Université de Montréal

Maximum flow-based formulation for the optimal location of electric vehicle charging stations

par

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Résumé

Due à l'augmentation de la force des changements climatiques, il devient critique d'éliminer les combustibles fossiles. Les véhicules électriques sont un bon moyen de réduire notre dépendance à ces matières polluantes, mais leur adoption est généralement limitée par le manque d'accessibilité à des stations de recharge. Dans cet article, notre but est d'agrandir l'infrastrucure liée aux stations de recharge pour fournir une meilleure qualité de service aux usagers (et une meilleure accessibilité aux stations). Nous nous attaquons spéficiquement au context urbain. Nous proposons de représenter un modèle d'assignation de demande de recharge à des stations sous la forme d'un problème de flux maximum. Ce modèle nous sert de base pour évaluer la satisfaction des usagers étant donné l'infrastruture disponible. Par la suite, nous incorporons le model de flux maximum à un programme en nombre entier mixte qui a pour but d'évaluer l'installation de nouvelles stations et d'étendre leur disponibilité en ajoutant plus de bornes de recharge. Nous présentons notre méthodologie dans le cas de la ville de Montréal et montrons que notre approche est en mesure résoudre des instances réalistes. Nous concluons en montrant l'importance de la variation dans le temps et l'espace de la demande de recharge lorsque l'on résout des instances de taille réelle.

Mots clés: Véhicules électriques, Flux maximum, Programme en nombre entier mixte, Station de recharge

Abstract

With the increasing effects of climate change, the urgency to step away from fossil fuels is greater than ever before. Electric vehicles (EVs) are one way to diminish these effects, but their widespread adoption is often limited by the insufficient availability of charging stations. In this work, our goal is to expand the infrastructure of EV charging stations, in order to provide a better quality of service in terms of user satisfaction (and availability of charging stations). Specifically, our focus is directed towards urban areas. We first propose a model for the assignment of EV charging demand to stations, framing it as a maximum flow problem. This model is the basis for the evaluation of the user satisfaction by a given charging infrastructure. Secondly, we incorporate the maximum flow model into a mixed-integer linear program, where decisions on the opening of new stations and on the expansion of their capacity through additional outlets is accounted for. We showcase our methodology for the city of Montreal, demonstrating the scalability of our approach to handle real-world scenarios. We conclude that considering both spacial and temporal variations in charging demand is meaningful when solving realistic instances.

Keywords: Electric vehicles, Maximum flow, Mixed-integer programming, Charging station placement

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MILP	Mixed-integer linear programming
EV	Electric vehicle
OD	Origin-destination
kW	Kilowatt

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Chapter 1

Introduction

In the last decade, the popularity of electric vehicles (EVs) has increased year after year. This is partially due to countries trying to reduce their greenhouse gas emissions and to provide financial incentives. This is particularly the case in Canada where fully electric car buyers are eligible to receive up to $5000 (GC [2023]). Similarly, in the province of Quebec, the government is adding a up to $7000 when purchasing a brand new EV (GQ [2023]). Beyond monetary incentives, Hydro-Québec, the publicly owned electricity company in the province, is also interested in other means to increase EV adoption. One such way is to improve EV infrastructure. This is because EV users fear not having enough range to reach their destination due to the vehicle's limited battery size, often referred to as range anxiety. Studies show that this range anxiety may affect potential buyers when deciding between an internal combustion engine and an EV (Pevec et al. [2020]). Naturally, the solution is to build more public stations and add outlets to increase availability. Carley et al. [2013] mention that, although perceived disadvantages of EVs are a strong deterrent to purchases, range anxiety could be addressed by increasing the number and visibility of public charging stations. The challenge is to identify the optimal locations for investment, and to determine where the funds should be spent.

Unlike typical gas stations, EV users may spend from minutes to hours at a charging station. This implies we have to take into account potential congestion depending on the speed of the station. Moreover, besides the selection of locations to open stations, we also have to select the charging speed and number of outlets to install. Level 1 outlets are usually not appropriate for public stations due to their speed and are usually instead installed on houses for personal use. Level 2 outlets are more expensive but fast enough to charge an empty battery within a few hours. This is sufficient for users who travel short distances and can leave their vehicle for extended periods of time. Level 3 outlets are fast enough to fully charge a vehicle in under an hour but are significantly more expensive. Usually level 2 and level 3 outlets are viable for public stations but their number per station will differ.

Beyond their price, level 3 outlets are quite energy consuming and can be taxing for the energy grid when multiple are installed on a station or in the same area. This can be a further consideration when picking a location.

A location should cover (i.e., make feasible) the trips of as many users as possible. Typically, a user will perform long trips between cities (intercity) or short trips between commonly visited locations (intracity). For long trips, a user may have to charge multiple times between their origin and destination. For this case, stations should be built between cities on highways and should be fast enough as to not delay the long trip. For short trips, users will typically move between home, work and public places. They are more likely to leave their vehicle to charge for longer periods of time, while they are at a certain location. For this case, stations should be built as close as possible to that final destination as to reduce the walking distance. After all, the willingness of drivers to buy an EV will likely depend on the distance between them and the nearest station. However, like mentioned above, not only does the station needs to be near, but it should not be occupied when needed. This means we have to estimate the demand for a given station and monitor how this demand varies over time. Firstly, demand can peak during certain hours of the day when everyone attempts to charge at the same time. Secondly, since our goal is to encourage EV adoption, at some point the number of vehicles in need of charging will increase. Based on this estimated demand, we can choose to add outlets to existing stations or, if a station is over-saturated or a region is not well-served, to build a new station nearby.

In this work, we focus on the optimal placement and sizing of EV charging stations in the intracity case, specifically, for the island of Montreal. To do so, we start by using data provided by Hydro-Québec to estimate the charging capacity of stations (supply). This is done using (charging) session data, describing how long a user has been charging at a specific outlet. It also gives us the amount of kilowatts (kWs) consumed by the vehicle. With this information, we are able to not only estimate the demand in terms of energy, but to also measure how it evolves over a day. To estimate the demand in each borough, we use, on top of the previous data, a 2018 OD survey[1] by the ARTM about trips between Montreal boroughs. The session data is used to estimate how many users charge within a borough. Next, we use the OD pairs to find from which borough they likely came from. This leaves us with an estimated demand in each borough.

Once the demand has been estimated, we aim to match (assign) the demand to stations. This allow us to evaluate the demand supplied by each station, potentially, identifying bottlenecks, namely, over-demanded regions. Another use of this approach is to finding under-supplied areas where stations are simply too few for the population. We consider these areas as prime candidate locations for future stations. This matching is defined as a maximum flow problem and solved using a linear program. The linear program is then extended to a

[1] https://www.artm.quebec/planification/enqueteod/

mixed-integer linear program where binary variables describe if a station is built or not on a candidate location. This program also comes with the possibility of adjusting the number of outlets for each station based on the demand near the station.

There are certain key differences between this work and most of the existing literature. Firstly, unlike the flow-based models related to Kuby and Lim [2005], our maximum flow model is adapted from Ford and Fulkerson [1958]. To the best of our knowledge, this is the first maximum flow model of its kind used within the context of the EV station location and sizing problem. Secondly, our work accounts for various features found in other works like station supply capacities (e.g., Upchurch et al. [2009]), planning considering multiple periods (e.g., Zhang et al. [2017]) and inclusion of already existing infrastructure (e.g., Yang [2018]). Moreover, we also focus on tackling real-world large-scale instances (e.g., as in Shahraki et al. [2015]) and in the development of exact methods (e.g., as in Cavadas et al. [2015]). We thus combine together into a single model important key modeling and methodologic aspects. Thirdly, we demonstrate our ability to solve large-scale instances with hundreds of stations and the aggregated power demand of thousands of users based real-world data. The current literature can solve these instances with heuristics, but our maximum-flow formulation enables its effective exact solving by off-the-shelf optimization solvers.

Importantly, this work provides a way to clearly identify weakness in an EV charging network with respect to over-demanded regions. However, any solution found through our methodology should be cautiously analyzed. Due to a lack of data, for example in terms of the exact location of the demand, we make certain assumptions in our testing. As such, before deploying our network expansion decisions, a discrete-event simulation should be used to accurately estimate the demand it can satisfy.

Thesis organization. The thesis is organized in the following way. In Chapter 2, we provide an extensive literature review of both maximum flow and charging infrastructure planning. Chapter 3 is our paper and can be divided in multiple sections. In Section 1, we give a brief introduction to our topic. In Section 3.2, we provide an overview of the existing literature on EV charging infrastructure planning, focusing particularly on station placement. In Section 3.3, we present the linear model for charging station network evaluation in terms of satisfied demand and the mixed-integer program, including station location and sizing decisions. In Section 3.4, we describe our case study for the island of Montreal and test our models on realistic instances. Section 3.5 concludes the paper and proposes potential future research. Finally, Chapter 4 concludes with a more in depth description of future research topics.

Chapter 2

Extended literature review

This chapter is divided into two sections. First, we go over the maximum flow literature. This is closely related to our work since the basis for our problem representation is a maximum flow which is a type of problem that tends to be fast to solve. Second, we review the literature related to charging infrastructure planning. Since our problem is about charging station location and sizing, we extend the existing literature by providing a new approach to maximum flow models.

2.1. Maximum Flow

The maximum flow problem was introduced by Ford and Fulkerson as a mean to calculate the optimal distribution of products within a given network (Ford and Fulkerson [1956]). The network is composed of a source node and a sink node from where products (i.e., flow) leave and arrive, respectively. Each arc within the network possesses a capacity, which is the maximum amount of flow that can travel on the arc, and a traversal time (sometimes referred to as cost) that represents the amount of time it takes for the said flow to travel on the arc. This model is called maximum static flow since everything is considered to happen within the same period. Ford and Fulkerson expanded upon their own idea by adding periods which are discretized over a time horizon. Each period is a copy of the network where flow can be held over to the next period (Ford and Fulkerson [1958]). This is known as a maximum dynamic flow. Halpern [1979] proposed to extend this idea by allowing the capacity of arcs to change between each period. This can model a system that evolves over time. For example, Chalmet et al. [1982] proposed a maximum dynamic flow model to evacuate a building as fast as possible. The model evolves over time as people exit the building and certain sections are cut off by hazards. This allows them to find bottlenecks in both time and space.

Since then, the field of maximum flow problems has undergone extensive research. Kotnyek [2003] gives a comprehensive overview of many different maximum flow algorithms to account for more complex and variations of the problem. Recently, a lot of the research on maximum flow algorithms has been dedicated to evacuation problems (e.g., Hoppe and Tardos [1995], Hoppe and Tardos [2000], Mamada et al. [2004], Baumann and Skutella [2006], Baumann and Köhler [2007], Baumann and Skutella [2009], Schmidt and Skutella [2014]). An example of a maximum flow problem closer to our research topic is Seo and Asakura [2021]. Their study focuses on a multi-objective shared autonomous vehicle problem, where travel time, infrastructure and fleet costs are minimized. The key difference is that in this problem, the number of vehicles to satisfy the demand is minimized, while we try to satisfy the demand for a maximum number of vehicles.

2.2. Charging Infrastructure Planning

Recently, there has been extensive research on EV charging station placement problems. Ko et al. [2017] gives a detailed description of the main aspects of this problem: location modeling, objective/constraints and demand estimation. To these characteristics, other key elements can be added, namely, *(i)* the inclusion of intracity travel, intercity travel or both, since travel range is an important characteristics of EVs, and *(ii)* the consideration of a discretized planning horizon in periods, since the charging demand and EV adoption is expected to vary over time. As the ultimate goal is to solve those problems, it also becomes relevant to describe the methodology used, which typically revolves around MILP formulations plugged directly into off-the-shelf solvers, or heuristics. Hence, guided by the enumerated aspects, we summarize related literature in Table 2.1.

Reference	Demand Estimation	Intracity or Intercity	Objective	Location Modelling	Methodology	Periods
This work	OD pairs	Intracity	Maximize flow	Flow-Based	MILP	Yes
Anjos et al. [2020]	Node-Based & OD pairs	Both	Maximize EV purchases	Maximum Covering	Heuristic	Yes
Baouche et al. [2014]	OD pairs	Intracity	Minimize costs & distance	Maximum Covering	MILP	No
Bouguerra and Layeb [2019]	Node-Based	Intracity	Minimize costs	Set Covering	MILP	No
Capar et al. [2013]	OD pairs	Intercity	Maximize flow	Maximum Covering	MILP	No
Cavadas et al. [2015]	OD pairs	Intracity	Maximize demand & Minimize distance	Maximum Covering	MILP	Yes
Chung and Kwon [2015]	OD pairs	Intercity	Maximize flow	Maximum Covering	MILP	Yes
Dong et al. [2014]	GPS trajectories	Intercity	Minimize missed trips	Maximum Covering	Heuristic	No
Filippi et al. [2023]	Node-Based	Intracity	Minimize costs & distance	Maximum Covering	MILP	Yes
Flath et al. [2014]	OD pairs	Intracity	Minimize costs	N/A	Heuristic	Yes

Frade et al. [2011]	Node-Based	Intracity	Minimize costs & Maximize coverage	Maximum Covering	MILP	Yes
Gan et al. [2020]	GPS trajectories	Intracity	Maximize profits	Maximum Covering	Heuristic	No
Hosseini and MirHassani [2015]	OD pairs	Intercity	Maximize flow	Maximum Covering	MILP or Heuristic	No
Hosseini et al. [2017]	OD pairs	Intercity	Maximize flow	Maximum Covering	MILP or Heuristic	Yes
Huang et al. [2015]	OD pairs	Intercity	Minimize costs	Maximum Covering	MILP	No
Kadri et al. [2020]	OD pairs	Intercity	Maximize flow	Maximum Covering	MILP or Heuristic	Yes
Kim and Kuby [2012]	OD pairs	Intercity	Maximize flow	Maximum Covering	MILP	No
Kim and Kuby [2013]	OD pairs	Intercity	Maximize flow	Maximum Covering	Heuristic	No
Kuby and Lim [2005]	OD pairs	Intercity	Maximize flow	Maximum Covering	MILP or Heuristic	No
Lam et al. [2014]	Node-Based	Intercity	Minimize costs	Maximum Covering	MILP or Heuristic	No
Lamontagne et al. [2023]	Simulation	Intracity	Maximize EV purchases	Maximum Covering	MILP or Heuristic	Yes
Li and Huang [2014]	OD pairs	Intercity	Minimize costs	Maximum Covering	Heuristic	No
Li et al. [2016]	OD pairs	Intercity	Minimize costs	Set Covering	MILP or Heuristic	Yes
Lim and Kuby [2010]	OD pairs	Intercity	Maximize flow	Maximum Covering	Heuristic	No
MirHassani and Ebrazi [2013]	OD pairs	Intercity	Minimize costs	Maximum Covering	MILP	No
Moghaddam et al. [2019]	Simulation	Intercity	Minimize costs, waiting & travel time	N/A	Heuristic	Yes
Shahraki et al. [2015]	GPS trajectories	Intracity	Maximize coverage	Maximum Covering	MILP	No
Tu et al. [2016]	GPS trajectories	Intracity	Maximize demand	Maximum Covering	Heuristic	Yes
Upchurch et al. [2009]	OD pairs	Intercity	Maximize flow	Maximum Covering	MILP	Yes
Upchurch and Kuby [2010]	Node-Based or OD pairs	Both	Minimize distance or Maximize flow	p-Median or Maximum Covering	Heuristic	No
Wang and Lin [2009]	OD pairs	Intercity	Minimize costs	Maximum Covering	MILP	No
Wang and Lin [2013]	OD pairs	Intercity	Maximize flow or Minimize costs	Maximum Covering	MILP	No
Xie et al. [2018]	OD pairs	Intercity	Minimize costs	Set Covering	MILP or Heuristic	Yes
Yang et al. [2017]	GPS trajectories	Intracity	Minimize costs	Maximum Covering	MILP	No

Yang [2018]	OD pairs	Intracity	Maximize demand	Maximum Covering	MILP	No
Zhang et al. [2015]	OD pairs	Intercity	Minimize costs	Set Covering	MILP	No
Zhang et al. [2017]	OD pairs	Intercity	Maximize flow	Maximum Covering	Heuristic	Yes
Zhang et al. [2020]	GPS trajectories	Intracity	Multi-Criteria	Maximum Covering	Heuristic	No
Zhong et al. [2022]	Simulation	Intracity	Minimize costs	Set Covering	Heuristic	No

Table 2.1. Summary of key characteristics of EV charging station location models

The demand estimation indicates when and where the charging demand is coming from. The most common method is based on the use of OD pair data to subsequently model how users move between locations. This kind of data often comes from surveys (e.g., Baouche et al. [2014], Zhang et al. [2015], Cavadas et al. [2015]). Another option is to use node-based demand. In this case, each node in a graph represents a population with demand based on one or more criteria (e.g., population density) (Frade et al. [2011], Lam et al. [2014], Bouguerra and Layeb [2019], Filippi et al. [2023]). Sometimes, GPS data of vehicles, usually taxis, is publicly available and can give a very accurate representation of movements within cities (Shahraki et al. [2015], Tu et al. [2016], Yang et al. [2017], Zhang et al. [2020], Gan et al. [2020]). Dong et al. [2014] has GPS data which extends much further than just a city, which is uncommon. It is also possible to use a simulation with discrete choice models to estimate user preferences over available stations (Lamontagne et al. [2023]). In our work, we use OD pairs. However, they only cover travels between boroughs. To improve the coverage, we generate random locations within the boroughs.

The location modelling usually falls into one of two categories: node-based or flow-based (Upchurch and Kuby [2010]). In the node-based approach, users are assigned to one or more locations and are able to charge their vehicle if there is a charging station nearby. A common node-based modelling is maximum coverage which tries to find locations that minimize the distance and/or maximize the number of users able to charge at those locations (e.g., Frade et al. [2011], Tu et al. [2016], Yang [2018]). A key aspect is that the models are provided with a set of candidate locations, and the goal is to find an optimal subset of locations to open a station in accordance with a given objective and a set of constraints. Another common option for node-based modelling is set coverage. The aim is to cover all the demand with the least number of charging stations (Zhang et al. [2015], Li et al. [2016], Xie et al. [2018], Bouguerra and Layeb [2019]). Unlike maximum coverage, a set of candidate locations is not needed and the nodes of the network are used instead. For the flow-based modelling, flow is assigned origin-destination (OD) pairs, and facilities (charging stations in this context) must capture as much flow as possible. This is another variant of maximum coverage proposed by Hodgson [1990]. Using the flow-based modelling, Kuby and Lim [2005] are the first to propose the Fuel Refuelling Location Problem (FRLP) which seeks to locate a fixed number of refuelling stations on a network so as to maximize the total flow volume refuelled. Over time, improvements have been made to solve the FRLP more efficiently (Lim and Kuby [2010],Capar et al. [2013]). Others have minimized the costs rather than maximize the flow (Wang and Lin [2009], Wang and Lin [2013], MirHassani and Ebrazi [2013], Li and Huang [2014]). Some researches have added a capacity on stations to limit how many vehicles can charge (Upchurch et al. [2009], Hosseini and MirHassani [2015], Hosseini et al. [2017]). A limitation of the base FRLP is that users cannot deviate from shortest path possible for the associated OD. In Kim and Kuby [2012], Kim and Kuby [2013], Huang et al. [2015] and Hosseini et al. [2017] users are allowed to stray from the shortest path to charge their vehicle. In our work, since we consider intracity travels, and hence, short trips, we do not consider the routing of EVs.

Not all EV infrastructure studies focus on the opening of new charging locations. Some try to optimize the already existing infrastructure. For instance, Flath et al. [2014] created driving profiles from data and adapted the charging prices to spread out the demand over time. Hu et al. [2016] minimized costs in a cooperative and competitive environment. More concretely, they aimed to minimize the load variance to reduce the impacts of peak demand on the energy grid and keep the prices down. Moghaddam et al. [2019] proposed an algorithm that incentivizes users to charge at periods of low usage to distribute the demand more uniformly over multiple periods.

The objective of optimal placement of charging stations can vary a lot, but it is often closely related to the location modelling. Flow-based models often maximize the total amount of flow in the network (e.g., Kuby and Lim [2005], Capar et al. [2013], Chung and Kwon [2015], Kadri et al. [2020]). However, some of them, instead, minimize costs like in Wang and Lin [2009], Lam et al. [2014] and Li and Huang [2014], or maximize EV purchases like in Anjos et al. [2020]. For node-based modelling, the objective is often one of four options or a combination of them: maximization of the covered area (Frade et al. [2011], Shahraki et al. [2015]), maximization of the satisfied EV demand (Cavadas et al. [2015], Tu et al. [2016], Yang [2018]), minimization of the costs (Zhang et al. [2015], Yang et al. [2017], Li et al. [2016], Xie et al. [2018], Bouguerra and Layeb [2019], Zhong et al. [2022]) or minimization of the distance between users and stations (Baouche et al. [2014], Cavadas et al. [2015], Filippi et al. [2023]). Some other objectives include the maximization of EV purchases (Lamontagne et al. [2023]), the optimization of profitability and reduction of the probability of outages (Gan et al. [2020]) and, in a similar vein, the maximization of the long-term profit of infrastructure investors (Vashisth et al. [2022]). In Zhang et al. [2020], the objective contains five different sub-objectives: maximization of the covered area, maximization of the satisfied EV demand and minimization of the distance between users and stations but also the reduction on the waiting time and encouragement for charging at the origin or the destination. Dong et al. [2014] aimed to minimize the number of failed trips by the EV users in an attempt to mitigate their concern of being stranded with an empty battery.

A key aspect of the EV charging station placement problem is whether the problem is intercity or intracity. The intercity case focuses on long distance travels between cities. This means that users may charge one or more times midway during a trip (e.g., Chung and Kwon [2015], Li et al. [2016], Xie et al. [2018]). Sometimes the intercity case allows for round trips and users can charge on their way back (e.g., Kuby and Lim [2005], Wang and Lin [2013], Kadri et al. [2020]). The intracity case focuses on a city or a smaller region where users are more likely to charge near homes, workplaces or public areas (Hardman et al. [2018]). Examples of concrete case studies include: Frade et al. [2011] who focused on the neighbourhood of Avenidas Novas in Lisbon, Baouche et al. [2014] who focused on the city of Lyon and Cavadas et al. [2015] who focused on the city of Coimbra. Upchurch and Kuby [2010] showed that their model works for the city of Orlando or the state of Florida, meaning it can do either intracity or intercity. To the best of our knowledge, Anjos et al. [2020] are the only authors to handle both intra and intercity cases simultaneously.

Some research includes a time component to the EV charging station placement problem. This is important as it is expected that the charging demand and infrastructure supply varies over time. Importantly, there are two ways in which periods have been modelled: in strategical planning, periods can represent years, and in tactical planing, periods can represent hours. These modelling of the time serves different purposes. If time is modelled over a number of years, the goal is to show the adoption of EV vehicles and the evolution of the EV infrastructure (Chung and Kwon [2015], Li et al. [2016], Zhang et al. [2017], Xie et al. [2018], Anjos et al. [2020], Lamontagne et al. [2023]). If time is modelled over a number of hours, the goal is to reflect high and low demand over certain times of the day and evaluate the infrastructure service quality (Frade et al. [2011], Cavadas et al. [2015], Tu et al. [2016], Filippi et al. [2023]). Zhang et al. [2015] proposed their own approach to account for temporal utilization of charging stations without using periods.

In this work, we propose a flow-based mixed-integer linear program (MILP) for the EV charging station placement and sizing problem. Baouche et al. [2014] investigated a case study of the city of Lyon for their intracity model. They also used OD data to estimate demand, yet the approach is fundamentally different, with their optimization model ensuring that all demand is covered at minimum cost, and without complementing the OD data with EV session data. Filippi et al. [2023] emphasized the importance of accounting for spatial and temporal variations in demand, which aligns with the considerations in our study. They adopt a node-based demand model and focus on minimizing installation costs and customers travel distance, subject to satisfying all the demand (which can be assigned to any opened station). This contrasts with our approach in two key ways: firstly, we focus on maximizing the satisfied energy demand; secondly, we use OD data and we limit the feasibility of the charging stations to points near the origin or destination, rather than node-based demand modelling. Lamontagne et al. [2023] proposed a MILP for the maximization of EV adoption in the long-term. Their model does not consider the stations' capacity, which is a crucial factor for our tactical problem, maximizing satisfied demand. Cavadas et al. [2015] considered an intracity case study as well as a time dimension along with station capacities in their model. Their work differentiates from ours as they used a node-based model rather than flow-based, aiming to minimize walking distance, and using a predetermined number of outlets per station. Finally, MILP approaches have encountered the issue of scalability (e.g., Zhang et al. [2017], Anjos et al. [2020] and Lamontagne et al. [2023]). However, by leveraging on the maximum flow model for the estimation of the satisfied demand, we are able to solve our MILP for instances based on the real Montreal demand and existing EV infrastructure.

Chapter 3

Maximum flow-based formulation for the optimal location of electric vehicle charging stations

by

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This article will be submitted to a journal after minor modifications.

Pierre-Luc Parent is the main author of the paper. He was involved in all stages of the research, including proposing a methodology to solve the problem, conducting a literature review, formulating the model, implementing the code, analyzing the results and writing the paper.

ABSTRACT. With the increasing effects of climate change, the urgency to step away from fossil fuels is greater than ever before. Electric vehicles (EVs) are one way to diminish these effects, but their widespread adoption is often limited by the insufficient availability of charging stations. In this work, our goal is to expand the infrastructure of EV charging stations, in order to provide a better quality of service in terms of user satisfaction (and availability of charging stations). Specifically, our focus is directed towards urban areas. We first propose a model for the assignment of EV charging demand to stations, framing it as a maximum flow problem. This model is the basis for the evaluation of the user satisfaction by a given charging infrastructure. Secondly, we incorporate the maximum flow model into a mixed-integer linear program, where decisions on the opening of new stations and on the expansion of their capacity through additional outlets is accounted for. We showcase our methodology for the city of Montreal, demonstrating the scalability of our approach to handle real-world scenarios. We conclude that considering both spacial and temporal variations in charging demand is meaningful when solving realistic instances.
Keywords: Electric vehicles, Maximum flow, Mixed-integer programming, Charging station placement

3.1. Introduction

Transportation accounts for 28% of greenhouse gas (GHG) emission in the US (EPA [2023]) and likewise in the UK (27%) and in Canada (28%) (EEA [2022],GC [2022]). For countries where a large percentage of electricity is generated from renewable sources, as is the case in Canada, studies show that electric vehicles (EVs) are a good alternative to fuel-based vehicles as a measure to curtail GHG emissions (Woo et al. [2017], Axsen et al. [2015]). To boost EV adoption, expansion and improvements to the already existing charging infrastructure must be made. This is because the willingness of car users to opt for an EV is closely linked to the EVs' travel range and the availability of charging stations (Pevec et al. [2020]). The addition of new charging stations can alleviate range anxiety, especially for prospective EV owners (Carley et al. [2013]). As such, Hydro-Québec, a publicly owned company responsible for most of the electric grid in the province of Quebec, is investing into more and faster charging stations. In fact, from 2017 to 2022, 1,800 new charging stations were added in the province of Quebec. Simultaneously, there was a surge in EV purchases, escalating from 3,347 in 2017 to 34,082 in 2022 (ST [2022]). This increase in EV purchases is mostly likely influenced by government policies (e.g., GC [2023], GQ [2023]), yet it may also be due to the introduction, in urban areas, of charging stations near homes, workplaces or public areas, as this is known to be a crucial incentive for EV adoption (Hardman et al. [2018]). Homes can sometimes be covered by privately owned chargers (Bailey et al. [2015]), but this does not apply to every EV owner. To top it off, public charging infrastructure has been shown to improve EV adoption (Coffman et al. [2017]). This unfortunately leads to the "chicken and egg" dilemma (Anjos et al. [2020]) where investors are only willing to supply more infrastructure if adoption is high, but EV purchases are dependent on widespread charging availability. As such, initial investment must come from governments and public institutions.

The motivation for this work is to support decision-makers who need to understand where infrastructure improvements are needed. Given a set of candidate locations and existing stations, they must choose where to build new stations and how many outlets should be installed or added. As mentioned previously, in a city, users mostly carry out intracity trips between home, workplace and public areas. We assume that it is possible to satisfy these users by providing them with access to charging stations near these places.

Contributions. Motivated by the context presented above, this paper tackles the challenge of optimally locating and sizing EV charging stations in urban areas to maximize the satisfied charging demand. Charging demand refers to the need of EV users to charge their vehicle at public stations. Satisfying the charging demand implies the availability in both time and space of a charging station according to its capacity. Maximizing the satisfied charging demand is an important tactical planning problem faced by EV infrastructure providers like Hydro-Québec, which regularly take decisions on the expansion of their infrastructure to meet the growing charging demand based on current usage. Our first contribution is the formulation of a linear programming model to efficiently evaluate the satisfied demand for a group of existent charging stations. Even though the satisfied demand can be determined from data on existing stations, this model serves two purposes: *(i)* it also allows us to compute the unsatisfied demand and *(ii)* it enables us to evaluate the satisfied demand for any set of stations. In this way, given a list of candidate locations for new stations, our second contribution is the integration of location and sizing decisions in the formulation, resulting in a mixed-integer linear program, maximizing the satisfied demand. Lastly, our third contribution involves detailing a case study of the island of Montreal. We base our research on real charging session data and origin-destination (OD) trips across Montreal boroughs. With it, we validate the effectiveness of our approach to solve large-scale instances and we conduct an analysis of the solutions it produces.

Our methodology differs from most papers in the literature in three key ways, underlying the novelty of our contributions.

(1) We solve the problem of determining the charging demand, i.e., the assignment of the EV users (demand) to stations, by formulating it as a maximum flow problem. Maximum flow problems have the advantage of being solvable efficiently. Importantly, our maximum flow problem is based on Ford and Fulkerson [1958] which is different from the flow-based model commonly found in Kuby and Lim [2005] and other papers about the station location problem. To the best of our knowledge, this is the first maximum flow model of its kind used within the context of the EV station location and sizing problem.

(2) Existing EV station placement methods can handle capacities (e.g., Upchurch et al. [2009]), multiple periods (e.g., Zhang et al. [2017]), already existing infrastructure (e.g., Yang [2018]), large instances (e.g., Shahraki et al. [2015]) or exact solutions (e.g., Cavadas et al. [2015]). The existing literature usually tackles only one or two of these aspects at a time. This paper integrates all these aspects in the maximum flow formulation.

(3) Thanks to our partnership with Hydro-Québec, we have access to real-world data, including the existing station locations and charging sessions with timestamps and energy consumption for every user. We use this information to generate realistic instances for testing our methodology, and demonstrate our ability to solve large-scale instances with hundreds of stations and the aggregated power demand of thousands of users.

Paper organization. The paper is organized in the following way. In Section 3.2, we provide an overview of the existing literature on EV charging infrastructure planning, focusing particularly on station placement. In Section 3.3, we present the linear model for charging station network evaluation in terms of satisfied demand and the mixed-integer program, including station location and sizing decisions. In Section 3.4, we describe our case study for the island of Montreal and test our models on realistic instances. Section 3.5 concludes the paper and proposes potential future research.

3.2. Related Literature

In this section, we begin with a brief review of the literature pertaining to the optimization of charging infrastructure utilization. Then, we delve into the literature's approaches to estimate charging demand, a crucial element for the optimal placement of charging stations. Subsequently, we discuss different location models, objective functions, intracity and intercity case studies, and temporal modeling considerations. Lastly, we position our work within the reviewed literature.

Research has been conducted on optimizing the existing charging infrastructure, particularly, through charging price decisions aimed at managing the distribution of demand (e.g., Flath et al. [2014], Hu et al. [2016], Moghaddam et al. [2019]). However, in our case study of the island of Montreal, prices cannot be changed and the power grid is prepared to handle even the most severe winter day. Therefore, we focus our review on charging station placement.

Decisions on the expansion and opening of charging stations requires the knowledge of its potential use. Thus, the estimation of charging demand is important, as it indicates when and where the demand for charging originates. The most common method is based on the use of OD data to subsequently model how users move between locations. This kind of data often comes from surveys (e.g., Baouche et al. [2014], Zhang et al. [2015], Cavadas et al. [2015]). This is the approach adopted in this paper. Nonetheless, it is important to note that our data only covers travels between boroughs (i.e., it is not granular) and it is not exclusive to EVs. Hence, we complement the demand estimation with other available data.

The location modelling usually falls into one of two categories: node-based or flow-based (Upchurch and Kuby [2010]). In the node-based approach, either a list of candidate locations is provided and the goal is to maximize the coverage (e.g., Frade et al. [2011], Tu et al. [2016], Yang [2018]), or population nodes are used as candidate locations and the goal is to satisfy all the demand at minimum cost, i.e., cost of opening stations (Zhang et al. [2015], Li et al. [2016], Xie et al. [2018], Bouguerra and Layeb [2019]). For the flow-based modelling, flow is assigned OD pairs, and facilities (charging stations in this context) must capture as much flow as possible. This is another variant of maximum coverage proposed by Hodgson [1990]. Using the flow-based modelling, Kuby and Lim [2005] are the first to propose the Fuel Refuelling Location Problem (FRLP) which seeks to locate a fixed number of refuelling stations on a network so as to maximize the total flow volume refuelled. In our work, since we consider intracity travels, and hence, short trips, we do not consider the routing of EVs. We define a maximum flow problem in the sense of Ford and Fulkerson [1958] for the location modelling. The key difference with the FRLP is that we treat flow as a variable rather than a parameter. In the FRLP each OD is assigned a flow volume on the shortest path between the origin and destination. A binary variable is then multiplied to validate whether each flow volume is present or not when maximizing the objective. This is fundamentally different since we view flow as a variable which can enter and leave both OD pairs and stations using flow constraints.

In the literature, the objective of the charging station placement problems varies significantly, but it is often closely related to the location modelling choice. Flow-based models often maximize the total amount of flow in the network (e.g., Kuby and Lim [2005], Capar et al. [2013], Chung and Kwon [2015], Kadri et al. [2020]), which is also our case. For node-based modelling, different objectives have been used, such as maximization of the satisfied EV demand (Cavadas et al. [2015], Tu et al. [2016], Yang [2018]) and a minimization of the costs (Zhang et al. [2015], Li et al. [2016], Yang et al. [2017], Xie et al. [2018], Bouguerra and Layeb [2019], Zhong et al. [2022] Filippi et al. [2023]).

A key aspect of the EV charging station placement problem is whether the problem is intercity or intracity. The intercity case focuses on long distance travels between cities, with users potentially charging once or more during a trip (e.g., Chung and Kwon [2015], Li et al. [2016], Xie et al. [2018]). The intracity case focuses on a city, with users typically charging near homes, workplaces or public areas (Hardman et al. [2018]). Frade et al. [2011], Baouche et al. [2014] and Cavadas et al. [2015] are all examples of works on intracity problems. To the best of our knowledge, Anjos et al. [2020] are the only authors to handle both intra and intercity cases simultaneously.

Some works include a time component to the EV charging station placement problem. Importantly, time periods have been modelled with two different goals: in strategical planning, periods can represent years, and in tactical planing, periods can represent hours. If time is modelled over a number of years, the goal is to focus on the adoption of EVs and the evolution of the EV infrastructure (Chung and Kwon [2015], Li et al. [2016], Zhang et al. [2017], Xie et al. [2018], Anjos et al. [2020], Lamontagne et al. [2023]). If time is modelled over a number of hours, the goal is to reflect high and low demand over certain times and evaluate the infrastructure service quality (Frade et al. [2011], Cavadas et al. [2015], Tu et al. [2016], Filippi et al. [2023]).

In this paper, we propose a flow-based (in the sense of Ford and Fulkerson [1958]) mixed-integer linear program (MILP) for the EV charging station placement and sizing problem. Baouche et al. [2014] investigated a case study of the city of Lyon for their intracity model. They also use OD data to estimate demand, yet the approach is fundamentally different from ours, with their optimization model ensuring that all demand is covered at minimum cost, and without complementing the OD data with EV session data. Filippi et al. [2023] emphasized the importance of accounting for spatial and temporal variations in demand, which aligns with the considerations in our study. They adopted a node-based demand model and focused on minimizing installation costs and customers travel distance, subject to satisfying all the demand (which can be assigned to any opened station). This contrasts with our approach in two key ways: firstly, we focus on maximizing the satisfied energy demand; secondly, we use OD data and we limit the feasibility of the charging stations to points near the origin or destination, rather than node-based demand modelling. Lamontagne et al. [2023] proposed a MILP for the maximization of EV adoption in the long-term. Their model does not consider the stations' capacity, which is a crucial factor for our tactical problem, maximizing satisfied demand. Cavadas et al. [2015] considered an intracity case study as well as a time dimension along with station capacities. Their work differentiates from ours as they use a node-based model rather than flow-based, aiming to minimize walking distance, and using a predetermined number of outlets per station. Finally, MILP approaches have encountered the issue of scalability (e.g., Zhang et al. [2017], Anjos et al. [2020] and Lamontagne et al. [2023]). However, by leveraging on the maximum flow model for the estimation of the satisfied demand, we are able to solve our MILP for instances based on the real Montreal demand and existing EV infrastructure.

3.3. Mathematical Formulation

3.3.1. Problem Statement

Our problem involves determining the optimal location and sizing (number of outlets) of EV charging stations in an urban context. The urban area under study can possess existing stations, but it is not required for the correctness of our model. The decision-maker's goal is to maximize the satisfied daily (charging) demand subject to a budget constraint for the infrastructure costs. Certainly, to address this problem, it is crucial to model how current EV users utilize the available charging stations, either existing or newly installed. Hence, we next describe the available information about EV users and the assumptions made in our work.

Since we consider the urban case, we expect EV users to travel between home and work, home and childcare, home and leisure areas, and so on, which are relatively short distances within urban settings. Therefore, we can assume that they do not charge along a path but rather at its origin or destination. Hence, the problem of determining how the charging demand is spread over the available stations becomes a matching problem, where we aim to match EV users to stations close to their origin or destination. Maximizing the number of matchings is equivalent to determining the maximum demand that can be satisfied. Given that, in our case study, users have access to an app providing in real-time the information about station occupancy (The Electric Circuit[1]), it is reasonable to optimize the assignment with this objective function.

Another important aspect of our problem is the consideration of time. Over a day, EV users do not necessarily travel and charge at the same time, nor do charging sessions have the same duration. For instance, we should expect peaks of demand in the evenings in residential areas, and significant charging duration differences between level 2 and level 3 charging stations. Therefore, we discretize the day into a finite number of periods over which the demand varies, and we consider the assignment of users to stations for each of these periods.

In our case study, we have access to the origin-destination matrix for the urban area under investigation, along with charging session data for existing stations.

3.3.2. Linear Model: Assigning Users to Stations

In this section, we describe our framework to determine the assign EV charging demand to stations. To this end, we first provide a graph modeling and then, a linear programming formulation.

[1] https://lecircuitelectrique.com/en/mobile-app/

Graph transformation. Let us go into more detail in the description of our assignment problem as it consists of a crucial building block for our methodology. We use a bipartite graph to describe potential matches (assignments). The left side of the bipartite graph is composed of EV users and the right side of stations. Instead of using individual EV users as vertices, it is more efficient to group them based on their trips: users with the same origin-destination (OD) pair are grouped together. The edges must represent feasible stations for each OD pair. To decide if a station is feasible for a given OD pair, we define a parameter R which describes the maximum radius around the origin or the destination of an OD pair. If a station is within either radius, then it is feasible for that OD pair and an edge is created.

The issue with this maximum matching approach is that there can be more than one user per OD pair and a station can charge more than one user at a time; note that a station can have more than one outlet. To fix this, we convert the bipartite graph into a flow graph, and the matching problem into a maximum flow problem. To do so, the edges of the bipartite graph are transformed into arcs from the vertices representing OD pairs to the vertices representing stations, a source vertex and a sink vertex are introduced, an arc from the source to each OD pair vertex is added, and an arc from each station vertex to the sink is added. Finally, to define a maximum flow problem over the resulting graph, restrictions regarding the amount of flow that can pass through each arc and the cost of using those arcs must also be defined. In our problem, there is no cost associated with the use of the arcs. It would be possible to add a cost based on the distance between an origin or destination and a station but this is out of the scope of this paper, since we assume that stations are within a short walking distance. On the other hand, we do define maximum flow capacities to the arcs between the source and the OD pairs, and between the stations and the sink. The former maximum flow capacity represents, at a given period, the amount of charging flow demand for every user travelling on each OD. The latter, the maximum flow capacity on the arc from a station to the sink, represents the maximum amount of charging flow supply available at that station within a period. See Figure 3.1 for an illustration. A key aspect here is that the charging flow is relative to a period. As such, the (flow) graph can be replicated for a certain number of periods T, where the graph remains the same but the arcs' maximum capacities can change between periods. Specifically, the maximum flow capacities on the arcs from the source to the OD pairs may vary, allowing for the representation of fluctuating number of users travelling on OD pairs at different times of the day. In this way, if we determine the maximum flow from the source to the sink of our graph over a finite time horizon (in our case study, 24 hours), we determine the maximum (daily) charging demand that can be satisfied by the current infrastructure.

Note that, in order to identify if a potential station location is interesting, we can add a list of candidate locations to the flow graph, following a similar approach as with existing stations. For instance, when generating the flow graph, it is possible to encounter OD pairs

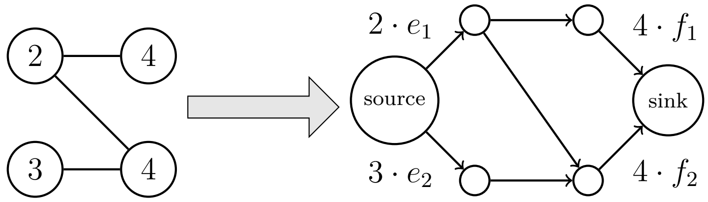


Figure 3.1. Converting a bipartite graph to a flow graph

Example: The bipartite graph on the left has two ODs, each with 2 and 3 EV users, and two stations, each with 4 outlets; the edges represent the feasible stations. On the right, we have the transformed flow graph, where in some of the arcs we have their maximum flow capacity related to EV demand and station supply; the conversion factors e_1 and e_2 map users to flow, while f_1 and f_2 map outlet supply to flow.

with no arc connected to a station. The demand from such an OD is referred to as *impossible demand*. Therefore, it would make sense to have in the list of potential new stations one or more locations close to the said OD pair; if we open at least one of these stations, it would guarantee an increase in the overall satisfied demand.

Formulation. We are ready to provide the linear program corresponding to the maximum flow of the described graph.

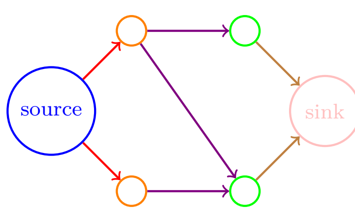


Figure 3.2. Flow model notation

From left to right, ■ source (vertex 1), ■ set L, ■ set O, ■ set M, ■ set $S_1 \cup S_2$, ■ set $R_1 \cup R_2$, ■ sink (vertex N)

Our notation is summarized in Table 3.1; the elements corresponding to new stations and outlets are only used in the next section. Figure 3.2 provides a visual summary of the notation used for the sets of arcs (continuation of the example of Figure 3.1).

Table 3.1. Notation

Type	Notation	Description
Sets	T	Set of time periods
	V	Set of vertices $\{1,2,...,N\}$ where 1 is the source and N is the sink
	O	Subset of vertices representing OD pairs
	S_1	Subset of vertices representing existing stations
	S_2	Subset of vertices representing candidate locations
	L	Subset of arcs representing the charging flow demand of users per OD pairs
	M	Subset of arcs between OD pairs and stations
	R_1	Subset of arcs representing the charging flow supply at an existing station
	R_2	Subset of arcs representing the charging flow supply at a candidate location
Parameters	A_e^t	Charging flow demand of an OD in period t for $e \in L$
	C_e	Charging flow supply at an existing station $e \in R_1$
	$I_e^{(2)}, I_e^{(3)}$	Cost of installing an outlet to a new level 2 or 3 station in location $e \in R_2$
	$J_e^{(2)}, J_e^{(3)}$	Cost of building a new level 2 or 3 station in location $e \in R_2$
	K_e	Cost of adding an outlet to an existing station $e \in R_1$
	G	Budget
	P_e	Amount of charging flow supply for a single outlet at an existing station $e \in R_1$
	$Q^{(2)}, Q^{(3)}$	Amount of charging flow supply for a single outlet at a new level 2 or 3 station
	$Y^{(2)}, Y^{(3)}$	Maximum number of outlets in a level 2 or 3 station
	Y_e	Maximum number of outlets in location $e \in R_1$
Variables	a_e^t	Amount of charging flow demand generated by an OD in period t for edge $e \in L$
	b_e^t	Amount of charging flow going from an OD to a station in period t for edge $e \in M$
	c_e^t	Amount of charging flow supply going through an existing station in period t for edge $e \in R_1$
	d_e^t	Amount of charging flow supply going through a candidate location in period t for edge $e \in R_2$
	x_e	Number of outlets to add to an existing station $e \in R_1$
	$y_e^{(2)}, y_e^{(3)}$	Number of outlets of a new level 2 or 3 station $e \in R_2$
	$z_e^{(2)}, z_e^{(3)}$	Binary variable indication whether or not to build a new level 2 or 3 station $e \in R_2$

Our maximum flow problem is the following linear program:

$$\max_{a,b,c} \sum_{t \in T} \sum_{e \in L} a_e^t \tag{3.3.1a}$$

$$s.t.\ a_{(1,v)}^t = \sum_{e \in M: e=(v,i)} b_e^t \qquad \forall v \in O, \forall t \in T \tag{3.3.1b}$$

$$\sum_{e \in M: e=(i,v)} b_e^t = c_{(v,N)}^t \qquad \forall v \in S_1, \forall t \in T \tag{3.3.1c}$$

$$0 \leq a_e^t \leq A_e^t \qquad \forall e \in L, \forall t \in T \tag{3.3.1d}$$

$$0 \leq b_e^t \qquad \forall e \in M, \forall t \in T \tag{3.3.1e}$$

$$0 \leq c_e^t \leq C_e \qquad \forall e \in R_1, \forall t \in T. \tag{3.3.1f}$$

The objective function (3.3.1a) is the sum of charging flow leaving the source. Since the flow constraints (3.3.1b) and (3.3.1c) guarantee that the amount of flow reaching the sink is equal to the amount leaving the source, this objective function is equivalent to the total amount of charging flow in the graph. Indeed, the flow constraints (3.3.1b) and (3.3.1c) ensure that the amount of charging flow entering into the OD pairs is the same amount leaving and the amount of charging flow entering the stations is the same amount leaving, respectively. Constraints (3.3.1d) limit the flow from the source to each OD pair to the charging demand for that specific OD pair within a given period. This charging demand is equal to the number of EV users travelling on that OD pair multiplied by the charging flow demand per user. For example, let a flow f be the average kW/s consumption of an EV and p the length of a period in seconds, the charging flow demand of a user is $f \cdot p$. If we multiply that value by the number of EV users in an OD, we obtain the complete requested charging flow for that OD. Constraints (3.3.1f) limit the amount of charging flow supply that can be provided at each station.

3.3.3. Mixed-Integer Model: Placing Stations and Outlets

In the previous section, we described our model for the assignment of the demand to stations. Next, our objective is to introduce new stations and outlets that remain available throughout all periods, with the aim of diminishing the existing unsatisfied demand. It is worth noting that our approach assumes that all stations and outlets are built from the beginning, rather than gradually over time. We integrate into Program (3.3.1) the decisions related to the opening of new stations and addition of outlets, leading to the following mixed-integer program:

$$
\begin{aligned}
\max_{a,b,c,d,x,y,z} \; & \sum_{t \in T} \sum_{e \in L} a_e^t && && (3.3.2a)\\
s.t. \; & \sum_{e \in R_2} \left(I_e^{(2)} y_e^{(2)} + J_e^{(2)} z_e^{(2)} + I_e^{(3)} y_e^{(3)} + J_e^{(3)} z_e^{(3)} \right) + \sum_{e \in R_1} K_e x_e \leq G && && (3.3.2b)\\
& (3.3.1b) - (3.3.1e) && && \\
& \sum_{e \in M: e=(i,v)} b_e^t = d_{(v,N)}^t && \forall v \in S_2, \forall t \in T && (3.3.2c)\\
& 0 \leq c_e^t \leq C_e + P_e x_e && \forall e \in R_1, \forall t \in T && (3.3.2d)\\
& 0 \leq d_e^t \leq Q^{(2)} y_e^{(2)} + Q^{(3)} y_e^{(3)} && \forall e \in R_2, \forall t \in T && (3.3.2e)\\
& x_e \leq Y_e - \frac{C_e}{P_e} && \forall e \in R_1 && (3.3.2f)\\
& y_e^{(2)} \leq Y^{(2)} z_e^{(2)} && \forall e \in R_2 && (3.3.2g)\\
& y_e^{(3)} \leq Y^{(3)} z_e^{(3)} && \forall e \in R_2 && (3.3.2h)\\
& z_e^{(2)} + z_e^{(3)} \leq 1 && \forall e \in R_2 && (3.3.2i)\\
& x_e \in \mathbb{N} && \forall e \in R_1 && (3.3.2j)\\
& y_e^{(2)}, y_e^{(3)} \in \mathbb{N}, z_e^{(2)}, z_e^{(3)} \in \{0,1\} && \forall e \in R_2. && (3.3.2k)
\end{aligned}
$$

The objective function is the same as before. The constraint (3.3.2b) enforces the costs of new stations and outlets to be below a given budget. Constraints (3.3.2c) are the same as Constraints (3.3.1c) but for candidate stations instead. Constraints (3.3.2d) adapt Constraints (3.3.1f) to account for newly added outlets. The number of new outlets is multiplied by a factor P_e to convert them into flow. Constraints (3.3.2e) limit the maximum amount of flow that can travel to candidate stations by the amount of new level 2 or level 3 outlets. Constraints (3.3.2f) limit the maximum amount of newly added outlets to an existing station by subtracting the already existing number of outlets from the maximum number possible. The parameters Y_e and the variables x_e are relative to the level of the station e they are associated with. If a station is level 2, only level 2 outlets can be installed and the same applies to level 3. Constraints (3.3.2g) and (3.3.2h) limit the number of outlets according to whether a new level 2 or level 3 station is built. Due to Constraints (3.3.2i), a new station can only be level 2 or level 3 but not both. In the rare case where there are level 2 and level 3 outlets at an existing station, that station is considered as two separate stations in the same location. Constraints (3.3.2j) and (3.3.2k) set the domains for variables x, y, z.

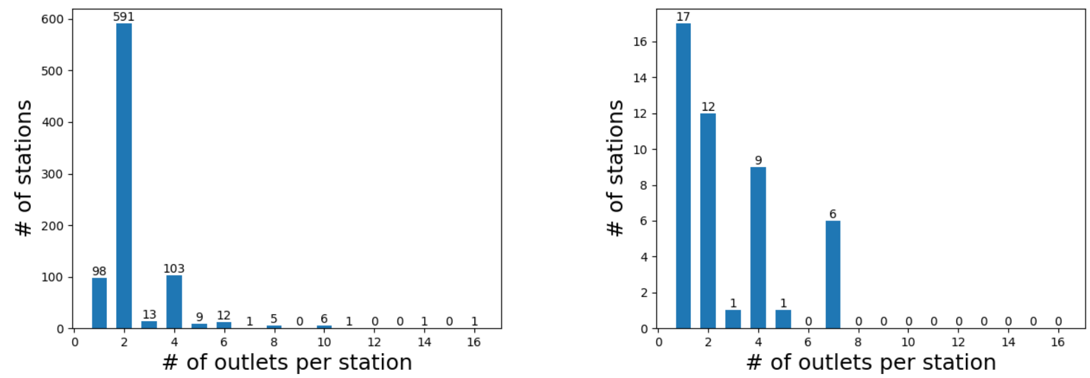
3.4. Computational Experiments

In this section, we aim to validate the use of our linear program to estimate station demand and the efficiency of solving our mixed-integer model for real-world instances. Hence, we start in Section 3.4.1 by detailing our case study of the island of Montreal. Then, in Section 3.4.2, we show experimental results of our linear model when it comes to matching users to existing stations. Finally, in Section 3.4.3, we provide experimental results for solving our mixed-integer program, modelling the addition of new stations and outlets. All experiments are run on an Intel i7-10700F CPU @ 2.90 GHz with 8 cores and 16 GB of RAM. We use CPLEX Optimization Studio V22.1.0 on a single thread per instance and a 30 minute time limit.

3.4.1. Montreal Case

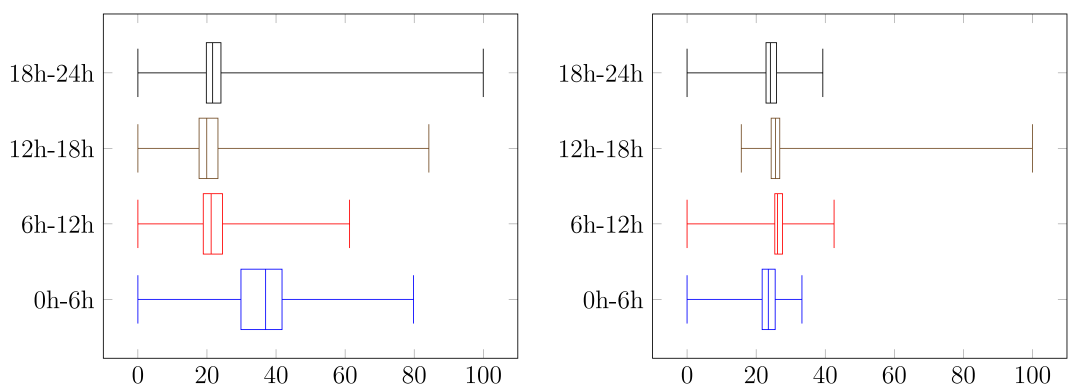
Data. We focus our experiments on the island of Montreal. For this case study, we obtained data about the location, level and number of outlets for existing public stations of the *Le Circuit électrique* and the time, duration and average kilowatts per second (kW/s) for charging sessions at these stations. This data does not include privately owned stations, but these could be added if we had the data, without changing our methodology. In our data, there are 841 level 2 stations and 41 level 3 stations on the island. The maximum number of outlets within a station is 16 and 7 for level 2 and level 3 stations, respectively. Figure 3.3 provides the distribution of outlets per station.

We tested two sets of periods: *(1)* a single time horizon of 24 hours and *(2)* the same horizon discretized into 6-hour periods. The goal is to find the impact of relaxing the demand over 24 hours, i.e., of assuming that the demand can be satisfied at any moment



(a) Level 2 (b) Level 3

Figure 3.3. Distribution of the number of outlets per station



(a) Level 2 (b) Level 3

Figure 3.4. Distribution of the daily average percentage of energy supplied per station per 6-hour period

of the day. Figure 3.4 shows that users leave their vehicles to charge overnight at level 2 stations. However, level 3 stations are used equally each day, since they are sufficiently fast, making overnight vehicle charging wasteful. As such, accounting for fluctuations of the charging demand over the day will result in more accurate modeling of the satisfied demand, which is expected to better inform the placement and sizing of stations. Indeed, using a 24-hour relaxation should overestimate the demand in comparison with the same discretized horizon, since users can be forced to charge at inconvenient times.

We also used the publicly available data of the 2018 OD survey[2] of the Montreal region collected by the ARTM. This data provides the average number of trips between each pair of Montreal boroughs within a day. These trips are not limited to EVs, but can be used

[2] https://www.artm.quebec/planification/enqueteod/

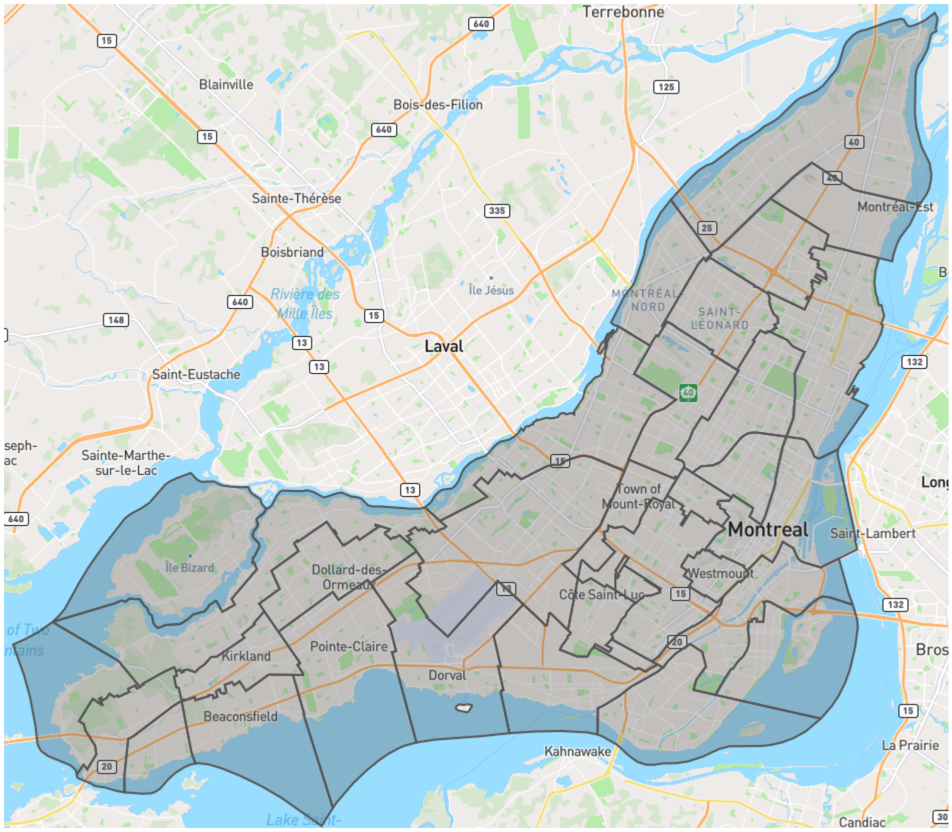


Figure 3.5. Map of Montreal boroughs

as a reasonable approximation of users' movements. We only consider the boroughs within Montreal which leaves us with 32 different boroughs (see Figure 3.5).

Generation of instances. Based on the described data, we now detail the process used to generate instances, namely, the sets and parameters of Table 3.1. To begin, we need to decide on two parameters: R for the radius and W for the number of points. The parameter R stands for the radius between a point and a station. This value indicates the maximum distance a user is willing to walk between their origin or destination and a charging station. In our framework, each OD pair could have its own radius, however, we use the same radius for all of them. We limit our radius between 400 and 700 meters based on research done on the acceptable walking distance for public transit stops and stores (Yang and Diez-Roux [2012], Millward et al. [2013], Gunn et al. [2017], Sugiyama et al. [2019]). The parameter W stands for the number of randomly generated (latitude, longitude) points in an instance. These randomly generated points serve to create OD pairs, where each point is both an origin and a destination. We generate points relative to the density of EVs users in each borough. In principle, we want to have a number of points W capable of covering the entirety of the urban area relative to the radius R. In practice however, this would require far too many points or an unrealistically large radius. As such, we try a different number of points to

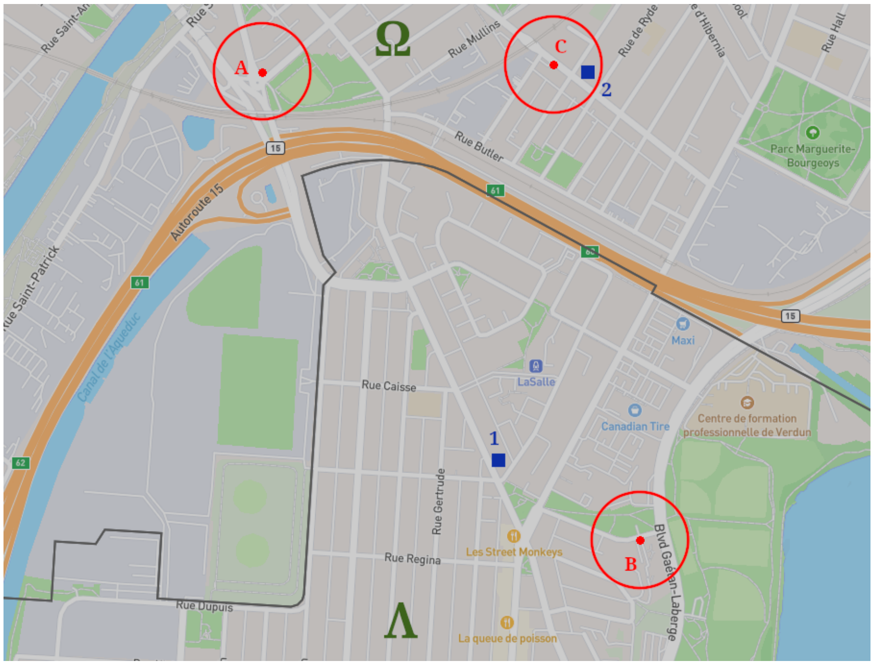


Figure 3.6. Example of an instance

test the sensibility of our model. For the rest of this section, we provide a trivial example to explain each step of our instance generation process. Figure 3.6 is a simple map with two borrows (Ω and Λ); containing three randomly generated points in red and two stations represented by the blue squares. Each point has the same radius R. We can convert Figure 3.6 into a bipartite graph. Figure 3.7 gives a visual representation of the transformation.

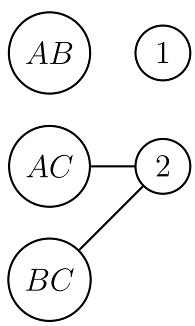


Figure 3.7. The bipartite graph representation of Figure 3.6

Our model provides the option to use any kind of data as the flow. We use kW since our dataset contains the kW/s per session. To calculate the maximum flow capacity of existing stations, we use the session data to estimate the average kW/s per session for each outlet. We sum up the outlets per station to get the maximum kW/s per station. This value can be multiplied by the length of the periods to obtain the maximum flow capacity per period.

To calculate the C_e parameters of our example, we take the average of kW/s per session from Table 3.2 for each station: 5kW/s for station 1 and 4kW/s for station 2. If we let our time horizon be a single 24-hour period, then the total flow supply is $5 \cdot 3600 \cdot 24 = 432000$kW and $4 \cdot 3600 \cdot 24 = 345600$kW for stations 1 and 2, respectively. For our example, we assume that each station only has one outlet.

Table 3.2. Example of session data

Station	Duration (s)	kW/s
1	110	5
2	100	3
2	10	5

To calculate the maximum flow capacity per OD, we start by calculating the daily average kW that is being used within a period for each station. We sum this supplied kW for each borough; this gives us the amount of supply per borough. From there, we need to convert the supply into its original demand per borough. We use the Montreal OD data to calculate the percentage of people travelling between two boroughs. This forms a set of linear equations: $\sum_{i \in H} q_i p_i^j = r_j \; \forall j \in H$ where H is the set of Montreal boroughs. In this set of equations, r_j represents the total amount of supply at each station within borough j. The coefficient p_i^j is the percentage of people travelling from borough i to j. Our variable is q_i which is the total amount of demand in the borough i. To calculate the amount of demand on each OD, we simply compute $q_i \cdot p_i^j + q_j \cdot p_j^i$ for each borough i and j. This assumes all travel is bidirectional within the OD, which is a fair assumption for intracity trips since the majority of trips occur between home, work and public places. This implies most people leave in the morning and come back at night which is bidirectional. If we want to account for unidirectional trips, we would simply need to duplicate each OD vertex. We distribute the previously computed demand uniformly between each OD pair with the same origin and destination. We also account for trips within the same borough by computing $q_i \cdot p_i^i$.

Table 3.3. Example of an OD matrix

	Ω	Λ
Ω	50%	50%
Λ	25%	75%

To calculate the A_e^t parameters of our example, we can take the average session duration from Table 3.2 per station over our 24-hour period. We assume for this example that all sessions are from the same day. This implies $110 \cdot 5 = 550$kW for station 1 and $100 \cdot 3 + 10 \cdot 5 = 350$kW for station 2. We sum all supplied kW within the same borough. Since station 1 and 2 are in different boroughs, the Λ borough has 550kW of supply and the Ω borough has 350kW. Using the OD matrix 3.3, we can write two equations: $0.5q_\Omega + 0.25q_\Lambda = 350$ and $0.5q_\Omega + 0.75q_\Lambda = 550$. Solving this set of equations gives us $q_\Omega = 500$ and $q_\Lambda = 400$. This implies that we have 500kW of demand in borough Ω and 400kW in Λ. The resulting demand

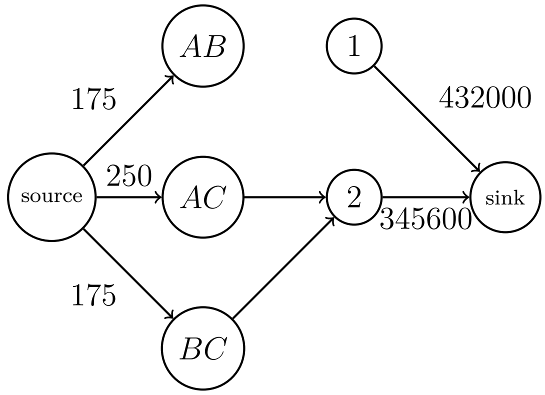


Figure 3.8. Example of a flow graph

flow is as follow: between Ω and Ω, $0.5q_\Omega = 250$kW, between Ω and Λ, $0.5q_\Omega + 0.25q_\Lambda = 350$kW and between Λ and Λ, $0.75q_\Lambda = 300$kW. We combine the flow from Ω to Λ and from Λ to Ω, since we make the assumption that our flow is bidirectional. Finally, we split these flows between each point to obtain the flow for each OD pair: AC has 250kW, AB has 175kW and BC has 175kW. It is worth noting that in this example, we lost 300kW of demand because we cannot represent the flow between Λ and Λ since we only have a single point in that borough. In practice, we generate a critical mass of points to guarantee at least 2 points in each borough. This gives us the final flow graph in Figure 3.8.

We do not have a list of potential locations, so we use impossible demand to create candidate locations. Recall that impossible demand is the demand generated by an OD vertex with no edges to any station in the bipartite graph. This demand cannot be satisfied, and as such, considering a candidate location near it is likely a good possibility. To do so, we add two candidate locations for each impossible OD demand: one at the origin and one at the destination. Note that any OD vertex within the defined radius R of a candidate location gets an outgoing arc to that location, including the OD vertex related with the impossible demand. In our example, OD AB does not have any valid station. As such, we would add two candidate locations: one at A and one at B. This means both ODs AC and BC would also gain a new station. The kW/s given to the new station is based on an average across all stations of the same level. If we assume all stations in our example are of the same level then a new station would have $\approx$4.33kW/s.

We do not have data about the cost of adding outlets or building new stations since the price can vary widely based on the location and electricity grid availability. As such, for our testing, we use arbitrary costs guided by reasonable considerations. We attribute to the addition of a level 2 outlet (to an existing station or a new one) a cost of 1. A level 3 outlet is twice that. Building a new level 2 station is 10 and a level 3 is 100. To account for these arbitrary costs, we run our experiments on multiple budgets ranging from 0 to 700 to perform a sensibility analysis of our MILP.

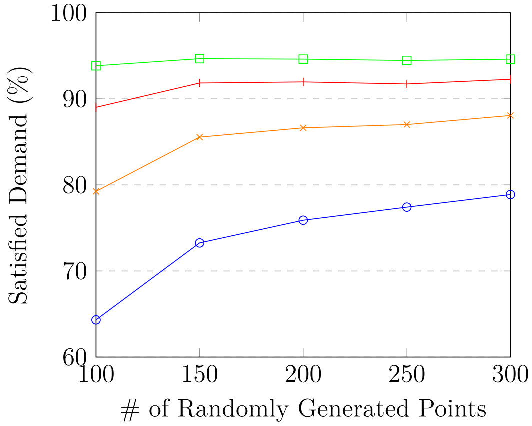
Having described our process for utilizing the data to generate the flow graphs and established the budget constraint, we now proceed to outline the various instances we create in accordance with the aforementioned procedure. We consider instances with $R \in \{400,500,600,700\}$, where the unit is meters, and with $W \in \{100,150,200,250,300\}$. For each combination of the R and W values, we generated 5 instances, where only the location of the W random OD points differ. Each instance contains the same 882 stations, which corresponds to set S_1, while the set of candidate locations S_2 can differ since it depends on the OD pairs (recall the description above). For $W = 100, 150, 200, 250$ and 300, the instances have 4,950, 11,175, 19,900, 31,125 and 44,850 ODs (i.e., the cardinality of set O) and an average of 43.4, 65.8, 92.2, 117.4 and 141.8 candidate locations, respectively. In the next sections, we provide average results over the 5 generated instances for each (R,W) pair. All the results are shown as a percentage of the total (average) kW charging demand, i.e., $\sum_{t \in T} \sum_{e \in L} A_e^t$.

3.4.2. Assigning Users to Stations

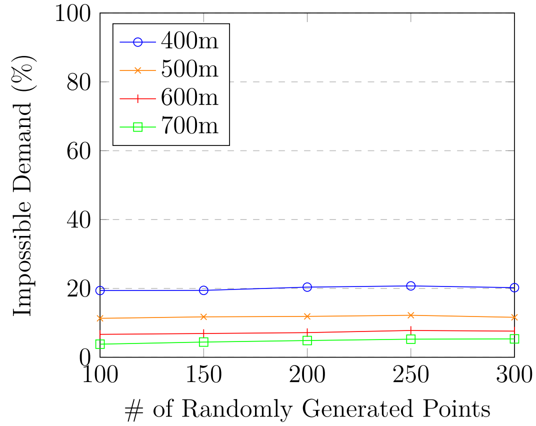
Our first tests are meant to evaluate the impact of the radius R and of the number of points W on Program (3.3.1). The goal is to assess the sensitivity of satisfied and impossible demand to those parameters, analyze the service of the current infrastructure, as well as to identify a suitable value of W that balances the model accuracy and computational efficiency when incorporating location and sizing decision. Note that larger values of W allow a greater diversity of OD scenarios, but lead to larger optimization problems.

In Figures 3.9 and 3.10 (the y-axis of figure (a) begins at 60% for better readability), we present our results for the satisfied and impossible demand[3] separately for two cases: one where a single period is considered, and the other where a day is discretized into 4 periods. In both cases, we observe that the satisfied demand increases as the number of points (W) increases, and then stabilizes. This is to be expected since the closer the number of points gets to the real number of EV users, the more accurately we depict the coverage of the territory by the existing infrastructure. Importantly, it appears that a relatively low value of W suffices to capture the prevailing charging station coverage, which has a direct impact on the number of variables and constraints of Program (3.3.2), analyzed in the next section. Similarly, an increase in the radius R results in an increase in satisfied demand. This can be attributed to the fact that a larger radius provides each OD with a greater number of station options to choose from. However, in contrast to the number of points W, opting for a larger radius distances us from reality, given that fewer individuals are expected to be willing to walk 700 meters compared to 400 meters. The larger radius can also be perceived as a

[3]Recall that unsatisfied demand is distinct from both the satisfied and the impossible demand. Unsatisfied demand arises when the supply is insufficient.



(a) Percentage of satisfied demand



(b) Percentage of impossible demand

Figure 3.9. Single period results for Program (3.3.1)

compromise in service quality (coverage). In any case, any increase in the number of points or radius is limited by the amount of demand that can be satisfied. If the stations provide enough supply, the limiting factor becomes the impossible demand. It is worth noting that since our demand generation is closely related to the amount of supply (recall Section 3.4.1), it makes sense that we are able to satisfy most of the demand for these instances. On the flip side, increasing the radius reduces the impossible demand. This can be explained by the fact that a larger radius increases the covered area, meaning ODs are less likely to have no nearby stations. A less intuitive result is the fact that increasing the number of points does not affect the impossible demand. The reason behind this comes from the stochastic nature of our points' generation. When an instance with 100 points has 20% of impossible demand, adding 100 more points will still statistically lead to an instance with 20% of impossible demand.

The main difference between the two cases in these figures is in a slightly lower satisfied demand when we consider the four 6-hour periods. This is because the unsatisfied demand in each period cannot be satisfied in others, modeling an implicit constraint on when users are willing to charge. In the 24 hours instances, users can charge at any point of the day which is unrealistic. With four blocks of 6 hours, we can more easily reflect when users are looking for a station. However, it should be noted that adopting a more granular discretization would lead to more variables and constraints in our models. Moreover, this might also necessitate the incorporation of charging over consecutive periods (if time blocks are small). The impossible demand is exactly the same in both cases. This is because the impossible demand does not change within a day (we have the same OD pairs over the time horizon).

Tables 3.5 and 3.6 give a brief overview of the amount of demand satisfied at each period. The purpose of these results is to analyze their similarity with regards to the actual

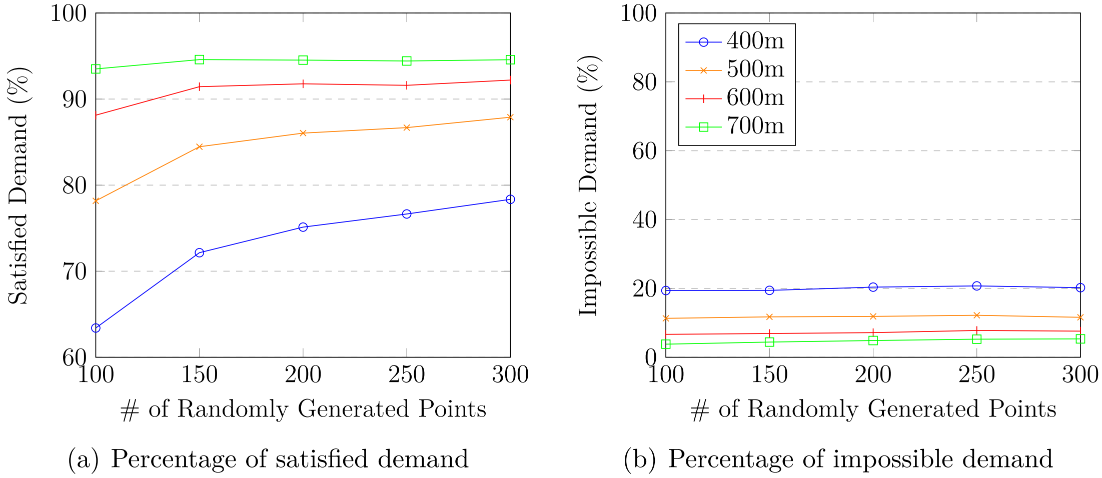


Figure 3.10. Multi-period results for Program (3.3.1)

satisfied demand of Table 3.4. In other words, we aim to understand if the charging habits over the time periods are similar. In our results, we note that level 3 stations tend to mirror the satisfied demand of level 2 stations. This is because we do not model charging habits, i.e., users preferences. For instance, during the period 0h-6h, level 3 satisfies more demand than in the other periods because in our instances (and session data) there is more demand over this period and there is no user preference making users to favour level 2. This could be improved to better match reality by either changing the capacity of level 3 stations during certain periods or introducing flow costs associated with the use of certain stations. For the tables with the other radii, we refer to Appendix A; the result trend is analogous. Figure 3.11 shows a solved instance of Montreal where all points with unsatisfied and impossible demand are visible. The impossible demand becomes more prevalent as we go west, away from downtown. The closer we get to downtown, the impossible demand turns to unsatisfied demand. Near downtown, the unsatisfied demand is nonexistent with no points visible since all demand is satisfied.

Table 3.4. Percentage of demand per station for each time period (Figure 3.4)

Period	Level 2	Level 3
18h-24h	23.64%	23.91%
12h-18h	21.92%	28.35%
6h-12h	22.46%	24.82%
0h-6h	31.96%	22.91%

Table 3.5. Percentage of satisfied demand assigned to level 2 stations per time period with a 400 meter radius

	# of Randomly Generated Points				
Period	100	150	200	250	300
18h-24h	24.57%	24.24%	23.90%	23.66%	23.59%
12h-18h	23.44%	22.68%	22.20%	21.94%	21.74%
6h-12h	23.78%	23.08%	22.76%	22.49%	22.32%
0h-6h	28.22%	30.00%	31.14%	31.91%	32.36%

Table 3.6. Percentage of satisfied demand assigned to level 3 stations per time period with a 400 meter radius

	# of Randomly Generated Points				
Period	100	150	200	250	300
18h-24h	23.45%	20.63%	22.92%	24.65%	21.91%
12h-18h	20.27%	20.57%	19.98%	17.30%	18.97%
6h-12h	21.77%	21.46%	20.55%	20.96%	19.30%
0h-6h	34.51%	37.34%	36.55%	37.09%	39.81%

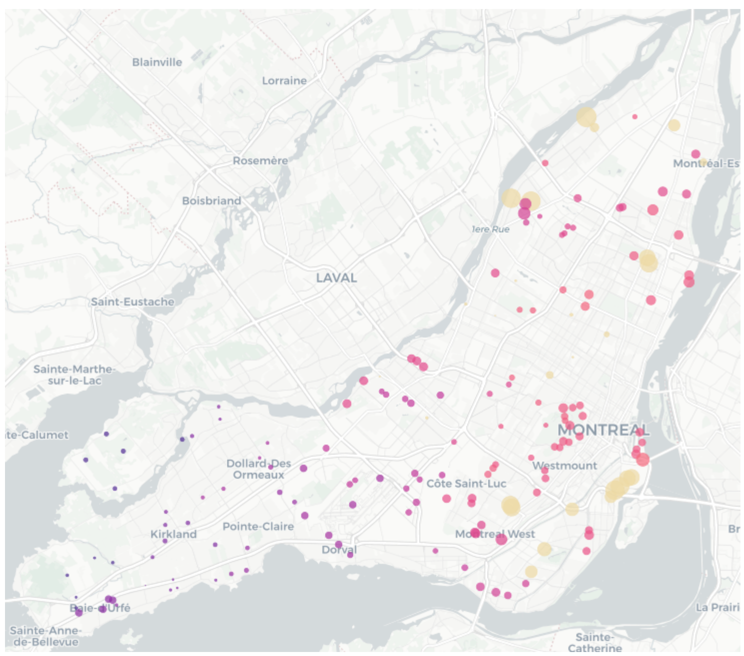


Figure 3.11. Map of the unsatisfied and impossible demand per point (R=400, W=300).

The points' size represents the unsatisfied demand and the colour represents the impossible demand (the larger and darker, the higher). Remark that points can have both unsatisfied and impossible demand as they are related to a set of OD pairs.

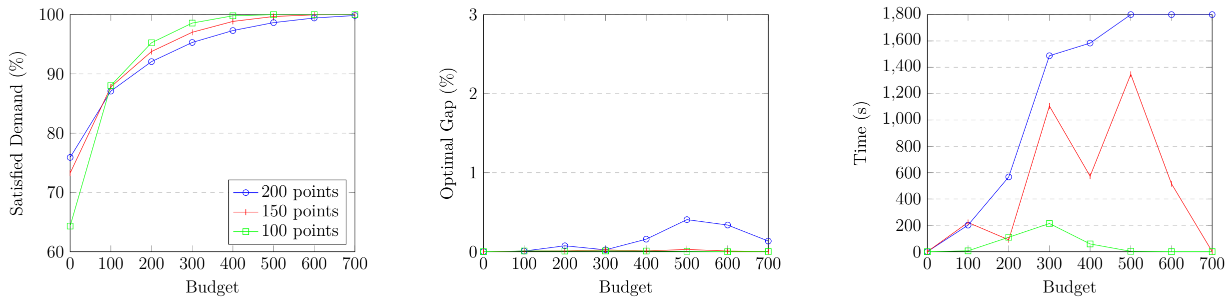
3.4.3. Adding and Expanding Stations

Our second set of tests is meant to evaluate the computational performance when solving our model, Program (3.3.2), for the installation of new stations and outlets. For these experiments, we exclusively focus on instances with $R = 400$ meter radius, as it offers the greatest flexibility to users. In other words, this radius allows for a concentration of stations in close proximity to users. We take the instances with $W = 100$, 150 and 200 random points to analyze the sensitivity of the optimal objective value (satisfied demand) to the number of points, while keeping the size of Program (3.3.2) reasonable. As explained in Section 3.4.1, we use the impossible demand to identify a list of candidate locations S_2.

Figures 3.12 and 3.13 provide our results for the case with a single period and for the case with 4 periods. We can remark two trivial cases. The first is when the budget is 0, which simply results in the linear model since no station or outlet can be added. The second is when the budget is so large ($G \geq 700$) that we can add as many stations and outlets as necessary to satisfy all the demand, both unsatisfied and impossible. The interesting cases lie in the middle, where instances with different numbers of points W have similar percentages. This can be explained by the fact that the impossible demand is relatively stable among instances (recall Figures 9(b) and 10(b) for $R = 400$m) and, as such, adding a budget reduces impossible demand by similar amounts. We can observe that solving instances with an increasing number of points W, tens to lead to higher computational times and optimality gaps. Therefore, going beyond a W value exceeding 200 points is anticipated to substantially increase computational times. The dips in the graphs are related to the high variance between instances (refer to the appendix A for detailed results).

In Figures 3.12 and 3.13, the main difference between the single and the multi-period cases is on the solving times and optimality gaps, with the multi-period one performing worst on both metrics. This is not surprising as the multi-period instances result in larger mixed-integer programs. With respect to the percentage of satisfied demand, the single and multi-period instances seems similar. In fact, the satisfied demand in the multi-period case is overall lower by at most 4.23% and on average 1% less. Although these differences are small, it is important to note that this comparison is not entirely fair, given that the instances are fundamentally distinct due to their utilization of different time discretizations. For this reason, we evaluate the location and sizing decisions of solutions derived from single-period instances within the more realistic multi-period program, resulting in Figure 3.14. We observe a discrepancy of up to 5.28% and an average of 2.75% less satisfied demand compared to the multi-period solution (Figure 3.13). This shows that accounting for the time component of the charging location and sizing problem is meaningful.

While analyzing the solutions to our instances, we observed that for the Montreal case, our model tends to prioritize the addition of level 2 outlets to existing stations and the addition of level 2 stations, as we increase the budget. In fact, among the solved instances, no solution included the addition of level 3 outlets or stations. This can be explained by the fact that the existing infrastructure is mostly sufficient to sustain the already existing demand. As such, the majority of the budget is spent providing a sparse amount of supply in

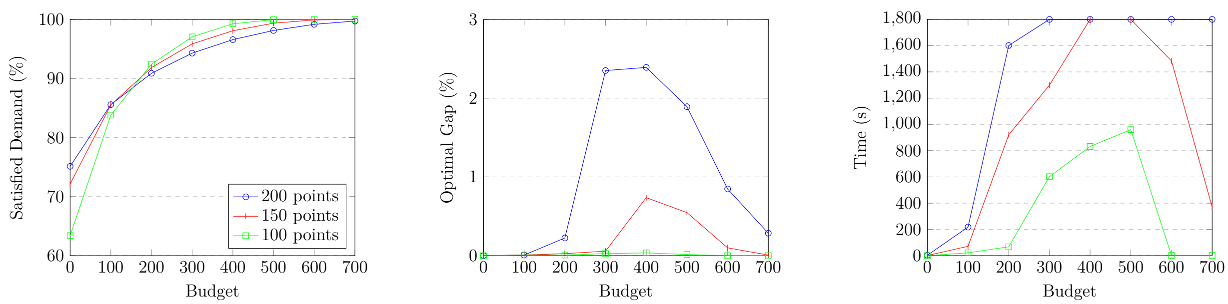


(a) Percentage of satisfied demand

(b) Optimality gap

(c) Computational time

Figure 3.12. Single period results for Program (3.3.2)

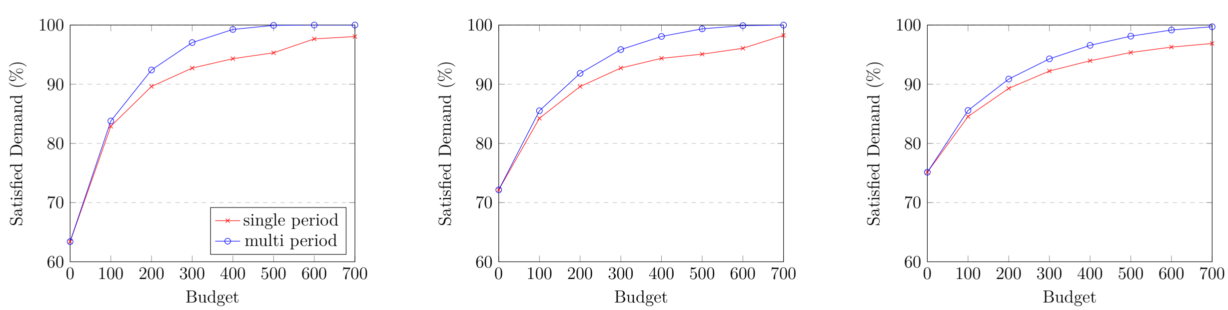


(a) Percentage of satisfied demand

(b) Optimality gap

(c) Computational time

Figure 3.13. Multi-period results for Program (3.3.2)



(a) 100 points

(b) 150 points

(c) 200 points

Figure 3.14. Single period results with multi-periods evaluation

undersupplied areas. Based on our cost parameters, level 3 stations are simply too expensive for this specific use case. Figure 3.15 is an example of this behavior.

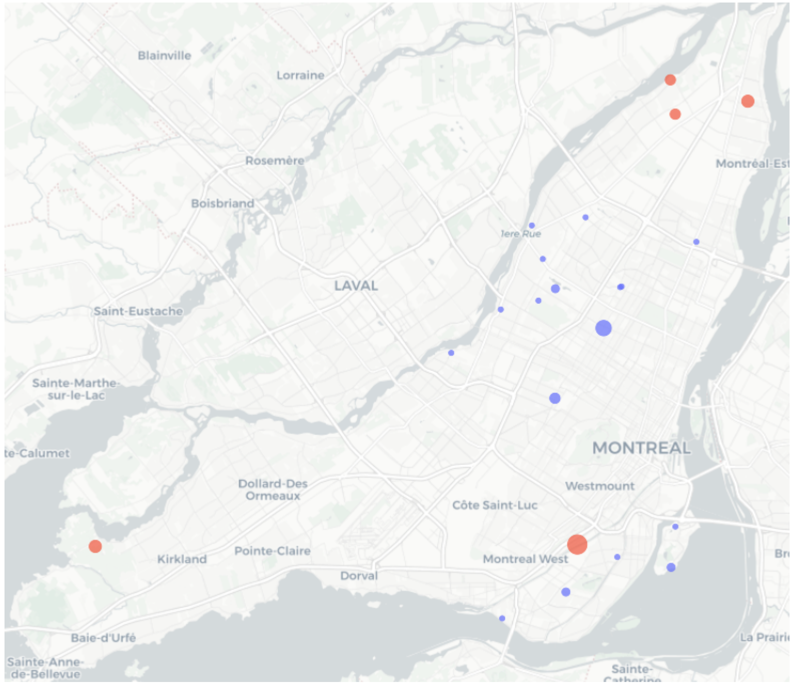


Figure 3.15. Map of Montreal with new stations and outlets (R=400, W=200, G=100)

The blue points represents existing level 2 stations and the orange ones are new level 2 stations. The size of the points corresponds to the number of outlets added. No level 3 stations are expanded or opened.

3.5. Conclusion

In this work, we propose a maximum flow-based formulation for modelling the problem of locating and sizing EV charging stations in an intracity context. By solving our mathematical programming model, we are able to optimize the placement of new stations as well as to evaluate their expected usage (i.e., the charging demand they satisfy), within a densely populated city. Performing such an evaluation can be computationally expensive due to the large number of users, existing stations and candidate locations. However, our key contribution on the transformation of the charging demand assignment to stations into a maximum flow problem, allows the scalability of our approach. To the best of our knowledge, we present one of the few approaches capable of handling realistically-sized instances without using a heuristic algorithm. Other examples of this are Shahraki et al. [2015] and Yang et al. [2017] which both provide MILPs for realistically-sized instances using taxi GPS trajectories as their demand estimation. However, our approach is multi-period which means it can evolve over time. Multi-period models with large instances tend to become intractable for MILP. Examples of this are Zhang et al. [2017] and Anjos et al. [2020] which both provide thorough MILPs but have to rely on heuristics to solve them for realistically-sized instances.

Our mathematical programming models are run based on real-world data. Concretely, the linear model can evaluate the quality of service provided by an already existing network of stations in terms of satisfied demand. Moreover, it can be used to identify impossible demand and hence, locations where the installation of new stations would be desirable. The mixed-integer model can take a list of candidate locations and return the most promising ones or increase the capacity of the already existing infrastructure given a budget. We also demonstrate the impact of discretizing a 24-hour period to account for highs and lows in demand over a day.

For the application of our approach to the case study of the island of Montreal, the only data needed was the charging session records of all the existing infrastructure and the daily OD travels across all boroughs. We show that the current infrastructure would not require a large increase in supply to satisfy all the demand at the moment. However, despite providing enough supply, the network does not provide uniform quality of service across the island, resulting in certain regions having limited access to public charging facilities.

In practice, our methodology should be especially useful for infrastructure owners to identify limitations in their provision of charging, namely, regions with impossible and unsatisfied demand. The direct use of our optimal expansion decisions must be cautiously analyzed for each specific application as simplifications were made for sake of tractability.

Further research could focus on the integration of our maximum flow model into EV charging stations placement problems using other objectives such as cost minimization. This would result in a bilevel program with a maximum flow problem at the lower level. Another important aspect would be the integration of power grid constraints, which could restrict candidate locations. Concerning the modeling of the assignment of the demand to stations, an aspect for future consideration is to account for user preferences over stations (or locations), instead of assuming that the demand is effectively spread (e.g., through a real-time app). This could be potentially done by assigning weights to the arcs of our flow network. Another line on research could be to further explore the best way to discretize a day to properly reflect reality. On the same topic, although we consider a time horizon for our flow model, we do not allow for flow to travel between consecutive periods, which could occur in practice. Finally, expanding our approach to handle both the intracity and intercity cases would allow for a more complete modelling of the optimal location and sizing of EV charging stations problem.

Acknowledgements

We are grateful to Jean-Luc Dupré from the *Direction Mobilité* of *Hydro-Québec* for sharing his knowledge about our case study, and to Steven Lamontagne for his insights about the project.

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All the map figures presented in this paper were created using `geojson.io`[4] and the open source graphing library for Python `plotly`[5].

[4]https://github.com/mapbox/geojson.io
[5]https://plotly.com/python/

Chapter 4

Conclusion

In this work, we propose a maximum flow-based formulation for modelling the problem of locating and sizing EV charging stations in an intracity context. By solving our mathematical programming model, we are able to optimize the placement of new stations as well as to evaluate their expected usage (i.e., the charging demand they satisfy), within a densely populated city. Performing such an evaluation can be computationally expensive due to the large number of users, existing stations and candidate locations. However, our key contribution on the transformation of the charging demand assignment to stations into a maximum flow problem, allows the scalability of our approach. To the best of our knowledge, we present one of the few approaches capable of handling realistically-sized instances without using a heuristic algorithm. Other examples of this are Shahraki et al. [2015] and Yang et al. [2017] which both provide MILPs for realistically-sized instances using taxi GPS trajectories as their demand estimation. However, our approach is multi-period which means it can evolve over time. Multi-period models with large instances tend to become intractable for MILP. Examples of this are Zhang et al. [2017] and Anjos et al. [2020] which both provide thorough MILPs but have to rely on heuristics to solve them for realistically-sized instances.

Our mathematical programming models are run based on real-world data. Concretely, the linear model can evaluate the quality of service provided by an already existing network of stations in terms of satisfied demand. Moreover, it can be used to identify impossible demand and hence, locations where the installation of new stations would be desirable. The mixed-integer model can take a list of candidate locations and return the most promising ones or increase the capacity of the already existing infrastructure given a budget. We also demonstrate the impact of discretizing a 24-hour period to account for highs and lows in demand over a day.

For the application of our approach to the case study of the island of Montreal, the only data needed was the charging session records of all the existing infrastructure and the daily OD travels across all boroughs. We show that the current infrastructure would not require a large increase in supply to satisfy all the demand at the moment. However, despite providing enough supply, the network does not provide uniform quality of service across the island, resulting in certain regions having limited access to public charging facilities.

Further research could focus on the integration of our maximum flow model into EV charging stations placement problems using other objectives such as cost minimization. This would result in a bilevel program with a maximum flow problem at the lower level. The lower level would assign users to different stations or candidate locations and the upper level would be selecting which stations to expand and which locations to build. Other papers like Baouche et al. [2014] and Filippi et al. [2023] balance costs with distance between users and stations. In our model, this would mean to remove the parameter R and include distance into the upper level's objective function. Another important aspect would be the integration of power grid constraints, which could restrict candidate locations. The idea behind this restriction is that allocating too many stations or outlets in a particular area would overload the power grid. This would either be infeasible or cause further expenses.

Concerning the modelling of the assignment of the demand to stations, an aspect for future consideration is to account for user preferences over stations (or locations), instead of assuming that the demand is effectively spread (e.g., through a real-time app). This could be potentially done by assigning weights to the arcs of our flow network. Our weights could represent the cost of charging at a given location. This would be similar to Flath et al. [2014] where they adapt the charging prices to spread out the demand over time. Another line on research could be to further explore the best way to discretize a day to properly reflect reality. This is shown in the discrepancy between level 2 and level 3 demand. Our data shows, Level 2 stations are used more at night since users leave their vehicle to charge over night. However, this is not the case for level 3 stations and this could be modelled better. On the same topic, although we consider a time horizon for our flow model, we do not allow for flow to travel between consecutive periods, which could occur in practice. Currently, a single user can be split between multiple periods which can lead to unintended consequences. For instance, a user could charge between 5h and 7h, but the model would sometime place the demand between 1h and 2h and 11h and 12h. Allowing users to charge between periods using heldover constraints as defined in Halpern [1979], would create a more consistent flow. Finally, expanding our approach to handle both the intracity and intercity cases would allow for a more complete modelling of the charging station placement problem similar to what Anjos et al. [2020] have done.

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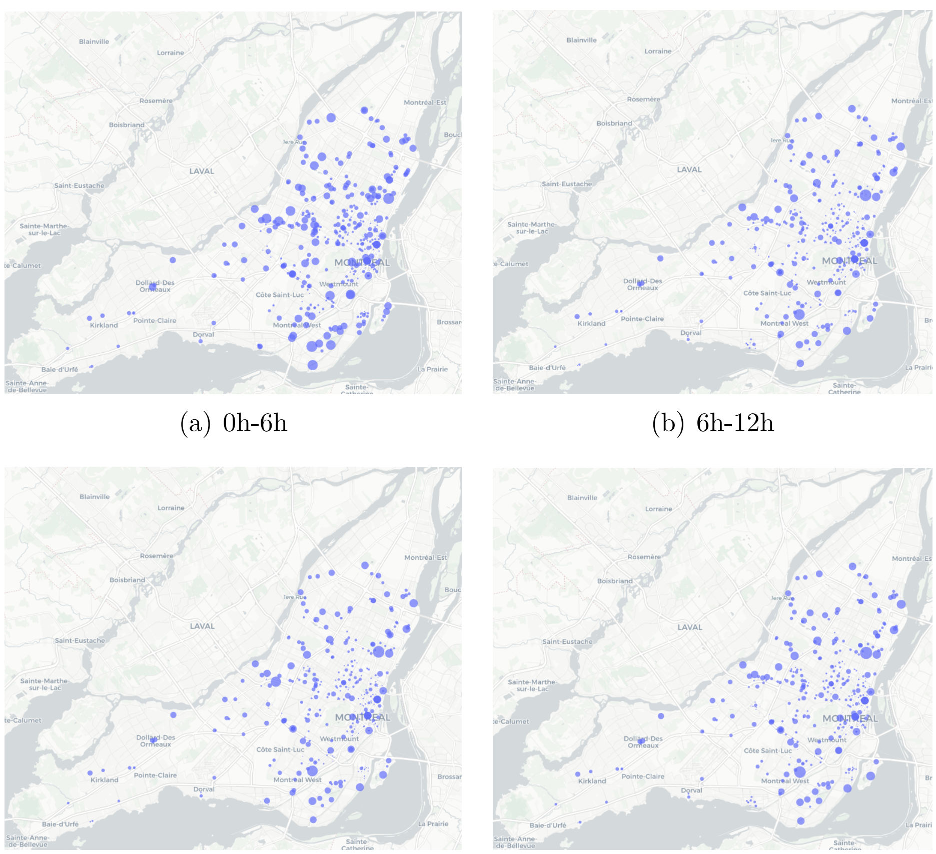
Chapter A

Appendix

Detailed Results for Section 3.4.2



(a) 0h-6h

(b) 6h-12h

(c) 12h-18h

(d) 18h-24h

Figure A.1. Example of satisfied demand over time periods (R=700, W=300)

Table A.1. Percentage of satisfied demand assigned to level 2 stations per time period with a 500 meter radius

	# of Randomly Generated Points				
Period	100	150	200	250	300
18h-24h	24.17%	23.90%	23.59%	23.44%	23.41%
12h-18h	22.60%	22.04%	21.76%	21.64%	21.54%
6h-12h	23.13%	22.54%	22.32%	22.19%	22.16%
0h-6h	30.10%	31.52%	32.34%	32.72%	32.89%

Table A.2. Percentage of satisfied demand assigned to level 3 stations per time period with a 500 meter radius

	# of Randomly Generated Points				
Period	100	150	200	250	300
18h-24h	23.23%	21.46%	22.17%	23.71%	22.20%
12h-18h	21.24%	20.31%	20.05%	18.73%	19.53%
6h-12h	21.34%	21.92%	20.79%	19.89%	20.36%
0h-6h	34.19%	36.31%	36.98%	37.67%	37.91%

Table A.3. Percentage of satisfied demand assigned to level 2 stations per time period with a 600 meter radius

	# of Randomly Generated Points				
Period	100	150	200	250	300
18h-24h	23.68%	23.39%	23.44%	23.36%	23.25%
12h-18h	22.02%	21.52%	21.54%	21.62%	21.43%
6h-12h	22.60%	22.10%	22.21%	22.17%	22.02%
0h-6h	31.70%	32.99%	32.82%	32.85%	33.30%

Table A.4. Percentage of satisfied demand assigned to level 3 stations per time period with a 600 meter radius

	# of Randomly Generated Points				
Period	100	150	200	250	300
18h-24h	24.14%	23.61%	22.71%	23.02%	23.81%
12h-18h	21.55%	22.09%	21.20%	19.58%	21.10%
6h-12h	21.67%	22.71%	21.15%	20.68%	21.80%
0h-6h	32.64%	31.60%	34.94%	36.71%	33.29%

Table A.5. Percentage of satisfied demand assigned to level 2 stations per time period with a 700 meter radius

	# of Randomly Generated Points				
Period	100	150	200	250	300
18h-24h	23.27%	23.35%	23.27%	23.34%	23.21%
12h-18h	21.45%	21.45%	21.44%	21.58%	21.52%
6h-12h	22.06%	22.05%	22.10%	22.11%	22.13%
0h-6h	33.22%	33.16%	33.19%	32.97%	33.13%

Table A.6. Percentage of satisfied demand assigned to level 3 stations per time period with a 700 meter radius

	# of Randomly Generated Points				
Period	100	150	200	250	300
18h-24h	23.99%	22.76%	23.70%	22.87%	23.95%
12h-18h	21.88%	21.19%	21.55%	19.99%	20.41%
6h-12h	22.40%	21.77%	21.77%	21.44%	21.12%
0h-6h	31.73%	34.28%	32.98%	35.69%	34.52%

Detailed Results for Section 3.4.3

Table A.7. Satisfied demand for Program (3.3.2) over a single period

	Budget							
	0	100	200	300	400	500	600	700
100 Points								
Instance 1	62.38%	89.14%	95.48%	98.85%	99.86%	99.99%	100.00%	100.00%
Instance 2	67.84%	88.43%	95.79%	98.73%	99.90%	99.99%	100.00%	100.00%
Instance 3	57.72%	84.21%	93.54%	97.88%	99.64%	99.99%	100.00%	100.00%
Instance 4	67.81%	92.85%	97.90%	99.79%	100.00%	100.00%	100.00%	99.99%
Instance 5	65.85%	85.46%	93.54%	97.62%	99.60%	100.00%	100.00%	100.00%
150 Points								
Instance 1	76.49%	90.04%	95.07%	97.73%	99.24%	99.87%	99.99%	100.00%
Instance 2	69.97%	86.27%	93.51%	97.23%	98.90%	99.75%	99.99%	100.00%
Instance 3	67.54%	83.65%	91.04%	95.15%	97.85%	99.24%	99.88%	99.99%
Instance 4	76.47%	91.74%	95.81%	98.11%	99.44%	99.91%	100.00%	100.00%
Instance 5	75.85%	87.30%	93.43%	96.86%	98.81%	99.63%	99.96%	99.99%
200 Points								
Instance 1	77.00%	88.84%	92.87%	95.91%	97.61%	98.84%	99.53%	99.92%
Instance 2	74.86%	86.25%	91.79%	95.35%	97.42%	98.66%	99.47%	99.88%
Instance 3	71.38%	83.16%	89.46%	93.48%	96.01%	97.92%	99.02%	99.67%
Instance 4	79.68%	91.13%	94.77%	96.90%	98.45%	99.27%	99.79%	99.97%
Instance 5	76.56%	86.06%	91.46%	94.88%	97.07%	98.54%	99.32%	99.81%

Table A.8. Computing time (s) for Program (3.3.2) over a single period

	Budget							
	0	100	200	300	400	500	600	700
100 Points								
Instance 1	0.078	7.156	447.078	80.485	95.047	0.546	0.437	0.469
Instance 2	0.063	7.625	10.079	27.375	19.578	1.016	0.5	0.406
Instance 3	0.078	6.672	16.609	29.156	91.062	7.297	0.343	0.375
Instance 4	0.063	5.282	55.266	14.563	0.281	0.328	0.328	0.422
Instance 5	0.063	6.047	18.282	926.094	88.515	6.063	0.406	0.344
150 Points								
Instance 1	0.14	413.594	103.828	1800	861.797	1536.781	23.718	0.735
Instance 2	0.172	33.578	133.328	64.75	657.157	655.625	830.922	0.906
Instance 3	0.172	418.547	42.125	1800	587.137	1800	1031.094	2.453
Instance 4	0.171	14.468	78.828	959.875	503.063	1800	1.578	0.75
Instance 5	0.172	228.953	92.187	312.156	258.016	942.156	693.094	0.969
200 Points								
Instance 1	0.328	254.719	1800	1800	1800	1800	1800	1800
Instance 2	0.282	11.719	413.36	1224.313	1428.156	1800	1800	1800
Instance 3	0.421	665.766	468.86	1800	1800	1800	1800	1800
Instance 4	0.36	12.235	61.734	1800	1090	1800	1800	1800
Instance 5	0.344	64.187	94.297	813.891	1800	1800	1800	1800

Table A.9. Optimal gap for Program (3.3.2) over a single period

	Budget							
	0	100	200	300	400	500	600	700
100 Points								
Instance 1	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%
Instance 2	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%
Instance 3	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%
Instance 4	0.00%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%	0.01%
Instance 5	0.00%	0.01%	0.00%	0.01%	0.01%	0.00%	0.00%	0.00%
150 Points								
Instance 1	0.00%	0.01%	0.01%	0.02%	0.01%	0.01%	0.01%	0.00%
Instance 2	0.00%	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%
Instance 3	0.00%	0.01%	0.01%	0.07%	0.01%	0.10%	0.01%	0.01%
Instance 4	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%
Instance 5	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%	0.01%
200 Points								
Instance 1	0.00%	0.01%	0.33%	0.02%	0.45%	0.84%	0.36%	0.08%
Instance 2	0.00%	0.00%	0.01%	0.01%	0.01%	0.48%	0.24%	0.09%
Instance 3	0.00%	0.01%	0.01%	0.01%	0.23%	0.19%	0.58%	0.33%
Instance 4	0.00%	0.01%	0.01%	0.06%	0.01%	0.42%	0.13%	0.02%
Instance 5	0.00%	0.01%	0.01%	0.01%	0.09%	0.09%	0.38%	0.14%

Table A.10. Satisfied demand for Program (3.3.2) over multiple periods

	Budget							
	0	100	200	300	400	500	600	700
100 Points								
Instance 1	62.28%	84.66%	92.94%	97.24%	99.36%	99.97%	100.00%	100.00%
Instance 2	66.20%	84.82%	93.56%	97.35%	99.56%	99.98%	100.00%	100.00%
Instance 3	57.30%	79.99%	89.82%	95.76%	98.73%	99.91%	100.00%	100.00%
Instance 4	67.32%	89.24%	96.26%	99.12%	99.97%	100.00%	100.00%	100.00%
Instance 5	63.91%	80.23%	89.48%	95.71%	98.66%	99.87%	100.00%	100.00%
150 Points								
Instance 1	74.87%	87.65%	93.26%	96.79%	98.59%	99.60%	99.97%	100.00%
Instance 2	69.85%	84.17%	91.67%	96.09%	98.19%	99.49%	99.93%	100.00%
Instance 3	66.03%	80.83%	88.18%	93.23%	96.55%	98.57%	99.63%	99.97%
Instance 4	75.62%	89.24%	94.38%	97.23%	98.87%	99.73%	99.99%	100.00%
Instance 5	74.39%	85.71%	91.73%	95.92%	98.17%	99.40%	99.87%	99.99%
200 Points								
Instance 1	76.12%	87.07%	91.73%	94.82%	97.00%	98.13%	99.34%	99.78%
Instance 2	74.03%	84.38%	90.70%	94.28%	96.50%	98.13%	99.17%	99.76%
Instance 3	70.63%	81.66%	87.76%	91.96%	95.03%	97.21%	98.50%	99.46%
Instance 4	78.51%	89.43%	93.77%	96.27%	97.88%	99.00%	99.67%	99.87%
Instance 5	76.33%	85.31%	90.39%	94.09%	96.45%	98.14%	99.12%	99.72%

Table A.11. Computing time (s) for Program (3.3.2) over multiple periods

	Budget							
	0	100	200	300	400	500	600	700
100 Points								
Instance 1	0.313	9.657	34.235	720.25	1037.297	1308.797	1.922	1.781
Instance 2	0.344	20.422	52.828	1800	222.047	946.719	1.844	1.906
Instance 3	0.422	10.203	41.359	210.234	833.25	743.031	2.234	2.25
Instance 4	0.375	19.437	24.532	97.797	266.703	1.406	1.625	2.188
Instance 5	0.36	51.422	185.687	181.938	1800	1800	2.25	2.343
150 Points								
Instance 1	0.922	152.844	1006.859	1421.375	1800	1800	1800	9.109
Instance 2	1.39	55.203	455.813	802.36	1800	1800	1800	29.891
Instance 3	1.047	43.203	1116.75	1800	1800	1800	1800	1800
Instance 4	1.969	34.469	224.922	669.594	1800	1800	218.344	4.781
Instance 5	1.563	73.453	1800	1800	1800	1800	1800	9.969
200 Points								
Instance 1	2.156	496.344	1800	1800	1800	1800	1800	1800
Instance 2	2.046	158.906	1765.328	1800	1800	1800	1800	1800
Instance 3	6.375	97.516	1800	1800	1800	1800	1800	1800
Instance 4	2.984	117.078	839.344	1800	1800	1800	1800	1800
Instance 5	3.75	221.328	1800	1800	1800	1800	1800	1800

Table A.12. Optimal gap for Program (3.3.2) over multiple periods

	Budget							
	0	100	200	300	400	500	600	700
100 Points								
Instance 1	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%
Instance 2	0.00%	0.01%	0.01%	0.09%	0.01%	0.01%	0.00%	0.00%
Instance 3	0.00%	0.01%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%
Instance 4	0.00%	0.01%	0.01%	0.01%	0.01%	0.00%	0.00%	0.00%
Instance 5	0.00%	0.01%	0.01%	0.01%	0.14%	0.05%	0.00%	0.00%
150 Points								
Instance 1	0.00%	0.01%	0.01%	0.01%	0.95%	0.40%	0.03%	0.00%
Instance 2	0.00%	0.01%	0.01%	0.01%	0.44%	0.44%	0.07%	0.00%
Instance 3	0.00%	0.01%	0.01%	0.23%	1.56%	1.17%	0.37%	0.03%
Instance 4	0.00%	0.01%	0.01%	0.01%	0.66%	0.27%	0.01%	0.00%
Instance 5	0.00%	0.01%	0.09%	0.03%	0.07%	0.45%	0.13%	0.01%
200 Points								
Instance 1	0.00%	0.01%	0.31%	2.34%	2.33%	1.90%	0.66%	0.22%
Instance 2	0.00%	0.01%	0.01%	2.26%	2.24%	1.82%	0.84%	0.24%
Instance 3	0.00%	0.01%	0.34%	3.21%	3.69%	2.85%	1.52%	0.54%
Instance 4	0.00%	0.01%	0.01%	1.15%	0.97%	0.99%	0.33%	0.13%
Instance 5	0.00%	0.01%	0.46%	2.79%	2.73%	1.89%	0.88%	0.28%