Université de Montréal

Recherche de nouvelle physique à basse énergie à l'aide de théories efficaces

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Recherche de nouvelle physique à basse énergie à l'aide de théories efficaces

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Résumé

L'existence de physique au-delà du modèle standard semble de plus en plus évidente, notamment en raison d'observations d'anomalies dans plusieurs phénomènes. De plus, certains phénomènes tels que la matière noire, la gravité et l'asymétrie baryonique dans l'univers sont inexpliqués. Ce mémoire s'intéresse à la recherche de nouvelle physique à basse énergie par l'approche des EFT et se penche sur les prédictions à basse énergie d'une EFT en particulier : la SMEFT. Le premier article présenté s'intéresse aux anomalies présentes dans les données expérimentales liées aux mésons B et teste les prédictions de la SMEFT pour la désintégration $b \to c\tau^-\overline{\nu}_\tau$ jusqu'à la dimension massive 6. Le but est de vérifier si la symétrie du modèle standard, soit $SU(3)_C \times SU(2)_L \times U(1)_Y$, est réalisée linéairement à haute énergie par la nouvelle physique. Le deuxième article détermine les relations de correspondance entre les opérateurs LEFT jusqu'à la dimension massive 6 et les opérateurs SMEFT jusqu'à la dimension massive 8.

Mots Clés : Modèle standard, Nouvelle physique, Brisure spontanée de symétrie, Théorie efficace de champ, Coefficients de Wilson, Théorie efficace de champ du modèle standard, Théorie efficace de champ de basse énergie

Abstract

The existence of physics beyond the Standard Model seems more and more obvious, partially because of observations of anomalies in many phenomena. Moreover, some phenomena such as dark matter, gravity and the baryon asymmetry in the universe are unexplained. This thesis addresses the search of New Physics at low energies using the EFT approach and looks into the predictions of low-energy predictions of one EFT in particular: SMEFT. The first paper presented addresses anomalies present in experimental data related to B mesons and tests the predictions of SMEFT for $b \to c\tau^-\overline{\nu}_\tau$ decays up to mass dimension 6. The goal is to check if the Standard Model symmetry, being $SU(3)_C \times SU(2)_L \times U(1)_Y$, is linearly realized at high energy by the New Physics. The second paper determines the matching conditions between LEFT operators up to mass dimension 6 and SMEFT operators up to mass dimension 8.

Keywords: Standard model, New physics, Spontaneous symmetry breakdown, Effective field theory, Wilson coefficients, Standard model effective field theory, Low-energy effective field theory

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Liste des sigles et des abréviations

MS	Modèle standard
NP	Nouvelle physique
BSS	Brisure spontanée de symétrie
vev	Valeur d'attente dans le vide, de l'anglais *Vacuum Expectation Value*
EFT	Théorie efficace de champ, de l'anglais *Effective Field Theory*
WC	Coefficients de Wilson, de l'anglais *Wilson Coefficients*
SMEFT	Théorie efficace de champ du modèle standard, de l'anglais *Standard Model Effective Field Theory*
LEFT	Théorie efficace de champ de basse énergie, de l'anglais *Low-Energy Effective Field Theory*

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Chapitre 1

Le modèle standard

Dans ce chapitre, beaucoup de détails sont donnés sur le modèle standard. L'importance d'en discuter en large est due au fait que plusieurs des paramètres et les normalisations de plusieurs des champs concernés sont impactés par la nouvelle physique, tel que discuté à la sous-section 2.3.1 (et dans le deuxième article présenté dans ce mémoire).

1.1. Particules et lagrangien

Le modèle standard (MS) fournit une explication extrêmement précise de pratiquement toutes les observations venant d'accélérateurs de particules. Il inclut les particules du tableau

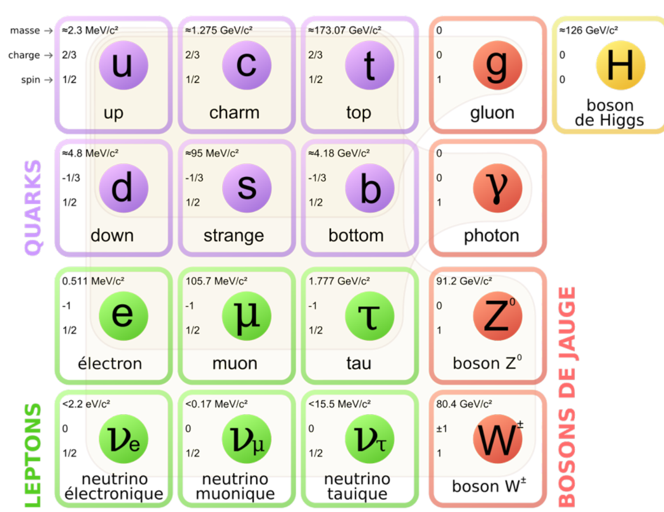


Fig. 1.1. Les particules du MS.

présenté à la figure 1.1 et leurs interactions sont décrites par le lagrangien suivant :

$$\begin{aligned}\mathcal{L}_{\text{MS}} = &-\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + (D_\mu H)^\dagger(D^\mu H) + \sum_{\psi\in\{q,u,d,l,e\}} \overline{\psi}i\not{D}\psi \\ &- \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 - \left[(Y_e)_{pr}\bar{l}_p e_r H + (Y_u)_{pr}\overline{q}_p u_r \tilde{H} + (Y_d)_{pr}\overline{q}_p d_r H + \text{h.c.}\right].\end{aligned} \tag{1.1.1}$$

Les différents paramètres et champs qui y ont été introduits seront expliqués dans les prochaines sous-sections. Ce lagrangien est construit de sorte à avoir comme symétrie le groupe de jauge $SU(3)_C \times SU(2)_L \times U(1)_Y$ où les sous-groupes $SU(3)_C$ et $SU(2)_L \times U(1)_Y$ décrivent respectivement les interactions fortes et électrofaibles. Ainsi, la dérivée covariante de la théorie est $D_\mu \equiv \partial_\mu + ig_sT^AG^A_\mu + igt^IW^I_\mu + ig'YB_\mu$, où les T^A (pour $A \in \{1,2,3,4,5,6,7,8\}$), les t^I (pour $I \in \{1,2,3\}$) et Y sont les générateurs respectifs des sous-groupes $SU(3)_C$, $SU(2)_L$ et $U(1)_Y$. Ils sont appelés respectivement générateurs de *couleur*, d'*isospin faible* et d'*hypercharge*. Les T^A et les t^I sont nuls lorsqu'ils agissent sur des particules qui ne ressentent pas les interactions fortes et faibles, respectivement. Dans le cas contraire, leurs expressions sont les suivantes:

$$T^1 = \frac{1}{2}\begin{bmatrix}0&1&0\\1&0&0\\0&0&0\end{bmatrix}, \quad T^2 = \frac{1}{2}\begin{bmatrix}0&-i&0\\i&0&0\\0&0&0\end{bmatrix}, \quad T^3 = \frac{1}{2}\begin{bmatrix}1&0&0\\0&-1&0\\0&0&0\end{bmatrix}, \quad T^4 = \frac{1}{2}\begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix},$$

$$T^5 = \frac{1}{2}\begin{bmatrix}0&0&-i\\0&0&0\\i&0&0\end{bmatrix}, \quad T^6 = \frac{1}{2}\begin{bmatrix}0&0&0\\0&0&1\\0&1&0\end{bmatrix}, \quad T^7 = \frac{1}{2}\begin{bmatrix}0&0&0\\0&0&-i\\0&i&0\end{bmatrix}, \quad T^8 = \frac{1}{2\sqrt{3}}\begin{bmatrix}1&0&0\\0&1&0\\0&0&-2\end{bmatrix},$$

$$t^1 = \frac{1}{2}\begin{bmatrix}0&1\\1&0\end{bmatrix}, \quad t^2 = \frac{1}{2}\begin{bmatrix}0&-i\\i&0\end{bmatrix}, \quad t^3 = \frac{1}{2}\begin{bmatrix}1&0\\0&-1\end{bmatrix}.$$

Il est à noter que les coefficients devant les termes cinétiques des bosons de jauge sont $-\frac{1}{4}$. Ceci sert à assurer que les équations du mouvement des bosons de jauge soient les bonnes. Note : si le coefficient du terme cinétique d'un boson de jauge donné n'avait pas été $-\frac{1}{4}$, il aurait toujours été possible de redéfinir la normalisation du champ du boson de jauge afin que le coefficient devienne $-\frac{1}{4}$.

1.1.1. Fermions et boson de Higgs dans les représentations de $SU(2)_L$

Seuls les fermions de chiralité gauche ressentent les interactions faibles. On place donc les fermions de chiralités gauche et droite respectivement dans des doublets (sur lesquels les t^I agissent) et singulets de $SU(2)_L$. Concrètement, les champs fermioniques ψ apparaissant dans $\mathcal{L}_{\text{MS}}$ prennent les expressions suivantes (dans $SU(2)_L$):

$$l_p = \begin{bmatrix} \nu_{Lp} \\ e_{Lp} \end{bmatrix}, \quad q_p = \begin{bmatrix} u_{Lp} \\ d_{Lp} \end{bmatrix}, \quad e_p = \begin{bmatrix} e_{Rp} \end{bmatrix}, \quad u_p = \begin{bmatrix} u_{Rp} \end{bmatrix}, \quad d_p = \begin{bmatrix} d_{Rp} \end{bmatrix}.$$

Ici, les indices p représentent des indices générationnels, allant de 1 à 3. Par exemple, e_{R1}, e_{R2} et e_{R3} représentent respectivement un électron, un muon et un tau (chacun de chiralité droite). On note l'absence des neutrinos de chiralité droite, ce qui s'avère être un fait de la nature. Il est à noter que pour que les termes décrivant des interactions (appelées interactions de *Yukawa*) entre les fermions et le champ H vérifient la symétrie du sous-groupe $SU(2)_L \times U(1)_Y$, il faut que ce champ soit un doublet de $SU(2)_L$:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} h^+ \\ h^0 \end{bmatrix},$$

où h^0 représente le boson de Higgs et h^+ représente un champ analogue ayant une charge électrique de $+1$.

1.1.2. Fermions dans les représentations de $SU(3)_C$

Parmi les fermions, seuls les quarks ressentent les interactions fortes. On place donc les quarks et les leptons respectivement dans des triplets (sur lesquels les T^A agissent) et singulets de $SU(3)_C$. Concrètement, les champs fermioniques ψ apparaissant dans $\mathcal{L}_{\text{MS}}$ prennent les expressions suivantes (dans $SU(3)_C$):

$$q = \begin{bmatrix} {\color{red}q} \\ {\color{green}q} \\ {\color{blue}q} \end{bmatrix}, \quad l = \begin{bmatrix} l \end{bmatrix}.$$

Ici, q et l représentent respectivement un quark et un lepton quelconque (ou une composante du doublet de $SU(2)_L$ correspondant dans le cas d'une chiralité gauche). Les couleurs sont ici utilisées pour représenter la *couleur*, soit la charge correspondant aux interactions fortes (note : les quarks ne sont pas réellement colorés; ce n'est qu'une définition).

1.1.3. Bosons de jauge

Les champs apparaissant dans les termes cinétiques des bosons de jauge sont les suivants (où les f^{ABC} sont les constantes de structures de $SU(3)_C$ et les ϵ^{IJK} sont les symboles de Levi-Civita et les constantes de structures de $SU(2)_L$):

$$\begin{aligned} G^A_{\mu\nu} &= \partial_\mu G^A_\nu - \partial_\nu G^A_\mu - g_s f^{ABC} G^B_\mu G^C_\nu, \\ W^I_{\mu\nu} &= \partial_\mu W^I_\nu - \partial_\nu W^I_\mu - g f^{IJK} W^J_\mu W^K_\nu, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu. \end{aligned}$$

Les champs G_μ^A représentent les gluons et les champs W_μ^I et B_μ représentent les bosons de gauge électrofaibles.

1.2. Brisure spontanée de symétrie

Dans le lagrangien (1.1.1), seul le doublet H a un terme de masse. Dans les faits, il est connu que les fermions, ainsi que les bosons W et Z, ont des masses. Leur apparition se fait par le *mécanisme de Higgs*, qui est un processus de brisure spontanée de symétrie (BSS) par lequel le boson de Higgs acquiert une valeur d'attente dans le vide (vev, de l'anglais *vacuum expectation value*). Concrètement:

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} h^+ \\ h^0 \end{bmatrix} \to \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ h^0 + v \end{bmatrix},$$

où la valeur de v est celle qui minimise le potentiel de Higgs

$$V(h^0) = \frac{\lambda}{4} \left[(h^0)^2 - v^2 \right]^2 .$$

Ceci est lourd d'implications physiques. Premièrement, il est à noter que la symétrie $SU(3)_C \times SU(2)_L \times U(1)_Y$ est brisée car le champ H, qui porte un isospin faible et une hypercharge, est remplacé par un champ qui ne porte aucune de ces charges.

1.2.1. Apparition des masses des fermions

Lorsque la transformation du champ H est substituée dans le lagrangien (1.1.1), les termes décrivant des interactions de Yukawa génèrent des masses fermioniques. Par exemple,

$$-(Y_e)_{pr} \bar{l}_p e_r H \to -\frac{(Y_e)_{pr}}{\sqrt{2}} \bar{e}_{Lp} e_{Rr} h^0 - \frac{(Y_e)_{pr} v}{\sqrt{2}} \bar{e}_{Lp} e_{Rr}.$$

Le premier terme après BSS décrit une interaction de Yukawa avec le boson de Higgs après BSS. Le deuxième terme génère les masses des leptons chargés.

1.2.2. Apparition des masses des bosons électrofaibles

Une autre conséquence de la BSS est l'apparition de masses pour les bosons électrofaibles, ce qui n'est pas surprenant considérant que la symétrie $SU(3)_C \times SU(2)_L \times U(1)_Y$ est brisée. Pour le voir, il faut d'abord noter que les bosons électrofaibles avant BSS (les W_μ^I et B_μ) ne correspondent pas aux bosons électrofaibles qui sont connus (après BSS), soit les bosons $W^\pm$ ($W_\mu^\pm$), le boson Z (Z_μ) et le photon (A_μ). Soit les transformations

$$W_\mu^1 \to \frac{W_\mu^+ + W_\mu^-}{\sqrt{2}}, \qquad W_\mu^2 \to \frac{i(W_\mu^+ - W_\mu^-)}{\sqrt{2}},$$
$$W_\mu^3 \to \cos\theta_W \, Z_\mu + \sin\theta_W \, A_\mu, \quad B_\mu \to -\sin\theta_W \, Z_\mu + \cos\theta_W \, A_\mu,$$

où θ_W est l'*angle de Weinberg*. Maintenant, le champ H (avant BSS) est un doublet d'isospin faible et porte une hypercharge de $Y = +1/2$, mais ne porte aucune couleur. Sa dérivée covariante est donc (avant BSS)

$$D_\mu H = \partial_\mu H + igt^I W_\mu^I H + \frac{ig'}{2} B_\mu H$$

et (après BSS)

$$D_\mu H \to \frac{1}{\sqrt{2}} \partial_\mu h^0 + \frac{i}{2} \begin{bmatrix} & gW_\mu^+ \\ -\frac{(g\cos\theta_W + g'\sin\theta_W)}{\sqrt{2}} Z_\mu + \frac{(g'\cos\theta_W - g\sin\theta_W)}{\sqrt{2}} A_\mu \end{bmatrix} (h^0 + v).$$

Ainsi, le terme cinétique $(D_\mu H)^\dagger (D^\mu H)$ génère les termes de masses

$$\frac{g^2 v^2}{4} W_\mu^- W^{+\mu} + \frac{1}{2} \frac{(g\cos\theta_W + g'\sin\theta_W)^2 v^2}{4} Z_\mu Z^\mu + \frac{1}{2} \frac{(g'\cos\theta_W - g\sin\theta_W)^2 v^2}{4} A_\mu A^\mu.$$

Maintenant, le photon a une masse nulle. Ceci force une valeur pour l'angle de Weinberg: $\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$ et $\sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$. Les masses des bosons de jauge électrofaibles sont alors

$$M_W{}^2 = \frac{g^2 v^2}{4}, \quad M_Z{}^2 = \frac{(g^2 + g'^2) v^2}{4}, \quad M_A{}^2 = 0.$$

Le terme cinétique $(D_\mu H)^\dagger (D^\mu H)$ génère également le terme cinétique du boson de Higgs après BSS:

$$\frac{1}{2} (\partial_\mu h^0)(\partial^\mu h^0).$$

Le coefficient $\frac{1}{2}$ sert à assurer que l'équation du mouvement du boson de Higgs soit la bonne, c'est-à-dire qu'elle se réduise, en l'absence d'interaction, à l'équation de Klein-Gordon. Comme mentionné plus haut pour les bosons de jauge, si le coefficient du terme cinétique du boson de Higgs n'avait pas été le bon, il aurait toujours été possible de redéfinir la normalisation du champ correspondant afin que le coefficient prenne la bonne valeur.

1.2.3. Interactions entre fermions et bosons de jauge

Le lagrangien (1.1.1) ne semble a priori inclure aucune des interactions entres les fermions et les bosons de jauge. En réalité, elles sont cachées dans les termes cinétiques des fermions. En effet, la dérivée covariante après BSS est

$$D_\mu = \partial_\mu + ig_s T^A G_\mu^A + \frac{ig}{\sqrt{2}} (t^+ W_\mu^+ + t^- W_\mu^-) + ig_Z (t^3 - \sin^2\theta_W\, Q) Z_\mu + ieQA_\mu,$$

où $Q \equiv t^3 + Y$ est le générateur de charge électrique et $t^\pm \equiv t^1 \pm it^2$, $g_Z \equiv \sqrt{g^2 + g'^2}$ et $e \equiv g\sin\theta_W$ est la charge électrique élémentaire. Ainsi, les termes cinétiques des fermions dans (1.1.1) génèrent les couplages

$$\bar{l}_p i \not{D} l_p \to \begin{bmatrix} -\dfrac{g}{\sqrt{2}}(\overline{\nu}_{Lp}\not{W}^{+}e_{Lp} + \overline{e}_{Lp}\not{W}^{-}\nu_{Lp}) \\ -\dfrac{g_Z}{2}\overline{\nu}_{Lp}\not{Z}\nu_{Lp} + g_Z\left(\dfrac{1}{2} - \sin^2\theta_W\right)\overline{e}_{Lp}\not{Z}e_{Lp} + e\,\overline{e}_{Lp}\not{A}e_{Lp} \end{bmatrix},$$

$$\overline{q}_p i \not{D} q_p \to \begin{bmatrix} -g_s(\overline{u}_{Lp}T^A\not{G}^A u_{Lp} + \overline{d}_{Lp}T^A\not{G}^A d_{Lp}) - \dfrac{g}{\sqrt{2}}(\overline{u}_{Lp}\not{W}^{+}d_{Lp} + \overline{d}_{Lp}\not{W}^{-}u_{Lp}) \\ +\dfrac{g_Z}{2}\left(-\dfrac{1}{2} + \dfrac{2}{3}\sin^2\theta_W\right)\overline{u}_{Lp}\not{Z}u_{Lp} + g_Z\left(\dfrac{1}{2} - \dfrac{1}{3}\sin^2\theta_W\right)\overline{d}_{Lp}\not{Z}d_{Lp} \\ -\dfrac{2e}{3}\,\overline{u}_{Lp}\not{A}u_{Lp} + \dfrac{e}{3}\,\overline{d}_{Lp}\not{A}d_{Lp} \end{bmatrix},$$

$$\overline{e}_p i \not{D} e_p \to -g_Z\sin^2\theta_W\,\overline{e}_{Rp}\not{Z}e_{Rp} + e\,\overline{e}_{Rp}\not{A}e_{Rp},$$

$$\overline{u}_p i \not{D} u_p \to -g_s\overline{u}_{Rp}T^A\not{G}^A u_{Rp} + \frac{2g_Z}{3}\sin^2\theta_W\,\overline{u}_{Rp}\not{Z}u_{Rp} - \frac{2e}{3}\,\overline{u}_{Rp}\not{A}u_{Rp},$$

$$\overline{d}_p i \not{D} d_p \to -g_s\overline{d}_{Rp}T^A\not{G}^A d_{Rp} - \frac{g_Z}{3}\sin^2\theta_W\,\overline{d}_{Rp}\not{Z}d_{Rp} + \frac{e}{3}\,\overline{d}_{Rp}\not{A}d_{Rp},$$

qui correspondent aux interactions connues entre les fermions et les bosons de jauge.

1.2.4. Réalisation de la symétrie du MS

Empiriquement, la symétrie qui est vérifiée est $SU(3)_C \times U(1)_Q$, du moins à basse énergie. Concrètement, ceci se manifeste par la conservation de la couleur et de la charge électrique lors de tout processus. Ainsi, le MS est une construction dans laquelle symétrie $SU(3)_C \times SU(2)_L \times U(1)_Y$ est réalisée par le doublet $SU(2)_L$ de Higgs (H) et des bosons de jauge G^A, W^I et B, qui sont reliés aux particules connues de la manière décrite dans cette section. On parle alors d'une *réalisation linéaire*. Ce concept est discuté en plus grands détails dans [**1**] et [**2**].

Chapitre 2

Les théories efficaces de champ

2.1. Motivation

Étant donné une théorie de champ sous-jacente à l'étude de certains phénomènes (par exemple le MS pour la physique des particules), une *théorie efficace de champ* (EFT, de l'anglais *Effective Field Theory*) consiste à approximer à basse énergie les effets de la physique au-delà de cette théorie, appelée nouvelle physique (NP). Pour cela, elle fait appel à des opérateurs décrivant des états initiaux et finaux de processus et ignore les détails de leurs états intermédiaires, qui peuvent être approximés comme étant ponctuels à basse énergie sous l'hypothèse que cette physique inconnue est lourde par rapport à l'échelle énergétique considérée. Cette hypothèse est généralement admise car sinon cette NP aurait déjà été détectée. Une EFT est définie par un ensemble de particules, souvent celles de la théorie acceptée, et une symétrie. En effet, même si la nature de la NP est par définition inconnue, une EFT définie à une certaine échelle énergétique doit respecter les symétries présentes à cette échelle. Tous les opérateurs qui concernent ces particules et qui vérifient la symétrie correspondante sont considérés, avec chacun un coefficient inconnu. Puisque chaque terme d'un lagrangien doit avoir une dimension massive de 4, les coefficients des opérateurs de dimensions massives supérieures sont supprimés par des puissances d'un paramètre Λ, appelé *échelle de supression*, qui a des dimensions de masse et qui est beaucoup plus grand que toute masse considérée dans l'EFT. Ainsi, le lagrangien d'une EFT prend la forme

$$\mathcal{L} = \sum_{i=4}^{\infty} \sum_{O \in \mathcal{O}_i} \frac{C_O}{\Lambda^{i-4}} O,$$

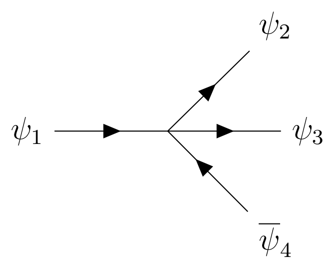
où $\mathcal{O}_i$ est l'ensemble des opérateurs de l'EFT ayant une dimension massive i et les coefficients C_O sont des coefficients sans dimension physique appelés *coefficients de Wilson* (WC, de l'anglais *Wilson Coefficient*). Ainsi, l'approche des EFT est perturbative et les opérateurs de dimensions massives supérieures contribuent aux ordres supérieurs du développement. Note : dans chaque référence bibliographique de ce mémoire, les lagrangiens des EFT prennent la forme

$$\mathcal{L} = \sum_{i=4}^{\infty} \sum_{O \in \mathcal{O}_i} C_O^{\text{ref}} O,$$

avec les $C_O^{\text{ref}} = \dfrac{C_O}{\Lambda^{i-4}}$ comme WC, qui ont des dimensions massives $4 - i$. Ceci n'est qu'une différence de notation et cela ne change rien à la physique en tant que telle.

2.2. Exemple : théorie de Fermi

Avant que les interactions faibles ne soient bien comprises, les désintégrations fermioniques à basse énergie étaient décrites par la *théorie de Fermi*, qui est une EFT. Soit, en guise d'exemple, une désintégration $\psi_1 \to \psi_2 \psi_3 \overline{\psi}_4$, où la masse de ψ_1 est plus grande que celles des trois autres fermions combinés, afin que la désintégration soit permise. Le processus est décrit par



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Le lagrangien efficace correspondant est

$$\mathcal{L}_{\text{eff}} = -G_F \begin{bmatrix} C^{V,LL}(\overline{\psi}_{2L}\gamma_\mu\psi_{1L})(\overline{\psi}_{3L}\gamma^\mu\psi_{4L}) + C^{V,LR}(\overline{\psi}_{2L}\gamma_\mu\psi_{1L})(\overline{\psi}_{3R}\gamma^\mu\psi_{4R}) \\ + C^{V,RL}(\overline{\psi}_{2R}\gamma_\mu\psi_{1R})(\overline{\psi}_{3L}\gamma^\mu\psi_{4L}) + C^{V,RR}(\overline{\psi}_{2R}\gamma_\mu\psi_{1R})(\overline{\psi}_{3R}\gamma^\mu\psi_{4R}) \\ + C^{S,LL}(\overline{\psi}_{2R}\psi_{1L})(\overline{\psi}_{3R}\psi_{4L}) + C^{S,LR}(\overline{\psi}_{2R}\psi_{1L})(\overline{\psi}_{3L}\psi_{4R}) \\ + C^{S,RL}(\overline{\psi}_{2L}\psi_{1R})(\overline{\psi}_{3R}\psi_{4L}) + C^{S,RR}(\overline{\psi}_{2L}\psi_{1R})(\overline{\psi}_{3L}\psi_{4R}) \\ + C^{T,L}(\overline{\psi}_{2R}\sigma_{\mu\nu}\psi_{1L})(\overline{\psi}_{3R}\sigma^{\mu\nu}\psi_{4L}) + C^{T,R}(\overline{\psi}_{2L}\sigma_{\mu\nu}\psi_{1R})(\overline{\psi}_{3L}\sigma^{\mu\nu}\psi_{4R}) \end{bmatrix}, \quad (2.2.1)$$

où $G_F = 1{,}166 \times 10^{-5}\ \text{GeV}^{-2}$ est la constante de Fermi et les WC (sans dimension physique) C sont a priori inconnus, par définition d'une EFT. Les indices V, S et T sur les WC réfèrent aux types des structures de Dirac (vectorielle, scalaire et tensorielle) des termes qu'ils multiplient. Les valeurs des WC sont déterminées phénoménologiquement, en comparant leurs prédictions avec des données expérimentales.

2.2.1. Détermination phénoménologique des WC

Il est à noter que les quatre premiers termes du lagrangien (2.2.1) conservent la chiralité, tandis que les six autres termes ne la conservent pas. Or, dans la limite $m_3 \to 0$ et $m_4 \to 0$, le fermion ψ_3 et l'antifermion $\overline{\psi}_4$ ne peuvent avoir respectivement qu'une chiralité gauche et droite. Ainsi, la désintégration dans cette limite n'est permise que si les interactions faibles ne conservent pas la chiralité. Une façon de tester expérimentalement si la chiralité est conservée ou non par les interactions faibles est de comparer les taux des désintégrations $\pi^+ \to \nu_e \overline{e}$ et $\pi^+ \to \nu_\mu \overline{\mu}$. Les seules variables physiques qui distinguent ces deux processus sont les masses des leptons produits. Les résultats expérimentaux donnent

$$\frac{\Gamma(\pi^+ \to \nu_e \overline{e})}{\Gamma(\pi^+ \to \nu_\mu \overline{\mu})} \simeq 1{,}2 \times 10^{-4} = O\left(\frac{{m_e}^2}{{m_\mu}^2}\right).$$

Ainsi, les taux de désintégration semblent diminuer à mesure que le lepton chargé produit est léger, puisque la masse de ce dernier est la seule variable qui change. Les interactions faibles conservent donc la chiralité. Ainsi,

$$C^{S,LL} = C^{S,LR} = C^{S,RL} = C^{S,RR} = C^{T,L} = C^{T,R} = 0$$

et le lagrangien efficace (2.2.1) se réduit donc à

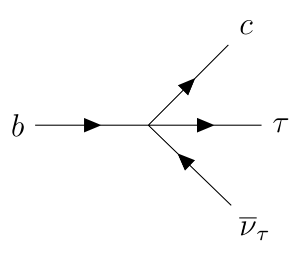
$$\mathcal{L}_{\text{eff}} = -G_F \left[\begin{aligned} & C^{V,LL}(\overline{\psi}_{2L}\gamma_\mu\psi_{1L})(\overline{\psi}_{3L}\gamma^\mu\psi_{4L}) + C^{V,LR}(\overline{\psi}_{2L}\gamma_\mu\psi_{1L})(\overline{\psi}_{3R}\gamma^\mu\psi_{4R}) \\ & + C^{V,RL}(\overline{\psi}_{2R}\gamma_\mu\psi_{1R})(\overline{\psi}_{3L}\gamma^\mu\psi_{4L}) + C^{V,RR}(\overline{\psi}_{2R}\gamma_\mu\psi_{1R})(\overline{\psi}_{3R}\gamma^\mu\psi_{4R}) \end{aligned} \right].$$

Maintenant, des données expérimentales sur les distributions angulaires des désintégrations $\mu \to \nu_\mu e \overline{\nu}_e$ permettent de déterminer que $C^{V,LL} = 1/\sqrt{2}$ et $C^{V,LR} = C^{V,RL} = C^{V,RR} = 0$. Ainsi, le boson chargé qui médie les interactions faibles ne couple qu'aux fermions de chiralité gauche. Le lagrangien de la théorie de Fermi se réduit donc à

$$\mathcal{L}_{\text{eff}} = -\frac{G_F}{\sqrt{2}}(\overline{\psi}_{2L}\gamma_\mu\psi_{1L})(\overline{\psi}_{3L}\gamma^\mu\psi_{4L}).$$

2.2.2. Physique se cachant derrière la théorie de Fermi

Aujourd'hui, il est connu que la théorie de Fermi est une approximation de basse énergie des interactions faibles. Par exemple, soit la désintégration $b \to c\tau\overline{\nu}_\tau$. La théorie de Fermi décrit ce processus comme



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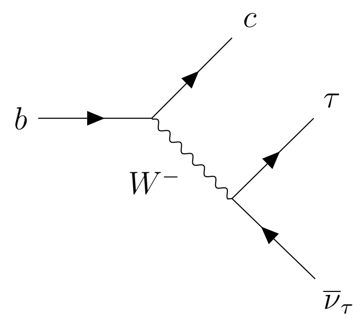
Le vrai processus est



.

Puisque le boson W est massif, ses effets à basse énergie peuvent être considérés comme ponctuels. Autrement dit, le propagateur peut à toute fin pratique être remplacé par un vertex. Le premier diagramme est donc une approximation du deuxième.

2.3. Théorie efficace de champ du modèle standard

Bien que le MS fasse des prédictions empiriques qui ont été vérifiées avec une précision remarquable, des anomalies présentes dans des données expérimentales, allant jusqu'à 4σ, suggèrent l'existence de physique au-delà du MS. Il est également sujet à des problèmes théoriques. Il est entre autres incapable d'expliquer la matière noire, la gravité et l'asymétrie baryonique dans l'univers. La *théorie efficace de champ du modèle standard* (SMEFT, de l'anglais *Standard Model Effective Field Theory*) fait l'hypothèse que la NP est significativement plus faible que le MS à basse énergie, sinon elle aurait déjà été détectée. Elle fait également l'hypothèse que la symétrie du MS est réalisée linéairement à haute énergie par la NP (cette dernière doit vérifier cette symétrie à haute énergie ; par contre rien ne garantit que cette réalisation est linéaire). En d'autres mots, la SMEFT a le MS comme ordre principal de son développement perturbatif, les mêmes particules que celles du MS et $SU(3)_C \times SU(2)_L \times U(1)_Y$ comme symétrie ; et le mécanisme de BSS est le même que dans le MS. Le lagrangien le plus général sous ces conditions est

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{MS}} + \sum_{i=5}^{\infty} \sum_{Q \in \mathcal{Q}_i} \frac{C_Q}{\Lambda^{i-4}} Q,$$

où $\mathcal{L}_{\text{MS}}$ est le lagrangien du MS, soit (1.1.1), Λ est l'échelle de supression et pour tout $i > 4$, $\mathcal{Q}_i$ est l'ensemble des opérateurs de dimension massive i formés avec les particules du MS et qui vérifient la symétrie $SU(3)_C \times SU(2)_L \times U(1)_Y$, c'est-à-dire qui décrivent des processus conservant à la fois la couleur, l'isospin faible et l'hypercharge. Des bases complètes des opérateurs SMEFT de dimension massive 6, 7 et 8 sont données dans [3], [4] et [5] (respectivement).

2.3.1. Effets sur les champs et paramètres du MS

En plus de nouvelles interactions efficaces aux ordres supérieurs, la SMEFT implique des redéfinitions des normalisations des champs et des corrections aux paramètres du MS. La raison à cela est que les opérateurs de dimensions supérieurs contenant des champs de Higgs contribuent, après BSS, des termes où le champ de Higgs a été remplacé par sa vev. Certains de ces termes interfèrent alors avec les termes du lagrangien (1.1.1). Par exemple, le terme $\frac{1}{\Lambda^2}C_{HD}(H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$ du lagrangien SMEFT ajoute notamment, après BSS, une perturbation au terme cinétique du boson de Higgs :

$$\frac{1}{\Lambda^2}C_{HD}(H^\dagger D_\mu H)^*(H^\dagger D^\mu H) \to \frac{{v_T}^2}{4\Lambda^2}C_{HD}(\partial_\mu h^0)(\partial^\mu h^0),$$

où v_T est la vev du boson de Higgs, corrigée aux ordres supérieurs. Ainsi, pour que le terme cinétique du boson de Higgs soit adéquatement normalisé, c'est-à-dire qu'il ait 1/2 comme coefficient, il faut redéfinir la normalisation du champ de Higgs.

Dans le second article présenté dans ce mémoire, ces corrections et redéfinitions de champs sont calculées jusqu'à la dimension massive 8, et les contributions des ordres supérieurs sont ignorées. Note : cette analyse est une extension d'une partie du travail fait dans [6], où ces corrections et redéfinitions sont calculées jusqu'à la dimension massive 6.

2.4. Théorie efficace de champ de basse énergie

La *théorie efficace de champ de basse énergie* (LEFT, de l'anglais *Low-Energy Effective Field Theory*) cherche à décrire les effets de la NP à basse énergie. Pour ce faire, la contrainte posée par la symétrie $SU(3)_C \times SU(2)_L \times U(1)_Y$ est relâchée et la symétrie moins restrictive $SU(3)_C \times U(1)_Q$ est imposée à la place ; la motivation derrière ceci étant que la physique connue à basse énergie obéit à cette dernière. Ainsi, la LEFT a les particules légères du MS, c'est-à-dire toutes à l'exception du quark t et des bosons W, Z et de Higgs, sa symétrie est $SU(3)_C \times U(1)_Q$ et les termes de l'ordre principal, correspondant aux opérateurs de dimension massive 4, sont ceux générés par le MS après BSS. En d'autres mots, le lagrangien est

$$\mathcal{L}_{\text{LEFT}} = \Lambda\left[C_{\underset{pr}{\nu}}(\nu_{Lp}^T C\nu_{Lr}) + \text{h.c.}\right] + \mathcal{L}_{\text{LEFT}}^{(\dim 4)} + \sum_{i=5}^{\infty}\sum_{\mathcal{O}\in O_i}\frac{C_{\mathcal{O}}}{\Lambda^{i-4}}\,\mathcal{O},$$

où Λ est l'échelle de supression, O_i est l'ensemble des opérateurs de dimension massive i (pour tout $i > 4$) qui décrivent des processus n'impliquant que des particules légères du MS et qui conservent à la fois la couleur et la charge électrique (c'est-à-dire qui vérifient la symétrie $SU(3)_C \times U(1)_Q$). $\nu_{Lp}^T C\nu_{Lr}$ est le seul opérateur LEFT de dimension massive 3 et représente le terme de masse du neutrino. La partie incluant les opérateurs de dimension massive 4 est

$$\mathcal{L}_{\text{LEFT}}^{(\dim 4)} = -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_{\psi\in\{u_L,u_R,d_L,d_R,\nu_L,e_L,e_R\}} \overline{\psi} i \not{D} \psi$$
$$- \Big[(M_e)_{pr}\overline{e}_{Lp}e_{Rr} + (M_u)_{pr}\overline{u}_{Lp}u_{Rr} + (M_d)_{pr}\overline{d}_{Lp}d_{Rr} + \text{h.c.}\Big],$$

où la dérivée covariante de la théorie est $D_\mu = \partial_\mu + ig_s T^A G^A_\mu + ieQA_\mu$ et les M_ψ sont les matrices de masses des différents fermions dans le modèle standard. Une base complète des opérateurs LEFT de dimension massive 6 est donnée dans [6].

2.5. Application aux désintégrations $b \to c\tau\nu$

Des anomalies par rapport au MS dans les données expérimentales d'observables liées à la désintégration $b \to c\tau\overline{\nu}_\tau$ suggère de la NP à l'œuvre dans ce processus. Il est donc possible d'utiliser ces données pour tester l'hypothèse, sous-jacente à la SMEFT, selon laquelle la symétrie du MS est réalisée linéairement à haute énergie. Les termes du lagrangien LEFT décrivant ce processus sont

$$\begin{aligned}
\mathcal{L}_{\text{LEFT}}^{b\to c\tau\overline{\nu}_\tau} &= \frac{1}{\Lambda^2}\begin{bmatrix} C^{V,LL*}_{\substack{\nu edu\\3332}}(\overline{c}_L\gamma_\mu b_L)(\overline{\tau}_L\gamma^\mu\nu_{L\tau}) + C^{V,LR*}_{\substack{\nu edu\\3332}}(\overline{c}_R\gamma_\mu b_R)(\overline{\tau}_L\gamma^\mu\nu_{L\tau}) \\ + C^{S,RR*}_{\substack{\nu edu\\3332}}(\overline{c}_R b_L)(\overline{\tau}_R\nu_{L\tau}) + C^{S,RL*}_{\substack{\nu edu\\3332}}(\overline{c}_L b_R)(\overline{\tau}_R\nu_{L\tau}) \\ + C^{T,RR*}_{\substack{\nu edu\\3332}}(\overline{c}_R\sigma_{\mu\nu}b_L)(\overline{\tau}_R\sigma^{\mu\nu}\nu_{L\tau}) \end{bmatrix} \\
&= \frac{1}{\Lambda^2}\begin{bmatrix} C^{V,LL*}_{\substack{\nu edu\\3332}}\mathcal{O}^{V,LL\dagger}_{\substack{\nu edu\\3332}} + C^{V,LR*}_{\substack{\nu edu\\3332}}\mathcal{O}^{V,LR\dagger}_{\substack{\nu edu\\3332}} \\ + C^{S,RR*}_{\substack{\nu edu\\3332}}\mathcal{O}^{S,RR\dagger}_{\substack{\nu edu\\3332}} + C^{S,RL*}_{\substack{\nu edu\\3332}}\mathcal{O}^{S,RL\dagger}_{\substack{\nu edu\\3332}} + C^{T,RR*}_{\substack{\nu edu\\3332}}\mathcal{O}^{T,RR\dagger}_{\substack{\nu edu\\3332}} \end{bmatrix}.
\end{aligned}$$

En se référant aux résultats du deuxième article présenté dans ce mémoire, les relations de correspondances SMEFT des WC de ces cinq termes sont les suivantes :

$$\begin{aligned}
\frac{1}{\Lambda^2}C^{V,LL*}_{\substack{\nu edu\\3332}} &= \frac{2}{\Lambda^2}\left[C^{(3)}_{\substack{lq\\3332}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{l^2q^2H^2\\3332}} - iC^{(5)}_{\substack{l^2q^2H^2\\3332}}\right)\right] - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{33}[W_q]^{\text{eff}*}_{23}, \\
\frac{1}{\Lambda^2}C^{V,LR*}_{\substack{\nu edu\\3332}} &= \frac{{v_T}^2}{4\Lambda^4}C_{\substack{l^2udH^2\\2333}}{}^* - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{33}[W_R]^{\text{eff}*}_{23}, \\
\frac{1}{\Lambda^2}C^{S,RR*}_{\substack{\nu edu\\3332}} &= \frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{lequ\\3332}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{lequH^2\\3332}} - C^{(2)}_{\substack{lequH^2\\3332}}\right)\right], \\
\frac{1}{\Lambda^2}C^{S,RL*}_{\substack{\nu edu\\3332}} &= \frac{1}{\Lambda^2}\left[C_{\substack{ledq\\3332}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{ledqH^2\\3332}} - C^{(2)}_{\substack{ledqH^2\\3332}}\right)\right], \\
\frac{1}{\Lambda^2}C^{T,RR*}_{\substack{\nu edu\\3332}} &= \frac{1}{\Lambda^2}\left[C^{(3)}_{\substack{lequ\\3332}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{lequH^2\\3332}} - C^{(4)}_{\substack{lequH^2\\3332}}\right)\right].
\end{aligned}$$

Ce que sont exactement des relations de correspondance et comment elles sont déterminées est expliqué dans la prochaine section. L'important à savoir pour la présente discussion est que chaque terme du lagrangien LEFT est généré après BSS par un ou des termes du lagrangien SMEFT. Dans les cinq équations ci-dessus, chaque membre gauche et un terme du lagrangien LEFT et chaque membre droit est la relation correspondante entre les termes impliqués du lagrangien SMEFT.

La seule contribution au terme du lagrangien LEFT $\frac{1}{\Lambda^2} C^{V,LR*}_{\substack{\nu edu \\ 3332}} \mathcal{O}^{V,LR\dagger}_{\substack{\nu edu \\ 3332}}$ à la dimension massive 6 de la SMEFT vient du terme qui implique un échange de boson W, soit $-\frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{33}[W_R]^{\text{eff}*}_{23}$. Plus précisément, cette contribution vaut

$$-\frac{\overline{g}^2 {v_T}^2}{4{M_W}^2\Lambda^2} C^{(3)\ *}_{\substack{Hud \\ 23}}.$$

Il faut cependant réaliser que ce coefficient contribue à la fois aux termes qui correspondent aux opérateurs LEFT $\mathcal{O}^{V,LR}_{\substack{\nu edu \\ 1132}}$, $\mathcal{O}^{V,LR}_{\substack{\nu edu \\ 2232}}$ et $\mathcal{O}^{V,LR}_{\substack{\nu edu \\ 3332}}$, avec les mêmes coefficients. On dit alors de cette contribution qu'elle est *universelle pour les saveurs leptoniques*. Seuls les termes du lagrangien SMEFT qui ne vérifient pas cette propriété peuvent expliquer des processus qui impliquent deux quarks et deux leptons et dont la probabilité varie selon la génération des leptons impliqués. Par exemple, la désintégration $b \to c\tau\overline{\nu}_\tau$ n'a pas la même rareté que les désintégrations $b \to ce\overline{\nu}_e$ et $b \to c\mu\overline{\nu}_\mu$; elle ne peut donc pas être expliquée par un terme universel pour les saveurs leptoniques. Or, parmi les cinq termes du lagrangien LEFT qui interviennent dans cette désintégration, chacun reçoit une contribution non universelle pour les saveurs leptoniques à la dimension massive 6, à l'exception de $\frac{1}{\Lambda^2} C^{V,LR*}_{\substack{\nu edu \\ 3332}} \mathcal{O}^{V,LR\dagger}_{\substack{\nu edu \\ 3332}}$, qui ne reçoit de telles contributions qu'à la dimension massive 8. Donc, si la SMEFT est la bonne théorie efficace à haute énergie, il est attendu que le coefficient de ce terme soit significativement plus petit que ceux des quatre autres. Le premier article présenté dans ce mémoire examine si les données expérimentales existantes sur les observables liées à cette désintégration sont compatibles avec cette prédiction.

2.6. Correspondances entre les termes des lagrangiens SMEFT et LEFT

Chaque terme du lagrangien SMEFT contribue, après BSS, à un ou plusieurs termes du lagrangien LEFT. Inversement, chaque terme du lagrangien LEFT est généré, lors de la BSS, par un ou plusieurs termes du lagrangien SMEFT. Les relations ainsi comprises entre les WC sont appelées *relations de correspondance* et leurs contributions viennent en deux types. Le premier consiste en des termes du lagrangien SMEFT dans lesquels chaque doublet de Higgs présent acquière une vev. Le deuxième, moins direct, implique des contributions de processus avec des particules virtuelles lourdes dont les propagateurs sont approximés par des vertex. Par exemple, le terme du lagrangien LEFT impliquant l'opérateur $\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}} : (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt})$ reçoit des contributions directes des termes du lagrangien SMEFT impliquant les opérateurs $Q_{\underset{prst}{ll}} : (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$, $Q^{(1)}_{\underset{prst}{l^4H^2}} : (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger H)$, $Q^{(2)}_{\underset{prst}{l^4H^2}} : (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu\tau^I l_t)(H^\dagger\tau^I H)$ et $Q^{(2)}_{\underset{stpr}{l^4H^2}} : (\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger\tau^I H)$. Plus précisément, lorsque les doublets de Higgs acquièrent chacun une vev, les termes de lagrangien correspondants génèrent les relations suivantes :

$$\frac{1}{\Lambda^2}C_{\underset{prst}{ll}}Q_{\underset{prst}{ll}} \to \frac{1}{\Lambda^2}C_{\underset{prst}{ll}}\begin{bmatrix}(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ +(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})\end{bmatrix},$$

$$\frac{1}{\Lambda^4}C^{(1)}_{\underset{prst}{l^4H^2}}Q^{(1)}_{\underset{prst}{l^4H^2}} \to \frac{{v_T}^2}{2\Lambda^4}C^{(1)}_{\underset{prst}{l^4H^2}}\begin{bmatrix}(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ +(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})\end{bmatrix},$$

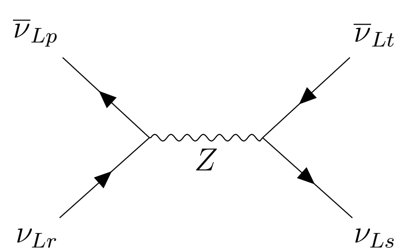
$$\frac{1}{\Lambda^4}C^{(2)}_{\underset{prst}{l^4H^2}}Q^{(2)}_{\underset{prst}{l^4H^2}} \to \frac{{v_T}^2}{2\Lambda^4}C^{(2)}_{\underset{prst}{l^4H^2}}\begin{bmatrix}-(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ -(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})\end{bmatrix},$$

$$\frac{1}{\Lambda^4}C^{(2)}_{\underset{stpr}{l^4H^2}}Q^{(2)}_{\underset{stpr}{l^4H^2}} \to \frac{{v_T}^2}{2\Lambda^4}C^{(2)}_{\underset{stpr}{l^4H^2}}\begin{bmatrix}-(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) - (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ +(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})\end{bmatrix}.$$

Ainsi, le terme $\frac{1}{\Lambda^2}\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}}$ du lagrangien LEFT reçoit la contribution SMEFT

$$\frac{1}{\Lambda^2}\left[C_{\underset{prst}{ll}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right].$$

Il reçoit également des contributions indirectes, décrites par les processus



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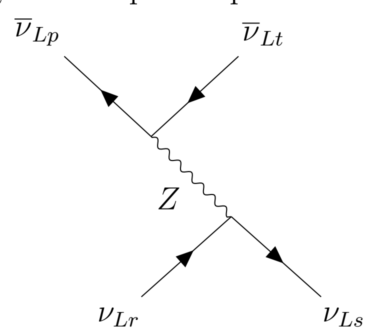

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La somme des amplitudes correspondantes, en approximant à basse énergie le propagateur d'un boson de jauge de masse M par $\frac{ig_{\mu\nu}}{M^2}$, est, avec un signe de différence entre les deux termes car les deux diagrammes sont liés par un échange de deux fermions et en tenant compte des corrections de la SMEFT aux couplages jusqu'à la dimension massive 8, qui sont déterminées dans le second article présenté dans ce mémoire,

$$-\frac{i\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{\nu_L}]^{\text{eff}}_{st}(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + \frac{i\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pt}[Z_{\nu_L}]^{\text{eff}}_{sr}(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lt})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lr}).$$

Cette amplitude à basse énergie peut être obtenue à partir du terme de lagrangien efficace

$$-\frac{\overline{g_Z}^2}{2{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{ab}[Z_{\nu_L}]^{\text{eff}}_{cd}(\overline{\nu}_{La}\gamma_\mu\nu_{Lb})(\overline{\nu}_{Lc}\gamma^\mu\nu_{Ld}) = -\frac{\overline{g_Z}^2}{2{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{ab}[Z_{\nu_L}]^{\text{eff}}_{cd}\,\mathcal{O}^{V,LL}_{\underset{abcd}{\nu\nu}}.$$

Maintenant, la transformation de Fierz de l'opérateur $\mathcal{O}^{V,LL}_{\underset{abcd}{\nu\nu}}$ est $\mathcal{O}^{V,LL}_{\underset{adcb}{\nu\nu}}$. Il s'agit du même opérateur, à un échange de saveurs près. Ce terme de lagrangien se réécrit donc

$$-\frac{\overline{g_Z}^2}{4{M_Z}^2}\left([Z_{\nu_L}]^{\text{eff}}_{ab}[Z_{\nu_L}]^{\text{eff}}_{cd} + [Z_{\nu_L}]^{\text{eff}}_{ad}[Z_{\nu_L}]^{\text{eff}}_{cb}\right)\mathcal{O}^{V,LL}_{\underset{abcd}{\nu\nu}}.$$

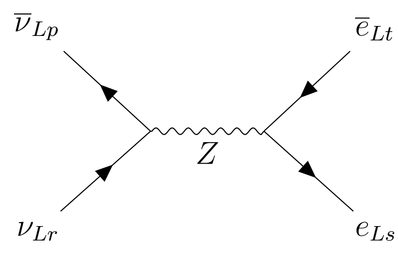
En tenant compte de toutes ces contributions, la relation de correspondance est

$$\frac{1}{\Lambda^2}C^{V,LL}_{\underset{prst}{\nu\nu}} = \left\{\begin{array}{l} \dfrac{1}{\Lambda^2}\left[C_{\underset{prst}{ll}} + \dfrac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right] \\ -\dfrac{\overline{g_Z}^2}{4{M_Z}^2}\left([Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{\nu_L}]^{\text{eff}}_{st} + [Z_{\nu_L}]^{\text{eff}}_{pt}[Z_{\nu_L}]^{\text{eff}}_{sr}\right) \end{array}\right\}.$$

Un autre exemple important est l'opérateur $\mathcal{O}^{V,LL}_{\underset{prst}{\nu e}}$: $(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})$. Il reçoit des contributions directes des opérateurs SMEFT $Q_{\underset{prst}{ll}}$: $(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$, $Q^{(1)}_{\underset{prst}{l^4H^2}}$: $(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger H)$ et $Q^{(2)}_{\underset{prst}{l^4H^2}}$: $(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu\tau^I l_t)(H^\dagger\tau^I H)$, obtenues à partir du lagrangien efficace

$$\frac{1}{\Lambda^2}\left[\left(C_{\underset{prst}{ll}} + C_{\underset{stpr}{ll}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} + C^{(1)}_{\underset{stpr}{l^4H^2}} + C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right].$$

Il reçoit également des contributions des processus



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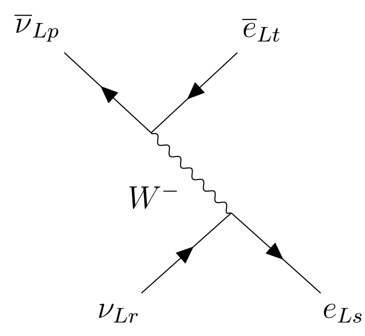

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La somme des amplitudes correspondantes est

$$-\frac{ig_{\mu\nu}\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{e_L}]^{\text{eff}}_{st}(\overline{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\nu e_{Lt}) + \frac{ig_{\mu\nu}\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pt}[W_l]^{\text{eff}*}_{sr}(\overline{\nu}_{Lp}\gamma^\mu e_{Lt})(\overline{e}_{Ls}\gamma^\nu\nu_{Lr}),$$

ou, en effectuant une transformation de Fierz sur les spineurs de Dirac dans le deuxième terme,

$$-\frac{i\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{e_L}]^{\text{eff}}_{st}(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) - \frac{i\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pt}[W_l]^{\text{eff}*}_{sr}(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}).$$

Cette amplitude peut être dérivée du lagrangien efficace

$$\left[-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{e_L}]^{\text{eff}}_{st} - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pt}[W_l]^{\text{eff}*}_{sr}\right](\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}).$$

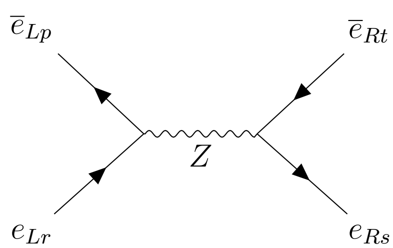
En tenant compte de toutes ces contributions, la relation de correspondance est

$$\frac{1}{\Lambda^2}C^{V,LL}_{\substack{\nu e\\ prst}} = \left\{\begin{array}{l} \frac{1}{\Lambda^2}\left[\left(C_{\substack{ll\\ prst}} + C_{\substack{ll\\ stpr}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^4H^2\\ prst}} + C^{(1)}_{\substack{l^4H^2\\ stpr}} + C^{(2)}_{\substack{l^4H^2\\ prst}} - C^{(2)}_{\substack{l^4H^2\\ stpr}}\right)\right] \\ -\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{e_L}]^{\text{eff}}_{st} - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pr}[W_l]^{\text{eff}*}_{st} \end{array}\right\}.$$

Un autre exemple important est l'opérateur $\mathcal{O}^{V,LR}_{\substack{ee\\ prst}}$: $(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Rs}\gamma^\mu e_{Rt})$. Il reçoit des contributions directes des opérateurs SMEFT $Q_{\substack{le\\ prst}}$: $(\bar{l}_p\gamma_\mu l_r)(\overline{e}_s\gamma^\mu e_t)$, $Q^{(1)}_{\substack{l^2e^2H^2\\ prst}}$: $(\bar{l}_p\gamma_\mu l_r)(\overline{e}_s\gamma^\mu e_t)(H^\dagger H)$ et $Q^{(2)}_{\substack{l^2e^2H^2\\ prst}}$: $(\bar{l}_p\gamma_\mu\tau^I l_r)(\overline{e}_s\gamma^\mu e_t)(H^\dagger\tau^I H)$, obtenues à partir du lagrangien efficace

$$\frac{1}{\Lambda^2}\left[C_{\substack{le\\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2e^2H^2\\ prst}} + C^{(2)}_{\substack{l^2e^2H^2\\ prst}}\right)\right].$$

Il reçoit également des contributions des processus



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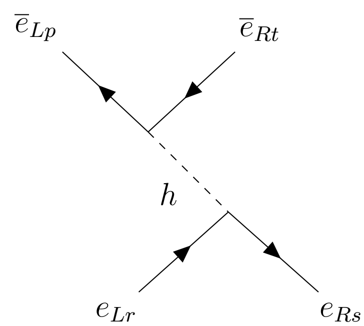

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La somme des amplitudes correspondantes, en approximant à basse énergie respectivement les propagateurs d'un boson de jauge de masse M et d'un boson de Higgs par $\frac{ig_{\mu\nu}}{M^2}$ et $\frac{-i}{{m_h}^2}$, est

$$-\frac{ig_{\mu\nu}\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st}(\overline{e}_{Lp}\gamma^\mu e_{Lr})(\overline{e}_{Rs}\gamma^\nu e_{Rt}) + \frac{i}{{m_h}^2}(Y_e)^{\text{eff}}_{pt}(Y_e)^{\text{eff}*}_{rs}(\overline{e}_{Lp}e_{Rt})(\overline{e}_{Rs}e_{Lr}),$$

ou, en effectuant une transformation de Fierz sur les spineurs de Dirac dans le deuxième terme,

$$-\frac{i\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st}(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Rs}\gamma^\mu e_{Rt}) - \frac{i}{2{m_h}^2}(Y_e)^{\text{eff}}_{pt}(Y_e)^{\text{eff}*}_{rs}(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Rs}\gamma^\mu e_{Rt}).$$

Cette amplitude peut être dérivée du lagrangien efficace

$$\left[-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st} - \frac{1}{2{m_h}^2}(Y_e)^{\text{eff}}_{pt}(Y_e)^{\text{eff}*}_{rs}\right](\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Rs}\gamma^\mu e_{Rt}).$$

En tenant compte de toutes ces contributions, la relation de correspondance est

$$\frac{1}{\Lambda^2}C^{V,LR}_{\substack{ee\\prst}} = \left\{\begin{array}{l} \frac{1}{\Lambda^2}\left[C_{\substack{le\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2e^2H^2\\prst}} + C^{(2)}_{\substack{l^2e^2H^2\\prst}}\right)\right] \\ -\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st} - \frac{1}{2{m_h}^2}(Y_e)^{\text{eff}}_{pt}(Y_e)^{\text{eff}*}_{rs} \end{array}\right\}.$$

Dans le deuxième article présenté dans ce mémoire, les relations de correspondance entre les WC des termes jusqu'à aux dimension massive 6 et 8 des lagrangiens LEFT et SMEFT (respectivement) sont données. Note : ceci est une extension d'une partie du travail de [6], où ces relations de correspondances sont calculées jusqu'à la dimension massive 6 du lagrangien SMEFT.

Chapitre 3

Beyond SMEFT with $b \to c\,\tau^- \bar{\nu}$

Par C.P. Burgess, Serge Hamoudou, Jacky Kumar et David London. Publié dans *Physical Review D* [**17**].

Dans cet article, j'ai établi les correspondances entre les opérateurs LEFT « non SMEFT » et les opérateurs SMEFT qui les génèrent au niveau des arbres et à l'ordre de grandeur principal (résultats présentés dans le tableau I de l'article). J'ai également, indépendamment de Jacky Kumar, effectué les ajustements (*fit* en anglais) de coefficients dont les résultats sont présentés dans le tableau II et nous avons obtenu les mêmes résultats.

Abstract

Electroweak interactions assign a central role to the gauge group $SU(2)_L \times U(1)_Y$, which is either realized linearly (SMEFT) or nonlinearly (*e.g.*, HEFT) in the effective theory obtained when new physics above the electroweak scale is integrated out. Although the discovery of the Higgs boson has made SMEFT the default assumption, nonlinear realization remains possible. The two can be distinguished through their predictions for the size of certain low-energy dimension-6 four-fermion operators: for these, HEFT predicts $O(1)$ couplings, while in SMEFT they are suppressed by a factor $v^2/\Lambda_{\rm NP}^2$, where v is the Higgs vev. One such operator, $O_V^{LR} \equiv (\bar{\tau}\gamma^\mu P_L \nu)\,(\bar{c}\gamma_\mu P_R b)$, contributes to $b \to c\,\tau^- \bar{\nu}$. We show that present constraints permit its non-SMEFT coefficient to have a HEFTy size. We also note that the angular distribution in $\bar{B} \to D^*(\to D\pi')\,\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$ contains enough information to extract the coefficients of all new-physics operators. Future measurements of this angular distribution can therefore tell us if non-SMEFT new physics is really necessary.

Introduction — The Standard Model (SM) of particle physics provides a spectacular description of the physics so far found at the Large Hadron Collider (LHC). But it also cannot be complete because it leaves several things unexplained (like neutrino masses, dark matter and dark energy, etc.), and it makes some of cosmology's initial conditions (such as primordial fluctuations and baryon asymmetry) seem unlikely. To have hitherto escaped detection, any new particles must either couple extremely weakly or be very massive (or possibly both).

This – together with the eventual need for something to unitarize gravity at high energies – underpins the widespread belief that the SM is the leading part of an effective field theory (EFT) describing the low-energy limit of something more fundamental. EFTs are largely characterized by their particle content and symmetries (see, *e.g.*, Refs. [**1**, **2**]). Since the discovery of the Higgs boson, the known particle content at energies above the top-quark mass, m_t, suffices to linearly realize the electroweak gauge group $SU(2)_L \times U(1)_Y$. Whether the known particles actually *do* linearly realize this symmetry is what distinguishes SMEFT, which lineary realizes it (see, *e.g.*, Refs. [**3**, **4**]) from alternatives like HEFT, which do not, despite also including a 'Higgs' scalar (see, *e.g.*, Refs. [**5**–**13**]).

The question of whether the symmetry is realized linearly or nonlinearly can only be answered experimentally. One proposal for doing this [**14**] seeks new particles whose presence requires nonlinear realization. In the present paper, we show how to use indirect b-physics signals to extract evidence for nonlinearly-realized new physics.

How symmetries are realized in an EFT comes up when power-counting how effective interactions are suppressed at low energies. For instance, an effective interaction like $g_z \, Z_\mu(\bar{u}\gamma^\mu P_R u) \in \mathcal{L}_{\rm eff}$, which describes a non-standard $Z\bar{u}_R u_R$ coupling, naively arises at mass-dimension 4 when $SU(2)_L \times U(1)_Y$ is nonlinearly realized [**15**, **16**], but instead arises at dimension-6 through an operator $\Lambda_h^{-2}(H^\dagger D_\mu H)(\bar{u}\gamma^\mu P_R u)$ when it is linearly realized, implying a coupling $g_z \sim v^2/\Lambda_h^2$ that is suppressed by the ratio of Higgs vev v to a UV scale Λ_h.

The assumption underlying SMEFT is that the scale Λ_h appearing here is the same order of magnitude as the scale Λ that suppresses all other dimension-6 operators If $\Lambda_h \sim \Lambda$ then the lower bound on Λ required to have generic dimension-6 SMEFT operators not be detected also implies an upper bound on the effective dimension-4 non-SMEFT coupling g_z. While this assumption is not unreasonable, it *is* an assumption, since nothing in the power-counting of EFTs requires the scale Λ_h that accompanies powers of a field like H to be the same as the scale Λ that appears with derivatives [**1**, **2**]. (For example, these scales are very different in supergravity theories, and this is why it is consistent to have complicated target-space metrics appearing in the kinetic energies of fields while working only to two-derivative order. Similar observations have also been made for SMEFT [**17**].)

Because it is an assumption, it should be tested. It is ultimately an experimental question which kind of symmetry realization provides a better description of Nature. Our purpose in this paper is to identify how to do so using a class of B-physics measurements. Despite being at relatively low energies, B-meson properties suggest themselves for this purpose because they can be precisely studied and because there are at present several observables that seem to disagree with the predictions of the SM.

SMEFT vs LEFT at low energies — A complicating issue arises when using B physics to distinguish SMEFT from non-SMEFT effective interactions because the EFT relevant at such low-energies necessarily already integrates out many of the heavier SM particles ($W^{\pm}$, Z^0, H, t). But once these particles are removed the remaining EFT necessarily nonlinearly realizes $SU(2)_L \times U(1)_Y$, while linearly realizing its $U(1)_{em}$ subgroup. This is why heavy top-quark loops can generate otherwise SM-forbidden effective interactions such as $\delta\mathcal{L}_{\rm eff} \ni \delta M_W^2 W_\mu^* W^\mu + \delta M_Z^2 Z_\mu Z^\mu$ that violate the SM condition $M_W = M_Z \cos\theta_W$, or more broadly contribute to oblique corrections or modification of gauge couplings [15].

The exercise of separating these more mundane sources of symmetry breaking from those coming from higher energies has been studied in the literature. For instance, the theory obtained below the W mass has been called LEFT (low-energy effective field theory) or WET (weak effective field theory), and in Ref. [18], Jenkins, Manohar and Stoffer (JMS) present a complete and non-redundant basis of operators in this theory up to dimension 6. For the particularly interesting class of dimension-6 four-fermion operators that conserve baryon and lepton number, they also identify how these effective interactions can be obtained (at tree level) from the similarly complete and non-redundant list of operators given for SMEFT in Ref. [19]. (For a fuller discussion of the relationships amongst these various EFTs see Ref. [20] and references therein.)

JMS find that most dimension-6 LEFT operators can be generated in this way starting from dimension-6 operators in SMEFT. However, a handful of dimension-6 LEFT operators are not invariant under $SU(2)_L \times U(1)_Y$, and so are not contained amongst dimension-6 SMEFT operators. Tree graphs can also generate these 'non-SMEFT' operators, but in this case only do so starting from SMEFT operators with mass dimension greater than 6. It is these non-SMEFT operators that interest us in our applications to B physics.

The existence of non-SMEFT operators affects the search for new physics at low energies, such as when analyzing discrepancies from the SM using four-fermion effective operators in LEFT. One current example is in observables involving the decay $b \to c\,\tau^-\bar\nu$. Assuming only left-handed neutrinos, five four-fermion $b \to c\,\tau^-\bar\nu$ operators are possible:

$$
\begin{aligned}
O_V^{LL,LR} &\equiv (\bar\tau\gamma^\mu P_L\nu)\,(\bar c\gamma_\mu P_{L,R}b)\,, \\
O_S^{LL,LR} &\equiv (\bar\tau P_L\nu)\,(\bar c P_{L,R}b)\ , \\
O_T &\equiv (\bar\tau\sigma^{\mu\nu}P_L\nu)\,(\bar c\sigma_{\mu\nu}P_Lb)\ ,
\end{aligned}
\tag{1}
$$

where $P_{L,R}$ are the left-handed and right-handed projection operators. As we will see below, O_V^{LR} is a non-SMEFT operator: it is generated at tree level starting from a dimension-8 SMEFT operator. Because of this, the coefficient of O_V^{LR} would naively be suppressed by the small factor v^2/Λ^4 if SMEFT were true at UV scales. It is usually excluded when seeking new physics in $b \to c\,\tau^-\bar{\nu}$ (see, *e.g.*, Refs. [21, 22]).

To test how the gauge symmetries are realized, one must measure the coefficients of such non-SMEFT operators, and see if their size is consistent with SMEFT power counting. If the SMEFT-predicted suppression in the coefficients is not present it would point to a more complicated realization of $SU_(2)_L \times U(1)_Y$ in the UV than is usually assumed.

The first step in performing such an analysis is to identify all the non-SMEFT dimension-6 operators in LEFT. We list these in Table I, along with the higher-dimension SMEFT operators from which they can be obtained at tree level. Operators appearing in the 'LEFT operator' column are denoted by $\mathcal{O}$ and are as defined in Ref. [18]. Operators appearing in the 'Tree-level SMEFT origin' column are denoted by Q. The one with dimension 6 (the operator Q_{Hud}) is as defined in Ref. [19]. The dimension-8 SMEFT operators have been tabulated in Refs. [23, 24]; our nomenclature for these operators is taken from Ref. [24]. JMS also identified these non-SMEFT operators, simply saying they had no direct dimension-6 SMEFT counterpart, and our list agrees with their findings.

Of course, there is nothing sacred about tree level, and in principle loops can also generate effective operators as one evolves down to lower energies (as the example of non-SM gauge-boson masses generated by top-quark loops mentioned above shows). Whether such loops are important in any particular instance depends on the size of any loop-suppressing couplings and the masses that come with them. As the top-quark example also shows, generating non-SMEFT operators from loops involving SMEFT operators necessarily involves a dependence on $SU(2)_L \times U(1)_Y$-breaking masses, implying a suppression (and a lowering of operator dimension) when these masses are small. The authors of Ref. [25] have computed how SM loops dress individual SMEFT operators, and show that such loops do not generate non-SMEFT operators in LEFT at the one-loop level.

Ref. [26] computes the running of the LEFT operators that are unsuppressed by such factors, arising due to dressing by photon and gluon loops, and shows that non-SMEFT dimension-6 operators of this type also can arise from the mixing of dimension-5 dipole operators of the form $(\bar{\psi}\sigma^{\mu\nu}\psi)X_{\mu\nu}$ in LEFT, where $X_{\mu\nu} = G_{\mu\nu}, F_{\mu\nu}$ are gauge field strengths. The two required insertions of these dipole operators ensure that they do not change the tree-level counting of powers of $1/\Lambda$, in their coefficients.

Applications to B physics — Although the Table shows quite a few non-SMEFT operators that can, in principle, be used to search for non-SMEFT new physics, one of these

LEFT operator	Tree-level SMEFT origin	Dims.
Semileptonic operators		
$\mathcal{O}_{\nu edu}^{V,LR}$: $(\overline{\nu}_{Lp}\gamma^\mu e_{Lr})(\overline{d}_{Rs}\gamma_\mu u_{Rt}) + \text{h.c.}$	Q_{Hud} : $i(\tilde{H}^\dagger D_\mu H)(\overline{u}_p\gamma^\mu d_r) + \text{h.c.}$	$6\to 6$
	$Q_{\ell^2udH^2}$: $(\overline{\ell}_p d_r H)(\tilde{H}^\dagger \overline{u}_s \ell_t) + \text{h.c.}$	$6\to 8$
$\mathcal{O}_{ed}^{S,RR}$: $(\overline{e}_{Lp}e_{Rr})(\overline{d}_{Ls}d_{Rt})$	$Q_{\ell eqdH^2}^{(3)}$: $(\overline{\ell}_p e_r H)(\overline{q}_s d_t H)$	$6\to 8$
$\mathcal{O}_{eu}^{S,RL}$: $(\overline{e}_{Rp}e_{Lr})(\overline{u}_{Ls}u_{Rt})$	$Q_{\ell equH^2}^{(5)}$: $(\overline{\ell}_p e_r H)(\tilde{H}^\dagger \overline{q}_s u_t)$	$6\to 8$
$\mathcal{O}_{ed}^{T,RR}$: $(\overline{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\overline{d}_{Ls}\sigma_{\mu\nu}d_{Rt})$	$Q_{\ell eqdH^2}^{(4)}$: $(\overline{\ell}_p \sigma_{\mu\nu} e_r H)(\overline{q}_s \sigma^{\mu\nu} d_t H)$	$6\to 8$
Four-lepton operators		
$\mathcal{O}_{ee}^{S,RR}$: $(\overline{e}_{Lp}e_{Rr})(\overline{e}_{Ls}e_{Rt})$	$Q_{\ell^2e^2H^2}^{(3)}$: $(\overline{\ell}_p e_r H)(\overline{\ell}_s e_t H)$	$6\to 8$
Four-quark operators		
$\mathcal{O}_{uddu}^{V1,LR}$: $(\overline{u}_{Lp}\gamma^\mu d_{Lr})(\overline{d}_{Rs}\gamma_\mu u_{Rt}) + \text{h.c.}$	Q_{Hud} : $i(\tilde{H}^\dagger D_\mu H)(\overline{u}_p\gamma^\mu d_r) + \text{h.c.}$	$6\to 6$
	$Q_{q^2udH^2}^{(5)}$: $(\overline{q}_p d_r H)(\tilde{H}^\dagger \overline{u}_s q_t) + \text{h.c.}$	$6\to 8$
$\mathcal{O}_{uddu}^{V8,LR}$: $(\overline{u}_{Lp}\gamma^\mu T^A d_{Lr})(\overline{d}_{Rs}\gamma_\mu T^A u_{Rt}) + \text{h.c.}$	$Q_{q^2udH^2}^{(6)}$: $(\overline{q}_p T^A d_r H)(\tilde{H}^\dagger \overline{u}_s T^A q_t) + \text{h.c.}$	
$\mathcal{O}_{uu}^{S1,RR}$: $(\overline{u}_{Lp}u_{Rr})(\overline{u}_{Ls}u_{Rt})$	$Q_{q^2u^2H^2}^{(5)}$: $(\overline{q}_p u_r \tilde{H})(\overline{q}_s u_t \tilde{H})$	$6\to 8$
$\mathcal{O}_{uu}^{S8,RR}$: $(\overline{u}_{Lp}T^A u_{Rr})(\overline{u}_{Ls}T^A u_{Rt})$	$Q_{q^2u^2H^2}^{(6)}$: $(\overline{q}_p T^A u_r \tilde{H})(\overline{q}_s T^A u_t \tilde{H})$	
$\mathcal{O}_{dd}^{S1,RR}$: $(\overline{d}_{Lp}d_{Rr})(\overline{d}_{Ls}d_{Rt})$	$Q_{q^2d^2H^2}^{(5)}$: $(\overline{q}_p d_r H)(\overline{q}_s d_t H)$	$6\to 8$
$\mathcal{O}_{dd}^{S8,RR}$: $(\overline{d}_{Lp}T^A d_{Rr})(\overline{d}_{Ls}T^A d_{Rt})$	$Q_{q^2d^2H^2}^{(6)}$: $(\overline{q}_p T^A d_r H)(\overline{q}_s T^A d_t H)$	

Table I: Non-SMEFT four-fermion operators in LEFT and the dimension-8 SMEFT operators to which they are mapped at tree level. In the 'LEFT operator' column, the subscripts p, r, s, t are weak-eigenstate indices; they are suppressed in the operator labels. The superscripts '1' and '8' of four-quark operators denote the colour transformation of the quark pairs. In the 'Tree-level SMEFT origin' column, ℓ and q denote left-handed $SU(2)_L$ doublets, while e, u and d denote right-handed $SU(2)_L$ singlets. Here, $\tilde{H} = i\sigma_2 H^*$ denotes the conjugate of the Higgs doublet H.

is particularly interesting: the operator O_V^{LR} of Eq. (1),

$$\mathcal{O}_{\nu\tau bc}^{V,LR} \equiv (\bar{\tau}_L\gamma^\mu\nu_L)(\bar{c}_R\gamma_\mu b_R) + \text{h.c.}\ , \tag{2}$$

that contributes to the decay $b \to c\,\tau^-\bar{\nu}$ [27].

Notice that Table I offers two possible SMEFT operators from which this operator can be obtained at tree level, one of which is the dimension-6 SMEFT operator Q_{Hud}. Naively this seems to imply that $\mathcal{O}_{\nu\tau bc}^{V,LR}$ is actually a SMEFT operator after all. But there is a subtlety here: Q_{Hud} is a lepton-flavour-universal operator that generates equal effective couplings for the operators $\mathcal{O}_{\nu ebc}^{V,LR}$, $\mathcal{O}_{\nu\mu bc}^{V,LR}$ and $\mathcal{O}_{\nu\tau bc}^{V,LR}$ [28]. An effective operator that generates *only* $\mathcal{O}_{\nu\tau bc}^{V,LR}$ without the other two violates lepton-flavour universality, and this can only come from the dimension-8 operator given in the Table. (A similar reasoning applies also to $\mathcal{O}_{uddu}^{V1,LR}$ and $\mathcal{O}_{uddu}^{V8,LR}$, where superscripts '1' and '8' give the colour transformation of the quark pairs.) Furthermore, at the 1-loop level, $\mathcal{O}_{\nu\tau bc}^{V,LR}$ does not mix with any other LEFT operators [26].

The five four-fermion operators given in Eq. (1) imply that the most general LEFT effective Hamiltonian describing $b \to c\,\tau^-\bar{\nu}$ decay with left-handed neutrinos is

$$\begin{aligned} \mathcal{H}_{eff} &= \frac{4G_F}{\sqrt{2}} V_{cb}\, O_V^{LL} - \frac{C_V^{LL}}{\Lambda^2}\, O_V^{LL} - \frac{C_V^{LR}}{\Lambda^2}\, O_V^{LR}\,, \\ &- \frac{C_S^{LL}}{\Lambda^2}\, O_S^{LL} - \frac{C_S^{LR}}{\Lambda^2}\, O_S^{LR} - \frac{C_T}{\Lambda^2}\, O_T\ . \end{aligned} \tag{3}$$

The first term is the SM contribution; the remaining five terms are the various new-physics contributions. Within LEFT, these are all dimension-6 operators and so, in the absence of other information, for a given new-physics scale Λ, their dimensionless coefficients (the Cs) are all at most $O(1)$. By contrast, the coefficient C_V^{LR} is instead proportional to v^2/Λ_h^2 if the new physics is described at higher energies by SMEFT (since $\mathcal{O}_V^{LR}$ then really descends from a Higgs-dependent interaction with dimension 8), and so is predicted to be small if $\Lambda_h \sim \Lambda$.

The beauty of $b \to c\,\tau^-\bar{\nu}$ decays is that, in principle, they provide sufficiently many observables to measure each of the couplings in Eq. (3) separately, thereby allowing a test of the prediction that C_V^{LR} should be negligible (assuming that the presence of new physics is confirmed). If the effective couplings do not follow the SMEFT pattern, non-SMEFT new physics must be involved.

What is currently known about C_V^{LR}? At present several observables have been measured that involve the decay $b \to c\,\tau^-\bar{\nu}$. These include

$$\begin{aligned} \mathcal{R}(D^{(*)}) &\equiv \frac{\mathcal{B}(B \to D^{(*)}\tau\nu)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}\,, \mathcal{R}(J/\psi) \equiv \frac{\mathcal{B}(B_c \to J/\psi\tau\nu)}{\mathcal{B}(B_c \to J/\psi\mu\nu)}\,, \\ F_L(D^*) &\equiv \frac{\Gamma(B \to D_L^*\tau\nu)}{\Gamma(B \to D^*\tau\nu)}\,,\ P_\tau(D^*) \equiv \frac{\Gamma^{+1/2} - \Gamma^{-1/2}}{\Gamma^{+1/2} + \Gamma^{-1/2}}\ , \end{aligned} \tag{4}$$

where $\Gamma^\lambda \equiv \Gamma(B \to D^*\tau^\lambda\nu)$. $P_\tau(D^*)$ measures the τ polarization asymmetry while $F_L(D^*)$ measures the longitudinal D^* polarization. These observables are useful for distinguishing new-physics models with different Lorentz structures and (interestingly) the measurements of most of these observables seem to be in tension with the predictions of the SM. Refs. [21,22]) perform fits to the data using the interactions of Eq. (3) (though with a different operator normalization than is used here), but with O_V^{LR} assumed not to be present (precisely because it is a non-SMEFT operator).

We make two observations about how to use these measurements to probe the size of $\mathcal{O}_V^{LR}$, one using existing data and one using new observables – proposed elsewhere [29] – to exploit future data to access more information about the effective coefficients appearing in Eq. (3).

Fits to current data — We have repeated the fit of Refs. [21, 22]), though this time including $\mathcal{O}_V^{LR}$ for comparison. The values of the experimental observables used in the fit are those found in Ref. [22]. One observable that is not used is $\mathcal{B}(B_c \to \tau\nu)$. This decay has not yet been measured, but it has been argued that its branching ratio has an upper limit in order to be compatible with the B_c lifetime. Unfortunately this upper bound varies enormously in different analyses, from 10% [30] to 60% [21]. Because of this uncertainty, we do not use this upper bound as a constraint, but simply compute the prediction for $\mathcal{B}(B_c \to \tau\nu)$ in each new physics scenario. For the theoretical predictions of the observables in the presence of new physics, we use the program `flavio` [31] and the fit itself is done using `MINUIT` [32–34].

Because the data is not yet rich enough to permit an informative simultaneous fit to all five effective couplings[1] we instead perform fits in which only one or two of the effective couplings are nonzero. We choose $\Lambda = 5$ TeV and consider the following three scenarios for nonzero new-physics coefficients: either C_V^{LL} or C_V^{LR} are turned on by themselves, or both C_V^{LL} and C_V^{LR} are turned on together. The results of fits using these three options are presented in Table II, and Fig. I presents the (correlated) allowed values of C_V^{LL} and C_V^{LR} for the joint fit. We see that the scenario that adds only C_V^{LL} provides an excellent fit to

New-physics coeff.	Best fit	p value (%)	pull$_{\rm SM}$
C_V^{LL}	-3.1 ± 0.7	51	4.1
C_V^{LR}	2.8 ± 1.2	0.3	2.3
(C_V^{LL},C_V^{LR})	(-3.0 ± 0.8,0.6 ± 1.2)	35	3.7

Table II: Fit results for the scenarios in which C_V^{LL}, C_V^{LR} or both C_V^{LL} and C_V^{LR} are allowed to be nonzero. At the best-fit point the prediction for $\mathcal{B}(B_c \to \tau\nu)$ is ∼2.8% for all scenarios.

the data. On the other hand, the fit is poor when C_V^{LR} alone is added (though it is still much better than for the SM itself). The fit remains acceptable when both C_V^{LL} and C_V^{LR} are allowed to be nonzero. In all scenarios, $\mathcal{B}(B_c \to \tau\nu)$ is predicted to be $< 3\%$, which easily satisfies all constraints.

It is clear that the current data is insufficient to constrain the value for C_V^{LR} in a useful way. Both the SMEFT prediction $C_V^{LR} \sim v^2/\Lambda^2 = O(10^{-3})$ and $C_V^{LR} \sim \mathcal{O}(1)$ are consistent with the joint fit with both C_V^{LL} and C_V^{LR} nonzero; the best-fit value $C_V^{LR} = 0.6 \pm 1.2$ is consistent with both zero and large $O(1)$ values[2]. At present, the data are consistent with the non-SMEFT coefficient C_V^{LR} being much larger than the SMEFT prediction.

It is worth noting that the same is *not* true for other non-SMEFT operators. From the operators listed in Table I, consider for example the specific operators $(\bar{\mu}_L \mu_R)(\bar{s}_L b_R)$ and $(\bar{\mu}_R \mu_L)(\bar{s}_R b_L)$ in the class $\mathcal{O}_{ed}^{S,RR}$, or the $\mathcal{O}_{ed}^{T,RR}$ operators of the type $(\bar{\mu}_L \sigma_{\mu\nu} \mu_R)(\bar{s}_L \sigma^{\mu\nu} b_R)$ and $(\bar{\mu}_R \sigma_{\mu\nu} \mu_L)(\bar{s}_R \sigma^{\mu\nu} b_L)$. These all contribute in a chirally unsupressed way to the decay

[1]Fits involving the other new-physics coefficients were performed in Ref. [22]. We have redone these fits in order to verify that we reproduce the results of this paper.

[2]We note that the central values satisfy $C_V^{LR}/C_V^{LL} \simeq -0.2$. In Ref. [35], it was assumed that the $b \to c\tau^-\bar{\nu}$ anomaly could be explained by the addition of a W' with general couplings. When they performed a fit with LL and LR couplings, they also found a ratio of $LR/LL \simeq -0.2$.

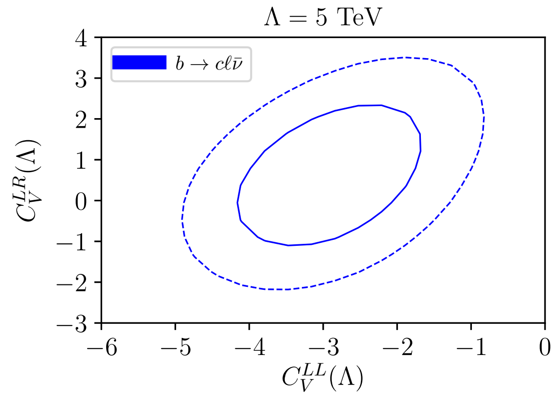

Fig. I: (Correlated) allowed values of C_V^{LL} and C_V^{LR} at 1σ (inner region) and 2σ (outer region).

$b \to s\mu^+\mu^-$ (unlike the case in the SM), and so the addition of any of these operators can dramatically change the prediction for $\mathcal{B}(B_s^0 \to \mu^+\mu^-)$. But the measured value $\mathcal{B}(B_s^0 \to \mu^+\mu^-) = (2.9 \pm 0.4) \times 10^{-9}$ [36] is close to the SM prediction, so that the coefficients of these operators cannot be larger than order $O(10^{-4})$, consistent with SMEFT expectations. Things are similar for the analogous operators contributing to $b \to se^+e^-$, for which the upper limit of $\mathcal{B}(B_s^0 \to e^+e^-) < 9.4 \times 10^{-9}$ [36] constrains the coefficients of these operators to be $< O(10^{-3})$, again consistent with SMEFT.

Future prospects — The above discussion shows that the non-SMEFT operator O_V^{LR} can have a large effective coupling, $C_V^{LR} \sim O(1)$ without causing observational difficulties with $b \to c\tau^-\bar{\nu}$ decays, though there are large errors. But even if the experimental errors on the currently measured observables were to improve dramatically, the five observables of Eq. (4) are never enough to measure all of these parameters in the most general case. This is simply because these five measurements cannot pin down all ten of the parameters that can appear in the five complex couplings given in Eq. (3).

Fortunately, there are potentially many more observables whose measurement can remedy this situation. Ref. [29] has proposed to measure the angular distribution in $\bar{B} \to D^*(\to D\pi')\,\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$. This decay includes three final-state particles whose four-momenta can be measured: D, π' and π^-. Using this information, the differential decay rate can be constructed. This depends on two non-angular variables, q^2 and E_π, as well as a number of angular variables. Here, q^2 is the invariant mass-squared of the $\tau^-\bar{\nu}_\tau$ pair and E_π is the energy of the π^- in the τ decay. The idea is then to separate the data into q^2-E_π bins, and then to perform an angular analysis in each of these bins. Each angular distribution consists of twelve different angular functions; nine of these terms are CP-conserving, and three are CP-violating. There are therefore a large number of observables in this differential decay rate; the exact number depends on how many q^2-E_π bins there are.

Eq. (1) lists five new-physics operators, but only four of these actually contribute to $\bar{B} \to D^* \tau^- \bar{\nu}_\tau$. To see why, consider the following linear combinations of the two scalar operators:

$$\begin{aligned} O_{LS} &\equiv O_S^{LR} + O_S^{LL} = (\bar{\tau} P_L \nu)\,(\bar{c}\, b)\,, \\ O_{LP} &\equiv O_S^{LR} - O_S^{LL} = (\bar{\tau} P_L \nu)\,(\bar{c}\gamma_5 b)\ . \end{aligned} \tag{5}$$

Of these, only O_{LP} contributes to $\bar{B} \to D^* \tau^- \bar{\nu}_\tau$.

With complex coefficients, there are therefore eight unknown theoretical parameters in the remaining four effective interactions. Observables are functions of these parameters, as well as q^2 and E_π. Thus, if the angular distribution in $\bar{B} \to D^*(\to D\pi')\,\tau^-(\to \pi^-\nu_\tau)\bar{\nu}_\tau$ can be measured, it may be possible to extract all of the new physics coefficients from a fit to observations. If the real or imaginary part of C_V^{LR} were found to be much larger than the SMEFT expectation, it would suggest the presence of non-SMEFT physics at higher energies.

Note that the decay $b \to c\,\mu^- \bar{\nu}$ can also be analyzed in a similar way (even though there is no hint of new physics in this reaction (but see Ref. [37] for an alternative point of view)). The angular distribution for $b \to c\,\mu^- \bar{\nu}$ described in Ref. [38] provides enough observables to perform a fit for the coefficients of all dimension-6 new-physics operators, including the non-SMEFT one.

In summary, we reproduce here the list of non-SMEFT four-fermion operators and identify their provenance, assuming that they arise at tree level starting from even-higher-dimension SMEFT operators, in order to pin down the SMEFT estimate for the size of their effective couplings. We show that fits to current observations allow one of these couplings – that of the semileptonic $b \to c\,\tau^- \bar{\nu}$ operator $\mathcal{O}_V^{LR}$ – to be $\mathcal{O}(1)/\Lambda^2$ for $\Lambda \sim 5$ TeV, which is consistent with couplings that are several orders of magnitude larger than would be predicted by SMEFT. We also identify a sufficiently large class of $b \to c\,\tau^- \bar{\nu}$ observables whose measurement would in principle allow all of the relevant effective couplings to be determined, including that of $\mathcal{O}_V^{LR}$. There is a good prospect that these measurements can be done in the future.

Finally, suppose it were eventually established that non-SMEFT new physics is present in $b \to c\,\tau^- \bar{\nu}$. The obvious question then is: What could this non-SMEFT new physics be? Although serious exploration of models probably awaits evidence for such a signal, some preliminary attempts have been made in the literature. One example is Ref. [39], which studies the non-SMEFT operators in $b \to s\mu^+\mu^-$ and $b \to c\,\tau^- \bar{\nu}$ in the context of HEFT, and argues that such operators can be generated by a nonstandard Higgs sector containing additonal strongly-interacting scalars. We regard a more systematic exploration of non-SMEFT physics in the UV to be well worthwhile, and look forward to that happy day when experimental results are what drives it.

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Chapitre 4

Dimension-8 SMEFT Matching Conditions for the Low-Energy Effective Field Theory

Par Serge Hamoudou, Jacky Kumar et David London. Soumis à *Journal Of High Energy Physics*.

Dans cet article, j'ai effectué l'entièreté des calculs et des comparaisons de résultats avec les références citées.

Abstract

In particle physics, the modern view is to categorize things in terms of effective field theories (EFTs). Above the weak scale, we have the SMEFT, formed when the heavy new physics (NP) is integrated out, and for which the Standard Model (SM) is the leading part. Below M_W, we have the LEFT (low-energy EFT), formed when the heavy SM particles ($W^{\pm}$, Z^0, H, t) are also integrated out. In order to determine how low-energy measurements depend on the underlying NP, it is necessary to compute the matching conditions of LEFT operators to SMEFT operators. These matching conditions have been worked out for all LEFT operators up to dimension 6 in terms of SMEFT operators up to dimension 6. However, this is not sufficient for all low-energy observables. In this paper we present the complete matching conditions of all such LEFT operators to SMEFT operators up to dimension 8.

1 Introduction

Despite its enormous success in accounting for almost all experimental data to date, the Standard Model (SM) of particle physics still has no explanation for a number of other key observations, such as neutrino masses, the baryon asymmetry of the universe, dark matter, etc. For this reason, it is widely believed that there must exist physics beyond the SM. And since the LHC has not discovered any new particles up to a scale of $O(\text{TeV})$, this new physics (NP) is likely to be very massive.

When the NP is integrated out, one obtains an effective field theory (EFT), of which it is now generally believed that the SM is simply the leading part. This EFT must obey the SM gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. Since the discovery of the Higgs boson, the default assumption is that this symmetry is realized linearly, i.e., the symmetry is broken via the Higgs mechanism, resulting in the Standard Model EFT, or SMEFT (see, *e.g.*, Refs. [1, 2]). The SMEFT has been studied extensively: a complete and non-redundant list of dimension-6 operators is given in Ref. [3], the dimension-7 operators can be found in Ref. [4], and the dimension-8 operators are tabulated in Refs. [6, 7].

The LEFT (low-energy effective field theory) describes the physics below the W mass, and is produced when the heavy SM particles (W, Z, t, H) are also integrated out. (This is also called the WET (weak effective field theory).) In Ref. [9], Jenkins, Manohar and Stoffer (JMS) present a complete and non-redundant basis of LEFT operators up to dimension 6, including those that violate B and L. The matching to dimension-6 SMEFT operators at tree level is also given. The one-loop contributions of dimension-6 SMEFT operators can be taken into account through the renormalization-group running of the coefficients of the LEFT. This is computed in Refs. [11–14]. With this information, if a discrepancy with the SM is observed in a process that uses a particular LEFT operator, we will know which dimension-6 SMEFT operators are involved.

However, this is not always sufficient. Information about the contributions from higher-dimension operators may be important if the process in question is suppressed in the SM and/or is very precisely measured. Examples of observables for which dimension-8 SMEFT effects must be taken into account include electroweak precision data from LEP [15], lepton-flavour-violating processes [16, 17], meson-antimeson mixing ($\Delta F = 2$) [18, 19], and electric dipole moments [20]. (Dimension-8 SMEFT operators have also been discussed in the context of high-energy processes, see Refs. [21–28].)

The analysis of Refs. [16, 17] involves (i) the identification of all LEFT operators relevant for lepton flavour violation, and (ii) the computation of the tree-level matching of these operators to SMEFT operators up to dimension 8. In the present paper, we apply step (ii) to all LEFT operators. The idea is simply that, when it is necessary to consider the contributions of dimension-8 SMEFT operators in the analysis of an experimental result, this information can be found here. (Note that a complete analysis of the relationship between LEFT operators and dimension-8 SMEFT operators must also take into account the renormalization-group running of SMEFT operators from the NP scale down to low energies. For bosonic SMEFT operators up to dimension 8, this has been calculated in Refs. [29, 30].)

In our analysis, we follow closely the approach of Ref. [9], and extend it to include dimension-8 SMEFT operators. Below, we often refer to this paper simply by the initials of its authors, as JMS.

We begin in Sec. 2 with some preliminary remarks comparing our analysis with that of JMS, and discuss in general terms how matching conditions are computed. In Sec. 3, we present the setup, showing how the presence of dimension-8 SMEFT operators affects the symmetry breaking, the generation of masses, and the couplings of the gauge and Higgs bosons to fermions. The computations required to derive the complete matching conditions are described in Sec. 4; the results are presented in Appendix D. We conclude in Sec. 5. Appendices A, B, C give a variety of information relevant to the details of the analysis.

2 Preliminaries

In Ref. [9], JMS compute the tree-level SMEFT matching conditions for the LEFT operators. The matching conditions for operators that conserve both B and L involve only even-dimension SMEFT operators, and are given up to dimension 6. For operators that violate B and/or L, the matching conditions can involve even- or odd-dimension SMEFT operators (but not both), depending on the operator, and are computed to dimension 6 or dimension 5. In the present paper, we extend this analysis: we compute these matching conditions up to dimension 8 (dimension 7) if even-dimension (odd-dimension) SMEFT operators are involved. (In this paper, when we refer to "computing the matching conditions up to dimension 8," both of these possibilities are understood.) If one eliminates the dimension-8 or dimension-7 contributions, the results of JMS are reproduced. This makes it easy to compare the results. Also, we present the elements of our analysis in much the same order as JMS.

In the LEFT Lagrangian, we consider only operators up to dimension 6 (like JMS):

$$\mathcal{L}_{\rm LEFT} = \mathcal{L}_{\rm LEFT}^{\rm Neutrino\ mass} + \mathcal{L}_{\rm QCD+QED} + \sum_{n=5}^{6} \sum_{\mathcal{O}\in \dim n} \frac{C_{\mathcal{O}}}{\Lambda^{n-4}}\,\mathcal{O}\ . \tag{1}$$

For the SMEFT, all operators up to dimension 8 are included:

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_{n=5}^{8} \sum_{Q\in \dim n} \frac{C_{Q}}{\Lambda^{n-4}}\,Q\ . \tag{2}$$

Still, there are two differences in our notation:

- Our convention is to have dimensionless Wilson coefficients (WCs). For instance, for the dimension-6 SMEFT lagrangian, we write

$$\mathcal{L}^{(6)}_{\rm SMEFT} = \sum_{\mathcal{O}\in {\rm dim}\,6} \frac{C_{\mathcal{O}}}{\Lambda^2}\,\mathcal{O} ~. \tag{3}$$

  This convention is different from that of JMS, which uses dimensionful WCs.
- In the unbroken phase, the SM lagrangian is

$$\begin{aligned}
\mathcal{L}_{\rm SM} \;\; = \;\; & -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
& + \sum_{\psi=q,u,d,l,e} \overline{\psi} i \not{D} \psi + (D_\mu H)^\dagger (D^\mu H) - \lambda \left(H^\dagger H - \frac{1}{2}v^2 \right)^2 \\
& - \left[\bar{l}_p e_r (Y_e)_{pr} H + \overline{q}_p u_r (Y_u)_{pr} \tilde{H} + \overline{q}_p d_r (Y_d)_{pr} H + {\rm h.c.} \right] \\
& + \frac{\theta_3 g_s^2}{32\pi^2} G^A_{\mu\nu}\tilde{G}^{A\mu\nu} + \frac{\theta_2 g^2}{32\pi^2} W^I_{\mu\nu}\tilde{W}^{I\mu\nu} + \frac{\theta_1 g'^2}{32\pi^2} B_{\mu\nu}\tilde{B}^{\mu\nu} ~.
\end{aligned} \tag{4}$$

  This uses the same notation as JMS, with one exception: our Yukawa matrices (the Ys) are the hermitian conjugates of those of JMS.

In Eq. (4), the fields q_r and l_r are (left-handed) $SU(2)_L$ doublets, while u_r, d_r and e_r are (right-handed) $SU(2)_L$ singlets, where $r = 1,\, 2,\, 3$ is a generation (weak-eigenstate) index. The physical (mass-eigenstate) states are the same for the charged leptons, the left- and right-handed u-type quarks, and the right-handed d-type quarks. For the left-handed d-type quarks, the relation between the weak and mass eigenstates is

$$d_{Lr} = V_{rd} d_L + V_{rs} s_L + V_{rb} b_L \equiv V_{rx} d_{Lx} ~, \tag{5}$$

where the left-hand side is a weak eigenstate, and the right-hand side is a linear combination of mass eigenstates. The V_{rx} are elements of the unitary mixing matrix, which is the Cabibbo-Kobayashi-Maskawa (CKM) matrix in the SM. Note: as in JMS, our LEFT matching conditions are given in the weak eigenstate basis. They can be written in terms of the physical states by using the above relation.

In our analysis, we make reference to several different sets of operators. The LEFT operators are taken from JMS [9], the dimension-6 SMEFT operators are found in Ref. [3], and we use Ref. [7] for the dimension-8 SMEFT operators. In all cases, we use the same notation for the operators and their WCs as is used in the references. For the dimension-7 SMEFT operators, we use a basis that is equivalent that of Ref. [4], but with a different notation. For convenience, in the Appendices, we present tables of all the operators used in this paper. These include LEFT operators (Appendix A), along with dimension-5 to 8 SMEFT operators (Appendix B).

It is useful to give an example that illustrates the various issues involved in deriving matching conditions. Consider the charged-current four-fermion operator

$$\mathcal{O}_{\nu edu}^{V,LL} = (\bar{\nu}_{Lp}\gamma^{\mu}e_{Lr})(\bar{d}_{Ls}\gamma_{\mu}u_{Lt}) + \text{h.c.}, \text{ coefficient :}\frac{1}{\Lambda^2}C_{\substack{\nu edu \\ prst}}^{V,LL} \ . \tag{6}$$

We begin by examining the matching to the SM. That is, $\mathcal{O}_{\nu edu}^{V,LL}$ is taken to be an operator of the Fermi theory, whose coefficient has magnitude $4G_F/\sqrt{2}$. The SM Lagrangian consists only of operators of at most dimension 4. This four-fermion operator can be generated in the SM when a W is exchanged between the two fermion currents, and the W is integrated out. The SM matching condition is then

$$\frac{1}{\Lambda^2}C_{\substack{\nu edu \\ prst}}^{V,LL} = -\frac{g^2}{2M_W^2}\,[W_l]_{pr}[W_q]^*_{ts} \ . \tag{7}$$

Here, W_l and W_q are the couplings of the W to the lepton and quark pair, respectively. In the SM, $[W_l]_{pr} = \delta_{pr}$ and $[W_q]_{ts} = \delta_{ts}$. Knowing that the coefficient has magnitude $4G_F/\sqrt{2}$, this leads to the well-known relation

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} \ . \tag{8}$$

The matching to SMEFT at dimension 6 was computed by JMS. It is

$$\frac{1}{\Lambda^2}C_{\substack{\nu edu \\ prst}}^{V,LL} + \text{h.c.} = \frac{2}{\Lambda^2}C_{\substack{lq \\ prst}}^{(3)} - \frac{\bar{g}^2}{2M_W^2}\,[W_l]^{\text{eff}}_{pr}[W_q]^{\text{eff}*}_{ts} + \text{ c.c.} \tag{9}$$

Since the SMEFT includes dimension-6 terms, it contains the four-fermion operator. That is, there is a *direct* contribution to the matching conditions, $C_{\substack{lq \\ prst}}^{(3)}$. As was the case in the SM, $C_{\substack{\nu edu \\ prst}}^{V,LL}$ can also be generated by the exchange of a W between the two fermion currents. This is represented by the second term above. Although this resembles the term in Eq. (7), there are several differences:

(1) In the presence of dimension-6 SMEFT operators, the coupling constant is modified: $g \to \bar{g}$. This is due to the fact that, when one adds dimension-6 corrections to the kinetic terms of the gauge bosons, these fields and the coupling constants must be redefined in order to ensure that the kinetic term is properly normalized.

(2) In the SM, the W coupling to fermions is fixed by the fermion kinetic term, $\bar{\psi}\not{D}\psi$. In SMEFT, there are dimension-6 corrections, such as $H^\dagger H\bar{\psi}\not{D}\psi$. These will change the magnitudes of the couplings, and permit inter-generational couplings, hence the 'eff' superscript on W_l and W_q.

The bottom line is that many dimension-6 SMEFT operators are implicitly present in the second term of Eq. (9) above. Collectively, these operators form the *indirect* contributions. They must be carefully taken into account in the matching conditions. (Note that, if one expands the effective parameters appearing in the matching conditions, many terms will appear; those that are of higher order than dimension 8 are to be ignored.)

3 Setup

The Lagrangian for the SM in the unbroken phase is given in Eq. (4). When the Higgs field acquires a vev, given by the minimum of the Higgs potential, the symmetry is broken, and masses are generated for the $W^\pm$, the Z^0 and the fermions. One can easily compute the masses of the physical gauge bosons, as well as their couplings to the physical fermions, in terms of the parameters of $\mathcal{L}_{\rm SM}$, in particular g, g' and v.

When one includes higher-order SMEFT operators of dimension 6, 8, etc., this whole process must be recalculated in order to take into account these new operators. One must make field redefinitions so that the kinetic terms are properly normalized, the minimum of the Higgs potential (i.e., the Higgs vev) must be recomputed, corrections to $\sin\theta_W$ must be taken into account, etc. One sees the effects of these additional operators in the redefinitions of the coupling constants, the couplings of gauge bosons to fermions, and other quantities that appear in both the direct and indirect contributions to the matching conditions.

In this section, we present the main effects of including SMEFT operators up to dimension 8. We emphasize those results that are important for the matching conditions. These results are in agreement with the predictions of the geometric formulation of the SMEFT [8].

3.1 Higgs sector

After the Higgs acquires a vev, we redefine the Higgs field as follows:

$$H \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ [1+c_{H,\rm kin}]h + v_T \end{pmatrix} \ . \tag{10}$$

Here, v_T and $c_{H,\rm kin}$ are respectively determined by minimizing the Higgs potential and by normalizing the Higgs kinetic term.

3.1.1 Higgs vev. In the presence of SMEFT operators up to dimension 8, the Higgs potential is

$$V(H) = \lambda \left(H^2 - \frac{1}{2} v^2 \right)^2 - \frac{1}{\Lambda^2} C_H H^6 - \frac{1}{\Lambda^4} C_{H^8} H^8 \ , \tag{11}$$

where only the real part of the second component of the Higgs doublet, H, is taken to be nonzero. We define the physical Higgs vev, v_T, to be $v_T \equiv \sqrt{2}\, H^{\rm min}$, where $H^{\rm min}$ minimizes the Higgs potential. This implies that

$$v_T = v \left(1 + \frac{3v^2}{8\lambda\Lambda^2}\, C_H + \frac{v^4}{4\lambda\Lambda^4} \left[\frac{63}{32\lambda} \, [C_H]^2 + C_{H^8} \right] \right) \ . \tag{12}$$

v_T is the physical parameter that appears in the matching relations, and whose value can in principle be determined by a fit to the data.

3.1.2 Higgs kinetic term. Including SMEFT contributions up to dimension 8, the Higgs kinetic term is

$$\mathcal{L}_{\text{SMEFT}}^{\text{Higgs kinetic}} = \frac{1}{2}\left[1+\frac{2v_T^2}{\Lambda^2}\left(\frac{1}{4}C_{HD}-C_{H\Box}\right)+\frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)}+C_{H^6}^{(2)}\right)\right](1+c_{H,\text{kin}})^2(\partial_\mu h)(\partial^\mu h)\ . \tag{13}$$

In order for this term to be properly normalized, one must have

$$c_{H,\text{kin}} = \frac{v_T^2}{\Lambda^2}\left(C_{H\Box}-\frac{1}{4}C_{HD}\right)-\frac{v_T^4}{8\Lambda^4}\left(C_{H^6}^{(1)}+C_{H^6}^{(2)}\right)+\frac{3v_T^4}{2\Lambda^4}\left(C_{H\Box}-\frac{1}{4}C_{HD}\right)^2\ . \tag{14}$$

This is essentially a redefiniton of the normalization of the Higgs field.

3.1.3 Higgs mass. Taking into account the SMEFT contributions up to dimension 8, the Higgs boson mass term is

$$\mathcal{L}_{\text{SMEFT}}^{\text{Higgs mass}} = \frac{1}{2}\left[\lambda v^2-3\lambda v_T^2+\frac{15v_T^4}{4\Lambda^2}\,C_H+\frac{7v_T^6}{2\Lambda^4}\,C_{H^8}\right](1+c_{H,\text{kin}})^2h^2\ . \tag{15}$$

This gives the following expression for the Higgs boson mass:

$$m_h{}^2 = (1+c_{H,\text{kin}})^2 v_T^2\left[2\lambda-\frac{3v_T^2}{\Lambda^2}\,C_H-\frac{3v_T^4}{\Lambda^4}\,C_{H^8}\right]\ . \tag{16}$$

3.2 Fermion mass matrices & Yukawa couplings

Before symmetry breaking, the SMEFT Lagrangian up to dimension 8 contains the following terms for charged leptons and quarks:

$$-(Y_\psi)_{pr}\,\bar\chi_p\psi_r\,\overline{H}+\frac{1}{\Lambda^2}C_{\underset{pr}{\psi H}}\,\bar\chi_p\psi_r\,\overline{H}(H^\dagger H)+\frac{1}{\Lambda^4}C_{\underset{pr}{\chi\psi H^5}}\,\bar\chi_p\psi_r\,\overline{H}(H^\dagger H)^2+\text{h.c.}\ , \tag{17}$$

where $\psi\in\{e,u,d\}$ (right-handed $SU(2)_L$ singlets), $\chi\in\{l,q\}$ (left-handed $SU(2)_L$ doublets), $\overline{H}=H$ if $\psi\in\{e,d\}$ and $\overline{H}=\tilde{H}$ if $\psi\in\{u\}$. Here, the first term (dimension 4) belongs to the SM and the last two terms are SMEFT operators (respectively dimension 6 and 8). Lepton-number-violating terms are also present:

$$\frac{1}{\Lambda}C_{\underset{pr}{5}}\,\epsilon^{ij}\epsilon^{kl}(l_{ip}^TCl_{kr})\,H_jH_l+\frac{1}{\Lambda^3}C_{\underset{pr}{l^2H^4}}\,\epsilon^{ij}\epsilon^{kl}(l_{ip}^TCl_{kr})\,H_jH_l(H^\dagger H)+\text{h.c.}\ . \tag{18}$$

When the Higgs gets a vev, both mass matrices and Yukawa coupling terms are generated.

3.2.1 Fermion mass matrices. The SMEFT mass terms for charged leptons and quarks up to dimension 8 are

$$\begin{aligned}
\mathcal{L}_{\rm SM}^{\rm Fermion\ mass} &= -\frac{v_T}{\sqrt{2}}\left[(Y_e)_{pr}\overline{e}_{Lp}e_{Rr} + (Y_u)_{pr}\overline{u}_{Lp}u_{Rr} + (Y_d)_{pr}\overline{d}_{Lp}d_{Rr}\right] + \text{h.c.},\\
\mathcal{L}_{\rm SMEFT,6}^{\rm Fermion\ mass} &= \frac{v_T^3}{2\sqrt{2}\,\Lambda^2}\left[C_{\underset{pr}{eH}}\overline{e}_{Lp}e_{Rr} + C_{\underset{pr}{uH}}\overline{u}_{Lp}u_{Rr} + C_{\underset{pr}{dH}}\overline{d}_{Lp}d_{Rr}\right] + \text{h.c.},\\
\mathcal{L}_{\rm SMEFT,8}^{\rm Fermion\ mass} &= \frac{v_T^5}{4\sqrt{2}\,\Lambda^4}\left[C_{\underset{pr}{leH^5}}\overline{e}_{Lp}e_{Rr} + C_{\underset{pr}{quH^5}}\overline{u}_{Lp}u_{Rr} + C_{\underset{pr}{qdH^5}}\overline{d}_{Lp}d_{Rr}\right] + \text{h.c.}
\end{aligned} \tag{19}$$

This gives the following mass matrices:

$$[M_\psi]_{pr} = \frac{v_T}{\sqrt{2}}\left[(Y_\psi)_{pr} - \frac{v_T^2}{2\Lambda^2}\,C_{\underset{pr}{\psi H}} - \frac{v_T^4}{4\Lambda^4}\,C_{\underset{pr}{\chi\psi H^5}}\right]\ . \tag{20}$$

The SMEFT neutrino mass terms are

$$\mathcal{L}_{\rm SMEFT}^{\rm Neutrino\ mass} = \frac{{v_T}^2}{2\Lambda}\left[C_{\underset{pr}{5}} + \frac{{v_T}^2}{2\Lambda^2}C_{\underset{pr}{l^2H^4}}\right]\overline{\nu}_{Lp}^T C\nu_{Lr} + \text{h.c.}\ . \tag{21}$$

This gives the following mass matrices:

$$[M_\nu]_{pr} = -\frac{{v_T}^2}{\Lambda}\left[C_{\underset{pr}{5}} + \frac{{v_T}^2}{2\Lambda^2}C_{\underset{pr}{l^2H^4}}\right]\ . \tag{22}$$

3.2.2 Yukawa couplings. The SM, dimension-6 and dimension-8 SMEFT Yukawa coupling terms for charged leptons and quarks are

$$\begin{aligned}
\mathcal{L}_{\rm SM}^{\rm Yukawa} &= -\frac{(1+c_{H,\rm kin})}{\sqrt{2}}\left[(Y_e)_{pr}\overline{e}_{Lp}e_{Rr}h + (Y_u)_{pr}\overline{u}_{Lp}u_{Rr}h + (Y_d)_{pr}\overline{d}_{Lp}d_{Rr}h\right] + \text{h.c.},\\
\mathcal{L}_{\rm SMEFT,6}^{\rm Yukawa} &= \frac{3(1+c_{H,\rm kin})v_T^2}{2\sqrt{2}\,\Lambda^2}\left[C_{\underset{pr}{eH}}\overline{e}_{Lp}e_{Rr}h + C_{\underset{pr}{uH}}\overline{u}_{Lp}u_{Rr}h + C_{\underset{pr}{dH}}\overline{d}_{Lp}d_{Rr}h\right] + \text{h.c.},\\
\mathcal{L}_{\rm SMEFT,8}^{\rm Yukawa} &= \frac{5(1+c_{H,\rm kin})v_T^4}{4\sqrt{2}\,\Lambda^4}\left[C_{\underset{pr}{leH^5}}\overline{e}_{Lp}e_{Rr}h + C_{\underset{pr}{quH^5}}\overline{u}_{Lp}u_{Rr}h + C_{\underset{pr}{qdH^5}}\overline{d}_{Lp}d_{Rr}h\right] + \text{h.c.}
\end{aligned} \tag{23}$$

This gives the following Yukawa couplings (up to dimension 8):

$$(Y_\psi)_{pr}^{\rm eff} = \frac{1+c_{H,\rm kin}}{\sqrt{2}}\left[\frac{\sqrt{2}}{v_T}\,[M_\psi]_{pr} - \frac{v_T^2}{\Lambda^2}\,C_{\underset{pr}{\psi H}} - \frac{v_T^4}{\Lambda^4}\,C_{\underset{pr}{\chi\psi H^5}}\right]\ . \tag{24}$$

There are also momentum-dependent Yukawa couplings (i.e., ordinary Yukawa couplings with additional derivatives) occurring at dimension 6 in SMEFT. However, as we will see in Sec. 4.1.3, these can be neglected in our analysis.

The SMEFT Yukawa coupling terms for neutrinos are

$$\mathcal{L}_{\rm SMEFT}^{\rm Neutrino\ Yukawa} = \frac{v_T}{\Lambda}(1+c_{H,\rm kin})\left[C_{\underset{pr}{5}} + \frac{{v_T}^2}{\Lambda^2}C_{\underset{pr}{l^2H^4}}\right]h\,\overline{\nu}_{Lp}^T C\nu_{Lr} + \text{h.c.}\ . \tag{25}$$

This gives the following Yukawa couplings:

$$(Y_\nu)^{\text{eff}}_{pr} = (1 + c_{H,\text{kin}}) \left[\frac{1}{v_T} [M_\nu]_{pr} - \frac{{v_T}^3}{2\Lambda^3} C_{\underset{pr}{l^2 H^4}} \right] . \tag{26}$$

These Yukawa couplings enter the matching conditions of certain four-fermion operators in LEFT.

3.3 Electroweak gauge boson masses & mixing and coupling constants

3.3.1 Kinetic terms. Including the SMEFT contributions up to dimension 8, the kinetic terms of the electroweak gauge bosons after symmetry breaking are

$$\mathcal{L}^{\text{Electroweak kin}}_{\text{SMEFT}} = -\frac{1}{4} \left\{ \begin{array}{l} \left[1 - \frac{2v_T^2}{\Lambda^2} C_{HW} - \frac{v_T^4}{\Lambda^4} C^{(1)}_{W^2H^4} \right] W^I_{\mu\nu} W^{I\mu\nu} \\ - \frac{v_T^4}{\Lambda^4} C^{(3)}_{W^2H^4} W^3_{\mu\nu} W^{3\mu\nu} \\ + \left[1 - \frac{2v_T^2}{\Lambda^2} C_{HB} - \frac{v_T^4}{\Lambda^4} C^{(1)}_{B^2H^4} \right] B_{\mu\nu} B^{\mu\nu} \\ + \left[\frac{2v_T^2}{\Lambda^2} C_{HWB} + \frac{v_T^4}{\Lambda^4} C^{(1)}_{WBH^4} \right] W^3_{\mu\nu} B^{\mu\nu} \end{array} \right\} . \tag{27}$$

Here there are two issues that must be resolved. First, the kinetic terms must be properly normalized. Second, the $W^3_{\mu\nu} B^{\mu\nu}$ mixing term must be removed.

Proper normalization of the kinetic terms can be achieved by redefining the coupling constants and the normalization of the gauge fields:

$$\begin{aligned} \overline{g} &= \left[1 + \frac{v_T^2}{\Lambda^2} C_{HW} + \frac{v_T^4}{2\Lambda^4} C^{(1)}_{W^2H^4} + \frac{3v_T^4}{2\Lambda^4} [C_{HW}]^2 \right] g , \\ \overline{g}' &= \left[1 + \frac{v_T^2}{\Lambda^2} C_{HB} + \frac{v_T^4}{2\Lambda^4} C^{(1)}_{B^2H^4} + \frac{3v_T^4}{2\Lambda^4} [C_{HB}]^2 \right] g' , \end{aligned} \tag{28}$$

$$\begin{aligned} W^I_\mu &= \left[1 + \frac{v_T^2}{\Lambda^2} C_{HW} + \frac{v_T^4}{2\Lambda^4} C^{(1)}_{W^2H^4} + \frac{3v_T^4}{2\Lambda^4} [C_{HW}]^2 \right] \mathcal{W}^I_\mu , \\ B_\mu &= \left[1 + \frac{v_T^2}{\Lambda^2} C_{HB} + \frac{v_T^4}{2\Lambda^4} C^{(1)}_{B^2H^4} + \frac{3v_T^4}{2\Lambda^4} [C_{HB}]^2 \right] \mathcal{B}_\mu . \end{aligned} \tag{29}$$

At this stage, there is still a $\mathcal{W}^3_{\mu\nu}\mathcal{B}^{\mu\nu}$ mixing term, as well as a separate $W^3_{\mu\nu}W^{3\mu\nu}$ term. These can both be removed by defining

$$\begin{bmatrix} \mathcal{W}^3_\mu \\ \mathcal{B}_\mu \end{bmatrix} = \begin{bmatrix} 1+\frac{{v_T}^4}{2\Lambda^4}C^{(3)}_{W^2H^4}+\frac{3{v_T}^4}{8\Lambda^4}[C_{HWB}]^2 & -\frac{{v_T}^2}{2\Lambda^2}\begin{pmatrix} C_{HWB}+\frac{{v_T}^2}{2\Lambda^2}C^{(1)}_{WBH^4} \\ +\frac{{v_T}^2}{\Lambda^2}C_{HWB}C_{HW} \\ +\frac{{v_T}^2}{\Lambda^2}C_{HWB}C_{HB} \end{pmatrix} \\ -\frac{{v_T}^2}{2\Lambda^2}\begin{pmatrix} C_{HWB}+\frac{{v_T}^2}{2\Lambda^2}C^{(1)}_{WBH^4} \\ +\frac{{v_T}^2}{\Lambda^2}C_{HWB}C_{HW} \\ +\frac{{v_T}^2}{\Lambda^2}C_{HWB}C_{HB} \end{pmatrix} & 1+\frac{3{v_T}^4}{8\Lambda^4}[C_{HWB}]^2 \end{bmatrix} \begin{bmatrix} \overline{\mathcal{W}}^3_\mu \\ \overline{\mathcal{B}}_\mu \end{bmatrix}. \quad (30)$$

With this, we have

$$\mathcal{L}^{\text{Electroweak kin}}_{\text{SMEFT}} = -\frac{1}{2}\mathcal{W}^+_{\mu\nu}\mathcal{W}^{-\mu\nu} - \frac{1}{4}\overline{\mathcal{W}}^3_{\mu\nu}\overline{\mathcal{W}}^{3\mu\nu} - \frac{1}{4}\overline{\mathcal{B}}_{\mu\nu}\overline{\mathcal{B}}^{\mu\nu}\ , \quad (31)$$

where $\mathcal{W}^\pm_{\mu\nu} \equiv \partial_\mu\mathcal{W}^\pm_\nu - \partial_\nu\mathcal{W}^\pm_\mu$, $\mathcal{W}^\pm_\mu \equiv \frac{1}{\sqrt{2}}(\mathcal{W}^1_\mu \mp i\mathcal{W}^2_\mu)$, $\overline{\mathcal{W}}^3_{\mu\nu} \equiv \partial_\mu\overline{\mathcal{W}}^3_\nu - \partial_\nu\overline{\mathcal{W}}^3_\mu$, $\overline{\mathcal{B}}_{\mu\nu} \equiv \partial_\mu\overline{\mathcal{B}}_\nu - \partial_\nu\overline{\mathcal{B}}_\mu$, and we have dropped the cubic and quartic self-coupling terms of the gauge bosons.

Note that we still have the freedom to perform the following rotation:

$$\begin{bmatrix} \overline{\mathcal{W}}^3_\mu \\ \overline{\mathcal{B}}_\mu \end{bmatrix} = \begin{bmatrix} \cos\overline{\theta}_W & \sin\overline{\theta}_W \\ -\sin\overline{\theta}_W & \cos\overline{\theta}_W \end{bmatrix} \begin{bmatrix} \mathcal{Z}_\mu \\ \mathcal{A}_\mu \end{bmatrix}\ . \quad (32)$$

In terms of the new fields, we have

$$\mathcal{L}^{\text{Electroweak kin}}_{\text{SMEFT}} = -\frac{1}{2}\mathcal{W}^+_{\mu\nu}\mathcal{W}^{-\mu\nu} - \frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}\ . \quad (33)$$

For completeness, we also present the results for gluons. Including the SMEFT contributions up to dimension 8, the gluon kinetic term is

$$\mathcal{L}^{\text{Gluons kin}}_{\text{SMEFT}} = -\frac{1}{4}\left[1 - \frac{2v_T^2}{\Lambda^2}C_{HG} - \frac{v_T^4}{\Lambda^4}C^{(1)}_{G^2H^4}\right]G^A_{\mu\nu}G^{A\mu\nu}\ . \quad (34)$$

In order to properly normalize this kinetic term, we make redefinitions similar to those in Eqs. (28) and (29):

$$\overline{g}_s = \left[1 + \frac{v_T^2}{\Lambda^2}C_{HG} + \frac{v_T^4}{2\Lambda^4}C^{(1)}_{G^2H^4} + \frac{3v_T^4}{2\Lambda^4}[C_{HG}]^2\right]g_s\ , \quad (35)$$

$$G^A_\mu = \left[1 + \frac{v_T^2}{\Lambda^2}C_{HG} + \frac{v_T^4}{2\Lambda^4}C^{(1)}_{G^2H^4} + \frac{3v_T^4}{2\Lambda^4}[C_{HG}]^2\right]\mathcal{G}^A_\mu\ . \quad (36)$$

3.3.2 Mass terms. The SMEFT contributions up to dimension 8 to the mass terms of the electroweak gauge bosons after symmetry breaking are

$$\mathcal{L}_{\text{SMEFT}}^{\text{Electroweak mass}} = \frac{v_T^2}{8}\left\{\begin{array}{l}\left[1+\frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)}-C_{H^6}^{(2)}\right)\right]g^2(W_\mu^1 W^{1\mu}+W_\mu^2 W^{2\mu}) \\ \left[\begin{array}{l}1+\frac{v_T^2}{2\Lambda^2}C_{HD} \\ \quad+\frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)}+C_{H^6}^{(2)}\right)\end{array}\right](gW_\mu^3-g'B_\mu)(gW^{3\mu}-g'B^\mu)\end{array}\right\}. \tag{37}$$

We can write W_μ^1 and B_μ in terms of $\mathcal{W}_\mu^\pm$, $\mathcal{Z}_\mu$ and $\mathcal{A}_\mu$ using the transformations described in Sec. 3.3.1. In order to ensure a massless photon, we require that the mixing angle $\overline{\theta}_W$ of Eq. (32) satisfy

$$\begin{aligned}\cos\overline{\theta}_W &= \frac{1}{\sqrt{\overline{g}^2+\overline{g}'^2}}\left[\begin{array}{l}\overline{g}+\frac{\overline{g}v_T^4}{8\Lambda^4}\frac{(6\overline{g}^2\overline{g}'^2-\overline{g}^4-5\overline{g}'^4)}{(\overline{g}^2+\overline{g}'^2)^2}[C_{HWB}]^2+\frac{v_T^4}{2\Lambda^4}\frac{\overline{g}\overline{g}'^2}{\overline{g}^2+\overline{g}'^2}C_{W^2H^4}^{(3)} \\ -\frac{\overline{g}'v_T^2}{2\Lambda^2}\frac{\overline{g}^2-\overline{g}'^2}{\overline{g}^2+\overline{g}'^2}\left(C_{HWB}+\frac{v_T^2}{2\Lambda^2}C_{WBH^4}^{(1)}+\frac{v_T^2}{\Lambda^2}C_{HWB}[C_{HW}+C_{HB}]\right)\end{array}\right] \\ \sin\overline{\theta}_W &= \frac{1}{\sqrt{\overline{g}^2+\overline{g}'^2}}\left[\begin{array}{l}\overline{g}'+\frac{\overline{g}'v_T^4}{8\Lambda^4}\frac{(6\overline{g}^2\overline{g}'^2-\overline{g}'^4-5\overline{g}^4)}{(\overline{g}^2+\overline{g}'^2)^2}[C_{HWB}]^2-\frac{v_T^4}{2\Lambda^4}\frac{\overline{g}^2\overline{g}'}{\overline{g}^2+\overline{g}'^2}C_{W^2H^4}^{(3)} \\ +\frac{\overline{g}v_T^2}{2\Lambda^2}\frac{\overline{g}^2-\overline{g}'^2}{\overline{g}^2+\overline{g}'^2}\left(C_{HWB}+\frac{v_T^2}{2\Lambda^2}C_{WBH^4}^{(1)}+\frac{v_T^2}{\Lambda^2}C_{HWB}[C_{HW}+C_{HB}]\right)\end{array}\right]\end{aligned} \tag{38}$$

up to dimension 8.

Note that, while in the SM we have $\sin\theta_W = g'/\sqrt{g^2+g'^2}$ and $\cos\theta_W = g/\sqrt{g^2+g'^2}$, these relations no longer hold in the presence of SMEFT operators. Similalry, in the SM, $e = gg'/\sqrt{g^2+g'^2}$. Including SMEFT operators, this becomes

$$\overline{e} = \frac{\overline{g}\overline{g}'}{\sqrt{\overline{g}^2+\overline{g}'^2}}\left[\begin{array}{l}1-\frac{\overline{g}\overline{g}'v_T^2C_{HWB}}{(\overline{g}^2+\overline{g}'^2)\Lambda^2}-\frac{\overline{g}\overline{g}'v_T^4C_{WBH^4}^{(1)}}{2(\overline{g}^2+\overline{g}'^2)\Lambda^4}+\frac{\overline{g}'^2v_T^4C_{W^2H^4}^{(3)}}{2(\overline{g}^2+\overline{g}'^2)\Lambda^4} \\ -\frac{\overline{g}\overline{g}'v_T^4C_{HWB}(C_{HW}+C_{HB})}{(\overline{g}^2+\overline{g}'^2)\Lambda^4}+\frac{3\overline{g}^2\overline{g}'^2v_T^4[C_{HWB}]^2}{2(\overline{g}^2+\overline{g}'^2)^2\Lambda^4}\end{array}\right]. \tag{39}$$

The masses of the W and Z are given by

$$M_W^2 = \frac{\overline{g}^2v_T^2}{4}\left[1+\frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)}-C_{H^6}^{(2)}\right)\right], \tag{40}$$

$$M_Z^2 = \frac{\overline{g}_Z{}^2v_T^2}{4}\left[1+\frac{v_T^2}{2\Lambda^2}C_{HD}+\frac{v_T^4}{4\Lambda^4}\left(C_{H^6}^{(1)}+C_{H^6}^{(2)}\right)\right], \tag{41}$$

where

$$\bar{g}_Z = \sqrt{\bar{g}^2+\bar{g}'^2}\left[1+\frac{\bar{g}\bar{g}'v_T^2 C_{HWB}}{(\bar{g}^2+\bar{g}'^2)\Lambda^2}+\frac{\bar{g}\bar{g}'v_T^4 C^{(1)}_{WBH^4}}{2(\bar{g}^2+\bar{g}'^2)\Lambda^4}+\frac{\bar{g}^2 v_T^4 C^{(3)}_{W^2H^4}}{2(\bar{g}^2+\bar{g}'^2)\Lambda^4}\right.$$
$$\left.+\frac{\bar{g}\bar{g}'v_T^4 C_{HWB}(C_{HW}+C_{HB})}{(\bar{g}^2+\bar{g}'^2)\Lambda^4}+\left(1-\frac{\bar{g}^2\bar{g}'^2}{(\bar{g}^2+\bar{g}'^2)^2}\right)\frac{[C_{HWB}]^2}{2\Lambda^4}\right] . \quad (42)$$

In the SM, the "charge" to which the Z^0 couples is $I_{3L} - Q_{em}\sin^2\theta_W$. When one adds SMEFT operators up to dimension 6, the mixing angle is changed, $\theta_W \to \bar{\theta}_W$, but the Z^0 coupling still has the same form: it couples to $I_{3L} - Q_{em}\sin^2\bar{\theta}_W$ [9]. However, when SMEFT operators up to dimension 8 are included, this no longer holds. Instead, the Z^0 couples to $I_{3L} - Q_{em}\sin^2\bar{\theta}_Z$, where

$$\sin^2\bar{\theta}_Z = \sin^2\bar{\theta}_W + \frac{v_T^4}{4\Lambda^4}[C_{HWB}]^2(\sin^2\bar{\theta}_W - \cos^2\bar{\theta}_W) . \quad (43)$$

(This was also noted in Ref. [22].)

3.4 Couplings of electroweak gauge bosons to fermions

As shown in Eq. (33), the physical electroeak gauge bosons are $\mathcal{A}_\mu$, $\mathcal{W}^\pm_\mu$ and $\mathcal{Z}_\mu$. Their effective couplings to fermions, as well as those of the gluon $\mathcal{G}^A_\mu$, take the following form:

$$\mathcal{L} = -\bar{g}_s\mathcal{G}^A_\mu j^{A\mu}_{\mathcal{G}} - \bar{e}\mathcal{A}_\mu j^\mu_{\mathcal{A}} - \frac{\bar{g}}{\sqrt{2}}\{\mathcal{W}^+_\mu j^{+\mu}_{\mathcal{W}} + \mathcal{W}^-_\mu j^{-\mu}_{\mathcal{W}}\} - \bar{g}_Z\mathcal{Z}_\mu j^\mu_{\mathcal{Z}} , \quad (44)$$

in which the corresponding currents are

$$\begin{aligned}
j^{A\mu}_{\mathcal{G}} &= \bar{u}_{Lp}\gamma^\mu T^A u_{Lr} + \bar{d}_{Lp}\gamma^\mu T^A d_{Lr} + \bar{u}_{Rp}\gamma^\mu T^A u_{Rr} + \bar{d}_{Rp}\gamma^\mu T^A d_{Rr} , \\
j^\mu_{\mathcal{A}} &= -\bar{e}_{Lp}\gamma^\mu e_{Lr} + \frac{2}{3}\bar{u}_{Lp}\gamma^\mu u_{Lr} - \frac{1}{3}\bar{d}_{Lp}\gamma^\mu d_{Lr} - \bar{e}_{Rp}\gamma^\mu e_{Rr} + \frac{2}{3}\bar{u}_{Rp}\gamma^\mu u_{Rr} - \frac{1}{3}\bar{d}_{Rp}\gamma^\mu d_{Rr} , \\
j^{+\mu}_{\mathcal{W}} &= [W_l]^{\rm eff}_{pr}\bar{\nu}_{Lp}\gamma^\mu e_{Lr} + [W_q]^{\rm eff}_{pr}\bar{u}_{Lp}\gamma^\mu d_{Lr} + [W_R]^{\rm eff}_{pr}\bar{u}_{Rp}\gamma^\mu d_{Rr} + [W_l^{\not{L}}]^{\rm eff}_{pr}(\nu^T_{Lp}C\gamma^\mu e_{Rr}) , \quad (45) \\
j^{-\mu}_{\mathcal{W}} &= [W_l]^{\rm eff*}_{rp}\bar{e}_{Lp}\gamma^\mu \nu_{Lr} + [W_q]^{\rm eff*}_{rp}\bar{d}_{Lp}\gamma^\mu u_{Lr} + [W_R]^{\rm eff*}_{rp}\bar{d}_{Rp}\gamma^\mu u_{Rr} + [W_l^{\not{L}}]^{\rm eff*}_{rp}(\bar{\nu}_{Lp}\gamma^\mu C\bar{e}^T_{Rr}) , \\
j^\mu_{\mathcal{Z}} &= \begin{bmatrix} [Z_{\nu_L}]^{\rm eff}_{pr}\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr} + [Z_{e_L}]^{\rm eff}_{pr}\bar{e}_{Lp}\gamma^\mu e_{Lr} + [Z_{u_L}]^{\rm eff}_{pr}\bar{u}_{Lp}\gamma^\mu u_{Lr} + [Z_{d_L}]^{\rm eff}_{pr}\bar{d}_{Lp}\gamma^\mu d_{Lr} \\ + [Z_{e_R}]^{\rm eff}_{pr}\bar{e}_{Rp}\gamma^\mu e_{Rr} + [Z_{u_R}]^{\rm eff}_{pr}\bar{u}_{Rp}\gamma^\mu u_{Rr} + [Z_{d_R}]^{\rm eff}_{pr}\bar{d}_{Rp}\gamma^\mu d_{Rr} \end{bmatrix} .
\end{aligned}$$

Since $SU(3)_C \times U(1)_{em}$ remains unbroken, the currents involving gluons and photons are fully determined by QCD and QED. This is not the case for the $\mathcal{W}^\pm_\mu$ and $\mathcal{Z}_\mu$ gauge bosons: the fermion currents to which the $\mathcal{W}^\pm_\mu$ and $\mathcal{Z}_\mu$ couple are given by the following (up to dimension 8):

$$
\begin{aligned}
[W_l]^{\rm eff}_{pr} &= \delta_{pr} + \frac{v_T^2}{\Lambda^2}\, C^{(3)}_{\substack{Hl\\pr}} + \frac{v_T^4}{2\Lambda^4}\left(C^{(2)}_{\substack{l^2H^4D\\pr}} - iC^{(3)}_{\substack{l^2H^4D\\pr}} \right) \,, \\
[W_q]^{\rm eff}_{pr} &= \delta_{pr} + \frac{v_T^2}{\Lambda^2}\, C^{(3)}_{\substack{Hq\\pr}} + \frac{v_T^4}{2\Lambda^4}\left(C^{(2)}_{\substack{q^2H^4D\\pr}} - iC^{(3)}_{\substack{q^2H^4D\\pr}} \right) \,, \\
[W_R]^{\rm eff}_{pr} &= \frac{v_T^2}{2\Lambda^2}\, C^{(3)}_{\substack{Hud\\pr}} + \frac{v_T^4}{4\Lambda^4}\, C_{\substack{udH^4D\\pr}} \,, \\
[W_l^{\not L}]^{\rm eff}_{pr} &= -\frac{{v_T}^3}{2\sqrt{2}\,\Lambda^3} C_{\substack{leH^3D\\pr}} \,, \\
[Z_{\nu_L}]^{\rm eff}_{pr} &= \frac{1}{2}\delta_{pr} - \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{Hl\\pr}} - C^{(3)}_{\substack{Hl\\pr}} \right) - \frac{v_T^4}{4\Lambda^4}\left(C^{(1)}_{\substack{l^2H^4D\\pr}} - 2C^{(2)}_{\substack{l^2H^4D\\pr}} \right) \,, \\
[Z_{e_L}]^{\rm eff}_{pr} &= \frac{1}{2}g_L^e\delta_{pr} - \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{Hl\\pr}} + C^{(3)}_{\substack{Hl\\pr}} \right) - \frac{v_T^4}{4\Lambda^4}\left(C^{(1)}_{\substack{l^2H^4D\\pr}} + 2C^{(2)}_{\substack{l^2H^4D\\pr}} \right) \,, \\
[Z_{u_L}]^{\rm eff}_{pr} &= \frac{1}{2}g_L^u\delta_{pr} - \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{Hq\\pr}} - C^{(3)}_{\substack{Hq\\pr}} \right) - \frac{v_T^4}{4\Lambda^4}\left(C^{(1)}_{\substack{q^2H^4D\\pr}} - 2C^{(2)}_{\substack{q^2H^4D\\pr}} \right) \,, \\
[Z_{d_L}]^{\rm eff}_{pr} &= \frac{1}{2}g_L^d\delta_{pr} - \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\substack{Hq\\pr}} + C^{(3)}_{\substack{Hq\\pr}} \right) - \frac{v_T^4}{4\Lambda^4}\left(C^{(1)}_{\substack{q^2H^4D\\pr}} + 2C^{(2)}_{\substack{q^2H^4D\\pr}} \right) \,, \\
[Z_{e_R}]^{\rm eff}_{pr} &= \frac{1}{2}g_R^e\delta_{pr} - \frac{v_T^2}{2\Lambda^2}\, C_{\substack{He\\pr}} - \frac{v_T^4}{4\Lambda^4}\, C^{(1)}_{\substack{e^2H^4D\\pr}} \,, \\
[Z_{u_R}]^{\rm eff}_{pr} &= \frac{1}{2}g_R^u\delta_{pr} - \frac{v^2}{2\Lambda^2}\, C_{\substack{Hu\\pr}} - \frac{v_T^4}{4\Lambda^4}\, C^{(1)}_{\substack{u^2H^4D\\pr}} \,, \\
[Z_{d_R}]^{\rm eff}_{pr} &= \frac{1}{2}g_R^d\delta_{pr} - \frac{v^2}{2\Lambda^2}\, C_{\substack{Hd\\pr}} - \frac{v_T^4}{4\Lambda^4}\, C^{(1)}_{\substack{d^2H^4D\\pr}} \,.
\end{aligned}
\tag{46}
$$

Here, we have defined $g_L^e \equiv -1 + 2\sin^2\overline{\theta}_Z$, $g_L^u \equiv 1 - \frac{4}{3}\sin^2\overline{\theta}_Z$, $g_L^d \equiv -1 + \frac{2}{3}\sin^2\overline{\theta}_Z$, $g_R^e \equiv 2\sin^2\overline{\theta}_Z$, $g_R^u \equiv -\frac{4}{3}\sin^2\overline{\theta}_Z$, and $g_R^d \equiv \frac{2}{3}\sin^2\overline{\theta}_Z$, where $\sin^2\overline{\theta}_Z$ is defined in Eq. (43).

4 Matching conditions

There are four categories of LEFT operators up to dimension 6: (i) four-fermion operators, (ii) magnetic dipole moment operators, (iii) three-gluon operators, and (iv) neutrino mass terms. The matching conditions for operators that conserve both B and L involve only even-dimension SMEFT operators, and are given up to dimension 6 in JMS. For operators that violate B and/or L, the matching conditions involve either even- or odd-dimension SMEFT operators, depending on the operator; these are given to dimension 6 or dimension 5 in JMS. In this section, we present the tree-level matching conditions for all of these operators up to dimension 8 in SMEFT.

4.1 Four-fermion operators

As was described in Sec. 2, there are generally two types of contributions to the SMEFT matching conditions of four-fermion LEFT operators: direct and indirect contributions.

4.1.1 Direct contributions. The dimension-6 SMEFT direct contribution is the LEFT operator itself, in which all left- and right-handed particles are replaced by the left-handed $SU(2)_L$ doublets and right-handed $SU(2)_L$ singlets to which they respectively belong. The dimension-8 contributions involve the dimension-6 SMEFT operator multiplied by a pair of Higgs fields. When the Higgs gets a vev, this generates the four-fermion LEFT operator.

The details of the computation are best illustrated with an example. Consider the LEFT operator

$$\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}} \equiv (\overline{\nu}_{Lp}\gamma_\mu \nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu \nu_{Lt}) \ . \tag{47}$$

It is generated by the dimension-6 SMEFT operator $Q_{\underset{prst}{ll}} \equiv (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$. This can be seen by separating the SMEFT operator into components:

$$\frac{1}{\Lambda^2} C_{\underset{prst}{ll}} Q_{\underset{prst}{ll}} \to \frac{1}{\Lambda^2} C_{\underset{prst}{ll}} \begin{bmatrix} (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \end{bmatrix} . \tag{48}$$

The first term is $\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}}$.

One dimension-8 SMEFT operator that is among the matching conditions is $Q^{(1)}_{\underset{prst}{l^4H^2}} \equiv (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger H)$. Because the $SU(2)_L$ doublets l and H are involved, there are two additional dimension-8 SMEFT operators that must be included: $Q^{(2)}_{\underset{prst}{l^4H^2}} \equiv (\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu \tau^I l_t)(H^\dagger \tau^I H)$ and $Q^{(2)}_{\underset{stpr}{l^4H^2}} \equiv (\bar{l}_p\gamma_\mu \tau^I l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger \tau^I H)$. When the Higgs gets a vev, these three operators can also generate $\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}}$:

$$\begin{aligned}
\frac{1}{\Lambda^4} C^{(1)}_{\underset{prst}{l^4H^2}} Q^{(1)}_{\underset{prst}{l^4H^2}} &\to \frac{v_T^2}{2\Lambda^4} C^{(1)}_{\underset{prst}{l^4H^2}} \begin{bmatrix} (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \end{bmatrix} , \\
\frac{1}{\Lambda^4} C^{(2)}_{\underset{prst}{l^4H^2}} Q^{(2)}_{\underset{prst}{l^4H^2}} &\to \frac{v_T^2}{2\Lambda^4} C^{(2)}_{\underset{prst}{l^4H^2}} \begin{bmatrix} -(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ -(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \end{bmatrix} , \\
\frac{1}{\Lambda^4} C^{(2)}_{\underset{stpr}{l^4H^2}} Q^{(2)}_{\underset{stpr}{l^4H^2}} &\to \frac{v_T^2}{2\Lambda^4} C^{(2)}_{\underset{stpr}{l^4H^2}} \begin{bmatrix} -(\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) - (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \\ +(\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{\nu}_{Ls}\gamma^\mu\nu_{Lt}) + (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt}) \end{bmatrix} .
\end{aligned} \tag{49}$$

We therefore see that the direct contribution to the matching condition of the LEFT operator $\frac{1}{\Lambda^2}\mathcal{O}^{V,LL}_{\underset{prst}{\nu\nu}}$, up to dimension 8, is

$$\frac{1}{\Lambda^2}\left[C_{\underset{prst}{ll}} + \frac{v_T^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}} \right)\right] . \tag{50}$$

The direct contributions to the matching conditions of the other LEFT four-fermion operators are calculated similarly.

4.1.2 Indirect contributions. A four-fermion operator can also be generated when a boson is exchanged between two fermion currents and this boson is integrated out. This produces an indirect contribution [e.g., see Eq. (7)].

Consider once again the LEFT operator $\underset{prst}{\mathcal{O}^{V,LL}_{\nu\nu}}$ of Eq. (47). The indirect contributions arise from the Z-exchange diagrams of Fig. 1, when the Z^0 is integrated out. We note that (i) there is a relative minus sign between the two diagrams, and (ii) when one Fierz transforms (see Appendix C) the amplitude of the second diagram, one obtains the amplitude of the first diagram, but with an exchange of generation indices $r \leftrightarrow t$. The indirect contribution to the matching condition of this operator, up to dimension 8, is

$$-\frac{\overline{g}_Z^2}{4M_Z^2}\left([Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{\nu_L}]^{\text{eff}}_{st} + [Z_{\nu_L}]^{\text{eff}}_{pt}[Z_{\nu_L}]^{\text{eff}}_{sr}\right) \ , \tag{51}$$

where $\overline{g}_Z$ and $[Z_{\nu_L}]^{\text{eff}}_{pr}$ are defined in Eqs. (42) and (46), respectively.

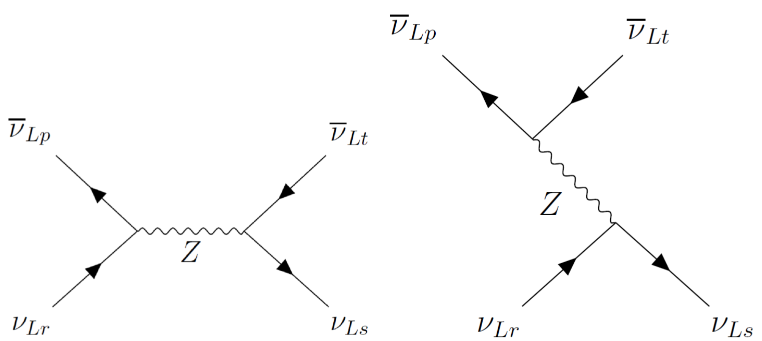

Fig. 1: Z-exchange contributions to $\mathcal{O}^{V,LL}_{\nu\nu}$ with flavour indices $prst$.

Another example is the LEFT operator $\underset{prst}{\mathcal{O}^{V,LL}_{\nu e}} \equiv (\overline{\nu}_{Lp}\gamma_\mu\nu_{Lr})(\overline{e}_{Ls}\gamma^\mu e_{Lt})$. Here the indirect contributions arise from the Z- and W-exchange diagrams of Fig, 2, when the heavy gauge bosons are integrated out. The indirect contribution to the matching condition of this operator, up to dimension 8, is

$$-\frac{\overline{g}_Z^2}{M_Z^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{e_L}]^{\text{eff}}_{st} - \frac{\overline{g}^2}{2M_W^2}[W_l]^{\text{eff}}_{pr}[W_l]^{\text{eff}*}_{st} \ , \tag{52}$$

where $\overline{g}$ and $[W_l]^{\text{eff}}_{pr}$ are defined in Eqs. (28) and (46), respectively.

The indirect contributions from gauge-boson exchange to the matching conditions of the other LEFT four-fermion operators are calculated similarly. Most such operators can be generated via diagrams with the exchange of a Z^0. A small subset of these also involve W-exchange diagrams. And a few LEFT operators can be generated only via the exchange of a W.

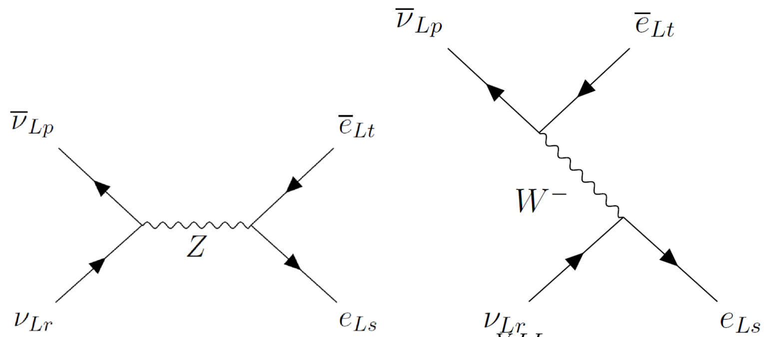

Fig. 2: Z- and W-exchange contributions to $\mathcal{O}_{\nu e}^{V,LL}$ with flavour indices $prst$.

Finally, the matching conditions of certain LEFT operators receive indirect contributions from Higgs exchange. As an example, consider the operator $\mathcal{O}_{\substack{ee\\prst}}^{V,LR} \equiv (\overline{e}_{Lp}\gamma_\mu e_{Lr})(\overline{e}_{Rs}\gamma^\mu e_{Rt})$.

The indirect contributions come from the diagrams of Fig. 3. The Z- and h-exchange contributions are computed similarly to the previous examples. The indirect contribution to the matching condition is

$$-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{e_R}]^{\text{eff}}_{st} - \frac{1}{2{m_h}^2}\,(Y_e)^{\text{eff}}_{pt}(Y_e)^{\text{eff}*}_{rs}\ . \tag{53}$$

The Yukawa coupling is [Eq. (24), repeated for convenience]

$$(Y_e)^{\text{eff}}_{pr} = \frac{1+c_{H,\text{kin}}}{\sqrt{2}}\left[\frac{\sqrt{2}}{v_T}\,[M_e]_{pr} - \frac{v_T^2}{\Lambda^2}\,C_{\substack{eH\\pr}} - \frac{v_T^4}{\Lambda^4}\,C_{\substack{\chi eH^5\\pr}}\right]\ .$$

The first term is $\sim m_e/v_T$ and is negligible. For this reason, JMS, which works only to dimension 6, argues that the h-exchange indirect contributions to the matching conditions are unimportant. However, when one works to dimension 8, there is a non-negligible contribution resulting from the square of the second term.

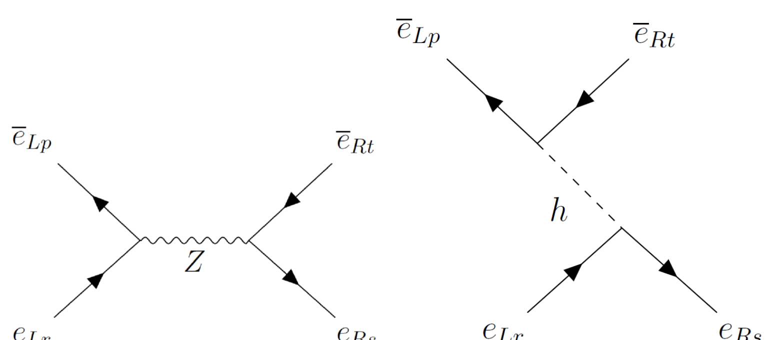

Fig. 3: Z- and h-exchange contributions to $\mathcal{O}_{\nu e}^{V,LL}$ with flavour indices $prst$.

4.1.3 Subdominant contributions. When one computes the matching conditions of LEFT operators to higher dimensions in SMEFT, one necessarily must take into account smaller contributions. For example, up to dimension-4 SMEFT operators, the smallest coefficient of a four-fermion LEFT operator is $O(1/v^2)$. At dimension 6, it is $O(1/\Lambda^2)$, and it falls to $O(v^2/\Lambda^4)$ at dimension 8 (technically, $O(v_T^2/\Lambda^4)$).

In the above indirect contributions, we have approximated the gauge-boson (or Higgs) propagators as $1/v^2$. However, there are corrections proportional to q^2/v^2, giving a contribution to the coefficient of $O(q^2/v^4)$. For $q \sim m_b$ (the largest low-energy scale), this is still $\ll 1/\Lambda^2$, so that it is unimportant for matching conditions up to dimension 6 (which is why JMS do not mention it). However, it *can be* of the same order as $O(v^2/\Lambda^4)$, the typical size of dimension-8 SMEFT contributions.

Now, this contribution is momentum-dependent, which means that it is process-dependent. For example, it may be important for $q \sim m_b$, but is not for $q \sim m_\mu$. For this reason, it is not included in our matching conditions, which are process-independent. Still, it should be included in any analysis of particular observables that considers dimension-8 SMEFT contributions. (Similarly, loop-level dimension-6 SMEFT contributions must also be taken into account [**11**–**14**].)

But this raises the question: what about momentum-dependent SMEFT contributions? Indirect contributions involve a propagator, approximated as $1/v^2$, and two vertices. If both vertices in a given diagram contribute the factor v^2/Λ^2, this saturates the dimension-8 contribution, so any momentum-dependent correction will be smaller. But if one vertex is ~ 1 (the SM) and the other vertex is $\sim v^2/\Lambda^2$, this yields a net contribution of $O(1/\Lambda^2)$, which contributes to the matching conditions at dimension 6. In this case, a subdominant contribution to the second vertex of $\sim qv/\Lambda^2$ would lead to a momentum-dependent SMEFT contribution, and could potentially be important.

It turns out that such a subdominant contribution does not arise in diagrams with the exchange of a gauge boson. It is only diagrams with Higgs exchange that contain such a vertex term. (For example, these can be generated from operators in the dimension-6 SMEFT class $\psi^2 H^2 D$ (see Appendix B.1).) The subdominant SMEFT vertex is $\sim mv/\Lambda^2$ (the m appears due to the Dirac equation), while the other vertex is the SM Higgs contribution, $\sim m/v$. The net contribution is

$$\frac{1}{v^2}\,\frac{vm}{\Lambda^2}\,\frac{m}{v} \sim \frac{m^2}{v^2\Lambda^2} \ll \frac{v^2}{\Lambda^4}\ . \tag{54}$$

The bottom line is that there are no sizeable momentum-dependent SMEFT contributions to the matching conditions.

Finally, another possibility is in the Higgs-exchange contribution to the LEFT operator $\mathcal{O}^{S,LL}_{\substack{\nu\nu\\prst}}$ through lepton-number-violating Yukawa couplings. This corresponds to the processes shown in Fig. 4, which have the total amplitude

$$\frac{i}{{m_h}^2}\left[(Y_\nu)^{\rm eff}_{pr}\,(Y_\nu)^{\rm eff}_{st} + (Y_\nu)^{\rm eff}_{pt}\,(Y_\nu)^{\rm eff}_{sr}\right](\nu^T_{Lp}C\nu_{Lr})(\nu^T_{Ls}C\nu_{Lt})\ . \tag{55}$$

The key point here is that the Higgs coupling $(Y_\nu)^{\rm eff}_{pr}$ is not proportional to the neutrino mass (which is tiny). There is also a contribution from the dimension-7 term, whose coefficient

$\sim v^3/\Lambda^3$ [see Eq. (26)]. This then leads to a coefficient

$$\frac{1}{v^2}\frac{v^3}{\Lambda^3}\frac{v^3}{\Lambda^3} = \frac{v^4}{\Lambda^6} \ll \frac{v^2}{\Lambda^4} \,, \tag{56}$$

i.e., it is still negligible.

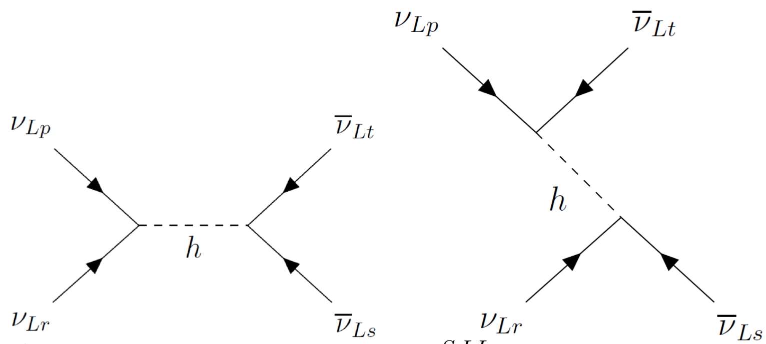

Fig. 4: h-exchange contributions to $\mathcal{O}_{\nu\nu}^{S,LL}$ with flavour indices $prst$.

4.2 Results

The matching conditions for all four-fermion LEFT operators up to dimension 8 in SMEFT are determined using the techniques described above for computing the direct and indirect contributions. For the LEFT magnetic dipole moment operators, three-gluon operators and neutrino mass terms, the calculations are straightforward, as there are no indirect contributions. The SMEFT matching conditions for all LEFT operators up to dimension 8 are given in the tables in Appendix D.

In the literature, the matching conditions of LEFT operators to dimension-7 SMEFT operators have been calculated in Ref. [10]. The results obtained there are in agreement with ours. The matching conditions of LEFT operators to dimension-8 SMEFT operators has only been performed in Refs. [16,17], where the focus was on LEFT operators that lead to lepton flavour violation. Our results agree with this analysis. Matching conditions to dimension-8 SMEFT operators have also been computed in Ref. [27], but in the context of high-energy processes. Although LEFT operators were not involved, there is still some overlap, and we agree here as well. Finally, the contributions of dimension-8 SMEFT operators to the SM parameters, as described in Sec. 3, was also examined in Ref. [16], and we are in agreement.

5 Conclusions

The modern thinking is that the Standard Model is the leading part of an effective field theory, produced when the heavy new physics is integrated out. This EFT is usually assumed to be the SMEFT, which includes the Higgs boson. The SMEFT has been well-studied – all operators up to dimension 8 have been worked out.

When the heavy particles of the SM ($W^{\pm}$, Z^0, H, t) are also integrated out, one obtains the LEFT (low-energy EFT), applicable at scales $\leq m_b$. In order to establish how low-energy measurements are affected by the underlying NP, it is necessary to determine how the LEFT operators depend on the SMEFT operators (the matching conditions).

In Ref. [9], Jenkins, Manohar and Stoffer (JMS) present a complete and non-redundant basis of LEFT operators up to dimension 6, and compute the matching to SMEFT operators up to dimension 6. However, if the low-energy observable in question is suppressed in the SM and/or is very precisely measured, this may not be sufficient. Indeed, it has been pointed out that dimension-8 SMEFT contributions may be important for electroweak precision data from LEP, lepton-flavour-violating processes, meson-antimeson mixing, and electric dipole moments.

In this paper, we extend the analysis of JMS: for all LEFT operators, we work out the complete matching conditions to SMEFT operators up to dimension 8. There are direct contributions to these matching conditions for all LEFT operators, and four-fermion operators also receive indirect contributions due to the exchange of a $W^{\pm}$, Z^0 and/or H.

Should the analysis of a LEFT observable require information about dimension-8 SMEFT contributions, that information can be found here.

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A LEFT operators up to dimension 6

The following two tables are taken from Ref. [9].

$\boldsymbol{\nu\nu}$ + h.c.	
$\mathcal{O}_{\nu}$	$(\nu_{Lp}^T C \nu_{Lr})$

$\boldsymbol{(\nu\nu)X}$ + h.c.	
$\mathcal{O}_{\nu\gamma}$	$(\nu_{Lp}^T C \sigma^{\mu\nu}\nu_{Lr})F_{\mu\nu}$

$\boldsymbol{(\overline{L}R)X}$ + h.c.	
$\mathcal{O}_{e\gamma}$	$\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{u\gamma}$	$\bar{u}_{Lp}\sigma^{\mu\nu}u_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{d\gamma}$	$\bar{d}_{Lp}\sigma^{\mu\nu}d_{Rr}\, F_{\mu\nu}$
$\mathcal{O}_{uG}$	$\bar{u}_{Lp}\sigma^{\mu\nu}T^A u_{Rr}\, G_{\mu\nu}^A$
$\mathcal{O}_{dG}$	$\bar{d}_{Lp}\sigma^{\mu\nu}T^A d_{Rr}\, G_{\mu\nu}^A$

$\boldsymbol{X^3}$	
$\mathcal{O}_{G}$	$f^{ABC}G_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$
$\mathcal{O}_{\widetilde{G}}$	$f^{ABC}\widetilde{G}_\mu^{A\nu}G_\nu^{B\rho}G_\rho^{C\mu}$

$\boldsymbol{(\overline{L}L)(\overline{L}L)}$	
$\mathcal{O}_{\nu\nu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{\nu}_{Ls}\gamma_\mu\nu_{Lt})$
$\mathcal{O}_{ee}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$
$\mathcal{O}_{\nu e}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$
$\mathcal{O}_{\nu u}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{\nu d}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{eu}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{ed}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{\nu edu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu u_{Lt})$ + h.c.
$\mathcal{O}_{uu}^{V,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$
$\mathcal{O}_{dd}^{V,LL}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{ud}^{V1,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$
$\mathcal{O}_{ud}^{V8,LL}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Ls}\gamma_\mu T^A d_{Lt})$

$\boldsymbol{(\overline{R}R)(\overline{R}R)}$	
$\mathcal{O}_{ee}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{eu}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{uu}^{V,RR}$	$(\bar{u}_{Rp}\gamma^\mu u_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{dd}^{V,RR}$	$(\bar{d}_{Rp}\gamma^\mu d_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ud}^{V1,RR}$	$(\bar{u}_{Rp}\gamma^\mu u_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ud}^{V8,RR}$	$(\bar{u}_{Rp}\gamma^\mu T^A u_{Rr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$

$\boldsymbol{(\overline{L}L)(\overline{R}R)}$	
$\mathcal{O}_{\nu e}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{ee}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{\nu u}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{\nu d}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{eu}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{ed}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ue}^{V,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{de}^{V,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$
$\mathcal{O}_{\nu edu}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt})$ + h.c.
$\mathcal{O}_{uu}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{uu}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{u}_{Rs}\gamma_\mu T^A u_{Rt})$
$\mathcal{O}_{ud}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{ud}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$
$\mathcal{O}_{du}^{V1,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$
$\mathcal{O}_{du}^{V8,LR}$	$(\bar{d}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{u}_{Rs}\gamma_\mu T^A u_{Rt})$
$\mathcal{O}_{dd}^{V1,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$
$\mathcal{O}_{dd}^{V8,LR}$	$(\bar{d}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$
$\mathcal{O}_{uddu}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt})$ + h.c.
$\mathcal{O}_{uddu}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A u_{Rt})$ + h.c.

$\boldsymbol{(\overline{L}R)(\overline{L}R)}$ + h.c.	
$\mathcal{O}_{ee}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{e}_{Ls}e_{Rt})$
$\mathcal{O}_{eu}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{u}_{Ls}u_{Rt})$
$\mathcal{O}_{eu}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{u}_{Ls}\sigma_{\mu\nu}u_{Rt})$
$\mathcal{O}_{ed}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Ls}d_{Rt})$
$\mathcal{O}_{ed}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{d}_{Ls}\sigma_{\mu\nu}d_{Rt})$
$\mathcal{O}_{\nu edu}^{S,RR}$	$(\bar{\nu}_{Lp}e_{Rr})(\bar{d}_{Ls}u_{Rt})$
$\mathcal{O}_{\nu edu}^{T,RR}$	$(\bar{\nu}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{d}_{Ls}\sigma_{\mu\nu}u_{Rt})$
$\mathcal{O}_{uu}^{S1,RR}$	$(\bar{u}_{Lp}u_{Rr})(\bar{u}_{Ls}u_{Rt})$
$\mathcal{O}_{uu}^{S8,RR}$	$(\bar{u}_{Lp}T^A u_{Rr})(\bar{u}_{Ls}T^A u_{Rt})$
$\mathcal{O}_{ud}^{S1,RR}$	$(\bar{u}_{Lp}u_{Rr})(\bar{d}_{Ls}d_{Rt})$
$\mathcal{O}_{ud}^{S8,RR}$	$(\bar{u}_{Lp}T^A u_{Rr})(\bar{d}_{Ls}T^A d_{Rt})$
$\mathcal{O}_{dd}^{S1,RR}$	$(\bar{d}_{Lp}d_{Rr})(\bar{d}_{Ls}d_{Rt})$
$\mathcal{O}_{dd}^{S8,RR}$	$(\bar{d}_{Lp}T^A d_{Rr})(\bar{d}_{Ls}T^A d_{Rt})$
$\mathcal{O}_{uddu}^{S1,RR}$	$(\bar{u}_{Lp}d_{Rr})(\bar{d}_{Ls}u_{Rt})$
$\mathcal{O}_{uddu}^{S8,RR}$	$(\bar{u}_{Lp}T^A d_{Rr})(\bar{d}_{Ls}T^A u_{Rt})$

$\boldsymbol{(\overline{L}R)(\overline{R}L)}$ + h.c.	
$\mathcal{O}_{eu}^{S,RL}$	$(\bar{e}_{Lp}e_{Rr})(\bar{u}_{Rs}u_{Lt})$
$\mathcal{O}_{ed}^{S,RL}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Rs}d_{Lt})$
$\mathcal{O}_{\nu edu}^{S,RL}$	$(\bar{\nu}_{Lp}e_{Rr})(\bar{d}_{Rs}u_{Lt})$

Table 1: The non-four-fermions LEFT operators up to dimension 6 and the dimension-6 four-fermion LEFT operators conserving B and L.

$\Delta L = 4 + \text{h.c.}$	
$\mathcal{O}_{\nu\nu}^{S,LL}$	$(\nu_{Lp}^T C \nu_{Lr})(\nu_{Ls}^T C \nu_{Lt})$

$\Delta L = 2 + \text{h.c.}$	
$\mathcal{O}_{\nu e}^{S,LL}$	$(\nu_{Lp}^T C \nu_{Lr})(\bar{e}_{Rs} e_{Lt})$
$\mathcal{O}_{\nu e}^{T,LL}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} \nu_{Lr})(\bar{e}_{Rs} \sigma_{\mu\nu} e_{Lt})$
$\mathcal{O}_{\nu e}^{S,LR}$	$(\nu_{Lp}^T C \nu_{Lr})(\bar{e}_{Ls} e_{Rt})$
$\mathcal{O}_{\nu u}^{S,LL}$	$(\nu_{Lp}^T C \nu_{Lr})(\bar{u}_{Rs} u_{Lt})$
$\mathcal{O}_{\nu u}^{T,LL}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} \nu_{Lr})(\bar{u}_{Rs} \sigma_{\mu\nu} u_{Lt})$
$\mathcal{O}_{\nu u}^{S,LR}$	$(\nu_{Lp}^T C \nu_{Lr})(\bar{u}_{Ls} u_{Rt})$
$\mathcal{O}_{\nu d}^{S,LL}$	$(\nu_{Lp}^T C \nu_{Lr})(\bar{d}_{Rs} d_{Lt})$
$\mathcal{O}_{\nu d}^{T,LL}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} \nu_{Lr})(\bar{d}_{Rs} \sigma_{\mu\nu} d_{Lt})$
$\mathcal{O}_{\nu d}^{S,LR}$	$(\nu_{Lp}^T C \nu_{Lr})(\bar{d}_{Ls} d_{Rt})$
$\mathcal{O}_{\nu edu}^{S,LL}$	$(\nu_{Lp}^T C e_{Lr})(\bar{d}_{Rs} u_{Lt})$
$\mathcal{O}_{\nu edu}^{T,LL}$	$(\nu_{Lp}^T C \sigma^{\mu\nu} e_{Lr})(\bar{d}_{Rs} \sigma_{\mu\nu} u_{Lt})$
$\mathcal{O}_{\nu edu}^{S,LR}$	$(\nu_{Lp}^T C e_{Lr})(\bar{d}_{Ls} u_{Rt})$
$\mathcal{O}_{\nu edu}^{V,RL}$	$(\nu_{Lp}^T C \gamma^\mu e_{Rr})(\bar{d}_{Ls} \gamma_\mu u_{Lt})$
$\mathcal{O}_{\nu edu}^{V,RR}$	$(\nu_{Lp}^T C \gamma^\mu e_{Rr})(\bar{d}_{Rs} \gamma_\mu u_{Rt})$

$\Delta B = \Delta L = 1 + \text{h.c.}$	
$\mathcal{O}_{udd}^{S,LL}$	$\epsilon_{\alpha\beta\gamma}(u_{Lp}^{\alpha T} C d_{Lr}^{\beta})(d_{Ls}^{\gamma T} C \nu_{Lt})$
$\mathcal{O}_{duu}^{S,LL}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C u_{Lr}^{\beta})(u_{Ls}^{\gamma T} C e_{Lt})$
$\mathcal{O}_{uud}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(u_{Lp}^{\alpha T} C u_{Lr}^{\beta})(d_{Rs}^{\gamma T} C e_{Rt})$
$\mathcal{O}_{duu}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C u_{Lr}^{\beta})(u_{Rs}^{\gamma T} C e_{Rt})$
$\mathcal{O}_{uud}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(u_{Rp}^{\alpha T} C u_{Rr}^{\beta})(d_{Ls}^{\gamma T} C e_{Lt})$
$\mathcal{O}_{duu}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C u_{Rr}^{\beta})(u_{Ls}^{\gamma T} C e_{Lt})$
$\mathcal{O}_{dud}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C u_{Rr}^{\beta})(d_{Ls}^{\gamma T} C \nu_{Lt})$
$\mathcal{O}_{ddu}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C d_{Rr}^{\beta})(u_{Ls}^{\gamma T} C \nu_{Lt})$
$\mathcal{O}_{duu}^{S,RR}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C u_{Rr}^{\beta})(u_{Rs}^{\gamma T} C e_{Rt})$

$\Delta B = -\Delta L = 1 + \text{h.c.}$	
$\mathcal{O}_{ddd}^{S,LL}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C d_{Lr}^{\beta})(\bar{e}_{Rs} d_{Lt}^{\gamma})$
$\mathcal{O}_{udd}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(u_{Lp}^{\alpha T} C d_{Lr}^{\beta})(\bar{\nu}_{Ls} d_{Rt}^{\gamma})$
$\mathcal{O}_{ddu}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C d_{Lr}^{\beta})(\bar{\nu}_{Ls} u_{Rt}^{\gamma})$
$\mathcal{O}_{ddd}^{S,LR}$	$\epsilon_{\alpha\beta\gamma}(d_{Lp}^{\alpha T} C d_{Lr}^{\beta})(\bar{e}_{Ls} d_{Rt}^{\gamma})$
$\mathcal{O}_{ddd}^{S,RL}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C d_{Rr}^{\beta})(\bar{e}_{Rs} d_{Lt}^{\gamma})$
$\mathcal{O}_{udd}^{S,RR}$	$\epsilon_{\alpha\beta\gamma}(u_{Rp}^{\alpha T} C d_{Rr}^{\beta})(\bar{\nu}_{Ls} d_{Rt}^{\gamma})$
$\mathcal{O}_{ddd}^{S,RR}$	$\epsilon_{\alpha\beta\gamma}(d_{Rp}^{\alpha T} C d_{Rr}^{\beta})(\bar{e}_{Ls} d_{Rt}^{\gamma})$

Table 2: The dimension-6 four-fermion LEFT operators violating B and/or L.

B SMEFT operators used in this paper

B.1 Even-dimensional operators

These tables list the dimension-6 [3] and dimension-8 [7] SMEFT operators that contribute to the matching conditions, separated into various categories.

Classes H^n and H^nD^2	
Operator	**WC**
$(H^\dagger H)^3$	$\frac{1}{\Lambda^2}C_H$
$(H^\dagger H)^4$	$\frac{1}{\Lambda^4}C_{H^8}$
$(H^\dagger H)\Box(H^\dagger H)$	$\frac{1}{\Lambda^2}C_{H\Box}$
$(H^\dagger D_\mu H)^*(H^\dagger D^\mu H)$	$\frac{1}{\Lambda^2}C_{HD}$
$(H^\dagger H)^2(D_\mu H^\dagger D^\mu H)$	$\frac{1}{\Lambda^4}C^{(1)}_{H^6}$
$(H^\dagger H)(H^\dagger \tau^I H)(D_\mu H^\dagger \tau^I D^\mu H)$	$\frac{1}{\Lambda^4}C^{(2)}_{H^6}$

Classes X^3H^n	
Operator	**WC**
$f^{ABC}G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\frac{1}{\Lambda^2}C_G$
$f^{ABC}\tilde{G}^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\frac{1}{\Lambda^2}C_{\tilde{G}}$
$f^{ABC}(H^\dagger H)G^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\frac{1}{\Lambda^2}C^{(1)}_{G^3H^2}$
$f^{ABC}(H^\dagger H)\tilde{G}^{A\nu}_\mu G^{B\rho}_\nu G^{C\mu}_\rho$	$\frac{1}{\Lambda^2}C^{(2)}_{G^3H^2}$

Classes ψ^2H^n	
Operator	**WC**
$(H^\dagger H)(\bar{l}_p e_r H)$	$\frac{1}{\Lambda^2}C_{\underset{pr}{eH}}$
$(H^\dagger H)(\bar{q}_p u_r \tilde{H})$	$\frac{1}{\Lambda^2}C_{\underset{pr}{uH}}$
$(H^\dagger H)(\bar{q}_p d_r H)$	$\frac{1}{\Lambda^2}C_{\underset{pr}{dH}}$
$(H^\dagger H)^2(\bar{l}_p e_r H)$	$\frac{1}{\Lambda^4}C_{\underset{pr}{leH^5}}$
$(H^\dagger H)^2(\bar{q}_p u_r \tilde{H})$	$\frac{1}{\Lambda^4}C_{\underset{pr}{quH^5}}$
$(H^\dagger H)^2(\bar{q}_p d_r H)$	$\frac{1}{\Lambda^4}C_{\underset{pr}{qdH^5}}$

Classes X^2H^n	
Operator	**WC**
$(H^\dagger H)(G^A_{\mu\nu}G^{A\mu\nu})$	$\frac{1}{\Lambda^2}C_{HG}$
$(H^\dagger H)(W^I_{\mu\nu}W^{I\mu\nu})$	$\frac{1}{\Lambda^2}C_{HW}$
$(H^\dagger H)(B_{\mu\nu}B^{\mu\nu})$	$\frac{1}{\Lambda^2}C_{HB}$
$(H^\dagger \tau^I H)(W^I_{\mu\nu}B^{\mu\nu})$	$\frac{1}{\Lambda^2}C_{HWB}$
$(H^\dagger H)^2 G^A_{\mu\nu}G^{A\mu\nu}$	$\frac{1}{\Lambda^4}C^{(1)}_{G^2H^4}$
$(H^\dagger H)^2 W^I_{\mu\nu}W^{I\mu\nu}$	$\frac{1}{\Lambda^4}C^{(1)}_{W^2H^4}$
$(H^\dagger \tau^I H)(H^\dagger \tau^J H)W^I_{\mu\nu}W^{J\mu\nu}$	$\frac{1}{\Lambda^4}C^{(3)}_{W^2H^4}$
$(H^\dagger H)(H^\dagger \tau^I H)W^I_{\mu\nu}B^{\mu\nu}$	$\frac{1}{\Lambda^4}C^{(1)}_{WBH^4}$
$(H^\dagger H)^2 B_{\mu\nu}B^{\mu\nu}$	$\frac{1}{\Lambda^4}C^{(1)}_{B^2H^4}$

Classes ψ^2XH^n	
Operator	**WC**
$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H W^I_{\mu\nu}$	$\frac{1}{\Lambda^2}C_{\underset{pr}{eW}}$
$(\bar{l}_p\sigma^{\mu\nu}e_r)H B_{\mu\nu}$	$\frac{1}{\Lambda^2}C_{\underset{pr}{eB}}$
$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\tilde{H} G^A_{\mu\nu}$	$\frac{1}{\Lambda^2}C_{\underset{pr}{uG}}$
$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I \tilde{H} W^I_{\mu\nu}$	$\frac{1}{\Lambda^2}C_{\underset{pr}{uW}}$
$(\bar{q}_p\sigma^{\mu\nu}u_r)\tilde{H} B_{\mu\nu}$	$\frac{1}{\Lambda^2}C_{\underset{pr}{uB}}$
$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H G^A_{\mu\nu}$	$\frac{1}{\Lambda^2}C_{\underset{pr}{dG}}$
$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I H W^I_{\mu\nu}$	$\frac{1}{\Lambda^2}C_{\underset{pr}{dW}}$
$(\bar{q}_p\sigma^{\mu\nu}d_r)H B_{\mu\nu}$	$\frac{1}{\Lambda^2}C_{\underset{pr}{dB}}$
$(\bar{l}_p\sigma^{\mu\nu}e_r)\tau^I H(H^\dagger H)W^I_{\mu\nu}$	$\frac{1}{\Lambda^4}C^{(1)}_{\underset{pr}{leWH^3}}$
$(\bar{l}_p\sigma^{\mu\nu}e_r)H(H^\dagger \tau^I H)W^I_{\mu\nu}$	$\frac{1}{\Lambda^4}C^{(2)}_{\underset{pr}{leWH^3}}$
$(\bar{l}_p\sigma^{\mu\nu}e_r)H(H^\dagger H)B_{\mu\nu}$	$\frac{1}{\Lambda^4}C_{\underset{pr}{leBH^3}}$
$(\bar{q}_p\sigma^{\mu\nu}T^A u_r)\tilde{H}(H^\dagger H)G^A_{\mu\nu}$	$\frac{1}{\Lambda^4}C_{\underset{pr}{quGH^3}}$
$(\bar{q}_p\sigma^{\mu\nu}u_r)\tau^I \tilde{H}(H^\dagger H)W^I_{\mu\nu}$	$\frac{1}{\Lambda^4}C^{(1)}_{\underset{pr}{quWH^3}}$
$(\bar{q}_p\sigma^{\mu\nu}u_r)\tilde{H}(H^\dagger \tau^I H)W^I_{\mu\nu}$	$\frac{1}{\Lambda^4}C^{(2)}_{\underset{pr}{quWH^3}}$
$(\bar{q}_p\sigma^{\mu\nu}u_r)\tilde{H}(H^\dagger H)B_{\mu\nu}$	$\frac{1}{\Lambda^4}C_{\underset{pr}{quBH^3}}$
$(\bar{q}_p\sigma^{\mu\nu}T^A d_r)H(H^\dagger H)G^A_{\mu\nu}$	$\frac{1}{\Lambda^4}C_{\underset{pr}{qdGH^3}}$
$(\bar{q}_p\sigma^{\mu\nu}d_r)\tau^I H(H^\dagger H)W^I_{\mu\nu}$	$\frac{1}{\Lambda^4}C^{(1)}_{\underset{pr}{qdWH^3}}$
$(\bar{q}_p\sigma^{\mu\nu}d_r)H(H^\dagger \tau^I H)W^I_{\mu\nu}$	$\frac{1}{\Lambda^4}C^{(2)}_{\underset{pr}{qdWH^3}}$
$(\bar{q}_p\sigma^{\mu\nu}d_r)H(H^\dagger H)B_{\mu\nu}$	$\frac{1}{\Lambda^4}C_{\underset{pr}{qdBH^3}}$

Classes ψ^2H^nD	
Operator	**WC**
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{l}_p\gamma^\mu l_r)$	$\frac{1}{\Lambda^2}C^{(1)}_{\underset{pr}{Hl}}$
$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{l}_p\tau^I\gamma^\mu l_r)$	$\frac{1}{\Lambda^2}C^{(3)}_{\underset{pr}{Hl}}$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$	$\frac{1}{\Lambda^2}C_{\underset{pr}{He}}$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{q}_p\gamma^\mu q_r)$	$\frac{1}{\Lambda^2}C^{(1)}_{\underset{pr}{Hq}}$
$(H^\dagger i\overleftrightarrow{D}^I_\mu H)(\bar{q}_p\tau^I\gamma^\mu q_r)$	$\frac{1}{\Lambda^2}C^{(3)}_{\underset{pr}{Hq}}$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{u}_p\gamma^\mu u_r)$	$\frac{1}{\Lambda^2}C_{\underset{pr}{Hu}}$
$(H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{d}_p\gamma^\mu d_r)$	$\frac{1}{\Lambda^2}C_{\underset{pr}{Hd}}$
$i(\tilde{H}^\dagger D_\mu H)(\bar{u}_p\gamma^\mu d_r)$	$\frac{1}{\Lambda^2}C_{\underset{pr}{Hud}}$
$i(\bar{l}_p\gamma^\mu l_r)(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\underset{pr}{l^2H^4D}}$
$i(\bar{l}_p\gamma^\mu\tau^I l_r)[(H^\dagger\overleftrightarrow{D}^I_\mu H)(H^\dagger H)+(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger\tau^I H)]$	$\frac{1}{\Lambda^4}C^{(2)}_{\underset{pr}{l^2H^4D}}$
$i\epsilon^{IJK}(\bar{l}_p\gamma^\mu\tau^I l_r)(H^\dagger\overleftrightarrow{D}^J_\mu H)(H^\dagger\tau^K H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\underset{pr}{l^2H^4D}}$
$i(\bar{e}_p\gamma^\mu e_r)(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\underset{pr}{e^2H^4D}}$
$i(\bar{q}_p\gamma^\mu q_r)(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\underset{pr}{q^2H^4D}}$
$i(\bar{q}_p\gamma^\mu\tau^I q_r)[(H^\dagger\overleftrightarrow{D}^I_\mu H)(H^\dagger H)+(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger\tau^I H)]$	$\frac{1}{\Lambda^4}C^{(2)}_{\underset{pr}{q^2H^4D}}$
$i\epsilon^{IJK}(\bar{q}_p\gamma^\mu\tau^I q_r)(H^\dagger\overleftrightarrow{D}^J_\mu H)(H^\dagger\tau^K H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\underset{pr}{q^2H^4D}}$
$i(\bar{u}_p\gamma^\mu u_r)(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\underset{pr}{u^2H^4D}}$
$i(\bar{d}_p\gamma^\mu d_r)(H^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\underset{pr}{d^2H^4D}}$
$i(\bar{u}_p\gamma^\mu d_r)(\tilde{H}^\dagger\overleftrightarrow{D}_\mu H)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\underset{pr}{udH^4D}}$

Table 3: The even-dimensional non-four-fermion SMEFT operators appearing in this paper.

Classes $(\overline{L}L)(\overline{L}L)H^n$	
Operator	**WC**
$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	$\frac{1}{\Lambda^2}C_{\substack{ll\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{qq\\prst}}$
$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\frac{1}{\Lambda^2}C^{(3)}_{\substack{qq\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{lq\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	$\frac{1}{\Lambda^2}C^{(3)}_{\substack{lq\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^4H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu\tau^I l_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^4H^2\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^4H^2\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^4H^2\\prst}}$
$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{q^4H^2\\prst}}$
$\epsilon^{IJK}(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^J q_t)(H^\dagger\tau^K H)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{q^4H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^2q^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu q_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^2q^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{l^2q^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{l^2q^2H^2\\prst}}$
$\epsilon^{IJK}(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^J q_t)(H^\dagger\tau^K H)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{l^2q^2H^2\\prst}}$

Classes $(\overline{R}R)(\overline{R}R)H^n$	
Operator	**WC**
$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	$\frac{1}{\Lambda^2}C_{\substack{ee\\prst}}$
$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	$\frac{1}{\Lambda^2}C_{\substack{uu\\prst}}$
$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C_{\substack{dd\\prst}}$
$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	$\frac{1}{\Lambda^2}C_{\substack{eu\\prst}}$
$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C_{\substack{ed\\prst}}$
$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{ud\\prst}}$
$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\frac{1}{\Lambda^2}C^{(8)}_{\substack{ud\\prst}}$
$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{e^4H^2\\prst}}$
$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{u^4H^2\\prst}}$
$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{d^4H^2\\prst}}$
$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{e^2u^2H^2\\prst}}$
$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{e^2d^2H^2\\prst}}$
$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{u^2d^2H^2\\prst}}$
$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{u^2d^2H^2\\prst}}$

Classes $(\overline{L}R)(\overline{L}R)H^n$	
Operator	**WC**
$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{quqd\\prst}}$
$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$	$\frac{1}{\Lambda^2}C^{(8)}_{\substack{quqd\\prst}}$
$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{lequ\\prst}}$
$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)$	$\frac{1}{\Lambda^2}C^{(3)}_{\substack{lequ\\prst}}$
$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^2udH^2\\prst}}$
$(\bar{q}_p^j u_r)(\tau^I\epsilon)_{jk}(\bar{q}_s^k d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^2udH^2\\prst}}$
$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{q^2udH^2\\prst}}$
$(\bar{q}_p^j T^A u_r)(\tau^I\epsilon)_{jk}(\bar{q}_s^k T^A d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{q^2udH^2\\prst}}$
$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{lequH^2\\prst}}$
$(\bar{l}_p^j e_r)(\tau^I\epsilon)_{jk}(\bar{q}_s^k u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{lequH^2\\prst}}$
$(\bar{l}_p^j\sigma_{\mu\nu}e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{lequH^2\\prst}}$
$(\bar{l}_p^j\sigma_{\mu\nu}e_r)(\tau^I\epsilon)_{jk}(\bar{q}_s^k\sigma^{\mu\nu}u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{lequH^2\\prst}}$
$(\bar{l}_p e_r H)(\bar{l}_s e_t H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{l^2e^2H^2\\prst}}$
$(\bar{l}_p e_r H)(\bar{q}_s d_t H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{leqdH^2\\prst}}$
$(\bar{l}_p\sigma_{\mu\nu}e_r H)(\bar{q}_s\sigma^{\mu\nu}d_t H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{leqdH^2\\prst}}$
$(\bar{q}_p u_r\tilde{H})(\bar{q}_s u_t\tilde{H})$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{q^2u^2H^2\\prst}}$
$(\bar{q}_p T^A u_r\tilde{H})(\bar{q}_s T^A u_t\tilde{H})$	$\frac{1}{\Lambda^4}C^{(6)}_{\substack{q^2u^2H^2\\prst}}$
$(\bar{q}_p d_r H)(\bar{q}_s d_t H)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{q^2d^2H^2\\prst}}$
$(\bar{q}_p T^A d_r H)(\bar{q}_s T^A d_t H)$	$\frac{1}{\Lambda^4}C^{(6)}_{\substack{q^2d^2H^2\\prst}}$

Classes $(\not{B})\psi^4H^n$	
Operator	**WC**
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(d_p^{\alpha T}Cu_r^\beta)(q_{js}^{\gamma T}Cl_{kt})$	$\frac{1}{\Lambda^2}C_{\substack{duq\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(q_{jp}^{\alpha T}Cq_{kr}^\beta)(u_s^{\gamma T}Ce_t)$	$\frac{1}{\Lambda^2}C_{\substack{qqu\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jn}\epsilon^{km}(q_{jp}^{\alpha T}Cq_{kr}^\beta)(q_{ms}^{\gamma T}Cl_{nt})$	$\frac{1}{\Lambda^2}C_{\substack{qqq\\prst}}$
$\epsilon^{\alpha\beta\gamma}(d_p^{\alpha T}Cu_r^\beta)(u_s^{\gamma T}Ce_t)$	$\frac{1}{\Lambda^2}C_{\substack{duu\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(d_p^{\alpha T}Cu_r^\beta)(q_{js}^{\gamma T}Cl_{kt})(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{lqudH^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}(\epsilon\tau^I)^{jk}(d_p^{\alpha T}Cu_r^\beta)(q_{js}^{\gamma T}Cl_{kt})(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{lqudH^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(q_{jp}^{\alpha T}Cq_{mr}^\beta)(u_s^{\gamma T}Ce_t)(H^{m\dagger}H_k)$	$\frac{1}{\Lambda^4}C_{\substack{eq^2uH^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}\epsilon^{mn}(q_{mp}^{\alpha T}Cq_{jr}^\beta)(q_{ks}^{\gamma T}Cl_{nt})(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{lq^3H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}(\epsilon\tau^I)^{mn}(q_{mp}^{\alpha T}Cq_{jr}^\beta)(q_{ks}^{\gamma T}Cl_{nt})(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{lq^3H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}(\epsilon\tau^I)^{jk}\epsilon^{mn}(q_{mp}^{\alpha T}Cq_{jr}^\beta)(q_{ks}^{\gamma T}Cl_{nt})(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{lq^3H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}(d_p^{\alpha T}Cu_r^\beta)(u_s^{\gamma T}Ce_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C_{\substack{eu^2dH^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}\epsilon^{mn}(l_{jp}^TCq_{mr}^\alpha)(u_s^{\beta T}Cu_t^\gamma)\tilde{H}_k\tilde{H}_n$	$\frac{1}{\Lambda^4}C_{\substack{lqu^2H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}\epsilon^{mn}(l_{jp}^TCq_{mr}^\alpha)(d_s^{\beta T}Cd_t^\gamma)H_kH_n$	$\frac{1}{\Lambda^4}C_{\substack{lqd^2H^2\\prst}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{jk}\epsilon^{mn}(e_p^TCd_r^\alpha)(q_{js}^{\beta T}Cq_{mt}^\gamma)H_kH_n$	$\frac{1}{\Lambda^4}C_{\substack{eq^2dH^2\\prst}}$

Classes $(\overline{L}L)(\overline{R}R)H^n$	
Operator	**WC**
$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$	$\frac{1}{\Lambda^2}C_{\substack{le\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$	$\frac{1}{\Lambda^2}C_{\substack{lu\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C_{\substack{ld\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$	$\frac{1}{\Lambda^2}C_{\substack{qe\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{qu\\prst}}$
$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$	$\frac{1}{\Lambda^2}C^{(8)}_{\substack{qu\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$	$\frac{1}{\Lambda^2}C^{(1)}_{\substack{qd\\prst}}$
$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$\frac{1}{\Lambda^2}C^{(8)}_{\substack{qd\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^2e^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{e}_s\gamma^\mu e_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^2e^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^2u^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{u}_s\gamma^\mu u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^2u^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{l^2d^2H^2\\prst}}$
$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{d}_s\gamma^\mu d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{l^2d^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^2e^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{e}_s\gamma^\mu e_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^2e^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^2u^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{u}_s\gamma^\mu u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^2u^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{q^2u^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu\tau^I T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{q^2u^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{q^2d^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{d}_s\gamma^\mu d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{q^2d^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(3)}_{\substack{q^2d^2H^2\\prst}}$
$(\bar{q}_p\gamma_\mu\tau^I T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(4)}_{\substack{q^2d^2H^2\\prst}}$

Classes $(\overline{L}R)(\overline{R}L)H^n$	
Operator	**WC**
$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	$\frac{1}{\Lambda^2}C_{\substack{ledq\\prst}}$
$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)(H^\dagger H)$	$\frac{1}{\Lambda^4}C^{(1)}_{\substack{leqdH^2\\prst}}$
$(\bar{l}_p e_r)\tau^I(\bar{d}_s q_t)(H^\dagger\tau^I H)$	$\frac{1}{\Lambda^4}C^{(2)}_{\substack{leqdH^2\\prst}}$
$(\bar{l}_p d_r H)(\tilde{H}^\dagger\bar{u}_s l_t)$	$\frac{1}{\Lambda^4}C_{\substack{l^2udH^2\\prst}}$
$(\bar{l}_p e_r H)(\tilde{H}^\dagger\bar{u}_s q_t)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{lequH^2\\prst}}$
$(\bar{q}_p d_r H)(\tilde{H}^\dagger\bar{u}_s q_t)$	$\frac{1}{\Lambda^4}C^{(5)}_{\substack{q^2udH^2\\prst}}$
$(\bar{q}_p T^A d_r H)(\tilde{H}^\dagger\bar{u}_s T^A q_t)$	$\frac{1}{\Lambda^4}C^{(6)}_{\substack{q^2udH^2\\prst}}$

Table 4: The even-dimensional four-fermion SMEFT operators appearing in this paper.

B.2 Odd-dimensional operators

There is only one dimension-5 SMEFT operator: $\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})H_j H_l$. The basis for the dimension-7 operators used here is equivalent to those given in Refs. [4, 5].

Classes $\psi^2 H^n$	
Operator	**WC**
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})H_j H_l$	$\frac{1}{\Lambda}C_{\underset{pr}{5}}$
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})H_j H_l(H^\dagger H)$	$\frac{1}{\Lambda^3}C_{\underset{pr}{l^2H^4}}$

Class $\psi^2 H^3 D$	
Operator	**WC**
$i\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C\gamma^\mu e_r)H_j H_k(D_\mu H)_l$	$\frac{1}{\Lambda^3}C_{\underset{pr}{leH^3D}}$

Class $\psi^2 H^2 X$	
Operator	**WC**
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C\sigma_{\mu\nu} l_{kr})H_j H_l B^{\mu\nu}$	$\frac{1}{\Lambda^3}C_{\underset{pr}{l^2H^2B}}$
$\epsilon^{ij}(\epsilon\tau^I)^{kl}(l_{ip}^T C\sigma_{\mu\nu} l_{kr})H_j H_l W^{I\mu\nu}$	$\frac{1}{\Lambda^3}C_{\underset{pr}{l^2H^2W}}$

Class $\psi^4 H$	
Operator	**WC**
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})(\bar{e}_s l_{jt})H_l$	$\frac{1}{\Lambda^3}C_{\underset{prst}{l^3eH}}$
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C l_{kr})(\bar{d}_s q_{lt})H_j$	$\frac{1}{\Lambda^3}C^{(1)}_{\underset{prst}{l^2dqH}}$
$\epsilon^{ij}\epsilon^{kl}(l_{ip}^T C\sigma_{\mu\nu} l_{kr})(\bar{d}_s\sigma^{\mu\nu} q_{lt})H_j$	$\frac{1}{\Lambda^3}C^{(2)}_{\underset{prst}{l^2dqH}}$
$\epsilon^{ij}(l_{ip}^T C l_{kr})(\bar{q}_s^k u_t)H_j$	$\frac{1}{\Lambda^3}C_{\underset{prst}{l^2quH}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{ij}(q_{kp}^{\alpha T} C q_{ir}^\beta)(\bar{l}_s^k d_t^\gamma)\tilde{H}_j$	$\frac{1}{\Lambda^3}C_{\underset{prst}{q^2ldH}}$
$\epsilon^{\alpha\beta\gamma}(d_p^{\alpha T} C d_r^\beta)(\bar{l}_s d_t^\gamma)H$	$\frac{1}{\Lambda^3}C_{\underset{prst}{d^3lH}}$
$\epsilon^{\alpha\beta\gamma}(u_p^{\alpha T} C d_r^\beta)(\bar{l}_s d_t^\gamma)\tilde{H}$	$\frac{1}{\Lambda^3}C_{\underset{prst}{ud^2lH}}$
$\epsilon^{ij}(l_{ip}^T C\gamma_\mu e_r)(\bar{d}_s\gamma^\mu u_t)H_j$	$\frac{1}{\Lambda^3}C_{\underset{prst}{leduH}}$
$\epsilon^{\alpha\beta\gamma}\epsilon^{ij}(d_p^{\alpha T} C d_r^\beta)(\bar{e}_s q_{it}^\gamma)\tilde{H}_j$	$\frac{1}{\Lambda^3}C_{\underset{prst}{eqd^2H}}$

Table 5: The odd-dimensional SMEFT operators appearing in this paper.

C Useful Fierz identities

The following Fierz identities are needed to derive the matching conditions given in this paper.

C.1 For $(\overline{L}L)(\overline{L}L)$ and $(\overline{R}R)(\overline{R}R)$ operators

In the case of a four-lepton operator, the identities take the form

$$(\bar{\nu}_{Lp}\gamma^\mu e_{Lt})(\bar{e}_{Ls}\gamma_\mu \nu_{Lr}) = (\bar{\nu}_{Lp}\gamma^\mu \nu_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt}). \tag{57}$$

In the case of a four-quark operator, color has to be taken into consideration. This is done through the identity $\delta_{\alpha\lambda}\delta_{\kappa\beta} = 2T^A_{\alpha\beta}T^A_{\kappa\lambda} + \frac{1}{3}\delta_{\alpha\beta}\delta_{\kappa\lambda}$. The identities are (for instance)

$$(\overline{u}_{Lp}\gamma^\mu d_{Lt})(\overline{d}_{Ls}\gamma_\mu u_{Lr}) = 2(\overline{u}_{Lp}\gamma^\mu T^A u_{Lr})(\overline{d}_{Ls}\gamma_\mu T^A d_{Lt}) + \frac{1}{3}(\overline{u}_{Lp}\gamma^\mu u_{Lr})(\overline{d}_{Ls}\gamma_\mu d_{Lt}). \tag{58}$$

C.2 For $(\overline{L}L)(\overline{R}R)$ operators

In the case of a four-lepton operator or a two-lepton and two-quark operator, the Fierz identities take the form

$$(\overline{\nu}_{Lp}e_{Rt})(\overline{e}_{Rs}\nu_{Lr}) = -\frac{1}{2}(\overline{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\overline{e}_{Rs}\gamma_\mu e_{Rt}). \tag{59}$$

In the case of a four-quark operator, the identities take the following forms:

$$(\overline{u}_{Lp}d_{Rt})(\overline{d}_{Rs}u_{Lr}) = -(\overline{u}_{Lp}\gamma^\mu T^A u_{Lr})(\overline{d}_{Rs}\gamma_\mu T^A d_{Rt}) - \frac{1}{6}(\overline{u}_{Lp}\gamma^\mu u_{Lr})(\overline{d}_{Rs}\gamma_\mu d_{Rt})\ , \tag{60}$$

$$(\overline{u}_{Lp}T^A d_{Rt})(\overline{d}_{Rs}T^A u_{Lr}) = -\frac{2}{9}(\overline{u}_{Lp}\gamma^\mu u_{Lr})(\overline{d}_{Rs}\gamma_\mu d_{Rt}) + \frac{1}{6}(\overline{u}_{Lp}\gamma^\mu T^A u_{Lr})(\overline{d}_{Rs}\gamma_\mu T^A d_{Rt}). \tag{61}$$

C.3 For fermion number-violating operators

The needed identities take the form

$$(\nu^T_{Lr}C\nu_{Lr})(\overline{e}_{Rs}e_{Lt}) = -\frac{1}{2}(\nu^T_{Lp}Ce_{Lt})(\overline{e}_{Rs}\nu_{Lr}) - \frac{1}{8}(\nu^T_{Lp}C\sigma_{\mu\nu}e_{Lt})(\overline{e}_{Rs}\sigma^{\mu\nu}\nu_{Lr})\ , \tag{62}$$

$$(\nu^T_{Lr}C\nu_{Lr})(\overline{e}_{Ls}e_{Rt}) = -\frac{1}{2}(\nu^T_{Lp}C\gamma_\mu e_{Rt})(\overline{e}_{Rs}\gamma^\mu\nu_{Rr})\ , \tag{63}$$

$$(\nu^T_{Lr}C\nu_{Lr})(e^T_{Ls}Ce_{Lt}) = -\frac{1}{2}(\nu^T_{Lp}Ce_{Lt})(e^T_{Ls}C\nu_{Lr}) - \frac{1}{8}(\nu^T_{Lp}C\sigma_{\mu\nu}e_{Lt})(e^T_{Ls}C\sigma^{\mu\nu}\nu_{Lr})\ . \tag{64}$$

D Matching conditions

D.1 $\nu\nu+$h.c. operator

LEFT WC	Matching
$\Lambda C_{\underset{pr}{\nu}}$	$\frac{{v_T}^2}{2\Lambda}\left[C_{\underset{pr}{5}} + \frac{{v_T}^2}{2\Lambda^2}C_{\underset{pr}{l^2H^4}}\right]$

D.2 $(\nu\nu)X+$ h.c. and $(\overline{L}R)X+$ h.c. operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C_{\underset{pr}{\nu\gamma}}$	$\frac{{v_T}^2}{2 g_Z \Lambda^3}\left[g C_{\underset{pr}{l^2H^2B}} - \frac{g'}{2}\left(C_{\underset{pr}{l^2H^2W}} - C_{\underset{rp}{l^2H^2W}} \right)\right]$
$\frac{1}{\Lambda^2} C_{\underset{pr}{e\gamma}}$	$\frac{v_T}{\sqrt{2}\, g_Z\, \Lambda^2}\left[\left(g C_{\underset{pr}{eB}} - g' C_{\underset{pr}{eW}} \right) + \frac{{v_T}^2}{2\Lambda^2}\left(g C_{\underset{pr}{leBH^3}} - g' C^{(1)}_{\underset{pr}{leWH^3}} - g' C^{(2)}_{\underset{pr}{leWH^3}} \right)\right]$
$\frac{1}{\Lambda^2} C_{\underset{pr}{u\gamma}}$	$\frac{v_T}{\sqrt{2}\, g_Z\, \Lambda^2}\left[\left(g C_{\underset{pr}{uB}} + g' C_{\underset{pr}{uW}} \right) + \frac{{v_T}^2}{2\Lambda^2}\left(g C_{\underset{pr}{quBH^3}} + g' C^{(1)}_{\underset{pr}{quWH^3}} - g' C^{(2)}_{\underset{pr}{quWH^3}} \right)\right]$
$\frac{1}{\Lambda^2} C_{\underset{pr}{d\gamma}}$	$\frac{v_T}{\sqrt{2}\, g_Z\, \Lambda^2}\left[\left(g C_{\underset{pr}{dB}} - g' C_{\underset{pr}{dW}} \right) + \frac{{v_T}^2}{2\Lambda^2}\left(g C_{\underset{pr}{qdBH^3}} - g' C^{(1)}_{\underset{pr}{qdWH^3}} - g' C^{(2)}_{\underset{pr}{qdWH^3}} \right)\right]$
$\frac{1}{\Lambda^2} C_{\underset{pr}{uG}}$	$\frac{v_T}{\sqrt{2}\, \Lambda^2}\left[C_{\underset{pr}{uG}} + \frac{{v_T}^2}{2\Lambda^2} C_{\underset{pr}{quGH^3}} \right]$
$\frac{1}{\Lambda^2} C_{\underset{pr}{dG}}$	$\frac{v_T}{\sqrt{2}\, \Lambda^2}\left[C_{\underset{pr}{dG}} + \frac{{v_T}^2}{2\Lambda^2} C_{\underset{pr}{qdGH^3}} \right]$

The non-physical ratios g/g_Z and g'/g_Z appearing here can be expressed in terms of the corrected coupling constants $\overline{g}$ and $\overline{g}'$ and the SMEFT WC's, using the following equations:

$$\frac{g}{g_Z} = \frac{\overline{g}}{\sqrt{\overline{g}^2 + \overline{g}'^2}} \left[\begin{array}{l} 1 + \frac{\overline{g}'^2 {v_T}^2}{(\overline{g}^2 + \overline{g}'^2)\Lambda^2}(C_{HB} - C_{HW}) \\ + \frac{\overline{g}'^2 {v_T}^4}{2(\overline{g}^2 + \overline{g}'^2)\Lambda^4}(C^{(1)}_{B^2H^4} - C^{(1)}_{W^2H^4}) \\ + \frac{{v_T}^4}{2(\overline{g}^2 + \overline{g}'^2)^2\Lambda^4} \left(\begin{array}{l} 3\overline{g}'^4\,[C_{HB}]^2 - \overline{g}'^2[4\overline{g}^2 + \overline{g}'^2]\,[C_{HW}]^2 \\ + 2\overline{g}'^2[2\overline{g}^2 - \overline{g}'^2]\, C_{HW}\, C_{HB} \end{array} \right) \end{array} \right], \tag{65}$$

$$\frac{g'}{g_Z} = \frac{\overline{g}'}{\sqrt{\overline{g}^2 + \overline{g}'^2}} \left[\begin{array}{l} 1 + \frac{\overline{g}^2 {v_T}^2}{(\overline{g}^2 + \overline{g}'^2)\Lambda^2}(C_{HW} - C_{HB}) \\ + \frac{\overline{g}^2 {v_T}^4}{2(\overline{g}^2 + \overline{g}'^2)\Lambda^4}(C^{(1)}_{W^2H^4} - C^{(1)}_{B^2H^4}) \\ + \frac{{v_T}^4}{2(\overline{g}^2 + \overline{g}'^2)^2\Lambda^4} \left(\begin{array}{l} 3\overline{g}^4\,[C_{HW}]^2 - \overline{g}^2[4\overline{g}'^2 + \overline{g}^2]\,[C_{HB}]^2 \\ + 2\overline{g}^2[2\overline{g}'^2 - \overline{g}^2]\, C_{HW}\, C_{HB} \end{array} \right) \end{array} \right]. \tag{66}$$

D.3 X^3 operators

LEFT WC	Matching
$\frac{1}{\Lambda^2}\mathcal{C}_G$	$\frac{1}{\Lambda^2}\left[C_G + \frac{{v_T}^2}{2\Lambda^2} C^{(1)}_{G^3H^2}\right]$
$\frac{1}{\Lambda^2}\mathcal{C}_{\tilde{G}}$	$\frac{1}{\Lambda^2}\left[C_{\tilde{G}} + \frac{{v_T}^2}{2\Lambda^2} C^{(2)}_{G^3H^2}\right]$

D.4 $(\overline{L}L)(\overline{L}L)$ operators

LEFT WC	Matching
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{\nu\nu}}$	$\frac{1}{\Lambda^2}\left[C_{\underset{prst}{ll}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{4{M_Z}^2}\left([Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{\nu_L}]^{\text{eff}}_{st} + [Z_{\nu_L}]^{\text{eff}}_{pt}[Z_{\nu_L}]^{\text{eff}}_{sr}\right)$
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{ee}}$	$\frac{1}{\Lambda^2}\left[C_{\underset{prst}{ll}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} + C^{(2)}_{\underset{prst}{l^4H^2}} + C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{4{M_Z}^2}\left([Z_{e_L}]^{\text{eff}}_{pr}[Z_{e_L}]^{\text{eff}}_{st} + [Z_{e_L}]^{\text{eff}}_{pt}[Z_{e_L}]^{\text{eff}}_{sr}\right)$
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{\nu e}}$	$\frac{1}{\Lambda^2}\left[\left(C_{\underset{prst}{ll}} + C_{\underset{stpr}{ll}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{l^4H^2}} + C^{(1)}_{\underset{stpr}{l^4H^2}} + C^{(2)}_{\underset{prst}{l^4H^2}} - C^{(2)}_{\underset{stpr}{l^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{e_L}]^{\text{eff}}_{st} - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pt}[W_l]^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{uu}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\underset{prst}{qq}} + C^{(3)}_{\underset{prst}{qq}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{q^4H^2}} - C^{(2)}_{\underset{prst}{q^4H^2}} - C^{(2)}_{\underset{stpr}{q^4H^2}} + C^{(3)}_{\underset{prst}{q^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{2{M_Z}^2}[Z_{u_L}]^{\text{eff}}_{pr}[Z_{u_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\underset{prst}{dd}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\underset{prst}{qq}} + C^{(3)}_{\underset{prst}{qq}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\underset{prst}{q^4H^2}} + C^{(2)}_{\underset{prst}{q^4H^2}} + C^{(2)}_{\underset{stpr}{q^4H^2}} + C^{(3)}_{\underset{prst}{q^4H^2}}\right)\right]$ $-\frac{{\overline{g_Z}}^2}{2{M_Z}^2}[Z_{d_L}]^{\text{eff}}_{pr}[Z_{d_L}]^{\text{eff}}_{st}$

LEFT WC	Matching
$\frac{1}{\Lambda^2} C^{V1,LL}_{\substack{ud \\ prst}}$	$\frac{1}{\Lambda^2}\left[\begin{array}{c}\left(C^{(1)}_{\substack{qq\\prst}} + C^{(1)}_{\substack{qq\\stpr}} - C^{(3)}_{\substack{qq\\prst}} - C^{(3)}_{\substack{qq\\stpr}} + \frac{2}{3}C^{(3)}_{\substack{qq\\ptsr}} + \frac{2}{3}C^{(3)}_{\substack{qq\\srpt}}\right) \\ + \frac{{v_T}^2}{2\Lambda^2}\left(\begin{array}{c} C^{(1)}_{\substack{q^4H^2\\prst}} + C^{(1)}_{\substack{q^4H^2\\stpr}} + C^{(2)}_{\substack{q^4H^2\\prst}} - C^{(2)}_{\substack{q^4H^2\\stpr}} - C^{(3)}_{\substack{q^4H^2\\prst}} - C^{(3)}_{\substack{q^4H^2\\stpr}} \\ + \frac{2}{3}C^{(3)}_{\substack{q^4H^2\\ptsr}} + \frac{2}{3}C^{(3)}_{\substack{q^4H^2\\srpt}} + \frac{2i}{3}C^{(5)}_{\substack{q^4H^2\\ptsr}} - \frac{2i}{3}C^{(5)}_{\substack{q^4H^2\\srpt}}\end{array}\right)\end{array}\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_L}]^{\text{eff}}_{pr}[Z_{d_L}]^{\text{eff}}_{st} - \frac{\overline{g}^2}{6{M_W}^2}[W_q]^{\text{eff}}_{pt}[W_q]^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2} C^{V8,LL}_{\substack{ud \\ prst}}$	$\frac{4}{\Lambda^2}\left[\left(C^{(3)}_{\substack{qq\\ptsr}} + C^{(3)}_{\substack{qq\\srpt}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^4H^2\\ptsr}} + C^{(3)}_{\substack{q^4H^2\\srpt}} - iC^{(5)}_{\substack{q^4H^2\\prst}} + iC^{(5)}_{\substack{q^4H^2\\stpr}}\right)\right]$ $-\frac{\overline{g}^2}{{M_W}^2}[W_q]^{\text{eff}}_{pt}[W_q]^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\substack{\nu u \\ prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{lq\\prst}} + C^{(3)}_{\substack{lq\\prst}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2q^2H^2\\prst}} - C^{(2)}_{\substack{l^2q^2H^2\\prst}} + C^{(3)}_{\substack{l^2q^2H^2\\prst}} - C^{(4)}_{\substack{l^2q^2H^2\\prst}}\right)\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{u_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\substack{\nu d \\ prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{lq\\prst}} - C^{(3)}_{\substack{lq\\prst}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2q^2H^2\\prst}} - C^{(2)}_{\substack{l^2q^2H^2\\prst}} - C^{(3)}_{\substack{l^2q^2H^2\\prst}} + C^{(4)}_{\substack{l^2q^2H^2\\prst}}\right)\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\text{eff}}_{pr}[Z_{d_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\substack{eu \\ prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{lq\\prst}} - C^{(3)}_{\substack{lq\\prst}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2q^2H^2\\prst}} + C^{(2)}_{\substack{l^2q^2H^2\\prst}} - C^{(3)}_{\substack{l^2q^2H^2\\prst}} - C^{(4)}_{\substack{l^2q^2H^2\\prst}}\right)\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{u_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\substack{ed \\ prst}}$	$\frac{1}{\Lambda^2}\left[\left(C^{(1)}_{\substack{lq\\prst}} + C^{(3)}_{\substack{lq\\prst}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2q^2H^2\\prst}} + C^{(2)}_{\substack{l^2q^2H^2\\prst}} + C^{(3)}_{\substack{l^2q^2H^2\\prst}} + C^{(4)}_{\substack{l^2q^2H^2\\prst}}\right)\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\text{eff}}_{pr}[Z_{d_L}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{V,LL}_{\substack{\nu edu \\ prst}} + \text{h.c.}$	$\frac{2}{\Lambda^2}\left[C^{(3)}_{\substack{lq\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{l^2q^2H^2\\prst}} - iC^{(5)}_{\substack{l^2q^2H^2\\prst}}\right)\right] - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pr}[W_q]^{\text{eff}*}_{ts} + \text{c.c.}$

D.5 $(\overline{R}R)(\overline{R}R)$ operators

LEFT WC	Matching
$\frac{1}{\Lambda^2} C^{V,RR}_{\substack{ee\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{ee\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2} C_{\substack{e^4H^2\\prst}}\right] - \frac{\overline{g_Z}^2}{4{M_Z}^2}\left([Z_{e_R}]^{\rm eff}_{pr}[Z_{e_R}]^{\rm eff}_{st} + [Z_{e_R}]^{\rm eff}_{pt}[Z_{e_R}]^{\rm eff}_{sr}\right)$
$\frac{1}{\Lambda^2} C^{V,RR}_{\substack{eu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{eu\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2} C_{\substack{e^2u^2H^2\\prst}}\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_R}]^{\rm eff}_{pr}[Z_{u_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,RR}_{\substack{ed\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{ed\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2} C_{\substack{e^2d^2H^2\\prst}}\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_R}]^{\rm eff}_{pr}[Z_{d_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,RR}_{\substack{uu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{uu\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2} C_{\substack{u^4H^2\\prst}}\right] - \frac{\overline{g_Z}^2}{2{M_Z}^2}[Z_{u_R}]^{\rm eff}_{pr}[Z_{u_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,RR}_{\substack{dd\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{dd\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2} C_{\substack{d^4H^2\\prst}}\right] - \frac{\overline{g_Z}^2}{2{M_Z}^2}[Z_{d_R}]^{\rm eff}_{pr}[Z_{d_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V1,RR}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{ud\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2} C^{(1)}_{\substack{u^2d^2H^2\\prst}}\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_R}]^{\rm eff}_{pr}[Z_{d_R}]^{\rm eff}_{st} - \frac{\overline{g}^2}{6{M_W}^2}[W_R]^{\rm eff}_{pt}[W_R]^{\rm eff*}_{rs}$
$\frac{1}{\Lambda^2} C^{V8,RR}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{ud\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2} C^{(2)}_{\substack{u^2d^2H^2\\prst}}\right] - \frac{\overline{g}^2}{{M_W}^2}[W_R]^{\rm eff}_{pt}[W_R]^{\rm eff*}_{rs}$

D.6 $(\overline{L}L)(\overline{R}R)$ operators

LEFT WC	Matching
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{\nu e\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{le\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2e^2H^2\\prst}} - C^{(2)}_{\substack{l^2e^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\rm eff}_{pr}[Z_{e_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{ee\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{le\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2e^2H^2\\prst}} + C^{(2)}_{\substack{l^2e^2H^2\\prst}}\right)\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\rm eff}_{pr}[Z_{e_R}]^{\rm eff}_{st} - \frac{1}{2{m_h}^2}(Y_e)^{\rm eff}_{pt}(Y_e)^{\rm eff*}_{rs}$
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{\nu u\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{lu\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2u^2H^2\\prst}} - C^{(2)}_{\substack{l^2u^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\rm eff}_{pr}[Z_{u_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{eu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{lu\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2u^2H^2\\prst}} + C^{(2)}_{\substack{l^2u^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\rm eff}_{pr}[Z_{u_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{\nu d\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{ld\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2d^2H^2\\prst}} - C^{(2)}_{\substack{l^2d^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{\nu_L}]^{\rm eff}_{pr}[Z_{d_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{ed\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{ld\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{l^2d^2H^2\\prst}} + C^{(2)}_{\substack{l^2d^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{e_L}]^{\rm eff}_{pr}[Z_{d_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{ue\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{qe\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2e^2H^2\\prst}} - C^{(2)}_{\substack{q^2e^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_L}]^{\rm eff}_{pr}[Z_{e_R}]^{\rm eff}_{st}$
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{de\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{qe\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2e^2H^2\\prst}} + C^{(2)}_{\substack{q^2e^2H^2\\prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{d_L}]^{\rm eff}_{pr}[Z_{e_R}]^{\rm eff}_{st}$

LEFT WC	Matching
$\frac{1}{\Lambda^2} C^{V1,LR}_{\substack{uu \\ prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{qu \\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2u^2H^2 \\ prst}} - C^{(2)}_{\substack{q^2u^2H^2 \\ prst}}\right)\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_L}]^{\text{eff}}_{pr}[Z_{u_R}]^{\text{eff}}_{st} - \frac{1}{6{m_h}^2}(Y_u)^{\text{eff}}_{pt}(Y_u)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2} C^{V1,LR}_{\substack{du \\ prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{qu \\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2u^2H^2 \\ prst}} + C^{(2)}_{\substack{q^2u^2H^2 \\ prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{d_L}]^{\text{eff}}_{pr}[Z_{u_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{V8,LR}_{\substack{uu \\ prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{qu \\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2u^2H^2 \\ prst}} - C^{(4)}_{\substack{q^2u^2H^2 \\ prst}}\right)\right] - \frac{1}{{m_h}^2}(Y_u)^{\text{eff}}_{pt}(Y_u)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2} C^{V8,LR}_{\substack{du \\ prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{qu \\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2u^2H^2 \\ prst}} + C^{(4)}_{\substack{q^2u^2H^2 \\ prst}}\right)\right]$
$\frac{1}{\Lambda^2} C^{V1,LR}_{\substack{ud \\ prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{qd \\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2d^2H^2 \\ prst}} - C^{(2)}_{\substack{q^2d^2H^2 \\ prst}}\right)\right] - \frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{u_L}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{V1,LR}_{\substack{dd \\ prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{qd \\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2d^2H^2 \\ prst}} + C^{(2)}_{\substack{q^2d^2H^2 \\ prst}}\right)\right]$ $-\frac{\overline{g_Z}^2}{{M_Z}^2}[Z_{d_L}]^{\text{eff}}_{pr}[Z_{d_R}]^{\text{eff}}_{st} - \frac{1}{6{m_h}^2}(Y_d)^{\text{eff}}_{pt}(Y_d)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2} C^{V8,LR}_{\substack{ud \\ prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{qd \\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2d^2H^2 \\ prst}} - C^{(4)}_{\substack{q^2d^2H^2 \\ prst}}\right)\right]$
$\frac{1}{\Lambda^2} C^{V8,LR}_{\substack{dd \\ prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{qd \\ prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2d^2H^2 \\ prst}} + C^{(4)}_{\substack{q^2d^2H^2 \\ prst}}\right)\right] - \frac{1}{{m_h}^2}(Y_d)^{\text{eff}}_{pt}(Y_d)^{\text{eff}*}_{rs}$
$\frac{1}{\Lambda^2} C^{V,LR}_{\substack{\nu edu \\ prst}} + \text{h.c.}$	$\frac{{v_T}^2}{4\Lambda^4} {C_{\substack{l^2udH^2 \\ tprs}}}^{*} - \frac{\overline{g}^2}{2{M_W}^2}[W_l]^{\text{eff}}_{pr}[W_R]^{\text{eff}*}_{ts} + \text{c.c.}$
$\frac{1}{\Lambda^2} C^{V1,LR}_{\substack{uddu \\ prst}} + \text{h.c.}$	$\frac{{v_T}^2}{2\Lambda^4}\left(\frac{1}{6}{C^{(5)}_{\substack{q^2udH^2 \\ tprs}}}^{*} + \frac{2}{9}{C^{(6)}_{\substack{q^2udH^2 \\ tprs}}}^{*}\right)$ $-\frac{\overline{g}^2}{2{M_W}^2}[W_q]^{\text{eff}}_{pr}[W_R]^{\text{eff}*}_{ts} - \frac{1}{6{m_h}^2}(Y_u)^{\text{eff}}_{pt}(Y_d)^{\text{eff}*}_{rs} + \text{c.c.}$
$\frac{1}{\Lambda^2} C^{V8,LR}_{\substack{uddu \\ prst}} + \text{h.c.}$	$\frac{{v_T}^2}{2\Lambda^4}\left({C^{(5)}_{\substack{q^2udH^2 \\ tprs}}}^{*} - \frac{1}{6}{C^{(6)}_{\substack{q^2udH^2 \\ tprs}}}^{*}\right)$ $-\frac{1}{{m_h}^2}(Y_u)^{\text{eff}}_{pt}(Y_d)^{\text{eff}*}_{rs} + \text{c.c.}$

D.7 $(\overline{L}R)(\overline{L}R)$ operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{ee\\prst}}$	$\frac{{\upsilon_T}^2}{2\Lambda^4}\, C^{(3)}_{\substack{l^2e^2H^2\\prst}} + \frac{1}{2{m_h}^2}\,(Y_e)^{\text{eff}}_{pr}(Y_e)^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{eu\\prst}}$	$\frac{1}{\Lambda^2}\left[-C^{(1)}_{\substack{lequ\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(-C^{(1)}_{\substack{lequH^2\\prst}} - C^{(2)}_{\substack{lequH^2\\prst}}\right)\right] + \frac{1}{{m_h}^2}\,(Y_e)^{\text{eff}}_{pr}(Y_u)^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{T,RR}_{\substack{eu\\prst}}$	$\frac{1}{\Lambda^2}\left[-C^{(3)}_{\substack{lequ\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(-C^{(3)}_{\substack{lequH^2\\prst}} - C^{(4)}_{\substack{lequH^2\\prst}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{ed\\prst}}$	$\frac{{\upsilon_T}^2}{2\Lambda^4} C^{(3)}_{\substack{leqdH^2\\prst}} + \frac{1}{{m_h}^2}\,(Y_e)^{\text{eff}}_{pr}(Y_d)^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{T,RR}_{\substack{ed\\prst}}$	$\frac{{\upsilon_T}^2}{2\Lambda^4} C^{(4)}_{\substack{leqdH^2\\prst}}$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{\nu edu\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{lequ\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{lequH^2\\prst}} - C^{(2)}_{\substack{lequH^2\\prst}}\right)\right]$
$\frac{1}{\Lambda^2} C^{T,RR}_{\substack{\nu edu\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(3)}_{\substack{lequ\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{lequH^2\\prst}} - C^{(4)}_{\substack{lequH^2\\prst}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S1,RR}_{\substack{uu\\prst}}$	$\frac{{\upsilon_T}^2}{2\Lambda^4}\, C^{(5)}_{\substack{q^2u^2H^2\\prst}} + \frac{1}{2{m_h}^2}\,(Y_u)^{\text{eff}}_{pr}(Y_u)^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{S8,RR}_{\substack{uu\\prst}}$	$\frac{{\upsilon_T}^2}{2\Lambda^4}\, C^{(6)}_{\substack{q^2u^2H^2\\prst}}$
$\frac{1}{\Lambda^2} C^{S1,RR}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(1)}_{\substack{quqd\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{q^2udH^2\\prst}} - C^{(2)}_{\substack{q^2udH^2\\prst}}\right)\right] + \frac{1}{{m_h}^2}\,(Y_u)^{\text{eff}}_{pr}(Y_d)^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{S8,RR}_{\substack{ud\\prst}}$	$\frac{1}{\Lambda^2}\left[C^{(8)}_{\substack{quqd\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(C^{(3)}_{\substack{q^2udH^2\\prst}} - C^{(4)}_{\substack{q^2udH^2\\prst}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S1,RR}_{\substack{dd\\prst}}$	$\frac{{\upsilon_T}^2}{2\Lambda^4}\, C^{(5)}_{\substack{q^2d^2H^2\\prst}} + \frac{1}{2{m_h}^2}\,(Y_d)^{\text{eff}}_{pr}(Y_d)^{\text{eff}}_{st}$
$\frac{1}{\Lambda^2} C^{S8,RR}_{\substack{dd\\prst}}$	$\frac{{\upsilon_T}^2}{2\Lambda^4}\, C^{(6)}_{\substack{q^2d^2H^2\\prst}}$
$\frac{1}{\Lambda^2} C^{S1,RR}_{\substack{uddu\\prst}}$	$\frac{1}{\Lambda^2}\left[-C^{(1)}_{\substack{quqd\\stpr}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(-C^{(1)}_{\substack{q^2udH^2\\stpr}} - C^{(2)}_{\substack{q^2udH^2\\stpr}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S8,RR}_{\substack{uddu\\prst}}$	$\frac{1}{\Lambda^2}\left[-C^{(8)}_{\substack{quqd\\prst}} + \frac{{\upsilon_T}^2}{2\Lambda^2}\left(-C^{(3)}_{\substack{q^2udH^2\\stpr}} - C^{(4)}_{\substack{q^2udH^2\\stpr}}\right)\right]$

D.8 $(\overline{L}R)(\overline{R}L)+$ h.c. operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{eu \\ prst}}$	$\frac{{v_T}^2}{2\Lambda^4} C^{(5)}_{\substack{lequH^2 \\ prst}} + \frac{1}{{m_h}^2} (Y_e)^{\text{eff}}_{pr} (Y_u)^{\text{eff}*}_{ts}$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{ed \\ prst}}$	$\frac{1}{\Lambda^2} \left[C_{\substack{ledq \\ prst}} + \frac{{v_T}^2}{2\Lambda^2} \left(C^{(1)}_{\substack{leqdH^2 \\ prst}} + C^{(2)}_{\substack{leqdH^2 \\ prst}} \right) \right] + \frac{1}{{m_h}^2} (Y_e)^{\text{eff}}_{pr} (Y_d)^{\text{eff}*}_{ts}$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{\nu edu \\ prst}} + \text{h.c.}$	$\frac{1}{\Lambda^2} \left[C_{\substack{ledq \\ prst}} + \frac{{v_T}^2}{2\Lambda^2} \left(C^{(1)}_{\substack{leqdH^2 \\ prst}} - C^{(2)}_{\substack{leqdH^2 \\ prst}} \right) \right]$

D.9 $\Delta L = 4+$ h.c. operator

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,LL}_{\substack{\nu\nu \\ prst}}$	0

D.10 $\Delta L = 2 +$ h.c. operators

LEFT WC (+ c.c.)	**Matching (+ c.c.)**
$\frac{1}{\Lambda^2} C^{S,LL}_{\underset{prst}{\nu e}}$	$\frac{v_T}{2\sqrt{2}\,\Lambda^3}\left[\left(C_{\underset{prst}{l^3eH}} + C_{\underset{rpst}{l^3eH}}\right) + \frac{1}{2}\left(C_{\underset{tpsr}{l^3eH}} + C_{\underset{trsp}{l^3eH}}\right)\right]$
$\frac{1}{\Lambda^2} C^{T,LL}_{\underset{prst}{\nu e}}$	$\frac{v_T}{16\sqrt{2}\,\Lambda^3}\left(C_{\underset{tpsr}{l^3eH}} - C_{\underset{trsp}{l^3eH}}\right)$
$\frac{1}{\Lambda^2} C^{S,LR}_{\underset{prst}{\nu e}}$	$\frac{\overline{g}^2}{2{M_W}^2}\left([W_l^{\not{L}}]^{\text{eff}}_{pt}[W_l]^{\text{eff}*}_{rs} + [W_l^{\not{L}}]^{\text{eff}}_{rt}[W_l]^{\text{eff}*}_{ps}\right)$
$\frac{1}{\Lambda^2} C^{S,LL}_{\underset{prst}{\nu u}}$	0
$\frac{1}{\Lambda^2} C^{T,LL}_{\underset{prst}{\nu u}}$	0
$\frac{1}{\Lambda^2} C^{S,LR}_{\underset{prst}{\nu u}}$	$\frac{v_T}{2\sqrt{2}\,\Lambda^3}\left(C_{\underset{prst}{l^2quH}} + C_{\underset{rpst}{l^2quH}}\right)$
$\frac{1}{\Lambda^2} C^{S,LL}_{\underset{prst}{\nu d}}$	$\frac{v_T}{2\sqrt{2}\,\Lambda^3}\left(C^{(1)}_{\underset{prst}{l^2dqH}} + C^{(1)}_{\underset{rpst}{l^2dqH}}\right)$
$\frac{1}{\Lambda^2} C^{T,LL}_{\underset{prst}{\nu d}}$	$\frac{v_T}{2\sqrt{2}\,\Lambda^3}\left(C^{(2)}_{\underset{prst}{l^2dqH}} - C^{(2)}_{\underset{rpst}{l^2dqH}}\right)$
$\frac{1}{\Lambda^2} C^{S,LR}_{\underset{prst}{\nu d}}$	0
$\frac{1}{\Lambda^2} C^{S,LL}_{\underset{prst}{\nu edu}}$	$-\frac{v_T}{\sqrt{2}\,\Lambda^3} C^{(1)}_{\underset{prst}{l^2qdH}}$
$\frac{1}{\Lambda^2} C^{T,LL}_{\underset{prst}{\nu edu}}$	$-\frac{v_T}{\sqrt{2}\,\Lambda^3} C^{(2)}_{\underset{prst}{l^2qdH}}$
$\frac{1}{\Lambda^2} C^{S,LR}_{\underset{prst}{\nu edu}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\underset{prst}{l^2quH}}$
$\frac{1}{\Lambda^2} C^{V,RL}_{\underset{prst}{\nu edu}}$	$-\frac{\overline{g}^2}{2{M_W}^2}[W_l^{\not{L}}]^{\text{eff}}_{pr}[W_q]^{\text{eff}*}_{ts}$
$\frac{1}{\Lambda^2} C^{V,RR}_{\underset{prst}{\nu edu}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\underset{prst}{leduH}}$

D.11 $\Delta B = \Delta L = 1+$ h.c. operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,LL}_{\substack{udd\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C_{\substack{qqq\\rpst}} + C_{\substack{qqq\\srpt}} - C_{\substack{qqq\\rspt}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{lq^3H^2\\rpst}} + C^{(1)}_{\substack{lq^3H^2\\srpt}} - C^{(1)}_{\substack{lq^3H^2\\rspt}} + C^{(2)}_{\substack{lq^3H^2\\rpst}} + C^{(2)}_{\substack{lq^3H^2\\srpt}} - C^{(2)}_{\substack{lq^3H^2\\rspt}} - C^{(3)}_{\substack{lq^3H^2\\rpst}} - C^{(3)}_{\substack{lq^3H^2\\srpt}} + C^{(3)}_{\substack{lq^3H^2\\rspt}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S,LL}_{\substack{duu\\prst}}$	$\frac{1}{\Lambda^2}\left[\left(C_{\substack{qqq\\rpst}} + C_{\substack{qqq\\srpt}} - C_{\substack{qqq\\rspt}}\right) + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{lq^3H^2\\rpst}} + C^{(1)}_{\substack{lq^3H^2\\srpt}} - C^{(1)}_{\substack{lq^3H^2\\rspt}} - C^{(2)}_{\substack{lq^3H^2\\rpst}} - C^{(2)}_{\substack{lq^3H^2\\srpt}} + C^{(2)}_{\substack{lq^3H^2\\rspt}} + C^{(3)}_{\substack{lq^3H^2\\rpst}} + C^{(3)}_{\substack{lq^3H^2\\srpt}} - C^{(3)}_{\substack{lq^3H^2\\rspt}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{uud\\prst}}$	$\frac{{v_T}^2}{2\Lambda^4} C_{\substack{eq^2dH^2\\tspr}}$
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{duu\\prst}}$	$-\frac{1}{\Lambda^2}\left[\left(C_{\substack{qqu\\prst}} + C_{\substack{qqu\\rpst}}\right) + \frac{{v_T}^2}{2\Lambda^2} C_{\substack{eq^2uH^2\\rpst}}\right]$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{uud\\prst}}$	$\frac{{v_T}^2}{2\Lambda^4} C_{\substack{lqu^2H^2\\tspr}}$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{duu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{duq\\prst}} + \frac{{v_T}^2}{2\Lambda^2}\left(C^{(1)}_{\substack{lqudH^2\\prst}} + C^{(2)}_{\substack{lqudH^2\\prst}}\right)\right]$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{dud\\prst}}$	$\frac{1}{\Lambda^2}\left[-C_{\substack{duq\\prst}} + \frac{{v_T}^2}{2\Lambda^2} C^{(2)}_{\substack{lqudH^2\\prst}}\right]$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{ddu\\prst}}$	$\frac{{v_T}^2}{2\Lambda^4} C_{\substack{lqd^2H^2\\tspr}}$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{duu\\prst}}$	$\frac{1}{\Lambda^2}\left[C_{\substack{duu\\prst}} + \frac{{v_T}^2}{2\Lambda^2} C_{\substack{eu^2dH^2\\prst}}\right]$

D.12 $\Delta B = -\Delta L = 1+$ h.c. operators

LEFT WC (+c.c.)	Matching (+c.c.)
$\frac{1}{\Lambda^2} C^{S,LL}_{\substack{ddd\\prst}}$	0
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{udd\\prst}}$	$-\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\substack{q^2ldH\\prst}}$
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{ddu\\prst}}$	0
$\frac{1}{\Lambda^2} C^{S,LR}_{\substack{ddd\\prst}}$	$-\frac{v_T}{2\sqrt{2}\,\Lambda^3} \left(C_{\substack{q^2ldH\\prst}} - C_{\substack{q^2ldH\\rpst}} \right)$
$\frac{1}{\Lambda^2} C^{S,RL}_{\substack{ddd\\prst}}$	$-\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\substack{eqd^2H\\prst}}$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{udd\\prst}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\substack{ud^2lH\\prst}}$
$\frac{1}{\Lambda^2} C^{S,RR}_{\substack{ddd\\prst}}$	$\frac{v_T}{\sqrt{2}\,\Lambda^3} C_{\substack{d^3lH\\prst}}$

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Conclusion

Le modèle standard est universellement accepté comme le point de départ de tout développement menant à une théorie complète de la physique des particules. En effet, il fait des prédictions qui ont été vérifiées expérimentalement avec une précision remarquable. La correction au facteur de Landé de l'électron en est un exemple. Cependant, il ne peut pas être complet car il échoue à expliquer certains phénomènes. Par exemple, l'asymétrie baryonique de l'univers, la matière sombre et la gravité sont des phénomènes qui échappent au modèle standard. Sinon, à une échelle de mesure plus pragmatique, des données d'observables liées entre autres à différents mésons présentent des anomalies par rapport au MS qui vont jusqu'à 4σ dans certains cas.

Pour toutes ces raisons, il est aujourd'hui généralement admis par la communauté scientifique qu'il doit exister de la physique au-delà du modèle standard. La nature de cette dernière est cependant inconnue puisqu'elle n'a jamais été observée. Ainsi, les théories efficaces de champ sont un outil parfaitement adapté à l'étude de cette nouvelle physique, puisqu'elles permettent par leur nature de considérer toutes les possibilités sous-jacentes à une hypothèse posée. Par exemple, la SMEFT fait l'hypothèse que la symétrie du modèle standard, soit $SU(3)_C \times SU(2)_L \times U(1)_Y$, est réalisée linéairement à haute énergie.

Le premier article présenté dans ce mémoire teste cette hypothèse dans le cadre de la désintégration $b \to c\tau\overline{\nu}_\tau$, en comparant les données expérimentales liées à cette désintégration avec les prédictions correspondantes de la SMEFT. L'intérêt de considérer spécifiquement cette désintégration est que les données expérimentales liées aux mésons B sont riches en anomalies par rapport au MS. Le lagrangien efficace la décrivant est

$$\begin{aligned}\mathcal{L}^{\text{eff}} = &\left(-\frac{4G_F}{\sqrt{2}} + \frac{C_V^{LL}}{\Lambda^2}\right)(\overline{c}_L\gamma_\mu b_L)(\overline{\tau}_L\gamma^\mu\nu_{L\tau}) + \frac{C_V^{LR}}{\Lambda^2}(\overline{c}_R\gamma_\mu b_R)(\overline{\tau}_L\gamma^\mu\nu_{L\tau}) \\ &+ \frac{C_S^{LL}}{\Lambda^2}(\overline{c}_R b_L)(\overline{\tau}_R\nu_{L\tau}) + \frac{C_S^{LR}}{\Lambda^2}(\overline{c}_L b_R)(\overline{\tau}_R\nu_{L\tau}) + \frac{C^T}{\Lambda^2}(\overline{c}_R\sigma_{\mu\nu} b_L)(\overline{\tau}_R\sigma^{\mu\nu}\nu_{L\tau}),\end{aligned}$$

où les coefficients C_V^{LL}, C_V^{LR}, C_S^{LL}, C_S^{LR} et C_T sont inconnus. Si la nouvelle physique est décrite par la SMEFT (c'est-à-dire qu'elle réalise linéairement la symétrie du modèle standard à haute énergie), il est attendu que la valeur du coefficient C_V^{LR} soit au plus dans l'ordre de grandeur de ${v_T}^2/\Lambda^2$, tandis que les valeurs des quatre autres coefficients peuvent prendre des valeurs allant jusqu'à l'ordre de grandeur de 1. Les analyses faites dans cet article produisent des valeurs pour C_V^{LR} qui peuvent être beaucoup plus grandes que ${v_T}^2/\Lambda^2$. Ceci permet donc la possibilité que la physique décrivant la désintégration $b \to c\tau\overline{\nu}_\tau$ ne soit pas décrite par la SMEFT. Toutefois, la possibilité inverse n'est pas exclue non plus. De plus, étant donné le manque de mesures précises d'observables liées à cette désintégration, il n'a pas été possible de faire une analyse simultanée avec les cinq coefficients. Le mieux qui a été fait est une analyse avec à la fois C_V^{LL} et C_V^{LR}. Dans le futur, lorsque plus de données expérimentales précises liées à $b \to c\tau\overline{\nu}_\tau$ existeront, il sera possible de déterminer de façon plus convaincante si la physique décrivant cette désintégration réalise linéairement la symétrie du MS à haute énergie ou non.

À ce jour, il existe plusieurs analyses des prédictions de la SMEFT qui vont jusqu'à la dimension massive 6 (consulter [6], [7], [8], [9] et [10] pour des exemples). Toutefois, dans le cas de certaines données expérimentales, la précision est telle qu'il faut également considérer les effets des dimensions massives supérieures. Ceci a été noté entre autres dans [11], [12], [13], [14], [15] et [16]. Chacun de ces articles établit les relations de correspondances entre les opérateurs LEFT et SMEFT nécessaires à l'analyse effectuée et allant jusqu'à la dimension massive 8 de la SMEFT. Toutefois, aucun n'établit une liste complète des relations de correspondance entre chaque opérateur LEFT jusqu'à la dimension massive 6 et les opérateurs SMEFT correspondants jusqu'à la dimension massive 8. Ce travail est accompli, au niveau des arbres, dans le deuxième article présenté dans ce mémoire. Ceci est une extension du travail de [6], où ces relations sont déterminées au niveau des arbres jusqu'à la dimension massive 6 de la SMEFT. Ainsi, cet article se veut un outil général présentant une liste complète des relations de correspondance qui pourront être utilisées dans le futur pour effectuer des analyses phénoménologiques ayant pour but de tester les hypothèses de la SMEFT avec une plus grande précision.

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