A bilevel approach for the collaborative transportation planning problem

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Abstract

The integration of the outbound and the inbound logistics of a company leads to a large transportation network, allowing to detect backhauling opportunities to increase the efficiency of the transportation. In collaborative networks, backhauling is used to find profitable services in the return trip to the depot and to reduce empty running of vehicles. This work investigates the vertical collaboration between a shipper and a carrier for the planning of integrated inbound and outbound transportation. Based on the hierarchical nature of the relation between the shipper and the carrier and their different goals, the problem is formulated as a bilevel Vehicle Routing Problem with Selective Backhauls (VRPSB). At the upper level, the shipper decides the minimum cost delivery routes and the set of incentives offered to the carrier to perform integrated routes. At the lower level, the carrier decides which incentives are accepted and on which routes the backhaul customers are visited. We devise a mathematical programming formulation for the bilevel VRPSB, where the routing and the pricing problems are optimized simultaneously, and propose an equivalent reformulation to reduce the problem to a single-level VRPSB. The impact of collaboration is evaluated against non-collaborative approaches and two different side payment schemes. The results suggest that our bilevel approach leads to solutions with higher synergy values than the approaches with side payments.

Key words: bilevel optimization, vertical collaboration, vehicle routing problem with selective backhauls, exact reformulation

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1. Introduction

Among the different logistics operations, transportation comprises the major portion of the total costs and it is strongly associated with a negative impact on the environment [Wang et al., 2019]. Thus, promoting sustainable initiatives for transportation is becoming a target for many companies and supply chains. Reducing empty running is one of the most popular initiatives to increase the efficiency of vehicles, which impacts directly on reducing costs, fuel consumption, and pollutant emissions [Evangelista et al., 2017]. Traditionally, the vehicles travel empty when returning to their original location, and this empty distance may represent up to 25% of the total route distance [Juan et al., 2014; Turkensteen and Hasle, 2017]. An efficient way to reduce empty running is to provide pickup loads for vehicles that would return empty to their depot. This is known as backhauling. For a company, backhauling allows to reduce the total costs of transportation by creating integrated outbound-inbound routes instead of dedicated delivery and dedicated pickup routes. For example, Sainsbury's uses backhauling to create integrated routes such that, after delivery all requests at stores, the vehicles collect stock at warehouses in the return trip to the distribution centre [Early, 2011]. Differently, Tesco uses backhauling under the context of reverse logistics. After delivering the requests to a store, a vehicle can collect, at that store, returned products to be delivered at the distribution centre.

Backhauling is also widely applied in the context of collaborative transportation. For instance, Nestlé and United Biscuits, competitors in the food market, have arranged a collaboration to improve their logistics operations. The companies, which share a common depot, make their own delivery transport but the vehicles collect loads from each others customers in the return trip. This backhauling strategy allowed to reduce empty running from 22% to 13% in four years [Early, 2011]. In the study of Juan et al. [2014] different carriers collaborate with each other through backhauling by allowing each carrier to service customers from other carrier's depot. For the overall network, the collaboration have provided average reductions of 16% on the total distance costs and of 24% on the environmental costs. Both of the above examples refer to horizontal collaboration, where the participants in the collaborative network are stakeholders at the same level in a supply chain. Vertical collaboration, on the other hand, refers to the case where participants are stakeholders at different levels in a supply chain, and usually involves an hierarchical relation between them. An example of vertical collaboration between a set of retailers and a service logistics provider (LSP) is investigated in Cruijssen et al. [2005]. The retailers need to serve all their customers, either using outsourcing or collaborating with the LSP. The LSP assumes the leading of the collaboration, and offers to retailers reduced tariffs to serve their stores. Each tariff represents the cost reduction that the LSP is able to offer to a retailer, and it depends on the degree of synergy achieved with collaboration.

In the present work, we investigate a case of vertical collaboration between a shipper and a carrier, where the shipper is the leading entity of the collaboration. The shipper aims to promote the creation of integrated routes such that transportation costs are minimized, whereas the carrier aims to maximize the revenues collected during its backhaul trips. The shipper must then offer incentives to the carrier to motivate it to perform integrated routes. Based on the conflicting and hierarchical nature of the objectives of the shipper and carrier, we propose a bilevel formulation for the collaborative transportation planning problem. A bilevel optimization model is composed of two levels: the upper level describes the problem of a leader and the lower level describes the problem of a follower. The main characteristic of the bilevel problem is that the lower level is part of the constraints of the upper level problem. Thus, this represents a sequential game where first, in the upper level, the leader (shipper) takes a decision, and afterwards, in the lower level, the follower (carrier) observes the strategy of the leader and solves its optimization problem. The collaborative problem is formulated as a Bilevel Vehicle Routing Problem with Selective Backhauls (VRPSB) and solved by reducing it to an equivalent single-level mixed integer program. The properties of the bilevel problem are analyzed, and the efficiency of the formulation is evaluated and compared against traditional modelling approaches. It is worth mentioning that, in practice, side payments (incentives) are often used to induce collaboration, which represent the portion of gains that the shipper is willing to offer to the carrier. Usually, side payments are obtained after solving the routing problem of the shipper. The bilevel VRPSB tackled in this work is distinct from these side payment approaches. First, the incentives to visit backhaul customers are obtained while solving the routing problem and not afterwards. Second, these are defined based on a combination of the shipper and carrier goals, allowing the incorporation of their individual rationality. Therefore, the incentives obtained by solving our bilevel model can be seen as individual rational-side payments.

The main contributions of this work are relevant for both literature and practice. First, the bilevel formulation proposed allows to explicitly model the interactions and the goals of both participants in the network, as well as it ensures the individual rationality. Second, the collaborative problem is formulated such that it can solve simultaneously the routing and pricing problems, where routing decisions are taken jointly by both participants, and pricing decisions are taken by the shipper when offering incentives to the carrier. Third, a thorough analysis on the properties of the bilevel approach, and a comparison with other alternative approaches (e.g., side payments), allow to gather several managerial insights on the potential application of the bilevel approach to form prominent collaborative networks. Fourth, the problem studied fits well real cases where several backhauling opportunities may arise. Particularly in the forestry industry, the idea of motivating carriers to use their empty vehicles to perform backhauling for shippers is widely applied [Marques et al., 2020; Audy et al., 2012].

The remainder of the paper is structured as follows. A literature review on the collaborative vehicle routing is presented in Section 2. Section 3 defines the bilevel VRPSB, describing the mathematical formulation and assumptions of the problem. The properties of the bilevel model are investigated and used to derive an equivalent single-level formulation in Section 4. The computational experiments are presented and discussed in Section 5, covering the generation of the data sets used in this work, the managerial insights obtained with the different transportation planning strategies, and the evaluation of the computational performance of models and solution method. Section 6 concludes this paper, presenting the main insights and limitations of this work, as well as suggestions for future work in the research field.

2. Background literature

The main motivation for a player to join a collaborative network is to reduce its total costs (or increase its profits). Therefore, each player expects that its costs (profits) are lower (higher) in collaboration than in the case where they perform individually (*stand alone solution*). This is designated as the *individual rationality*. The difference between a solution for the entire collaborative network and the stand alone solution is known as the *coalition gain* [Cuervo et al., 2016], whereas the augmented percentage profit defines the *synergy value* [Cruijssen et al., 2007].

Collaboration can be one of the three types: *i)* horizontal collaboration, if players are at the same level of the network, *ii)* vertical, if players are at different levels of the network, and *iii)* lateral, if there is a combination of both. A recent review on collaborative vehicle routing concludes that horizontal collaboration is the most investigated type in the literature [Gansterer and Hartl, 2018].

Thus, collaboration can be achieved differently depending on the business context. This work focus on type *ii)* hence, next, we review the literature on different business models considering vertical collaboration, the use of cooperative game theory to incentive collaboration and the application of bilevel programming to model leadership in VRP and collaborative problems.

A case of vertical collaboration is studied in Ergun et al. [2007], which considers a shipper interested in identifying repeatable and continuous tours for carriers, in order to minimize their re-positioning needs and, consequently, the routing costs. By combining inbound and outbound routes, the shipper can negotiate discounts with the carrier and thus pay less for the overall transportation. The problem is modeled as a time-constrained lane covering problem and the results show savings between 5.5% and 13%. Bailey et al. [2011] study the problem of a carrier seeking for collaborative shipments with potential partners in a transportation network, receiving a revenue for each shipment. The collaboration can occur with a shipper, which offers a pickup-delivery task close to the backhaul routes of the carrier of interest, or with other carriers, who do not have sufficient capacity to fulfill all their tasks. In a case-study, the authors demonstrate that the carrier can reach savings between 13% and 28% compared with the stand alone solutions. A problem investigated in Xu et al. [2017], that involves a manufacturer with a private fleet and outsourcing options, shows that collaboration with a third party logistics (3PL) may allow to reduce the total costs in 10%. Cruijssen et al. [2005] propose an alternative to outsourcing, which they designate as insinking. In opposition to outsourcing, which is decided by shippers, the *insinking* allows a logistics service provider (LSP) to motivate shippers to be their customers. In this case, the LSP selects the shippers it wishes to serve in order to build strong synergies. Based on the synergy value, the carrier then determines customized tariffs to motivate the shippers to collaborate.

Collaboration may be tackled with cooperative game theoretical tools. In this context, typically, the participants problems are aggregated into one large optimization problem, and afterwards, the benefits (savings or profits) are determined and shared among the participants. This usually requires solving a pricing problem. Moreover, the collaboration should attend specific criteria, such as individual rationality (i.e., each participant cannot perform better individually than in collaboration). Several methods to allocate the profits from the collaboration have been investigated in the literature. Among them, the Shapley value [Shapley, 1953] is the most commonly used. This method distributes the profits among the players, taking into account the contribution of each player to the overall coalition gain. For example, Krajewska et al. [2008] applies the Shapley value to fairly allocate the profits in a collaborative Pickup and Delivery Problem with Time Windows (PDPTW), while Pradenas et al. [2013] use it in a collaborative Vehicle Routing Problem with Backhauls and Time Windows (VRPBTW). Cruijssen et al. [2005] have also used the Shapley value to determine customized tariffs to offer the participants in the collaborative Vehicle Routing Problem with Time Windows (VRPTW). Another approach to solve the profit sharing problem is to optimize the routes of a leading participant selfishly and then provide side payments for each of the remaining participants [Özener et al., 2011]. These side payments represent a part of the profits generated for a leading entity that should be sufficient to compensate the losses of one or multiple participants. Dahl and Derigs [2011] investigated different compensation schemes to solve a collaborative problem between carriers that are allowed to share orders. These compensation schemes distinguish between orders executed by private fleet or by a partner fleet, and orders served by dedicated vehicles or inserted in already existing routes. In Liu et al. [2010], the compensation scheme covers the side payments received by a carrier if it executes orders from their partners and the penalty costs if the carrier needs to outsource. Nevertheless, using side payments is not always a guarantee that an efficient collaboration is created [Özener et al., 2011].

Recently, Defryn et al. [2019] have put in evidence that the traditional modelling of collaborative vehicle routing problems presents some fragility when it comes to consider different goals of participants in the collaboration. Due to the capabilities of bilevel optimization to explicitly consider the different optimization problems of players, it is expected that bilevel models are able to overcome this difficulty. To the best of our knowledge, only Xu et al. [2018] developed a bilevel formulation for a collaborative VRP. The bilevel formulation considers a centralized logistics platform that allocates vehicles to customers orders (upper level) and a set of vehicle owners that execute those orders (lower level). The upper level problem aims to minimize the variable and the fixed routing costs, whereas the lower level aims to minimize the total empty distances. This work highlights the benefits of balancing the different objectives of different players in a transportation network, but the coalition gain or profit sharing are not discussed.

Finally, it is worth mentioning that although bilevel optimization models are still emergent in the field of collaborative vehicle routing, they are of very much use in pricing problems (e.g., define the price of vaccines to sell in the market [Lunday and Robbins, 2019], prices of shared transportation [Qiu and Huang, 2016], storage price for outbound containers in dry ports [Qiu et al., 2015], setting revenue shares of retailers and prices of suppliers in marketplaces [de Matta et al., 2017]. Bilevel formulations can also be found to model the VRP. The first one is described in Marinakis et al. [2007], where the upper level focuses on the assignment of customers to vehicles and the lower level focuses on the routing decisions. A similar approach is considered in Du et al. [2017], where the assignment of customers to locations is decided in the upper level and the routing decisions are taken at the lower level. Nikolakopoulos [2015] proposes a bilevel formulation for the VRP where the upper level minimizes the number of vehicles and the lower level minimizes the duration of the routes. A common aspect of these works is the use of metaheuristics to solve the problem, in particular genetic algorithms. Thus, considering that we propose to solve simultaneously a routing and a pricing problem, a bilevel optimization model seems well suited for the purpose of this work. Hence, the main focus of this paper is to propose an innovative formulation to handle the collaborative problem, thus providing an efficient alternative to side payments strategies.

Our work differs in several aspects from the above literature. First, we study a problem of vertical collaboration, where the upper level problem belongs to the shipper and the lower level problem belongs to the carrier, and where the goals of each player are different. Most of the literature on vertical collaborative transportation considers a single goal of minimizing the costs or maximizing the savings for the entire network, while our work considers the different goals of the different decision makers. Second, we assume that the shipper is the leading entity of the collaboration and the incentives offered are based on the response function of the carrier. In opposition, Cruijssen et al. [2005] considers that the carrier is the leading entity and the tariffs offered to shippers are based on the Shapley value. Third, we aim to demonstrate the advantage of using a bilevel formulation to handle the collaborative problem instead of a traditional planning with side payments. We further develop an exact reformulation to solve the problem up to optimality. The work of Xu et al. [2018] provides only one single example of the capabilities of the bilevel formulation, and it describes a genetic algorithm to solve the hierarchical problem. Finally, our study contributes to the scientific literature with a thorough analysis on the benefits of the bilevel approach, as well as its limitations, against traditional modelling strategies.

3. Problem description

This section describes in detail the problem investigated in this work. First, the collaborative problem investigated in this work is presented, describing the perspectives of both players - the shipper and the carrier, and how their problems relate to each other. Next, the bilevel formulation for the collaborative VRPSB is presented, where the upper level is the cost minimization problem of the shipper and the lower level is the profit maximization problem of the carrier. The section concludes with an illustrative example of a collaborative network formed by a shipper and a carrier.

3.1. The collaborative transportation planning problem

The transportation network is composed of a common depot, a set of linehaul customers (customers of the shipper) and a set of backhaul customers (suppliers of the shipper). The fleet of vehicles of the carrier are located at the common depot.

The shipper does not own a fleet of vehicles but, on the basis of a contract with a carrier, sends regular shipments to meet the demand of all its customers (outbound routes). The shipper has also requests to be picked up at different suppliers, for which typically it is the supplier who sends a full truck load vehicle to the depot of the shipper (inbound routes).

The shipper recognizes that integrating some inbound trips in the outbound routes of the carrier, may lead to reduce its total routing costs. For the carrier, this strategy may also bring benefits, because guaranteeing a full truck load in the return trip to the depot reduces empty backhaul distances. Thus, to motivate the carrier to collaborate and perform an integrated outbound-inbound route, the shipper must pay an additional incentive. However, the carrier, which may serve other requests to other shippers, may not be willing to collaborate. For example, the carrier can get a better incentive from another service or the distance to perform any integrated outbound-inbound route exceeds the maximum distance allowed.

A main distinct feature of this problem is the way we consider the competition between incentives. The incentives for backhauling offered by the shipper compete with the costs of pure inbound routes (for the shipper) and the external incentives (for the carrier). The competition with the former follows the rational principle that the cost of integrating a backhaul customer in a delivery route must be lower than the cost of a pure inbound route to visit this customer. Therefore, an incentive offered by the shipper is always upper bounded by the cost of outsourcing a dedicated inbound vehicle. The competition with the latter comes from the fact that, after deliveries, a vehicle of the carrier has a remaining distance that can be used to provide external services while returning to the depot. We assume that the total remaining distance is used to perform a single backhauling service, either for the shipper or for an external entity.

Another feature of our problem is that outbound and inbound routes are treated differently. Many companies still present separated departments to deal with inbound and outbound logistics (e.g. Marques et al. [2020]). For the shipper, the priority are the outbound logistics, whereas the inbound material typically arrives at the shipper by routes performed by its suppliers. Thus, in our problem setting, the backhauling is opportunistic in the perspective of both players. This means that while outbound routes are previously defined, inbound routes are subjected to competition with outsourcing costs and external incentives. In other words, outbound routes are mandatory and inbound routes are opportunistic.

3.2. Mathematical programming formulation

The following sets are used in the formulation. Set $V = \{0 \cup L \cup B\}$ represents all nodes in the network, where $\{0\}$ is the depot, $L = \{1, ..., n\}$ is the subset of linehaul customers and $B = \{n + 1, ..., n + m\}$ is the subset of backhaul customers. Set $K = \{1, ..., k\}$ denotes the delivery vehicles of the carrier. Each arc (i, j) in the network has an Euclidean distance d_{ij} and an associated symmetric cost c_{ij}, such that $c_{ij} = c_{ji}$ and $i \neq j$. The unitary cost of distance travelled for the shipper is c^U_{ij} and for the carrier is c^L_{ij}. The cost of a dedicated inbound vehicle is $2c^U_{i0}$, which pays the load and no-load distances between a backhaul customer i and the depot $\{0\}$. Each linehaul customer i requires a given quantity q_i to be delivered and the depot requires a minimum amount of raw-materials Q_0 to be collected at backhaul customers. All vehicles have similar capacity C. The total distance travelled by one delivery vehicle cannot exceed the maximum distance allowed D_{max}. The expected unitary profit per unit of distance of an external service outside the collaboration performed by the carrier is given by ϕ.

The routing problem is modelled using a single commodity flow formulation, since only one type of product can be carried on each arc, for delivery or for pickup load. The routing problem is modelled as a VRPSB with the exception that pure inbound routes are also allowed. Allowing the creation of all type of routes (only outbound, only inbound and integrated outbound-inbound) brings more benefits to the optimization than forcing backhaul customers to be visited in integrated routes [Marques et al., 2020].

The profit sharing problem is combined with the routing planning through the incentives for backhauling offered by the shipper to the carrier, which leads to the Collaborative VRPSB. Finally, the collaborative VRPSB can be formulated as a mixed-integer bilevel VRPSB, where the upper level describes the problem of the shipper (Problem (1)-(9)) and the lower level describes the problem of the carrier (Problem (14)-(18)).

The upper level decision variables are:

$$x_{ij}^k := \begin{cases} 1, & \text{if vehicle } k \text{ travels on arc } (i,j) \\ 0, & \text{otherwise} \end{cases} \qquad \forall k \in K, \forall i,j \in V \setminus B$$

$Z_b :=$ incentive offered to visit backhaul customer b, $\quad \forall b \in B$

$O_b :=$ number of visits to backhaul customer b by dedicated inbound vehicles, $\quad \forall b \in B$

$y_{ij} :=$ load in a vehicle between customers i and j, $\quad \forall i,j \in V \setminus B$.

The lower level decision variables are:

$$\hat{x}_{ij}^k := \begin{cases} 1, & \text{if vehicle } k \text{ travels on arc } (i,j) \\ 0, & \text{otherwise} \end{cases} \qquad \forall k \in K, \forall i \in L, \forall j \in B$$

$$Z_{ext}^k := \text{external incentive offered in route } k, \qquad \forall k \in K.$$

3.2.1. Upper level problem

The objective function of the shipper is the minimization of the total cost of the routing plan as in Equation (1) below. The total cost comprises three aspects: *i)* the cost associated to the total distance travelled to visit all linehaul customers, *ii)* the total incentives payed to the carrier to visit backhaul customers, and *iii)* the total cost of outsourcing dedicated inbound vehicles. Note that the upper and lower problems interact through the two variables present in the second term of the objective function, namely Z_b and $\hat{x}_{ij}^k$. Moreover, this term of the function is nonlinear, but it can be linearized as we will see later (through constraints (10)-(13)).

$$\min \sum_{i \in V \setminus B} \sum_{j \in L} \sum_{k \in K} c_{ij}^U \cdot x_{ij}^k + \sum_{i \in L} \sum_{b \in B} \sum_{k \in K} Z_b \cdot \hat{x}_{ib}^k + \sum_{b \in B} O_b \cdot 2c_{b0}^U \tag{1}$$

All routes start at the depot, as expressed by Constraints (2) and the flow constraints in delivery routes, given by Constraints (3), guarantee the connectivity of the locations visited in each route.

$$\sum_{i \in L} x_{0i}^k \leq 1, \qquad \forall k \in K \tag{2}$$

$$\sum_{i\in V\setminus B} x_{ij}^k = \sum_{i\in V\setminus B} x_{ji}^k + \sum_{b\in B} \hat{x}_{jb}^k, \qquad \forall j \in L, \forall k \in K \tag{3}$$

Equations (4) guarantee that all linehaul customers are visited exactly once by only one vehicle.

$$\sum_{i\in V\setminus B} \sum_{k\in K} x_{ij}^k = 1, \qquad \forall j \in L \tag{4}$$

The total load carried on each vehicle must decrease gradually as linehaul customers are visited in each route, which is ensured by Equations (5). These constraints are also subtour elimination constraints. Furthermore, Constraints (6) guarantees that the capacity of each vehicle is never exceeded on each route.

$$\sum_{i\in V\setminus B} y_{ij} = \sum_{i\in V\setminus B} y_{ji} + q_j, \qquad \forall j \in L \tag{5}$$

$$y_{ij} \leq x_{ij}^k \cdot C, \qquad \forall i, j \in V \setminus B, \forall k \in K \tag{6}$$

The demand of linehaul customers is fully satisfied with Equations (7) and the minimum demand of the depot is satisfied with Constraints (8). It is assumed that, each time a vehicle visits a backhaul customer (either on integrated or on pure inbound routes), it returns full to the depot, so that the quantity delivered at the depot matches exactly the total capacity of the vehicle. This rationale is based on the common practice of vehicles travelling in full truck load, such as in the forestry industry [Marques et al., 2020]. It is worth mentioning that, although the decision of which backhaul customer(s) to visit belongs to the carrier, it is the shipper who decides the maximum number of visits. This avoids the shipper paying a double incentive (e.g., the carrier would like to visit a backhaul twice if the incentive is attractive) when only one visit is sufficient to meet the demand of the shipper. Mathematically, this will be guaranteed by adopting the optimistic version of our model discussed later in Section 4 .

$$\sum_{j\in L} y_{0j} = \sum_{j\in L} q_j \tag{7}$$

$$\sum_{i\in L} \sum_{b\in B} \sum_{k\in K} \left(\hat{x}_{ib}^k + O_b\right) \geq \left\lceil \frac{Q_0}{C} \right\rceil \tag{8}$$

The domain of the upper level variables is as follows:

$$x_{ij}^k \in \{0,1\}, Z_b, y_{ij} \geq 0, O_b \in \{0, ..., \lceil \frac{Q_0}{C} \rceil\}, \forall i, j \in V \setminus B, k \in K, b \in B. \tag{9}$$

To linearize the objective function of the upper level, we use the McCormick constraints [McCormick, 1976]. Thus, we introduce a new variable $A_b^k = Z_b \cdot \sum_{i \in L} \hat{x}_{ib}^k$ and derive the following constraints. If $\sum_{i \in L} \hat{x}_{ib}^k = 0$, then the inequality (10) ensures that A_b^k is also zero (A_b^k is higher than a negative number from inequality (12) and cannot be negative due to the Equation (13)). On the other hand, if $\sum_{i \in L} \hat{x}_{ib}^k = 1$, the inequality (10) ensures that A_b^k is lower than M (large number), and inequalities (11) and (12) guarantee that $A_b^k = Z_b$. Note that we can make $M = max\{2c_{i0}^U\}, \forall i \in B$, which is the upper bound of each inventive Z_b.

$$A_b^k \leq M \cdot \sum_{i \in L} \hat{x}_{ib}^k, \qquad \forall b \in B, \forall k \in K \tag{10}$$

$$A_b^k \leq Z_b, \qquad \forall b \in B, \forall k \in K \tag{11}$$

$$A_b^k \geq Z_b - (1 - \sum_{i \in L} \hat{x}_{ib}^k) \cdot M, \qquad \forall b \in B, \forall k \in K \tag{12}$$

$$A_b^k \geq 0, \qquad \forall b \in B, \forall k \in K \tag{13}$$

3.2.2. Lower level problem

The objective function of the carrier is described by Equation (14), which is the maximization of the total profits collected with all routes. The profit collected with integrated routes is determined as the difference between the total incentives accepted and the total travelling cost of including the backhaul customers in delivery routes. The profit collected with the external services is equivalent to the total net external incentives accepted. Finally, the profit collected with a delivery route corresponds to the difference between the total cost charged to the shipper and the total effective cost of the deliveries routes paid by the carrier. Note that this term could be removed since it is constant (it does not include decision variables of the lower level).

$$\begin{aligned} \max \sum_{i \in L} \sum_{b \in B} \sum_{k \in K} Z_b \cdot \hat{x}_{ib}^k - \sum_{i \in L} \sum_{b \in B} \sum_{k \in K} \hat{x}_{ib}^k \cdot (c_{ib}^L + c_{b0}^L) + \\ \sum_{k \in K} Z_{ext}^k \cdot (1 - \sum_{i \in L} \sum_{b \in B} \hat{x}_{ib}^k) + \sum_{i \in V \setminus B} \sum_{j \in L} \sum_{k \in K} (c_{ij}^U - c_{ij}^L) \cdot x_{ij}^k \end{aligned} \tag{14}$$

Constraints (15) forces the precedence constraint of a typical VRPSB, where backhaul customers can only be linked to a last linehaul customer in a route. First, delivery routes are the priority in the VRPSB, since these can only be performed by delivery vehicles. Second, the load to be collected at any backhaul customer can fill all the capacity of a vehicle. Thus, no mixing of delivery and pickup loads is possible.

$$\hat{x}_{bj}^{k} = 0, \qquad \forall b \in B, j \in L, k \in K \tag{15}$$

Constraints (16) enforce that the maximum distance D_{max} is never exceeded in any route.

$$\sum_{i,j \in V \setminus B} d_{ij} \cdot x_{ij}^{k} + \sum_{i \in L} \sum_{b \in B} \sum_{k \in K} \hat{x}_{ib}^{k} \cdot (d_{ib} + d_{b0}) \leq D_{max}, \qquad \forall k \in K \tag{16}$$

The external incentive, given by Equations (17), must be equal to the remaining distance of a delivery route multiplied by the unitary profit ϕ. This is an important aspect considered by the carrier, since the shortest are the delivery routes, the higher the distance remaining to perform additional services.

$$Z_{ext}^{k} = (D_{max} - \sum_{i \in V \setminus B} \sum_{j \in L} d_{ij} \cdot x_{ij}^{k}) \cdot \phi, \qquad \forall k \in K \tag{17}$$

The domain of the lower level variables is as follows:

$$\hat{x}_{ib}^{k} \in \{0, 1\}, \qquad \forall i \in L, \forall b \in B, \forall k \in K \tag{18}$$

The nonlinear terms in the objective function of the lower level are linearized through the definition of a new variable $G^k = Z_{ext}^{k} \cdot (1 - \sum_{i \in L} \sum_{b \in B} \hat{x}_{ib}^{k})$ and the respective McCormick constraints (19)-(22). Note that M can be set to the upper bound of the maximum external service, i.e. $M = D_{max} \cdot \phi$.

$$G^k \leq M \cdot (1 - \sum_{i \in L} \sum_{b \in B} \hat{x}_{ib}^{k}), \qquad \forall k \in K \tag{19}$$

$$G^k \leq Z_{ext}^{k}, \qquad \forall k \in K \tag{20}$$

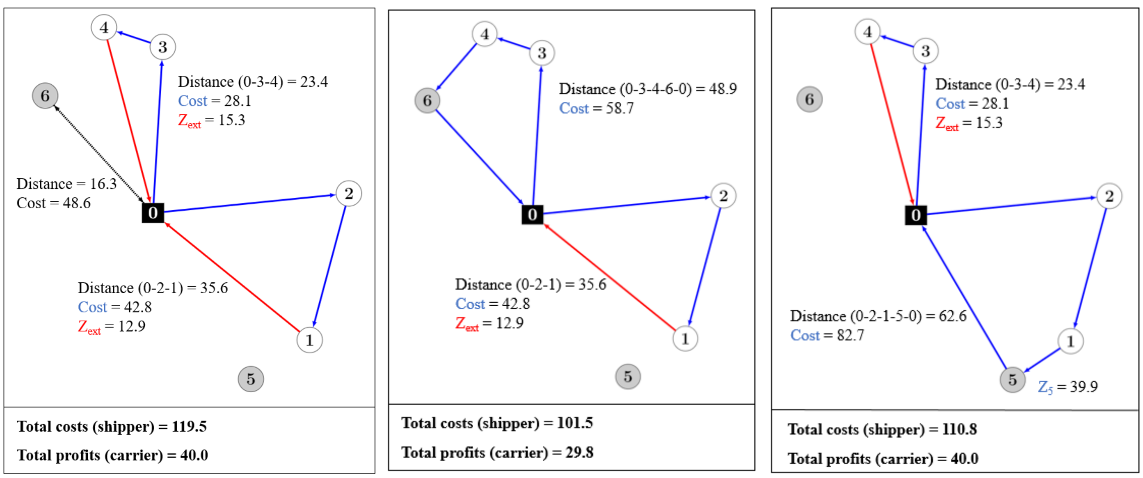
$$G^k \geq Z_{ext}^{k} - \sum_{i \in L} \sum_{b \in B} \hat{x}_{ib}^{k} \cdot M, \qquad \forall k \in K \tag{21}$$

$$G^k \geq 0, \qquad \forall k \in K \tag{22}$$

3.3. Numerical example

In the following numerical example, we demonstrate the rationale upon the bilevel model is build to model collaboration. Consider a shipper that needs to send requests to customers 1, 2, 3 and 4, and requires a full truck load from one of the backhaul customers 5 or 6. The carrier has two vehicles available.

Figure 1 illustrates the bilevel solution and two non-collaborative solutions, for the purpose. The non-collaborative solutions correspond to the separated planning (VRP, optimization of inbound and outbound routes independently) and the integrated planning (VRPSB, all routes are optimized under the perspective of the shipper only). The non-collaborative models are detailed in A.



(a) VRP (b) VRPSB (c) Bilevel VRPSB

Figure 1: Routing plans obtained with each planning approach. The square is the depot, the white circles are linehaul customers and the grey circles are backhaul costumers. Blue lines represent the (part of) the routes performed by the carrier to serve the requests of the shipper. Red lines represent the part of the routes performed by the carrier to serve external services. Black line represents the pure inbound route outsourced to a supplier.

With a VRP (separated planning) (Figure 1a), the solution of the shipper includes the least cost routes for deliveries and an inbound route with the backhaul nearest to the depot. The carrier benefits with the external incentives from both delivery routes. This is designated as the stand alone solution.

With a VRPSB (integrated planning) (Figure 1b), where the shipper controls the fleet of vehicles, it would select backhaul customer 6 instead of backhaul customer 5, due to the lower backhaul distance, i.e. $d_{46} + d_{60} < d_{15} + d_{50}$. However, the carrier would lose profits in visiting a backhaul customer in a route that generates a higher external incentive.

Under a bilevel approach (Figure 1c), the shipper sets only an incentive for backhaul customer 5. No incentive is provided for backhaul customer 6 because it would be higher or, at least, equal to the cost of an inbound route. The carrier, in turns, would integrate the backhaul visit in its longest route, where the backhaul incentive competes with a lower external incentive, leaving the shortest route to guarantee a higher external incentive.

In this example, it was demonstrated how the bilevel VRPSB incorporates the rational response of the carrier into the problem of the shipper, by guaranteeing that the profits achieved with collaboration are not lower than its stand alone solution. In Section 5.3, we compare the bilevel approach with a traditional VRPSB with side payments, where the shipper plans all the delivery and integrated routes and decides on the side payment of each integrated route.

4. Methodology

There are no out of the box solvers to tackle our bilevel model. One classic way to handle mathematically a bilevel problem is to reformulate it into a single-level optimization problem [Sinha et al., 2018] and then solve it with an exact method to find the global optimum. The goal of the reformulation is to transform the rational set of the lower level problem into a set of constraints that are further embedded into the upper level problem. Popular reformulation techniques are based on the Karush-Kuhn-Tucker (KKT) conditions [Zeng and An, 2014] and duality theory [Garcia-Herreros et al., 2016]. However, these methods can only apply when the lower level is concave (maximization), for which these optimality conditions are necessary and sufficient. Recently, solvers for mixed integer bilevel programming are available: MibS [2] and bilevel [3], whose technical details are respectively discussed in Tahernejad et al. [2020] and Fischetti et al. [2017]. In both solvers, only the integer upper level variables can appear in the lower level, since such assumption guarantees the existence of an optimal solution [Vicente et al., 1996].

The bilevel VRPSB presents both binary and continuous decision variables, and the upper level continuous variable Z_b appears in the objective function of the lower level. Therefore, the general purpose techniques mentioned above are not suitable for the problem at hand. In order to overcome

[2] available at `https://github.com/coin-or/MibS`
[3] available at `https://msinnl.github.io/pages/bilevel.html`

the lack of solution approaches to our problem, we will focus in its specific structure. Therefore, we first present the properties of the bilevel VRPSB and then the reformulation method used to build an equivalent single-level mixed integer linear programming (MILP) problem. In this way, the obtained single level problem can be solved by off-the-shelf solvers.

4.1. Properties of the bilevel VRPSB

One particular characteristic of bilevel problems is its intrinsic hierarchical structure. The upper level is the dominant player and the first to select an action. Afterwards, the lower level observes the decisions of the upper level and optimizes its own objective function. Each strategy selected by the lower level is called a *rational response* and the set of all responses is known as the *rational set* [Colson et al., 2007; Safari et al., 2014; Sun et al., 2008]. Knowing the rational set, the upper level can anticipate the response of the lower level and decide the final strategy that minimizes its costs.

The bilevel VRPSB states that, for each strategy of the shipper, i.e. for a fixed set of upper decision variables, the carrier accepts or withdraws each incentive offered by the shipper, in each route. If any incentive offered is not accepted in a route, the carrier performs an external service. The profit for the carrier of an integrated route P_b^k is determined as the difference between the incentive Z_b and the additional travelling cost to visit backhaul customer b, as follows:

$$P_b^k = Z_b - \sum_{i \in L} \hat{x}_{ib}^k \cdot (c_{ib}^L + c_{b0}^L), \qquad \forall k \in K, \forall b \in B. \tag{23}$$

The profit of an external service P_E^k equals the external incentive on that route, as follows:

$$P_E^k = Z_{ext}^k, \qquad \forall k \in K. \tag{24}$$

Given a fixed input of upper decision variables, the optimal response of the lower level problem, F^k, corresponds to the optimal solution of the carrier, as follows:

$$F^k = \max\{P_E^k, \max_{b \in B}\{P_b^k\}\}, \qquad \forall k \in K. \tag{25}$$

Equation (25) ensures that the carrier will always choose the most profitable service to perform in each route. If one or more incentives are offered by the shipper, the carrier compares the profits obtained with visits to backhaul customers P_b^k with the profits obtained with an external service P_E^k, and selects the most rewarding one. When the rational response is not singleton, i.e. more than one response of the lower level may be obtained for a single strategy of the upper level, two different approaches can be applied - the *optimistic* and the *pessimistic* approaches [Colson et al., 2007; Sinha et al., 2018]. The optimistic approach assumes that the lower level will select the rational response that is more favourable to the upper level. In opposition, the pessimistic approach assumes that the lower level will select the least favourable response. The optimistic approach is the most investigated in literature and allows to slightly reduce the non-cooperative nature of bilevel models [Kozanidis et al., 2013; Sinha et al., 2018]. For our model, the reformulation is built under an optimistic approach. Hence, whenever the carrier is presented with a backhaul incentive and an external incentive equally profitable (i.e. $\max_{b \in B} P_b^k = P_E^k$), the carrier will select to visit a backhaul customer, if the demand of the depot is not yet totally satisfied. Note that if there are two vehicles, k and k', wishing to visit a backhaul b^* (i.e., $\max_{b \in B} P_b^k = P_{b^*}^k$ and $P_{b^*}^{k'} = \max_{b \in B} P_b^{k'}$), but only one is sufficient to satisfy the demand of the depot, then the shipper could be better-off by decreasing the incentive to visit b^*, until it is only preferred by one vehicle. In this way, the shipper is guaranteed not to pay more incentives than the necessaries.

It is also worth mentioning that, although we use an optimistic approach, the problem of the lower level considers also its most optimistic case. In fact, as the upper level does not know exactly the problem of the lower level, it is assumed that the lower level can always achieve the highest possible profits in each route, considering that all the remaining distance can be used to provide external services. Thus, the incentives offered by the upper level are set to cover the highest external incentive in each route.

4.2. Single-level reformulation

The detailed description of the reformulation technique applied in this work is presented in the following steps.

Step 1. The integer variables of the lower level problem, $\hat{x}_{ib}^k$, are replaced by the corresponding flow variables of the upper level, x_{ij}^k. By extending the entire set of locations to $\{i, j\} \in V = \{0\} \cup L \cup B$, all constraints of the lower level can be re-written using only the decision variables of the upper level, as follows.

$$x_{bj}^k = 0, \qquad \forall b \in B, j \in L, k \in K \tag{15'}$$

$$\sum_{i,j \in V} d_{ij} \cdot x_{ij}^k \leq D_{max}, \qquad \forall k \in K \tag{16'}$$

$$Z_{ext}^k = (D_{max} - \sum_{i \in V} \sum_{j \in L} d_{ij} \cdot x_{ij}^k) \cdot \phi, \qquad \forall k \in K. \tag{17''}$$

Step 2. The optimal response of the lower level in Equation (25) can be divided into the Equations (26) and (27), which together translates the maximization of the profits in all routes. The former represents the profit obtained by accepting to visit backhaul customers and the latter represent the profits obtained with external services.

$$F^k \geq P_b^k, \qquad \forall k \in K, \forall b \in B \tag{26}$$

$$F^k \geq P_E^k, \qquad \forall k \in K. \tag{27}$$

Step 3. To enforce that, in each route, the carrier decides in favour for the incentive that maximizes its profits, a set of disjunctive constraints are established, using M as a large number (e.g., $M = D_{max} \cdot \phi$). Constraints (28) introduce a binary variable H_E^k that takes the value of 1 if the external incentive Z_{ext}^k is higher than the profit of visiting any backhaul b in route k (i.e. $P_E^k > P_b^k$) and 0 otherwise. Similarly, Constraints (29) introduce a binary variable H_b^k that takes the value of 1 if $P_b^k > P_E^k$ and $P_b^k > P_i^k, \forall i \in B, b \neq i$, and 0 otherwise.

$$F^k - P_E^k \leq (1 - H_E^k) \cdot M, \qquad \forall k \in K \tag{28}$$

$$F^k - P_b^k \leq (1 - H_b^k) \cdot M, \qquad \forall k \in K, \forall b \in B. \tag{29}$$

Step 4. If the external incentive is accepted for a given route ($H_E^k = 1$), all the arcs between backhauls and linehaul customers must be zero, which is enforced by Constraints (30). Otherwise, the incentive offered for a backhaul b is preferred ($H_b^k = 1$), for which Constraints (31) ensure that, for route k the arcs containing the remaining backhauls are all set to zero, since each vehicle visits only one backhaul. Constraints (32) guarantee that if $H_b^k = 1$, then, at least, one arc linking a linehaul to a backhaul customer must exist. Equation (33) ensures that only a single incentive may be accepted by the carrier in each route. The domain of the new decision variables are presented in (34).

$$(1 - H_E^k) \geq x_{ib}^k, \qquad \forall k \in K, \forall i \in L, \forall b \in B \tag{30}$$

$$(1 - H_b^k) \geq x_{ij}^k, \qquad \forall k \in K, \forall i \in L, \forall j \neq b \in B \tag{31}$$

$$\sum_{i \in L} x_{ib}^k \geq H_b^k, \qquad \forall k \in K, \forall b \in B \tag{32}$$

$$\sum_{b \in B} H_b^k + H_E^k = 1, \qquad \forall k \in K \tag{33}$$

$$H_b^k, H_E^k \in \{0, 1\}, \qquad \forall b \in B, k \in K \tag{34}$$

Concluding the reformulation procedure, the resultant model is a single level VRPSB with the following (MILP) formulation:

$$\min \sum_{i \in V} \sum_{j \in L} \sum_{k \in K} c_{ij}^U \cdot x_{ij}^k + \sum_{b \in B} \sum_{k \in K} A_b^k + \sum_{b \in B} O_b \cdot 2c_{b0}^U \tag{35}$$

$$\begin{aligned} s.t.&(2) - (9) \\ &(15'), (16) \text{ and } (17'') \\ &(23) - (24) \\ &(26) - (34) \end{aligned} \tag{36}$$

Theorem 1. *Any optimal solution of Problem* (35) *is also optimal to the optimistic version of the bilevel VRPSB. Moreover, this optimality equivalence continues to hold when adding Constraint* (37) *to the upper level of the bilevel VRPSB and Problem* (35)*.*

5. Computational experiments

The computational experiments performed in this section cover three main analysis. The first set of experiments aims to evaluate the solutions obtained with the bilevel approach for the collaborative transportation problem. Bilevel solutions are compared with stand alone solutions and the synergy values of collaboration are determined. The second set of experiments aims to analyse potential side payments schemes, and to compare them with our bilevel approach. We argue that our bilevel approach, as it incorporates the rational response of the carrier into the problem of the shipper, provides balanced solutions as well as higher synergy than side payments schemes. The third set of experiments analyses the scalability of our approach to solve the bilevel model (single level reformulation) in comparison with solving non-collaborative models.

Each set of experiments reports the results for two different scenarios: one that allows multiple visits to the same backhaul customer, and one that forbids more than one visit to the same backhaul customer. The motivation behind is to analyze the efficiency of the bilevel approach to deal with different contexts of backhauling. Most of the literature considers that backhaul customers can only be visited once, but in practice, they may have enough availability of raw-material, which allows multiple pickups [Marques et al., 2020; Santos et al., 2020]. The bilevel model presented in Section 3.2 allows multiple backhaul visits. To forbid multiple visits to the same backhaul customer, it is sufficient to include constraint (37) and change the dimension of the variable to $O_b \in \{0, 1\}$. Hence, Constraint 37 ensures that each backhaul customer can only be visited once by all type of vehicles. It is worth mentioning that the decision on limiting the number of backhaul visits is responsibility of the shipper. Hence, if a backhaul customer is profitable to be visited in two routes but only one is necessary, the carrier would have to accept the external incentive of one of the two routes.

$$\sum_{i \in L} \sum_{k \in K} \hat{x}_{ib}^{k} + O_b \leq 1, \qquad \forall b \in B \tag{37}$$

All data sets used in the computational experiments are collected from Augerat et al. [1995] and adapted to the VRPSB, as described next. All models are coded in Python 3.6.3 and solved with Gurobi, on a computer equipped with the processor Intel Core i7 of 2.20GHz and 16 GB of RAM.

5.1. Data sets

The original data sets define the number of locations (depot and linehaul customers) and respective coordinates, the demand of linehaul customers, the number of vehicles and respective capacity. In particular, we have used all instances up to 22 locations from data set P and randomly selected instances up to 50 locations from data set B.

The adapted data sets comprise the following modifications: *i)* the backhaul customers locations correspond to the 5 to 17 last locations in the original data set, and *ii)* the number of vehicles available is determined as the minimum number required to meet all the demand of linehaul customers without exceeding vehicle capacity, i.e. $|K| = \lceil \frac{\sum_{i \in L} q_i}{C} \rceil$ in each data set. The adapted instances used in this work are reported in Table 1.

The remaining parameters of the bilevel VRPSB are set as follows. The depot demand is a multiple of the capacity of a vehicle, such that $Q_0 = [C, 2C, \ldots, |K|C]$. The maximum distance allowed is 150 for instances with less than 20 nodes, and 250 for the remaining ones. These limits are sufficient to create integrated inbound-outbound routes. The unitary cost per unit of distance is set to 1.2 € for the shipper and 1.0 € for the carrier, following the study in Yu and Dong [2013]. The unitary profit obtained with an external service equals the unitary profit obtained with a delivery route, such that $\phi = 0.2$ €\unit of distance.

Throughout the experiments, 10 small instances with varying Q_0 are used, since optimal solutions are obtained in reasonable time. The motivation to use different Q_0 values is to determine if increasing the number of required visits to backhaul customers increases the performance of the collaboration. The last set of experiments uses also medium size instances. We anticipate that large instances are not tested, as the exact method developed in this work is not efficient to solve them. However, the motivation of our work is not on the efficiency of the solution method but rather on the efficiency of the bilevel formulation to handle the collaborative problem.

Table 1: Adapted instances used in this work

Original	Adapted	$\|L\|$	$\|B\|$	$\|K\|$	D_{max}
P-n16-k8	A	10	5	6	150
P-n19-k2	B	13	5	2	150
P-n20-k2	C	14	5	2	150
P-n22-k2	D	16	5	2	150
P-n22-k8	E	16	5	6	150
B-n31-k5	F	25	5	4	250
B-n31-k5	G	20	10	3	250
B-n38-k6	H	25	12	4	250
B-n41-k6	I	25	15	4	250
B-n45-k6	J	27	17	4	250
B-n45-k6	K	39	5	6	250
B-n45-k6	L	35	9	5	250
B-n50-k6	M	44	5	6	250
B-n50-k6	N	40	9	6	250

5.2. Bilevel versus traditional planning

With the bilevel VRPSB model proposed in this work, we aim to demonstrate that it is efficient to handle the collaborative problem, while incorporating the different goals of the players, and solving the routing and the pricing problems simultaneously. The impact of collaboration is determined by two measures. The first is the network costs (NC), which is given by the difference between the total costs of the shipper and the total profits of the carrier (Equation 38). The second is the synergy value (SV), which provides the percentage gain that a collaborative network can reach compared with the stand alone solution (Equation 39).

$$NC = Costs - Profits \tag{38}$$

$$SV = \frac{NC_{VRP} - NC_{collab}}{NC_{VRP}} \tag{39}$$

Tables 2 and 3 present a comparison between stand alone solutions obtained with the VRP model and collaborative solutions obtained with the bilevel VRPSB model, for the scenarios investigated. Both read as follows: total costs of the shipper (column "Costs"), total profits of the carrier (column "Profits"), costs of outsourcing inbound vehicles to suppliers (column "Out."), network costs (column "NC"), total incentives offered by the shipper and accepted by the carrier (column "Incent."), and the synergy value of the collaborative solution (column "SV").

Table 2 shows that seven out of ten instances do not promote collaboration, i.e. the optimal solution for the shipper is to outsource all necessary vehicles to the suppliers. Despite disappointing, this outcome is reasonable since the same backhaul customer can be visited as many times as the depot demand requires. Thus, when the cost of visiting the backhaul customer closest to the depot with an inbound vehicle is relatively low, and no backhauling incentive is lower, the shipper always tend to outsource all necessary vehicles to that closest backhaul customer. Nonetheless, when collaboration occurs, synergy values can reach about 11%.

From Table 3, it is possible to observe very different results compared with the previous scenario of unlimited visits to backhaul customers. When the number of visits is limited to one, the incentives proposed by the shipper compete with more diverse options than only the nearest backhaul customer. This seems to increase heavily the potential for collaboration, as shown in eight out of ten instances. Moreover, any solution of the scenario with limited visits provides equal or higher synergy values than in the scenario with unlimited backhaul visits.

Table 2: Stand alone and collaborative solutions in a scenario with unlimited visits to backhaul customers.

		VRP (stand alone) solutions				Bilevel VRPSB (collaborative) solutions					
Inst.	Q_0	Costs	Profits	Outs.	NC	Costs	Profits	Incent.	Outs.	NC	SV
A	C	275	180	68	95	265	180	58	0	85	10.0%
	2C	342	180	135	162	324	180	117	0	144	11.3%
	3C	410	180	203	230	387	180	180	0	207	10.0%
	4C	478	180	270	298	454	185	247	0	270	9.4%
	5C	545	180	338	365	521	187	314	0	334	8.6%
B	C	192	60	38	132	192	60	0	38	132	0.0%
	2C	230	60	76	170	230	60	0	76	170	0.0%
C	C	204	60	38	144	204	60	0	38	144	0.0%
	2C	242	60	76	182	242	60	0	76	182	0.0%
D	C	207	60	38	147	207	60	0	38	147	0.0%
	2C	245	60	76	185	245	60	0	76	185	0.0%
E	C	323	180	53	143	323	180	0	53	143	0.0%
	2C	376	180	106	196	376	180	0	106	196	0.0%
	3C	429	180	159	249	429	180	0	159	249	0.0%
	4C	482	180	212	302	482	180	0	212	302	0.0%
	5C	535	180	265	355	535	180	0	265	355	0.0%
F	C	499	200	113	299	478	200	88	0	278	7.0%
	2C	611	200	226	411	581	200	177	0	381	7.4%
	3C	724	200	338	524	685	200	281	0	485	7.4%
	4C	837	200	451	637	798	200	281	113	598	6.1%
G	C	429	150	91	279	424	150	86	0	274	1.7%
	2C	520	150	182	370	515	150	86	91	365	1.2%
	3C	611	150	273	461	606	150	86	182	456	1.0%
H	C	444	200	51	244	444	200	0	51	244	0.0%
	2C	495	200	101	295	495	200	0	101	295	0.0%
	3C	546	200	152	346	546	200	0	152	346	0.0%
	4C	596	200	203	396	596	200	0	203	396	0.0%
I	C	469	200	35	269	469	200	0	35	269	0.0%
	2C	504	200	70	304	504	200	0	70	304	0.0%
	3C	538	200	105	338	538	200	0	105	338	0.0%
	4C	573	200	140	373	573	200	0	140	373	0.0%
J	C	379	200	50	179	379	200	0	50	179	0.0%
	2C	429	200	100	229	429	200	0	100	229	0.0%
	3C	479	200	150	279	479	200	0	150	279	0.0%
	4C	529	200	200	329	529	200	0	200	329	0.0%
Average											2.3%

Table 3: Stand alone and collaborative solutions in a scenario with limited visits to backhaul customers.

		VRP (stand alone) solutions				Bilevel VRPSB (collaborative) solutions					
Inst.	Q_0	Costs	Profits	Outs.	NC	Costs	Profits	Incent.	Outs.	NC	SV
A	C	275	180	68	95	265	180	58	0	85	10.0%
	2C	345	180	137	165	324	180	117	0	144	12.5%
	3C	416	180	209	236	387	180	180	0	207	12.4%
	4C	488	180	281	308	457	180	245	0	277	10.1%
	5C	562	180	355	382	528	180	248	68	348	8.9%
B	C	192	60	38	132	192	60	0	38	132	0.0%
	2C	264	60	110	204	246	60	53	38	186	8.5%
C	C	204	60	38	144	204	60	0	38	144	0.0%
	2C	276	60	110	216	255	60	50	38	195	9.6%
D	C	207	60	38	147	207	60	0	38	147	0.0%
	2C	270	60	101	210	253	60	45	38	193	8.3%
E	C	323	180	53	143	323	180	0	53	143	0.0%
	2C	393	180	123	213	393	180	69	53	213	0.2%
	3C	466	180	196	286	466	180	69	126	286	0.1%
	4C	542	180	272	362	542	180	69	202	362	0.1%
	5C	623	180	352	443	622	180	69	283	442	0.1%
F	C	499	200	113	299	478	200	88	0	278	7.0%
	2C	616	200	230	416	581	200	177	0	381	8.4%
	3C	733	200	348	533	685	200	281	0	485	9.1%
	4C	854	200	468	654	802	200	281	117	602	7.8%
G	C	429	150	91	279	424	150	86	0	274	1.7%
	2C	534	150	197	384	515	150	86	91	365	5.0%
	3C	647	150	310	497	609	150	179	91	459	7.6%
H	C	444	200	51	244	444	200	0	51	244	0.0%
	2C	502	200	108	302	502	200	0	108	302	0.0%
	3C	584	200	191	384	583	200	77	108	383	0.3%
	4C	672	200	278	472	671	200	83	191	471	0.2%
I	C	469	200	35	269	469	200	0	35	269	0.0%
	2C	508	200	75	308	508	200	0	75	308	0.0%
	3C	554	200	121	354	554	200	0	121	354	0.0%
	4C	601	200	168	401	601	200	0	168	401	0.0%
J	C	379	200	50	179	379	200	0	50	179	0.0%
	2C	439	200	109	239	439	200	0	109	239	0.0%
	3C	499	200	169	299	499	200	0	169	299	0.0%
	4C	561	200	232	361	561	200	0	232	361	0.0%
Average											3.7%

We were expecting to see an increase in the synergy value with increasing Q_0, but such was not verified for all instances. Therefore, we cannot conclude on the impact of increasing the demand of the depot. Overall, the results show that any solution of the bilevel model leads to costs for the shipper that are always equal or lower than the stand alone solution. Similarly, any solution of the bilevel model leads to profits for the carrier that are always equal or higher than the stand alone solution. Based on these results, we argue that our bilevel approach is efficient to deal with the collaborative problem where different goals are considered and decisions are taken hierarchically. We must also emphasise that, although the solution of the bilevel VRPSB seems to only benefit the shipper, in fact the carrier also gains using a bilevel approach, since the upper level considers the most optimistic case of the lower level problem, as implicit by the bilevel formulation (demonstrated in Section 4.1).

5.3. Bilevel versus compensation schemes

One alternative to model a collaborative transportation problem involves a leading participant optimizing selfishly the routing problem, and then compensating another participant with a side payment, so that the latter does not lose with the collaboration [Özener et al., 2011]. Some works in the literature determine the side payments as a fixed value (e.g., Caballini et al. [2016], Defryn et al. [2016]), whereas other compute the side payment as a value dependent on the distance (e.g., Liu et al. [2010], Dahl and Derigs [2011], Archetti et al. [2016]).

In this work, we use the integrated problem (VRPSB) to model selfishly the routing problem, and we propose two different compensation schemes. In the first scheme, the side payment corresponds to the difference in the profits of the carrier between the stand alone solution (VRP) and the integrated solution (VRPSB). The first side payment is designated as SP_{Δ} and their values are presented in C, for each VRPSB solution (before side payment). The second scheme computes the side payment as a value proportional to the backhaul distance. This is designated as $SP(sp)$, where sp stands for the proportion used, as it is computed as Equation (40). The first compensation scheme provides the side payment after the routing, whereas the second determines the side payment while solving the routing problem.

In this section, we compare the performance of the collaboration of our bilevel approach with compensation schemes. Table 4 report the objective functions of the shipper and the carrier, for the different side payments, along with the synergy value of the solutions. To facilitate the comparison, the synergy values of the bilevel solutions are also reported. Since the results obtained with both scenarios (unlimited and limited backhauls visits) follow the same trend, the results for the last case are presented in D.

$$SP(sp) = sp \cdot (d_{lb} + d_{b0}), \qquad \forall\ sp = \{0.50, 0.75\}, l \in L, b \in B \tag{40}$$

In the bilevel model we assume an optimistic approach such that when the profit of an external incentive equals the profit of a given backhaul visit, the carrier performs an integrated route. The same assumption applies to the case of using side payments. Therefore, as expected, a side payment SP_{Δ} would always motivate the carrier to collaborate because the profits gained are the same as in the stand alone solution. However, for most of the instances reported in Table 4, the costs of the shipper are higher than its stand alone solution. In these cases, the network costs surpass the stand alone solution, which result in negative synergy values. These results also put in evidence the lack of efficiency of determining the side payment only after solving the routing optimization problem.

With respect to the schemes where the side payments are determined while solving the routing problem, the results show that when using the highest proportion ($sp = 0.75$) the solutions tend to be non-collaborative ($SV = 0.0\%$). On the other hand, using the lowest proportion ($sp = 0.50$), many solutions do not attend the individual rationality of the carrier, since the profits are lower than in the stand alone solution (e.g., instances F, G, J). In such cases, collaboration would not take place. Nonetheless, Table 4 also show that, when collaboration occurs, several solutions provided by these side payments allow the carrier to collect more profits than with the bilevel approach, but this comes at higher costs for the shipper.

Generally, the results demonstrate that any solution of the bilevel approach has higher or equal synergy value than any solution obtained through the compensation schemes analyzed. Based on these results, we argue that the bilevel model proposed can more effectively capture the interactions of the different players than the compensation schemes, guaranteeing a more balanced solution with a higher synergy value.

Finally, as also observed from the results in the previous section, synergy values obtained with any side payments are higher, on average, for a scenario with limited visits than a scenario with unlimited visits to backhaul customers.

Table 4: Comparison of synergy values from bilevel solutions and from the integrated planning with different side payment schemes, for the case of unlimited visits to backhaul customers.

			SP_{Δ}			$SP(0.50)$			$SP(0.75)$		Bilevel
Inst.	Q_0	Costs	Profits	SV	Costs	Profits	SV	Costs	Profits	SV	SV
A	C	266	180	9.3%	267	181	9.3%	275	180	0.0%	**10.0%**
	2C	324	180	11.3%	329	185	11.3%	342	180	0.0%	**11.3%**
	3C	388	180	9.5%	394	186	9.5%	410	180	0.0%	**10.0%**
	4C	451	180	9.0%	461	190	9.0%	478	180	0.0%	**9.4%**
	5C	514	180	8.6%	528	194	8.6%	545	180	0.0%	**8.6%**
B	C	198	60	-4.7%	192	60	0.0%	192	60	0.0%	**0.0%**
	2C	236	60	-3.7%	230	60	0.0%	230	60	0.0%	**0.0%**
C	C	213	60	-6.1%	204	60	0.0%	204	60	0.0%	**0.0%**
	2C	254	60	-6.6%	242	60	0.0%	242	60	0.0%	**0.0%**
D	C	215	60	-5.5%	207	60	0.0%	207	60	0.0%	**0.0%**
	2C	259	60	-7.4%	245	60	0.0%	245	60	0.0%	**0.0%**
E	C	334	180	-7.6%	323	180	0.0%	323	180	0.0%	**0.0%**
	2C	404	180	-13.9%	376	180	0.0%	376	180	0.0%	**0.0%**
	3C	457	180	-11.0%	429	180	0.0%	429	180	0.0%	**0.0%**
	4C	510	180	-9.0%	482	180	0.0%	482	180	0.0%	**0.0%**
	5C	563	180	-7.7%	535	180	0.0%	535	180	0.0%	**0.0%**
F	C	478	200	7.0%	475	197	7.0%	487	210	7.0%	**7.0%**
	2C	581	200	7.4%	576	195	7.4%	600	210	5.1%	**7.4%**
	3C	685	200	7.4%	688	198	6.5%	713	210	4.0%	**7.4%**
	4C	802	200	5.4%	801	198	5.4%	826	210	3.3%	**6.1%**
G	C	424	150	1.7%	420	146	1.6%	429	150	0.0%	**1.7%**
	2C	517	150	0.7%	511	146	1.2%	520	150	0.0%	**1.2%**
	3C	608	150	0.5%	602	146	1.0%	611	150	0.0%	**1.0%**
H	C	469	200	-10.0%	444	200	0.0%	444	200	0.0%	**0.0%**
	2C	520	200	-8.3%	495	200	0.0%	495	200	0.0%	**0.0%**
	3C	570	200	-7.1%	546	200	0.0%	546	200	0.0%	**0.0%**
	4C	621	200	-6.2%	596	200	0.0%	596	200	0.0%	**0.0%**
I	C	469	200	0.0%	469	200	0.0%	469	200	0.0%	**0.0%**
	2C	504	200	0.0%	504	200	0.0%	504	200	0.0%	**0.0%**
	3C	538	200	0.0%	538	200	0.0%	538	200	0.0%	**0.0%**
	4C	573	200	0.0%	573	200	0.0%	573	200	0.0%	**0.0%**
J	C	398	200	-10.2%	373	175	-10.2%	379	181	-10.2%	**0.0%**
	2C	473	200	-18.8%	423	175	-8.0%	429	181	-8.0%	**0.0%**
	3C	522	200	-15.5%	473	175	-6.6%	479	181	-6.6%	**0.0%**
	4C	572	200	-13.1%	523	175	-5.6%	529	181	-5.6%	**0.0%**
Average				-2.7%			1.4%			-0.3%	**2.3%**

5.4. Computational limits of the collaborative VRPSB

The VRPB is strongly NP-hard since it generalizes the VRP [Toth and Vigo, 2002]. Thus, since the VRPSB generalizes the VRP (note that if there are no bakchauls, we have a simply a VRP), VRPSB is also NP-hard. In addition, bilevel optimization problems are proven to be strongly NP-hard and a mere assessment of the optimal solution is also NP-hard, even for the simplest linear bilevel program [Jeroslow, 1985].

In this section, we aim to evaluate the practical difficulty of solving the collaborative transportation planning formulations proposed. More precisely, we aim to empirically analyze at what extent the collaborative formulation increases the practical difficulty of the problem.

Instances from F to N are tested with a computing time limit of 3600 seconds. Tables 5 and 6 provide the upper bound (UB), the computing time required to achieve the best solution and the percentage gap obtained with each model, for both scenarios (limited and unlimited backhaul visits). To avoid repetition, the UB is only displayed for instances not already covered by Tables 2 and 3.

The results show that the bilevel formulation is effectively the main reason behind the complexity of solving the collaborative problem. The computing time to solve an instance with the bilevel model tends to be higher than with the non-collaborative models, as well as the percentage gap. Nevertheless, the complexity of the bilevel approach does not seem to be much different than for the traditional VRPSB for some instances (e.g., instances L and M). Moreover, the bilevel model solves simultaneously a routing and a pricing problem, which is expected to be more computational challenging than solving a routing problem only.

In general, the computing time to solve the bilevel problem tends to increase with increasing size of the instance. It seems that the number of linehauls customers have more influence than the number of backhaul customers, but some instances present exceptions. For example, instance N has more linehaul customers than instance L but it is solved in a much shorter time. On the other hand, the exact method seems suitable to solve bilevel instances with relatively high number of backhaul customers. These type of instances fit well real industries that have a wide range of backhauling opportunities, such as the forestry [Marques et al., 2020].

Furthermore, it seems more challenging to solve the problems in a scenario with limited visits to backhaul customers, than in a scenario with unlimited visits. On average, both the computing time and the percentage gap are higher for the scenario with limit visits. These results support that limiting the number of visits brings additional complexity the bilevel model, since the incentives for backhauling also compete with diverse options for pure inbound routes other than the least costly one.

Finally, we point out that the focus of this work is on the modelling aspects of the collaborative problem rather than on the solution methods. Using the properties of the bilevel optimization, we have demonstrated an effective way to solve a hierarchical collaborative problem. The rationale used to design the reformulation method could be applied to design a metaheuristic, and thus guarantee higher efficiency when solving the problem.

6. Conclusions and future research

This work investigates an innovative formulation for a collaborative transportation planning between a shipper and a carrier. The shipper offers incentives to the carrier in order to create cost-effective integrated inbound-outbound routes. These incentives compete with each other with other potential incentives offered to the carrier by external companies. The problem of the shipper is a cost minimization VRPSB and the problem of the carrier is a VRP with profits. Based on the hierarchical nature of the players and on the conflicting objectives, the collaborative problem is formulated as a bilevel optimization problem. The upper level describes the problem of the shipper and the lower level describes the problem of the carrier. To solve the bilevel problem, we convert it in an equivalent single-level mixed integer linear program by exploring problem-specific characteristics of the lower level, and then, standard linearization techniques.

This work conducts an extensive analysis on the properties of the bilevel approach to handle the collaborative problem. The bilevel model is compared with traditional non-collaborative routing problems and with different side payment schemes, in order to assess the impact of the collaboration and the approach applied. In addition, the impact of limiting the number of backhaul visits is also evaluated. Finally, the computational limits of the collaborative formulation is compared against traditional single-level routing problems.

The results of this work put in evidence the advantages of the bilevel approach to handle a collaborative transportation planning, although the computational effort tend to be higher than than the traditional non-collaborative formulations. Therefore, one main direction for future work includes the development of an efficient metaheuristic to solve the problem. Such metaheuristic could encompass a genetic algorithm or another method of the family of evolutionary algorithms,

Table 5: Computational performance of each model, considering unlimited backhaul visits.

		VRP			VRPSB			Bilevel		
Inst.	Q_0	UB	gap	$time$	UB	gap	$time$	UB	gap	$time$
F	C		0.0%	10		0.0%	25		0.0%	92
	2C		0.0%	11		0.0%	19		0.0%	1579
	3C		0.0%	10		0.0%	43		0.0%	145
	4C		0.0%	9		0.0%	76		0.0%	118
G	C		0.0%	14		0.0%	21		0.0%	24
	2C		0.0%	16		0.0%	39		0.0%	73
	3C		0.0%	6		0.0%	30		0.0%	61
H	C		0.0%	5		0.0%	17		0.0%	25
	2C		0.0%	5		0.0%	5		0.0%	27
	3C		0.0%	5		0.0%	10		0.0%	24
	4C		0.0%	6		0.0%	16		0.0%	27
I	C		0.0%	13		0.0%	36		0.0%	35
	2C		0.0%	24		0.0%	30		0.0%	70
	3C		0.0%	18		0.0%	25		0.0%	57
	4C		0.0%	15		0.0%	45		0.0%	45
J	C		0.0%	10		0.0%	37		0.0%	377
	2C		0.0%	11		0.0%	34		0.0%	98
	3C		0.0%	15		0.0%	19		0.0%	131
	4C		0.0%	14		0.0%	19		0.0%	457
K	C	472	0.0%	287	455	0.0%	1072	472	0.5%	3600
	2C	522	0.0%	289	499	0.0%	161	522	0.0%	650
	3C	571	0.0%	162	549	0.0%	652	571	0.0%	2568
	4C	621	0.0%	114	599	0.0%	223	621	0.0%	416
L	C	452	0.0%	169	435	0.0%	1079	452	0.0%	1224
	2C	502	0.0%	283	477	0.0%	1248	502	0.0%	1949
	3C	552	0.0%	825	527	0.5%	3600	552	0.0%	1531
	4C	601	0.0%	877	577	0.0%	1216	601	0.0%	796
M	C	542	1.9%	3600	527	2.7%	3600	543	2.1%	3600
	2C	591	1.0%	3600	567	1.1%	3600	595	2.5%	3600
	3C	640	0.0%	3545	617	1.6%	3600	646	3.3%	3600
	4C	692	1.8%	3600	666	0.0%	2215	702	7.3%	3600
N	C	474	0.0%	22	460	0.0%	11	474	0.0%	444
	2C	523	0.0%	22	500	0.0%	14	523	0.0%	985
	3C	573	0.0%	22	548	0.0%	4	573	0.0%	206
	4C	622	0.0%	21	597	0.0%	34	622	0.0%	167
Average			0.1%	504		0.2%	654		0.4%	926

Table 6: Computational performance of each model, considering limited backhaul visits.

		VRP			VRPSB			Bilevel		
Inst.	Q_0	UB	gap	$time$	UB	gap	$time$	UB	gap	$time$
F	C		0.0%	9		0.0%	17		0.0%	45
	2C		0.0%	12		0.0%	15		0.0%	114
	3C		0.0%	11		0.0%	68		0.0%	242
	4C		0.0%	23		0.0%	22		0.0%	37
G	C		0.0%	12		0.0%	5		0.0%	79
	2C		0.0%	15		0.0%	32		0.0%	40
	3C		0.0%	7		0.0%	10		0.0%	121
H	C		0.0%	4		0.0%	7		0.0%	33
	2C		0.0%	12		0.0%	21		0.0%	41
	3C		0.0%	4		0.0%	11		0.0%	362
	4C		0.0%	8		0.0%	17		0.0%	509
I	C		0.0%	11		0.0%	28		0.0%	451
	2C		0.0%	12		0.0%	5		0.0%	202
	3C		0.0%	23		0.0%	12		0.0%	224
	4C		0.0%	26		0.0%	17		0.0%	146
J	C		0.0%	22		0.0%	16		0.0%	338
	2C		0.0%	14		0.0%	13		0.0%	1026
	3C		0.0%	34		0.0%	14		0.0%	587
	4C		0.0%	29		0.0%	24		0.0%	173
K	C	472	0.0%	290	455	0.0%	368	472	0.0%	873
	2C	538	0.0%	162	503	0.0%	217	538	0.0%	1800
	3C	649	0.0%	73	578	0.0%	1374	637	1.7%	3600
	4C	773	0.0%	81	654	0.8%	3600	734	1.2%	3600
L	C	452	0.0%	173	435	0.0%	633	452	4.8%	3600
	2C	512	0.0%	338	477	0.5%	3600	512	1.3%	3600
	3C	578	0.0%	422	532	0.0%	635	580	11.4%	3600
	4C	661	0.0%	324	591	0.0%	695	658	0.6%	3600
M	C	542	2.0%	3600	527	2.2%	3600	578	11.8%	3600
	2C	612	0.0%	2160	567	0.0%	2308	623	4.4%	3600
	3C	725	0.7%	3600	627	1.3%	3600	701	3.5%	3600
	4C	843	1.6%	3600	690	0.0%	1014	789	3.3%	3600
N	C	474	0.0%	22	460	0.0%	8.5	474	0.0%	1062
	2C	529	0.0%	19	500	0.0%	10	529	0.0%	799
	3C	600	0.0%	22	548	0.0%	9	600	0.0%	235
	4C	690	0.0%	20	600	0.0%	7	677	0.0%	1909
Average			0.1%	434		0.1%	629		1.3%	1356

since these are the most used advanced methods to solve bilevel optimization problems [Sinha et al., 2018]. Furthermore, several successful metaheuristic approaches for VRP exist (e.g., Du et al. [2017]). Hence it is worth exploring how they could integrate pricing decisions and the follower's optimal solution (recall that a bilevel feasible solution requires that the follower selects an optimal solution). The bilevel model proposed in this work is designed under an optimistic approach for both players. Because the objectives of each player are different and they collaborate in a hierarchical structure, considering the most optimistic case of the lower level can be seen as a robust strategy to achieve robust solutions for the upper level. An interesting future line of research could encompass investigating the collaborative problem under a pessimistic approach and compare it the optimistic.

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A. Traditional models

Traditionally, the transportation planning does not consider collaboration. In this section, two traditional formulations are presented for the transportation planning, namely a separated model (which is described by a typical VRP) and an integrated model (which is described by a VRPSB). Both models are simplifications of the bilevel VRPSB presented in Section 3.2 that exclude the lower level problem from the formulation. Consequently, the variable and all constraints related to the incentives are also excluded in the non-collaborative models. On the other hand, the maximum distance allowed per route is taking into account in both models, since the carrier would never accept to exceed this distance, even for a single delivery route. In addition, the objective function of the carrier is treated as an expression in both non-collaborative models. The mathematical formulation of the traditional models are presented.

Separated model

The separated model describes the problem where inbound and outbound routes are planned separately and integrated routes are not allowed. The problem is formulated as an Open VRP and the objective function in (41) minimizes the total routing costs of delivery vehicles, plus the constant expression given by the minimum number of vehicles necessary to satisfy the depot demand. This states that the demand of the depot can only be satisfied by dedicated inbound vehicles. The complete formulation of the separated model is presented as follows:

$$\min \sum_{i \in V} \sum_{j \in L} \sum_{k \in K} c_{ij}^{U} \cdot x_{ij}^{k} + \sum_{b \in B} O_b \cdot 2c_{b0}^{U} \tag{41}$$

$$\text{s.t.}(2) - (4)$$

$$(5) - (37)$$

$$(16)$$

$$x_{ij}^{k}, O_b \in \{0,1\}, y_{ij} \geq 0, \forall i, j \in V = \{0\} \cup L, b \in B, k \in K, w \in \mathbb{Z}_0^{+} \tag{42}$$

Integrated model

The integrated model describes the problem where inbound and outbound routes are planned jointly by the shipper. This model considers that the shipper assumes control over all vehicles of the carrier used in the network. As the lower level variables are not considered, the shipper no more compete with others for backhaul routes. Instead, the shipper assumes that the unitary cost to visit a backhaul customer is the same as visiting a linehaul customer. Thus, the objective function in (43) is to minimize the total routing costs and outsourcing of dedicated inbound vehicles. The constraints of the integrated model are the same as those from the upper level problem with the additional backhaul customers constraints of the lower level problem. Note that the variables of the lower level are substituted by variables of the upper level in the backhaul customers constraints.

$$\min \sum_{i \in V} \sum_{j \in L} \sum_{k \in K} c^U_{ij} \cdot x^k_{ij} + \sum_{b \in B} O_b \cdot 2c^U_{b0} \tag{43}$$

$$\text{s.t.}(2) - (37)$$

$$(15') \text{ and } (16')$$

$$x^k_{ij}, O_b \in \{0,1\}, y_{ij} \geq 0, \forall i, j \in V, b \in B, k \in K, w \in \mathbb{Z}^+_0 \tag{44}$$

B. Proof and theorem of the single-level reformulation

Theorem 2. *Any optimal solution of Problem* (35) *is also optimal to the optimistic version of the bilevel VRPSB. Moreover, this optimality equivalence continues to hold when adding Constraint* (37) *to the upper level of the bilevel VRPSB and Problem* (35)*.*

Proof. Next, we will show the correctness of each step in the single level reformulation.

Start by noticing that if a leader's optimal solution has $x_{ib}^k \neq \hat{x}_{ib}^k$, for some i, j and k, then changing x_{ib}^k to $\hat{x}_{ib}^k$ still results in an optimal solution for the leader: the modified leader's strategy is feasible, the follower's feasible region does not change, and none of the objective functions, (1) and (14), changes. Consequently, we can restrict x_{ij}^k to mimic the follower's reaction as done in **Step 1**.

Note that the follower's problem can be decompose in $|K|$ maximization problems, one for each vehicle, since there is no lower level linking constraint with the different vehicles. Hence, we can focus on each of these optimization problems, namely, on the profit F^k that can be obtained by each vehicle $k \in K$. Recalling that F^k can be modeled accordingly with Equation (25), **Step 2** and **Step 3** linearize it through a set of 4 constraints (Constraints (26) to (29)), and new binary variables, H_E^k and H_b^k, are added to model the type of incentive accepted (external or backhaul). With these newly introduced variables, in **Step 4**, we can ensure that the x_{ib}^k reflect the follower's reaction.

In this way, we can conclude that any optimal solution of Problem (35) is also optimal to the optimistic version of the bilevel VRPSB.

If Constraint (37) is added to the upper level of the bilevel VRPSB and, its version to the single-level reformulation

$$\sum_{i \in L} \sum_{k \in K} x_{ib}^k + O_b \leq 1, \qquad \forall b \in B \tag{45}$$

is added to Problem (35), the theorem continues to hold. This is because the same reasoning holds: the leader can change its x_{ib}^k to match $\hat{x}_{ib}^k$ and, since this constraint must be guaranteed by the leader, nothing changes in the description of the optimal reaction for the follower.

□

It is worth mentioning that the reduced single-level problem maintains the hierarchical nature of the bilevel problem, allowing the leader to move first and the follower to react next to the decisions of the leader.

C. Side payments after routing (SP_Δ) for the two different scenarios

		Unlimited visits			Limited visits		
Inst.	Q_0	Costs	Profits	SP_Δ	Costs	Profits	SP_Δ
A	C	250	164	16	250	164	16
	2C	293	149	31	293	149	31
	3C	339	131	49	341	134	46
	4C	386	115	65	395	117	63
	5C	433	100	80	458	104	76
B	C	184	46	14	184	46	14
	2C	222	46	14	232	39	21
C	C	200	47	13	200	47	13
	2C	233	39	21	248	41	19
D	C	204	49	11	204	49	11
	2C	239	40	20	246	41	19
E	C	314	160	20	314	160	20
	2C	367	143	37	367	143	37
	3C	420	143	37	438	146	34
	4C	473	143	37	514	146	34
	5C	526	143	37	594	146	34
F	C	450	172	28	450	172	28
	2C	526	145	55	526	145	55
	3C	611	126	74	611	126	74
	4C	723	121	80	726	121	79
G	C	398	124	26	398	124	26
	2C	464	97	53	464	97	53
	3C	556	97	53	557	97	53
H	C	436	167	33	436	167	33
	2C	486	167	33	493	167	33
	3C	537	167	33	552	168	32
	4C	588	167	33	626	149	51
I	C	469	200	0	469	200	0
	2C	504	200	0	508	200	0
	3C	538	200	0	553	170	30
	4C	573	200	0	599	170	30
J	C	361	163	37	361	163	37
	2C	403	130	70	403	130	70
	3C	453	130	70	455	131	69
	4C	503	130	70	515	131	69

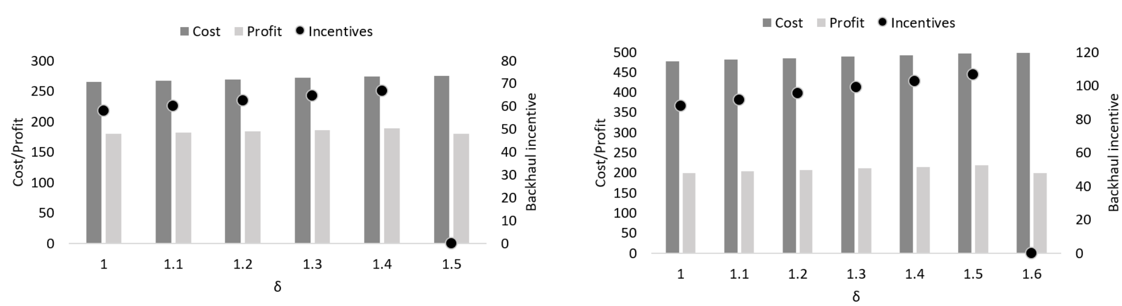
D. Synergy values obtained with the bilevel model and side payments strategies, for the case of limited visits to backhaul customers

		SP_Δ			$SP(0.50)$			$SP(0.75)$			Bilevel
Inst.	Q_0	Costs	Profits	SV	Costs	Profits	SV	Costs	Profits	SV	SV
A	C	266	180	9.3%	267	181	9.3%	275	180	0.0%	**10.0%**
	2C	324	180	12.5%	329	185	12.5%	345	180	0.0%	**12.5%**
	3C	387	180	12.4%	396	189	12.4%	416	180	0.0%	**12.4%**
	4C	458	180	9.7%	468	189	9.6%	488	180	0.0%	**10.1%**
	5C	534	180	7.3%	542	189	7.8%	562	180	0.0%	**8.9%**
B	C	198	60	-4.7%	192	60	0.0%	192	60	0.0%	**0.0%**
	2C	253	60	5.0%	257	68	6.9%	264	60	0.0%	**8.5%**
C	C	213	60	-6.1%	204	60	0.0%	204	60	0.0%	**0.0%**
	2C	267	60	4.3%	270	66	5.9%	276	60	0.0%	**9.6%**
D	C	215	60	-5.5%	207	60	0.0%	207	60	0.0%	**0.0%**
	2C	265	60	2.2%	265	72	8.3%	270	60	0.0%	**8.3%**
E	C	334	180	-7.6%	323	180	0.0%	323	180	0.0%	**0.0%**
	2C	404	180	-5.0%	393	180	0.0%	393	180	0.0%	**0.2%**
	3C	471	180	-1.8%	466	180	0.0%	466	180	0.0%	**0.1%**
	4C	547	180	-1.4%	542	180	0.0%	542	180	0.0%	**0.1%**
	5C	628	180	-1.2%	623	180	0.0%	623	180	0.0%	**0.1%**
F	C	478	200	7.0%	475	197	7.0%	487	210	7.0%	**7.0%**
	2C	581	200	8.4%	576	195	8.4%	602	221	8.4%	**8.4%**
	3C	685	200	9.1%	693	195	6.6%	719	221	6.6%	**9.1%**
	4C	805	200	7.4%	811	195	5.8%	836	221	5.8%	**7.8%**
G	C	424	150	1.7%	420	146	1.6%	429	150	0.0%	**1.7%**
	2C	517	150	4.4%	512	144	4.3%	526	161	5.0%	**5.0%**
	3C	609	150	7.6%	607	145	7.2%	630	169	7.2%	**7.6%**
H	C	469	200	-10.0%	444	200	0.0%	444	200	0.0%	**0.0%**
	2C	526	200	-8.1%	502	200	0.0%	502	200	0.0%	**0.0%**
	3C	583	200	0.3%	570	187	0.3%	580	196	0.3%	**0.3%**
	4C	678	200	-1.3%	658	187	0.2%	667	196	0.2%	**0.2%**
I	C	469	200	0.0%	469	200	0.0%	469	200	0.0%	**0.0%**
	2C	508	200	0.0%	508	200	0.0%	508	200	0.0%	**0.0%**
	3C	583	200	-8.1%	554	200	0.0%	554	200	0.0%	**0.0%**
	4C	629	200	-6.9%	601	200	0.0%	601	200	0.0%	**0.0%**
J	C	398	200	-10.2%	373	175	-10.2%	379	181	-10.2%	**0.0%**
	2C	473	200	-14.1%	427	178	-4.5%	434	184	-4.8%	**0.0%**
	3C	524	200	-8.6%	483	158	-8.6%	494	185	-3.4%	**0.0%**
	4C	584	200	-6.3%	542	158	-6.3%	554	185	-2.2%	**0.0%**
Average				0.0%			2.4%			0.6%	**3.7%**

E. Impact of increasing the lower bound of backhaul incentive

In this section, we investigate the trade-off between the objectives of upper and lower levels, by changing the lower bounds of incentives offered by the shipper. In the previous computational experiments, the lower bound of an incentive offered by the shipper is bounded by the external incentive. In these experiments, we define a parameter δ that represents the percentage increase in the incentive bound in comparison with the external incentive. Departing from the original bound ($\delta = 1$), gradual increases are tested for the same instance and the trade-off between costs and profits are investigated.

The results are reported in Figures 2a and 2b, respectively for instances A and F. It is possible to observe that increasing the lower bound of the incentive to visit a backhaul customer leads to an increase in both costs and profits, up to a break-point after which the outsourcing cost of an inbound route is more attractive for the shipper than offering an incentive to the carrier.



(a) Instance A (b) Instance F

Figure 2: Trade-off between objectives of upper level (cost) and lower level (profit)

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