Université de Montréal

Essays in Macroeconomics

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A mon père Louis, mon épouse Victoria et mon enfant Lukas

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Résumé

La littérature macroéconomique a connu deux évolutions majeures au cours de la dernière décennie. Premièrement, l'introduction des inégalités dans le modèle néo-keynésien de base pour comprendre le rôle des inégalités dans la transmission de la politique monétaire. Deuxièmement, comment le contrôle des capitaux et la politique macroprudentielle peuvent contribuer à atténuer la sévérité et la fréquence des crises financières. Cette thèse, divisée en trois chapitres, contribue à cette vaste et récente littérature d'étude des inégalités, de la stabilité financière et de la politique monétaire. Le premier chapitre se concentre exclusivement sur une analyse positive. Les deux derniers se penchent à la fois sur les analyses positive et normative.

Le premier chapitre fournit les conditions dans lesquelles un modèle néo-keynésien à deux agents, également connu sous le nom de TANK, se rapproche d'un modèle néo-keynésien à agents hétérogènes (HANK) en termes de réponse agrégée à un choc de politique monétaire ? Dans ce chapitre, je montre que la réponse dépend de la source des rigidités nominales. Si les prix sont rigides, la réponse est oui, comme le montrent Debortoli et Gali (2018). Si les salaires sont rigides, la réponse est non, comme le montre cet article. Pour ce faire, je montre que le modèle TANK avec uniquement des rigidités salariales est équivalent, en termes de variables agrégées, au modèle néo-keynésien à agent représentatif. Dans le modèle TANK avec à la fois des rigidités de prix et de salaire, je montre numériquement que TANK n'est pas une bonne approximation de HANK.

Le deuxième chapitre étudie le rôle de l'hétérogénéité des ménages dans la sévérité des crises financières appelées sudden stops et ses implications pour la politique prudentielle de contrôle des capitaux. J'utilise des données sur les les sudden stops et la participation aux marchés financiers pour documenter qu'un niveau plus faible de participation aux marchés financiers est associé à une baisse plus importante des prix des actifs. Pour expliquer le rôle que joue la participation aux marchés financiers dans la baisse des prix des actifs, je construis un modèle de cycle économique avec une contrainte collatérale et une participation limitée aux marchés financiers. L'hétérogénéité à l'accès au marché financier génère des inégalités de revenus et de consommation dans le modèle. La mesure dans laquelle la participation limitée aux marchés financiers amplifie la baisse du prix des actifs dépend de la cyclicité des inégalités de consommation. Conformément à mes conclusions empiriques utilisant des données d'enquêtes auprès des ménages du Mexique, le modèle génère une baisse des inégalités de consommation pendant la crise financière

ce qui amplifie la baisse des prix des actifs, de la production et de la consommation. Je montre que l'impôt sur la dette devrait être plus élevé dans une économie à participation limitée aux marchés financiers. Ce qui rationalise l'utilisation du contrôle des capitaux dans les marchés émergents. Enfin, mes conclusions suggèrent qu'il est possible de lutter contre l'instabilité financière sans accroître les inégalités.

Le dernier chapitre étudie conjointement la politique monétaire optimal et du contrôle des capitaux dans un environnement motivé à la fois par la stabilité financière et la stabilité des prix. Ce chapitre est une extension du deuxième chapitre où j'introduis la rigidité des prix des biens de consommation. Je montre qu'en l'absence des frictions de crédit (c'est-à-dire que la contrainte de garantie n'est jamais contraignante), l'autorité monétaire dans le cadre sa politique monétaire discrétionnaire est incitée à dévier de la mise en œuvre de la stabilité des prix (la coïncidence divine ne tient pas). De plus, je montre qu'en cas d'instabilité financière due aux frictions du crédit, l'autorité monétaire dans le cadre de sa politique monétaire discrétionnaire ne devrait adopter une politique monétaire prudentielle qu'en cas de liberté des mouvements de capitaux. Cette politique monétaire prudentielle est exacerbée par les inégalités des ménages. En l'absence d'un prêt de fonds de roulement, la politique monétaire procyclique n'est jamais optimale.

Keywords: Crise financière, prix des actifs, inégalités, politique optimale, contrôle des capitaux, salaires rigides, prix rigides, agents hétérogènes, politique monétaire.

Abstract

Two main developments have occurred in the last decade in the macroeconomics literature. First, the introduction of inequality in the standard New Keynesian model to understand the role of inequality in the transmission of a monetary policy. Second, how capîtal control and macroprudential policy can help to alleviate the severity and the frequency of financial crises. This thesis, divided into three chapters, contributes to this vast and recent literature studying inequality, financial stability, and monetary policy. The first chapter focuses exclusively on positive analysis. The last two one study both positive and normative analysis.

The first chapter provides the conditions under which a Two-Agent New Keynesian model, also known as TANK, approximates a Heterogeneous Agents New Keynesian (HANK) model in terms of its aggregate response to a monetary policy shock? In this chapter, I show that the answer depends on the source of nominal rigidities. If prices are sticky, the answer is yes, as shown by Debortoli and Gali (2018). If wages are sticky, the answer is no, as shown in this paper. To make this point, I show that the TANK model with only wage rigidities is equivalent, in terms of aggregate variables, to the representative agent New Keynesian model. For TANK with both price and wage rigidities, I show numerically that TANK does not approximate HANK well.

The second chapter studies the role of household heterogeneity in the severity of sudden stop crises and its implications for prudential capital control policy. I use data on sudden stop events and financial market participation to document that a lower level of financial market participation is associated with a higher drop in asset prices. To explain the role that financial market participation plays in the drop in asset prices, I build an equilibrium business cycle model with a collateral constraint and with limited financial market participation. The heterogeneity in access to the financial market generates income and consumption inequality in the model. The extent to which the limited financial market participation amplifies the drop in the asset price depends on the cyclicality of consumption inequality. Consistent with my empirical findings using household survey data from Mexico, the model generates a drop in consumption inequality during the financial crisis that amplifies the drop in asset prices, output, and consumption. I show that the optimal time-consistent debt tax should be higher in a limited financial market participation economy, which rationalizes the use of capital control in emerging markets. Finally, my findings suggest it is possible to address financial instability without raising inequality.

The last chapter studies the joint design of monetary policy and capital control in an environment with a motive for both financial stability and price stability. I extend the second chapter and introduce price rigidity. I show that, in the absence of credit friction (i.e., the collateral is never binding), the monetary authority under the discretionary monetary policy has an incentive to deviate implementing price stability (the divine coincidence does not hold). In addition, I show that in the case of financial instability due to credit frictions, the monetary authority under the discretionary monetary policy should adopt a prudential monetary policy only if capitals flows are free. This prudential monetary policy is exacerbated by household inequality. In the absence of a working capital loan procyclical monetary policy is never optimal.

Keywords: Financial crisis, asset prices, inequality, optimal policy, capital controls sticky wages, sticky prices, heterogeneous agents, monetary policy.

Chapter 1

Monetary Policy, Sticky wages, and Household heterogeneity

1.1 Introduction

What is the transmission mechanism of a monetary policy shock in an economy? What are the responses of aggregate variables, such as GDP and consumption, to a monetary policy shock? Those are some classic questions addressed in the large monetary economics literature within a Representative Agent New Keynesian (i.e., RANK) model. In recent years, Heterogeneous Agents New Keynesian models often referred to as HANK models, have gained attention and have substantially revised our understanding of the answers to these questions[0]. However, the HANK model features a continuous joint distribution of income and wealth and lacks tractability. In this paper, I study the extent to which a Two-Agent New Keynesian (TANK) model with limited heterogeneity can approximate a HANK model in terms of the response of aggregate variables, such as GDP and consumption, to a monetary policy shock. I argue that whether TANK models provide a good approximation to HANK models depends on the considered nominal rigidities.

Understanding the condition under which the TANK model approximates the HANK model is important for the following reasons. First, the HANK model does not have a closed-form solution since it requires to keep track of the wealth distribution as a state variable. We can only rely on a nontrivial numerical solution to solve for the equilibrium of HANK economies. This lack of analytical tractability poses challenges for the identification of the economic mechanisms underlying the results[1]. Second, nominal

[0] In seminal work, Kaplan et al. (2018) studied the transmission of monetary policy in HANK model to household consumption. In contrast to the RANK model, they found that in HANK, monetary policy works mostly through the income effect as opposed to the inter-temporal substitution effect. The substitution effect captures the extent to which households save less (or borrow more) to increase consumption when the real interest rate declines. The income effect captures the general equilibrium effect where the decline in the real interest rate affects labor demand and thus labor income.

[1] Acharya and Dogra (2020a) study a full tractable HANK model with CARA utility function.

rigidities is the source of monetary non-neutrality in these models. Rigidities in the price of consumption and wages have been documented (Taylor (1999)). Thus, it is important to know if the approximation of HANK by TANK in terms of aggregate fluctuations depends on the considered nominal rigidity.

To answer this question, I use a general equilibrium framework as in Debortoli and Galí (2018) in which I introduce sticky wages and a monopolistically competitive labor market. I introduce sticky wages for three reasons. First, the New Keynesian literature has empirically documented that wages are as sticky as prices (Taylor (1999)). Second, besides their empirical relevance, wage rigidities have been shown to be qualitatively and quantitatively important for the modeling of economies with a role for monetary policy. When wages are rigid, output exhibits persistence in its response to a monetary shock, which is in line with the response observed in the data (Christiano et al. (2005)). For models with heterogenous agents where a fraction of households live as "hand-to-mouth", wage rigidities play an important role in keeping the volatility of real income in line with the one observed in the data. Third, sticky wages have been shown to preserve the "standard aggregate demand logic" and the relevance of the Taylor principle for a plausible calibration of the share of hand-to-mouth households (Bilbiie (2008), Colciago (2011), Ascari et al. (2011)) [2].

In this paper, I first show that under sticky wages (and flexible prices), a TANK model is equivalent to a RANK model in terms of its response to demand and supply shocks. The intuition behind this result is as follows. It is worth noting that the differences between RANK and TANK models are twofold. First, in the TANK model, there is a fixed fraction of households that cannot borrow. Second, profit is not redistributed uniformly because profit is generally shared between asset holders. At the equilibrium, there is no trade in bond in the equilibrium of the RANK and TANK models as asset holders are identical. Therefore, the only source of difference between RANK and TANK is how the firms' profit are distributed. If a firm's profit is uniformly redistributed to all households in the TANK model, then TANK will be equivalent to RANK whatever the type of nominal rigidity is in the economy.

In monopolistic competition, the firm's profit is proportional to the price markup and output. Given that the profit rate (profit over output) depends only on the markup, the consumption inequality defined as a ratio of the consumption of unconstrained (asset holders) and constrained agents (those who do not participate in the financial market), is proportional to the price markup. If prices are flexible (and wages are sticky), the firm can always adjust its price for a constant price markup. Every firm faces the same nominal wage set by the wage union. So, following an aggregate shock, there is no change in consumption inequality. Then TANK is equivalent to RANK. So why is HANK not

[2] For a plausible proportion of Hand-to-Mouth, TANK models lead to a situation where the Taylor principle is no longer a necessary condition for equilibrium determinacy, and the Standard Aggregate Demand Logic (an increase in real interest rate leads to a decrease in aggregate consumption) does not hold.

equivalent to RANK when prices are flexible? The reason is simple. In HANK, the consumption inequality does not only come from the ownership of the firms but also from the idiosyncratic labor income risk and debt choice. To summarize, TANK is equivalent to RANK under sticky wages and flexible prices because the consumption gap between constrained and unconstrained agents is constant over time so there is no change in the consumption inequality.

With inequality, not only the type of nominal rigidity matters but also the source of nominal rigidity because redistribution matters (see Auclert (2017) who shed light on the role of redistribution in the transmission of the monetary policy). Wage markup uniformly impacts every household in my work because wages are set outside the firms. On the contrary, price markup is not distributed uniformly. In general, only asset holders gain from a price markup since they own the firms.

Second, I find quantitatively as Debortoli and Galí (2018) that under sticky prices, TANK approximates HANK well in terms of its response to an aggregate shock. Unlike Debortoli and Galí (2018), under sticky prices and sticky wages, I find that TANK can no longer approximate HANK. Building on the intuition above, sticky wages mute the response of the gap between the consumption of hand to mouth and other households in response to an aggregate shock.

The extent to which the heterogeneity[3] affects aggregate fluctuations has been studied by many authors including (Werning (2015); Acharya and Dogra (2020a); Bilbiie (2019)). Debortoli and Galí (2018) also offer a better understanding of how the heterogeneity affects aggregate fluctuations in response to a demand shock (monetary policy shock and preference shock) and supply shock (technology shock). Based on the structure of constrained and unconstrained agents in HANK, Debortoli and Galí (2018) upon a first order linear approximation show analytically that the HANK framework is different from the RANK framework along three dimensions: the change in the consumption gap between constrained and unconstrained agents, the change in the consumption dispersion within unconstrained agents and the change in the share of constrained agents. It is not possible to analytically compute the three statistics since it requires to know the wealth distribution at each point in time. For this reason, Debortoli and Galí (2018) build on Bilbiie (2008) a Two Agents New Keynesian model often referred to as TANK. Three statistics summarize the state of the economy in the TANK model. The advantage of the TANK framework is that the three statistics (the change in the consumption gap between constrained and unconstrained agents, the change in the consumption dispersion within unconstrained agents and the change in the share of constrained agents) can be computed analytically. Note that, by construction, all three are zero in RANK while TANK allows to focus on the consumption gap between constrained and unconstrained agents. Under sticky prices Debortoli and Galí (2018) find that TANK approximates well HANK in

[3]In most of the HANK literature, the assumed heterogeneity is often of the form postulated by Aiyagari (1994), where each household faces uninsurable idiosyncratic labor income risk and a borrowing limit.

terms of aggregate fluctuations both for demand and supply shock.

My paper mainly relates to the recent literature on HANK Werning (2015); Gornemann et al. (2016); Bilbiie (2019); Auclert (2017); Kaplan et al. (2018); Luetticke (2018); Bayer et al. (2019); Acharya and Dogra (2020a). I contribute to this literature by studying sticky wages. To my knowledge, Hagedorn et al. (2019b), Hagedorn et al. (2019a) are the first papers introducing sticky wages in a general HANK framework. While they focus on the fiscal multiplier and forward guidance, I offer detailed comparison between a non-tractable HANK model and a tractable TANK model. My work is closely related to Debortoli and Galí (2018) who are the first to offer a better understanding of the difference between HANK and TANK in terms of its response to aggregate shocks. I introduce sticky wages, which play a key role in the comparison between TANK and HANK.

My work is also related to earlier literature on two agents model as Bilbiie (2008); Colciago (2011); Ascari et al. (2011). While they focus on the comparison between RANK and TANK, I build on their work by comparing a Two-Agent New Keynesian model to a Heterogeneous Agents New Keynesian model.

The rest of the paper is organized as follows. In Section 3.2 I present the HANK model. Section 1.3 presents the TANK framework. Section 1.4 presents my finding and discusses some findings in Debortoli and Galí (2018) and Section 3.4 concludes.

1.2 Model

I build a dynamic stochastic model with household heterogeneity. The household faces labor income risk and a borrowing limit à la Aiyagari (1994). There is a monopolistic competitive firm that faces sticky prices. Wages are sticky in the spirit of Erceg et al. (2000).

1.2.1 Household

There is a continuum of ex-ante identical households of measure one indexed by their liquid asset B, their share of the equity fund F and their uninsurable labor income risk e. Labor income risk follows a markov process. Households self-insure against the labor income risk by saving in the liquid asset B. By purpose, the household side is kept as close as possible to Debortoli and Galí (2018). A household i choses C_{it} to maximize his expected discounted utility $\mathbb{E}\sum_{t=0}^{\infty}\beta^t U(C_{it}, N_{it})$, where $U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta}$, subject to its current budget constraint:

$$C_{it} + Q_t F_{it} + \frac{B_{i,t}}{P_t} = \frac{B_{it-1}(1+i_{t-1})}{P_t} + w_t N_t e_{it} + \left[Q_t + (1-\delta)D_t\right] F_{it-1} + T_{it} - e_{it}\frac{\theta_w}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 \mathbb{Z}_t,$$

where $\mathbb{Z}_t$ is an aggregate variable (aggregate output Y_t for instance) taken as given by households. The budget constraint is as in Debortoli and Galí (2018) except for the wage adjustment cost. The right hand side of the budget constraint is composed of bond income, labor income, equity income, transfer income and the wage adjustment cost. A share $1-\delta$ of firm's profit D_t is claimed by equity fund holders. The remaining share δ of firm's profit is transferred to household from a specific rule described below. The real wage is w_t and Q_t is the price of the equity fund. Each household is subject to a borrowing limit of the form:

$$\frac{B_{i,t}}{P_t} \geq -\Psi Y,$$

where Y is the yearly output. The equity share F_{it} is assumed to be non negative; that is there is no short selling. As Debortoli and Galí (2018), we assume that: $Q_t F_{it} = \max[0, v_t A_{it}]$, where A_{it} is the net worth given by: $A_{it} = Q_t F_{it} + \frac{B_{it}}{P_t}$ and $v_t \in [0\ 1]$. With $\int_0^1 F_{it} di = 1$, one can show that $F_{it} = \frac{A^+_{it}}{A^+_t}$ where $A^+_{it} = \max[0, A_{it}]$ and $A^+_t = \int_0^1 A^+_{it} di$.

The transfer is assumed to follow the below rule:

$$T_{it} = \left[1 + \tau^a_t \left(\frac{A^+_{it}}{A^+_t} - 1\right) + \tau^e_t \left(e_{it} - 1\right)\right] \delta D_t.$$

From this transfer rule, three cases are considered: the first one is the Wealth-based rule ($W - rule$) where $\tau^a_t = 1$ and $\tau^e_t = 0$; the second one is the productivity-based rule ($P - rule$) where $\tau^a_t = 0$ and $\tau^e_t = 1$; and the third one is the Uniform-based rule ($U - rule$) where $\tau^a_t = 0$ and $\tau^e_t = 0$. In the $W - rule$ the illiquid profit δD_t is distributed only to current share holders. The $P - rule$ shares the illiquid profits among households proportionally to their labor productivity. In the $U - rule$, the illiquid profit is equally shared between all households (See Debortoli and Galí (2018) for more details).

Let's $b_{it} = \frac{B_{it}}{P_t}$, $1 + r_t = \frac{1+i_t}{\Pi_{t+1}}$, and $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. The household's problem gives the standard Euler equation with inequality:

$$U_c(C_{it}, N_t) \geq \beta(1 + r_t)\mathbb{E}(U_c(C_{it+1}, N_{t+1}). \tag{1.1}$$

This standard Euler equation holds with equality for unconstrained agent (household for whom the credit limit is not binding). One additional unit of consumption today increases its utility by $U_c(c_t)$. If the household saves this unit of consumption in a the riskless bond, it gains tomorrow $(1 + r_t)U_c(c_{t+1})$, where r_t is the riskless real interest rate. At the optimum the cost of saving should be equal to its discounted benefit for unconstrained agents.

Following Debortoli and Galí (2018), we assume that the equity share price Q_t is the discounted expected of all futures return of the equity share:

$$Q_t = \mathbb{E}_t(\Lambda^Q_t \left[Q_{t+1} + (1 - \delta)D_{t+1}\right]), \tag{1.2}$$

where Λ^Q_t is the stochastic discounted factor. The relevant stochastic discounted factor is given by: $\Lambda_t^Q = \beta \frac{U_c(C^+_{t+1/t})}{U_c(C^+_t)}$ where C^+_t and $C^+_{t+1/t}$ are consumption in period t and t+1 of households with positive net wealth in period t, weighted by their share holding or their wealth.

1.2.2 Employment Agencies

We assume as in Erceg et al. (2000) that there exists a perfectly-competitive employment agencies which hire the differentiated labor of consumers and aggregate them using the CES technology.

$$N_t = \left[\int_0^1 e_{it} \left(N_{it}\right)^{1-\frac{1}{\epsilon_w}} di\right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{1.3}$$

Where ϵ_w is the elasticity of substitution across labor services and e_{it} is the uninsurable idiosyncratic labor income risk. The profit function of the agency is given by: $W_t N_t - \int_0^1 W_{it} N_{it} e_{it}$. The solution to the profit maximization (in appendix A.1.1) gives the labor demand:

$$N_{it} = \left[\frac{W_t}{W_{it}}\right]^{\epsilon_w} N_t. \tag{1.4}$$

Equation 1.4 states that with the same nominal wage among household that is $W_{it} = W_{jt}$ for every i, j then the labor supply is the same across household. This implies that the observed difference of labor income across household comes from the idiosyncratic risk. The volatility of this labor income crucially depends on the variance of the idiosyncratic risk.

Competition implies that profits are null in equilibrium. We obtain the wage index W_t by replacing the solution N_{it} into the profit function, which yields the following wage index:

$$W_t = \left[\int_0^1 \left(W_{it}\right)^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}}. \tag{1.5}$$

1.2.3 Wage setting

Following Hagedorn et al. (2019b), we assume that there is a middleman who sets the nominal wage $\hat{W}_t$ for an effective unit of labor such that $\hat{W}_t = W_{it}$ and $N_{it} = \hat{N}_t$. Since wages are sticky, a change in current wages with respect to past wages is subject to an adjustment cost à la Rotemberg (1982). This adjustment cost for a household is proportional to the realization of the idiosyncratic risk and is given by: $\Theta_{it}(W_{it}, W_{it-1}, Z_t) = e_{it}\frac{\theta_w}{2}\left(\frac{W_{it}}{W_{it-1}} - 1\right)^2 Z_t$ where Z_t is aggregate output.

For the middleman the benefit for an effective labor is given by: $\int_0^1 \hat{W}_t \hat{N}_t e_{it} di - \int_0^1 \Theta_{it}(\hat{W}_t, \hat{W}_{t-1}, Z_t) di$ with $\hat{N}_t(\hat{W}_t, W_t, Z_t) = \left[\frac{W_t}{\hat{W}_t}\right]^{\epsilon_w} N_t$. The cost is given by $\int_0^1 \frac{g\left(\hat{N}_t(\hat{W}_t, W_t, Z_t)\right)}{u'(C_t)} di$, where $g(N_t) = \frac{N_t^{1+\eta}}{1+\eta}$ is labor dis-utility and $u'(C_t)$ is the aggregate marginal utility which is present in the cost because we abstract it from the benefit function.

The middleman solves the following problem:

$$\max_{\hat{W}_t} \sum_{t=0}^{\infty} \beta^t \left[\int_0^1 \hat{W}_t \hat{N}_t e_{it} di - \int_0^1 \Theta_{it}(\hat{W}_t, \hat{W}_{t-1}, Z_t) di - \int_0^1 \frac{g\left(\hat{N}_t(\hat{W}_t, W_t, Z_t)\right)}{u'(C_t)} di \right] \tag{1.6}$$

$$s.t \quad \hat{N}_t(\hat{W}_t, W_t, Z_t) = \left[\frac{W_t}{\hat{W}_t}\right]^{\epsilon_w} N_t.$$

Solving the problem in 1.6 using $\hat{W}_t = W_t$ yields the wage Phillips curve:

$$\theta_w \Pi_t^w (\Pi_t^w - \bar{\Pi}^w) = w_t(1-\epsilon_w) + \epsilon_w N_t^\eta C_t^\sigma + \beta \theta_w \Pi_{t+1}^w (\Pi_{t+1}^w - \bar{\Pi}^w) \frac{\mathbb{Z}_{t+1}}{\mathbb{Z}_t}, \tag{1.7}$$

where $\Pi_t^w = \frac{W_t}{W_{t-1}}$ is the nominal wage inflation, and $w_t = \frac{W_t}{P_t}$ is the real wage. Let denote μ_t the real wage markup. We have : $\frac{W_t}{P_t} = \mu_t MRS_t$, where $MRS_t = -\frac{U_N}{U_C}$ is the marginal rate of substitution.

Condition 1.7 is the non linear version of New-Keynesian Wage Phillips Curve. Note that when wage is fully flexible (ie $\theta_w = 0$), $w_t = \frac{\varepsilon_w}{\varepsilon_w - 1} N_t^\eta C_t^\sigma$ that is the real wage is equal to the product of the labor wedge and the marginal rate of substitution between consumption and labor. The labor wedge $\frac{\varepsilon_w}{\varepsilon_w - 1}$ is equivalent to the steady state real wage markup raised in this set up because of monopolistic labor market. When wage is fully rigid (ie $\theta_w \longrightarrow \infty$) $\Pi_t^w = 1$, for all t. The middleman sets wage once for all and never updates it.

Linearize 1.7 around the deterministic steady state yields:

$$\pi_t^w = \beta \mathbb{E} \pi_{t+1}^w - \frac{\epsilon_w w}{\theta_w} \hat{\mu}_t^w. \tag{1.8}$$

Condition 1.8 is the linear version of New-Keynesian Wage Phillips Curve. The log deviation of the firm's real markup from its steady state is denoted by $\hat{\mu}_t^w$. It states that if household real wage markup is below their natural level (equivalent to the steady-state level), the middleman resets nominal wage up which increases wage inflation. When wages are fully rigid (ie $\theta_w \longrightarrow \infty$) $\pi_t^w = 0$, for all t and the wage inflation will be 0 for any shock.

1.2.4 Firms

There are monopolistically competitive intermediate good producing firms and perfectly final goods producing firms that aggregate differentiated intermediate goods into a single good Y_t. Using the Rotemberg (1982) adjustment cost $AC_t = \frac{\theta}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - \pi\right)^2 Y_t$, the solution to the firm's problem gives the following condition:

$$\pi_t^p = \beta \mathbb{E} \pi_{t+1}^p - \frac{\varepsilon - 1}{\theta} \hat{\mu}_t^p. \tag{1.9}$$

Condition 1.9 is the linear version of the New-Keynesian Price Phillips Curve. The elasticity of substitution across good is denoted by ϵ, π_t^p is the price inflation, and $\hat{\mu}_t$ is the log deviation of firm real markup from his steady state. It states that if firms' markup are below their natural level (equivalent to the steady state level), it resets prices up, which increases inflation.

1.2.5 Monetary authority

The monetary authority uses taylor-rule to set nominal interest as follows:.

$$\hat{i}_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t, \tag{1.10}$$

where v_t represents exogenous monetary policy shocks and it follows an AR(1) process.

1.3 Two Agents: TANK framework

We assume two types of agents. The first type is the time-invariant unconstrained agents U of measures $1-\lambda$, and the second type is the time-invariant constrained agents K of measures λ. The time-invariant unconstrained agents U are not constrained on bond market and the time-invariant constrained agents K do not participate to the bond market. The agents U own firms and claim the a part of aggregate profit $(1-\delta)D_t$ as in Debortoli and Gali (2018). There is no idiosyncratic risk in TANK: $e_{it} = 1$.
The household budget constraint for constrained agents U is given by:

$$C_{1t} + \frac{1}{1+r_t} b_{1,t+1} = b_{1t} + \frac{1}{P_t} W_{1t} N_{1t} + \left[Q_t + (1-\delta)D_t\right] F_{it-1} + T_{1t} - \frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_{1t}}{W_{1t-1}} - 1\right)^2 \mathbb{Z}_t \tag{1.11}$$

The household budget constraint for constrained agents K is given by:

$$C_{2t} = \frac{1}{P_t} W_{2t} N_{2t} + T_{1t} - \frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_{1t}}{W_{2t-1}} - 1\right)^2 \mathbb{Z}_t. \tag{1.12}$$

Since there is no heterogeneity between unconstrained agents, at the equilibrium: $F_{it-1} = \frac{1}{1-\lambda}$ and $b_{1,t+1} = 0$ the budget constraints of households are given by: $C_t^U = w_t N_t + \frac{1-\delta}{1-\lambda} D_t + T_t^U - \frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 \mathbb{Z}$ and $C_t^K = w_t N_t + T_t^K - \frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_t}{W_{t-1}} - 1\right)^2 \mathbb{Z}_{\approx}$.Where w_t is the real wage. Following Debortoli, Gali (2018), $T_t^U = \left(1 + \frac{\tau\lambda}{1-\lambda}\right)\delta D_t$ and $T_t^K = (1-\tau)\,\delta D_t$. Note that $(1-\lambda)T_t^U + \lambda T_t^K = \delta D_t$ For $\tau = 1$, all the profits end up in the hands of unconstrained agents (W-rule), for $\delta = 1$ and $\tau = 0$, all profits are sharing equally between unconstrained and constrained households(U-rule and P-rule).
The Euler equation of unconstrained agents is given by:$Z_t {C_t^U}^{-\sigma} = \beta(1+r_t)\mathbb{E}\left[Z_{t+1} {C_{t+1}^U}^{-\sigma}\right]$.
The linearization of this Euler equation yields:

$$\hat{c}_t^U = \mathbb{E}\hat{c}_{t+1}^U - \frac{1}{\sigma}\hat{r}_t - \frac{1}{\sigma}\mathbb{E}\Delta z_{t+1} \tag{1.13}$$

Equation 1.13 depends on the percentage change of unconstrained agents' consumption. The goal is to have a version of equation 1.13 that depends only on aggregate variables. The new equation will be the Dynamics Investment-Saving (DIS) curve. In HANK equilibrium, it was impossible to aggregate the economy because of the uninsurable idiosyncratic labor risk. The TANK framework allows us to aggregate because there is no endogenous transition between unconstrained and constrained states. Indeed the state in which the agents belong is permanent and exogenous. The following three lemma help us to aggregate the economy.

Lemma 1.3.1. *Two measures are necessary and sufficient to aggregate the consumption in the economy. The two measures are: the consumption of unconstrained agents and a measure of consumption inequality. That is:*

$$\hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1-\lambda\gamma}\hat{\gamma}_t \tag{1.14}$$

Proof. By definition, the aggregate consumption is: $C_t = (1-\lambda)C_t^U + \lambda C_t^K$ or equivalently $C_t = C_t^U(1-\lambda\gamma_t)$, where $\gamma_t = \frac{C_t^U - C_t^K}{C_t^U}$. Linearize around the steady state gives: $\hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1-\lambda\gamma}\hat{\gamma}_t$. The dynamics of $\hat{c}_t^U$ expressed in equation 1.14 is known using the Euler equation for unconstrained agents. Equation 1.14 tells us two things. First, the dynamics of the aggregate consumption depends on two factors: the dynamics of the consumption of unconstrained agents and the dynamics of the consumption inequality (the consumption inequality is the ratio of the consumption of constrained agents to the consumption of unconstrained agents). Second, if the consumption inequality is constant over time, then the dynamic of the aggregate consumption is the same as the dynamics of the consumption of unconstrained agents.We need now to find an analytical expression for $\hat{\gamma}_t$. Next lemma describes the change in consumption inequality $\hat{\gamma}_t$.

Lemma 1.3.2. *The change in consumption inequality around the steady state is proportional to the percentage change in real price markup around the steady state. That is:*

$$\hat{\gamma}_t = \Psi_1 \hat{\mu}_t^p, \tag{1.15}$$

where $\Psi_1 = -\gamma_m m$ *;* $\Psi_1 > 0$ *and* $\hat{\gamma}_t = \gamma_t - \gamma$

Proof (see the complete proof in appendix A.2.2). At the equilibrium, $C_t^U - C_t^K = D_t\left(\frac{1-(1-\tau)\delta}{1-\lambda}\right)$, Where D_t is firm's profit. The profit $D_t = Y_t - w_t N_t - AC_t = \left[\left(1-\tilde{AC}_t\right) - (1-\alpha)\, m_t\right] Y_t$, where $m_t = \frac{w_t}{MPN}$ is the inverse of the real price markup. The markup determines the profit which affects the consumption inequality. Up to first order approximation, $\hat{\gamma}_t = \Psi_1 \hat{\mu}_t^p$, where $\Psi_1 > 0$ and $\hat{\mu}_t^p$ is the real price markup deviation from its steady value $\mu^p = \frac{\epsilon_p}{\epsilon_p - 1}$. In addition, using the definition of the price markup: $\hat{\mu}_t^p = -\tilde{w}_t - \frac{\alpha}{1-\alpha}\tilde{y}_t$
Equation 1.15 states that when the real markup increases the consumption gap between unconstrained and constrained agents increases. As expected, higher price markup negatively affects the Hand-to-Mouth's consumption.

Lemma 1.3.3. *The percentage change in real price is proportional to the output gap and the real wage gap.*

$$\hat{\mu}_t^p = -\tilde{w}_t - \frac{\alpha}{1-\alpha}\tilde{y}_t. \tag{1.16}$$

Proof: By definition, $M_t^p = \frac{w_t}{MPN_t}$ then: $log(M_t^p) = \log(w_t) - \log(MPN_t) = (w_t) - log(1-\alpha) + \frac{\alpha}{1-\alpha}logY_t - \frac{1}{1-\alpha}logA_t$. This implies that: $\hat{\mu}_t^p = -\tilde{w}_t - \frac{\alpha}{1-\alpha}\tilde{y}_t$.

Equation 1.16 states that given output gap, the price markup has a negative correlation with the real wage gap. Higher real wage implies a higher marginal cost for firms which lowers the firm's markup. In sticky wage equilibrium, $\hat{\mu}_t^p = 0$, thus $\tilde{w}_t = -\frac{\alpha}{1-\alpha}\tilde{y}_t$. A higher real wage gap induces a decline in the output gap. This decline is through the decline in labor supply. The presence of hand-to-mouth does not affect the relationship between the real wage gap and the output gap in sticky wage equilibrium.

1.3.1 Derivation of IS curve

Now, with lemma1 to lemma3 in hands, we can characterize the Dynamics Investment-Saving curve in function of aggregate variables. Combining equations 1.14, 1.15, and 1.16 gives $\hat{c}_t^U = \hat{c}_t - \frac{\lambda}{1-\lambda\gamma}\Psi_1\left(\tilde{w}_t + \frac{\alpha}{1-\alpha}\tilde{y}_t\right)$

$$\hat{c}_t^U = \hat{c}_t - \Psi_3\tilde{y}_t - \Psi_2\tilde{w}_t, \tag{1.17}$$

where $\Psi_3 = \frac{\lambda}{1-\lambda\gamma}\frac{\alpha}{1-\alpha}\Psi_1$ and $\Psi_2 = \Psi_1\frac{\lambda}{1-\lambda\gamma}$. Using the fact that $\hat{c}_t^U = \mathbb{E}\hat{c}_{t+1}^U - \frac{1}{\sigma}\mathbb{E}\hat{r}_{t+1}^b - \frac{1}{\sigma}\mathbb{E}\Delta z_{t+1}$ and $\hat{y}_t = \hat{c}_t = \tilde{y}_t + \hat{y}_t^n$ where $\hat{y}_t^n = \frac{1+\eta}{\eta+\alpha+\sigma(1-\alpha)}a_t$. a_t denotes the technology shock.

$$\tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma(1-\Psi_3)}\left(\hat{r}_t^b - \hat{r}_t^n\right) - \frac{\Psi_2}{1-\Psi_3}\mathbb{E}\left[\tilde{w}_{t+1} - \tilde{w}_t\right], \tag{1.18}$$

where $\hat{r}_t^n = -\mathbb{E}\Delta z_{t+1} + \sigma\Psi_a\mathbb{E}\Delta a_{t+1}$ with $\Psi_a = \frac{1+\eta}{\eta+\alpha+\sigma(1-\alpha)}$. Note that $\mathbb{E}\Delta z_{t+1} = (\rho_z - 1)z_t$ and $\mathbb{E}\Delta a_{t+1} = (\rho_a - 1)a_t$. Condition 1.18 is the DIS equation. The aggregation alters the demand side of the model. First, the sensitivity of output gap to the real interest rate is now $\sigma(1-\Psi_3)$ instead of σ. For plausible parameters, $0 < (1-\Psi_3) < 1$ implies that the output gap tends to be more responsive to the change in real interest rates. The presence of Hand-to-mouth consumers amplifies the response of the output gap to the change in real interest rates because they react more to a change in their labor income which increases when the output increases. Indeed, the HtM consumers have a higher marginal propensity to consume. Second, the output gap is proportional to the change in the real wage gap.

1.3.2 Derivation of Wage and Price Phillips Curve

In this section, I characterize the wage and price Phillips curve.

Wage Phillips Curve From equation 1.8 I have: $\pi_t^w = \beta\mathbb{E}\pi_{t+1}^w - \frac{\epsilon_w w}{\theta_w}\hat{\mu}_t^w$, where $\hat{\mu}_t^w$ is the steady state log deviation of the real wage markup. I also have $w_t = M_t^w N_t^\eta C_t^\sigma$. By linearizing the equation, we get: $\tilde{w}_t = \hat{\mu}_t^w + \sigma\tilde{c}_t + \frac{\eta}{1-\alpha}\tilde{y}_t$. Since the gap in consumption is equal to the gap in output (that is $\tilde{c}_t = \tilde{y}_t$), we have; $\tilde{w}_t = \hat{\mu}_t^w + \left[\sigma + \frac{\eta}{1-\alpha}\right]\tilde{y}_t$. The real wage markup is given by:

$$\hat{\mu}_t^w = -\left[\sigma + \frac{\eta}{1-\alpha}\right]\tilde{y}_t + \tilde{w}_t. \tag{1.19}$$

If the real wage is above its natural level in only sticky prices equilibrium (that is $\hat{\mu}_t^w = 0$), $\tilde{w}_t = \left[\sigma + \frac{\eta}{1-\alpha}\right]\tilde{y}_t$. There is positive correlation between real wage gap and output gap. The presence of Hand-to-Mouth, does not turn off this correlation. The Wage Phillips curve is given by:

$$\pi_t^w = \beta \mathbb{E} \pi_{t+1}^w - \lambda_w \tilde{w}_t + k_w \tilde{y}_t, \tag{1.20}$$

Where $\lambda_w = \frac{\epsilon_w w}{\theta_w}$, $k_w = \lambda_w \left[\sigma + \frac{\eta}{1-\alpha}\right]$. When the real wage is about its natural level, the middleman resets the wage down. There is a negative relation between wage inflation and the real wage gap. But if the output is above its natural level, the middleman resets up the wage. So there is a positive relation between wage inflation and output gap.
Price Phillips Curve: Combining 1.9 and 1.23 gives: $\pi_t = \beta \mathbb{E} \pi_{t+1} + \frac{\varepsilon m}{\theta} \left(\tilde{w}_t + \frac{\alpha}{1-\alpha} \tilde{y}_t\right)$

$$\pi_t^p = \beta \mathbb{E} \pi_{t+1}^p + \lambda_p \tilde{w}_t + k_p \tilde{y}_t, \tag{1.21}$$

Where $\lambda_p = \frac{\varepsilon - 1}{\theta}$ and $Kp = \left(\frac{\alpha}{1-\alpha}\right) \lambda_p$. This is a version of price NKPC. Contrary to the wage Phillips curve, When the real wage is above its natural level, the firms adjust up their price because of a higher marginal cost. So there is a positive relationship between price inflation and the real wage gap. If the output is above its natural level, the firms adjust up their prices. So there is a positive co-movement between the inflation and the output.
Wage identity equation: In this framework of sticky wage and sticky prices we have an important identity equation that makes the link between wage inflation and price inflation. By definition, $\Delta \tilde{w}_t = \Delta w_t - \Delta w_t^n$. Which implies that $\Delta \tilde{w}_t = (w_t - w_{t-1}) - (p_t - p_{t-1}) - \Delta w_t^n$. So the wage identity condition can be written as follows.

$$\Delta \tilde{w}_t = \pi_t^w - \pi_t^p - \Delta w_t^n. \tag{1.22}$$

Note that $\Delta w_t^n = 0$ in the case of a demand shock. For a supply shock, $\Delta w_t^n = \frac{1-\alpha \Psi_a}{1-\alpha} \Delta a_t$.
Brief detour: Neutrality of Monetary policy From 1.16 and 1.19 , I get:

$$\begin{cases} \hat{\mu}_t^p = -\tilde{w}_t - \frac{\alpha}{1-\alpha} \tilde{y}_t \\ \hat{\mu}_{1t}^w = -\left[\sigma + \frac{\eta}{1-\alpha}\right] \tilde{y}_t + \tilde{w}_t \end{cases} \tag{1.23}$$

It is clear from that system of equation that in both flexible wage and price equilibrium ($\hat{\mu}_t^p = 0$ and $\hat{\mu}_t^w = 0$) $\tilde{w}_t = 0$ and $\tilde{y}_t = 0$ which means that the output gap and the real wage gap are zero. Thus, in absence of nominal rigidity, a monetary policy shock does not have any real effect on real variables: the so- called the neutrality of monetary policy in the absence of nominal rigidity.

1.3.3 Characterization of the equilibrium in TANK

In this section, I characterize the equilibrium in the TANK framework. The first proposition summarizes the equations in TANK equilibrium. The second proposition shows one of our main results: the observational equivalence between RANK and TANK in a flexible price and sticky wage environment.

Proposition 1. *Under sticky wages and prices, the TANK model can be summarized with the following system of 5 equations*

$$\begin{cases} \pi_t^p = \beta \mathbb{E}\pi_{t+1}^p + \lambda_p \tilde{w}_t + k_p \tilde{y}_t & \text{Price NKPC} \\ \pi_t^w = \beta \mathbb{E}\pi_{t+1}^w - \lambda_w \tilde{w}_t + k_w \tilde{y}_t & \text{Wage NKPC} \\ \tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma(1-\Psi_3)}\left[\hat{r}_t^b + \mathbb{E}\Delta z_{t+1} - \sigma\Psi_a \mathbb{E}\Delta a_{t+1}\right] - \frac{\Psi_2}{1-\Psi_3}\mathbb{E}\left[\tilde{w}_{t+1} - \tilde{w}_t\right] & \text{DIS} \\ \hat{i}_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t & \text{Taylor rule} \\ \Delta\tilde{w}_t = \pi_t^w - \pi_t^p - \frac{1-\alpha\Psi_a}{1-\alpha}\Delta a_t & \text{Identity} \end{cases} \tag{1.24}$$

Where $\lambda_p = \frac{\varepsilon - 1}{\theta}$,$Kp = \left(\frac{\alpha}{1-\alpha}\right)\lambda_p$, $\lambda_w = \frac{\epsilon_w w}{\theta_w}$ and $k_w = \lambda_w \left[\sigma + \frac{\eta}{1-\alpha}\right]$
The proof of the proposition is straightforward (see equations 1.10, 1.18, 1.20, 1.21, and 1.22). There are two differences with the standard "3 equations" model. There is one Phillips curve for each source of nominal rigidity and the aggregation only alters the demand side of the model: the Dynamic IS equation and the change is proportional to the change in the wage gap. In the price NKPC, when real wage is above their natural level, firms reset price up because it increases their marginal cost. While in the wage NKPC, when real wage is above their natural level, the middleman resets nominal wage down to smooth the adjustment cost. The presence of Hand-to-Mouth(HtM) does not play any direct role in the first two equations. In the DIS the dynamics of the real wage gap matters for the aggregate response of the output gap to aggregate shock. In the absence of HtM (ie $\Psi_2 = \Psi_3 = 0$) this dynamics is no longer relevant. The reason is due to the fact that the HtM relies on their current income so the real wage. The change in their real wage appears to be very important for the aggregate response of an aggregate shock. In only sticky prices or only sticky wages, this real wage tends to be more volatile. Sticky wages and sticky prices together limit that volatility and thus the role of HtM in TANK.
Next proposition establishes our first main result. It states that RANK and TANK are equivalent in terms of aggregate fluctuations to aggregate shock.

Proposition 2. *Under sticky wages, RANK and TANK are equivalent. The TANK model can be summarized with the following system of 4 equations*

$$\begin{cases} \pi_t^w = \beta \mathbb{E}\pi_{t+1}^w + \lambda_w \left[\sigma + \frac{\eta+\alpha}{1-\alpha}\right]\tilde{y}_t & \text{Wage NKPC} \\ \tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma}\left[\hat{r}_t^b + \mathbb{E}\Delta z_{t+1} - \sigma\Psi_a \mathbb{E}\Delta a_{t+1}\right] & \text{DIS} \\ \hat{i}_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t & \text{Taylor rule} \\ -\frac{\alpha}{1-\alpha}\Delta\tilde{y}_t = \pi_t^w - \pi_t^p & \text{Identity} \end{cases} \tag{1.25}$$

The system is independent of the share of HtM. Therefore, RANK is equivalent to TANK. The proof is as follows. First, use the previous proposition 3 and eliminate the Price NKPC equation (this equation does not hold under flexible price). Second, use the following relation $\tilde{w}_t = -\frac{\alpha}{1-\alpha}\tilde{y}_t$ and $\Psi_3 = \frac{\alpha}{1-\alpha}\Psi_2$. In monopolistic competition, the firm's profit is proportional to the price markup and output. Given that the profit rate (profit over output) depends only on the markup, the consumption inequality defined as a ratio of the consumption of unconstrained (asset holders) and constrained agents (those who do not participate in the financial market), is proportional to the price markup. If prices are flexible (and wages are sticky), the firm can always adjust its price for a constant price markup. Every firm faces the same nominal wage set by the wage union. So, following an aggregate shock, there is no change in consumption inequality. Then TANK is equivalent to RANK. In the case of sticky prices, we show in appendix A.2.1 that we are back to Debortoli and Galí (2018) case.

1.4 Findings: HANK vs TANK

In this section, I first present my calibration result with the method used to solve for the model. Second I outline some main findings with a discussion of Debortoli and Galí (2018) findings.

1.4.1 Calibration

In the calibration, for comparability reason, I stay as close as possible to Debortoli and Galí (2018). My model has two more parameters than Debortoli and Galí (2018), which are: elasticity of substitution across labor and the wage adjustment cost. I set the elasticity of substitution across labor to be $\epsilon_w = 10$ (the same as the elasticity of substitution among goods) and the wage adjustment cost to be $\theta_w = 150$ which is half of the value used in Hagedorn et al. (2019b)[4]. Time is set to be a quarter. I calibrate the discount factor β for the steady state risk-less real interest rate to be 3% per year. The borrowing limit is set to be $\Psi = 0.5$. This implies the share of constraints agents to be 21.7% (Wealth-based), 22.7% (Labor-based) and 26.8% (Uniform-based). The remaining parameters are the same as in (Debortoli and Galí (2018) section 3.5).

1.4.2 Numerical method

To solve for the TANK model, I use the system of equations in the proposition 3. This can be solved manually by guessing that each variable (control and state variables) is a linear function of exogenous state variables (monetary policy shock v_t, preference shock z_t, and technology shock a_t) and the endogenous state variable (real wage gap $\tilde{w}_{t-1}$). Let $Y_t = [\pi_t^p, \pi_t^w\ \tilde{y}_t\ \hat{r}_t]'$ be the vector of the control variables and $X_t = [\tilde{w}_{t-1}\ a_t\ z_t\ v_t]'$ be the vector of state variables (endogenous and exogenous states). $Y_t = g_x X_t$ and $X_{t+1} = h_x X_t$ where g_x is a matrix 4x4 and h_x is a matrix 4x4. With few seconds the matrix g_x and h_x can be computed using Schmitt-Grohé and Uribe (2004) toolbox.

The HANK model is solved in two steps. First, I solve for the stationary distribution in which every aggregate shock is equal to zero. Second, I solve for the dynamics in which I analyze the effect of each of the aggregate shock. I use Endogenous grid point method (see Carroll (2006)) to solve for the stationary distribution. The dynamics is solved by following Bayer and Luetticke (2018) and Bayer et al. (2019). The method is different from Debortoli and

[4] This implies that wage is changing roughly every 2 quarters. This too low value is by purpose to show that our numerical result is not driven by a high degree of wage stickiness.

Galí (2018) [5]with almost the same result. This is an evidence of robustness of their finding. I discretize the idiosyncratic risk process using Tauchen (1986) with 11 grid points. I use 80 grid points for the net worth A_t and the minimum point is the corresponding borrowing limit. See appendix A.3 for the details on the algorithm used to solve for the HANK model. Next section presents some of my main finding.

Table 1: Parameters

Parameter	Description	Target/source
$\beta = \begin{cases} 0.9778 & W-rule \\ 0.9773 & P-rule \\ 0.9799 & U-rule \\ 0.9909 & RANK/TANK \end{cases}$	Discount factor	avg real interest $\bar{r} = 3\%$
$\Psi = 0.5$	Borrowing limit	share of constr. 21.7% -26.8%
$\sigma = 1$	Risk aversion	Standard value
$\alpha = 0$	Curvature Prod. function	Standard value
$\eta = 1$	Frisch elasticity	Standard value
$\epsilon_w = 10$	Elasticity of substitution across labor	Standard value
$\epsilon_p = 10$	Elasticity of substitution among good	Profit share 10%
$\theta_p = 105.63$	Price adjustment cost	Debortoli and Galí (2018)
$\theta_w = 150$	Wage adjustment cost	slope of 0.06
$\rho_e = 0.9777$	Persistence of idiosyncratic shock	Debortoli and Galí (2018)
$\rho_v = \rho_z = 0.5$	Persistence of pref. and monetary policy shock	Debortoli and Galí (2018)
$\rho_a = 0.9$	Persistence of technology shock	Debortoli and Galí (2018)
$\phi_\pi = 1.5 \quad \phi_y = 0.5/4$	Interest rate coefficients	Debortoli and Galí (2018)
$\tau_a = 1; \quad \tau_e = 0$	Wealth-based	Debortoli and Galí (2018)
$\tau_a = 0; \quad \tau_e = 1$	Labor -based	Debortoli and Galí (2018)
$\tau_a = 0; \quad \tau_e = 0$	Uniform -based	Debortoli and Galí (2018)

1.4.3 Findings

In this section I compare RANK, TANK and HANK model in terms of aggregate fluctuation following a monetary policy shock. Debortoli and Galí (2018) use three main outcome to compare RANK, TANK, and HANK model. First, they compare the path of the impulse response of aggregate variable (output, price inflation, real interest rate, etc.) following a demand shock and a supply shock. They conclude that the path in TANK closely follows the one in HANK. Second, they compare the path of the cumulative of the impulse response of the aggregate variable over 16 quarters after an aggregate shock for different values of the interest rate coefficients ϕ_π and ϕ_y. They conclude that the path in TANK track well the one in HANK. Third they simulate the model and compare some second moment (standard deviation, correlation) of the simulated time series of output (and output gap) and the heterogeneity factors of the models. They also conclude that in general, TANK approximates well HANK model[6].

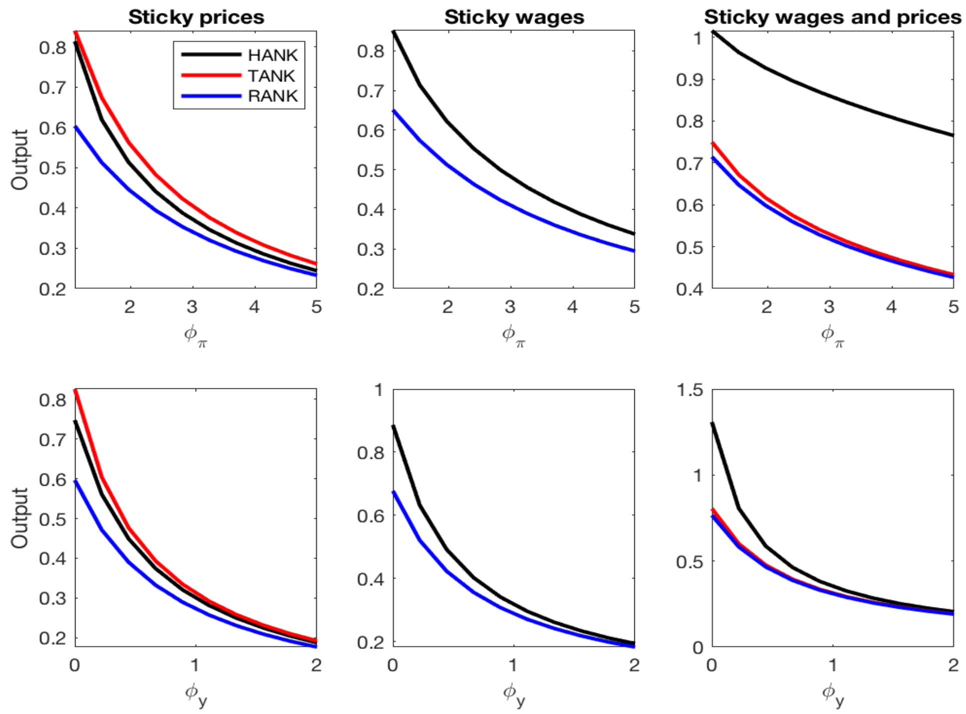
The figure 1.1 makes it clear that in sticky prices, the TANK model approximates well the HANK model. In the presence of sticky wages and sticky prices TANK is far away from HANK. RANK instead approximates very well TANK model. It is another evidence that TANK is not so different from RANK in terms of aggregate fluctuations from monetary policy shock in the presence of both sticky wages and sticky prices.

My findings call for caution when comparing TANK to another model. In fact 10 years ago when TANK model becomes popular, Bilbiie (2008) emphasizes that under sticky prices for a plausible share of Hand to Mouth, the Taylor Principle is violated. Colciago (2011) introduces

[5]They use the method outiline in Reiter (2009).

[6]See Debortoli and Galí (2018) section 5.2 for their general comment about TANK and RANK.

Figure 1.1: Change in environment, Monetary policy rule



Note: The figure compares the cumulative response over 16 quarters in sticky prices (first column), in sticky wages (second column) in sticky wages and prices(third column) for different values of ϕ_π (first row) and ϕ_y (second row) for RANK, TANK, and HANK.

sticky wages in TANK find that the Taylor principle is restored. Nowadays, HANK is becoming popular and Debortoli and Galí (2018) show that under sticky prices TANK approximates well HANK. In this paper, I show that this is no longer the case once we account for sticky wages. The reason why TANK becomes less powerful is that in the presence of both sticky wages and sticky prices, the real wage of the permanent Hand to Mouth Consumer fluctuates less. Also, TANK is equivalent to RANK under only sticky wages for the reasons I described.

1.4.4 Discussion

My paper highlights three main results. First, if prices are sticky (and wages are flexible), TANK approximates HANK. Second, if wages are sticky (and prices are flexible), TANK is equivalent to RANK. Then TANK cannot approximate HANK. Third, if prices and wages are sticky, TANK does not approximate HANK. It somewhat approximates RANK. It is crucial to know how the heterogeneity in HANK affects the aggregate consumption compare to the RANK to understand all those results.

The heterogeneity in HANK affects aggregate consumption in three dimensions: the Change in the consumption gap between unconstrained and constrained agents. This is the consumption inequality dynamics; the change in the consumption dispersion within the unconstrained agents; and the change in the share of constrained agents. Debortoli and Gali (2018) show that the first dimension is the most important difference between RANK and HANK. The key difference between RANK and HANK is the dynamics of consumption inequality (consumption gap between unconstrained and constrained agents). In TANK only the first dimension is (or maybe) present. By construction, the second and the third dimension are not present in TANK (or are always zero).

Given that the most important dimension of the heterogeneity in HANK is the consumption inequality dynamics and that the latter is present also in TANK, TANK can approximate HANK if it tracks well the consumption dynamics in HANK following an aggregate shock. Now, let explain what the sources of the consumption inequality in HANK and TANK are and discuss how the type of nominal rigidity affects the consumption inequality in TANK. In HANK, consumption inequality comes from labor income risk, firm's ownership, and debt choice. On the contrary, in TANK, the consumption inequality comes only from the firm's ownership, which can be understood as follows.

Economy 1: Suppose a continuum of identical agents of mass $1 - \alpha$ who consume and save into a risk-free asset. They have two sources of income: same labor income and same firm ownership income (firm's profit). Because every agent is identical, the choice of bond is zero at the equilibrium (the bond is not traded). So, every household will consume all his income at the equilibrium.Therefore, there is no consumption inequality.

Economy 2: Now, let's introduce in Economy 1 another type of agent of measure α who do not have access to the financial market (they cannot borrow/save). Both agents still have the same labor income and own the same share of firms. Then at the equilibrium, the consumption in economy 1 is the same as the one in economy two because again, the asset is not traded. Again, there is no consumption inequality, and the aggregate consumption will be the same. Despite the difference in the participation in the financial market, economy 1 and economy 2 are the same. This is because there is no trade in the bond and no difference in the budget constraint. So, let's call this result no-trade result.

Economy 3: Let modify economy 2. Let assume now that there is heterogeneity in the budget constraint so that only asset holders own the firms. Households who do not participate in the financial market receive only the same labor income as those who participate in the financial market. At the equilibrium, we still have no trade in the bond market. But now there Is a difference in the consumption of both types of agents. This will generate consumption inequality, a function of firm's profit.

In monopolistic competition, the firm's profit is given by the price markup. Given that the profit rate (profit over GDP) depends only on the markup, one can show that the consumption inequality defined as a ratio of the consumption of unconstrained (asset holders) and constrained agents (those who do not participate in the financial market) is proportional to the price markup.Note that our economy 3 is our TANK model, and Economy 1 is our RANK.

If prices are flexible (wages are sticky), the firm can always adjust its price for a constant price markup (every firm faces the same nominal wage set by the wage union). So, following an aggregate shock, there is no change in consumption inequality. Then TANK is equivalent to RANK (result 2). So why is HANK not equivalent to RANK when prices are flexible?

The reason is simple. In HANK, the consumption inequality does not only come from the firm's ownership but also the labor idiosyncratic income risk and debt choice. If prices are sticky (wages are flexible), following an aggregate shock, the change in consumption inequality in TANK tracks well the one in HANK. So, TANK approximates HANK (result 1). Since prices are sticky, firms cannot adjust their prices to keep a constant markup, and then there is a movement in the consumption inequality. Numerically, this movement in the consumption inequality tracks well the one in HANK. Hence TANK can approximate well HANK. When both prices and wages are sticky, result 2 dominates result 1 so that TANK approximates RANK instead of HANK.

My results highlight the importance of the source of nominal rigidity. In a the standard New Keynesian literature, it is assumed that wages subject to some rigidities are set outside the firms. Wage unions have some labor market power which allow them to have some wage markup over the flexible wages. So every firm in the economy are homogeneous in wages. On the contrary, in the good market, firms have some monopolistic power which allows them to set prices subject to some rigidities. Firms are then ex-ante heterogenous in price setting. Firms can only influence their profit via prices because they take wages are given. If firms were heterogenous in wages and can only set wages instead of prices, then TANK cannot be equivalent to RANK even if prices are flexible. Because what will determine Firm's profit now is how firms set wages.

1.5 Conclusion

This paper studies the implications of idiosyncratic income risk for the aggregate consumption responses to a monetary policy shock. In particular, the paper contrasts the properties of Two Agents New Keynesian (TANK) models with those of Heterogeneous Agents New Keynesian (HANK) models in a sticky wage environment in terms of aggregate fluctuations.

First, I show that under sticky wages (and flexible prices) the TANK model is equivalent to the RANK model. This equivalence means that the equations summarize the state of the economy in a TANK model do not depend on the share of "hand-to-mouth" consumers. In this environment, the consumption inequality is constant over time so not relevant following an aggregate shock. It follows that the TANK model cannot approximate HANK models. Second, under sticky prices (and flexible wages), the TANK model can approximates well HANK model as shown by Debortoli and Galí (2018). But in a sticky prices and sticky wages environment, the TANK model can no longer approximate HANK models in terms of aggregate fluctuations.

My findings call for caution when comparing TANK to other models in a nominal rigidity environment. In fact, how TANK model performs greatly depends on the type of nominal rigidities. With inequality, not only the type of nominal rigidity matters but also the source of nominal rigidity because redistribution matters. Wage markup is uniformly redistributed to every household in my work because wages are set outside the firms. On the contrary, price markup is not redistributed uniformly. In general, only asset holders gain from a price markup since they earn the firms.

In addition, central banks worldwide that aims to integrate income inequality in their quantitative framework, should investigate the source of nominal rigidity in their economy.

Chapter 2

Sudden stops and asset prices: the role of financial market participation*

2.1 Introduction

Output and consumption are more volatile in emerging markets than in advanced economies. Since the 1980s, emerging markets have on average faced a higher drop in the asset price and output relative to advanced economies during a sudden stop crisis[1]. Several studies have been done to understand sudden stops and how macroprudential policy or capital control can reduce their severity, but accounting for household heterogeneity in the financial market is a topic that remains less investigated. This paper studies the role that limited financial market participation plays in the severity of sudden stop crises and its implications for prudential capital control. I show that the effect of household heterogeneity in access to the financial market depends crucially on the cyclicality of consumption inequality.

My paper is motivated by three main empirical facts that I document in Section 2.2. First, across countries, there is heterogeneity in financial market participation. In emerging markets, the proportion of households that have access to financial markets is small. Those households consume their available labor income and do not hold any assets. Second, there is a negative correlation between the level of financial market participation and the drop in the asset price during sudden stops. Historically, countries with a low level of financial market participation faced a higher drop in the asset price during sudden stops. Third, consumption inequality is procyclical during Mexico's 1995 sudden stop crisis. Indeed, during Mexico's 1995 crisis, households who participated in the financial market faced a higher drop in their consumption

*I'm indebted to my advisor, Guillaume Sublet, and co-advisor, Julien Bengui, for their invaluable guidance and support. I would like to thank my colleagues in the workshop group, organized by my advisor Guillaume Sublet at the University of Montreal, for their comments

[1]A sudden stop is an economic crisis that features current account reversal (i.e., massive capital outflows).

compared to those who did not participate in the financial market.

To quantity the importance of inequality in the severity of sudden stops, I enrich a standard dynamic stochastic general equilibrium model that features an occasionally binding collateral constraint with limited household heterogeneity. The model features two types of households. The first type comprises households who participate in the financial market and have access to the capital and bond market. These households are called *asset holders*. The second type of household comprises those who do not participate in either the capital or bond market. These households, called *"hand-to-mouth"* consumers, consume all of their labor income plus any additional transfers. The small open economy faces shocks to its productivity, the real interest rate, and the price of imported inputs. The model economy nests the model in Mendoza (2010) with a fixed supply of capital.

The collateral constraint limits the total private debt to a fraction of the market value of capital. Total private debt is composed of private debt with one-year maturity plus a within-period working capital loan. The within-period working capital loan generates a contemporaneous drop in output when the collateral constraint binds, as it does when successive negative aggregate shocks hit the economy. If the constraint binds, the economy faces a financial crisis called a *sudden stop*. Here two main credit channels are in place. The first is the endogenous financing premium on debt, equity, and working capital as borrowing costs rise when the collateral constraint binds. The second is the Fisher (1933) debt-deflation mechanism. When the collateral constraint binds, asset holders subject their assets to a fire sale to smooth consumption and meet their obligations. The fire sale of assets leads to a decline in the capital price, which further tightens the collateral constraint. This non-linear feedback between the price of capital and the collateral value (borrowing capacity) exacerbates a financial crisis.

Qualitatively, the credit channel I describe is the same in a model with or without household heterogeneity. Quantitatively, I show that the severity of the drop in the asset price depend on the cyclicality of consumption inequality[2]. If consumption inequality is constant, then the level of financial market participation does not affect the severity or the drop in the asset price during the sudden stop. This shows that the pro(cyclicality), not the level of inequality, exacerbates the severity of a sudden stop crisis. If consumption inequality decreases during the crisis, the asset price drops more, and vice versa. Since consumption inequality decreased during Mexico's 1995 sudden stop episode, limited financial market participation exacerbated the severity of the crisis.

In the model, macroeconomic policy plays a role because of two externalities: a pecuniary externality and an aggregate demand externality. First, asset holders take the asset price as given. Their choice of debt affects, in the aggregate, the asset price, which determines the value of the collateral. This general equilibrium effect is called a pecuniary externality. The social planner who takes this pecuniary externality into account may substantially reduce the severity of a financial crisis because of a binding collateral constraint. Second, the aggregate demand externality arises because asset holders take as given the choice of labor supply of hand-to-mouth consumers. Note that this aggregate demand externality is only present in the case of limited financial market participation. In the optimal policy, I choose to use a tax on foreign debt — a capital control — to decentralize the planner solution.

[2] A crisis is more (less) severe when the drop in output and consumption is high (low).

The quantitative results show that limited financial market participation amplifies the drop in output and asset holders' consumption. The fall in the asset price is 23% larger in a limited financial market participation economy compared to a full financial market participation economy. The model is consistent with three main business cycle fact in emerging markets. First, consumption is more volatile than output. Second, the trade balance is countercyclical. Third, the real interest rate is countercyclical.

To study the optimal policy, I introduce a financial shock. A financial shock — a drop to the loan-to-value ratio — consists of a drop during a financial crisis in the fraction of the total value of physical assets that households can pledge as collateral. The introduction of a financial shock to study the optimal policy is consistent with the data. The loan-to-value ratio is consistently low during a sudden stop crisis. With a financial shock, the asset price drops by 57% and asset holders' consumption drops by 25% in a limited financial market participation economy, whereas the asset price drops by 40% and the asset holders' consumption drops by 15% in a full financial market participation economy. The fall in the asset price is now 42% higher in a limited financial market participation economy compared to a full financial market participation economy. As expected, the results suggest that the financial shock exacerbates the financial crisis.

The optimal time-consistent constrained efficient allocation suggests three main points. First, the optimal time-consistent solution effectively reduces the frequency and severity of the financial crisis in both the full and limited financial market participation economies. Second, the average tax on foreign debt needed to decentralize the optimal time-consistent solution is higher in a limited financial market participation economy. This suggests that more capital control is needed in emerging markets, which have a low level of financial market participation relative to advanced economies. This second lesson rationalizes the prevalent use of capital control in emerging markets. In fact, data on capital controls suggest that emerging markets control more capital flows than advanced economies. Third, the optimal time-consistent solution suggests that there is no trade-off between financial stability and consumption inequality in the case of limited financial market participation. While in the very short run (at the time of the financial crisis), the social planner may tolerate a slight increase in consumption inequality, average consumption inequality is no higher in the optimal time-consistent equilibrium than it is in the competitive economy.

My paper mainly relates to the literature that studies the aggregate effects of a sudden stop (see, for example, Arellano and Mendoza (2002), Chari et al. (2005), Mendoza (2006), Calvo et al. (2006). Mendoza (2010), and Korinek and Mendoza (2014)). My paper is closely related to Mendoza (2010), who studies how an endogenous binding collateral can trigger the economy within standard business cycle moments. My contribution to this literature is twofold. First, I introduce limited financial market participation where a fixed share of households do not participate in the financial market. This characterization of the economy is closer to that of emerging markets and helps us to explain the observed gap in the decline in the asset price during sudden stops between emerging markets and advanced economies. In addition, my work studies the optimal time-consistent solution and rationalizes the prevalent use of capital control in emerging markets.

My work is also related to recent literature that studies the optimal policy in a financial crisis model. These papers include Caballero and Krishnamurthy (2004), Bianchi (2011), Bengui (2014), Bengui and Bianchi (2018), Bianchi and Mendoza (2018), and Arce et al. (2019). I contribute to this literature by taking into account household heterogeneity in the financial market and show that it is possible to address financial instability without raising inequality.

The rest of the paper is organized as follows. In Section 2.2, I present the data and the empirical facts. Section 3.2 presents the model. Sections 2.4 and 2.5 present my findings and discuss the results, and Section 3.4 concludes.

2.2 Data and empirical facts related to asset prices and inequality

In this section, I present the data and the empirical facts related to financial market participation, sudden stops, and inequality. I use three sources of macro and micro data. The first is panel data on financial market participation, which cover low-income countries, emerging markets, and advanced economies. The second is aggregate data on the drop in asset prices during sudden stops. The third is micro survey data on household consumption, income, and wealth in Mexico.

Financial market participation. I use the IMF's Financial Development Index Database. The index database provides nine indexes for 180 countries for every year since 1980. I focus on two indexes that measure the ability of individuals and firms to access financial services: the Financial Institution Access index (FIA) and the Financial Market Access index (FMA). FIA measures the number of bank branches per 100,000 adults and the number of ATMs per 100,000 adults. FMA measures the percentage of market capitalization outside of the top 10 largest companies and the total number of issuers of debt (domestic and external, non-financial corporations) per 100,000 adults. All indexes are between 0 and 1 where 1 means full access to financial services.

Sudden stops. I use the sudden stops data constructed by Korinek and Mendoza (2014). A sudden stop is defined as a large capital outflow as measured by a year-over-year increase in the current account/GDP ratio by more than two standard deviations above the average change in this ratio. I use the stock market index provided in the data as a measure of asset prices. The data include emerging markets and advanced economies over the period 1980-2012.

Consumption, income, and wealth. I use Mexico's National Survey of Household Income and Expenditure (ENIGH). This is a representative household survey that covers rural and urban areas and has been conducted every two years since 1992. More than 10,000 households are interviewed at each survey. The survey has detailed information about household consumption items as well as household income and wealth. I define "hand-to-mouth" consumers as households who hold zero liquid wealth. I define consumption inequality as the ratio of *asset holders'* consumption to the consumption of hand-to-mouth households.

I document three facts:

Fact 1: There is heterogeneity in financial market participation across countries. Also, on average, emerging markets have a lower level of financial market participation relative to advanced economies. Figure 2.1 displays the median of the financial market access index from

1980 to 2017 for each country. Figure 2.2 displays the median of the financial institution access index from 1980 to 2017 for each country. Darker red areas indicate higher financial market participation. North American countries, western European countries, Japan, Australia, and a few other countries have relatively high access to financial services.

Fact 2: There is a negative correlation between the level of financial market participation and the drop in asset prices during a sudden stop episode. Table 2.1 presents the results from a panel regression of asset prices on a dummy variable that takes the value of 1 if a given country in a given year is in a sudden stop crisis, financial market access, and the cross-product between the dummy and financial market access. The cross-product captures the marginal effect of the level of financial market participation on the drop in asset prices during sudden stops. The first column considers advanced economies and emerging markets. The second column considers only advanced economies, and the third column considers only emerging markets. All regressions include country and year fixed effects. I control for capital flows. The estimated coefficient on the cross-product is positive, which means that a country with a higher level of financial market participation has a lower drop in the asset price during a sudden stop episode.

Fact 3: Consumption inequality is procyclical during Mexico's 1995 sudden stop crisis. Figure 2.3 plots the consumption inequality dynamics in Mexico from 1992 to 2000. Consumption inequality is defined as the ratio of *asset holders'* consumption to the consumption of "hand-to-mouth" households. Prior to the sudden stop crisis in 1995, consumption inequality increases. It then decreases during the crisis from 1994 to 1996 and starts to increase again beginning in 1996, evidence that asset holder consumers, are hit relatively harder by the sudden stop crisis than "hand-to-mouth" consumers.

I use these three facts to discipline my model. In my model, the financial crisis and consumption inequality are endogenous, whereas the level of financial market participation is exogenous. I present the model in the next section.

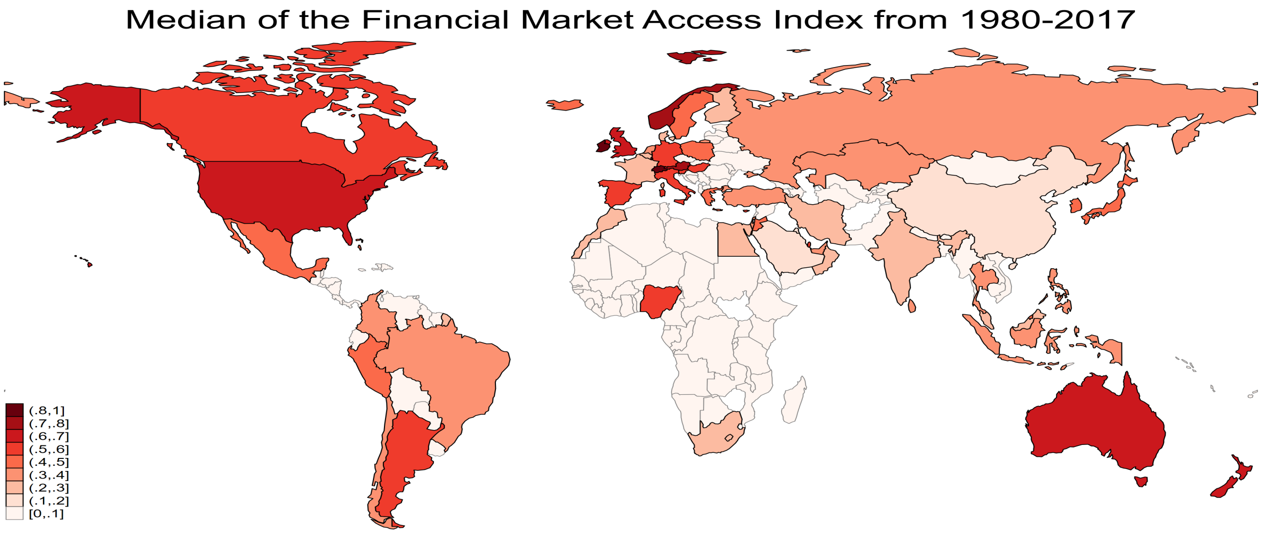


Figure 2.1: Financial market accessibility across countries

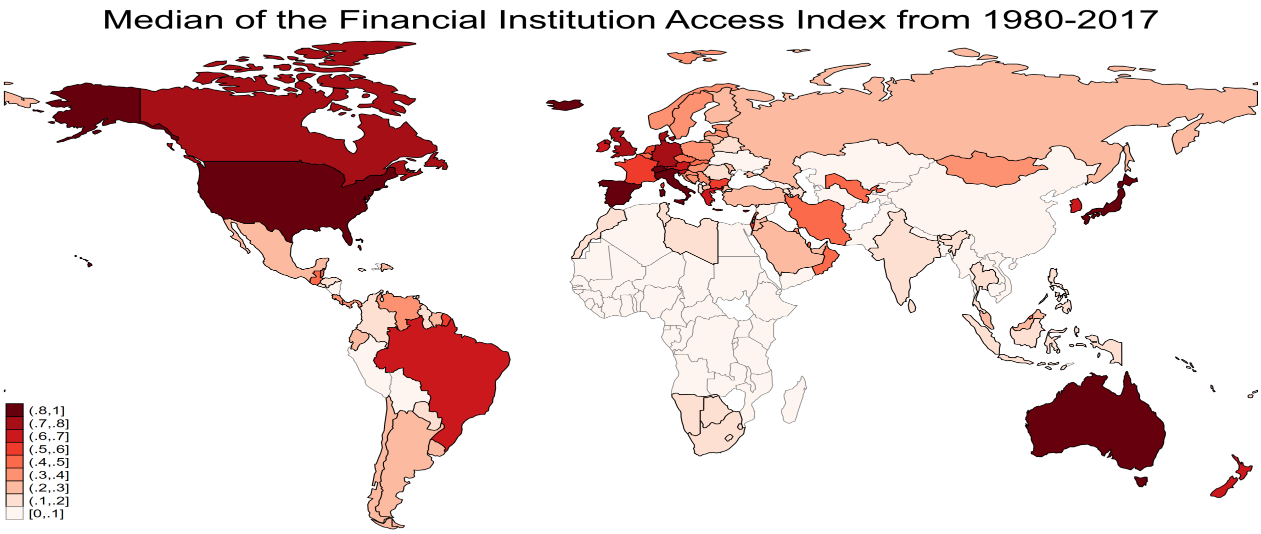


Figure 2.2: Financial institution accessibility across countries

Table 2.1: Panel regression of equity price growth on sudden stops

	(1)	(2)	(3)
Asset price growth	Aggregate	Advanced Economies	Emerging Markets
Sudden stops (SS)	-0.438***	-0.334***	-0.649***
Financial market participation (FMA)	-0.110	0.0104	-0.269
FMAxSS	0.416**	0.295*	0.985**
Observations	631	366	265
R-squared	0.352	0.533	0.421
Number of countries	29	15	14

Note: Regressions are done with country and year fixed effects. SS is a dummy variable that takes the value of 1 if a country is in a sudden stop for a given year. Data on sudden stops are from Korinek and Mendoza (2014). FMA is the Financial Market Access index from the IMF. FMAxSS is a cross-product of FMA and SS. I control for capital flows. The data cover the period 1980-2012. I drop the sudden stop events that have an increase in asset prices. *** $p<0.01$, ** $p<0.05$, * $p<0.1$

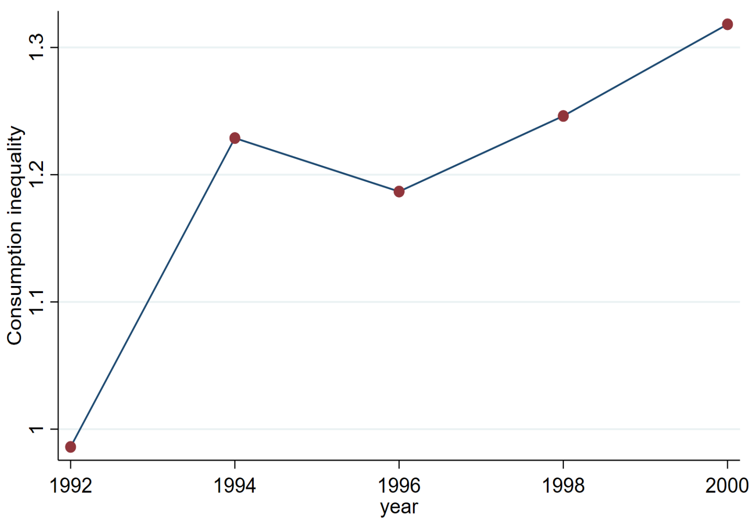


Figure 2.3: Consumption inequality in Mexico: consumption inequality is defined as the ratio of *asset holder* consumers' consumption to “hand-to-mouth" consumers' consumption.

2.3 Model with collateral constraint and household heterogeneity

I build a small open economy model with household heterogeneity and a collateral constraint. The sudden stop crisis is driven by an occasionally binding collateral constraint. There are two types of households. The first type comprises asset holder consumers who have access to the financial market through their holding of both physical assets and foreign bonds. The second type are "hand-to-mouth" consumers who do not hold any assets — neither physical assets nor foreign bonds. They consume all of their labor income plus any additional transfers from the government. In this section, I assume that asset holder consumers make production and consumption decisions.[3]

2.3.1 Firm and asset holder households' optimization problem

There is a continuum of identical *asset holder* households of measure $1-\theta \in (0,1]$. The preferences of an *asset holder* consumer indexed by 1 are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{1t} - G(L_{1t})), \tag{2.1}$$

where $\mathbb{E}_0$ is the expectations operator; β is the discount factor; C_{1t} is consumption, and L_{1t} is labor supply; $u(.)$ is the utility function which is a standard concave, twice continuously differentiable function that satisfies the Inada condition; and G(L) is a convex, strictly increasing, and continuously differentiable function that measures the disutility of labor. These preferences (known as GHH preferences due to Greenwood et al. (1988)) remove the wealth effect on labor supply, which prevents a counterfactual increase in labor supply during crises.

Households produce final goods using three inputs, which are physical assets k_t, intermediate goods v_t, and labor L_t. Total labor L_t in the economy is given by $(1-\theta)L_{1t} + \theta L_{2t}$, where L_{2t} is the labor supply of a "hand-to-mouth" consumer. The production technology is such that $y = A_t F(k_t, L_t, v_t)$, where F is a twice continuously differentiable, concave production function and $A_t = A\exp(\epsilon_t^A)$ is TFP subject to a random shock ϵ_t^A. This shock follows a stationary Markov process. Intermediate goods are traded in competitive world markets at a price p_t^v. The price $p_t^v = p\exp(\epsilon_t^v)$ is subject to a random shock ϵ_t^v that follows a stationary Markov process. *Asset holder* households borrow on the foreign bond market at the real interest rate $R_t^v = R\exp(\epsilon_t^r)$, where ϵ_t^r is a random shock that follows a stationary Markov process. The budget constraint of *asset holder* households is given by

$$(1-\theta)\, C_{1t} + \frac{b_{t+1}}{R_t} + q_t k_{t+1} = F(k_t, L_t, v_t) - p_t^v v_t - \theta w_t L_{2t} - \phi r_t (w_t L_t + p_t^v v_t) + b_t + q_t k_t - T_t. \tag{2.2}$$

In equation 3.2, q_t is the price of the physical asset k_t, $r_t = R_t - 1$ is the net real interest rate, and w_t is the real wage. On the right-hand side of (3.2), the term $\phi r_t (w_t L_t + p_t^v v_t)$ represents

[3] In appendix B.1, I show that a separate problem between firm and asset holder consumers has the same outcome.

the interest payment abroad on the working capital loan. The working capital loan is a fraction ϕ of the total cost of intermediate inputs and labor in advance of sales. The term $\theta w_t L_{2t}$ represents the total labor income paid to "hand-to-mouth" households. The term T_t is the total lump-sum taxes paid by all *asset holder* households. Lump-sum taxes are used to calibrate the average consumption inequality.

The total private debt in the economy is restrained to a fraction κ of the market value of the end-of-period physical asset given by

$$\frac{b_{t+1}}{R_t} - \phi R_t \left(w_t L_t + p_t^v v_t\right) \geq -\kappa q_t k_{t+1}. \tag{2.3}$$

On the left-hand side of (3.3), total private debt (in negative terms) is the sum of private debt with one-year maturity and the within-period working capital loan. On the right-hand side of (3.3), the term $\kappa q_t k_{t+1}$ represents a fraction κ of the market value of the end-of-period physical asset. Only *asset holder* households who borrow in the foreign bond market face this collateral constraint. Although I do not derive the collateral constraint from an optimization problem, Bianchi and Mendoza (2018) show that this type of constraint could be obtained as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ of the market value of an asset owned by a defaulting debtor.

The *asset holder* households choose consumption, borrowing, capital, labor, and intermediate inputs to maximize their utility (3.1) subject to their budget constraint (3.2) and their borrowing constraint (3.3), taking prices as given. Their optimality conditions are given by

$$u'(t) = \beta R_t \mathbb{E}_t u'(t+1) + \mu_t, \tag{2.4}$$

$$q_t u'(t) = \beta \mathbb{E}_t \left[\left(d_{t+1} + q_{t+1}\right) u'(t+1)\right] + k q_t \mu_t, \tag{2.5}$$

$$A_t F_l \left(k_t, L_t, v_t\right) = G'(L_{1t}) + \phi \left(r_t + R_t \frac{\mu_t}{u'(t)}\right) w_t, \tag{2.6}$$

$$A_t F_v \left(k_t, L_t, v_t\right) = p_t^v + \phi \left(r_t + R_t \frac{\mu_t}{u'(t)}\right) p_t^v, \tag{2.7}$$

where $\mu_t \geq 0$ is the Lagrange multiplier on the borrowing constraint, $u'(t)$ is the partial derivative of $u(C_{1t} - G(L_{1t}))$ with respect to C_{1t}, and $d_{t+1} = A_{t+1} F_k \left(k_{t+1}, L_{t+1}, v_{t+1}\right)$.

The first two optimality conditions are the Euler equations for bonds and physical assets, respectively. The last two optimality conditions are the intratemporel conditions on the labor market and intermediate good market, respectively.

Condition (3.4) states that if the collateral constraint is not binding ($\mu_t = 0$), the marginal benefit of borrowing to increase today's consumption is equal to the expected marginal cost of repaying back tomorrow. If the collateral costraint binds, the shadow price of relaxing the collateral constraint is positive ($\mu_t > 0$), so the marginal benefit of borrowing is greater than its expected marginal cost. Condition (3.5) states that the marginal cost of buying one additional unit of physical asset at price q_t is equal to its expected marginal benefit. If the collateral constraint binds, the marginal cost exceeds the marginal benefit by $k q_t \mu_t$.

Condition (3.6) states that the marginal productivity of labor demand is equal to the marginal disutility of labor supply plus the financing cost of labor from the working capital loan. The financing cost is higher when the collateral constraint binds. Condition (3.7) states that the marginal productivity of the intermediate input is equal to its price plus the financing cost of the intermediate input from the working capital loan. The financing cost of the intermediate input is higher when the collateral constraint binds.

2.3.2 Hand-to-mouth households' optimization problem

There is a continuum of identical "hand-to-mouth" households of measure $\theta \in [0, 1)$. The preferences of a "hand-to-mouth" consumer indexed by 2 are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{2t} - G(L_{2t})), \tag{2.8}$$

where C_{2t} is consumption, L_{2t} is labor supply, and $u(.)$ is the same utility function as in section 3.2.1. The budget constraint of "hand-to-mouth" households is given by

$$\theta C_{2t} = \theta w_t L_{2t} + T_t. \tag{2.9}$$

The *hand-to-mouth* households choose consumption and labor to maximize their utility (3.8) subject to their budget constraint (3.9), taking prices as given. Their optimality condition is given by

$$G'(L_{2t}) = w_t. \tag{2.10}$$

Condition (3.10) states that the marginal disutility of labor supply for *asset holder* consumers is equal to the real wage rate.

2.3.3 Competitive equilibrium

In this this section, I define the competitive equilibrium and the main credit channel through which sudden stops arise in this type of framework. The aggregate resource of the economy is given by

$$C_t + \frac{b_{t+1}}{R_t} - b_t + \phi r_t \left(w_t L_t + p_t^v v_t\right) = F\left(1, L_t, v_t\right) - p_t^v v_t, \tag{2.11}$$

where $C_t = (1-\theta) C_{1t} + \theta C_{2t}$ is aggregate consumption, the term $\frac{b_{t+1}}{R_t} - b_t + \phi r_t \left(w_t L_t + p_t^v v_t\right)$ represents the trade balance, and the term $F\left(1, L_t, v_t\right) - p_t^v v_t$ represents GDP.

A competitive equilibrium in this model is a stochastic sequence $Q_t = \{C_{1t}, C_{2t}, L_{1t}, L_{2t}, v_t, b_{t+1}\}_{t \geq 0}$ and prices $P_t = \{q_t, w_t\}_{t \geq 0}$ such that:

1. Given P_t, Q_t solves households' and firms' problems;
2. w_t and q_t are determined competitively that is: $G'(L_t) = w_t$ and q_t solves equation 3.5;

3. markets clear:

(a) labor market: $L_t = L_{1t} = L_{2t}$,

(b) capital market: $K_t = 1$,

(c) aggregate resource: equation (3.15) is satisfied.

2.3.4 Equity premium and consumption inequality wedge

In this section, I characterize the equity premium and show how limited financial market participation distorts the equity premium.

Let $\frac{\lambda_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]}$ be the inverse of the stochastic discount factor in the economy with full financial market participation ($\theta = 0$) where $\lambda_t^R = \beta R_t \mathbb{E}_t \lambda_{t+1}^R + \mu_t^R$. Let $\frac{\lambda_t}{E_t[\beta\lambda_{t+1}]}$ be the inverse of the stochastic discount factor in the economy with limited financial market participation ($\theta > 0$) where $\lambda_t = \beta R_t \mathbb{E}_t \lambda_{t+1} + \mu_t$. Using the definition of asset returns and conditions 3.4 and 3.5, the expected excess returns of bonds $E_t\left[R_{t+1}^q - R_t\right]$ can be decomposed into a liquidity premium, an inequality wedge, and the risk premium as follows:

$$E_t\left[R_{t+1}^q - R_t\right] = (1-\kappa) \underbrace{\frac{\mu_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]}}_{\textbf{liquidity premium}} -(1-\kappa)\underbrace{\left(\frac{\lambda_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]} - \frac{\lambda_t}{E_t\left[\beta\lambda_{t+1}\right]}\right)}_{\textbf{consumption inequality wedge}} - \underbrace{\frac{Cov\left(\lambda_{t+1}, R_{t+1}^q\right)}{E_t\left[\lambda_{t+1}\right]}}_{\textbf{risk premium}} \tag{2.12}$$

The term $\frac{\lambda_t^R}{E_t\left[\beta\lambda_{t+1}^R\right]} - \frac{\lambda_t}{E_t[\beta\lambda_{t+1}]}$, which I call the *consumption inequality wedge*, is the difference of the inverse of the stochastic discount factor between the economy with full financial market participation $\theta = 0$ and the economy with limited financial market participation $\theta > 0$. For $\kappa = 1$, the liquidity premium and the consumption inequality wedge do not affect the equity premium. If the agents can pledge all of their assets as collateral ($\kappa = 1$), when the collateral constraint binds, the agents can always offset it by increasing their physical assets by one unit. The relevant case is when the agents cannot pledge all of their assets as collateral ($\kappa < 1$).

The liquidity premium raises the equity premium when the collateral constraint binds for $\kappa < 1$. The rise in the equity premium decreases asset prices and the collateral constraint tightens even more. This mechanism is known as the debt-deflation mechanism and is at the core of the financial crisis generated by the model I present. In the model, the collateral constraint binds endogenously when the leverage is relatively high, and an exogenous negative aggregate shock (high real interest rate, low productivity shock, and/or high imported price) hits the economy.

The limited financial market participation add a new term to the equity premium in equation (2.12), which is the consumption inequality wedge. When the consumption inequality wedge is negative, it raises the equity premium and the asset price decreases more, leading to a high decline in the asset price. Inversely, if the consumption inequality wedge is positive, the equity premium decreases, leading to a low decline in the asset price. The effect of the consumption inequality wedge depends on the cyclicality of consumption inequality.

I now characterize how the cyclicality of consumption inequality affects the consumption inequality wedge. Suppose that $u(C_t - G(L_t)) = \frac{\left(C_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma}}{1-\sigma}$ where ω is the labor elasticity. Then $\lambda_t = a_t^{\sigma} \lambda_t^R$ with $a_t = \frac{(1-\theta)\omega + (\omega\theta - 1)\frac{c_{2t}}{c_{1t}}}{\omega - \frac{c_{2t}}{c_{1t}}}$. The ratio $\frac{c_{1t}}{c_{2t}}$ is defined as consumption inequality.

Claim 1: If consumption inequality is constant over time then financial market participation does not matter for sudden stop crises. The drop in the asset price is the same in the economy with full financial market participation as in the economy with limited financial market participation.

The proof follows from the definition of the consumption inequality wedge, which is zero for a constant consumption inequality since $\frac{\lambda_t^R}{E_t\left[\beta \lambda_{t+1}^R\right]} = \frac{\lambda_t}{E_t[\beta \lambda_{t+1}]}$. The consequence of claim 1 is that the level of consumption inequality does not affect the drop in the asset price, hence the severity of the crisis. The next claim completes this claim by showing that the cyclicality of the consumption inequality indeed matters.

Claim 2: Let's suppose perfect foresight (no uncertainty); that is, $\mathbb{E}_t[X_{t+1}] = X_{t+1}$. If the consumption inequality is lower (higher) during the financial crisis, the economy will generate a higher (lower) amplification effect.

Under perfect foresight, the consumption inequality can then be rewritten as $\frac{\lambda_t^R}{\beta \lambda_{t+1}^R}\left(1 - \left(\frac{a_t}{a_{t+1}}\right)^{\sigma}\right)$. Suppose now that at time t, the collateral constraint binds and $\frac{c_{1t}}{c_{2t}} < \frac{c_{1,t+1}}{c_{2,t+1}}$ (that is lower consumption inequality) this implies that $a_t > a_{t+1}$. It follows that the consumption inequality wedge is negative, leading to a high equity premium.

Claims 1 and 2 have shown that, given the limited financial market participation, what matters is the cyclicality of consumption inequality. As shown in fact 3 of Section 2, the consumption inequality is procyclical, leading quantitatively to a higher drop in asset prices. Werning (2015) and Acharya and Dogra (2020b) have argued that the cyclicality of income inequality could affect the aggregate outcome variables in a monetary policy with a household heterogeneity framework. In my framework, where there are no nominal rigidities, I find that the cyclicality of consumption inequality does affect the severity of a financial crisis. In the next section, I present the quantitative results.

2.4 Quantitative results

This section studies the model's quantitative implications using numerical simulation. First, I present the calibration and then discuss the results.

2.4.1 Calibration

A period in the model represents a year. The calibration uses data from Mexico. The results are presented in Table 2.2. The functions forms for preference and technology are the following:

$$u(C_t - G(L_t)) = \frac{\left(C_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}, \quad \omega > 1$$
$$F(k_t, L_t, v_t) = A_t k_t^{\gamma} L_t^{\alpha} v_t^{\eta}.$$

The preference parameters for risk aversion and the elasticity of substitution are set to standard values from the literature: $\sigma = 2$. The average real interest rate is set to 4%, also standard in the literature.

Labor supply elasticity ω is set equal to 1.846, as in Mendoza (2010). Mendoza (2010) uses data for the period 1993:1-2005:11 and finds that the annualized average ratio of GDP to gross output (gdp/y) is 0.896 and the ratio of imported inputs to GDP (pv/gdp) is 0.114. The average share of imported inputs in gross output is 0.102, which implies that η= 0.102. The labor share on GDP for Mexico is 0.66, which implies that $\alpha = 0.592$. The value of $\gamma = 0.042$ is set so that the equity premium is zero at the deterministic steady state. The steady state asset price is set to 1.

The shocks are modeled as a joint discrete Markov process that approximates the statistical moments of their actual time-series processes. The Markov process is defined by a set E of all combinations of realizations of the shocks, each combination given by a triple $e = (\epsilon^A, \epsilon^R, \epsilon^P)$ and by a matrix of transition probabilities of moving from e_t to e_{t+1}. I closely follow Mendoza (2010) to set the transition probability between the different states. In the data, $\epsilon^A, \epsilon^R, \epsilon^P$ are AR(1) processes with standard deviations and first-order autocorrelations,respectively, 0.537, 0.572, and 0.737. Since the three shocks are nearly independent, except for a statistically significant correlation between ϵ^A and ϵ^R of about -0.67, the Markov process is constructed using the parsimonious structure of the two-point symmetric simple persistence rule as in Mendoza (2010) . Each shock has two realizations equal to plus/minus one standard deviation of each shock in the data ($\epsilon_1^A = -\epsilon_2^A = 0.0134$, $\epsilon_1^R = -\epsilon_2^R = 0.0196$, $\epsilon_1^P = -\epsilon_2^P = 0.0335$), so E contains eight triples. The simple persistence rule produces an 8x8 matrix, which yields autocorrelations of the shocks and a correlation between ϵ^A and ϵ^R that match those in the data. Mendoza (2010) points out, however, that the procedure requires that the AR(1) coefficients of the shocks that are correlated with each other (ϵ^A and ϵ^R) be the same, which is in line with the data where $\rho(eR) = 0.572$ and $\rho(eA) = 0.537$.

The two parameters remaining are β and κ. The value of β is set to 0.92 to match the average net foreign asset of 20% of GDP. The value of κ is set to 0.43 to match the frequency of the financial crisis of 4%. Average private debt is 19.7 % of GDP, and the frequency of the financial crisis is 4.6%.

2.4.2 Sudden stops: the dynamics of asset prices, output, debt, and consumption

This section presents the quantitative results. I first describe the difference in the debt policy function between the economies with limited and full financial market participation. Second, I show the dynamics of some aggregate variables when the collateral constraint binds. I finish this section by showing some long-run moments.

Policy functions for gross private debt. Figure 2.4 presents the next period private debt b_{t+1} as a function of current private debt b_t. The solid magenta line represents the policy function for a negative aggregate shock that has a high real interest rate and low productivity.

Table 2.2: Parameter values

Parameters set independent	Value	Source/Target
Risk aversion	$\sigma = 2$	Standard value
Share of labor in gross output	$\alpha = 0.592$	Mexico GDP labor share 0.66
Share of input in gross output	$\eta = 0.10229$	Mexico data
Share of asset in output	$\gamma = 0.043$	steady state asset return
Frisch elasticity	$\omega = 1.846$	Mendoza (2010)
Working capital coefficient	$\phi = 0.13$	Working capital/ GDP ratio = 10%
Share of Hand-to-Mouth	$\theta = 0.5$	Mexico data
Transfer	$T_t = 0.14$	Avr cons ineq of 1.25
Parameters set simulation	Value	Target
Discount factor	$\beta = 0.920$	Net foreign asset of 20%
Fraction of collateral value	$\kappa = 0.43$	Financial crisis of 4%

The dash blue line represents the policy function for a positive aggregate shock that has a low real interest rate and high productivity. In panel (a), the economy with limited financial market participation ($\theta = 0.5$) is shown. In panel (b), the economy with full financial market participation ($\theta = 0$) is shown .

In both panels, the next period debt as a function of the current debt for a negative aggregate shock has a V-shape. This V-shape is due to the collateral constraint, which is more likely to bind for a high debt and negative aggregate shock. If the debt level is high (that is, the bond is more negative), households are forced to deleverage when a negative shock hits the economy. For a positive shock, the policy function is almost linear since the collateral constraint is less likely to bind.

(a) Limited financial market participation (b) Full financial market participation

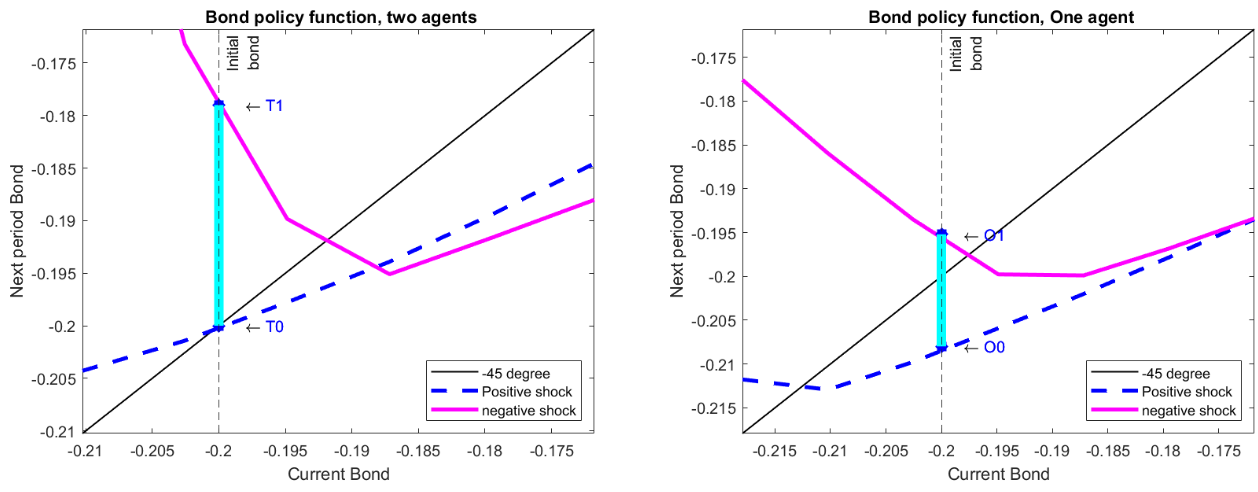


Figure 2.4: Policy function for private debt

To understand the quantitative difference between the two economies (panel (a) and panel (b)), consider the cyan line (T0,T1) and (O0,O1). Suppose that both economies start at the same bond, which is equal to -0.2. In a limited financial market participation economy, for a positive shock, households will choose T0, otherwise they will choose T1. Households then expect to reduce the debt level by 0.02 from a positive shock to a negative shock. In the same way, in a full financial market participation economy, households expect to reduce the debt by 0.015 from a positive shock to a negative shock. The limited financial market participation economy will generate a higher amplification because households are expected to reduce their debt more when the collateral constraint binds.

Financial crises. I now analyze the ability of the model to generate financial crises and the role of limited financial market participation in generating a higher drop in the asset price. For that purpose, I simulated the model for 100,000 periods and constructed a nine-year event windows centered at the crisis year. A financial crisis is defined as when the collateral constraint binds and the trade balance is two standard deviations above its mean. Using the current account instead of the trade balance gives the same result.

Figure 2.5 shows the average of output, consumption of asset holders, consumption of hand-to-mouth consumers, private debt, asset prices, and exogenous shock across the nine-year event windows for the two economies. I normalize the average of the variables to 1 at t-1. For panels (a) to (e), the solid blue line represents the limited financial market participation economy where the share of hand-to-mouth consumers is set to 50%. The dashed magenta line represents the economy with full financial market participation; that is, the share of hand-to-mouth consumers is set to 0. Panel (f) shows the percentage differences relative to the unconditional averages of aggregate exogenous shocks that hit the economy.

I follow Bianchi and Mendoza (2018) to construct comparable event windows for the two economies. First, I simulate the limited financial market economy for 100,000 periods and identify financial crises using the definition described above. Second, I construct nine-year event windows centered at the crisis year, denoted date t, by computing averages for each variable across the cross section of crisis events at each date. The result of this procedure is the solid blue line in Figure 2.5. Third, I take the initial bond position at t-4 of the limited financial market participation economy crisis and the sequences of aggregate exogenous shocks in the 9-year window in this economy, and I pass them through the policy functions of the full financial market participation economy. Finally, I compute the average, as in the previous case. The result of this procedure is the dashed magenta line in Figure 2.5 .

Panels (a) , (b), and (e) show that output, consumption of the asset holders, asset prices fall more in the economy with limited financial market participation relative to the economy with full financial market participation. In the full financial market participation economy, output falls short by 0.5 percentage points (5% vs. 5.5%), consumption of asset holders falls short by 2 percentage points (9% vs. 11%), and asset prices fall short by 3 percentage points (13.5% vs. 16.75%). The fall in the asset price is then 23% higher in a limited financial market participation economy than in a full financial market participation economy.

Panel (f) shows that prior to the crisis, the real interest rate is below the average real interest rate by almost 200 basis points, which corresponds to one standard deviation. At date t of the crisis, the real interest rate rises sharply to almost 200 basis points above its average. It then decreases slowly to converge to the average value four years after the crisis. Contrary to the real interest rate, TFP rises prior to the crisis and decreases to one standard deviation

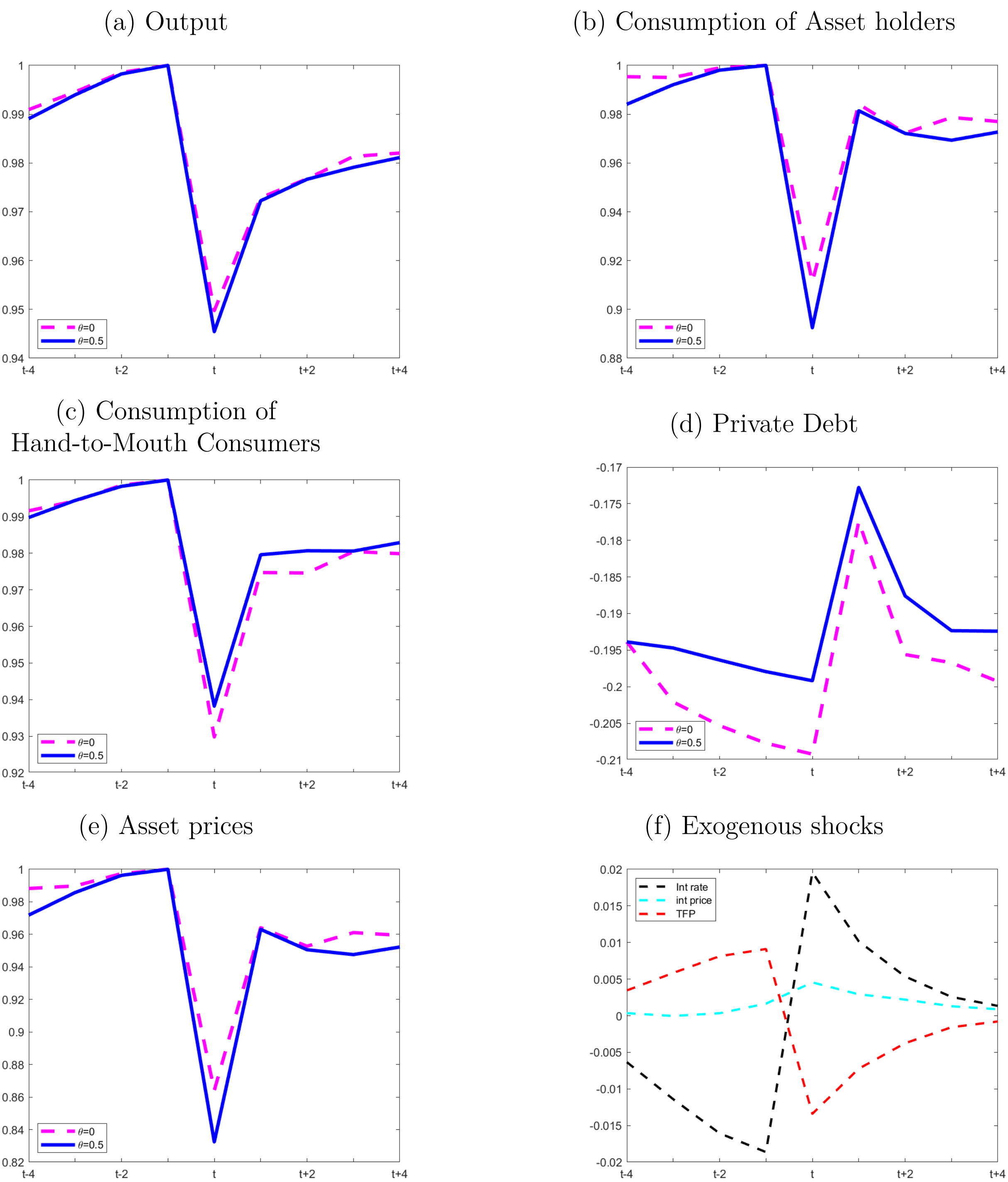


Figure 2.5: Financial crisis dynamics

below its average. The input import price shows small fluctuations along the financial crisis dynamics.

To summarize, the financial crises show a higher amplification effect in a limited financial market participation economy relative to a full financial market participation economy. The asset prices fall more and the asset holders face higher burdens. I next present why the asset price falls more when every household is not participating in the financial market.

Asset prices and inequality. In Section 2.3.4, I show that, given the share of hand-to-mouth consumers, the cyclicality of consumption inequality is important to determine the relative amplification effect in a limited financial market participation economy. As shown by Figure 2.6, a higher drop in the asset price in a limited financial market participation economy is followed by a drop in consumption inequality during the crisis year. Second, the overall pattern of the asset prices is qualitatively similar to the consumption inequality dynamics. A drop in consumption inequality is associated with a higher drop in the asset price in a limited financial market participation economy because of the burden on asset holders. Indeed, when asset holders face a higher burden following an aggregate shock, they will be willing to sell more of their assets to meet their obligations. By doing so, they increase the supply of the asset, leading to a decrease in its prices.

It is worth noting that, in the model, I did not calibrate the drop in the consumption inequality. Instead, I use a constant transfer (a constant lump-sum tax on asset holders) to hand-to-mouth consumers to calibrate the average consumption inequality of 1.27 in the economy. Consistent with Mexico's data, the model endogenously generates the drop in consumption inequality during the financial crisis. The model is able to generate a drop in consumption inequality during the financial crisis because the labor supply, which determines the labor income of hand-to-mouth consumers, has shown a small decline. Mendoza (2010) suggests that the labor decline is not the main cause of the decline in GDP during Mexico's 1995 sudden stop crisis.

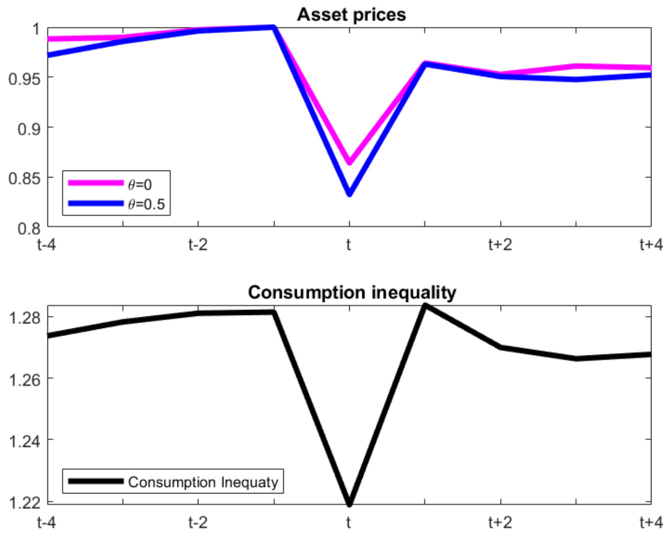


Figure 2.6: Asset prices and inequality

Long-run moments. Table 2.3 presents the business cycle moments. The column with $\theta = 0$ represents the economy with full financial market participation. The column with $\theta = 0.5$

Table 2.3: Business cycle moments

	Standard deviation			Correlation with output		
	$\theta = 0$	$\theta = 0.5$	data	$\theta = 0$	$\theta = 0.5$	data
GDP	2.68	2.73	2.72	1.00	1.00	1.00
Consumption	3.55	3.51	3.39	0.94	0.95	0.89
Trade balance/GDP	1.33	1.26	2.1	-0.51	-0.49	-0.68
Asset prices	5.63	6.68	14.64	0.89	0.89	0.57
Interest rate	1.95	1.95	1.95	-0.64	-0.65	-0.59

represents the economy with limited financial market participation where the share of hand-to-mouth consumers is 50% of the population. The statistics in the data column come from Mendoza (2010). In the table, only the standard deviation of the real interest rate has been used to calibrate the exogenous process of the real interest rate. The results show that the models do a good job of matching the standard deviation and the correlation of key aggregate variables such as output and consumption.

The emerging market is characterized by three main business cycles. First, consumption is more volatile than output. The models with full and limited financial market participation replicate well the standard deviation of output and consumption quantitatively. Second, the trade balance is countercyclical. In fact, the correlation between output and the trade balance ratio to GDP is negative. Third, the real interest rate is countercyclical. In the limited financial market participation economy, the correlation between the real interest rate and output is -0.65, which is comparable to what is observed in the data: -0.59.

The models underestimate the volatility of asset prices. While the data suggest that the volatility of asset prices in Mexico is six times the volatility of output (in the data, the volatility of asset prices and output is 14.64 and 2.72, respectively), the estimates for both economies are only about two times the volatility. By introducing a financial shock in the model where the parameter κ is not constant over time, one can significantly increase the volatility of asset prices. The results also suggest that the economy with limited financial market participation displays more volatility in the asset price relative to the economy with full financial market participation.

Even though the model falls short in displaying the volatility of asset prices relative to the volatility of output, as is observed in the data, it is, however, able to show that asset prices are more volatile than output. In contrast, Mendoza (2010) has found that asset prices are less volatile than output because in his framework, expectation does not play a direct role in the determination of asset prices. Indeed, with an investment and capital adjustment cost in his framework, the asset price is equal to one plus the marginal adjustment cost. So, without a change in the stock of capital, the asset price does not change. Mendoza (2010) has then excluded any direct role for expectation where a change in the expectation of the supply of assets could substantially affect the asset price even though at the equilibrium, the stock of the asset is constant.

2.5 Financial crisis with a financial shock

The framework presented in Section 3.2 does not have a financial shock. In this section, I introduce a financial shock, as in Bianchi and Mendoza (2018) to analyze the optimal policy. The parameter κ that represents the fraction of the total value of physical assets the households can pledge as collateral is not constant anymore. But it takes two values: a high value κ_h regime and a low value κ_l regime (time of crisis) with a switching probability between both regimes. This is consistent with the data, which suggest that the loan-to-value ratio decreases during a financial crisis. According to the loan-to-value ratio in Mexico in the 1990s, I set $\kappa_h = 0.7$ and $\kappa_l = 0.55$. The probability of staying in the low regime is set to zero to reflect the fact that the average duration of a sudden stop is one year. I then use the probability of staying in the high regime to calibrate the frequency of the financial crisis. In addition to the financial shock, a small change is made to make the model comparable to Bianchi and Mendoza (2018), who analyze the optimal time-consistent problem with a representative agent. I assume there is no shock on the imported input price. The beginning-of-period asset K_t is used as collateral instead of the end-of-period asset K_{t+1}, and there is no labor in the working capital loan. See in Appendix B.2.1 for the full model.

2.5.1 Optimal prudential capital control and financial crisis

In this section, I analyze the optimal time-consistent policy presented in Appendix B.2.2. In the optimal policy, I choose to use a tax on foreign debt — a capital control — to decentralize the planner's solution. The taxes collected are redistributed in the form of lump-sum transfers to asset households. This section answers two main questions. First, how effective is an optimal tax on debt in reducing the severity and frequency of a financial crisis in a limited financial market participation economy? Second, can we rationalize the prevalent use of a capital control in emerging market characterized by a low level of financial markets participation?

The optimal time-consistent solution suggests two main lessons. First, the optimal time-consistent solution effectively reduces the frequency and severity of the financial crisis in both the full and limited financial market participation economies. Second, the average tax on foreign debt needed to decentralize the optimal time-consistent solution is higher in a limited financial market participation economy (1.2% vs. 6%). This suggests that more capital control is needed in emerging markets, which have a low level of financial market participation relative to advanced economies. This second lesson rationalizes the use of capital control in the world. In fact, data on capital control suggest that emerging markets control more capital flows than advanced economies.

Figure 2.7 shows the average of output, consumption of asset holders, consumption of hand-to-mouth consumers, bonds, asset prices, and the exogenous shock across the nine-year event windows for the competitive equilibrium with a limited financial market participation economy and the optimal policy. I normalize the average of the variables to 1 at t-1. In the first five panels, the solid blue line represents the limited financial market participation economy where the share of hand-to-mouth consumers is set to 50%. The dashed magenta line represents the optimal solution economy with limited financial market participation with the same share of hand-to-mouth consumers. The last panel shows the percentage differences relative to the unconditional averages of aggregate exogenous shocks that hit the economy.

The top three panels shows that the decline in output, asset holder consumption, and hand-to-mouth consumption is substantially higher in the competitive equilibrium relative to the social planner's solution during the financial crisis. Indeed, output drops from 4.5% to 2.5%, asset holder consumption drops from 24% vs. 2.3%, and hand-to-mouth consumption drops from 6% to 3.3%. Therefore, the planner's solution is effective in reducing the severity of the financial crisis.

The bottom left panel shows the debt dynamics around the financial crisis. At t-4, both the competitive equilibrium (CE) and the social planner (SP) start with the same stock of debt. But at t-3, while the SP reduces debt by 4 percentage points, the CE builds up the debt. This trend continues until t-1, where the difference in the debt between the CE and SP is more than 3 percentage points. Therefore, the debt dynamics suggest that the household overborrows in the competitive equilibrium. Because of this overborrowing, the CE experiences a larger adjustment in the debt when a financial crisis hits the economy.

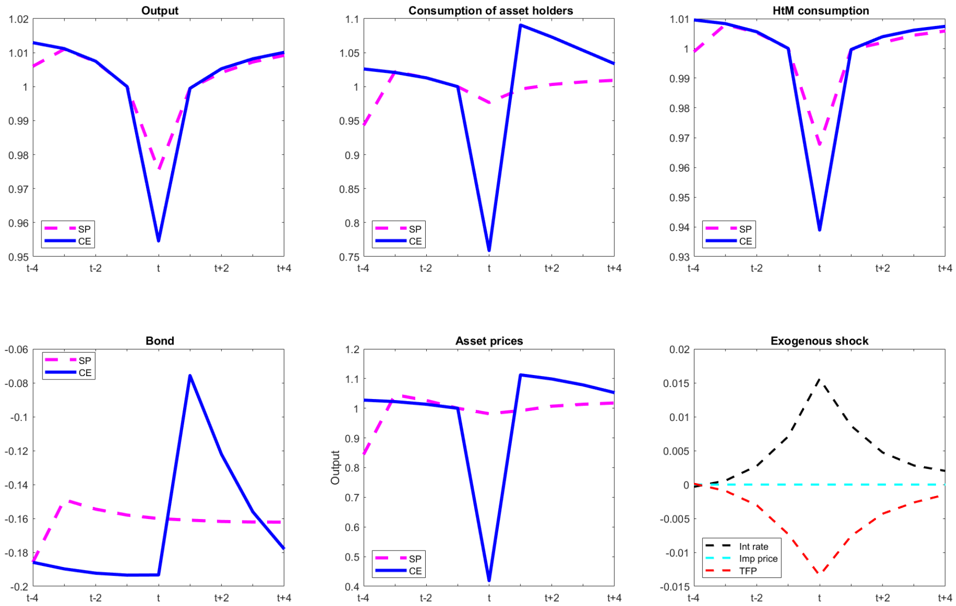


Figure 2.7: Financial crisis with the optimal policy. SP stands for the social planner and CE for the competitive equilibrium.

The bottom middle panel shows the asset prices dynamics around the financial crisis. The CE experiences a larger decline in the asset price relative to the SP. The decline in the asset price is 59% in the CE, and only 2% in the SP. The SP effectively reduces the decline in the asset price substantially because it does not reduce the debt too much when the financial crisis hits the economy. By taking into account the pecuniary externality, the SP did not experience a sharp decline in the asset price. The sharp decline observed in the asset price for the CE is because asset holders substantially deleverage when a financial crisis hits the economy and therefore are more willing to sell their physical assets to meet their obligations. The excess supply of assets leads to a sharp decline in the asset price.

At t-4, the real interest rate and TFP are at their average. During the financial crisis, the real interest rate increases by 150 basis points, which reflects the scarcity of the availability of the foreign asset. The real interest rate is slightly below its average level four years after the crisis. TFP decreases by one standard deviation during the financial crisis, and as with the real interest rate, it is slightly below its average level four years after the crisis.

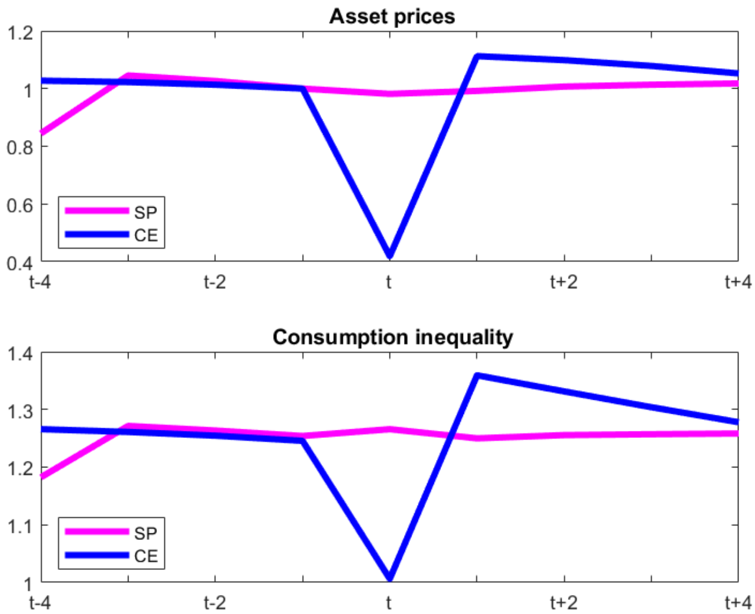


Figure 2.8: Inequality and asset prices. SP stands for the social planner and CE for the competitive equili- brium

2.5.2 Optimal prudential capital control and inequality

Section 2.5.1 shows that prudential capital control is very effective at reducing the severity of a financial crisis. This section answers the following question: Does the inequality increase when using a prudential capital control (a tax on foreign debt)? The optimal time-consistent solution suggests that it is possible to address financial stability without raising (consumption) inequality (see Figure 2.8). While in the very short run (at the time of the financial crisis), the social planner may accept a slight increase in consumption inequality, it appears that long-run (average) consumption inequality is lower in the optimal time-consistent equilibrium relative to the competitive economy.

2.6 Conclusion

This paper presents a financial crisis model in a limited financial market participation economy with a collateral constraint. Participation in the financial market is limited because a fixed share of households do not hold any liquid wealth. That is, they do not participate in either the bond market or the stock market. When negative aggregate shocks hit the economy, the collateral constraint binds, and households are forced to deleverage. The extent to which limited financial market participation amplifies or dampens the economy's response to the aggregate shocks depends on the cyclicality of the consumption inequality. In the model, the consumption inequality is endogenous.

Consistent with the empirical evidence I document using Mexico's household survey data, the model generates a decline in consumption inequality during the financial crisis. This decline in consumption inequality amplifies the economy's response to the aggregate shocks during the financial crisis. Moreover, the optimal time-consistent solution shows that the average tax on foreign assets is higher in a limited financial market participation economy than in a full financial market participation economy. This finding rationalizes the prevalent use of capital control in emerging markets.

Chapter 3

Monetary Policy, Financial Stability, and Inequality

3.1 Introduction

Aftermath the global financial crisis, capital control has been recommended as a macroprudential tool to alleviate the severity and the frequency of financial instability. In addition, many advanced economies and emerging markets have adopted an inflation targeting[0] that aims to achieve price stability. This paper studies the joint design of monetary policy and capital control in an environment with inequality and a motive for both financial stability and price stability. I show that in the case of financial instability due to credit frictions, the monetary authority under the discretionary monetary policy should adopt a prudential monetary policy only if capitals flows are free. This prudential monetary policy is exacerbated by inequality.

To study the joint design of monetary policy and capital control, I extend one of my previous paper[1] to incorporate price rigidity. I enrich a standard dynamic stochastic general equilibrium model that features an occasionally binding collateral constraint with limited household heterogeneity. The model features two types of households. The first type comprises households who participate in the financial market and have access to the capital and bond market. These households are called *asset holders*. The second type of household comprises those who do not participate in either the capital or bond market. These households, called *"hand-to-mouth"* consumers, consume all of their labor income plus any additional transfers. The small open economy faces shocks to its productivity, the real interest rate, and the price of imported inputs. I also introduce a financial shock. A financial shock — a drop to the loan-to-value ratio — consists of a drop during a financial crisis in the fraction of the total value of physical assets that households can pledge as collateral. The model economy nests the model in Bianchi and

[0]The central bank forecasts the future path of inflation and compares it with the target inflation rate and the difference between the forecast and the target determines how much monetary policy has to be adjusted. The difference between the forecast and the target determines how much monetary policy has to be adjusted.

[1]The paper is titled Sudden stops, asset prices: the role of financial market participation and can be downloaded here

Mendoza (2018).

I characterize the optimal monetary policy under discretion in absence of credit frictions. The result suggests that there is a trade-off between price stability and output stabilization. The monetary authority under the discretionary monetary policy has an incentive to deviate implementing price stability (the divine coincidence does not hold). A central bank deviates from its price stability objective because of a concern of inequality. My result is consistent with Acharya et al. (2020) who find in a Heterogeneous Agents New Keynesian (HANK) framework, that a concern of inequality leads the monetary-maker to weigh more an economic activity stabilization than a price stabilization.

I also characterize the optimal monetary policy under discretion and with free capital flow (i.e., no capital control). Whether or not the central banks should conduct a contractionary monetary policy or an expansionary monetary policy during the financial crisis is ambiguous. But, in the absence of a working capital loan, the central banks should conduct an expansionary monetary policy during the financial crisis. By lowering domestic nominal interest during the crisis, investors demand lower premium on their domestic physical asset which raised the asset price and relax the collateral constraint. In normal time (i.e., when the collateral does not bind), If the monetary authority anticipates financial crises in the future, they are more likely to conduct an expansionary monetary policy. By lowering domestic nominal interest, it lowers the demand for foreign bond and reduce vulnerability to capital inflows in the future. Coulibaly (2018) finds similar result in two consumption goods model, tradable and non-tradable goods. He shows that a sufficient condition to conduct an expansionary monetary policy in normal time is when the intra-temporel elasticity of substitution is greater than the inter-temporel elasticity substitution.

The presence of household heterogeneity distorts the discretionary monetary policy with free capital flow in three dimensions. First, in the absence of credit friction, the central banks have an incentive to deviate from the price stability for inequality concern. Second, inequality amplifies the ex-ante financial motive response for monetary policy. The monetary policy should be more expansionary in normal time to mitigate the distributional impacts of the financial crisis. Third, inequality may affect qualitatively the monetary policy during the financial crisis. The monetary policy is less likely to be contractionary during the financial crisis.

My paper mainly relates to the literature that studies the aggregate effects of a sudden stop (see, for example, Arellano and Mendoza (2002), Chari et al. (2005), Mendoza (2006), Calvo et al. (2006). Mendoza (2010), and Korinek and Mendoza (2014)). My paper is closely related to Mendoza (2010), who studies how an endogenous binding collateral can trigger the economy within standard business cycle moments. I have three contributions to this literature. First, I introduce limited financial market participation where a fixed share of households do not participate in the financial market. This characterization of the economy is closer to that of emerging markets and helps us to explain the observed gap in the decline in the asset price during sudden stops between emerging markets and advanced economies. Second, my work studies the optimal time-consistent solution and rationalizes the prevalent use of capital control in emerging markets. Finally, I introduce price rigidity to study the optimal monetary policy under discretion.

My work is also related to recent literature that studies the optimal policy in a financial crisis model. These papers include Caballero and Krishnamurthy (2004), Bianchi (2011), Bengui (2014), Bengui and Bianchi (2018), Bianchi and Mendoza (2018), and Arce et al. (2019). I contribute to this literature by taking into account household heterogeneity in the financial market and show that it is possible to address financial instability without raising inequality. My paper also relates to the literature on financial crises and macroprudential policy. This literature has shown how capital controls can correct pecuniary externalities that generate excessive systemic risk (e.g., Lorenzoni (2008); Bianchi (2011); Dávila and Korinek (2018)). I contribute to this literature by showing monetary policy can serve as a macroprudential policy tool.

My work is also related to the literature that studies the optimal monetary policy when the economy is prone to sudden stops (e.g., Coulibaly (2018); Devereux et al. (2019); Devereux et al. (2015); Bianchi and Coulibaly (2021) ; Davis and Presno (2017); Chang et al. (2015)). I contribute to this literature by studying how inequality exacerbate the prudential monetary policy. Coulibaly (2018) in two consumption goods model has shown that procyclical monetary policy is optimal when both goods are complements. In my model, consumers have access to only one consumption good and procyclical monetary policy is never optimal in the absence of working capital loans.

The rest of the paper is organized as follows. Section 3.2 presents the model. Sections B.2.2 presents my theoretical findings and Section 3.4 concludes.

3.2 Model with collateral constraint and household heterogeneity

I build a small open economy model with household heterogeneity and a collateral constraint. The sudden stop crisis is driven by an occasionally binding collateral constraint. There are two types of households. The first type comprises asset holder consumers who have access to the financial market through their holding of both physical assets and foreign bonds. The second type are "hand-to-mouth" consumers who do not hold any assets — neither physical assets nor foreign bonds. They consume all of their labor income plus any additional transfers from the government. Asset holders act as an entrepreneur who produces an intermediate good. The intermediate good is sold to retailers, which differentiate the good at no cost and sell to the final-good producer. I assume that each retailer set on a monopolistically competitive market, the price of its own differentiated good subject to a convex adjustment cost a la Rotemberg (1982) . The retailers' profits are redistributed to asset holders. The final good producer set the price of the aggregate good on a perfectly competitive market.

3.2.1 Entrepreneur and asset holder households' optimization problem

There is a continuum of identical *asset holder* households of measure 1. The preferences of an *asset holder* consumer indexed by 1 are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}), \tag{3.1}$$

where $\mathbb{E}_0$ is the expectations operator; β is the discount factor; C_{1t} is consumption; $u(.)$ is the utility function which is a standard concave, twice continuously differentiable function that satisfies the Inada condition.

Households produce final goods using three inputs, which are physical assets k_t, intermediate goods v_t, and labor demnand L_t. The production technology is such that $y = A_t F(k_t, L_t, v_t)$, where F is a twice continuously differentiable, concave production function and $A_t = A \exp(\epsilon_t^A)$ is TFP subject to a random shock ϵ_t^A. This shock follows a stationary Markov process. Intermediate goods are traded in competitive world markets at a price p_t^v. The price $p_t^v = p \exp(\epsilon_t^v)$ is subject to a random shock ϵ_t^v that follows a stationary Markov process. *Asset holder* households borrow on the foreign bond market at the real interest rate $R_t^v = R \exp(\epsilon_t^r)$, where ϵ_t^r is a random shock that follows a stationary Markov process. The budget constraint of *asset holder* households is given by

$$P_t c_{1t} + \frac{B_{t+1}}{R_t} + P_t q_t k_{t+1} = P_t^e F(k_t, L_t, v_t) - P_t p_t^v v_t - P_t w_t L_t + B_t + P_t q_t k_t - T_t. \tag{3.2}$$

In equation 3.2, q_t is the price of the physical asset k_t, R_t is the nominal interest rate, and w_t is the real wage. P_t is consumption price and P_t^e is the intermediate good price. The entrepreneur sell to retailers the intermediate good that they produce at price P_t^e. The term $P_t w_t L_{2t}$ represents the total nominal labor income paid to "hand-to-mouth" households. The term T_t is the total lump-sum taxes paid by all *asset holder* households. Lump-sum taxes are used to calibrate the average consumption inequality.

The total private debt in the economy is restrained to a fraction κ of the market value of the beginning-of-period physical asset given by

$$\frac{B_{t+1}}{R_t} - \phi P_t p_t^v v_t \geq -\kappa_t P_t q_t k_t. \tag{3.3}$$

On the left-hand side of (3.3), total private debt (in negative terms) is the sum of private debt with one-year maturity and the within-period working capital loan. On the left-hand side of (3.3), the term $\phi P_t p_t^v v_t$ represents the working capital loan. The working capital loan is a fraction ϕ of the total cost of intermediate inputs in advance of sales. On the right-hand side of (3.3), the term $\kappa_t P_t q_t k_t$ represents a fraction κ_t of the market value of the end-of-period physical asset. Only *asset holder* households who borrow in the foreign bond market face this collateral constraint. Although I do not derive the collateral constraint from an optimization problem, Bianchi and Mendoza (2018) show that this type of constraint could be obtained as an implication of incentive-compatibility constraints on borrowers if limited enforcement prevents lenders from collecting more than a fraction κ of the market value of an asset owned by a defaulting debtor.

The term κ_t represents the financial shock and can be interpreted as the fraction of the total value of physical assets the households can pledge as collateral. It takes two values: a high value κ_h regime and a low value κ_l regime (time of crisis) with a switching probability between both regimes. This is consistent with the data, which suggest that the loan-to-value ratio decreases during a financial crisis. According to the loan-to-value ratio in Mexico in the 1990s, I set $\kappa_h = 0.7$ and $\kappa_l = 0.55$. The probability of staying in the low regime is set to zero to reflect the fact that the average duration of a sudden stop is one year. I then use the probability of staying in the high regime to calibrate the frequency of the financial crisis. The beginning-of-period asset k_t is used as collateral instead of the end-of-period asset k_{t+1}, and there is no labor in the working capital loan.

The *asset holder* households choose consumption, borrowing, capital, labor, and intermediate inputs to maximize their utility (3.1) subject to their budget constraint (3.2) and their borrowing constraint (3.3), taking prices as given. Their optimality conditions are given by

$$u'(t) = \beta R \mathbb{E}_t \left[u'(t+1)\right] + \mu_t, \tag{3.4}$$

$$q_t u'(t) = \beta \mathbb{E}_t \left[\left(d_{t+1} + q_{t+1}\right) u'(t+1) + \kappa_{t+1} q_{t+1} \mu_{t+1}\right], \tag{3.5}$$

$$X_t F_l \left(k_t, L_t, v_t\right) = w_t, \tag{3.6}$$

$$X_t F_v \left(k_t, L_t, v_t\right) = p_t^v + \phi \frac{\mu_t}{u'(t)} p_t^v, \tag{3.7}$$

where $X_t = \frac{P_t^e}{P_t}$ is the inverse of the retailer markup, $\mu_t \geq 0$ is the Lagrange multiplier on the borrowing constraint, $u'(t)$ is the partial derivative of $u(c_{1t})$ with respect to c_{1t}, and $d_{t+1} = X_{t+1} F_k \left(t+1\right))$.

The first two optimality conditions are the Euler equations for bonds and physical assets, respectively. The last two optimality conditions are the intratemporel conditions on the labor market and intermediate good market, respectively.

Condition (3.4) states that if the collateral constraint is not binding ($\mu_t = 0$), the marginal benefit of borrowing to increase today's consumption is equal to the expected marginal cost of repaying back tomorrow. If the collateral constraint binds, the shadow price of relaxing the collateral constraint is positive ($\mu_t > 0$), so the marginal benefit of borrowing is greater than its expected marginal cost. Condition (3.5) states that the marginal cost of buying one additional unit of physical asset at price q_t is equal to its expected marginal benefit. If the collateral constraint is expected to bind, the marginal cost exceeds the marginal benefit by $\mathbb{E}_t \left[\kappa_{t+1} q_{t+1} \mu_{t+1}\right]$.

Condition (3.6) states that the marginal productivity of labor demand is equal to the marginal disutility of labor supply plus the financing cost of labor from the working capital loan. The financing cost is higher when the collateral constraint binds. Condition (3.7) states that the marginal productivity of the intermediate input is equal to its price plus the financing cost of the intermediate input from the working capital loan. The financing cost of the intermediate input is higher when the collateral constraint binds.

3.2.2 Hand-to-mouth households' optimization problem

There is a continuum of identical "hand-to-mouth" households of measure 1. The preferences of a "hand-to-mouth" consumer indexed by 2 are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{2t} - G(L_{2t})), \tag{3.8}$$

where C_{2t} is consumption, L_{2t} is labor supply, and $u(.)$ is the same utility function as in section 3.2.1. G(L) is a convex, strictly increasing, and continuously differentiable function that measures the disutility of labor. These preferences (known as GHH preferences due to Greenwood et al. (1988)) remove the wealth effect on labor supply, which prevents a counterfactual increase in labor supply during crises.The budget constraint of "hand-to-mouth" households is given by

$$P_t C_{2t} = P_t w_t L_{2t} + T_t. \tag{3.9}$$

The *hand-to-mouth* households chooses consumption and labor to maximize their utility (3.8) subject to their budget constraint (3.9), taking prices as given. Their optimality condition is given by

$$G'(L_{2t}) = w_t. \tag{3.10}$$

Condition (3.10) states that the marginal disutility of labor supply for *asset holder* consumers is equal to the real wage rate.

3.2.3 Final good producers

The final good producer combines the differentiated goods produced by retailers using a CES production technology. The retailers are indexed by $j \in [01]$.

$$Y_t = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3.11}$$

where $\varepsilon > 1$ is the elasticity of substitution between retailers' goods. The competitive final good producer chooses the demand for each differentiated good $y_{j,t}$ to maximizes his profit given by $P_t Y_t - \int_0^1 p_{j,t} y_{j,t} dj$. The optimization of final good producer' profit gives the iso-elastic demand curve faced by each retailers

$$y_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\frac{1}{\varepsilon}} Y_t, \tag{3.12}$$

where P_t is the standard price of the final good given by $P_t = \left(\int_0^1 y_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$.

3.2.4 Retailers with price-stickiness

There are monopolistically competitive differentiated good producing firms. Each retailer sets his price $p_{j,t}$ and faces a convex adjustment cost a la Rotemberg (1982), which is given by $\mathcal{A}_t = \frac{\theta}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2 Y_t$. Each retailer j maximizes his present discounted value of profits taking as given the price P_t, aggregate output Y_t, the price of the intermediate good P_t^e, and the stochastic discount factor $\beta^t \mathcal{M}_t$.

$$\max_{p_{j,t}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \mathcal{M}_t \left\{ \left(\frac{p_{j,t}}{P_t} - \frac{P_t^e}{P_t}\right) y_{j,t} - \frac{\theta}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2 Y_t \right\} \quad st \quad y_{j,t} = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t. \quad (3.13)$$

The real marginal cost is represented by $\frac{P_t^e}{P_t}$. By taking the first order condition and using the symmetric price $p_{jt} = P_t$, I get the standard non-linear New Keynesian Phillips Curve (NKPC):

$$\pi_t (1 + \pi_t) = \frac{\varepsilon}{\theta}\left(X_t - \frac{\varepsilon - 1}{\varepsilon}\right) + \beta \mathbb{E}_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \pi_{t+1} (1 + \pi_{t+1}) \frac{Y_{t+1}}{Y_t}\right], \quad (3.14)$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the inflation and $X_t = \frac{P_t^e}{P_t}$ is the inverse of the retailer's markup. Equation 3.14 states when retailers anticipate higher inflation in the future, they adjust up today their price to smooth the cost of adjustment. If price is fully flexible (i.e. $\theta = 0$), retailers always set prices for a constant markup, which depends only on the elasticity of substitution between retailers' goods. The constant markup is given by $\frac{\varepsilon}{\varepsilon - 1}$. To achieve the constant markup, retailers set prices to equate current marginal revenue to current marginal cost. If price is fully rigid (i.e. $\theta \to \infty$) retailers set once and for all their prices to equate average marginal cost to average marginal revenue.

3.2.5 Competitive equilibrium

In this section, I define the competitive equilibrium and the main credit channel through which sudden stops arise in this type of framework. The aggregate resource of the economy is given by

$$c_t + \frac{b_{t+1}}{R_t} - b_t = F(1, L_t, v_t) - p_t^v v_t - \mathcal{A}_t, \quad (3.15)$$

where b_{t+1} is the real bond holdings, $c_t = c_{1t} + c_{2t}$ is aggregate consumption, the term $\frac{b_{t+1}}{R_t} - b_t$ represents the trade balance, and the term $F(1, L_t, v_t) - p_t^v v_t - \mathcal{A}_t$ represents GDP.

A competitive equilibrium in this model is a stochastic sequence $Q_t = \{C_{1t}, C_{2t}, L_t, L_{2t}, v_t, b_{t+1}\}_{t \geq 0}$, inflation and markup $\{\pi_t, X_t\}_{t \geq 0}$ and prices $P_t = \{q_t, w_t\}_{t \geq 0}$ such that:

1. Given P_t, Q_t solves households' and firms' problems;
2. w_t and q_t are determined competitively that is: $G'(L_t) = w_t$, and q_t solves equation (3.5);
3. the New Keynesian Phillips curve (NKPC) (3.14) holds:

4. markets clear:

   (a) labor market: $L_t = L_{2t}$,

   (b) capital market: $k_t = 1$,

   (c) aggregate resource: equation (3.15) is satisfied.

3.2.6 Monetary policy instrument

The central bank sets a domestic nominal interest rate i_t on domestic bond B_t^d to control the inflation rate. I assume that only asset holders have access to the domestic bond. With This assumption, Equation 3.3 holds at the equilibrium since. At the equilibrium $B_t^d = 0$. The no-arbitrage condition between domestic and foreign bond is given by:

$$\beta \mathbb{E}_t \left[R_t - \frac{(1+i_t)}{1+\pi_{t+1}} u'(t+1) \right] + \mu_t = 0, \tag{3.16}$$

where R_t is the foreign interest rate taking as given, i_t is the domestic nominal interest rate, and μ_t is the gain of relaxing the borrowing constraint on foreign asset. Note that only the foreign bond is subject to a collateral constraint. The no-arbitrage is the combination of the Euler equation of the foreign bond and the domestic bond. Equation 3.16 allows to recover back the optimal domestic nominal interest rate.

I assume that the central banks set the nominal in three different regimes, which are: inflation targeting using a the Taylor rule, discretionary monetary policy without capital control, and discretionary monetary policy with capital control. The Taylor rule on the nominal domestic interest rate is given by

$$1 + i_t = (1 + \bar{i}) \left(\frac{1+\pi_t}{1+\bar{\pi}} \right)^{\phi_\pi}, \tag{3.17}$$

where $\bar{i}$ is the average net domestic nominal interest rate and $\bar{\pi}$ is the target inflation. In the baseline inflation targeting regime, I assume that the target inflation is 0 . The discretionary monetary policy with and without capital control is set via an optimal time-consistent problem presented in the following section.

3.3 Optimal Time-consistent Planner's Problem

In this section, I analyze the optimal time-consistent solution. The planner chooses the optimal current allocations taking as given the future policy functions. I study Two main regimes. The first is the discretionary monetary policy without capital and the second is the discretionary monetary policy with capital control. In the discretionary monetary policy with capital control, I choose to use a tax on foreign debt — a capital control — to decentralize the planner's solution and the no-arbitrage condition in equation 3.16 to get back the optimal domestic nominal interest rate. The taxes collected are redistributed in the form of lump-sum transfers to asset households. This section presents the optimization problem to answer two main questions.

First, how effective is an optimal discretionary monetary policy to reduce both the severity and frequency of a financial crisis in a limited financial market participation economy? Second, what are the benefit of the joint design of monetary policy and capital control in a household heterogeneity environment?

Input wedge: In general, the production function is inefficient (different from the first best allocations) under an arbitrary monetary policy or capital control policy. Moving the equilibrium allocations from the first best can be conveniently summarized in the input wedge, defined below:

$$\varphi_t \equiv \frac{F_v(1, l_t, v_t)}{p_t^v} - \frac{F_l(1, l_t, v_t)}{G'(l_t)}, \tag{3.18}$$

where F_v F_l are the marginal product of imported input and the marginal product of labor respectively. The input wedge in equation 3.18 is defined as the difference between the relative benefit of labor the imported input and the relative benefit of labor. The relative benefit is the ratio of the value of employment to the cost of supplying labor. Note that in my framework the relevant wedge is the input wedge and not the labor wedge usually used in the literature [2] . The input wedge is the relevant wedge because of the presence of the two inputs in the production function, which are labor $l_t$and imported input v_t.

At a first-best allocation $\varphi_t = 0$ (see appendix C.2). A positive input wedge, $\varphi_t > 0$, reflects the relative benefit of the imported input exceed the relative benefit of labor. In this case, the economy experiences a recession. Conversely, a negative labor wedge, $\varphi_t < 0$, reflects relative benefit of the imported input is too low compared to the the relative benefit of labor . In this case, the economy experiences a boom.

Combining (3.6) and (3.7) and using the definition of the input wedge, the real marginal cost X_t of retailers satisfies the following equation.

$$\left[\varphi_t + \frac{F_l(1, l_t, v_t)}{G'(l_t)}\right] X_t = 1 - \frac{1}{\epsilon} + \phi \frac{\mu_t}{u'(c_{1t})}, \tag{3.19}$$

where $\frac{1}{\epsilon}$ represents wage subsidy to offset the monopolistic distortion.

Following Klein et al. (2008), Bianchi and Mendoza (2018) I focus on Markov stationary policy rules that are expressed as functions of the payoff-relevant state variables (b, s). A Markov perfect equilibrium is characterized by a fixed point in these policy rules, at which the policy rules of future planners that the current planner takes as given to solve its optimization problem match those that the current planner finds optimal to choose. Hence, the planner does not have the incentive to deviate from other planners' policy rules, thereby making these rules time consistent. Let $\mathcal{B}(b, s)$ be the policy functions for foreign bond holding of futures planners. Taking as given $\left\{\mathcal{B}(b, s), \mathcal{C}_1(b, s),\ \mathcal{L}(b, s),\ \mathbf{v}(b, s),\ \mu(b, s),\ \mathcal{Q}(b', s'),\ \pi(b, s),\ \mathcal{X}(b', s')\right\}$, the social planner solves problem 3.20 in the discretionary monetary policy without capital control regime. In the discretionary monetary policy with capital control regime, the social planner solves the same problem where the foreign bond Euler equation is not bind.

[2]see Coulibaly (2018), Bianchi and Coulibaly (2021).

The foreign bond Euler equation implementability constraint has the multiplier $\gamma \geq 0$. The asset pricing implementability constraint has the multiplier $\xi \geq 0$. The "hand-to-mouth" consumer resource constraint has the multiplier δ. The firm price setting implementability constraint has the multiplier ϑ. The economy's resource constraint has the multiplier $\lambda \geq 0$. The collateral constraint has the multiplier $\mu^* \geq 0$. The asset holders' slackness condition has the multiplier $\varsigma \geq 0$.

Definition: The recursive constrained-efficient equilibrium is defined by the policy function b'(b, s) with associated decision rules $c_1(b,s),\ l(b,s),\ v(b,s),\ \mu(b,s)$, pricing function q(b, s), inflation and markup functions $\pi(b,s), X(b,s)$, value function $\mathcal{V}(b,s)$, the conjectured function characterizing the decision rule of future planners $\mathcal{B}(b,s)$ and the associated decision rules $\mathcal{C}_1(b,s),\ \ \mathcal{L}(b,s),\ \ \mathbf{v}(b,s),\ ,\ \ \mu(b,s)$, asset prices $\mathcal{Q}(b,s)$, and price inflation and the inverse of markup $\pi(b,s)$, X(b,s) such that these conditions hold:

1. Social planner optimizes: $\mathcal{V}(b,s)$ and the policy functions $\Big\{ b'(b,s), c_1(b,s),\ l(b,s),\ v(b,s), \mu(b,s),$ $q(b,s), \pi(b,s),\ X(b,s) \Big\}$ solves the problem 3.20 given $\Big\{ \mathcal{B}(b,s), \mathcal{C}_1(b,s),\ \mathcal{L}(b,s),\ \mathbf{v}(b,s), \mu(b,s),\ \mathcal{Q}(b,s),\ \pi(b,s), \mathcal{X}(b',s') \Big\}$
2. The policy functions are time consistent: The conjectured policy functions that represent optimal choices of future planners match the corresponding recursive functions that represent optimal plans of the current planner, which are: $b'(b,s) = \mathcal{B}(b,s),\ c_1(b,s) = \mathcal{C}_1(b,s)$, $l(b,s) = \mathcal{L}(b,s),\ v(b,s) = \mathbf{v}(b,s),\ \mu(b,s) = \mu(b,s)$, $q(b,s) = \mathcal{Q}(b,s),\ \pi(b,s) = \pi(b,s)$, $X(b,s) = \mathcal{X}(b,s)$.

$$
\begin{aligned}
\mathcal{V}(b,s) &= \max_{c_1,c_2,b',l,v,q,\pi,\mu} \Big\{ u(c_1) + \omega u(c_2 - G(l) + \beta \mathbb{E}_{s',s} \mathcal{V}(b',s') \Big\} \\
u'(c_1) &= \beta R \mathbb{E}_{s',s} \left[u'(\mathcal{C}_1(b',s')\right] + \mu, \\
q u'(c_1) &= \beta \mathbb{E}_{s',s} \left[u'(b',s') \left[\mathcal{X}(b',s') F_k \left(1, \mathcal{L}(b',s'), \mathbf{v}(b',s')\right) + \mathcal{Q}(b',s') \right] + \kappa' \mu(b',s') \mathcal{Q}(b',s') \right] \\
\left[\varphi + \frac{F_l(1,l,v)}{G'(l)} \right] X &= 1 + \phi \frac{\mu}{u'(c_1)} \\
c_2 &= G'(l) l + t \\
\pi(1+\pi) &= \frac{\varepsilon}{\theta} \left(X - \frac{\varepsilon - 1}{\varepsilon} \right) + \beta \mathbb{E}_{s',s} \left[\frac{u'(\mathcal{C}_1(b',s') F\left(1, \mathcal{L}(b',s'), \mathbf{v}(b',s')\right)}{u'(c_1) F(1,l,v)} \pi(b',s') \left(1 + \pi(b',s')\right) \right] \\
c_1 + c_2 + \frac{b'}{R} - \frac{b}{1+\pi} &= \left(1 - \frac{1}{2} \theta \pi^2 \right) F(1,l,v) - p^v v \\
\frac{b'}{R_t} - \phi p^v v &\geq -\kappa q \\
\mu \left(\frac{b'}{R} - \phi p^v v + \kappa q \right) &= 0. \quad \mu \geq 0
\end{aligned}
\tag{3.20}
$$

Let define some auxiliary variables. $\Omega(b',s') \equiv \beta \mathbb{E}_{s',s} \left[R u'(\mathcal{C}_1(b',s')\right]$,

$\Delta(b',s') \equiv \beta \mathbb{E}_{s',s} \left[u'(b',s') \left[\mathcal{X}(b',s') F_k \left(1, \mathcal{L}(b',s'), \mathbf{v}(b',s')\right) + \mathcal{Q}(b',s') \right] + \kappa' \mu(b',s') \mathcal{Q}(b',s') \right]$, and

$$\Gamma((b', s', c_1, l, v, \mu) \equiv \frac{\varepsilon}{\theta}\left(X - \frac{\varepsilon - 1}{\varepsilon}\right) + \beta \mathbb{E}_{s',s}\left[\frac{u'(\mathcal{C}_1(b', s')F\left(1, \mathcal{L}(b', s'), \mathbf{v}(b', s')\right)}{u'(c_1)F\left(1, l, v\right)}\pi(b', s')\left(1 + \pi(b', s')\right)\right]$$

Lemma 3.3.1. *Suppose there exist a wage and imported input price subsidies to offset the monopolistic distortions at the flexible prices allocations. Let τ_w and τ_v be the wage and the imported input price subsidy respectively such that and $\tau_w = \tau_v = \frac{1}{\varepsilon}$. Then, in the absence of a credit friction (i.e., $\mu_t = 0$ for all t), the constraint-efficient flexible prices allocations coincide with the competitive equilibrium allocations with the optimal relative weight given by $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$.*

Proof: See appendix C.3.2.

The optimal relative weight ω implies that the resource constraint of "hand-to-mouth" consumer is not bind. It means that the social planner can optimally chooses the lump sum transfer t to replicate the competitive allocations of "hand-to-mouth" consumer. If the social planner do not have this instrument, lemma (3.3.1) says that the constraint-efficient flexible prices allocations won't coincide with the competitive equilibrium allocations. A number of reasons may make this optimal weight infeasible in reality. Consider a relative weight that implies that the social planner should tax hand-to-mouth to redistributed to asset holders. For political reasons, it may not be feasible.

3.3.1 Discretionary monetary policy with free capital flows

In this section I characterize the optimal monetary policy under discretion when there is no capital control. I also discuss how does inequality affect this optimal monetary policy. The optimal monetary policy under discretion solves problem 3.20. The first proposition characterizes the optimal monetary policy under discretion in absence of credit frictions and the second proposition generalizes the first one and conclude that there is exists a comprise between price stability, financial stability and inequality. Proposition

Proposition 3. *Suppose there exist a wage and imported input price subsidies to offset the monopolistic distortions at the flexible prices allocations. Let τ_w and τ_v be the wage and the imported input price subsidy respectively such that and $\tau_w = \tau_v = \frac{1}{\varepsilon}$. In the absence of credit friction (i.e., $\mu_t = 0$ for all t) the optimal monetary policy under discretion strictly stabilizes inflation (i.e $\pi_t = 0$ for all t) and the optimal relative weight is given by $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$. Further if at the equilibrium, when the relative weight $\omega \neq \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$, the optimal monetary policy under discretion deviates from price stability (i.e $\pi_t \neq 0$).*

The proof is straightforward and comes from lemma (3.3.1). By setting always the inflation at its target, the social planner will replicate the flexible-price allocations. Lemma (3.3.1) establishes that the constraint-efficient flexible prices allocations coincide with the competitive equilibrium allocations with the optimal relative weight given by $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$. This ends the proof. It is important to understand that in the absence of household heterogeneity, it is well know in the New Keynesian literature that price stability is optimal in absence of credit friction. Central banks do not have any incentive to deviates from the price stability policy when the collateral constraint is not binding. The first part of the proposition says that, when it is possible for the central bank and the fiscal authority to coordinate and optimally choose the lump sum transfer, it is optimal for the central banks to not deviate from its target inflation. The second part of the proposition says whenever the coordination is not possible or the collateral binds, the central bank has an incentive to deviate from the target inflation.

Proposition 3 breakdowns what Blanchard and Galí (2007) call the divine coincidence observed in the standard New Keynesian models in the absence of credit friction. A central bank deviates from its price stability objective because of a concern of inequality. My result is consistent with Acharya et al. (2020) who find in a Heterogeneous Agents New Keynesian (HANK) framework, that a concern of inequality leads the monetary-maker to weight more an economic activity stabilisation than a price stabilization.

Proposition

Proposition 4. *Suppose there exist a wage and imported input price subsidies to offset the monopolistic distortions at the flexible prices allocations. Let τ_w and τ_v be the wage and the imported input price subsidy respectively such that and $\tau_w = \tau_v = \frac{1}{\varepsilon}$. In the presence of credit friction, the optimal monetary policy under discretion is given by*

$$\underbrace{\theta\Phi y_t\pi_t}_{\text{Price stability motive}} = \tilde{w}u'(c_{1t}) + \underbrace{\left\{\sigma\frac{\kappa_t q_t}{c_{1t}}\tilde{w} - \phi\alpha p_t^v v_t\right\}\mu_t^*}_{\text{Ex-post financial stability motive}} + \underbrace{\sigma\frac{u'(c_{1t})}{c_{1t}}\tilde{w}\gamma_t}_{\text{Ex-ante financial stability motive}} + \underbrace{\eta G''(l)l^2\delta_t}_{\text{Inequality motive}} \tag{3.21}$$

$$\text{where } \Phi = \Phi_0 + \beta\mathbb{E}_t\left[\Phi_1\pi_{t+1}\right], \text{ with } \Phi_0 = \underbrace{\frac{\varepsilon}{\theta}\left[-\alpha v_t\frac{F_{vv}(t)}{p_t^v}s_t z_t^{-2} + \eta l_t\frac{F_{vl}(t)}{p_t^v}s_t z_t^{-2} + \sigma\tilde{\omega}\frac{\phi\mu_t}{c_{1t}}z_t^{-1}\right]}_{>0}\frac{\lambda_t}{1+2\pi_t},$$

$\delta_t = -\omega u'(c_{2t} - G(l_t)) + u'(c_{1t}) + \sigma\frac{u'(c_{1t})}{c_{1t}}\gamma_t + \sigma\frac{\kappa_t q_t}{c_{1t}}\mu_t^* - \theta\frac{\Gamma_c\lambda_t}{1+2\pi_t}\pi_t y_t$, *and* $\gamma_t = 0$ *if for all* t $\mu_t^* = 0$

Proof: See appendix C.3.3.

In Proposition 4 , the social planner's Lagrange multiplier on the collateral constraint μ_t^* captures the adjustment in the monetary policy when the economy is in crisis. The multiplier on the foreign bond Euler equation γ_t captures the adjustment in the monetary policy when the monetary authority anticipates a financial crisis in the future. The Lagrange multiplier on the resource constraint σ_t of the hand-to-mouth consumers captures the inequality motive. In the absence this type of consumers, the multiplier $\sigma_t = 0$. Proposition 4 is a generalization of Proposition 3. In the absence of credit friction (i.e., $\mu_t^* = \gamma_t = 0$) and in the absence of household heterogeneity (i.e., $\delta_t = 0$), price stability will perfectly stabilize output with the given optimal relative weight. In the presence of credit friction and inequality,policymakers face a comprise between price stability, financial stability and inequality.

Ex-post financial motive: The second term on the right side of 3.21 captures the ex-post financial motive in the setting of a monetary policy. It implies that monetary authority has incentive to deviate from price stability when the collateral constraint binds. The degree to which the financial crisis affect the optimal monetary policy depends on two outcomes, which are a weighted value of the value of the collateral given by $\sigma\frac{\kappa_t q_t}{c_{1t}}\tilde{w}$ and a weighted value of the working capital loan $\phi \alpha p_t^v v_t$. It is quite intuitive to see why the sensitivity of the optimal monetary policy during the financial crisis depends on these two outcomes. First, it is worth noting that when the collateral constraint binds, the foreign loan is equal to the current value of the collateral minus the current value of the working capital loan, weighted by the foreign interest rate. Second, the monetary authority internalizes the fact that the borrowing decision affect the current asset price. So the monetary authority can affect the current value of the collateral and the working capital loan.

During the financial crisis whether the central banks should conduct a contractionary monetary policy or an expansionary monetary policy is ambiguous. In the absence of a working capital loan (i.e., $\phi = 0$), during the financial crisis, the central banks should conduct an expansionary monetary policy. By lowering domestic nominal interest during the crisis, investors demand lower premium on their domestic physical asset which raised the asset price and relax the collateral constraint.

Ex-ante financial motive: The third term on the right side of 3.21 captures the ex-ante financial motive in the setting of a monetary policy. It implies that there is a role for monetary policy as a macro-prudential tool.The monetary authority can 'lean against the wind' in advance of a financial crisis, when policy is made under discretion (absence of commitment). Departing from inflation stabilization may have a benefit even if the economy is not currently borrowing-constrained. Devereux et al. (2019)shows in a representative agent model that the monetary authority should not try to 'lean against the wind' in advance of a financial crisis, when policy is made under discretion because they use future-asset price as opposed to a current-asset price collateral constraint. In a flexible price framework without working capital loan Ottonello et al. (2021) show that the desirability of macroprudential policies critically depends on the specific form of collateral used in debt contracts. They argued that the equilibrium is inefficient when current prices affect collateral but there is no inefficiency when only future prices affect collateral.

In normal time (i.e., when the collateral does not bind), if the monetary authority anticipates financial crises in the future (i.e., γ_t positive), they are more likely to conduct an expansionary monetary policy since the coefficient on γ_t in 3.21 is positive. By lowering domestic nominal interest, it lowers the demand for foreign bond and reduce vulnerability to capital inflows in the future. Coulibaly (2018) finds similar result in two consumption goods model, that are tradable and non-tradable goods. He shows that a sufficient condition to conduct an expansionary monetary policy in normal time is when the intra-temporel elasticity of substitution is greater than the inter-temporel elasticity substitution.

Inequality motive: The presence of household heterogeneity distort the price stability in three dimensions. First, in the absence of credit friction, the central banks have an incentive to deviate from the price stability for inequality concern. Second, inequality amplifies the ex-ante financial motive response for monetary policy. The monetary policy should be more expansionary in normal time to mitigate the distributional impacts of the financial crisis. Third, inequality may affect qualitatively the ex-post financial motive response for monetary policy. The monetary policy is less likely to be contractionary during the financial crisis.

3.3.2 Discretionary monetary policy with capital control

In this section I characterizes the optimal monetary policy under discretion when capital flows are taxed. I also discuss how does inequality affect this optimal monetary policy. The optimal monetary policy under discretion solves problem 3.20 without the foreign bond Euler equation implementability constraint (i.e., the multiplier $\gamma_t = 0$ for all t).

Corollary 3.3.1.1. *In the presence of capital control, monetary policy should not be used as a macroprudential tool.*

In the presence of capital control, the foreign bond Euler equation implementability constraint is never always bind (i.e., the multiplier $\gamma_t = 0$ for all t). There is no ex-ante financial motive for the monetary policy. I conclude then that the monetary policy should not be used as a macroprudential tool. Capital control through a tax on foreign debt can efficiently act as macroprudential tool.

3.4 Conclusion

This paper studies the joint design of monetary policy and capital control in an environment with a motive for both financial stability and price stability. I build an equilibrium business cycle model with a current-price collateral constraint, household heterogeneity due to a limited financial market participation, and nominal rigidity. I show that, in the absence of credit friction (i.e., the collateral is never binding), the monetary authority under the discretionary monetary policy has an incentive to deviate implementing price stability (the divine coincidence does not hold). In addition, I show that in the case of financial instability due to credit frictions, the monetary authority under the discretionary monetary policy should adopt a prudential monetary policy only if capitals flows are free. This ex-ante prudential monetary policy is exacerbated by household inequality. In the absence of a working capital loan procyclical monetary policy is never optimal.

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Appendix A

Appendix to Chapter 1

A.1 Firm problem

A.1.1 Profit Maximisation: Employment Agency

$$\max_{N_{it}} W_t N_t - \int_0^1 W_{it} N_{it} e_{it} \tag{A.1}$$

$$s.t \qquad N_t = \left[\int_0^1 e_{it} \left(N_{it} \right)^{1-\frac{1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{A.2}$$

CPO:
$[Nit] : W_t \frac{\partial N_t}{\partial N_{it}} - W_{it} e_{it} = 0$ where $\frac{\partial N_t}{\partial N_{it}} = e_{it} N_{it}^{-\frac{1}{\epsilon_w}} N_t^{\frac{1}{\epsilon_w}}$. Then it follows the the demand for the i-th consumer's labor in the main text 1.4 .

A.1.2 Price decision of Intermediate good produce

$$\max_{p_{j,t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s Q_{t+s/t} \left\{ \left(\frac{p_{j,t+s}}{P_{t+s}} - m_{j,t+s} \right) y_{j,t+s} - \frac{\theta}{2} \left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \pi \right)^2 Y_{t+s} \right\} \tag{A.3}$$

$$st \quad y_{j,t+s} = \left(\frac{p_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon} Y_{t+s} \tag{A.4}$$

FOC

$$\mathbb{E}_t \beta^s Q_{t+s/t} \left[\frac{1}{P_{t+s}} y_{j,t+s} - \frac{\varepsilon}{P_{t+s}} \left(\frac{p_{j,t+s}}{P_{t+s}} - m_{j,t+s} \right) y_{j,t+s} \left(\frac{p_{j,t+s}}{P_{t+s}} \right)^{-\varepsilon - 1} Y_{t+s} - \theta \frac{1}{p_{j,t+s-1}} \left(\frac{p_{j,t+s}}{p_{j,t+s-1}} - \pi \right) Y_{t+s} \right]$$
$$+ \mathbb{E}_t \beta^{s+1} Q_{t+s+1/t} \left[\theta \frac{p_{j,t+s+1}}{p_{j,t+s}^2} \left(\frac{p_{j,t+s+1}}{p_{j,t+s}} - \pi \right) Y_{t+s+1} \right] = 0$$

Using symetric price $p_{j,t} = P_t$ we have $y_{j,t} = Y_t$. Using the definition for the inflation $\Pi_{t+s} =$

$\frac{P_{t+s}}{P_{t+s-1}}$ and reaarring the FOC we get:

$$\mathbb{E}_t\left[\left[1-\varepsilon m_{t,s}\right)-\theta\left(\Pi_{t+s}-\Pi\right)\Pi_{t+s}\right]-\beta\theta\Lambda_{t,t+s+1}\left[\Pi_{t+s+1}\left(\Pi_{t+s+1}-\Pi\right)\frac{Y_{t+s+1}}{Y_{t+s}}\right]\right] \quad \text{(A.5)}$$

where $\Lambda_{t,t+s+1} = \frac{Q_{t+s/t}}{Q_{t+s+1/t}}$. The above equation is true for every s. For $s = 0$ and the steady inflation $\Pi = 1$ we have :

$$\Pi_t\left(\Pi_t-1\right)=\frac{1}{\theta}-\frac{\varepsilon}{\theta}\left(1-m_t\right)+\beta\mathbb{E}_t\left[\Lambda_{t,t+1}\Pi_{t+1}\left(\Pi_{t+1}-1\right)\frac{Y_{t+1}}{Y_t}\right] \quad \text{(A.6)}$$

Log linearize the above around the steady state we get:
First taylor approximation of the LHS

$$\begin{aligned}\Pi_t\left(\Pi_t-1\right) &\simeq \Pi\left(\Pi-1\right)+\left(2\Pi-1\right)\left(\Pi_t-\Pi\right)\\ &= 0+\left(\Pi_t-1\right)\\ &= \pi_t\end{aligned}$$

First taylor approximation of the RHS

$$\begin{aligned}&\simeq 0+\frac{\varepsilon}{\theta}(m_t-m)+\beta\mathbb{E}\left[\Lambda\frac{Y}{Y}\left(2\Pi-1\right)\left(\Pi_{t+1}-\Pi\right)\right)\right]\\ &= \frac{\varepsilon}{\theta}(m_t-m)+\beta\mathbb{E}\left(\Pi_{t+1}-\Pi\right)\\ &= \frac{\varepsilon}{\theta}(m_t-m)+\beta\mathbb{E}\left(\Pi_{t+1}-1\right)\\ &= \frac{\varepsilon m}{\theta}\hat{m}_t+\beta\mathbb{E}\pi_{t+1}\end{aligned}$$

Equating both side we get:

$$\pi_t = \beta\mathbb{E}\pi_{t+1}+\frac{\varepsilon m}{\theta}\hat{m}_t \quad \text{(A.7)}$$

Note that $m = \frac{\varepsilon-1}{\varepsilon}$. $\hat{m}_t$ is the log deviation of the marginal cost from his steady state and $-\hat{m}_t$ is the log deviation of firm markup from his steady state. Equation 1.9 says that if firm markup is below their natural level then price will increase (vis-versa).

A.2 Household problem

A.2.1 TANK: sticky prices

$$\begin{cases}\pi_t^p = \beta\mathbb{E}\pi_{t+1}^p+\lambda_p\tilde{w}_t+k_p\tilde{y}_t & \text{NKPC}\\ \tilde{y}_t = \mathbb{E}\tilde{y}_{t+1}-\frac{1}{\sigma(1+\Psi_3)}\left[\hat{r}_t^b+\mathbb{E}\Delta z_{t+1}-\sigma\Psi_a\mathbb{E}\Delta a_{t+1}\right]+\frac{\Psi_2}{1+\Psi_3}\mathbb{E}\left[\tilde{w}_{t+1}-\tilde{w}_t\right] & \text{DIS}\\ \hat{i}_t = \rho+\phi_\pi\pi_t+\phi_y\hat{y}_t+v_t & \text{Taylor rule}\end{cases} \quad \text{(A.8)}$$

Using the following relation $\tilde{w}_t = \left[\sigma + \frac{\eta}{1-\alpha}\right]\tilde{y}_t$ and $\Psi_3 = \frac{\alpha}{1-\alpha}\Psi_2$, it is straightforward to end up with the following system of equations:

$$\begin{cases} \pi_t^p = \beta\mathbb{E}\pi_{t+1}^p + \lambda_p\left[\sigma + \frac{\eta+\alpha}{1-\alpha}\right]\tilde{y}_t & \text{Wage NKPC} \\ \tilde{y}_t = \mathbb{E}\tilde{y}_{t+1} - \frac{1}{\sigma}\frac{1}{1+(\sigma+\frac{\eta+\alpha}{1-\alpha})\Psi_2}\left[\hat{r}_t^b + \mathbb{E}\Delta z_{t+1} - \sigma\Psi_a\mathbb{E}\Delta a_{t+1}\right] & \text{DIS} \\ \hat{i}_t = \rho + \phi_\pi\pi_t + \phi_y\hat{y}_t + v_t & \text{Taylor rule} \end{cases} \tag{A.9}$$

As opposed to the sticky wage framework, the system is not independent of the proportion of hand to mouth meaning that the RANK is not equivalent to TANK under sticky prices. From the DIS equation, if $1 + (\sigma + \frac{\eta+\alpha}{1-\alpha})\Psi_2 < 0$, a positive shock on nominal interest rate lead to boom: what Bilbiee (2008) refers to the Inverted Aggregate Demand Logic (IADL) region.

A.2.2 Proof of Lemma 2

The goal of this section is to linearize ?? to find an anlytical expression $\hat{\gamma}_t$. [1]
$C_t^U - C_t^K = D_t\left(\frac{1-(1-\tau)\delta}{1-\lambda}\right)$ and $(1-\lambda)C_t^U = (1-\lambda)w_tN_t + (1-\delta\lambda(1-\tau))D_t - (1-\lambda)AC_t^w$

$$\gamma_t = \frac{D_t(1-(1-\tau)\delta)}{(1-\lambda)w_tN_t + (1-\delta\lambda(1-\tau))D_t - (1-\lambda)AC_t^w} \tag{A.10}$$

The profit $D_t = Y_t - w_tN_t - AC_t$. Since $m_t = \frac{w_t}{MPN}$ we have $w_tN_t = (1-\alpha)m_tY_t$ .So $D_t = \frac{1}{(1-\alpha)m_t}w_tN_t\left(1 - \tilde{AC}_t\right) - w_tN_t$ where $\tilde{AC}_t = \frac{\theta}{2}\left(\Pi_t^p - \Pi^p\right)^2$. γ_t can be rewritten as :

$$\gamma_t = \frac{\left[\frac{1}{(1-\alpha)m_t}\left(1-\tilde{AC}_t\right) - 1\right](1-(1-\tau)\delta)}{1-\lambda+(1-\delta\lambda(1-\tau))\left[\frac{1}{(1-\alpha)m_t}\left(1-\tilde{AC}_t\right)-1\right] - (1-\lambda)\frac{AC_t^w}{W_tN_t}} \tag{A.11}$$

The steady state value of γ_t is given by: $\gamma = \frac{[1-(1-\alpha)m](1-(1-\tau)\delta)}{(1-\lambda)(1-\alpha)m+(1-\delta\lambda(1-\tau))[1-(1-\alpha)m]}$. Let's γ_m be the first partial derivative of γ_t evaluated at the steady state.

$$\gamma_m = -\frac{(1-\alpha)(1-\lambda)(1-\delta\lambda(1-\tau))}{[(1-\lambda)(1-\alpha)m + (1-\delta\lambda(1-\tau))[1-(1-\alpha)m]]^2} \tag{A.12}$$

So linearizing γ_t, we get:

$$\hat{\gamma}_t = \Psi_1\hat{\mu}_t^p, \tag{A.13}$$

where $\Psi_1 = -\gamma_m m$; $\Psi_1 > 0$ and $\hat{\gamma}_t = \gamma_t - \gamma$

[1] See Debortoli and Galí (2018) Section 5

A.3 Numerical method

A.3.1 Stationnary distribution

Prelimilary:
Construct a $ne = 11$ grid point for e and $na = 80$grid point for the asset A. I use a log-space (not a linear) grid point for the asset.
Define $C_{ij} + A_j = X_{ij}$. Where X_{ij} is a cash on hand for an household with idiosyncratic risk e_i and an asset A_j. X_{ij} is composed of labor income, bond income, equity income and income and transfer income minus the wage adjustment cost.
$X_{ij} = w.Ne_i + (1+r)A_j + [-(1+r)Q + Q + (1-\delta)D]\frac{A_j^+}{A^+} + T_{ij} - AC_i^w$ where $T_{ij} = \left[1 + \tau^a\left(\frac{A_j^+}{A^+} - 1\right) + \tau^e(e_i - 1)\right]\delta D$ $A_j^+ = \max[0, A_j]$. A^+ is the total asset hold by positive asset holders
Compute some aggregate steady state variables which do not require a distribution given my model.

$$mc = \frac{\epsilon_p - 1}{\epsilon_p} \tag{A.14}$$

$$\mu_w = \frac{\epsilon_w}{\epsilon_w - 1} \tag{A.15}$$

$$N = [(1-\alpha)mc.\mu_w]^{1/(\sigma(1-\alpha)+\alpha+\eta))} \tag{A.16}$$

$$w = (1-\alpha)mc.N^{-\alpha} \tag{A.17}$$

$$Y = N^{1-\alpha} \tag{A.18}$$

$$D = Y - w.N \tag{A.19}$$

$$Q = \beta\left[Q + (1-\delta)D\right] \tag{A.20}$$

1. Guess A^+
   (a) Guess β
      i. Guess the consumption $C_{ij} = w.Ne_i + [-rQ + (1-\delta)D]\frac{A_j^+}{A^+} + T_{ij}$ for every i and j. Let's denote it C_{guess}
         A. Update 1(a)i using the Euler Equation (with equality) Let's denote it C_{new}
         B. Compute the policies function A_{ij}^*
         C. Identify binding constraints
         D. Interpolate the policies function A_{ij}^* and C_{new} on A grid . Denote the policy function A_{star}
         E. Update 1(a)iD by taking into account the binding constraints . For binding constraints we have $C_{ij} = X_{ij} - min(A_j)$. Let denote this C_{star}
         F. if $\max(abs(C_{guess} - C_{star}))$ close to zero enough . stop if not update $C_{guess} = C_{star}$ and go back to step 1(a)iA
      ii. Use A_{star} to compute the stationary distribution μ

(b) Check the asset market $\sum_i \sum_j A^{ij}_{star}\mu_{ij} = Q$ If the asset market verified, stop. If not go back to step 1a

2. Compute $A^+_{new} = \sum_i \sum_j A_j \mu_{ij}(A_j > 0)$. If A^+_{new} enough close to A^+, stop if not go back to step 1

A.3.2 Aggregate fluctuations

I closely follow Bayer et al. (2019) to solve for the aggregate fluctuation. The HANK model can be summarized in a system of equations of the form

$$\mathbb{E}\left[X_t, X_{t+1}, Y_t, Y_{t+1}\right] = 0 \tag{A.21}$$

where X_t is a set of state variables and Y_t is a set of control variables. This can be solved using Schmitt-Grohé and Uribe (2004) toolbox. Bayer et al. (2019) propose a matlab file (a variant of Schmitt-Grohé and Uribe (2004) algorithm to solve for the system). Note that the number of state variables here is 80x11-1 + 3. 80x11-1 is the number of state variables from the joint distribution of asset and labor income risk. 3 is the number of aggregate states variable $(R_t, w_{t-1}, v_t/z_t/a_t)$. The number of control variables is 80x11+7. 80x11 for each level of consumption and 7 for the number of aggregate control variables ($y_t, \pi^p_t, \pi^w_t, Q_t, D_t, N_t, A^+_t$). So A.21 is a system of 2x80x11-1+3+7 = 1769 equations. Bayer et al. (2019) proposes a method to reduce the dimension of the state and the control variables. For details on the reduction of the dimensionality please see (Bayer and Luetticke (2018) and Bayer et al. (2019)

1. I solve my system without applying of the reduction of the dimension proposed by (Bayer and Luetticke (2018) and Bayer et al. (2019) . That is the full system of 1769 equations.

2. I solve my system by reducing just the dimension of the state variable. This gives a system of (80+11)-2 + 3 +80x11+7= 979 equations

3. I solve the system by reducing both the state space and the control space. This gives a system of 164 equations.

I find (almost) no difference in the impulse response of aggregate control variables.

A.4 Monetary policy (MP) shock

A.4.1 MP shock: RANK

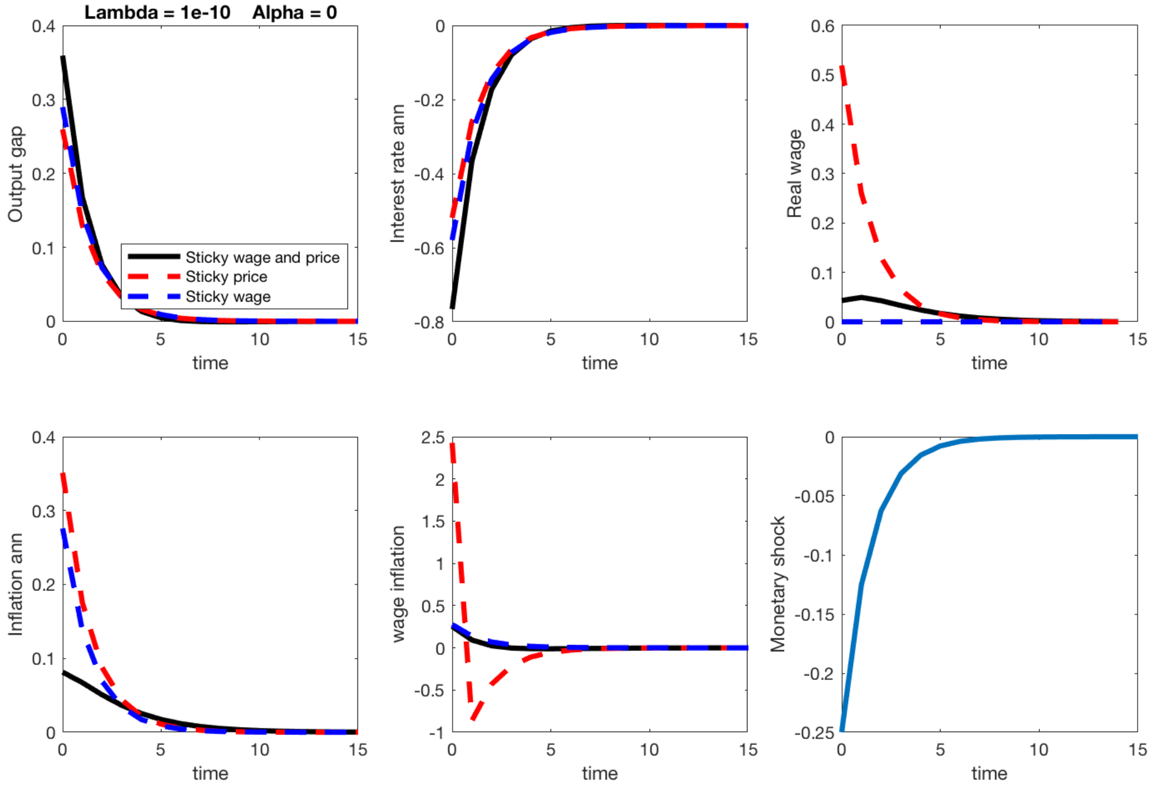


Figure A.1: Impulse response of a monetary policy shock in a RANK model

A.4.2 Solution for stationary distribution

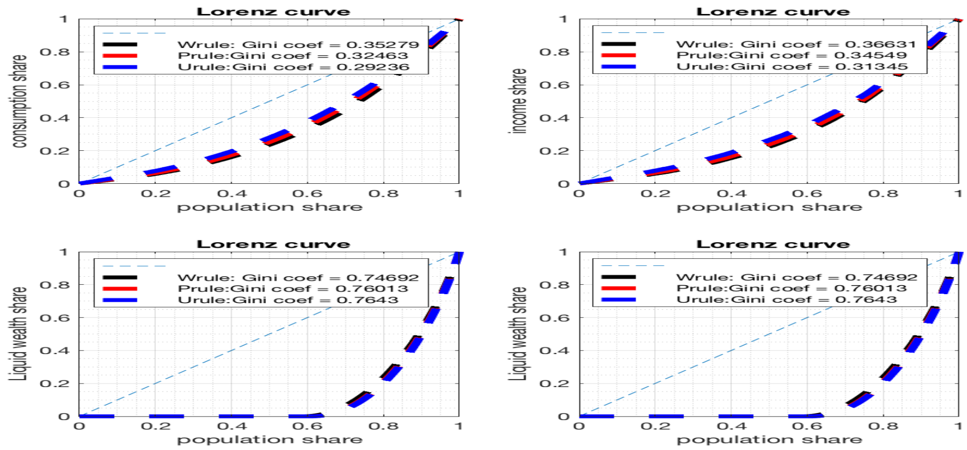


Figure A.2: Lorenz Curve

A.4.3 MP shock: TANK

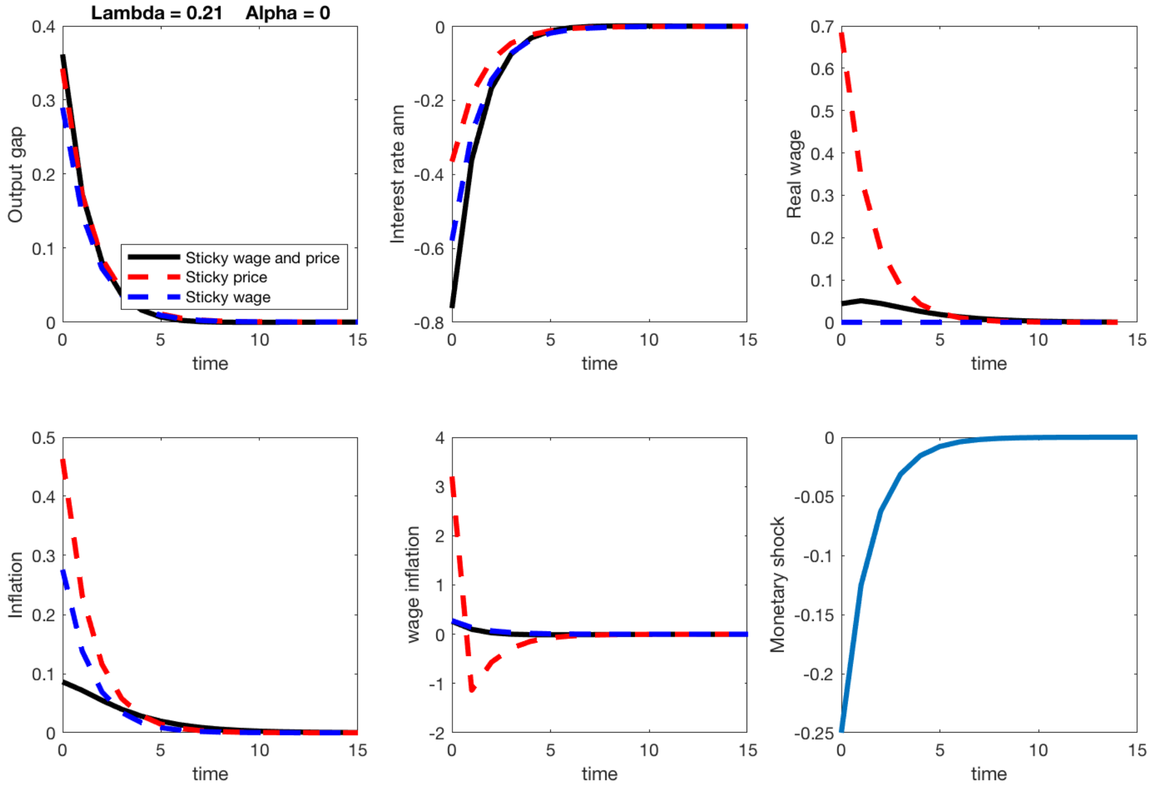
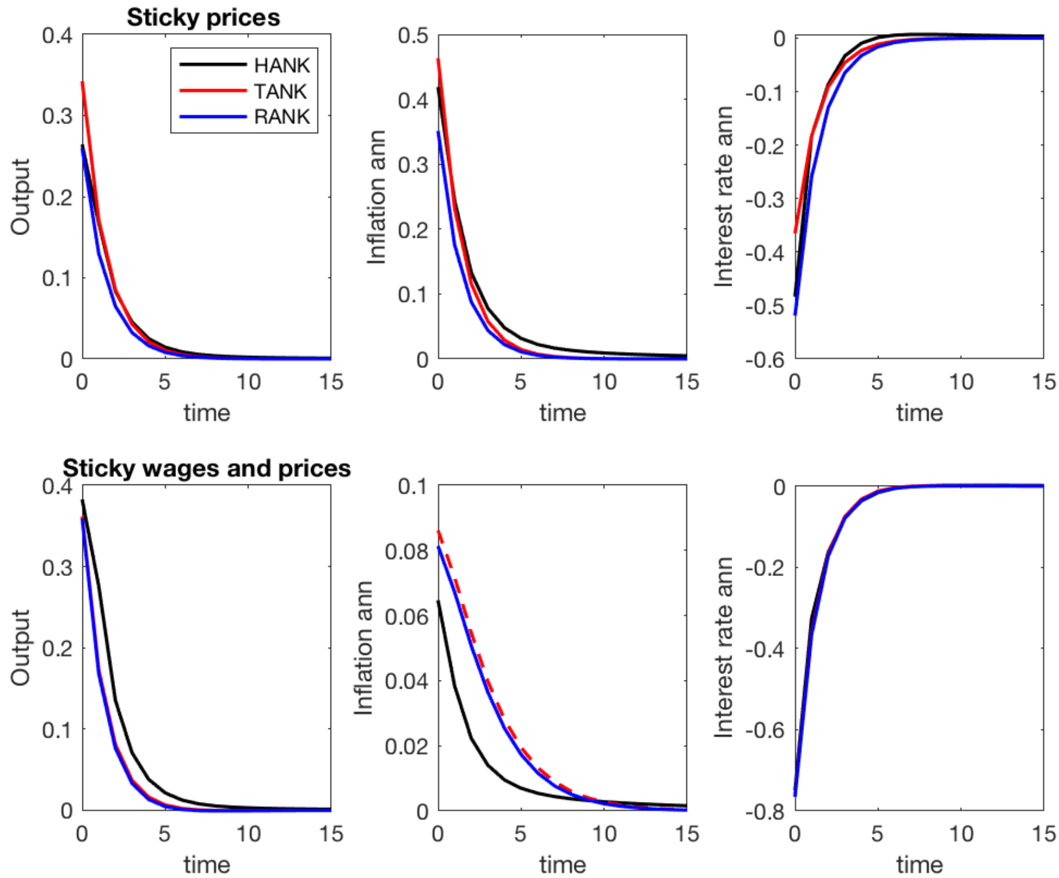


Figure A.3: Impulse response of a monetary policy shock in a TANK model

A.4.4 Aggregate fluctuations: IRF of MP shock

Figure A.4: Impulse response of a monetary policy shock: Comparison across models



$$\max_{c_{it},W_{it}} \mathbb{E}\sum_{t=0}^{\infty}\beta^t u(c_{it},N_{it}) \tag{A.22}$$

$$c_{it}+\frac{1}{1+r_t}b_{i,t+1} = b_{it}+\frac{1}{P_t}W_{it}n_{it}e_{it}+T_{it}-\frac{1}{P_t}\frac{\theta_w}{2}\left(\frac{W_{it}}{W_{it-1}}-1\right)^2\mathbb{Z} \tag{A.23}$$

$$N_{it}e_{it} = \left[\frac{W_t}{W_{it}}\right]^{\epsilon_w}N_t \tag{A.24}$$

$$b_{it+1} \geq 0 \tag{A.25}$$

Where $b_{it}=\frac{B_{it}}{P_t}$, $1+r_t=\frac{1+i_t}{\Pi_{t+1}}$ and $\Pi_{t+1}=\frac{P_{t+1}}{P_t}$
With the latter specification the states variables are W_{it-1}, b_{it} and e_{it} the uninsurable idiosyncratic labor shock. The choices variables are c_{it}, W_{it} which is equivalent to choosing b_{it+1}, W_{it}

Recursive formulation :

$$V(b_i, W_{-1i}, e) = \max_{C_i, W_i} \{u(C_i, N_i) + \beta \mathbb{E} V(b', W_i, e)/e'\} \quad (A.26)$$

$$\frac{1}{1+r} b_i' = \frac{1}{P} W_i N_i e_i + T_i + b_i - \frac{1}{P}\frac{\theta_w}{2}\left(\frac{W_i}{W_{-1i}} - 1\right)^2 \mathbb{Z} - C_i \quad (A.27)$$

$$N_i e_i = \left[\frac{W}{W_i}\right]^{\epsilon_w} N \quad (A.28)$$

$$b' \geq 0 \quad (A.29)$$

$$V(b, W_{-1}, e) = \max_{b', W} u(-\frac{1}{1+r} b_i' + \frac{1}{P} W_i N_i e_i + T_i + b_i - \frac{1}{P}\frac{\theta_w}{2}\left(\frac{W_i}{W_{-1i}} - 1\right)^2 \mathbb{Z}, N_i) \quad (A.30)$$

$$+ \beta \mathbb{E} V(b', R(a+d), e')/e \quad (A.31)$$

$$N_i e_i = \left[\frac{W}{W_i}\right]^{\epsilon_w} N \quad (A.32)$$

$$b' \geq 0 \quad (A.33)$$

FOC Let's λ_t be the multiplier associated to the bond

$$[b'] \quad : -\frac{1}{1+r} U_c + \beta \mathbb{E} V_b(b', W) + \lambda_t = 0 \quad (A.34)$$

$$[W_i] \quad : \left[\frac{N_i e_i}{P} + \frac{W_i e_i}{P}\frac{\partial N_i}{\partial W_i} - \frac{\theta_w}{P}\frac{1}{W_{-1i}}\left(\frac{W_i}{W_{-1i}} - 1\right)\mathbb{Z}\right] U_c \quad (A.35)$$

$$+ \frac{\partial N_i}{\partial W_i} U_N + \beta \mathbb{E} V_W(b', W) = 0 \quad (A.36)$$

Envelope condition

$$V_b = U_c \quad (A.37)$$

$$V_W = \left[\frac{\theta_w}{P}\frac{Wi}{W_{-1i}^2}\left(\frac{W_i}{W_{-1i}} - 1\right)\mathbb{Z}\right] U_c \quad (A.38)$$

with $\frac{\partial N_i}{\partial W_i} = -\epsilon_w \frac{N_t}{W_i e_i}\left[\frac{W}{W_i}\right]^{\epsilon_w}$

Optimality condition we combine both FOC and envelope condition. we get:

$$U_c(C_i, N_i) \geq \beta(1+r_t)\mathbb{E}(U_c C_i^{'}, N_i^{'}) \quad (A.39)$$

$$\left[\frac{N_i e_i}{P} - \epsilon_w \frac{N_t}{P}\left[\frac{W}{W_i}\right]^{\epsilon_w} - \frac{\theta_w}{P}\frac{1}{W_{-1i}}\left(\frac{W_i}{W_{-1i}} - 1\right)\mathbb{Z}\right] U_c(C_i, N_i) + \beta \left[\frac{\theta_w}{P}\frac{W_i^{'}}{W_i^2}\left(\frac{W_i^{'}}{W_i} - 1\right)\mathbb{Z}\right] U_c(C_i^{'}, N_i^{'}) - \epsilon_w \frac{N_t}{W_i e_i}\left[\right.$$

The optimality condition gives the standard Euler equation: One unit of consumption today cost $U_c(c_{t+1})$. If household gives up this unit of consumption by saving it in a the riskless bond, he will gain tomorrow $(1+r_t)U_c(c_{t+1})$ where r_t is the riskless bond real interest rate. At the optimum the cost of saving should be equal to its discounted benefice.

In the place of the usual intra-temporal condition (real wage is equal to the marginal rate of substitution) we now have an optimal wage setting equation. In a similar way as in the well known sticky prices model, we can define the wage markup as the wedge between the real wage and the marginal rate of substitution. Let's M_t^w be the wage markup, we have : $\frac{W_{it} e_{it}}{P_t} = -M_{it}^w \frac{U_{n_i}}{U_{c_i}}$. The optimal wage equation can be rewritten as :

Appendix B

Appendix to Chapter 2

B.1 Separate problem

B.1.1 Problem of households who are not hand-to-mouth consumers

$$\max_{C_{1t}, b_{1t+1}, s_{t+1}, N_{1t}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U(C_{1t} - G(N_{1t}))$$

$$C_{1t} + \frac{b_{1t+1}}{R_t} + q_t s_{t+1} = w_t N_{1t} + b_{1t} + (d_t + q_t) s_t$$

$$\frac{b_{1t+1}}{R_t} \geq -\kappa q_t s_{t+1}$$

$$EE1: \quad \overbrace{U'(t)}^{\textbf{marginal benefit of borr.}} = \overbrace{\beta R_t E_t U'(t+1)}^{\textbf{marginal cost of borr.}} + \overbrace{\mu_t^{nh}}^{\textbf{shadow price of relaxing the constr.}}$$

$$EE2: \quad \underbrace{q_t U'(t)}_{\textbf{marginal cost of buy.}} = \underbrace{\beta E_t \left[(d_{t+1} + q_{t+1}) U'(t+1) \right]}_{\textbf{marginal benef. of buy.}} + \underbrace{k q_t \mu_t^{nh}}_{\textbf{gain of relax. the constr.}}$$

$$Lab: \quad \underbrace{G'(N_{1t})}_{\textbf{marginal disutility of labor}} = \underbrace{w_t}_{\textbf{real wage}}$$

B.1.2 Firm's problem

$$\max_{d_t, k^f_{t+1}, b^f_{t+1}, v_t, L_t} \mathbb{E} \sum_{t=0}^{\infty} \beta^t U'(C_{1t} - G(N_{1t}))d_t$$

$$d_t + \frac{b^f_{t+1}}{R_t} + i_t = F\left(k^f_t, L_t, v_t\right) - (1 + \theta r_t)\left(w_t L_t + p^v_t v_t\right) + b^f_t$$

$$i_t = k^f_{t+1} - k^f_t + \delta k^f_t + \left(k^f_{t+1} - k^f_t\right)\psi\left(\frac{k^f_{t+1} - k^f_t}{k^f_t}\right)$$

$$\frac{b^f_{t+1}}{R_t} - \theta R_t\left(w_t L_t + p^v_t v_t\right) \geq -\kappa^f q_t k^f_{t+1}$$

Optimality conditions for firm

$$\left[b^f_{t+1}\right] :: U'(t) = R_t E_t\left[U'(t+1)\right] + U'(t)\mu^f_t$$

$$\left[k^f_{t+1}\right] :: U'(t)\frac{\partial i_t}{\partial k_{t+1}} = E_t\left[U'(t+1)\left\{F_k\left(k^f_t, L_t, v_t\right) - \frac{\partial i_{t+1}}{\partial k_{t+1}}\right\}\right]$$

$$+k^f q_t U'(t)\mu^f_t$$

$$\left[L_t\right] :: F_l\left(k^f_t, L_t, v_t\right) = \left(1 + \phi r_t + \phi R_t \mu^f_t\right) w_t$$

$$\left[v_t\right] :: F_v\left(k^f_t, L_t, v_t\right) = \left(1 + \phi r_t + \phi R_t \mu^f_t\right) p^v_t$$

$$KT :: \mu^f_t\left(\frac{b^f_{t+1}}{R_t} - \phi R_t\left(w_t L_t + p_t v_t\right) + k^f q_t k^f_{t+1}\right)$$

$$\mu^f_t \geq 0$$

B.1.3 Market equilibrium for the separate problem

Labor market: $L_t = (1-\lambda) N_{1t} + N_{2t}$
Stock market : $s_t = \frac{1}{1-\lambda}$
Good market : $C_t = (1-\lambda) C_{1t} + \lambda C_{2t}$ Bond market:$b_{t+1} = (1-\lambda) b_{1t+1} + b^f_{t+1}$
Aggregate capital: $k_{t+1} =, k^f_{t+1}$

Definition : A competitive equilibrium is a set of allocations $Q_t = \left\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, s_{t+1}, b_{1t+1}, b^f_{t+1}, k^f_{t+1}, d_t\right\}$ and prices $P_t = \{p_t, w_t, R_t, q_t\}$ such that:

1. Given P_t, Q_t solves households' and firms' problem ;
2. w_t and q_t are determined competitively G'(L$_t$) $= w_t$; $\frac{\partial i_t}{\partial k_{t+1}} = q_t$
3. Markets are clear.

B.1.4 Equivalence result: Separate versus Non-separate firm household problem

If the allocation $\left\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, s_{t+1}, b_{1t+1}, b^f_{t+1}, k^f_{t+1}, d_t, w_t, q_t, \mu^{nh}_t\right\}$ is a competitive equilibrium in the economy with separate asset holder consumers and firm problems, then

$\{C_{1t}, C_{2t}, N_{1t}, N_{2t}, L_t, v_t, b_{t+1}, k_{t+1}, w_t, q_t, \mu_t\}$ is a competitive equilibrium in the economy with non-separate firm and asset holders consumer problem with $b_{t+1} = (1-\lambda) b_{1t+1} + b^f_{t+1}$ and $k_{t+1} = k^f_{t+1}$. (The converse is also true.)

The proof follows Bianchi and Mendoza (2018), who show it in the representative agent economy. The equivalence result still holds because there is no heterogeneity among firm owners.

B.2 Model with financial shock

B.2.1 Firm-Asset holder households' optimization problem

There is a continuum of identical *asset holder* households of measure $1-\theta \in (0,1]$. The preferences of an *asset holder* consumer indexed by 1 are given by

$$\max_{C_{1t}, b_{t+1}, k_{t+1}, v_t, L_{1t}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(C_{1t} - G(L_{1t}))$$

$$\begin{aligned}
(1-\theta) C_{1t} + \frac{b_{t+1}}{R_t} + q_t k_{t+1} &= F(k_t, L_t, v_t) - p^v_t v_t - \theta w_t L_{2t} + b_t + q_t k_t - T_t \\
\frac{b_{t+1}}{R_t} - \phi(p^v_t v_t) &\geq -\kappa_t q_t k_t.
\end{aligned}$$

The optimal solution gives

$$\begin{aligned}
u'(t) &= \beta R_t \mathbb{E}_t u'(t+1) + \mu_t, \\
q_t u'(t) &= \beta \mathbb{E}_t \left[(d_{t+1} + q_{t+1}) u'(t+1) + \kappa_{t+1} q_{t+1} \mu_{t+1} \right], \\
A_t F_l(k_t, L_t, v_t) &= G'(L_{1t}) \\
A_t F_v(k_t, L_t, v_t) &= p^v_t + \phi \left(\frac{\mu_t}{u'(t)} \right) p^v_t.
\end{aligned}$$

B.2.2 Time-consistent Planner's Problem

$$V(b,s) = \max_{c_1, c_2, b', L, v, q} \left\{ \theta u(c_2 - G(L) + (1-\theta) u(c_1 - G(L)) + \beta \mathbb{E}_{s',s} V(b', s') \right\}$$

$$\begin{aligned}
(1-\theta) c_1 + \frac{b'}{R} &= F(1, L, v) - p^v v - \theta w L + b - T. \\
\theta c_2 &= \theta w L + T \\
-\phi(p^v v) + \frac{b'}{R} &\geq -\kappa q \\
A F_v(1, L, v) &= p^v + \phi \left(\frac{\mu}{u'(c_1 - G(L))} \right) p^v \\
A F_l(1, L, v) &= G'(L) \\
w &= G'(L) \\
q u'(c_1 - G(L)) &= \beta \mathbb{E} \left[(\mathbb{D}(b', s') + \mathbb{Q}(b', s')) u'(\mathbb{C}(b', s')) + \kappa' \mathbb{Q}(b', s') \mu(b', s') \right]
\end{aligned}$$

Appendix C

Appendix to Chapter 3

C.1 Competitive equilibrium

The competitive equilibrium is summarized by the following equations

$$
\begin{aligned}
u'(t) &= \beta \mathbb{E}_t \left[\frac{R_t}{1+\pi_{t+1}} u'(t+1) \right] + \mu_t, \\
q_t u'(t) &= \beta \mathbb{E}_t \left[\left\{ X_{t+1} F_k \left(1, L_{t+1}, v_{t+1}\right) + q_{t+1} \right\} u'(t+1) + \kappa_{t+1} q_{t+1} \mu_{t+1} \right], \\
X_t F_l \left(1, L_t, v_t\right) &= G'(L_t), \\
X_t F_v \left(1, L_t, v_t\right) &= p_t^v + \phi \frac{\mu_t}{u'(t)} p_t^v, \\
c_{2t} &= G'(L_t) L_t + t_t. \\
\pi_t \left(1+\pi_t\right) &= \frac{\varepsilon}{\theta} \left(X_t - \frac{\varepsilon - 1}{\varepsilon} \right) + \beta \mathbb{E}_t \left[\frac{\mathcal{M}_{t+1}}{\mathcal{M}_t} \pi_{t+1} \left(1+\pi_{t+1}\right) \frac{Y_{t+1}}{Y_t} \right], \\
c_{1t} + c_{2t} + \frac{b_{t+1}}{R_t} - \frac{b_t}{1+\pi_t} &= F\left(1, L_t, v_t\right) - p_t^v v_t - \mathcal{A}_t, \\
\mu_t \left(\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t k_t \right) &= 0. \quad \mu_t >= 0 \\
\textit{Taylor Rule regime} \quad 1 + i_t &= i \left(\frac{1+\pi_t}{1+\pi} \right)^{\phi_\pi} \\
\beta \mathbb{E}_t \left[R_t - \frac{(1+i_t)}{1+\pi_{t+1}} u'(t+1) \right] + \mu_t &= 0
\end{aligned}
$$

I solve for the competitive equilibrium in which price is fully stable (i.e $\pi_t = 0$).

C.2 First best allocation

The first best allocation solves problem (C.1)

$$\max_{c_{1t},c_{2t},b_{t+1},l_t,v_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[u(c_{1t}) + \omega u(c_{2t} - G(l)) \Big], \tag{C.1}$$

$$s.t \quad c_{1t} + c_{2t} + \frac{b_{t+1}}{R_t} - b_t = F(1, l_t, v_t) - p_t^v v_t$$

Let λ_t the multiplier on the aggregate resource constraint. The first best optimality conditions are given by

$$c_{1t} :: u'(c_{1t}) - \lambda_t = 0 \tag{C.2}$$

$$c_{2t} :: \omega u'(c_{2t} - G(l_t)) - \lambda_t = 0 \tag{C.3}$$

$$l_t :: -\omega G'(l_t) u'(c_{2t} - G(l_t)) + F_l(1, l_t, v_t) \lambda_t = 0 \tag{C.4}$$

$$v_t :: \Big[F_v(1, l_t, v_t) - p_t^v \Big] \lambda_t = 0 \tag{C.5}$$

$$b_{t+1} :: -\frac{1}{R_t} \lambda_t + \beta \lambda_{t+1} = 0 \tag{C.6}$$

Combining conditions (C.2)-(C.5) I obtain that the input wedge is zero at a first best allocation

$$\varphi_t \equiv \frac{F_v(1, l_t, v_t)}{p_t^v} - \frac{F_l(1, l_t, v_t)}{G'(l_t)} = 0$$

In addition the first best relative weight ω is equal to the relative marginal utility. That is: $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$

C.3 Discretionary monetary policy

Under the discretionary monetary policy, the central banks solves the following problem. Let define some auxiliary variables. $\Omega(b', s') \equiv \beta \mathbb{E}_{s',s} [R u'(\mathcal{C}_1(b', s')]$, $X_t = \Big[\varphi_t + \frac{F_l(1, l_t, v_t)}{G'(l_t)} \Big]^{-1} \Big\{ 1 - \frac{1}{\epsilon} + \phi \frac{\mu_t}{u'(c_{1t})} \Big\}$,
$\Delta(b', s') \equiv \beta \mathbb{E}_{s',s} \left[u'(b', s') \left[\mathcal{X}(b', s') F_k(1, \mathcal{L}(b', s'), \mathbf{v}(b', s')) + \mathcal{Q}(b', s') \right] + \kappa' \mu(b', s') \mathcal{Q}(b', s') \right]$, and

,
$$\Gamma((b', s', c_1, l, v, \mu) \equiv \frac{\varepsilon}{\theta} \left(\phi \frac{\mu_t}{u'(c_{1t}) \varphi_t} - \frac{\varepsilon - 1}{\varepsilon} \right) + \beta \mathbb{E}_{s',s} \left[\frac{u'(\mathcal{C}_1(b', s') F(1, \mathcal{L}(b', s'), \mathbf{v}(b', s'))}{u'(c_1) F(1, l, v)} \pi(b', s') \left(1 + \pi(b', s')\right) \right]$$

$$\mathcal{V}(b,s) = \max_{c_1,c_2,b',l,v,q,\pi,\mu} \left\{ u(c_1) + \omega u(c_2 - G(l) + \beta \mathbb{E}_{s',s} \mathcal{V}(b',s') \right\} \tag{C.7}$$

$$u'(c_1) = \Omega(b',s') + \mu, \qquad : \gamma_t \tag{C.8}$$

$$qu'(c_1) = \Delta(b',s') \qquad : \xi_t \tag{C.9}$$

$$c_2 = G'(l)l + t \qquad : \delta_t \tag{C.10}$$

$$\pi\,(1+\pi) = \Gamma((b',s',c_1,l,v) \qquad : \vartheta_t \tag{C.11}$$

$$c_1 + c_2 + \frac{b'}{R} - b = \left(1 - \frac{1}{2}\theta\pi^2\right) F\,(1,l,v) - p^v v \qquad : \lambda_t \tag{C.12}$$

$$\frac{b'}{R_t} - \phi p^v v \geq -\kappa q \qquad : \mu_t^* \tag{C.13}$$

$$\mu\left(\frac{b'}{R} - \phi p^v v + \kappa q\right) = 0. \qquad : \varsigma_t \tag{C.14}$$

The social planner's optimality conditions are given by

$$c_{1t} :: u'(c_{1t}) - \gamma_t u''(c_{1t}) - \xi_t q_t u''(c_{1t}) + \vartheta_t \Gamma_3(t+1) - \lambda_t = 0 \tag{C.15}$$

$$c_{2t} :: \omega u'(c_{2t} - G(l_t)) - \delta_t - \lambda_t = 0 \tag{C.16}$$

$$b_{t+1} :: \beta \mathbb{E}_{s',s} \mathcal{V}_b(b',s') + \gamma_t \Omega_1(t+1) + \xi_t \Delta_1(t+1) + \vartheta_t \Gamma_1(t+1) - \frac{1}{R}\lambda_t + \frac{1}{R}\mu_t^* + \frac{1}{R}\varsigma_t \mu_t = 0 \tag{C.17}$$

$$l_t :: -\omega G'(l_t) u'(c_{2t} - G(l_t)) + \delta_t \left(G''(l)l + G'(l)\right) + \vartheta_t \Gamma_4 + \lambda_t (1 - \frac{1}{2}\theta\pi_t^2) F_l\,(1,l,v) = 0 \tag{C.18}$$

$$v_t :: \vartheta_t \Gamma_5(t+1) + \lambda_t \left\{ \left(1 - \frac{1}{2}\theta\pi_t^2\right) F_v\,(1,l,v) - p_t^v \right\} - \phi \mu_t^* p_t^v - \phi p_t^v \varsigma_t \mu_t = 0 \tag{C.19}$$

$$q_t :: -\xi_t u'(c_{1t}) + \kappa_t \mu_t^* + \kappa_t \varsigma_t \mu_t = 0 \tag{C.20}$$

$$\mu_t :: \gamma_t + \varsigma_t \left(\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t\right) + \vartheta_t \Gamma_6(t+1) = 0 \tag{C.21}$$

$$\pi_t :: -(1+2\pi_t)\vartheta_t - \theta \pi_t F\,(1,l,v)\,\lambda_t = 0 \tag{C.22}$$

$$KT :: \mu_t^* \left(\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t\right) = 0 \tag{C.23}$$

$$EC :: \mathcal{V}_b(b_t, s_t) = \lambda_t \tag{C.24}$$

C.3.1 Lemma 1

Lemma C.3.1. *It is optimal to set* $\varsigma_t \mu_t = 0$ *for all* t.

Proof: Suppose $\mu_t* > 0$, by the KT condition in equation (C.23) $\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t = 0$. Then $\mu_t \left(\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t\right)$ is equal to zero. So Condition (C.14) is satisfied. It is then optimal to set $\varsigma_t = 0$. Suppose now that $\mu_t* = 0$, by the KT condition in equation (C.23) $\frac{b_{t+1}}{R_t} - \phi p_t^v v_t + \kappa_t q_t > 0$. Then $\mu_t = 0$.

C.3.2 Proof of lemma 3.3.1

Suppose there exist a wage and imported input price subsidies to offset the monopolistic distortions at the flexible prices allocations. Let τ_w *and* τ_v *be the wage and the imported input price subsidy respectively such that and* $\tau_w = \tau_v = \frac{1}{\varepsilon}$*. Then, in the absence of a credit friction, the constraint-efficient flexible prices allocations coincide with the competitive equilibrium allocations with the optimal relative weight given by* $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$*.*

Proof: Under flexible prices, Condition (C.22) shows that the multiplier $\vartheta_t = 0$. In addition, in the absence of credit friction, $\mu_t* = \mu = 0$, which implies from condition (C.20) that the multiplier $\xi = 0$. Suppose that the bond Euler equation implementability constraint is not bind that is $\gamma_t = 0$.
Combining (C.15), (C.17), and (C.24) gives the foreign bond Euler equation $u'(c_{1t}) = \beta R_t \mathbb{E}_{s',s} u'(c_{1t+1})$. Setting $\gamma_t = 0$, is then optimal. In addition, from condition it is optimal to set the multiplier $\nu_t = 0$ so that $F_v(1, l_t, v_t) - p_t^v = 0$. Now setting the relative weight $\omega = \frac{u'(c_{1t})}{u'(c_{2t} - G(l_t))}$ leads $\delta_t = 0$. Given that, condition (C.18) shows that $F_l(1, l_t, v_t) = G'(l_t)$.

C.3.3 Proof of proposition 4

The proof uses the social planner's optimality conditions under discretionary monetary policy. I combine conditions (C.16) in (C.18)

$$-\lambda_t G'(l) + \delta_t G''(l) l + \vartheta_t \Gamma_4(t+1) + \lambda_t \left(1 - \frac{1}{2}\theta \pi_t^2\right) F_l(1, l, v) = 0 \tag{C.25}$$

I rearrange (C.25) and (C.19) to get

$$\alpha \lambda_t \left(1 - \frac{1}{2}\theta \pi_t^2\right) F = \lambda_t G'(l) l_t - \delta_t G''(l) l^2 - \vartheta_t \Gamma_4 l_t \tag{C.26}$$

$$\eta \lambda_t \left\{ \left(1 - \frac{1}{2}\theta \pi_t^2\right) F \right\} = -\vartheta_t \Gamma_5 v_t + \phi \mu_t^* p_t^v v_t + \lambda_t p_t^v v_t \tag{C.27}$$

I then substitute (C.26) into (C.27) to obtain

$$-\vartheta_t(\alpha \Gamma_5 v_t - \eta \Gamma_4 l_t) + \phi \alpha \mu_t^* p_t^v v_t + \left\{ \alpha p_t^v v_t - \eta l_t G'(l) \right\} \lambda_t + \eta \delta_t G''(l) l^2 = 0 \tag{C.28}$$

Let $\tilde{w} \equiv \eta l_t G'(l) - \alpha p_t^v v_t$. Now, I use (C.25) to eliminate the lagrange multiple λ_t in (C.28), which gives

$$-\vartheta_t(\alpha \Gamma_5 v_t - \eta \Gamma_4 l_t + \tilde{w} \Gamma_3) + \phi \alpha \mu_t^* p_t^v v_t - \tilde{w} u'(c_{1t}) + \tilde{w} u''(c_{1t}) \gamma_t + \xi_t \tilde{w} q_t u''(c_{1t}) + \eta \delta_t G''(l) l^2 = 0 \tag{C.29}$$

I use conditions (C.15) and (C.16) to obtain $\delta_t = \omega u'(c_{2t} - G(l_t)) - u'(c_{1t}) + \gamma_t u''(c_{1t}) + \xi_t q_t u''(c_{1t}) - \vartheta_t \Gamma_3(t+1)$. Condition (C.20) gives $\xi_t = \frac{\kappa_t}{u'(c_{1t})} \mu_t^*$ and condition (C.22) $\vartheta_t = -\theta \frac{\lambda_t}{1 + 2\pi_t} \pi_t F(1, l, v)$.

Let $\Phi \equiv (\alpha\Gamma_5 v_t - \eta\Gamma_4 l_t + \tilde{w}\Gamma_3)\frac{\lambda_t}{1+2\pi_t}$. I finally substitute those expressions into (C.29) to get

$$\theta\Phi y_t\pi_t = \tilde{w}u'(c_{1t}) - \left\{\phi\alpha p_t^v v_t + \frac{\kappa_t}{u'(c_{1t})}\tilde{w}q_t u''(c_{1t})\right\}\mu_t^* - \tilde{w}u''(c_{1t})\gamma_t - \eta G''(l)l^2\delta_t \tag{C.30}$$

It can be shown using conditions (C.15) and (C.17) that γ_t is equal to zero if $\mu_t^* = 0$ for all t. Further, if the collateral is expected to bind in the future $\gamma_t \neq 0$. The multiplier γ_t captures the prudential motives for the discretionary monetary policy.

Now let denote $\sigma \equiv -\frac{u''(c_{1t})c_{1t}}{u'(c_{1t})}$ the risk aversion or the inverse of the elasticity of intertemporel of substitution. let $s_t \equiv 1 - \frac{1}{\epsilon} + \phi\frac{\mu_t}{u'(c_{1t})}$ and $z_t \equiv \varphi_t + \frac{F_l(1,l_t,v_t)}{G'(l_t)}$ so $X_t = z_t^{-1}s_t$. I can rewrite Φ to obtain

$$\Phi = \Phi_0 + \beta\mathbb{E}_t\left[\Phi_1\pi_{t+1}\right]$$

where $\Phi_0 = \underbrace{\frac{\varepsilon}{\theta}\left[-\alpha v_t\frac{F_{vv}(t)}{p_t^v}s_t z_t^{-2} + \eta l_t\frac{F_{vl}(t)}{p_t^v}s_t z_t^{-2} + \sigma\tilde{\omega}\frac{\phi\mu_t}{c_{1t}}z_t^{-1}\right]}_{>0}\frac{\lambda_t}{1+2\pi_t}$ and

$$\Phi_1 = \sigma\frac{\tilde{\omega}}{c_{1t}}\frac{u'(c_{1t+1})}{u'(c_{1t})}\frac{y_{t+1}}{y_t}\frac{(1+\pi_{t+1})\lambda_t}{1+2\pi_t}$$

Finally the optimal monetary policy under discretion satisfies:

$$\theta\Phi y_t\pi_t = \tilde{w}u'(c_{1t}) + \left\{\sigma\frac{\kappa_t q_t}{c_{1t}}\tilde{w} - \phi\alpha p_t^v v_t\right\}\mu_t^* + \sigma\frac{u'(c_{1t})}{c_{1t}}\tilde{w}\gamma_t - \eta G''(l)l^2\delta_t \tag{C.31}$$