 \\ \vdots \\ \vdots \\ \hline \end{array}$ | $\begin{aligned} & 2 \\ & 0 \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{gathered} \text { I } \\ 0 \\ i \end{gathered}$ | $\left\lvert\, \begin{gathered} 0 \\ 0 \\ 0 \\ \hline \end{gathered}\right.$ | $\begin{aligned} & 1 \\ & 0 \\ & \vdots \\ & \vdots \end{aligned}$ | $\left\lvert\, \begin{aligned} & \mathrm{t} \\ & 0 \\ & i \end{aligned}\right.$ | $\begin{aligned} & \overrightarrow{0} \\ & \stackrel{1}{i} \end{aligned}$ | $\begin{aligned} & \stackrel{0}{6} \\ & \stackrel{0}{6} \end{aligned}$ | $\begin{aligned} & \hat{6} \\ & \stackrel{i}{i} \end{aligned}$ | $\begin{aligned} & \text { ion } \\ & 0 \\ & \hline \end{aligned}$ | $\begin{gathered} \underset{\sim}{6} \\ \underset{i}{1} \end{gathered}$ | $\begin{aligned} & n \\ & \stackrel{n}{n} \\ & i \end{aligned}$ | ¢ |
| $\left.\begin{aligned} & \underset{寸}{寸} \\ & \underset{~}{~} \end{aligned} \right\rvert\,$ |  | $\begin{aligned} & \overrightarrow{0} \\ & \text { a } \\ & \underset{\sim}{ \pm} \end{aligned}$ | $\underset{\sim}{c}$ |  |  |  |  | $\frac{3}{2}$ |  |  | $\begin{array}{\|c} \hline 0 \\ \stackrel{\infty}{2} \\ \stackrel{3}{9} \\ \hline \end{array}$ |  | $\begin{gathered} c \\ \\ \vdots \\ \vdots \\ \hline \end{gathered}$ | $\begin{aligned} & \frac{m}{2} \\ & \frac{2}{1} \end{aligned}$ | $\begin{aligned} & \substack{\infty \\ n \\ \underset{y}{2} \\ \hline} \end{aligned}$ | $\begin{aligned} & \frac{n}{3} \\ & \text { din } \end{aligned}$ | $\begin{aligned} & \text { A } \\ & 2 \\ & \end{aligned}$ | $0$ | $\begin{gathered} \frac{m}{\infty} \\ \substack{9} \end{gathered}$ | $\left(\begin{array}{l} \infty \\ m \\ \cdots \\ \underset{\sim}{2} \end{array}\right.$ | $\stackrel{\substack{\underset{\infty}{\infty} \\ \underset{1}{4} \\ \hline \\ \hline}}{ }$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{6} \\ & \stackrel{1}{1} \end{aligned}$ | $\left\|\begin{array}{c} \underset{\sim}{c} \\ \underset{\sim}{\infty} \end{array}\right\|$ | $\begin{aligned} & 0 \\ & \stackrel{0}{6} \\ & \stackrel{0}{\circ} \end{aligned}$ | $\dot{8}$ | $\stackrel{+}{0}$ |
| － |  | $\left\|\begin{array}{c} \approx \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\underset{i c}{\text { an }}$ |  |  |  |  | $\begin{array}{l\|l\|} \hline & n \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ | $\begin{array}{l\|l\|} \hline 6 & 0 \\ 0 & 0 \\ 6 & 0 \\ 0 & 0 \\ 0 & 0 \end{array}$ | $\begin{array}{l\|l\|} \hline 0 & \overline{0} \\ 0 & 0 \\ 0 \\ 0 & 0 \\ 0 \\ 0 \\ \hline \end{array}$ |  |  |  | $\begin{aligned} & \infty \\ & \stackrel{0}{2} \\ & \stackrel{3}{6} \\ & 0 \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{2} \\ & 0 \\ & 0 \\ & \hline 0 \end{aligned}$ | $$ | $\begin{aligned} & 9 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |  |  |  | $\begin{aligned} & \vec{F} \\ & \stackrel{\infty}{\underset{~}{A}} \end{aligned}$ |  | $\begin{aligned} & \overrightarrow{0} \\ & \stackrel{0}{2} \\ & \stackrel{\rightharpoonup}{n} \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\infty} \\ & \underset{\sim}{\infty} \\ & \vdots \end{aligned}$ | $\stackrel{\rightharpoonup}{t}$ |
| $\stackrel{J}{=}$ | $\frac{9}{3}$ | $\underline{0}$ | $\underset{\mathrm{c}}{\mathrm{E}} \underset{\mathrm{E}}{\mathrm{E}}$ | $=\frac{\infty}{\bar{c}}=$ | $\stackrel{\infty}{\bar{c}}$ | a\|c| | $\stackrel{\rightharpoonup}{3}$ | $\underset{0}{4} \underset{y}{c}$ | $\underset{\sim}{2} \underset{\sim}{2}$ | Cix | $\underset{\sim}{t} \mid$ | $\begin{gathered} 9 \\ \hline 0 \end{gathered}$ | $9$ | Eve | $\begin{aligned} & \text { a } \\ & \underline{3} \\ & \hline \end{aligned}$ | $\frac{8}{2}$ | $\begin{gathered} \bar{m} \\ \underline{e} \\ \hline \end{gathered}$ | $\frac{\stackrel{c}{m}}{2}$ | $\frac{m}{2}$ | $\stackrel{\frac{4}{2}}{\mathrm{~m}}$ | $\begin{gathered} \dot{m} \\ \underline{a} \end{gathered}$ | $\stackrel{\circ}{2}$ | $\begin{gathered} \hat{m} \\ \underline{\omega} \end{gathered}$ | $\stackrel{\infty}{e}$ | $\stackrel{m}{m}$ | ㅇㅏㅢ |


| $\stackrel{+}{\infty}$ | $\left\|\begin{array}{c} \infty \\ \stackrel{n}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ \infty \\ \infty \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \infty \\ \infty \\ \underset{\infty}{\infty} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ 0 . \\ 0 \end{array}\right\|$ | $\|\stackrel{\circ}{\dot{C}}\|$ | $\begin{aligned} & \vec{i} \\ & \stackrel{\rightharpoonup}{2} \end{aligned}$ | $\stackrel{c}{i}$ | $\stackrel{N}{\hat{6}}$ | $\stackrel{\text { Nin }}{\underset{\sim}{2}}$ | $\left\|\begin{array}{l} \underset{\sim}{n} \\ \infty \end{array}\right\|$ | $\stackrel{\hat{\partial}}{\infty}$ | $\left\|\begin{array}{c} \hat{0} \\ \stackrel{8}{\circ} \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ \dot{j} \\ \dot{\alpha} \end{array}\right\|$ | $\begin{gathered} \underset{i}{c} \\ \dot{\alpha} \end{gathered}$ | $\underset{\sim}{x}$ | $\stackrel{\substack{\infty \\ \infty}}{\infty}$ | $\begin{aligned} & \text { M } \\ & \underset{\infty}{\infty} \end{aligned}$ | $\begin{aligned} & 0 \\ & \substack{\dot{\infty} \\ \infty} \end{aligned}$ | $$ | $\underset{\infty}{\infty}$ | $\vdots \underset{\infty}{\substack{\infty}}$ | $\begin{aligned} & \stackrel{\bullet}{n} \\ & \underset{\sim}{n} \end{aligned}$ | $0$ | $\underset{\sim}{4}$ | $\left\lvert\, \begin{gathered} \mathrm{m} \\ \stackrel{5}{5} \end{gathered}\right.$ | n |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\frac{a}{\lambda}$ | $\left\lvert\, \begin{gathered} \infty \\ \stackrel{\infty}{\lambda} \\ \hline \end{gathered}\right.$ | $\left\|\begin{array}{c} a \\ \stackrel{\rightharpoonup}{n} \\ i \end{array}\right\|$ | $\left\|\begin{array}{c} \text { m} \\ \stackrel{y}{e} \\ \stackrel{n}{n} \end{array}\right\|$ |  | $\frac{\infty}{0}$ | $\begin{array}{\|c} \stackrel{\rightharpoonup}{0} \\ \stackrel{0}{n} \end{array}$ |  | $\left.\begin{gathered} \frac{8}{n} \\ \frac{1}{n} \end{gathered} \right\rvert\,$ | $\left.\frac{7}{2} \right\rvert\,$ |  | $\left\lvert\, \begin{gathered} 0 \\ 0 \\ \\ \end{gathered}\right.$ | $\left\lvert\, \begin{gathered} \hat{n} \\ \frac{\infty}{\infty} \\ i \end{gathered}\right.$ | $\left\|\begin{array}{c} \text { d } \\ \text { on } \\ i n \end{array}\right\|$ | $\begin{gathered} 0 \\ \substack{0 \\ m \\ i} \end{gathered}$ |  | $\begin{gathered} 0 \\ \frac{0}{m} \\ \frac{\infty}{n} \end{gathered}$ | $\frac{0}{2}$ | $\frac{o}{\frac{o}{0}}$ | $\stackrel{e}{2}$ | $\begin{aligned} & \check{e} \\ & \stackrel{0}{\infty} \\ & \stackrel{n}{n} \end{aligned}$ |  | $\begin{gathered} 4 \\ \stackrel{\rightharpoonup}{0} \\ i \\ \hline \end{gathered}$ |  | $\begin{gathered} \underset{\sim}{\infty} \\ \infty \\ \sim \\ \sim \end{gathered}$ | $\left\|\begin{array}{c} \vec{\infty} \\ \underset{\infty}{\infty} \\ \stackrel{\rightharpoonup}{n} \end{array}\right\|$ | $\stackrel{\infty}{0}$ |
| $\begin{array}{\|c} \hline 0 \\ \stackrel{0}{4} \\ \stackrel{9}{9} \end{array}$ | $\left\|\begin{array}{c} \underset{0}{c} \\ \underset{\substack{c}}{ } \end{array}\right\|$ | $\left\|\begin{array}{c} \overrightarrow{0} \\ \frac{\underset{\sim}{e}}{} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \underset{\sim}{f} \\ \underset{\sim}{7} \\ \hline \end{gathered}\right.$ | $\begin{aligned} & \text { 兴 } \\ & \vdots \\ & \text { n } \end{aligned}$ | $\begin{aligned} & \stackrel{ल}{m} \\ & \stackrel{\rightharpoonup}{\mathrm{a}} \end{aligned}$ | $\begin{gathered} \substack{9 \\ \vdots \\ \vdots \\ \mathbf{c} \\ \hline} \end{gathered}$ |  | $m$ 0 0 0 |  |  | $\mathfrak{n}$ | $\left\lvert\, \begin{gathered} \mathrm{J} \\ \underset{\sim}{\mathrm{~N}} \end{gathered}\right.$ |  | $\begin{aligned} & \underset{\sim}{\sigma} \\ & \underset{\sim}{m} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{gathered} n \\ 0 \\ 0 \\ 2 \\ 2 \\ \end{gathered}$ | $\left\|\begin{array}{c} 0 \\ \underset{\sim}{\infty} \\ \underset{\sim}{0} \end{array}\right\|$ | $$ |  | $\hat{i}$ | $\begin{aligned} & \infty \\ & \stackrel{\sim}{2} \\ & \stackrel{c}{2} \end{aligned}$ | $\hat{h} \hat{h}$ |  |  | $\begin{aligned} & \text { N} \\ & \text { n } \\ & \text { and } \end{aligned}$ | $\left\|\begin{array}{l} \infty \\ \stackrel{\circ}{0} \\ \vdots \\ \stackrel{9}{0} \end{array}\right\|$ | $\begin{aligned} & \text { M } \\ & \stackrel{\rightharpoonup}{4} \\ & \stackrel{y}{4} \end{aligned}$ |
| $\stackrel{+}{2}$ | $\left\lvert\, \begin{aligned} & 0 \\ & 6 \\ & 0 \\ & 0 \end{aligned}\right.$ | $\begin{aligned} & \vec{\infty} \\ & \underset{\infty}{8} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\stackrel{\sim}{\infty}$ | $\begin{aligned} & 0 \\ & \stackrel{\infty}{0} \end{aligned}$ | $\stackrel{\substack{\infty \\ \stackrel{\infty}{\bullet} \\ \stackrel{y}{2} \\ \hline}}{ }$ | $\stackrel{\rightharpoonup}{a}$ | $E \begin{gathered} n \\ \infty \\ = \\ = \end{gathered}$ | $\begin{gathered} c \\ 2 \\ 2 \\ 2 \\ \end{gathered}$ |  | $\begin{gathered} \infty \\ \underset{\sim}{m} \\ \underset{\sim}{2} \end{gathered}$ | $\underset{\substack{\vec{⿳} \\ \underset{\sim}{2} \\ \hline}}{ }$ | $\begin{aligned} & 0 \\ & \stackrel{0}{2} \\ & \end{aligned}$ |  | $$ | $\stackrel{9}{9}$ | $\begin{array}{\|c} 8 \\ 0 \\ 0 \end{array}$ | $\begin{aligned} & \stackrel{e}{0} \\ & \stackrel{0}{0} \end{aligned}$ | $\stackrel{8}{7}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{巳} \\ & \underset{\sim}{j} \end{aligned}$ | $\underset{\sim}{c}$ |  | $\begin{gathered} \hat{6} \\ \underset{\sim}{2} \end{gathered}$ | $\begin{gathered} 0 \\ \stackrel{e}{n} \\ \underset{\sim}{2} \end{gathered}$ | $\stackrel{\sim}{\infty}$ | $\xrightarrow{2}$ |
| $\left\|\begin{array}{l} \frac{士}{2} \\ \stackrel{n}{n} \end{array}\right\|$ | $\begin{gathered} \frac{9}{7} \\ 6 \end{gathered}$ | $\left\|\begin{array}{l} \infty \\ \infty \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ n \\ \infty \end{array}\right\|$ | E | $\stackrel{F}{\vec{G}}$ | $\stackrel{\infty}{\infty} \underset{\substack{\infty \\ 子 \\ \hdashline}}{ }$ |  | $\begin{array}{c\|c\|} \hline & \stackrel{a}{2} \\ \dot{a} \\ \dot{m} \end{array}$ | $\underset{\sim}{\infty}\left\|\begin{array}{c} \infty \\ \underset{\sim}{c} \\ \underset{\sim}{n} \end{array}\right\|$ | $\underset{\sim}{4} \mid \stackrel{0}{0}$ |  | $\frac{m}{m}$ | $0 \begin{gathered} n \\ 6 \\ 0 \end{gathered}$ | Sin | $\hat{i}$ | $\begin{array}{\|c} \hat{y} \\ \hline \end{array}$ | ה | $\stackrel{\rightharpoonup}{+}$ |  | $\begin{aligned} & 0 \\ & \infty \\ & 0 \\ & 0 \end{aligned}$ |  | $\frac{6}{6}$ | $\underset{f}{6} \underset{\sim}{\infty}$ | $\begin{gathered} 9 \\ \substack{4 \\ 子} \end{gathered}$ | $\begin{aligned} & \hat{0} \\ & \underset{子}{7} \end{aligned}$ | $\stackrel{\sim}{\circ}$ |
| $\begin{gathered} \text { a } \\ \underset{\sim}{2} \\ \hline \end{gathered}$ | $\left\|\begin{array}{c} \dot{\sim} \\ \underset{\sim}{n} \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}\right.$ | $\left\|\begin{array}{c} n \\ \infty \\ \infty \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \vec{n} \\ \underset{n}{n} \end{gathered}\right.$ | $\underset{\sim}{7}$ | $\frac{\tilde{m}}{\underset{\sim}{\sim}}$ |  |  | $\begin{array}{c\|c} 0 \\ \\ \\ i \end{array}$ | $\begin{array}{l\|l\|} 0 & n_{1} \\ i & 0 \\ i & 0 \end{array}$ |  | $\overline{\bar{n}}$ | $\begin{gathered} \text { 合 } \\ i \\ i \end{gathered}$ |  | $\stackrel{i}{\mathrm{~m}}$ | $\frac{\infty}{\frac{\infty}{n}}$ |  | $\begin{aligned} & 0 \\ & \stackrel{0}{6} \\ & \end{aligned}$ |  | $\begin{aligned} & \hat{n} \\ & 0 \\ & i \end{aligned}$ | $\begin{array}{\|c} \hline \infty \\ \substack{\infty \\ 0} \end{array}$ | $\begin{gathered} 0 \\ 0 \\ \hline \end{gathered}$ |  | $\begin{aligned} & \infty \\ & \stackrel{\infty}{n} \\ & \underset{m}{2} \end{aligned}$ | $\begin{aligned} & \underset{F}{7} \\ & \underset{\sim}{2} \end{aligned}$ |  |
| 热 | $\stackrel{\infty}{\infty} \underset{\sim}{2} \mid$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | 융 | $\begin{gathered} \mathfrak{F} \\ 0 \end{gathered}$ | $\begin{array}{\|c} \infty \\ \stackrel{\infty}{0} \\ \hline \end{array}$ | $0$ | $0$ | $$ |  | $\begin{array}{c\|c} 2 \\ 0 & 0 \\ 0 \\ \hline \end{array}$ | $\begin{array}{\|l\|l\|} \hline 8 \\ 0 & 0 \\ 0 \\ 0 & 0 \end{array}$ | $\begin{gathered} \overrightarrow{0} \\ 0 \\ 0 \end{gathered}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{3} \\ & \hline \end{aligned}$ |  | $\underset{b}{6}$ | $\begin{gathered} \infty \\ \stackrel{\sim}{3} \\ 0 \end{gathered}$ | Co | $\stackrel{n}{n}$ |  | $\begin{aligned} & 5 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & \hline \\ & \hline \end{aligned}$ | $\begin{gathered} 8 \\ 8 \\ 0 \\ \hline \end{gathered}$ | $\frac{2}{5}$ | En | $\begin{aligned} & \text { त } \\ & 0 \\ & 0 \end{aligned}$ | 층 |
| 为 | $\frac{\infty}{\infty}$ | $\begin{aligned} & 2 \\ & \underset{0}{2} \end{aligned}$ | $0$ |  | $\begin{array}{\|c} \underset{\sim}{\sim} \\ \tilde{0} \end{array}$ | $0$ | $2$ | $=\frac{m}{2}$ | $\begin{array}{l\|l\|} \hat{2} & \hat{0} \\ 0 & 0 \end{array}$ | $$ |  | $\left\|\begin{array}{l} \overrightarrow{0} \\ 0 \end{array}\right\|$ | 昌 | $\begin{gathered} \mathrm{w} \\ \stackrel{\omega}{0} \end{gathered}$ | $\underbrace{9}_{0}$ | $\begin{aligned} & 0 \\ & \hline \end{aligned}$ |  | $\begin{gathered} 0 \\ \stackrel{0}{2} \\ 0 \end{gathered}$ |  | $\begin{array}{c\|c} \hat{n} \\ \hline \end{array}$ | $\begin{gathered} \hat{e} \\ \stackrel{\rightharpoonup}{2} \\ \hline \end{gathered} \underset{\sim}{\infty}$ | $8$ | $\frac{2}{8}$ | $\dot{e}$ | $\left\lvert\, \begin{gathered} \tilde{c} \\ 0 \\ \hline \end{gathered}\right.$ | － |
| $\begin{array}{\|l\|l} \hline \overrightarrow{0} \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{aligned} & 0 \\ & 7 \\ & 6 \end{aligned}$ | $\begin{gathered} \tilde{c}_{2} \\ \underset{c}{c} \end{gathered}$ | $\begin{array}{\|c} \infty \\ \infty \\ 0 \\ 0 \\ 0 \end{array}$ | $0$ | $8$ |  |  |  |  |  | $\begin{array}{l\|l\|} \hline \end{array}$ | $\begin{gathered} \stackrel{\infty}{\infty} \\ \underset{\sim}{\infty} \\ \infty \end{gathered}$ | $\begin{gathered} 9 \\ \stackrel{y}{2} \\ 5 \end{gathered}$ | $\begin{aligned} & \overline{0} \\ & \stackrel{0}{n} \end{aligned}$ |  | $\begin{gathered} \hat{0} \\ \text { Co } \\ 0 \end{gathered}$ |  | $6$ |  |  | $e_{i}^{6}$ | $\begin{gathered} 6 \\ \hline \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{gathered} t \\ \hline \end{gathered}$ | $\begin{gathered} 9 \\ \\ \hdashline \\ \hline \end{gathered}$ | $\begin{aligned} & \infty \\ & \frac{\infty}{R} \\ & \dot{q} \end{aligned}$ | 宕 |
| $\begin{array}{\|l\|l} \hline 5 \\ 0 \\ 0 \\ \hline \end{array}$ | $\left[\begin{array}{l} 7 \\ \infty \\ 0 \\ 0 \\ C \end{array}\right.$ | $\begin{aligned} & \text { ed } \\ & \text { I } \end{aligned}$ | $\begin{array}{\|c} 8 \\ \vdots \\ \pm \\ \hline \end{array}$ | $\begin{aligned} & \hat{6} \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |  |  |  | $\begin{array}{l\|l} n \\ 0 & 2 \\ 0 \\ 0 & 2 \\ \hline \end{array}$ |  |  | $\begin{array}{\|l} \hline 0 \\ \dot{8} \\ \underset{\sim}{8} \end{array}$ | 8 0 0 0 0 |  | $\stackrel{\rightharpoonup}{2} \stackrel{ \pm}{ \pm}$ | $\begin{aligned} & \stackrel{8}{0} \\ & \underset{i}{i} \\ & \hat{S} \end{aligned}$ |  |  | $\begin{gathered} 6 \\ \infty \\ \infty \\ \infty \\ \infty \end{gathered}$ |  |  |  |  | $c_{i}^{x}$ |  | 菏 |
| $\begin{array}{\|l} \hline 8 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & 1 \\ & \underset{C}{1} \\ & \text { in } \end{aligned}$ | $\begin{gathered} \hat{9} \\ \underset{\substack{2}}{ } \end{gathered}$ | $\begin{aligned} & \text { y } \\ & \hline \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \vdots \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{array}{l\|l\|} \hline 0 & \overrightarrow{0} \\ \\ \hline \end{array}$ | $\begin{array}{l\|l\|} \hline 0 \\ \hat{0} \\ \hat{n} \\ \hat{n} \\ \hline \end{array}$ | $\begin{array}{c\|c} 0 \\ \hat{6} \\ i \\ \hline \end{array}$ |  |  | $\begin{array}{\|l} \hline 0.0 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & \circ \\ & 0 \\ & \hline \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 9 \\ & \hline \\ & \vdots \end{aligned}$ | $\begin{aligned} & \overrightarrow{0} \\ & 0 \\ & 0 \end{aligned}$ |  | $\begin{gathered} 6 \\ \vdots \\ \vdots \end{gathered} \frac{x}{c}$ |  |  | $\begin{gathered} \text { ה } \\ \vdots \\ \hline \end{gathered}$ | $$ |  | $\begin{aligned} & 8 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { O } \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | － |
| $\begin{aligned} & \vec{\circ} \\ & \infty \\ & \underset{\sim}{\infty} \end{aligned}$ |  | $\left(\begin{array}{c} \frac{2}{c} \\ \frac{2}{7} \end{array}\right.$ | $\begin{gathered} 0 \\ 2 \\ \\ \hline \end{gathered}$ |  |  | $\begin{gathered} \text { g } \\ \text { g } \end{gathered}$ |  |  | $\begin{array}{l\|l} 7 & \overline{0} \\ 0 & 0 \\ 0 & 0 \end{array}$ |  |  |  | $\begin{aligned} & \text { O} \\ & \text { d } \\ & \end{aligned}$ |  |  | $\begin{aligned} & \vec{~} \\ & \vec{i} \\ & \vec{~} \end{aligned}$ |  |  |  |  |  | $\underset{\sim}{\underset{\sim}{c}} \underset{\substack{2 \\ \hline}}{2}$ |  |  |  | （1） |
| $$ | $\begin{aligned} & \hat{\infty} \\ & 0 \\ & \stackrel{\theta}{0} \\ & \stackrel{0}{0} \end{aligned}$ | $0 \begin{gathered} 7 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\begin{array}{\|l\|} \hline \\ \hline 0 \\ \infty \\ \infty \\ 0 \\ \hline \end{array}$ | $\begin{gathered} n \\ 0 \\ 0 \\ 0 \\ \dot{c} \\ \dot{c} \\ 0 \end{gathered}$ |  | $\hat{a}$ | $3 \begin{gathered} 0 \\ 2 \\ 0 \\ \vdots \\ \vdots \end{gathered}$ | $\begin{aligned} & m \\ & \vdots \\ & \vdots \end{aligned}$ | $\underset{y}{c}\left\|\begin{array}{c} \bar{\infty} \\ \infty \\ \end{array}\right\|$ |  |  |  |  | $\begin{array}{c\|c} C_{i} & \underset{\sim}{\alpha} \\ \hline \end{array}$ |  |  | $\begin{array}{l\|l} 6 \\ \hline & 2 \\ 0 \\ 0 & 2 \\ 0 & 2 \end{array}$ |  |  |  |  |  |  |  | $\begin{gathered} 0 \\ 7 \\ 8 \\ 0 \\ 9 \end{gathered}$ | 哭 |
| Э | $\stackrel{\underset{\sim}{\Xi}}{\substack{3}}$ | $\frac{9}{\square}$ | $\frac{7}{2}$ | $\frac{f}{2} \frac{9}{c}$ | $\left\|\frac{0}{9}\right\|$ | 曷 | $\underset{\sim}{2}$ |  | $\left.\frac{9}{2} \right\rvert\, \frac{0}{9}$ | $\frac{8}{2} \frac{\bar{r}}{9}$ | $\left[\begin{array}{c} \frac{\pi}{2} \\ \frac{2}{9} \end{array}\right.$ | $\begin{gathered} m \\ 9 \\ \hline \end{gathered}$ | $\stackrel{+}{2}$ | $$ |  | $\begin{gathered} n \\ 0 \\ 0 \end{gathered}$ |  |  |  | $\frac{8}{9}$ | ETA |  |  | $\stackrel{E}{2}$ | $\stackrel{\infty}{\stackrel{\infty}{\Xi}}$ | O |


| $\begin{aligned} & \hat{\infty} \\ & \stackrel{\rightharpoonup}{b} \end{aligned}$ | $\left\|\begin{array}{c} m \\ \stackrel{c}{d} \\ \stackrel{\rightharpoonup}{\infty} \end{array}\right\|$ | $\left\|\begin{array}{c} t \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} e \\ \vdots \\ i \\ n \end{array}\right\|$ | $\begin{aligned} & \infty \\ & 0 \\ & \substack{n} \end{aligned}$ | $\underset{\sim}{\underset{\sim}{i}} \mid$ | $\left\|\begin{array}{c} \infty \\ \underset{\sim}{d} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \underset{\sim}{\infty} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \underset{\sim}{n} \\ \underset{\sim}{n} \end{gathered}\right.$ | $\left\|\begin{array}{l} \text { N } \\ 8 \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{n} \\ \mid \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ \stackrel{0}{m} \\ \mid \end{array}\right\|$ | $\left\|\begin{array}{l} \infty \\ \stackrel{e}{e} \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \underset{\sim}{\dot{m}} \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ 0 \\ \underset{\sim}{*} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ 0 \\ \stackrel{0}{子} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\alpha}{\sigma} \\ \dot{\sigma} \end{array}\right\|$ | $\left\|\begin{array}{c} \hat{\omega} \\ \dot{\sigma} \end{array}\right\|$ | $\begin{gathered} n \\ \dot{n} \\ i \end{gathered}$ | $\left\|\begin{array}{c} \infty \\ \infty \\ 0 \\ i n \end{array}\right\|$ | $\stackrel{m}{i}$ | $\begin{aligned} & \text { 荷 } \\ & \dot{0} \end{aligned}$ | 荷 | $\begin{array}{\|c} \stackrel{~}{\mathrm{i}} \\ \stackrel{1}{2} \end{array}$ | $\begin{gathered} n \\ n \\ م \\ \hline \end{gathered}$ | $$ | $\stackrel{\sim}{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|\begin{array}{c} n \\ \infty \\ \stackrel{n}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{0}{\dot{c}} \\ \underset{n}{n} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \stackrel{y}{\infty} \\ \infty \\ \stackrel{n}{n} \end{gathered}\right.$ | $\left\|\begin{array}{c} N \\ \infty \\ \infty \\ i n \end{array}\right\|$ | $\stackrel{N}{\hat{0}} \overrightarrow{\hat{n}}$ | $\left\|\begin{array}{l} \vec{n} \\ \frac{m}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \stackrel{0}{n} \\ \stackrel{n}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} 2 \\ \stackrel{2}{2} \\ \mid \end{array}\right\|$ | $\begin{aligned} & \text { or } \\ & \underset{\sim}{y} \end{aligned}$ | $\left\|\begin{array}{c} \frac{\infty}{3} \\ \frac{\lambda}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \frac{2}{2} \\ \frac{2}{n} \end{array}\right\|$ | $\left.\begin{aligned} & \underset{\sim}{\infty} \\ & \stackrel{\sim}{n} \end{aligned} \right\rvert\,$ | $\left\|\begin{array}{c} \overrightarrow{\mathbf{e}} \\ \stackrel{0}{m} \\ \mathbf{n} \end{array}\right\|$ | $\left\|\begin{array}{c} Z_{t}^{2} \\ \stackrel{y}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \check{n} \\ \vdots \\ \stackrel{7}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} \stackrel{t}{x} \\ \stackrel{\rightharpoonup}{f} \end{array}\right\|$ | $\left.\begin{gathered} m \\ 0 \\ i n \\ n \end{gathered} \right\rvert\,$ | $\left\|\begin{array}{\|c} \frac{0}{n} \\ \frac{n}{n} \end{array}\right\|$ | $\begin{gathered} \underset{y}{w} \\ \stackrel{y}{6} \\ \stackrel{y}{n} \end{gathered}$ | $\frac{2}{2}$ | $\begin{array}{\|} n \\ 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $\left.\begin{array}{\|c} \hat{n} \\ \hat{0} \\ \stackrel{n}{n} \end{array} \right\rvert\,$ | $\begin{gathered} \tilde{N} \\ \frac{N}{n} \\ \hline \end{gathered}$ | $\left.\begin{array}{\|c} \infty \\ \frac{\infty}{\infty} \\ \infty \\ i n \end{array} \right\rvert\,$ | $\left\|\begin{array}{l} \vec{e} \\ \infty \\ \dot{\infty} \\ i \end{array}\right\|$ | $\left\|\begin{array}{c} z_{0} \\ \infty \\ \vdots \\ n \end{array}\right\|$ | cron |
| $\left\lvert\, \begin{gathered} \substack{2 \\ e \\ 0 \\ \stackrel{0}{2} \\ \hline} \end{gathered}\right.$ | $\begin{aligned} & \stackrel{\circ}{2} \\ & \stackrel{2}{2} \\ & \hline \end{aligned}$ | $\begin{gathered} \tilde{i} \\ \text { in } \\ \text { in } \end{gathered}$ | $\left\lvert\, \begin{gathered} n \\ \underset{\sim}{2} \\ \underset{\sim}{2} \end{gathered}\right.$ | $\left\|\begin{array}{c} \infty \\ 0 \\ 0 \\ \vdots \\ \end{array}\right\|$ | $\left\|\begin{array}{c} \bar{\infty} \\ \dot{0} \\ \underset{\sim}{c} \end{array}\right\|$ | $\left\|\begin{array}{c} \vec{e} \\ \stackrel{e}{0} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{2} \\ \alpha_{0} \\ \underset{\sim}{7} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{y}{2} \\ 2 \\ \underset{\sim}{c} \end{array}\right\|$ | $\left\|\begin{array}{c} \overrightarrow{0} \\ 0 \\ 0 \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ 2 \\ 2 \\ 2 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} \stackrel{n}{0} \\ 0 \\ 0 \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \mathbf{\sim} \\ \underset{\sim}{\sim} \\ \underset{\sim}{2} \end{gathered}\right.$ | $\left\|\begin{array}{c} \underset{7}{\tilde{m}} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\substack{c}}{\underset{\sim}{9}} \end{array}\right\|$ | $\begin{gathered} 0 \\ \stackrel{0}{0} \\ \underset{\sim}{2} \end{gathered}$ | $\left\|\begin{array}{l} \hat{N} \\ \hat{e} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \stackrel{0}{7} \\ \underset{\sim}{2} \\ \hline \end{array}\right\|$ |  | $\left\|\begin{array}{c} 0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} \frac{n}{2} \\ \frac{\alpha}{2} \\ \stackrel{c}{c} \end{array}\right\|$ |  |  | $\left\|\begin{array}{c} 0 \\ \stackrel{y}{2} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{0} \\ 0 \\ 0 \\ \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{c} \\ \underset{\sim}{\tilde{m}} \\ \underset{\sim}{2} \end{array}\right\|$ | － |
| $\left\|\begin{array}{l} \circ \\ \infty \\ \underset{\sim}{\infty} \end{array}\right\|$ | $\begin{gathered} \underset{\sim}{c} \\ \stackrel{y}{2} \end{gathered}$ | $\mathfrak{n}$ | $\left\|\begin{array}{c} \underset{0}{J} \\ \underset{J}{2} \end{array}\right\|$ | $\frac{n}{0}$ | $\left\|\begin{array}{c} 8 \\ \stackrel{\rightharpoonup}{4} \\ \stackrel{i}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \tilde{\sim} \\ 0 \\ 0 \end{array}\right\|$ | $\stackrel{\circ}{\circ}$ | $\left\|\begin{array}{c} \overrightarrow{9} \\ \underset{0}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \stackrel{\sim}{0} \\ \underset{1}{2} \end{array}\right\|$ | $\stackrel{\circ}{\underset{\sim}{9}}$ | $\stackrel{\text { n }}{\underset{\sim}{c}} \mid$ | $\begin{aligned} & \stackrel{m}{e} \\ & \underset{m}{2} \end{aligned}$ | $\underset{\sim}{7}$ | $\hat{\dot{G}}$ | $\|\stackrel{ \pm}{=}\|$ | $\left\|\begin{array}{c} \infty \\ \stackrel{\infty}{⿱} \\ \stackrel{n}{2} \end{array}\right\|$ | $\stackrel{\underset{\sim}{\infty}}{\underset{\sim}{\infty}} \mid$ | $\left\lvert\, \begin{aligned} & \underset{\infty}{\infty} \\ & \stackrel{\infty}{2} \end{aligned}\right.$ | $\left\|\begin{array}{l} n \\ n \\ \infty \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{aligned} & \text { 守 } \\ & 0 \\ & 0 \end{aligned}$ | $\left\lvert\, \begin{gathered} \substack{4 \\ \underset{\sim}{4} \\ \hline} \end{gathered}\right.$ | $\left\lvert\, \begin{gathered} \underset{\sim}{2} \\ \underset{\sim}{2} \end{gathered}\right.$ | $\left.\begin{array}{\|c\|} \hline 8 \\ 8 \\ \infty \\ \infty \end{array} \right\rvert\,$ | $\begin{array}{\|c\|} \hline 0 \\ \\ \end{array}$ | $C$ |
| $\begin{array}{\|l\|l} \frac{3}{\circ} \\ \hdashline \\ \hline \end{array}$ | $0 \begin{gathered} 0 \\ 0 \\ m \\ m \end{gathered}$ | $\begin{gathered} o \\ \stackrel{c}{6} \\ i \end{gathered}$ | $\frac{i n}{n}$ | $\left\|\begin{array}{l} \vec{~} \\ \overrightarrow{0} \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} \hat{m} \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} 2 \\ \hat{0} \\ 0 \end{array}\right\|$ | $\begin{aligned} & \overrightarrow{7} \\ & \stackrel{0}{0} \end{aligned}$ | $\left\|\begin{array}{c} 0 \\ \underset{0}{0} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \stackrel{O}{O} \\ - \end{gathered}\right.$ | $\left\lvert\, \begin{aligned} & \infty \\ & \substack{\infty \\ 0 \\ 0} \end{aligned}\right.$ |  | $\underset{\sim}{\underset{\sim}{n}}$ | 蠏 | $\begin{gathered} 0 \\ \infty \\ \hline \end{gathered}$ | $\begin{gathered} \because \\ - \\ -1 \end{gathered}$ | $\left\lvert\, \begin{gathered} \infty \\ \underset{\sim}{\infty} \\ \hline \end{gathered}\right.$ | $\stackrel{\rightharpoonup}{\otimes}$ | $\begin{aligned} & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\left.\begin{array}{\|c\|} \hline i \\ i n \\ i \end{array} \right\rvert\,$ | $\left\|\begin{array}{c} \hat{f} \\ \text { in } \end{array}\right\|$ | $$ | $\left\|\begin{array}{l} \hat{\infty} \\ \infty \\ \underset{\infty}{2} \end{array}\right\|$ | $\frac{m}{2}$ |  | $\begin{array}{\|l\|} \hline 0 \\ 0 \\ \dot{O} \\ 子 \end{array}$ | चै |
| $\stackrel{\stackrel{n}{9}}{f} \mid$ | $0$ | $\stackrel{0}{2}$ | $\left\lvert\, \begin{gathered} \underset{\substack{c}}{ } \end{gathered}\right.$ | $\left\|\begin{array}{c} 9 \\ 7 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ \vdots \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ \stackrel{0}{9} \\ 0 \end{array}\right\|$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $\left\|\begin{array}{c} 9 \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \stackrel{\infty}{\infty} \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\underset{0}{\pi}$ | $\stackrel{9}{0}$ |  | $\stackrel{ \pm}{n}$ |  | $\left\lvert\, \begin{gathered} \text { 导 } \\ \underset{\sim}{2} \end{gathered}\right.$ | $\overrightarrow{\vec{v}_{0}}$ | $\left\lvert\, \begin{gathered} 0 \\ \hline 6 \\ -2 \end{gathered}\right.$ | $\left\|\frac{\mathrm{q}}{\underset{\mathrm{i}}{ }}\right\|$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 1 \\ i \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{c} \\ \sim \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{\underset{\sim}{\underset{\sim}{c}}} \\ \hline \end{array}\right\|$ | $\left\|\begin{array}{c} \overline{0} \\ \underset{\sim}{c} \end{array}\right\|$ | $\left\|\begin{array}{l} \text { r } \\ \text { 峖 } \end{array}\right\|$ | $\left\|\begin{array}{l} n \\ \mathrm{n} \\ \mathrm{~m} \end{array}\right\|$ | त |
| $\left\lvert\, \begin{gathered} c \\ 6 \\ \substack{0} \end{gathered}\right.$ | $5$ | $\stackrel{2}{2}$ | $0$ | $\begin{array}{\|l\|} \hline 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $\left\|\begin{array}{l} 9 \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{array}{\|l\|} \hline 8 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & \text { a } \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $0$ | $\begin{aligned} & 0 \\ & \hline 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{array}{ll} 8 \\ 0 \\ 0 \\ 0 \end{array}$ | $\begin{aligned} & 3 \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{array}{\|c\|} \hline \left.\begin{array}{c} 1 \\ 0 \\ 0 \end{array} \right\rvert\, \end{array}$ | $\begin{array}{\|c\|} \hline 1 \\ 0 \\ 0 \\ \hline \end{array}$ | $\left\|\frac{6}{0}\right\|$ | $\begin{array}{\|c\|} \hline \frac{\infty}{0} \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \stackrel{n}{n} \\ \underset{\sim}{n} \end{array}$ | $\begin{array}{\|c\|} \hline \stackrel{0}{0} \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\left\|\begin{array}{c} 9 \\ 0 \\ 0 \end{array}\right\|$ | － |
| $\left\|\begin{array}{l} \tilde{z} \\ \tilde{0} \end{array}\right\|$ | $\underset{\sim}{n}$ | $\frac{8}{0}$ | $\begin{aligned} & 0 \\ & \infty \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 8 \\ & 8 \\ & 0 \end{aligned}$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\stackrel{1}{6}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \mathbf{J}_{0} \\ 0 \end{gathered}$ | 증 | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\therefore$ | $\overline{0}$ | $\left\lvert\, \begin{aligned} & 2 \\ & 0 \\ & 0 \end{aligned}\right.$ | $\left.\begin{aligned} & 8 \\ & 0 \\ & 0 \end{aligned} \right\rvert\,$ | $\left\|\begin{array}{l} \hat{0} \\ 0 \\ 0 \end{array}\right\|$ | $\begin{array}{\|l\|} \hline 0 \\ 0 \\ 0 \end{array}$ | $\left\|\begin{array}{c} 0 \\ \frac{2}{0} \end{array}\right\|$ | $\left\|\begin{array}{l} \text { 웅 } \\ 0 \end{array}\right\|$ | $\left\|\frac{2}{5}\right\|$ | $\left\|\begin{array}{c} \text { तु } \end{array}\right\|$ | $\begin{array}{\|l\|} \hline 8 \\ 8 \\ 0 \end{array}$ | $\begin{array}{\|l\|} \hline 0 \\ 0 \\ 0 \end{array}$ | 苞 |
| $\left.\begin{array}{c} f \\ \infty \\ f \\ f \end{array}\right]$ |  | $\begin{gathered} c \\ c \\ c \end{gathered}$ | $\left.\begin{array}{\|c} \vec{n} \\ \stackrel{7}{c} \\ 0 \\ 0 \end{array} \right\rvert\,$ | $\left.\begin{array}{l\|l} \hat{2} \\ \hat{2} \\ \hat{i} \end{array} \right\rvert\,$ | $\left\|\begin{array}{c} + \\ \hdashline \\ \hdashline \end{array}\right\|$ | $\left\|\begin{array}{c} ⿱ 士 口 \\ \substack{n \\ i} \end{array}\right\|$ | $\left\|\begin{array}{c} \stackrel{\rightharpoonup}{m} \\ \stackrel{c}{c} \end{array}\right\|$ | $\frac{y}{n}$ | $\left\|\begin{array}{c} \vec{Z} \\ \underset{i}{n} \end{array}\right\|$ | $\left\lvert\, \begin{aligned} & \vec{~} \\ & \hdashline \\ & 0 \\ & \hline \end{aligned}\right.$ | $\begin{gathered} \text { 寺 } \\ \hdashline \\ \hline \end{gathered}$ |  | $\left[\begin{array}{l} 9 \\ \\ \hline \end{array}\right.$ | $\begin{gathered} \hat{\infty} \\ \\ \hline \end{gathered}$ |  | $\begin{aligned} & \text { ? } \\ & 7 \\ & 7 \\ & 7 \end{aligned}$ | $\begin{array}{\|l\|} \hline \infty \\ \stackrel{+}{7} \\ \stackrel{y}{2} \end{array}$ | $\left\|\begin{array}{c} c_{\infty}^{\prime} \\ \infty \\ \underset{\sim}{\infty} \end{array}\right\|$ | $\left.\begin{array}{\|c\|} \infty \\ \stackrel{n}{m} \\ \vdots \end{array} \right\rvert\,$ | $\left.\begin{array}{\|c\|} \hline \hat{3} \\ \dot{2} \\ \mathrm{n} \end{array} \right\rvert\,$ | $\left\lvert\, \begin{gathered} \mathbf{c}_{\substack{1 \\ \infty \\ \infty \\ m}} \end{gathered}\right.$ | $\begin{array}{\|c\|} \hline \text { in } \\ \hline \frac{1}{2} \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \hat{\sim} \\ \text { 肙 } \end{array}$ | $\begin{aligned} & \vec{m} \\ & \stackrel{\rightharpoonup}{f} \end{aligned}$ |  | 令 |
| $\begin{aligned} & \infty \\ & \underset{\sim}{7} \\ & \stackrel{2}{2} \end{aligned}$ | $\stackrel{\rightharpoonup}{2}$ |  | $\begin{array}{\|c\|} \hline 0 \\ 子 \\ 2 \\ \hline \end{array}$ |  | O. | $\stackrel{\circ}{0}$ | $\left[\begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \end{array}\right]$ | $\left\lvert\, \begin{gathered} \pm \\ \underset{\sim}{c} \\ \hline \end{gathered}\right.$ | $\left\|\begin{array}{c} 0 \\ \infty \\ \infty \\ \end{array}\right\|$ | $\frac{\infty}{\underset{\sim}{c}}$ | $\left\|\begin{array}{c} \hat{5} \\ \text { nem } \end{array}\right\|$ | $\left[\begin{array}{c} c \\ a \\ \text { f } \end{array}\right.$ | $\underset{\sim}{r}$ | $0$ | $5$ | $\begin{aligned} & 0 \\ & \stackrel{0}{1} \\ & + \\ & \infty \end{aligned}$ | $\begin{aligned} & r_{i}^{n} \\ & \stackrel{a}{\infty} \end{aligned}$ | $\left\|\begin{array}{l}  \pm \\ a \\ \stackrel{\rightharpoonup}{\infty} \end{array}\right\|$ | $\left.\begin{gathered} \vec{\infty} \\ 0 \\ 0 \end{gathered} \right\rvert\,$ | $\left.\begin{array}{\|c\|} \hline \underset{n}{c} \\ \vdots \\ 0 \end{array} \right\rvert\,$ | $\begin{array}{\|c\|} \substack{0 \\ \underset{\alpha}{\infty} \\ \hline} \end{array}$ | $\left.\begin{array}{\|c\|} \hline 0 \\ 0 \\ 0 \\ 0 \\ i \end{array} \right\rvert\,$ | $\left\|\begin{array}{c} \tilde{a} \\ \vdots \\ \underset{\sim}{2} \end{array}\right\|$ | $\begin{aligned} & \circ \\ & \infty \\ & \infty \\ & \underset{\sim}{\infty} \end{aligned}$ |  |  |
| $\left\lvert\, \begin{gathered} \infty \\ \underset{\sim}{c} \\ \vdots \end{gathered}\right.$ | $\begin{gathered} \infty \\ \stackrel{\infty}{x} \\ \stackrel{1}{2} \end{gathered}$ | $\begin{array}{\|c\|c} \infty \\ \infty \\ \infty \\ \hline \end{array}$ | $\begin{array}{\|c\|} \hline \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & \mathbf{~} \\ & \hline \mathbf{~} \\ & \hline \end{aligned}$ | $\begin{array}{\|c} 6 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & \overrightarrow{0} \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 2 \\ & 8 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{gathered} 0 \\ 0 \\ 0 \\ i \end{gathered}$ |  | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & \underset{8}{8} \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & \because \\ & \hline \\ & \hline \end{aligned}$ | $\frac{i n}{i}$ |  | $\begin{aligned} & \text { än } \\ & \text { oi } \end{aligned}$ | $\begin{gathered} 0.0 \\ { }_{1}^{1} \\ \hline \end{gathered}$ | $\begin{gathered} \hat{9} \\ \underset{c}{1} \end{gathered}$ | $\begin{array}{\|c} \bar{n} \\ \stackrel{1}{2} \end{array}$ | $\begin{aligned} & \underset{\sim}{N} \\ & \underset{i}{n} \end{aligned}$ | $\left\|\begin{array}{c} 0 \\ 0 \\ \vdots \end{array}\right\|$ | $\begin{aligned} & 6 \\ & 0 \\ & i \end{aligned}$ | $\begin{gathered} n \\ \stackrel{n}{i} \end{gathered}$ | $\left.\begin{aligned} & \mathbf{0} \\ & 0 \\ & i \\ & i \end{aligned} \right\rvert\,$ |  | $\begin{array}{\|c\|} \hline \\ \vdots \\ \vdots \end{array}$ | $\stackrel{\sim}{4}$ |
| $\left\|\begin{array}{c} \vec{c} \\ 0 \\ 0 \\ \stackrel{\rightharpoonup}{x} \end{array}\right\|$ | $\underbrace{~}_{i}$ | $\begin{aligned} & \circ \\ & 0 \\ & 2 \\ & 0 \\ & ? \end{aligned}$ | $\begin{gathered} 0 \\ \stackrel{0}{c} \\ r_{i} \end{gathered}$ | $\left\|\begin{array}{c} \infty \\ \underset{0}{0} \\ \hdashline \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \vec{~} \\ \underset{寸}{寸} \end{gathered}\right.$ | $\stackrel{\underset{\sim}{\underset{\sim}{f}}}{\substack{0}}$ | $\begin{gathered} \infty \\ \substack{\infty \\ \\ i} \end{gathered}$ | $0 \begin{gathered} 1 \\ \infty \\ \vdots \\ \hline \end{gathered}$ | $\left(\begin{array}{c} \infty \\ \vdots \\ \vdots \\ \vdots \end{array}\right.$ | $\begin{gathered} c \\ \substack{1 \\ ~} \\ \hline \end{gathered}$ | $\begin{aligned} & \infty \\ & \underset{\sim}{n} \\ & \text { הָ } \end{aligned}$ |  |  | $\frac{2}{5}$ | $\begin{gathered} o \\ \substack { o \\ \begin{subarray}{c}{1{ o \\ \begin{subarray} { c } { 1 } } \end{gathered}$ | $\begin{aligned} & \because \\ & \stackrel{n}{\infty} \\ & \vdots \\ & \hline \end{aligned}$ | $\begin{aligned} & 9 \\ & \frac{9}{6} \\ & 0 \\ & 8 \end{aligned}$ | $\begin{gathered} \bar{\infty} \\ \frac{1}{n} \end{gathered}$ | $\begin{aligned} & 2 \\ & \infty \\ & \infty \\ & n \end{aligned}$ | $\begin{array}{\|c\|} \hline \\ \infty \\ \infty \\ 0 \end{array}$ | $\begin{array}{\|c\|} \hline 0 \\ 0 \\ 0 \\ \end{array}$ |  | $\begin{array}{\|c} \mathbf{B}_{6} \\ \infty \\ \infty \end{array}$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\underset{\substack{0}}{\cong}$ |  |
| $\begin{array}{\|l\|l} \hline \frac{1}{x} \\ \frac{ \pm}{\beth} \\ \hline \end{array}$ | $\begin{aligned} & \hline \stackrel{c}{c} \\ & \stackrel{1}{2} \\ & 0 \\ & = \end{aligned}$ | $\begin{aligned} & \text { m } \\ & \infty \\ & \text { en } \\ & = \end{aligned}$ |  |  | $\begin{array}{\|c} \hline 0 \\ \hat{0} \\ 0 \\ \vdots \end{array}$ | $\begin{aligned} & \hat{n} \\ & 2 \\ & \underset{\sim}{n} \end{aligned}$ | $\begin{array}{\|l\|} \hline \underset{\sim}{\sim} \\ \underset{\sim}{\infty} \\ \\ \hline \end{array}$ | $\begin{gathered} \overrightarrow{2} \\ \alpha \\ \infty \\ \end{gathered}$ |  | $2 \begin{aligned} & n \\ & \vdots \\ & \vdots \\ & \vdots \\ & \end{aligned}$ | $\underset{\substack{\infty \\ \\ \\ \\ \hline}}{ }$ |  | $\begin{array}{\|c} \hline \\ \vdots \\ \vdots \\ \vdots \\ \hline \end{array}$ | $\begin{array}{\|l} \hline 0 \\ 0 \\ 0 \\ 0 \end{array}$ |  |  |  | $\begin{aligned} & \stackrel{士}{6} \\ & \stackrel{\rightharpoonup}{さ} \\ & \underset{I}{2} \end{aligned}$ |  | $\begin{gathered} \infty \\ 0 \\ \infty \\ \underset{~}{\infty} \\ \end{gathered}$ | $\begin{aligned} & \hline \tilde{2} \\ & \underset{\sim}{\tilde{a}} \\ & \hline \end{aligned}$ | $\begin{array}{\|c} 9 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 1 \\ & 0 \\ & 0 \\ & 0 \\ & 1 \end{aligned}$ | O－ |
| $\frac{0}{2}$ | St | $\frac{6}{c} \frac{\infty}{\infty}$ | $\begin{gathered} \frac{x}{x} \\ \underline{\underline{a}} \\ \hline \end{gathered}$ | $\stackrel{a}{2}=\frac{\vec{a}}{2}$ | $\frac{\tilde{2}}{\bar{e}}$ | $\frac{2}{2}$ | $\left[\begin{array}{l} \frac{士}{2} \\ \frac{2}{2} \end{array}\right.$ | $\stackrel{n}{2}$ | $\frac{0}{2}$ | $\frac{9}{2}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{9} \\ & \hline \end{aligned}$ | $\frac{2}{2} \frac{2}{3}$ | Ele | 发管 | $\begin{gathered} 5 \\ \hline 0 \\ \hline 0 \end{gathered}$ | $\begin{aligned} & \stackrel{2}{0} \\ & \stackrel{0}{0} \end{aligned}$ | $\begin{aligned} & \text { ta } \\ & \stackrel{0}{9} \\ & \hline \end{aligned}$ | E | $\begin{array}{\|c} 0 \\ 0 \\ 0 \\ \hline 0 \end{array}$ | $\left\lvert\, \begin{gathered} \hat{0} \\ \underset{\sim}{2} \end{gathered}\right.$ | $\begin{aligned} & \infty \\ & \frac{0}{2} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { and } \\ & \text { n } \end{aligned}$ | $\begin{gathered} \circ \\ \stackrel{\rightharpoonup}{9} \end{gathered}$ | $=$ | $\frac{\pi}{2}$ | $\frac{m}{2}$ |


| $\stackrel{c}{\dot{n}} \underset{\hat{c}}{ }$ |  | $\left\lvert\, \begin{aligned} & \hat{j} \\ & \dot{d} \end{aligned}\right.$ | $\left\|\begin{array}{c} n \\ \vdots \\ \stackrel{n}{0} \end{array}\right\|$ | $\left\|\frac{0}{2}\right\|$ | 希 | $\left.\frac{0}{9} \right\rvert\,$ | $\begin{gathered} c \\ \infty \\ \dot{\sim} \\ \stackrel{y}{2} \end{gathered}$ | $\left.\begin{gathered} \underset{i}{i} \\ \stackrel{\sim}{2} \end{gathered} \right\rvert\,$ | $\frac{8}{i}$ | $\dot{\hat{\circ}}$ | $\underset{\text { 犬̈ }}{\text { H}}$ | $\begin{gathered} \underset{\sim}{\otimes} \\ \underset{\sim}{2} \end{gathered}$ | $\stackrel{9}{9}$ |  |  | $\begin{gathered} \underset{\sim}{n} \\ \stackrel{\rightharpoonup}{n} \end{gathered}$ | $\left\|\begin{array}{c} 0 \\ \dot{G} \\ \hline \end{array}\right\|$ | $\begin{array}{\|c\|} \infty \\ 0 \\ 0 \end{array}$ | $\frac{n}{n}$ | $\left\|\begin{array}{c} n \\ n \\ 0 \\ n \\ n \end{array}\right\|$ | $\frac{n}{z}$ | $\left\lvert\, \begin{gathered} \infty \\ \underset{7}{\infty} \end{gathered}\right.$ | $\left\lvert\, \begin{aligned} & \hat{q} \\ & \dot{q} \end{aligned}\right.$ | $\left\|\begin{array}{c}  \\ \hline 0 \end{array}\right\|$ | N\| | $\left\|\begin{array}{c} \vec{a} \\ \hat{0} \end{array}\right\|$ | － |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|\begin{array}{c} \frac{1}{\infty} \\ \frac{\infty}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} o \\ 0 \\ 0 \\ \infty \\ n \\ n \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ i n \\ i n \end{array}\right\|$ | $\left\|\begin{array}{c} 0 \\ \infty \\ \infty \\ i n \end{array}\right\|$ | $\begin{aligned} & \vec{\infty} \\ & \underset{\sim}{\infty} \\ & \stackrel{y}{n} \end{aligned}$ | $\begin{gathered} \frac{\tilde{m}}{\infty} \\ \frac{\infty}{n} \end{gathered}$ | $\stackrel{y}{n} \stackrel{\substack{0 \\ d}}{\substack{2}}$ | $\stackrel{i}{9} \frac{8}{8}$ | $\dot{8}$ | $\dot{8}$ | $\begin{array}{\|c} \frac{\pi}{o} \\ \frac{i}{7} \end{array}$ | $\stackrel{\circ}{E}$ | $\begin{gathered} \circ \\ \stackrel{0}{\circ} \\ \frac{1}{n} \end{gathered}$ |  |  | $\begin{aligned} & \stackrel{0}{0} \\ & \stackrel{e}{\infty} \\ & i n \end{aligned}$ |  | $\left\lvert\, \begin{gathered} \underset{\text { an }}{\substack{n}} \mid \end{gathered}\right.$ | $\left\|\begin{array}{c} \stackrel{n}{\hat{\infty}} \\ \frac{\infty}{n} \end{array}\right\|$ | $\left\|\begin{array}{c} 4 \\ \substack{\infty \\ m} \\ i \end{array}\right\|$ | $\begin{gathered} \infty \\ \infty \\ \infty \\ \frac{\infty}{n} \end{gathered}$ | $\stackrel{n}{\stackrel{n}{2}} \underset{\substack{\infty}}{ }$ | $\left.\begin{gathered} \infty \\ \frac{\infty}{\infty} \\ \infty \\ m \end{gathered} \right\rvert\,$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & =0 \end{aligned}$ | $\frac{8}{2}$ | $\begin{aligned} & \text { Nㅡㅇ } \end{aligned}$ | $\left\|\begin{array}{c} \frac{m}{e} \\ \stackrel{e}{n} \\ \stackrel{n}{2} \end{array}\right\|$ | 宮 |
|  | $\left\|\begin{array}{c} c \\ \underset{\sim}{訁} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \vec{A} \\ \mathbf{0} \\ \underset{\sim}{0} \\ \underset{\sim}{2} \end{gathered}\right.$ | $\left\|\begin{array}{c} 7 \\ \underset{\sim}{2} \\ \underset{\sim}{2} \end{array}\right\|$ | $\begin{gathered} \text { à } \\ \stackrel{\rightharpoonup}{2} \\ \underset{\sim}{2} \end{gathered}$ | $\begin{array}{\|c} \mathfrak{N} \\ \underset{\sim}{2} \end{array}$ | $\begin{gathered} n \\ \stackrel{n}{2} \\ \underset{\sim}{2} \end{gathered}$ | $\stackrel{i}{2} \underset{2}{2}$ | $\hat{C}_{2}^{2} \hat{2}_{2}^{\circ}$ | 虽茴 | $\begin{aligned} & \infty \\ & \vdots \\ & \vdots \\ & 2 \\ & 2 \end{aligned}$ | $\hat{h}$ |  |  |  |  | $\left\|\begin{array}{l} \stackrel{\rightharpoonup}{0} \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\alpha}{\underset{\sim}{2}} \\ \underset{\sim}{2} \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{n} \\ \underset{\sim}{2} \\ \mid \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \infty \\ \underset{\sim}{2} \\ \underset{\sim}{2} \end{gathered}\right.$ | $\begin{aligned} & \infty \\ & \stackrel{\sim}{\circ} \\ & \text { and } \end{aligned}$ | $\begin{aligned} & \underset{0}{t} \\ & \stackrel{\rightharpoonup}{2} \\ & \stackrel{\rightharpoonup}{9} \end{aligned}$ | $\left\|\begin{array}{c} \stackrel{a}{0} \\ \stackrel{2}{2} \\ \stackrel{2}{2} \end{array}\right\|$ | $\begin{aligned} & \stackrel{\circ}{\infty} \\ & \stackrel{1}{0} \\ & \stackrel{\oplus}{m} \end{aligned}$ | $\begin{aligned} & \underset{\sim}{\infty} \\ & \stackrel{\rightharpoonup}{2} \\ & \stackrel{\omega}{2} \end{aligned}$ | $\begin{aligned} & \circ \\ & \stackrel{\circ}{\infty} \\ & \stackrel{\infty}{\sim} \end{aligned}$ | $\left\|\begin{array}{c} \underset{1}{\hat{2}} \\ \underset{\sim}{2} \end{array}\right\|$ | $\begin{aligned} & 8 \\ & \hline 6 \\ & \stackrel{\rightharpoonup}{9} \end{aligned}$ |
| $$ | $\begin{aligned} & 7 \\ & x_{2} \\ & \pm \end{aligned}$ | $\left\|\begin{array}{l} \tilde{n} \\ n \\ n \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{\mathrm{N}} \\ \text { n } \end{array}\right\|$ | $\stackrel{n}{n} \underset{=}{n}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{0} \\ & \stackrel{1}{\prime} \end{aligned}$ | $\left\|\begin{array}{l} \mathrm{J} \\ \mathbf{J} \end{array}\right\|$ | $\begin{aligned} & \underset{\sim}{\infty} \\ & \underset{\sim}{n} \end{aligned}$ |  | $\begin{gathered} \hat{E} \\ \\ \\ \\ \hline \end{gathered}$ | $\begin{aligned} & \underset{\sim}{2} \\ & \stackrel{y}{n} \end{aligned}$ | $\begin{aligned} & \overrightarrow{-} \\ & \underset{-}{2} \end{aligned}$ | $$ |  |  | $\begin{aligned} & 0 \\ & \stackrel{9}{4} \\ & \dot{r} \end{aligned}$ | $\begin{array}{\|c\|} \hline \frac{0}{n} \\ m \end{array}$ | $\left\|\begin{array}{l} \bar{n} \\ \vdots \\ 0 \end{array}\right\|$ | $\begin{aligned} & \infty \\ & \infty \\ & \underset{~}{\infty} \end{aligned}$ | $\begin{aligned} & \stackrel{n}{0} \\ & \stackrel{0}{0} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{aligned} & \pm \\ & \mathbf{g}_{1} \\ & \underset{~}{2} \end{aligned}$ | $\begin{aligned} & \infty \\ & \stackrel{\infty}{\infty} \\ & \underset{\sim}{2} \end{aligned}$ | $\begin{array}{\|l} 0 \\ \stackrel{\rightharpoonup}{\lambda} \end{array}$ | $\begin{aligned} & 8 \\ & 8 \\ & \hline \end{aligned}$ | $\left.\begin{gathered} \pm \\ \infty \\ \infty \\ \hline \end{gathered} \right\rvert\,$ | $\frac{2}{2}$ | $\left\|\begin{array}{l} r \\ 0 \\ 0 \end{array}\right\|$ | n |
| $\left\lvert\, \begin{aligned} & \text { 각 } \\ & 子 \end{aligned}\right.$ | $\left(\begin{array}{c} x \\ x \\ - \end{array}\right.$ | $\left\lvert\, \begin{aligned} & \infty \\ & \substack{\infty \\ +} \end{aligned}\right.$ | $\begin{aligned} & \tilde{m} \\ & \dot{\sim} \end{aligned}$ |  | $\begin{aligned} & 0 \\ & \hline 0 \\ & 0 \end{aligned}$ | $\frac{0}{9}$ | n | $\begin{array}{l\|l} \hat{6} & \stackrel{\rightharpoonup}{2} \\ \underset{\sim}{2} \\ \hline \end{array}$ | $\begin{array}{\|c} \infty \\ \dot{c} \\ \text { f } \end{array}$ | $\vec{\sim}$ |  | $\hat{i}$ |  |  | $\begin{aligned} & 0 \\ & \stackrel{0}{0} \\ & m \end{aligned}$ | $\left\|\begin{array}{c} 0 \\ \stackrel{0}{c} \\ m \end{array}\right\|$ | $\left\|\begin{array}{c} \infty \\ 0 \\ 0 \\ i n \end{array}\right\|$ | $\begin{aligned} & 8 \\ & 0 \\ & \text { co } \end{aligned}$ | $\begin{gathered} \hat{n} \\ \underset{\sim}{i} \end{gathered}$ | $\underset{\sim}{7}$ | $\begin{gathered} \infty \\ \infty \\ m \\ m \end{gathered}$ | $\begin{gathered} \underset{c}{c} \\ \text { in } \end{gathered}$ | cicie | $\stackrel{\ddot{n}}{\stackrel{\sim}{n}}$ | $\begin{gathered} \overrightarrow{0} \\ \dot{r} \end{gathered}$ | $\left\|\begin{array}{c} \tilde{0} \\ 0 \\ 0 \end{array}\right\|$ | त |
| $\overrightarrow{\mathrm{N}}\|\overrightarrow{\mathrm{~m}}\|$ | $\left\lvert\, \begin{gathered} \tilde{x} \\ \infty \\ m \end{gathered}\right.$ | $\stackrel{\circ}{\dot{C}} \underset{\sim}{i}$ | $\frac{\sqrt{7}}{7}$ | $\left\lvert\, \begin{aligned} & \vec{t} \\ & \vdots \\ & 子 \\ & 子 \end{aligned}\right.$ | $\left.\begin{aligned} & \infty \\ & \stackrel{8}{6} \\ & \dot{b} \end{aligned} \right\rvert\,$ | 喜 |  | $\stackrel{\circ}{\circ}$ | $\stackrel{\infty}{\infty}$ |  |  | $\underset{i}{f} \underset{\sim}{c}$ |  |  |  | $\left\|\begin{array}{c} \infty \\ \infty \\ \infty \\ \text { i } \end{array}\right\|$ | $\left\|\begin{array}{c} \underset{\sim}{n} \\ 7 \end{array}\right\|$ | $\left\lvert\, \begin{gathered} \infty \\ c \\ 8 \\ 7 \\ 子 \end{gathered}\right.$ | $\begin{gathered} \infty \\ \stackrel{\infty}{c} \\ \underset{\sim}{c} \end{gathered}$ | $\underset{\sim}{\underset{\sim}{*}}$ | $\vec{a} \underset{i}{c}$ | $\stackrel{\hat{S}}{\mathrm{i}}$ | $\underbrace{\underset{O}{O}}_{i}$ | $\begin{gathered} \circ \\ \stackrel{2}{2} \\ \sim \end{gathered}$ | $\begin{aligned} & \overrightarrow{0} \\ & \overrightarrow{7} \end{aligned}$ | $\left\|\begin{array}{c} o \\ 0 \\ \dot{r} \\ i \end{array}\right\|$ | $\underset{\substack{\text { in }}}{\substack{\text { in }}}$ |
| $\frac{\vec{a}}{6}$ | $\begin{gathered} \substack{n \\ 0 \\ 0 \\ 0} \end{gathered}$ | $\begin{aligned} & \tilde{y} \\ & \vdots \\ & 0 \\ & \hline \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{gathered} \stackrel{N}{0} \\ 0 \\ 0 \end{gathered}$ | $\stackrel{\otimes}{\infty}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \\ & 8 \end{aligned}$ |  | $\begin{array}{l\|l} 8 \\ 8 & \stackrel{0}{9} \\ \hline \end{array}$ |  | $\stackrel{e}{6}$ |  |  |  |  | $0$ | $\left\|\begin{array}{c} \vec{N} \\ 0 \end{array}\right\|$ | $\begin{gathered} 0 \\ \infty \\ 0 \end{gathered}$ | $0 \begin{gathered} \infty \\ \vdots \\ \hline \end{gathered}$ | $\begin{aligned} & \overrightarrow{7} \\ & 0 \end{aligned}$ | $\begin{aligned} & \stackrel{\rightharpoonup}{\infty} \\ & 0 \end{aligned}$ | 흥 |  | 당 | $\begin{array}{l\|l\|l\|l\|l\|l\|} \hline 0 \\ \hline \end{array}$ | $\begin{aligned} & \frac{m}{7} \\ & 0 \end{aligned}$ | $\begin{aligned} & \text { 응 } \\ & \hline \end{aligned}$ | $\stackrel{\infty}{\square}$ |
| $\left\lvert\, \begin{aligned} & x \\ & \cline { 1 - 1 } \\ & = \end{aligned}\right.$ | $\begin{aligned} & \because \\ & = \\ & = \end{aligned}$ | $\frac{0}{7}$ | $\begin{aligned} & \text { é } \\ & 0 \end{aligned}$ | 잉 | $\stackrel{i}{m}$ | $\hat{E}_{2}$ | $\begin{array}{c\|c} \substack{0 \\ 0 \\ 0} \\ 0 \\ 0 \end{array}$ | $\begin{array}{c\|c} 8 & \stackrel{9}{3} \\ \hline-0 \end{array}$ |  |  | $\begin{aligned} & 2 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{array}{l\|l} 6 & 8 \\ \hline 0 & 8 \\ \hline \end{array}$ |  |  | $\begin{aligned} & \mathbf{~} \\ & \mathbf{~} \\ & 0 \end{aligned}$ | $\left\|\begin{array}{l}  \pm \\ \frac{ \pm}{0} \end{array}\right\|$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \end{array}\right\|$ | $0 \begin{gathered} n \\ \infty \\ 0 \\ 0 \end{gathered}$ | $\stackrel{7}{2}$ | $\stackrel{\underset{c}{c}}{\substack{2}}$ | $=\frac{\infty}{0}$ | $0 .$ |  | $\stackrel{\rightharpoonup}{\mathrm{s}}$ | $\begin{array}{\|c} \hline 0 \\ 0 \\ 0 \\ \hline \end{array}$ | $\begin{aligned} & n \\ & n \\ & 0 \end{aligned}$ | － |
|  | $\begin{gathered} \ddagger \\ \stackrel{7}{4} \\ \underset{T}{\prime} \end{gathered}$ | $\begin{aligned} & 8 \\ & 0 \\ & 0 \\ & 8 \end{aligned}$ |  | $B_{i}^{2}$ | $\begin{gathered} m \\ 0 \\ i \\ \hline \end{gathered}$ | $\begin{array}{c\|c} \substack{c \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline \\ \hline} \end{array}$ |  | $\stackrel{M}{2}$ |  |  |  |  |  |  | $\begin{aligned} & \overrightarrow{\mathrm{E}} \\ & \mathbf{0} \end{aligned}$ | $\begin{array}{\|c\|} \hline 9 \\ \vdots \\ \hline \end{array}$ | $\left\|\begin{array}{c} \mathbf{A} \\ \infty \\ \dot{寸} \end{array}\right\|$ | $\begin{gathered} \overrightarrow{0} \\ \underset{\sim}{\infty} \\ \underset{\sim}{\infty} \end{gathered}$ |  | $\stackrel{c}{c}$ | $\stackrel{C}{C}$ | Co | $5$ |  |  | $\begin{aligned} & 7 \\ & 9 \\ & 7 \\ & \hline \end{aligned}$ | 2 |
| $$ | $\begin{aligned} & \text { n} \\ & \stackrel{1}{\Delta} \\ & \hat{\Delta} \end{aligned}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | ${ }_{2}^{x}$ |  |  | $\begin{aligned} & x_{2}^{2} \\ & \\ & \hline \end{aligned}$ |  | $\begin{gathered} a \\ \underset{y y}{c} \\ e \\ e \end{gathered}$ | $$ |  |  |  |  | $\begin{gathered} \text { y } \\ \dot{\sim} \\ \vdots \end{gathered}$ | $\left\|\begin{array}{c} 9 \\ 0 \\ \infty \\ \underset{f}{9} \end{array}\right\|$ | $\begin{gathered} \infty \\ n \\ n \\ \underset{\sim}{2} \end{gathered}$ | $\begin{aligned} & \text { 志 } \\ & \text { 心 } \\ & \hline \end{aligned}$ | $\begin{aligned} & 9 \\ & 9 \\ & 9 \end{aligned}$ |  |  | $\hat{C}_{2}^{2}$ | $\frac{9}{7}$ | $Q_{i}=\stackrel{\infty}{\infty}$ | $\begin{array}{\|c} \substack{0 \\ \infty \\ 0 \\ \\ \hline} \end{array}$ | $\begin{aligned} & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | － |
| $\begin{array}{\|l\|} \hline 8 \\ 0 \\ = \\ \hline \end{array}$ |  | $\begin{aligned} & \hat{0} \\ & 0 \\ & i \end{aligned}$ | $\begin{gathered} m \\ \vdots \\ \vdots \end{gathered}$ | $\begin{gathered} n \\ \hline \end{gathered}$ | : | $\begin{array}{l\|l} 0 & 0 \\ 0 \\ 0 & 0 \\ \hline \end{array}$ | $\begin{array}{ccc} 6 \\ 0 & 0 \\ & 0 \\ \hline \end{array}$ | $\begin{array}{c\|c} \infty & 0 \\ \infty & \infty \\ & 0 \\ \hline \end{array}$ |  |  |  |  |  | $\stackrel{\rightharpoonup}{\mathrm{a}} \mathrm{i}$ | $\begin{gathered} \stackrel{\circ}{4} \\ \underset{1}{2} \end{gathered}$ | $\begin{gathered} m \\ m \\ \hdashline \\ \hline \end{gathered}$ | $\frac{ㅇ ㅗ ㄴ ~}{9}$ | $\left.\begin{array}{\|c\|} \hline \\ \hline 1 \\ \hline 1 \end{array} \right\rvert\,$ | $\begin{gathered} \text { But } \\ \hline 0 \\ \hline \end{gathered}$ | $\underset{\substack{\text { din } \\ \hline \\ \hline}}{ }$ | $\begin{gathered} \infty \\ \hline \end{gathered} \underset{\substack{0 \\ \hline \\ \hline \\ \hline \\ \hline}}{ }$ | $0$ | $\begin{gathered} i \\ e \\ 0 \\ 0 \\ 0 \end{gathered}$ |  | $\begin{aligned} & \stackrel{N}{n} \\ & \stackrel{1}{9} \end{aligned}$ | $\begin{aligned} & n \\ & 0 \\ & i \end{aligned}$ | $\stackrel{7}{4}$ |
| $\left\lvert\, \begin{gathered} t \\ \vdots \\ \vdots \\ x_{1} \\ x_{1} \end{gathered}\right.$ | $\left.\begin{gathered} x \\ \stackrel{x}{4} \\ \underset{\sim}{n} \end{gathered} \right\rvert\,$ | $\left.\begin{array}{\|c} n \\ \stackrel{n}{\infty} \\ \underset{c}{\infty} \end{array} \right\rvert\,$ |  |  | $\begin{gathered} \circ \\ \stackrel{\circ}{6} \\ \stackrel{y}{1} \end{gathered}$ | ： |  | $\begin{gathered} 0 \\ \stackrel{0}{0} \\ \stackrel{y}{\circ} \\ \hline \end{gathered}$ | $\begin{aligned} & \infty \\ & \hline \\ & \hline \\ & \hline \end{aligned}$ | $\stackrel{y}{c}$ |  |  |  | $\underset{\substack{\hat{N} \\ \underset{\sim}{n} \\ \hline}}{ }$ | $\begin{gathered} \underset{\sim}{\hat{1}} \\ \stackrel{\text { ®n}}{ } \end{gathered}$ | $\left\|\begin{array}{c} \overrightarrow{0} \\ \underset{9}{9} \end{array}\right\|$ | $\begin{aligned} & \text { y } \\ & \frac{1}{2} \\ & \text { p } \end{aligned}$ | $\begin{array}{\|l\|l} \substack{\mathbf{N}_{1} \\ \vec{t} \\ \hline} \end{array}$ | $\frac{\vec{a}}{\frac{0}{8}}$ | $\begin{gathered} \text { Su} \\ \hline \\ \hline \end{gathered}$ |  |  | $\stackrel{\rightharpoonup}{\mathrm{T}}$ |  | $\begin{aligned} & \stackrel{n}{=} \\ & \square \end{aligned}$ | $\begin{array}{\|c} \text { ? } \\ \stackrel{4}{0} \\ \stackrel{\rightharpoonup}{2} \end{array}$ | 解 |
| $$ | $\begin{aligned} & x \\ & \\ & \end{aligned}$ | $\begin{aligned} & \substack{0 \\ 子 \\ 0 \\ 0 \\ 0 \\ 0} \end{aligned}$ | $\begin{aligned} & 7 \\ & \frac{7}{6} \\ & \stackrel{0}{6} \end{aligned}$ |  | $\begin{aligned} & \text { き } \\ & \text { 弇 } \end{aligned}$ |  |  |  |  |  |  |  | $\begin{aligned} & \bar{n} \\ & \hat{0} \\ & \end{aligned}$ | $\begin{aligned} & \text { 志 } \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  | $$ | $\begin{aligned} & 2 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |  |  | $\underset{y}{c}$ |  |  |  | $\stackrel{0}{2}$ |  | $\begin{aligned} & n \\ & =0 \\ & 0 \\ & \hline 0 \end{aligned}$ | $\stackrel{\text { n }}{\substack{\text { a } \\ 0 \\ 0 \\ 0}}$ |
| $\stackrel{ \pm}{2}$ |  | $\begin{aligned} & \frac{\infty}{2} \\ & \frac{2}{2} \end{aligned}$ |  | $\stackrel{n}{2} \frac{\infty}{2}$ |  |  |  |  |  |  |  | ex |  | त्व |  | $\begin{array}{\|c} \text { ते } \\ \hline \end{array}$ | $\begin{aligned} & 0 \\ & 0 \\ & \vdots \\ & \hline \end{aligned}$ | 를 |  |  | $\begin{gathered} \underset{\sim}{c} \\ \stackrel{y y}{2} \\ \hline \end{gathered}$ |  | $\underset{\sim}{\sim}$ | $\stackrel{e}{0}$ | $\begin{aligned} & \text { y } \\ & \text { di } \end{aligned}$ | $\begin{aligned} & \stackrel{\infty}{さ} \\ & \vec{a} \\ & \hline \end{aligned}$ | 9 |


| tel250 | 1704.283 | -90.538 | -0.245 | 134.592 | -43.879 | 0.436 | 0.523 | 4.798 | 5.758 | 5.830 | 1393604 | 516212 | 659.0 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| tel251 | 1711.056 | -83.765 | -0.241 | 120.198 | -41.142 | 0.270 | 0.324 | 4.109 | 4.931 | 5.923 | 1393592 | 516109 | 631.8 |
| tel252 | 1718.119 | -76.702 | -0.250 | 116.103 | -37.851 | 0.179 | 0.215 | 2.656 | 3.187 | 4.320 | 1393620 | 515882 | 599.1 |
| tcl253 | 1728.562 | -66.259 | -0.254 | 102.677 | -33.474 | 0.157 | 0.188 | 2.211 | 2.653 | 5.149 | 1393630 | 515688 | 555.6 |
| tel254 | 1735.326 | -59.495 | -0.300 | 93.882 | -30.607 | 0.105 | 0.126 | 1.605 | 1.926 | 5.150 | 1393764 | 515470 | 527.1 |
| tel255 | 1680.904 | -113.917 | -0.392 | 178.092 | -58.060 | 0.262 | 0.314 | 5.184 | 6.221 | 11.876 | 1394030 | 518483 | 800.0 |
| tel256 | 1682.802 | -112.019 | -0.373 | 178.171 | -58.086 | 0.158 | 0.190 | 4.707 | 5.648 | 13.149 | 1393974 | 518749 | 800.2 |
| tel257 | 1663.419 | -131.402 | -0.283 | 200.019 | -65.209 | 0.430 | 0.516 | 7.355 | 8.825 | 12.085 | 1393714 | 519044 | 871.0 |
| tel258 | 1660.377 | -134.444 | -0.216 | 204.962 | -66.820 | 0.121 | 0.145 | 7.503 | 9.004 | 12.249 | 1393519 | 519227 | 887.1 |
| tel259 | 1671.238 | -123.583 | -0.140 | 193.336 | -63.030 | 0.028 | 0.034 | 6.283 | 7.540 | 13.774 | 1393302 | 519281 | 849.4 |
| tel260 | 1673.755 | -121.066 | -0.160 | 191.561 | -62.452 | 0.040 | 0.048 | 5.564 | 6.677 | 14.227 | 1393359 | 519501 | 843.6 |
| tel261 | 1677.467 | -117.354 | -0.192 | 187.685 | -61.188 | 0.220 | 0.264 | 5.255 | 6.306 | 15.140 | 1393450 | 519757 | 831.1 |

## Appendix B

Masaya microgravity data
Table B1
Daily gravity variations at station A7 in 1997

| Local time | GMT time | Ref. time ${ }^{\text {* }}$ | Raw Scintrex (mGal)** | Scintrex ( $\mu \mathrm{Gal}$ ) ${ }^{\text {\% }}$ | RawLCR (mGal) ${ }^{\text {N** }}$ | LCR ( $\mu \mathrm{Gal})^{*}$ | Tides ( $\mu \mathrm{Gal}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7:10 |  | - |  |  |  |  | -58 |
| $7: 15$ |  |  |  |  |  |  | -59.8 |
| 7:20 |  |  |  |  |  |  | -61.4 |
| 7:25 |  |  |  |  |  |  | -62.8 |
| 7:30 |  |  |  |  |  |  | -64.1 |
| 7:35 |  |  |  |  |  |  | -65.2 |
| 7:40 |  |  |  |  |  |  | -66.1 |
| 7:45 |  |  |  |  |  |  | -68.8 |
| 7.50 |  |  |  |  |  |  | -67.4 |
| 7.55 |  |  |  |  |  |  | -67.7 |
| 8:00 |  |  |  |  |  |  | -67.9 |
| 8:05 |  |  |  |  |  |  | -67.9 |
| 8:10 |  |  |  |  |  |  | -67.6 |
| 8:15 |  |  |  |  |  |  | -67.3 |
| 8:20 |  |  |  |  |  |  | -66.8 |
| 8.25 |  |  |  |  |  |  | -66 |
| 8:30 |  |  |  |  |  |  | -65.1 |
| 8:35 |  |  |  |  |  |  | -64 |
| 8:40 |  |  |  |  |  |  | -62.7 |
| 8:45 |  |  |  |  |  |  | -61.3 |
| 8:50 |  |  |  |  |  |  | -59.6 |
| 8:55 |  |  |  |  |  |  | -57.8 |
| 9:00 |  |  |  |  |  |  | -55.8 |
| 9:05 |  |  |  |  |  |  | -53.7 |
| 9:11 | 15:11 | $0: 00$ | 2456.692 | 0 |  |  | -51.3 |
| 9:16 | 15:16 | 0:05 | 2456.698 | 6 |  |  | -48.9 |
| 9:21 | 15:21 | 0:10 | 2456.700 | 8 |  |  | -46.2 |




| 15:01 | 21:01 | 5.50 | 2456.725 | 33 |  |  | 149.7 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 15:06 | 21:06 | 5:55 | 2456.723 | 31 | 1721.941 | 31 | 148.3 |
| 15:11 | 21:11 | 6:00 | 2456.724 | 32 | 1721.943 | 32 | 146.7 |
| 15:16 | 21:16 | 6:05 | 2456.719 | 27 |  |  | 144.9 |
| 15:21 | 21:21 | 6:10 | 2456.711 | 19 |  |  | 142.9 |
| 15:26 | 21:26 | 6:15 | 2456.724 | 32 |  |  | 140.8 |
| 15:29 | 21:29 | 6:18 | 2456.725 | 33 |  |  | 138.5 |
| 15.34 | 21:34 | 6:23 | 2456.715 | 23 |  |  | 136.1 |
| 15:39 | 21:39 | 6:28 | 2456.722 | 30 |  |  | 133.5 |
| 15:44 | 21:44 | 6:33 | 2456.721 | 29 | 1721.970 | 44 | 130.8 |
| 15:49 | 21:49 | 6:38 | 2456.725 | 33 | 1721.974 | 46 | 128 |
| $15: 54$ | 21:54 | 6:43 | 2456.723 | 31 | 1721.973 | 44 | 125 |
| 15:59 | 21:59 | 6:48 | 2456.716 | 24 |  |  | 121.8 |
| 16:04 | 22:04 | 6:53 | 2456.727 | 35 |  |  | 118.6 |
| 16:09 | 22:09 | 6:58 | 2456.721 | 29 |  |  | 115.2 |
| 16:14 | 22:14 | 7:03 | 2456.720 | 28 |  |  | 111.7 |
| 16:19 | 22:19 | 7:08 | 2456.717 | 25 |  |  | 108.1 |
| 16:24 | 22:24 | 7:13 | 2456.724 | 32 | 1722.001 | 49 | 104.5 |
| 16:29 | 22:29 | 7:18 | 2456.722 | 30 | 1722.002 | 46 | 100.7 |
| 16:34 | 22:34 | 7:23 | 2456.725 | 33 | 1722.001 | 43 | 96.8 |
| 16:39 | 22:39 | 7:28 | 2456.718 | 26 |  |  | 92.9 |
| 16:45 |  |  |  |  |  |  | 88.9 |
| 16:50 |  |  |  |  |  |  | 84.8 |
| 16.55 |  |  |  |  |  |  | 80.7 |
| 17:00 |  |  |  |  |  |  | 76.5 |
| 17:05 |  |  |  |  |  |  | 72.3 |
| 17:10 |  |  |  |  |  |  | 68.1 |
| 17:15 |  |  |  |  |  |  | 59.5 |

*Relative to starting time
** Already tides-corrected
*** Not tides-corrected

## Table B2

## Daily gravity variations at station A7 in 1998

98-02-25

| Time Local | Time GMT | Ref Time* | Raw LCR ${ }^{* *}$ (mGal) | A7 ( $\mu \mathrm{Gal}$ ) ${ }^{\text {* }}$ | Tides ( $\mu \mathrm{Gal}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 10:01 | 16:01 | 0:00 | 1718.515 | 0 | 129 |
| 10:14 | 16:14 | 0:13 | 1718.497 | -8 | 138 |
| 10:24 | 16:24 | 0:23 | 1718.497 | -3 | 144 |
| 10:33 | 16:33 | 0:32 | 1718.492 | -3 | 149 |
| 10:43 | 16:43 | 0:42 | 1718.486 | -5 | 153 |
| 10:54 | 16:54 | 0:53 | 1718.484 | -4 | 156 |
| 11:05 | 17:05 | 1:04 | 1718.492 | 7 | 158 |
| 11:18 | 17:18 | 1:17 | 1718.487 | 3 | 159 |
| 11:30 | 17:30 | 1:29 | 1718.492 | 7 | 159 |
| 11:42 | 17:42 | 1:41 | 1718.497 | 10 | 157 |
| 11:56 | 17:56 | 1:55 | 1718.500 | 9 | 153 |
| 12:08 | 18:08 | 2:07 | 1718.508 | 12 | 148 |
| 12:20 | 18:20 | 2:19 | 1718.511 | 8 | 142 |
| 12:35 | $18: 35$ | 2:34 | 1718.524 | 12 | 132 |
| 12:45 | 18:45 | 2:44 | 1718.532 | 13 | 125 |
| 12:55 | 18:55 | 2:54 | 1718.538 | 11 | 116 |
| 13:10 | 19:10 | 3:09 | 1718.555 | 14 | 103 |
| 13:20 | 19:20 | 3:19 | 1718.572 | 21 | 93 |
| 13:32 | $19: 32$ | 3:31 | 1718.584 | 21 | 81 |
| 13:48 | 19:48 | 3:47 | 1718.605 | 24 | 63 |
| 14:05 | 20:05 | 4:04 | 1718.623 | 23 | 44 |
| 14:22 | 20:22 | 4:21 | 1718.642 | 23 | 25 |
| $14: 35$ | 20;35 | 4:34 | 1718.659 | 24 | 10 |
| 14:45 | 20:45 | 4:44 | 1718.674 | 28 | -2 |
| 15:08 | 21:08 | 5:07 | 1718.694 | 24 | -27 |
| 15:22 | 21:22 | 5:21 | 1718.717 | 32 | -41 |
| 15:34 | 21:34 | 5:33 | 1718.722 | 26 | -52 |
| 15:47 | 21:47 | 5.46 | 1718.722 | 14 | -64 |
| 16:05 | 22:05 | 6:04 | 1718.744 | 23 | -77 |
| 16:18 | 22:18 | 6:17 | 1718.752 | 22 | -86 |
| 16:31 | 22:31 | 6:30 | 1718.758 | 21 | -92 |
| 16:40 | 22:40 | 6:39 | 1718.757 | 17 | -96 |
| 16:55 | 22:55 | 6:54 | 1718.759 | 14 | -101 |
| 17:04 | 23:04 | 7:03 | 1718.762 | 16 | -102 |
| 17:16 | 23:16 | 7:15 | 1718.763 | 16 | -103 |
| 17:28 | 23:28 | $7: 27$ | 1718.757 | 10 | -103 |
| 17:43 | 23:43 | 7:42 | 1718.753 | 9 | -100 |
| 17:52 | 23:52 | 7.51 | 1718.744 | 3 | -97 |
| 18:07 | 0:07 | 8:06 | 1718.741 | 7 | -91 |
| 18:20 | 0:20 | 8:19 | 1718.731 | 4 | -83 |
| 18:30 | 0:30 | 8:29 | 1718.730 | 10 | -77 |
| 18:43 | 0.43 | 8:42 | 1718.712 | 1 | -67 |
| 18:56 | 0.56 | 8:55 | 1718.701 | 1 | -55 |


| 19:07 | 1:07 | 9:06 | 1718.686 | -2 | -45 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 19:19 | 1:19 | 9:18 | 1718.686 | 10 | -32 |
| 19:29 | 1:29 | 9:28 | 1718.670 | 5 | -21 |
| 19:42 | 1:42 | 9:41 | 1718.649 | -1 | -6 |
| 20:02 | 2:02 | 10:01 | 1718.625 | -1 | 18 |
| 20:18 | 2:18 | 10:17 | 1718.598 | -7 | 38 |
| 20:33 | 2:33 | 10:32 | 1718.579 | -8 | 57 |
| 20:51 | 2:51 | 10:50 | 1718.560 | -4 | 80 |
| 21:08 | 3:08 | 11:07 | 1718.537 | -6 | 101 |
| 21:22 | 3:22 | 11:21 | 1718.528 | 2 | 118 |
| 21:37 | 3:37 | 11:36 | 1718.506 | -3 | 135 |
| $21: 56$ | 3:56 | 11:55 | 1718.487 | -3 | 154 |
| 22:09 | 4:09 | 12:08 | 1718.470 | -8 | 166 |
| 22:24 | 4:24 | 12:23 | 1718.469 | 3 | 178 |
| 22:34 | 4:34 | 12:33 | 1718.462 | 4 | 185 |
| 22:48 | 4:48 | 12:47 | 1718.457 | 7 | 193 |
| 23:00 | 5:00 | 12:59 | 1718.452 | 7 | 199 |
| 23:10 | 5:10 | 13:09 | 1718.449 | 8 | 203 |
| 23:21 | 5:21 | 13:20 | 1718.446 | 8 | 205 |
| 98-03-06 |  |  |  |  |  |
| 9.58 | 15:58 | 0:00 | 1719.097 | 0 | 34 |
| 10:11 | 16:11 | 0:13 | 1719.102 | 1 | 30 |
| 10:26 | 16:26 | 0:28 | 1719.107 | 1 | 25 |
| 10.43 | 16:43 | $0: 45$ | 1719.118 | 7 | 20 |
| 10.57 | 16:57 | 0.59 | 1719.121 | 4 | 15 |
| 11:15 | 17:15 | 1:17 | 1719.130 | 7 | 9 |
| 11:33 | 17:33 | 1:35 | 1719.135 | 7 | 3 |
| 11:47 | 17:47 | 1:49 | 1719.145 | 12 | -2 |
| 12:00 | 18:00 | 2:02 | 1719.149 | 13 | -5 |
| 12:17 | 18:17 | 2:19 | 1719.152 | 11 | -10 |
| 12:31 | 18:31 | 2:33 | 1719.155 | 11 | -13 |
| 12:47 | 18:47 | 2:49 | 1719.159 | 12 | -16 |
| 13:05 | 19:05 | 3:07 | 1719.168 | 18 | -18 |
| 13:21 | $19: 21$ | 3:23 | 1719.173 | 22 | -20 |
| $13: 32$ | 19:32 | $3 \cdot 34$ | 1719.172 | 20 | -20 |
| $13: 52$ | 19:52 | 3:54 | 1719.170 | 19 | -20 |
| 14:14 | 20:14 | 4:16 | 1719.168 | 19 | -17 |
| 14:25 | 20:25 | 4:27 | 1719.166 | 20 | -16 |
| $14: 43$ | 20:43 | 4:45 | 1719.160 | 17 | -12 |
| 14:58 | 20:58 | 5:00 | 1719.159 | 20 | -8 |
| 15:13 | 21:13 | 5:15 | 1719.154 | 20 | -3 |
| 15:28 | 21:28 | 5:30 | 1719.149 | 21 | 3 |
| 15:46 | 21:46 | 5:48 | 1719.142 | 21 | 10 |
| 16:03 | 22:03 | 6:05 | 1719.129 | 16 | 18 |
| 16:18 | 22:18 | 6:20 | 1719.121 | 15 | 25 |
| $16: 33$ | 22:33 | 6:35 | 1719.107 | 9 | 33 |
| 16:58 | 22:58 | 7:00 | 1719.103 | 19 | 47 |
| 17:16 | 23:16 | 7:18 | 1719.100 | 25 | 56 |
| 17:31 | 23:31 | 7:33 | 1719.087 | 20 | 64 |
| 17:49 | 23:49 | 7.51 | 1719.076 | 18 | 74 |


| $18: 02$ | $0: 02$ | $8: 04$ | 1719.069 | 18 | 80 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $18: 14$ | $0: 14$ | $8: 16$ | 1719.061 | 16 | 86 |
| $18: 31$ | $0: 31$ | $8: 33$ | 1719.056 | 18 | 93 |
| $19: 01$ | $1: 01$ | $9: 03$ | 1719.043 | 15 | 103 |
| $19: 13$ | $1: 13$ | $9: 15$ | 1719.044 | 20 | 107 |
| $19: 33$ | $1: 33$ | $9: 35$ | 1719.038 | 18 | 111 |
| $19: 48$ | $1: 48$ | $9: 50$ | 1719.034 | 16 | 114 |
| $20: 02$ | $2: 02$ | $10: 04$ | 1719.033 | 16 | 115 |
| $20: 19$ | $2: 19$ | $10: 21$ | 1719.032 | 16 | 115 |
| $20: 34$ | $2: 34$ | $10: 36$ | 1719.034 | 17 | 115 |
| $20: 49$ | $2: 49$ | $10: 51$ | 1719.037 | 18 | 112 |
| $21: 05$ | $3: 05$ | $11: 07$ | 1719.041 | 19 | 109 |
| $21: 22$ | $3: 22$ | $11: 24$ | 1719.040 | 13 | 105 |
| $21: 39$ | $3: 39$ | $1: 41$ | 1719.042 | 10 | 99 |
| $22: 00$ | $4: 00$ | $12: 02$ | 1719.052 | 11 | 91 |
| $22: 20$ | $4: 20$ | $12: 22$ | 1719.067 | 18 | 81 |
| $22: 38$ | $4: 38$ | $12: 40$ | 1719.071 | 12 | 72 |
| $22: 53$ | $4: 53$ | $12: 55$ | 1719.080 | 13 | 64 |
| $23: 07$ | $5: 07$ | $13: 09$ | 1719.088 | 13 | 56 |
| $98-03-13$ |  |  |  |  |  |
| $10: 50$ | $16: 50$ | $0: 00$ | 1719.063 | 0 | 124 |
| $11: 07$ | $17: 07$ | $0: 17$ | 1719.055 | 2 | 134 |
| $11: 12$ | $17: 12$ | $0: 22$ | 1719.049 | -2 | 136 |
| $11: 27$ | $17: 27$ | $0: 37$ | 1719.046 | 1 | 143 |
| $11: 44$ | $17: 44$ | $0: 54$ | 1719.038 | -2 | 148 |
| $12: 08$ | $18: 08$ | $1: 18$ | 1719.038 | 1 | 151 |
| $12: 29$ | $18: 29$ | $1: 39$ | 1719.046 | 8 | 149 |
| $12: 48$ | $18: 48$ | $1: 58$ | 1719.051 | 9 | 145 |
| $13: 16$ | $19: 16$ | $2: 26$ | 1719.059 | 5 | 133 |
| $13: 31$ | $19: 31$ | $2: 41$ | 1719.071 | 9 | 124 |
| $13: 46$ | $19: 46$ | $2: 56$ | 1719.083 | 10 | 114 |
| $14: 05$ | $20: 05$ | $3: 15$ | 1719.104 | 16 | 100 |
| $14: 27$ | $20: 27$ | $3: 37$ | 1719.119 | 13 | 80 |
| $14: 48$ | $20: 48$ | $3: 58$ | 1719.136 | 9 | 61 |
| $14: 55$ | $20: 55$ | $4: 05$ | 1719.145 | 12 | 54 |
| $15: 16$ | $21: 16$ | $4: 26$ | 1719.156 | 2 | 33 |
| $15: 32$ | $21: 32$ | $4: 42$ | 1719.185 | 15 | 17 |
| $15: 50$ | $21: 50$ | $5: 00$ | 1719.201 | 14 | -1 |
| $16: 09$ | $22: 09$ | $5: 19$ | 1719.211 | 6 | -18 |
| $16: 35$ | $22: 35$ | $5: 45$ | 1719.228 | 1 | -40 |
| $17: 05$ | $23: 05$ | $6: 15$ | 1719.252 | 5 | -60 |
| $17: 16$ | $23: 16$ | $6: 26$ | 1719.258 | 5 | -66 |
| $17: 31$ | $23: 31$ | $6: 41$ | 1719.270 | 9 | -73 |
| $17: 55$ | $23: 55$ | $7: 05$ | 1719.274 | 6 | -80 |
| $18: 08$ | $0: 08$ | $7: 18$ | 1719.272 | 3 | -82 |
| $18: 23$ | $0: 23$ | $7: 33$ | 1719.276 | 6 | -82 |
| $18: 40$ | $0: 40$ | $7: 50$ | 1719.277 | 9 | -80 |
| $18: 57$ | $0: 57$ | $8: 07$ | 1719.258 | -5 | -76 |
| $19: 19$ | $1: 19$ | $8: 29$ | 1719.253 | 0 | -66 |
| $19: 38$ | $1: 38$ | $8: 48$ | 1719.240 | -3 | -55 |


| $19: 56$ | $1: 56$ | $9: 06$ | 1719.230 | 0 | -43 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $20: 18$ | $2: 18$ | $9: 28$ | 1719.212 | 1 | -25 |
| $20: 36$ | $2: 36$ | $9: 46$ | 1719.191 | -4 | -8 |
| $20: 57$ | $2: 57$ | $10: 07$ | 1719.166 | -8 | 13 |
| $21: 18$ | $3: 18$ | $10: 28$ | 1719.142 | -11 | 34 |
| $21: 36$ | $3: 36$ | $10: 46$ | 1719.124 | -11 | 53 |
| $21: 57$ | $3: 57$ | $11: 07$ | 1718.998 | -15 | 74 |
| $22: 15$ | $4: 15$ | $11: 25$ | 1718.980 | -16 | 92 |
| $22: 21$ | $4: 21$ | $11: 31$ | 1718.973 | -17 | 97 |
| $22: 38$ | $4: 38$ | $11: 48$ | 1718.964 | -11 | 112 |
| $22: 55$ | $4: 55$ | $12: 05$ | 1718.943 | -20 | 125 |
| $23: 14$ | $5: 14$ | $12: 24$ | 1718.934 | -16 | 137 |
| $23: 28$ | $5: 28$ | $12: 38$ | 1718.930 | -12 | 145 |
| $23: 43$ | $5: 43$ | $12: 53$ | 1718.929 | -7 | 151 |
| $23: 58$ | $5: 58$ | $13: 08$ | 1718.916 | -15 | 156 |
| $0: 09$ | $6: 09$ | $13: 19$ | 1718.922 | -7 | 158 |

* Relative to starting time
** Not tides-corrected
Table B3
Result of the microgravity line at Masaya caldera for the LCR

| VAR | ONS BETY | EEN 04/03 | /97 AND | 7/03/97 | Beaulieu | and H. | mer) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | MUSEO-AI | $\triangle$ MUSEO | A3-AI | $\triangle \mathrm{A} 3$ | A5-AI | $\triangle \mathrm{A} 5$ | A7-Al | $\triangle \mathrm{A} 7$ | B2-Al | $\triangle$ B2 | B1A-A1 | $\triangle$ BIA |
| 04/03/97 |  |  | 0.869 | 0.000 |  |  | -56.291 | 0.000 | -57.805 | 0.000 | -57.382 | 0.000 |
| 10/03/97 |  |  |  |  |  |  | -56.327 | -0.036 | -57.836 | -0.031 | -57.413 | -0.031 |
| 13/03/97 |  |  | 0.854 | -0.015 |  |  | -56.308 | -0.017 | -57.832 | -0.027 | -57.415 | -0.033 |
| 17/03/97 |  |  | 0.865 | -0.004 |  |  | -56.370 | -0.079 | -57.870 | -0.065 | -57.439 | -0.057 |
| VARIATIO | IONS BETW | EN 27/01 | /98 AND | 4/03/98 | auli | , | d C | liams |  |  |  |  |
|  | MUSEO-Al | $\triangle$ MUSEO | A3-Al | $\triangle \mathrm{A} 3$ | A5-A1 | $\triangle \mathrm{AS}$ | A7-Al | $\triangle \mathrm{A} 7$ | B2-A1 | $\triangle$ B2 | BIA-A1 | $\triangle$ BIA |
| 98-01-27 | -2.106 | 0.000 | 0.831 | 0.000 |  |  | -56.352 | 0.000 | -57.858 | 0.000 | -57.433 | 0.000 |
| 98-02-18 | -2.161 | -0.055 | 0.833 | 0.002 | -17.314 | 0.000 | -56.372 | -0.020 | -57.873 | -0.015 | -57.446 | -0.013 |
| 98-02-24 | -2.162 | -0.056 | 0.857 | 0.026 | -17.286 | 0.028 | -56.368 | -0.016 | -57.871 | -0.013 | -57.456 | -0.023 |
| 98-02-27 | -2.164 | -0.058 | 0.867 | 0.036 | -17.257 | 0.057 | -56.306 | 0.046 | -57.808 | 0.050 | -57.401 | 0.032 |
| 98-03-01 | -2.161 | -0.055 | 0.855 | 0.024 | -17.289 | 0.024 | -56.359 | -0.007 | -57.854 | 0.004 | -57.442 | -0.009 |
| 98-03-02 | -2.149 | -0.043 | 0.852 | 0.021 | -17.285 | 0.029 | -56.352 | 0.000 | -57.856 | 0.002 | -57.444 | -0.011 |
| 98-03-03 | -2.154 | -0.048 | 0.850 | 0.019 | -17.282 | 0.031 | - 56.369 | -0.017 | -57.853 | 0.005 | -57.443 | -0.010 |
| 98-03-05 | -2.155 | -0.049 | 0.854 | 0.023 | -17.286 | 0.027 | -56.360 | -0.008 | -57.870 | -0.012 | -57.441 | -0.008 |
| 98-03-09 | -2.144 | -0.038 | 0.847 | 0.016 | -17.281 | 0.033 | -56.368 | -0.016 | -57.838 | 0.020 | -57.417 | 0.016 |
| 98-03-14 | -2.134 | -0.028 | 0.857 | 0.026 | -17.284 | 0.029 | -56.347 | 0.005 | -57.847 | 0.011 | -57.439 | -0.006 |
| VARIATI | IONS BETV | EEN 24/02 | 2/98 AND | 09/03/98 | A.Beaulieu |  |  |  |  |  |  |  |
|  | MUSEO-A1 | $\triangle$ MUSEO | A3-A1 | $\triangle \mathrm{A} 3$ | A5-A1 | $\triangle \mathrm{A} 5$ | A7-AI | $\triangle \mathrm{A} 7$ | B2-A1 | $\triangle$ B2 | BlA-Al | $\triangle$ BIA |
| 98-02-24 | -2.162 | 0.000 | 0.857 | 0.000 | -17.286 | 0.000 | -56.368 | 0.000 | -57.871 | 0.000 | -57.456 | 0.000 |
| 98-(1)-01 | -2.161 | 0.000 | 0.855 | -0.003 | -17.289 | -0.00t | -56,359 | 0.009 | -57.854 | 0.017 | -57.442 | 0.014 |
| 98-03-02 | -2.149 | 0.013 | 0.852 | -0.005 | -17.285 | 0.001 | -56.352 | 0.016 | -57.856 | 0.016 | -57.444 | 0.013 |
| 98-03-03 | -2.154 | 0.007 | 0.850 | -0.007 | -17.282 | 0.003 | -56.369 | -0.001 | -57.853 | 0.019 | -57.443 | 0.013 |
| 98-03-05 | -2.155 | 0.007 | 0.854 | -0.003 | -17.286 | -0.001 | - 56.360 | 0.008 | -57.870 | 0.002 | -57.441 | 0.015 |
| 98-03-09 | -2.144 | 0.018 | 0.847 | -0.010 | -17.281 | 0.005 | -56.368 | 0.000 | -57.838 | 0.033 | .57.417 | 0.040 |
| VARIATI | IONS BETV | EEN 1993 | AND 199 | 8 (A.Beau | leu, H. Ry | mer and | . William | Jones) |  |  |  |  |
|  | MUSEO-AI | $\triangle$ MUSEO | A3-AI | $\triangle \mathrm{A} 3$ | A5-A1 | $\triangle \mathrm{A} 5$ | A7-A1 | $\triangle \mathrm{A} 7$ | B2-A1 | $\triangle$ B2 | BIA-AI | $\triangle \mathrm{B1A}$ |
| 93-02-01 |  |  | 0.840 | 0.000 | -17.296 | 0.000 | -56.282 | 0.000 |  |  |  |  |


|  |  |  | $\begin{array}{l\|l} \hline 0 & 0 \\ 0 \\ 0 & 0 \\ 0 \end{array}$ | $\left\|\begin{array}{l\|l\|} \hline 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}\right\|$ | $\begin{array}{l\|l\|} \hline & m \\ 0 & 0 \\ 0 & 0 \end{array}$ | $\begin{gathered} n \\ \hline 8 \\ \hline 8 \\ \hline \end{gathered}$ | $6$ | $\begin{gathered} 5 \\ i \\ i \\ i \\ \hline \end{gathered}$ | $\begin{gathered} g \\ \hline \end{gathered}$ |  |  |  |  |  |  | $\begin{gathered} n \\ \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\left\|\begin{array}{c} 2 \\ y \\ r \\ r \end{array}\right\|$ |  |  |  |  |  |  | $\dot{c}$ |  |  |  |  | $\underset{i n}{f}$ |  |  |
| $\left\|\begin{array}{c} 8 \\ 8 \\ 0 \\ 0 \end{array}\right\|$ |  | $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \end{array}\right\|$ |  | $\left\|\begin{array}{c\|c\|} \hline & 0 \\ 0 \\ 0 \\ 0 & 0 \\ 0 \end{array}\right\|$ | $8$ |  |  |  | 合 | $e_{0}^{\infty} \underset{\substack{\infty \\ 0 \\ 0 \\ 0}}{ }$ | $\stackrel{\infty}{\infty}\left\|\begin{array}{c} \infty \\ 0 \\ 0 \\ 0 \end{array}\right\|$ |  |  | $\begin{gathered} i \\ i \\ \vdots \\ \vdots \\ i \\ \hline \end{gathered}$ | Bis | $\begin{gathered} 0 \\ 0 \\ 0 \\ 6 \end{gathered}=$ |
| $\left\|\begin{array}{c} \infty \\ \infty \\ \infty \\ \stackrel{\infty}{n} \\ \stackrel{n}{2} \end{array}\right\|$ |  | $\left\|\begin{array}{c} \infty \\ i \\ n \end{array}\right\|$ |  |  |  | $\underset{\sim}{\infty}$ |  | $\underset{\sim}{\infty}$ | $\underset{c}{\infty} \underset{\sim}{\infty} \underset{\sim}{\infty} \underset{\sim}{\infty}$ |  |  | $\begin{gathered} + \\ \hline 6 \\ \hline \end{gathered}$ | $\mathfrak{n}$ |  |  |  |
| $\left\|\begin{array}{l} \infty \\ 0 \\ 0 \\ 0 \end{array}\right\|$ | $\begin{aligned} & \infty \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\left\|\begin{array}{l} \stackrel{3}{8} \\ 0 \\ 0 \end{array}\right\|$ |  |  |  | $\begin{gathered} 0 \\ 0 \\ 0 \\ \hline \end{gathered}$ | $\begin{gathered} \infty \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hline \end{gathered}$ | $\begin{gathered} 8 \\ \hline 1 \\ \hline 1 \\ \hline 1 \\ \hline 1 \end{gathered}$ | $\begin{gathered} 8 \\ 0 \\ 0 \\ \hline 1 \\ \hline \end{gathered}$ |  |  |  |  | $\begin{gathered} \infty \\ 0 \\ \hline \\ \hline \end{gathered}$ |  |  |
| $\left\|\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{array}\right\|$ |  | $\left\|\begin{array}{l} m \\ m \\ n \\ n \end{array}\right\|$ |  |  | $$ | 0 |  | $\underset{C}{2}$ | $\underset{\sim}{N}$ | 0 |  |  |  | $\hat{c}_{6}^{2}$ |  | $\infty_{c}^{\infty}$ |
| $\left\|\begin{array}{l} 3 \\ 8 \\ 8 \end{array}\right\|$ | $20$ | $\left\|\begin{array}{l} 0 \\ 0 \\ 0 \\ 1 \end{array}\right\|$ | $\left\|\begin{array}{l} \vec{a} \\ 0 \\ 0 \end{array}\right\|$ |  |  |  |  |  | $\bigcirc$ | 0 | - | $0_{0}$ | $5$ | $50$ | $\begin{gathered} 0 \\ \hline 6 \\ 0 \end{gathered}$ | $\begin{aligned} & 2 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \\ & 0 \end{aligned}$ |
| $\left.\begin{array}{\|c} \overline{9} \\ \end{array} \right\rvert\,$ | $\begin{array}{\|} \underset{7}{2} \\ \underset{\sim}{1} \\ \stackrel{1}{2} \\ \hline \end{array}$ | $\left\|\begin{array}{c} 0 \\ 0 \\ \underset{\sim}{2} \end{array}\right\|$ | $\begin{aligned} & \text { 즐 } \\ & \end{aligned}$ |  |  |  |  |  | $\pm$ | ${ }_{\sim}^{2}$ |  |  |  |  | $\underset{\sim}{\sim}$ | $\underset{\sim}{\sim}$ |
| $\stackrel{x}{\infty}$ | $2$ |  |  |  |  |  | $\begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{gathered}$ | $0 .$ | $\begin{array}{ll} 5 \\ 0 \\ 0 \end{array}$ |  | $\begin{array}{lll} 1 & 2 \\ 0 & 0 \\ 0 & 0 \\ 0 \end{array}$ | $\left.\begin{array}{c} n \\ 0 \\ 0 \end{array}\right]$ |  | $\begin{aligned} & 0 \\ & 0 \\ & 0 \end{aligned}$ | $\begin{aligned} & 9 \\ & \hline 0 \\ & 0 \end{aligned}$ |  |
| $\left\|\begin{array}{l} \infty \\ \infty \\ x \\ \end{array}\right\|$ |  |  |  |  | $\left\|\begin{array}{c} 4 \\ \infty \\ 0 \\ 0 \end{array}\right\|$ | $\begin{array}{c\|c} \substack{\infty \\ \infty \\ \infty \\ 0} \\ \hline \end{array}$ | $\mathrm{c}_{\substack{0}}^{\substack{0 \\ 0 \\ 0 \\ 0}}$ | $\infty_{0}^{\infty} \times$ | $x_{\infty}^{\infty} \left\lvert\, \begin{gathered} \infty \\ \infty \\ 0 \end{gathered}\right.$ | $\underbrace{\infty}_{\substack{\infty \\ \infty}}$ | $\begin{array}{c\|c} 6 \\ \infty & 0 \\ \infty & \infty \\ - & \infty \\ 0 \end{array}$ |  | $\stackrel{0}{\circ}$ | $0_{0}$ | $1 \begin{array}{cc} 1 \\ 0 & 6 \\ 0 & 0 \\ 0 \end{array}$ |  |
|  |  |  |  |  |  |  | $\left\|\begin{array}{l} 8 \\ 0 \\ 0 \end{array}\right\|$ | $0 . \begin{gathered} 0 \\ 0 \\ 0 \\ 0 \\ \hline \end{gathered}$ |  | $0$ | $\begin{array}{l\|l} \infty & 0 \\ 0 & 6 \\ 0 & 6 \\ 0 & 0 \end{array}$ | $c_{1}^{6}$ |  |  |  |  |
|  |  |  |  |  |  |  |  | $\stackrel{\rightharpoonup}{c}$ | $\xrightarrow{\square}$ | - |  | $\bigcirc$ |  | 筞 |  |  |
|  |  |  | $0$ | $\begin{aligned} & 2 \\ & \hline \end{aligned}$ | $\begin{aligned} & n \\ & \substack{2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0} \end{aligned}$ |  |  |  |  |  |  |  |  |  |  |  |

## Appendix C

Scintrex-LCR comparison in 1997

During the 1997 survey, the microgravity stations at Masaya were monitored using two meters, the Scintrex CG-3 \#9101184 and the LCR model G-513. Some differences were observed between the results for the two meters. The Scintrex instrument does not show exactly the same trend as the LCR instrument for the three stations next to the Santiago crater (A7, B2, And B1A) (Fig C1). The Scintrex measurements began on 10 March. Between 10-13 March, there is some difference in the variation of microgravity between stations near the crater (A7, B2 and B1A) (Fig. C1 \& Table Cla). Station A7 shows an increase of $23 \mu \mathrm{Gal}$, B2 a decrease of $20 \mu \mathrm{Gal}$ and B1A a decrease of $35 \mu \mathrm{Gal}$. Between 13-17 March, station A7 decreases by $28 \mu \mathrm{Gal}$, B2 decreases by $26 \mu \mathrm{Gal}$ and B1A increases by $15 \mu \mathrm{Gal}$. We now compare the temporal variation trend between the two instruments for each station near the crater. To do this, the values obtained on 10 March for the two instruments are used as a reference. Then the temporal variation for the two instruments may be compared for 10,13 and 17 March. For station A7, the two meters show the same trend, but they have different values (Fig. C1). This is particularly so for 17 March, where the LCR shows a greater decrease. Again, station B2 shows a similar pattern, but here the main difference is for 10 March, where the Scintrex shows a larger decrease. Between 13 and 17 March, the temporal variation observed by the two instruments is practically the same. For station B1A, the results for the LCR instrument and the Scintrex instrument are different. The LCR shows practically no variations between 10 and 13 March and a decrease of $24 \mu \mathrm{Gal}$ between 13 and 17 March. With the Scintrex, there is a decrease of $35 \mu \mathrm{Gal}$ between 10 and 13 March and an increase of $15 \mu \mathrm{Gal}$ between 13 and 17 March. For the two

## FIGURE C1

LCR-Scintrex gravity changes at Santiago crater between 10-17 March 1997.


Table C1a: Results of the microgravity line at Masaya for the Scintrex (mGal) Variation between 10-17 March 1997 for Scintrex and LCR meters

|  | $\mathrm{A} 7-\mathrm{A} 1$ | $\Delta \mathrm{~A} 7$ | $\mathrm{~B} 2-\mathrm{A} 1$ | $\Delta \mathrm{~B} 2$ | $\mathrm{~B} 1 \mathrm{~A}-\mathrm{A} 1$ | $\Delta \mathrm{~B} 1 \mathrm{~A}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| $10 / 03 / 97$ | -56.372 | 0 | -57.882 | 0 | -57.459 | 0 | LCR |
|  | -56.301 | 0 | -57.789 | 0 | -57.365 | 0 | Scintrex |
| $13 / 03 / 97$ | -56.353 | 0.019 | -57.878 | 0.004 | -57.461 | -0.002 | LCR |
|  | -56.278 | 0.023 | -57.809 | -0.020 | -57.400 | -0.035 | Scintrex |
| $17 / 03 / 97$ | -56.415 | -0.043 | -57.916 | -0.034 | -57.485 | -0.026 | LCR |
|  | -56.306 | -0.005 | -57.845 | -0.056 | -57.385 | -0.020 | Scintrex |

Table C1b: Raw data at École Polytechnique and differences between Scintrex and LCR

|  | Scintrex | LCR |
| :--- | :--- | :--- |
| Station | $\mathbf{3 0 / 0 4 / 9 7}$ | $\mathbf{0 1 / 0 4 / 9 7}$ |
| U1 | 4069.110 | 4891.296 |
| U2 | 4069.310 | 4891.509 |
| U3 | 4073.321 | 4895.646 |
| U4 | 4073.673 | 4895.994 |
| U1(end) | 4069.034 | 4891.302 |$\quad$| Scintrex |  |  |  | La Coste |
| :--- | :---: | :---: | :---: | :---: |
| Station | $01 / 04 / 97$ | $\mathbf{3 0 / 0 4 / 9 7}$ | Difference |  |
| U4-U1 | 4.698 | 4.563 | 0.135 |  |
| U3-U1 | 4.35 | 4.211 | 0.139 |  |
| U2-U1 | 0.213 | 0.200 | 0.013 |  |
| U4-U3 | 0.348 | 0.352 | -0.004 |  |
| U1(fin)-U1 | 0.006 | -0.076 |  |  |

Table C1c: Differences between A1 and other stations in mGal for the Scintrex and the LCR

| Lravimeter |  | LCR |  |  | Scintrex |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Station | $10 / 03 / 97$ | $13 / 03 / 97$ | $17 / 03 / 97$ | $10 / 03 / 97$ | $13 / 03 / 97$ | $17 / 03 / 97$ |  |
| A3-Al | nd | 0.855 | 0.866 | nd | 0.874 | 0.857 |  |
| A7-Al | -56.372 | -56.353 | -56.415 | -56.301 | -56.278 | -56.306 |  |
| B2-Al | -57.882 | -57.878 | -57.916 | -57.789 | -57.809 | -57.845 |  |
| B1A-Al | -57.459 | -57.461 | -57.485 | -57.365 | -57.400 | -57.385 |  |
| Al(end)-Al | 0.008 | 0.027 | -0.016 | -0.052 | 0.008 | -0.009 |  |

Difference Scintrex-LCR

| Station | $10 / 03 / 97$ | $13 / 03 / 97$ | $17 / 03 / 97$ |
| :---: | :---: | :---: | :---: |
| A3 | nd | 19 | 9 |
| A7 | 71 | 75 | 109 |
| B2 | 93 | 69 | 71 |
| B1A | 94 | 61 | 100 |

other stations (A7 and B2), the temporal variation of microgravity is also not the same, but the direction of variation is the same, and the trend for the two lines is similar.

Another experiment was conducted with both instruments at the École Polytechnique in Montreal to again check any differences in the measurement of the two instruments. This was done only one time for each gravimeter in the perimeter of the École Polytechnique building. The Scintrex gave good results since the closure at the end of the survey was $6 \mu \mathrm{Gal}$. But for the LCR, the closure was $76 \mu \mathrm{Gal}$ (Table Clb). This could be explained by the fact that the Scintrex is more robust in an unstable environment such as the $6^{\text {th }}$ floor of the École Polytechnique, where there is a lot of vibration due to people, the exterior wind and machines. In a stable environment, the LCR model G-513 has proven to be very accurate. For example, during the time of this survey, a Bouguer survey was conducted at another volcano (Telica, Nicaragua) with the LCR, and the average difference at the closure of each gravity line was about $17 \mu \mathrm{Gal}$. The Telica survey was not conducted as carefully as a microgravity survey. In a microgravity survey, the drift at the end of the day for the LCR model G-513 is generally of the order of $10 \mu \mathrm{Gal}$ if the correct procedure is followed (Rymer, 1989). Thus, this high value of $76 \mu \mathrm{Gal}$ for the LCR at the École Polytechnique Building is not a result of drift. The greater stability of the Scintrex compared to the LCR in noisy environments is due to the rejection system of the Scintrex which eliminates values that are too far from the mean. The LCR cannot do his, and it was difficult to achieve a good measurement since there was a lot of vibration in the mechanism of the gravity meter. The difference between the two
instruments was of the order of $140 \mu \mathrm{Gal}$ between station U1\&U3 and U1\&U4 (Table Clb ). The difference is not a matter of calibration, since the largest difference among the stations was less than 5 mGal (discussed below). Another observation is that the difference between the two instruments between U1\&U2 and U3\&U4 is very small. It is possible that the environment significantly affected our measurements in this case. Because U1 and U2 were measured in a building, they were affected in the same way by the building interference. The same observation could be made for U3 and U4 which were outside the building with less noise. When we compare a station inside and outside the building, however, the response of the two instruments is different. This difference is probably due to the different responses to the vibration for each gravity meter.

In the case of the difference between the measurements for the two gravity meters (Scintrex and LCR) at Masaya, we observe that the LCR always gave larger values (Table C1c). At Masaya, the difference among the stations was of the order of 56-58 mGal. If there is a difference in the calibration of the instruments, this could be the factor of difference. An update in the calibration factor of the LCR reduce the difference between the two gravity meters by a certain amount. The last calibration check was done in 1995 for the LCR (H. Rymer Personal Communication, 1997). There was no correction factor made for the Scintrex at Masaya, but it was done at Scintrex Ltd before it was loaned to us. There also could be some error due to manipulation and tare, but it should not exceed $10-20 \mu \mathrm{Gal}$. In general, the two instruments are in good agreement.

