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Multi-attribute deterministic and stochastic Two Echelon Location Routing Problems

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Résumé

Les problèmes de localisation-routage à deux échelons (2E-LRP) sont devenus un domaine de recherche important dans le domaine de la logistique et de la gestion de la chaîne d'approvisionnement. Le 2E-LRP représente un problème d'optimisation dans les systèmes de distribution non dirigés, visant à organiser le transport de marchandises entre les plateformes et les clients par le biais d'installations intermédiaires appelées satellites. Ce problème implique de prendre des décisions simultanées concernant l'emplacement d'un ou deux niveaux d'installations (plateformes et/ou satellites) et de créer un ensemble limité d'itinéraires aux deux échelons afin de répondre efficacement à toutes les demandes des clients. Récemment, la communauté scientifique s'est intéressée de plus en plus à l'étude et à la résolution de problèmes plus réalistes. Cet intérêt provient de la reconnaissance du fait que les systèmes de distribution du monde réel sont caractérisés par une multitude de complexités et d'incertitudes qui ont un impact significatif sur l'efficacité opérationnelle, la rentabilité et la satisfaction des clients. Les chercheurs ont reconnu la nécessité d'aborder ces complexités et incertitudes pour développer des solutions pratiques et efficaces.

Cette thèse comprend trois études différentes, chacune correspondant à un article de recherche autonome. Dans les trois articles, nous nous concentrons sur différents 2E-LRP riches qui comprennent plusieurs attributs en interaction. Ces variantes du problème sont appelées problèmes de localisation-routage à deux échelons et à attributs multiples (2E-MALRP). Pour analyser l'influence des incertitudes sur les solutions optimales et les processus de prise de décision, nous considérons à la fois les perspectives déterministes et stochastiques. Cette approche nous permet de mieux comprendre le comportement de ces problèmes complexes.

Le premier document de recherche abordé dans cette thèse se concentre sur un problème de localisation-routage déterministe à deux échelons et à attributs multiples avec synchronisation de la flotte dans les installations intermédiaires (2E-MALRPS). Le cadre du problème comprend divers facteurs, notamment la demande de marchandises multiples dépendant du temps, les fenêtres temporelles, le manque de capacité de stockage dans les installations intermédiaires et la nécessité de synchroniser les flottes opérant à différents échelons. Dans le 2E-MALRPS, tous les paramètres, tels que les demandes des clients, les temps de trajet et les coûts, sont connus avec certitude. Dans cet article, nous introduisons le cadre du problème,

présentons une formulation de programmation en nombres entiers mixtes et proposons un cadre de découverte de discrétisation dynamique comme méthode de résolution du problème.

Le deuxième article de cette thèse traite du problème de localisation-routage à deux échelons en cas de demandes stochastiques et corrélées (2E-MLRPSCD). Contrairement au 2E-MALRPS, le 2E-MLRPSCD prend en compte les incertitudes liées aux demandes des clients, ainsi que la corrélation entre ces demandes. Nous formulons le problème sous la forme d'un modèle de programmation stochastique en deux étapes. Au cours de la première étape, des décisions sont prises concernant la conception des installations satellites, tandis qu'au cours de la deuxième étape, des décisions de recours déterminent la manière dont les demandes observées sont servies. Nous proposons une métaheuristique de couverture progressive comme méthode de résolution. Dans cette approche, nous incorporons deux structures de population dans le cadre de la couverture progressive. Ces structures renforcent la diversité des décisions de conception obtenues pour chaque sous-problème de scénario et fournissent des informations pertinentes pour améliorer la qualité de la solution. En outre, nous introduisons et comparons trois nouvelles stratégies différentes pour accélérer la recherche de l'espace de solution pour le problème stochastique.

Finalement, le troisième article présenté dans cette thèse se concentre sur un problème de localisation-routage multi-attributs à deux échelons avec des temps de trajet stochastiques (2E-MALRPSTT). Le 2E-MALRPSTT combine un problème multi-attributs riche avec des éléments stochastiques, en particulier en considérant des temps de trajet stochastiques. Pour traiter le problème stochastique complet, un cadre de couverture progressive (PH) est proposé en s'appuyant sur les lignes directrices méthodologiques définies dans notre deuxième article pour le 2E-MLRPSCD. En outre, une heuristique basée sur la décomposition est introduite pour accélérer le cadre PH, et deux nouvelles stratégies d'agrégation sont présentées pour accélérer le processus de consensus concernant les décisions de la première étape.

Les contributions présentées dans cette thèse couvrent divers aspects de la modélisation et des méthodologies de solution pour les 2E-MALRP riches, à la fois d'un point de vue déterministe et d'un point de vue stochastique. Les trois articles inclus dans cette thèse démontrent l'efficacité des approches proposées à travers des campagnes expérimentales étendues, mettant en évidence leur efficacité de calcul et la qualité des solutions, en particulier dans les cas difficiles. En abordant les aspects déterministes et stochastiques de ces 2E-MALRP, cette thèse vise à contribuer à l'ensemble des connaissances en optimisation de la logistique et de la chaîne d'approvisionnement, à répondre aux besoins importants de la littérature actuelle et à fournir des informations importantes pour les systèmes de distribution à deux échelons dans divers contextes.

Mots-clés : localisation-routage à deux échelons, transport, logistique urbaine, synchronisation, temps de trajet stochastiques, demande stochastique, découverte de la discrétisation dynamique, couverture progressive.

Abstract

The Two-Echelon Location-Routing Problems (2E-LRPs) have emerged as a prominent research area within the field of logistics and supply chain management. The 2E-LRP represents an optimization problem in undirected distribution systems, aiming to streamline freight transportation between platforms and customers through intermediate facilities known as satellites. This problem involves making simultaneous decisions concerning the location of one or two levels of facilities (platforms and/or satellites) and creating a limited set of routes at both echelons to effectively serve all customer demands. In recent years, there has been a growing interest among the scientific community in studying and solving more realistic problem settings. This interest arises from the recognition that real-world distribution systems are characterized by a multitude of complexities and uncertainties that significantly impact operational efficiency, cost-effectiveness, and customer satisfaction. Researchers have acknowledged the need to address these complexities and uncertainties to develop practical and effective solutions.

This dissertation comprises three distinct studies, each serving as a self-contained research article. In all three articles, we focus on different rich 2E-LRPs that encompass multiple interacting attributes. These problem variants are referred to as two-echelon multi-attribute location-routing problems (2E-MALRPs). To analyze the influence of uncertainties on optimal solutions and decision-making processes, we consider both deterministic and stochastic perspectives. This approach allows us to gain insights into the behavior of these complex problem settings.

The first research paper addressed in this thesis focuses on a deterministic two-echelon multi-attribute location-routing problem with fleet synchronization at intermediate facilities (2E-MALRPS). The problem setting encompasses various factors, including time-dependent multicommodity demand, time windows, lack of storage capacity at intermediate facilities, and the need for synchronization of fleets operating at different echelons. In the 2E-MALRPS, all parameters, such as customer demands, travel times, and costs, are known with certainty. In this paper, we introduce the problem setting, present a mixed-integer programming formulation, and propose a dynamic discretization discovery framework as the solution method to address the problem.

The second paper in this thesis addresses the two-echelon multicommodity location-routing problem with stochastic and correlated demands (2E-MLRPSCD). In contrast to the 2E-MALRPS, the 2E-MLRPSCD takes into account uncertainties related to customer demands, as well as the correlation among these demands. We formulate the problem as a two-stage stochastic programming model. In the first stage, decisions are made regarding the design of satellite facilities, while in the second stage, recourse decisions determine how the observed demands are allocated and served. We propose a progressive hedging metaheuristic as the solution method. In this approach, we incorporate two population structures within the progressive hedging framework. These structures enhance the diversity of the design decisions obtained for each scenario subproblem and provide valuable insights for improving the solution quality. Additionally, We also introduce and compare three different novel strategies to accelerate the search for the solution space for the stochastic problem.

Finally, the third paper presented in this thesis focuses on a multi-attribute two-echelon location-routing problem with stochastic travel times (2E-MALRPSTT). The 2E-MALRPSTT combines a rich multi-attribute problem setting with stochastic elements, specifically considering stochastic travel times. To address the complete stochastic problem, a progressive hedging metaheuristic is proposed building on the methodological guidelines defined in our second paper for the 2E-MLRPSCD. Furthermore, a decomposition-based heuristic is introduced to accelerate the PH framework, and two novel selection strategies are presented to expedite the consensus process regarding the first-stage decisions.

The contributions presented in this thesis encompass various aspects of modeling and solution methodologies for rich 2E-MALRPs from both deterministic and stochastic perspectives. The three articles included in this thesis demonstrate the effectiveness of the proposed approaches through extensive experimental campaigns, highlighting their computational efficiency and solution quality, particularly in challenging instances. By addressing the deterministic and stochastic aspects of these 2E-MALRPs, this thesis aims to contribute to the broader body of knowledge in logistics and supply chain optimization, fill important gaps in the present literature and provide valuable insights for two-echelon distribution systems in diverse settings.

Keywords: two-echelon location-routing, transportation, city logistics, synchronization, stochastic travel times, stochastic demand, dynamic discretization discovery, progressive hedging.

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List of acronyms and abbreviations

2E-LRP	Two-Echelon Location-Routing Problem
2E-LRPTW	Two-Echelon Location-Routing Problem with Time Windows
2E-LRPSD	Two-Echelon Location-Routing Problem with Stochastic Demands
2E-MALRP	Two-Echelon Multi-Attribute Location-Routing Problem
2E-MALRPS	Two-Echelon Multi-Attribute Location-Routing Problem with fleet Synchronization
2E-MALRPSTT	Two-Echelon Multi-Attribute Location-Routing Problem with Stochastic Travel Times
2E-MLRPSCD	Two-Echelon Multicommodity Location-Routing Problem with Stochastic and Correlated Demands
2E-VRP	Two-Echelon Vehicle Routing Problem
2E-VRPTW	Two-Echelon Vehicle Routing Problem with Time Windows

ADA	Mean Deterministic Approximation
AISR	Average Invalid Stability Requirement
ALNS	Adaptive Large Neighborhood Search
C-DDD	Dynamic Discretization Discovery method using the complete model
CFLP	Capacitated Facility Location Problem
CFLPTW	Capacitated Facility Location Problem with Time Windows
DBS	Diversity-Based Strategy
DDD	Dynamic Discretization Discovery method
DSS	Deterministic Scenario Subproblem
FIFLOR	Flow-Intercepting Facility Location-Routing Problem
H-DDD	Dynamic Discretization Discovery method using the hybrid model
HTF	Hybrid Time-space Formulation

LDS	Longest-Distance Strategy
LRP	Location-Routing Problem
LRPSD	Location-Routing Problem with Stochastic Demands
LRPSTT	Location-Routing Problem with Stochastic Travel Times
LRPTW	Location-Routing Problem with Time Windows
LRPTWTDD	Location-Routing Problem with Time Windows and Time-Dependent origin-destination Demands
MDA	Maximum Deterministic Approximation
MDVRP	Multi-Depot Vehicle Routing Problem
MDVRPTW	Multi-Depot Vehicle Routing Problem with Time Windows
MIP	Mixed Integer Programming
PH	Progressive Hedging
RD	Relative Difference

SNDP	Service Network Design Problem
SSND	Scheduled Service Network Design Problem
TS	Tabu Search Method
TSP	Travelling salesman problem
TTP	Truck and Trailer Problem
VAR	Variance
VND	Variable Neighborhood Descent
VRP	Vehicle Routing Problem
VRPTW	Vehicle Routing Problem with Time Windows
VSR	Valid Stability Requirement
VSS	Value of the Stochatic Solution

To Laura, Sofia and my family

«*Throughout the infinite, the forces are in perfect balance, and hence the energy of a single thought may determine the motion of a universe*[1]*.* »

1. Nikola Tesla

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Chapter 1

INTRODUCTION

Freight distribution and logistics play a vital role in contemporary society, making it one of the largest industries. However, designing and planning efficient distribution systems remains a complex and crucial task for enabling sustainable and effective logistic operations (Crainic et al., 2023a). At the strategic planning level, the design of these logistic networks involves determining the optimal number, locations, and capacities of platform facilities to meet the evolving demands of customers. Additionally, it requires establishing efficient vehicle routes to reach and serve each customer's demand. This has led to a growing body of literature on *location-routing problems* (*LRPs*), which integrate these location and routing decisions under the same modeling framework.

The increasing commodification and massification of production and consumption have led to new approaches in customer delivery, resulting in the scale and complexity of freight distribution. To address these challenges, the concept of *two-tier logistics* has been introduced. This approach aims to promote efficient and sustainable freight transportation by adopting indirect shipping alternatives. It involves using two levels of intermediate facilities, such as satellites, to transship and consolidate freight before reaching its final destination. The *two-echelon location routing problem* (*2E-LRP*) arises as a modeling framework capable of integrating the location and routing decisions in planning such two-level systems (Cuda et al., 2015).

The fundamental concept underlying two-echelon location-routing problems is to minimize line-haul distances and enhance consolidation. These systems consist of two types of facilities: large distribution centers or platforms, and intermediate transdock-type facilities known as satellites. Freight flows from platforms to satellite facilities using high-capacity vehicles (first echelon) to be consolidated and transferred to smaller, generally environment-friendly vehicles for distribution to the final destinations (second echelon).

A significant number of studies have been undertaken to tackle distribution planning challenges in freight logistics (Cattaruzza et al., 2017). However, the majority of existing research on 2E-LRPs has primarily concentrated on the fundamental problem variant, neglecting the incorporation of additional attributes. This simplification can frequently lead to unrealistic scenarios, as real-life logistics applications often involve multiple attributes that can potentially yield improvements (see, for instance, Prodhon and Prins 2014; Cuda et al. 2015).

In recent years, there has been a notable shift in the field towards embracing more detailed and complex problem settings that encompass various aspects of real-world applications. This trend has necessitated the inclusion of multiple attributes, leading to the formulation of highly intricate problem settings (Crainic et al., 2009; Gonzalez-Feliu, 2009; Drexl, 2013; Buijs et al., 2014). However, despite the growing interest in the field, there remains a scarcity of literature specifically dedicated to the complexity and dimensionality of these rich 2E-LRPs. The consideration of multiple interacting attributes in 2E-LRPs is an area that has received limited attention, and in some cases, no formal models or solution methodologies tailored to address these intricate problem settings have been developed thus far (Sluijk et al., 2022).

Against this background, the primary goal of this thesis is to make a meaningful contribution to the design and analysis of rich 2E-LRPs by taking into account multiple interacting attributes. To accomplish this objective, the present dissertation introduces a problem class known as the *two-echelon multi-attribute location-routing problem* (*2E-MALRP*). The 2E-MALRP encompasses various attributes explored in this study, such as time-dependent demands, non-substitutable demands, fleet synchronization, and uncertainty. Our aim is to address the knowledge gaps in the existing literature by providing advancements in the modeling of such systems and decision-making processes, as well as contributing to the body of solution methods for these problem formulations.

The remainder of the dissertation is organized as follows. In Chapter 2, we give a global definition of the problem class named the 2E-MALRP, a discussion of the relevance and potential applications of the attributes considered and literature review of the research works related to the 2E-MALRP. The following chapters correspond to the three self-contained studies that present the contributions made to the 2E-MALRP. In Article 1, we introduce the *two-echelon multi-attribute location-routing problem with synchronization constraints*, *2E-MALRPS*, and present a mixed-integer programming formulation on a hybrid time-space network combining continuous and discrete time representations. We also present an exact solution framework that iteratively refines a reduced time-space network, solving the 2E-MALRPS formulation defined on the reduced network to extract bounds and achieve temporal granularity refinements, in order to guide the method toward to optimal solution of the original problem. The paper generalizes the dynamic discretization discovery method to complex problem settings involving several levels of location, routing, and synchronization decisions. We perform thorough analyses to assess the impact of the problem attributes and requirements on the system behaviour and algorithm performance.

In Article 2, we introduce the stochastic *the two-echelon multicommodity location-routing problem with stochastic and correlated demands* (*2E-MLRPSCD*). This study presents a two-stage stochastic programming formulation to effectively model the problem. In the first stage, we determine the locations of satellite facilities as design decisions, while in the second stage, we make recourse decisions to distribute the observed demands effectively. To tackle this problem, we propose a progressive-hedging metaheuristic, which incorporates two population structures to enhance the diversity of design decisions for each scenario subproblem. Additionally, we introduce and compare three novel strategies aimed at accelerating the search for the solution space in the context of this stochastic problem. The efficiency and effectiveness of all proposed strategies to produce high-quality solutions under a variety of problem characteristics and demand correlations are assessed through a set of extensive computational experiments.

In Article 3, we introduce and investigate *the two-echelon multi-attribute location-routing problem with stochastic travel times* (*2E-MALRPSTT*). This problem is formulated using a two-stage scenario-based stochastic programming approach, which effectively incorporates the interactions among multiple attributes. To address the complete stochastic problem, we propose a progressive-hedging metaheuristic, which decomposes the problem into multiple scenario subproblems. These subproblems are iteratively solved and adjusted until a consensus is reached regarding the first-stage decisions. To tackle each scenario subproblem, we propose a decomposition-based heuristic that accelerates the solution process for the multi-attribute problem arising from scenario decomposition. Additionally, we introduce two novel aggregation strategies to further expedite the consensus over the first-stage decisions. The effectiveness of the proposed approaches is assessed through extensive experimental campaigns, highlighting their computational efficiency and solution quality. Finally, Chapter 3 gives the general conclusion regarding the proposed research project.

Chapter 2

PROBLEM SETTING AND LITERATURE REVIEW

This chapter focuses on the research related to the problem class studied in this dissertation. Therefore, we first introduce the problem class named the *Two-Echelon Multi-Attribute Location-Routing Problem* (*2E-MALRP*). Then a complete literature review is provided to situate the 2E-MALRPS within the relevant literature on the 2E-LRP and LRP, pointing out the gaps in knowledge with respect to time-dependent demand, origin-destination demand, fleet synchronization and uncertainty. Section 2.1 introduces system description and the pertinent terminology related to the 2E-MALRP, followed by the literature review related to the 2E-MALRPS in Section 2.2.

2.1. System description

We conduct our investigation on a two-echelon location-routing problem that encompasses multiple interacting attributes. This novel problem class, referred to as the *Two-Echelon Multi-Attribute Location-Routing Problem* (*2E-MALRP*), involves various sets of entities, including *suppliers* (the demand origins), *platforms* (the primary facilities), *satellites* (the intermediate facilities), and *customers* (the demand destinations). *Platforms* are large-scale facilities responsible for storing, sorting, and consolidating inbound freight from supply points, utilizing various transportation modes. On the other hand, *satellites* are medium- to small-sized facilities designed as multimodal trans-docking infrastructures with limited or no storage capacity, enabling efficient transshipment operations. They play a crucial role in the final stages of transportation, facilitating the delivery of freight to customers.

Freight delivery is carried out by two homogeneous fleets of vehicles with limited load capacities, designated for the first and second echelons, respectively, and capable of transporting any demand. The first echelon is defined between each platform and a series of satellite facilities. The second echelon is defined by the connections between each satellite

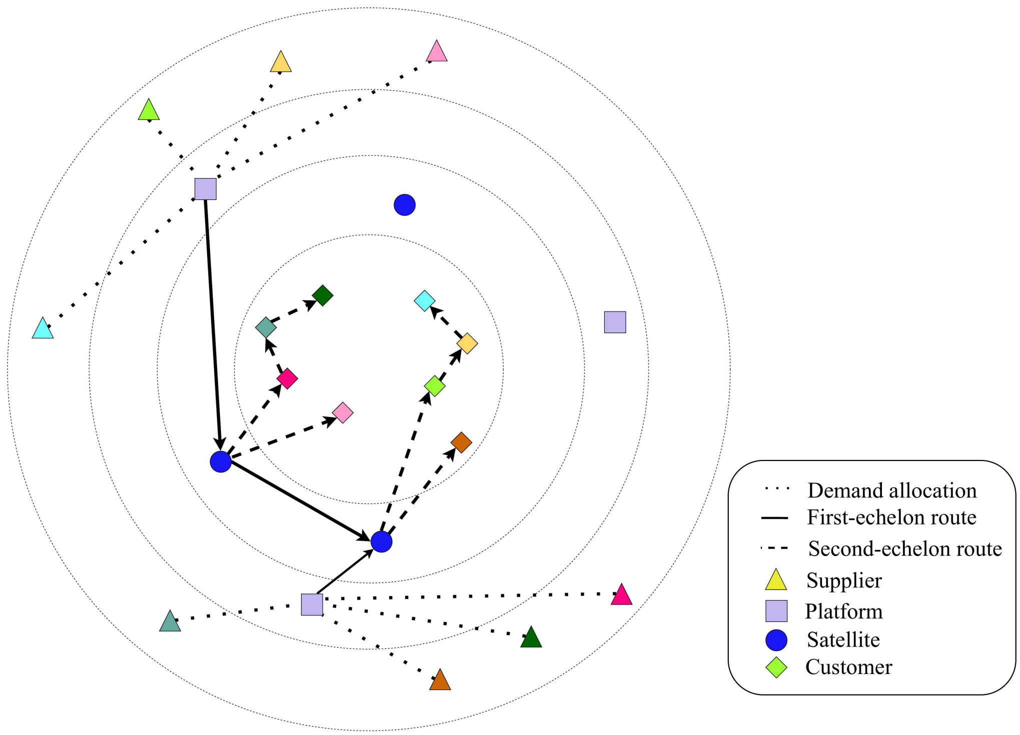

Figure 1 – Two-echelon distribution system topology

facility and the set of customers. Freight flows from platforms to customers through a set of satellite facilities, where first-echelon vehicles consolidate and transship the demand to smaller second-echelon vehicles before reaching their final destination. It is worth mentioning that more than one fleet may exist for each echelon. However, the consideration of heterogeneous fleets increases the complexity of the underlying problem without significant changes in the modeling and solution methods required to address them. Without loss of generality, this thesis utilizes homogeneous fleets for each echelon.

Demand is defined between suppliers and customers. Each individual demand is characterized by its origin, destination and volume. The system may include time-dependency on the demand from the origin and the destination. This time-dependent characteristic are usually present in terms of availability times at the origin and a due time at the destination.

As illustrated in Figure 1, the objective is to assign each origin-destination (OD) demand to an open platform, where it can potentially be consolidated with other commodities. Subsequently, a first-echelon vehicle is responsible for transporting the consolidated loads to a sequence of designated satellites, where the demand flows are transferred. The loads delivered at satellites are transshipped and consolidated further into second-echelon vehicles, which ultimately carry out the final deliveries to the respective destinations.

The scope of the 2E-MALRP is primarily concerned with the design and strategic planning of transportation and logistic systems, supporting the inbound freight flow to customers, from suppliers, through platforms and satellite facilities. The problem entails selecting facilities at one or both levels, allocating suppliers to platforms, satellites to platforms, and customers to satellites, as well as routing and scheduling vehicles at each echelon to deliver freight from platforms to customers via satellite facilities. However, the problem considers a number of attributes that have not been previously considered together, including: (i) multicommodity, origin-to-destination (OD) demands; (ii) time dependent demand; (iii) fleet synchronization; (iv) uncertainty. We will provide a more detailed description of these attributes in the following:

Multicommodity, origin-to-destination (OD) demands: Multicommodity characteristic correspond to many representation in different applications in transportation, telecomnuications and logistics. Multicommodity in logistic refer to the demand differentiation, which is often a crucial aspect of many applications (e.g. postal services, parcel delivery) where packages have customer-specific demands with a known origin and destination, and its differentiation can be used to tailor services to better meet the needs of customers (Crainic and Hewitt, 2021).

Time dependent demand: Demand is typically influenced by a diverse range of time-related factors governed by one or multiple entities. In general, the transportation of demand can be contingent upon various temporal considerations, including production schedules, operating hours, delivery policies, city regulations, customer availability, and more (Crainic and Hewitt, 2021). Time dependency in this dissertation is defined to represent the temporal components in demand, which may not necessarily capture the dynamic nature of specific temporal attributes (e.g. cost fluctuations over time). These temporal dependencies are often expressed as the permissible time intervals during which demand is either available at its origin or due at its destination.

Fleet Synchronization: Modern distribution systems often require the coordination of multiple shippers or itineraries to ensure timely and accurate deliveries in a cost-effective manner (Sluijk et al., 2022). Synchronization constraints in logistics operations can manifest in various forms, including precedence constraints on routing decisions (where two vehicles are required or allowed to arrive at a specific location in a particular order) or space-time coordination (where two or more vehicles must or can arrive at the same location simultaneously) (Schiffer and Walther, 2017). Operations and resources within the system can be constrained by spatial or temporal relationships with other tasks. For instance, a vehicle may be unable to initiate a delivery until the corresponding freight is available at its location or until a specific time is reached upon the vehicle's arrival. These spatial and temporal dependencies impose restrictions on the sequencing and timing of operations, requiring careful coordination to optimize the efficiency and effectiveness of the distribution system.

Uncertainty: Freight transportation activities frequently encounter situations in which the system's behavior is significantly influenced by unpredictable events originating from various sources (Gendreau et al., 2014). Uncertainties in transportation manifest in different ways, such as traffic disruptions that affect delivery travel times, unavailability of customers during visits, or changes in demand that impact planned vehicle or facility capacities. Planning and designing these networks for the medium and long term often involve forecasting these random events. However, effective management of transportation operations necessitates strategies to handle such uncertainties and mitigate their adverse effects. This dissertation presents Article 1 with the hypothesis that uncertainty can be approximated, while Article 2 and Article 3 explicitly consider the uncertainty in the problem setting.

The core objective of the 2E-MALRP is to minimize the total cost of the system, composed of the cost of selecting/opening facilities at one or both levels and the transportation costs, while satisfying the demand and the capacities of the system elements. However, based on the multiples attributes that can be simultaneously considered to be optimized, either the objective function or the type of constraints to be satisfied may change.

2.2. Literature review

This Section focuses on the research related to the *Two-Echelon Multi-Attribute Location-Routing Problem (*2E-MALRP*)* studied in this dissertation. Therefore, to provide some useful insights, a brief description and an extensive review of different research classes of LRPs and 2E-LRPs are included.

2.2.1. Scope of the literature review

This section aims to situate the 2E-MALRP within the relevant literature on the multi-attribute 2E-LRP and LRP, pointing out the gaps in knowledge with respect to time dependent demand, multicommodity origin-destination demand, fleet synchronization and uncertainty. This literature review emphasizes the developments of LRPs and 2E-LRPs defined by the interplay of two or more of these attributes within the same problem setting. To this end, we classify the family of problems defined by each of the attributes introduced in Section 2.1. We thus define the following six classes of problems to group contribution on both LPRs and 2E-LRPs:

- The standard LRP
- The standard 2E-LRP
- Multicommodity LRP and 2E-LRP
- LRP and 2E-LRP with temporal constraints
- Stochastic LRP and 2E-LRP
- LRP and 2E-LRP with fleet synchronization

The first two sections introduce the definitions and literature review of the standard LRP and 2E-LRP, respectively. To facilitate the discussion, we provide the mathematical formulations of the standard LRP and 2E-LRP. The following four sections present the literature review of contributions on LRPs and 2E-LRPs for the remaining classes involving considerations of different attributes. We classify the articles with similar characteristics into each of the six classes above. Needless to say, some articles might be included in several classes, and so we only discuss them within the class that we believe describes the best the problem under consideration. It is worth noting that the review of contributions in closely related problems, such as the *Vehicle Routing Problem* (*VRP*) and the *Two-Echelon Vehicle Routing Problem* (*2E-VRP*), which do not involve any location decision, is outside the scope of this literature review. Hence, for an overview of the different contributions in VRPs and 2E-VRPs we refer to interest readers the recent surveys by Desaulniers et al. (2014), Archetti et al. (2016), Schneider and Drexl (2017) and Sluijk et al. (2022).

2.2.2. The standard LRP

The LRP is a well-known class of combinatorial optimization problems used in a wide range of applications. It combines elements of two *NP*-hard problems: the Facility Location-Allocation Problem and the Vehicle Routing Problem (VRP) (Prodhon and Prins, 2014; Cuda et al., 2015; Nagy and Salhi, 2006). One can formally define the standard LRP as a complete, weighted graph $G = (V, A)$, with vertices $V = P \cup C$ divided into two disjoint sets: platforms P and customers C. Each platform location $p \in P$ is associated with a limited storage capacity Q_p and a fixed opening cost F_p. Each customer $c \in C$ is associated with a demand with volume $vol(c)$. This demand is defined as a single commodity for all customers, which can be delivered and handled for all vehicles and platforms in the system.

The arc-set A represents the direct links between locations, i.e., the vertices in V. A non-negative unit cost ζ_{ij} is associated with each arc $(i,j) \in A$. Demand is served by a homogeneous fleet of vehicles H with a limited capacity *cap*. The standard LRP can be defined by four sets of decision variables:

— $y_p \in \{0,1\}, p \in P$: location variable, 1 if a platform is opened/selected in location p, 0 otherwise;

— $f_{pc} \in \{0,1\}, p \in P, c \in C$: allocation variable, 1 if customer c is allocated to platform p, 0 otherwise;

— $x_{ijh} \in \{0,1\}, (i,j) \in A, h \in H$: vehicle flow variable, 1 if arc (i,j) is used by vehicle h, and 0 otherwise;

— $u_{ih} \geq 0, i \in V, h \in H$: Accumulated demand by vehicle h at a location i;

Using this notation, one can formulate the standard LRP as a mixed-integer programming formulation.

$$\min \sum_{p \in P} F_p y_p + \sum_{h \in H} \sum_{(i,j) \in A} \zeta_{ij} x_{ijh} \tag{2.2.1}$$

subject to

$$\sum_{h\in H}\sum_{i\in V,i\neq j} x_{ijh} = 1 \quad \forall j \in C \tag{2.2.2}$$

$$\sum_{i\in V,i\neq j} x_{ijh} = \sum_{i\in V,i\neq j} x_{jih} \quad \forall h \in H, j \in V \tag{2.2.3}$$

$$\sum_{i\in P}\sum_{j\in C} x_{ijh} \leq 1 \quad \forall h \in H \tag{2.2.4}$$

$$\sum_{e\in C} x_{ieh} + \sum_{e\in V\setminus j} x_{ejh} \leq 1 + f_{ij} \quad h \in H, i \in P, j \in C \tag{2.2.5}$$

$$u_{ih} + vol(j) \leq u_{jh} + (1 - x_{ijh})M \quad \forall h \in H, i \in V, j \in C, i \neq j \tag{2.2.6}$$

$$\sum_{i\in V}\sum_{j\in C,i\neq j} vol(j)\ x_{ijh} \leq cap \quad \forall h \in H \tag{2.2.7}$$

$$\sum_{j\in C} vol(j)\ f_{ij} \leq Q_i y_i \quad \forall i \in P \tag{2.2.8}$$

The objective function of the standard LRP (2.2.1) is to minimize the total cost of the system, composed of the cost of selecting/opening platform facilities and the transportation costs. Constraints (2.2.2) guarantee that every customer is served by a single route. Constraints (2.2.3) and (2.2.4) ensure the continuity of each route and a return to the platform from which it has started. Constraints (2.2.5) specify that a customer can be assigned to a single platform if a route connecting them is used. Constraints (2.2.6) are subtour elimination constraints. Constraints (2.2.7) and (2.2.8) are associated with vehicle and facility capacity, respectively.

Because of its practical relevance, the LRP has attracted a major attention of the research community resulting in a wide variety of high-quality solution approaches (Drexl and Schneider, 2015). Nagy and Salhi (2006), make a compilation of all the early works and surveys that appeared before and in 2005, and provide a discussion of methods and mathematical models. The most recent surveys, namely those of Lopes et al. (2013) and Prodhon and Prins (2014), cover the variants of the LRP including the standard version, following different strategies for its categorization. Lopes et al. (2013) perform a taxonomical analysis of LRPs including an overview of the algorithmic approaches and main objectives functions, while Prodhon and Prins (2014) give a more detailed description of the problems and its respective research contributions in the literature of the LRP and closely related problems as the truck and trailer problem (TTP). Moreover, some surveys are responsible for a review of specific aspects such as the multi-echelon LRP (Cuda et al., 2015), the standard LRP (Schneider and Drexl, 2017), and the multiples variants of the LRP (Mara et al., 2021b), which in particular are mutually complementary.

There is a broad variety of solution frameworks that have been proposed for the LRP. The most successful exact methods for the LRP are based on cut-and-column generation relying on the decomposition of the problem into a set of multi-depot vehicle-routing problems (MDVRPs) and their exact solutions. Essentially, the works of Baldacci et al. (2011) and Contardo et al. (2014a) arise as the most prominent contributions by exploiting the aforementioned methodology on set-partitioning formulations to solve the large part of the benchmark instances in the related literature. The most successful (meta)heuristic methods use a variety of paradigms. We highlight that the wide majority of metaheuristics methods are hybridizations of several concepts rather than one pure implementation of a classical metaheuristic. Successful solution approaches combine efficient intensification procedures via local searches with diversifications methods such as crossovers, shaking, restarts, or decomposition phases (see, e.g. Hemmelmayr et al. 2012; Escobar et al. 2013; Contardo et al. 2014b). One notices that time windows are used in most cases when temporal dependencies are addressed (Farham et al., 2018), while issues related to non-substitutable demand and fleet synchronization have been scarcely addressed from an optimization point of view (Govindan et al., 2014; Boccia et al., 2018). Contributions on rich, multi-attribute LRPs, and the influence that the simultaneous consideration of several interacting attributes may have on the decision-making, are still very limited (Mara et al., 2021b).

2.2.3. The standard 2E-LRP

The two-echelon location routing problem (2E-LRP) is defined as an undirected distribution system determined to optimize the freight transportation between platforms and customers through intermediate facilities, also known as satellites. The standard 2E-LRP can be formally defined as a weighted directed graph $G = (V, A)$ representing the physical network on which the problem is defined. The set of vertices $V = P \cup Z \cup C$ is composed by three disjoint sets: platforms P, satellites Z, and customers C. A fixed selection cost F_p and a capacity Q_p are defined for each possible platform location $p \in P$. A fixed selection cost F_z and capacity Q_z are also defined for each potential satellite site $z \in Z$. Each customer $c \in C$ is associated with a demand with volume $vol(c)$. Demand is defined as a single commodity for all customers, which can be handled and carried by all vehicles and facilities in the system.

The arc-set $A = A^1 \cup A^2$ represents the direct links between the vertices in V. The set A^1 includes the arcs of the first echelon, corresponding to the connections between platforms P and satellites Z, as well as the arcs connecting pairs of satellites. The set A^2 includes the arcs of the second echelon, that is, the connections between satellites Z and customers C, and the arcs connecting pairs of customers. A non-negative unit cost ζ_{ij} is associated with each arc $(i,j) \in A$. Two homogeneous fleets of vehicles $H = H^1 \cup H^2$, with limited load capacities cap_1 and cap_2, are available for the first and second echelon, respectively. The standard 2E-LRP can be defined by the following sets of decision variables:

— $y_i \in \{0,1\}, i \in (P \cup Z)$: location variable, 1 if a facility is opened/selected in location i, 0 otherwise;

— $f_{zc} \in \{0,1\}, z \in Z, c \in C$: allocation variable, 1 if customer c is allocated to satellite z, 0 otherwise;

— $r_{ijh} \in \{0,1\}, (i,j) \in A^1, h \in H^1$: first-echelon vehicle flow variable, 1 if arc (i,j) is used by first-echelon vehicle h, and 0 otherwise;

— $x_{ijh} \in \{0,1\}, (i,j) \in A^2, h \in H^2$: second-echelon vehicle flow variable, 1 if arc (i,j) is used by second-echelon vehicle h, and 0 otherwise;

— $u_{ih} \geq 0, i \in (P \cup Z), h \in H^1$: Accumulated demand by first-echelon vehicle h at a location i;

— $v_{ih} \geq 0, i \in (Z \cup C), h \in H^2$: Accumulated demand by second-echelon vehicle h at a location i;

— $w_{pzh} \geq 0, p \in P, z \in Z, h \in H^1$: Demand flow from platform p to satellite z with first-echelon vehicle h;

Using this notation, one can formulate the standard 2E-LRP as a mixed-integer programming formulation.

$$\min \sum_{i \in (P \cup Z)} F_i y_i + \sum_{h \in H^1} \sum_{(i,j) \in A^1} \zeta_{ij} r_{ijh} + \sum_{h \in H^2} \sum_{(i,j) \in A^2} \zeta_{ij} x_{ijh} \tag{2.2.9}$$

subject to

$$\sum_{h \in H^2} \sum_{i \in (Z \cup C), i \neq j} x_{ijh} = 1 \quad \forall j \in C \tag{2.2.10}$$

$$\sum_{i \in (Z \cup C), i \neq j} x_{ijh} = \sum_{i \in (Z \cup C), i \neq j} x_{jih} \quad \forall h \in H^2, j \in (Z \cup C) \tag{2.2.11}$$

$$\sum_{i \in Z} \sum_{j \in C} x_{ijh} \leq 1 \quad \forall h \in H^2 \tag{2.2.12}$$

$$v_{ih} + vol(j) \leq v_{jh} + (1 - x_{ijh})M \quad \forall h \in H^2, i \in (Z \cup C), j \in C, i \neq j \tag{2.2.13}$$

$$\sum_{h \in H^1} \sum_{i \in (P \cup Z), i \neq j} r_{ijh} = y_j \quad \forall j \in Z \tag{2.2.14}$$

$$\sum_{i \in (P \cup Z), i \neq j} x_{ijh} = \sum_{i \in (P \cup Z), i \neq j} x_{jih} \quad \forall h \in H^1, j \in (P \cup Z) \tag{2.2.15}$$

$$\sum_{i \in P} \sum_{j \in Z} r_{ijh} \leq 1 \quad \forall h \in H^1 \tag{2.2.16}$$

$$u_{ih} + 1 \leq u_{jh} + (1 - r_{ijh})M \quad \forall h \in H^1, i \in (P \cup Z), j \in Z, i \neq j \tag{2.2.17}$$

$$\sum_{j\in(Z\cup C),c\neq j} x_{cjh} + \sum_{j\in C} x_{zjh} - f_{zc} \leq 1 \quad \forall h \in H^2, c \in C, z \in Z \tag{2.2.18}$$

$$\sum_{z\in Z} f_{zc} = 1 \quad \forall c \in C \tag{2.2.19}$$

$$\sum_{p\in P}\sum_{h\in H^1} w_{pzh} - \sum_{c\in C} vol(c) f_{zc} = 0 \quad \forall z \in Z \tag{2.2.20}$$

$$\sum_{h\in H^1}\sum_{z\in Z} w_{pzh} \leq Q_p y_p \quad \forall p \in P \tag{2.2.21}$$

$$\sum_{c\in C} vol(c)\ f_{zc} \leq Q_z y_z \quad \forall z \in Z \tag{2.2.22}$$

$$\sum_{p\in P}\sum_{z\in Z} w_{pzh} \leq cap_1 \quad \forall h \in H^1 \tag{2.2.23}$$

$$\sum_{i\in C}\sum_{j\in(Z\cup C),i\neq j} vol(i)\ x_{ijh} \leq cap_2 \quad \forall h \in H^2 \tag{2.2.24}$$

The objective function of the standard 2E-LRP (2.2.9) is to minimize the total system cost, which includes the fixed cost of selecting facilities at both echelons and the variable routing costs at each echelon. Constraints (2.2.10) ensure that each customer is visited by a second-echelon vehicle exactly once. Constraints (2.2.11) and (2.2.12) ensure the continuity of each route and a return to the platform from which it started. Constraints (2.2.13) are subtour elimination constraints for second-echelon routes. Constraints (2.2.14), (2.2.15), (2.2.16), and (2.2.17) are vehicle flow constraints on the first echelon, imposing the same conditions as constraints (2.2.10), (2.2.11), (2.2.12), and (2.2.13) do on second-echelon vehicles. Constraints (2.2.18) link allocation and routing variables. Constraints (2.2.19) ensure that each customer is allocated to a satellite facility. Constraints (2.2.20) represent flow conservation at satellite locations. Constraints (2.2.21) and (2.2.22) pertain to facility capacity for platform and satellite facilities, respectively. Constraints (2.2.23) and (2.2.24) guarantee that the demand carried by first- and second-echelon vehicles does not exceed their capacity, respectively.

The 2E-LRP has been the object of numerous studies since the work of Jacobsen and Madsen (1980). The most recent surveys on the matter cover different perspectives concerning the 2E-LRP. Prodhon and Prins (2014) and Cuda et al. (2015) provide complete surveys of heuristics and exact methods for the 2E-LRP. Both surveys present the modelling and algorithmic implications addressing two-echelon systems where location decisions are considered in one or the two level of facilities.

A limited number of contributions have addressed the standard 2E-LRP in which location decisions are restricted to a single level of facilities (see, Nguyen et al. 2012a,b). In contrast, a considerable majority of the algorithms have been developed for the 2E-LRP with location decisions in both echelons, most of them heuristics. (Boccia et al., 2010) present a specialized tabu search (TS) heuristic for the 2E-LRP, decomposing the distribution system in four sub-problems, a capacitated facility location problem (CFLP) and a multi-depot vehicle routing problem (MDVRP) for each echelon. The global solution for the 2E-LRP is obtained by a *bottom-up* approach, where the algorithm firstly finds solution for the second echelon and then optimize the first echelon solution based on the solution obtained for the second echelon. Boccia et al. (2021) and Crainic et al. (2011b), later present deeper insights in the design of two-echelon systems by extending some known VRP formulations and by proposing a diverse set of mixed-integer programming (MIP) models.

Contardo et al. (2012) introduce a new modelling framework for the 2E-LRP that decomposes the problem into two LRPs and exploit the previous works of Belenguer et al. (2011) and Contardo and Martinelli (2014) to derive a compact formulation. The work presents the value of strengthening its exact method through the separation algorithms used at each echelon and the addition of valid inequalities derived from the LRP. The authors also introduce an adaptive large neighbourhood search (ALNS) heuristic adapted from Hemmelmayr et al. (2012) for solving large-sized problems, which outperforms the previous solution methods in the 2E-LRP literature.

Concerning the solution methods, it is noteworthy that due to the complexity of 2E-LRP, exact methods have been limited to small- and medium-sized instances (Contardo et al., 2012). The effectiveness of these methods strongly depends on the quality of the lower bounds provided by the linear relaxation of the models. Large-scale and industrial applications are normally handled by heuristics. Most of the existing research examines hybridization of heuristic or metaheuristic approaches (e.g. TS, ALNS) to this problem based on the decomposition of the 2E-LRP into sub-problems which are then solved sequentially. Decomposition methods benefit from and exploit the multi-level structure of the system while not ignoring their interdependence. We observe that clustering techniques as well as granularity have not been fully exploited for the 2E-LRP although having delivered some promising results on LRPs (Prodhon and Prins, 2014).

The growing literature on 2E-LRPs reflects the increasing importance of these types of problems in logistics and supply chain management. Recent studies tend to propose variants and extensions of the 2E-LRP by considering one additional attributes, including pick-up and delivery, heterogeneous fleets, time windows, multiple products, or multiple objectives (see, for instance, Farham et al. 2018; Ouhader and Elkyal 2016; Vidović et al. 2016; Gianessi and Alfandari 2015; Govindan et al. 2014), or focused on the scalability to address large-scale problems (Winkenbach et al., 2016). To the best of our knowledge, only two contributions have addressed problem settings with at least two attributes, which are relevant for this research: Bala et al. (2017) address a 2E-LRP with synchronized production schedules and time windows, while Mirhedayatian et al. (2019) consider a pick-up and delivery with a fleet synchronization setting.

2.2.4. Multicommodity LRP and 2E-LRP

This section presents the review of the advances of LRPs class involving demand differentiation, most notably the literature involving the differentiation of demand by means of their know origin and destination. We refer to this problem class as the multicommodity LRP, which inherits the same characteristics as those of the standard LRP. The problem setting concerns the definition of location and routing decisions where demand needs to be transported from (potentially multiple) supply points (origin of the demand) to multiple delivery points (destination of the demand). Unlike the standard LRP, the routing decisions (and therefore the location decisions) are completely dependent to the differentiation of demand. This particularity is due to the fact that the facility-to-facility and customer-to-facility assignments must be made considering either the position of the supplier and the final destination (Boccia et al., 2018). One refers to the multicommodity 2E-LRP, when intertwined facility-selection and vehicle-routing decisions are to be taken for systems involving two sets of facilities and vehicles must be routed between the first and the second level of facilities.

A very limited number of scientific contributions have been devoted to the consideration of demand differentiation, with most contributions addressing 2E-LRPs. Contributions on LRPs are yet to be addressed in the literature. Gianessi and Alfandari (2015) present a variant of the 2E-LRP called the multicommodity-ring LRP. This problem extends a LRP by allowing the connection between all the satellites to carry out the exchange of any type of request. A matheuristic that decomposes the problem into several subproblems (location, allocation, network design and routing) is presented that sequentially solves each sub-problem and uses its output as input for the subsequent problem. Boccia et al. (2018) tackle a multi-commodity LRP by introducing the flow-intercepting facility location-routing problem inspired by city logistic applications. The authors present a branch-and-cut algorithm strengthened by valid inequalities and a heuristic procedure to obtain good-quality upper bounds.

Considering demand differentiation implies an increase in the computational complexity in different aspects, since it suggests the transition from a dedicated handling of the demands to the inclusion of flexibility at different levels in the way in which these commodities can be treated (Archetti et al., 2016). The existing literature lacks a standardized comparative benchmark test set due to the distinctive characteristics exhibited by each problem setting. Consequently, determining the effectiveness of methods employed in these studies is subjective and reliant on their specific contexts. In this regard, the literature contributes to the field of non-exact methods by emphasizing the significance of combining heuristics that explore neighborhoods sequentially and employing decomposition strategies (Boccia et al., 2018), as well as utilizing hybrid approaches that leverage sub-problem division strategies

(Gianessi and Alfandari, 2015). However, exact methods can become considerably intricate when incorporating demand differentiation based on origin-destination pairs, as this significantly increases the number of decision variables. To overcome this complexity and mitigate its impact on solution quality, two studies highlight the importance of incorporating specialized cuts adapted from related problems and linear relaxations of the model, along with the integration of heuristics to provide effective upper bounds (Boccia et al., 2018). To the best of our knowledge, the study of multicommodity 2E-LRPs that consider additional attributes has yet to be addressed in the literature.

2.2.5. LRP and 2E-LRP with temporal constraints

Demand is usually sensitive to a wide variety of time regulations by one or multiple actors. In general, demand transportation may depend on production times, opening hours, delivery policies, city regulations, customer availability, among others (Crainic et al., 2023a). These time dependencies are often expressed as cost changes over time or as time windows defined as the lower and upper bounds where demand is available or due at its origin and/or destination. This section thus presents the literature review of the LRPs and 2E-LRPs, where time windows constraints are defined in either the origin and/or destination of demand, pointing out the gaps in knowledge with respect to time dependencies.

The literature on the LRP involving the consideration of time constraints is very scarce. Contributions in the field usually address time windows as the main temporal constraint. The LRP with time windows (LRPTW) extends the standard LRP by assuming that customers can only be serviced within certain time window. The scheduling of the services becomes then crucial for the feasibility of a given solution. Given the complexity of the problem, solving the LRPTW with pure exact methods is often an extremely arduous task, due the large amount of computational effort required to address the temporal alternatives (Farham et al., 2018). To overcome these limitations, researchers have proposed several approaches for addressing the Location-Routing Problem with Time Windows (LRPTW). Ponboon et al. (2016) developed a branch-and-price method to tackle the LRPTW. Building upon this work, Farham et al. (2018) extended the method by incorporating new valid inequalities and additional acceleration features. Furthermore, Capelle et al. (2019) introduced the LRPTW with pick-ups and deliveries and successfully addressed it using column generation. Overall, contributions to branch-and-price applied to the LRPTW rely on the set-partitioning formulation of the problem. The main strategy consists in decomposing the problem in such a way that the pricing problem aims at finding feasible vehicle routes for each candidate depot location (platforms and satellites), whereas the master problem assures the location, demand satisfaction, and the respect of the depot capacities. Koç et al. (2016) address the fleet size and mix LRP with time windows. The work contributes to the structuring of different mathematical formulations for the problem. For this, the authors consider the addition of valid inequalities derived from variants of the LRP to either reduce the size of the formulation through the aggregation of variables or to tighten the linear relaxation bounds by disaggregating some of the constraints.

Metaheuristics for the LRPTW have also been implemented with success, mainly due to their ability to find feasible solutions and escape local optima. Burks (2006) presents a penalization scheme in an adaptive TS for the *theatre distribution problem*. The algorithm utilizes penalization for time window constraints violations in the objective function, to gradually reduce infeasibility. Gündüz (2011) proposes a TS that decomposes the problem into a capacitated facility location problem with time windows and a MDVRP with time windows that are solved in an iterative manner. The comparison of solution algorithms for the LRPTW is often hindered by variations in problem formulations across different articles and the absence of standardized benchmark datasets. Despite advancements in this field, the majority of research on LRPs predominantly emphasizes time windows as the primary temporal constraint. Similarly, studies that investigate time dependency on the origin of the demand and other time-sensitive aspects of the system remain largely unexplored.

The literature on 2E-LRP with time constraints is very limited as well. Govindan et al. (2014) introduced a bi-objective 2E-LRP with time windows, for the simultaneous minimization of distribution costs and greenhouses gas emissions, for perishable food supply chain distribution. Bala et al. (2017) address the 2E-LRP with synchronized production schedules and time windows. Wang et al. (2018) introduce a bi-objective model for the 2E-LRP with time windows through a clustering-based algorithm hinged on locations and purchase behavior. Lu et al. (2019) address the 2E-LRP heightening multimodal freight consolidation. Li et al. (2019) propose a 2E-LRP considering real-time trans-shipment capacity, varying with transshipment and consolidation operations. Darvish et al. (2019) address the 2E-LRP incorporating multiple periods and maximal due date on customer demands incorporating flexible decisions in terms of location of intermediate facilities on each time period. Mirhedayatian et al. (2019) propose a MIP formulation and a decomposition-based heuristic for a 2E-LRP with fleet synchronization and pick-up and delivery. The investigation of rich, multi-attribute 2E-LRPs that encompass multiple interacting sources of time-dependency, such as time-dependent non-substitutable demand and fleet synchronization, has received limited attention in the literature.

2.2.6. Stochastic LRPs and 2E-LRP

Freight transportation activities often encounter situations where the behavior of the system is closely tied to unpredictable events. Examples of transportation issues arising from uncertainties include traffic disruptions affecting delivery travel times, unavailability of customers during visits, and changes in demand impacting planned vehicle or facility capacities. Stochastic optimization refers to a collection of methods for minimizing or maximizing an objective function when uncertainty is present (King and Wallace, 2012). Uncertainty is typically incorporated into optimization models through the cost function or the constraint set, taking into account the randomness and variability of input data. In the context of LRPs, where strategic and tactical decisions are crucial for designing the distribution system, considering uncertainty is essential to achieve a more accurate and realistic modeling of the problem and improve the flexibility or robustness of the solutions. The review of the advances of each specific algorithmic and modelling frameworks used in stochastic optimization is out of the scope of this dissertation. Instead, this section emphasizes the review of the developments of LRPs and 2E-LRPs considering uncertainty, in particular those considering stochastic demand and stochastic travel times. For a comprehensive survey of LRPs addressing different stochastic aspects not covered in this review, we refer interested readers to recent surveys by Cuda et al. (2015), Schiffer et al. (2019), and Mara et al. (2021b).

The LRP with stochastic demands (LRPSD) involves uncertainty stemming from random changes affecting the demands. In terms of decision-making and information processing, the planning decisions must be defined based on an evaluation/estimation of their impact on operations, including the available recourse actions to adapt the plan to the observed demands. Contributions in the field build on the assumption that demand is uncertain in the sense that all its request fluctuations are statistically independent, and thus not correlated (Marinakis et al., 2016).

Multiple contributions have been proposed to address the LRPSD since the work of Laporte et al. (1989). The literature is notably characterized by the extensive use of local-search-based metaheuristic frameworks to address the underlying transportation problems, where location, allocation, and routing decisions are treated by different heuristics (see, Albareda-Sambola et al. 2007; Huang 2015; Marinakis 2015; Marinakis et al. 2016; Schiffer and Walther 2018; Zhang et al. 2019). On the other hand, most recent contributions present a different approach for the LRPSD with application of algorithms hybridizing a Monte Carlo simulation with a iterated local search metaheuristic (Quintero-araujo et al., 2019) and multi-objective frameworks (Rabbani et al., 2019). Despite the advances in the field, the literature on LRPSD remains limited, particularly in the case of non-substitutable demands. The case where stochastic demands are statistically independent remains the most predominant setting studied in the literature. Research concerning correlation features and its impact on the decision making process has yet to be addressed. Important contributions are also still required to deepen the understanding of the impact of richer problem settings and their influence on location decisions under demand uncertainty.

The literature on 2E-LRP with stochastic demands (2E-LRPSD) is very limited. To the best of our knowledge only two studies have addressed the 2E-LRPSD. Snoeck et al.

(2018) have presented a stochastic mixed-integer linear programming formulation to model a 2E-LRPSD arising from a practical application. Ben Mohamed et al. (2023) propose a logic-based Benders decomposition framework combined with a branch-and-cut-and-price for a multi-period 2E-LRPSD. However, particular developments are required in the field, especially in relation to explicitly consider demand correlations, non-substituable demand considerations, and meeting the modeling and algorithmic challenges these considerations imply.

Contributions in LRPs and 2E-LRPs involving stochastic travel times are also very scarce. Despite its practical relevance, very few LRP studies have considered uncertain travel times. Herazo-Padilla et al. (2015) integrated a discrete-event simulation with ant colony optimization for the LRP with stochastic travel speed and transportation cost. Gao et al. (2016) introduced an ant colony algorithm to solve a LRP with stochastic travel times expressed as random and cyclic traffic factors. Heuristic frameworks in the field share a similar strategy of decoupling location and routing decisions, where location decisions are defined by the iterative test of a pre-defined set of potential facility locations evaluated under the realization of the uncertain travel times. On the other hand, research on 2E-LRP integrating uncertain travel times is yet to be addressed. The literature clearly lacks research on alternative methodologies capable of integrating stochastic travel times into rich multi-attribute LRPs and 2E-LRPs involving different time-sensitive features, such as time-dependent demands and fleet synchronization. Particularly, addressing the modeling and algorithmic challenges that come with such integration.

2.2.7. LRP and 2E-LRP with synchronization constraints

This section provides an overview of the literature on LRPs and 2E-LRPs that involve synchronization constraints. We begin by discussing the advances and impact of considering synchronization constraints in the logistics and transportation industry. Subsequently, we review the most recent contributions on LRPs with synchronization constraints.

Synchronization in logistics refers to the process in which two or more operations adjust their temporal and/or spatial state of motion to achieve a common behavior. This enables more efficient management of scarce resources such as storage capacity, product availability, and vehicle capacity (Crainic et al., 2009). Examples of such applications in logistics include systems that employ cross-docking principles or urban logistics applications, where freight is typically transferred between synchronized vehicles due to limited or no storage availability. The nature of distribution systems established across multiple echelons inherently requires strict spatial coordination (although it may not need to be explicitly addressed) and the consideration of temporal synchronization (Andersen et al., 2009; Crainic et al., 2009). In contrast, single-tiered systems do not exhibit such characteristics unless specific circumstances arise (Crainic et al., 2009; Nguyen et al., 2013; Ait Haddadene et al., 2016). Consequently, when storage capacity or time becomes a bottleneck for the system, temporary synchronization plays a crucial role in enabling continuous freight or service consolidation throughout the network.

Transportation problems considering synchronization constraints have currently become a subject undergoing intense study in a broad range of routing and scheduling variants (Sluijk et al., 2022). Among the most influential proposals in this domain is the work of Crainic et al. (2009), which, to the best of our knowledge, is the first work to highlight the importance of time-sensitive operations and synchronization in facilities, along with the challenges that arise from city logistics. To the best of our knowledge, the work of Mirhedayatian et al. (2019) is the only contribution that has addressed a 2E-LRP with fleet synchronization and pick-up and delivery for an application in postal service. The work introduces a decomposition-based heuristic composed by a location-allocation phase determined by the interplay between a constructive method and an allocation model, and a routing phase where vehicle schedules and synchronization are handled with a set-partitioning formulation. Particular developments are required for the 2E-LRP, particularly with respect to the modelling and algorithmic challenges that arise when addressing multiple time-sensitive attributes.

2.3. Algorithmic discussion

From the review of the literature, we can highlight that LRP variants have been the subject of numerous studies characterized by an increasing trend to adopt more than one attribute simultaneously, in order to achieve a more accurate approximation of the reality of industrial applications (Drexl, 2012). Location-routing variants in this domain remain sparse due to the numerous amounts of practical constraints (see, e.g. Gianessi and Alfandari 2015; Rahmani et al. 2016; Bala et al. 2017; Govindan et al. 2014; Hamidi et al. 2014; Crainic and Montreuil 2016) as each specific attribute have a particular relevance in various types of freight transport systems. However, drawbacks addressing rich LRPs still underlie in the complexity obtained by handling multiple attributes (Sadjady and Davoudpour, 2012), either due of the size of the resulting system or by the hard interrelation between the considered attributes (Crainic et al., 2009).

A considerable part of the literature is characterized by a predominant use of heuristic methods. This fact is mainly motivated by the trade-off between good quality solutions and computational effort (compared with the exact methods) (Rahmani et al., 2016; Bala et al., 2017; Masson et al., 2013; Grangier et al., 2016). These approaches often adopt a decomposition strategy, breaking down the problem into a location-allocation problem and a vehicle routing problem. In cases where multiple echelons are considered, authors often incorporate further system decomposition based on echelons. The resulting subproblems are then solved sequentially (e.g., location-allocation first, routing second) or iteratively (with feedback from one subproblem influencing the others). In some cases, both allocation and routing decisions are made simultaneously. The most effective heuristics in the literature are distinguished by the combination of tailored diversification and intensification phases, which primarily rely on local search procedures integrated into well-known heuristics and metaheuristics frameworks. The success of these heuristics hinges on their ability to leverage specialized neighborhoods to efficiently explore the solution space. However, designing suitable neighborhoods for rich-LRPs presents a formidable challenge due to the intricate nature, heterogeneity, and interdependence of the various components inherent in the problem definition (Gianessi and Alfandari, 2015).

There are a few contributions in relation to exact methods, due the high complexity and the huge number of variables necessary to carry out an efficient enumeration of the resulting formulation. Some of the proposed methods often extend ideas from related vehicle routing problems and combine dynamic cut and column generation with bounding procedures. Decomposition schemes happen to play a key role in the way in which problems can be expressed as the interaction of multiple known subproblems (Gianessi and Alfandari, 2015; Aksen and Altinkemer, 2008; Nikbakhsh and Zegordi, 2010; Crainic et al., 2009). However, maintaining the interrelation between the resulting subproblems happens to be the most complicating aspect to avoid the exclusion of optimal solutions in the solution space (Gianessi and Alfandari, 2015; Drexl, 2012; Guastaroba et al., 2016). The authors in this field address decomposition schemes by means of a sequential interaction between the subproblems (Gianessi and Alfandari, 2015); or in an integrated manner, either by solving the subproblem by using the output of the others as inputs, or in an exact manner by exploiting Benders or Dantzing-Wolfe decomposition schemes (Koç et al., 2016; Aksen and Altinkemer, 2008; Nikbakhsh and Zegordi, 2010; Contardo et al., 2012). The latter have demonstrated beneficial potential in small- and medium- sized instances, but still with a huge computational effort (in time and memory usage).

2.4. Conclusions

Among the works in this field, there are gaps in relation to models and solution methods for each class of problems (mentioned in their respective section) as well as transportation issues that have not been addressed yet. Mentioning every gap in the reviewed literature, however, is outside the scope of this document, therefore, we highlight the following gaps that are relevant for this research and which we seek to fill throughout the research project.

Problem class: Numerous references in the literature address problem settings involving multiple attributes, and authors have categorized them using various names and descriptions. The lack of consensus in the literature regarding the classification of these types of problems makes it challenging to follow the advances and trends in this research field. Although multiple contributions have introduced different classification schemes for various types of VRPs, there is a clear need for a problem class to classify LRPs and 2E-LRPs involving multiple interacting attributes. Therefore, developing a problem class with a more unified perspective on the problem settings could be helpful.

New benchmarks and performance analysis: Literature in this field remains sparse due to the wide variety of attributes and side constraints that each problem definition may have. Solution methods are often compared with related LRP variants or tested under specialized instances. Sometimes, related works differ in the size of the instances that the methods can solve. Hence, developing a comprehensive set of benchmark instances with a suitable range of sizes (for practical and theoretical contributions) can lead to more deeper insights on the usefulness of algorithmic performance under different conditions.

Modeling and solution frameworks: The consideration of multiple interacting attributes is both practically relevant and scientifically challenging in various logistic applications (Sluijk et al., 2022). There has been a growing interest in these complex problem settings that take into account time-sensitive aspects, demand differentiation, and uncertainty, primarily motivated by their potential impact on the logistics industry (Crainic et al., 2023a). However, formal models and solution frameworks to address these challenges have yet to be developed and analyzed for LRPs. Research in these directions can contribute to the introduction of robust and efficient solution methods for dealing with these complex mathematical models. Additionally, such research can provide the tools needed to analyze the behavior of multiple interacting attributes in location and routing decisions.

The aim of this thesis is twofold. First, from a modeling perspective, we introduce novel mathematical formulations that incorporate multiple interacting attributes to enhance the representation of temporal components. Furthermore, we extend and enhance existing models by integrating stochastic considerations with location routing decisions. In our three papers, we analyze various vehicle-flow formulations and time-space formulations suitable for multi-attribute problem settings, which have not been explored previously.

Another significant innovation in our work lies in the algorithmic aspect. The existing literature on multi-attribute 2E-LRPs is limited, and the problem settings we consider in each of the three papers are highly challenging. This is due to their combinatorial nature and the large scale resulting from the presence and interrelation of multiple interacting attributes. Since solving the standalone formulations is typically impractical under these conditions, they cannot be successfully applied. To address these challenges, we introduce a diverse set of solution frameworks for each problem setting, ensuring an effective approach to solving them

First Article.

Multi-attribute Two-echelon Location Routing: Formulation and Dynamic Discretization Discovery Approach

by

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The main contributions of David Escobar Vargas for this articles are presented.

- *David Escobar Vargas:* Conceptualization, Methodology, Software, Data Analysis, Writing - original draft.
- *Teodor Gabriel Crainic:* Conceptualization, Methodology, Writing - review & editing.

Résumé. Nous étudions le système de localisation-routage à deux échelons sous de fortes contraintes de synchronisation, en plus de plusieurs autres attributs en interaction. Stimulé, en particulier, par les applications de logistique urbaine, le système que nous étudions concerne un plan de distribution à deux échelons composé d'un ensemble d'installations de plateformes et d'un ensemble d'installations intermédiaires (ou satellites) pour livrer le fret des zones d'approvisionnement à l'extérieur de la ville aux clients à l'intérieur de la ville. Le problème comprend une demande multi-marchandises dépendante du temps, des fenêtres temporelles, un manque de capacité de stockage dans les installations intermédiaires et la synchronisation, dans ces installations, des flottes opérant à différents échelons. Le problème nécessite la sélection des installations aux deux niveaux, le placement des fournisseurs sur les plateformes et des clients sur les satellites, ainsi que le routage et la programmation des véhicules à chaque échelon, afin de livrer le fret des plateformes aux clients, par l'intermédiaire des satellites. Le manque de capacité de stockage des installations partagées, les satellites, exige une programmation rigoureuse des itinéraires des véhicules et de la demande, c'est-à-dire des heures de départ des plateformes et des satellites, ainsi que la synchronisation des itinéraires des véhicules dans les satellites pour des opérations de transbordement efficaces. Nous introduisons le cadre du problème, présentons une formulation de programmation en nombres entiers mixtes et une méthode de solution exacte basée sur le *dynamic discretization discovery* pour le problème. Nous effectuons des analyses approfondies pour évaluer l'impact des attributs et des exigences du problème sur le comportement du système et les performances de l'algorithme.
Mots clés : transport, localisation-routage à deux échelons dépendant du temps, synchronisation, découverte de discrétisation dynamique, logistique urbaine

Abstract. We study the two-echelon location-routing system under tight synchronization constraints, in addition to several other interacting attributes. Prompted, in particular, by city-logistics applications, the system we address concerns a two-echelon distribution layout composed of a set of platform facilities and a set of intermediate satellite facilities to deliver freight from supply zones outside the city to customers within. The problem setting involves time-dependent multicommodity demand, time windows, lack of storage capacity at intermediate facilities, and synchronization at these facilities of the fleets operating on different echelons. The main decisions in the problem include the selection of facilities at both levels, the allocation of suppliers to platforms and customers to satellites, as well as the routing and scheduling of vehicles at each echelon. These decisions are made in order to facilitate the delivery of freight from platforms to customers through the satellites. The lack of storage capacity of the shared facilities, the satellites, requires tight scheduling of the vehicle routes and demand itineraries, i.e., departure times from the platforms and satellites, and the synchronization of vehicle routes at satellites for efficient transshipment operations. We introduce the problem setting, present a mixed-integer programming formulation, and a dynamic discretization discovery-based exact solution method for the problem. We perform thorough analyses to assess the impact of the problem attributes and requirements on the system behaviour and algorithm performance.
Keywords: transportation, time-dependent two-echelon location-routing, synchronization, dynamic discretization discovery, city logistics

1. Introduction

Locating/selecting facilities and building vehicle routes are two of the most critical problems arising in planning and managing transportation and logistics systems. The *Location Routing Problem* (*LRP*) class combines the two problems (under suitably defined harmonizing cost and demand values) into a single formulation providing a more accurate and refined representation of the impact of facility selections on the functioning and performance of the resulting system. One refers to the *Two-Echelon Location Routing Problem*, *2E-LRP*, when intertwined facility-selection and vehicle-routing decisions are to be taken for systems involving two sets of facilities and vehicles must be routed between the first and the second level of facilities, and between facilities on the second level and customers making up the "third" level. The interest and vast application areas of LRP and 2E-LRP are emphasized by a very large body of literature and the numerous surveys synthesizing the field (e.g., Prodhon and Prins, 2014; Schneider and Drexl, 2017; Mara et al., 2021b).

However, the literature addressing richer problem settings characterized by several interacting attributes is still very limited (Sluijk et al., 2022). Existing 2E-LRP models and methods are unable to keep pace with the rapidly evolving planning challenges that require more comprehensive and rich problem representations. This gap is particularly evident in applications such as Physical Internet (Crainic et al., 2023b) and City Logistics (Crainic et al., 2021b), which involve multiple origin-to-destination commodities, time-dependent demand, scheduled and synchronized activities, and other complex considerations. The complexity arising from the need for specialized modelling and heuristics to handle the interactions among these diverse attributes restricts the applicability of existing 2E-LRP models and methods, especially when considering time-dependent aspects (e.g., Bala et al., 2017). Our objective is to contribute towards filling these gaps in the knowledge, through contributions to the modelling of such systems and decisions, as well as to the body of solution methods for these formulations.

We address a 2E-LRP with multiple interacting attributes, including time-dependent multicommodity origin-to-destination (OD) demand, time windows, limited storage capacity at intermediate facilities, and synchronization at the intermediary facilities of the fleets operating on different echelons. This new *Two-Echelon Multi-Attribute Location-Routing Problem with fleet Synchronization* at intermediate facilities (*2E-MALRPS*) thus requires 1) the selection of facilities on both levels, 2) the routing, scheduling, and synchronization of vehicles at second-echelon (intermediate) facilities, and 3) the allocation of OD demands to the selected facilities and their delivery using sequences of synchronized routes. The goal of the 2E-MALRPS is to minimize the total cost of the system, composed of the facility-selection cost at both levels and the transportation (fleet-utilization) cost, while satisfying the demand and the capacities of the system elements.

The time dependencies of demand and scheduled operations require time to be explicitly represented in the formulation. Time-space networks constitute a widely-known modelling technique to efficiently capture and handle temporal information (e.g., Ford and Fulkerson, 1962; Crainic and Hewitt, 2021). Most contributions in the literature use a classic time-space representation in which the duration of the plan to be built is discretized, according to a given granularity, into a number of consecutive periods, and the nodes in the physical network representing the system studied are duplicated at the points in time defining the periods. While such a modelling provides the means to adequately represent the level of detail of time-dependent activities and planning, the size of the corresponding formulations grows very rapidly with the refinement of the time discretization, making exact solution methods impractical. We address this challenge through the model-building and algorithm-design aspects. On the modelling front, we propose a hybrid formulation, where the nodes standing for facilities, which can be active at any moment while the system works, are duplicated at all periods, while customer nodes appear only at relevant periods (when they can be reached within their time windows), while a continuous time representation is used to capture the timing of vehicles arriving and departing to/from intermediary facilities and customers.

On the algorithmic front, we propose an exact solution method based on the *Dynamic Discretization Discovery* (*DDD*) strategy introduced by Boland et al. (2017) for the *Scheduled Service Network Design* (*SSND*) problem. In that context, the DDD consists in iteratively solving a mixed-integer model formulated on a sparser version of the time-space network, also known as the *reduced time-space network*, and refining this network (i.e., refining the granularity of the time representation), until an optimal solution is found. The refinement procedure is the core of the DDD, in which all the nodes and arcs within the reduced network undergoing refinement must be reviewed and adjusted. Refinement generates an iterative network growth. It is hoped, and achieved (e.g., Boland et al., 2017), that growth will be significantly smaller than a full time-space network while adequately representing the time attributes of the problem in hand.

The iterative refinement process inherent in the DDD enables the dynamic discretization of relevant time moments in the network, resulting in a systematic reduction of computational complexity. This characteristic offers promising opportunities for addressing complex time-dependent problems by simplifying the representation of time (Vu et al., 2020). However, effectively tackling the two-echelon definition and the combinatorial nature of the 2E-MALRPS, which integrates location, routing, and time aspects, requires extending and adapting the original DDD methodology. These extensions are crucial for guiding the refinement process, controlling network growth, and preserving attribute interaction. To achieve these goals, we define specific properties and procedures for computing bounds, handling solution degeneracy, refining granularity, and controlling iterative growth of the time-space network.

The proposed DDD solution method thus enables an efficient time-space representation of the system, while overcoming the scalability limitations of the explicit representation of time. The results of the computational study illustrate the behaviour and very good performance of the proposed modelling approach and solution method, and emphasize the importance of explicitly accounting for the time attributes of the problem elements, and of the associated fleet synchronization requirements within time-sensitive distribution systems.

The paper is organized as follows. The problem definition is given in Section 2, and an overview of related literature in Section 3. Section 4 is dedicated to the hybrid modelling

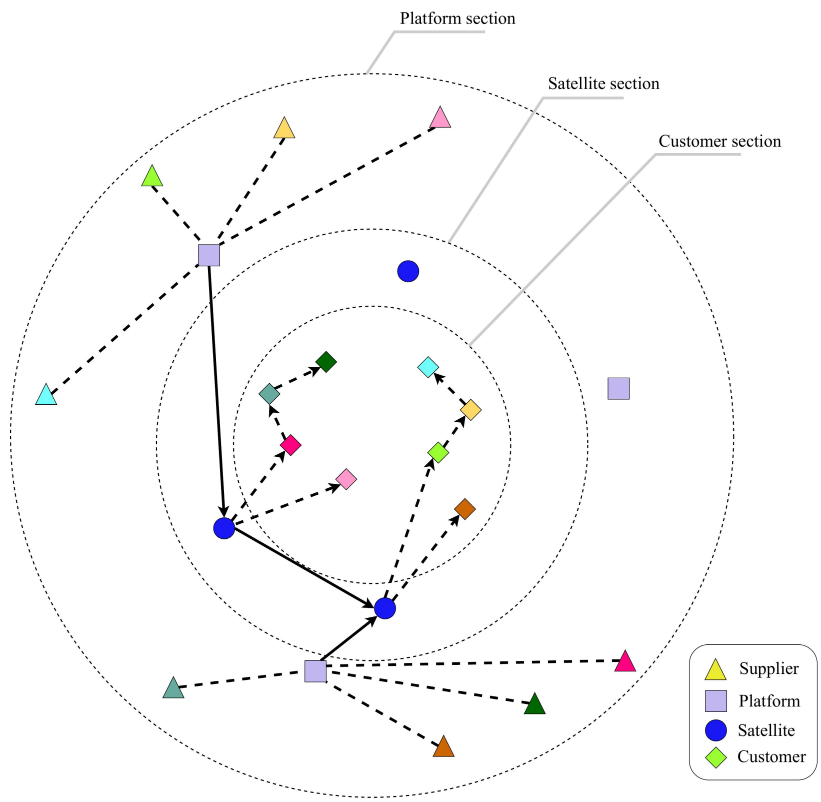


Figure 2 – Two-echelon distribution system topology

of time we use and the 2E-MALRPS formulation. Section 5 describes the DDD solution method we propose. Computational results are presented and analysed in Section 6. We summarize our work in Section 7.

2. Problem Setting

The two-echelon system is composed of sets of suppliers (demand origins), platforms (primary facilities), satellites (intermediate facilities), customers (demand destinations), and two vehicle garages. *Platforms* are large-sized infrastructures where one performs the storage, sorting, and consolidation of the inbound freight provided by supply points through various modes of transportation. *Satellites*, on the other hand, are medium- to small-sized facilities, located within the city limits and providing reduced or null storage capacity, where first and second-echelon vehicles meet and freight is transshipped and consolidated for the second part of transportation to customers. Freight transportation is performed by two fleets of homogeneous and limited-capacity vehicles, each operating within a specific echelon and able to transport the products making up the OD demand. Vehicles are assumed to be available at the garage of their corresponding echelon, where they start and end their routes.

The *multi-commodity, origin-to-destination (OD) demand* is defined between suppliers and customers, each individual demand being characterized by origin, destination, volume, availability time, which is then adjusted for each platform, and due time window at destination. As depicted in Figure 2, each OD demand has to be assigned to an open platform and an open satellite. OD demands are consolidated at platforms, and are then moved by first-echelon vehicles to their selected satellites for the second part of their journeys. Once delivered at satellites, demand flows are transshipped and consolidated into second-echelon vehicles, which perform the deliveries to the final destinations. First and second echelon vehicle routes visit sequences of satellites and customers, respectively. To simplify the presentation of the system, garage nodes are not displayed in any of the illustrations of the paper.

The problem requires the selection of facilities at both levels, the allocation of suppliers to platforms and of customers to satellites, as well as the routing and scheduling of vehicles at each echelon to deliver the freight from platforms to customers, through satellite facilities. Vehicle routes and the OD-demand itineraries, determining the sites (facilities and customers) visited and the visit schedules, i.e., arrival, waiting, and departure times, must be determined within the time restrictions imposed by each OD demand (at platforms and customer), as well as by the need to synchronize vehicles at satellites due to the time dependency of demand and the lack of satellite storage capacity. Transportation costs are assumed to be equal to the travel times of the corresponding inter-site movements, while, to simplify the presentation but without loss of generality, waiting times at the various sites do not yield additional costs. The main objective of the resulting 2E-MALRPS is to minimize the total cost of the system, composed of the cost of selecting/opening facilities at both levels and the transportation costs, while satisfying the demand and the capacities of the system elements.

3. Literature Review

This section aims to situate the 2E-MALRPS within the relevant literature on the multi-attribute 2E-LRP and LRP, pointing out the gaps in knowledge with respect to time dependencies, time windows, origin-destination demand, and fleet synchronization. Given the available space, the review of the advances of each specific attribute is out of the scope of this paper. Therefore, our focus is on multi-attribute 2E-LRP and LRP, which encompass applications characterized by the interplay of two or more of these attributes within the same problem setting. A brief discussion on time-space formulations is also provided, focusing on dynamic discretization schemes used as solution frameworks. For an overview of the different problem variants in single- and two-echelon distribution systems and location routing problems that are out of the scope of this work, we refer the interested reader to recent surveys by Albareda-Sambola and Rodríguez-Pereira (2019); Crainic et al. (2021a); Mara et al. (2021b), and Sluijk et al. (2022). We also refer readers to well-known surveys by Lopes et al. (2013) and Prodhon and Prins (2014), which provide a broad compilation of the early works and surveys for single-echelon LRPs with less attribute considerations.

The LRP has been the object of numerous studies since Maranzana (1964), research spanning a wide range of problem settings. Most LRPs focus on a single key attribute to represent the time or demand requirements of the problem under consideration. One notices that, time windows are used in most cases when temporal dependencies are addressed (Farham et al., 2018), while issues related to non-substitutable demand and fleet synchronization have scarcely been addressed from an optimization point of view (Boccia et al., 2018). Contributions to rich, multi-attribute LRPs, and the influence that the simultaneous consideration of several interacting attributes may have on the decision-making, are still very limited (Mara et al., 2021b).

The literature on multi-attribute 2E-LRP, a more challenging problem setting due to the inter-echelon relationships, is much more recent and rather sparse. Research in the field has largely centred on the study of time-sensitive applications, focusing on customer time windows (e.g., Wang et al., 2018) and multi-period settings only (e.g., Darvish et al., 2019). Rich, multi-attribute 2E-LRPs incorporating multiple interacting sources of time-dependency, such as time-dependent non-substitutable demand and fleet synchronization, have been scarcely investigated. To the best of our knowledge, two contributions only have addressed problem settings with at least two attributes, which are relevant for this research: Bala et al. (2017) address a 2E-LRP with synchronized production schedules and time windows, while Mirhedayatian et al. (2019) consider a pick-up and delivery with fleet synchronization setting.

Location-routing and routing problems are related. Although surveying the Two-Echelon Vehicle Routing Problem (2E-VRP) literature (see, e.g., Crainic et al., 2021a; Sluijk et al., 2022) is out of the scope of this paper, it is worth noticing that more 2E-VRP variants with time-dependency and synchronization constraints were explored compared to the 2E-LRP. Yet, similarly to the 2E-VRP, the study of rich, multi-attribute 2E-VRP settings is still largely lacking.

From a modelling perspective, 2E-LRP contributions tend to share a compact-type structure, also known as flow formulations, to model the problem setting, with temporal and demand decisions being recorded through additional variables or indexes linked with the vehicle routing/flow variables (Contardo et al., 2012; Mara et al., 2021b). The complexity of the 2E-LRP makes exact solution methods for these formulations impractical in most cases, in particular as the problem size or the number of attributes being considered grows, even when combining formulation-strengthening valid inequalities and column-generation mechanisms (Farham et al., 2018) (Albareda-Sambola and Rodríguez-Pereira, 2019). Meta-heuristics are thus generally proposed (e.g., Mirhedayatian et al., 2019; Abbassi et al., 2020).

It is noteworthy that the scientific literature on problems with tight or complex time considerations has widely adopted the use of time-space network representations (Crainic and Hewitt, 2021). Also noteworthy is the growing research effort addressing the iterative refining of the discretization of time-space networks, in order to efficiently deal with the trade-offs between the precision of the model, brought by refined time discretizations, and the significant computational challenges of addressing the corresponding large formulations. Introduced by Boland et al. (2017) for the Service Network Design Problem (SNDP), research on the *Dynamic Discretization Discovery* approach focuses mostly on the design of reduced time-space networks through a sparse discretization of time, which are iteratively refined to derive lower and upper bounds to the problem without the use of a highly detailed time-space representation. Very few efforts target topics outside the SNDP. Thus, Vu et al. (2020) propose a DDD-inspired solution method for the time-dependent travelling salesman problem with time windows, while Lagos et al. (2022) address the impact of time discretization on a continuous-time inventory-routing problem.

The 2E-LRP requires particular developments with respect to the modelling and algorithmic challenges that arise when addressing multiple interacting time-sensitive attributes, including the interest of alternative exact solution frameworks. This research work aims towards filling these gaps in the literature by proposing a hybrid time-space formulation and bv adapting and enhancing the guidelines of the DDD methodology for the 2E-MALRPS.

4. 2E-MALRPS Modelling

Section 4.1 is dedicated to the formal problem definition, while Section 4.2 introduces the time-space representation. Finally, Section 4.3 presents the 2E-MALRPS formulation.

4.1. Problem definition and notation

Let $\mathcal{G}^{\text{ph}} = (\mathcal{V}^{\text{ph}}, \mathcal{A}^{\text{ph}})$ be the weighted directed graph representing the physical network on which the problem is defined. The set of vertices $\mathcal{V}^{\text{ph}} = \mathcal{Q}^{\text{ph}} \cup \mathcal{P}^{\text{ph}} \cup \mathcal{Z}^{\text{ph}} \cup \mathcal{E}^{\text{ph}} \cup \mathcal{C}^{\text{ph}}$ is made up of five disjoint sets standing for the physical sites (known or among which locations are to be decided) of suppliers $\mathcal{Q}^{\text{ph}}$, potential platform sites $\mathcal{P}^{\text{ph}}$, possible satellite sites $\mathcal{Z}^{\text{ph}}$, vehicle garages $\mathcal{E}^{\text{ph}}$, and customers $\mathcal{C}^{\text{ph}}$. A fixed selection (opening) cost F_p and a capacity Θ_p are defined for each possible platform location $p \in \mathcal{P}^{\text{ph}}$. A fixed selection (opening) cost F_z is also defined for each potential satellite site.

The arc-set $\mathcal{A}^{\text{ph}} = \mathcal{A}_1^{\text{ph}} \cup \mathcal{A}_2^{\text{ph}}$ represents the direct links between locations, i.e., the vertices in $\mathcal{V}^{\text{ph}}$. A non-negative unit cost ζ_{ij} and a travel time τ_{ij} are associated with each arc $(i,j) \in \mathcal{A}^{\text{ph}}$. The set $\mathcal{A}_1^{\text{ph}}$ includes the arcs of the first echelon, corresponding to the connections between suppliers $\mathcal{Q}^{\text{ph}}$ and platforms $\mathcal{P}^{\text{ph}}$, between the latter and satellites $\mathcal{Z}^{\text{ph}}$, as well as the arcs connecting pairs of satellites and the first-echelon garage to platforms and satellites. The set $\mathcal{A}_2^{\text{ph}}$ includes the arcs of the second echelon, that is, the connections between satellites

$\mathcal{Z}^{\text{ph}}$ and the final customers $\mathcal{C}^{\text{ph}}$, and the arcs connecting pairs of customers, and the second-echelon garage to satellites and customers.

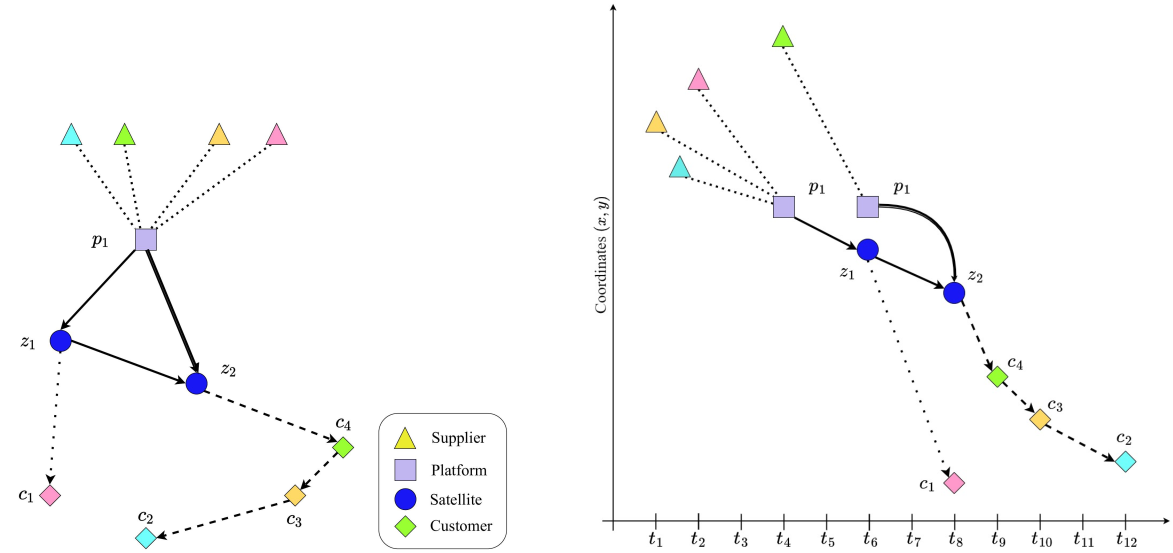
Due to the lack of storage capacity at satellites and the time dependency of demand, interacting vehicles from the first and second echelons must be synchronized at satellites, at certain points in time, where first and second echelon vehicles may wait for a maximum time W^2_{max}. Moreover, it is assumed that each customer $c \in \mathcal{C}^{\text{ph}}$ has a (hard) time window $[a_c, b_c]$ (the time interval in which service must start at the node) and a service time σ_c. The distribution plan and the corresponding time-sensitive service network are built for a given schedule length Ψ (e.g., a day or a week). The system, and the distribution plan, follow a cyclic and repetitive logistics operation over a certain planning horizon (e.g., a month or a season), during which demand and the temporal properties of the system do not change. Therefore, all transportation activities take place between time 0 and the given schedule length Ψ.

Let $\mathcal{K}$ denote the set of OD demands that must be transported from suppliers to customers. For each commodity $k \in \mathcal{K}$, let $vol(k)$ be its volume, $O(k) \in \mathcal{Q}^{\text{ph}}$ the associated supplier node, $D(k) \in \mathcal{C}^{\text{ph}}$ the associated customer node, and α^{pk} the time when commodity k would become ready for transportation if assigned to be shipped from platform $p \in \mathcal{P}^{\text{ph}}$. This parameter takes into consideration the time required for the transportation of each commodity from the supplier to the given platform. An *itinerary* for a given commodity specifies a possible scheduled journey from the moment when it becomes available at the supplier node, until its delivery at its final destination, through a platform and a satellite, including the specific time instances associated to each arrival and departure at each site. More formally, an itinerary r for commodity $k \in \mathcal{K}$ is a tuple $\{(v_i, \mu_i, \nu_i) : i \in r\}$, where $v_i \in \mathcal{V}^{\text{ph}}$ is the i-th node visited, μ_i the arrival time to v_i, and ν_i the departure time from the node.

Two homogeneous fleets of vehicles $\mathcal{H} = \mathcal{H}^1 \cup \mathcal{H}^2$, with limited load capacities cap_1 and cap_2, are available for the first and second echelon, respectively. Vehicle capacities are fixed. Vehicles can deliver any demand and are parked in strategically-located garages, $\mathcal{E}^{\text{ph}}_1$ for vehicles operating in the first echelon, and $\mathcal{E}^{\text{ph}}_2$ for vehicles operating in the second echelon.

The 2E-MALRPS consists in the selection of platform and satellite facilities, the allocation of demand from suppliers to platforms and of customers to satellites, as well as the construction of a limited set of routes for the first and second echelons in such a way that: (i) all the customer demands are satisfied on time; (ii) the load capacity of each vehicle is not exceeded; (iii) each customer is visited by one vehicle only; (iv) the total demand assigned to a facility (platforms and satellites) does not exceed its capacity at any time moment; and (vi) the sum of the fixed selection costs and the variable routing costs is minimized.

Figure 3 illustrates the dynamics of the system from a physical and a temporal point of view. Figure 3a shows a feasible solution where four OD demands are dispatched to their



(a) **(b)**

Figure 3 – Example of a feasible solution for the 2E-MALRPS. (a) spatial layout of the feasible solution for the 2E-MALRPS. (b) time-space representation of the feasible solution for the 2E-MALRPS.

destinations by means of platform facility p_1 and satellites z_1 and z_2, the full and dotted lines illustrating the first- and second-echelon vehicle movements, respectively. Operations are illustrated from a temporal point of view in Figure 3b, starting with the three OD demands, each with its own availability time, all being assigned to platform p_1 and ready to be shipped at time t_4. The fourth OD demand, available at time t_4, is also assigned to platform p_1, but it is ready to be shipped at time t_6. A first-echelon vehicle (single full line) arrives at platform p_1, picks-up part of the available demand, and proceeds to visit satellite z_1 at time t_6 and satellite z_2 at time t_8. A different first-echelon vehicle (thick full line), then picks-up the remaining demand at a later time, t_6, and arrives at satellite z_2 at time t_8. Two second-echelon vehicles leave their garage to arrive on time at satellites z_1 and z_2 to enable the freight transfer from the first-echelon vehicles and, then, deliver on time the freight to the appropriate customers. Multiple fleet synchronization activities take thus place at the two satellites. A first synchronization at satellite z_1, at time t_6 and a second synchronization takes places at satellite z_2, at time t_8. Vehicles returns to their respective garages once their routes are completed.

4.2. Time-space network

Time is a key aspect of the system. Time-dependent OD demands constrain the timing of operations by means of availability restrictions at platforms, the need to pass through satellites of limited capacity (if any), and customer time windows, creating an interdependency in time throughout the whole distribution process. Excess in travelling or waiting times may thus become prohibitive or result in operational infeasibilities. One must therefore carefully model time.

There are two major modelling alternatives to represent these timing decisions. The first type considers an implicit representation by focusing on the time of operations, e.g., when vehicles arrive and depart facilities and customers to pick up or drop freight. This representation leads to a compact, *continuous-time* representation with a polynomial number of variables (indexed by the arcs and nodes of the network). The itinerary of each commodity can be constructed by matching the vehicle flow (visits of vehicles to sites), allocation (what commodity is allocated to what vehicle), and time variables (the time a site is visited).

The second modelling alternative is the "classic" *discretization* approach according to a Δ granularity. The schedule length Ψ is partitioned into Δ time periods, each physical node $i \in \mathcal{V}^{\text{ph}}$ being duplicated at each time period. For simplicity of presentation, but without loss of generality, we assume in the following that all time periods defined by the discretization granularity Δ are of equal length. This mechanism leads to a *time-space network*, where every node is a pair (i, t) with i representing a physical node in $\mathcal{V}^{\text{ph}}$ and $0 \leq t \leq \Psi$ a moment in time. Physical arcs (i, j) in the original system now take the form $((i, t_i),(j, t_j))$ meaning that travel is performed between nodes i and j departing at time t_i and arriving at time t_j. In contrast to the continuous-time representation, the discrete representation is explicit, in the sense that the nodes and arcs in the time-space network encode the timing decisions explicitly. Synchronization and other timing actions and requirements are thus expressed as decisions and constraints in the resulting formulation.

The proposed formulation is thus built on the 2E-MALRPS time-space network $\mathcal{G} = (\mathcal{V}, \mathcal{A})$, with the sets $\mathcal{V} = \mathcal{Q} \cup \mathcal{E} \cup \mathcal{P} \cup \mathcal{Z} \cup \mathcal{C}$ standing for the nodes in time and space for suppliers $\mathcal{Q}$, vehicle garages $\mathcal{E} = \mathcal{E}^1 \cup \mathcal{E}^2$, platforms $\mathcal{P}$, satellites $\mathcal{Z}$, and customers $\mathcal{C}$.

Let $\mathcal{T}(\Delta)$ be the (ordered) set of time periods given by discretizing the schedule length Ψ according to the granularity Δ. Let also $\mathcal{T}_i(\Delta) \subseteq \mathcal{T}(\Delta)$ represent the set of time periods at which node $i \in \mathcal{V}^{\text{ph}}$ is *relevant* in $\mathcal{G}$ because vehicles or commodity flows may access it at that time. Each system component has its own set of relevant periods: suppliers appear once only, the time realizations of customers $i \in \mathcal{C}^{\text{ph}}$ must satisfy $\mathcal{T}_i(\Delta) \subseteq [a_i, b_i]$, and copies in time are made at all periods for satellites and platforms, as these must be available for the complete schedule length.

Figure 4 illustrates this hybrid time-space network structure for two OD demands, from the same supplier to two customers, passing through a platform and a satellite. All periods are relevant for the platform and satellite, while two nodes only are relevant for each of the two customers. This mechanism allows to reduce the cardinality of $\mathcal{V}$ by considering the spatial and time positions (i,t) of each node $i \in \mathcal{V}$ at time periods $t \in \mathcal{T}_i(\Delta)$ only. Let $\mathcal{V}_i$ stand for the set of time-space nodes $\{(i, t) : i \in \mathcal{V}^{\text{ph}},\ t \in \mathcal{T}_i(\Delta)\}$, and [$a_i$,$b_i$] be the

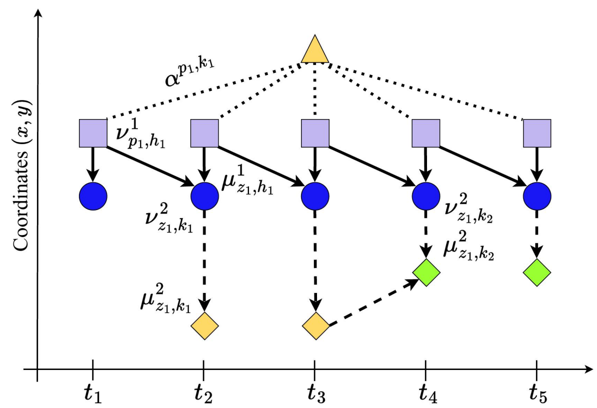


Figure 4 – Hybrid representation of time.

time interval during which node $i \in \mathcal{V}$ is relevant in $\mathcal{G}$, i.e., $a_i = \min\{t : t \in \mathcal{T}_i(\Delta)\}$ and $b_i = \max\{t : t \in \mathcal{T}_i(\Delta)\}$.

Similar to the physical network, the set of arcs $\mathcal{A} = \mathcal{A}^1 \cup \mathcal{A}^2$ stands for connections between time-space nodes representing the various system components. At the first echelon, $\mathcal{A}^1$, one finds the connections between suppliers and platforms, platforms and satellites, pairs of satellites, as well as from the first-echelon garage to platforms and from satellites to the former. The second echelon arcs in $\mathcal{A}^2$ stand for the connections from satellites to customers, between pairs of the latter, as well as from the second-echelon garage to satellites and from customer to the former. An arc $((i,t),(j,t')) \in \mathcal{A}$ is then defined for arc $(i,j) \in \mathcal{A}^{\text{ph}}$ with $t \in \mathcal{T}_i(\Delta)$ and $t' = t + \tau_{ij} \in \mathcal{T}_j(\Delta)$. To simplify the notation, the travel time of inbound arcs to customer nodes are considered to embed the service time at customers. Recall that commodities must be assigned to a single platform and satellite and the flow should not be split. Consequently, let $\mathcal{P}_0(k) = \{(p,t),(j,t') : p \in \mathcal{P}, j \in \mathcal{Z}, t \geq \alpha^{pk}\}$ be the set of platform-to-satellite arcs commodity k can be assigned to if passing through platform p to travel to a reachable satellite.

4.3. The 2E-MALRPS formulation

This section presents a mixed-integer formulation for the 2E-MALRPS that combines continuous and discrete time modelling strategies to represent time. The formulation consists of a *standalone* time-space formulation, determined by constraints ((4.2)-(4.22)), and a series of continuous-time constraints ((4.23)-(4.29)). In this formulation, waiting times at nodes $v_i \in \mathcal{V}$ are implicitly represented by the time difference between the departure of a vehicle and its prior arrival at the node, and enforced by the set of continuous-time constraints. Although these continuous-time constraints may appear redundant, they serve to preserve the precision of time-related decisions, compensating for any potential loss of accuracy caused by coarse discretization granularity.

Define the following decision variables on $\mathcal{G}$:
$y_i \in \{0{,}1\}$: 1, if facility i is open, 0 otherwise (location);
$x_{ij} \in \{0{,}1\}$: 1 if arc $(i{,}j)$ is selected, 0 otherwise (vehicle routing);
$f^k_{ijh} \in \{0{,}1\}$: 1, if commodity k goes through arc $(i{,}j)$ with vehicle h, 0 otherwise;
$\gamma^k_{ij} \in \{0{,}1\}$: 1, if commodity k goes through arc $(i{,}j)$, 0 otherwise;
$\mu^1_{ih} \geq 0$: Arrival time of first-echelon vehicle h at vertex i;
$\mu^2_{ik} \geq 0$: Arrival time of commodity k at vertex i;
$\nu^1_{ih} \geq 0$: Departure time of first-echelon vehicle h from vertex i;
$\nu^2_{ik} \geq 0$: Departure time of commodity k from vertex i .
The continuous-time variables needed to track vehicle schedules over the time-space network in order to deliver the OD demand k_1 are illustrated in Figure 4, which also shows that only customer-to-customer connections are not handled by continuous time variables. Let M be a large integer number. The hybrid 2E-MALRPS formulation then becomes:

$$\min \sum_{i \in \mathcal{P}^{\mathrm{ph}}} F_i y_i + \sum_{i \in \mathcal{Z}^{\mathrm{ph}}} F_i y_i + \sum_{(i,j) \in \mathcal{A}^1} \zeta_{ij} x_{ij} + \sum_{(i,j) \in \mathcal{A}^2} \zeta_{ij} x_{ij} \tag{4.1}$$

subject to

$$\sum_{j \in \mathcal{V}_c} \sum_{i \in ((\mathcal{C} \setminus \mathcal{V}_c) \cup \mathcal{Z})} x_{ij} = 1 \quad \forall c \in \mathcal{C}^{\mathrm{ph}} \tag{4.2}$$

$$\sum_{j \in \mathcal{V}_c} \sum_{i \in ((\mathcal{C} \setminus \mathcal{V}_c) \cup \mathcal{Z})} x_{ij} = \sum_{j \in \mathcal{V}_c} \sum_{i \in ((\mathcal{C} \setminus \mathcal{V}_c) \cup \mathcal{E}^2)} x_{ji} \quad \forall c \in \mathcal{C}^{\mathrm{ph}} \tag{4.3}$$

$$\sum_{i \in ((\mathcal{C} \setminus \mathcal{V}_c) \cup \mathcal{Z})} x_{ij} \geq \sum_{l \in \mathcal{V}_c} \sum_{i \in ((\mathcal{C} \setminus \mathcal{V}_c) \cup \mathcal{E}^2)} x_{li} \quad \forall j \in \mathcal{V}_c, c \in \mathcal{C}^{\mathrm{ph}} \tag{4.4}$$

$$\sum_{i \in \mathcal{E}^2} x_{ij} \leq \sum_{i \in \mathcal{C}} x_{ji} \quad \forall j \in \mathcal{Z} \tag{4.5}$$

$$\sum_{j \in \mathcal{V}_z} \sum_{i \in \mathcal{E}^2} x_{ij} = \sum_{j \in \mathcal{V}_z} \sum_{i \in \mathcal{C}} x_{ji} \quad \forall z \in \mathcal{Z}^{\mathrm{ph}} \tag{4.6}$$

$$\sum_{i \in ((\mathcal{Z} \setminus \mathcal{V}_z) \cup \mathcal{P})} x_{ij} \leq y_z \quad \forall j \in \mathcal{V}_z, z \in \mathcal{Z}^{\mathrm{ph}} \tag{4.7}$$

$$\sum_{i \in ((\mathcal{Z} \setminus \mathcal{V}_z) \cup \mathcal{P})} x_{ij} \leq \sum_{i \in ((\mathcal{Z} \setminus \mathcal{V}_z) \cup \mathcal{E}^1)} x_{ji} \quad \forall j \in \mathcal{V}_z, z \in \mathcal{Z}^{\mathrm{ph}} \tag{4.8}$$

$$\sum_{i \in \mathcal{E}^1} x_{ij} = \sum_{i \in \mathcal{Z}} x_{ji} \quad \forall j \in \mathcal{P} \tag{4.9}$$

$$\sum_{i \in \mathcal{V}_p} \sum_{j \in \mathcal{Z}} x_{ij} \leq |\mathcal{H}^1| \, y_p \quad \forall p \in \mathcal{P}^{\mathrm{ph}} \tag{4.10}$$

$$\sum_{h \in \mathcal{H}^1} \sum_{i \in P_0(k)} f^k_{ijh} = 1 \quad \forall k \in \mathcal{K}, j \in \mathcal{Z} \tag{4.11}$$

$$\sum_{h \in \mathcal{H}^1} \sum_{i \in \mathcal{V}_p} \sum_{j \in \mathcal{Z}} f^k_{ijh} \leq y_p \quad \forall k \in \mathcal{K}, p \in \mathcal{P}^{\mathrm{ph}} \tag{4.12}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})}\sum_{j\in\mathcal{V}_c}\gamma_{ij}^k - \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)}\sum_{j\in\mathcal{V}_c}\gamma_{ji}^k = 0$$

$$\forall c\in\mathcal{C}^{\mathrm{ph}}, k\in\mathcal{K}, D(k)\neq c \tag{4.13}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})}\sum_{j\in\mathcal{V}_c}\gamma_{ij}^k - \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)}\sum_{j\in\mathcal{V}_c}\gamma_{ji}^k \leq 1 - \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)}\sum_{j\in\mathcal{V}_c} x_{ji}$$

$$\forall c\in\mathcal{C}^{\mathrm{ph}}, k\in\mathcal{K}, D(k)\neq c \tag{4.14}$$

$$\sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})}\sum_{j\in\mathcal{V}_c}\gamma_{ij}^k - \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{E}^2)}\sum_{j\in\mathcal{V}_c}\gamma_{ji}^k = \sum_{i\in((\mathcal{C}\setminus\mathcal{V}_c)\cup\mathcal{Z})}\sum_{j\in\mathcal{V}_c} x_{ij}$$

$$\forall c\in\mathcal{C}^{\mathrm{ph}}, k\in\mathcal{K}, D(k)= c \tag{4.15}$$

$$\sum_{h\in\mathcal{H}^1}\sum_{j\in\mathcal{V}_z}\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{P})} f_{ijh}^k = \sum_{h\in\mathcal{H}^1}\sum_{j\in\mathcal{V}_z}\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{E}^1)} f_{jih}^k + \sum_{j\in\mathcal{V}_z}\sum_{l\in\mathcal{C}}\gamma_{jl}^k$$

$$\forall z\in\mathcal{Z}^{\mathrm{ph}}, k\in\mathcal{K} \tag{4.16}$$

$$\sum_{h\in\mathcal{H}^1}\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{P})} f_{ijh}^k \leq \sum_{h\in\mathcal{H}^1}\sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{E}^1)} f_{jih}^k + \sum_{l\in\mathcal{C}}\gamma_{jl}^k$$

$$\forall j\in\mathcal{V}_z, z\in\mathcal{Z}^{\mathrm{ph}}, k\in\mathcal{K} \tag{4.17}$$

$$\sum_{k\in\mathcal{K}} vol(k) \sum_{i\in((\mathcal{Z}\setminus\mathcal{V}_z)\cup\mathcal{P})} f_{ijh}^k \leq cap_1 \quad \forall j\in\mathcal{V}_z, z\in\mathcal{Z}^{\mathrm{ph}}, h\in\mathcal{H}^1 \tag{4.18}$$

$$\sum_{k\in\mathcal{K}} vol(k)\sum_{i\in\mathcal{Z}}\gamma_{ij}^k \leq cap_2 \quad \forall j\in\mathcal{C}, \tag{4.19}$$

$$\sum_{k\in\mathcal{K}} vol(k)\sum_{h\in\mathcal{H}^1}\sum_{i\in\mathcal{V}_p}\sum_{j\in\mathcal{Z}} f_{ijh}^k \leq \Theta_p\, y_p \quad \forall p\in\mathcal{P}^{\mathrm{ph}} \tag{4.20}$$

$$\sum_{h\in\mathcal{H}^1} f_{ijh}^k \leq x_{ij} \quad \forall k\in\mathcal{K}, (i,j)\in\mathcal{A}^1 \tag{4.21}$$

$$\gamma_{ij}^k \leq x_{ij} \quad \forall k\in\mathcal{K}, (i,j)\in\mathcal{A}^2 \tag{4.22}$$

$$\nu_{ih}^1 \geq \alpha^{ik}\sum_{j\in\mathcal{Z}} f_{ijh}^k \quad \forall h\in\mathcal{H}^1, k\in\mathcal{K}, i\in\mathcal{P} \tag{4.23}$$

$$\nu_{jk}^2 \geq \mu_{jh}^1 - (2-\gamma_{jD(k)}^k - \sum_{i\in(\mathcal{Z}\cup\mathcal{P}), i\neq j} f_{ijh}^k)M \quad \forall k\in\mathcal{K}, h\in\mathcal{H}^1, j\in\mathcal{Z} \tag{4.24}$$

$$\nu_{jh}^1 \geq \mu_{jk}^2 - (2-\gamma_{jD(k)}^k - \sum_{i\in(\mathcal{Z}\cup\mathcal{P}), i\neq j} f_{ijh}^k)M \quad \forall k\in\mathcal{K}, h\in\mathcal{H}^1, j\in\mathcal{Z} \tag{4.25}$$

$$\mu_{ih}^1 + \tau_{ij} - \mu_{jh}^1 \leq (1-f_{ijh}^k)M \quad \forall k\in\mathcal{K}, h\in\mathcal{H}^1, (i,j)\in\mathcal{A}^1 \tag{4.26}$$

$$\nu_{ih}^1 + \tau_{ij} - \mu_{jh}^1 \leq (1-f_{ijh}^k)M \quad \forall h\in\mathcal{H}^1, k\in\mathcal{K}, (i,j)\in\mathcal{A}^1 \tag{4.27}$$

$$\mu_{ik}^2 + \tau_{ij} - \mu_{jk}^2 \leq (1-\gamma_{ij}^k)M \quad \forall k\in\mathcal{K}, (i,j)\in\mathcal{A}^2 \tag{4.28}$$

$$\nu_{ik}^2 + \tau_{ij} - \mu_{jk}^2 \leq (1-f_{ijh}^k)M \quad \forall k\in\mathcal{K}, h\in\mathcal{H}^2, (i,j)\in\mathcal{A}^2 \tag{4.29}$$

$$\nu_{ih}^1 - \mu_{ih}^1 \leq W_{max}^2 \quad \forall h\in\mathcal{H}^1, i\in\mathcal{Z} \tag{4.30}$$

$$\nu_{ik}^2 - \mu_{ik}^2 \leq W_{max}^2 \quad \forall k\in\mathcal{K}, i\in\mathcal{Z} \tag{4.31}$$

$$a_{D(k)} \leq \mu_{D(k)k}^2 \leq b_{D(k)} \quad \forall k\in\mathcal{K} \tag{4.32}$$

The objective function (4.1) minimizes the total cost of the system, which includes the fixed cost of selecting (opening) facilities on both echelons and the variable travel costs of the vehicles for demand transportation. Constraints (4.2) ensure that each customer is visited by a second-echelon vehicle exactly once. Constraints (4.3) represent vehicle-flow conservation at customer locations. Constraints (4.4) ensure that departure times of second-echelon vehicles from customers occur after the service time at the current customer. Constraints (4.5) enforce the requirement that for each outbound connection of a second-echelon vehicle from a satellite to a customer, there must be an inbound connection from a second-echelon garage to the satellite, including the waiting times at the satellite. Constraints (4.6) represent conservation constraints on routing variables for each satellite facility in the second echelon. Constraints (4.7) enable multiple visits to satellites in different time periods by ensuring that each open satellite is visited at most once for each time period, while constraints (4.8) ensure that for each outbound connection of a first-echelon vehicle from a satellite to another satellite or garage, there is an inbound connection from a platform or a different satellite. Constraints (4.9) and (4.10) enforce the routing conservation of first-echelon routing variables and restrict the maximum number of outbound connections from platform facilities in terms of the fleet size, respectively.

Constraints (4.11) and (4.12) ensure that each demand is not split and departs from the assigned open platform after its availability time, respectively. Constraints (4.13) and (4.14) impose flow conservation for commodities in nodes different to their destination customer. Constraints (4.15) guarantee that each commodity flow reaches its destination customer. Constraints (4.16) and (4.17) enforce flow conservation and spatial and temporal synchronization at time-space satellites, considering waiting times. Constraints (4.18) and (4.19) ensure that the total flow assigned to each route does not exceed the vehicle capacity for the first and second echelons, respectively. Similarly, constraints (4.20) impose that the assigned routes to each platform do not exceed the facility capacity. Constraints (4.21) and (4.22) establish the relationship between flow and routing variables.

Constraints (4.23) guarantee the feasibility of the schedule with respect to demand availability. Constraints (4.24) and (4.25) relate the arrival times at satellites of first- and second-echelon vehicles to ensure fleet synchronization at satellite facilities. Constraints (4.26) and (4.27) handle the arrival and departure times of first-echelon vehicles, while constraints (4.28) and (4.29) handle the arrival and departure times of second-echelon vehicles. Constraints (4.30) and (4.31) ensure that waiting times at satellite facilities respect the maximum permitted waiting time at each echelon. Constraints (4.32) ensure that second-echelon vehicles arrive within the customer time windows.

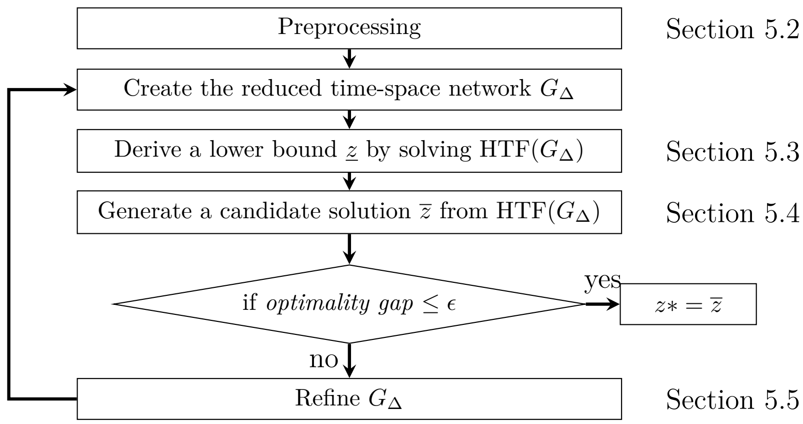


Figure 5 – Dynamic discretization discovery for the 2E-MALRPS

5. Dynamic Discretization Discovery for the 2E-MALRPS

The temporal dimension of the time-space 2E-MALRPS model provides a detailed and precise representation of the problem setting (Ford and Fulkerson, 1962). On the other hand, the pseudo-polynomial size of the integer formulation also makes head-on solution methods less scalable as the time granularity gets finer. We therefore propose a *Dynamic Discretization Discovery (DDD)* solution method for the time-space model to address this methodological challenge, building on the method introduced by Boland et al. (2017) for service network design problems. Notice that, demand in the 2E-MALRPS generates compulsory time moments, which must be explicitly included in the time-space network. While this appears to somehow facilitate the discretization of time, it greatly increases the difficulty of the problem and precludes a straightforward application of the method proposed in the service network design literature (Crainic and Hewitt, 2021).

The DDD algorithm we propose is illustrated in Figure 5. It iteratively refines a reduced time-space network, solving at each iteration the integer program defined by the hybrid formulation on that sparse network to extract lower and upper bounds for the 2E-MALRPS. The process is repeated until the problem is solved to optimality, or up to a specified tolerance ϵ. This tolerance is defined as the bound on the optimality gap percentage, which is utilized to determine when to stop the algorithm.

This section provides first the notation and foundations of the proposed DDD (Section 5.1), followed by the descriptions of the main algorithmic components as shown in Figure 5. To simplify the notation, we refer to the solution of the hybrid formulation as HTF($\mathcal{G}_\Delta$) for a reduced time-space network $\mathcal{G}_\Delta$.

5.1. Preliminary notation and problem analysis

Without loss of generality, let $\mathcal{G}_{\bar{\Delta}}$ be the *complete 2E-MALRPS time-space network*. Here, $\mathcal{T}(\bar{\Delta})$ denotes the full discretization of the length of the planning time span (i.e., schedule length), with $\mathcal{T}(\bar{\Delta}) = \bigcup_{i \in \mathcal{V}^{\text{ph}}} \mathcal{T}_i(\bar{\Delta}_i)$ standing for the set of the temporal time periods required to capture the relevant time moments $\mathcal{T}_i(\bar{\Delta}_i)$ for each $i \in \mathcal{V}^{\text{ph}}$. Similarly, let $\bar{\Delta}$ and $\bar{\Delta}_i$ denote the maximum number of time periods needed to encompass all relevant time moments across the system and for each $i \in \mathcal{V}^{\text{ph}}$, respectively. The proposed DDD solution method relaxes $\mathcal{T}(\bar{\Delta})$ through a granularity parameter $\Delta, 1 < \Delta < \bar{\Delta}$, and decreasing the number of relevant time periods for each vertex.

Let $\mathcal{G}_\Delta = (\mathcal{V}_\Delta, \mathcal{A}_\Delta)$, be the *reduced time-space network* defined by a reduced set of relevant time instants $\mathcal{T}_i(\Delta) \cap [a_i,b_i]$, for each node $i \in \mathcal{V}^{\text{ph}}$, where $[a_i,b_i]$ stands for the original interval of relevance of the node. Recall (Section 4.2) that, $[a_i,b_i]$ is the time window of each customer $i \in \mathcal{C}^{\text{ph}}$, the complete schedule length $[0, \Psi]$ for satellites and garages $i \in \mathcal{Z}^{\text{ph}} \cup \mathcal{E}^{\text{ph}}$, and the interval between the earliest potential commodity arrival $a_i = min_{k \in K}\{\alpha^{ik}\}$ and the schedule length $b_i = \Psi$ for platforms $i \in \mathcal{P}^{\text{ph}}$.

Inbound arcs to customers no longer embed waiting times in a reduced time-space network $\mathcal{G}_\Delta$. We rather consider a set of time-space nodes before each time window to represent early arrivals and waiting times at customers. The reduced time-space network is thus an aggregated network derived from $\mathcal{G}$, where $|\mathcal{G}_\Delta| \leq |\mathcal{G}|$. Consequently, the length τ_{ij} of arc $(i,j) \in \mathcal{A}^{\text{ph}}$ is also "aggregated" in terms of Δ, ensuring that there is a time-space arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$ for each arc $(i,j) \in \mathcal{A}^{\text{ph}}$, with $t' \leq t + \tau_{ij}$. The aggregation does not change the arc travel costs, but it does impact travel times. Arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$ is then considered to be *too short* when $t' < t + \tau_{ij}$, as it might model negative $(t' < t)$ or zero $(t' = t)$ travel times. These considerations are summarized in the following four properties:

- **Property 1.** A set of time-space nodes $(i,t) \in \mathcal{V}_\Delta$ exists for each node $i \in \mathcal{V}^{\text{ph}}$ and set of relevant time instants $\mathcal{T}_i(\Delta) \cap [a_i,b_i]$.
- **Property 2.** For each node $(i,t) \in \mathcal{V}_\Delta$ of a reduced time-space network and for each arc $(i,j) \in \mathcal{A}^{\text{ph}}$, there is a time-space arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$ with $t' \leq t + \tau_{ij}$. If arc $((i,t),(j,t')) \in \mathcal{A}_\Delta$, there is no time-space node $(j,t'') \in \mathcal{V}_\Delta$ with $t' < t'' \leq t + \tau_{ij}$.
- **Property 3.** A *waiting-time* arc exists out of each time representation of a satellite $(i,t) \in \mathcal{Z}_\Delta$ towards a later node $(i,t') \in \mathcal{Z}_\Delta$ with $t' \leq t + W^2_{max}$.
- **Property 4.** For each customer $i \in \mathcal{C}^{\text{ph}}$, at least one time-space node (i,t) with $t < a_i$ exists in $\mathcal{C}_\Delta$ to handle early inbound connections exclusively.

Turning to the commodity flows through time and space, define *itinerary* r for commodity $k \in \mathcal{K}$ from its origin $O(k)$ to its destination $D(k)$ through the network $\mathcal{G}$ as a path $r = (v_i, t_i)_{i=1}^{l}$ connecting the node of the initial vehicle arrival and the departure of the commodity from a platform (i.e., $v_1 \in \mathcal{P}^{\text{ph}}$), to the node of the arrival and departure time at its destination (i.e., $v_l = D(k)$), including the times of arrival and departure at each intermediary node in $\mathcal{G}$. Let $R^k_{\mathcal{G}}$ be the set of feasible itineraries for commodity $k \in \mathcal{K}$, and $R_{\mathcal{G}} = \cup_k R^k_{\mathcal{G}}$.

The transfer between the first and second echelons of itinerary $r \in R^k$ occurs at a satellite $j \in \mathcal{Z}^{\text{ph}}$, where $v_j \in \mathcal{Z}, 1 < j < l-1$. The transfer timing is then framed by the four nodes $v_{j+w} \in \mathcal{Z}^{\text{ph}}$ with $w = \{0,1,2,3\}$, i.e., t_j, t_{j+1} and t_{j+2}, t_{j+3} represent the arrival time to and departure time from the transfer satellite at each echelon, respectively. Hence, an itinerary $r = (v_i, t_i)_{i=1}^l$ in $\mathcal{G}$ is made up of the sequence of arcs $((v_i, t_i), (v_{i+1}, t_{i+1})) \in \mathcal{A}$ for every $i = 1, \ldots, l$, except for $i = j+1$, as the arc $((v_{j+1}, t_{j+1}), (v_{j+2}, t_{j+2}))$ would connect the departure and arrival times of first and second echelon routes, respectively.

The proposed DDD solution method relies on addressing the formulation (4.1) - (4.32) defined on a sequence of reduced time-space networks $\mathcal{G}_\Delta$ for various granularity levels $1 < \Delta < \bar{\Delta}$. The following lemmas ensure that 1) the 2E-MALRPS formulated on a reduced time-space network is a relaxation for the original problem and 2) the solutions obtained for the hybrid formulation on a reduced time-space network represent lower bounds for the 2E-MALRPS, regardless of the granularity of the discretization. The proofs of the lemmas may be found in the supplementary material Appendix A.1.

Lemma 1. Let $\mathcal{G}_\Delta$ be a reduced time-space network that satisfies properties 1, 2, 3, and 4. Then, for each commodity $k \in \mathcal{K}$ and itinerary $r = (v_i, t_i)_{i=1}^l \in R^k_{\mathcal{G}}$, there exists an itinerary $r' = (v_i, t'_i)_{i=1}^l \in R^k_{\mathcal{G}_\Delta}$ such that $t'_i \leq t_i$ for every $i = 1 \ldots l$.

Lemma 2. If a reduced time-space network $\mathcal{G}_\Delta$ satisfies properties 1, 2, 3, and 4, then the optimal solution of the 2E-MALRPS formulated on the reduced time-space network $\mathcal{G}_\Delta$ is a lower bound for the solution of the 2E-MALRPS on the complete time-space network $\mathcal{G}$.

Lemma 3. The proposed DDD algorithm ends with an optimal solution for the 2E-MALRPS.

Note that, the set of properties address fundamental aspects of the time-space representation of the problem. Therefore, the lemmas and proofs hold not only for the hybrid formulation (constraints (4.2) -(4.32)), but also for a "classic" time-space formulation that can be defined by isolating constraints (4.2) -(4.21).

5.2. Initial reduced network

The first step of the DDD algorithm tries to prune arcs and tighten the possible relevance-period sets, and then creates an initial reduced network $\mathcal{G}_\Delta$ satisfying Properties 1-4.

The time dependency of demand may imply that particular couples of locations and time instants might not be reachable when going from the origin to the destination of a demand. This may be used to tighten both the availability time at platforms and the customer time window of the particular demand and, thus, the set of relevant periods. The preprocessing analysis is performed on the physical network ($\mathcal{G}^{\mathrm{ph}}$), through a breadth-first search strategy. The procedure iteratively explores the set of commodities, enumerating the possible platform-satellite combinations that could link the corresponding origin and destination at all time periods when the commodity could be available at the platform. The resulting set of feasible partial paths (including garage connections at each echelon) for each commodity defines the possible unreachable time periods for platforms and customers, which can then be excluded from the network.

The initial reduced time-space network $\mathcal{G}_{\Delta}$ is then generated by duplicating each node and arc in $\mathcal{G}^{\mathrm{ph}}$ at each relevant time period, verifying that Properties 1-4 are satisfied.

5.3. A 2E-MALRPS lower bound on $\mathcal{G}_{\Delta}$

The definition of good quality lower bounds is fundamental for the DDD, as it guides the search through the solution space. The specifics of 2E-MALRPS makes this process challenging.

As previously indicated, lower bounds are obtained by solving the integer program defined by the hybrid formulation HTF($\mathcal{G}_{\Delta}$) (Section 4.3) on the reduced time-space network $\mathcal{G}_{\Delta}$. This solution may not be feasible for the original 2E-MALRPS, however. Indeed, even though it is feasible in terms of capacity constraints and location/allocation decisions, vehicle schedules might be infeasible when evaluated with the original travel times, due to the "aggregation" of the arc travel times. Compounding the challenge, the routing aspect of the problem involves a high number of interconnection between nodes of the time-space network, as each arc in the physical network may be represented by multiple time-space arcs, the multiplicity depending on the granularity of the time discretization. This tends to increase the size of the reduced time-space networks one has to address. Moreover, together with the presence of short arcs, it also results in numerous candidate solutions on the reduced time-space network to share the same solution structure, with similar objective function values and location/routing decisions while differing in vehicle schedules. Hence, although the reduced network is refined multiple times, one might still derive the same solution structure and costs with different sets of time-space arcs. This solution degeneracy increases the complexity of the optimization problem addressed at each iteration. The issue is particularly serious for the second tier of the system, first-echelon route degeneracy being avoided due to the presence of additional variables and constraints to keep track of time.

We therefore propose a procedure to handle degenerate solutions (second-echelon routes) within the reduced time-space networks, in order to mitigate the computational impact arising from the growth of the continuous refinement of the network (Algorithm 1). The fundamental idea when observing a degenerate solution is to dive towards a level of granularity where the degeneracy is no longer present, without solving the HTF($\mathcal{G}_\Delta$) for each intermediate value of Δ.

Algorithm 1: Degeneracy($\mathcal{G}_\Delta, Sol$)

input: $\mathcal{G}_\Delta, Sol = (z, x_{\mathcal{G}_\Delta}, f_{\mathcal{G}_\Delta}, y_{\mathcal{G}_\Delta})$

1 **if** $Sol \neq \varnothing$ **then**
2     $Sol_{local} \leftarrow HTF(\mathcal{G}_\Delta, Sol)$;
3   **if** $z(Sol_{local}) \neq z(Sol)$ **then**
4         $Sol \leftarrow HTF(\mathcal{G}_\Delta)$;
5   **else**
6         $Sol \leftarrow Sol_{local}$;
7   **end**
8 **else**
9     $Sol \leftarrow HTF(\mathcal{G}_\Delta)$;
10 **end**

Let us define a *solution structure* as the location, allocation, and routing decisions (but not the schedules) of the solution. A solution structure is then considered degenerate when two consecutive iterations of the DDD may identify it, even though the reduced network has been refined at least once. The procedure searches for degeneracy at each iteration of the DDD. It first aims to identify whether the structure of the current solution is degenerate by comparing it to that of the previous iteration (granularity level). If the answer is yes, the degeneracy mitigation procedure is activated by fixing the location and allocation decisions (line 2 and line 6) for the following iterations (the DDD continues normally otherwise). Thus, solving the HTF($\mathcal{G}_\Delta$) at the next granularity refinement level with fixed nodes and arcs focuses on routes and schedules only, which is significantly less computationally heavy compared to addressing the complete formulation. The solution structures are compared again and either degeneracy is still present, i.e., the same routes are found with the refined granularity, or is no longer observed. In the latter case, the fixing of location and allocations decisions is removed and the DDD continues normally. The diving with fixed location and allocation decisions continues otherwise until either degeneracy is no longer observed, in which case the current solution structure is feasible for the 2E-MALRPS, or the HTF($\mathcal{G}_\Delta$) returns an "infeasible solution" for the current level of granularity. The latter message indicates that the current structure cannot generate a feasible solution for the time-space network at the current or more refined level of granularity. Hence, the procedure has identified the granularity level where, with unfrozen location and allocation decisions, a new solution structure is found, without solving the HTF($\mathcal{G}_\Delta$) on the full networks for all the intermediate granularity levels.

5.4. A 2E-MALRPS upper bound

To determine a feasible upper bound for the 2E-MALRPS, the optimal solution obtained on the reduced time-space network $\mathcal{G}_\Delta$ must remain feasible when evaluated with the original travel times for all arcs. As indicated above, the variables tracking the vehicle schedules ensure the feasibility of the temporal decisions of the first-echelon routes, despite the presence of short arcs. Hence, a *feasibility-analysis* procedure is employed to evaluate the routes of the second echelon with the original travel times and verify the feasibility of customer time windows. This procedure utilizes the vehicle schedules defined by the optimal solution obtained from the reduced time-space network $\mathcal{G}_\Delta$ as the starting point for second-echelon routes. The procedure then proceeds by traversing the sequence of nodes for each second-echelon route while evaluating each connection using the original travel times. This iterative evaluation serves to identify whether all routes are feasible for the 2E-MALRPS or if any of the customer time windows are unreachable and, thus, infeasible for the 2E-MALRPS. It is important to note that the evaluation of each route acts as a means to map the routes generated by the reduced time-space network into the complete time-space formulation, thereby producing a feasible solution for the 2E-MALRPS when the schedules of both fleets are combined. When the feasibility of the solution obtained on $\mathcal{G}_\Delta$ is verified, then 1) the solution is feasible for the 2E-MALRPS and its value is an upper bound on the optimal-solution value of the original problem, and 2) if the solution value of HTF($\mathcal{G}_\Delta$) is better (lower) than the current best upper bound (if any), then the value of the best upper bound is updated and the procedure terminates.

When the HTF($\mathcal{G}_\Delta$) solution is found to be infeasible for the 2E-MALRPS, the algorithm stops and forwards the solution structure to the refinement step of the DDD. Notice that, in this case, the DDD procedure will iteratively refine the reduced network until the solution is proven infeasible or a DDD stopping criteria is reached. Notice also that, the upper-bounding procedure we propose provides the means to the DDD algorithm to identify potentially good-quality upper bounds even when the reduced time-space network is not well-refined.

5.5. Refinement

The final step of the proposed DDD solution method refines the reduced time-space network. Refinement is performed whenever the HTF($\mathcal{G}_\Delta$) solution, obtained on the reduced time-space network $\mathcal{G}_\Delta$, is found to have short arcs which violate one or more 2E-MALRPS temporal constraints when evaluated with the original travel times. Recall, Lemma 2 and Section 5.4, that a HTF($\mathcal{G}_\Delta$) solution is a lower bound for the 2E-MALRPS and, thus, insights on how to refine the reduced time-space network $\mathcal{G}_\Delta$ may be derived from the short arcs found in that solution. Moreover, given that the hybrid time representation provides the means to avoid refining the arcs involving platforms and satellite facilities, one may focus

Algorithm 2: Refinement($\mathcal{G}_\Delta, Sol$)

input: $\mathcal{G}_\Delta, Sol = (z^*,(\mathcal{V}^*_\Delta, \mathcal{A}^*_\Delta))$

1 **for** $((i,t),(j,t')) \in \mathcal{A}^*_\Delta$ **do**

2   **if** $t' \leq t + \tau_{ij}$ ***and*** $i,j \in \mathcal{C}^{ph} \cup \mathcal{E}^{ph}$ **then**

3   **if** $isFeasible(\mathcal{G}_\Delta, (i, t' - \tau_{ij}))$ **then**

4   AddNode($\mathcal{G}_\Delta, (i, t' - \tau_{ij})$);

5   Restore($\mathcal{G}_\Delta, (i, t' - \tau_{ij})$);

6   **end**

7   **if** $isFeasible(\mathcal{G}_\Delta, (j, t + \tau_{ij}))$ **then**

8   AddNode($\mathcal{G}_\Delta, (j, t + \tau_{ij})$);

9   Restore($\mathcal{G}_\Delta, (j, t + \tau_{ij})$);

10   **end**

11   **end**

12 **end**

the refinement on arcs connecting customers to other customers or garages. Refining the reduced network in terms of these short arcs, extracted from the lower bound obtained at each iteration, strengthens the reduced time-space network and improves the precision of the 2E-MALRPS lower bounds in future iterations.

The proposed refinement procedure extends such short arcs in $\mathcal{G}_\Delta$, while ensuring that Properties 1-4 are valid at the nodes involved in the extension (Algorithm 2). Recall that, short arcs $((i,t),(j,t'))$ within the integer solution of HTF($\mathcal{G}_\Delta$) display short travel times $t' < t + \tau_{ij}$ relative to the pair of nodes $((i,t),(j,t + \tau_{ij}))$ in the original problem. The refinement procedure aims to extend each short arc from both of its extreme points (line 3 and line 7), thereby adding two new time-space customer nodes and arcs to the reduced network (Algorithm 3). (Notice that, due to the presence of solution degeneracy, refining a short arc in terms of one of its extreme points does not exclude its counterpart to potentially appear in the following iterations of the DDD.) Subsequently, Algorithm 3 is employed to incorporate each time-space node and its corresponding valid time-space arcs into the reduced network. The original travel time of each arc is utilized in this process, and the waiting times are updated accordingly (Algorithm 4), along with the arc connections of the reduced network (Algorithm 5) for each newly added node. To uphold Property 4, Algorithm 4 ensures that any newly introduced time-space customer node is added within the customer's time window and can connect with other time-space customers within and prior to (but never after) their respective time windows. Finally, Algorithm 5 guarantees the preservation of Property 2 once a new time-space node and its corresponding time-space arcs are added to the reduced time-space network.

Algorithm 3: AddNode((i, t'))

input: $\mathcal{G}_\Delta$, (i,t') *with* $t_b < t' < t_{b+1}$
if $(i,t') \notin \mathcal{G}_\Delta$ **then**
  $\mathcal{G}_\Delta \leftarrow \mathcal{G}_\Delta \cup (i,t')$;
  for $((i,t_b),(j,t)) \in \mathcal{A}_\Delta$ **do**
    $AddArc(((i,t'),(j,t)))$;
  end
  $UpdateWaitingArcs((i,t'))$;
end

Algorithm 4: UpdateWaitingArcs((i, t'))

input: $\mathcal{G}_\Delta$, (i,t') *with* $t' < a_i$
if $i \in \mathcal{C}^{ph}$ **then**
  for $(i,t) \in \mathcal{T}_i(\Delta)$ **do**
    if $t \geq a_i$ **then**
      $AddArc(((i,t'),(i,t)))$;
    end
  end
end

Algorithm 5: Restore($\mathcal{G}_\Delta, (i, t')$)

input: $\mathcal{G}_\Delta$, (i,t') *with* $t_b < t' < t_{b+1}$
if $(i,t') \notin \mathcal{G}_\Delta$ **then**
  for $((i,t_b),(j,t)) \in \mathcal{A}_\Delta$ **do**
    $t" \leftarrow \arg\max\{d \in \mathcal{T}_j(\Delta) \mid d \leq t' + \tau_{ij}\}$;
    if $t" \neq t$ **then**
      $Delete(((i,t'),(j,t)))$;
      $AddArc(((i,t'),(j,t")))$;
    end
  end
  for $((j,t),(i,t_b)) \in \mathcal{A}_\Delta$ *with* $t + \tau_{ij} \geq t'$ **do**
    $Delete(((j,t),(i,t_b)))$;
    $AddArc(((j,t),(i,t')))$;
  end
end

6. Computational results

This section presents and discusses the results of experiments conducted to asses the performance of the proposed mathematical formulation and solution method for the 2E-MALRPS. We first introduce the instances used in the computational study in Section 6.1.

We then present the performance of the proposed hybrid formulation using CPLEX in Section 6.2, followed by the performance analysis of our DDD algorithm in Section 6.3. The results of a series of experiments illustrating the sensitivity analysis of the DDD and the effects of problem instance characteristics are then analysed in Section 6.4.

The experiments were conducted on a single machine with Intel(R) Core(TM) i7-7800X with 128 GB of RAM running Linux. The mathematical formulation and the proposed solution method are implemented in C++ using IBM ILOG CPLEX concert technology 20.1. The formulation is solved to optimality or to an optimality gap tolerance of less than or equal to 1% as the stopping criterion for instances with 50 OD demands.

The tables of this section display average results for the instance sets (detailed results in the supplementary material Appendix A.2) used in the associated experiments, indicating the numbers of potential platforms ($|\mathcal{P}^{\text{ph}}|$), satellites ($|\mathcal{Z}^{\text{ph}}|$), OD demands (OD), and instances in the set (NI). For the formulations considered in each experiment, the tables provide at least the number of instances for which feasible (FUB) and optimal (OUB) solutions were found within the given time limit, the run time in seconds (CPUsec), and the optimality gap (OG). The lower bound values provided by the proposed solution method correspond to, either the optimal solution of the hybrid model on the reduced network or, when the optimally gap tolerance is not reached within the given time limit, the best linear-relaxation value obtained throughout the optimization process.

6.1. Instances

No instances are available in the literature involving the integrated treatment of the attributes considered in the 2E-MALRPS, time-dependent origin-destination demands, fleet synchronization, and vehicle garages, in particular We thus define 5 new sets of instances, extending those with 15, 30, and 50 OD-demands (ODs in the following) introduced by Dellaert et al. (2019) for the 2E-VRPTW, while instances with 5 and 10 ODs are created by selecting customers with the minimum distance to satellites from the 100-customer instances of Dellaert et al. (2019).

Each set of instances consists of 60 instances, which are divided into four categories (CA, CB, CC, and CD) to examine the behaviour of the proposed DDD solution method under different variations of time and capacity. These categories introduce diverse distinctions by varying the time window width and demand values, thereby capturing the influence of the temporal component on the system and solution method. The instances represent a circular urban area, segmented into three concentric sections for platforms, satellites, and customers. OD demands are generated by randomly placing supplier points within the platform section (see Figure 2), assigning a unique customer (OD) and demand to each supply point. The availability time for each OD at each platform is determined by rounding up the Euclidean distance between the supply point and the platform. These availability times are incorporated into the temporal components of the system as a global temporal offset ρ based on the latest availability time. This offset ρ is added to each customer's time window to account for the additional time when the demand is available at the platforms, resulting in the time window range $[a_i + \rho, b_i + \rho]$. The maximum waiting time at satellites is set to 4, while the service time at customers is set to 2. Load capacities for vehicles given in Dellaert et al. (2019) are considered to be fixed, where first-level vehicles have a capacity of $cap_1 = 200$ and second-echeclon vehicles have a capacity of $cap_2 = 50$. Travel costs are computed as the ceiling of Euclidean distances. Schedule lengths are set to $\Psi = 100$ for small-sized instances with 5 and 10 ODs, and $\Psi = 200$ for medium- and large-sized instances with 15 ODs or more. The new 2E-MALRPS instances are available at `https://github.com/davescovar/2emalrpslib`

6.2. Hybrid formulation performance

We first focus on the effectiveness of the hybrid formulation, as well as on the impact of the granularity of the time discretization on its behaviour in terms of solution quality and computational efficiency. The sets of small- and medium-sized instances (5, 10, and 15 ODs) are used for benchmarking and the formulations are solved directly using CPLEX. The hybrid formulation is compared to a distinct standalone time-space formulation defined by isolating constraints (4.2) - (4.22) and the objective function (4.1). The hybrid and standalone time-space formulations are solved for a complete time-space network with $\bar{\Delta}$ time periods, as well as for different granularity values.

Tables 1 and 2 display the comparative performance results on the complete time-space network ($\Delta = \bar{\Delta}$) and the reduced time-space networks with granularity values $\Delta = 50$ and $\Delta = 25$. The run-time limit equals 2.5 hours. In addition to the information described earlier on, each table provides the average root gap (RG) computed as the percentage difference between the initial lower bound, obtained by the LP relaxation of the respective model at the root of the branch-and-bound tree, and the best integer solution obtained for the instance. Two additional performance measures present the average cost increment percentage (Dif UB) and CPU time reduction percentage (Dif CPUsec) obtained by each formulation using the reduced time-space network compared with the results of using the complete time-space network. (Recall that, we obtain reduced time-space networks with coarse discretizations by aggregating nodes and arcs of the complete time-space network. While this significantly reduces computation times, it also very often yields solutions which are infeasible with respect to the initial instance with a finer discretization. We did not consider these results when computing the last two performance measures.)

Instances				Hybrid								
				Δ					$\Delta = 50$		$\Delta = 25$	
$\|\mathcal{P}^{\text{ph}}\|$	$\|\mathcal{Z}^{\text{ph}}\|$	**OD**	**NI**	**FUB**	**OUB**	**CPUsec**	**RG(%)**	**OG(%)**	**Dif UB**	**Dif CPUsec**	**Dif UB**	**Dif CPUsec**
2	3	5	20	20	20	4561.33	16.38	0.00	0.71	49.70	1.54	83.94
3	5	5	20	20	20	3759.97	18.34	0.00	0.31	39.91	0.84	72.72
6	4	5	20	20	20	3118.99	21.99	0.00	1.38	40.05	3.19	75.45
2	3	10	20	20	15	4356.59	16.88	4.14	2.24	67.19	2.71	92.60
3	5	10	20	20	15	8586.08	17.86	9.13	2.79	64.20	3.28	87.36
6	4	10	20	20	15	8960.59	18.84	9.61	1.44	66.34	1.72	86.11
2	3	15	20	11	0	9000.00	36.43	29.30	10.41	4.31	11.78	36.83
3	5	15	20	0	0	9000.00	31.01	24.13	N.A	0.00	N.A	31.89
6	4	15	20	0	0	9000.00	37.85	31.19	N.A	0.00	N.A	33.19

Table 1 – Performance of the hybrid time-space formulation

Instances				Standalone								
				Δ					$\Delta = 50$		$\Delta = 25$	
$\|\mathcal{P}^{\text{ph}}\|$	$\|\mathcal{Z}^{\text{ph}}\|$	**OD**	**NI**	**FUB**	**OUB**	**CPUsec**	**RG(%)**	**OG(%)**	**Dif UB**	**Dif CPUsec**	**Dif UB**	**Dif CPUsec**
2	3	5	20	20	20	1521.48	19.79	0.00	1.17	25.81	1.95	46.96
3	5	5	20	20	20	1290.38	21.55	0.00	0.86	26.36	1.22	45.24
6	4	5	20	20	20	2306.77	30.51	0.00	2.73	25.90	3.48	54.74
2	3	10	20	20	14	3208.01	21.57	3.19	2.54	71.70	3.07	81.01
3	5	10	20	20	14	4897.51	21.25	4.32	3.10	75.26	3.78	83.08
6	4	10	20	20	15	5255.78	21.94	5.03	1.64	74.93	2.14	84.85
2	3	15	20	17	1	9000.00	37.50	27.76	3.80	41.00	5.21	76.97
3	5	15	20	17	0	8961.78	31.59	23.06	3.71	39.48	4.66	79.89
6	4	15	20	16	0	9000.00	38.94	31.15	3.92	37.83	5.05	73.88

Table 2 – Performance of the standalone time-space formulation

The results reported in Tables 1 and 2 show the expected performance similarity with respect to the upper bounds, but remarkable differences in the lower-bound and run-time values. When compared to the standalone formulation, the linear relaxation of the hybrid model provides much better lower bounds (average improvement of some 14%), but is usually slower at proving optimality. We observe that the overall lower bound improvements by the hybrid formulation result from the combination of the vehicle index in the commodity-flow variables and the redundant continuous-time constraints to keep track of time, which help reduce noise in the mathematical model.

The experiments on 15-OD instances show a significant reduction in the number of feasible and optimal solutions for both formulations. Multiple factors contribute to this behaviour. The use of time-space representation of the system clearly influences the quality of the solutions and makes it more challenging to tackle problems with larger numbers of ODs or longer schedule lengths with either of the two formulations. The standalone time-space model, despite providing good quality LP-relaxations, suffers from scalability issues provoked by the size of the time-space representation of the network, which, in turn, leads to larger and, thus, harder to solve, integer programs. The hybrid formulation shares a similar issue in terms of scalability due to the nature of its time-space structure, and the greater model size resulting from the inclusion of continuous-time constraints. The latter significantly increases the hybrid formulation size, which, when evaluated on the complete time-space network, results in longer exploration times of the solution space and, as a consequence, fewer optimal solutions being discovered for instances with 15 ODs. Nonetheless, the formulation usually gains from the redundancy that these continuous temporal constraints add, improving the root gap for all instance sets.

The discretization granularity plays a key role in the trade off between accuracy and performance of time-space models. On the one hand, as seen in the results of Tables 1 and 2, a finely discretized time-space network provides an accurate representation of the system, but at the price of a large integer problem. On the other hand, the sizes of the time-space representation and resulting integer problem, as well as the solution time, can be reduced by coarsening the discretization (see also Section 5), at the price of a decrease in solution quality or even feasibility. Thus, one observes average computing-time reductions of 77% and 50% for the hybrid and the standalone time-space formulation, respectively, with corresponding solution-quality losses of 1.3% and 1.9% in cost increments.

The results show the superiority of the hybrid model, even in the case a straightforward solution approach using a commercial solver. It consistently yields less-degraded solutions compared to the classical time-space formulation. Furthermore, it exhibits a robust behaviour when applied with coarser granularity values. This results from the redundancy induced through the continuous-time constraints, which, when paired with coarser granularity values, reduce model size while retaining accuracy on a significant number of arcs in the system. This, in turn, it improves the general performance of the hybrid formulation.

6.3. Performance of the dynamic discretization discovery solution method

We investigate the performance of the DDD solution method introduced in Section 5 for the hybrid 2E-MALRPS model, and compare it to that of the DDD adapted to address the standalone time-space formulation defined previously, by allowing the refinement and the degeneracy procedures to be executed on the complete time-space network (as opposed to being limited to the customer section, as designed when the hybrid formulation is used).

As discussed in Section 5, the use of continuous-time constraints helps the model to retain its precision under very coarse discretizations. Therefore, computational tests are performed using the coarsest discretization granularity ($\Delta = 2$) possible, to decrease the size of the underlying network and enable a further reduction in the time required to solve the integer programme. The stopping criteria are a maximum run time of 2.5 hours for small (5 and 10 ODs) and medium-sized (15 ODs) instances, 5 hours for 30-OD instances, and 8 hours for 50-OD instances and an optimality gap of less or equal to 1% for instances with 50 OD demands.

Instances			H-DDD					C-DDD				
$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	**OD**	**NI**	**FUB**	**OUB**	**OG(%)**	**CPUsec**	**NI**	**FUB**	**OUB**	**OG(%)**	**CPUsec**
2	3	5	20	20	20	0.00	1.95	20	20	20	0.00	947.99
3	5	5	20	20	20	0.00	1.64	20	20	20	0.00	1090.95
6	4	5	20	20	20	0.00	1.58	20	20	20	0.00	917.45
2	3	10	20	20	20	0.00	24.94	20	20	20	0.00	3502.23
3	5	10	20	20	20	0.00	176.99	20	20	20	0.00	3568.11
6	4	10	20	20	20	0.00	112.46	20	20	20	0.00	3948.30
2	3	15	20	20	20	0.00	238.17	20	20	4	17.86	15741.92
3	5	15	20	20	20	0.00	374.10	20	20	3	17.91	16296.16
6	4	15	20	20	20	0.00	545.73	20	20	0	27.43	18000.00
2	3	30	20	20	12	4.03	14506.63	20	20	0	36.45	18000.00
3	5	30	20	20	7	4.64	15381.70	20	20	1	33.23	17430.12
6	4	30	20	20	7	4.07	16156.92	20	20	0	39.07	18000.00
2	3	50	20	20	0	15.06	28800.00	20	20	0	47.57	28800.00
2	3	50	20	20	0	24.90	28800.00	20	0	0	N.A	28800.00
2	3	50	20	20	0	14.41	28800.00	20	0	0	N.A	28800.00

Table 3 – Performance of DDD solution method for 5, 10, 15, 30, and 50 OD demands

Table 3 summarizes the results of the experiments for the proposed DDD algorithm using the hybrid model (H-DDD) and the DDD adapted for the standalone formulation (C-DDD). These results show that the DDD algorithm clearly outperforms the commercial solver (results in Section 6.2).

The H-DDD also presents a considerable general improvement in the solution quality and run times compared to the C-DDD. H-DDD identifies the optimal solution for 60 out of 60 instances with 15 ODs, compared with only 7 for the C-DDD. Furthermore, H-DDD is increasingly more robust as the problem size increases, providing feasible solutions for 120 out of 120 instances, as compared to only 80 for the C-DDD for the instances with 30 and 50 ODs. In terms of computational time, the H-DDD is on average 77% faster than the C-DDD in identifying optimal solutions. This notable performance gain is attributed to the use of continuous-time constraints along with the time-space representation of the problem. Despite the large size of the hybrid formulation (compared with the standalone time-space model), the additional redundancy provided by these continuous-time constraints allows the model to prevent the degeneracy of the first-echelon and part of the second-echelon arcs and reduce the growth of the underlying network. This in turn accelerates the convergence of the H-DDD to optimal solutions.

Degeneracy is still present on the customer side of the problem, however, as the method relies on the time-space representation to derive a feasible integer solution, which reduces the lower-bound increase rate at each iteration. The proposed degeneracy procedure behaves as expected and considerably lowers the impact of degeneracy on the performance of the proposed DDD, especially for instances with broad time windows (which thus provide more scheduling-decision alternatives). Our analysis tends to show, however, that the degeneracy procedure primarily contributes to attaining good upper bounds faster, rather than

$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	**OD**	**NI**	**FUB**	**OUB**	**OG(%)**	**CPUsec**
2	3	5	20	20	20	0.00	0.77
3	5	5	20	20	20	0.00	2.71
6	4	5	20	20	20	0.00	6.52
2	3	10	20	20	20	0.00	30.13
3	5	10	20	20	20	0.00	260.04
6	4	10	20	20	20	0.00	573.43
2	3	15	20	20	20	0.00	102.23
3	5	15	20	20	20	0.00	993.43
6	4	15	20	20	20	0.00	1729.48

Table 4 – H-DDD performance on instances without availability times

$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	**OD**	**NI**	**FUB**	**OUB**	**OG(%)**	**CPUsec**
2	3	5	20	20	20	0.00	0.74
3	5	5	20	20	20	0.00	3.72
6	4	5	20	20	20	0.00	9.89
2	3	10	20	20	20	0.00	1015.592
3	5	10	20	20	19	0.17	1770.973
6	4	10	20	20	15	1.90	4075.948
2	3	15	20	20	20	0.00	124.01
3	5	15	20	20	17	0.50	1557.42
6	4	15	20	20	18	0.68	3931.11

Table 5 – H-DDD performance on instances without availability times and synchronization

improving the lower bound at a fast rate (see the supplementary material Appendix A.3). Consequently, while the proposed DDD solution method yields provably high-quality upper bounds for larger instances, the slow incremental rate of the lower bounds makes assessing the true quality of the solution more difficult, in particular as the number of ODs increase. This is illustrated on instances with 50 ODs, where the optimality of most solutions obtained by the H-DDD remains unproven.

6.4. Sensitivity analysis

We performed a sensitivity analysis of two main time-related components of the 2E-MALRPS, the availability times of demands and the synchronization at satellites, with respect to the behaviour of the proposed H-DDD solution method. These two characteristics directly determine the degree of tightness of vehicle operations on both echelons and, thus, the performance of the system in servicing the end customers. Our goal is to study the impact of these interacting attributes and gain insights into the system behaviour in less time-sensitive environments. The study was performed on a subset of instances with 5, 10, and 15 ODs that the H-DDD solves to optimality (reducing the noise due to optimality gaps). Tables 4 and 5 summarize the results of the experiments performed with H-DDD using the coarsest discretization granularity ($\Delta = 2$) possible.

The figures in Table 4 show that the H-DDD algorithm is able to provide good quality solutions, even when the demand availability time is not binding or is not important/considered in the problem setting. The H-DDD achieves optimality for all instances considered with or without demand availability times. In terms of computational efficiency, the solution method presents a better performance when availability times are present, finding the optimal solution some 60% faster than when these temporal elements are not considered. The lower performance in this latter case follows from the absence of precise time moments (due the larger number of availability options) which results in larger solution spaces and more complex integer problems at each iteration of the H-DDD. This impacts the quality of lower bounds and increases the runtime needed to achieve optimality (see Section 6.3). It is worth mentioning that the multi-commodity aspect limits the performance improvement when demand availability time is considered. In single commodity cases, prioritizing the earliest availability times for each platform allows for greater flexibility in synchronizing satellite operations and customer arrivals. In the multi-commodity case with multiple availability times for different OD demands, however, flexibility in other temporal aspects of the system is limited (see the supplementary material Appendix A.4).

Note that, tight availability times at the origin of demand and (narrow) time windows at destination imposes a certain *de facto* level of synchronization, in particular for small instances. Hence, to better study the impact of synchronization requirements on the H-DDD performance, we disabled both synchronization and availability times. Table 5 sums up the results of this analysis.

The results indicate that omitting also synchronization increases significantly the computation times, not only with respect to the full problem setting (by some 70%), but also compared to when availability times are disabled only. This follows from the fact that the lack of significant temporal constraints delimiting the area of relevance of satellite facilities yields a large number of second-echelon route alternatives and, thus, the generation of more degenerate solutions. The even larger performance degradation for the 10-OD instances comes from the additional impact of large time windows on the behaviour just explained.

In terms of cost, solutions on instances with disabled availability times with/without disabled synchronization are cheaper than solutions on instances with all temporal aspects enabled, with an average cost reduction of some 4% and 12%, respectively. This reduction in operational costs can be attributed to a series of low-cost routes that can only be used at certain times of the schedule length, but are unavailable when time limits are tight. Moreover, disabling the availability times opens up a lot more possibilities for demand consolidation at satellite facilities. The system fixed costs related to facility usage are reduced as a result of the higher level of consolidation, which results in a smaller number of facilities selected. This is even more significant for instances with late availability times and early due times (e.g., the instance types CA and CB), where a larger number of adjacent platforms and satellites must be opened and connected to satisfy demand on time.

7. Conclusions and Perspectives

This paper introduces the multi-attribute two-echelon location-routing problem with synchronization constraints, 2E-MALRPS, and presents a mixed-integer programming formulation on a hybrid time-space network combining continuous and discrete time representations We also present an exact solution method that iteratively refines a reduced time-space network, solving the 2E-MALRPS formulation defined on the reduced network to extract bounds and temporal granularity refinements, in order to guide the method towards to optimal solution of the original problem. The paper generalizes the dynamic discretization discovery method to complex problem settings involving several levels of location, routing, and synchronization decisions.

The computational study demonstrates the effectiveness of the mathematical formulation and the DDD solution method. Comparative analyses reveal that the proposed hybrid formulation outperforms a classic time-space formulation (which only considers discrete time modelling) in all performance measures. This superiority is observed when utilizing a commercial solver directly as well as when applying the DDD solution method. The results highlight the effectiveness of the proposed DDD in handling large instances, achieved through the degeneracy mitigation procedure and the integration of discrete and continuous-time representations to maintain the accuracy of time-related decisions.

To conclude, it is observed that neglecting availability times and synchronization requirements in time-driven applications can result in an inaccurate depiction of the distribution system. Sensitivity to time is evident, as lower distribution costs do not always guarantee feasible operations due to the inclusion of routes that may be infeasible when availability times or synchronization are not taken into account. The hybrid 2E-MALRPS formulation and DDD solution method offer high-quality solutions for decision-makers and improve performance for time-critical applications. Future research can focus on heuristic methods or specialized modelling approaches to enhance the convergence of the DDD by improving upper and/or lower bounds in each iteration, as well as investigating the impact of variations in the properties defining the reduced network.

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Second Article.

The Two-Echelon Multicommodity Location-Routing Problem with Stochastic and Correlated Demands

by

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— *David Escobar Vargas:* Conceptualization, Methodology, Software, Data analysis, Writing - original draft.
— *Teodor Gabriel Crainic:* Conceptualization, Methodology, Writing - review & editing.
— *Walter Rei:* Conceptualization, Methodology, Writing - review & editing.
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Résumé. Cette étude présente le problème stochastique de localisation-routage à deux échelons avec des demandes multicommodité stochastiques et corrélées. Nous proposons une formulation de programmation stochastique en deux étapes, les décisions de conception des installations du deuxième échelon définissant la première étape, tandis que les décisions de recours, prises lors de la deuxième étape, déterminent la manière dont les demandes observées sont distribuées. L'objectif global est d'optimiser le coût des décisions de conception de la première étape plus le coût de distribution total prévu dans la deuxième étape. Pour résoudre cette formulation, nous proposons une métaheuristique de couverture progressive avec une série d'améliorations algorithmiques pour accélérer l'exploration de l'espace de solution. Ces améliorations comprennent 1) des structures de population pour obtenir des solutions alternatives et diverses pour les sous-problèmes du scénario qui doivent être résolus tout au long du processus de recherche ; 2) des stratégies alternatives pour définir les solutions de référence qui sont utilisées pour guider et accélérer la recherche globale ; et 3) une procédure de réinitialisation qui réduit le risque que la méthode soit piégée dans des optimum locaux. Nous évaluons l'efficacité et l'efficience de toutes les stratégies proposées par le biais d'expériences informatiques approfondies, en évaluant leur capacité à générer des solutions de haute qualité pour différentes caractéristiques du problème et corrélations de la demande.
Mots clés : Problème de localisation-routage à deux échelons, demande stochastique, demandes multi-marchandises avec origine-destination, couverture progressive

Abstract. This study introduces the stochastic two-echelon multicommodity location routing problem with stochastic and correlated demands. We propose a two-stage stochastic programming formulation, with second-echelon facilities design decisions defining the first stage, while recourse decisions, which are made in the second stage, establish how the observed demands are distributed. The overall objective is to optimize the cost of the first-stage design decisions plus the total expected routing cost incurred in the second stage. To solve this formulation, we propose a progressive hedging metaheuristic with a series of algorithmic enhancements to accelerate the exploration of the solution space. These enhancements include: 1) population structures to obtain alternative and diverse solutions for the scenario subproblems that need to be solved throughout the search process; 2) alternative strategies to define the reference solutions which are used to guide and accelerate the overall search; and 3) a reset procedure that reduces the risk of the method becoming trapped in local optima. We assess the efficiency and effectiveness of all proposed strategies through extensive computational experiments, evaluating their capability to generate high-quality solutions across various problem characteristics and demand correlations.
Keywords: Two-echelon location-routing problem, stochastic demand, multicommodity origin-destination demands, progressive hedging

1. Introduction

The two-echelon location-routing problem (2E-LRP) is an important class of combinatorial optimization problems with a wide rage of applications in the freight transportation industry. At its core, the concept is to design a two-layer freight transportation system that enables indirect freight transportation between platforms (distribution centers) and customers through a set of intermediate facilities named satellites. The 2E-LRP has been defined as the preferred methodology for efficiently capturing the simultaneous decisions concerning the location of one or two levels of facilities (platforms and/or satellites) and creating a limited set of routes at both echelons to effectively serve all customer demands (Cuda et al., 2015). Despite the growing number of scientific contributions and advances in this field, most research on 2E-LRP has focused on models and solution methods for "classic" problem variants and deterministic cases, while uncertain factors are often overlooked (Gendreau et al., 2014).

Considering demand uncertainty and its interrelation is of great significance when planning decisions are involved. In logistics planning, which encompasses strategic and tactical choices in distribution network design, obtaining accurate information about customer demand variations is essential for long-term planning (Lium et al., 2009). Several sources of uncertainty related to demand can be observed, such as variations in volume, inaccuracies in forecasted values, or unexpected demand fluctuations between specific origin-destination pairs (Crainic et al., 2011a). While studies in LRPs with stochastic demands often assume statistically independent request fluctuations, variability and correlation are observed in many logistics contexts (Verma and Campbell, 2019). Different demand values often display degrees of positive or negative correlation relative to other customer demands (Bucci et al., 2006). Seasonal demand variations serve as an example, though they may not entail high uncertainty due to their predictability. Correlations and more intricate covariation gain importance during planning, especially when systematic relationships exist among customer demands (e.g., regions, product types, time periods) (Heath and Jackson, 1994; Thapalia et al., 2012; Verma and Campbell, 2019; Mirhedayatian et al., 2019). One can thus assume that demand correlation exhibits a *mixed* nature, rather than being purely positive or uncorrelated. To the best of our knowledge, 2E-LRPs considering correlated and uncertain demands, specifically involving non-substitutable demand with a known origin and destination, remains unexplored. This study is aimed at deepening the understanding of the effects of the integrated treatment of uncertain and correlated non-substitutable demands on location and routing decisions. Our goal is to provide a methodology to respond to the modelling and algorithmic challenges and, thus, to contribute toward filling the gaps in the literature.

This paper address a 2E-LRP with stochastic and correlated multicommodity, origin-to-destination (OD) demands. We thus introduce the *Two-Echelon Multicommodity Location-Routing Problem with Stochastic and Correlated Demands* (*2E-MLRPSCD*) as a unified view of the attributes considered. The problem centers around design decisions concerning the selection of satellite facilities and the allocation of multicommodity origin-destination (OD) demands to these satellites, while also encompassing the definition of a limited set of routes at both echelons to efficiently fulfill the demand. To address the uncertainty, we propose a stochastic programming approach oriented towards devising a singular system design capable of maintaining cost-effectiveness in the presence of diverse demand realizations. Specifically, we present a two-stage stochastic programming formulation, with satellite facility design decisions defining the first stage, while recourse decisions, which are made in the second stage, establish how the observed demands are distributed. We thus represent the demand uncertainty through a finite set of scenarios, which must approximate the uncertainty inherent in the planning context. However, employing scenario-based uncertainty modeling yield large-scale models that may prove impractical to address using standalone exact solution methods (King and Wallace, 2012).

The proposed work thus introduces a progressive hedging-based metaheuristic for addressing the 2E-MLRPSCD, building on the work of Crainic et al. (2011a) for the network design problem. From a methodological standpoint, the classic progressive-hedging (PH) algorithm iteratively solves deterministic subproblems derived from the scenario-based decomposition of the stochastic program. The PH metaheuristic iterates by adjusting the mathematical formulation of scenario subproblems using aggregated solutions until reaching an optimal solution when a general consensus among non-scenario-dependent decisions is observed. However, the classic structure of the PH metaheuristic and the metaheuristic methods derived from it lack alternative aggregation methods to effectively derive key insights from the subproblem solutions. To address this, we present a specialized PH-based metaheuristic with a series of algorithmic enhancements. These enhancements include: population structures to obtain alternative and diverse solutions for the scenario subproblems that need to be solved throughout the search process; 2) alternative strategies to define the reference solutions which are used to guide and accelerate the overall search; and 3) a reset procedure that reduces the risk of the method becoming trapped in local optima. In the computational study, we analyze the cost sensitivity, infrastructure usage, and a comparison between the uncertain and deterministic definition of the demand to derive insights of the effectiveness of the proposed solution method.

The remaining parts of the paper are organized as follows. Section 2 is dedicated to describing the problem definition. An overview of the related scientific literature is provided in Section 3. Section 4 presents the system modelling and the proposed mathematical formulation. The solution method we developed is described in Section 5. Computational results are then presented and analyzed in Section 6. Finally, in Section 7, we conclude and highlight some general avenues for future research.

2. Problem definition

This section introduces the 2E-MLRPSCD, which involves addressing a 2E-LRP with stochastic and correlated multicommodity origin-to-destination (OD) demands. The section is divided into two parts. Section 2.1 presents the physical problem setting of the 2E-MLRPSCD. Section 2.2 outlines representation of stochastic and correlated demands as well as the main lines, objective and requirements, of the problem.

2.1. The 2E-MLRPSCD setting

The two-echelon system consists of three main components: *platforms* (primary facilities serving as demand origins), *satellites* (intermediate facilities), and *customers* (demand destinations).

Formally, the 2E-MLRPSCD is represented as a complete weighted directed graph $N = (V, A)$, with vertices $V = P \cup Z \cup C$, divided into three disjoint sets: platforms P, satellites Z, and customers C. Platforms are large-sized facilities with a potential set of commodities to be distributed to customers. Satellites are medium- to small-sized multimodal infrastructures that serve as intermediate facilities, allowing the consolidation and sorting of freight between the two transportation echelons involved in distributing goods to customers. Each satellite location $z \in Z$ is associated with a limited storage capacity Q_z and a fixed opening cost F_z.

Demand is defined between platforms and customers, each individual demand being characterized by an origin, a destination and a requested volume to be delivered. Let K denote the set of origin-destination (OD) demands. For the deterministic version of the 2E-MLRPSCD, each OD demand $k \in K$, is thus characterized by a volume vol_k, an origin $O(k)$ associated with a platform node in P, and a destination $D(k)$ associated with a customer node in C. Additionally, a fixed allocation cost Δ_{pzk} represents the cost of serving OD demand $k \in K$ through platform $p \in P$ and satellite $z \in Z$.

Each arc $(i,j) \in A = A^1 \cup A^2$ is associated with a non-negative cost ζ_{ij} for a vehicle to travel between i and j. Let A^1 denote the set of arcs of the first echelon, corresponding to the connections between platforms P and satellites Z and between satellites. The set A^2 includes the arcs of the second echelon, that is, the connection of the satellites Z with the final customers C and between customers.

Freight delivery is performed by two homogeneous fleets of vehicles $H = H^1 \cup H^2$ with limited load capacities cap_1 and cap_2, which are respectively available for the first and second echelon, and are able to transport any demand. Vehicles are assumed to be available at each existing facility for each echelon, where vehicles start and end their routes.

The considered problem involves the selection of satellite facilities, the allocation of OD demands to satellites, as well as the routing of vehicles at each echelon to deliver the freight from platforms to customers, going through satellite facilities. As depicted in Figure 6, each OD demand that is made available at its originating platform has to be moved by a first-echelon vehicle to a given satellite to be then transferred to a second-echelon vehicle. Loads

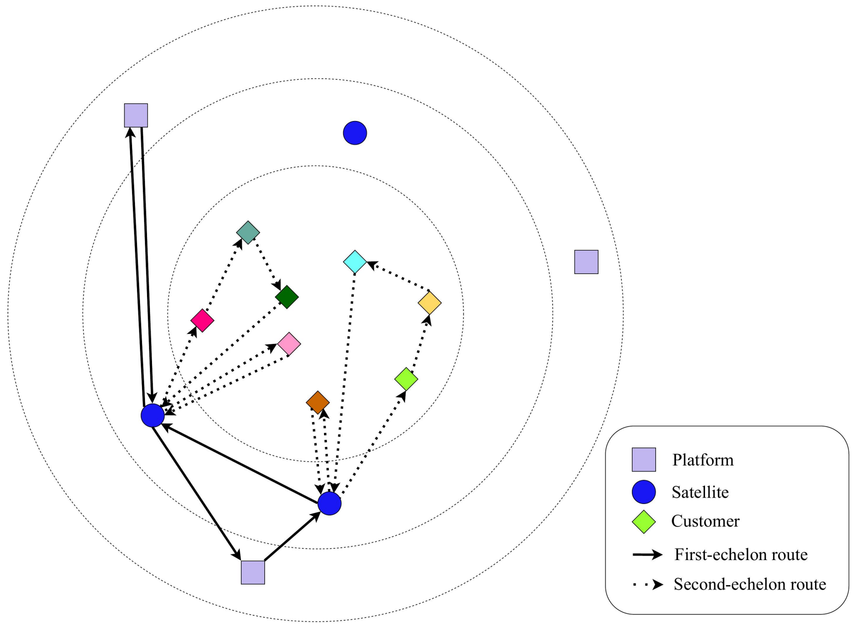

Figure 6 – Topology of the 2E-MLRPSCD.

delivered at satellites are then transshipped and consolidated into second-echelon vehicles, which will perform the deliveries to the final destinations.

2.2. The stochastic setting

The 2E-MLRPSCD involves uncertainty in the volume of demand stemming from random changes occurring between correlated OD pairs. We assume that probability distributions exist to describe the variation in the random events affecting the volume of demands. Moreover, the problem setting involves correlation among OD pairs, where each OD pair can be either positively or negatively correlated with other distinct OD pairs. The problem is characterized by two sets of OD pairs used to represent the correlation; OD pairs within each set are positively correlated, while all correlations between OD pairs in different sets are strongly negative (i.e., low demands in one set result in high demands in the other).

The 2E-MLRPSCD problem setting addresses strategic and tactical planning decisions in multiple application fields. In terms of decision-making and information processing, the design and allocation decisions during the planning stage must be defined based on an evaluation/estimation of their impact on operations, including the available recourse actions to adapt the plan to the observed demands. The recourse actions in the present case involve the definition of the optimal routes to fulfill observed ("realized") customer demands including, when necessary, the use of external outsourcing services with high additional operational costs.

The 2E-MLRPSCD then consists in the selection of the locations of the satellite facilities, the allocation of OD demands to satellites, as well as the construction of a limited set of routes for the first and second echelons vehicles in such a way that: (i) the demand of each platform is assigned to an open satellite; (ii) every route of the first echelon starts and ends at the same platform; (iii) every route of the second echelon starts and ends at the same satellite; (iv) all the customers' demands are satisfied either by the system or an outsource service; (v) the load capacity of each vehicle is not exceeded; (vi) each customer served by the system is visited by only one vehicle; (vii) the total demand assigned to a satellite facility must not exceed its capacity; and (viii) the sum of the fixed location and allocation costs and the expected routing costs (the recourse action) is minimized.

3. Literature review

The 2E-MLRPSCD belongs to the *Location-Routing Problem* (*LRP*) category, which constitutes an important problem class that contains a vast number of contributions and ongoing works in the literature. LRPs fundamentally appear in the context of the planning process that seeks to open one or more platforms from a given set of pre-defined locations, define the customer assignments to them and establish a variety of routes required to meet the demands of each customer considered. Studies dedicated to the LRPs and the 2E-LRP are increasingly gaining attention, in particular on realistic multi-attribute problem settings (Escobar-Vargas and Crainic, 2023). This section aims to situate the 2E-MLRPSCD within the relevant literature on both the 2E-LRP and LRP, specially pointing out the gaps in knowledge concerning how to deal with stochastic demands in this setting. A brief discussion on the progressive-hedging strategy is also provided, focusing on the challenges and gaps of the application of this method when tackling integer programming problems. Works on 2E-LRP and LRP dedicated to their deterministic versions or stochastic aspects other than demand uncertainty are out of the scope of this study. Therefore, we refer the interested readers to the recent surveys by Cuda et al. (2015), Schiffer et al. (2019) and Mara et al. (2021a).

Because of its practical relevance, the LRP has attracted much attention from the research community resulting in a wide variety of high-quality solution approaches for its deterministic versions since its introduction in Maranzana (1964). While studies on demand uncertainty are still scarce, more attention has been devoted to this variant spurred by the desire to solve more realistic distribution planning problems (Cuda et al., 2015; Escobar-Vargas and Crainic, 2023). Because of the complexity of considering demand uncertainty in LRP, most studies have focused on proposing heuristics methods to solve the problem setting considered. The literature is notably characterized by the extensive use of local-search-based metaheuristic frameworks to address the underlying transportation problems to guide two- or multi-stage heuristics, where location, allocation and routing decisions are treated by different heuristics at different stages (see, Albareda-Sambola et al., 2007; Huang, 2015; Marinakis, 2015; Marinakis et al., 2016; Zhang et al., 2019). A different approach is proposed by Quintero-araujo et al. (2019), where a simheuristic algorithm is proposed to deal with the LRP with stochastic demands. This simheuristic algorithm then hybridizes a Monte Carlo simulation with an iterated local search metaheuristic. In spite of the advances in the field, the literature in LRPs with demand uncertainty is quite limited, especially in the case of non-substituable demands. The case where stochastic demands are statistically independent remains the most predominant setting studied in the literature. Research concerning correlation features and their impact on the decision making process has yet to be addressed. Important contributions are also still required to deepen the understanding of the impact of richer problem settings and their influence on location decisions under uncertainty.

The literature on 2E-LRP with uncertain demands is very limited. To the best of our knowledge only Snoeck et al. (2018) have presented a stochastic mixed-integer linear programming formulation to model a two-echelon capacitated location-routing problem with uncertain demands arising from a practical application. However, particular developments are required in the field, especially in relation to explicitly consider demand correlations, non-substituable demand considerations, and meeting the modelling and algorithmic challenges these considerations imply.

Aside from the modelling aspects, there is a fundamental need for more effective solution procedures for 2E-LRP with uncertainty considerations. Concerning exact and approximate solution frameworks, decomposition-based methods have shown very promising results for solving two- and multi-stage stochastic optimization models (Atakan and Sen, 2018). The effectiveness of such methods rely on how the stochastic problem can be decomposed. Two general decomposition strategies are usually applied here. The first strategy decomposes the model according to the scenarios used to formulate the uncertain phenomena, while the second strategy separates the model according to the decision stages that define the optimization model. The progressive hedging algorithm is one of the most used dual decomposition frameworks in the field. Rockafellar and Wets (1991) developed progressive hedging to solve convex stochastic programs. The algorithm involves decomposing the stochastic problem by scenario, solving each of the resulting scenario subproblems independently, and then determining the stochastic problem's solution based on the consensus (or averaging) of all scenario subproblems solutions. However, converging to a globally optimal solution for mixed-integer stochastic programs in a computationally-efficient manner is challenging, primarily because of the non-convex nature of the feasible set (Atakan and Sen, 2018). To overcome such computational burden several studies have proposed different heuristic frameworks following the progressive hedging algorithm to allow the application of the method to integer programming formulations (see, Løkketangen and Woodruff, 1996; Haugen et al., 2001; Crainic et al., 2011a; Lamghari and Dimitrakopoulos, 2016; Alvarez et al., 2021). To reach a consensus for the complete integer stochastic problem, the standard structure of these PH-based metaheuristics, usually relies on obtaining the best solution possible (not necessarily optimal) for each scenario subproblem. This strategy enables the use of the best decisions to guide the search of the solution space. Nonetheless, considering the single best solution possible for each scenario subproblem can also reduce the overall diversity of solutions of the complete stochastic problem, which is crucial at each iteration of the PH. The current work seeks to close these gaps in the literature by extending and enhancing a progressive-hedging-based metaheuristic to the 2E-LRP, by introducing a specialised set of heuristics to allow the consideration of diverse alternative solutions for each scenario subproblem, as well as a set of novel techniques to accelerate consensus for the stochastic problem.

4. Modelling

Section 4.1 introduces the initial outline of the modelling approach, followed by the proposed mathematical formulation in Section 4.2.

4.1. Modelling uncertainty

The 2E-MLRPSCD is formulated as a two-stage stochastic program to account for the strategic planning decisions. The proposed two-stage model consists of a first stage, where the location of satellite facilities and the OD demand to satellite allocation decisions are made while facing the demand uncertainty, and a second stage, where the vehicle routes for both echelons are determined when customer demands are observed. Additionally, the option of resorting to ad-hoc, outsourced capacity when necessary is also part of the second stage, where an operational cost R is associated with the percentage of the demand volume that is served by an outsourced service.

The demand uncertainty is represented through known distributions, while correlations are given by matrices.

We model the demand uncertainty and correlation in this system through the generation of a set of scenarios, obtained by sampling probability distributions, each scenario representing a possible realization of the random event affecting the demands. Let S denote the set of scenarios, where scenario $s \in S$, represents a possible realization of the random events which sets the demand values of each customer, while reflecting the assumed correlations. Let ρ_s be the probability of occurrence of scenario s, such that $\sum_{s \in S} \rho_s = 1$. Then for a given $s \in S$, there is a demand volume fixed to $vol_k(s)$ for all $k \in K$, such that $vol_k(s) \geq 0$.

4.2. Two-stage formulation for the 2E-MLRPSCD

This section presents the Mixed-Integer Programming (MIP) formulation for the 2E-MLRPSCD, as a two-stage stochastic programming problem using a three-index vehicle-flow formulation. Two sets of decision variables are defined. First-stage variables address the satellite location and OD demand to satellite allocation decisions. Vehicle-routing decisions at both echelons are made in the second stage. Following the general trend in the literature, we save space and present the formulation directly in terms of the set of scenarios S. This yields second-stage variables indexed by scenario, while first-stage ones are not (they are not supposed to be modified in the second-stage). The following definitions describe the decision variables that constitute the extensive form of the proposed two-stage formulation:

— $y_i \in \{0,1\}, i \in Z$: location variable, 1 if a satellite is opened in location i, 0 otherwise;

— $f_{pzk} \in \{0,1\}, p \in P, z \in Z, k \in K$: allocation variable, 1 if satellite z is allocated to platform p to serve the demand k, 0 otherwise;

— $u^s_{pzkh} \in \{0,1\}, p \in P, z \in Z, k \in K, h \in H^1, s \in S$: vehicle allocation variable, 1 if vehicle h is allocated to serve satellite z from platform p with demand k for scenario s, 0 otherwise;

— $v^s_{zch} \in \{0,1\}, z \in Z, c \in C, h \in H^2, s \in S$: vehicle allocation variable, 1 if vehicle h is allocated to serve the customer c with satellite z for scenario s, 0 otherwise;

— $x^s_{ijh} \in \{0,1\}, (i,j) \in A, h \in H, s \in S$: vehicle flow variable, 1 if arc (i,j) is used by vehicle h for scenario s, and 0 otherwise;

— $w^s_{zkh} \geq 0, z \in Z, k \in K, h \in H^2, s \in S$: percentage of demand k served by a satellite z with a vehicle h for scenario s;

— $o^s_k \geq 0, k \in K, s \in S$: percentage of demand k that is outsourced for scenario s;

— $b^s_{kh} \geq 0, k \in K, h \in H^1, s \in S$: percentage of demand k dispatched with a vehicle h for scenario s;

— $L^s_{zh} \geq 0, z \in Z, h \in H^1, s \in S$: integer variable used to record the position of the satellite z in the route allocated to the first-echelon vehicle h for scenario s;

— $N^s_{ch} \geq 0, c \in C, h \in H^2, s \in S$: integer variable used to record the position of the customer c in the route allocated to the second-echelon vehicle h for scenario s;

The extensive two-stage formulation of the 2E-MLRPSCD then becomes:

$$\min \sum_{s \in S} \rho_s \left(\sum_{h \in H} \sum_{(i,j) \in A} \zeta_{ij} x^s_{ijh} + \sum_{k \in K} R o^s_k \right) + \sum_{i \in Z} F_i y_i + \sum_{i \in P} \sum_{j \in Z} \sum_{k \in K} \Delta_{ijk} f_{ijk} \tag{4.1}$$

subject to

$$\sum_{j \in (P \cup Z), i \neq j} x^s_{ijh} \leq 1 \quad \forall i \in (P \cup Z), h \in H^1, s \in S \tag{4.2}$$

$$\sum_{i \in (P \cup Z), i \neq j} x^s_{ijh} - \sum_{i \in (P \cup Z), i \neq j} x^s_{jih} = 0 \quad \forall j \in (P \cup Z), h \in H^1, s \in S \tag{4.3}$$

$$L^s_{ih} - L^s_{jh} + |Z| \, x^s_{ijh} \leq |Z| - 1 \quad \forall i,j \in Z, i \neq j, h \in H^1, s \in S \tag{4.4}$$

$$\sum_{h \in H^2} \sum_{j \in (Z \cup C), i \neq j} x^s_{ijh} = 1 \quad \forall i \in C, s \in S \tag{4.5}$$

$$\sum_{i \in (Z \cup C), i \neq j} x^s_{ijh} - \sum_{i \in (Z \cup C), i \neq j} x^s_{jih} = 0 \quad \forall j \in (Z \cup C), h \in H^2, s \in S \tag{4.6}$$

$$\sum_{h \in H^2} \sum_{j \in C} x^s_{ijh} \leq |H^2| y_i \quad \forall i \in Z, s \in S \tag{4.7}$$

$$N^s_{ih} - N^s_{jh} + |C| \, x^s_{ijh} \leq |C| - 1 \quad \forall i, j \in C, i \neq j, h \in H^2, s \in S \tag{4.8}$$

$$\sum_{j \in (Z \cup C), i \neq j} x^s_{ijh} + \sum_{j \in (Z \cup C), l \neq j} x^s_{ljh} - v^s_{lih} = 0 \quad \forall i \in C, l \in Z, h \in H^2, s \in S \tag{4.9}$$

$$\sum_{h \in H^2} \sum_{i \in Z} v^s_{ijh} = 1 \quad \forall j \in C, s \in S \tag{4.10}$$

$$\sum_{i \in P} \sum_{h \in H^1} u^s_{ijkh} = \sum_{h \in H^2} v^s_{jD(k)h} \quad \forall j \in Z, k \in K, s \in S \tag{4.11}$$

$$\sum_{h \in H^2} \sum_{i \in Z} w^s_{ijh} + o^s_j = 1 \quad \forall j \in K, s \in S \tag{4.12}$$

$$w^s_{ijh} \leq v^s_{iD(j)h} \quad \forall i \in Z, j \in K, h \in H^2, s \in S \tag{4.13}$$

$$b^s_{kh} \geq w^s_{ikl} - (2 - v^s_{iD(k)l} - \sum_{p \in P} u^s_{pikh}) M$$

$$\forall h \in H^1, l \in H^2, i \in Z, k \in K, s \in S \tag{4.14}$$

$$\sum_{k \in K} vol_k(s) \sum_{h \in H^2} w^s_{ikh} \leq Q_i \quad \forall i \in Z, s \in S \tag{4.15}$$

$$\sum_{k \in K} vol_k(s) \sum_{i \in Z} w^s_{ikh} \leq cap_2 \quad \forall h \in H^2, s \in S \tag{4.16}$$

$$\sum_{k \in K} b^s_{kh} \leq cap_1 \quad \forall h \in H^1, s \in S \tag{4.17}$$

$$\sum_{h \in H^1} u^s_{ijkh} = f_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S \tag{4.18}$$

$$\sum_{h \in H^2} v^s_{zD(k)h} = \sum_{p \in P} f_{pzk} \quad \forall z \in Z, k \in K, s \in S \tag{4.19}$$

$$y_i \in \{0,1\} \quad \forall i \in Z \tag{4.20}$$

$$f_{pzk} \in \{0,1\} \quad \forall p \in P, z \in Z, k \in K \tag{4.21}$$

$$u^s_{pzkh} \in \{0,1\} \quad \forall p \in P, z \in Z, k \in K, h \in H^1, s \in S \tag{4.22}$$

$$v^s_{zch} \in \{0,1\} \quad \forall z \in Z, c \in C, h \in H^2, s \in S \tag{4.23}$$

$$x^s_{ijh} \in \{0,1\} \quad \forall (i,j) \in A, h \in H, s \in S \tag{4.24}$$

$$w^s_{zkh} \geq 0 \quad \forall z \in Z, k \in K, h \in H^2, s \in S \tag{4.25}$$

$$o^s_k \geq 0 \quad \forall k \in K, s \in S \tag{4.26}$$

$$b^s_{kh} \geq 0 \quad \forall k \in K, h \in H^1, s \in S \tag{4.27}$$

$$L^s_{zh} \geq 0 \quad \forall z \in Z, h \in H^1, s \in S \tag{4.28}$$

$$N_{ch}^{s} \geq 0 \quad \forall c \in C, h \in H^2, s \in S \tag{4.29}$$

The objective function (4.1) seeks to minimize the sum of the expected total routing and outsourced costs and the total fixed cost of opening satellites and allocating them to the platforms. Constraints (4.2) ensure that each available vehicle is assigned to at most one platform. Constraints (4.3) are the flow conservation constraints for platforms and satellite facilities. Constraints (4.4) are the sub-tour elimination constraints for the first-echelon vehicles. Constraints (4.5) ensure that every customer is served by a single second-echelon vehicle. Constraints (4.6) are the flow conservation constraints at satellites and at customers. Constraints (4.7) state that second echelon vehicles can only be used from located satellites. Constraints (4.8) are sub-tour elimination constraints for the second-echelon vehicles. Constraints (4.9) link the allocation and routing variables. Constraints (4.10) impose that each customer has to be assigned to a satellite.

Constraints (4.11) are the flow conservation constraints for each commodity k at each satellite z. Constraints (4.12) ensure that the portion of the customer demand served by a satellite and the portion served by an outsourced service meet the complete demand for each customer. Constraints (4.13) ensure that each satellite can only serve its assigned customers. Constraints (4.14) ensure that, for each commodity k, the portion of the demand that is serviced via the located satellite corresponds to the inbound portion that originates from the associated platform. Constraints (4.15) impose that flow leaving an open satellite z is less or equal than its storage capacity. Constraint (4.16) and (4.17) guarantee that the commodity flow carried by each vehicle, in the first and second echelon, respectively, is less than or equal to its own capacity. Constraints (4.18) and (4.19) link allocation and vehicle allocation variables for the first echelon and second echelon vehicles, respectively. Constraints (4.20)-(4.29) impose the integrality and non-negativity of each decision variables in the model.

5. A progressive hedging-based metaheuristic for the 2E-MLRPSCD

This section presents a progressive hedging-based metaheuristic to address the 2E-MLRPSCD, building on the work of Crainic et al. (2011a) for the stochastic network design problem.

As its name implies, the methodology is derived from the progressive hedging (PH) algorithm introduced by Rockafellar and Wets (1991) for multi-stage stochastic optimization problems. From a methodological perspective, the ‘classic’ progressive-hedging algorithm, iteratively solves the set of deterministic subproblems, which result from the scenario-based decomposition of the extensive formulation. At each iteration, the PH metaheuristic solves each scenario-specific deterministic subproblem separately, thus producing a series of solutions that may differ from one another. The search then proceeds by computing a reference solution (the expected value of the best scenario-specific solutions is traditionally used), which also serves to assess the overall level of consensus among the scenario-specific solutions. The formulations of the scenario subproblems are then adjusted to incentivize agreement (i.e., to make subproblems move toward the same implementable solution). This general process is repeated until either a consensus solution is found or another stopping criterion is reached (e.g., a computation time limit).

The PH algorithm is known to converge to optimality for continuous pseudo-polynomial convex formulations. This is not the case for mixed-integer programs, such as the 2E-MLRPSCD (see, Atakan and Sen, 2018). A significant algorithmic challenge also arises from the computational load of solving a series of NP-hard problems (one for each scenario) at each iteration of the PH metaheuristic. There is a clear need of an efficient guiding strategy and procedure to direct the algorithm toward finding a consensus solution more quickly. We thus introduce a PH metaheuristic with a set of algorithmic and methodological enhancements aimed at hopefully accelerating the search for an efficient implementable solution. These enhancements encompass: (1) a set of population structures to obtain alternative and diverse solutions for the scenario subproblems, (2) a set of novel scenario-selection strategies that effectively derive key insights from subproblem solutions to identify potential consensus, (3) a specialized heuristic to define a high-quality reference solution in the first PH iteration, and (4) a reset procedure to prevent the PH metaheuristic from getting trapped in local optima. This section presents the structure of our proposed PH metaheuristic and the novel strategies developed to accelerate the consensus.

5.1. General structure

The proposed PH metaheuristic, illustrated in Figure 7, follows the general structure of the method proposed by Crainic et al. (2011a). The algorithm starts with the scenario decomposition of the extensive formulation introduced in Section 4.2. This results in a set of subproblems that take the form of a deterministic 2E-MLRPSCD for each scenario $s \in S$. Unlike the standard structure of Crainic et al. (2011a), the proposed PH metaheuristic is set up to define a group of alternative solutions for each scenario subproblem, instead of using the single best solution, aiming to broaden the design options, specifically for location and allocation decisions.

We introduce two population structures to handle the group of alternative solutions for each scenario subproblem: a set of *local populations*, one for each scenario subproblem, and a single *global population* for the complete problem.

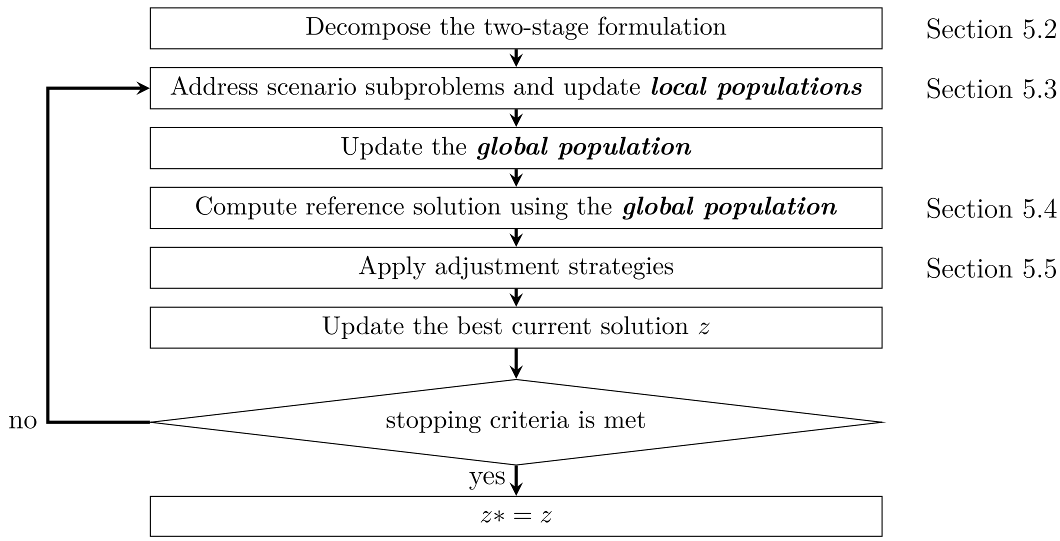

Figure 7 – Progressive Hedging-based metaheuristic for the 2E-MLRPSCD

Each *local population* serves to organize scenario-specific alternative solutions. These *local populations* start empty and are updated at each iteration of the PH with the objective of maintaining the best representative solutions for each scenario subproblem. To assess the value of the solutions obtained for each scenario at each PH iteration, a ranking measure is defined. Each scenario-specific solution is ranked based on its quality and contribution to *diversity*. This ranking is performed with respect to the solutions already present in the *local population* and determines whether a solution should be included in the *local population* at each PH iteration. This ranking prioritizes *diversity* in solutions, favoring those that exhibit the most dissimilarity with respect to the first-stage decision variables compared to other solutions already present in the same *local population*.

The *global population* is constructed at each iteration of the PH, based on the best subset of solutions from each *local population*. A general reference solution is then determined based on a selected subset of solutions from the *global population* defined by one of the proposed scenario-selection strategies. This reference solution is used to guide the search by adjusting the costs in the objective function of each scenario subproblem, aiming to reach a consensus on the first-stage decisions across all scenarios.

Finally, the algorithm ends when a consensus is reached on the first-stage decisions or when external stopping criteria are met, while saving the best feasible solution obtained at each iteration of the PH. In the following sections, we provide a more in-depth description of each step of the proposed PH metaheuristic.

5.2. Scenario decomposition for the 2E-MLRPSCD

The decomposition strategy applied to the extensive formulation, requires the first-stage decisions to be reformulated (detailed reformulation in the supplementary material Appendix B.1). Specifically, these decisions need to be defined as scenario-dependent and constraints must be added to ensure that first-stage variables are the “same” in all scenario-subproblems. Let y_i^s and f_{ijk}^s be the reformulation of the first-stage variables for each scenario $s \in S$, for the location and allocation decisions, respectively. In doing so, constraints (4.7), (4.18) and (4.19) are reexpressed according to the scenario-specific location and allocation first-stage decisions. This reformulation explicitly includes the following non-anticipativity constraints, which prevent the first-stage decision variables to be set to different scenario-specific values:

$$y_i^s = \bar{y}_i \quad \forall i \in Z, s \in S, \tag{5.1}$$

$$f_{ijk}^s = \bar{f}_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S, \tag{5.2}$$

$$\bar{y}_i \in \{0,1\} \quad \forall i \in Z, \tag{5.3}$$

$$\bar{f}_{ijk} \in \{0,1\} \quad \forall i \in P, j \in Z, k \in K. \tag{5.4}$$

The non-anticipativity constraints (5.1) and (5.2) ensure that the first-stage solutions will be the same for all the scenarios, with variables $\bar{y}_i$ and $\bar{f}_{ijk}$ serving as the reference variables for the first-stage decisions. This ensure that a single set of facility location and allocation decisions are made for all the scenarios (thus preventing tailored scenario-specific decisions to be made). Then, following the decomposition scheme, originally proposed by Rockafellar and Wets (1991), constraints (5.1) and (5.2) are relaxed using an augmented Lagrangean method, which results in the following relaxed reformulation of the extensive model:

$$\begin{aligned} \min \sum_{s \in S} \rho_s \Bigg(& \sum_{h \in H} \sum_{(i,j) \in A} \zeta_{ij} x_{ijh}^s + \sum_{k \in K} Ro_k^s + \sum_{i \in Z} \left(F_i + \lambda_i^s + \frac{1}{2}\gamma + \gamma \bar{y}_i \right) y_i^s \\ & + \sum_{i \in P} \sum_{j \in Z} \sum_{k \in K} \left(\Delta_{ijk} + \mu_{ijk}^s + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \right) f_{ijk}^s \Bigg) \end{aligned} \tag{5.5}$$

subject to

$$(4.2) - (4.6)$$

$$(4.8) - (4.17)$$

$$\sum_{h \in H^2} \sum_{j \in C} x_{ijh}^s \leq |H^2| y_i^s \quad \forall i \in Z, s \in S \tag{5.6}$$

$$\sum_{h\in H^1} u^s_{ijkh} = f^s_{ijk} \quad \forall i\in P, j\in Z, k\in K, s\in S \tag{5.7}$$

$$\sum_{h\in H^2} v^s_{zD(k)h} = \sum_{p\in P} f^s_{pzk} \quad \forall z\in Z, k\in K, s\in S \tag{5.8}$$

$$y^s_i \in \{0,1\} \quad \forall i\in Z, s\in S \tag{5.9}$$

$$f^s_{ijk} \in \{0,1\} \quad \forall i\in P, j\in Z, k\in K, s\in S. \tag{5.10}$$

The objective function now involves the Lagrangean multipliers λ^s_i and μ^s_{ijk} for the relaxed non-anticipativity constraints corresponding to the location and allocation decisions, respectively, and a penalty term γ. Constraints (5.6) state that second echelon vehicles can only be used from located satellites. Constraints (5.7) and (5.8) link the facility allocation variables with the vehicle allocation variables. Constraints (5.9) and (5.10) impose the integrality and non-negativity of each decision variables in the model.

For a given overall design $\bar{y}_i$ and $\bar{f}_{ijk}$, the relaxed reformulation then undergoes a scenario-based decomposition (the initial values for the overall design are discussed in Section 5.5). This decomposition yields individual scenario subproblems, each adopting the structure of a deterministic scenario-specific problem with modified fixed costs. For a particular scenario subproblem, the Lagrangean multipliers λ^s_i and μ^s_{ijk}, along with the term γ, penalize the discrepancies between the values of the location and allocation decision in the local design and those present in the current overall design. The following sections examine the proposed strategies to extract the overall design and allocation decisions and the approach to adjust the fixed costs of the scenario subproblem to guide the search toward consensus of the first-stage variables.

5.3. Subproblem algorithm

This section presents the algorithm proposed to address the scenario subproblems. The objective of the proposed subproblem algorithm is twofold, (1) generate a set of candidate solutions to represent each scenario subproblem; (2) rank and define the set of candidate solutions in terms of diversity and quality. In what follows, we describe the strategies that are proposed to achieve these two objectives.

First, we solve the MIP defined for each scenario $s\in S$ to identify a sufficient number of high-quality alternative solutions for each subproblem. The MIP for each scenario subproblem consists of the objective function (5.5), the constraint set: (4.2)-(4.6), (4.8)-(4.17), (5.6)-(5.10), as well as a complete *a priori* enumeration of the subtour elimination constraints.

To handle the set of alternative solutions defined for each scenario subproblem we introduce two types of solution population, a *local population* for each scenario subproblem, and a *global population* for the complete problem. Each *local population* serves to organize the scenario-specific alternative solutions found at each iteration of the PH. It is chraracterized by a total number ψ_T of individual alternative solutions associated with each scenario subproblem, including a reduced number ψ_E of elite solutions.

At each iteration of the PH, our approach involves the individual evaluation of all the feasible solutions found for the MIP defined by each scenario subproblem $s \in S$. This evaluation serves to determine whether an individual solution should be retained in the *local population* of its respective scenario subproblem. To evaluate each scenario-specific solution, we define a ranking measure. This ranking is determined based on the contribution of each individual solution in terms of both quality and diversity relative to the other solutions present in the same *local population*.

The ranking of each individual solution is determined by a *fitness* measure. We define this *fitness* measure by combining both the objective value and the *diversity contribution* of a given solution Sol_i. In this context, the *diversity contribution* or $\Xi(Sol_i)$ refers to the average distance between solution Sol_i and the set of solutions $\mathcal{N}$ present in the respective *local population*, as calculated according to equation (5.11), where $|\mathcal{N}| \leq \psi_T$. This *diversity contribution* aims to favor solutions that exhibit the greatest dissimilarity with respect to the first-stage decision variables when compared to other solutions already present in the same *local population*.

To measure the dissimilarity between two distinct solutions Sol_i and Sol_j, we propose a normalized Hamming distance $\sigma(Sol_i, Sol_j)$, inspired by the work of Vidal et al. (2012). We define $\sigma(Sol_i, Sol_j)$ as a measure of the dissimilarities between: (1) the satellite allocation decisions $\xi_k(Sol_i)$ and (2) the negative correlation score $\phi_k(Sol_i)$ for each OD demand $k \in K$. For each OD pair $k \in K$, the negative correlation score $\phi_k(Sol_i)$ represents the number of different OD pairs allocated to the same satellite that share a negative correlation with k. It is worth noting that the negative correlation score $\phi_k(Sol_i)$ is introduced to take advantage of negative correlations between OD pairs, as suggested by the opportunities arising from consolidating negatively correlated demands and their impact on system efficiency (King and Wallace, 2012). The proposed Hamming distance is defined according to equation (5.12), where $\mathbf{1}$ (*cond*) is an indicator function that returns the value 1 if condition *cond* is true, and 0 otherwise.

$$\Xi(Sol_i) = \frac{1}{|\mathcal{N}|} \sum_{Sol_j \in \mathcal{N}} \sigma(Sol_i, Sol_j) \tag{5.11}$$

$$\sigma(Sol_i, Sol_j) = \frac{1}{2|K|} \sum_{k \in K} \mathbf{1}(\xi_k(Sol_i) \neq \xi_k(Sol_j)) + \mathbf{1}(\phi_k(Sol_i) \neq \phi_k(Sol_j)) \tag{5.12}$$

We then define a *biased fitness* function $BF(Sol_i)$ computed according to equation (5.13), where, $RKQ(Sol_i)$ and $RKD(Sol_i)$ define the ranks of the solution Sol_i with respect to the *local population*, in terms of the objective function (5.5) and the *diversity contribution* $\Xi(Sol_i)$, respectively,

$$BF(Sol_i) = RKQ(Sol_i) + \left(1 + \frac{\psi_E}{\psi_T}\right) RKD(Sol_i). \tag{5.13}$$

This ranking process is performed while solving each scenario subproblem until all *local populations* have been updated. It is important to mention that the *local populations* are designed to be updated rather than being built from scratch at each iteration of the PH. This allows each *local population* to serve as a 'memory' for the PH, as it can retain well-ranked solutions from previous iterations.

5.4. Defining the reference solution

Once the ranking process presented in Section 5.3 is completed, one can build the *global population* of size ψ_G by including all the subset of elite solutions from all *local populations*. Given that all alternative solutions for each scenario subproblem must be considered in the *global population*, we have that $\psi_G \geq \psi_E|S|$. This *global population*, defines the base that is used to obtain the reference solution for the 2E-MLRPSCD in the subsequent steps of the PH metaheuristic.

This section presents four scenario-selection strategies to determine the reference solution at each iteration of the PH. These selection strategies include a classic strategy, which is an adaptation of the original method traditionally used in PH-based methods (Crainic et al., 2009), and three novel selection strategies. Moreover, a specialized heuristic is also introduced to define the reference solution for the first PH iteration. In the following sections, we provide a comprehensive description of each of these strategies.

5.4.1. Classic strategy. This approach represents the steps of the 'classic' selection strategy proposed by Crainic et al. (2011a). Fundamentally, the classic strategy follows the guidelines of the Rockafellar and Wets (1991), by defining an aggregation operator to combine the scenario solutions into a single solution, given a weight for each scenario $s \in S$. This classic strategy defines the reference solution using the single best solution obtained for each scenario subproblem. We describe the classic strategy by means of the population structures introduced in this work (see, Section 5.3) to maintain a consistent notation throughout the paper.

To describe the classic strategy, let Λ_s be the set of alternative solutions present in the *global population* for each scenario $s \in S$. For this scenario-selection strategy, we have that $|\Lambda_s| = 1, \ \forall s \in S$, to emulate the use of the single best solution for each scenario subproblem.

Let ν be the index of iterations performed by the proposed PH metaheuristic. Let $y_{ai}^{s\nu}$ and $f_{aijk}^{s\nu}$ be the value of each alternative first-stage variable $a \in \Lambda_s$ defined for each subproblem associated with each scenario $s \in S$. Similarly, let $\bar{y}_i^{\nu}$ and $\bar{f}_{ijk}^{\nu}$ be the reference solution for iteration ν of the PH. The values of $\bar{y}_i^{\nu}$ and $\bar{f}_{ijk}^{\nu}$ can then be computed by equations (5.14) and (5.15) based on the content of the *global population* at iteration ν and the probability of occurrence ρ_s of each scenario $s \in S$,

$$\bar{y}_i^{\nu} = \sum_{s \in S} \sum_{a \in \Lambda_s} \rho_s y_{ai}^{s\nu} \quad \forall i \in Z, \tag{5.14}$$

$$\bar{f}_{ijk}^{\nu} = \sum_{s \in S} \sum_{a \in \Lambda_s} \rho_s f_{aijk}^{s\nu} \quad \forall i \in P, j \in Z, k \in K. \tag{5.15}$$

Notice that when $\bar{y}_i^{\nu} \in \{0,1\}$, $\forall i \in Z$, and $\bar{f}_{ijk}^{\nu} \in \{0,1\}$, $\forall i \in P, j \in Z, k \in K$, at a given iteration ν, this means that the method has reached consensus for the first-stage decision values. The PH metaheuristic has found thus an implementable solution for the stochastic problem. In most cases, however, the integrality requirements of the first-stage variables are not enforced, i.e., with $0 < \bar{y}_i^{\nu} < 1$ and $0 < \bar{f}_{ijk}^{\nu} < 1$, implying that the current reference solution is infeasible. Although these values are not feasible for the complete stochastic problem, they can still be used to indicate a trend of facility usage and allocation over the system. Therefore, if $\bar{y}_i^{\nu} \approx 0$, then one can interpret this as a trend towards not opening the facility i, whereas $\bar{y}_i^{\nu} \approx 1$ indicates the reverse (i.e., a trend towards opening the facility). Finally, the same observations can be made regarding the reference solution values associated with the allocation decisions.

5.4.2. Probabilistic strategy. Similar to the classic strategy presented in Section 5.4.1, we define an aggregation operator to combine the scenario solution into a single solution. The reference solution is defined by means of the given weights determined by the probability of occurrence ρ_s and the set of alternative solutions Λ_s of each scenario $s \in S$. Unlike the classic strategy, this strategy uses more than one scenario-specific solutions to define the reference solution, meaning that $|\Lambda_s| \geq 1$, $\forall s \in S$. Let $y_{ai}^{s\nu}$ and $f_{aijk}^{s\nu}$ be the value of each alternative first-stage variable $a \in \Lambda_s$ defined for each subproblem associated with each scenario $s \in S$ for iteration ν. The values of the reference solution defined by $\bar{y}_i^{\nu}$ and $\bar{f}_{ijk}^{\nu}$ can then be computed by equations (5.16) and (5.17) based on the content of the *global population* for each scenario $s \in S$ at iteration ν,

$$\bar{y}_i^{\nu} = \sum_{s \in S} \frac{\rho_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} y_{ai}^{s\nu} \quad \forall i \in Z \tag{5.16}$$

$$\bar{f}_{ijk}^{\nu} = \sum_{s \in S} \frac{\rho_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} f_{aijk}^{s\nu} \quad \forall i \in P, j \in Z, k \in K. \tag{5.17}$$

The probabilistic strategy enables the consideration of a broader range of options for the first-stage decision variables available in the *global population* for each scenario subproblem $s \in S$. At the same time, the aggregation operator defined to combine the scenario solution into a single solution is versatile enough to be used with other selection strategies and can be adapted to behave like the classic strategy simply by setting $|\Lambda_s| = 1, \ \forall s \in S$. We, therefore, use the same aggregation operator with the remaining scenario-selection strategies in this section.

5.4.3. Social strategy. The idea behind the social strategy is to define the set of solutions with the best *social score* among the *global population*. In this context, we define $\pi(Sol_i, Sol_j)$ as a normalized Hamming distance, which is computed using equation (5.18). This metric evaluates the similarity between two distinct solutions, Sol_i and Sol_j. We thus define $\chi_z(Sol_i)$ and $\kappa_k(Sol_i)$ as the functions that return the location decision of each $z \in Z$ and allocation of each OD demand $k \in K$, respectively, of a given solution Sol_i,

$$\pi(Sol_i, Sol_j) = \frac{1}{|K|} \sum_{k \in K} \mathbf{1}(\kappa_k(Sol_i) = \kappa_k(Sol_j)) + \frac{1}{|Z|} \sum_{z \in Z} \mathbf{1}(\chi_z(Sol_i) = \chi_z(Sol_j)). \quad (5.18)$$

The *social score* for a given solution Sol_i is defined by summing the values of equation (5.18) between the solution Sol_i and all other solutions Sol_j in the *global population*, where $i \neq j$. Using these *social score* values, the solutions in the *global population* can be ranked. It is important to note that this rank favors solutions that share the most similarities with other solutions, meaning that solutions with the most commonalities in both location and allocation first-stage decisions within the *global population* will be ranked higher.

There are two main approaches for determining the reference solution based on the rank of each scenario subproblem. The first approach involves selecting a reduced set of elite solutions from the complete *global population* to define the reference first-stage decisions. This can be achieved by following the steps defined for the probabilistic strategy, as described in Section 5.4.2. The second approach involves identifying a single elite solution, whose first-stage decisions will be used as the reference solution. However, preliminary experiments conducted using these two strategies have shown that using a single elite solution as the reference solution can cause the PH metaheuristic to become trapped in local optima; consequently, we exclusively utilize the first strategy.

5.4.4. Decision-based scenario clustering strategy. The *decision-based scenario clustering strategy* is proposed to identify scenario groups that lead to mutually acceptable solutions (i.e., solutions that remain efficient when considering all the subproblems associated with the scenarios included in the group). Fundamentally, the proposed strategy uses a *dissimilarity function* inspired by the *opportunity cost*, originally proposed by Hewitt et al. (2022). This *opportunity cost* is defined as a measure to quantify the impact of implementing the first-stage decisions associated with a given scenario s_1 when another scenario s_2 occurs. This measure relies on the existence of a single solution for each of the scenarios involved. This characteristic prevents the direct application of the *opportunity cost* defined by Hewitt et al. (2022) for our PH metaheuristic, which uses a set of alternative solutions for each scenario subproblem to determine the *reference solution*. Therefore, this section introduces the proposed *decision-based scenario clustering strategy* to leverage the alternative solutions associated with each scenario subproblem.

The proposed strategy aims to define a specialized *opportunity cost* measure based on the subset of solutions in the *global population* associated with each scenario. Therefore, let Λ_s be the set of indices of the solutions in the *global population* that are associated with scenario $s \in S$. Let $g_i^{\nu}((\hat{y}_n^{\nu*}, \hat{f}_n^{\nu*}); s_j)$ be the updated value of the objective function (5.5), evaluated with the set of the best first-stage decision variables $\hat{y}_n^{\nu*}$ and $\hat{f}_n^{\nu*}$ at iteration ν, obtained for the solution $n \in \Lambda_{s_i}$ with scenario s_i, when a scenario s_j occurs. The *opportunity cost*, denoted by $\theta^{\nu}(s_i|s_j)$, represents the value of the decision associated with scenario s_i under the assumption that scenario s_j actually occurs. This quantity is calculated using equation (5.19) as the minimum value obtained by evaluating all the combinations of solutions associated with each pair of distinct scenarios s_i and s_j in S with $i \neq j$.

$$\theta^{\nu}(s_i|s_j) = \min_{n \in \Lambda_{s_i};\, m \in \Lambda_{s_j}} \{g_i^{\nu}((\hat{y}_n^{\nu*}, \hat{f}_n^{\nu*}); s_j) - g_j^{\nu}((\hat{y}_m^{\nu*}, \hat{f}_m^{\nu*}); s_j)\} \tag{5.19}$$

Based on the *opportunity costs* determined for each scenario within the *global population*, one can then define an *opportunity cost dissimilarity function* by equation (5.20) for each pair of scenarios $s_i,\ s_j \in S$ with $i \neq j$, which represents the loss incurred by optimizing under the assumption that scenario s_i happens, when scenario s_j occurs instead, and vice versa.

$$d^{\nu}(s_i|s_j) = \theta^{\nu}(s_i|s_j) + \theta^{\nu}(s_j|s_i) \tag{5.20}$$

A *Normalized Spectral Clustering* is then used to determine which scenarios are close to each other in terms of *opportunity cost distance function* building upon the approach proposed by Hewitt et al. (2022). This process yields a set of clusters $CL = \{cl_1, cl_2, \ldots, cl_{|CL|}\}$ of the scenarios. Once the clusters are determined, we define a set of representative scenarios Υ, where each representative scenario corresponds to the *medoid* of each cluster (i.e., the scenario with the minimum average opportunity cost dissimilarity function to all other scenarios within the same cluster). Subsequently, we assign the probability η_i to each representative scenario $s \in cl_i$ for all $cl_i \in CL$, computed as the sum of the probabilities ρ_s of all scenarios within the same cluster, as shown in equation (5.21). Finally, the reference solution for the location and allocation decisions can be determined by computing equations (5.22) and (5.23) for each representative scenario $s \in \Upsilon$ and the subset of solutions Λ_s associated with each scenario.

$$\eta_i = \sum_{s \in cl_i} \rho_s \quad \forall cl_i \in CL \tag{5.21}$$

$$\bar{y}_i^{\nu} = \sum_{s \in \Upsilon} \frac{\eta_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} y_{ai}^{s\nu} \quad \forall i \in Z \tag{5.22}$$

$$\bar{f}_{ijk}^{\nu} = \sum_{s \in \Upsilon} \frac{\eta_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} f_{aijk}^{s\nu} \quad \forall i \in P, j \in Z, k \in K \tag{5.23}$$

5.4.5. First iteration reference solution. A major goal of our PH metaheuristic is to efficiently guide the process searching for solution consensus. An initial reference solution $\bar{y}_i$ and $\bar{f}_{ijk}$ must be defined to enable the scenario-based decomposition of the extensive formulation, as described in Section 5.2. These initial values are used to define the solutions for the scenario subproblems obtained in the first iteration of the PH. However, no 'memory' is available in the PH to assess the quality of these solutions since the *local populations* are empty at this stage. Defining a high-quality reference solution at the end of the first iteration is crucial in this case, considering that the consensus search is performed by successively adjusting the objective function costs of the scenario subproblems to gradually encourage agreement. The quality of the decisions defined in the first iteration will greatly influence the subsequent ones.

We propose an heuristic to define the reference solution to be applied in the first iteration of the PH metaheuristic and the initial *global population*. Let us recall that the *global population* is composed of at least one elite solution picked from the *local population* of each scenario subproblem. The 'quality' of the initial reference solution will thus be a function of both the quality of the solutions present in the initial *global population* and the specific selection strategy that is used to obtain the point.

The proposed heuristic generates an initial *global population* by comparing two independent population generation strategies. Let GP_1 and GP_2 be two independent *populations*, each constructed using one of the proposed heuristic strategies. GP_1 is populated with the set of elite alternative solutions from the *local population* of each scenario subproblem, while GP_2 is populated with the single best solution found for each subproblem. Once GP_1 and GP_2 are populated, we let the given selection strategy determine the reference solution of the first-stage decisions for each population. Notice that the resulting reference solutions may contain decision variables with continuous values. To address this, the proposed heuristic *approximates* each reference solution by rounding each continuous value to the nearest discrete value. Each *approximation* now represents an integer solution for the first-stage decisions, which can be evaluated in the extensive formulation. After evaluating each *approximation*, the proposed heuristic selects the reference solution leading to the best objective function to determine which of the two populations should be defined as the first *global population*.

To keep the solutions used to define the best reference solution in subsequent iterations, one must update each *local population* accordingly, as the *global population* is built from scratch at each iteration. Note that *local populations* are only modified when GP_2 is selected as the best initial *global population*. In such case, each *local population* is updated to retain only the best single scenario-specific solution, rather than the complete set of alternative solutions.

It is worth mentioning that preliminary experiments conducted using the proposed heuristic at each iteration of the PH led to the method relying on the *approximation* of the reference solution as the main guiding strategy. This caused the method to become trapped in local optima. Consequently, after the first iteration, the PH continues to work with the set of alternative solutions for each scenario subproblem.

5.5. Consensus procedure

This section describes the heuristics to adjust the costs of the scenario subproblems aiming to guide the PH method towards a consensus for the first-stage solutions over all the scenario subproblems. We build on the work of Crainic et al. (2011a) and present two adjustment heuristics to modify the location and allocation costs in the scenario subproblems, specifically, a global adjustment designed for the overall search and a local adjustment to influence the search for each scenario subproblem.

The proposed global adjustment begins with the reference solution defined by $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ at iteration ν to identify trends among the scenario solutions. The costs are defined according to the objective function (5.5). In this context, we define the costs $\bar{B}_i^{s\nu} = \left(F_i + \lambda_i^s + \frac{1}{2}\gamma + \gamma\bar{y}_i\right)$ and $\bar{E}_{ijk}^{s\nu} = \left(\Delta_{ijk} + \mu_{ijk}^s + \frac{1}{2}\gamma + \gamma\bar{f}_{ijk}\right)$ as the location and allocation costs of the scenario subproblem, respectively.

As mentioned previously, low values of $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ indicate that most of the scenario solutions share the decision to keep the given facility closed, while high values mean that the facility is open in the majority of the scenario solutions. Therefore, we introduce a parameter $\beta > 1$ as the adjustment rate of the costs, and threshold parameters $0 \leq \epsilon^y \leq 0.5$ and $0 \leq \epsilon^f \leq 0.5$ to determine when the values $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ should be considered either high or low. Specifically, when $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ are lower than ϵ^y and ϵ^f, the fixed costs are increased to incentivize the subproblems to avoid opening the corresponding facility and performing the associated allocation. On the other hand, when $\bar{y}_i^\nu$ and $\bar{f}_{ijk}^\nu$ are higher than $1 - \epsilon^y$ and $1 - \epsilon^f$, the fixed costs are decreased to encourage the subproblems to include the facility in the network design and perform the allocation. We define this procedure with equations (5.24) and (5.25):

$$\bar{B}_i^{\nu} = \begin{cases} \beta B_i^{\nu-1} & \text{if } \bar{y}_i^{\nu-1} < \epsilon^y, \\ \frac{1}{\beta} B_i^{\nu-1} & \text{if } \bar{y}_i^{\nu-1} > 1 - \epsilon^y, \\ B_i^{\nu-1} & \text{otherwise;} \end{cases} \tag{5.24}$$

$$\bar{E}_{ijk}^{\nu} = \begin{cases} \beta \bar{E}_{ijk}^{(\nu-1)} & \text{if } \bar{f}_{ijk}^{(\nu-1)} < \epsilon^f, \\ \frac{1}{\beta} \bar{E}_{ijk}^{(\nu-1)} & \text{if } \bar{f}_{ijk}^{(\nu-1)} > 1 - \epsilon^f, \\ \bar{E}_{ijk}^{(\nu-1)} & \text{otherwise.} \end{cases} \tag{5.25}$$

The second adjustment strategy is performed at the level of each scenario subproblem $s \in S$, where the costs of variables with large differences between the value of the current reference solution at iteration ν, are further adjusted using the equations (5.26) and (5.27). In this context, we define $0.5 < \delta^y < 1$ and $0.5 < \delta^f < 1$ as the thresholds that prescribe when a local adjustment has to be applied for the location and allocation variables, respectively:

$$\bar{B}_i^{s\nu} = \begin{cases} \beta B_i^{\nu} & \text{if } |y_i^{s(\nu-1)} - \bar{y}_i^{\nu-1}| \geq \delta^y \text{ and } y_i^{s(\nu-1)} = 1, \\ \frac{1}{\beta} B_i^{\nu} & \text{if } |y_i^{s(\nu-1)} - \bar{y}_i^{\nu-1}| \geq \delta^y \text{ and } y_i^{s(\nu-1)} = 0, \\ B_i^{\nu} & \text{otherwise;} \end{cases} \tag{5.26}$$

$$\bar{E}_{ijk}^{s\nu} = \begin{cases} \beta \bar{E}_{ijk}^{\nu} & \text{if } |f_{ijk}^{s(\nu-1)} - \bar{f}_{ijk}^{(\nu-1)}| \geq \delta^f \text{ and } f_{ijk}^{s(\nu-1)} = 1, \\ \frac{1}{\beta} \bar{E}_{ijk}^{\nu} & \text{if } |f_{ijk}^{s(\nu-1)} - \bar{f}_{ijk}^{(\nu-1)}| \geq \delta^f \text{ and } f_{ijk}^{s(\nu-1)} = 0, \\ \bar{E}_{ijk}^{\nu} & \text{otherwise.} \end{cases} \tag{5.27}$$

Given that there is no reference solution in the original extensive formulation (Section 4.2), we set the values of the initial overall design variables $\bar{y}_i$ and $\bar{f}_{ijk}$ to determine the initial fixed costs for each scenario subproblem. We, therefore, define the initial overall design in terms of the location costs $\bar{B}_i^{s\nu}$ and allocation costs $\bar{E}_{ijk}^{s\nu}$ at iteration $\nu = 0$ (i.e., before the PH starts its first iteration). The location and allocation costs of the scenario subproblems are initialized with their original costs. Therefore, we set $\bar{B}_i^{s(0)} = F_i,\ \forall i \in Z, s \in S$, and $\bar{E}_{ijk}^{s(0)} = \Delta_{ijk},\ \forall i \in P, j \in Z, k \in K, s \in S$. Note that the values of the initial overall design $\bar{y}_i$ and $\bar{f}_{ijk}$ will be updated based on the reference solution obtained at the end of the first iteration of the PH to adjust the costs for each scenario subproblem.

The proposed PH is designed to terminate once either a consensus solution is found, or another stopping criterion is reached (e.g., a limit on computation time). A consensus solution is determined when all first-stage decisions $\bar{y}_i^{\nu}$ and $\bar{f}_{ijk}^{\nu}$ have reached a general consensus at a given iteration ν. However, consensus on all first-stage decisions may not be observed at the end of each iteration of the PH metaheuristic. When such a situation occurs, the PH is designed to define a feasible solution for the 2E-MLRPSCD by using the extensive formulation presented in Section 4.2. The approach to define a feasible solution consists of fixing the location and allocation variables for which consensus is obtained by the PH metaheuristic, and then solving the restricted mixed-integer program defined by the extensive formulation. The results of solving the proposed formulation yield a feasible solution for all the design decisions. One can then update the best solution obtained and continue with the PH metaheuristic.

5.6. Reset procedure

As described previously, the proposed PH algorithm relies on the *global population* to determine the reference solution at each iteration. This *global population* is constituted by the collection of elite solutions from the *local population* of each scenario subproblem. As the search progresses, certain solutions may remain in the *local population* of each scenario subproblem for several iterations of the PH. Consequently, the *global population* may also end up comprising the same set of solutions over consecutive iterations of the PH. If the *global population* remains unchanged over several iterations, the reference solution may become trapped on a series of values that hinder the overall search for a consensus solution. To mitigate such occurrences, we propose a reset procedure that partially reinitializes the overall search process. Specifically, the reset procedure is triggered when the values of the reference solution do not change for ι consecutive iterations. When triggered, the reset procedure clears the contents of all *local populations* and repopulates them with solutions from their corresponding scenario subproblems obtained in the ongoing iteration. It is important to note that while this process defines a new set of alternative solutions for the *global population*, the current costs corresponding to the first-stage decisions in each scenario subproblem, which were updated over the previous PH iterations, remain unchanged.

6. Computational results

This section presents the results of the computational experiments that were conducted to assess: (1) the stability of the scenario generation procedure, (2) the performance of the proposed PH-based metaheuristic, (3) the effectiveness of the proposed acceleration procedures for the 2E-MLRPSCD and (4) evaluate the need to explicitly consider stochastic demands when solving the considered problem setting. We first introduce the instances and the scenario generation procedure used in the computational study in Section 6.1. The computational experiments to evaluate the stability of the scenario generation process are presented in Section 6.2. The results of a series of experiments illustrating the performance

category	distribution	mean	standard deviation
CA	left-skewed lognormal	2.7	0.4
CB	symmetrical lognormal	2.7	0.1
CC	left-skewed lognormal	3.25	0.4
CD	symmetrical lognormal	3.25	0.1

Table 6 – Instance category description

of the proposed PH metaheuristic and the effects of problem instance characteristics are then analyzed in Section 6.3, followed by the value of the stochastic solution in Section 6.4.

The experiments were conducted on a single machine with Intel(R) Core(TM) i7-7800X processor, with 128 GB of RAM running Linux. The mathematical formulation and the proposed solution method are implemented in C++ using IBM ILOG CPLEX concert technology 20.1. The MIPs used within the solution method were solved with an optimality gap tolerance of 1% as the stopping criterion. Finally, the computation times reported are in seconds. The tables of this section display the summarized results for the associated experiments, while more detailed results are provided in the supplementary material in Appendix B.2.

6.1. Instances

We define our testbed based on the instances introduced by Dellaert et al. (2019) for the 2E-VRPTW, since no instances were available in the literature involving the integrated treatment of all of the attributes considered in the 2E-MLRPSCD. The instances introduced by Dellaert et al. (2019) simulate an urban area constituted of platforms, satellites, and customers. The original instances generated do not consider stochastic correlated OD demands, which are explicitly included in the 2E-MLRPSCD. Furthermore, the original instances included delivery time windows, which are not considered in the present setting. Therefore, adjustments were made to the original instances to obtain the testbed for the present study. These adjustments involved the introduction of stochastic and correlated OD demands and the exclusion of the temporal components in the original instances.

Our instance set consists of 60 instances, each with 15 OD demands. We randomly assigned to each platform facility a unique set of OD demands. The same load capacities for vehicles set in Dellaert et al. (2019) were used. The first-level vehicles have thus a capacity of $cap_1 = 200$ and the second-level vehicles have a capacity of $cap_2 = 50$. Travel costs are computed as the ceiling of the Euclidean distances.

Scenarios are generated using the copula-based method proposed by Kaut (2014) to adhere to the statistical properties defined for the stochastic OD demands. This procedure requires the target marginal distribution for each OD demand (which can be specified using

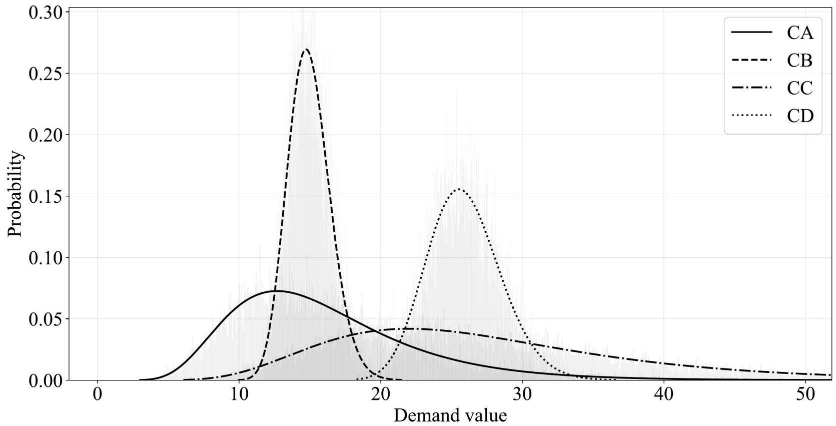

Figure 8 – Instance category distribution for scenario generation.

a set of marginal distributions available in the method), and the correlation matrix between OD pairs as inputs.

To determine the marginal distribution for all OD demands, we considered the original demand values in the set of instances defined by Dellaert et al. (2019). We identified the distribution that provided the closest fit (among the marginal distributions available in the copula-based method) to the value of the OD demands in the complete instance set. This led us to select a lognormal distribution (with similar mean and standard deviation values) as the best fit for the given demand values. We then defined a set of four lognormal distributions with different mean and standard deviation values to capture the impact on the variation of the marginal distributions representing the demand. Table 6 introduces the proposed instance set, categorized into four groups: CA, CB, CC, and CD. As illustrated in Figure 8, a lognormal distribution with consistent mean and standard deviation values, as defined in Table 6, is utilized for each instance category.

To define demand correlation, two testbed instances are proposed: one considering demand correlation and the other without demand correlation. In the correlated case, correlation matrices are randomly generated. Correlations between OD pairs are determined using a standard normal distribution with a range of [-0.6, 0.6] for each correlation value.

A properly defined correlation matrix must be positive semidefinite (Xu and Evers, 2003). Before applying the scenario generation method, this condition is verified for each correlation matrix obtained (i.e., correlation matrices that do not meet this condition are ignored by the copula-based method). Scenarios are generated, once the positive semidefinite condition is verified. The copula-based heuristic uses the mean and standard deviation of the distributions for each instance category and the correlation matrix defined for each instance to generate a predefined number of scenarios $|S|$ with equal probability. This means that the probability of occurrence ρ_s for scenario $s \in S$ is $\rho_s = 1/|S|$.

6.2. Scenario stability

This section presents the computational experiments conducted to assess the stability of the chosen scenario generation procedure with and without demand correlation. Assessing scenario stability aims to guarante that there is no significant influence by the scenario trees utilized with respect to the results obtained when solving the considered stochastic problem (Kaut and Wallace, 2007). In our case, we used the copula-based method introduced by Kaut (2014) to generate the scenario set. This method, in contrast to other ones (such as sampling methods), has a high probability of producing identical scenario trees when consecutive runs are conducted with the same correlation and distribution inputs. Using 'standard' in-sample and out-of-sample stability tests (see, Kaut and Wallace, 2007) are inappropriate, as these stability tests could overestimate the quality of the scenario generation method (Guo et al., 2019). Therefore, our scenario stability tests build on the work of Zhang et al. (2021) to derive a valid variant of the 'standard' approach for our problem setting.

Based on the guidelines proposed by Zhang et al. (2021), stability tests require creating and evaluating a subset of scenario trees for each problem instance, with a fixed scenario tree size. To test the stability of a scenario tree of size $|S|$, it is necessary to define a set of $2m+1$ scenario trees of sizes $|S|-m$, $|S|-(m-1)$,...,$|S|$,...,$|S|+m$, where m is a positive integer. Let $Z_{|S|+i}$ denote the optimal (or best-known) solution for each $i \in [-m,m]$ of the $2m+1$ scenario trees obtained for each problem instance. The proposed PH metaheuristic is used to solve the 2E-MLRPSCD resulting from each of the $2m+1$ scenario trees, resulting in $2m+1$ solutions $Z_{|S|+i}$, one for each scenario tree. These solutions are then evaluated by calculating the objective function $F(Z_{|S|+i})$ for each of the $2m+1$ scenario trees, yielding a set of $2m+1$ objective function values for each solution $Z_{|S|+i}$. Finally, for each problem instance, the maximum ($F^{+}(Z_{|S|+i})$), minimum ($F^{-}(Z_{|S|+i})$), and variance ($\sigma_{|S|+i}$) are defined based on the objective function values of each solution $Z_{|S|+i}$. Stability is then determined by computing the relative difference (RD) between the maximum and minimum values and the variance (VAR) of each scenario tree, as follows:

$$RD = \max_{i\in[-m,m]} \left\{ \frac{F^{+}(Z_{|S|+i}) - F^{-}(Z_{|S|+i})}{F^{+}(Z_{|S|+i})} \times 100\% \right\} \tag{6.1}$$

$$VAR = \max_{i\in[-m,m]} \{\sigma_{|S|+i}\} \tag{6.2}$$

Table 7 presents a summary of the relative difference (RD) and variance (VAR) values obtained for the $2m+1$ scenario trees defined for each problem instance. The table shows the number of scenarios for each scenario tree ($|S|$), as well as the minimum (MIN), average (AVR), and maximum (MAX) values for the relative difference and variance. In order to

$\|S\|$	RD (%)					VAR		
	VSR	**AISR**	**MIN**	**AVERAGE**	**MAX**	**MIN**	**AVERAGE**	**MAX**
10	38	3.96	0	1.79	8.58	0	1041.89	15222.89
20	47	3.17	0	1.13	5.93	0	335.62	3351.09
30	54	2.65	0	0.82	3.55	0	190.04	1944.3
40	53	2.25	0	0.58	2.25	0	108.05	1142.38
50	53	2.01	0	0.45	2.01	0	74.62	933.34
100	60	n.a	0	0.28	1.44	0	32.52	525.93

Table 7 – Stability tests: summarized results of relative difference and variance for different scenario sizes.

assess stability, a criterion of RD $\leq 2\%$ is defined. Two additional performance measures are used to present the number of instances satisfying the stability criterion (valid stability requirement or **VSR**) and the average RD of the instances failing to meet the criterion (average invalid stability requirement or **AISR**). Experiments are performed using multiple scenario trees with varying numbers of scenarios ($|S|$) and m set to 4, based on the work of Guo et al. (2019). To reduce noise in the results, only the best objective function obtained for each instance by the proposed PH metaheuristic is used for the stability tests.

The results reported in Table 7 show the expected reduction of the relative difference with respect to the increased number of scenarios considered. It is worth mentioning that due to the randomness and heuristic nature of the scenario generation procedure, small fluctuations exist in the relative error for some instances, which do not affect the validity of the proposed stability tests. In general, the use of 30 scenarios represents the best balance between solution stability and scenario size, as it provides a relative error of less than 2% for 53 out of 60 instances, with an average relative error of 2.6% for the remaining instances. Although larger scenario trees are desirable for achieving a smaller relative error, solving the resulting subproblems for the entire instance set using CPLEX at each iteration of the PH metaheuristic becomes exceedingly challenging.

Figure 9 displays the relative difference values of each instance type as a function of the number of scenarios used for the stability testing. One can observe that the dispersion of the demand distributions significantly affects the stability of solutions. Notably, instances of type CA and CC, characterized by more dispersed demand distributions, exhibit more fluctuations in the relative difference values. This behavior can be attributed to the copula-based method generating a diverse set of scenarios, leading to increased volatility in the objective function values and greater fluctuations regarding the recourse actions.

In conclusion, the results presented in Table 7 indicates that using $|S| = 30$ achieves the best balance between solution stability and the resulting complexity of the two-stage formulation. Additionally, Figure 9 shows that increasing the number of scenarios beyond $|S| = 30$ only marginally improves the relative difference percentage in terms of stability. This in turn indicates that using scenario trees with $|S| > 30$ is not favorable due to the

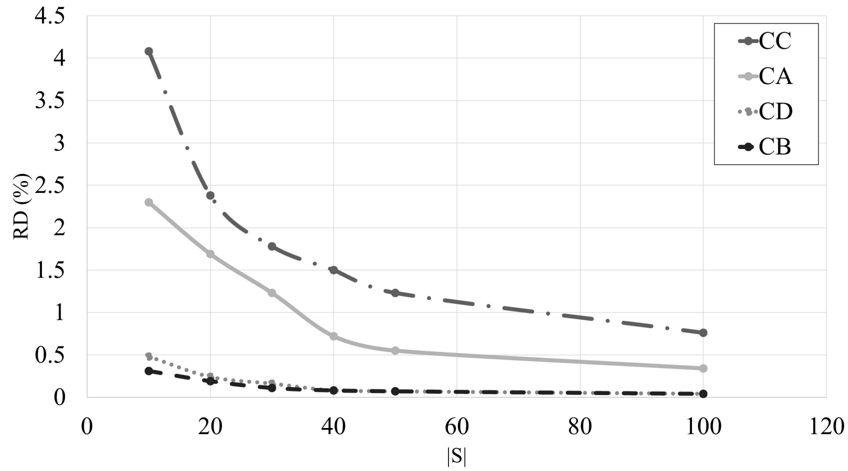

Figure 9 – Stability test: Relative difference for each instance type vs scenario size.

significantly increased computational burden required to address this greater number of scenarios at each iteration of the PH metaheuristic.

6.3. Performance of the PH metaheuristic

This section presents a performance analysis of the proposed PH metaheuristic. Computational tests were conducted to compare the performance of the PH metaheuristic to that of CPLEX when solving the complete stochastic model. The results presented in this section focus on the quality of the upper bound obtained and the computational time needed by each solution method. The stopping criteria for all solution methods were set to a maximum running time of 2 hours. Additionally, the PH metaheuristic was limited to a maximum of 60 iterations. CPLEX was used with the default parameter settings, with a thread limit of 6 imposed when solving the overall stochastic model and a thread limit of 1 specified when solving the scenario subproblems within the PH metaheuristic.

To address the stochastic problem, computational tests were conducted by solving the complete two-stage stochastic model with CPLEX or by employing the proposed PH metaheuristic. Additionally, experiments were performed using a *classic strategy* as the baseline for the PH metaheuristic. This approach represents the steps of the 'classic' PH metaheuristic proposed by Crainic et al. (2011a) (Section 5.4.1). In all tables, the results obtained using CPLEX and the PH metaheuristic are labeled as 'CPLEX' and 'PH', respectively. Moreover, the PH metaheuristic results are differentiated based on the specific version of the procedure used that is: the classical approach (CL), the probabilistic strategy (PS), the social strategy (SS), and the cluster-based strategy (CBS). Each table presents the average optimality gap expressed as a percentage (OG), the average computational time in seconds, and the average upper bound differences between PH and CPLEX (Diff. UB).

Table 8 presents the results of solving instances with no demand correlation. One can clearly observe that solving the overall stochastic problem using CPLEX is challenging. CPLEX achieves an average optimality gap of 21% within the time limit of 2 hours. The PH metaheuristic outperforms CPLEX. The classic approach (CL), achieves an average improvement of 14.5% in solution quality over CPLEX. Moreover, the classic approach is able of reaching consensus and generating high-quality upper bounds for 53 of the 60 instances within the 2-hour limit. One notices that the exclusive use of the best quality solutions of each scenario subproblem to define the reference solution, is not effective enough to reach consensus over the complete set of first-stage decisions. This general adverse effect is particularly evident when analysing the results obtained when solving the CA and CC instances. For these instances, the demand values are sparse, which leads to scenario subproblems which, when solved, tend to produce more diverse first-stage decisions.

Compared to the classic approach, the proposed strategies can achieve consensus for the entire set of instances. The probabilistic strategy, which builds on the classic approach, demonstrates significant improvements in both solution quality and runtime. Specifically, the probabilistic strategy achieves an average improvement of 16.2% compared with CPLEX and a 35.3% decrease of average runtime compared with the classic approach. To reach consensus efficiently, our PH metaheuristic benefits from including alternative solutions for each scenario subproblem. This approach increases the number of complementary first-stage decisions that are used to define the aggregation at each iteration. Notwithstanding the general improvements made by PH utilising the probabilistic strategy, the social and cluster strategies are able of leveraging more efficiently the existing alternative solutions to further improve the overall performance of the algorithm.

The social strategy consistently yields reduced runtimes, with the largest average decrease of 68.3% compared to the classic approach. The consensus-driven approach, which ranks the global population, is also able to help PH metaheuristic reaching consensus faster and cut down on computation time. However, reaching general consensus faster does not necessarily guarantee good-quality solutions. An illustration of this can be seen when comparing the results obtained with the cluster-based strategy to those obtained with the social strategy. Although the runs of the PH using the social strategy produces results, on average, 40% faster, the use of the cluster-based strategy results in consensus solutions of higher quality. On average, the optimality gap achieved by the PH metaheuristic with the cluster-based strategy is 2.6%, compared to 6.8% when using the social strategy.

Computational tests on instances with demand correlation are reported in Table 9. Similar to the results obtained on instances with no demand correlation, solving the complete two-stage formulation with CPLEX leads to the worst optimality gap while reaching the maximum time limit on all instances. On the other hand, the classic approach presents a significant performance improvement, where consensus is reached for 40 out of the 60

Instance type	CPLEX		PH											
			CL			PS			SS			CBS		
	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)
CA	30.91	7200	-7.02	28.24	4458.16	-13.54	23.71	1323.66	-38.65	8.09	884.99	-44.98	3.76	2295.22
CB	23.66	7200	-32.54	1.54	3568.78	-32.54	1.54	1350.34	-32.62	1.47	1081.83	-32.66	1.44	1239.97
CC	17.42	7200	-9.07	10.07	4651.69	-9.30	9.87	4243.97	-9.04	10.21	2326.02	-19.35	1.95	2442.96
CD	15.79	7200	-9.19	8.11	4520.49	-9.32	8.01	4210.59	-10.11	7.38	1151.61	-14.92	3.47	2200.99
Averages	***21.94***	***7200***	***-14.46***	***11.99***	***4299.78***	***-16.17***	***10.78***	***2782.14***	***-22.61***	***6.79***	***1361.11***	***-27.98***	***2.65***	***2044.79***

Table 8 – Summarized results on instances with no demand correlation.

Instance type	CPLEX		PH											
			CL			PS			SS			CBS		
	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)	diff. ub	OG (%)	time (s)
CA	28.70	7200	1.49	29.77	7200.00	-29.94	10.14	2393.54	-38.05	4.71	2178.57	-42.06	2.35	1663.38
CB	24.35	7200	-33.52	0.86	4071.12	-33.76	0.67	1384.67	-33.76	0.67	1989.33	-33.88	0.58	1920.12
CC	13.03	7200	-11.17	3.57	4622.19	-12.50	2.47	2095.32	-12.71	2.29	2167.33	-13.64	1.48	2091.91
CD	11.77	7200	-7.76	5.10	4809.67	-10.45	2.84	1609.31	-11.35	2.02	2061.57	-11.87	1.56	1781.87
Averages	***19.46***	***7200***	***-12.74***	***9.82***	***5175.74***	***-21.66***	***4.03***	***1870.71***	***-23.97***	***2.42***	***2099.20***	***-25.36***	***1.49***	***1864.32***

Table 9 – Summarized results on instances with demand correlation.

instances within the given time limit. The probabilistic and social strategies both show considerably improved performances in solution quality when compared to CPLEX, with an optimality gap of 4% and 2.4%, respectively. That being said, the cluster-based strategy outperforms all selection strategies in terms of both time and solution quality. It obtains the best solutions for 50 out of the 60 instances with an average runtime reduction of 64% when compared to the classic approach.

The performance of the PH metaheuristic with each proposed aggregation strategy shows significant variations when tested on instances with and without demand correlation. Scenario trees generated assuming that demands are entirely uncorrelated often result in scenarios with a large number of high demand values. These scenarios have more predominant solution structures, as the first-stage decisions defined under high demand values are more likely to fit scenarios with lower demand values. This effect is less likely to occur for instances where negative demand correlation is considered. The likelihood of producing scenarios with high demand values decreases as the degree of negative correlation between demands increases. As a result, there is increased diversity in demand values for the complete set of scenarios. This diversity leads to a more varied set of first-stage decisions, which, in turn, poses challenges for the PH metaheuristic to reach consensus. This general effect explains the improved performance of the proposed acceleration strategies when demand correlation is considered.

6.4. Value of the stochastic solution

This section reports on the value of the stochastic solution (VSS), which is a bound to assess the added value of using the stochastic model compared to the deterministic formulation for the 2E-MLRPSCD problem. Experiments are performed using both instances with and without demand correlation. Following the general trend in the literature, we use the

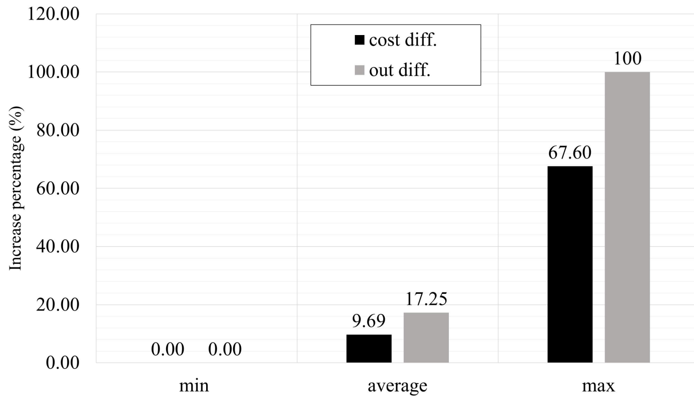

Figure 10 – Comparison of the deterministic versus the stochastic formulation of the 2E-MLRPSCD on instance with demand correlation.

deterministic formulation (DF) with the mean approximation of the demand, where the stochastic demands are estimated as their mean values obtained from the scenario sets that are considered. The integrated design and routing decisions are then determined based on the average value of demands. Results for the deterministic formulation of the 2E-MLRPSCD are obtained by solving each instance using CPLEX under a 2-hour time limit. The computational tests conducted using the DF are then compared to the solutions obtained by solving the 2E-MLRPSCD (i.e., the results reported in Section 6.3).

Feasible solutions for all instances can be obtained by using the PH metaheuristic and the DF. Therefore, we conduct experiments by comparing the results obtained from the DF approach against the stochastic approach with respect to the general cost increase percentages. Figures 10 and 11 present the results of these experiments by illustrating the increasing percentage of the objective function (Cost Diff.) and the use of outsourced services (Out Diff.) for the instances with and without demand correlation, respectively. The results are organized to depict the minimum, average, and maximum cost increase associated with the solutions of the deterministic formulation, using the stochastic approach as the baseline. Tables 10 and 11 present the density of location and allocation decisions on satellite facilities for each approach and each instance type for instances with and without demand correlation, respectively. Each table shows the number of satellites ($|Z|$) for each instance type, and the average value of increased percentages of the objective function (Cost Diff.) of the DF against the PH metaheuristic. Moreover, two additional measures are presented at each table to quantify the spatial homogeneity of the distribution of located satellite facilities and customer allocation to them. These measures correspond to: 1) the *satellite location density* (SLD), which represents the average number of open satellites for each instance type, and 2) the maximum and minimum *customer allocation density* (CAD), which represents the average number of customers assigned to each open satellite.

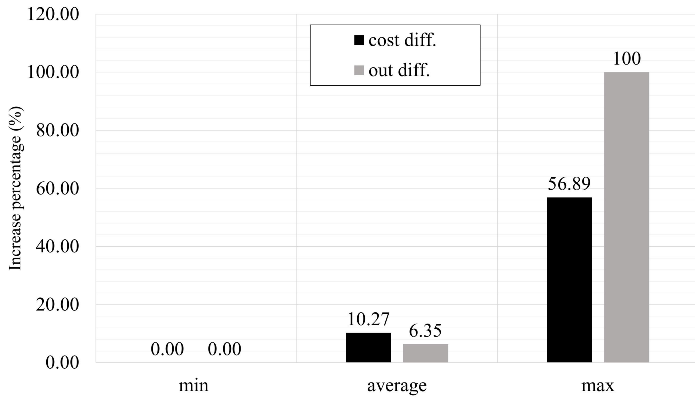

Figure 11 – Comparison of the deterministic versus the stochastic formulation of the 2E-MLRPSCD on instance with no demand correlation.

Instance type	$\|Z\|$	cost diff.	PH			DF		
			SLD	CAD		SLD	CAD	
				max	min		max	min
CA	3	14.43	2.60	10.40	2.00	1.60	13.40	1.60
	5	47.46	2.60	5.60	3.80	1.20	12.00	3.00
	4	39.34	2.60	9.00	0.20	1.00	15.00	0.00
CB	3	0.00	1.20	12.00	3.00	1.20	12.00	3.00
	5	0.28	1.20	12.00	3.00	1.20	12.00	3.00
	4	0.00	1.00	15.00	0.00	1.00	15.00	0.00
CC	3	0.75	3.00	6.40	3.00	3.00	5.00	5.00
	5	2.07	3.60	5.40	0.40	3.20	5.00	0.40
	4	4.93	3.80	6.20	1.60	4.00	5.00	2.20
CD	3	3.53	3.00	6.20	4.00	3.00	5.00	5.00
	5	1.97	3.60	5.60	1.40	3.60	4.80	1.40
	4	4.43	3.40	6.20	1.20	4.00	5.00	2.40
Averages		***9.93***	***2.60***	***8.33***	***1.97***	***2.33***	***9.10***	***2.25***

Table 10 – Location/allocation density by instance-types with demand correlation

The results presented in Figure 10 and Figure 11 indicate that the stochastic formulation consistently outperforms the deterministic formulation in terms of overall solution quality. The deterministic formulation incurs significantly higher design costs and outsourced services. This discrepancy can be attributed to the fact that the deterministic formulation, fails to capture important demand variations, resulting in higher expected costs. This is particularly evident in design planning decisions, where the deterministic formulation produces facility configurations that are insufficient to conduct the necessary vehicle operations during the second stage for all scenarios. This issue is evident in both instance sets, where the deterministic formulation used on average 17% and 6% more outsourced services for instances with and without demand correlation, respectively.

Instance type	$\|Z\|$	Dif. UB	PH			DF		
			SLD	CAD		SLD	CAD	
				MAX	MIN		MAX	MIN
CA	3	16.64	1.20	12.40	2.00	1.60	13.40	1.60
	5	37.24	1.20	13.20	1.80	1.20	12.00	3.00
	4	35.09	2.60	9.60	0.20	1.00	15.00	0.00
CB	3	0.06	1.20	13.20	3.00	1.20	13.20	3.00
	5	0.00	1.20	13.20	3.00	1.00	13.20	3.00
	4	0.05	1.00	15.00	0.00	1.00	15.00	0.00
CC	3	7.15	3.00	7.80	3.00	3.00	5.00	5.00
	5	5.42	2.8	8.60	2.20	3.20	5.00	0.40
	4	9.32	2.80	6.80	2.60	4.00	5.00	2.20
CD	3	0.39	3.00	8.60	3.00	3.00	5.00	5.00
	5	4.46	3.20	6.40	3.20	3.60	4.80	1.40
	4	7.48	3.20	7.00	2.00	4.00	5.00	2.40
Averages		***10.27***	***2.20***	***10.15***	***2.17***	***2.32***	***9.30***	***2.25***

Table 11 – Location/allocation density by instance-types with no demand correlation

Distributions covering a wide range of low demand values, exemplified by instance type CA, demonstrate a clear distinction between instances with and without demand correlation. When demand correlation is considered for instance type CA, there is a tendency to employ more satellite facilities to accommodate the greater diversity of scenarios. In contrast, instances without demand correlation tend to utilize fewer satellite facilities, focusing on addressing scenarios with high demand values within the scenario set, which are typically low when compared to other instance types. For instance types CC and CD, where distributions yield higher demand values, the results usually show a high number of open satellite facilities with a more homogeneous set of customer allocations. Regardless of demand correlation, high demand values force both approaches to lean towards higher satellite usage to handle the high-value demand variations. Interestingly, a very narrow and low-value demand distribution, as in instance type CB, allowed the deterministic (based on the use of the average demands) approach to better approximate the demand distribution, which in turn lead to the deterministic approach to produce results comparable to those of the stochastic approach. These observations hold even when demand correlation is not considered, with a general increase in customer allocation density across each instance set due to reduced demand variability within each scenario set.

One can conclude that the stochastic approach addressed by the proposed PH metaheuristic is generally more cost-effective for both design and routing decisions. The deterministic formulation approach produces solutions that lack operational efficiency, especially for the second stage, since it does not sufficiently account for uncertainty at the design planning stage. Unless the demand distribution is narrow and low enough, the deterministic formulation approach proves unsuitable for designing distribution networks with uncertain demands, with or without correlation. Therefore, a stochastic approach should be used to warrant an effective distribution system design involving location routing decisions under uncertainty.

7. Conclusions

We introduced the two-Echelon multicommodity location-routing problem with stochastic and correlated Demands (2E-MLRPSCD). The problem is formulated as a two-stage stochastic program where, the location of satellite facilities and the customer-to-satellite allocation decisions are made in the first stage, while the vehicle routes for both echelons are decided in the second stage, when customers demands are observed. To address the proposed two-stage model, we present a specialized PH-based metaheuristic with a series of novel enhancements. These include: 1) population structures of alternative and diverse solutions for the scenario subproblems; 2) strategies to define the reference solutions, which are used to guide the overall search; and 3) a reset procedure that reduces the risk of the method becoming trapped in local optima.

A series of numerical experiments were performed, involving a set of instances with varying characteristics, which computationally showed that the proposed enhancements significantly improved the overall performance of the PH method built for the 2E-MLRPSCD, both in terms of the quality of the solution obtained and the computation times of the algorithm. Moreover, the numerical results also clearly showed the added value of explicitly considering the uncertainty in demand and its interrelations. The solutions obtained by solving the stochastic problem outperformed the ones obtained by applying a deterministic approximation approach (where the average values were used for the customer demands).

Several interesting avenues for future research may be identified. There is a need to design novel heuristic and exact methods to more efficiently address the set of scenario subproblems that must be solved at each iteration of the PH metaheuristic. There are also interesting extensions to the considered problem that could be studied. Specifically, solving the problem with additional sources of uncertainty (e.g., travel times uncertainty) would certainly be worthwhile for a wide gamut of applications.

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Third Article.

The multi-attribute two-echelon location-routing problem with stochastic travel times

by

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The main contributions of David Escobar Vargas for this articles are presented.

— *David Escobar Vargas:* Conceptualization, Methodology, Software, Data analysis, Writing - original draft.

— *Teodor Gabriel Crainic:* Conceptualization, Methodology, Writing - review & editing.
— *Walter Rei:* Conceptualization, Methodology, Writing - review & editing.

RÉSUMÉ. Cet article présente et étudie le problème de localisation-routage à deux échelons et à attributs multiples avec des temps de trajet stochastiques. Le problème est formulé à l'aide d'une approche de programmation stochastique en deux étapes, qui intègre efficacement les interactions entre plusieurs attributs. Dans la première étape, les décisions de conception des plateformes et des installations satellites sont prises en compte, tandis que la deuxième étape consiste à déterminer les décisions de routage de la demande sur la base des temps de trajet observés. Pour traiter le problème stochastique complet, un cadre de couverture progressive (PH) est proposé. En outre, une heuristique basée sur la décomposition est introduite pour accélérer le cadre PH, et deux nouvelles stratégies d'agrégation sont présentées pour accélérer le processus de consensus concernant les décisions de la première étape. Des expériences informatiques complètes sont effectuées pour évaluer l'efficacité de la métaheuristique basée sur la décomposition dans la résolution de problèmes déterministes multi-attributs riches. Des analyses comparatives démontrent que le cadre de PH proposé, ainsi que les deux nouvelles stratégies d'agrégation, sont plus performants que les approches les plus récentes lorsqu'ils prennent en compte des temps de trajet stochastiques.
Mots clés : Problème de localisation-routage à deux échelons et à attributs multiples, temps de déplacement stochastiques, couverture progressive.

ABSTRACT. This paper investigates the two-echelon multi-attribute location-routing problem with stochastic travel times. The problem setting we address revolves around the interplay between uncertain travel times and multiple interacting attributes. The problem is formulated as a two-stage stochastic program, with the first stage handling design decisions for facilities, while the second stage determines routing decisions based on observed travel times. A Progressive-Hedging (PH) metaheuristic is proposed to address the stochastic formulation. To tackle the inherent complexity stemming from the multi-attribute nature of the problem and the stochastic considerations, we introduce a decomposition-based heuristic and two novel scenario-selection heuristic strategies to enhance the effectiveness of the proposed PH metaheuristic and accelerate the exploration of the solution space. Comprehensive computational experiments are conducted to assess the effectiveness of the metaheuristic in addressing rich multi-attribute stochastic problem settings. Comparative analyses show that the proposed solution method, outperforms state-of-the-art approaches when considering stochastic travel times.
Keywords: Two-Echelon Multi-Attribute Location-Routing Problem, stochastic travel times, progressive hedging

1. Introduction

The *Two-Echelon Location Routing Problem* (*2E-LRP*) is a well-established modelling framework in logistics and supply chain management (Sluijk et al., 2022). The problem addresses both design and routing decisions for distribution systems encompassing two distinct hierarchical tiers: primary *platform* facilities (distribution centers) constituting the first echelon, and end *customers* (demand points) making up the second echelon. The core concept of the 2E-LRP involves using intermediary facilities called *satellites* to consolidate and distribute goods rather than directly serving customers from platforms. The 'standard' definition of the 2E-LRP is composed of (1) design decisions, which consist of selecting/opening a number of platform and satellite facilities; (2) the allocation of customers to the selected facilities; and (3) routing decisions, which concern the definition of a limited set of vehicle routes on both tiers to meet customer demands using the system design. The objective of this 'standard' 2E-LRP problem consists of minimizing the total system cost defined by the fixed cost associated with the design and assignment decisions and the total distribution cost, while respecting both vehicle and facility capacities.

Despite the growing research in the field, the literature largely focuses on the deterministic 'standard' 2E-LRP. Limited attention has been given to emerging logistic challenges that require more comprehensive multi-attribute settings (Mara et al., 2021b). The significance of these problem settings has been underscored by Escobar-Vargas and Crainic (2023), wherein a multi-attribute 2E-LRP with fleet synchronization is introduced, delving into the modelling and algorithmic challenges inherent to the interplay of multiple interacting attributes. As businesses adopt time-critical operations amid uncertain travel times due to various random events (e.g., weather, traffic, etc.), understanding the interplay between multi-attribute considerations and stochastic dynamics becomes crucial (Ben Mohamed et al., 2023). Developing effective modeling and solution frameworks is a necessary step in addressing these challenges.

To address this research gap, this study introduces the *Two-Echelon Multi-Attribute Location-Routing Problem with Stochastic Travel Times* (*2E-MALRPSTT*). The problem incorporates several attributes included in the work of Escobar-Vargas and Crainic (2023), e.g., time-dependent multicommodity, origin-to-destination (OD) demand, time windows, and fleet synchronization at intermediate facilities. A significant difference with the approach taken by Escobar-Vargas and Crainic (2023), is that our problem formulation includes uncertain travel times. This addition necessitates a mild synchronization requirement to mitigate the potential impact of tighter temporal constraints on the decisions related to system design.

Stochastic programming is a widely used method for managing uncertainty involving planning decisions (Guo et al., 2019). The goal of stochastic programming in this context is to determine a design that remains cost-effective when different travel time realizations are encountered. Our study then focuses on a decision and information process where an initial design is established before travel time values are known. This design is then used for optimizing vehicle routes when travel times are realized with the eventual use of an outsourced service to serve the demand, when needed. The problem is structured as a two-stage stochastic program. The first stage involves determining platform and satellite locations, as well as customer allocations. In the second stage, vehicle routes are determined based on observed travel times. Distribution costs are influenced by these times, aiming to minimize the total design and expected distribution costs.

We propose a progressive-hedging (PH) metaheuristic to address the 2E-MALRPSTT, building upon the work of Escobar-Vargas et al. (2023) for the 2E-LRP with stochastic and correlated demands. In general, the PH metaheuristic decomposes the stochastic problem into multiple deterministic subproblems based on the considered set of scenarios, which are iteratively solved to define a reference solution that is used to adjust the formulation until non-scenario dependent decisions have reached a general consensus. The work of Escobar-Vargas et al. (2023) introduces a set of algorithmic enhancements to the PH metaheuristic to improve diversity for the generation of the reference solution and the scenario-selection strategies to accelerate the search of the solution space defined by the stochastic problem. Extending this method to the 2E-MALRPSTT is not a straightforward process, due to the inherent complexity of the deterministic multi-attribute 2E-LRP, resulting from the scenario decomposition of the 2E-MALRPSTT and the challenge of addressing large-scale mixed-integer program (MIP) to model associated with it at each iteration of the PH.

To address this issue, our work introduces a decomposition-based heuristic approach for the scenario subproblems, which corresponds to a deterministic 2E-MALRPSTT. Each scenario subproblem consists of two main components: a location-routing problem with time windows and time-dependent origin-destination demands (LRPTWTDD) and a multi-depot vehicle routing problem with time windows (MDVRPTW). The heuristic operates on each echelon, considering both coordinated and integrated problems to determine the location and routing decisions for the complete deterministic 2E-MALRPSTT. The resulting optimization problems from the decomposition are solved using the commercial solver CPLEX. A computational study demonstrates the behaviour and robust performance of the proposed heuristic for both the deterministic 2E-MALRPSTT and when paired with the PH-based metaheuristic.

The paper is organized as follows: Section 2 provides the problem definition. An overview of related literature is presented in Section 3. Section 4 presents the system modelling and the proposed mathematical formulation. Section 5 describes the PH metaheuristic we propose. Computational results are presented and analyzed in Section 6.

2. Problem definition

This section presents the 2E-MALRPSTT, a multi-attribute 2E-LRP involving time-dependent multicommodity, origin-destination demands, fleet synchronization, time windows and uncertain travel times. For the sake of clarity, this section is structured into two parts. Section 2.1 presents the physical components of the 2E-MALRPSTT. Section 2.2 outlines the representation of the stochastic travel times, along with the objectives and requirements characterizing the problem setting.

2.1. The 2E-MALRPSTT setting

The 2E-MALRPSTT is formally defined on a complete weighted directed graph $G = (V, A)$. The set of vertices $V = Q \cup P \cup Z \cup C \cup E$ consists of five disjoint sets representing different components of the network: suppliers Q (the origins of demand), potential platform sites P (primary facilities), potential satellite sites Z (intermediate facilities), customers C (demand destination), and vehicle garages E. *Platforms* are large-scale infrastructures responsible for receiving, sorting, and consolidating inbound freight from suppliers, while *satellites* are intermediate depots of medium to small size with limited storage capacity, where the two transportation echelons meet and freight is transshipped and consolidated. Each potential platform location $p \in P$, is associated with a fixed selection (opening) cost F_p and a capacity Θ_p. Similarly, a fixed selection cost F_z is defined for each potential satellite location $z \in Z$.

The *multi-commodity, origin-destination (OD) demand* refers to the transportation requirements between suppliers and customers, represented by set K. Each OD demand $k \in K$ is characterized by an origin supplier $O(k) \in Q$, a destination customer $D(k) \in C$, a volume $vol(k)$, and an availability time α^{pk}, indicating when commodity k will be ready for transportation if assigned to ship from platform $p \in P$. Additionally, a fixed allocation cost Δ_{pzk} is established to account for the design costs incurred when serving OD demand $k \in K$ using a platform $p \in P$ and a satellite $z \in Z$.

Freight transportation is executed using two fleets of homogeneous vehicles, one for each echelon, denoted as $H = H^1 \cup H^2$. The vehicles of these fleets have limited load capacities, cap_1 and cap_2, respectively. Vehicles are assumed to be parked in strategically positioned garages $E = E_1 \cup E_2$, for the first echelon vehicles (E_1) and the second echelon vehicles (E_2).

The arc set $A = A_1 \cup A_2$ represents the direct links between the vertices in V. For the deterministic version of the 2E-MALRPSTT, each arc $(i,j) \in A$ is associated with a non-negative unit cost ζ_{ij} and a travel time τ_{ij}. The set A_1 consists of arcs belonging to the first echelon, including connections between platforms P and satellites Z, arcs connecting first-echelon garages to platforms and satellites as well as connections between satellites. On the other hand, the set A_2 comprises arcs of the second echelon, representing connections between satellites Z to customers C, connections between pairs of customers, and connections between second-echelon garages and satellites and customers.

The distribution plan and corresponding time-sensitive network are developed for a given schedule length Ξ. The system, along with the distribution plan, operates cyclically and repetitively over a planning horizon, during which demand and temporal properties remain unchanged (Wang et al., 2019). Thus, all transportation activities occur within the interval 0 to Ξ. Platform facilities have the capability to hold demands for a maximum duration of W^1_{max} without extra costs. Due to satellite storage limitations and the time-dependency of demand, synchronization of first- and second-echelon vehicles is necessary when transporting the same OD demand. In this context, *fleet synchronization* dictates that the departure time of the second-echelon vehicle should be later than or equal to the arrival time of the first-echelon vehicle at the satellite. Hence, each second-echelon vehicle should wait for the arrival of all first-echelon vehicles that carry at least one of the assigned commodities. Moreover,

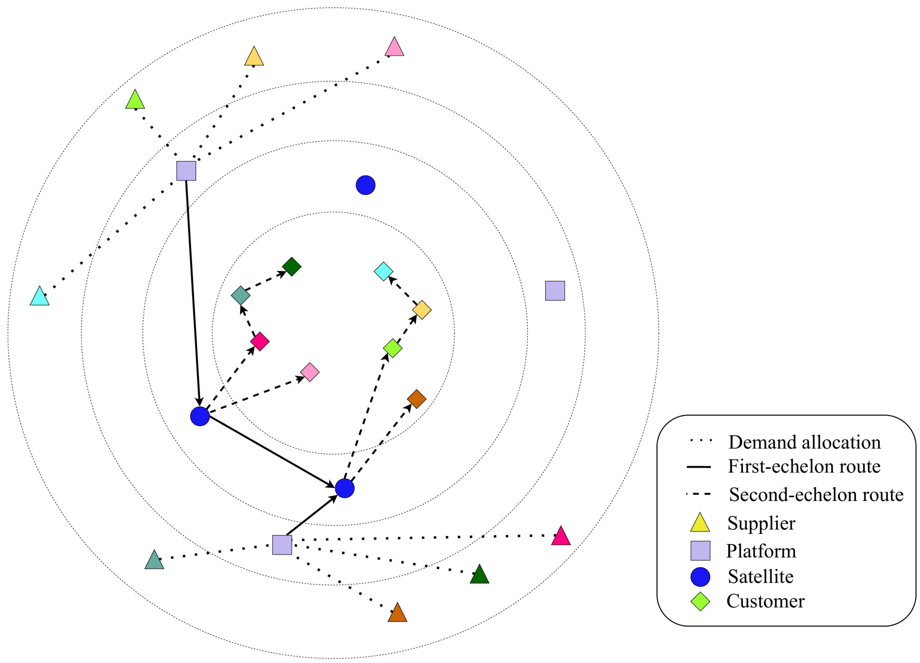

Figure 12 – Two-echelon distribution system topology

each customer $c \in C$ has a (hard) time window $[a_c, b_c]$ representing the interval within which service must start and end.

The problem involves multiple aspects, including the selection of platforms and satellites, the allocation of customer-to-satellite and satellite-to-platform, and the routing of the two vehicle fleets to meet all OD demands. As shown in Figure 12, each OD demand is linked to an open platform. At platforms, demand consolidates with other goods and is transported by a first-echelon vehicle to the designated satellites, where demand flows are transferred. Subsequently, satellites transship and consolidate received loads into second-echelon vehicles, delivering goods to final destinations. For the sake of clarity, the garage nodes are intentionally omitted from all illustrations presented in this paper.

2.2. The stochastic setting

The 2E-MALRPSTT involves uncertainty in travel times. We assume the existence of probability distributions to characterize the variability of the travel times. Given that the 2E-MALRPSTT is inspired by long and medium-term planning of freight distribution applications, it becomes imperative to differentiate the sequence in which decisions are to be taken based on the point at which values of the stochastic travel times become known.

In terms of decision-making and information processing, design and allocation decisions are made as part of the planning stage based on an evaluation/estimation of their impact on operations, along with available actions for adapting the plan to observed travel times. These adaptive measures, or recourse actions, involve determining the routes to fulfill customer demands and, if necessary, adding ad-hoc capacity. The recourse actions in the present case involve the definition of the optimal routes to fulfill customer demands with the observed ("realized") travel times including, when necessary, the use of external outsourcing services with high additional operational costs.

The 2E-MALRPSTT then consists of the selection of platform and satellite facilities, the allocation of demand from suppliers-to-platforms, satellites-to-platforms, and customers-to-satellites, as well as the creation of a limited set of routes for the first and second echelon vehicles in such a way that: (i) each supplier must be assigned to an available platform and satellite; (ii) all routes within the first echelon must start and end at the same vehicle garage (E_1); (iii) all routes within the second echelon must start and end at the same vehicle garage (E_2); (iv) every customer demand must be satisfied within the designated time window; (v) the load capacity of each vehicle must not be exceeded; (vi) each customer must be visited by a single vehicle or by an outsourced service; (vii) the total demand assigned to each platform facility should not surpass its capacity at any given time; (viii) the departure time of demand from platforms/satellites and the start time of vehicles at each echelon must be determined; (ix) The objective is to minimize the combined sum of the fixed selection costs and the expected routing costs (i.e., the recourse action).

3. Literature review

This work addresses a two-echelon location routing problem (2E-LRP) that considers multiple deterministic interacting attributes and stochastic travel times. In this section, our objective is to position the problem within the broader class of location routing problems and provide a literature review specifically focusing on multi-attribute 2E-LRPs and LRPs. We aim to identify the existing knowledge gaps related to time dependencies, time windows, origin-destination demand, fleet synchronization, and stochastic travel times. It is important to clarify that providing an exhaustive review of each individual attribute and their treatment in the literature, as well as their incorporation into related problem classes such as the *vehicle routing problem* (*VRP*) and the *two-echelon vehicle routing problem* (*2E-VRP*), falls beyond the scope of this section. Therefore, our literature review primarily concentrates on 2E-LRPs and LRPs that consider two or more of the attributes addressed in this study. Furthermore, we explore the advancements made in the field with regard to progressive-hedging-based methods. For a comprehensive overview of studies on 2E-LRP and LRP that integrate attributes beyond the scope of this work, as well as contributions related to VRPs and 2E-VRPs, we refer to recent surveys conducted by Schiffer et al. (2019), Albareda-Sambola and Rodríguez-Pereira (2019), Crainic et al. (2021a), Mara et al. (2021b) and Sluijk et al. (2022).

The literature on LRPs encompasses a wide range of attributes considered, varying in number and type. Since the seminal work of Maranzana (1964), LRPs have attracted numerous contributions that explore rich and complex problem settings. Despite the extensive literature on LRPs, the integration of multiple interacting attributes is predominantly represented by studies focusing on individual considerations, such as time windows at customer locations as the primary temporal constraint (e.g., Ponboon et al., 2016; Farham et al., 2018) or multi-commodity demands (e.g., Govindan et al., 2014; Boccia et al., 2018). However, there is a notable scarcity of literature on LRPs with stochastic travel times. Herazo-Padilla et al. (2015) integrated discrete-event simulation with ant colony optimization to address LRPs with stochastic travel times and transportation costs. Gao et al. (2016) proposed an ant colony algorithm for solving LRPs with stochastic travel times represented as random and cyclic traffic factors. The study of LRPs considering various time-sensitive attributes, such as time-dependent demands or synchronization, as well as the associated modelling and algorithmic challenges, remains largely unexplored in the field.

Research on multi-attribute 2E-LRPs is currently limited. Unlike the LRP, the 2E-LRP represents a more recent and complex problem setting that has garnered increasing attention from the scientific community. Most contributions in this field have primarily focused on studying time-sensitive logistic problems, where time is typically modeled using customer time windows (e.g., Govindan et al., 2014; Wang et al., 2018) and considering multi-period considerations (e.g., Darvish et al., 2019). However, the investigation of rich, multi-attribute 2E-LRPs that incorporate multiple interacting sources of time dependency has received little attention. Only three research studies have specifically addressed a multi-attribute 2E-LRP with two attributes relevant to this research. Bala et al. (2017) tackles a 2E-LRP with synchronized production schedules and time windows, while Mirhedayatian et al. (2019) considered a pick-up and delivery setting with fleet synchronization. The most recent work by Escobar-Vargas and Crainic (2023) addresses a 2E-LRP with time-dependent origin-destination demands, time windows and fleet synchronization. However, to the best of our knowledge, there is currently no existing study on 2E-LRP involving stochastic travel times.

From a methodological perspective, studies on LRPs with stochastic travel times typically employ a similar solution strategy that involves decoupling location and routing decisions. This approach often relies on defining location decisions through iterative testing of a pre-defined set of potential facility locations and routing alternatives evaluated under the realization of uncertain travel times (Herazo-Padilla et al., 2015; Gao et al., 2016). Decomposition-based methods, such as this one, have demonstrated promising potential for solving two- and multi-stage stochastic optimization models (Atakan and Sen, 2018).

Decomposition strategies performed with respect to scenarios or time periods (stages) in the decision model, such as the progressive-hedging strategy (Rockafellar and Wets, 1991), have become an important research avenue in the field. The progressive-hedging algorithm, is one of the most widely used dual decomposition frameworks. However, the non-convex nature of the feasible set defined by stochastic mixed integer programs and the complexity of proving the convergence of PH for such applications (Atakan and Sen, 2018) have led to the development of PH-based heuristics that follow the guidelines of the original algorithm to overcome these limitations (e.g., Løkketangen and Woodruff, 1996; Haugen et al., 2001; Crainic et al., 2011a; Lamghari and Dimitrakopoulos, 2016; Alvarez et al., 2021).

These methods typically employ heuristic solution approaches to address the deterministic problems resulting from scenario decomposition in the stochastic problem. They iteratively use the best solution defined for each subproblem to guide the search of the solution space. A recent contribution by Escobar-Vargas et al. (2023) presents a richer extension of the PH-based algorithm introduced by Crainic et al. (2011a), incorporating two population structures, the use of alternative solutions for each scenario subproblem, and three scenario-selection methods to accelerate consensus. The objective of this study is to address the existing research gaps by extending and enhancing the PH-based metaheuristic proposed by Escobar-Vargas et al. (2023) for the 2E-MALRPSTT. This is achieved through the introduction of two novel scenario-selection heuristics to accelerate consensus in the stochastic problem and a decomposition-based heuristic employed to tackle the deterministic subproblems arising from scenario decomposition.

4. Modelling

Section 4.1 is dedicated to presenting the proposed modelling approach, while Section 4.2 outlines the mathematical formulation proposed for the 2E-MALRPSTT.

4.1. Modelling uncertainty

The 2E-MALRPSTT is formulated as a two-stage stochastic program. The proposed two-stage formulation consists of a first stage where the selection of satellite and platform facilities and the allocation of OD demands to them are made. In the second stage, routing decisions at both echelons are determined when travel times are observed. Additionally, the option of using outsourced capacity when necessary is also part of the second stage, where an operational cost R is incurred for each unit of volume in $vol(k)$.

Uncertainty in travel times is modeled with a finite set of scenarios generated by sampling probability distributions. Let S represent the set of scenarios, where each scenario $s \in S$ signifies a specific realization of the random events that determine the travel time values for each arc within the system. Let ρ_s be the probability of occurrence of scenario s, with $\sum_{s \in S} \rho_s = 1$. Consequently, for a given scenario $s \in S$, there exists a fixed travel time value τ_{ij}^s for all arcs $(i,j) \in A$, with $\tau_{ij}^s \geq 0$.

4.2. Two-stage stochastic formulation for the 2E-MALRPSTT

This section introduces the Mixed-Integer Programming (MIP) formulation for the 2E-MALRPSTT. The problem is structured as a two-stage stochastic programming problem employing a three-index vehicle-flow formulation. To model fleet synchronization and the multiple visits made by first-echelon vehicles to satellite facilities, a set of clone satellites $\tilde{Z}_z \in \tilde{Z}$ is introduced for each satellite $z \in Z$. Each clone satellite in $\tilde{Z}_z$ corresponds to a physical satellite $z \in Z$, which, is replicated based on the number of visits allowed at the facility. The total number of OD demands $|K|$ serves as an upper limit of visits at each satellite. Two sets of decision variables are defined: (1) the first-stage variables to address satellite location and OD demand-to-satellite allocation decisions, and (2) the second-stage variables pertain to vehicle-routing decisions at both echelons. Following the prevailing convention in the literature, we adopt a concise presentation by formulating the problem directly in terms of the set of scenarios S. This leads to second-stage variables being indexed by scenario, while first-stage variables are not indexed as they remain fixed in the second stage. We employ satellite facilities $z \in Z$ to denote the second-stage variables consistently throughout the paper. However, in this extensive formulation, we represent these second-stage variables in terms of the clone satellites $\tilde{Z}$ to capture the behaviour of the 2E-MALRPSTT. The following definitions describe the decision variables on the extensive form of the proposed two-stage formulation:

— $y_i \in \{0,1\}, i \in (P \cup Z)$: location variable, 1 if a facility i is open, 0 otherwise;

— $f_{pzk} \in \{0,1\}, p \in P, z \in Z, k \in K$: allocation variable, 1 if demand k is allocated to platform p and satellite z;

— $x^s_{ijh} \in \{0,1\}, (i,j) \in A, h \in H, s \in S$: vehicle flow variable, 1 if arc (i,j) is used by vehicle h in scenario s, and 0 otherwise;

— $\phi^s_{pzkh} \in \{0,1\}, p \in P, z \in Z, k \in K, h \in H^1, s \in S$: vehicle allocation variable, 1 if demand k is allocated to satellite z and platform p with a given vehicle h in scenario s, 0 otherwise;

— $\psi^s_{zch} \in \{0,1\}, z \in Z, c \in C, h \in H^2, s \in S$: vehicle allocation variable, 1 if costumer c is allocated to satellite z with a given vehicle h at scenario s, 0 otherwise;

— $\mu^1_{ihs} \geq 0, i \in (P \cup Z), h \in H^1, s \in S$: arrival time of vehicle h at vertex i in scenario s;

— $\mu^2_{ihs} \geq 0, i \in (Z \cup C), h \in H^2, s \in S$: arrival time of vehicle h at vertex i in scenario s;

— $\nu^1_{ihs} \geq 0, i \in (P \cup Z), h \in H^1, s \in S$: departure time of vehicle h at vertex i in scenario s;

— $\nu^2_{ihs} \geq 0, i \in (Z \cup C), h \in H^2, s \in S$: departure time of vehicle h at vertex i in scenario s;

— $o^s_k \geq 0, k \in K, s \in S$: outsourced demand k in scenario s;

— M: large integer number;

The extensive two-stage formulation of the 2E-MALRPSTT then becomes:

$$\min \sum_{i\in(Z\cup P)} F_i y_i + \sum_{i\in P}\sum_{j\in Z}\sum_{k\in K} \Delta_{ijk} f_{ijk} + \sum_{s\in S}\rho_s\Big(\sum_{h\in H}\sum_{(i,j)\in A}\zeta_{ij}^s x_{ijh}^s + R\sum_{k\in K} o_k^s\Big) \tag{4.1}$$

subject to

$$\sum_{h\in H^1}\sum_{i\in\tilde{Z}_z}\sum_{j\in(E_1\cup\tilde{Z}\setminus\tilde{Z}_z)} x_{ijh}^s \le |H^1| y_z \quad \forall z\in Z, s\in S \tag{4.2}$$

$$\sum_{j\in\tilde{Z}} x_{ijh}^s - \sum_{j\in E_1} x_{jih}^s = 0 \quad \forall i\in P, h\in H^1, s\in S \tag{4.3}$$

$$\sum_{j\in(E_1\cup\tilde{Z}),i\neq j} x_{ijh}^s - \sum_{j\in(P\cup\tilde{Z}),i\neq j} x_{jih}^s = 0 \quad \forall i\in\tilde{Z}, h\in H^1, s\in S \tag{4.4}$$

$$\sum_{j\in P} x_{ijh}^s - \sum_{j\in\tilde{Z}} x_{jih}^s = 0 \quad \forall i\in E_1, h\in H^1, s\in S \tag{4.5}$$

$$\sum_{i\in E_1}\sum_{j\in P} x_{ijh}^s \le 1 \quad \forall h\in H^1, s\in S \tag{4.6}$$

$$\sum_{h\in H^2}\sum_{j\in(E_2\cup C),D(k)\neq j} x_{D(k)jh}^s + o_k^s = 1 \quad \forall k\in K, s\in S \tag{4.7}$$

$$\sum_{j\in E_2} x_{jih}^s - \sum_{j\in C} x_{ijh}^s = 0 \quad \forall i\in\tilde{Z}, h\in H^2, s\in S \tag{4.8}$$

$$\sum_{j\in(\tilde{Z}\cup C),i\neq j} x_{jih}^s - \sum_{j\in(C\cup E_2),i\neq j} x_{ijh}^s = 0 \quad \forall i\in C, h\in H^2, s\in S \tag{4.9}$$

$$\sum_{j\in\tilde{Z}} x_{ijh}^s - \sum_{j\in C} x_{jih}^s = 0 \quad \forall i\in E_2, h\in H^2, s\in S \tag{4.10}$$

$$\sum_{i\in E_2}\sum_{j\in\tilde{Z}} x_{ijh}^s \le 1 \quad \forall h\in H^2, s\in S \tag{4.11}$$

$$\mu_{ihs}^1 \ge \alpha^{ik}\sum_{j\in\tilde{Z}}\phi_{ijkh}^s \quad \forall i\in P, h\in H^1, k\in K, s\in S \tag{4.12}$$

$$\nu_{ihs}^1 \le (\alpha^{ik} + W_{max}^1) + (1 - \sum_{j\in\tilde{Z}}\phi_{ijkh}^s)M \quad \forall i\in P, h\in H^1, k\in K, s\in S \tag{4.13}$$

$$\mu_{ihs}^1 + \tau_{ij}^s - \mu_{jhs}^1 \le (1 - x_{ijh}^s)M \quad \forall h\in H^1, (i,j)\in A_1, s\in S \tag{4.14}$$

$$\nu_{ihs}^1 + \tau_{ij}^s - \nu_{jhs}^1 \le (1 - x_{ijh}^s)M \quad \forall h\in H^1, (i,j)\in A_1, s\in S \tag{4.15}$$

$$\mu_{ihs}^2 + \tau_{ij}^s - \mu_{jhs}^2 \le (1 - x_{ijh}^s)M \quad \forall h\in H^2, (i,j)\in A_2, s\in S \tag{4.16}$$

$$\nu_{ihs}^2 + \tau_{ij}^s - \nu_{jhs}^2 \le (1 - x_{ijh}^s)M \quad \forall h\in H^2, (i,j)\in A_2, s\in S \tag{4.17}$$

$$\nu_{jbs}^2 \ge \mu_{jhs}^1 - (2 - \psi_{jD(k)b}^s - \sum_{i\in P}\phi_{ijkh}^s)M$$
$$\forall h\in H^1, b\in H^2, k\in K, j\in\tilde{Z}, s\in S \tag{4.18}$$

$$a_i \le \mu_{ihs}^2 \le b_i \quad \forall i\in C, h\in H^2, s\in S \tag{4.19}$$

$$\sum_{j \in (C \cup E_2), i \neq j} x^s_{ijh} + \sum_{j \in C} x^s_{zjh} - \psi^s_{zih} \leq 1$$

$$\forall i \in C, z \in \tilde{Z}, h \in H^2, s \in S \tag{4.20}$$

$$cap_1 \; x^s_{ijh} - \sum_{k \in K} vol(k) \, \phi^s_{ijkh} \geq 0$$

$$\forall h \in H^1, i \in P, j \in \tilde{Z}, s \in S \tag{4.21}$$

$$\sum_{h \in H^1} \sum_{i \in P} \phi^s_{ijkh} = \sum_{l \in H^2} \psi^s_{jD(k)l}$$

$$\forall j \in \tilde{Z}, k \in K, s \in S \tag{4.22}$$

$$\sum_{h \in H^1} \sum_{i \in P} \sum_{j \in \tilde{Z}} \phi^s_{ijkh} + o^s_k = 1 \quad \forall k \in K, s \in S \tag{4.23}$$

$$\sum_{h \in H^1} \sum_{k \in K} \sum_{j \in \tilde{Z}} vol(k) \, \phi^s_{ijkh} \leq \Theta_i \, y_i \quad \forall i \in P, s \in S \tag{4.24}$$

$$\sum_{k \in K} \sum_{i \in P} \sum_{j \in \tilde{Z}} vol(k) \, \phi^s_{ijkh} \leq cap_1 \quad \forall h \in H^1, s \in S \tag{4.25}$$

$$\sum_{k \in K} vol(k) \sum_{j \in (E_2 \cup C), D(k) \neq j} x^s_{D(k)jh} \leq cap_2 \quad \forall h \in H^2, s \in S \tag{4.26}$$

$$\sum_{h \in H^1} \phi^s_{ijkh} \leq f_{izk} \quad \forall i \in P, j \in \tilde{Z}_z, k \in K, z \in Z, s \in S \tag{4.27}$$

$$\sum_{h \in H^2} \psi^s_{iD(k)h} \leq \sum_{p \in P} f_{pzk} \quad \forall i \in \tilde{Z}_z, k \in K, z \in Z, s \in S. \tag{4.28}$$

The objective function (4.1) minimizes the total transportation costs of the distribution network computed as the sum of the fixed cost of the selected facilities and the expected routing costs (the recourse action) of the demand flows through the resulting network. Constraints (4.2) impose that outbound arcs from every open satellite must respect the total number of first-echelon vehicles. Constraints (4.3)-(4.5) are the flow conservation constraints for platforms, satellites, and first-echelon garages, respectively. Constraints (4.6) ensure that each active vehicle is assigned to one platform only. Constraints (4.7) ensure that every customer is served either by the distribution system with a single second-echelon vehicle or by an outsourced service. Constraints (4.8)-(4.10) are the flow conservation constraints for satellites, customers, and second-echelon garages, respectively. Constraints (4.11) ensure that each active vehicle is assigned to one satellite only. Constraints (4.14)-(4.15) handle the arrival and departure times of first-echelon vehicles.

Constraints (4.12) and (4.13) guarantee schedule feasibility with respect to demand availability and maximum holding time at platform facilities. Constraints (4.16)-(4.17) handle the arrival and departure times of second-echelon vehicles. Constraints (4.18) relate the departure time and the arrival time of first- and second-echelon vehicles, respectively, to guarantee fleet synchronization at satellite facilities. Constraints (4.19) ensure that second-echelon vehicles arrive within the customer time windows. Constraints (4.20) and (4.21) link allocation and routing variables. Constraints (4.22) are the flow conservation constraints at satellites. Constraints (4.23) ensure that each origin supplier is allocated either to an open platform or to an outsourced service. Constraints (4.24) ensure that the multicommodity flow going out from platforms is less than or equal to the platform capacity. Constraints (4.25) and (4.26) impose that the multicommodity flow carried by each vehicle, in the first and second echelon, respectively, is less than its capacity. Constraints (4.27) and (4.28), respectively link allocation and vehicle allocation variables for the first-echelon and second-echelon vehicles.

5. A progressive-hedging-based metaheuristic for the 2E-MALRPSTT

This section presents a progressive-hedging-based metaheuristic to address the 2E-MALRPSTT, building upon the work of Escobar-Vargas et al. (2023). The algorithm, as depicted in Figure 13, begins with the scenario-based decomposition of the extensive formulation. This approach yields a collection of scenario subproblems with modified fixed costs associated with the first-stage decision variables. At each iteration of the PH metaheuristic, each scenario-specific subproblem is individually addressed. For each scenario subproblem, there is a *local population* of size ξ_L (including a reduced number ξ_E of elite solutions) responsible for storing the set of alternative solutions that are obtained when addressing it. To populate the *local population*, each alternative solution is ranked in terms of its quality (i.e., objective function value) and contribution to diversity (i.e., dissimilarity with respect to the first-stage decision variables) over the set of solutions that are already included in the *local population*. *Local populations* are maintained from one iteration to the next. The best (elite) selected solutions of each *local population* are then gathered at each iteration into a *global population* of solutions for the complete problem, of size ξ_G. This *global population* is then used to define a general reference solution using a decision-based scenario clustering strategy (see, Escobar-Vargas et al., 2023). This reference solution is used to guide the search by adjusting the costs in the objective function of each scenario subproblem, aiming to reach a consensus on the first-stage decisions over all the scenarios. The algorithm ends when a consensus is reached on the first-stage decisions or when external stopping criteria are met while saving the best feasible solution obtained on each iteration.

There are multiple algorithmic challenges that arise when tackling the 2E-MALRPSTT using a PH method. On the one hand, the scenario subproblems defined by the decomposition of the extensive formulation are significantly complex to be addressed by a standalone commercial solver (Escobar-Vargas and Crainic, 2023). On the other hand, the decision-based

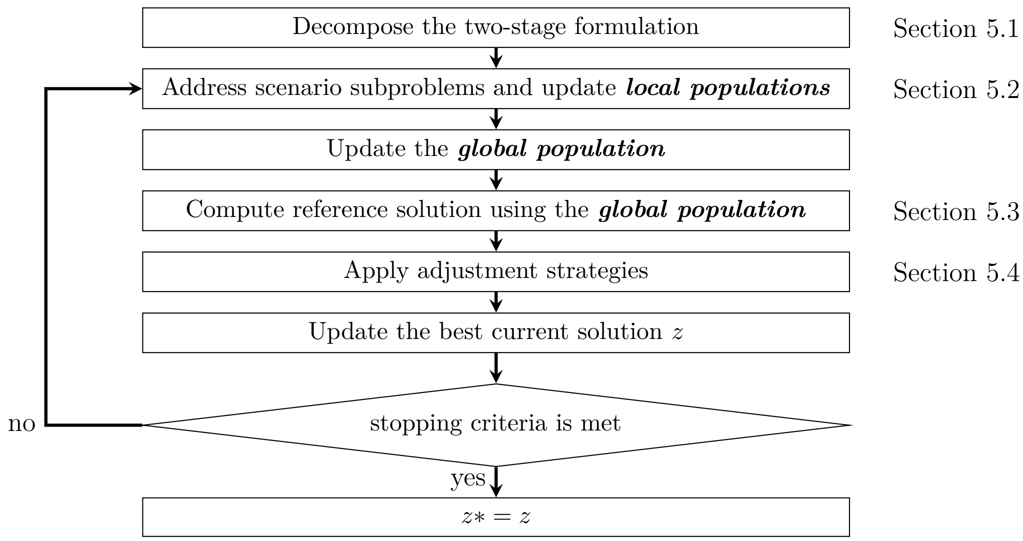

Figure 13 – Progressive Hedging-based metaheuristic for the 2E-MALRPSTT

scenario clustering strategy proposed by Escobar-Vargas et al. (2023) appears to be insufficient to account for the large dimension of the stochastic parameters in the 2E-MALRPSTT. We thus propose two algorithmic enhancements for the PH metaheuristic to address these challenges. These enhancements include: (1) a decomposition-based heuristic to address the scenario subproblems defined at each iteration of the PH, and (2) two novel scenario-selection heuristics to further enhance the capabilities of the decision-based strategy for the 2E-MALRPSTT. This section provides a comprehensive, step-by-step, description of the algorithmic structure of the proposed PH metaheuristic.

5.1. Scenario decomposition for the 2E-MALRPSTT

This section introduces the decomposition methodology applied to the extensive two-stage formulation (Section 4.2). The decomposition approach requires a reformulation of the first-stage decisions (detailed reformulation in the supplementary material Appendix C.1). Specifically, these decisions need to be redefined as scenario-dependent, and constraints enforcing consistency in the first-stage variables across all scenarios must be included. Let y_i^s and f_{ijk}^s be the reformulation of the location and allocation decisions, respectively, for each scenario $s \in S$. This also reformulates constraints (4.2), (4.24), (4.27), and (4.28), tailoring them to the scenario-specific location and allocation first-stage decisions. A set of non-anticipativity constraints is added to the model to prevent the first-stage decision variables from taking distinct scenario-specific values:

$$y_i^s = \bar{y}_i \quad \forall i \in (P \cup Z), s \in S, \tag{5.1}$$

$$f_{ijk}^s = \bar{f}_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S, \tag{5.2}$$

$$\bar{y}_i \in \{0,1\} \quad \forall i \in (P \cup Z), \tag{5.3}$$

$$\bar{f}_{ijk} \in \{0,1\} \quad \forall i \in P, j \in Z, k \in K. \tag{5.4}$$

Let $\bar{y}_i$ and $\bar{f}_{ijk}$ be the reference variables for the first-stage decisions preventing the adoption of distinct scenario-specific decisions. Then, following the decomposition scheme, originally proposed by Rockafellar and Wets (1991), constraints (5.1) and (5.2) are relaxed using an augmented Lagrangian method, which results in the following relaxed reformulation of the extensive model:

$$\begin{aligned} \min \sum_{s \in S} \rho_s \Bigg(\sum_{(i,j) \in A} \sum_{h \in H} \zeta_{ij} x_{ijh}^s + \sum_{k \in K} Ro_k^s + \sum_{i \in (P \cup Z)} \Big(F_i + \lambda_i^s + \frac{1}{2}\gamma + \gamma \bar{y}_i \Big) y_i^s \\ + \sum_{i \in P} \sum_{j \in Z} \sum_{k \in K} \Big(\Delta_{ijk} + \pi_{ijk}^s + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \Big) f_{ijk}^s \Bigg) \end{aligned} \tag{5.5}$$

subject to

$$(4.3) - (4.23)$$

$$(4.25) - (4.26)$$

$$\sum_{h \in H^1} \sum_{i \in \tilde{Z}_z} \sum_{j \in (E_1 \cup \tilde{Z} \setminus \tilde{Z}_z)} x_{ijh}^s \leq |H^1| y_z^s \quad \forall z \in Z, s \in S \tag{5.6}$$

$$\sum_{h \in H^1} \sum_{k \in K} \sum_{j \in \tilde{Z}} vol(k)\, \phi_{ijkh}^s \leq \Theta_i\, y_i^s \quad \forall i \in P, s \in S \tag{5.7}$$

$$\sum_{h \in H^1} \phi_{ijkh}^s \leq f_{izk}^s \quad \forall i \in P, j \in \tilde{Z}_z, k \in K, z \in Z, s \in S \tag{5.8}$$

$$\sum_{h \in H^2} \psi_{iD(k)h}^s \leq \sum_{p \in P} f_{pzk}^s \quad \forall i \in \tilde{Z}_z, k \in K, z \in Z, s \in S \tag{5.9}$$

$$y_i^s \in \{0,1\} \quad \forall i \in (P \cup Z), s \in S \tag{5.10}$$

$$f_{ijk}^s \in \{0,1\} \quad \forall i \in P, j \in Z, k \in K, s \in S. \tag{5.11}$$

The objective function now involves the Lagrangian multipliers λ_i^s and π_{ijk}^s for the relaxed non-anticipativity constraints corresponding to the location and allocation decisions, respectively, and a penalty term γ. Constraints (5.6) impose that outbound arcs from every open satellite must respect the total number of first-echelon vehicles. Constraints (5.7) ensure that the multicommodity flow going out from platforms is less than or equal to the platform capacity. Constraints (5.8) and (5.9) link the facility allocation variables with the vehicle allocation variables. Constraints (5.10) and (5.11) impose the integrality and non-negativity of each decision variable in the model.

The relaxed reformulation can be then decomposed by scenario for a given reference design $\bar{y}_i$ and $\bar{f}_{ijk}$ (the initial values for the reference design are discussed in Section 5.4), resulting in a set of deterministic and scenario-specific subproblems with modified fixed costs. In this context, the Lagrangian multipliers λ_i^s and π_{ijk}^s and the parameter γ, penalize the discrepancies between the values of the location and allocation decision in the local design of a single scenario subproblem and those present in the current overall design. In the following sections, we explore the proposed methods for deriving the overall design and allocation decisions, as well as the approach for adjusting the fixed costs of each scenario subproblem. These strategies aim to guide the search process toward achieving consensus among the first-stage variables.

5.2. Subproblem algorithm

This section presents the proposed heuristic method to address the individual scenario subproblems resulting from the scenario decomposition of the extensive formulation. It should be noted that each scenario $s \in S$ is associated with a deterministic scenario subproblem (DSS), which corresponds to a deterministic 2E-MALRPSTT. Each DSS is a NP-hard problem of very large dimensions (Escobar-Vargas and Crainic, 2023). Therefore, addressing each DSS independently using a standalone commercial solver is impractical. To overcome this limitation, we introduce a decomposition-based heuristic to address each DSS.

The proposed solution method focuses on decomposing the deterministic 2E-MALRPSTT defined by each DSS into two separate optimization subproblems. The first step in the decomposition process involves defining a relaxed DSS, obtained by ignoring the fleet synchronization requirements imposed on the satellite facilities, which create a strong interdependency between the scheduling of vehicles in the first and second echelons. Once these synchronization constraints are ignored, the relaxed DSS can be decomposed into two main components: (1) a Location-Routing Problem with Time Windows and Time-Dependent OD demands (LRPTWTDD) to model the second-echelon problem, and (2) a Multi-Depot Vehicle Routing Problem with Time Windows (MDVRPTW) to model the first-echelon problem. The heuristic employs a bottom-up approach, where the first-echelon solution is built and optimized based on the second-echelon solution. The proposed heuristic operates on each echelon separately and addresses two coordinated and integrated problems to determine the location and routing decisions for the entire DSS.

The proposed decomposition heuristic, illustrated in Figure 14, begins with a preprocessing procedure aimed at structuring the available data for the 2E-MALRPSTT. This organization ensures that each subproblem resulting from the decomposition of the DSS can

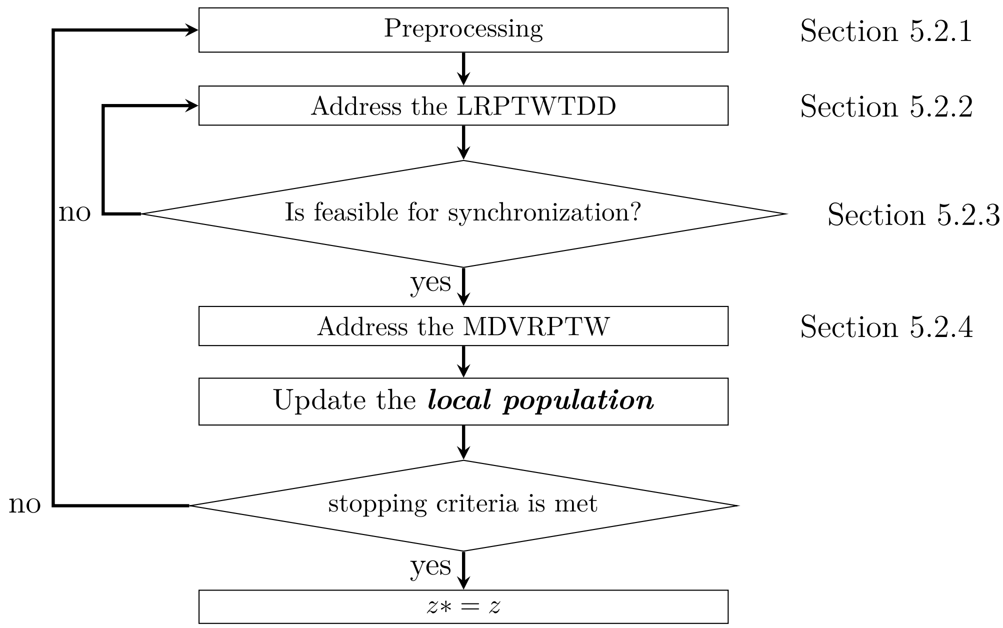

Figure 14 – Decomposition-based heuristic for the DSS

be independently addressed. To further reduce the complexity of each subproblem, our approach employs distinct and reduced subsets of platform facilities, which define the set of subproblems (for both the first and second echelons) to be addressed in each iteration of the algorithm. Hence, the decomposition-based heuristic first defines a set comprising all the possible combinations of platforms, based on the complete set of platforms in P. At each iteration of the decomposition heuristic, the methodology selects and fixes a single reduced subset of platforms from this comprehensive set. The first-echelon subproblem, corresponding to a LRPTWTDD and defined by this reduced platform subset, is then addressed. This LRPTWTDD is iteratively solved and 'refined' in terms of fleet synchronization requirements until a feasible solution that satisfies the fleet synchronization constraints defined by the 2E-MALRPSTT is obtained. Once this feasible solution is achieved, the second subproblem, defined by the MDVRPTW, is addressed to determine the scheduling decisions for the first-echelon vehicles based on the solution obtained from the LRPTWTDD. During each iteration of the algorithm, the best solution is updated if improvements are made, continuing until all possible combinations of platforms have been explored or another stopping criterion is reached (e.g., a limit on the computation time).

5.2.1. Preprocessing. The objective of the proposed preprocessing procedure is twofold: extend the availability time of all OD demands to satellite facilities and reduce the complexity of each scenario subproblem. Two strategies are introduced for each objective. The first strategy consists of the extension of availability times from platforms to satellite facilities. This extension allows the LRPTWTDD to have a feasible set of availability times for each OD demand directly for second-echelon routes, without the explicit consideration of the scheduling decisions of the first-echelon vehicles. The procedure consists of calculating the availability time of each OD demand $k \in K$ at each satellite $z \in Z$ based on their minimum 'departure' time (i.e., either when the demand is available or when a first-echelon vehicle can pick it) from each platform $p \in P$, and the travel time between each platform and the satellite under consideration in a given scenario $s \in S$. One can define $\alpha^{pzks} \geq 0$ as the extended availability time of demand $k \in K$ at each satellite $z \in Z$ from a platform $p \in P$ in scenario $s \in S$. Notice that while availability times are not originally scenario-dependent, it extension to satellites must be indexed by scenario to incorporate the realization of the travel times defined by each scenario.

The second strategy involves generating multiple reduced subsets of platform facilities to reduce the complexity associated with each LRPTWTDD. Handling the complete platform set P for the LRPTWTDD becomes demanding as the set size increases. We thus introduce the subset $P^{'} \subseteq P$, comprising a limited number of platform facilities. To do so, one must first define the set $\mathcal{P}$ that encompasses all feasible platform combinations ${}_{i}\mathcal{C}_{|P|}$, where i ranges from 1 to $|P|$ as a fixed facility count, and $|P|$ indicates the total number of platforms in P. This comprehensive set $\mathcal{P}$ is structured in such a way that, at each iteration of the decomposition heuristic, a single subset $P^{'}$ is selected from $\mathcal{P}$ to define the set of subproblems (for both the first and second echelons) to be addressed. This process of selecting a subset P' is repeated iteratively until all subsets of platform combinations within $\mathcal{P}$ have been explored.

5.2.2. Mathematical formulation for the LRPTWTDD. We reformulate the extensive formulation to address the LRPTWTDD. This new formulation results from ignoring the fleet synchronization constraints and the vehicle routing decision on the first echelon. The LRPTWTDD consists of the definition of the location decisions of platforms and satellite facilities as well as the routing decisions of second-echelon vehicles to meet customer demands (either by the system or with an outsourced service). Instead of the explicit consideration of first-echelon routes, this formulation only uses direct connections between platforms and satellite facilities as a means to represent the capacity allocation decisions from first-echelon vehicles that are required to meet the routing decisions on second-echelon vehicles. For this, we assume that each direct connection between platforms and satellites represents an aggregated travel time composed of the travel times between a given platform and satellite, as well as its connection to and from the vehicle garages.

The proposed formulation is composed of the variables and constants presented in Section 4.2 and Section 5.2.1. Three main changes are introduced to reformulate the extensive model (Section 4.2). First, the set of satellite copies $\tilde{Z}$ defined in Section 4.2 is replaced by the original satellite set Z, due to the lack of synchronization constraints in the problem. Second, a new vehicle allocation variable ψ^{s}_{pzkh} is introduced, whose value is equal to one if the OD demand $k \in K$ is served by a vehicle $h \in H^2$ from a satellite $z \in Z$ and a platform $p \in P^{'}$. Third, vehicle variables x^s_{ijh}, originally defined for first-echelon routing decision, are used to model the capacity allocation decisions from first-echelon vehicles, to represent the amount of capacity (provided by first-echelon vehicles) required to meet the routes defined by second-echelon vehicles. We also define a constant $\zeta^{'}_{pzs}$ as the aggregated costs associated with each direct connection x^s_{pzh} for first-echelon vehicles. Each aggregated cost $\zeta^{'}_{pzs}$ is composed of the sum of the travel time between a single platform $p \in P^{'}$ and a single satellite $z \in Z$ for each scenario $s \in S$, and the travel cost associated to the connection between each facility and its corresponding vehicle garage. It is worth mentioning that although the variables in the formulation are indexed by scenario, the LRPTWTDD is addressed individually for each scenario subproblem. The formulation for the LRPTWTDD is then defined as follows:

$$\min \sum_{h \in H^1} \sum_{i \in P^{'}} \sum_{j \in Z} \zeta^{'}_{ijs} x^s_{ijh} + \sum_{h \in H^2} \sum_{(i,j) \in A^2} \zeta^s_{ij} x^s_{ijh} + \sum_{k \in K} Ro^s_k$$

$$+ \sum_{i \in (P^{'} \cup Z)} \left(F_i + \lambda^s_i + \frac{1}{2}\gamma + \gamma \bar{y}_i \right) y^s_i + \sum_{i \in P^{'}} \sum_{j \in Z} \sum_{k \in K} \left(\Delta_{ijk} + \pi^s_{ijk} + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \right) f^s_{ijk} \tag{5.12}$$

subject to

$$\sum_{h \in H^1} \sum_{j \in Z} x^s_{ijh} \leq |H^1|\, y^s_i \quad \forall i \in P^{'} \tag{5.13}$$

$$\sum_{h \in H^1} x^s_{ijh}\, cap_1 \geq \sum_{k \in K} \sum_{h \in H^2} vol(k)\, \psi^s_{ijkh} \quad \forall j \in Z, i \in P^{'} \tag{5.14}$$

$$\sum_{h \in H^2} \sum_{k \in K} \sum_{j \in Z} vol(k)\, \psi^s_{ijkh} \leq \Theta_i\, y^s_i \quad \forall i \in P^{'} \tag{5.15}$$

$$\sum_{h \in H^2} \sum_{j \in C} x^s_{ijh} \leq |H^2| y^s_i \quad \forall i \in Z \tag{5.16}$$

$$\sum_{h \in H^2} \sum_{j \in (E_2 \cup C), D(k) \neq j} x^s_{D(k)jh} + o^s_k = 1 \quad \forall k \in K \tag{5.17}$$

$$\sum_{j \in E_2} x^s_{jih} - \sum_{j \in C} x^s_{ijh} = 0 \quad \forall i \in Z, h \in H^2 \tag{5.18}$$

$$\sum_{j \in (Z \cup C), i \neq j} x^s_{jih} - \sum_{j \in (C \cup E_2), i \neq j} x^s_{ijh} = 0 \quad \forall i \in C, h \in H^2 \tag{5.19}$$

$$\sum_{j \in Z} x^s_{ijh} - \sum_{j \in C} x^s_{jih} = 0 \quad \forall i \in E_2, h \in H^2 \tag{5.20}$$

$$\mu^2_{zhs} \geq \alpha^{pzks} \psi^s_{pzkh} \quad \forall p \in P^{'}, z \in Z, h \in H^2, k \in K \tag{5.21}$$

$$\mu^2_{ihs} + \tau^s_{ij} - \mu^2_{jhs} \leq (1 - x^s_{ijh}) M \quad \forall h \in H^2, (i,j) \in A_2 \tag{5.22}$$

$$a_i \leq \mu^2_{ihs} \leq b_i \quad \forall i \in C, h \in H^2 \tag{5.23}$$

$$\sum_{j \in (C \cup E_2), D(k) \neq j} x^s_{D(k)jh} + \sum_{j \in C} x^s_{zjh} - \sum_{p \in P'} \psi^s_{pzkh} \leq 1 \quad \forall k \in K, z \in Z, h \in H^2 \tag{5.24}$$

$$\sum_{k \in K} vol(k) \sum_{j \in (E_2 \cup C), D(k) \neq j} x^s_{D(k)jh} \leq cap_2 \quad \forall h \in H^2 \tag{5.25}$$

$$\sum_{h \in H^2} \psi^s_{pzkh} \leq f^s_{pzk} \quad \forall p \in P', z \in Z, k \in K. \tag{5.26}$$

The objective function (5.12) minimizes the design and routing costs associated with each scenario subproblem. Constraints (5.13) impose that outbound arcs from every open platform must respect the total number of first-echelon vehicles. Constraints (5.14) ensure that the capacity of each active first-echelon vehicle is not exceeded by the demand allocated to each satellite facility. Constraints (5.15) ensure that the multicommodity flow going out from platforms is less than the platform capacity. Constraints (5.16) impose that outbound arcs from every open satellite must respect the total number of second-echelon vehicles. Constraints (5.17) ensure that every customer is served either by the distribution system with a single second-echelon vehicle or by an outsourced service. Constraints (5.18) - (5.20) are the flow conservation constraints for satellites, customers, and second-echelon garages, respectively.

Constraints (5.21) impose a schedule of second-echelon vehicles based on demand availability at each satellite facility. Constraints (5.22) handle the arrival times of second-echelon vehicles. Constraints (5.23) ensure that second-echelon vehicles arrive within the customer time windows. Constraints (5.24) link allocation and routing variables for second-echelon vehicles. Constraints (5.25) ensure that demand carried by each second-echelon vehicle is less than or equal to its capacity. Constraints (5.26) link allocation and vehicle allocation variables for the second-echelon vehicles.

5.2.3. Evaluation and refinement. The proposed mathematical formulation for the LRPTWTDD lacks the presence of synchronization constraints for first-echelon vehicles. This implies that an optimal solution obtained by the formulation defined for the LRPTWTDD might not be feasible for the 2E-MALRPSTT when evaluated with the complete constraint sets. Hence, a *refinement* of the mathematical formulation is proposed, where valid inequalities are included in the LRPTWTDD formulation when synchronization constraints are found infeasible. The objective of our procedure is to iteratively identify violations of the synchronization constraints and the posterior *refinement* of the LRPTWTDD model with valid inequalities until a feasible solution for the 2E-MALRPSTT can be obtained.

To evaluate the feasibility of synchronization constraints, one must define a set of *route time windows* representing the temporal lower and upper bounds where fleet synchronization can take place for first-echelon vehicles at each satellite facility. The objective of the proposed procedure, Algorithm 6, is to define these *route time windows* for each second-echelon route for a given solution for the LRPTWTDD, and evaluate the feasibility of the routes in terms of the fleet synchronization requirements. Let Γ_z be the set of all second-echelon routes using satellite $z \in Z$ and serving a subset of at least one customer using a vehicle $h \in H^2$, with $\Gamma = \bigcup_{z \in Z} \Gamma_z$ being the set of all possible second-echelon routes serving at least one customer. Then, let $[lbt_l, ubt_l]$ be the *route time windows* for each second-echelon route $l \in \Gamma$. To compute these *route time windows* for each route $l \in \Gamma$, the proposed procedure first defines the departure time dep_l of each route at its respective satellite facility, based on the arrival time of the vehicle on the first customer visited in the route. For each route $l \in \Gamma$, the algorithm proceeds by traversing the sequence of customer nodes in $l = c_0, c_1, \ldots, c_{|l|-1}$, to compute the *temporal slack* allowed for each customer within the route. This *temporal slack* is defined by the difference between the lower/upper bound of each customer time window and the actual arrival time of the second-echelon vehicle. The upper bound ubt_l of each route $l \in \Gamma$ can be defined by the departure time recorded at the satellite facility plus the minimum *temporal slack* defined by the upper bound of the customer time windows. Similarly, the lower bound lbt_l of each route $l \in \Gamma$ can be defined by the departure time at the satellite minus the minimum *temporal slack* allowed by the lower bound of the customer time windows.

Algorithm 6: Evaluation($\Gamma_z, s \in S$)

for $l \in \Gamma_z$ **do**
    $h = \text{getVehicle}(l, H^2)$;
    $lbt_l = 0$;
    $ubt_l = \Xi$;
    $dep_l = \mu_{c_0\,hs} - \tau^s_{z\,c_0}$;
    $lwSlack = \varnothing$;
    $upSlack = \varnothing$;
    for $c \in l$ **do**
        $lwSlack \leftarrow \mu_{chs} - a_c$;
        $lwSlack \leftarrow b_c - \mu_{chs}$;
    end
    $lbt_l = \arg\min(lwSlack)$;
    $ubt_l = \arg\min(uwSlack)$;
end

After the definition of the *route time windows* for each second-echelon route $l \in \Gamma$, one can identify *scheduling conflicts* between them. We refer to *scheduling conflicts* as the presence of two non-overlapping *route time windows* sharing the same satellite facility, and served by the same single first-echelon vehicle $h \in H^1$. Two *route time windows* are then considered in conflict when neither of its bounds overlaps. The presence of these *scheduling conflicts* thus implies that fleet synchronization is not feasible for a single first-echelon vehicle.

A heuristic method is introduced to avoid *scheduling conflicts* for the LRPTWTDD. The objective of the proposed procedure is then to find *scheduling conflicts* and add a set of valid inequalities to the LRPTWTDD formulation, such that these *scheduling conflicts* can be avoided. The proposed procedure starts by evaluating pairs of *route time windows* for each open satellite facility to store both the routes involved and the pairs of customers with the minimum *temporal slack*. We assume that the presence of customers with very low temporal slack is most likely to provoke *scheduling conflicts* in *route time windows*. Hence, for each satellite $z \in Z$, we define the constant value κ_l to represent the OD demand associated with the customer with the minimum *temporal slack* in each route $l \in \Gamma_z$:

$$\begin{aligned} \sum_{h \in H^1} x^s_{pzh} \geq \omega - (2 - \sum_{h \in H^2} \psi^s_{pz\kappa_i h} - \sum_{h \in H^2} \psi^s_{pz\kappa_j h})M \\ \forall p \in P', i,j \in \Gamma_z, z \in Z, i \neq j. \end{aligned} \tag{5.27}$$

The valid inequality (5.27) then imposes that for each pair of routes allocated to the same satellite $z \in Z$ with *scheduling conflicts*, there must be at least ω first-echelon vehicles active to serve the routes under consideration. By the definition of *scheduling conflicts*, the value of ω is usually set to 2. However, there may be particular cases when *scheduling conflicts* are found in a given satellite $z \in Z$, such that the capacity of all first-echelon vehicles assigned to it is insufficient to accommodate at least one of the two demands κ_l of either of the two routes in conflict. In such a case, the value ω must be greater than the total number of first-echelon vehicles active in the given solution.

The valid inequalities (5.27), which are defined to address all the *scheduling conflicts* for a given solution, can be included in the LRPTWTDD formulation. This refined formulation can then be used to derive a new integer solution. The process of evaluating and refining the LRPTWTDD formulation is repeated iteratively until no *scheduling conflicts* are found in the solution. Once the solution derived by the LRPTWTDD formulation is proven to be feasible for the 2E-MALRPSTT, one can use the best integer solution obtained as the input for the MDVRPTW formulation to determine the routing decisions for the first-echelon routes.

5.2.4. Mathematical formulation for the MDVRPTW. This section describes a three-index flow formulation for the MDVRPTW, representing the subproblem defined by the first echelon of the 2E-MALRPSTT. The proposed formulation relies on the solution of the LRPTWTDD to determine the set of open platform locations and the feasible *route time windows* for each open satellite facility, where fleet synchronization can take place for first-echelon vehicles. It is important to note that each satellite $z \in Z$ can be associated with more than one *route time window*, as they are defined by each second-echelon route $l \in \Gamma_z$ allocated to the satellite. Therefore, we assume that each route $l \in \Gamma_z$ represents the final destination of each first-echelon route, where $[lbt_l, ubt_l]$ represents the time windows for each route. In this representation, each $l \in \Gamma_z$ is considered as a physical point at the same location as the satellite $z \in Z$ to which it is allocated. Similarly, we assume that the demand for each route $l \in \Gamma$ is characterized by a total volume $dem(l)$, which is composed of the demand volumes of the set of OD demands $k \in K$ assigned to the route. The problem setting involves selecting the routing and scheduling decisions for first-echelon vehicles to fulfill the demand of all second-echelon routes $l \in \Gamma$ within their associated time windows. For simplicity, we define the following sets to model the system layout, as well as the sets of values derived from the solution of the LRPTWTDD.

— $\bar{P}'$: set of open platforms defined by the solution of the LRPTWTDD;

— K': set of demands served by the system based on the solution of the LRPTWTDD;

— A_1': first echelon arcs composed by connections between platforms and routes $l \in \Gamma$ as well as connections between different routes $l \in \Gamma$;

The formulation can then be defined as follows:

$$\min \sum_{h \in H^1} \sum_{(i,j) \in A_1'} \zeta_{ij}^s x_{ijh}^s \tag{5.28}$$

subject to

$$\sum_{h \in H^1} \sum_{j \in \Gamma} x_{ijh}^s \leq |H^1| \quad \forall i \in \bar{P}' \tag{5.29}$$

$$\sum_{h \in H^1} \sum_{j \in (\Gamma \cup E_1), i \neq j} x_{ijh}^s = 1 \quad \forall i \in \Gamma \tag{5.30}$$

$$\sum_{j \in E_1} x_{jih}^s - \sum_{j \in \Gamma} x_{ijh}^s = 0 \quad \forall h \in H^1, i \in \bar{P}' \tag{5.31}$$

$$\sum_{j \in (E_1 \cup \Gamma), i \neq j} x_{ijh}^s - \sum_{j \in (\bar{P}' \cup \Gamma), i \neq j} x_{jih}^s = 0 \quad \forall h \in H^1, i \in \Gamma \tag{5.32}$$

$$\sum_{i \in \Gamma} \sum_{j \in (E_1 \cup \Gamma), i \neq j} dem(i)\, x_{ijh}^s \leq cap_1 \quad \forall h \in H^1 \tag{5.33}$$

$$\mu_{ihs}^1 + \tau_{ij}^s - \mu_{jhs}^1 \leq (1 - x_{ijh}^s)M \quad \forall h \in H^1, (i,j) \in A_1' \tag{5.34}$$

$$\nu_{ihs}^1 + \tau_{ij}^s - \nu_{jhs}^1 \leq (1 - x_{ijh}^s)M \quad \forall h \in H^1, (i,j) \in A_1' \tag{5.35}$$

$$\mu_{ihs}^1 \geq \alpha^{ik} \sum_{j \in \Gamma} \phi_{ijkh}^s \quad \forall i \in \bar{P}', h \in H^1, k \in K' \tag{5.36}$$

$$\nu_{ihs}^1 \leq (\alpha^{ik} + W_{max}^1) + (1 - \sum_{j \in \Gamma} \phi_{ijkh}^s)M \quad \forall i \in \bar{P}', h \in H^1, k \in K' \tag{5.37}$$

$$lbt_i \leq \mu_{ihs}^1 \leq ubt_i \quad \forall i \in \Gamma, h \in H^1 \tag{5.38}$$

$$cap_1 x_{ijh}^s - \sum_{k \in K'} vol(k)\, \phi_{ijkh}^s \geq 0 \quad \forall h \in H^1, i \in \bar{P}', j \in \Gamma \tag{5.39}$$

The objective function (5.28) minimizes the total routing cost of first-echelon routes. Constraints (5.29) ensure that the total number of active outbound connections does not exceed the total fleet size. Constraints (5.30) impose that each route $l \in \Gamma$ is visited once by a single vehicle. Constraints (5.31) and (5.32) are flow conservation constraints for platforms and routes $l \in \Gamma$, respectively. Constraints (5.33) ensure that the capacity of each first-echelon vehicle is not exceeded. Constraints (5.34) and (5.35) are responsible for recording the arrival and departure time of each first-echelon vehicle, respectively. Constraints (5.36) assures that the availability times for each commodity are respected, while constraints (5.37) assures that each vehicle departs before the maximum waiting at each platform is reached. Constraints (5.38) ensure that each fisrt-echelon vehicle visits each route $l \in \Gamma$ within their time window. Constraints (5.39) are responsible to link flow and routing variables.

5.3. Defining the reference solution

This section introduces a decision-based scenario clustering method used to determine the reference solution at each iteration of the PH metaheuristic. The proposed strategy aims to quantify the proximity of scenario solutions using an *opportunity cost* measure, introduced by Escobar-Vargas et al. (2023). This *opportunity cost* is defined as a measure to assess the impact of implementing the first-stage decisions associated with a given scenario s_1 when another scenario s_2 occurs (Hewitt et al., 2022).

It is worth noting that the *global population* built to define the reference solution at each iteration of the PH contains more than one solution associated with each scenario subproblem. Therefore, let Λ_s be the set of indices of the solutions in the *global population* that are associated with a scenario $s \in S$. Let $g_i^\iota((\hat{y}_n^{\iota *}, \hat{f}_n^{\iota *}); s_j)$ be the updated value of the objective function (5.5), evaluated with the set of the best first-stage decision variables $\hat{y}_n^{\iota *}$ and $\hat{f}_n^{\iota *}$ at iteration ι, obtained for the solution $n \in \Lambda_{s_i}$ with scenario s_i, assuming that scenario s_j actually occurs. The *opportunity cost* is then denoted by $\theta^\iota(s_i|s_j)$ and calculated according to equation (5.40) for each pair of scenarios s_i and s_j in S with $i \neq j$.

$$\theta^\iota(s_i|s_j) = \min_{n \in \Lambda_{s_i};\, m \in \Lambda_{s_j}} \{g_i^\iota((\hat{y}_n^{\iota *}, \hat{f}_n^{\iota *}); s_j) - g_j^\iota((\hat{y}_m^{\iota *}, \hat{f}_m^{\iota *}); s_j)\} \tag{5.40}$$

Using the *opportunity costs* calculated for each scenario within the *global population*, an *opportunity cost dissimilarity function* can be defined by equation (5.41) for each pair of scenarios s_i and s_j in the set S, where $i \neq j$. This dissimilarity function quantifies the loss incurred when optimizing under the assumption that scenario s_i occurs, given that scenario s_j actually occurs.

$$d^\iota(s_i|s_j) = \theta^\iota(s_i|s_j) + \theta^\iota(s_j|s_i) \tag{5.41}$$

A *Normalized Spectral Clustering* method, based on the methodology proposed by Hewitt et al. (2022), is used to identify scenarios that are similar according to the opportunity cost dissimilarity function. This clustering approach facilitates the formation of a distinct set of clusters CL, denoted as $CL = \{cl_1, cl_2, \ldots, cl_{|CL|}\}$, where each cluster $cl_i \in CL$ represents a group of closely related scenarios. With the clusters identified, we proceed to establish a set of representative scenarios, denoted as Υ, that will define the reference solution. The 'classic' approach outlined by Escobar-Vargas et al. (2023) uses the *medoid* (i.e., the scenario whose average *opportunity cost dissimilarity function* to all other scenarios within the cluster is minimum) of each cluster to define the set of representative scenarios. In addition to this method, we introduce two alternative heuristic strategies for determining the representative scenario within each cluster:

(1) *The longest-distance strategy*: Select one scenario from each cluster whose average opportunity-cost dissimilarity function to all other scenarios within the cluster is maximum. This strategy is designed to establish a reference solution derived from scenarios that exhibit the least similarity within their respective clusters.

(2) *The diversity-based strategy*: Builds upon the work of (Escobar-Vargas et al., 2023) by selecting the *medoids* of each cluster. In addition, this strategy includes two additional scenarios to improve the diversity of design decisions considered for defining the reference solution. This approach incorporates the most central scenario from the entire scenario set (i.e., the scenario with the minimum average opportunity-cost dissimilarity function from all scenarios in the *global population*). Similarly, we also include the scenario with the maximum average opportunity-cost dissimilarity function from all scenarios in the *global population*, representing the scenario with the least similarities.

Once the set of representative scenarios Υ is established, a probability of occurrence η_s can be assigned to each representative scenario $s \in \Upsilon$, such that $\sum_{s \in \Upsilon} \eta_s = 1$. Finally, the reference solution for the location and allocation decisions can be determined by computing equations (5.42) and (5.43) accounting for each representative scenario $s \in \Upsilon$ and the subset of solutions Λ_s associated with each scenario. Here, $y_{ai}^{s\iota}$ and $f_{aijk}^{s\iota}$ represent the first-stage variables of the alternative solution $a \in \Lambda_s$ for a specific scenario $s \in \Upsilon$.

$$\bar{y}_i^{\iota} = \sum_{s \in \Upsilon} \frac{\eta_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} y_{ai}^{s\iota} \quad \forall i \in Z \tag{5.42}$$

$$\bar{f}_{ijk}^{\iota} = \sum_{s \in \Upsilon} \frac{\eta_s}{|\Lambda_s|} \sum_{a \in \Lambda_s} f_{aijk}^{s\iota} \quad \forall i \in P, j \in Z, k \in K \tag{5.43}$$

5.4. Consensus procedure

This section describes the heuristics employed to adapt the fixed costs of the design decisions in accordance with the reference solution specified in Section 5.3. The primary objective of these heuristics is to facilitate the exploration for consensus among all scenario subproblems. Two adjustment heuristics are implemented to modify the location and allocation costs of the scenarios. Specifically, a global adjustment is utilized to guide the overall search process, while a local adjustment is employed to influence the search within each individual scenario subproblem.

The global adjustment starts from the reference solution defined by $\bar{y}_i^{\iota}$ and $\bar{f}_{ijk}^{\iota}$, at iteration ι to identify trends among the scenario solutions and set the cost defined by the objective function in Section 5.1. A low value of either $\bar{y}_i^{\iota}$ or $\bar{f}_{ijk}^{\iota}$ indicates that both design decisions are taken by a very few number of scenario subproblems, while a high value implies that a great majority of scenario subproblems agree on the same design decision. Therefore, one can define a parameter $\beta > 1$ as the adjustment rate of the costs and threshold parameters $0 \leq \epsilon^y \leq 0.5$ and $0 \leq \epsilon^f \leq 0.5$ to determine when the values $\bar{y}_i^{\iota}$ and $\bar{f}_{ijk}^{\iota}$ are either high or low, respectively. The costs are defined according to the objective function (5.5). We thus define the costs $\bar{B}_i^{s\iota} = \left(F_i + \lambda_i^s + \frac{1}{2}\gamma + \gamma\bar{y}_i\right)$ and $\bar{E}_{ijk}^{s\iota} = \left(\Delta_{ijk} + \pi_{ijk}^s + \frac{1}{2}\gamma + \gamma\bar{f}_{ijk}\right)$ as the location and allocation costs of the scenario subproblem, respectively. Then, when $\bar{y}_i^{\iota}$ and $\bar{f}_{ijk}^{\iota}$, are lower than ϵ^y and ϵ^f, the fixed costs are increased to guide the subproblems to avoid opening the given facility. On the other hand, when $\bar{y}_i^{\iota}$ and $\bar{f}_{ijk}^{\iota}$ are higher than $1 - \epsilon^y$ and $1 - \epsilon^f$, the fixed costs are decreased to encourage the scenario subproblems to include the facility within the network design. We define this procedure by equations (5.44) and (5.45):

$$\bar{B}_i^{\iota} = \begin{cases} \beta B_i^{\iota-1} & \text{if } \bar{y}_i^{\iota-1} < \epsilon^y, \\ \frac{1}{\beta} B_i^{\iota-1} & \text{if } \bar{y}_i^{\iota-1} > 1 - \epsilon^y, \\ B_i^{\iota-1} & \text{otherwise;} \end{cases} \tag{5.44}$$

$$\bar{E}_{ijk}^{\iota} = \begin{cases} \beta \bar{E}_{ijk}^{(\iota-1)} & \text{if } \bar{f}_{ijk}^{(\iota-1)} < \epsilon^f, \\ \frac{1}{\beta} \bar{E}_{ijk}^{(\iota-1)} & \text{if } \bar{f}_{ijk}^{(\iota-1)} > 1 - \epsilon^f, \\ \bar{E}_{ijk}^{(\iota-1)} & \text{otherwise.} \end{cases} \tag{5.45}$$

The second adjustment strategy is performed at the level of each scenario $s \in S$, where the costs of variables with large differences relative to the value of the current reference solution at iteration ι, are adjusted using equations (5.46) and (5.47). In this context, we define $0.5 < \delta^y < 1$ and $0.5 < \delta^f < 1$ as the thresholds defining when a local adjustment has to be applied for the location and allocation variables, respectively:

$$\bar{B}_i^{s\iota} = \begin{cases} \beta B_i^{\iota} & \text{if } |y_i^{s(\iota-1)} - \bar{y}_i^{\iota-1}| \geq \delta^y \text{ and } y_i^{s(\iota-1)} = 1, \\ \frac{1}{\beta} B_i^{\iota} & \text{if } |y_i^{s(\iota-1)} - \bar{y}_i^{\iota-1}| \geq \delta^y \text{ and } y_i^{s(\iota-1)} = 0, \\ B_i^{\iota} & \text{otherwise;} \end{cases} \tag{5.46}$$

$$\bar{E}_{ijk}^{s\iota} = \begin{cases} \beta \bar{E}_{ijk}^{\iota} & \text{if } |f_{ijk}^{s(\iota-1)} - \bar{f}_{ijk}^{(\iota-1)}| \geq \delta^f \text{ and } f_{ijk}^{s(\iota-1)} = 1, \\ \frac{1}{\beta} \bar{E}_{ijk}^{\iota} & \text{if } |f_{ijk}^{s(\iota-1)} - \bar{f}_{ijk}^{(\iota-1)}| \geq \delta^f \text{ and } f_{ijk}^{s(\iota-1)} = 0, \\ \bar{E}_{ijk}^{\iota} & \text{otherwise.} \end{cases} \tag{5.47}$$

One needs to establish the values of the initial reference design $\bar{y}_i$ and $\bar{f}_{ijk}$. We define the initial reference design in terms of the location costs $\bar{B}_i^{s\iota}$ and allocation costs $\bar{E}_{ijk}^{s\iota}$ at iteration $\iota = 0$. The location and allocation costs of the scenario subproblems are initialized with their original costs. Therefore, we set $\bar{B}_i^{s(0)} = F_i$, $\forall i \in (P \cup Z), s \in S$, and $\bar{E}_{ijk}^{s(0)} = \Delta_{ijk}$, $\forall i \in P, j \in Z, k \in K, s \in S$.

The proposed PH metaheuristic is designed to terminate either upon reaching a consensus solution or upon satisfying another stopping criterion, such as a computation time limit. A consensus solution is achieved when all first-stage decisions $\bar{y}_i^{\iota}$ and $\bar{f}_{ijk}^{\iota}$ reach a general consensus at a particular iteration ι. However, achieving consensus on all first-stage decisions might not occur at the end of each iteration of the PH metaheuristic. In such cases, the PH metaheuristic is configured to establish a feasible solution for the 2E-MALRPSTT by means of the extended formulation outlined in Section 4.2. The approach to defining a feasible solution consists of fixing the location and allocation variables for which consensus is obtained by the PH metaheuristic, and then addressing the resulting problem with the subproblem algorithm. The solution obtained provides a feasible solution for all design decisions. One can then update the best solution obtained and continue with the PH metaheuristic.

6. Computational results

This section presents and discusses the results of experiments conducted to assess several aspects, including the stability of the scenario generation procedure, the performance of the proposed PH metaheuristic for the 2E-MALRPSTT, the effectiveness of the heuristics used for the selection strategy, and the significance of explicitly considering stochastic travel times. We begin by introducing the instances utilized in the computational study, which are described in Section 6.1. The performance of the decomposition-based heuristic for the 2E-MALRPSTT is presented in Section 6.2. The stability of the scenarios is investigated and discussed in Section 6.3. Furthermore, the performance analysis of our PH metaheuristic is presented in Section 6.4, followed by the analysis of the value of the stochastic solution in Section 6.5.

The experiments were conducted on a single machine with an Intel(R) Core(TM) i7-7800X processor with 128 GB of RAM running Linux. The mathematical formulation and the proposed solution frameworks are implemented in C++ using IBM ILOG CPLEX concert technology 20.1. The tables in this section display the summarized results for the associated experiments, while more detailed results are provided in the supplementary material in C.2 and C.3.

6.1. Instances

This section presents the set of instances defined for the 2E-MALRPSTT. To the best of our knowledge, no instances are available in the literature considering the set of attributes addressed in this work. Hence, we define our testbed based on the instances introduced by Escobar-Vargas and Crainic (2023) for the 2E-MALRPS. The instances introduced by Escobar-Vargas and Crainic (2023), simulate an urban area constituted by platforms, satellites, and customers. In spite of the similarities in the problem settings, the original instances proposed by Escobar-Vargas and Crainic (2023) are designed for a deterministic problem and thus lack any uncertain aspect. We thus adapt these instances to accommodate the consideration of stochastic travel times.

We introduce two instance sets composed of 20 instances with 10 and 15 OD demands with stochastic travel times. Temporal aspects such as time-dependent OD demands, customer time windows and service times are fixed as the values given by Escobar-Vargas and Crainic (2023). Load capacities for vehicles are considered to be fixed, where first-level vehicles have a capacity of $cap_1 = 200$ and second-level vehicles have a capacity of $cap_2 = 50$. The instance set is divided into four categories CA, CB, CC, and CD, differing in the time window width and customer-demand values (Dellaert et al., 2019).

Scenarios are created using the copula-based approach introduced by Kaut (2014) to ensure adherence to the statistical characteristics specified for the stochastic travel times. This process necessitates identifying the appropriate marginal distribution for each travel time, which can be selected from a range of marginal distribution options available within the method. We adopt the strategy outlined by Escobar-Vargas et al. (2023) for the 2E-LRP with stochastic and correlated demands to determine the marginal distributions for all travel times within each instance.

We use the four categories, CA, CB, CC, and CD, to test a unique probability distribution for all travel times within each category. The aim is to capture four distinct events that influence travel time values. Following the guidelines of Escobar-Vargas et al. (2023), we employ the lognormal distribution to model stochastic travel times. As illustrated in Figure 15,

category	distribution	mean	standard deviation
CA	left-skewed lognormal	2.76	0.92
CB	left-skewed lognormal	3.37	0.26
CC	symmetrical lognormal	3.10	0.20
CD	symmetrical lognormal	2.90	0.32

Table 12 – Instance category description

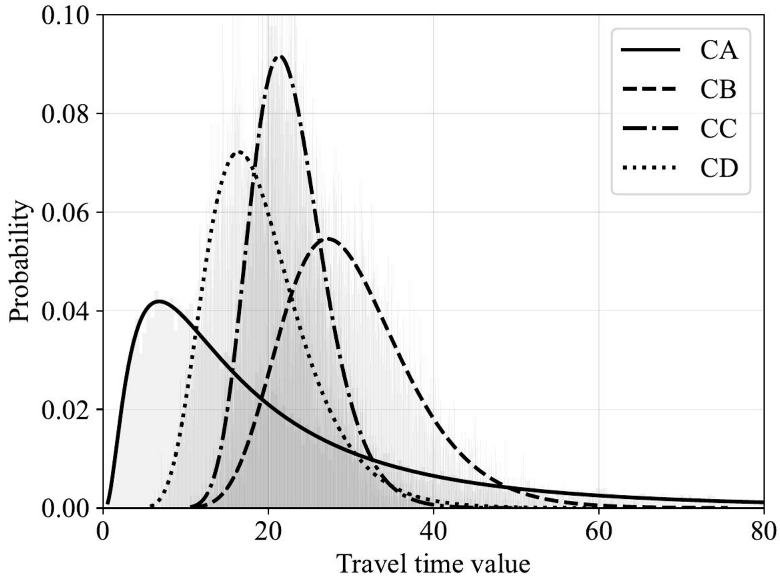

Figure 15 – Probability distribution

each instance category is associated with a distinct probability distribution characterized by different parameter settings. The details of the proposed distributions for categories CA, CB, CC, and CD, which vary in terms of travel time values, as well as the distribution used to generate random scenarios for each instance, are presented in Table 12.

6.2. Performance of heuristic method for the scenario subproblems

We initially assess the effectiveness of the proposed decomposition-based heuristic for the DSS. To the best of our knowledge, only the work of Escobar-Vargas and Crainic (2023) has introduced an exact solution method for addressing a multi-attribute 2E-LRP, which shares most of the attributes of the deterministic 2E-MALRPSTT but imposes stricter synchronization requirements. Similarly, the work of Dellaert et al. (2019) deals with a closely related problem setting, involving a 2E-VRP with multi-commodity demands and time windows. This problem setting provides a suitable benchmark to evaluate our heuristic method. We thus compare the performance of the decomposition-based heuristic against the results obtained by Escobar-Vargas and Crainic (2023) and Dellaert et al. (2019) in terms of solution quality and computational efficiency. To perform the experiments on instances provided by

Dellaert et al. (2019), our decomposition-based heuristic is adapted to enable each vehicle to start and finish its route at facilities, instead of vehicle garages. Similarly, a set of broad availability times is defined for each OD demand to adapt it to the work of Dellaert et al. (2019), where no availability times are being considered.

Table 13 displays the comparative performance results. It provides the number of OD demands (OD) for each instance set, the source of each instance, the instance type (INT), the average optimality gap percentage (GAP) between the upper bound obtained by the decomposition-based heuristic and the lower bound provided by Escobar-Vargas and Crainic (2023) and Dellaert et al. (2019) for each instance. Additionally, it provides the average difference percentage between the upper bounds (UB Diff) and the runtime in seconds (CPU Diff) of the solution methods involved. We use the results obtained by the decomposition-based heuristic as the reference for comparing with the solution methods of Escobar-Vargas and Crainic (2023) and Dellaert et al. (2019). Hence, a negative value in the *UB Diff* or the *CPU Diff* column indicates an improvement by the proposed decomposition-based heuristic when compared to the works of Escobar-Vargas and Crainic (2023) or Dellaert et al. (2019).

The results reported in Table 13 present a general improvement in both solution quality and run-time for both instance sets. Compared to the results of Escobar-Vargas and Crainic (2023), the decomposition-based heuristic can obtain the optimal solutions in 60 out of 60 instances with up to 15 OD demands. For instances with 30 OD demands, the proposed heuristic is capable of improving the best-known upper bounds of six instances from the results reported by Escobar-Vargas and Crainic (2023), while matching the best-known solutions of 40 out of the 60 instances. In terms of run-time, the decomposition heuristic presents an expected improvement in the computational time required to obtain the optimal solution of instances with up to 30 OD demands. We observe that the decomposition of the deterministic multi-attribute 2E-LRP into two reduced problem, enables a more efficient exploration of the solution space, by allowing a faster convergence to optimality for each one of the mathematical formulations, which in turn leads to a significant reduction of computational time required to address the deterministic multi-attribute 2E-LRP.

A similar behaviour was obtained for the decomposition-based heuristic when compared to the results reported by Dellaert et al. (2019). The proposed method can obtain the optimal solution for all instances with up to 30 OD demands. Nonetheless, experiments on instances with 30 OD demands are not able to match the run-time of the branch-and-price method proposed by Dellaert et al. (2019). Multiple factors contribute to this behaviour. On the one hand, our heuristic needs to explore the complete solution space defined by the combination of platform facilities, even when a good quality solution is found at early iterations of the method. On the other hand, the lack of an explicit consideration of availability times yields a larger integer problem to cover all the potential outbound connections of each OD demand. The latter, in combination with the weak LP relaxation of the compact

Instance	INT	OD											
		5			10			15			30		
		GAP	UB DIFF	CPU DIFF	GAP	UB DIFF	CPU DIFF	GAP	UB DIFF	CPU DIFF	GAP	UB DIFF	CPU DIFF
Escobar&Crainic	CA	0.00	0.00	-25.37	0.00	0.00	-275.65	0.00	0.00	-417.59	1.42	0.43	-1606.69
	CB	0.00	0.00	-11.67	0.00	0.00	-2516.36	0.00	0.00	-3.56	5.39	-0.07	-1848.43
	CC	0.00	0.00	33.50	0.00	0.00	-209.92	0.00	0.00	23.91	8.62	0.55	-2061.53
	CD	0.00	0.00	-24.77	0.00	0.00	-78.09	0.00	0.00	62.43	2.65	0.20	-1823.24
Dellaert	CA	N.A	N.A	N.A	N.A	N.A	N.A	14.54	-22.30	-59570.96	1.59	-11.73	91.77
	CB	N.A	N.A	N.A	N.A	N.A	N.A	18.91	-15.19	-22791.08	2.11	-14.10	93.70
	CC	N.A	N.A	N.A	N.A	N.A	N.A	14.44	-20.39	-11390.61	3.70	-8.55	-153.92
	CD	N.A	N.A	N.A	N.A	N.A	N.A	18.23	-17.58	-43054.35	5.40	-18.92	97.64

Table 13 – Decomposition based heuristic: Summary results on Escobar and Dellaert instances.

formulation, leads to greater run times to converge to an optimal solution on each of the mathematical formulations.

Overall, it is evident that the proposed solution method is able to generate high-quality upper bounds within reasonable computation times. The heuristic also exhibits robust behaviour, demonstrating its ability to handle various instance types and delivering significant enhancements in both performance and solution quality when compared to the exact approaches presented by Escobar-Vargas and Crainic (2023) and Dellaert et al. (2019). The computational experiments further indicate that the proposed heuristic consistently performs well across complex problem settings characterized by multiple interacting attributes. This characteristic is particularly advantageous for decomposition-based methods, such as the PH metaheuristic, where multiple complex scenario subproblems must be iteratively solved.

6.3. Scenario stability

This section presents the computational experiments conducted to evaluate the stability of the selected scenario generation procedure. To ensure meaningful results within reasonable computation times, we examine the number and quality of scenarios that should be considered when using the copula-based scenario generation method (Kaut, 2014), which is employed in this study. In this context, we apply the 'standard' concept of in-sample and out-of-sample stability evaluation proposed by Kaut and Wallace (2007) to assess the performance of the scenario generation approach, which involves generating and comparing multiple sets of scenarios. Notice that, due to the characteristics of the copula-based scenario generation method, which produces identical or similar scenario sets with identical inputs in different runs, the conventional in-sample stability test is not applicable in this context (Guo et al., 2019).

This study adopts a modified version of the 'standard' approach proposed by Zhang et al. (2021) to assess the stability and quality of the scenario set. The modified approach utilizes two measures, namely the *relative difference* (RD) and *variance* (VAR). To represent a case with $|S|$ scenarios of travel times, a set of $2m+1$ scenario trees is constructed, each with sizes $|S|-m$, $|S|-(m-1)$,...,$|S|$,...,$|S|+m$, where m is a positive integer. The proposed PH

S	RD (%)					VAR		
	VSR	**AISR**	**MIN**	**AVERAGE**	**MAX**	**MIN**	**AVERAGE**	**MAX**
5	1	4.30	1.94	4.18	8.92	20.89	35.66	61.26
10	4	2.90	1.55	2.69	3.92	6.52	28.62	53.90
20	15	2.36	0.61	1.68	2.52	9.85	20.82	44.82
30	17	2.05	0.00	1.11	2.11	7.66	20.54	43.60
50	20	n.a.	0.31	1.06	1.84	8.05	18.95	35.22
100	20	n.a.	0.06	0.24	0.50	8.21	14.56	29.50

Table 14 – Stability tests: summarized results of relative difference and variance for different scenario sizes.

metaheuristic is employed to address the 2E-MALRPSTT for each of the $2m+1$ scenario trees, resulting in $2m+1$ solutions denoted as $Z_{|S|+i}$, representing one solution for each scenario tree. These solutions are evaluated by computing the objective function value $F(Z_{|S|+i})$ for each of the $2m+1$ scenario trees, resulting in a set of $2m+1$ objective function values for each solution $Z_{|S|+i}$. Finally, for each problem instance, the maximum $(F^{+}(Z_{|S|+i}))$, minimum $(F^{-}(Z_{|S|+i}))$, and variance $(\sigma_{|S|+i})$ of the objective function values are determined for each solution $Z_{|S|+i}$. The calculations for the *relative difference* and *variance* are defined by Equations (6.1) and (6.2), respectively.

$$RD = \max_{i \in [-m,m]} \left\{ \frac{F^{+}(Z_{|S|+i}) - F^{-}(Z_{|S|+i})}{F^{+}(Z_{|S|+i})} \times 100\% \right\} \tag{6.1}$$

$$VAR = \max_{i \in [-m,m]} \{\sigma_{|S|+i}\} \tag{6.2}$$

Table 14 presents a summary of the relative difference (RD) and variance (VAR) values obtained for the $2m+1$ scenario trees constructed for each problem instance. The table provides information on the number of scenarios in each scenario tree ($|S|$), as well as the minimum (MIN), average (AVG), and maximum (MAX) values of RD and VAR. For stability assessment, a criterion of RD $\leq$ 2% is defined. Two additional performance measures are used: the number of instances that satisfy the stability criterion (Valid Stability Requirement or VSR) and the average RD of instances that fail to meet the criterion (Average Invalid Stability Requirement or AISR). The experiments are conducted for instances with 15 OD demands using multiple scenario trees with varying numbers of scenarios ($|S|$), and the value of m is set to 4 based on the studies by Guo et al. (2019) and Zhang et al. (2021). To ensure reliable results, only the best objective function obtained for each instance using the proposed PH metaheuristic is used for the stability tests.

The results presented in Table 14 validate the expected improvement in approximation quality with an increased number of scenarios. However, this improvement comes at the cost of increased computational effort. While larger scenario trees are desirable to achieve

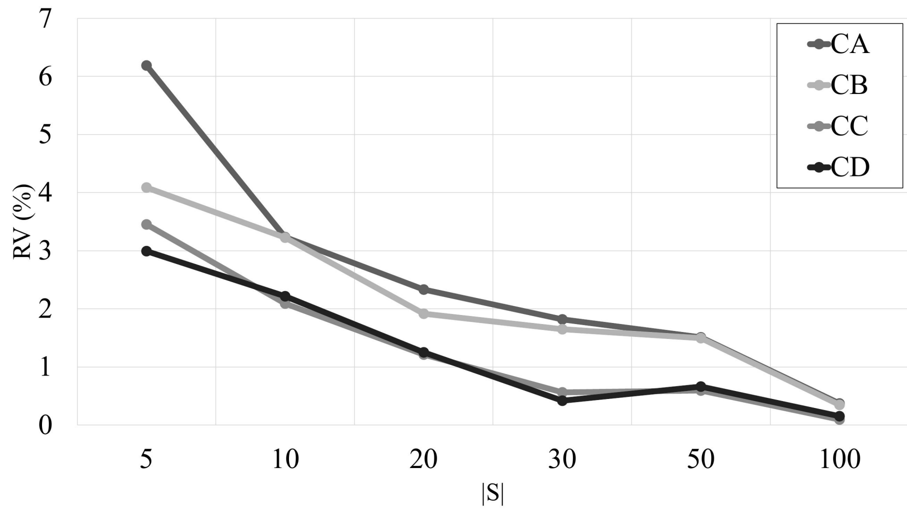

Figure 16 – Stability test: Relative difference for each instance type vs scenario size.

smaller relative errors, solving the resulting subproblems for the entire set of instances at each iteration of the PH metaheuristic becomes progressively more challenging. In the instances examined in this study, using a minimum of 20 scenarios yields sufficiently accurate solutions. Specifically, it results in a relative error of less than 2% for 15 out of the 20 instances, with the remaining instances having an average relative error of 2.4%. Alternatively, instances encompassing 30 scenarios can be solved with reasonable computational effort while ensuring adequately precise solutions. The outcomes demonstrate a relative error of less than 2% for 17 out of the 20 instances, while reducing the average relative error to 2% for the remaining instances. Based on these findings, we conclude that stability is achieved, enabling the consideration of instances with $|S| = 30$ in the subsequent computational study.

Figure 16 illustrates the correlation between the number of scenarios employed for stability assessment and the associated relative difference values for each instance category. The results clearly indicate that the stability of solutions is significantly affected by the dispersion of travel time distributions. Notably, instances classified as CA and CB, characterized by a higher degree of dispersion in their travel time distributions, exhibit much more fluctuations in the relative difference values compared to instance type CC and CD. This behaviour can be attributed to the copula-based method utilized, which generates a diverse array of scenarios, leading to heightened volatility in objective function values and increased variability in the recourse actions.

6.4. Performance of the PH metaheuristic

Computational experiments were conducted to assess the performance of the PH metaheuristic using various heuristic strategies to define the reference solution. Moreover, a

INSTANCE TYPE	PH									CPLEX	
	CS			DBS			LDS				
	OG(%)	ITER.	TIME (seg.)	OG(%)	ITER	TIME (seg.)	OG(%)	ITER.	TIME (seg.)	OG(%)	TIME (seg.)
CA	3.66	6.80	237.08	0.49	9.40	600.09	1.42	7.40	332.49	19.30	28800.00
CB	2.78	7.00	187.59	0.58	9.80	598.84	0.96	8.80	594.81	18.85	28800.00
CC	1.96	6.20	195.76	0.41	7.00	415.79	0.56	5.20	328.43	16.74	28800.00
CD	2.19	7.40	366.10	0.18	7.20	491.84	0.56	7.40	411.55	18.47	28800.00
Averages	2.65	6.85	246.63	0.41	8.35	526.64	0.87	7.20	416.82	18.34	28800.00

Table 15 – Summarized results on instances with 10 OD demands

INSTANCE TYPE	PH									CPLEX	
	CS			DBS			LDS				
	OG(%)	ITER	TIME (seg.)	OG(%)	ITER	TIME (seg.)	OG(%)	ITER	TIME (seg.)	OG(%)	TIME (seg.)
CA	2.97	6.40	2081.61	1.06	10.80	2894.03	1.40	8.20	2441.52	39.29	57600.00
CB	1.57	9.00	2158.48	0.49	10.80	2938.00	0.78	9.20	2511.86	30.06	57600.00
CC	2.42	8.60	2285.84	1.04	9.40	3088.12	1.27	7.40	2442.64	33.93	57600.00
CD	2.83	8.20	2417.04	1.71	8.20	2964.89	2.07	8.00	2406.67	38.29	57600.00
Averages	2.45	8.05	2235.74	1.08	9.80	2971.26	1.38	8.20	2450.67	35.39	57600.00

Table 16 – Summarized results on instances with 15 OD demands

comparison was made between the results obtained by the PH metaheuristic and those obtained by CPLEX when solving the complete extensive model. A comprehensive set of experiments was performed, incorporating diverse stopping criteria. In the experiments involving the PH metaheuristic, a maximum time limit of 2 hours was imposed. Conversely, for the experiments with the complete extensive model using CPLEX, time limits of 8 hours and 16 hours were set for instances with 10 OD demands and 15 OD demands, respectively. CPLEX was employed with default parameter settings, using a thread limit of 6 for solving the extensive model and a thread limit of 1 for solving the scenario subproblems within the PH metaheuristic.

The tables in this section are structured to present the results obtained using CPLEX and the PH metaheuristic, labeled as 'CPLEX' and 'PH' respectively. Additionally, within the PH results, distinctions are made based on the specific selection strategy employed: the classical clustering strategy (CS) proposed by Escobar-Vargas et al. (2023), the diversity-based strategy (DBS), and the longest-distance strategy (LDS). Each table provides the average optimality gap expressed as a percentage (OG), the average computational time in seconds, and the average number of iterations (ITER) for the PH metaheuristic.

Tables 15 and 16 provide the summarized results obtained from instances with 10 and 15 OD demands respectively. Addressing the complete stochastic problems which involve a diverse range of travel time values and multiple interacting time-sensitive attributes poses a significant challenge. This challenge becomes apparent when solving the complete stochastic problem with 10 and 15 OD demands, where CPLEX achieved average optimality gaps of 18% and 35% respectively, while consistently reaching the maximum time limit. In contrast, all configurations of the proposed PH metaheuristic outperformed CPLEX in all computational measures.

The proposed PH metaheuristic demonstrates significant improvements in both solution quality and run times. When considering instances with 10 and 15 OD demands, all three configurations of the PH metaheuristic achieve consensus for all first-stage decisions, resulting in an average time reduction of 95% and an average upper bound improvement of 25% compared to CPLEX. This notable performance enhancement is attributed to the use of the decomposition-based heuristic to accelerate the resolution of each scenario subproblem and the use of specialized strategies to define the reference solution at each iteration of the PH metaheuristic to accelerate the search for consensus.

It is noteworthy that the behaviour of the PH metaheuristic varies depending on the selected selection strategy. Interestingly, the diversity-based strategy demonstrates a higher level of robustness compared to the classical clustering strategy and the longest-distance strategy. In terms of upper bounds, the diversity-based strategy exhibits the most notable improvement, with an average optimality gap of 0.4% and 1.08% for instances featuring 10 and 15 OD demands, respectively.

Regarding the number of iterations, the diversity-based strategy, on average, requires 16% more iterations than the other two strategies to reach a consensus on the complete set of first-stage decisions. This is primarily due to the increased diversity of scenarios considered when determining the reference solution at each iteration of the PH metaheuristic, which allows for a wider range of first-stage solutions to be considered in subsequent iterations. This reduces the presence of strong trends in the first-stage decisions during the early iterations of the PH. These slight variations tend to increase the number of iterations needed to achieve consensus over the complete set of first-stage decisions. This phenomenon becomes apparent in experiments conducted on instance types CA and CB, where the travel time values for each scenario are set to display a broader range. In these instances, the diversity-based strategy achieves an optimality gap of less than 1% for 8 out of the 10 instances with 10 OD demands and 9 out of the 10 instances with 15 OD demands. However, it requires a higher number of iterations compared to instances of type CC and CD to achieve consensus over the complete set of first-stage decisions.

It is important to emphasize that both the classic clustering strategy and the longest-distance strategy offer significant benefits when compared to the diversity-based strategy. First, the classic clustering strategy demonstrates the lowest runtime for instances with both 10 and 15 OD demands, resulting in an average reduction of 32% in comparison to the other strategies. Second, the longest-distance strategy achieves good-quality upper bounds while requiring, on average, 19% less runtime than the diversity-based strategy. It is worth noting that the longest-distance strategy exhibits greater robustness in instances where the travel time values are more evenly distributed, such as instance types CC and CD. This is attributed to the use of a significantly different set of scenarios for defining the reference solution at each iteration of the PH metaheuristic. This reduces the number of alternative scenario solutions that can be used to define the reference solutions in subsequent iterations of the PH metaheuristic, thereby accelerating the search for consensus. However, this effect also reduces the strategy's effectiveness in avoiding local optima when the travel time values are sparse, as observed in instance types CA and CB. In these instances, the reset procedure proposed by Escobar-Vargas et al. (2023) is extensively used, which in turn increases the runtime necessary to reach consensus.

6.5. Value of the stochastic problem

This section aims to evaluate the value of the stochastic solution (VSS) for the 2E-MALRPSTT. The evaluation involves comparing the solutions obtained from the stochastic program with those derived from a deterministic approximation problem. The deterministic approximation problem represents a simplified version of the complete stochastic formulation, where a single scenario is used, and the random variables are set to their corresponding expected values. To calculate the expected cost of the deterministic approximation problem, each scenario tree defined for each instance is evaluated while keeping the first-stage decisions from the deterministic approximation problem fixed. The VSS is then determined as the difference between the optimal value of the deterministic approximation problem and the solution obtained from the stochastic model using the proposed PH metaheuristic.

The deterministic approximation is obtained using two different approaches: the Mean Deterministic Approximation (ADA) and the Maximum Deterministic Approximation (MDA). In the ADA approach, the stochastic travel times are estimated as the mean value across all considered scenarios, while in the MDA approach, the stochastic travel times are estimated as the maximum value among all considered scenarios. Results for the deterministic approximation of the 2E-MALRPSTT are obtained by solving each instance with the complete stochastic model using CPLEX under a 2-hour time limit. Subsequently, the computational tests performed using the ADA and MDA approaches are compared to the solutions obtained by the proposed PH metaheuristic, as reported in Section 6.4.

Tables 17 and 18 present a comparison between the results of the stochastic problem and the two deterministic approximation approaches for instances with 10 and 15 OD demands, respectively. The tables are structured to highlight the cost differences associated with the deterministic approximation solutions, using the stochastic approach as the baseline. Each table includes information about the instance names and the best-known solutions (BKS) obtained using the proposed PH metaheuristic. Additionally, the tables provide the optimal solution (UB), the outsourced cost difference (out. diff.), and the percentage increase in the complete operational cost (cost diff.) of the deterministic approximation approaches compared to the PH metaheuristic.

INSTANCE	BKS	ADA				MDA			
		UB	Cost diff.	Out diff.	TIME (seg.)	UB	Cost diff.	TIME (seg.)	Out diff.
Ca1-2,3,10	342.17	393.00	12.93	7.76	1.65	387.50	11.70	12.62	3.47
Ca2-2,3,10	249.67	294.00	15.08	20.99	5.24	308.33	19.03	12.44	3.83
Ca3-2,3,10	329.30	370.17	11.04	17.32	2.40	386.67	14.84	15.52	4.52
Ca4-2,3,10	307.50	345.67	11.04	15.06	5.76	364.83	15.71	14.49	1.31
Ca5-2,3,10	304.00	342.50	11.24	20.81	1.99	348.67	12.81	10.67	5.34
Cb1-2,3,10	356.70	393.33	9.31	6.86	1.64	410.83	13.18	11.32	4.12
Cb2-2,3,10	281.67	320.83	12.21	5.03	2.62	343.33	17.96	15.07	3.89
Cb3-2,3,10	370.83	409.17	9.37	18.22	3.24	421.17	11.95	8.02	1.18
Cb4-2,3,10	327.50	367.00	10.76	11.59	2.00	384.17	14.75	8.32	4.14
Cb5-2,3,10	319.00	358.83	11.10	18.20	4.16	342.33	6.82	11.38	4.25
Cc1-2,3,10	361.80	399.50	9.44	14.24	3.15	384.00	5.78	4.96	4.39
Cc2-2,3,10	279.17	316.33	11.75	17.08	1.10	301.33	7.36	2.35	5.16
Cc3-2,3,10	351.33	387.33	9.29	10.81	1.94	371.83	5.51	7.77	2.11
Cc4-2,3,10	331.00	371.50	10.90	8.41	3.12	356.50	7.15	9.62	2.66
Cc5-2,3,10	307.67	347.33	11.42	20.96	1.31	331.33	7.14	5.25	2.66
Cd1-2,3,10	343.50	383.33	10.39	9.50	3.24	367.67	6.57	2.94	4.42
Cd2-2,3,10	266.50	306.67	13.10	18.95	3.15	290.33	8.21	3.74	2.59
Cd3-2,3,10	330.67	374.50	11.70	18.28	2.95	353.83	6.55	6.47	3.18
Cd4-2,3,10	317.67	356.17	10.81	8.20	3.67	340.50	6.71	4.58	2.28
Cd5-2,3,10	295.17	335.00	11.89	11.34	3.04	319.00	7.47	3.63	2.52
Averages			11.24	13.98	2.87		10.36	8.56	3.40

Table 17 – Value of the stochastic solutions for instances with 10 OD demands

INSTANCE	BKS	ADA				MDA			
		UB	Cost diff.	Out diff.	TIME (seg.)	UB	Cost diff.	TIME (seg.)	Out diff.
Ca1-2,3,15	778.00	906.25	14.15	14.74	5.99	900.17	13.57	9.76	6.85
Ca2-2,3,15	699.40	879.25	20.45	8.37	4.44	848.33	17.56	15.07	7.83
Ca3-2,3,15	724.40	819.41	11.60	10.12	5.00	803.33	9.83	9.97	8.29
Ca4-2,3,15	646.30	797.41	18.95	14.49	8.31	789.00	18.09	15.33	6.90
Ca5-2,3,15	686.40	864.25	20.58	16.36	8.25	860.50	20.23	14.51	6.15
Cb1-2,3,15	1043.00	1208.41	13.69	17.23	7.48	1188.50	12.24	9.24	7.83
Cb2-2,3,15	1636.40	1864.25	12.22	12.12	6.23	1883.67	13.13	9.03	5.99
Cb3-2,3,15	1225.00	1272.75	3.75	13.20	4.13	1287.67	4.87	16.43	6.00
Cb4-2,3,15	960.80	1131.25	15.07	16.28	4.35	1134.17	15.29	11.51	8.85
Cb5-2,3,15	931.10	1063.41	12.44	11.85	5.94	1038.67	10.36	6.85	5.55
Cc1-2,3,15	855.00	1008.75	15.24	12.62	4.94	936.17	8.67	6.03	5.28
Cc2-2,3,15	831.20	962.50	13.64	10.59	7.50	926.17	10.25	14.88	8.69
Cc3-2,3,15	876.50	1007.67	13.02	12.40	8.06	972.33	9.86	5.35	8.67
Cc4-2,3,15	793.60	904.33	12.24	12.78	6.81	881.00	9.92	5.39	4.14
Cc5-2,3,15	733.33	866.00	15.32	12.76	6.63	822.50	10.84	10.12	6.62
Cd1-2,3,15	769.90	884.17	12.92	19.96	5.60	854.50	9.90	5.24	6.18
Cd2-2,3,15	655.90	777.83	15.68	20.40	5.51	708.17	7.38	3.76	7.18
Cd3-2,3,15	658.90	784.67	16.03	14.36	7.49	748.67	11.99	11.47	7.62
Cd4-2,3,15	693.30	851.83	18.61	13.80	8.09	765.00	9.37	17.85	5.88
Cd5-2,3,15	802.90	929.83	13.65	10.82	6.28	888.17	9.60	6.05	8.76
Averages			14.46	13.76	6.35		11.65	10.19	6.96

Table 18 – Value of the stochastic solutions for instances with 15 OD demands

The results reveal important cost increases when uncertainty is not accounted for in the problem setting. Overall, the stochastic approach consistently outperformes both deterministic approximation approaches, demonstrating an average upper bound improvement of 9% across all instances. However, among the two deterministic approximation approaches, the MDA approach exhibited the best overall performance, by achieving an average cost increase of 11% and an average increase in outsourced services of 9% for instances with 10 and 15 OD demands. This can be attributed to the robust design decisions made by the MDA approach, which are based on the scenario with the highest travel time values for each scenario tree. This led to a reduced number of outsourced services but also less effective design decisions, resulting in suboptimal vehicle operation during the recourse stage for the complete set of scenarios. In contrast, the ADA approach demonstrated less variation in the cost difference throughout the complete instance set but required, on average, 4% more outsourced services compared to the MDA approach. It is evident that the definition of design decisions based on the mean approximation of travel times often resulted in insufficient first-stage decisions to accommodate scenarios with generally higher travel time values. As a consequence, this increased the overall cost of outsourced services required to serve customers that could not be served by a vehicle route within the system during the recourse stage due to violations of customer time windows or fleet synchronization constraints.

It can be concluded that the deterministic approximation used to address the 2E-MALRPSTT lacks the necessary contextual information to incorporate the diverse variations present in the scenario sets defined for each instance. This limitation becomes apparent when considering the complete instance set, where multiple time-sensitive attributes interact, posing challenges in defining flexible design decisions when only a single scenario is considered. In contrast, the stochastic approach proves to be more cost-effective for both design and routing decisions, clearly outperforming the deterministic approximation in terms of solution quality. These observations provide strong justification for utilizing the stochastic approach, to ensure a cost-effective system design that incorporates location routing decisions under uncertainty, particularly in complex multi-attribute problem settings.

7. Conclusions

This paper introduces and investigates the two-echelon multi-attribute location-routing problem with stochastic travel times. The problem is formulated using a two-stage stochastic programming approach, which effectively incorporates multiple interacting attributes. In the first stage, the design phase involves making location and allocation decisions for both platform and satellite facilities. The recourse (second) stage, includes routing and scheduling decisions in a rich multi-attribute distribution system after the realization of uncertain travel times. To address the complete stochastic problem, an enhanced PH metaheuristic is proposed. We also propose a decomposition-based heuristic to address each scenario-specific subproblem, which accelerates the solution process for the multi-attribute problem arising from scenario decomposition. Additionally, two novel scenario-selection strategies are introduced in computing the reference solution to further accelerate the consensus over the first-stage decisions.

Comprehensive computational experiments show the effectiveness of the decomposition-based heuristic in addressing rich multi-attribute deterministic problem settings. Comparative analyses reveal that the proposed PH metaheuristic, along with the two novel selection strategies, outperforms state-of-the-art approaches when stochastic travel times are taken into account. The results underscore the effectiveness of the decomposition-based heuristic and the PH metaheuristic in handling diverse sets of scenario sets with different problem characteristics.

For future research, when targeting larger instances in terms of network size or the number of scenarios, it would be beneficial to develop or adapt efficient solution methods to further accelerate the solution of each scenario-specific subproblem at each iteration of the PH metaheuristic. Additionally, exploring alternative selection strategies to enhance the search for consensus could prove valuable. The modelling approach and solution frameworks proposed in this paper for the specific problem setting may also find applications in other settings that share one or more of the considered attributes.

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Chapter 3

CONCLUSIONS

This thesis addresses a significant class of the two-echelon location-routing problem known as two-echelon multi-attribute location-routing problems (2E-MALRPs). This problem class provides a unified perspective on a wide range of transportation issues, drawing inspiration from city logistics principles. Each problem addressed in this thesis aims to encompass crucial attributes, including time-dependent multicommodity, origin-to-destination (OD) demand, time windows, fleet synchronization at intermediate facilities, correlated uncertain demands, and uncertain travel times. Motivated by the growing interest in more realistic problem settings and recognizing the lack of efficient modeling and solution frameworks for the general versions of these problems, we have embraced the challenge of this research endeavor.

In Chapter 2, a comprehensive literature review is given, focused on the related works on the MA-2ELRP. The classification and scope of the considered contributions are structured to present the advances of different LRPs and 2E-LRPs considering multiple attributes and the VRPs with synchronization constraints on facilities. The classification presents a generalized perspective of each problem variant and highlights the most prominent contributions followed by an algorithmic discussion of the whole considered literature body. A narrowed list of gaps in the literature is then presented, focusing on the most relevant aspects that this research project can contribute to fill.

In Article 1 we introduce the two-echelon multi-attribute location-routing problem with synchronization constraints, 2E-MALRPS, and present a mixed-integer programming formulation on a hybrid time-space network combining continuous and discrete time representations. We also present an exact solution framework that iteratively refines a reduced time-space network, solving the 2E-MALRPS formulation defined on the reduced network to extract bounds and achieve temporal granularity refinements, in order to guide the method toward to optimal solution of the original problem. The paper generalizes the dynamic discretization discovery method to complex problem settings involving several levels of location, routing, and synchronization decisions.

In Article 2, we address the two-echelon multicommodity location-routing problem with stochastic and correlated Demands (2E-MLRPSCD). This study presents a two-stage stochastic programming formulation to effectively model the problem. In the first stage, we determine the locations of satellite facilities as design decisions, while in the second stage, we make recourse decisions to distribute the observed demands effectively. To tackle this problem, we propose a PH metaheuristic framework, which incorporates two population structures to enhance the diversity of design decisions for each scenario subproblem. Additionally, we introduce and compare three novel strategies aimed at accelerating the search for the solution space in the context of this stochastic problem.

In Article 3, we introduce and investigate the two-echelon multi-attribute location-routing problem with stochastic travel times (2E-MALRPSTT). This problem is formulated using a two-stage scenario-based stochastic programming approach, which effectively incorporates the interactions among multiple attributes. In the first stage, the design phase involves making location and allocation decisions for both platform and satellite facilities. These design decisions are then fixed in the recourse (second) stage, where routing and scheduling decisions are made in a rich multi-attribute distribution system after the realization of uncertain travel times. To address the complete stochastic problem, we propose a Progressive Hedging (PH) metaheuristic, which decomposes the problem into scenario-specific subproblems. These subproblems are iteratively solved and adjusted until a consensus is reached regarding the first-stage decisions. To tackle each scenario subproblem, we propose a decomposition-based heuristic that accelerates the solution process for the multi-attribute problem arising from scenario decomposition. Additionally, we introduce two novel scenario-selection strategies to further expedite the consensus over the first-stage decisions.

The contributions presented in this thesis encompass various aspects of modeling and solution methodologies for rich two-echelon multi-attribute location-routing problems (2E-MALRPs) from both deterministic and stochastic perspectives. The three articles included in this thesis show the effectiveness of the proposed approaches through extensive experimental campaigns, highlighting their computational efficiency and solution quality, particularly in challenging instances. We believe that decomposition-based algorithms deserve significant consideration in the field, and we have explored this idea in our papers. The proposed models are flexible and can accommodate numerous real-life attributes. Moreover, the solution frameworks proposed in each paper are general enough to be extended to related problem settings.

Although this area of research is gaining considerable attention, we believe that there are several topics that have yet to be studied. Exploring more general and challenging versions of the problems to assess the limits of the proposed methods is an important research avenue. Additionally, numerous variants of the fundamental two-echelon location-routing problems (2E-LRPs) have been studied from both applications and theoretical perspectives. Most of these versions could be extended to incorporate the multi-attribute aspect to further study the influence of a broad set of attributes on location and routing decisions.

Another important step towards a more systematic growth of the field is targeting larger problem instances in terms of network size or the number/type of scenarios or both. To achieve this, it would be advantageous to develop or adapt efficient solution frameworks (whether exact, metaheuristic, or hybrid) to further accelerate the resolution of each scenario subproblem at each iteration of the PH metaheuristic. Moreover, delving into more intricate forms of correlation among stochastic variables, such as temporal and spatial correlations, could yield valuable managerial insights and offer an opportunity to assess the impact of these considerations on the solution methodologies introduced in this thesis.

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Chapter A

SUPPLEMENTARY MATERIAL FIRST ARTICLE

A.1. Mathematical Proofs of Lemmas of Section 5.1

Lemma 1. Let $\mathcal{G}_\Delta$ be a reduced time-space network that satisfies properties 1, 2, 3, and 4. Then, for each commodity $k \in \mathcal{K}$ and itinerary $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^{k}$, there exists an itinerary $r' = (v_i, t'_i)_{i=1}^{l} \in R_{\mathcal{G}_\Delta}^{k}$ such that $t'_i \leq t_i$ for every $i = 1 \ldots l$.

Proof Lemma 1. We conduct the proof by induction on i for the itinerary $r = (v_i, t_i)_{i=1}^{l}$ in $R_{\mathcal{G}}^{k}$. For $i = 1$, let $t'_1 = a_{v_1}$ be the temporal lower bound of v_1. By Property 1, we have that the time-space node (v_1, t'_1) with $t'_1 = a_{v_1}$ yields to $t'_1 \leq t_1$, as there is no $(v_1, t''_1) \in \mathcal{G}_\Delta$ with $t''_1 < a_{v_1}$. From the time-space node (v_1, t'_1), we can map the remainder of the itinerary $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^{k}$ by defining an *equivalent* time-space node (v_i,t'_i) in $\mathcal{G}_\Delta$, for each $(v_i,t_i) \in R_{\mathcal{G}}^{k}$ with $t'_i = argmax\{d \in \mathcal{T}_i(\Delta) \mid d \leq t_i\}$. By Property 2, there is an arc $((v_i, t'_i),(v_{i+1}, t'_{i+1})) \in \mathcal{A}_\Delta$ with $t'_{i+1} \leq t'_i + \tau_{v_i v_{i+1}} \leq t_i + \tau_{v_i v_{i+1}}$, while Property 3 and 4, enables early waiting times at satellites as well early arrival at customers, respectively.

Assuming that $i = w$ is true, we can prove that our condition holds for $i = w+1$. Hence, by the inductive assumption, there is an itinerary $r = [(v_1,t'_1), (v_2,t'_2),..., (v_w,t'_w)]$ with $(v_w,t'_w) \in \mathcal{G}_\Delta$ and $t'_w \leq t_w$. By Property 1, the set of integer time points $\mathcal{T}_{v_i(\Delta)}$ representing the time moments at which node $i = w+1$ becomes relevant must exist in $\mathcal{V}_\Delta$. By Property 2, 3 and 4, arc $((v_w, t'_w),(v_{w+1}, t'_{w+1})) \in \mathcal{A}_\Delta$ with $t'_{w+1} \leq t'_w + \tau_{v_w v_{w+1}} \leq t_w + \tau_{v_w v_{w+1}}$. By Property 3, there must exist a waiting time at satellites with a lesser of equal value to the original waiting time, while Property 4, ensures that there is an early arrival point in time for customers. Consequently, we can ensure that the defined conditions can be verified for each connection within $r = (v_i, t_i)_{i=1}^{l} \in R_{\mathcal{G}}^{k}$, and thus for each itinerary in $R_{\mathcal{G}}$. $\square$

Lemma 2. If a reduced time-space network $\mathcal{G}_\Delta$ satisfies properties 1, 2, 3, and 4, then the optimal solution of the 2E-MALRPS formulated on the reduced time-space network $\mathcal{G}_\Delta$ is a lower bound for the solution of the 2E-MALRPS on the complete time-space network $\mathcal{G}$.

Proof Lemma 2. To prove this lemma, we will show that each time-space arc representing the optimal solution for the 2E-MALRPS in a complete time-space network, can be mapped onto a reduced time-space network, with an equal or lesser operational cost.

Consider $Z^*_{\mathcal{G}} = (x^*_{\mathcal{G}},f^*_{\mathcal{G}},y^*_{\mathcal{G}})$ an optimal integer solution of the 2E-MALRPS in a complete time-space network ($\mathcal{G}$), with $A^*_{\mathcal{G}} = \{((v_i,t_i),(v_j,t_j)) \in \mathcal{A}_{\mathcal{G}} \mid x_{(v_i,t_i),(v_j,t_j)} = 1\}$. Let $R_{\mathcal{G}}$ be the set of itineraries $r \in R_{\mathcal{G}}$ dispatching each commodity $k \in \mathcal{K}$ from its origin $O(k)$ to its destination $D(k)$ throughout the system with the arcs in $A^*_{\mathcal{G}}$. In what follows, we will show that each arc in $A^*_{\mathcal{G}}$ can be mapped to a unique set of arcs $A_{\mathcal{G}_\Delta}$ of a reduced time-space network ($\mathcal{G}_\Delta$), so we can construct $Z_{\mathcal{G}_\Delta} = (x_{\mathcal{G}_\Delta},f_{\mathcal{G}_\Delta},y_{\mathcal{G}_\Delta})$ in respect to each arc in $A_{\mathcal{G}_\Delta}$.

By Lemma 1, for each arc $((v_i,t_i)(v_j,t_j)) \in A^*_{\mathcal{G}}$ in $R_{\mathcal{G}}$ (excluding garages connections), there exists $(v_i,{t'}_i) \in \mathcal{G}_\Delta$ with ${t'}_i \leq t_i$ and a ${t'}_j$ such that $((v_i,{t'}_i)(v_j,{t'}_j)) \in A_{\mathcal{G}_\Delta}$. Hence, for each itinerary $r = (v_i,t_i)_{i=1}^l \in R_{\mathcal{G}}$, there is an *equivalent* itinerary $r' = (v_i,t'_i)_{i=1}^l \in R_{\mathcal{G}_\Delta}$ such that $t'_i \leq t_i$. Because the number of both platform and satellite facilities must hold for each $r \in R_{\mathcal{G}}$ mapped to $\mathcal{G}_\Delta$, we have that $y_{\mathcal{G}_\Delta} = y^*_{\mathcal{G}}$. Now we can track each commodity flow from its origin to its destination in $R_{\mathcal{G}_\Delta}$ to derive both routing and flow decisions to $x_{\mathcal{G}_\Delta}$ and $f_{\mathcal{G}_\Delta}$. Notice that by Lemma 1 and Property 3, fleet synchronization within each $r \in R_{\mathcal{G}_\Delta}$ holds, but takes place at the same or earlier point in time on the same satellite.

Recall that every route in the first and second echelon must start and end at a vehicle garage. We have that for each path $r \in R_{\mathcal{G}}$, every origin, satellite serving as the transfer point and destination of each commodity $k \in \mathcal{K}$ are known. Notice that, for some of these time-space nodes, there exists an unique time-space arc in $A^*_{\mathcal{G}}$ that represents the leg used for each route as the *start point* after a vehicle leaves the garage or the *end point* before the vehicle returns to the garage. By Lemma 1 and Properties 1 and 2, there must exist a time-space node $(e_1,{t'}_{e_1}) \in \mathcal{E}^1_\Delta$ and $(e_2,{u'}_{e_2}) \in \mathcal{E}^2_\Delta$, such that $((z,{t'}_z),(e_1,{t'}_{e_1}))$ and $((c,{t'}_c),(e_2,{t'}_{e_2}))$ exists in $A_{\mathcal{G}_\Delta}$ for each *end point* at the first and second echelon, with $(z,{t'}_z)$ and $(c,{t'}_c)$ in $R_{\mathcal{G}_\Delta}$, $z \in \mathcal{Z}^{\text{ph}}$ and $c \in \mathcal{C}^{\text{ph}}$. Similarly, there are time-space nodes $({e'}_1,{t'}_{{e'}_1}) \in \mathcal{E}^1_\Delta$ and $({e'}_2,{u'}_{{e'}_2}) \in \mathcal{E}^2_\Delta$, such that $(({e'}_1,{t'}_{{e'}_1}),(p,{t'}_p))$ and $(({e'}_2,{t'}_{{e'}_2}),(z',{t''}_{z'}))$ exists in $A_{\mathcal{G}_\Delta}$ for each *start point* at the first and second echelon, with $(p,{t'}_p)$ and $(z',{t''}_{z'})$ in $R_{\mathcal{G}_\Delta}$, $p \in \mathcal{P}^{\text{ph}}$ and $z' \in \mathcal{Z}^{\text{ph}}$. Thus, we can then derive the routing decisions to $x_{\mathcal{G}_\Delta}$ for the resulting inbound and outbound for each garage.

Observe that, for a given reduced network $\mathcal{G}_\Delta$, there is a unique set of time-space arcs that satisfy properties 1 and 2. This implies that first-echelon routes mapped from the complete time-space network would have the same cost but reach each destination in the reduced time-space network at the same or an earlier time moment. By property 3, we know that waiting times follow a similar behavior, wherein vehicles can wait until the same or a lesser time moment than the one defined in the complete time-space network. Furthermore, property 2 guarantees that there are no time-space arcs $((i,t),(j,t'))$ and $((i,t),(j,t"))$ where $t' < t"$. Consequently, for each time-space node $(i,t) \in \mathcal{V}_\Delta$ and each $(i,j) \in \mathcal{A}^{ph}$, there can be at most one time-space arc of the form $((i,t),(j,t'))$ that satisfies properties 1 and 2. Combining properties 2 and 3, it follows that $t' = \text{argmax}\{d \mid d \leq t + \tau_{ij}, (i,j) \in \mathcal{A}^{ph}, (j,d) \in \mathcal{V}_\Delta\}$. Therefore, for each time moment in which fleet synchronization takes place in the complete time-space network, there exists a time moment with an equal or lesser time value in the reduced network.

Now, the solution $Z_{\mathcal{G}_\Delta} = (x_{\mathcal{G}_\Delta}, f_{\mathcal{G}_\Delta}, y_{\mathcal{G}_\Delta})$ constructed in this way is feasible for the 2E-MALRPS onto the reduced time-space network ($\mathcal{G}_\Delta$) while routing and location costs holds. Therefore, we have that $Z_{\mathcal{G}_\Delta}$ is identical to $Z^*_{\mathcal{G}}$ with arrival and departure times taking earlier or equal values than the ones on the complete time-space network, but with the same operational cost. Consequently, the optimal solution for the 2E-MALRPS on a reduced time-space network ($\mathcal{G}_\Delta$) is a lower bound of the optimal solution obtained in a complete time-space network ($\mathcal{G}$). □

Lemma 3. The proposed DDD algorithm terminates with an optimal solution for the 2E-MALRPS.

Proof Lemma 3. The DDD algorithm terminates when the optimal integer solution of the hybrid formulation HTF($\mathcal{G}_\Delta$) on a reduced time-space network $\mathcal{G}_\Delta$ can be mapped onto a complete time-space network with the same cost. By Lemma 2, the solutions derived from HTF($\mathcal{G}_\Delta$) is as lower bounds for the 2E-MALRPS on the complete time-space network $\mathcal{G}$. Therefore, the solution obtained from HTF($\mathcal{G}_\Delta$), which remains feasible when mapped to the complete time-space network, is the optimal solution for the 2E-MALRPS.

During each iteration of the DDD algorithm, there may be situations where the solution obtained by HTF($\mathcal{G}_\Delta$) is not feasible for the 2E-MALRPS when evaluated using the original travel times, due to the presence of at least one short arc in the solution. When such a situation occurs, a refinement procedure is applied, which extends these short arcs found in the solution while ensuring that Properties 1-4 remain valid at the nodes involved in the extension. Given that the system comprises a finite number of time points and arcs, the DDD algorithm can eventually reach an iteration where all arcs in the reduced network have travel times corresponding to the actual travel time of the system. In this specific scenario, the solution obtained from HTF($\mathcal{G}_\Delta$) effectively satisfies the temporal requirements of the 2E-MALRPS and leads to the optimal solution, ultimately resulting in the termination of the algorithm.

A.2. Complete Result Tables

Tables 1 - 6 showcase the results of the proposed formulations under different granularity values. Each table display the comparative performance results on the complete time-space network ($\Delta = \bar{\Delta}$) and the reduced time-space networks with granularity values $\Delta = 50$ and $\Delta = 25$ with a time limit of 2.5 hours. Table 1, Table 3 and Table 5 presents experiments of the hybrid formulation with different granularity values on instances with 5, 10 and 15 OD, respectively. Results presented in Table 2, Table 4 and Table 6 extend the same experiments using the standalone time-space formulation. The tables display the instance **ID**, the schedule length (Ψ), the best upper bound (**UB**), the run-time (CPUsec), the lower bound (**LB**), the root gap (**RG(%)**), and cost increment percentage (**Dif UB**).

Tables 7 - 12 display the detailed results of the set of experiments focusing on the performance of the dynamic discretization discovery (DDD) solution framework for the 2E-MALRPS. Test results are shown for the instances with 5, 10 and 15 OD demands in Table 7. Results for the same instances with disabled availability times in Table 11 as well as disabled availability times and synchronization in Table 12. The experiments are performed using a coarse discretization granularity $\Delta = 2$. The stopping criteria are an optimality gap of less or equal to 1% and a maximum run time of 2.5 hours for small-sized instances (5 and 10 OD demands), 5 hours for instances with 30 OD demands and 8 hours for instances with 50 OD demands. The tables display the instance **ID**, the schedule length (Ψ), the best upper bound (**UB**), the run-time (CPUsec), the lower bound (**LB**), and the optimality gap (**OG(%)**).

ID	Ψ	Hybrid formulation ($\Delta = \bar{\Delta}$)				Hybrid formulation ($\Delta = 50$)			Hybrid formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,5	100	280	1.73	191.38	31.65	280	1.05	0.00	285	0.75	1.61
Ca1-3,5,5	100	222	82.53	199.00	10.36	222	32.55	0.00	223	22.52	0.54
Ca1-6,4,5	100	271	328.45	213.03	21.39	282	73.03	3.87	282	70.75	3.87
Ca2-2,3,5	100	152	7.81	132.86	12.59	152	5.53	0.00	152	3.29	0.13
Ca2-3,5,5	100	284	9.16	216.74	23.68	284	6.70	0.00	293	3.68	3.10
Ca2-6,4,5	100	150	87.36	112.00	25.33	156	67.39	4.00	156	24.61	4.13
Ca3-2,3,5	100	287	8.47	238.33	16.96	299	1.64	4.14	299	1.48	4.15
Ca3-3,5,5	100	220	72.43	173.73	21.03	224	29.99	2.00	224	29.54	2.00
Ca3-6,4,5	100	171	42.30	164.50	3.80	173	39.66	1.29	173	38.72	1.29
Ca4-2,3,5	100	358	6.66	252.23	29.55	358	6.32	0.00	374	0.84	4.27
Ca4-3,5,5	100	168	9.36	128.78	23.35	170	5.06	1.19	170	0.44	1.37
Ca4-6,4,5	100	161	99.82	125.28	22.19	164	43.65	1.98	164	25.31	1.99
Ca5-2,3,5	100	199	39.11	189.30	4.87	200	19.69	0.25	202	1.46	1.31
Ca5-3,5,5	100	186	134.80	161.93	12.94	186	104.69	0.00	186	96.60	0.22
Ca5-6,4,5	100	159	2620.27	152.67	3.98	165	1765.59	3.65	167	25.92	4.84
Cb1-2,3,5	100	152	2.49	140.00	7.89	153	1.83	0.39	153	1.20	0.53
Cb1-3,5,5	100	164	645.87	117.74	28.21	164	629.06	0.00	165	1.63	0.30
Cb1-6,4,5	100	305	581.10	186.03	39.01	305	412.05	0.00	311	5.93	1.87
Cb2-2,3,5	100	129	1.75	117.95	8.57	131	1.61	1.86	131	0.44	1.86
Cb2-3,5,5	100	154	4.12	134.00	12.99	154	2.04	0.00	155	1.46	0.84
Cb2-6,4,5	100	143	454.68	117.53	17.81	143	25.37	0.00	149	14.55	4.27
Cb3-2,3,5	100	332	0.38	248.84	25.05	343	0.37	3.10	343	0.37	3.10
Cb3-3,5,5	100	160	52.57	129.00	19.37	160	40.04	0.00	161	6.86	0.62
Cb3-6,4,5	100	198	158.15	141.85	28.36	201	54.56	1.46	204	8.71	2.78
Cb4-2,3,5	100	280	9000.00	171.05	38.91	280	2047.83	0.00	286	4.73	2.21
Cb4-3,5,5	100	142	27.91	122.00	14.08	142	20.69	0.28	144	13.48	1.34
Cb4-6,4,5	100	188	193.07	136.05	27.63	192	173.47	1.97	195	22.69	3.56
Cb5-2,3,5	100	129	17.97	115.31	10.61	130	12.31	0.78	131	1.82	1.32
Cb5-3,5,5	100	179	429.57	139.65	21.98	179	131.93	0.00	180	16.65	0.50
Cb5-6,4,5	100	199	44.91	137.67	30.82	206	41.15	3.37	207	35.11	3.82
Cc1-2,3,5	100	129	9000.00	114.69	11.10	129	3934.41	0.00	131	163.17	1.24
Cc1-3,5,5	100	135	9000.00	109.99	18.53	136	5153.75	0.89	136	261.11	0.89
Cc1-6,4,5	100	150	9000.00	108.20	27.86	150	5115.29	0.00	155	460.03	3.40
Cc2-2,3,5	100	122	9000.00	99.62	18.34	122	8764.65	0.00	122	21.64	0.00
Cc2-3,5,5	100	175	9000.00	120.46	31.17	175	3660.59	0.23	176	3355.92	0.29
Cc2-6,4,5	100	140	9000.00	114.84	17.97	140	7960.76	0.00	144	4019.37	2.86
Cc3-2,3,5	100	136	9000.00	116.66	14.22	136	2030.28	0.00	137	513.69	0.37
Cc3-3,5,5	100	142	9000.00	128.80	9.30	142	7102.57	0.00	144	323.32	1.06
Cc3-6,4,5	100	157	9000.00	110.13	29.86	157	5963.72	0.00	161	1371.77	2.48
Cc4-2,3,5	100	171	9000.00	120.69	29.42	171	2234.24	0.00	171	546.19	0.23
Cc4-3,5,5	100	154	9000.00	114.87	25.41	154	7494.61	0.19	154	2255.48	0.26
Cc4-6,4,5	100	138	9000.00	122.31	11.37	138	7090.72	0.00	142	2201.86	3.04
Cc5-2,3,5	100	123	9000.00	114.54	6.88	124	1568.97	0.81	124	151.12	0.89
Cc5-3,5,5	100	124	9000.00	104.60	15.64	124	4568.95	0.00	124	3832.12	0.00
Cc5-6,4,5	100	138	9000.00	119.21	13.61	138	6978.99	0.00	144	2600.04	3.84
Cd1-2,3,5	100	155	1140.22	136.21	12.13	155	239.64	0.00	156	29.06	0.90
Cd1-3,5,5	100	170	24.13	140.45	17.38	170	23.17	0.18	171	12.65	0.53
Cd1-6,4,5	100	188	870.99	127.01	32.44	188	407.19	0.00	191	198.22	1.70
Cd2-2,3,5	100	140	9000.00	127.24	9.11	142	2619.41	1.21	145	30.92	3.29
Cd2-3,5,5	100	158	1707.02	141.04	10.73	158	857.78	0.00	159	1204.89	0.57
Cd2-6,4,5	100	142	692.04	100.52	29.21	142	478.78	0.34	148	152.25	3.73
Cd3-2,3,5	100	142	9000.00	119.98	15.51	142	1312.74	0.28	145	89.65	1.83
Cd3-3,5,5	100	147	9000.00	118.75	19.22	148	2994.43	0.95	149	1324.92	1.09
Cd3-6,4,5	100	157	133.33	125.88	19.82	160	47.84	1.72	162	47.06	3.31
Cd4-2,3,5	100	202	9000.00	172.63	14.54	204	4954.89	0.84	204	102.22	1.04
Cd4-3,5,5	100	162	9000.00	138.26	14.65	163	3916.45	0.31	164	552.18	1.11
Cd4-6,4,5	100	147	2073.23	109.56	25.47	147	1149.21	0.00	151	397.82	2.38
Cd5-2,3,5	100	178	9000.00	160.73	9.70	179	2681.85	0.50	179	56.07	0.56
Cd5-3,5,5	100	178	9000.00	148.29	16.69	178	4878.72	0.00	178	442.24	0.22
Cd5-6,4,5	100	157	9000.00	138.29	11.92	163	2445.97	3.95	165	549.26	4.65
Averages			***3813.43***		***18.90***		***1907.12***	***0.80***		***462.46***	***1.86***

Table 1 – Direct solving the hybrid formulation on instances with 5 OD demands and multiple Δ time periods

ID	Ψ	Time-Space formulation ($\Delta = \bar{\Delta}$)				Time-Space formulation ($\Delta = 50$)			Time-Space formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,5	100	280	0.14	168.08	39.97	287	0.06	2.50	286	0.01	2.10
Ca1-3,5,5	100	222	0.88	190.89	14.01	225	0.48	1.35	224	0.47	0.89
Ca1-6,4,5	100	271	0.37	209.51	22.69	281	0.32	3.69	282	0.22	3.90
Ca2-2,3,5	100	152	0.27	120.73	20.57	152	0.27	0.00	153	0.26	0.65
Ca2-3,5,5	100	284	0.15	173.40	38.95	294	0.14	3.52	295	0.14	3.73
Ca2-6,4,5	100	150	1.26	90.82	39.45	156	1.23	4.00	157	0.71	4.46
Ca3-2,3,5	100	287	0.05	235.01	18.12	299	0.05	4.18	301	0.05	4.65
Ca3-3,5,5	100	220	0.46	160.73	26.94	224	0.28	1.82	225	0.27	2.22
Ca3-6,4,5	100	171	0.64	120.30	29.65	173	0.56	1.17	174	0.33	1.72
Ca4-2,3,5	100	358	0.07	189.88	46.96	373	0.09	4.19	376	0.09	4.79
Ca4-3,5,5	100	168	0.42	128.26	23.66	170	0.30	1.19	171	0.28	1.75
Ca4-6,4,5	100	161	1.55	123.71	23.16	164	1.48	1.86	165	1.45	2.42
Ca5-2,3,5	100	199	0.48	175.45	11.84	199	0.36	0.00	202	0.18	1.49
Ca5-3,5,5	100	186	0.32	120.45	35.24	189	0.27	1.61	187	0.19	0.53
Ca5-6,4,5	100	159	54.79	91.36	42.54	165	54.65	3.77	168	54.59	5.36
Cb1-2,3,5	100	152	0.23	131.27	13.64	152	0.12	0.00	153	0.16	0.65
Cb1-3,5,5	100	164	1.98	116.89	28.72	167	1.62	1.83	165	1.51	0.61
Cb1-6,4,5	100	305	0.31	178.02	41.63	314	0.24	2.95	312	0.14	2.24
Cb2-2,3,5	100	129	0.14	117.44	8.96	131	0.12	1.55	132	0.14	2.27
Cb2-3,5,5	100	154	0.22	133.35	13.41	155	0.17	0.65	156	0.11	1.28
Cb2-6,4,5	100	143	1.04	105.47	26.25	149	0.86	4.20	150	0.78	4.67
Cb3-2,3,5	100	332	0.06	236.65	28.72	342	0.04	3.01	344	0.00	3.49
Cb3-3,5,5	100	160	0.52	128.32	19.80	161	0.23	0.63	162	0.19	1.23
Cb3-6,4,5	100	198	0.35	136.20	31.21	201	0.23	1.52	204	0.16	2.94
Cb4-2,3,5	100	280	9000	161.05	42.48	290	1642.28	3.57	288	19.91	2.78
Cb4-3,5,5	100	142	0.82	118.46	16.58	142	0.61	0.00	144	0.47	1.39
Cb4-6,4,5	100	188	0.64	129.68	31.02	191	0.62	1.60	196	0.30	4.08
Cb5-2,3,5	100	129	0.47	111.73	13.38	130	0.43	0.78	131	0.07	1.53
Cb5-3,5,5	100	179	0.71	131.95	26.28	181	0.57	1.12	180	0.46	0.56
Cb5-6,4,5	100	199	0.43	136.02	31.65	205	0.41	3.02	208	0.35	4.33
Cc1-2,3,5	100	129	9000	112.13	13.08	130	4854.27	0.78	131	35.47	1.53
Cc1-3,5,5	100	135	3622.1	109.11	19.18	136	2544.38	0.74	137	22.82	1.46
Cc1-6,4,5	100	150	9000	81.69	45.54	155	3838.39	3.33	156	19.61	3.85
Cc2-2,3,5	100	122	51	99.08	18.79	123	50.91	0.82	123	50.75	0.81
Cc2-3,5,5	100	175	9000	119.59	31.66	175	4816.26	0.00	176	9.56	0.57
Cc2-6,4,5	100	140	9000	107.85	22.96	144	2401.17	2.86	145	84.68	3.45
Cc3-2,3,5	100	136	208.75	115.10	15.37	136	186.73	0.00	137	25.37	0.73
Cc3-3,5,5	100	142	3906.78	127.89	9.94	144	823.62	1.41	144	30.25	1.39
Cc3-6,4,5	100	157	9000	109.02	30.56	160	3907.48	1.91	161	9.49	2.48
Cc4-2,3,5	100	171	3160.12	120.89	29.31	171	1291.09	0.00	172	14.53	0.58
Cc4-3,5,5	100	154	9000	114.98	25.34	154	4762.04	0.00	155	259.42	0.65
Cc4-6,4,5	100	138	9000	88.36	35.97	142	335.82	2.90	142	5.56	2.82
Cc5-2,3,5	100	123	9000	114.32	7.06	124	2641.20	0.81	125	75.83	1.60
Cc5-3,5,5	100	124	246.22	103.09	16.86	124	142.84	0.00	124	20.03	0.00
Cc5-6,4,5	100	138	9000	104.58	24.22	143	7828.48	3.62	144	89.96	4.17
Cd1-2,3,5	100	155	1.31	134.59	13.17	155	1.27	0.00	157	1.25	1.27
Cd1-3,5,5	100	170	0.75	139.12	18.16	170	0.74	0.00	172	0.56	1.16
Cd1-6,4,5	100	188	99.91	121.25	35.50	191	99.41	1.60	191	99.38	1.57
Cd2-2,3,5	100	140	1.72	126.35	9.75	141	1.24	0.71	145	1.19	3.45
Cd2-3,5,5	100	158	6.96	136.47	13.62	158	6.84	0.00	160	6.83	1.25
Cd2-6,4,5	100	142	25.38	94.76	33.27	147	25.21	3.52	148	25.12	4.05
Cd3-2,3,5	100	142	3.05	117.30	17.40	142	3.01	0.00	145	2.93	2.07
Cd3-3,5,5	100	147	11.45	118.29	19.53	148	11.28	0.68	149	11.27	1.34
Cd3-6,4,5	100	157	600.47	121.23	22.78	159	282.06	1.27	163	20.45	3.68
Cd4-2,3,5	100	202	0.58	168.87	16.40	203	0.53	0.50	205	0.41	1.46
Cd4-3,5,5	100	162	1.7	137.76	14.96	162	1.70	0.00	165	1.69	1.82
Cd4-6,4,5	100	147	156.39	108.88	25.93	150	113.33	2.04	151	4.87	2.65
Cd5-2,3,5	100	178	1.13	158.80	10.79	178	0.84	0.00	180	0.72	1.11
Cd5-3,5,5	100	178	5.14	145.71	18.14	179	5.07	0.56	179	4.95	0.56
Cd5-6,4,5	100	157	191.79	134.67	14.22	163	146.44	3.82	165	84.29	4.85
Averages			***1706.21***		***23.95***		***713.89***	***1.59***		***18.40***	***2.22***

Table 2 – Direct solving the classic time-space formulation on instances with 5 OD demands and multiple Δ time periods

ID	Ψ	Hybrid Formulation ($\Delta = \bar{\Delta}$)				Hybrid Formulation ($\Delta = 50$)			Hybrid Formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,10	100	256	1683.71	225.71	11.83	258	732.01	0.78	261	17.53	1.92
Ca1-3,5,10	100	215	9000	181.20	15.72	216	3640.05	0.46	217	252.25	0.92
Ca1-6,4,10	100	305	9000	266.85	12.51	320	1605.60	4.69	320	707.46	4.69
Ca2-2,3,10	100	161	1904.23	126.71	21.30	162	1278.83	0.62	162	32.92	0.62
Ca2-3,5,10	100	340	6601.6	289.82	14.76	355	3600.04	4.23	356	768.02	4.49
Ca2-6,4,10	100	179	8211.97	149.64	16.40	182	4260.06	1.65	182	460.61	1.65
Ca3-2,3,10	100	344	7.26	303.32	11.82	363	3.56	5.23	363	1.81	5.23
Ca3-3,5,10	100	318	9000	274.63	13.64	333	4391.81	4.50	333	295.38	4.50
Ca3-6,4,10	100	236	9000	184.54	21.80	240	3018.73	1.67	240	787.23	1.67
Ca4-2,3,10	100	437	48.51	408.79	6.46	462	27.39	5.41	462	8.35	5.41
Ca4-3,5,10	100	213	9000	166.95	21.62	220	3908.61	3.18	220	994.43	3.18
Ca4-6,4,10	100	238	9000	204.74	13.97	243	4560.37	2.06	243	1530.76	2.06
Ca5-2,3,10	100	277	9000	245.16	11.49	282	819.25	1.77	284	14.58	2.46
Ca5-3,5,10	100	264	9000	240.84	8.77	272	3829.65	2.94	275	965.99	4.00
Ca5-6,4,10	100	187	9000	135.19	27.71	190	3427.89	1.58	192	592.42	2.60
Cb1-2,3,10	100	181	419.36	155.28	14.21	183	195.68	1.09	183	41.96	1.09
Cb1-3,5,10	100	211	9000	179.52	14.92	215	1429.47	1.86	219	1062.87	3.65
Cb1-6,4,10	100	261	9000	227.46	12.85	262	2925.71	0.38	267	1593.28	2.25
Cb2-2,3,10	100	199	1184.1	163.46	17.86	204	618.57	2.45	204	14.39	2.45
Cb2-3,5,10	100	268	3120.03	215.18	19.71	278	1980.90	3.60	282	544.55	4.96
Cb2-6,4,10	100	185	9000	145.09	21.58	187	4463.33	1.07	187	1485.47	1.07
Cb3-2,3,10	100	337	8.02	304.17	9.74	348	3.47	3.16	348	1.37	3.16
Cb3-3,5,10	100	202	9000	173.86	13.93	207	3557.87	2.42	207	368.58	2.42
Cb3-6,4,10	100	283	9000	257.07	9.16	286	2836.46	1.05	289	870.77	2.08
Cb4-2,3,10	100	251	37.24	213.21	15.06	254	13.10	1.18	258	7.34	2.71
Cb4-3,5,10	100	245	9000	190.39	22.29	254	3525.42	3.54	256	1348.33	4.30
Cb4-6,4,10	100	288	9000	238.55	17.17	291	2928.67	1.03	294	1268.66	2.04
Cb5-2,3,10	100	197	7056.22	168.90	14.26	202	144.08	2.48	203	18.16	2.96
Cb5-3,5,10	100	232	9000	205.91	11.24	236	3597.15	1.69	240	166.47	3.33
Cb5-6,4,10	100	337	9000	306.20	9.14	343	2600.04	1.75	344	1511.27	2.03
Cc1-2,3,10	100	200	9000	159.41	20.30	206	3559.44	2.91	206	163.17	2.91
Cc1-3,5,10	100	199	9000	129.07	35.14	204	3184.53	2.45	204	1601.11	2.45
Cc1-6,4,10	100	255	9000	212.97	16.48	261	2523.50	2.30	261	1400.03	2.30
Cc2-2,3,10	100	201	9000	141.67	29.52	209	3486.52	3.83	209	21.64	3.83
Cc2-3,5,10	100	244	9000	200.18	17.96	252	2611.12	3.17	252	1355.92	3.17
Cc2-6,4,10	100	188	9000	120.23	36.05	189	2489.66	0.53	189	1019.37	0.53
Cc3-2,3,10	100	211	9000	147.37	30.15	218	2244.13	3.21	218	513.69	3.21
Cc3-3,5,10	100	241	9000	198.54	17.62	249	2951.10	3.21	249	2203.32	3.21
Cc3-6,4,10	100	215	9000	164.31	23.58	217	2554.78	0.92	217	2371.77	0.92
Cc4-2,3,10	100	235	9000	188.90	19.62	241	2684.68	2.49	241	546.19	2.49
Cc4-3,5,10	100	236	9000	190.70	19.20	245	3183.95	3.67	245	2255.48	3.67
Cc4-6,4,10	100	246	9000	195.04	20.72	253	3443.32	2.77	253	2001.86	2.77
Cc5-2,3,10	100	194	9000	137.50	29.12	200	2610.25	3.00	200	151.12	3.00
Cc5-3,5,10	100	182	9000	135.97	25.29	188	3110.36	3.19	188	2332.12	3.19
Cc5-6,4,10	100	187	9000	151.80	18.82	187	2775.97	0.00	187	1300.04	0.00
Cd1-2,3,10	100	197	189.76	173.95	11.70	200	87.32	1.50	201	29.06	1.99
Cd1-3,5,10	100	215	9000	172.93	19.57	219	2100.18	1.83	220	712.65	2.27
Cd1-6,4,10	100	248	9000	218.19	12.02	251	3532.77	1.20	251	1298.22	1.20
Cd2-2,3,10	100	193	9000	142.18	26.33	194	951.20	0.52	197	30.92	2.03
Cd2-3,5,10	100	236	9000	199.03	15.66	243	1466.72	2.88	244	1204.89	3.28
Cd2-6,4,10	100	213	9000	161.68	24.09	215	3505.38	0.93	215	1252.25	0.93
Cd3-2,3,10	100	186	8423.65	158.57	14.75	188	458.37	1.06	190	89.65	2.11
Cd3-3,5,10	100	224	9000	180.00	19.64	230	3073.16	2.61	230	1324.92	2.61
Cd3-6,4,10	100	196	9000	161.72	17.49	197	856.91	0.51	197	757.06	0.51
Cd4-2,3,10	100	257	189.76	249.57	2.89	260	51.36	1.15	264	42.22	2.65
Cd4-3,5,10	100	241	9000	220.19	8.63	247	2001.15	2.43	250	1252.18	3.60
Cd4-6,4,10	100	189	9000	134.55	28.81	190	2300.01	0.53	190	1197.82	0.53
Cd5-2,3,10	100	207	2980	167.13	19.26	209	12.21	0.96	211	6.07	1.90
Cd5-3,5,10	100	213	9000	166.35	21.90	217	2255.85	1.84	218	442.24	2.29
Cd5-6,4,10	100	184	9000	153.48	16.59	188	3568.37	2.13	189	1549.26	2.65
Averages			***7301.09***		***17.86***		***2325.97***	***2.15***		***802.66***	***2.57***

Table 3 – Direct solving the hybrid formulation on instances with 10 OD demands and multiple Δ time periods

ID	Ψ	Time-Space formulation ($\Delta = \bar{\Delta}$)				Time-Space formulation ($\Delta = 50$)			Time-Space formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,10	100	256	2.81	198.06	22.63	260	1.39	1.54	261	0.87	1.92
Ca1-3,5,10	100	215	84.24	163.75	23.84	217	46.13	0.92	219	25.35	1.83
Ca1-6,4,10	100	305	7.62	266.12	12.75	320	2.71	4.69	320	2.62	4.69
Ca2-2,3,10	100	161	2210.6	120.84	24.94	162	1044.25	0.62	163	466.77	1.23
Ca2-3,5,10	100	340	1.21	289.54	14.84	355	0.68	4.23	359	0.42	5.29
Ca2-6,4,10	100	179	2537.13	147.69	17.49	182	1074.93	1.65	183	692.36	2.19
Ca3-2,3,10	100	344	0.3	303.28	11.84	363	0.13	5.23	365	0.08	5.75
Ca3-3,5,10	100	318	4.29	272.62	14.27	333	2.24	4.50	333	1.00	4.50
Ca3-6,4,10	100	236	4.15	175.41	25.67	240	2.38	1.67	241	1.18	2.07
Ca4-2,3,10	100	437	0.96	395.44	9.51	462	0.34	5.41	465	0.39	6.02
Ca4-3,5,10	100	213	485.53	162.78	23.58	220	219.65	3.18	220	171.13	3.18
Ca4-6,4,10	100	238	9.32	187.07	21.40	243	5.17	2.06	244	2.62	2.46
Ca5-2,3,10	100	277	8.18	241.47	12.83	284	4.74	2.46	287	1.86	3.48
Ca5-3,5,10	100	264	13.21	234.69	11.10	274	4.94	3.65	276	4.50	4.35
Ca5-6,4,10	100	187	9000	116.30	37.81	192	4539.17	2.60	192	2356.60	2.60
Cb1-2,3,10	100	181	3.35	118.59	34.48	183	1.67	1.09	184	1.26	1.63
Cb1-3,5,10	100	211	3913.71	163.51	22.51	218	1806.98	3.21	221	1020.80	4.52
Cb1-6,4,10	100	261	4.71	223.75	14.27	266	2.17	1.88	269	1.94	2.97
Cb2-2,3,10	100	199	0.99	149.31	24.97	204	0.56	2.45	204	0.41	2.45
Cb2-3,5,10	100	268	2.54	215.04	19.76	280	1.26	4.29	284	1.01	5.63
Cb2-6,4,10	100	185	1017	144.74	21.76	187	484.26	1.07	188	158.54	1.60
Cb3-2,3,10	100	337	0.7	302.22	10.32	348	0.31	3.16	349	0.16	3.44
Cb3-3,5,10	100	202	1061.83	170.30	15.69	207	562.08	2.42	209	430.68	3.35
Cb3-6,4,10	100	283	11.74	250.45	11.50	288	5.72	1.74	290	4.30	2.41
Cb4-2,3,10	100	251	1.04	212.99	15.14	258	0.59	2.71	259	0.36	3.09
Cb4-3,5,10	100	245	806.72	185.77	24.18	255	300.39	3.92	258	202.18	5.04
Cb4-6,4,10	100	288	2519.29	234.85	18.45	293	1067.52	1.71	296	549.69	2.70
Cb5-2,3,10	100	197	30.08	159.27	19.15	203	12.41	2.96	203	11.62	2.96
Cb5-3,5,10	100	232	1579.01	194.55	16.14	237	554.47	2.11	241	540.96	3.73
Cb5-6,4,10	100	337	4.77	304.82	9.55	344	2.18	2.03	347	1.35	2.88
Cc1-2,3,10	100	197	9000	153.53	23.23	203	282.33	2.96	203	179.71	2.96
Cc1-3,5,10	100	189	9000	118.86	40.27	194	285.40	2.58	194	186.83	2.58
Cc1-6,4,10	100	246	9000	210.99	17.26	251	329.03	1.99	253	153.20	2.77
Cc2-2,3,10	100	191	9000	140.21	30.25	199	325.79	4.02	199	182.08	4.02
Cc2-3,5,10	100	232	9000	184.16	24.52	239	213.75	2.93	241	91.92	3.73
Cc2-6,4,10	100	173	9000	117.67	37.41	174	321.26	0.57	174	170.18	0.57
Cc3-2,3,10	100	196	9000	143.42	32.03	203	271.74	3.45	204	159.23	3.92
Cc3-3,5,10	100	237	9000	198.00	17.84	246	261.09	3.66	246	115.71	3.66
Cc3-6,4,10	100	207	9000	157.89	26.56	209	233.51	0.96	210	99.00	1.43
Cc4-2,3,10	100	231	9000	179.82	23.48	237	267.75	2.53	238	206.18	2.94
Cc4-3,5,10	100	228	9000	175.35	25.70	237	248.87	3.80	237	100.82	3.80
Cc4-6,4,10	100	232	9000	189.96	22.78	238	276.99	2.52	240	127.53	3.33
Cc5-2,3,10	100	183	9000	127.46	34.30	189	326.35	3.17	190	134.12	3.68
Cc5-3,5,10	100	174	9000	120.60	33.74	180	239.61	3.33	181	170.28	3.87
Cc5-6,4,10	100	179	9000	147.09	21.34	179	235.01	0.00	180	181.83	0.56
Cd1-2,3,10	100	199	9000	167.26	15.09	202	209.24	1.49	203	106.09	1.97
Cd1-3,5,10	100	219	9000	170.03	20.92	224	212.45	2.23	225	149.05	2.67
Cd1-6,4,10	100	260	9000	214.67	13.44	263	237.26	1.14	265	123.12	1.89
Cd2-2,3,10	100	193	3145.09	140.47	27.22	194	88.04	0.52	197	23.99	2.03
Cd2-3,5,10	100	236	9000	191.03	19.05	244	214.38	3.28	246	86.34	4.07
Cd2-6,4,10	100	213	9000	156.23	26.65	215	303.55	0.93	216	139.53	1.39
Cd3-2,3,10	100	186	4709.4	152.16	18.19	190	143.60	2.11	191	81.38	2.62
Cd3-3,5,10	100	224	9000	178.49	20.32	230	257.46	2.61	231	142.98	3.03
Cd3-6,4,10	100	196	9000	138.59	29.29	197	229.62	0.51	197	89.97	0.51
Cd4-2,3,10	100	257	5.25	212.23	17.42	261	1.73	1.53	266	1.68	3.38
Cd4-3,5,10	100	241	9000	210.43	12.68	248	253.67	2.82	251	184.33	3.98
Cd4-6,4,10	100	189	9000	132.67	29.81	190	256.44	0.53	191	128.71	1.05
Cd5-2,3,10	100	207	41.39	156.58	24.36	210	10.43	1.43	211	7.01	1.90
Cd5-3,5,10	100	213	9000	161.67	24.10	218	329.92	2.29	219	128.80	2.74
Cd5-6,4,10	100	184	9000	140.24	23.78	189	222.80	2.65	189	154.58	2.65
Averages			***4453.80***		***21.59***		***314.01***	***2.43***		***174.32***	***2.99***

Table 4 – Direct solving the classic time-space formulation on instances with 10 OD demands and multiple Δ time periods

ID	Ψ	Hybrid formulation ($\Delta = \bar{\Delta}$)				Hybrid formulation ($\Delta = 50$)			Hybrid formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,15	200	338	9000.00	177.75	35.36	441	9000.00	23.36	432	4130.20	21.76
Ca1-3,5,15	200	N.A	9000.00	236.04	28.04	N.A	9000.00	N.A	342	1438.47	N.A
Ca1-6,4,15	200	N.A	9000.00	185.08	37.05	N.A	9000.00	N.A	300	4379.18	N.A
Ca2-2,3,15	200	330	9000.00	188.28	41.71	417	9000.00	20.86	419	3261.55	21.24
Ca2-3,5,15	200	N.A	9000.00	223.03	36.28	532	9000.00	N.A	350	9000.00	N.A
Ca2-6,4,15	200	N.A	9000.00	169.65	40.68	N.A	9000.00	N.A	384	9000.00	N.A
Ca3-2,3,15	200	366	9000.00	239.38	26.34	401	9000.00	8.73	401	4282.41	8.73
Ca3-3,5,15	200	N.A	9000.00	1.21	0.00	441	9000.00	N.A	441	1366.57	N.A
Ca3-6,4,15	200	N.A	9000.00	153.30	44.25	N.A	9000.00	N.A	383	9000.00	N.A
Ca4-2,3,15	200	420	9000.00	224.95	30.78	517	9000.00	18.76	447	9000.00	6.04
Ca4-3,5,15	200	N.A	9000.00	234.36	18.34	N.A	9000.00	N.A	418	9000.00	N.A
Ca4-6,4,15	200	N.A	9000.00	188.65	33.81	N.A	9000.00	N.A	568	4828.49	N.A
Ca5-2,3,15	200	N.A	9000.00	3.72	0.00	518	9000.00	N.A	547	9000.00	N.A
Ca5-3,5,15	200	N.A	9000.00	226.55	17.92	N.A	9000.00	N.A	384	4823.17	N.A
Ca5-6,4,15	200	N.A	9000.00	132.55	45.45	684	9000.00	N.A	687	9000.00	N.A
Cb1-2,3,15	200	N.A	9000.00	2.70	0.00	387	9000.00	N.A	422	9000.00	N.A
Cb1-3,5,15	200	N.A	9000.00	236.76	24.84	N.A	9000.00	N.A	406	673.45	N.A
Cb1-6,4,15	200	N.A	9000.00	171.47	44.86	N.A	9000.00	N.A	424	9000.00	N.A
Cb2-2,3,15	200	N.A	9000.00	2.20	0.00	487	1240.03	N.A	530	523.20	N.A
Cb2-3,5,15	200	N.A	9000.00	196.89	44.38	N.A	9000.00	N.A	359	9000.00	N.A
Cb2-6,4,15	200	N.A	9000.00	187.48	39.52	N.A	9000.00	N.A	415	4239.02	N.A
Cb3-2,3,15	200	402	9000.00	242.17	30.81	501	9000.00	19.76	550	9000.00	26.91
Cb3-3,5,15	200	N.A	9000.00	240.51	35.52	N.A	9000.00	N.A	665	9000.00	N.A
Cb3-6,4,15	200	N.A	9000.00	135.31	50.25	N.A	9000.00	N.A	546	5620.25	N.A
Cb4-2,3,15	200	380	9000.00	186.06	40.55	387	9000.00	1.81	593	6000.65	35.92
Cb4-3,5,15	200	N.A	9000.00	210.87	22.48	N.A	9000.00	N.A	562	7643.49	N.A
Cb4-6,4,15	200	N.A	9000.00	165.94	52.86	N.A	9000.00	N.A	443	5611.26	N.A
Cb5-2,3,15	200	N.A	9000.00	175.07	35.16	N.A	9000.00	N.A	688	6780.86	N.A
Cb5-3,5,15	200	N.A	9000.00	160.85	36.92	N.A	9000.00	N.A	558	5467.55	N.A
Cb5-6,4,15	200	N.A	9000.00	97.19	58.46	N.A	9000.00	N.A	611	5934.99	N.A
Cc1-2,3,15	200	N.A	9000.00	143.35	48.80	N.A	9000.00	N.A	728	7322.75	N.A
Cc1-3,5,15	200	N.A	9000.00	154.43	37.48	N.A	9000.00	N.A	549	7938.73	N.A
Cc1-6,4,15	200	N.A	9000.00	2.85	0.00	N.A	9000.00	N.A	637	6295.60	N.A
Cc2-2,3,15	200	N.A	9000.00	146.36	50.05	N.A	9000.00	N.A	702	5276.32	N.A
Cc2-3,5,15	200	N.A	9000.00	4.58	0.00	N.A	9000.00	N.A	617	7192.03	N.A
Cc2-6,4,15	200	N.A	9000.00	19.02	93.62	N.A	9000.00	N.A	608	6348.15	N.A
Cc3-2,3,15	200	N.A	9000.00	97.58	64.77	N.A	9000.00	N.A	699	5284.95	N.A
Cc3-3,5,15	200	N.A	9000.00	154.90	32.36	N.A	9000.00	N.A	644	6750.50	N.A
Cc3-6,4,15	200	N.A	9000.00	3.24	0.00	N.A	9000.00	N.A	575	5247.18	N.A
Cc4-2,3,15	200	N.A	9000.00	126.74	61.59	N.A	9000.00	N.A	436	5729.75	N.A
Cc4-3,5,15	200	N.A	9000.00	57.45	78.48	N.A	9000.00	N.A	458	5167.62	N.A
Cc4-6,4,15	200	N.A	9000.00	3.43	0.00	N.A	9000.00	N.A	570	6517.16	N.A
Cc5-2,3,15	200	N.A	9000.00	148.45	43.12	N.A	9000.00	N.A	581	5496.01	N.A
Cc5-3,5,15	200	N.A	9000.00	5.22	0.00	N.A	9000.00	N.A	617	6322.83	N.A
Cc5-6,4,15	200	N.A	9000.00	3.94	0.00	N.A	9000.00	N.A	601	6169.12	N.A
Cd1-2,3,15	200	330	9000.00	190.60	39.49	445	9000.00	25.84	445	5702.43	25.84
Cd1-3,5,15	200	N.A	9000.00	194.57	40.13	303	9000.00	N.A	303	5657.12	N.A
Cd1-6,4,15	200	N.A	9000.00	171.62	43.17	544	9000.00	N.A	544	1864.64	N.A
Cd2-2,3,15	200	303	9000.00	143.88	44.45	429	9000.00	29.37	429	5386.76	29.37
Cd2-3,5,15	200	N.A	9000.00	225.45	34.27	417	9000.00	N.A	417	6077.70	N.A
Cd2-6,4,15	200	N.A	9000.00	183.72	37.72	409	9000.00	N.A	408	2894.30	N.A
Cd3-2,3,15	200	305	9000.00	138.14	48.84	504	9000.00	39.48	504	2417.37	39.48
Cd3-3,5,15	200	N.A	9000.00	136.41	44.55	333	9000.00	N.A	333	6124.06	N.A
Cd3-6,4,15	200	N.A	9000.00	175.03	40.26	413	9000.00	N.A	430	6959.73	N.A
Cd4-2,3,15	200	400	9000.00	179.28	46.64	502	9000.00	20.32	502	5736.13	20.32
Cd4-3,5,15	200	N.A	9000.00	169.17	44.17	N.A	9000.00	N.A	465	6474.81	N.A
Cd4-6,4,15	200	N.A	9000.00	155.54	44.84	N.A	9000.00	N.A	586	4454.65	N.A
Cd5-2,3,15	200	388	9000.00	169.24	40.20	388	9000.00	0.00	388	4375.93	0.00
Cd5-3,5,15	200	N.A	9000.00	150.00	43.82	N.A	9000.00	N.A	586	7474.07	N.A
Cd5-6,4,15	200	N.A	9000.00	111.93	50.25	466	9000.00	N.A	625	6901.60	N.A
Averages			***9000.00***		***35.10***	***N.A***	***8870.67***	***18.94***		***5942.75***	***21.42***

Table 5 – Direct solving the hybrid formulation on instances with 15 OD demands and multiple Δ time periods

ID	Ψ	Time-Space formulation ($\Delta = \bar{\Delta}$)				Time-Space formulation ($\Delta = 50$)			Time-Space formulation ($\Delta = 25$)		
		UB	CPU (sec)	LB	RG (%)	UB	CPU (sec)	Dif. Ub (%)	UB	CPU (sec)	Dif. Ub (%)
Ca1-2,3,15	200	275	9000	175.58	36.15	285	5545.51	3.51	290	2743.71	5.17
Ca1-3,5,15	200	328	9000	229.32	30.09	347	5817.79	5.48	351	716.29	6.55
Ca1-6,4,15	200	294	9000	179.08	39.09	307	4260.08	4.23	312	3498.33	5.77
Ca2-2,3,15	200	323	9000	180.22	44.20	330	5522.43	2.12	346	3421.68	6.65
Ca2-3,5,15	200	350	9000	219.94	37.16	374	5197.28	6.42	374	2261.60	6.42
Ca2-6,4,15	200	286	9000	161.71	43.46	298	4206.75	4.03	298	2936.00	4.03
Ca3-2,3,15	200	325	9000	234.42	27.87	361	4141.41	9.97	361	3081.85	9.97
Ca3-3,5,15	200	N.A	9000	N.A	N.A	370	5298.97	N.A	370	1033.74	N.A
Ca3-6,4,15	200	275	9000	148.21	46.10	292	5012.42	5.82	295	743.37	6.78
Ca4-2,3,15	200	325	9000	218.81	32.67	335	6356.93	2.99	354	1907.54	8.19
Ca4-3,5,15	200	287	8235.57	234.09	18.43	299	2549.68	4.01	309	967.52	7.12
Ca4-6,4,15	200	285	9000	186.46	34.58	299	6152.51	4.68	303	1079.68	5.94
Ca5-2,3,15	200	N.A	9000	N.A	N.A	282	5985.84	N.A	300	2829.34	N.A
Ca5-3,5,15	200	276	9000	223.43	19.05	291	4415.18	5.15	292	2161.30	5.48
Ca5-6,4,15	200	243	9000	124.65	48.70	254	5047.90	4.33	254	2877.62	4.33
Cb1-2,3,15	200	N.A	9000	N.A	N.A	330	4216.92	N.A	335	1066.85	N.A
Cb1-3,5,15	200	315	9000	232.01	26.35	322	3619.18	2.17	331	1055.96	4.83
Cb1-6,4,15	200	311	9000	162.25	47.83	330	4437.30	5.76	332	1433.31	6.33
Cb2-2,3,15	200	N.A	9000	N.A	N.A	400	4924.76	N.A	400	797.08	N.A
Cb2-3,5,15	200	354	9000	188.23	46.83	363	4341.31	2.48	365	2786.88	3.01
Cb2-6,4,15	200	310	9000	189.34	38.92	324	5781.35	4.32	332	2763.45	6.63
Cb3-2,3,15	200	350	9000	250.12	28.54	364	4353.17	3.85	368	1646.80	4.89
Cb3-3,5,15	200	373	9000	235.42	36.88	394	4503.81	5.33	394	3081.23	5.33
Cb3-6,4,15	200	272	9000	131.81	51.54	287	3874.09	5.23	289	2529.11	5.88
Cb4-2,3,15	200	313	9000	185.83	40.63	333	6398.45	6.01	333	2143.75	6.01
Cb4-3,5,15	200	272	9000	211.23	22.34	287	6419.74	5.23	289	2935.72	5.88
Cb4-6,4,15	200	352	9000	167.40	52.44	368	3816.80	4.35	383	1880.40	8.09
Cb5-2,3,15	200	270	9000	180.19	33.26	280	3521.03	3.57	283	2464.00	4.59
Cb5-3,5,15	200	255	9000	154.02	39.60	266	9000.00	4.14	266	799.29	4.14
Cb5-6,4,15	200	234	9000	95.36	59.25	246	5572.18	4.88	246	852.20	4.88
Cc1-2,3,15	200	280	9000	130.72	53.32	286	4487.22	2.10	294	1500.38	4.76
Cc1-3,5,15	200	247	9000	157.25	36.33	261	5303.19	5.36	264	2736.41	6.44
Cc1-6,4,15	200	N.A	9000	N.A	N.A	315	4340.11	N.A	385	2960.93	N.A
Cc2-2,3,15	200	293	9000	146.72	49.92	299	6345.32	2.01	301	1343.04	2.66
Cc2-3,5,15	200	N.A	9000	N.A	N.A	300	5651.23	N.A	310	1229.22	N.A
Cc2-6,4,15	200	298	9000	13.10	95.60	317	5551.68	5.99	320	2028.28	6.88
Cc3-2,3,15	200	277	9000	94.16	66.01	292	5678.36	5.14	294	1574.91	5.78
Cc3-3,5,15	200	229	9000	151.77	33.73	234	5964.98	2.14	249	2264.92	8.03
Cc3-6,4,15	200	N.A	9000	N.A	N.A	333	4792.36	N.A	401	1903.99	N.A
Cc4-2,3,15	200	330	9000	117.27	64.46	343	3523.23	3.79	352	2330.30	6.25
Cc4-3,5,15	200	267	9000	N.A	80.14	278	5726.73	3.96	281	1041.89	4.98
Cc4-6,4,15	200	N.A	9000	N.A	0.00	N.A	9000.00	N.A	312	3169.75	N.A
Cc5-2,3,15	200	261	9000	145.64	44.20	275	5711.14	5.09	283	3114.25	7.77
Cc5-3,5,15	200	N.A	9000	N.A	N.A	N.A	9000.00	N.A	222	1125.20	N.A
Cc5-6,4,15	200	N.A	9000	N.A	N.A	N.A	9000.00	N.A	320	3094.40	N.A
Cd1-2,3,15	200	315	9000	188.39	40.19	335	5458.08	5.97	335	1173.86	5.97
Cd1-3,5,15	200	325	9000	196.65	39.49	346	5721.14	6.07	347	2221.93	6.34
Cd1-6,4,15	200	302	9000	170.41	43.57	320	5998.57	5.63	325	3351.01	7.08
Cd2-2,3,15	200	259	9000	135.81	47.57	269	9000.00	3.72	275	1728.63	5.82
Cd2-3,5,15	200	343	9000	221.52	35.42	350	5918.94	2.00	351	1425.62	2.28
Cd2-6,4,15	200	295	9000	181.65	38.42	311	4737.80	5.14	320	2921.11	7.81
Cd3-2,3,15	200	270	9000	131.55	51.28	288	4225.03	6.25	289	1734.90	6.57
Cd3-3,5,15	200	246	9000	137.15	44.25	257	3614.86	4.28	258	3076.52	4.65
Cd3-6,4,15	200	293	9000	171.98	41.30	301	5991.25	2.66	319	2732.01	8.15
Cd4-2,3,15	200	336	9000	184.15	45.19	353	4518.30	4.82	358	2614.95	6.15
Cd4-3,5,15	200	303	9000	172.97	42.91	318	4534.98	4.72	324	1145.43	6.48
Cd4-6,4,15	200	282	9000	158.66	43.74	302	9000.00	6.62	303	1270.97	6.93
Cd5-2,3,15	200	283	9000	157.03	44.51	298	6278.18	5.03	304	2244.60	6.91
Cd5-3,5,15	200	267	9000	152.71	42.80	282	6093.68	5.32	282	2049.28	5.32
Cd5-6,4,15	200	225	9000	103.12	54.17	236	5337.94	4.66	238	2991.61	5.46
Averages			***8987.26***		***42.36***		***5446.58***	***4.57***		***2076.60***	***5.97***

Table 6 – Direct solving the classic time-space formulation on instances with 15 OD demands and multiple Δ time periods

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	280	2.55	280	0	Ca1-2,3,10	100	256	36.36	256	0	Ca1-2,3,15	200	275	257.86	275	0
Ca1-3,5,5	100	222	0.48	222	0	Ca1-3,5,10	100	215	21.38	215	0	Ca1-3,5,15	200	309	288.40	309	0
Ca1-6,4,5	100	271	2.72	271	0	Ca1-6,4,10	100	305	24.14	305	0	Ca1-6,4,15	200	274	1427.18	274	0
Ca2-2,3,5	100	152	1.86	152	0	Ca2-2,3,10	100	161	14.07	161	0	Ca2-2,3,15	200	306	29.53	306	0
Ca2-3,5,5	100	284	2.30	284	0	Ca2-3,5,10	100	340	5.56	340	0	Ca2-3,5,15	200	335	442.19	335	0
Ca2-6,4,5	100	150	1.56	150	0	Ca2-6,4,10	100	179	14.42	179	0	Ca2-6,4,15	200	272	944.95	272	0
Ca3-2,3,5	100	287	2.05	287	0	Ca3-2,3,10	100	344	3.08	344	0	Ca3-2,3,15	200	319	36.86	319	0
Ca3-3,5,5	100	220	2.89	220	0	Ca3-3,5,10	100	318	20.26	318	0	Ca3-3,5,15	200	307	54.43	307	0
Ca3-6,4,5	100	171	1.61	171	0	Ca3-6,4,10	100	236	41.83	236	0	Ca3-6,4,15	200	267	5526.43	267	0
Ca4-2,3,5	100	358	1.24	358	0	Ca4-2,3,10	100	437	16.14	437	0	Ca4-2,3,15	200	309	31.34	309	0
Ca4-3,5,5	100	168	2.35	168	0	Ca4-3,5,10	100	213	462.45	213	0	Ca4-3,5,15	200	287	3585.06	287	0
Ca4-6,4,5	100	161	0.83	161	0	Ca4-6,4,10	100	238	31.8	238	0	Ca4-6,4,15	200	262	1147.66	262	0
Ca5-2,3,5	100	199	2.20	199	0	Ca5-2,3,10	100	277	27.64	277	0	Ca5-2,3,15	200	265	32.87	265	0
Ca5-3,5,5	100	186	2.16	186	0	Ca5-3,5,10	100	264	26.4	264	0	Ca5-3,5,15	200	262	25.99	262	0
Ca5-6,4,5	100	159	1.01	159	0	Ca5-6,4,10	100	187	18.09	187	0	Ca5-6,4,15	200	218	11.95	218	0
Cb1-2,3,5	100	152	1.03	152	0	Cb1-2,3,10	100	181	13.08	181	0	Cb1-2,3,15	200	305	280.14	305	0
Cb1-3,5,5	100	164	0.59	164	0	Cb1-3,5,10	100	211	20.3	211	0	Cb1-3,5,15	200	291	126.13	291	0
Cb1-6,4,5	100	305	1.57	305	0	Cb1-6,4,10	100	261	18.99	261	0	Cb1-6,4,15	200	295	98.39	295	0
Cb2-2,3,5	100	129	2.09	129	0	Cb2-2,3,10	100	199	13.42	199	0	Cb2-2,3,15	200	337	207.52	337	0
Cb2-3,5,5	100	154	1.07	154	0	Cb2-3,5,10	100	268	41.25	268	0	Cb2-3,5,15	200	292	53.31	292	0
Cb2-6,4,5	100	143	2.47	143	0	Cb2-6,4,10	100	185	234.99	185	0	Cb2-6,4,15	200	330	145.68	330	0
Cb3-2,3,5	100	332	2.85	332	0	Cb3-2,3,10	100	337	6.28	337	0	Cb3-2,3,15	200	354	1655.75	354	0
Cb3-3,5,5	100	160	1.22	160	0	Cb3-3,5,10	100	202	144.99	202	0	Cb3-3,5,15	200	266	74.13	266	0
Cb3-6,4,5	100	198	2.13	198	0	Cb3-6,4,10	100	283	1456.59	283	0	Cb3-6,4,15	200	298	44.46	298	0
Cb4-2,3,5	100	280	1.84	280	0	Cb4-2,3,10	100	251	106.53	251	0	Cb4-2,3,15	200	255	384.53	255	0
Cb4-3,5,5	100	142	0.96	142	0	Cb4-3,5,10	100	245	2355.73	245	0	Cb4-3,5,15	200	334	311.41	334	0
Cb4-6,4,5	100	188	1.97	188	0	Cb4-6,4,10	100	288	7.43	288	0	Cb4-6,4,15	200	252	51.74	252	0
Cb5-2,3,5	100	129	2.35	129	0	Cb5-2,3,10	100	197	42.39	197	0	Cb5-2,3,15	200	243	64.14	243	0
Cb5-3,5,5	100	179	1.90	179	0	Cb5-3,5,10	100	232	128.22	232	0	Cb5-3,5,15	200	223	699.52	223	0
Cb5-6,4,5	100	199	1.56	199	0	Cb5-6,4,10	100	337	50.75	337	0	Cb5-6,4,15	200	265	119.31	265	0
Cc1-2,3,5	100	129	2.03	129	0	Cc1-2,3,10	100	189	64.63	189	0	Cc1-2,3,15	200	233	594.93	233	0
Cc1-3,5,5	100	135	2.30	135	0	Cc1-3,5,10	100	180	41.67	180	0	Cc1-3,5,15	200	280	175.81	280	0
Cc1-6,4,5	100	150	2.40	150	0	Cc1-6,4,10	100	238	45.67	238	0	Cc1-6,4,15	200	284	131.55	284	0
Cc2-2,3,5	100	122	1.04	122	0	Cc2-2,3,10	100	187	29.35	187	0	Cc2-2,3,15	200	299	64.06	299	0
Cc2-3,5,5	100	175	0.94	175	0	Cc2-3,5,10	100	231	27.7	231	0	Cc2-3,5,15	200	273	378.17	273	0
Cc2-6,4,5	100	122	1.77	122	0	Cc2-6,4,10	100	163	18.8	163	0	Cc2-6,4,15	200	271	380.97	271	0
Cc3-2,3,5	100	136	2.11	136	0	Cc3-2,3,10	100	184	12.7	184	0	Cc3-2,3,15	200	223	670.41	223	0
Cc3-3,5,5	100	142	2.59	142	0	Cc3-3,5,10	100	225	79.81	225	0	Cc3-3,5,15	200	259	668.33	259	0
Cc3-6,4,5	100	157	0.90	157	0	Cc3-6,4,10	100	193	21.95	193	0	Cc3-6,4,15	200	307	90.63	307	0
Cc4-2,3,5	100	171	1.27	171	0	Cc4-2,3,10	100	225	24.11	225	0	Cc4-2,3,15	200	250	240.03	250	0
Cc4-3,5,5	100	154	1.25	154	0	Cc4-3,5,10	100	223	28.05	223	0	Cc4-3,5,15	200	268	69.52	268	0
Cc4-6,4,5	100	138	1.16	138	0	Cc4-6,4,10	100	230	25.04	230	0	Cc4-6,4,15	200	236	14.54	236	0
Cc5-2,3,5	100	123	1.66	123	0	Cc5-2,3,10	100	182	12.52	182	0	Cc5-2,3,15	200	247	69.91	247	0
Cc5-3,5,5	100	124	2.65	124	0	Cc5-3,5,10	100	168	27.72	168	0	Cc5-3,5,15	200	208	73.73	208	0
Cc5-6,4,5	100	138	1.41	138	0	Cc5-6,4,10	100	179	51.26	179	0	Cc5-6,4,15	200	293	12.48	293	0
Cd1-2,3,5	100	155	2.84	155	0	Cd1-2,3,10	100	197	13.29	197	0	Cd1-2,3,15	200	304	43.78	304	0
Cd1-3,5,5	100	170	1.68	170	0	Cd1-3,5,10	100	215	12.11	215	0	Cd1-3,5,15	200	277	47.97	277	0
Cd1-6,4,5	100	188	2.20	188	0	Cd1-6,4,10	100	248	44.06	248	0	Cd1-6,4,15	200	249	3.62	249	0
Cd2-2,3,5	100	140	1.21	140	0	Cd2-2,3,10	100	193	11.31	193	0	Cd2-2,3,15	200	319	4.06	319	0
Cd2-3,5,5	100	158	1.09	158	0	Cd2-3,5,10	100	236	12.85	236	0	Cd2-3,5,15	200	274	28.50	274	0
Cd2-6,4,5	100	142	0.97	142	0	Cd2-6,4,10	100	213	44.5	213	0	Cd2-6,4,15	200	257	2.57	257	0
Cd3-2,3,5	100	142	2.64	142	0	Cd3-2,3,10	100	186	20.14	186	0	Cd3-2,3,15	200	222	45.33	222	0
Cd3-3,5,5	100	147	2.16	147	0	Cd3-3,5,10	100	224	32.52	224	0	Cd3-3,5,15	200	268	100.99	268	0
Cd3-6,4,5	100	157	1.03	157	0	Cd3-6,4,10	100	196	25.17	196	0	Cd3-6,4,15	200	324	25.14	324	0
Cd4-2,3,5	100	202	2.73	202	0	Cd4-2,3,10	100	257	11.3	257	0	Cd4-2,3,15	200	289	31.08	289	0
Cd4-3,5,5	100	162	1.35	162	0	Cd4-3,5,10	100	241	20.37	241	0	Cd4-3,5,15	200	263	25.48	263	0
Cd4-6,4,5	100	147	1.11	147	0	Cd4-6,4,10	100	189	30.48	189	0	Cd4-6,4,15	200	271	10.09	271	0
Cd5-2,3,5	100	178	1.35	178	0	Cd5-2,3,10	100	207	20.54	207	0	Cd5-2,3,15	200	247	19.36	247	0
Cd5-3,5,5	100	178	0.82	178	0	Cd5-3,5,10	100	213	30.54	213	0	Cd5-3,5,15	200	208	252.93	208	0
Cd5-6,4,5	100	157	1.19	157	0	Cd5-6,4,10	100	184	43.23	184	0	Cd5-6,4,15	200	272	725.23	272	0
Averages			1.72		0	***Averages***			104.80		0	***Averages***			386.00		0

Table 7 – DDD results on instances with 5, 10 and 15 OD demands

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	280	78.14	280	0	Ca1-2,3,10	100	256	597.03	256	0	Ca1-2,3,15	200	277	18000.00	236.03	14.79
Ca1-3,5,5	100	222	92.85	222	0	Ca1-3,5,10	100	215	4068.76	215	0	Ca1-3,5,15	200	311	18000.00	283.20	8.94
Ca1-6,4,5	100	271	302.53	271	0	Ca1-6,4,10	100	305	2916.36	305	0	Ca1-6,4,15	200	289	18000.00	221.95	23.20
Ca2-2,3,5	100	152	191.19	152	0	Ca2-2,3,10	100	161	3620.48	161	0	Ca2-2,3,15	200	306	8160.277	306.00	0.00
Ca2-3,5,5	100	284	368.60	284	0	Ca2-3,5,10	100	340	4042.23	340	0	Ca2-3,5,15	200	339	18000.00	267.23	21.17
Ca2-6,4,5	100	150	439.69	150	0	Ca2-6,4,10	100	179	3685.74	179	0	Ca2-6,4,15	200	283	18000.00	210.13	25.75
Ca3-2,3,5	100	287	293.68	287	0	Ca3-2,3,10	100	344	1172.01	344	0	Ca3-2,3,15	200	319	4097.53	319.00	0.00
Ca3-3,5,5	100	220	134.13	220	0	Ca3-3,5,10	100	318	1678.52	318	0	Ca3-3,5,15	200	315	18000.00	298.27	5.31
Ca3-6,4,5	100	171	113.06	171	0	Ca3-6,4,10	100	236	2872.09	236	0	Ca3-6,4,15	200	286	18000.00	206.43	27.82
Ca4-2,3,5	100	358	136.83	358	0	Ca4-2,3,10	100	437	1076.81	437	0	Ca4-2,3,15	200	321	18000.00	275.13	14.29
Ca4-3,5,5	100	168	269.84	168	0	Ca4-3,5,10	100	213	969.23	213	0	Ca4-3,5,15	200	287	5999.64	287.00	0.00
Ca4-6,4,5	100	161	104.24	161	0	Ca4-6,4,10	100	238	3998.11	238	0	Ca4-6,4,15	200	272	18000.00	229.60	15.59
Ca5-2,3,5	100	199	406.72	199	0	Ca5-2,3,10	100	277	3044.70	277	0	Ca5-2,3,15	200	270	18000.00	266.46	1.31
Ca5-3,5,5	100	186	202.17	186	0	Ca5-3,5,10	100	264	948.10	264	0	Ca5-3,5,15	200	280	7166.95	280.00	0.00
Ca5-6,4,5	100	159	410.28	159	0	Ca5-6,4,10	100	187	768.66	187	0	Ca5-6,4,15	200	220	18000.00	170.64	22.44
Cb1-2,3,5	100	152	341.33	152	0	Cb1-2,3,10	100	181	2666.72	181	0	Cb1-2,3,15	200	305	8958.35	305.00	0.00
Cb1-3,5,5	100	164	381.53	164	0	Cb1-3,5,10	100	211	2702.84	211	0	Cb1-3,5,15	200	298	18000.00	264.82	11.14
Cb1-6,4,5	100	305	370.46	305	0	Cb1-6,4,10	100	261	2746.78	261	0	Cb1-6,4,15	200	301	18000.00	200.44	33.41
Cb2-2,3,5	100	129	283.59	129	0	Cb2-2,3,10	100	199	3759.89	199	0	Cb2-2,3,15	200	275	5621.68	275.00	0.00
Cb2-3,5,5	100	154	173.17	154	0	Cb2-3,5,10	100	268	2191.68	268	0	Cb2-3,5,15	200	348	18000.00	248.03	28.73
Cb2-6,4,5	100	143	162.15	143	0	Cb2-6,4,10	100	185	3813.43	185	0	Cb2-6,4,15	200	311	18000.00	237.86	23.52
Cb3-2,3,5	100	332	132.18	332	0	Cb3-2,3,10	100	337	3115.60	337	0	Cb3-2,3,15	200	337	18000.00	300.20	10.92
Cb3-3,5,5	100	160	355.92	160	0	Cb3-3,5,10	100	202	2510.81	202	0	Cb3-3,5,15	200	354	6756.64	354.00	0.00
Cb3-6,4,5	100	198	270.45	198	0	Cb3-6,4,10	100	283	4204.87	283	0	Cb3-6,4,15	200	278	18000.00	180.21	35.18
Cb4-2,3,5	100	280	399.33	280	0	Cb4-2,3,10	100	251	3211.56	251	0	Cb4-2,3,15	200	306	18000.00	249.17	18.57
Cb4-3,5,5	100	142	78.82	142	0	Cb4-3,5,10	100	245	4197.41	245	0	Cb4-3,5,15	200	270	18000.00	248.00	8.15
Cb4-6,4,5	100	188	294.48	188	0	Cb4-6,4,10	100	288	3570.49	288	0	Cb4-6,4,15	200	340	18000.00	231.25	31.98
Cb5-2,3,5	100	129	314.79	129	0	Cb5-2,3,10	100	197	4020.52	197	0	Cb5-2,3,15	200	265	18000.00	229.01	13.58
Cb5-3,5,5	100	179	314.78	179	0	Cb5-3,5,10	100	232	3327.24	232	0	Cb5-3,5,15	200	247	18000.00	197.69	19.96
Cb5-6,4,5	100	199	166.44	199	0	Cb5-6,4,10	100	337	3733.03	337	0	Cb5-6,4,15	200	238	18000.00	146.17	38.59
Cc1-2,3,5	100	129	3641.96	129	0	Cc1-2,3,10	100	189	6462.98	189	0	Cc1-2,3,15	200	271	18000.00	173.28	36.06
Cc1-3,5,5	100	135	5094.15	135	0	Cc1-3,5,10	100	180	7163.11	180	0	Cc1-3,5,15	200	245	18000.00	201.21	17.88
Cc1-6,4,5	100	150	1347.79	150	0	Cc1-6,4,10	100	238	6139.92	238	0	Cc1-6,4,15	200	302	18000.00	227.77	24.58
Cc2-2,3,5	100	122	2960.55	122	0	Cc2-2,3,10	100	187	6832.77	187	0	Cc2-2,3,15	200	296	18000.00	209.84	29.11
Cc2-3,5,5	100	175	1491.50	175	0	Cc2-3,5,10	100	231	6496.19	231	0	Cc2-3,5,15	200	315	18000.00	215.18	31.69
Cc2-6,4,5	100	122	5175.69	122	0	Cc2-6,4,10	100	163	6195.34	163	0	Cc2-6,4,15	200	301	18000.00	188.30	37.44
Cc3-2,3,5	100	136	1253.47	136	0	Cc3-2,3,10	100	184	6843.57	184	0	Cc3-2,3,15	200	290	18000.00	199.05	31.36
Cc3-3,5,5	100	142	3175.83	142	0	Cc3-3,5,10	100	225	6556.92	225	0	Cc3-3,5,15	200	230	18000.00	186.84	18.77
Cc3-6,4,5	100	157	2602.36	157	0	Cc3-6,4,10	100	193	5790.61	193	0	Cc3-6,4,15	200	284	18000.00	224.51	20.95
Cc4-2,3,5	100	171	5693.12	171	0	Cc4-2,3,10	100	225	5937.40	225	0	Cc4-2,3,15	200	326	18000.00	229.35	29.65
Cc4-3,5,5	100	154	2677.88	154	0	Cc4-3,5,10	100	223	5937.87	223	0	Cc4-3,5,15	200	269	18000.00	165.09	38.63
Cc4-6,4,5	100	138	2836.50	138	0	Cc4-6,4,10	100	230	5152.39	230	0	Cc4-6,4,15	200	276	18000.00	195.97	29.00
Cc5-2,3,5	100	123	1525.61	123	0	Cc5-2,3,10	100	182	6576.45	182	0	Cc5-2,3,15	200	243	18000.00	179.41	26.17
Cc5-3,5,5	100	124	5426.52	124	0	Cc5-3,5,10	100	168	7008.74	168	0	Cc5-3,5,15	200	261	18000.00	189.47	27.40
Cc5-6,4,5	100	138	2312.30	138	0	Cc5-6,4,10	100	179	5791.82	179	0	Cc5-6,4,15	200	226	18000.00	183.83	18.66
Cd1-2,3,5	100	155	126.76	155	0	Cd1-2,3,10	100	197	794.44	197	0	Cd1-2,3,15	200	301	18000.00	225.56	25.06
Cd1-3,5,5	100	170	221.75	170	0	Cd1-3,5,10	100	215	943.94	215	0	Cd1-3,5,15	200	311	18000.00	243.56	21.69
Cd1-6,4,5	100	188	289.63	188	0	Cd1-6,4,10	100	248	1027.98	248	0	Cd1-6,4,15	200	279	18000.00	219.46	21.34
Cd2-2,3,5	100	140	400.20	140	0	Cd2-2,3,10	100	193	1594.63	193	0	Cd2-2,3,15	200	259	18000.00	191.34	26.12
Cd2-3,5,5	100	158	431.44	158	0	Cd2-3,5,10	100	236	2202.01	236	0	Cd2-3,5,15	200	325	18000.00	285.64	12.11
Cd2-6,4,5	100	142	358.49	142	0	Cd2-6,4,10	100	213	3507.31	213	0	Cd2-6,4,15	200	280	18000.00	213.54	23.74
Cd3-2,3,5	100	142	91.05	142	0	Cd3-2,3,10	100	186	3333.83	186	0	Cd3-2,3,15	200	272	18000.00	195.50	28.12
Cd3-3,5,5	100	147	165.73	147	0	Cd3-3,5,10	100	224	3978.59	224	0	Cd3-3,5,15	200	227	18000.00	152.09	33.00
Cd3-6,4,5	100	157	435.13	157	0	Cd3-6,4,10	100	196	4341.92	196	0	Cd3-6,4,15	200	281	18000.00	212.89	24.24
Cd4-2,3,5	100	202	337.53	202	0	Cd4-2,3,10	100	257	1845.55	257	0	Cd4-2,3,15	200	338	18000.00	254.39	24.74
Cd4-3,5,5	100	162	325.12	162	0	Cd4-3,5,10	100	241	3545.78	241	0	Cd4-3,5,15	200	295	18000.00	206.79	29.90
Cd4-6,4,5	100	147	204.10	147	0	Cd4-6,4,10	100	189	4254.66	189	0	Cd4-6,4,15	200	266	18000.00	184.60	30.60
Cd5-2,3,5	100	178	351.91	178	0	Cd5-2,3,10	100	207	3537.62	207	0	Cd5-2,3,15	200	274	18000.00	198.78	27.45
Cd5-3,5,5	100	178	437.20	178	0	Cd5-3,5,10	100	213	892.31	213	0	Cd5-3,5,15	200	254	18000.00	193.76	23.72
Cd5-6,4,5	100	157	153.18	157	0	Cd5-6,4,10	100	184	4454.43	184	0	Cd5-6,4,15	200	214	18000.00	126.97	40.67
Averages			985.463		0	***Averages***			3672.88		0	***Averages***			16679.36		21.07

Table 8 – DDD results on instances with 5, 10 and 15 OD demands using the time-space formulation

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,30	200	530	2006.52	530	0.00	Ca1-2,3,50	200	808	28800	742.50	8.11
Ca1-3,5,30	200	490	12877.17	490	0.00	Ca1-3,5,50	200	649	28800	585.80	9.74
Ca1-6,4,30	200	417	18000	398.1389	4.52	Ca1-6,4,50	200	710	28800	606.11	14.63
Ca2-2,3,30	200	441	9001.31	441	0.00	Ca2-2,3,50	200	713	28800	410.33	42.45
Ca2-3,5,30	200	439	1214.24	439	0.00	Ca2-3,5,50	200	817	28800	510.96	37.46
Ca2-6,4,30	200	448	18000	436.5814	2.55	Ca2-6,4,50	200	784	28800	707.58	9.75
Ca3-2,3,30	200	570	15555.90	570	0.00	Ca3-2,3,50	200	770	28800	664.91	13.65
Ca3-3,5,30	200	428	12028.32	428	0.00	Ca3-3,5,50	200	573	28800	405.81	29.18
Ca3-6,4,30	200	445	12407.91	445	0.00	Ca3-6,4,50	200	745	28800	662.60	11.06
Ca4-2,3,30	200	508	15297.08	508	0.00	Ca4-2,3,50	200	685	28800	632.00	7.74
Ca4-3,5,30	200	423	18000	399.735	5.50	Ca4-3,5,50	200	650	28800	554.98	14.62
Ca4-6,4,30	200	392	10124.80	392	0.00	Ca4-6,4,50	200	793	28800	608.65	23.25
Ca5-2,3,30	200	429	11249.56	429	0.00	Ca5-2,3,50	200	812	28800	771.54	4.98
Ca5-3,5,30	200	499	18000	487.4232	2.32	Ca5-3,5,50	200	734	28800	488.00	33.51
Ca5-6,4,30	200	406	13173.54	406	0.00	Ca5-6,4,50	200	820	28800	710.29	13.38
Cb1-2,3,30	200	492	10787.85	492	0.00	Cb1-2,3,50	200	790	28800	683.60	13.47
Cb1-3,5,30	200	510	18000	454.869	10.81	Cb1-3,5,50	200	729	28800	578.56	20.64
Cb1-6,4,30	200	421	18000	389.26	7.54	Cb1-6,4,50	200	745	28800	597.25	19.83
Cb2-2,3,30	200	402	12971.62	402	0.00	Cb2-2,3,50	200	680	28800	440.46	35.23
Cb2-3,5,30	200	436	18000	421.65	3.29	Cb2-3,5,50	200	835	28800	518.71	37.88
Cb2-6,4,30	200	437	10177.10	437	0.00	Cb2-6,4,50	200	837	28800	742.92	11.24
Cb3-2,3,30	200	553	18000	494.76	10.53	Cb3-2,3,50	200	780	28800	588.23	24.59
Cb3-3,5,30	200	432	18000	409.66	5.17	Cb3-3,5,50	200	656	28800	535.00	18.45
Cb3-6,4,30	200	473	18000	453.13	4.20	Cb3-6,4,50	200	683	28800	614.19	10.07
Cb4-2,3,30	200	505	18000	433.44	14.17	Cb4-2,3,50	200	798	28800	432.00	45.86
Cb4-3,5,30	200	437	18000	410.73	6.01	Cb4-3,5,50	200	675	28800	522.44	22.60
Cb4-6,4,30	200	395	18000	378.37	4.21	Cb4-6,4,50	200	788	28800	670.66	14.89
Cb5-2,3,30	200	396	14560.44	396	0.00	Cb5-2,3,50	200	726	28800	639.87	11.86
Cb5-3,5,30	200	527	18000	505.02	4.17	Cb5-3,5,50	200	654	28800	514.33	21.36
Cb5-6,4,30	200	418	18000	370.18	11.44	Cb5-6,4,50	200	797	28800	717.83	9.93
Cc1-2,3,30	200	411	18000	411	0.00	Cc1-2,3,50	200	767	28800	663.57	13.49
Cc1-3,5,30	200	536	18000	485.83	9.36	Cc1-3,5,50	200	691	28800	629.23	8.94
Cc1-6,4,30	200	393	18000	361.20	8.09	Cc1-6,4,50	200	632	28800	562.32	11.03
Cc2-2,3,30	200	423	18000	374.56	11.45	Cc2-2,3,50	200	700	28800	547.60	21.77
Cc2-3,5,30	200	418	18000	375.69	10.12	Cc2-3,5,50	200	726	28800	543.50	25.14
Cc2-6,4,30	200	445	18000	411.58	7.51	Cc2-6,4,50	200	680	28800	573.80	15.62
Cc3-2,3,30	200	552	18000	506.62	8.22	Cc3-2,3,50	200	663	28800	521.80	21.30
Cc3-3,5,30	200	450	18000	409.81	8.93	Cc3-3,5,50	200	618	28800	458.83	25.75
Cc3-6,4,30	200	443	18000	404.85	8.61	Cc3-6,4,50	200	652	28800	493.58	24.30
Cc4-2,3,30	200	501	18000	454.30	9.32	Cc4-2,3,50	200	593	28800	498.80	15.89
Cc4-3,5,30	200	418	18000	370.64	11.33	Cc4-3,5,50	200	705	28800	544.39	22.78
Cc4-6,4,30	200	378	18000	341.14	9.75	Cc4-6,4,50	200	637	28800	371.41	41.69
Cc5-2,3,30	200	423	18000	402.61	4.82	Cc5-2,3,50	200	735	28800	472.97	35.65
Cc5-3,5,30	200	429	18000	396.69	7.53	Cc5-3,5,50	200	619	28800	461.00	25.53
Cc5-6,4,30	200	390	18000	364.45	6.55	Cc5-6,4,50	200	774	28800	689.63	10.9
Cd1-2,3,30	200	445	13823.36	445	0.00	Cd1-2,3,50	200	640	28800	560.32	12.45
Cd1-3,5,30	200	533	13043.42	533	0.00	Cd1-3,5,50	200	664	28800	554.97	16.42
Cd1-6,4,30	200	412	12720.21	412	0.00	Cd1-6,4,50	200	688	28800	604.82	12.09
Cd2-2,3,30	200	429	18000	388.24	9.50	Cd2-2,3,50	200	741	28800	606.21	18.19
Cd2-3,5,30	200	435	15479.58	435	0.00	Cd2-3,5,50	200	769	28800	562.75	26.82
Cd2-6,4,30	200	434	16873.03	434	0.00	Cd2-6,4,50	200	680	28800	607.92	10.6
Cd3-2,3,30	200	550	18000	481.25	12.50	Cd3-2,3,50	200	625	28800	542.50	13.2
Cd3-3,5,30	200	440	8435.23	440	0.00	Cd3-3,5,50	200	771	28800	651.42	15.51
Cd3-6,4,30	200	437	18000	420.65	3.74	Cd3-6,4,50	200	712	28800	603.70	15.21
Cd4-2,3,30	200	510	12095.64	510	0.00	Cd4-2,3,50	200	614	28800	438.82	28.53
Cd4-3,5,30	200	424	18000	389.02	8.25	Cd4-3,5,50	200	678	28800	605.38	10.71
Cd4-6,4,30	200	395	18000	384.57	2.64	Cd4-6,4,50	200	648	28800	583.91	9.89
Cd5-2,3,30	200	425	10783.34	425	0.00	Cd5-2,3,50	200	764	28800	697.99	8.64
Cd5-3,5,30	200	501	10556.08	501	0.00	Cd5-3,5,50	200	641	28800	556.51	13.18
Cd5-6,4,30	200	432	13661.84	432	0.00	Cd5-6,4,50	200	857	28800	767.53	10.44
Averages			15348.42		4.24	***Averages***			28800		18.88

Table 9 – DDD results on instances with 30 and 50 OD demands

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,30	200	542	18000	418.42	22.80	Ca1-2,3,50	200	1018	28800	607.13	40.36
Ca1-3,5,30	200	548	18000	409.38	25.29	Ca1-3,5,50	200	N.A	28800	N.A	N.A
Ca1-6,4,30	200	616	18000	336.00	45.45	Ca1-6,4,50	200	N.A	28800	N.A	N.A
Ca2-2,3,30	200	481	18000	353.93	26.42	Ca2-2,3,50	200	819	28800	409.14	50.04
Ca2-3,5,30	200	439	6602.54	439.00	0.00	Ca2-3,5,50	200	N.A	28800	N.A	N.A
Ca2-6,4,30	200	583	18000	433.39	25.66	Ca2-6,4,50	200	N.A	28800	N.A	N.A
Ca3-2,3,30	200	713	18000	354.52	50.28	Ca3-2,3,50	200	1010	28800	491.68	51.32
Ca3-3,5,30	200	557	18000	284.06	49.00	Ca3-3,5,50	200	N.A	28800	N.A	N.A
Ca3-6,4,30	200	594	18000	335.30	43.55	Ca3-6,4,50	200	N.A	28800	N.A	N.A
Ca4-2,3,30	200	573	18000	429.24	25.09	Ca4-2,3,50	200	890	28800	486.17	45.37
Ca4-3,5,30	200	488	18000	351.20	28.03	Ca4-3,5,50	200	N.A	28800	N.A	N.A
Ca4-6,4,30	200	520	18000	290.77	44.08	Ca4-6,4,50	200	N.A	28800	N.A	N.A
Ca5-2,3,30	200	518	18000	274.52	47.00	Ca5-2,3,50	200	1049	28800	577.54	44.94
Ca5-3,5,30	200	516	18000	375.33	27.26	Ca5-3,5,50	200	N.A	28800	N.A	N.A
Ca5-6,4,30	200	549	18000	375.75	31.56	Ca5-6,4,50	200	N.A	28800	N.A	N.A
Cb1-2,3,30	200	561	18000	345.97	38.33	Cb1-2,3,50	200	1050	28800	508.18	51.60
Cb1-3,5,30	200	702	18000	480.47	31.56	Cb1-3,5,50	200	N.A	28800	N.A	N.A
Cb1-6,4,30	200	528	18000	265.76	49.67	Cb1-6,4,50	200	N.A	28800	N.A	N.A
Cb2-2,3,30	200	582	18000	286.90	50.70	Cb2-2,3,50	200	849	28800	429.67	49.39
Cb2-3,5,30	200	553	18000	301.47	45.49	Cb2-3,5,50	200	N.A	28800	N.A	N.A
Cb2-6,4,30	200	576	18000	315.15	45.29	Cb2-6,4,50	200	N.A	28800	N.A	N.A
Cb3-2,3,30	200	616	18000	393.11	36.18	Cb3-2,3,50	200	1020	28800	508.25	50.17
Cb3-3,5,30	200	549	18000	387.74	29.37	Cb3-3,5,50	200	N.A	28800	N.A	N.A
Cb3-6,4,30	200	586	18000	373.50	36.26	Cb3-6,4,50	200	N.A	28800	N.A	N.A
Cb4-2,3,30	200	575	18000	428.42	25.49	Cb4-2,3,50	200	952	28800	423.85	55.48
Cb4-3,5,30	200	614	18000	434.47	29.24	Cb4-3,5,50	200	N.A	28800	N.A	N.A
Cb4-6,4,30	200	511	18000	342.67	32.94	Cb4-6,4,50	200	N.A	28800	N.A	N.A
Cb5-2,3,30	200	456	18000	241.95	46.94	Cb5-2,3,50	200	997	28800	554.78	44.36
Cb5-3,5,30	200	649	18000	433.26	33.24	Cb5-3,5,50	200	N.A	28800	N.A	N.A
Cb5-6,4,30	200	512	18000	297.06	41.98	Cb5-6,4,50	200	N.A	28800	N.A	N.A
Cc1-2,3,30	200	536	18000	330.78	38.29	Cc1-2,3,50	200	984	28800	592.02	39.84
Cc1-3,5,30	200	873	18000	517.15	40.76	Cc1-3,5,50	200	N.A	28800	N.A	N.A
Cc1-6,4,30	200	696	18000	380.85	45.28	Cc1-6,4,50	200	N.A	28800	N.A	N.A
Cc2-2,3,30	200	634	18000	400.56	36.82	Cc2-2,3,50	200	913	28800	500.54	45.18
Cc2-3,5,30	200	652	18000	417.32	35.99	Cc2-3,5,50	200	N.A	28800	N.A	N.A
Cc2-6,4,30	200	697	18000	360.73	48.24	Cc2-6,4,50	200	N.A	28800	N.A	N.A
Cc3-2,3,30	200	809	18000	455.79	43.66	Cc3-2,3,50	200	1097	28800	504.77	53.99
Cc3-3,5,30	200	669	18000	383.55	42.67	Cc3-3,5,50	200	N.A	28800	N.A	N.A
Cc3-6,4,30	200	681	18000	404.33	40.63	Cc3-6,4,50	200	N.A	28800	N.A	N.A
Cc4-2,3,30	200	832	18000	410.83	50.62	Cc4-2,3,50	200	806	28800	466.72	42.09
Cc4-3,5,30	200	574	18000	379.32	33.92	Cc4-3,5,50	200	N.A	28800	N.A	N.A
Cc4-6,4,30	200	521	18000	333.55	35.98	Cc4-6,4,50	200	N.A	28800	N.A	N.A
Cc5-2,3,30	200	516	18000	380.26	26.31	Cc5-2,3,50	200	908	28800	458.25	49.53
Cc5-3,5,30	200	576	18000	417.31	27.55	Cc5-3,5,50	200	N.A	28800	N.A	N.A
Cc5-6,4,30	200	644	18000	357.40	44.50	Cc5-6,4,50	200	N.A	28800	N.A	N.A
Cd1-2,3,30	200	588	18000	363.64	38.16	Cd1-2,3,50	200	902	28800	540.51	40.08
Cd1-3,5,30	200	655	18000	410.98	37.25	Cd1-3,5,50	200	N.A	28800	N.A	N.A
Cd1-6,4,30	200	544	18000	277.67	48.96	Cd1-6,4,50	200	N.A	28800	N.A	N.A
Cd2-2,3,30	200	637	18000	389.28	38.89	Cd2-2,3,50	200	1002	28800	606.04	39.52
Cd2-3,5,30	200	559	18000	348.86	37.59	Cd2-3,5,50	200	N.A	28800	N.A	N.A
Cd2-6,4,30	200	450	18000	393.57	12.54	Cd2-6,4,50	200	N.A	28800	N.A	N.A
Cd3-2,3,30	200	647	18000	468.87	27.53	Cd3-2,3,50	200	843	28800	376.43	55.35
Cd3-3,5,30	200	534	18000	348.58	34.72	Cd3-3,5,50	200	N.A	28800	N.A	N.A
Cd3-6,4,30	200	653	18000	403.99	38.13	Cd3-6,4,50	200	N.A	28800	N.A	N.A
Cd4-2,3,30	200	687	18000	497.36	27.60	Cd4-2,3,50	200	754	28800	405.51	46.22
Cd4-3,5,30	200	530	18000	352.39	33.51	Cd4-3,5,50	200	N.A	28800	N.A	N.A
Cd4-6,4,30	200	527	18000	303.49	42.41	Cd4-6,4,50	200	N.A	28800	N.A	N.A
Cd5-2,3,30	200	536	18000	365.00	31.90	Cd5-2,3,50	200	1044	28800	452.35	56.67
Cd5-3,5,30	200	646	18000	374.14	42.08	Cd5-3,5,50	200	N.A	28800	N.A	N.A
Cd5-6,4,30	200	565	18000	404.74	28.37	Cd5-6,4,50	200	N.A	28800	N.A	N.A
Averages			18000		36.25	***Averages***			28800		47.57

Table 10 – DDD results on instances with 30 and 50 OD demands using the time-space formulation

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	171	0.87	171	0	Ca1-2,3,10	100	226	24.25	226	0	Ca1-2,3,15	200	270	60.19	270	0
Ca1-3,5,5	100	185	2.34	185	0	Ca1-3,5,10	100	199	13.59	199	0	Ca1-3,5,15	200	279	106.11	279	0
Ca1-6,4,5	100	195	6.95	195	0	Ca1-6,4,10	100	247	19.67	247	0	Ca1-6,4,15	200	274	789.99	274	0
Ca2-2,3,5	100	136	1.04	136	0	Ca2-2,3,10	100	161	17.4	161	0	Ca2-2,3,15	200	306	18.54	306	0
Ca2-3,5,5	100	199	0.83	199	0	Ca2-3,5,10	100	260	10.76	260	0	Ca2-3,5,15	200	332	208.01	332	0
Ca2-6,4,5	100	150	4.96	150	0	Ca2-6,4,10	100	179	72.76	179	0	Ca2-6,4,15	200	272	724.68	272	0
Ca3-2,3,5	100	287	0.22	287	0	Ca3-2,3,10	100	344	0.88	344	0	Ca3-2,3,15	200	285	14.50	285	0
Ca3-3,5,5	100	198	1.89	198	0	Ca3-3,5,10	100	211	29.24	211	0	Ca3-3,5,15	200	226	182.90	226	0
Ca3-6,4,5	100	154	5.06	154	0	Ca3-6,4,10	100	220	19.04	220	0	Ca3-6,4,15	200	261	265.32	261	0
Ca4-2,3,5	100	197	0.46	197	0	Ca4-2,3,10	100	306	39.93	306	0	Ca4-2,3,15	200	309	36.50	309	0
Ca4-3,5,5	100	150	3.2	150	0	Ca4-3,5,10	100	213	31.29	213	0	Ca4-3,5,15	200	245	743.13	245	0
Ca4-6,4,5	100	154	6.32	154	0	Ca4-6,4,10	100	230	69.39	230	0	Ca4-6,4,15	200	249	128.39	249	0
Ca5-2,3,5	100	185	0.33	185	0	Ca5-2,3,10	100	256	2.85	256	0	Ca5-2,3,15	200	265	21.18	265	0
Ca5-3,5,5	100	141	2.1	141	0	Ca5-3,5,10	100	230	18.43	230	0	Ca5-3,5,15	200	262	92.58	262	0
Ca5-6,4,5	100	159	4.51	159	0	Ca5-6,4,10	100	187	46.47	187	0	Ca5-6,4,15	200	218	74.68	218	0
Cb1-2,3,5	100	139	0.8	139	0	Cb1-2,3,10	100	181	2.78	181	0	Cb1-2,3,15	200	275	54.05	275	0
Cb1-3,5,5	100	147	2.17	147	0	Cb1-3,5,10	100	207	24.51	207	0	Cb1-3,5,15	200	247	241.33	247	0
Cb1-6,4,5	100	199	3.83	199	0	Cb1-6,4,10	100	238	63.94	238	0	Cb1-6,4,15	200	295	1466.17	295	0
Cb2-2,3,5	100	129	0.35	129	0	Cb2-2,3,10	100	171	12.1	171	0	Cb2-2,3,15	200	268	15.24	268	0
Cb2-3,5,5	100	152	2.03	152	0	Cb2-3,5,10	100	241	30.76	241	0	Cb2-3,5,15	200	337	247.10	337	0
Cb2-6,4,5	100	143	5.88	143	0	Cb2-6,4,10	100	185	133.02	185	0	Cb2-6,4,15	200	290	432.00	290	0
Cb3-2,3,5	100	332	0.11	332	0	Cb3-2,3,10	100	337	6.46	337	0	Cb3-2,3,15	200	304	24.60	304	0
Cb3-3,5,5	100	133	2.51	133	0	Cb3-3,5,10	100	200	122.22	200	0	Cb3-3,5,15	200	226	9093.18	226	0
Cb3-6,4,5	100	198	3.56	198	0	Cb3-6,4,10	100	254	41.3	254	0	Cb3-6,4,15	200	266	972.32	266	0
Cb4-2,3,5	100	280	0.55	280	0	Cb4-2,3,10	100	251	25.72	251	0	Cb4-2,3,15	200	298	57.86	298	0
Cb4-3,5,5	100	135	3.71	135	0	Cb4-3,5,10	100	231	2651.79	231	0	Cb4-3,5,15	200	251	65.95	251	0
Cb4-6,4,5	100	188	7.16	188	0	Cb4-6,4,10	100	232	47.33	232	0	Cb4-6,4,15	200	264	348.13	264	0
Cb5-2,3,5	100	129	0.59	129	0	Cb5-2,3,10	100	184	6.35	184	0	Cb5-2,3,15	200	249	103.93	249	0
Cb5-3,5,5	100	146	1.57	146	0	Cb5-3,5,10	100	184	262.08	184	0	Cb5-3,5,15	200	243	120.09	243	0
Cb5-6,4,5	100	199	4.9	199	0	Cb5-6,4,10	100	224	39.64	224	0	Cb5-6,4,15	200	223	8273.80	223	0
Cc1-2,3,5	100	129	1.26	129	0	Cc1-2,3,10	100	189	181.9	189	0	Cc1-2,3,15	200	265	205.65	265	0
Cc1-3,5,5	100	135	4.2	135	0	Cc1-3,5,10	100	180	194.61	180	0	Cc1-3,5,15	200	233	4940.03	233	0
Cc1-6,4,5	100	150	7.94	150	0	Cc1-6,4,10	100	238	764.7	238	0	Cc1-6,4,15	200	280	2901.32	280	0
Cc2-2,3,5	100	122	0.78	122	0	Cc2-2,3,10	100	187	120.43	187	0	Cc2-2,3,15	200	284	165.56	284	0
Cc2-3,5,5	100	175	4.5	175	0	Cc2-3,5,10	100	231	173.82	231	0	Cc2-3,5,15	200	299	285.15	299	0
Cc2-6,4,5	100	122	8.8	122	0	Cc2-6,4,10	100	163	3248.49	163	0	Cc2-6,4,15	200	273	4266.17	273	0
Cc3-2,3,5	100	136	1.2	136	0	Cc3-2,3,10	100	184	32.61	184	0	Cc3-2,3,15	200	271	828.79	271	0
Cc3-3,5,5	100	142	3.89	142	0	Cc3-3,5,10	100	225	308.58	225	0	Cc3-3,5,15	200	223	651.09	223	0
Cc3-6,4,5	100	157	7.71	157	0	Cc3-6,4,10	100	193	700.1	193	0	Cc3-6,4,15	200	259	8204.47	259	0
Cc4-2,3,5	100	171	1.37	171	0	Cc4-2,3,10	100	225	62.13	225	0	Cc4-2,3,15	200	307	261.46	307	0
Cc4-3,5,5	100	154	4.56	154	0	Cc4-3,5,10	100	223	193.08	223	0	Cc4-3,5,15	200	250	1686.34	250	0
Cc4-6,4,5	100	138	8.23	138	0	Cc4-6,4,10	100	230	702	230	0	Cc4-6,4,15	200	268	1221.56	268	0
Cc5-2,3,5	100	123	1.09	123	0	Cc5-2,3,10	100	182	51.66	182	0	Cc5-2,3,15	200	236	51.99	236	0
Cc5-3,5,5	100	124	3.37	124	0	Cc5-3,5,10	100	168	1068.67	168	0	Cc5-3,5,15	200	247	348.89	247	0
Cc5-6,4,5	100	138	9.7	138	0	Cc5-6,4,10	100	179	5168.43	179	0	Cc5-6,4,15	200	208	767.43	208	0
Cd1-2,3,5	100	155	1.06	155	0	Cd1-2,3,10	100	197	3.03	197	0	Cd1-2,3,15	200	293	17.37	293	0
Cd1-3,5,5	100	170	1.18	170	0	Cd1-3,5,10	100	215	3.7	215	0	Cd1-3,5,15	200	304	260.80	304	0
Cd1-6,4,5	100	188	8.07	188	0	Cd1-6,4,10	100	248	125.57	248	0	Cd1-6,4,15	200	277	907.69	277	0
Cd2-2,3,5	100	140	0.56	140	0	Cd2-2,3,10	100	193	3.41	193	0	Cd2-2,3,15	200	241	10.60	241	0
Cd2-3,5,5	100	158	2.58	158	0	Cd2-3,5,10	100	231	19.82	231	0	Cd2-3,5,15	200	319	25.77	319	0
Cd2-6,4,5	100	142	8.59	142	0	Cd2-6,4,10	100	213	81.68	213	0	Cd2-6,4,15	200	274	597.98	274	0
Cd3-2,3,5	100	142	1.17	142	0	Cd3-2,3,10	100	186	4.79	186	0	Cd3-2,3,15	200	257	6.42	257	0
Cd3-3,5,5	100	147	2.78	147	0	Cd3-3,5,10	100	224	18.39	224	0	Cd3-3,5,15	200	222	248.68	222	0
Cd3-6,4,5	100	157	5.73	157	0	Cd3-6,4,10	100	196	29.91	196	0	Cd3-6,4,15	200	268	939.95	268	0
Cd4-2,3,5	100	202	0.82	202	0	Cd4-2,3,10	100	243	2.25	243	0	Cd4-2,3,15	200	324	57.97	324	0
Cd4-3,5,5	100	162	2.07	162	0	Cd4-3,5,10	100	241	15.96	241	0	Cd4-3,5,15	200	251	229.10	251	0
Cd4-6,4,5	100	147	7.59	147	0	Cd4-6,4,10	100	187	55.48	187	0	Cd4-6,4,15	200	263	537.24	263	0
Cd5-2,3,5	100	178	0.7	178	0	Cd5-2,3,10	100	207	1.59	207	0	Cd5-2,3,15	200	271	32.12	271	0
Cd5-3,5,5	100	178	2.67	178	0	Cd5-3,5,10	100	213	9.55	213	0	Cd5-3,5,15	200	247	92.34	247	0
Cd5-6,4,5	100	157	4.92	157	0	Cd5-6,4,10	100	184	39.76	184	0	Cd5-6,4,15	200	208	770.27	208	0
Averages			3.33		0	***Averages***			287.87		0	***Averages***			941.71		0

Table 11 – DDD results on instances with 5, 10 and 15 OD demands and disabled availability time

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	144	0.64	144	0	Ca1-2,3,10	100	214	4630.13	214	0	Ca1-2,3,15	200	267	63.59	267.00	0.00
Ca1-3,5,5	100	171	16.18	171	0	Ca1-3,5,10	100	201	3626.34	201	0	Ca1-3,5,15	200	276	222.42	276.00	0.00
Ca1-6,4,5	100	166	15.23	166	0	Ca1-6,4,10	100	225	9000	213.21	5.24	Ca1-6,4,15	200	272	934.90	272.00	0.00
Ca2-2,3,5	100	134	1.28	134	0	Ca2-2,3,10	100	208	9.22	208	0	Ca2-2,3,15	200	305	43.23	305.00	0.00
Ca2-3,5,5	100	181	3.87	181	0	Ca2-3,5,10	100	197	28.89	197	0	Ca2-3,5,15	200	328	286.45	328.00	0.00
Ca2-6,4,5	100	148	9.13	148	0	Ca2-6,4,10	100	204	9000.00	173.196	15.1	Ca2-6,4,15	200	270	758.88	270.00	0.00
Ca3-2,3,5	100	144	0.88	144	0	Ca3-2,3,10	100	160	22.62	160	0	Ca3-2,3,15	200	255	26.67	255.00	0.00
Ca3-3,5,5	100	171	2.42	171	0	Ca3-3,5,10	100	208	104.75	208	0	Ca3-3,5,15	200	224	692.66	224.00	0.00
Ca3-6,4,5	100	148	28.61	148	0	Ca3-6,4,10	100	171	7404.46	171	0	Ca3-6,4,15	200	260	963.44	260.00	0.00
Ca4-2,3,5	100	180	1.53	180	0	Ca4-2,3,10	100	202	67.71	202	0	Ca4-2,3,15	200	306	92.77	306.00	0.00
Ca4-3,5,5	100	147	2.21	147	0	Ca4-3,5,10	100	192	8053.48	192	0	Ca4-3,5,15	200	244	259.36	244.00	0.00
Ca4-6,4,5	100	150	10.61	150	0	Ca4-6,4,10	100	198	422.81	198	0	Ca4-6,4,15	200	218	8500.72	218.00	0.00
Ca5-2,3,5	100	173	0.60	173	0	Ca5-2,3,10	100	225	83.81	225	0	Ca5-2,3,15	200	263	138.70	263.00	0.00
Ca5-3,5,5	100	141	2.08	141	0	Ca5-3,5,10	100	194	455.02	194	0	Ca5-3,5,15	200	238	109.63	238.00	0.00
Ca5-6,4,5	100	159	8.52	159	0	Ca5-6,4,10	100	202	118.65	202	0	Ca5-6,4,15	200	201	8960.25	201.00	0.00
Cb1-2,3,5	100	136	0.53	136	0	Cb1-2,3,10	100	166	1418.14	166	0	Cb1-2,3,15	200	270	99.10	270.00	0.00
Cb1-3,5,5	100	149	14.15	149	0	Cb1-3,5,10	100	190	6046.12	190		Cb1-3,5,15	200	261	9000.00	237.80	8.89
Cb1-6,4,5	100	171	12.03	171	0	Cb1-6,4,10	100	203	9000	191.1448	5.84	Cb1-6,4,15	200	277	1211.64	277.00	0.00
Cb2-2,3,5	100	128	0.70	128	0	Cb2-2,3,10	100	170	111.28	170	0	Cb2-2,3,15	200	268	31.27	268.00	0.00
Cb2-3,5,5	100	135	1.92	135	0	Cb2-3,5,10	100	195	9000	188.292	3.44	Cb2-3,5,15	200	332	571.72	332.00	0.00
Cb2-6,4,5	100	142	8.86	142	0	Cb2-6,4,10	100	173	5035.45	173	0	Cb2-6,4,15	200	287	7900.56	287.00	0.00
Cb3-2,3,5	100	266	1.53	266	0	Cb3-2,3,10	100	286	558.57	286	0	Cb3-2,3,15	200	277	231.81	277.00	0.00
Cb3-3,5,5	100	129	5.32	129	0	Cb3-3,5,10	100	198	6481.42	198	0	Cb3-3,5,15	200	224	2767.83	224.00	0.00
Cb3-6,4,5	100	167	19.53	167	0	Cb3-6,4,10	100	237	9000	223.5621	5.67	Cb3-6,4,15	200	264	5908.05	264.00	0.00
Cb4-2,3,5	100	156	0.77	156	0	Cb4-2,3,10	100	220	5457.06	220	0	Cb4-2,3,15	200	297	75.03	297.00	0.00
Cb4-3,5,5	100	135	2.07	135	0	Cb4-3,5,10	100	215	810.03	215	0	Cb4-3,5,15	200	204	4248.78	204.00	0.00
Cb4-6,4,5	100	170	10.96	170	0	Cb4-6,4,10	100	262	9000	245.6774	6.23	Cb4-6,4,15	200	263	1304.74	263.00	0.00
Cb5-2,3,5	100	129	0.32	129	0	Cb5-2,3,10	100	185	7527.81	0	0	Cb5-2,3,15	200	229	38.39	229.00	0.00
Cb5-3,5,5	100	152	4.20	152	0	Cb5-3,5,10	100	176	180.39	0	0	Cb5-3,5,15	200	238	143.96	238.00	0.00
Cb5-6,4,5	100	158	10.2	158	0	Cb5-6,4,10	100	319	8244.13	0	0	Cb5-6,4,15	200	219	9000.00	206.82	5.56
Cc1-2,3,5	100	129	0.51	129	0	Cc1-2,3,10	100	189	33	189	0	Cc1-2,3,15	200	262	489.25	262.00	0.00
Cc1-3,5,5	100	135	2.15	135	0	Cc1-3,5,10	100	180	35.8	180	0	Cc1-3,5,15	200	233	348.71	233.00	0.00
Cc1-6,4,5	100	150	4.92	150	0	Cc1-6,4,10	100	238	99.58	238	0	Cc1-6,4,15	200	278	9000.00	265.05	4.66
Cc2-2,3,5	100	122	0.25	122	0	Cc2-2,3,10	100	187	23.43	187	0	Cc2-2,3,15	200	283	190.07	283.00	0.00
Cc2-3,5,5	100	174	2.89	174	0	Cc2-3,5,10	100	231	62.14	231	0	Cc2-3,5,15	200	259	307.38	259.00	0.00
Cc2-6,4,5	100	122	5.73	122	0	Cc2-6,4,10	100	163	869.12	163	0	Cc2-6,4,15	200	270	7602.87	270.00	0.00
Cc3-2,3,5	100	136	0.43	136	0	Cc3-2,3,10	100	184	15.29	184	0	Cc3-2,3,15	200	270	382.08	270.00	0.00
Cc3-3,5,5	100	142	2.12	142	0	Cc3-3,5,10	100	225	64.8	225	0	Cc3-3,5,15	200	222	1910.51	222.00	0.00
Cc3-6,4,5	100	157	5.87	157	0	Cc3-6,4,10	100	193	178.57	193	0	Cc3-6,4,15	200	258	9000.00	249.33	3.36
Cc4-2,3,5	100	171	0.71	171	0	Cc4-2,3,10	100	225	20.91	225	0	Cc4-2,3,15	200	307	398.04	307.00	0.00
Cc4-3,5,5	100	154	2.27	154	0	Cc4-3,5,10	100	223	72.43	223	0	Cc4-3,5,15	200	249	9000.00	246.09	1.17
Cc4-6,4,5	100	138	6.54	138	0	Cc4-6,4,10	100	230	230.58	230	0	Cc4-6,4,15	200	265	519.87	265.00	0.00
Cc5-2,3,5	100	123	0.46	123	0	Cc5-2,3,10	100	182	21.87	182	0	Cc5-2,3,15	200	218	20.72	218.00	0.00
Cc5-3,5,5	100	124	1.25	124	0	Cc5-3,5,10	100	168	56.6	168	0	Cc5-3,5,15	200	244	202.62	244.00	0.00
Cc5-6,4,5	100	138	7.90	138	0	Cc5-6,4,10	100	179	5088.73	179	0	Cc5-6,4,15	200	208	1732.72	208.00	0.00
Cd1-2,3,5	100	143	0.83	143	0	Cd1-2,3,10	100	179	19.88	179	0	Cd1-2,3,15	200	267	37.17	267.00	0.00
Cd1-3,5,5	100	165	0.57	165	0	Cd1-3,5,10	100	203	7.42	203	0	Cd1-3,5,15	200	278	264.90	278.00	0.00
Cd1-6,4,5	100	157	5.72	157	0	Cd1-6,4,10	100	211	176.35	211	0	Cd1-6,4,15	200	274	1929.13	274.00	0.00
Cd2-2,3,5	100	140	0.56	140	0	Cd2-2,3,10	100	176	16.62	176	0	Cd2-2,3,15	200	240	21.44	240.00	0.00
Cd2-3,5,5	100	150	2.11	150	0	Cd2-3,5,10	100	216	42	216	0	Cd2-3,5,15	200	275	150.03	275.00	0.00
Cd2-6,4,5	100	131	6.16	131	0	Cd2-6,4,10	100	190	7942.26	190	0	Cd2-6,4,15	200	273	647.53	273.00	0.00
Cd3-2,3,5	100	139	0.78	139	0	Cd3-2,3,10	100	171	14.85	171	0	Cd3-2,3,15	200	224	5.10	224.00	0.00
Cd3-3,5,5	100	144	2.00	144	0	Cd3-3,5,10	100	199	123.8	199	0	Cd3-3,5,15	200	221	394.04	221.00	0.00
Cd3-6,4,5	100	138	4.76	138	0	Cd3-6,4,10	100	175	206.05	175	0	Cd3-6,4,15	200	266	1460.48	266.00	0.00
Cd4-2,3,5	100	175	0.82	175	0	Cd4-2,3,10	100	223	19.12	223	0	Cd4-2,3,15	200	316	77.34	316.00	0.00
Cd4-3,5,5	100	160	2.05	160	0	Cd4-3,5,10	100	217	74.25	217	0	Cd4-3,5,15	200	247	174.70	247.00	0.00
Cd4-6,4,5	100	136	7.22	136	0	Cd4-6,4,10	100	186	215	186	0	Cd4-6,4,15	200	260	432.44	260.00	0.00
Cd5-2,3,5	100	144	0.78	144	0	Cd5-2,3,10	100	183	240.52	183	0	Cd5-2,3,15	200	270	18.32	270.00	0.00
Cd5-3,5,5	100	160	2.53	160	0	Cd5-3,5,10	100	191	93.78	191	0	Cd5-3,5,15	200	247	92.71	247.00	0.00
Cd5-6,4,5	100	157	9.31	157	0	Cd5-6,4,10	100	183	287.22	183	0	Cd5-6,4,15	200	208	853.88	208.00	0.00
Averages			4.78		0	***Averages***			2287.50		0.69	***Averages***			1870.84		0.394

Table 12 – DDD results on instances with 5, 10 and 15 OD demands with disabled availability time and synchronization

A.3. Degeneracy

Figure 1 illustrates the impact of the degeneracy procedure using a coarse granularity ($\Delta = 2$). The performance of the DDD is illustrated by contrasting the average optimality gap obtained by the DDD using the degeneracy procedure, for each instance type CA, CB, CC, and CD, to that of the DDD without the degeneracy procedure, identified as NCA, NCB, NCC, and NCD. The experimental results show that, using the degeneracy procedure leads to a general improvement of the optimality gap over the entire instance set. One also observes that, wider customer time windows, reduce the temporal preciseness of the reduced time-space network and provide broader refining options for short arcs, which, in turn drives degeneracy on the integer problem. Instances with broader and sparse time windows, most notably, instances of CC and CD types, tend to benefit from the degeneracy procedure, displaying optimality-gap improvements of 78% and 84%, respectively, compared to the results on same instances types without the degeneracy procedure. On the other hand, instances with tighter time windows, e.g., instances of CA and CB types, display improvements of 58% and 77%, respectively. The latter reflects the efficiency of the proposed degeneracy procedure in supporting the DDD to avoid being trapped on lower bounds values for several iterations and enabling tightening the general lower-bound values obtained by the DDD.

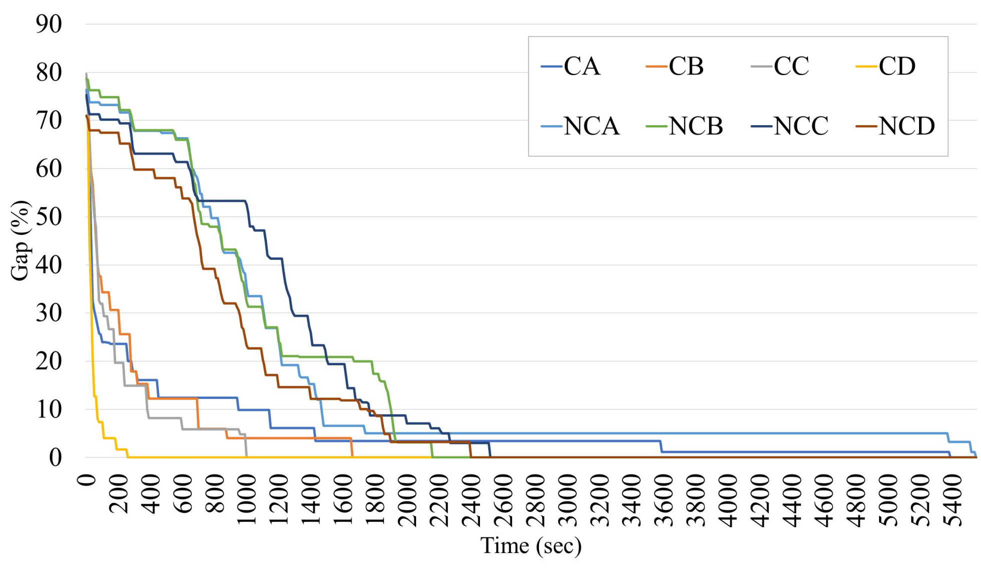

Figure 1 – Degeneracy: Average Optimality gap (%) versus run time and instance type.

A.4. Sensitivity analysis OD demands

We conducted a sensitivity analysis on the number of OD demands in the 2E-MALRPS to evaluate the performance of the proposed H-DDD solution framework. Our objective is to assess the impact of the number of OD demands on the problem and gain insights into system behavior. To conduct these experiments, we adapted a subset of instances originally defined with 5, 10, and 15 ODs to a single-commodity problem. This adaptation preserved the original number of destinations while assuming that all demands consist of the same commodity. Consequently, availability times are not differentiated by demand, implying that the single commodity is available on all platforms at the original availability times defined for each instance set.

Each instance set was solved to optimality using the H-DDD to compare the effects of the number of OD demands on solution quality and runtime. Table 13 summarizes the complete results (see Table 14) obtained using the H-DDD with the coarsest discretization granularity ($\Delta = 2$).

The results presented in Table 13 demonstrate a general decrease in solution quality as the average runtime of the H-DDD increases. Removing the multi-commodity aspect of the problem setting leads to an average cost reduction of 5% for the complete instance set. Despite the unchanged number of availability times, the larger number of departure options from platforms allows for solutions with earlier departure times. This enables the utilization of more cost-effective routes that are otherwise restricted in the original problem, where ODs have specific availability times. However, the runtime has an average increase of 16% due to the increased flexibility introduced by considering a single commodity. By allowing for more homogeneous departure times at platform facilities, more synchronization options are available. This, in turn, increases the number of possible synchronization alternatives and expands the feasible region defined by the integer problem.

The single-commodity case demonstrates a robust behavior of the H-DDD by obtaining the optimal solution for all instances considered. However, it is important to note that not considering the multi-commodity aspect may yield cheaper solutions that are not realistic or achievable when demand items are not substitutable.

$\lvert\mathcal{P}^{\mathbf{ph}}\rvert$	$\lvert\mathcal{Z}^{\mathbf{ph}}\rvert$	**OD**	**NI**	**FUB**	**OUB**	**OG(%)**	**CPUsec**
2	3	5	20	20	20	0.00	1.65
3	5	5	20	20	20	0.00	1.35
6	4	5	20	20	20	0.00	1.50
2	3	10	20	20	20	0.00	17.24
3	5	10	20	20	20	0.00	39.06
6	4	10	20	20	20	0.00	28.80
2	3	15	20	20	20	0.00	187.62
3	5	15	20	20	20	0.00	257.26
6	4	15	20	20	20	0.00	497.48

Table 13 – H-DDD performance on instances with a single OD demand

ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)	ID	Ψ	UB	CPU (sec)	LB	OG (%)
Ca1-2,3,5	100	271	2.17	271	0	Ca1-2,3,10	100	245	34.33	245	0	Ca1-2,3,15	200	273	186.86	273	0
Ca1-3,5,5	100	214	1.45	214	0	Ca1-3,5,10	100	197	14.48	197	0	Ca1-3,5,15	200	301	203.37	301	0
Ca1-6,4,5	100	268	1.73	268	0	Ca1-6,4,10	100	295	6.95	295	0	Ca1-6,4,15	200	267	1344.67	267	0
Ca2-2,3,5	100	152	2.30	152	0	Ca2-2,3,10	100	139	3.89	139	0	Ca2-2,3,15	200	306	47.03	306	0
Ca2-3,5,5	100	264	2.36	264	0	Ca2-3,5,10	100	327	6.57	327	0	Ca2-3,5,15	200	329	388.78	329	0
Ca2-6,4,5	100	148	1.21	148	0	Ca2-6,4,10	100	179	10.40	179	0	Ca2-6,4,15	200	272	865.41	272	0
Ca3-2,3,5	100	276	0.68	276	0	Ca3-2,3,10	100	343	16.23	343	0	Ca3-2,3,15	200	310	41.01	310	0
Ca3-3,5,5	100	218	0.48	218	0	Ca3-3,5,10	100	300	20.18	300	0	Ca3-3,5,15	200	300	3.52	300	0
Ca3-6,4,5	100	155	0.46	155	0	Ca3-6,4,10	100	221	24.09	221	0	Ca3-6,4,15	200	255	5452.21	255	0
Ca4-2,3,5	100	346	0.51	346	0	Ca4-2,3,10	100	422	14.15	422	0	Ca4-2,3,15	200	302	37.63	302	0
Ca4-3,5,5	100	155	1.38	155	0	Ca4-3,5,10	100	194	180.35	194	0	Ca4-3,5,15	200	284	2310.80	284	0
Ca4-6,4,5	100	150	1.62	150	0	Ca4-6,4,10	100	221	11.68	221	0	Ca4-6,4,15	200	250	690.85	250	0
Ca5-2,3,5	100	179	2.49	179	0	Ca5-2,3,10	100	274	16.09	274	0	Ca5-2,3,15	200	253	44.13	253	0
Ca5-3,5,5	100	176	1.01	176	0	Ca5-3,5,10	100	242	14.68	242	0	Ca5-3,5,15	200	256	33.07	256	0
Ca5-6,4,5	100	147	1.65	147	0	Ca5-6,4,10	100	178	7.47	178	0	Ca5-6,4,15	200	206	73.70	206	0
Cb1-2,3,5	100	152	0.63	152	0	Cb1-2,3,10	100	179	7.42	179	0	Cb1-2,3,15	200	296	208.41	296	0
Cb1-3,5,5	100	148	1.56	148	0	Cb1-3,5,10	100	210	16.46	210	0	Cb1-3,5,15	200	284	73.38	284	0
Cb1-6,4,5	100	286	2.21	286	0	Cb1-6,4,10	100	255	6.35	255	0	Cb1-6,4,15	200	290	40.79	290	0
Cb2-2,3,5	100	125	2.26	125	0	Cb2-2,3,10	100	181	6.38	181	0	Cb2-2,3,15	200	325	121.71	325	0
Cb2-3,5,5	100	148	1.50	148	0	Cb2-3,5,10	100	246	22.77	246	0	Cb2-3,5,15	200	291	28.48	291	0
Cb2-6,4,5	100	133	1.23	133	0	Cb2-6,4,10	100	173	150.65	173	0	Cb2-6,4,15	200	327	93.40	327	0
Cb3-2,3,5	100	329	2.35	329	0	Cb3-2,3,10	100	315	5.43	315	0	Cb3-2,3,15	200	351	1597.96	351	0
Cb3-3,5,5	100	147	1.23	147	0	Cb3-3,5,10	100	179	156.89	179	0	Cb3-3,5,15	200	265	19.51	265	0
Cb3-6,4,5	100	196	0.94	196	0	Cb3-6,4,10	100	269	92.69	269	0	Cb3-6,4,15	200	286	17.59	286	0
Cb4-2,3,5	100	275	0.80	275	0	Cb4-2,3,10	100	249	87.33	249	0	Cb4-2,3,15	200	249	274.38	249	0
Cb4-3,5,5	100	128	0.96	128	0	Cb4-3,5,10	100	239	69.13	239	0	Cb4-3,5,15	200	330	237.18	330	0
Cb4-6,4,5	100	182	1.49	182	0	Cb4-6,4,10	100	288	7.18	288	0	Cb4-6,4,15	200	242	18.49	242	0
Cb5-2,3,5	100	114	1.03	114	0	Cb5-2,3,10	100	194	28.25	194	0	Cb5-2,3,15	200	235	7.53	235	0
Cb5-3,5,5	100	172	2.39	172	0	Cb5-3,5,10	100	223	85.51	223	0	Cb5-3,5,15	200	218	613.70	218	0
Cb5-6,4,5	100	181	1.15	181	0	Cb5-6,4,10	100	317	39.61	317	0	Cb5-6,4,15	200	263	43.94	263	0
Cc1-2,3,5	100	119	1.45	119	0	Cc1-2,3,10	100	173	49.19	173	0	Cc1-2,3,15	200	208	389.06	208	0
Cc1-3,5,5	100	120	1.41	120	0	Cc1-3,5,10	100	157	21.06	157	0	Cc1-3,5,15	200	257	108.09	257	0
Cc1-6,4,5	100	142	2.35	142	0	Cc1-6,4,10	100	216	28.42	216	0	Cc1-6,4,15	200	256	65.96	256	0
Cc2-2,3,5	100	108	2.11	108	0	Cc2-2,3,10	100	163	14.38	163	0	Cc2-2,3,15	200	294	9.23	294	0
Cc2-3,5,5	100	162	1.16	162	0	Cc2-3,5,10	100	218	25.27	218	0	Cc2-3,5,15	200	249	321.93	249	0
Cc2-6,4,5	100	121	2.09	121	0	Cc2-6,4,10	100	162	6.43	162	0	Cc2-6,4,15	200	265	220.34	265	0
Cc3-2,3,5	100	117	0.94	117	0	Cc3-2,3,10	100	168	2.21	168	0	Cc3-2,3,15	200	211	487.11	211	0
Cc3-3,5,5	100	128	1.94	128	0	Cc3-3,5,10	100	224	65.13	224	0	Cc3-3,5,15	200	247	568.54	247	0
Cc3-6,4,5	100	137	1.12	137	0	Cc3-6,4,10	100	179	15.44	179	0	Cc3-6,4,15	200	303	28.15	303	0
Cc4-2,3,5	100	171	1.99	171	0	Cc4-2,3,10	100	211	23.50	211	0	Cc4-2,3,15	200	247	108.98	247	0
Cc4-3,5,5	100	154	0.86	154	0	Cc4-3,5,10	100	209	14.14	209	0	Cc4-3,5,15	200	259	10.44	259	0
Cc4-6,4,5	100	119	1.89	119	0	Cc4-6,4,10	100	221	20.91	221	0	Cc4-6,4,15	200	223	60.17	223	0
Cc5-2,3,5	100	107	1.18	107	0	Cc5-2,3,10	100	167	8.13	167	0	Cc5-2,3,15	200	221	4.08	221	0
Cc5-3,5,5	100	113	0.85	113	0	Cc5-3,5,10	100	167	12.75	167	0	Cc5-3,5,15	200	197	0.71	197	0
Cc5-6,4,5	100	118	1.12	118	0	Cc5-6,4,10	100	162	33.13	162	0	Cc5-6,4,15	200	268	56.68	268	0
Cd1-2,3,5	100	144	2.35	144	0	Cd1-2,3,10	100	196	3.34	196	0	Cd1-2,3,15	200	279	21.83	279	0
Cd1-3,5,5	100	170	1.46	170	0	Cd1-3,5,10	100	195	7.26	195	0	Cd1-3,5,15	200	268	20.72	268	0
Cd1-6,4,5	100	183	0.46	183	0	Cd1-6,4,10	100	241	27.82	241	0	Cd1-6,4,15	200	222	48.20	222	0
Cd2-2,3,5	100	128	2.08	128	0	Cd2-2,3,10	100	190	1.37	190	0	Cd2-2,3,15	200	293	64.14	293	0
Cd2-3,5,5	100	145	0.85	145	0	Cd2-3,5,10	100	235	11.52	235	0	Cd2-3,5,15	200	256	27.75	256	0
Cd2-6,4,5	100	123	1.94	123	0	Cd2-6,4,10	100	202	24.08	202	0	Cd2-6,4,15	200	235	49.67	235	0
Cd3-2,3,5	100	122	1.92	122	0	Cd3-2,3,10	100	176	4.87	176	0	Cd3-2,3,15	200	197	34.29	197	0
Cd3-3,5,5	100	131	1.80	131	0	Cd3-3,5,10	100	218	13.95	218	0	Cd3-3,5,15	200	245	17.78	245	0
Cd3-6,4,5	100	143	1.16	143	0	Cd3-6,4,10	100	196	11.49	196	0	Cd3-6,4,15	200	318	56.08	318	0
Cd4-2,3,5	100	195	1.86	195	0	Cd4-2,3,10	100	256	4.18	256	0	Cd4-2,3,15	200	272	35.89	272	0
Cd4-3,5,5	100	144	1.52	144	0	Cd4-3,5,10	100	230	8.86	230	0	Cd4-3,5,15	200	245	56.72	245	0
Cd4-6,4,5	100	129	2.18	129	0	Cd4-6,4,10	100	181	26.43	181	0	Cd4-6,4,15	200	241	56.37	241	0
Cd5-2,3,5	100	162	2.02	162	0	Cd5-2,3,10	100	202	14.18	202	0	Cd5-2,3,15	200	221	31.18	221	0
Cd5-3,5,5	100	174	0.91	174	0	Cd5-3,5,10	100	212	14.30	212	0	Cd5-3,5,15	200	186	100.73	186	0
Cd5-6,4,5	100	142	2.13	142	0	Cd5-6,4,10	100	163	24.84	163	0	Cd5-6,4,15	200	248	666.90	248	0
Averages			1.51		0.00	***Averages***			28.37		0.00	***Averages***			314.12		0.00

Table 14 – DDD results on instances with 5, 10 and 15 OD demands expressed as a single OD demand

Chapter B

SUPPLEMENTARY MATERIAL SECOND ARTICLE

B.1. Decomposition strategy for the two-stage stochastic formulation

This section presents the complete steps used to perform the decomposition approach that is applied to the stochastic 2E-MLRPSCD formulation introduced in Section 4.2. This decomposition approach utilizes an augmented Lagrangean strategy.

The decomposition strategy applied on the scenario-based formulation, along the scenarios included in S, requires the first-stage decisions to be reformulated. Specifically, these decisions need to be defined as scenario-dependent. In doing so, constraints (4.7), (4.18) and (4.19) are then reexpressed according to the scenario-specific location and allocation first-stage decisions. Therefore, one starts with the following alternative, but equivalent, formulation:

$$\min \sum_{s\in S} \rho_s \left(\sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in Z} F_i y^s_i + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Delta_{ijk} f^s_{ijk} \right) \tag{B.1.1}$$

Subject to

$$(4.2) - (4.6)$$

$$(4.8) - (4.17)$$

$$\sum_{h\in H^2} \sum_{j\in C} x^s_{ijh} \leq |H^2| y^s_i \quad \forall i \in Z, s \in S \tag{B.1.2}$$

$$\sum_{h\in H^1} u^s_{ijkh} = f^s_{ijk} \quad \forall i \in P, j \in Z, k \in K, s \in S \tag{B.1.3}$$

$$\sum_{h\in H^2} v^s_{zD(k)h} = \sum_{p\in P} f^s_{pzk} \quad \forall z\in Z, k\in K, s\in S \tag{B.1.4}$$

$$y^s_i = \bar{y}_i \quad \forall i\in Z, s\in S \tag{B.1.5}$$

$$f^s_{ijk} = \bar{f}_{ijk} \quad \forall i\in P, j\in Z, k\in K, s\in S \tag{B.1.6}$$

$$y^s_i \in \{0{,}1\} \quad \forall i\in Z, s\in S \tag{B.1.7}$$

$$f^s_{ijk} \in \{0{,}1\} \quad \forall i\in P, j\in Z, k\in K, s\in S \tag{B.1.8}$$

$$\bar{y}_i \in \{0{,}1\} \quad \forall i\in Z \tag{B.1.9}$$

$$\bar{f}_{ijk} \in \{0{,}1\} \quad \forall i\in P, j\in Z, k\in K \tag{B.1.10}$$

This reformulation now explicitly includes the set of non-anticipativity constraints which prevent the first-stage decision variables to be set to different scenario-specific values (i.e., the first-stage decisions must be implementable). Constraints (B.1.2)-(B.1.4) link the facility allocation variables with the vehicle allocation variables. Constraints (B.1.5) and (B.1.6) ensure that the first-stage solutions will be the same for all the scenarios (also known as the non-anticipativity constraints), where variables $\bar{y}_i$ and $\bar{f}_{ijk}$ serve as the reference first-stage variables. The latter ensure that a single set of facility location and allocation decisions are made for all the scenarios (thus preventing tailored scenario-specific decisions to be made). Then, following the decomposition scheme, originally proposed by Rockafellar and Wets (1991), constraints (B.1.5) and (B.1.6) are relaxed using an augmented Lagrangean method, which results in the following objective function:

$$\begin{aligned} \min \sum_{s\in S} \rho_s \Big(\sum_{h\in H^1} \sum_{(i,j)\in A^1} \zeta_{ij} x^s_{ijh} + \sum_{h\in H^2} \sum_{(i,j)\in A^2} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in Z} F_i y^s_i \\ + \sum_{i\in Z} \lambda^s_i (y^s_i - \bar{y}_i) + \frac{1}{2} \sum_{i\in Z} \gamma (y^s_i - \bar{y}_i)^2 + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Delta_{ijk} f^s_{ijk} \\ + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \mu^s_{ijk} (f^s_{ijk} - \bar{f}_{ijk}) + \frac{1}{2} \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \gamma (f^s_{ijk} - \bar{f}_{ijk})^2 \Big) \end{aligned} \tag{B.1.11}$$

The objective function now involves the lagrangean multipliers λ^s_i and μ^s_{ijk} for the relaxed constraints (B.1.5) and (B.1.6), respectively, and a penalty term γ. Given the binary requirements of the location and allocation variables, the objective function can be reduced as follows:

$$
\begin{aligned}
\min \sum_{s\in S} \rho_s \Big(& \sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in Z} \Big(F_i + \lambda^s_i + \frac{1}{2}\gamma + \gamma \bar{y}_i \Big) y^s_i \\
& + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Big(\Delta_{ijk} + \mu^s_{ijk} + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \Big) f^s_{ijk} + \frac{1}{2} \sum_{i\in Z} \gamma \bar{y}_i - \sum_{i\in Z} \lambda^s_i \bar{y}_i \\
& + \frac{1}{2} \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \gamma \bar{f}_{ijk} + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \mu^s_{ijk} \bar{f}_{ijk} \Big)
\end{aligned} \tag{B.1.12}
$$

Given the objective (B.1.12) and the constraint set: (4.2)-(4.6), (4.8)-(4.17), (B.1.2)-(B.1.4) and (B.1.7)-(B.1.10), if the reference point (or solution) $\bar{y}_i$ and $\bar{f}_{ijk}$ is fixed, then the relaxed formulation is decomposed by scenario. Specifically, for each $s \in S$, a deterministic 2E-MLRPSCD subproblem with modified fixed costs is obtained:

$$
\begin{aligned}
\min \sum_{s\in S} \rho_s \Big(& \sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in Z} \Big(F_i + \lambda^s_i + \frac{1}{2}\gamma + \gamma \bar{y}_i \Big) y^s_i \\
& + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Big(\Delta_{ijk} + \mu^s_{ijk} + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \Big) f^s_{ijk} \Big)
\end{aligned} \tag{B.1.13}
$$

Subject to

$$
(4.2) - (4.6), (4.8) - (4.17), (B.1.2) - (B.1.4) \text{ and } (B.1.7) - (B.1.10)\ .
$$

As previously stated, the proposed PH algorithm then proceeds by solving the previous scenario subproblems separately, thus obtaining scenario-specific first-stage solutions. These scenario-specific solutions are then used to compute the reference point. Using the reference point, the objective functions (B.1.13), for all $s \in S$, are modified to incentivise decision agreement among the subproblems (i.e., consensus). This general process in then repeated until consensus first-stage solution can be found.

B.2. Complete Result Tables

Instance	$\lvert S\rvert = \mathbf{10}$		$\lvert S\rvert = \mathbf{20}$		$\lvert S\rvert = \mathbf{30}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Ca1-2,3,15	1.52	121.57	0.85	35.95	0.70	24.46
Ca1-3,5,15	2.39	297.35	1.40	101.77	0.94	42.26
Ca1-6,4,15	2.38	216.61	1.02	38.49	0.87	29.87
Ca2-2,3,15	1.04	68.28	0.61	27.66	0.78	30.22
Ca2-3,5,15	1.12	41.96	0.75	20.36	0.61	14.39
Ca2-6,4,15	2.97	466.96	1.11	68.72	0.81	37.49
Ca3-2,3,15	4.37	1447.25	2.05	303.71	1.60	144.58
Ca3-3,5,15	5.37	1406.21	5.93	3351.09	3.55	993.23
Ca3-6,4,15	1.61	200.43	0.67	41.05	0.48	20.11
Ca4-2,3,15	0.62	20.00	2.43	9.75	1.50	17.16
Ca4-3,5,15	0.88	37.02	1.42	120.58	2.22	252.29
Ca4-6,4,15	3.22	547.49	1.59	142.76	0.94	44.33
Ca5-2,3,15	0.62	23.44	0.84	44.53	0.60	23.96
Ca5-3,5,15	4.35	1008.95	3.47	735.45	2.06	243.41
Ca5-6,4,15	2.03	236.19	1.22	89.12	0.73	26.19
Cb1-2,3,15	0.40	15.57	0.20	4.39	0.13	2.05
Cb1-3,5,15	0.09	0.37	0.06	0.15	0.08	0.25
Cb1-6,4,15	0.14	1.16	0.10	0.53	0.03	0.05
Cb2-2,3,15	0.41	10.15	0.22	3.21	0.17	1.83
Cb2-3,5,15	0.00	0.00	0.00	0.00	0.00	0.00
Cb2-6,4,15	0.64	12.51	0.28	2.92	0.16	0.81
Cb3-2,3,15	0.08	0.58	0.04	0.10	0.03	0.04
Cb3-3,5,15	0.18	1.12	0.12	0.47	0.05	0.09
Cb3-6,4,15	0.58	17.82	0.36	7.65	0.11	0.46
Cb4-2,3,15	0.78	28.85	0.48	12.22	0.34	6.38
Cb4-3,5,15	0.00	0.00	0.02	0.01	0.01	0.00
Cb4-6,4,15	0.11	0.54	0.02	0.03	0.01	0.01
Cb5-2,3,15	1.17	95.45	0.64	40.17	0.43	16.60
Cb5-3,5,15	0.00	0.00	0.18	1.73	0.08	0.33
Cb5-6,4,15	0.00	0.00	0.19	2.65	0.08	0.49
Cc1-2,3,15	4.45	3469.30	2.66	1206.38	1.65	467.20
Cc1-3,5,15	1.43	244.96	1.41	317.74	1.01	180.20
Cc1-6,4,15	4.38	4084.98	2.15	1037.43	1.71	657.59

Instance	$\lvert S \rvert = \mathbf{10}$		$\lvert S \rvert = \mathbf{20}$		$\lvert S \rvert = \mathbf{30}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Cc2-2,3,15	2.12	776.30	1.33	372.10	1.95	755.08
Cc2-3,5,15	4.22	2998.10	4.83	2131.22	2.09	869.33
Cc2-6,4,15	4.92	6358.39	1.94	833.12	1.28	362.06
Cc3-2,3,15	4.84	5836.54	2.64	1756.38	1.36	472.44
Cc3-3,5,15	7.12	5331.21	3.62	1524.46	1.82	394.82
Cc3-6,4,15	2.54	944.92	2.35	255.76	1.75	85.37
Cc4-2,3,15	3.48	2034.46	1.50	380.47	2.00	604.30
Cc4-3,5,15	8.58	15222.89	3.14	1245.38	1.90	393.44
Cc4-6,4,15	3.73	2509.57	1.41	327.63	1.40	573.11
Cc5-2,3,15	3.24	2604.01	2.75	1834.62	2.48	1944.30
Cc5-3,5,15	1.89	752.12	0.84	166.96	0.83	97.15
Cc5-6,4,15	4.24	2797.43	3.15	1473.54	3.52	1547.38
Cd1-2,3,15	0.90	57.21	0.00	8.58	0.38	12.02
Cd1-3,5,15	0.41	7.93	0.08	0.27	0.12	0.83
Cd1-6,4,15	2.22	3.04	1.58	13.41	0.75	0.42
Cd2-2,3,15	0.00	0.00	0.03	0.04	0.09	0.31
Cd2-3,5,15	0.00	0.00	0.00	0.00	0.00	0.00
Cd2-6,4,15	0.46	13.54	0.15	0.91	0.13	0.64
Cd3-2,3,15	0.00	0.00	0.00	0.00	0.00	0.00
Cd3-3,5,15	0.13	0.72	0.21	2.23	0.04	0.05
Cd3-6,4,15	0.31	4.59	0.36	5.77	0.01	0.00
Cd4-2,3,15	0.94	47.77	0.01	0.01	0.35	5.79
Cd4-3,5,15	0.01	0.00	0.06	0.24	0.01	0.00
Cd4-6,4,15	0.54	13.37	0.00	0.00	0.15	1.16
Cd5-2,3,15	0.50	22.22	0.32	7.09	0.19	3.02
Cd5-3,5,15	0.01	0.00	0.30	7.66	0.03	0.06
Cd5-6,4,15	0.82	53.81	0.46	20.86	0.11	0.78
Max.	***8.58***	***15222.89***	***5.93***	***3351.09***	***3.55***	***5457.38***
Averages	***1.79***	***1041.89***	***1.13***	***335.62***	***0.82***	***265.20***

Table 1 – Stability tests. Relative difference and variance for each instance and scenario set with 10, 20 and 30 scenarios.

Instance	$\|S\| = \mathbf{40}$		$\|S\| = \mathbf{50}$		$\|S\| = \mathbf{100}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Ca1-2,3,15	0.69	37.87	0.98	77.01	0.59	23.07
Ca1-3,5,15	0.22	2.13	0.12	0.70	0.08	0.35
Ca1-6,4,15	0.95	41.30	0.97	34.73	0.66	19.61
Ca2-2,3,15	0.23	5.28	0.09	0.71	0.07	0.33
Ca2-3,5,15	0.25	3.19	0.26	2.62	0.15	1.47
Ca2-6,4,15	1.04	58.61	0.49	18.64	0.32	5.33
Ca3-2,3,15	0.76	35.98	1.08	82.10	0.80	59.33
Ca3-3,5,15	1.75	127.10	1.07	45.46	0.88	33.46
Ca3-6,4,15	0.10	0.73	0.16	2.07	0.06	0.22
Ca4-2,3,15	1.1	50.22	0.82	10.88	0.4	9.54
Ca4-3,5,15	1.86	120.55	1.3	95.5	0.78	80.12
Ca4-6,4,15	0.7	78.45	0.35	35.44	0.11	22.1
Ca5-2,3,15	0.47	12.99	0.22	8.31	0.09	5.5
Ca5-3,5,15	1.95	122.28	1.05	99.75	0.83	70.47
Ca5-6,4,15	0.66	11.96	0.44	20.56	0.21	5.1
Cb1-2,3,15	0.27	6.17	0.22	4.07	0.12	1.23
Cb1-3,5,15	0.08	0.38	0.07	0.24	0.03	0.06
Cb1-6,4,15	0.05	0.18	0.03	0.06	0.02	0.02
Cb2-2,3,15	0.05	0.10	0.04	0.06	0.03	0.03
Cb2-3,5,15	0.00	0.00	0.00	0.00	0.00	0.00
Cb2-6,4,15	0.10	0.44	0.08	0.33	0.04	0.07
Cb3-2,3,15	0.00	0.00	0.00	0.00	0.00	0.00
Cb3-3,5,15	0.02	0.02	0.02	0.02	0.02	0.01
Cb3-6,4,15	0.05	0.12	0.05	0.09	0.03	0.05
Cb4-2,3,15	0.23	2.82	0.19	1.82	0.12	0.65
Cb4-3,5,15	0.05	0.12	0.04	0.05	0.01	0.00
Cb4-6,4,15	0.12	0.72	0.09	0.49	0.05	0.13
Cb5-2,3,15	0.15	1.87	0.12	1.27	0.06	0.32
Cb5-3,5,15	0.02	0.02	0.03	0.05	0.02	0.02
Cb5-6,4,15	0.07	0.35	0.05	0.17	0.02	0.04
Cc1-2,3,15	0.85	137.51	0.92	163.63	0.56	59.25
Cc1-3,5,15	1.88	575.19	1.28	252.90	0.79	96.69
Cc1-6,4,15	1.11	258.47	0.94	192.33	0.5	55.07
Cc2-2,3,15	1.31	345.14	1.07	228.73	0.58	65.14
Cc2-3,5,15	1.58	510.21	1.17	281.18	0.69	98.23

Instance	$\|S\| = \mathbf{40}$		$\|S\| = \mathbf{50}$		$\|S\| = \mathbf{100}$	
	RD	**VAR**	**RD**	**VAR**	**RD**	**VAR**
Cc2-6,4,15	2.25	1142.38	2.01	933.34	1.44	525.93
Cc3-2,3,15	1.39	346.35	1.15	236.46	0.74	94.22
Cc3-3,5,15	1.77	428.80	1.42	269.09	0.9	121.36
Cc3-6,4,15	1.37	500.51	1.07	319.98	0.6	102.52
Cc4-2,3,15	1.54	579.72	1.12	355.86	0.76	140.25
Cc4-3,5,15	0.83	81.46	0.90	172.08	0.23	11.19
Cc4-6,4,15	0.87	173.46	0.35	30.19	0.20	10.51
Cc5-2,3,15	1.18	417.88	0.49	90.28	0.47	88.57
Cc5-3,5,15	0.88	155.78	0.96	272.79	0.58	76.07
Cc5-6,4,15	0.63	95.11	0.83	126.08	0.61	64.69
Cd1-2,3,15	0.27	6.17	0.22	4.07	0.12	1.23
Cd1-3,5,15	0.08	0.38	0.07	0.24	0.03	0.06
Cd1-6,4,15	0.05	0.18	0.03	0.06	0.02	0.02
Cd2-2,3,15	0.05	0.10	0.04	0.06	0.03	0.03
Cd2-3,5,15	0.00	0.00	0.00	0.00	0.00	0.00
Cd2-6,4,15	0.10	0.44	0.08	0.33	0.04	0.07
Cd3-2,3,15	0.00	0.00	0.00	0.00	0.00	0.00
Cd3-3,5,15	0.02	0.02	0.02	0.02	0.02	0.01
Cd3-6,4,15	0.05	0.12	0.05	0.09	0.03	0.05
Cd4-2,3,15	0.23	2.82	0.19	1.82	0.12	0.65
Cd4-3,5,15	0.05	0.12	0.04	0.05	0.01	0.00
Cd4-6,4,15	0.12	0.72	0.09	0.49	0.05	0.13
Cd5-2,3,15	0.15	1.87	0.12	1.27	0.06	0.32
Cd5-3,5,15	0.02	0.02	0.03	0.05	0.02	0.02
Cd5-6,4,15	0.07	0.35	0.05	0.17	0.02	0.04
Max.	***2.25***	***1142.38***	***2.01***	***933.34***	***1.44***	***525.93***
Averages	***0.58***	***108.05***	***0.45***	***74.61***	***0.28***	***32.56***

Table 2 – Stability tests. Relative difference and variance for each instance and scenario set with 40, 50 and 100 scenarios.

Instance	|S|	CPLEX			PH																				LB
					Classic (AC)					Probability (AC)					Social (AC)					Cluster (AC)					
		UB	OG (%)	TIME	UB	ITER.	GAP	TIME (s)	DIFF UB	UB	ITER.	GAP	TIME (s)	DIFF UB	GAP (%)	ITER.	GAP	TIME (s)	DIFF UB	UB	ITER.	GAP	TIME (s)	DIFF UB	
Ca1-2,3,15	30	2984.10	19.10	7200.00	2445.2	21	1.28	7200.00	-22.04	2455.73	47	1.70	2678.67	-21.52	2424.70	10	0.44	2674.34	-23.07	2424.70	10	0.44	1826.92	-23.07	2414.00
Ca1-3,5,15	30	2863.63	28.19	7200.00	3098.63	10	33.64	7200.00	7.58	2062.03	40	0.28	3443.61	-38.87	2076.70	19	0.98	2893.36	-37.89	2076.70	19	0.98	1238.20	-37.89	2056.31
Ca1-6,4,15	30	2453.70	18.57	7200.00	3246.67	11	38.46	7200.00	24.42	2250.23	50	11.21	1984.56	-9.04	2206.40	20	9.45	2467.88	-11.21	2034.00	25	1.77	2560.43	-20.63	1997.93
Ca2-2,3,15	30	3175.90	35.07	7200.00	2288.27	19	9.89	7200.00	-38.79	2287.67	50	9.86	2486.71	-38.83	2184.50	15	5.61	2151.20	-45.38	2110.00	15	2.27	2147.58	-50.52	2062.04
Ca2-3,5,15	30	2927.17	37.28	7200.00	3442.67	10	46.67	7200.00	14.97	2734.90	13	32.87	1311.36	-7.03	1870.10	23	1.83	3552.67	-56.52	1870.10	19	1.83	1904.20	-56.52	1835.92
Ca2-6,4,15	30	2798.63	15.81	7200.00	3678.33	10	35.94	7200.00	23.92	2456.70	30	4.09	3044.98	-13.92	2388.40	20	1.35	1841.73	-17.18	2388.40	18	1.35	1412.05	-17.18	2356.17
Ca3-2,3,15	30	4290.00	43.59	7200.00	3502.47	7	30.91	7200.00	-22.48	2506.60	44	3.46	6328.41	-71.15	2617.20	10	7.54	2298.98	-63.92	2485.50	17	2.64	1243.50	-72.60	2419.99
Ca3-3,5,15	30	3513.17	53.96	7200.00	5039	10	67.90	7200.00	30.28	1629.00	11	0.70	1353.02	-115.66	1886.10	10	14.23	1908.30	-86.27	1629.10	10	0.70	1784.63	-115.65	1617.63
Ca3-6,4,15	30	3056.97	18.69	7200.00	2679.8	7	7.25	7200.00	-14.07	2679.80	20	7.25	2525.70	-14.07	2546.53	22	2.39	2232.41	-20.04	2546.53	12	2.39	1338.94	-20.04	2485.62
Ca4-2,3,15	30	3069.33	22.73	7200.00	2456.93	21	3.47	7200.00	-24.93	2456.93	20	3.47	3630.18	-24.93	2456.93	15	3.47	1238.29	-24.93	2377.38	15	0.24	1410.76	-29.11	2371.67
Ca4-3,5,15	30	2579.53	22.58	7200.00	3303.4	15	39.55	7200.00	21.91	2133.80	18	6.41	2137.43	-20.89	2179.53	10	8.37	3315.26	-18.35	2133.22	15	6.38	1798.50	-20.92	1997.04
Ca4-6,4,15	30	2295.33	16.46	7200.00	2905.87	11	34.01	7200.00	21.01	2896.67	20	33.80	1616.07	20.76	2124.33	26	9.74	2149.35	-8.05	2108.10	20	9.04	2462.50	-8.88	1917.52
Ca5-2,3,15	30	2689.60	20.44	7200.00	3221	10	33.57	7200.00	16.50	2213.17	13	3.32	1234.94	-21.53	2212.03	17	3.27	1301.36	-21.59	2212.03	15	3.27	1845.60	-21.59	2139.72
Ca5-3,5,15	30	4403.87	50.30	7200.00	4967.33	12	55.93	7200.00	11.34	3250.80	11	32.67	794.15	-35.47	2209.03	10	0.91	920.19	-99.36	2209.03	10	0.91	1020.78	-99.36	2188.88
Ca5-6,4,15	30	2754.57	27.74	7200.00	2164.9	9	8.05	7200.00	-27.24	2011.00	10	1.02	1333.33	-36.97	2011.00	18	1.02	1733.26	-36.97	2011.00	9	1.02	956.13	-36.97	1990.54
Cb1-2,3,15	30	2490.57	11.55	7200.00	2237.97	8	1.56	6492.91	-11.29	2214.00	11	0.50	1225.45	-12.49	2214.00	10	0.50	1836.89	-12.49	2214.00	10	0.50	1125.32	-12.49	2203.00
Cb1-3,5,15	30	2270.67	15.85	7200.00	1914.40	8	0.19	7200.00	-18.61	1914.40	11	0.19	1482.36	-18.61	1914.40	10	0.19	4457.70	-18.61	1914.40	10	0.19	3731.31	-18.61	1910.70
Cb1-6,4,15	30	2444.10	14.04	7200.00	2107.73	10	0.32	1327.03	-15.96	2107.73	11	0.32	1487.02	-15.96	2107.73	10	0.32	1992.89	-15.96	2107.73	9	0.32	1280.82	-15.96	2101.00
Cb2-2,3,15	30	3310.70	44.71	7200.00	1834.70	15	0.23	3134.15	-80.45	1834.70	11	0.23	1892.26	-80.45	1834.70	10	0.23	2281.38	-80.45	1834.70	10	0.23	2869.50	-80.45	1830.50
Cb2-3,5,15	30	2784.40	32.82	7200.00	1871.00	16	0.02	6747.27	-48.82	1871.00	11	0.02	1052.97	-48.82	1871.00	10	0.02	2411.31	-48.82	1871.00	10	0.02	2566.47	-48.82	1870.54
Cb2-6,4,15	30	2388.83	19.85	7200.00	1922.37	10	0.40	914.35	-24.26	1922.37	11	0.40	993.44	-24.26	1922.37	10	0.40	1338.19	-24.26	1922.37	10	0.40	1366.29	-24.26	1914.60
Cb3-2,3,15	30	3083.47	29.96	7200.00	2214.53	10	2.48	7200.00	-39.24	2176.20	11	0.76	976.29	-41.69	2176.20	10	0.76	2738.49	-41.69	2176.20	14	0.76	2500.10	-41.69	2159.60
Cb3-3,5,15	30	2159.47	21.29	7200.00	1741.23	8	2.38	6283.76	-24.02	1741.23	11	2.38	1866.33	-24.02	1741.23	10	2.38	1401.96	-24.02	1717.23	10	1.02	1613.95	-25.75	1699.76
Cb3-6,4,15	30	2294.77	15.46	7200.00	1958.00	7	0.92	3435.08	-17.20	1958.00	11	0.92	1650.32	-17.20	1958.00	10	0.92	2407.27	-17.20	1958.00	9	0.92	2084.66	-17.20	1939.90
Cb4-2,3,15	30	3545.03	35.82	7200.00	2297.60	12	0.97	2372.62	-54.29	2297.60	11	0.97	1711.90	-54.29	2297.60	10	0.97	1766.00	-54.29	2297.60	10	0.97	1437.97	-54.29	2275.22
Cb4-3,5,15	30	3029.67	39.58	7200.00	1848.53	12	0.97	7200.00	-63.90	1848.53	11	0.97	1218.64	-63.90	1848.53	10	0.97	2258.24	-63.90	1848.53	13	0.97	1864.30	-63.90	1830.68
Cb4-6,4,15	30	2429.53	16.64	7200.00	2043.77	15	0.91	1250.18	-18.87	2043.77	11	0.91	1601.55	-18.87	2043.77	10	0.91	1238.52	-18.87	2043.77	13	0.91	1411.92	-18.87	2025.23
Cb5-2,3,15	30	3009.00	25.98	7200.00	2229.60	13	0.10	3108.03	-34.96	2229.60	11	0.10	1276.73	-34.96	2229.60	10	0.10	1216.53	-34.96	2229.60	10	0.10	1786.83	-34.96	2227.26
Cb5-3,5,15	30	2740.53	21.38	7200.00	2166.17	12	0.54	3162.39	-26.52	2166.17	11	0.54	1014.90	-26.52	2166.17	10	0.54	1081.41	-26.52	2166.17	10	0.54	1614.34	-26.52	2154.58
Cb5-6,4,15	30	3095.37	20.26	7200.00	2488.53	11	0.82	1238.98	-24.39	2488.53	11	0.82	1319.86	-24.39	2488.53	10	0.82	1413.15	-24.39	2488.53	10	0.82	1548.08	-24.39	2468.15
Cc1-2,3,15	30	5603.50	14.19	7200.00	4852.63	30	0.91	4218.65	-15.47	4989.93	38	3.64	1181.42	-12.30	4852.63	22	0.91	4139.52	-15.47	4852.63	20	0.91	2433.63	-15.47	4808.26
Cc1-3,5,15	30	4440.17	8.88	7200.00	4062.17	35	0.40	2780.82	-9.31	4072.67	11	0.65	702.83	-9.02	4062.17	20	0.40	675.83	-9.31	4062.17	42	0.40	1211.85	-9.31	4046.10
Cc1-6,4,15	30	5946.87	20.44	7200.00	5166.13	30	8.41	4824.92	-15.11	4771.87	50	0.85	4242.11	-24.62	4771.87	31	0.85	1746.19	-24.62	4771.87	25	0.85	1805.19	-24.62	4731.53
Cc2-2,3,15	30	6379.13	22.82	7200.00	5030.10	25	2.12	7200.00	-26.82	4939.33	48	0.32	3394.34	-29.15	4939.83	30	0.33	2352.11	-29.14	4939.83	30	0.33	3432.54	-29.14	4923.41
Cc2-3,5,15	30	4915.20	10.80	7200.00	4594.93	30	4.58	4593.78	-6.97	4597.83	52	4.64	2397.99	-6.90	4594.93	34	4.58	2448.40	-6.97	4394.47	10	0.23	2871.15	-11.85	4384.47
Cc2-6,4,15	30	5688.00	17.22	7200.00	4904.73	30	4.00	4255.09	-15.97	4940.77	43	4.70	2377.38	-15.12	4940.77	25	4.70	2289.77	-15.12	4790.83	30	1.72	1481.57	-18.73	4708.53
Cc3-2,3,15	30	6205.03	14.50	7200.00	5378.20	45	1.36	4060.80	-15.37	5335.10	41	0.56	1868.77	-16.31	5372.33	30	1.25	1924.50	-15.50	5311.03	13	0.11	2647.65	-16.83	5305.06
Cc3-3,5,15	30	4768.47	7.40	7200.00	4524.87	42	2.41	4584.09	-5.38	4528.60	45	2.50	768.11	-5.30	4510.87	59	2.11	894.67	-5.71	4510.87	10	2.11	1057.68	-5.71	4415.60
Cc3-6,4,15	30	6054.90	13.79	7200.00	5655.20	55	7.70	4813.03	-7.07	5532.70	49	5.65	3651.34	-9.44	5532.70	26	5.65	3080.84	-9.44	5457.60	21	4.35	1954.79	-10.94	5219.93
Cc4-2,3,15	30	5404.10	14.82	7200.00	4616.20	40	0.28	5838.46	-17.07	4640.63	50	0.80	1665.85	-16.45	4640.63	37	0.80	3056.46	-16.45	4616.47	29	0.28	1449.05	-17.06	4603.46
Cc4-3,5,15	30	5874.30	6.85	7200.00	5567.70	35	1.72	3486.46	-5.51	5554.83	20	1.49	1089.74	-5.75	5554.13	20	1.48	892.86	-5.76	5554.13	20	1.48	2635.10	-5.76	5471.91
Cc4-6,4,15	30	6479.77	9.31	7200.00	6160.67	45	4.61	3846.69	-5.18	6160.60	48	4.61	3555.68	-5.18	6160.60	39	4.61	1625.86	-5.18	6109.70	39	3.82	1782.66	-6.06	5876.50
Cc5-2,3,15	30	6772.73	15.05	7200.00	5866.00	20	1.92	7200.00	-15.46	5755.40	33	0.03	1119.42	-17.68	5755.40	38	0.03	3281.59	-17.68	5753.67	22	0.00	2236.85	-17.71	5753.43
Cc5-3,5,15	30	5494.37	7.03	7200.00	5184.57	30	1.47	2933.12	-5.98	5230.00	55	2.33	1112.15	-5.05	5230.00	10	2.33	431.41	-5.05	5193.97	28	1.65	2294.64	-5.78	5108.12
Cc5-6,4,15	30	6062.60	12.43	7200.00	6008.70	25	11.64	4696.87	-0.90	5548.50	49	4.32	2302.72	-9.27	5548.50	32	4.32	3669.92	-9.27	5530.21	27	4.00	2084.36	-9.63	5309.02
Cd1-2,3,15	30	5729.57	19.76	7200.00	4759.13	40	3.40	6715.33	-20.39	4641.17	38	0.94	1928.31	-23.45	4641.17	28	0.94	2017.33	-23.45	4640.20	26	0.92	1420.60	-23.48	4597.35
Cd1-3,5,15	30	5421.03	9.26	7200.00	5019.83	33	2.01	5573.19	-7.99	5019.83	28	2.01	1233.64	-7.99	5019.83	21	2.01	2769.83	-7.99	5048.30	20	2.56	2183.80	-7.38	4919.04
Cd1-6,4,15	30	5354.53	8.68	7200.00	5365.00	50	8.86	5331.32	0.20	5066.50	50	3.49	2739.24	-5.68	5066.50	30	3.49	2630.84	-5.68	4985.60	22	1.92	1274.32	-7.40	4889.76
Cd2-2,3,15	30	4632.90	11.49	7200.00	4142.27	55	1.00	6875.08	-11.84	4126.57	30	0.63	1172.88	-12.27	4126.57	17	0.63	1055.86	-12.27	4126.57	18	0.63	2059.15	-12.27	4100.72
Cd2-3,5,15	30	4358.60	9.40	7200.00	4377.50	33	9.79	3482.54	0.43	4048.03	40	2.45	1059.64	-7.67	3982.87	40	0.85	2322.06	-9.43	3982.87	20	0.85	1620.93	-9.43	3948.84
Cd2-6,4,15	30	6549.37	23.40	7200.00	5607.30	25	10.53	4300.29	-16.80	5111.67	37	1.85	1498.48	-28.13	5100.00	22	1.63	1296.91	-28.42	5100.00	25	1.63	2244.71	-28.42	5017.04
Cd3-2,3,15	30	5597.53	11.50	7200.00	5035.73	55	1.63	4389.55	-11.16	5062.50	56	2.15	1329.84	-10.57	5035.73	31	1.63	1666.06	-11.16	4962.37	22	0.18	1175.30	-12.80	4953.63
Cd3-3,5,15	30	3760.10	8.48	7200.00	3509.07	25	1.93	3273.71	-7.15	3777.47	17	8.90	626.67	0.46	3509.07	34	1.93	3030.22	-7.15	3509.07	20	1.93	2332.00	-7.15	3441.24
Cd3-6,4,15	30	5108.20	9.94	7200.00	4605.23	50	0.11	3037.11	-10.92	4833.93	48	4.84	1716.41	-5.67	4680.30	42	1.71	2510.27	-9.14	4605.23	15	0.11	1772.61	-10.92	4600.21
Cd4-2,3,15	30	5671.47	9.47	7200.00	5151.37	40	0.33	3065.34	-10.10	5135.57	35	0.03	854.77	-10.44	5151.60	34	0.34	884.78	-10.09	5145.37	38	0.22	1366.18	-10.22	5134.20
Cd4-3,5,15	30	4684.00	6.32	7200.00	4435.50	20	1.08	4091.75	-5.60	4435.50	33	1.08	910.18	-5.60	4423.23	18	0.80	816.58	-5.90	4423.23	18	0.80	1757.45	-5.90	4387.76
Cd4-6,4,15	30	6216.10	13.44	7200.00	5675.07	18	5.19	2730.26	-9.53	5677.73	54	5.23	3471.61	-9.48	5643.77	36	4.66	3729.45	-10.14	5489.63	22	1.99	1332.61	-13.23	5380.66
Cd5-2,3,15	30	4816.20	8.25	7200.00	5074.80	20	12.93	7095.28	5.10	4444.10	40	0.57	1953.31	-8.37	4448.40	32	0.66	1585.84	-8.27	4448.40	35	0.66	2887.52	-8.27	4418.86
Cd5-3,5,15	30	5586.57	10.55	7200.00	5201.83	20	3.93	6513.27	-7.40	5203.53	37	3.97	854.99	-7.36	5233.80	22	4.52	1830.50	-6.74	5233.17	15	4.51	569.55	-6.75	4997.19
Cd5-6,4,15	30	5921.73	16.56	7200.00	5733.47	25	13.82	5670.99	-3.28	5173.13	51	4.49	2789.67	-14.47	5173.13	23	4.49	2777.04	-14.47	5172.83	23	4.48	2731.32	-14.48	4941.09
Averages			***19.46***	***7200.00***		***22.93***	***9.82***	***5175.74***	***-12.74***		***29.80***	***4.03***	***1870.71***	***-21.66***		***21.30***	***2.42***	***2099.20***	***-23.97***		***18.20***	***1.49***	***1864.32***	***-25.36***	

Table 3 – Complete results on instances with demand correlation

Instance	|S|	CPLEX			PH																				LB
					Classic (AC)					Probability (AC)					Social (AC)					Cluster (AC)					
		UB	OG (%)	TIME	UB	ITER.	GAP	TIME (s)	DIFF UB	UB	ITER.	GAP	TIME (s)	DIFF UB	GAP (%)	ITER.	GAP	TIME (s)	DIFF UB	UB	ITER.	GAP	TIME (s)	DIFF UB	
Ca1-2,3,15	30	2984.10	20.65	7200.00	2451.10	60	3.39	6247.8	-21.75	2451.1	10	3.39	1048.00	-21.75	2450.90	12	3.38	832.38	-21.76	2450.03	11	3.35	3082.30	-21.80	2367.96
Ca1-3,5,15	30	2364.10	15.60	7200.00	3196.97	11	37.59	6387.9	26.05	2256.73	10	11.58	1431.70	-4.76	2103.23	9	5.13	810.493	-12.40	2081.47	5	4.14	1409.11	-13.58	1995.30
Ca1-6,4,15	30	2400.70	9.80	7200.00	3874.17	6	44.11	4746.7	38.03	3874.83	15	44.12	1310.30	38.04	2400.70	9	9.80	683.951	0.00	2362.37	5	8.34	2889.59	-1.62	2165.38
Ca2-2,3,15	30	2923.40	22.43	7200.00	2288.97	60	0.93	6160.03	-27.72	2288.97	10	0.93	970.05	-27.72	2288.97	9	0.93	688.635	-27.72	2288.97	5	0.93	2157.66	-27.72	2267.69
Ca2-3,5,15	30	2930.10	36.84	7200.00	3700.10	42	49.98	6369.7	20.81	1945.47	9	4.87	827.00	-50.61	1945.47	9	4.87	661.852	-50.61	1937.60	12	4.49	1942.69	-51.22	1850.65
Ca2-6,4,15	30	4874.30	53.05	7200.00	3507.93	12	34.76	2365.7	-38.95	3512.17	10	34.84	1610.40	-38.78	2507.93	9	8.75	693.671	-94.36	2398.77	12	4.59	1047.22	-103.20	2288.58
Ca3-2,3,15	30	3766.53	33.33	7200.00	3359.80	11	25.26	6782.5	-12.11	3394.2	15	26.01	1708.20	-10.97	2572.43	10	2.38	707.498	-46.42	2568.27	13	2.22	1608.87	-46.66	2511.24
Ca3-3,5,15	30	2781.53	41.60	7200.00	2774.10	60	41.45	6217.32	-0.27	2780.03	10	41.57	1585.20	-0.05	1743.37	12	6.83	1604.34	-59.55	1703.87	10	4.67	2486.38	-63.25	1624.29
Ca3-6,4,15	30	2845.43	13.25	7200.00	3271.10	10	24.54	3426.41	13.01	3303.47	12	25.27	1462.20	13.87	2660.53	8	7.22	938.761	-6.95	2535.10	13	2.63	3406.30	-12.24	2468.54
Ca4-2,3,15	30	4978.20	52.31	7200.00	2983.53	12	20.43	1117.6	-66.86	2996.9	10	20.78	1760.30	-66.11	2495.70	9	4.87	995.521	-99.47	2395.37	10	0.89	1082.71	-107.83	2374.11
Ca4-3,5,15	30	2566.80	31.34	7200.00	2575.70	14	31.58	6247.8	0.35	2587.93	9	31.90	1242.30	0.82	2022.37	8	12.86	1031.23	-26.92	1911.30	9	7.79	3170.40	-34.30	1762.33
Ca4-6,4,15	30	2627.07	24.28	7200.00	2124.27	27	6.36	4253.8	-23.67	2148.9	10	7.43	1009.50	-22.25	2124.27	8	6.36	988.307	-23.67	2061.13	8	3.49	1006.32	-27.46	1989.18
Ca5-2,3,15	30	3514.90	39.77	7200.00	2270.13	16	6.74	2210.4	-54.83	2270.57	10	6.76	1555.90	-54.80	2248.23	10	5.84	993.08	-56.34	2220.20	13	4.65	2413.50	-58.31	2117.05
Ca5-3,5,15	30	4072.13	43.48	7200.00	5410.80	11	57.46	2291.9	24.74	5388.13	10	57.28	1587.10	24.42	2383.57	9	3.44	914.942	-70.84	2341.20	11	1.69	2375.60	-73.93	2301.60
Ca5-6,4,15	30	2773.77	25.87	7200.00	3373.43	18	39.04	2046.8	17.78	3365.73	9	38.90	746.79	17.59	3350.13	7	38.62	730.2	17.20	2108.87	14	2.49	4349.70	-31.53	2056.32
Cb1-2,3,15	30	2659.87	19.27	7200.00	2244.00	18	4.31	6936.7	-18.53	2244	8	4.31	2780.35	-18.53	2220.17	8	3.28	2414.88	-19.80	2214	8	3.01	3353.50	-20.14	2147.32
Cb1-3,5,15	30	2433.03	22.69	7200.00	1911.77	60	1.61	7014.12	-27.27	1911.77	8	1.61	2669.96	-27.27	1911.77	9	1.61	1132.95	-27.27	1911.77	8	1.61	1069.26	-27.27	1881.06
Cb1-6,4,15	30	2604.30	20.95	7200.00	2104.97	10	2.19	2179.68	-23.72	2104.97	10	2.19	1093.86	-23.72	2104.97	10	2.19	560.01	-23.72	2104.97	8	2.19	3409.17	-23.72	2058.82
Cb2-2,3,15	30	3532.13	48.26	7200.00	1838.90	11	0.62	3078.91	-92.08	1838.9	11	0.62	1059.20	-92.08	1838.90	8	0.62	1112.68	-92.08	1838.90	8	0.62	1061.65	-92.08	1827.43
Cb2-3,5,15	30	2196.27	15.93	7200.00	1875.53	12	1.55	3409.16	-17.10	1875.53	9	1.55	1409.11	-17.10	1875.53	8	1.55	2738.97	-17.10	1875.53	9	1.55	956.43	-17.10	1846.39
Cb2-6,4,15	30	2367.47	18.99	7200.00	1918.80	10	0.05	1842.186	-23.38	1918.8	9	0.05	839.76	-23.38	1918.80	9	0.05	408.065	-23.38	1918.80	10	0.05	3367.30	-23.38	1917.89
Cb3-2,3,15	30	2640.33	18.67	7200.00	2176.73	60	1.35	7178.6	-21.30	2176.73	8	1.35	1774.89	-21.30	2176.73	9	1.35	1157.3	-21.30	2176.73	9	1.35	1079.95	-21.30	2147.26
Cb3-3,5,15	30	2115.83	18.66	7200.00	1743.07	10	1.26	6263.76	-21.39	1743.07	8	1.26	1316.96	-21.39	1743.07	10	1.26	1581.79	-21.39	1743.07	9	1.26	728.67	-21.39	1721.06
Cb3-6,4,15	30	2282.67	15.48	7200.00	1948.83	9	1.00	1654.399	-17.13	1948.83	10	1.00	659.10	-17.13	1948.83	10	1.00	482.61	-17.13	1948.83	9	1.00	684.88	-17.13	1929.39
Cb4-2,3,15	30	4082.93	44.37	7200.00	2305.00	12	1.45	2374.48	-77.13	2305	11	1.45	1377.58	-77.13	2305.00	7	1.45	690.084	-77.13	2305.00	8	1.45	571.43	-77.13	2271.50
Cb4-3,5,15	30	2715.50	32.41	7200.00	1852.50	11	0.92	3409.12	-46.59	1852.5	12	0.92	1490.20	-46.59	1852.50	6	0.92	2107.93	-46.59	1852.50	10	0.92	319.97	-46.59	1835.40
Cb4-6,4,15	30	2463.40	19.20	7200.00	2048.60	9	2.84	1050.87	-20.25	2048.6	8	2.84	1059.52	-20.25	2048.60	5	2.84	412.606	-20.25	2043.77	9	2.62	462.02	-20.53	1990.32
Cb5-2,3,15	30	2324.17	4.85	7200.00	2224.17	10	0.57	2817.061	-4.50	2224.17	10	0.57	816.73	-4.50	2224.17	5	0.57	303.057	-4.50	2224.17	8	0.57	646.21	-4.50	2211.42
Cb5-3,5,15	30	3454.23	38.37	7200.00	2165.00	11	1.67	3070.23	-59.55	2165	9	1.67	1055.36	-59.55	2165.00	9	1.67	598.016	-59.55	2165.00	9	1.67	505.96	-59.55	2128.80
Cb5-6,4,15	30	2944.10	16.78	7200.00	2491.70	10	1.67	1252.35	-18.16	2491.7	10	1.67	852.59	-18.16	2491.70	9	1.67	526.508	-18.16	2491.70	9	1.67	383.21	-18.16	2450.21
Cc1-2,3,15	30	6755.67	28.89	7200.00	5905.97	35	18.66	4072.39	-14.39	5909.63	15	18.71	4027.86	-14.32	5852.17	15	17.91	586.5	-15.44	4853.07	20	1.01	4340.70	-39.20	4803.93
Cc1-3,5,15	30	4686.40	10.63	7200.00	4589.47	25	8.74	3282.42	-2.11	4590.87	12	8.77	2923.61	-2.08	4639.50	12	9.72	8426.5	-1.01	4197.03	18	0.21	2638.90	-11.66	4188.36
Cc1-6,4,15	30	6546.80	26.27	7200.00	5920.73	50	18.48	5031.55	-10.57	5985.27	22	19.36	5078.81	-9.38	5613.07	10	14.01	797.03	-16.63	4844.63	15	0.37	1864.10	-35.14	4826.78
Cc2-2,3,15	30	6371.13	21.31	7200.00	5465.37	60	8.26	6953.86	-16.57	5465.37	35	8.26	6871.83	-16.57	5380.20	15	6.81	743	-18.42	5021.93	12	0.16	1571.10	-26.87	5013.76
Cc2-3,5,15	30	5375.17	19.61	7200.00	4913.13	30	12.05	3379.43	-9.40	4913.13	22	12.05	3421.97	-9.40	5641.23	10	23.40	813.3	4.72	4394.47	14	1.67	1226.08	-22.32	4320.91
Cc2-6,4,15	30	6074.37	18.78	7200.00	5673.33	25	13.04	3652.01	-7.07	5638.43	21	12.50	3344.92	-7.73	5267.87	9	6.34	641.23	-15.31	4790.83	15	-2.98	2474.56	-26.79	4933.71
Cc3-2,3,15	30	5471.47	12.88	7200.00	4950.30	28	3.71	3548.62	-10.53	4950.3	20	3.71	3084.99	-10.53	4965.90	25	4.01	2301.35	-10.18	4888.47	10	2.49	2886.60	-11.93	4766.77
Cc3-3,5,15	30	4498.60	10.71	7200.00	4199.70	20	4.36	3728.62	-7.12	4199.7	15	4.36	3434.44	-7.12	4316.53	15	6.95	2488.85	-4.22	4178.30	20	3.87	3680.40	-7.67	4016.67
Cc3-6,4,15	30	5279.00	13.17	7200.00	5113.90	44	10.36	5278.12	-3.23	5083.4	15	9.83	5160.15	-3.85	4891.13	20	6.28	2974.21	-7.93	4763.67	11	3.77	2292.40	-10.82	4583.89
Cc4-2,3,15	30	5052.40	16.35	7200.00	4459.47	50	5.23	5291.41	-13.30	4358.07	22	3.02	5385.40	-15.93	4391.10	24	3.75	1059.7	-15.06	4358.90	15	3.04	3492.23	-15.91	4226.27
Cc4-3,5,15	30	4933.27	11.74	7200.00	4542.53	25	4.15	2868.98	-8.60	4542.53	20	4.15	2703.38	-8.60	5313.87	20	18.06	2897.4	7.16	4516.53	12	3.60	2104.56	-9.23	4354.14
Cc4-6,4,15	30	6616.47	15.65	7200.00	5959.33	60	6.35	4797.01	-11.03	5898.47	15	5.38	4069.75	-12.17	5898.47	16	5.38	4256.09	-12.17	5819.23	15	4.09	1948.19	-13.70	5581.06
Cc5-2,3,15	30	7474.70	23.35	7200.00	6552.07	52	12.55	7143.51	-14.08	6552.47	16	12.56	6323.33	-14.07	6461.73	15	11.33	1438.5	-15.68	5753.67	15	0.42	1890.50	-29.91	5729.47
Cc5-3,5,15	30	5987.80	14.84	7200.00	5626.80	48	9.37	6167.79	-6.42	5642.6	25	9.63	3015.36	-6.12	5939.43	25	14.14	1262.6	-0.81	5193.97	25	1.82	1649.15	-15.28	5099.44
Cc5-6,4,15	30	5791.60	17.16	7200.00	5695.13	20	15.75	4579.63	-1.69	5695.13	18	15.75	4813.79	-1.69	5055.53	23	5.09	4204.06	-14.56	5084.57	20	5.64	2584.92	-13.91	4797.96
Cd1-2,3,15	30	5258.73	18.66	7200.00	4531.03	30	5.60	6976.99	-16.06	4438.47	30	3.63	6325.37	-18.48	4459.93	12	4.09	1340.35	-17.91	4430.83	20	3.46	2351.80	-18.68	4277.49
Cd1-3,5,15	30	5331.43	11.50	7200.00	4861.63	25	2.94	4736.16	-9.66	4861.63	25	2.94	4393.57	-9.66	4893.50	16	3.58	915.922	-8.95	4861.43	20	2.94	2369.30	-9.67	4718.51
Cd1-6,4,15	30	5055.70	13.15	7200.00	4802.90	28	8.57	3436.45	-5.26	4804.97	21	8.61	3477.84	-5.22	4538.73	21	3.25	840.004	-11.39	4538.73	26	3.25	3791.40	-11.39	4391.08
Cd2-2,3,15	30	4655.93	14.05	7200.00	4151.63	22	3.61	5507.9	-12.15	4151.63	26	3.61	5158.61	-12.15	4152.63	15	3.63	580.424	-12.12	4128.23	15	3.06	2146.96	-12.78	4001.89
Cd2-3,5,15	30	4236.83	15.49	7200.00	3677.03	36	2.63	5657.06	-15.22	3677.03	27	2.63	2612.48	-15.22	3711.37	18	3.53	558.477	-14.16	3677.03	24	2.63	2859.20	-15.22	3580.45
Cd2-6,4,15	30	5619.10	19.41	7200.00	4872.67	30	7.07	4119.82	-15.32	4872.67	25	7.07	3574.08	-15.32	4777.10	18	5.21	511.523	-17.63	4777.37	10	5.21	983.80	-17.62	4528.24
Cd3-2,3,15	30	5592.87	12.24	7200.00	5093.23	22	3.63	4248.46	-9.81	5093.23	22	3.63	4049.79	-9.81	5013.23	21	2.09	570.811	-11.56	4962.37	25	1.09	1226.70	-12.71	4908.38
Cd3-3,5,15	30	4549.10	18.81	7200.00	4255.67	15	13.21	3730.45	-6.90	4228.13	15	12.65	3451.91	-7.59	4219.13	20	12.46	499.507	-7.82	3758.07	15	1.72	1683.16	-21.05	3693.29
Cd3-6,4,15	30	4693.47	12.33	7200.00	4325.80	27	4.87	3301.1	-8.50	4340.1	22	5.19	3942.10	-8.14	4359.63	21	5.61	475.383	-7.66	4325.80	15	4.87	1263.61	-8.50	4114.99
Cd4-2,3,15	30	5252.93	13.47	7200.00	5305.13	35	14.32	5221.69	0.98	5305.13	33	14.32	5664.35	0.98	4720.30	21	3.70	407.409	-11.28	4720.30	20	3.70	2519.15	-11.28	4545.53
Cd4-3,5,15	30	4919.30	10.14	7200.00	4638.87	33	4.71	3384.91	-6.05	4676.53	33	5.48	2776.62	-5.19	4795.10	15	7.81	1921.65	-2.59	4467.63	25	1.06	2510.20	-10.11	4420.37
Cd4-6,4,15	30	5189.00	17.41	7200.00	4790.93	22	10.55	2688.71	-8.31	4790.93	25	10.55	2706.05	-8.31	4824.40	20	11.17	1723.3	-7.56	4638.17	22	7.60	2914.40	-11.88	4285.58
Cd5-2,3,15	30	4668.23	17.77	7200.00	4495.93	18	14.62	4938.49	-3.83	4495.9	22	14.62	4979.94	-3.83	4495.67	22	14.61	1928.7	-3.84	3970.97	25	3.33	1700.50	-17.56	3838.75
Cd5-3,5,15	30	5189.83	15.06	7200.00	4718.13	46	6.57	4384.32	-10.00	4718.13	35	6.57	4237.30	-10.00	5241.27	15	15.90	2039.1	0.98	4708.97	13	6.39	2415.90	-10.21	4408.03
Cd5-6,4,15	30	6991.40	27.29	7200.00	6251.17	45	18.69	5474.89	-11.84	6251.17	37	18.69	5808.90	-11.84	5916.33	15	14.08	2961.55	-18.17	5172.83	25	1.73	2278.80	-35.16	5083.11
Averages			***21.94***	***7200.00***		***27.32***	***11.99***	***4299.78***	***-14.46***		***16.52***	***10.78***	***2782.14***	***-16.17***		***13.07***	***6.79***	***1361.11***	***-22.61***		***13.65***	***2.65***	***2044.79***	***-27.98***	

Table 4 – Complete results on instances with no demand correlation

Instance	CPLEX			PH		
	ub	**time (s)**	**OSU**	**best ub**	**dif. ub**	**OSU**
Ca1-2,3,15	2445.13	7.90	0	2424.70	0.84	0
Ca1-3,5,15	3081.47	30.41	144	2062.03	33.08	0
Ca1-6,4,15	3246.93	25.16	124	2034.00	37.36	0
Ca2-2,3,15	2287.67	9.71	0	2110.00	7.77	0
Ca2-3,5,15	3435.97	9.32	171	1870.10	45.57	0
Ca2-6,4,15	3657.30	1.01	151	2388.40	34.69	0
Ca3-2,3,15	3506.63	1.41	133	2485.50	29.12	0
Ca3-3,5,15	5027.43	12.93	372	1629.00	67.60	0
Ca3-6,4,15	5165.10	5.13	334	2546.53	50.70	0
Ca4-2,3,15	2456.93	7.64	0	2377.38	3.24	0
Ca4-3,5,15	3314.70	21.64	169	2133.22	35.64	0
Ca4-6,4,15	2900.53	6.92	116	2108.10	27.32	0
Ca5-2,3,15	3214.30	7.99	110	2212.03	31.18	0
Ca5-3,5,15	4952.87	11.26	224	2209.03	55.40	0
Ca5-6,4,15	3767.30	8.77	195	2011.00	46.62	0
Cb1-2,3,15	2214.00	11.46	0	2214.00	0.00	0
Cb1-3,5,15	1914.40	15.62	0	1914.40	0.00	0
Cb1-6,4,15	2107.73	8.33	0	2107.73	0.00	0
Cb2-2,3,15	1834.70	50.00	0	1834.70	0.00	0
Cb2-3,5,15	1871.00	14.76	0	1871.00	0.00	0
Cb2-6,4,15	1922.37	3.92	0	1922.37	0.00	0
Cb3-2,3,15	2176.20	35.62	0	2176.20	0.00	0
Cb3-3,5,15	1741.23	26.19	0	1717.23	1.38	0
Cb3-6,4,15	1958.00	6.72	0	1958.00	0.00	0
Cb4-2,3,15	2297.60	16.06	0	2297.60	0.00	0
Cb4-3,5,15	1848.53	25.20	0	1848.53	0.00	0
Cb4-6,4,15	2043.77	8.72	0	2043.77	0.00	0
Cb5-2,3,15	2229.60	3.60	0	2229.60	0.00	0
Cb5-3,5,15	2166.17	27.23	0	2166.17	0.00	0
Cb5-6,4,15	2488.53	9.18	0	2488.53	0.00	0
Cc1-2,3,15	4989.93	6.14	161	4852.63	2.75	140
Cc1-3,5,15	4102.73	2.15	54	4062.17	0.99	50
Cc1-6,4,15	5278.93	12.45	128	4771.87	9.61	138
Cc2-2,3,15	4939.33	7.90	115	4939.33	0.00	115
Cc2-3,5,15	4627.83	1.40	120	4394.47	5.04	119

Table 5 continued from previous page

Cc2-6,4,15	5468.07	6.05	123	4790.83	12.39	161
Cc3-2,3,15	5335.10	11.25	114	5311.03	0.45	114
Cc3-3,5,15	4663.03	4.46	147	4510.87	3.26	201
Cc3-6,4,15	5532.70	3.85	232	5457.60	1.36	232
Cc4-2,3,15	4640.63	5.04	129	4616.20	0.53	202
Cc4-3,5,15	5554.83	2.36	306	5554.13	0.01	313
Cc4-6,4,15	6168.00	2.28	175	6109.70	0.95	272
Cc5-2,3,15	5755.40	17.99	127	5753.67	0.03	127
Cc5-3,5,15	5238.67	6.54	151	5184.57	1.03	178
Cc5-6,4,15	5548.50	6.55	253	5530.21	0.33	263
Cd1-2,3,15	4641.17	7.76	171	4640.20	0.02	171
Cd1-3,5,15	5048.30	4.73	171	5019.83	0.56	196
Cd1-6,4,15	5356.30	6.10	215	4985.60	6.92	218
Cd2-2,3,15	4127.40	3.96	136	4126.57	0.02	135
Cd2-3,5,15	4048.03	3.77	172	3982.87	1.61	180
Cd2-6,4,15	5205.17	3.37	174	5100.00	2.02	181
Cd3-2,3,15	5962.50	2.60	262	4962.37	16.77	173
Cd3-3,5,15	3777.47	7.15	108	3509.07	7.11	116
Cd3-6,4,15	4879.17	3.45	204	4605.23	5.61	215
Cd4-2,3,15	5152.50	2.67	315	5135.57	0.33	313
Cd4-3,5,15	4446.83	1.89	189	4423.23	0.53	181
Cd4-6,4,15	5941.80	10.41	231	5489.63	7.61	229
Cd5-2,3,15	4467.23	2.29	151	4444.10	0.52	147
Cd5-3,5,15	5203.53	5.15	184	5201.83	0.03	180
Cd5-6,4,15	5173.13	6.50	173	5172.83	0.01	173
Averages		***9.97***	***123.90***		***9.93***	***90.55***

Table 5 – Complete results stochastic vs deterministic problem on instances with demand correlation

Instance	CPLEX			PH		
	ub	**time (s)**	**outsourced**	**Best UB**	**dif. Ub**	**outsourPH**
Ca1-2,3,15	2450.9	15.34	0	2450.03	0.04	0
Ca1-3,5,15	3239.7	86.33	159	2081.47	35.75	0
Ca1-6,4,15	3883.57	85.67	266	2362.37	39.17	0
Ca2-2,3,15	2288.97	18.03	0	2288.97	0.00	0
Ca2-3,5,15	3731.47	17.23	201	1937.6	48.07	0

Table 6 continued from previous page

Ca2-6,4,15	3532.57	2.58	172	2398.77	32.10	0
Ca3-2,3,15	3378.13	9.1	124	2568.27	23.97	0
Ca3-3,5,15	2768.27	29.68	132	1703.87	38.45	0
Ca3-6,4,15	3311.83	12.78	122	2535.1	23.45	0
Ca4-2,3,15	2451.23	11.01	0	2395.37	2.28	0
Ca4-3,5,15	2066.7	85.67	0	1911.3	7.52	0
Ca4-6,4,15	3569.27	12.84	236	2061.13	42.25	0
Ca5-2,3,15	5149.63	41.29	402	2220.2	56.89	0
Ca5-3,5,15	5368.9	13.69	401	2341.2	56.39	0
Ca5-6,4,15	3428.33	15.95	198	2108.87	38.49	0
Cb1-2,3,15	2220.17	27.38	0	2214	0.28	0
Cb1-3,5,15	1911.77	85.62	0	1911.77	0.00	0
Cb1-6,4,15	2104.97	21.01	0	2104.97	0.00	0
Cb2-2,3,15	1838.9	36.11	0	1838.9	0.00	0
Cb2-3,5,15	1875.53	85.63	0	1875.53	0.00	0
Cb2-6,4,15	1918.8	10.2	0	1918.8	0.00	0
Cb3-2,3,15	2176.73	85.62	0	2176.73	0.00	0
Cb3-3,5,15	1743.07	63.37	0	1743.07	0.00	0
Cb3-6,4,15	1948.83	24.48	0	1948.83	0.00	0
Cb4-2,3,15	2305	32.37	0	2305	0.00	0
Cb4-3,5,15	1852.5	85.63	0	1852.5	0.00	0
Cb4-6,4,15	2048.6	17.1	0	2043.77	0.24	0
Cb5-2,3,15	2224.17	8.27	0	2224.17	0.00	0
Cb5-3,5,15	2165	51.73	0	2165	0.00	0
Cb5-6,4,15	2491.7	16.19	0	2491.7	0.00	0
Cc1-2,3,15	5861.53	28.63	375	4853.07	17.20	350
Cc1-3,5,15	4329.97	4.44	199	4197.03	3.07	227
Cc1-6,4,15	6052.33	31.72	298	4844.63	19.95	375
Cc2-2,3,15	5380.2	25.48	326	5021.93	6.66	296
Cc2-3,5,15	4958.03	7.9	260	4394.47	11.37	200
Cc2-6,4,15	5737.77	8.97	210	4790.83	16.50	190
Cc3-2,3,15	4897.57	25.42	205	4888.47	0.19	170
Cc3-3,5,15	4347.07	11.99	122	4178.3	3.88	172
Cc3-6,4,15	4923.67	9.05	131	4763.67	3.25	176
Cc4-2,3,15	4359.77	9.35	128	4358.07	0.04	118
Cc4-3,5,15	4531.3	4.88	209	4516.53	0.33	129

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Cc4-6,4,15	6085.07	5.88	193	5819.23	4.37	256
Cc5-2,3,15	6514.1	34.44	315	5753.67	11.67	288
Cc5-3,5,15	5672.73	11.12	284	5193.97	8.44	236
Cc5-6,4,15	5185.87	13.11	207	5055.53	2.51	222
Cd1-2,3,15	4434.77	16.01	245	4430.83	0.09	224
Cd1-3,5,15	4893.4	14.17	165	4861.43	0.65	162
Cd1-6,4,15	4877.63	13.84	222	4538.73	6.95	200
Cd2-2,3,15	4152.9	9.18	165	4128.23	0.59	223
Cd2-3,5,15	3741.43	5.23	132	3677.03	1.72	147
Cd2-6,4,15	4892.3	6.69	291	4777.1	2.35	147
Cd3-2,3,15	5022.37	8.3	238	4962.37	1.19	342
Cd3-3,5,15	4389.13	9.91	256	3758.07	14.38	260
Cd3-6,4,15	4636.83	7.91	239	4325.8	6.71	234
Cd4-2,3,15	4722.3	5.71	273	4720.3	0.04	234
Cd4-3,5,15	4682.87	4.59	85	4467.63	4.60	239
Cd4-6,4,15	5017.27	26.39	145	4638.17	7.56	230
Cd5-2,3,15	3972	5.92	193	3970.97	0.03	228
Cd5-3,5,15	4754.73	18	285	4708.97	0.96	141
Cd5-6,4,15	6002.07	13.2	51	5172.83	13.82	300
Averages		***25.09***	***147.67***		***10.27***	***111.93***

Table 6 – Complete results stochastic vs deterministic problem on instances with no demand correlation

Chapter C

SUPPLEMENTARY MATERIAL THIRD ARTICLE

C.1. Decomposition strategy for the two-stage stochastic formulation

This section presents the decomposition approach applied to the stochastic formulation of the 2E-MALRPSTT introduced in Section 4.2. The decomposition approach adopts an augmented Lagrangean strategy. A key prerequisite for implementing decomposition strategies is to establish a block-diagonal structure within the stochastic formulation of the 2E-MALRPSTT. To accomplish this, we propose a reformulation of the stochastic formulation by replicating the first-stage variables, namely the location and allocation variables, for each scenario $s \in S$. Consequently, we reformulate the sets of constraints (4.2), (4.24), (4.27), and (4.28), which involve first-stage decision variables, resulting in an equivalent scenario-separable formulation.

$$\min \sum_{s \in S} \rho_s \left(\sum_{h \in H} \sum_{(i,j) \in A} \zeta_{ij} x^s_{ijh} + \sum_{k \in K} Ro^s_k + \sum_{i \in (Z \cup P)} F_i y^s_i + \sum_{i \in P} \sum_{j \in Z} \sum_{k \in K} \Delta_{ijk} f^s_{ijk} \right) \tag{C.1.1}$$

Subject to

$$(4.3) - (4.23)$$
$$(4.25) - (4.26)$$

$$\sum_{h \in H^1} \sum_{i \in \tilde{Z}_z} \sum_{j \in (E_1 \cup \tilde{Z} \setminus \tilde{Z}_z)} x^s_{ijh} \leq |H^1| y^s_z \quad \forall z \in Z, s \in S \tag{C.1.2}$$

$$\sum_{h\in H^1}\sum_{k\in K}\sum_{j\in\tilde{Z}} vol(k)\,\phi^s_{ijkh} \le \Theta_i\, y^s_i \quad \forall i\in P, s\in S \tag{C.1.3}$$

$$\sum_{h\in H^1} \phi^s_{ijkh} \le f^s_{izk} \quad \forall i\in P, j\in \tilde{Z}_z, k\in K, z\in Z, s\in S \tag{C.1.4}$$

$$\sum_{h\in H^2} \psi^s_{iD(k)h} \le \sum_{p\in P} f^s_{pzk} \quad \forall i\in \tilde{Z}_z, k\in K, z\in Z, s\in S \tag{C.1.5}$$

$$y^s_i = \bar{y}_i \quad \forall i\in (P\cup Z), s\in S \tag{C.1.6}$$

$$f^s_{ijk} = \bar{f}_{ijk} \quad \forall i\in P, j\in Z, k\in K, s\in S \tag{C.1.7}$$

$$y^s_i \in \{0{,}1\} \quad \forall i\in (P\cup Z), s\in S \tag{C.1.8}$$

$$f^s_{ijk} \in \{0{,}1\} \quad \forall i\in P, j\in Z, k\in K, s\in S \tag{C.1.9}$$

$$\bar{y}_i \in \{0{,}1\} \quad \forall i\in (P\cup Z) \tag{C.1.10}$$

$$\bar{f}_{ijk} \in \{0{,}1\} \quad \forall i\in P, j\in Z, k\in K \tag{C.1.11}$$

The alternative reformulation includes a set of additional constraints (C.1.2)-(C.1.5) to ensure the incorporation of first-stage variables for each scenario. To prevent the first-stage decisions from being dependent on specific scenarios, a set of non-anticipativity constraints (C.1.6) and (C.1.7) is introduced. These constraints utilize reference first-stage variables, namely $\bar{y}_i$ and $\bar{f}_{ijk}$. Following the separation scheme proposed by Rockafellar and Wets (1991), constraints (C.1.6) and (C.1.7) are relaxed using an augmented Lagrangean strategy. This relaxation leads to the formulation of the following objective function:

$$\begin{aligned}\min \sum_{s\in S}\rho_s\Bigg(&\sum_{h\in H}\sum_{(i,j)\in A}\zeta^s_{ij}x^s_{ijh} + \sum_{k\in K} Ro^s_k + \sum_{i\in(Z\cup P)} F_i y^s_i + \sum_{i\in P}\sum_{j\in Z}\sum_{k\in K}\Delta_{ijk} f^s_{ijk}\\ &+ \sum_{i\in(Z\cup P)}\lambda^s_i(y^s_i-\bar{y}_i) + \frac{1}{2}\sum_{i\in(Z\cup P)}\gamma(y^s_i-\bar{y}_i)^2 + \sum_{i\in P}\sum_{j\in Z}\sum_{k\in K}\pi^s_{ijk}(f^s_{ijk}-\bar{f}_{ijk})\\ &+\frac{1}{2}\sum_{i\in P}\sum_{j\in Z}\sum_{k\in K}\gamma(f^s_{ijk}-\bar{f}_{ijk})^2\Bigg)\end{aligned} \tag{C.1.12}$$

The objective function involves the Lagrangean multipliers λ^s_i and π^s_{ijk} associated with the relaxed constraints (C.1.6) and (C.1.7), respectively, as well as a penalty term γ. Considering the binary nature of the location and allocation variables, the objective function can be simplified as follows:

$$\min \sum_{s\in S} \rho_s \Bigg(\sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij}^s x_{ijh}^s + \sum_{k\in K} Ro_k^s + \sum_{i\in(Z\cup P)} \Big(F_i + \lambda_i^s + \frac{1}{2}\gamma + \gamma \bar{y}_i \Big) y_i^s$$
$$+ \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Big(\Delta_{ijk} + \pi_{ijk}^s + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \Big) f_{ijk}^s + \frac{1}{2} \sum_{i\in(Z\cup P)} \gamma \bar{y}_i - \sum_{i\in(Z\cup P)} \lambda_i^s \bar{y}_i \tag{C.1.13}$$
$$+ \frac{1}{2} \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \gamma \bar{f}_{ijk} + \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \pi_{ijk}^s \bar{f}_{ijk} \Bigg)$$

Based on a given value for the general location and allocation variables $\bar{y}_i$ and $\bar{f}_{ijk}$, the relaxed formulation decomposes by scenario. Then, for each scenario $s \in S$, there is a deterministic 2E-MALRPSTT subproblem with the following modified fixed costs:

$$\min \sum_{s\in S} \rho_s \Bigg(\sum_{h\in H} \sum_{(i,j)\in A} \zeta_{ij} x_{ijh}^s + \sum_{k\in K} Ro_k^s + \sum_{i\in(Z\cup P)} \Big(F_i + \lambda_i^s + \frac{1}{2}\gamma + \gamma \bar{y}_i \Big) y_i^s$$
$$+ \sum_{i\in P} \sum_{j\in Z} \sum_{k\in K} \Big(\Delta_{ijk} + \pi_{ijk}^s + \frac{1}{2}\gamma + \gamma \bar{f}_{ijk} \Big) f_{ijk}^s \Bigg) \tag{C.1.14}$$

Subject to

$$(4.3) - (4.23), (4.25) - (4.26), (C.1.2) - (C.1.5) \text{ and } (C.1.8) - (C.1.11) \tag{C.1.15}$$

The proposed PH framework follows by solving the complete set of scenario subproblems to define a reference solution for $\bar{y}_i$ and $\bar{f}_{ijk}$ to guide the method to reach a consensus for the general first-stage variables.

C.2. Complete Result Tables for the decomposition-based heuristic

Tables 1 - 4 presents the results of the proposed decomposition-based heuristic for the deterministic 2E-MALRPSTT. Each table display the comparative performance results the proposed heuristic against the solution frameworks presented by Escobar-Vargas and Crainic (2023) and Dellaert et al. (2019) with a time limit of 30 minutes. Table 1 and Table 2 presents the experiments of the proposed heuristic against the a dynamic discretization discovery framework proposed by Escobar-Vargas and Crainic (2023) with 5, 10, 15 and 30 OD, respectively. Results presented in Table 3 and 4 extend the same experiments using the solution frameworks presented by Dellaert et al. (2019), with 15 and 30 OD demands. The tables display the instance **ID**, the best upper bound (**UB**), the run-time (**CPUsec**), the lower bound (**LB**), the optimality gap (**OG(%)**), cost difference percentage (**UB Diff.**) and run-time difference percentage (**Time Diff.**).

C.3. Complete Result Tables for the PH-based meta-heuristic

Table 5 and Table 6 showcase the results of the proposed progresive-hedging-based framework on instances with 10 and 15 OD demands, respectively. Each table display the comparative performance results obtained using CPLEX and the PH framework, labeled as 'CPLEX' and 'PH' respectively. Additionally, within the PH framework results, distinctions are made based on the specific aggregation strategy employed: the classical strategy (CS), the diversity-based strategy (DBS), and the longest-distance strategy (LDS). Each table provides the average optimality gap expressed as a percentage (OG), the average computational time in seconds, and the average number of iterations (ITER.) for the PH framework. In the experiments involving the PH framework, a maximum time limit of 2 hours was imposed. Conversely, for the experiments with the complete stochastic model using CPLEX, time limits of 8 hours and 16 hours were set for instances with 10 OD demands and 15 OD demands, respectively. CPLEX was employed with default parameter settings, using a thread limit of 6 for solving the overall stochastic model and a thread limit of 1 for solving the scenario subproblems within the PH framework.

ID	Escobar-Vargas and Crainic (2023)			**DBH**					ID	Escobar-Vargas and Crainic (2023)			**DBH**				
	UB	**LB**	**CPU (sec)**	**UB**	**CPU (sec)**	**OG (%)**	**UB Diff.**	**Time Diff.**		**UB**	**LB**	**CPU (sec)**	**UB**	**CPU (sec)**	**OG (%)**	**UB Diff.**	**Time Diff.**
Ca1-2,3,5	280	280	2.55	280	1.96	0.00	0.00	-30.34	Ca1-2,3,10	256	256	36.36	256	10.15	0.00	0.00	-258.24
Ca1-3,5,5	222	222	0.48	222	2.71	0.00	0.00	82.27	Ca1-3,5,10	215	215	21.38	215	12.70	0.00	0.00	-68.35
Ca1-6,4,5	271	271	2.72	271	0.89	0.00	0.00	-203.92	Ca1-6,4,10	305	305	24.14	305	12.57	0.00	0.00	-92.10
Ca2-2,3,5	152	152	1.86	152	3.49	0.00	0.00	46.71	Ca2-2,3,10	161	161	14.07	161	8.38	0.00	0.00	-67.82
Ca2-3,5,5	284	284	2.3	284	2.32	0.00	0.00	0.80	Ca2-3,5,10	340	340	5.56	340	8.29	0.00	0.00	32.94
Ca2-6,4,5	150	150	1.56	150	1.20	0.00	0.00	-30.01	Ca2-6,4,10	179	179	14.42	179	14.84	0.00	0.00	2.81
Ca3-2,3,5	287	287	2.05	287	2.16	0.00	0.00	5.19	Ca3-2,3,10	344	344	3.08	344	13.01	0.00	0.00	76.32
Ca3-3,5,5	220	220	2.89	220	0.97	0.00	0.00	-197.76	Ca3-3,5,10	318	318	20.26	318	9.45	0.00	0.00	-114.46
Ca3-6,4,5	171	171	1.61	171	2.03	0.00	0.00	20.71	Ca3-6,4,10	236	236	41.83	236	9.64	0.00	0.00	-333.89
Ca4-2,3,5	358	358	1.24	358	2.25	0.00	0.00	44.94	Ca4-2,3,10	437	437	16.14	437	22.52	0.00	0.00	28.33
Ca4-3,5,5	168	168	2.35	168	2.22	0.00	0.00	-5.95	Ca4-3,5,10	213	213	462.45	213	15.67	0.00	0.00	-2850.42
Ca4-6,4,5	161	161	0.83	161	3.00	0.00	0.00	72.35	Ca4-6,4,10	238	238	31.8	238	17.56	0.00	0.00	-81.09
Ca5-2,3,5	199	199	2.2	199	0.56	0.00	0.00	-295.19	Ca5-2,3,10	277	277	27.64	277	6.02	0.00	0.00	-359.06
Ca5-3,5,5	186	186	2.16	186	3.34	0.00	0.00	35.40	Ca5-3,5,10	264	264	26.4	264	18.88	0.00	0.00	-39.85
Ca5-6,4,5	159	159	1.01	159	3.92	0.00	0.00	74.26	Ca5-6,4,10	187	187	18.09	187	16.46	0.00	0.00	-9.89
Cb1-2,3,5	152	152	1.03	152	0.68	0.00	0.00	-51.71	Cb1-2,3,10	181	181	13.08	181	20.06	0.00	0.00	34.80
Cb1-3,5,5	164	164	0.59	164	0.75	0.00	0.00	21.23	Cb1-3,5,10	211	211	20.3	211	21.80	0.00	0.00	6.89
Cb1-6,4,5	305	305	1.57	305	1.70	0.00	0.00	7.51	Cb1-6,4,10	261	261	18.99	261	8.95	0.00	0.00	-112.09
Cb2-2,3,5	129	129	2.09	129	3.93	0.00	0.00	46.82	Cb2-2,3,10	199	199	13.42	199	11.58	0.00	0.00	-15.86
Cb2-3,5,5	154	154	1.07	154	3.79	0.00	0.00	71.79	Cb2-3,5,10	268	268	41.25	268	19.91	0.00	0.00	-107.17
Cb2-6,4,5	143	143	2.47	143	2.70	0.00	0.00	8.39	Cb2-6,4,10	185	185	234.99	185	12.40	0.00	0.00	-1794.41
Cb3-2,3,5	332	332	2.85	332	1.68	0.00	0.00	-69.68	Cb3-2,3,10	337	337	6.28	337	19.17	0.00	0.00	67.24
Cb3-3,5,5	160	160	1.22	160	1.57	0.00	0.00	22.47	Cb3-3,5,10	202	202	144.99	202	16.97	0.00	0.00	-754.24
Cb3-6,4,5	198	198	2.13	198	2.83	0.00	0.00	24.70	Cb3-6,4,10	283	283	1456.59	283	10.10	0.00	0.00	-14317.47
Cb4-2,3,5	280	280	1.84	280	2.71	0.00	0.00	32.03	Cb4-2,3,10	251	251	106.53	251	11.33	0.00	0.00	-840.21
Cb4-3,5,5	142	142	0.96	142	1.86	0.00	0.00	48.47	Cb4-3,5,10	245	245	2355.73	245	12.39	0.00	0.00	-18911.97
Cb4-6,4,5	188	188	1.97	188	3.58	0.00	0.00	44.93	Cb4-6,4,10	288	288	7.43	288	22.49	0.00	0.00	66.96
Cb5-2,3,5	129	129	2.35	129	0.42	0.00	0.00	-453.11	Cb5-2,3,10	197	197	42.39	197	20.68	0.00	0.00	-105.02
Cb5-3,5,5	179	179	1.9	179	3.27	0.00	0.00	41.89	Cb5-3,5,10	232	232	128.22	232	15.79	0.00	0.00	-712.12
Cb5-6,4,5	199	199	1.56	199	2.21	0.00	0.00	29.26	Cb5-6,4,10	337	337	50.75	337	14.47	0.00	0.00	-250.72
Cc1-2,3,5	129	129	2.03	129	1.71	0.00	0.00	-18.86	Cc1-2,3,10	189	189	64.63	189	18.50	0.00	0.00	-249.30
Cc1-3,5,5	135	135	2.3	135	2.96	0.00	0.00	22.41	Cc1-3,5,10	180	180	41.67	180	17.18	0.00	0.00	-142.50
Cc1-6,4,5	150	150	2.4	150	3.25	0.00	0.00	26.25	Cc1-6,4,10	238	238	45.67	238	7.84	0.00	0.00	-482.42
Cc2-2,3,5	122	122	1.04	122	1.40	0.00	0.00	25.77	Cc2-2,3,10	187	187	29.35	187	6.32	0.00	0.00	-364.36
Cc2-3,5,5	175	175	0.94	175	3.72	0.00	0.00	74.71	Cc2-3,5,10	231	231	27.7	231	6.11	0.00	0.00	-353.26
Cc2-6,4,5	122	122	1.77	122	2.05	0.00	0.00	13.71	Cc2-6,4,10	163	163	18.8	163	8.06	0.00	0.00	-133.26
Cc3-2,3,5	136	136	2.11	136	3.88	0.00	0.00	45.61	Cc3-2,3,10	184	184	12.7	184	9.36	0.00	0.00	-35.67
Cc3-3,5,5	142	142	2.59	142	3.75	0.00	0.00	30.87	Cc3-3,5,10	225	225	79.81	225	15.37	0.00	0.00	-419.41
Cc3-6,4,5	157	157	0.9	157	2.00	0.00	0.00	55.02	Cc3-6,4,10	193	193	21.95	193	13.16	0.00	0.00	-66.82
Cc4-2,3,5	171	171	1.27	171	2.97	0.00	0.00	57.24	Cc4-2,3,10	225	225	24.11	225	7.23	0.00	0.00	-233.50
Cc4-3,5,5	154	154	1.25	154	1.92	0.00	0.00	34.76	Cc4-3,5,10	223	223	28.05	223	20.04	0.00	0.00	-39.95
Cc4-6,4,5	138	138	1.16	138	2.29	0.00	0.00	49.33	Cc4-6,4,10	230	230	25.04	230	12.17	0.00	0.00	-105.70
Cc5-2,3,5	123	123	1.66	123	2.27	0.00	0.00	26.98	Cc5-2,3,10	182	182	12.52	182	8.38	0.00	0.00	-49.35
Cc5-3,5,5	124	124	2.65	124	2.50	0.00	0.00	-6.10	Cc5-3,5,10	168	168	27.72	168	6.20	0.00	0.00	-347.46
Cc5-6,4,5	138	138	1.41	138	4.00	0.00	0.00	64.75	Cc5-6,4,10	179	179	51.26	179	22.70	0.00	0.00	-125.86
Cd1-2,3,5	155	155	2.84	155	4.12	0.00	0.00	31.10	Cd1-2,3,10	197	197	13.29	197	17.30	0.00	0.00	23.17
Cd1-3,5,5	170	170	1.68	170	0.61	0.00	0.00	-174.77	Cd1-3,5,10	215	215	12.11	215	18.66	0.00	0.00	35.11
Cd1-6,4,5	188	188	2.2	188	0.61	0.00	0.00	-259.45	Cd1-6,4,10	248	248	44.06	248	13.57	0.00	0.00	-224.66
Cd2-2,3,5	140	140	1.21	140	2.60	0.00	0.00	53.41	Cd2-2,3,10	193	193	11.31	193	12.57	0.00	0.00	10.06
Cd2-3,5,5	158	158	1.09	158	1.75	0.00	0.00	37.56	Cd2-3,5,10	236	236	12.85	236	10.98	0.00	0.00	-17.06
Cd2-6,4,5	142	142	0.97	142	1.59	0.00	0.00	38.98	Cd2-6,4,10	213	213	44.5	213	13.59	0.00	0.00	-227.48
Cd3-2,3,5	142	142	2.64	142	4.02	0.00	0.00	34.31	Cd3-2,3,10	186	186	20.14	186	7.97	0.00	0.00	-152.54
Cd3-3,5,5	147	147	2.16	147	2.31	0.00	0.00	6.50	Cd3-3,5,10	224	224	32.52	224	10.60	0.00	0.00	-206.92
Cd3-6,4,5	157	157	1.03	157	2.19	0.00	0.00	53.03	Cd3-6,4,10	196	196	25.17	196	21.73	0.00	0.00	-15.83
Cd4-2,3,5	202	202	2.73	202	1.30	0.00	0.00	-110.68	Cd4-2,3,10	257	257	11.3	257	14.88	0.00	0.00	24.04
Cd4-3,5,5	162	162	1.35	162	0.45	0.00	0.00	-199.82	Cd4-3,5,10	241	241	20.37	241	9.83	0.00	0.00	-107.31
Cd4-6,4,5	147	147	1.11	147	0.61	0.00	0.00	-82.84	Cd4-6,4,10	189	189	30.48	189	17.13	0.00	0.00	-77.93
Cd5-2,3,5	178	178	1.35	178	3.84	0.00	0.00	64.84	Cd5-2,3,10	207	207	20.54	207	22.05	0.00	0.00	6.85
Cd5-3,5,5	178	178	0.82	178	4.16	0.00	0.00	80.28	Cd5-3,5,10	213	213	30.54	213	12.84	0.00	0.00	-137.88
Cd5-6,4,5	157	157	1.19	157	2.70	0.00	0.00	55.96	Cd5-6,4,10	184	184	43.23	184	21.30	0.00	0.00	-102.96
Averages					*2.34*	*0.00*	*0.00*	*-7.08*						*13.96*	*0.00*	*0.00*	*-770.01*

Table 1 – Decomposition-based heuristic : results on Escobar-Vargas and Crainic (2023) instances with 5 and 10 OD demands.

ID	Escobar-Vargas and Crainic (2023)			DBH					ID	Escobar-Vargas and Crainic (2023)			DBH				
	UB	LB	CPU (sec)	UB	CPU (sec)	OG (%)	UB DIFF	Time Diff.		UB	LB	CPU (sec)	DBH	CPU (sec)	OG (%)	UB DIFF	Time Diff.
Ca1-2,3,15	275	275	257.86	275	425.84	0	0	39.45	Ca1-2,3,30	530	530	530	530	600.61	0.00	0.00	11.75692
Ca1-3,5,15	309	309	288.4	309	151.35	0	0	-90.55	Ca1-3,5,30	490	490	12877.17	490	667.84	0.00	0.00	-1828.19
Ca1-6,4,15	274	274	1427.18	274	248.23	0	0	-474.95	Ca1-6,4,30	417	398.1389	18000	417	1132.47	4.52	0.00	-1489.45
Ca2-2,3,15	306	306	29.53	306	131.89	0	0	77.61	Ca2-2,3,30	441	441	9001.31	441	726.44	0.00	0.00	-1139.09
Ca2-3,5,15	335	335	442.19	335	286.09	0	0	-54.56	Ca2-3,5,30	439	439	1214.24	449	956.59	2.23	2.23	-26.9339
Ca2-6,4,15	272	272	944.95	272	342.24	0	0	-176.11	Ca2-6,4,30	448	436.5814	18000	446	681.63	2.11	-0.45	-2540.72
Ca3-2,3,15	319	319	36.86	319	234.73	0	0	84.30	Ca3-2,3,30	570	570	15555.9	570	569.63	0.00	0.00	-2630.9
Ca3-3,5,15	307	307	54.43	307	361.99	0	0	84.96	Ca3-3,5,30	428	428	12028.32	428	1173.78	0.00	0.00	-924.752
Ca3-6,4,15	267	267	5526.43	267	209.27	0	0	-2540.77	Ca3-6,4,30	445	445	12407.91	448	782.91	0.67	0.67	-1484.85
Ca4-2,3,15	309	309	31.34	309	168.52	0	0	81.40	Ca4-2,3,30	508	508	15297.08	508	618.38	0.00	0.00	-2373.73
Ca4-3,5,15	287	287	3585.06	287	107.18	0	0	-3244.94	Ca4-3,5,30	423	399.735	18000	432	578.93	7.47	2.08	-3009.16
Ca4-6,4,15	262	262	1147.66	262	279.52	0	0	-310.59	Ca4-6,4,30	392	392	10124.8	392	747.36	0.00	0.00	-1254.74
Ca5-2,3,15	265	265	32.87	265	115.25	0	0	71.48	Ca5-2,3,30	429	429	11249.56	429	570.18	0.00	0.00	-1873
Ca5-3,5,15	262	262	25.99	262	339.60	0	0	92.35	Ca5-3,5,30	499	487.4232	18000	509	928.39	4.24	1.96	-1838.83
Ca5-6,4,15	218	218	11.95	218	406.27	0	0	97.06	Ca5-6,4,30	406	406	13173.54	406	732.74	0.00	0.00	-1697.84
Cb1-2,3,15	305	305	280.14	305	209.80	0	0	-33.52	Cb1-2,3,30	492	492	10787.85	492	822.42	0.00	0.00	-1211.72
Cb1-3,5,15	291	291	126.13	291	287.39	0	0	56.11	Cb1-3,5,30	510	454.869	18000	510	531.09	10.81	0.00	-3289.23
Cb1-6,4,15	295	295	98.39	295	292.19	0	0	66.33	Cb1-6,4,30	421	389.26	18000	421	1074.97	7.54	0.00	-1574.47
Cb2-2,3,15	337	337	207.52	337	230.32	0	0	9.90	Cb2-2,3,30	402	402	12971.62	402	908.26	0.00	0.00	-1328.19
Cb2-3,5,15	292	292	53.31	292	141.50	0	0	62.33	Cb2-3,5,30	436	421.65	18000	436	711.98	3.29	0.00	-2428.15
Cb2-6,4,15	330	330	145.68	330	187.29	0	0	22.21	Cb2-6,4,30	437	437	10177.1	437	1111.77	0.00	0.00	-815.393
Cb3-2,3,15	354	354	1655.75	354	324.52	0	0	-410.22	Cb3-2,3,30	553	494.76	18000	550	563.12	10.04	-0.55	-3096.49
Cb3-3,5,15	266	266	74.13	266	258.31	0	0	71.30	Cb3-3,5,30	432	409.66	18000	430	1035.61	4.73	-0.47	-1638.11
Cb3-6,4,15	298	298	44.46	298	38.27	0	0	-16.18	Cb3-6,4,30	473	453.13	18000	474	845.58	4.40	0.21	-2028.71
Cb4-2,3,15	255	255	384.53	255	324.84	0	0	-18.38	Cb4-2,3,30	505	433.44	18000	490	732.59	11.54	-3.06	-2357.05
Cb4-3,5,15	334	334	311.41	334	309.02	0	0	-0.77	Cb4-3,5,30	437	410.73	18000	444	948.92	7.49	1.58	-1796.9
Cb4-6,4,15	252	252	51.74	252	293.37	0	0	82.36	Cb4-6,4,30	395	378.37	18000	402	1096.99	5.88	1.74	-1540.86
Cb5-2,3,15	243	243	64.14	243	178.59	0	0	64.08	Cb5-2,3,30	396	396	14560.44	396	1070.36	0.00	0.00	-1260.33
Cb5-3,5,15	223	223	699.52	223	416.17	0	0	-68.09	Cb5-3,5,30	527	505.02	18000	527	942.86	4.17	0.00	-1809.08
Cb5-6,4,15	265	265	119.31	265	291.91	0	0	59.13	Cb5-6,4,30	418	370.18	18000	416	1089.72	11.01	-0.48	-1551.8
Cc1-2,3,15	233	233	594.93	233	378.71	0	0	-57.10	Cc1-2,3,30	411	411	18000	411	1174.17	0.00	0.00	-1433
Cc1-3,5,15	280	280	175.81	280	283.71	0	0	38.03	Cc1-3,5,30	536	485.83	18000	546	844.07	11.02	1.83	-2032.53
Cc1-6,4,15	284	284	131.55	284	247.54	0	0	46.86	Cc1-6,4,30	393	361.2	18000	400	656.68	9.70	1.75	-2641.04
Cc2-2,3,15	299	299	64.06	299	345.75	0	0	81.47	Cc2-2,3,30	423	374.56	18000	423	986.40	11.45	0.00	-1724.81
Cc2-3,5,15	273	273	378.17	273	225.30	0	0	-67.85	Cc2-3,5,30	418	375.69	18000	418	954.96	10.12	0.00	-1784.9
Cc2-6,4,15	271	271	380.97	271	387.99	0	0	1.81	Cc2-6,4,30	445	411.58	18000	442	618.59	6.88	-0.68	-2809.86
Cc3-2,3,15	223	223	670.41	223	348.90	0	0	-92.15	Cc3-2,3,30	552	506.62	18000	552	740.03	8.22	0.00	-2332.35
Cc3-3,5,15	259	259	668.33	259	304.49	0	0	-119.49	Cc3-3,5,30	450	409.81	18000	457	1057.85	10.33	1.53	-1601.56
Cc3-6,4,15	307	307	90.63	307	339.69	0	0	73.32	Cc3-6,4,30	443	404.85	18000	452	954.12	10.43	1.99	-1786.56
Cc4-2,3,15	250	250	240.03	250	404.29	0	0	40.63	Cc4-2,3,30	501	454.3	18000	501	1124.27	9.32	0.00	-1501.04
Cc4-3,5,15	268	268	69.52	268	311.49	0	0	77.68	Cc4-3,5,30	418	370.64	18000	418	1079.64	11.33	0.00	-1567.23
Cc4-6,4,15	236	236	14.54	236	399.92	0	0	96.36	Cc4-6,4,30	378	341.14	18000	378	513.44	9.75	0.00	-3405.79
Cc5-2,3,15	247	247	69.91	247	281.24	0	0	75.14	Cc5-2,3,30	423	402.61	18000	431	920.81	6.59	1.86	-1854.81
Cc5-3,5,15	208	208	73.73	208	239.15	0	0	69.17	Cc5-3,5,30	429	396.69	18000	429	635.36	7.53	0.00	-2733.06
Cc5-6,4,15	293	293	12.48	293	236.30	0	0	94.72	Cc5-6,4,30	390	364.45	18000	390	992.04	6.55	0.00	-1714.44
Cd1-2,3,15	304	304	43.78	304	397.61	0	0	88.99	Cd1-2,3,30	445	445	13823.36	445	665.86	0.00	0.00	-1976.02
Cd1-3,5,15	277	277	47.97	277	287.92	0	0	83.34	Cd1-3,5,30	533	533	13043.42	533	878.03	0.00	0.00	-1385.53
Cd1-6,4,15	249	249	3.62	249	247.94	0	0	98.54	Cd1-6,4,30	412	412	12720.21	412	654.48	0.00	0.00	-1843.55
Cd2-2,3,15	319	319	4.06	319	224.50	0	0	98.19	Cd2-2,3,30	429	388.24	18000	426	815.65	8.86	-0.70	-2106.82
Cd2-3,5,15	274	274	28.5	274	203.54	0	0	86.00	Cd2-3,5,30	435	435	15479.58	435	561.26	0.00	0.00	-2657.98
Cd2-6,4,15	257	257	2.57	257	404.89	0	0	99.37	Cd2-6,4,30	434	434	16873.03	434	566.37	0.00	0.00	-2879.16
Cd3-2,3,15	222	222	45.33	222	250.12	0	0	81.88	Cd3-2,3,30	550	481.25	18000	550	962.47	12.50	0.00	-1770.19
Cd3-3,5,15	268	268	100.99	268	114.49	0	0	11.79	Cd3-3,5,30	440	440	8435.23	440	879.11	0.00	0.00	-859.523
Cd3-6,4,15	324	324	25.14	324	429.27	0	0	94.14	Cd3-6,4,30	437	420.65	18000	445	910.57	5.47	1.80	-1876.78
Cd4-2,3,15	289	289	31.08	289	375.73	0	0	91.73	Cd4-2,3,30	510	510	12095.64	510	551.74	0.00	0.00	-2092.28
Cd4-3,5,15	263	263	25.48	263	333.59	0	0	92.36	Cd4-3,5,30	424	389.02	18000	424	948.69	8.25	0.00	-1797.35
Cd4-6,4,15	271	271	10.09	271	351.26	0	0	97.13	Cd4-6,4,30	395	384.57	18000	395	811.52	2.64	0.00	-2118.06
Cd5-2,3,15	247	247	19.36	247	330.30	0	0	94.14	Cd5-2,3,30	425	425	10783.34	425	997.80	0.00	0.00	-980.716
Cd5-3,5,15	208	208	252.93	208	372.02	0	0	32.01	Cd5-3,5,30	501	501	10556.08	511	921.29	1.96	1.96	-1045.79
Cd5-6,4,15	272	272	725.23	272	231.60	0	0	-213.13	Cd5-6,4,30	432	432	13661.84	432	663.57	0.00	0.00	-1958.84
Averages					281.34	0	0	-83.70						833.25	4.52	0.28	-1834.97

Table 2 – Decomposition-based heuristic: results on Escobar-Vargas and Crainic (2023) instances with 15 and 30 OD demands.

ID	Dellaert et al. (2019)			DBH				
	UB	**LB**	**CPU (sec)**	**UB**	**CPU (sec)**	**OG (%)**	**UB Diff.**	**Time Diff.**
Ce1-2,3,15	661.669	497.701	1846.9	553	10.3	10.00	-19.65	-17831.07
Ce2-2,3,15	642.485	447.528	1384.2	508	1.91	11.90	-26.47	-72371.20
Ce3-2,3,15	659.692	442.433	3217.1	536	1.64	17.46	-23.08	-196064.63
Ce4-2,3,15	681.745	429.047	542.9	568	6.69	24.46	-20.03	-8015.10
Ce5-2,3,15	600.304	447.307	399.6	491	10.88	8.90	-22.26	-3572.79
Cf1-2,3,15	716.019	494.759	2832.6	621	19.28	20.33	-15.30	-14591.91
Cf2-2,3,15	556.624	403.037	1408.3	475	20.88	15.15	-17.18	-6644.73
Cf3-2,3,15	616.333	474.07	177.6	610	7.72	22.28	-1.04	-2200.52
Cf4-2,3,15	615.837	407.561	786.9	515	1.50	20.86	-19.58	-78590.00
Cf5-2,3,15	578.521	396.025	553.3	471	4.6	15.92	-22.83	-11928.26
Cg1-2,3,15	631.036	466.829	1559.1	558	15.3	16.34	-13.09	-10090.20
Cg2-2,3,15	576.754	403.907	405	480	25.38	15.85	-20.16	-1495.74
Cg3-2,3,15	651.089	450.34	1284.5	524	28.19	14.06	-24.25	-4456.58
Cg4-2,3,15	659.014	411.939	2467.7	540	91.67	23.72	-22.04	-2591.94
Cg5-2,3,15	483.593	386.194	1982.4	395	5.16	2.23	-22.43	-38318.60
Ci1-2,3,15	638.939	499.34	421.2	545	2.33	8.38	-17.24	-17977.25
Ci2-2,3,15	574.943	389.781	1711.8	504	1.9	22.66	-14.08	-89994.74
Ci3-2,3,15	664.019	439.991	3328.4	606	4.3	27.39	-9.57	-77304.65
Ci4-2,3,15	642.778	430.357	938.8	519	9.76	17.08	-23.85	-9518.85
Ci5-2,3,15	612.198	419.241	728.4	497	3.54	15.65	-23.18	-20476.27
Averages					*13.62*	*16.53*	*-18.87*	*-34201.75*

Table 3 – Decomposition-based heuristic : results on Dellaert et al. (2019) instances with 15 OD demands.

ID	Dellaert et al. (2019)			DBH				
	UB	**LB**	**CPU (sec)**	**UB**	**CPU (sec)**	**OG (%)**	**UB Diff.**	**Time Diff.**
Ce1-2,3,30	1257.82	1038.63	76.5	1088	247.19	4.54	-15.61	69.05
Ce2-2,3,30	1189.67	1084.74	4.6	1120	106.12	3.15	-6.22	95.67
Ce3-2,3,30	1207.82	1179.87	0.8	1095	81.50	-7.75	-10.30	99.02
Ce4-2,3,30	1101.32	925.87	2.3	938	380.26	1.29	-17.41	99.40
Ce5-2,3,30	1210.97	1035.38	2.0	1110	46.62	6.72	-9.10	95.71
Cf1-2,3,30	1261.87	1091.47	59.9	1104	253.02	1.13	-14.30	76.33
Cf2-2,3,30	1082.73	1023.06	1.2	905	230.82	-13.05	-19.64	99.48
Cf3-2,3,30	1136.04	1018.91	2.4	931	302.73	-9.44	-22.02	99.21
Cf4-2,3,30	1172.14	1025.35	1.8	1120	30.28	8.45	-4.66	94.06
Cf5-2,3,30	1033.77	918.96	2.9	941	501.45	2.34	-9.86	99.42
Cg1-2,3,30	990.03	832.09	15.9	883	301.90	5.77	-12.12	94.73
Cg2-2,3,30	1170.76	943.96	1108.1	997	100.02	5.32	-17.43	-1007.88
Cg3-2,3,30	1038.19	1003.81	211.7	1010	153.64	0.61	-2.79	-37.79
Cg4-2,3,30	1103.88	988.41	68.3	1025	402.39	3.57	-7.70	83.03
Cg5-2,3,30	1052.60	991.62	1.7	1025	101.72	3.26	-2.69	98.33
Ci1-2,3,30	1036.89	854.03	1.0	863	112.50	1.04	-20.15	99.11
Ci2-2,3,30	1090.12	981.90	11.3	951	201.22	-3.25	-14.63	94.38
Ci3-2,3,30	1094.12	1055.48	2.1	904	402.06	-16.76	-21.03	99.48
Ci4-2,3,30	1076.57	948.46	15.1	895	344.36	-5.97	-20.29	95.62
Ci5-2,3,30	1093.93	942.05	2.0	923	522.76	-2.06	-18.52	99.62
Averages					*241.128*	*-0.55*	*-13.32*	*32.30*

Table 4 – Decomposition-based heuristic : results on Dellaert et al. (2019) instances with 30 OD demands.

INSTANCE	CS				DBS				LDS				CPLEX			LB
	UB	OG(%)	ITER	TIME (seg.)	UB	OG(%)	ITER	TIME (seg.)	UB	OG(%)	ITER	TIME (seg.)	UB	OG(%)	TIME (seg.)	
Ca1-2,3,10	348.90	2.44	6	332.17	342.17	0.52	10	580.87	342.30	0.56	7	292.83	384.17	11.40	28800	340.38
Ca2-2,3,10	272.70	8.58	6	296.50	249.67	0.15	9	501.27	253.20	1.54	8	277.98	338.67	26.39	28800	249.30
Ca3-2,3,10	337.10	2.89	9	44.13	329.30	0.59	8	495.79	329.30	0.59	10	392.25	417.83	21.66	28800	327.35
Ca4-2,3,10	317.00	3.65	7	278.75	307.50	0.67	10	554.98	317.00	3.65	7	376.36	395.50	22.77	28800	305.43
Ca5-2,3,10	304.80	0.75	6	233.84	304.00	0.49	10	867.54	304.80	0.75	5	323.04	353.00	14.31	28800	302.50
Cb1-2,3,10	361.70	2.48	6	200.47	356.70	1.11	11	738.50	356.70	1.11	8	591.34	415.00	15.00	28800	352.74
Cb2-2,3,10	283.20	0.66	7	275.04	281.67	0.12	9	508.32	283.20	0.66	10	790.44	376.67	25.31	28800	281.32
Cb3-2,3,10	376.30	2.82	6	136.47	370.83	1.39	9	690.13	371.30	1.51	9	639.85	466.83	21.67	28800	365.69
Cb4-2,3,10	337.00	3.06	8	146.56	327.50	0.25	10	502.14	329.00	0.71	8	491.87	406.50	19.64	28800	326.68
Cb5-2,3,10	335.10	4.85	8	179.40	319.00	0.05	10	555.09	321.40	0.80	9	460.55	365.00	12.65	28800	318.83
Cc1-2,3,10	362.30	0.23	8	167.80	361.80	0.09	8	487.93	361.80	0.09	5	247.98	446.83	19.10	28800	361.48
Cc2-2,3,10	290.00	3.79	5	203.48	279.17	0.06	7	390.19	281.10	0.75	5	319.77	343.17	18.70	28800	279.00
Cc3-2,3,10	356.40	1.86	5	143.37	351.33	0.45	6	444.05	351.40	0.46	5	583.26	418.33	16.39	28800	349.77
Cc4-2,3,10	333.80	1.09	7	248.87	331.00	0.25	6	273.48	331.00	0.25	6	318.06	387.33	14.76	28800	330.17
Cc5-2,3,10	312.90	2.84	6	215.31	307.67	1.19	8	483.27	307.90	1.26	5	173.07	356.67	14.76	28800	304.02
Cd1-2,3,10	348.90	2.23	6	665.37	343.50	0.69	10	690.51	343.90	0.80	7	244.72	428.50	20.39	28800	341.13
Cd2-2,3,10	272.70	2.27	7	230.78	266.50	0.00	6	369.10	267.70	0.45	9	724.93	333.50	20.09	28800	266.50
Cd3-2,3,10	334.00	1.15	10	470.75	330.67	0.15	5	513.69	332.10	0.58	7	310.82	415.67	20.57	28800	330.17
Cd4-2,3,10	329.00	3.50	8	188.09	317.67	0.05	9	579.21	320.10	0.81	6	349.65	397.67	20.16	28800	317.50
Cd5-2,3,10	300.60	1.81	6	275.48	295.17	0.01	6	306.69	295.60	0.15	8	427.62	332.17	11.14	28800	295.15
Averages		2.65	6.85	246.63		0.41	8.35	526.64		0.87	7.20	416.82		18.34	28800.00	

Table 5 – PH framework vs CPLEX: Complete results on instances with 10 OD demands

INSTANCE	CS				DBS				LDS				CPLEX			LB
	UB	OG(%)	ITER	TIME (seg.)	UB	OG(%)	ITER	TIME (seg.)	UB	OG(%)	ITER	TIME (seg.)	UB	OG(%)	TIME (seg.)	
Ca1-2,3,15	796.80	4.56	7	2425.37	778.00	2.25	11	3130.79	780.00	2.50	7	2672.578	1178.00	35.44	57600	760.50
Ca2-2,3,15	712.20	2.38	5	1876.40	699.40	0.60	9	2776.37	700.40	0.74	9	2579.585	1301.00	46.56	57600	695.22
Ca3-2,3,15	737.20	2.53	6	1906.57	724.40	0.81	11	3043.51	728.40	1.35	11	2396.755	1129.00	36.35	57600	718.57
Ca4-2,3,15	659.10	2.74	6	2032.69	646.30	0.81	12	2671.08	648.30	1.12	8	2156.146	1105.00	41.99	57600	641.05
Ca5-2,3,15	699.20	2.65	8	2167.00	686.40	0.84	11	2848.38	689.40	1.27	6	2402.552	1065.00	36.09	57600	680.64
Cb1-2,3,15	1066.70	2.56	9	2281.68	1043.00	0.35	14	3057.11	1047.00	0.73	10	2441.739	1663.83	37.53	57600	1039.38
Cb2-2,3,15	1646.60	1.56	8	1966.13	1636.40	0.95	11	2806.21	1640.40	1.19	9	2363.416	2308.67	29.79	57600	1620.86
Cb3-2,3,15	1235.20	1.49	9	2410.04	1225.00	0.66	10	2980.05	1227.00	0.83	9	2807.829	1650.67	26.28	57600	1216.86
Cb4-2,3,15	969.00	1.15	8	2101.81	960.80	0.31	10	2731.50	963.80	0.62	10	2723.807	1374.50	30.32	57600	957.81
Cb5-2,3,15	939.30	1.07	11	2032.74	931.10	0.20	9	3115.12	934.10	0.52	8	2222.487	1262.00	26.37	57600	929.26
Cc1-2,3,15	877.60	3.26	10	2083.49	855.00	0.70	10	2740.94	858.00	1.05	11	2636.894	1160.50	26.84	57600	849.03
Cc2-2,3,15	839.40	1.29	9	2491.88	831.20	0.31	10	3151.58	833.20	0.55	8	2540.088	1357.33	38.95	57600	828.61
Cc3-2,3,15	887.40	2.57	7	2437.85	876.50	1.35	8	3406.67	879.50	1.69	7	2582.208	1205.50	28.28	57600	864.63
Cc4-2,3,15	801.80	2.64	9	2056.27	793.60	1.63	11	3226.94	795.60	1.88	6	2474.916	1226.70	36.36	57600	780.67
Cc5-2,3,15	742.00	2.34	8	2359.69	733.33	1.18	8	2914.47	733.33	1.18	5	1979.102	1192.33	39.22	57600	724.65
Cd1-2,3,15	778.10	4.40	11	2473.94	769.90	3.39	10	3187.61	773.90	3.89	8	2522.777	1120.83	33.64	57600	743.83
Cd2-2,3,15	664.10	4.76	7	2335.20	655.90	3.57	7	2638.67	656.90	3.71	6	2111.047	1088.50	41.89	57600	632.50
Cd3-2,3,15	667.10	1.29	9	2683.48	658.90	0.06	9	2713.54	662.90	0.66	9	2270.572	1152.50	42.86	57600	658.50
Cd4-2,3,15	701.50	1.52	6	2752.54	693.30	0.35	8	3532.33	694.30	0.49	9	2657.465	1069.83	35.42	57600	690.87
Cd5-2,3,15	811.10	2.21	8	1840.06	802.90	1.21	7	2752.33	805.90	1.58	8	2471.508	1272.17	37.65	57600	793.19
Averages		2.45	8.05	2235.74		1.08	9.80	2971.26		1.38	8.20	2450.67		35.39	57600.00	

Table 6 – PH framework vs CPLEX: Complete results on instances with 15 OD demands