HOW LONG DO WE HAVE TO KEEP TRACK OF PAST FUNCTIONAL ELECTRICAL STIMULATION WHEN CALCULATING MUSCLE FORCE?

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1. Introduction

Some mathematical models exist to estimate the muscle force due to a train of electrical stimulations. Ding's model [1] estimates muscle force through a pulse summation, including force decay due to muscle fatigue. During prolonged simulations, it experiences computational complexity, limiting its applicability for optimal control problems. This study investigates the possibility of truncating this pulse summation to reduce the computational time while keeping an acceptable accuracy for solving optimal control problems.

2. Materials and Methods

Ding's model and muscle parameters given in [1] were used in this study. To reduce the computational complexity, we truncated the pulse summation term of the calcium rate $\dot{C}_N$ in Eq. (1). We only kept a predefined number n of past stimulations in the summation.

$$\dot{C}_N = \frac{1}{\tau_{c_i}} \sum_{i=1}^{n} R_i \, e^{\left(-\frac{t-t_i}{\tau_c}\right)} - \frac{C_n}{\tau_c} \qquad (1)$$

A sensitivity analysis on pulse train type (single, doublet, and triplet) and frequency (1 to 100 Hz) was performed through one-second simulations. In line with common accuracy of optimal control problems, the absolute error threshold was set to 1e-8. The error was then assessed between the no truncation (ground truth) and the simulations for the muscle force (N) with the truncated calcium rates (unitless).

Simulations were conducted using CocoFest [2], an open-source Python package. The initial value problems were integrated with a 1-ms time step using a 4th-order Runge–Kutta method. Calculations were done on one thread of a 3.7 GHz Intel Core i5 processor with 48 GB RAM.

3. Results

The proposed truncation technique can save up to 0.64 s execution time (9.6%) compared to the ground truth solution for a single pulse train and 0.73 s (9.7%) for a doublet pulse train. The absolute error evolves linearly with frequency and exponentially according to truncation (Fig. 1).

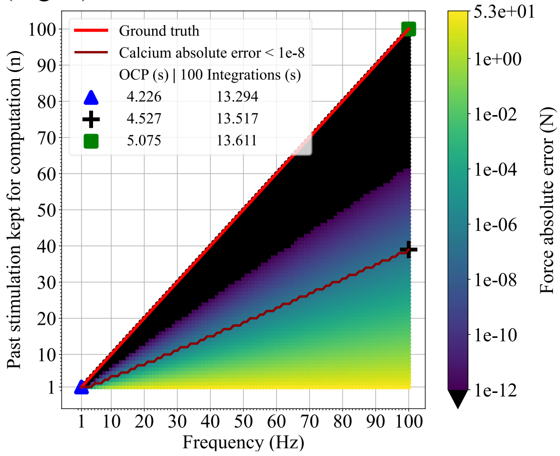

Figure 1: Truncation sensitivity analysis for a single pulse train.

4. Discussion and Conclusion

This study highlighted the relevance of keeping the full summation for high frequencies of doublets and triplets. As the calcium equation does not depend on any subject-specific parameter, this analysis is valid for every generic muscle model using Ding's model dynamics. Furthermore, we quantified the capacity of the proposed truncation technique to reduce the computational time while keeping an acceptable accuracy.

5. References

[1] Ding J. et al., Journal of Electromyography and Kinesiology; 575-588 (2003).
[2] CocoFest: DOI:10.5281/zenodo.10672335