Université de Montréal

Three Essays in Environmental Economics

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Résumé

Cette thèse est composée de trois chapitres qui traitent de la problématique de la régulation optimale des émissions de carbone pour atténuer le changement climatique.

Dans le premier chapitre, nous analysons les interactions stratégiques entre un cartel, exportant une ressource non renouvelable génératrice de pollution, et deux pays importateurs hétérogènes qui souhaitent atténuer les dommages dus à la pollution. Les pays importateurs diffèrent selon leur demande de la ressource et selon leur degré d'exposition au stock (mondial) de pollution. Les pays importateurs fixent de manière non coopérative des taxes carbone sur la consommation de la ressource polluante et le cartel exportateur fixe son prix à la production. En utilisant l'équilibre de Nash en boucle ouverte, nous obtenons des solutions explicites des trajectoires temporelles des taxes carbone, du prix au producteur et du stock de pollution. Nous montrons que lorsque les pays importateurs agissent de manière non coopérative, à un temps fini, le prix à la production bondit et le pays importateur le plus touché par la pollution cesse de demander la ressource. Nos résultats numériques basés sur la caractérisation explicite de l'équilibre non coopératif montrent qu'une plus grande symétrie par rapport aux coûts de la pollution conduit à une augmentation plus rapide du stock de pollution en début d'horizon temporel, mais à un stock de pollution de long terme plus faible et un bien-être total plus élevé.

Dans le deuxième chapitre, Nous analysons les effets des ajustements carbone bilatéraux aux frontières sur les taxes carbone dans un jeu non coopératif entre deux pays symétriques ouverts ayant des firmes en concurrence imparfaite en présence de pollution transfrontalière. Nous comparons également dans ce chapitre les résultats de ce jeu avec ceux de deux benchmarks (soient, le jeu non coopératif sans ajustements carbone aux frontières et la solution efficace). Nous constatons que lorsque les pays souffrent peu de la pollution, seuls des équilibres symétriques existent. En revanche, si les pays souffrent suffisamment de la pollution, seuls des équilibres asymétriques existent. Les taxes sur le carbone en équilibres symétriques sont plus élevées que les taxes efficaces, tandis que l'inverse est vrai pour les équilibres asymétriques. Dans tous les cas d'intérêt, le bien-être total à l'équilibre du jeu non coopératif avec ajustements carbone aux frontières est supérieur à celui du jeu non coopératif sans ajustements carbone aux frontières. Lorsque les coûts de la pollution sont suffisamment bas, il existe un niveau d'ajustement carbone aux frontières tel que les taxes d'équilibre non coopératif sont efficaces. Enfin, dans le cas où les pays souffrent suffisamment de la pollution, le niveau optimal d'ajustement carbone aux frontières peut être partiel ou total selon les paramètres du modèle.

Nous étudions enfin, dans le dernier chapitre, un jeu de pollution transfrontalière non coopératif entre respectivement deux pays et trois pays fixant des taxes carbone en présence d'ajustements carbone aux frontières et avec présence d'une concurrence imparfaite sur le marché international des biens polluants. Les pays sont asymétriques quant à leur volonté de payer pour la réduction des émissions mondiales de carbone. Dans nos modèles, seul le pays le plus touché impose un ajustement carbone aux frontières. Nous montrons que, contrairement à la littérature existante utilisant des modèles à deux pays avec un seul marché, le pays le plus touché préfère très généralement utiliser un ajustement carbone total aux frontières (c'est-à -dire un tarif qui ajuste exactement les écarts entre sa propre taxe carbone et celles des autres pays) à un ajustement carbone partiel aux frontières. De plus, un ajustement carbone total aux frontières est optimal pour le bien-être global dans la plupart des cas d'intérêt.

Mots-clés: Pollution transfrontalière, Taxes carbone, Ressources non renouvelables, Théorie des jeux, Fuite de carbone, Compétitivité, Ajustement carbone aux frontières.

Abstract

This thesis is composed of three chapters which concern the problem of the optimal regulation of carbon emissions to mitigate climate change.

In the first chapter, we analyze strategic interactions between a resource cartel exporting a non-renewable stock pollutant and two heterogeneous importing countries, who want to mitigate pollution damages. The importing countries differ with respect to market size and with respect to how strongly they are affected by the (global) stock of pollution. The importing countries non cooperatively set emissions taxes and the exporting cartel sets its producer price. Using open loop Nash equilibrium, we obtain explicit solutions for the time paths of the carbon taxes, the producer price and the stock of pollution. We show that when the countries act non cooperatively, at a finite time, the producer price jumps and the country that is most affected by pollution stops demanding the resource. Our numerical results based on the explicit characterization of the non-cooperative equilibrium yield that more symmetry with respect to the cost of pollution leads to faster increase of the stock of pollution initially, but to a lower long-term stock and higher total discounted welfare.

In the second chapter, we analyzes the effects of bilateral border tax adjustments on carbon taxes in a non-cooperative game between two symmetric open countries trading in an oligopolistic framework with cross-border pollution. We also contrast the results of this BTA game with those of two benchmarks (the non-cooperative game without BTA and the efficient solution). We note that when countries suffer little from pollution, only symmetric equilibria exist. By contrast, if countries suffer sufficiently from pollution,

only asymmetric equilibria exist. Carbon taxes in symmetric equilibria are higher than the efficient taxes, while the opposite is true for the asymmetric equilibria. In all cases of interest, the total welfare in the equilibrium of the non cooperative game with BTA is higher than that in the equilibrium of the non cooperative game without BTA. If the cost of pollution is sufficiently low, there is a level of BTA such that non cooperative equilibrium taxes are efficient. Finally, in the case where the countries suffer a lot from pollution, the optimal level of BTA can be partial or full depending on the parameters of the model.

Finally, in the last chapter, we study a non-cooperative transboundary pollution game between respectively two countries and three countries setting carbon taxes in the presence of a Border Tax Adjustment (BTA) and with imperfect competition in the international polluting goods market. Countries are asymmetric with respect to their willingness to pay for reductions of global emissions. In our models, only the most affected country imposes a BTA. We show that, unlike in the existing literature using two-country models with only one market, the most affected country generally prefers using a full BTA, a tariff that fully adjusts for the differences between its own carbon tax and those in other countries, to a partial BTA. Moreover, a full BTA is optimal for the global welfare in most cases of interest.

Keywords: Transboundary pollution, Carbon taxes, Non-renewable resources, Game theory, Carbon leakage, Competitiveness, Border tax adjustment.

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Chapter 1

Trade of an Exhaustible Resource with Multilateral Externality

1 Introduction

In the United States, the use of fossil fuel in 2017 was responsible for about 76%[1] of emissions of greenhouse gases in the atmosphere. Fuel taxes are a controversial but important instrument for influencing emissions. As the problem of setting carbon taxes is complicated by the presence of a resource-exporting cartel (the Organization of the Petroleum Exporting Countries (OPEC)) (Liski and Tahvonen (2004), Dullieux, Ragot, and Schubert (2011)), several authors have studied strategic interactions between a coalition of importing countries and a resource-exporting cartel of a polluting non-renewable resource (a stock pollutant) (e.g. Wirl (1994), Tahvonen (1996), Rubio and Escriche (2001), Liski and Tahvonen (2004), Dullieux, Ragot, and Schubert (2011), Kagan, Van der Ploeg, and Withagen (2015)). This literature studies differential bilateral monopoly games between a resource-importing country (or a coalition of importing countries) setting a carbon tax and a resource-exporting cartel setting a producer price. A main insight of this literature is that Markov perfect Nash equilibrium taxes have two components, a pure Pigouvian component (that corrects the externality) and a rent shifting component. By contrast, open loop Nash equilibrium taxes are purely Pigouvian (see e.g. Dullieux, Ragot, and

[1] United States Environmental Protection Agency.

Schubert (2011)).

The above literature has neglected the fact that importing countries do not necessarily form a coalition to determine their carbon taxes and that countries can be asymmetric with respect to how much they are affected by pollution[2] (their cost of pollution) and with respect to their demand for the resource.

In this paper, we extend a linear-quadratic version of the differential bilateral monopoly game to analyze strategic interactions between three parties, two importing countries and a resource-exporting cartel which owns the stock of the non-renewable, polluting resource. Our goal is to shed light on how asymmetries between the importing countries, with respect to the cost of pollution and/or market size, affect equilibrium taxes, producer prices, consumption levels, the evolution of the stock of pollution, and welfare. We assume as Rubio and Escriche (2001) that there is no decay of the stock of pollution. Our main result is an (almost) explicit characterization of the unique open loop Nash equilibrium of the differential game between the three players. Obtaining this characterization is not so straightforward when countries differ in their cost of pollution, because the Hamiltonian of the exporting cartel's problem is not concave and because the more affected country exits the market at a finite time. We show that taxes and the producer price are such that both countries consume the resource before some finite time Θ, and that the producer price then increases discontinuously to the price of a bilateral monopoly game between the cartel and the less affected country. Thus, the cartel "excludes" the more affected country (which sets a higher tax) from time Θ onwards. We characterize the equilibrium explicitly up to an implicit equation for Θ, which has an easily computable, unique solution. We have to restrict attention to the classical solution concept of open loop Nash equilibrium for tractability.[3] This means that the taxes set by the importing countries are purely Pigouvian, i.e., they equal the present value of the discounted marginal damages, along the equilibrium path.

Our analytical results allow interesting insights based on numerical simulations into how the asymmetries between the two countries affect the equilibrium (consumption paths,

[2]Kaitala and Pohjola (1995) in their game theory model for an international environmental negotiation problem, worked with countries that differ in their vulnerability to global warming.

[3]In the case of more than one importing country, it is difficult to have an explicit form of the equilibrium with the closed loop model (i.e. Markov perfect equilibria, see Chou and Long (2009)).

stock of pollution, etc.) and the discounted welfare of the two importing countries, and to compare the non cooperative case with two benchmark scenarios. These benchmarks are the equilibrium of the bilateral monopoly game between the coalition of the two importing countries and the cartel, and the social optimum, respectively.

The paper is organized as follows. In sections 2 and 3, we set up the model and compute the non cooperative open loop Nash equilibrium. In section 4, we compute two benchmarks. Section 5 presents numerical results. We conclude in the last section.

2 The Model

Our basic model extends the infinite horizon bilateral monopoly models of trade of an exhaustible and polluting natural resource (e.g. Tahvonen (1996), Rubio and Escriche (2001), Liski and Tahvonen (2004)). We consider a world with a resource-exporting cartel that acts as a monopoly and two resource-importing countries. The importing countries can be asymmetric, both with respect to the size of their market and with respect to their cost of pollution. The resource-exporting cartel extracts a stock of an exhaustible and polluting natural resource and sells it to the two resource-importing countries. In each importing country, the consumption of the natural resource increases utility but at the same time, it generates a multilateral externality due to the fact that it increases the global stock of pollution.

The representative consumer in importing country $i \in \{1, 2\}$ has the quasi-linear instantaneous utility

$$U_i(q_i(t), Z(t), m(t)) = Aq_i(t) - \frac{1}{2\beta_i}q_i^2(t) - \frac{\gamma_i}{2}Z(t)^2 - m(t),$$

where $q_i(t)$ denotes her consumption of the exhaustible resource at time t, $m(t)$ is the net payment she has to make at time t, $\beta_i > 0$ is country i's size of market parameter, $A > 0$ is the choke price (which is the same for both importing countries), and $\frac{\gamma_i}{2}Z(t)^2$ is the disutility due to the stock of pollution $Z(t)$ at time t.

The tax rate on consumption of the polluting resource set by importing country i at time t is denoted $T_i(t)$. As usual, we assume that tax revenues are returned to consumers as lump sum transfers. Consumers take the price of the resource, the stock of pollution and the lump sum transfers as given. So, their demand depends only on the current consumer price $p(t) + T_i(t)$, and is given by

$$q_i(t) = Q_i(p(t) + T_i(t)) = \max\{0, \beta_i(A - p(t) - T_i(t))\}, \tag{1.1}$$

where $p(t)$ is the producer price.

It follows that if the time paths of the tax in country i and of the producer price are $T_i : \mathbb{R}_+ \to \mathbb{R}_+$ and $p : \mathbb{R}_+ \to \mathbb{R}_+$, the discounted present value of the utility of the representative consumer in country i is

$$\int_0^\infty e^{-\rho t}\left(Aq_i(t) - \frac{1}{2\beta_i}q_i^2(t) - p(t)q_i(t) - \frac{\gamma_i}{2}Z(t)^2\right)dt, \tag{1.2}$$

where ρ is the discount rate[4].

After substituting $q_i(t)$ in (1.2) by expression (1.1) and after some simplifications, the discounted present value of utility in (1.2) becomes

$$\int_0^\infty e^{-\rho t}\left(\max\{\frac{\beta_i}{2}\left[(A-p(t))^2 - T_i^2(t)\right], 0\} - \frac{\gamma_i}{2}Z(t)^2\right)dt. \tag{1.3}$$

The cartel sets a uniform producer price of the natural resource for both importing countries in order to maximize its present discounted profits

$$\int_0^\infty e^{-\rho t}\left[\sum_{i=1}^{2} Q_i(p(t) + T_i(t))\,(p(t) - cZ(t))\right]dt, \tag{1.4}$$

where $cZ(t)$ is the marginal cost of extraction at time t of the natural exhaustible resource by the cartel ($c > 0$).

We assume that one unit of consumption creates one unit of pollution, that pollution does not decay, and that $Z(0) = 0$. Thus, for given p, T_1 and T_2, the stock of pollution evolves

[4] We assume the same discount rate for both importing countries and for the cartel.

according to the equation

$$\frac{dZ(t)}{dt} = \sum_{i=1}^{2} Q_i(p(t) + T_i(t)). \tag{1.5}$$

Following the related literature (e.g. Rubio and Escriche (2001), Liski and Tahvonen (2004), Chou and Long (2009)), we assume that the initial stock of the exhaustible resource is bigger than A/c. Hence, there will be only economic (no physical) exhaustion of the resource.

As it will not lead to any ambiguity, we omit from now on the time argument, and we write $\dot{Z}$ for $\frac{dZ}{dt}$, $\dot{Q}_i$ for $\frac{dQ_i(t)}{dt}$, and so on.

Throughout the paper, we use the notation

$$B = \beta_1 + \beta_2 \quad and \quad \Gamma = \gamma_1 + \gamma_2.$$

3 Non cooperative Open Loop Nash Equilibrium

In this section, we develop our main analytical results. We establish a closed form characterization of the unique open loop Nash equilibrium of the game where all three parties, i.e., both importing countries and the cartel, act non-cooperatively. Both countries and the cartel choose the entire time path of their control variable non-cooperatively and simultaneously at time $t = 0$, best-responding to the others' choices. Thus, (T_1^*, T_2^*, p^*) is an open loop Nash equilibrium if and only if for $i \in \{1, 2\}$

$$T_i^* \in \arg\max_{T_i} \int_0^\infty e^{-\rho t} \left(\max\{ \frac{\beta_i}{2} \left[(A - p^*)^2 - T_i^2 \right], 0\} - \frac{\gamma_i}{2} Z^2 \right) dt$$

subject to

$$\dot{Z} = Q_i(p^* + T_i) + Q_j(p^* + T_j^*) \quad i \neq j \in \{1, 2\},$$

and

$$p^* \in \arg\max_p \int_0^\infty e^{-\rho t} \left[\sum_{i=1}^2 Q_i(p + T_i^*)\,(p - cZ) \right] dt$$

subject to

$$\dot{Z} = \sum_{i=1}^2 Q_i(p + T_i^*).$$

Without loss of generality, we assume that $\gamma_1 > \gamma_2$.

In order to ensure that the two importing countries have a positive consumption at the beginning of the horizon (at time $t = 0$), we make the following assumption.

Assumption 1. *We assume that* $\sqrt{\beta_1 + \beta_2}\theta_2' > \sqrt{\beta_2}\theta_2$*, where* $\theta_2' = \frac{1}{2}(\rho + \sqrt{\Delta'})$*,* $\Delta' = \rho^2 + 2\beta_2(\rho c + \gamma_2)$*,* $\theta_2 = \frac{1}{2}(\sqrt{\Delta} + \rho)$ *and* $\Delta = \rho^2 + 2(B\rho c + \sum_{i=1}^2 \gamma_i \beta_i)$*.*

We start by considering the optimal control problem faced by the government of country i, if it takes as given some time paths p and T_j (where $j \neq i$), i.e., the problem

$$\max_{T_i} \int_0^\infty e^{-\rho t} \left(\max\{ \frac{\beta_i}{2} \left[(A-p)^2 - T_i^2 \right], 0\} - \frac{\gamma_i}{2} Z^2 \right) dt$$

subject to

$$\begin{cases} \dot{Z} = \max\{\beta_1(A - p - T_1), 0\} + \max\{\beta_2(A - p - T_2), 0\}, \\ Z(0) = 0. \end{cases}$$

Here, p (with values in $[0, A]$) and T_j are assumed to be measurable, but not necessarily continuous (in equilibrium, p will actually have a jump).

The current value Hamiltonian for this problem is

$$H_i = r_i \left(\max\{ \frac{\beta_i}{2} \left[(A-p)^2 - T_i^2 \right], 0\} - \frac{\gamma_i}{2} Z^2 \right) + \lambda_i \left(\max\{\beta_i(A - p - T_i), 0\} + \max\{\beta_j(A - p - T_j), 0\} \right),$$

where $r_i \in \{0, 1\}$ and λ_i is the co-state variable associated to the state variable Z.[5]

[5] Equivalently, $e^{-\rho t}\lambda_i(t)$ is the adjoint function for the standard Hamiltonian (rather than the current value Hamiltonian).

By Theorem 12 and footnote 9 on page 132 in Seierstad and Sydsaeter (1987), the following conditions must be satisfied t-a.e. by an optimal solution T_i, provided that the value of r_i corresponding to the optimal solution is 1:

$$T_i \in \arg\max(H_i), \tag{1.6}$$

$$\dot{\lambda}_i = \rho\lambda_i + \gamma_i Z, \tag{1.7}$$

$$\dot{Z} = \sum_{j=1}^{2} \max\{\beta_j(A - p - T_j), 0\}, \tag{1.8}$$

for absolutely continuous functions λ_i and Z.
We note that if the optimal value of T_i in (1.6) (maximizing $\max\{\frac{\beta_i}{2}[(A-p(t))^2 - \hat{T}_i^2], 0\} + \lambda_i \max\{\beta_i(A-p(t)-\hat{T}_i), 0\}$) is smaller than $A-p(t)$, we must have $T_i = -\lambda_i$. Furthermore, the optimal value is greater than or equal to $A - p(t)$ if and only if $A - p(t) \leq -\lambda_i(t)$, and we may set $T_i(t) = -\lambda_i(t)$ without loss of generality (as all values of $T_i(t) \geq A - p(t)$ maximize the Hamiltonian in this case, and the evolution of the state is unaffected by the exact choice). Thus, we can replace (1.6) by

$$T_i = -\lambda_i. \tag{1.9}$$

Combining (1.7) and (1.9), we obtain that the law of motion of T_i must satisfy

$$\dot{T}_i = \rho T_i - \gamma_i Z.$$

By integrating the above differential equation with respect to time, we obtain that the optimal time path for the tax rate in each importing country i (up to the degree of arbitrariness where $T_i(t) \geq A - p(t)$) is of the form

$$T_i = \gamma_i \int_t^\infty e^{-\rho(\tau - t)} Z d\tau. \tag{1.10}$$

As the maximized Hamiltonian (the Hamiltonian evaluated at $T_i(t)$) is concave with respect to the state, and as the transversality condition $\liminf_{t\to\infty} e^{-\rho t}\lambda_i(t)(\hat{Z}(t) - Z(t)) \geq 0$ holds for any admissible path $\hat{Z}$, Theorem 14 in Seierstad and Sydsaeter (1987) (Arrow's sufficiency Theorem) implies that T_i given by (1.10) is the unique optimal solution of country i's problem (up to the arbitrariness of T_i when $T_i(t) \geq A - p(t)$).

Equation (1.10) shows that in any open loop Nash equilibrium, the tax rate at each time t in each importing country is equal to the expected present value of the marginal damages along the equilibrium path. Thus, taxes are purely Pigouvian (Liski and Tahvonen (2004)). This confirms previous results (e.g. Dullieux, Ragot, and Schubert (2011)) in the literature that have shown that open loop Nash equilibrium taxes have no rent shifting component.

We then see from (1.10) that the tax rate in importing country 1 is always larger than the one in importing country 2 ($T_1 > T_2$), because we have assumed that $\gamma_1 > \gamma_2$.

We now consider the problem faced by the resource-exporting cartel (if it anticipates some tax paths $T_1 > T_2$), i.e., the problem

$$\max_p \int_0^\infty e^{-\rho t}[Q(p)\,(p - cZ)]\,dt$$

subject to

$$\begin{cases} \dot{Z} = Q(p), \\ Z(0) = 0, \end{cases}$$

$$\text{where } Q(p) = \begin{cases} \sum_{j=1}^{2} \beta_j (A - p - T_j) \quad if \quad p \leq A - T_1 \\ \beta_2 (A - p - T_2) \quad if \quad A - T_1 \leq p \leq A - T_2 \\ 0 \quad otherwise \end{cases}$$

Note that we may restrict attention to prices in $[0, A\,]$ (as there is never any extraction/demand if $p \geq A$).

The current value Hamiltonian for this problem is

$$H_e = r_e Q(p)(p - cZ) + \mu_e Q(p),$$

where $r_e \in \{0, 1\}$ and μ_e is the co-state variable associated to the stock of pollution.

H_e is not concave in p (due to the kink of $Q(p)$ at $p = A - T_1$), and Arrow's sufficiency conditions (that are based on concavity of the maximized Hamiltonian with respect to the state) are also not applicable. However, Theorems 12 and 16 (including footnote 27) in Chapter 3 of Seierstad and Sydsaeter (1987) imply that if p is an optimal solution then the following necessary conditions must be satisfied for $r_e = 1$ (it is straightforward to verify that time paths satisfying the necessary conditions in Theorems 12 and 16 for $r_e = 0$ cannot be optimal):

$$
\begin{aligned}
p &\in \arg\max(H_e) && (1.11)\\
\dot{\mu}_e &= \rho\mu_e + c\dot{Z} && (1.12)\\
\dot{Z} &= Q(p) && (1.13)\\
\lim_{t\to\infty} e^{-\rho t}\mu_e &= 0, && (1.14)
\end{aligned}
$$

where (1.11), (1.12) and (1.13) must hold t-a.e. We will show below that this set of necessary conditions has a unique solution. As Theorem 15 from Chapter 3 in Seierstad and Sydsaeter (1987) implies that an optimal solution exists,[6] we know that the unique solution to the necessary conditions is the optimal time path for the cartel.

Equation (1.12) which characterizes the scarcity rent for the exporting cartel expresses the fact that, the marginal unit of the stock of the resource demanded at time t increases the future extraction costs by $c\dot{Z}(t)$. This, in consequence, contributes to reduce future profits, and increases the scarcity rent and today's price.

The following lemma characterizes the producer price that maximizes the Hamiltonian H_e at any given time t, for given time paths $T_1 > T_2$.

Lemma 1. *At each time t, the price p^* that maximizes the Hamiltonian H_e is given by:*

$$
p^* = \begin{cases} p_1^* = \frac{1}{2B}\left(BcZ + BA - B\mu_e - \sum_{i=1}^{2}\beta_i T_i\right) & if \quad cZ - \mu_e \le D, \\ p_2^* = \frac{1}{2}\left(cZ + A - \mu_e - T_2\right) & if \quad cZ - \mu_e \ge D, \end{cases}
$$

where $D = A - \frac{\sqrt{B}}{\sqrt{B}-\sqrt{\beta_2}}\frac{\sum_{i=1}^{2}\beta_i T_i}{B} + \frac{\sqrt{\beta_2}}{\sqrt{B}-\sqrt{\beta_2}}T_2.$

[6] The key condition to verify is that the set $\{(Q(p)(p - cZ) + x, Q(p))|p \in [0, A], x \le 0\}$ is a convex subset of $\mathbb{R}^2$ for each Z, which is true because $p - cZ$ decreases when $Q(p)$ increases.

Proof. See Appendix.

Observe that $cZ - \mu_e$ is increasing over time. Indeed, $c\dot{Z} - m\dot{u}_e = -\rho\mu_e$ by (1.12), and $\mu_e < 0$ (otherwise (1.12) and (1.14) could not be jointly satisfied). On the other hand, D is decreasing in t, because Z and thus (by (1.10)) T_1 and T_2 are increasing. Proposition 1 shows that in any open loop Nash equilibrium, there is a unique finite time Θ for which $cZ - \mu_e = D$, so that the producer price jumps from p_1^* to p_2^*.

Proposition 1. *The producer price path must be of the following form in any open loop Nash equilibrium. There is a finite time $\Theta \geq 0$ such that $p = p_1^*$ for $t < \Theta$ and $p = p_2^*$ for $t > \Theta$. Thus, provided that $\Theta > 0$ (which will be the case if Assumption 1 holds) there is an initial period where both countries consume the resource. After time Θ, the cartel excludes country 1, and chooses the bilateral monopoly price p_2^*.*

Proof. See Appendix.

We now derive the unique open loop Nash equilibrium, using our insights about the structure of the producer price path (Lemma 1 and Proposition 1) and combining the conditions we identified above for both importing countries and for the cartel. We focus on the case of interest where $\Theta > 0$ (i.e., we do not have a bilateral monopoly between country 2 and the cartel from the beginning). The analysis will also reveal Assumption 1 as the necessary and sufficient condition for $\Theta > 0$.

Before time Θ, both importing countries have a positive demand. From (1.1), we obtain

$$Bp + \left(\sum_{i=1}^{2} \beta_i T_i\right) = BA - \left(\sum_{i=1}^{2} q_i\right) = BA - \dot{Z},$$

which is equivalent to

$$p = A - \frac{\left(\sum_{i=1}^{2} \beta_i T_i\right)}{B} - \frac{\dot{Z}}{B}. \tag{1.15}$$

Using (1.15) and the expression for the producer price before time Θ (see Lemma 1), we have

$$2A - \frac{2\left(\sum_{i=1}^{2} \beta_i T_i\right)}{B} - \frac{2\dot{Z}}{B} = A + cZ - \mu_e - \frac{1}{B}\left(\sum_{i=1}^{2} \beta_i T_i\right),$$

which is equivalent to

$$A - \frac{\left(\sum_{i=1}^{2} \beta_i T_i\right)}{B} - \frac{2\dot{Z}}{B} = cZ - \mu_e. \tag{1.16}$$

Differentiating equation (1.16) with respect to time, we obtain

$$-\frac{\left(\sum_{i=1}^{2} \beta_i \dot{T}_i\right)}{B} - \frac{2\ddot{Z}}{B} = c\dot{Z} - \dot{\mu}_e,$$

which is equivalent to

$$-\frac{2\ddot{Z}}{B} - c\dot{Z} = \frac{\left(\sum_{i=1}^{2} \beta_i \dot{T}_i\right)}{B} - \dot{\mu}_e. \tag{1.17}$$

Combining (1.9) and (1.7) with (1.12) and (1.17), we obtain

$$-\frac{2\ddot{Z}}{B} - c\dot{Z} = \rho(\frac{\left(\sum_{i=1}^{2} \beta_i T_i\right)}{B} - \mu_e) - \frac{\left(\sum_{i=1}^{2} \beta_i \gamma_i\right)}{B} Z - c\dot{Z},$$

which we rewrite as

$$-\frac{2\ddot{Z}}{B} + \frac{\left(\sum_{i=1}^{2} \beta_i \gamma_i\right)}{B} Z = \rho(\frac{\left(\sum_{i=1}^{2} \beta_i T_i\right)}{B} - \mu_e). \tag{1.18}$$

Combining (1.16) and (1.18) gives

$$-\frac{2\ddot{Z}}{B} + \frac{\left(\sum_{i=1}^{2} \beta_i \gamma_i\right)}{B} Z = \rho(A - \frac{2\dot{Z}}{B} - cZ),$$

which is equivalent to

$$-2\ddot{Z} + 2\rho\dot{Z} + (\rho Bc + \sum_{i=1}^{2} \beta_i \gamma_i) Z = \rho AB. \tag{1.19}$$

The resolution of the differential equation (1.19) yields

$$Z(t) = w_1 e^{\theta_1 t} + w_2 e^{\theta_2 t} + \frac{\rho AB}{\rho Bc + \sum_{i=1}^{2} \beta_i \gamma_i} \quad \text{for all} \quad t \in [0, \Theta[\quad \text{with} \quad w_1, w_2 \in \mathbb{R}, \tag{1.20}$$

where $\theta_1 = \frac{1}{2}(\rho - \sqrt{\Delta})$, $\theta_2 = \frac{1}{2}(\sqrt{\Delta} + \rho)$ and $\Delta = \rho^2 + 2(B\rho c + \sum_{i=1}^{2} \gamma_i \beta_i)$.

For $t > \Theta$, the evolution of the stock of pollution, country 2's tax, the producer price and the cartel's co-state variable must be of the following form (see Appendix 1 for how we obtain these expressions).

$$\begin{cases} Z(t) = w_1' e^{\theta_1' t} + \frac{A\rho}{\rho c + \gamma_2} \\ T_2(t) = \frac{\gamma_2}{\theta_2'} w_1' e^{\theta_1' t} + \frac{A\gamma_2}{\rho c + \gamma_2} \\ \mu_e(t) = -\frac{c\theta_1'}{\theta_2'} w_1' e^{\theta_1' t} \\ p(t) = \frac{c\rho - \gamma_2}{2\theta_2'} w_1' e^{\theta_1' t} + \frac{A\rho c}{\rho c + \gamma_2} \end{cases} \tag{1.21}$$

where $w_1' \in \mathbb{R}$, $\theta_1' = \frac{1}{2}(\rho - \sqrt{\Delta'})$, $\theta_2' = \frac{1}{2}(\rho + \sqrt{\Delta'})$ and $\Delta' = \rho^2 + 2\beta_2(\rho c + \gamma_2)$.

We find the unknowns by using the continuity of Z at time Θ, the $Z(0)$ and the fact that at time Θ, $H_e(p_1^*) = H_e(p_2^*)$ (see Appendix 1 for details). After some resolutions, we obtain

$$\begin{aligned} w_1' &= \frac{\beta_1 A \theta_2' (\gamma_2 - \gamma_1) e^{-\theta_1' \Theta}}{(\rho c + \gamma_2)(B\rho c + \sum_{i=1}^2 \gamma_i \beta_i - \sqrt{B\beta_2}(\rho c + \gamma_2))}, \\ w_1 &= \frac{Z_\Theta + \frac{AB\rho(e^{\theta_2 \Theta} - 1)}{B\rho c + \sum_{i=1}^2 \gamma_i \beta_i}}{e^{\theta_1 \Theta} - e^{\theta_2 \Theta}}, \\ w_2 &= \frac{Z_\Theta + \frac{AB\rho(e^{\theta_1 \Theta} - 1)}{B\rho c + \sum_{i=1}^2 \gamma_i \beta_i}}{e^{\theta_2 \Theta} - e^{\theta_1 \Theta}}, \end{aligned}$$

where $Z_\Theta = w_1' e^{-\theta_1' \Theta} + \frac{A\rho}{\rho c + \gamma_2}$ and Θ is the unique solution of the following equation

$$(\theta_1 e^{-\theta_1 \Theta} - \theta_2 e^{-\theta_2 \Theta})(Z_\Theta - \frac{AB\rho}{B\rho c + \sum_{i=1}^2 \gamma_i \beta_i}) - \frac{AB\rho\sqrt{\Delta}}{B\rho c + \sum_{i=1}^2 \gamma_i \beta_i} - \sqrt{\frac{B}{\beta_2}} w_1' \theta_1' e^{-\theta_2' \Theta}(e^{\theta_1 \Theta} - e^{\theta_2 \Theta}) = 0,$$

When the importing countries are heterogeneous in their costs of pollution, the importing country that has the highest cost of pollution stops demanding the resource at a finite time. However, the lowest cost importing country demands the resource until economic exhaustion.

This completes the characterization of the open loop Nash equilibrium when the parameters are such that $\Theta > 0$. It is straightforward to see that $\Theta > 0$ if assumption 1 is

satisfied. Otherwise, $\Theta = 0$, and we will have the uninteresting case where the more affected country never consumes anything and the game is bilateral monopoly game between country 2 and the exporting cartel.
The analytical characterization of this section will allow us to examine variables of interest, such as welfare and the equilibrium extraction path numerically, and to compare them against the benchmarks established in Section 4.

4 Benchmarks

Cooperation between the importing countries (bilateral monopoly)

In this section, we assume that the two resource importing countries cooperate to maximize their joint welfare. This is the case studied in the literature when we have only one importing country or a coalition of resource-importing countries (seeking to maximize utilitarian welfare) and a cartel of resource-exporting countries (see Dullieux, Ragot, and Schubert (2011), Rubio and Escriche (2001), Tahvonen (1996)). The following proposition summarizes our results in this section.

Proposition 2. *The open loop Nash equilibrium of the bilateral monopoly game between the coalition of importing countries and the cartel, called the cooperative equilibrium, is characterized as follows*

$$\begin{cases} Z^c(t) = \frac{\rho A}{\rho c+\Gamma}\left(1-e^{-\theta_c t}\right) \\ T_1(t) = T_2(t) = T^c(t) = \frac{A\Gamma}{\rho c+\Gamma}\left(1-\frac{2\rho}{\rho+\sqrt{\Delta_c}}e^{-\theta_c t}\right) \\ p^c(t) = \frac{\rho A}{\rho c+\Gamma}\left(c+\frac{\Gamma-c\rho}{\rho+\sqrt{\Delta_c}}e^{-\theta_c t}\right) \\ q_i^c(t) = \frac{\beta_i \rho A}{(\rho+\sqrt{\Delta_c})}e^{-\theta_c t} \qquad \text{for} \quad i \in \{1,2\} \end{cases} \tag{1.22}$$

with $\theta_c = \frac{1}{2}(\sqrt{\Delta_c}-\rho)$ *and* $\Delta_c = \rho^2 + 2B(c\rho+\Gamma)$*. In particular, both countries demand the resource until economic exhaustion.*

Proof. See Appendix.

Proposition 2 shows that the rate of taxation in the resource-importing countries, the stock of resource already extracted and the consumer price are all continuously increasing functions of time. When the two importing countries form a coalition, they set the same tax rate. The tax rate does not depend on how the natural resource is distributed between the two resource-importing countries.

Social Optimum

In this section, a social planner maximizes the sum of the joint welfare of the two resource-importing countries and of the profits of the resource-exporting cartel. For completeness, we also compute what the producer price and the optimal taxes would be in the case where the social optimum results as an efficient equilibrium in an economy with competitive producers (see details in the proof of Proposition 3 in the Appendix). The social optimum is characterized in the following proposition.

Proposition 3. *The social optimum is characterized as follows*

$$\begin{cases} Z^o(t) = \frac{\rho A}{\rho c+\Gamma}\left(1-e^{-\theta t}\right) \\ \mu_o = -\frac{\rho A}{\rho c+\Gamma}\left(\frac{\rho-\sqrt{\Delta}}{2B}+c\right)e^{-\theta t} - \frac{A\Gamma}{\rho c+\Gamma} \\ T^o(t) = \frac{A\Gamma}{\rho c+\Gamma}\left(1-\frac{\rho}{\rho+\theta}e^{-\theta t}\right) \\ q_i^o(t) = \frac{\beta_i \rho A(\sqrt{\Delta}-\rho)}{2B(\rho c+\Gamma)}e^{-\theta t} \qquad for \quad i\in\{1,2\} \\ p^o(t) = \frac{\rho A c}{\rho c+\Gamma}\left(1-\frac{\rho}{\rho+\theta}e^{-\theta t}\right) \end{cases}$$

with $\theta = \frac{1}{2}(\sqrt{\Delta}-\rho)$ *and* $\Delta = \rho^2 + 4B(c\rho+\Gamma)$.

Proof. See Appendix.

In the social optimum, the consumer price, the producer price, the tax rate and the stock of pollution are increasing functions of time while the demand decreases over time. Even if the importing countries are heterogeneous in the way in which they are affected by the stock of pollution, when they cooperate with each other and in the social optimum, their consumptions differ only if they have different market size. This can be explained by the fact that, when they cooperate, they only care about the overall cost of pollution.

5 Numerical comparisons

In this section, we make some comparisons between the open loop Nash equilibrium when all three parties act non-cooperatively and the two benchmarks scenarios of Section 4. First, the long term stock of pollution, or equivalently the limit amount of the extracted resource is larger in the non cooperative case than in the cooperative case. Consequently, from an environmental point of view, cooperation between the two importing countries is preferred to non cooperation. We also note that the final stock of pollution is the same in the cooperative equilibrium and in the social optimum. This leads us to the same result as Wirl (1994), which is that, from an environmental point of view, the cooperation between the two resource-importing countries and the resource-exporting cartel is not necessary, only the cooperation between importing countries matters.

Both in the case of cooperation between the resource-importing countries and in the social optimum, the country that is most affected by the pollution does not stop consuming the resource, in contrast to what we find for the non cooperative equilibrium. In both importing countries, consumption is initially higher in the social optimum compared to the cooperative equilibrium ($q_i^o(0) > q_i^c(0)$ for all $i \in \{1,2\}$) which is the consequence of the fact that the initial consumer price in the social optimum is lower than the one obtained in the cooperative equilibrium. However, there is a time t_1 after which the consumption in the social optimum remains smaller in the cooperative equilibrium (for all $i \in \{1,2\}$, we have: $q_i^o(t) > q_i^c(t)$ for all $t \in [0, t_1[$, $q_i^o(t) < q_i^c(t)$ for all $t \in]t_1, \infty[$ and $q_i^o(t_1) = q_i^c(t_1)$).

Our characterization of the non cooperative equilibrium is not tractable enough for simple analytical comparisons with the benchmark scenarios, but it allows us to easily make numerical comparisons for given parameters, and to obtain interesting insights into the evolution of consumption levels, the stock of pollution, and welfare. We assume as Chou and Long (2009), that $A = 100$, $c = 0.25$, $\rho = 0.05$. For the value of the slope of the marginal cost of pollution, we fix $\gamma_1 = 0.006$ and $\gamma_2 = 0.001$.[7] Later, we also study the effects of changing the asymmetry with respect to how much countries are affected by pollution, by considering different values of (γ_1, γ_2) with $\Gamma = 0.007$.

[7] Liski and Tahvonen (2004) worked with a similar range of parameters.

Table 1.1 – Welfare levels of importing countries and exporting cartel.

	Cooperative		**Social optimum**	
	Importing	Exporting	Importing	Exporting
B=1	3057.2	5663.5	4237.5	4741.4
B=2	3446.3	6323.7	4377	5582.5
B=3	3634.6	6638.8	4431.5	5998.5
B=4	3751.6	6833.3	4461.5	6260

Table 1.1 shows the discounted joint welfare of the resource-importing countries (left column) and the profits of the exporting cartel (right column) for the two benchmark scenarios, for different values of the total market size B. We see that the welfare level of the importing countries is always higher in the social optimum than in the cooperative equilibrium, while the exporting cartel is better off in the cooperative equilibrium than the competitive producers in the decentralized social optimum.

Table 1.2 – Welfare levels of the two importing countries in the non cooperative equilibrium.

	$\beta_1 = \beta_2$	$\beta_1 = 1.5\beta_2$	$\beta_1 = 2\beta_2$	$\beta_1 = 3\beta_2$
B=1	-1896.5, 1961.5	-1359.3, 1811.8	-913.7, 1656.2	-236.9, 1380.4
	$\beta_2 = \beta_1$	$\beta_2 = 1.5\beta_1$	$\beta_2 = 2\beta_1$	$\beta_2 = 3\beta_1$
B=1	-1896.5, 1961.5	-2306.7, 2036.1	-2524, 2056	-2745.8, 2058.5

Table 1.2 illustrates, for $\gamma_1 = 0.006$, how the welfare levels of the more affected country 1 (left entry) and of country 2 (right entry) in the non cooperative equilibrium depend on the relative market sizes of the two countries. Note that the total welfare of the coalition of both countries in either the cooperative equilibrium or in the social optimum is the same for all values of (β_1, β_2) shown in Table 1.2, because $B = \beta_1 + \beta_2 = 1$ is fixed (compare the formulas in Propositions 2 and 3, which depend only on B and Γ). We see that the total welfare of the two countries and the welfare of country 1 decrease sharply when the market size parameter of the less affected country increases. A situation where a country that does not suffer much from pollution has high consumer demand is particularly bad for the other country's welfare.

Table 1.3 – Welfare levels of the two importing countries in the cooperative equilibrium for $\beta_1 \geq \beta_2$.

	$\beta_1 = \beta_2$		$\beta_1 = 1.5\beta_2$		$\beta_1 = 2\beta_2$		$\beta_1 = 3\beta_2$	
	Import1	**Import2**	**Import1**	**Import2**	**Import1**	**Import2**	**Import1**	**Import2**
B=1	777.2	2280	1293.3	1763.9	1637.4	1419.8	2067.4	989.7
B=2	774.9	2671.4	1385	20613	1791.8	1654.5	2300.3	1146.1
B=3	766.7	2867.9	1424.3	2210.3	1862.7	1771.8	2410.7	1223.8

Tables 1.3 and 1.4 show the welfare levels of the two importing countries in the cooperative equilibrium respectively when $\beta_1 \geq \beta_2$ and when $\beta_2 \geq \beta_1$ (still for $\gamma_1 = 0.006$). Comparing these tables with Table 1.2 for $B = 1$, we observe that the country that is more strongly affected by pollution always gains from cooperation, while the less affected country only gains from cooperation (coordinating actions in the game with the cartel) if it has a sufficiently low relative market size. However, the total welfare of the importing countries is always larger in the cooperative equilibrium than in the non cooperative equilibrium (note also again that it only depends on B). We can then conclude that, the incentive of country 2 to cooperate with country 1 may depend on whether the coalition can pursue other objectives than maximizing utilitarian welfare and/or whether side payments are possible.

Table 1.4 – Welfare levels of the two importing countries in the cooperative equilibrium with $\beta_2 \geq \beta_1$.

	$\beta_2 = \beta_1$		$\beta_2 = 1.5\beta_1$		$\beta_2 = 2\beta_1$		$\beta_2 = 3\beta_1$	
	Import1	**Import2**	**Import1**	**Import2**	**Import1**	**Import2**	**Import1**	**Import2**
B=1	777.2	2280	261	2796.1	-83	3140.2	-513.1	3570.3
B=2	774.9	2671.4	164.7	3281.6	-242	3688.4	-750.4	4196.8
B=3	766.7	2867.9	109	3525.5	-329.3	3963.9	-877.3	4512

Figure 1.1 shows how Θ depends on $\frac{\gamma_1}{\gamma_2} \in (1, 6]$ for $\Gamma = 0.007$ and under the assumption that $\beta_1 = \beta_2 = 0.5$. The larger is the asymmetry between the two countries with respect to their cost of pollution, the earlier we enter the bilateral monopoly phase.

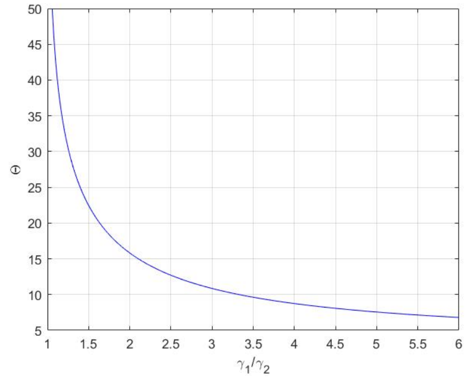


Figure 1.1 – Evolution of Θ with $\frac{\gamma_1}{\gamma_2}$.

Figure 1.2 shows the carbon taxes in the two importing countries when they do not cooperate and when they form a coalition. We see that, in importing country 2, the tax rate is always much higher in the cooperative equilibrium than in the non cooperative equilibrium. In importing country 1 the carbon tax is almost the same in both cooperative and non cooperative equilibria at the beginning of the horizon, then at a certain time, the non cooperative carbon tax becomes higher than the cooperative tax and it remains larger forever. This can be explained by the fact that, when the two importing countries cooperate, they both consume the resource until economic exhaustion. None of them leave the market at a finite time.

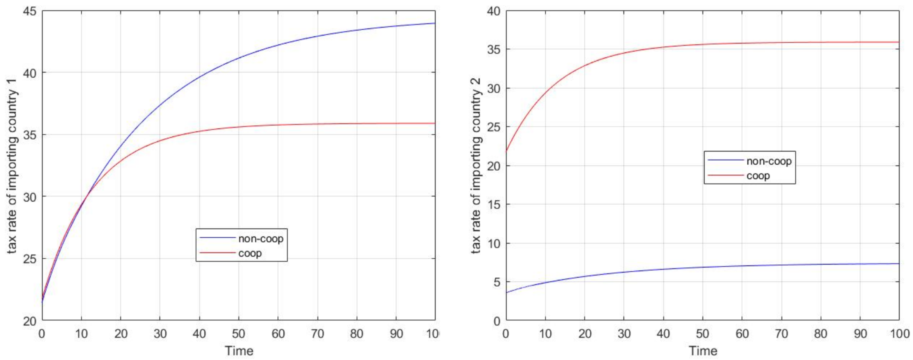


Figure 1.2 – Tax rates with $\gamma_1 = 0.006$ and $\gamma_2 = 0.001$.

Figure 1.3 shows that consumption in country 2 converges more quickly to zero in the co-operative equilibrium than in the non cooperative equilibrium. Moreover, its consumption is always higher in the non cooperative equilibrium than in the cooperative equilibrium. Consumption in country 1 is always higher in the cooperative equilibrium than in the non cooperative equilibrium. Concerning the stock of pollution, we see that at the beginning of the horizon, the stock of pollution in the social optimum is higher than in the non cooperative equilibrium.

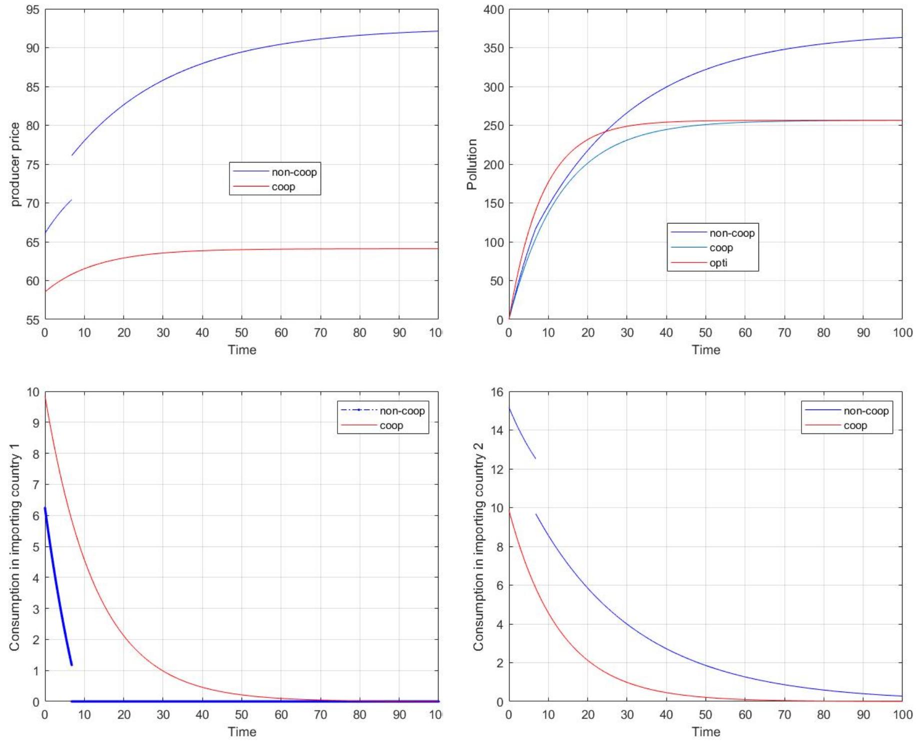


Figure 1.3 – Price, Stock of pollution and Consumption levels with $\gamma_1 = 0.006$ and $\gamma_2 = 0.001$.

In Figure 1.4, we show the welfare of country 2 and the profit of the exporting cartel, both for the non cooperative equilibrium and for a bilateral monopoly game between country 2 and the cartel. The latter scenario can be interpreted as a case where γ_1 is so high that Assumption 1 is violated, so that country 1 never consumes anything. We see that the cartel's profit is decreasing in the cost of pollution of the more affected country, γ_1. More interestingly, we see that country 2 may benefit from the presence of country 1. Its welfare may be higher in the non-cooperative game with three players than in the bilateral monopoly game. This happens when country 1 is sufficiently more affected than country 2, but not so much that it stays out of the market from the beginning.

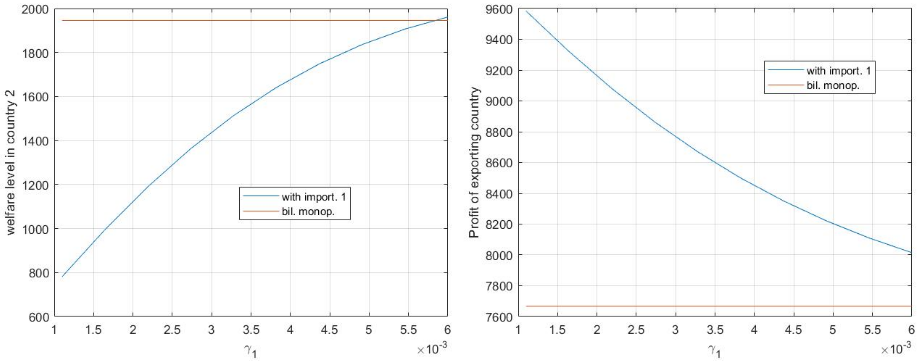


Figure 1.4 – Welfare of country 2 and that of the cartel with $\gamma_2 = 0.001$.

Throughout the rest of the section, we fix $B = 1$. The upper part of Figure 1.5 shows how the consumption paths in the non cooperative equilibrium depend on the relative market sizes, for $(\gamma_1, \gamma_2) = (0.006, 0001)$. We see that increasing the relative market size of country 2 implies that country 1 is excluded earlier and consumes less overall. Country 2 consumes more early on and more overall, and its consumption converges faster to 0 in the long term. Figure 1.7 shows the corresponding evolution of Z. For an initial period, the evolution of Z looks similar for all values of β_2, but in the middle and long term, Z is higher for higher values of β_2. Figure 1.8 shows that the differences become much smaller when γ_1 and γ_2 are much more similar.

For the rest of the section, we fix $\Gamma = 0.007$ and let γ_1 vary between 0.0037 and 0.006, while we fix $\beta_1 = \beta_2 = 0.5$. This allows us to study how the asymmetry with respect to how much the two countries are affected by pollution affects the non cooperative equilibrium (while the cooperative equilibrium always remains the same). The lower part of Figure 1.5 shows the consumption paths, with the expected pattern that country 1 consumes less and country 2 consumes more if the asymmetry increases. More interestingly, Figure 1.6 (in the Appendix) shows the evolution of Z. If countries are more symmetric, pollution increases faster for a relatively long period. However, in the long term, pollution is higher if countries are more asymmetric.

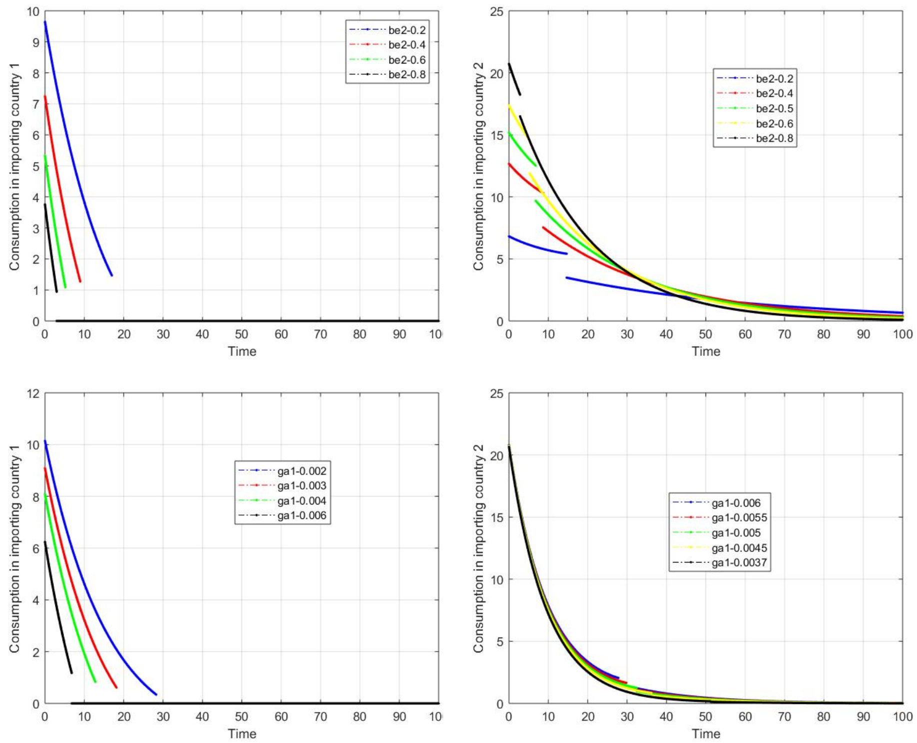


Figure 1.5 – Consumption levels.

In Table 1.5, we can see that as the countries become more symmetric in the cost of pollution, the welfare of country 1 increases while that of country 2 and of the exporting cartel decrease. However, the global welfare of all countries is increasing. Figure 1.9 in the Appendix shows how the welfare of countries depends on both (γ_1, γ_2) and (β_1, β_2) for $B = 1$ and $\Gamma = 0.007$.

Table 1.5 – Welfare levels of countries with $\Gamma = 0.007$.

	$\gamma_1 = 0.006$	$\gamma_1 = 0.0055$	$\gamma_1 = 0.005$	$\gamma_1 = 0.0045$	$\gamma_1 = 0.0037$
Importing country 1	-1896.5	-1466.6	-1015.1	-531.3	350.5
Importing country 2	1961.5	1912.3	1774.2	1529.7	850.7
Exporting cartel	8014.6	7831.3	7697.5	7611.1	7559.6

6 Conclusion

In this paper, we have extended a linear-quadratic version of the bilateral monopoly trade model between an importing country and an exporting cartel of an exhaustible and polluting resource (see Rubio and Escriche (2001), Liski and Tahvonen (2004), Dullieux, Ragot, and Schubert (2011), Kagan, Van der Ploeg, and Withagen (2015)) by analyzing the non cooperative open loop game between two asymmetric importing countries and a resource-exporting cartel which owns a stock of a polluting resource.

We have shown that when the countries act non cooperatively, at a finite time, the producer price jumps and the country that is most affected by pollution stops demanding the resource. However, the country with the lowest cost of pollution demands the resource until economic exhaustion. The cooperative equilibrium leads to a lower long term stock of pollution than the non cooperative equilibrium and to the same long term stock as the social optimum, and it is more conservative than the latter in the sense that extraction occurs more slowly.

Our numerical results based on the explicit characterization of the non-cooperative equilibrium have yielded a number of interesting insights. In particular, even a country that has moderately large consumer demand for the resource and is significantly less affected by pollution may be reluctant to join a coalition with the other importing country if the coalitions' objective is restricted to maximizing joint utilitarian welfare. In the non cooperative scenario, more symmetry with respect to the cost of pollution leads to faster increase of the stock of pollution initially, but to a lower long term stock and higher total discounted welfare.

Appendix

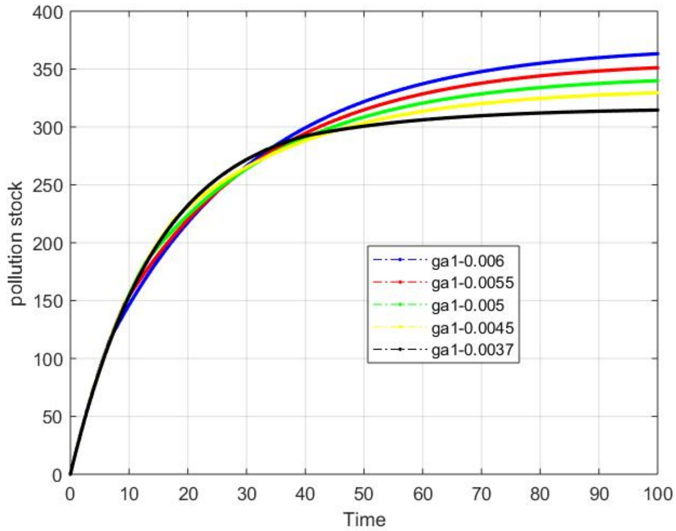


Figure 1.6 – Pollution stock with $\beta_1 = \beta_2 = 0.5$.

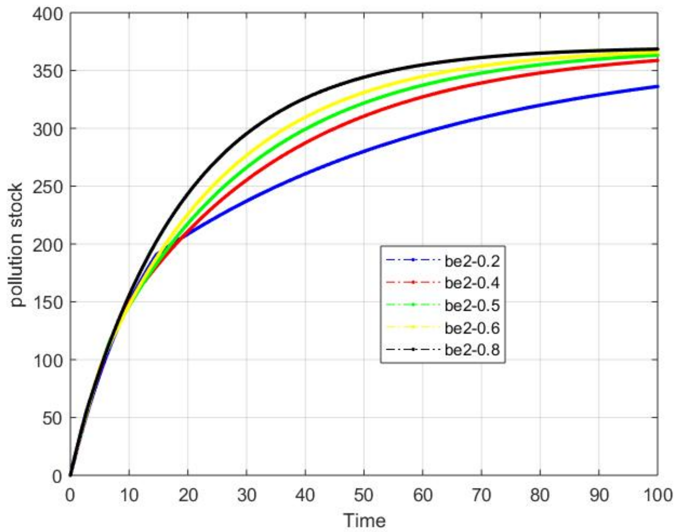


Figure 1.7 – Pollution stock with $\gamma_1 = 0.006$ and $\gamma_2 = 0.001$.

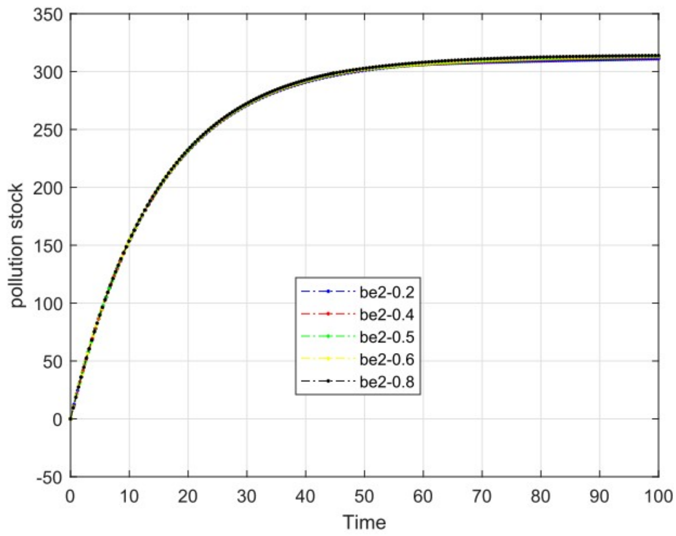


Figure 1.8 – Pollution stock with $\gamma_1 = 0.0036$ and $\gamma_2 = 0.0034$.

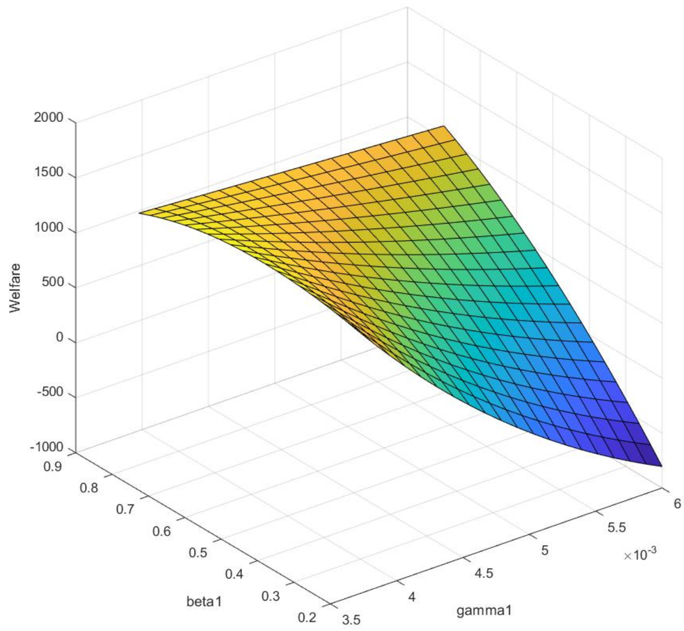


Figure 1.9 – Sum of welfare of countries.

Proof of Lemma 1:

The current value Hamiltonian H_e has the form

$$H_e = \begin{cases} H_e^1 = \sum_{j=1}^2 \beta_j(A-p-T_j)\,(p-cZ+\mu_e) \quad if \quad p \le A-T_1 \\ H_e^2 = \beta_2(A-p-T_2)\,(p-cZ+\mu_e) \quad if \quad A-T_1 \le p \le A-T_2 \\ 0 \quad otherwise. \end{cases}$$

H_e^1 and H_e^2 are both strictly concave in p, and we have:

$$\begin{aligned} \frac{\partial H_e^1}{\partial p} &= BA - 2Bp + cBZ - B\mu_e - \sum_{j=1}^{2} \beta_j T_j, \\ \frac{\partial H_e^2}{\partial p} &= \beta_2 A - 2\beta_2 p + c\beta_2 Z - \beta_2\mu_e - \beta_2 T_2. \end{aligned}$$

Thus, the unconstrained maximum of H_e^1 is $p_1^* = \frac{1}{2B}\left(BcZ + BA - B\mu_e - \sum_{i=1}^2 \beta_i T_i\right)$.
The unconstrained maximum of H_e^2 is $p_2^* = \frac{1}{2}\left(cZ + A - \mu_e - T_2\right)$.
We note first that H_e cannot be maximized at $p = A - T_1$. Indeed this would require $\frac{\partial H_e^1}{\partial p} \ge 0$ and $\frac{\partial H_e^2}{\partial p} \le 0$ at $p = A - T_1$, which is equivalent to

$$-A + cZ - \mu_e + 2T_2 \ge \frac{\sum_{j=1}^2 \beta_j T_j}{B} \quad and \tag{1.23}$$

$$-A + cZ - \mu_e + 2T_2 \le T_2. \tag{1.24}$$

If (1.23) and (1.24) hold simultaneously, we obtain $\frac{\sum_{j=1}^2 \beta_j T_j}{B} \le T_2$, i.e., $T_1 \le T_2$, which is impossible, because according to (1.10), $T_1 > T_2$.
We now show that $p = p_1^*$ maximizes H_e if and only if $H_e^1(p_1^*) \ge H_e^2(p_2*)$, and that $p = p_2^*$ maximizes H_e if and only if $H_e^1(p_1^*) \le H_e^2(p_2*)$. Note that $p_1^* < p_2^*$. If $p_2^* \le A - T_1$, then $H_e^2(p_2*) \le H_e^1(p_2^*) \le H_e^1(p_1^*)$, where at least one of the inequalities is strict (because $H_e^1(p) \ge H_e^2(p)$ for $p \le A - T_1$, and becauseH_e^1 is strictly concave), and p_1^* is the unique maximizer of H_e. If $p_1^* \ge A - T_1$, then $H_1^e(p_1^*) \le H_2^e(p_1*) \le H_e^2(p_2*)$, where at least one of the inequalities is strict (because $(A - T_1 - p) \le 0$ for $p \ge A - T_1$ and H_e^2 is strictly concave). Finally, if $p_1^* \le A - T_1$ and $p_2^* \ge A - T_1$, p_1^* maximizes H_e if $H_e^1(p_1^*) \ge H_e^2(p_2*)$, and p_2^* maximizes H_e if $H_e^1(p_1^*) \le H_e^2(p_2*)$. As $H_e^1(p_1^*) \gtreqless H_e^2(p_2*)$ if and only if $cZ - \mu_e \lesseqgtr A - \frac{\sqrt{B}}{\sqrt{B}-\sqrt{\beta_2}} \frac{\sum_{i=1}^2 \beta_i T_i}{B} + \frac{\sqrt{\beta_2}}{\sqrt{B}-\sqrt{\beta_2}} T_2$ this proves the Lemma 1.

Proof of Proposition 1:

As noted in the text, $cZ - \mu_e$ is increasing in t, and $D = A - \frac{\sqrt{B}}{\sqrt{B}-\sqrt{\beta_2}}\frac{\sum_{i=1}^{2}\beta_i T_i}{B} + \frac{\sqrt{\beta_2}}{\sqrt{B}-\sqrt{\beta_2}}T_2$ is decreasing in t. If $cZ - \mu^e \geq D$ at $t = 0$, we are done. If $cZ - \mu_e < D$ at $t = 0$, we must show that there is a finite time Θ for which $cZ - \mu_e = D$. Observe that p_2^* is the midpoint between $cZ - \mu_e$ and $A - T_2$, and that $p_1^* < p_2^*$. In particular, $Q(p) = \dot{Z}$ can only converge to 0 as $t \to \infty$ (which must be the case, because Z is a bounded, increasing function) if $cZ - \mu_e$ converges to $A - T_2$. On the other hand, $A - T_2 - D = \frac{\beta_1}{B-\sqrt{B\beta_2}}(T_1 - T_2) = \frac{\beta_1}{B-\sqrt{B\beta_2}}(\gamma_1 - \gamma_2)\int_t^{\infty} e^{-\rho(\tau-t)}Z(\tau)d\tau$ is strictly positive for any t and increasing (because Z is increasing). This proves the existence of a unique Θ for which $cZ - \mu_e = D$.

Appendix 1:

After time Θ, importing country 1 is already out of the market. However, by combining the first order conditions of the problem of importing country 2 and the expression of the producer price (p_2^*) that the exporting cartel sets in this phase, we have the following system of differential equations to solve:

$$\begin{cases} \dot{Z} = \frac{\beta_2}{2}(-cZ + \mu_e) + \frac{1}{2}\beta_2(A - T_2) \\ \dot{T_2} = \rho T_2 - \gamma_2 Z \\ \dot{\mu}_e = (\rho + \frac{c\beta_2}{2})\mu_e - \frac{c^2\beta_2 Z}{2} + \frac{c}{2}\beta_2(A - T_2) \end{cases}$$

The resolution of this system of differential equations gives the following results for all $t \in [\Theta, \infty)$

$$\begin{cases} Z(t) = w_1' e^{\theta_1' t} + \frac{A\rho}{\rho c+\gamma_2} \\ T_2(t) = \frac{\gamma_2}{\theta_2'} w_1' e^{\theta_1' t} + \frac{A\gamma_2}{\rho c+\gamma_2} \\ \mu_e(t) = -\frac{c\theta_1'}{\theta_2'} w_1' e^{\theta_1' t} \\ p(t) = \frac{c\rho-\gamma_2}{2\theta_2'} w_1' e^{\theta_1' t} + \frac{A\rho c}{\rho c+\gamma_2} \end{cases}$$

where $w_1' \in \mathbb{R}$, $\theta_1' = \frac{1}{2}(\rho - \sqrt{\Delta'})$, $\theta_2' = \frac{1}{2}(\rho + \sqrt{\Delta'})$ and $\Delta' = \rho^2 + 2\beta_2(\rho c + \gamma_2)$.

At the time Θ, the value of the Hamiltonian for the problem of the exporting cartel is such that, $H_e(p_1^*) = H_e(p_2^*)$. Using the two expressions for the formulas of Z at this time (see (1.20) and (1.21)) with this equality, we get these two following equations

$$w_1' e^{\theta_1' \Theta} = \frac{\beta_1 A \theta_2' (\gamma_2 - \gamma_1)}{(\rho c + \gamma_2)(B\rho c + \sum_{i=1}^2 \gamma_i \beta_i - \sqrt{B\beta_2}(\rho c + \gamma_2))}, \tag{1.25}$$

and

$$w_1 \theta_1 e^{\theta_1 \Theta} + w_2 \theta_2 e^{\theta_2 \Theta} = \sqrt{\frac{B}{\beta_2}} w_1' \theta_1' e^{\theta_1' \Theta}. \tag{1.26}$$

To find the values of the different unknowns w_1, w_2, w_1' and Θ, we use the initial value of the stock of pollution, the continuity of the stock of pollution at time Θ and the equations (1.25) and (1.26). Combining these equations, the system of equations to solve is the following

$$\begin{cases} w_1 + w_2 + \frac{\rho AB}{B\rho c + \sum_{i=1}^2 \gamma_i \beta_i} = 0, \\ w_1 e^{\theta_1 \Theta} + w_2 e^{\theta_2 \Theta} + \frac{\rho AB}{B\rho c + \sum_{i=1}^2 \gamma_i \beta_i} = Z_\Theta, \\ w_1' e^{\theta_1' \Theta} = \frac{\beta_1 A \theta_2' (\gamma_2 - \gamma_1)}{(\rho c + \gamma_2)(B\rho c + \sum_{i=1}^2 \gamma_i \beta_i - \sqrt{B\beta_2}(\rho c + \gamma_2))}, \\ w_1 \theta_1 e^{\theta_1 \Theta} + w_2 \theta_2 e^{\theta_2 \Theta} = \sqrt{\frac{B}{\beta_2}} w_1' \theta_1' e^{\theta_1' \Theta}, \end{cases}$$

where $Z_\Theta = w_1' e^{\theta_1' \Theta} + \frac{A\rho}{\rho c + \gamma_2}$.

Proof of Proposition 2:

The problem solved by the two importing countries in the cooperative equilibrium is:

$$\max_{T_1, T_2} \int_0^\infty e^{-\rho t} \sum_{i=1}^2 \left(\max\{ \frac{\beta_i}{2} \left[(A-p)^2 - T_i^2 \right], 0\} - \frac{\gamma_i}{2} Z^2 \right) dt$$

$$\text{subject to } \dot{Z}(t) = \textstyle\sum_{i=1}^2 \max\{\beta_i (A - p - T_i), 0\}$$

The current value Hamiltonian in this case is:

$H = r \sum_{i=1}^2 \left(\max\{ \frac{\beta_i}{2} \left[(A-p)^2 - T_i^2 \right], 0\} - \frac{\gamma_i}{2} Z^2 \right) + \lambda \left(\sum_{i=1}^2 \max\{\beta_i (A - p - T_i), 0\} \right)$, where $r \in \{0,1\}$ and λ is the co-state variable related to the stock of pollution.

As H is concave in T_1, T_2 and Z with $r=1$, the necessary conditions are also sufficient (see Theorems 12 and 13 in Chapter 3 of Seierstad and Sydsaeter (1987)), and are given by:

$$
\begin{aligned}
-\beta_i T_i - \beta_i \lambda &= 0 \quad \text{for} \quad i \in \{1,2\} && (1.27)\\
\dot{\lambda} &= \rho\lambda + Z\sum_{i=1}^{2}\gamma_i && (1.28)\\
\dot{Z} &= \sum_{i=1}^{2}\max\{\beta_i(A-p-T_i),0\} && (1.29)\\
\lim_{t\to\infty} e^{-\rho t}\lambda Z &= 0 && (1.30)
\end{aligned}
$$

By combining equations (1.27) and (1.28), we obtain

$$\dot{T_i} = \rho T_i - \Gamma Z \quad \text{for} \quad i \in \{1,2\}. \tag{1.31}$$

From equation (1.31), we can easily see that in equilibrium, $T_1 = T_2 = T^c$.

The problem solved by the planner in the exporting country is:

$$\max_{p} \int_0^{\infty} e^{-\rho t}\left[\left(\sum_{i=1}^{2}\max\{\beta_i(A-p-T_i),0\}\right)(p-cZ)\right]dt$$

$$\text{subject to } \dot{Z} = \textstyle\sum_{i=1}^{2}\max\{\beta_i(A-p-T_i),0\}$$

Given that $T_1 = T_2 = T^c$, the current value Hamiltonian is:

$H = r_e^c B\max\{(A-p-T^c),0\}(p-cZ) + \mu_c B\max\{(A-p-T^c),0\}$,

where $r_e^c \in \{0,1\}$ and μ_c is the co-state variable related to the stock of pollution.

Because of the fact that H_e is concave in p and Z for $r_e^c = 1$, the necessary conditions below are also sufficient (see Theorems 12 and 13 in Chapter 3 of Seierstad and Sydsaeter

(1987))

$$
\begin{aligned}
-B(p - cZ + \mu_c) + B(A - p - T^c) &= 0 &(1.32)\\
\dot{\mu_c} &= \rho\mu_c + c\dot{Z} &(1.33)\\
\dot{Z} &= B(A - p - T^c) &(1.34)\\
\lim_{t\to\infty} e^{-\rho t}\mu_c Z &= 0 &(1.35)
\end{aligned}
$$

From (1.32), we have:

$$
p = \frac{1}{2}\left(cZ + A - \mu_c - T^c)\right). \tag{1.36}
$$

When we replace p by its expression (1.36) into (1.34), we obtain

$$
\dot{Z} \;=\; \frac{B}{2}(\mu_c - cZ + A - T^c). \tag{1.37}
$$

To obtain the cooperative equilibrium, we have to solve the following system of differential equations:

$$
\begin{cases}
\dot{Z} = \frac{B}{2}(\mu_c - cZ + A - T^c)\\
\dot{T^c} = \rho T^c - \Gamma Z\\
\dot{\mu_c} = (\rho + \frac{cB}{2})\mu_c - \frac{cB}{2}(cZ - A) - \frac{cB}{2}T^c,
\end{cases}
$$

the solution of which is given in (1.22).

Proof of Proposition 3:

The problem solved by the social planner in this case is as follows:

$$
\max_{p+T_1, p+T_2} \int_0^\infty e^{-\rho t}\sum_{i=1}^{2}\left[A\beta_i(A-p-T_i) - \frac{1}{2\beta_i}(\beta_i(A-p-T_i))^2 - \beta_i cZ(A-p-T_i) - \frac{\gamma_i}{2}Z^2\right] dt
$$

subject to

$$
\dot{Z}(t) = \sum_{i=1}^{2}\beta_i(A - p - T_i)
$$

This problem can be rewritten as:

$$\max_{p+T_1,p+T_2} \int_0^\infty e^{-\rho t} \sum_{i=1}^{2} \left[\frac{\beta_i}{2}(A-p-T_i)\,(A+p+T_i-2cZ)-\frac{\gamma_i}{2}Z^2\right] dt$$

subject to

$$\dot{Z}(t) = \sum_{i=1}^{2} \beta_i(A-p-T_i)$$

The current value Hamiltonian is:
$H = r^o \left(\sum_{i=1}^2 \left[\frac{\beta_i}{2}(A-p-T_i)\,(A+p+T_i-2cZ)-\frac{\gamma_i}{2}Z^2\right]\right) + \mu_o(\sum_{i=1}^2 \beta_i(A-p-T_i))$,
where $r^o \in \{0,1\}$ and μ_o is the co-state variable associated to the stock of pollution.
For $r^o = 1$, H is concave in $p+T_1$, $p+T_2$ and Z. Thus, the following necessary conditions for the optimality are also sufficient (see Theorems 12 and 13 in Chapter 3 of Seierstad and Sydsaeter (1987)):

$$\begin{aligned}
-\frac{\beta_i}{2}(A+p-2cZ)+\frac{\beta_i}{2}(A-p-T_i)-\beta_i\mu_o &= 0 \quad \text{for} \quad i \in \{1,2\} \\
\dot{\mu}_o &= \rho\mu_o + c\dot{Z} + Z\sum_{i=1}^{2}\gamma_i \\
\dot{Z} &= \sum_{i=1}^{2}\beta_i(A-p-T_i) \\
\lim_{t\to\infty} e^{-\rho t}\mu_o Z &= 0
\end{aligned}$$

These conditions are equivalent to:

$$p+T_i = cZ-\mu_o \quad \text{for} \quad i \in \{1,2\} \tag{1.38}$$

$$\dot{\mu}_o = (\rho+Bc)\mu_o + (\Gamma - c^2B)Z + ABc \tag{1.39}$$

$$\dot{Z} = -cBZ + B\mu_o + AB \tag{1.40}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu_o Z = 0 \tag{1.41}$$

To compute the optimal tax rate and the price in a decentralized efficient equilibrium, we solve the problem of a competitive producer.

This problem is as follows

$$\max_{q} \int_0^{\infty} e^{-\rho t}\left[(p - cZ)\, q\right] dt$$

$$\text{subject to } \dot{Z} = q$$

The current value Hamiltonian is: $H = r_e^o\,(p - cZ) + \mu q$,

where $r_e^o \in \{0, 1\}$ and μ is the co-stated variable related to the stock of pollution. The first order conditions are

$$\begin{aligned} p &= cZ - \mu & (1.42)\\ \dot{\mu} &= \rho\mu + c\dot{Z} & (1.43)\\ \lim_{t\to\infty} e^{-\rho t}\mu Z &= 0 & (1.44) \end{aligned}$$

By combining (1.38) and (1.42), we obtain

$$T_i = \mu - \mu_o. \tag{1.45}$$

By differentiating (1.45) and by combining the result with (1.39), (1.40), (1.43) and (1.45), we find

$$\dot{T_i} = \rho T_i - \Gamma Z. \tag{1.46}$$

By differentiating (1.38) and by combining the result with (1.39), (1.40) and (1.46), we obtain

$$\dot{p} = -\rho(T_i + \mu). \tag{1.47}$$

By solving the system of differential equations constitutes with (1.39), (1.40), (1.46) and (1.47), we obtain that the stock of pollution, the optimal producer price and the optimal tax rate at each time t are

$$\begin{aligned} Z^o(t) &= \frac{\rho A}{\rho c + \Gamma}(1 - e^{-\theta t),}\\ p^o(t) &= \frac{\rho A c}{\rho c + \Gamma}\left(1 - \frac{\rho}{\rho + \theta} e^{-\theta t}\right),\\ T^o(t) &= \frac{A\Gamma}{\rho c + \Gamma}\left(1 - \frac{\rho}{\rho + \theta} e^{-\theta t}\right). \end{aligned}$$

Chapter 2

The Strategic Effects of a Bilateral Border Tax Adjustment in an Emissions Taxation Game

1 Introduction

Carbon emissions are the main driver of climate change. Climate change is a global problem. While full cooperation between all countries within an international environmental agreement would be the best way to mitigate global emissions, agreements implementing effective actions are difficult to achieve due to free-rider incentives. On the other hand, the effectiveness of unilateral actions aimed at reducing greenhouse gas emissions may be limited due to "carbon leakage". Stricter environmental policies for producers, for example higher carbon taxes, may trigger a relocation of the production of emissions-intensive goods to countries with laxer regulations. Consequently, reductions in domestic emissions may be partially, or even fully, offset by higher emissions abroad. Furthermore, stricter policies may reduce the international competitiveness of domestic firms.

Border adjustments have been widely proposed as a measure for eliminating or at least reducing the issues of carbon leakage and loss of competitiveness (e.g. Hecht and Peters (2019); Larch and Wanner (2017); Baksi and Chaudhuri (2017); Böhringer, Müller, and Schneider (2015); Eyland and Zaccour (2012, 2014); Anouliés (2015)).

Border adjustments can take the form of import tariffs, export rebates, or both. In the case of border adjustments through an import tariff, a country with strict carbon regulations imposes a tariff on imports from countries with lower levels of regulation to (partially or fully) adjust the carbon price for goods consumed in the domestic market. Export rebates are subsidies granted by countries with stricter regulation for exports to countries with laxer regulation, to adjust the carbon price and hence level the playing field in these foreign markets.

The effectiveness of border adjustments has been studied by two different strands of literature.

The first strand uses empirically calibrated computable general equilibrium models to estimate the mitigation of leakage effects through different forms of border adjustments (e.g. Böhringer, Rosendahl, and Schneider (2014); Böhringer, Müller, and Schneider (2015); Böhringer and Rutherford (2017)).

The second strand of literature studies the effects of border adjustments in strategic trade models with oligopolistic industries and transboundary pollution (e.g. Hecht and Peters (2019); Baksi and Chaudhuri (2017); Eyland and Zaccour (2012, 2014); Anouliés (2015)). This literature analyzes countries' strategic choices of emissions taxes to evaluate to what extent border adjustments, and in particular the threat of border adjustments, can lead to more efficient tax levels, when taxes are the only policy instrument and thus play a role in addressing both the environmental problem and oligopoly distortions. Another question in this context is whether full or partial adjustment of carbon prices is preferable, either from the perspective of the country that imposes the border adjustment, or for global welfare.

The latter literature has focused on two-country models and on duopoly competition between one domestic and one foreign firm in each market. Moreover, most work has restricted attention to Cournot competition and to import border adjustments, as these are generally considered more likely to be compatible with WTO rules[1].

The present paper contributes to this literature in two ways. First, we consider a model

[1]Hecht and Peters (2019) consider both Cournot and Bertrand competition in a model with differentiated products, and they consider both import border adjustments and border adjustments that combine an import tariff and an export rebate.

with an arbitrary number of identical firms in each country, all of which compete à la Cournot in both markets. This allows a more detailed analysis of how equilibrium emissions taxes in the presence of an import border tax adjustment (BTA) depend on the competitiveness of the industry. Secondly, the existing literature has assumed that countries differ in their vulnerability to (their cost of) pollution and has restricted attention to cases where it is clear that the more affected country sets the higher tax and hence charges the BTA, either by assuming this exogenously, or by making assumptions on the parameters that guarantee it. By contrast, we take a closer look at the strategic effects associated with a BTA when *either* country might be the one that sets the higher tax and hence charges the BTA. This is called the case of a "bilateral" BTA-policy in Elboghdadly and Finus (2020). To this end, we consider a model with two symmetric countries, called Home and Foreign, engaged in oligopolistic trade of a polluting good that is very similar to (a particular instance of) the free-trade model of Kennedy (1994) and introduce an import BTA into that setting. If the Government of country k sets a carbon tax rate t_k per unit of production of its firms, and if $t_k > t_l$ where t_l is the carbon tax rate of the other country (l), it imposes a per-unit adjustment tax $\delta(t_k - t_l)$ on its imports from country l. Following Eyland and Zaccour (2012, 2014), we say that the BTA is partial if $\delta < 1$ and full if $\delta = 1$.

We study the Nash equilibria of the game where both countries simultaneously choose their taxes, taking as given the level of the BTA (δ), and anticipating the Cournot equilibrium quantities and prices in both markets (which result for the given taxes and border tax adjustments). Assuming free-trade (i.e., no BTA), Kennedy (1994) has shown that equilibria are symmetric and that equilibrium taxes are always lower than efficient taxes, due to two effects, a transboundary externality effect and a rent-capture effect.

Our main findings are as follow. First, we obtain a complete characterization of the pure-strategy Nash equilibria of the game with arbitrary δ-BTA. We show that for any given δ, when the cost of pollution is sufficiently small, equilibria are symmetric and tax rates are *higher* than the efficient taxes. Thus, the threat of BTA pushes taxes above the efficient levels. There is a unique level of the cost of pollution for which the symmetric equilibrium is unique and efficient. For high level of pollution, the equilibria are asymmetric, and the taxes are uniquely determined and are both lower than the efficient taxes. The BTA always help to mitigate global level of pollution. When the countries suffer little from pollution there is a unique and generally partial BTA for which the non cooperative taxes are equal to the efficient taxes. Whenever the cost of pollution is such that the efficient tax is positive, the global welfare in the equilibrium with BTA (regardless whether equilibrium is symmetric or asymmetric) is higher than in the non cooperative equilibrium without BTA.

To the best of our knowledge, the other paper that study the effect of the bilateral BTA is Elboghdadly and Finus (2020). However, they are interested in the case where firms can endogenously choose their location and they focus on duopoly competition. Another paper by Al Khourdajie and Finus (2020) studied the ability of border tax adjustment to reduce free-riding in international environmental agreements.

The paper is organized as follows. In sections 2 and 3, we set up the model and discuss preliminaries results. Section 4 analyzes the cooperative tax rate and sections 5 and 6 present our main results about the non cooperative taxes and some comparisons. We conclude in section 7.

2 Model

We consider a world with two identical countries, denoted as Home (H) and Foreign (F). In each country, there are n firms who produce a homogeneous and polluting good at a constant marginal cost of $c \geq 0$. There is one market in each country, and the $2n$ firms compete à la Cournot in quantities in each market. The quantity sold by firm i from country l to the market k is denoted q^i_{lk}, where $i \in \{1, ..., n\}$ and $l, k \in \{H, F\}$. The inverse demand function in each country is $p(Q_l) = \alpha - \beta Q_l$, where $\alpha > c$, $\beta > 0$ and Q_l is the total quantity sold and consumed in country $l \in \{H, F\}$, i.e., $Q_l = \sum_{k\in\{H,F\}} \sum_{i\in\{1,..,n\}} q^i_{kl}$. We assume that one unit of production of the good generates one unit of pollution. The global level of pollution is denoted by Z (i.e., $Z = Q_H + Q_F$).

For each country, the cost of pollution is described by the function $D(Z) = \frac{\gamma}{2} Z^2$, where $\gamma \geq 0$ is a constant. The case that interests us is of course $\gamma > 0$. However, all of our

analysis applies to the case $\gamma = 0$ (no pollution problem) as well, and it will be convenient to refer to this case at some point. The higher is γ the more countries suffer from the pollution. To regulate pollution, the Government in each country imposes a carbon tax per unit of production of their firms. We denote the carbon tax rate imposed in country k by t_k. Moreover, each country sets a border tax adjustment (BTA) on imports if its tax rate is higher than the one set in the other country. The BTA set by country k is $\delta(t_k - t_l)1\!\!1_{\{t_k \geq t_l\}}$, where $\delta \in [0, 1]$, $k \neq l \in \{H, F\}$ and $1\!\!1_{\{t_k \geq t_l\}}$ is equal to 1 if $t_k \geq t_l$ and 0 otherwise. The parameter δ is exogenous. The case $\delta = 1$ corresponds to a *full* BTA and $\delta \in (0, 1)$ to a *partial* BTA (see Eyland and Zaccour (2012)).
The two countries play a two-stage game as follows: in the first stage, taking the parameter of the BTA as exogenous, countries simultaneously determine their carbon taxes, and in the second stage, firms simultaneously produce and sell to both markets. We compute the subgame perfect equilibrium of this game.

3 Preliminaries

In this section, we determine the industry equilibrium (firms' equilibrium quantities and profits) for any given vector $\mathbf{t} = (t_H, t_F)$ and any level of the BTA. We also state the formulas for each country's total welfare.

The industry equilibrium

We let $\mathbf{q} \in \mathbb{R}^{2n}$ denote the vector of quantities and $\mathbf{t} = (t_H, t_F)$ the vector of tax rates. The profit functions of firms $i \in \{1, 2, ..., n\}$ from country H and $j \in \{1, 2, ..., n\}$ from country F are:

$$\begin{aligned}
\pi_H^i(\mathbf{q}, \mathbf{t}) &= (A - \beta Q_H - t_H) q_{HH}^i + (A - \beta Q_F - t_H - \delta(t_F - t_H)1\!\!1_{\{t_F \geq t_H\}}) q_{HF}^i, \\
\pi_F^j(\mathbf{q}, \mathbf{t}) &= (A - \beta Q_H - t_F - \delta(t_H - t_F)1\!\!1_{\{t_H \geq t_F\}}) q_{FH}^j + (A - \beta Q_F - t_F) q_{FF}^j.
\end{aligned}$$

where $A = \alpha - c$.
If $\mathbf{t}$ and δ are such that all quantities in (2.1) below are positive, then these constitute

the Cournot equilibrium quantities (see Belleflamme and Peitz (2015) for details on the linear Cournot model):

$$\begin{aligned}
q^i_{HH} &= \frac{A-(1+n)t_H+nt_F+n\delta(t_H-t_F)\mathbb{1}_{\{t_H\geq t_F\}}}{\beta(1+2n)}, \\
q^i_{HF} &= \frac{A-(1+n)t_H+nt_F+\delta(1+n)(t_H-t_F)\mathbb{1}_{\{t_F\geq t_H\}}}{\beta(1+2n)}, \\
q^j_{FH} &= \frac{A+nt_H-(1+n)t_F-\delta(1+n)(t_H-t_F)\mathbb{1}_{\{t_H\geq t_F\}}}{\beta(1+2n)}, \\
q^j_{FF} &= \frac{A+nt_H-(1+n)t_F-n\delta(t_H-t_F)\mathbb{1}_{\{t_F\geq t_H\}}}{\beta(1+2n)}.
\end{aligned} \tag{2.1}$$

As all firms from a given country sell identical quantities, we omit the superscripts i and j from now on.

The total quantity consumed in country k is given by:

$$Q_k = \frac{n\left[2A-t_k-t_l-\delta(t_k-t_l)\mathbb{1}_{\{t_k\geq t_l\}}\right]}{\beta(1+2n)} \quad with \quad k\neq l. \tag{2.2}$$

The total quantity sold by firms from country k in market l is

$$X_{kl} = nq_{kl},$$

where q_{kl} is given by the formula in (2.1). The aggregate quantity produced by firms from country k is

$$X_k = \frac{n\left[2A-2(1+n)t_k+2nt_l+n\delta(t_k-t_l)\mathbb{1}_{\{t_k\geq t_l\}}+\delta(1+n)(t_k-t_l)\mathbb{1}_{\{t_l\geq t_k\}}\right]}{\beta(1+2n)} \quad with \quad k\neq l. \tag{2.3}$$

Regardless of the level of the border tax adjustment, the quantity sold by an individual firm in the domestic market is a decreasing function of the domestic tax rate. The total consumption in each country decreases with its own tax rate. Total consumption in the country that sets the higher tax rate is decreasing in δ. Production in each country is a decreasing function of its tax rate but is an increasing function of the other country tax rate.

Country k's net exports are

$$X_k - Q_k = \frac{-n(t_k - t_l)\left[(1+2n) - \delta(1+n)\right]}{\beta(1+2n)} \quad with \quad k \neq l.$$

Welfare

The welfare of country k is:

$$W_k(t_H, t_F) = CS_k + n\pi_k + TR_k + n\delta(t_k - t_l)\mathbb{1}_{\{t_k \geq t_l\}} q_{lk} - \frac{\gamma}{2} Z^2, \tag{2.4}$$

where $CS_k = \frac{\beta Q_k^2}{2}$ is the consumer surplus in country k, π_k is the equilibrium profit of a single firm from country k, TR_k denotes the revenue from the environmental tax, the fourth term is the revenue from BTA ($k \neq l \in \{H, F\}$) and the last term is the damage from global pollution.

4 The cooperative tax rate

We consider here the benchmark scenario where the two countries form a coalition and cooperatively set taxes to maximize their joint welfare. In line with the cooperative scenario, we assume that there is no BTA. Joint welfare is given by:

$$W^c(t_H, t_F) = \beta Q^2 + 2(p(Q) - c)Q - \gamma Z^2. \tag{2.5}$$

where $Q = Q_H = Q_F$ is the equilibrium level of consumption in either country and $Z = 2Q$.
Differentiating (2.5) with respect to taxes (using (2.2)), we obtain

$$\frac{\partial W^c(t_H, t_F)}{\partial t_H} = \frac{\partial W^c(t_H, t_F)}{\partial t_F} = \frac{-2n(p(Q) - c)}{\beta(1+2n)} + \frac{4n\gamma Z}{\beta(1+2n)} \tag{2.6}$$

which easily yields the following result.

Proposition 1. *Joint welfare is maximized if both countries set the tax rate*

$$t^c = t^c(A, \beta, n, \gamma) = \frac{A(8n\gamma - \beta)}{2n(\beta + 4\gamma)}, \tag{2.7}$$

and t^c is the unique symmetric tax rate maximizing joint welfare[2].

Proof. See Appendix.

We refer to t^c as the *cooperative* tax rate.

The quantity consumed in each country in this case is $Q^c = \frac{A}{\beta+4\gamma}$.

We make a few basic observations. First (nonsurprisingly), the cooperative tax rate is increasing in the parameter of the cost of pollution, i.e., $\frac{\partial t^c(A,\beta,n,\gamma)}{\partial \gamma} > 0$. Secondly, for any $\gamma > 0$, the cooperative tax rate is lower than the corresponding Pigouvian tax, which is equal to $2\gamma Z^c$, where $Z^c = 2Q^c$ is the global level of pollution. This is due to the fact that the tax is used to correct two market failures at the same time, the one due to the environmental externality and the under-production problem due to oligopolistic competition (see Kennedy (1994) for more explanations). Next, for any $\gamma > 0$, t^c is decreasing in β ($\frac{\partial t^c(A,\beta,n,\gamma)}{\partial \beta} < 0$), i.e., increasing in the parameter $\frac{1}{\beta}$ that is proportional to "market size". If $\gamma = 0$, then $t^c = -A/2n$ is independent of β, and exactly offsets the oligopoly distortion, i.e., we have $p(Q^c) = c$ in this case. By contrast, for any $\gamma > 0$, we have $p(Q^c) > c$, i.e., price is above marginal production cost. Finally, we note that t^c is actually negative if $8n\gamma < \beta$. All our main results below apply to cases where taxes are positive and hence really correspond to a tax rather than a subsidy, but we formally allow taxes to be negative unless otherwise mentioned.

5 The non cooperative taxation game

In this section, we study the non cooperative, simultaneous-move game where countries set their taxes to maximize their own welfare, taking firms' behavior (the industry equilibrium that will result depending on $\mathbf{t}$) as given.

[2]Due to the symmetry and the linearity properties of the model, joint welfare is also maximized for asymmetric tax profiles satisfying $t_H + t_F = 2t^c$ (as long as all firms produce positive quantities). This multiplicity is obviously irrelevant, as the socially optimal total quantity is unique.

5.1 Nash equilibrium without BTA

As a benchmark, we first derive the Nash equilibrium taxes in the simple case when there is no BTA ($\delta = 0$). Without BTA, our model is very similar to a particular case of the model studied by Kennedy (1994), for the case of perfectly transboundary (global) pollution. Consequently, we obtain the same effects (see below). If $\delta = 0$ (free trade), we have $Q_H = Q_F =: Q$, and the welfare of country k can be written as follows.

$$W_k^{no}(t_H, t_F) = \frac{\beta Q^2}{2} + ((p(Q) - c)X_k - \frac{\gamma}{2} Z^2. \tag{2.8}$$

Differentiating (2.8) with respect to t_k yields:

$$\frac{\partial W_k^{no}(t_H, t_F)}{\partial t_k} = \beta Q \frac{\partial Q}{\partial t_k} + p'(Q) X_k \frac{\partial Q}{\partial t_k} + (p(Q) - c)\frac{X_k}{\partial t_k} - \gamma Z (\frac{\partial Z}{\partial t_k}).$$

In particular, a straightforward computation (using (2.2) and (2.3)) yields,

$$\frac{\partial W_k^{no}(t_H, t_F)}{\partial t_k} |(t^c, t^c) = \frac{(-2n(1+n))(p(Q^c) - c)}{\beta(1+2n)} + \frac{2n\gamma Z^c}{\beta(1+2n)}. \tag{2.9}$$

We know that $\frac{\partial W^c(t_H,t_F)}{\partial t_k}|(t^c, t^c) = 0$. Then subtracting (2.6) from (2.9), we obtain:

$$\frac{\partial W_k^{no}(t_H, t_F)}{\partial t_k} |(t^c, t^c) = \frac{-2n^2(p(Q^c) - c)}{\beta(1+2n)} - \frac{2n\gamma Z^c}{\beta(1+2n)} \tag{2.10}$$

The first term in (2.10) is what Kennedy (1994) called the rent capture effect (RCE) and the second term is what he called the transboundary externality effect (TEE). Using the expression of the cooperative tax rate in (2.7) and the expressions of quantities in (2.1), we obtain:

$$RCE = \frac{-2n^2(p(Q^c) - c)}{\beta(1+2n)} = \frac{-8n^2\gamma A}{\beta(1+2n)(\beta+4\gamma)} \tag{2.11}$$

$$TEE = -\frac{2n\gamma Z}{\beta(1+2n)} = \frac{-4n\gamma A}{\beta(1+2n)(\beta+4\gamma)} \tag{2.12}$$

We can see that under free trade (without BTA), two forces reduce the non cooperative taxes below the efficient ones, for any $\gamma > 0$. First, countries ignore the environmental externality imposed on the other country (leading to the TEE). Secondly, given that market price is above marginal costs, countries have an incentive to lower their own tax to give an advantage to their producers and capture more rents from sales to foreign consumers. The following Proposition provides the explicit formula for the equilibrium tax rate.

Proposition 2. *The unique pure-strategy Nash equilibrium of the non-cooperative taxation game without BTA is* (t^{no}, t^{no})*, where*

$$t^{no} = t^{no}(A, \beta, n, \gamma) = \frac{A(4n\gamma - \beta(1+n))}{2n(2\gamma + \beta(1+n))}. \tag{2.13}$$

Proof. See Appendix.

We note that, like t^c, the non cooperative tax rate is an increasing function of γ and a decreasing function of β. Moreover, in line with the observations we made above, $t^c > t^{no}$ holds for any $\gamma > 0$. Finally, for $\gamma = 0$, we have $t^{no} = t^c = -A/2n$. Thus, if there is no pollution (only the oligopoly distortion), the equilibrium of the non-cooperative game is actually efficient (in the symmetric setting considered here).

5.2 Nash equilibrium with BTA

We now study the game where both countries set their taxes non-cooperatively in the presence of BTA. We begin with a discussion of country k's welfare function. Throughout, l denotes the other country.

The welfare of country k is given by

$$W_k(t_k, t_l) = \begin{cases} W_k^1(t_k, t_l) & if \quad t_k \geq t_l \\ W_k^2(t_k, t_l) & if \quad t_k \leq t_l \end{cases} \tag{2.14}$$

where

$$W_k^2(t_k, t_l) = \frac{\beta}{2}Q_k^2 + (p(Q_k) - c)X_{kk} + (p(Q_l) - c - \delta(t_l - t_k))X_{kl} - \frac{\gamma}{2}Z^2. \tag{2.15}$$

$$W_k^1(t_k, t_l) = \frac{\beta}{2}Q_k^2 + (p(Q_k) - c)X_{kk} + (p(Q_l) - c)X_{kl} + \delta(t_k - t_l)X_{lk} - \frac{\gamma}{2}Z^2. \tag{2.16}$$

and all quantities are replaced by the respective expressions in (2.1) and (2.2). The term $\frac{\beta}{2}Q_k^2 + (p(Q_k) - c)X_{kk}$ is the sum of the consumer surplus and of the profits and tax revenues associated with sales by domestic firms in the domestic market. If $t_k < t_l$, the profits and tax revenues for country k associated with sales in country l are $(p(Q_l) - c - \delta(t_l - t_k))X_{kl}$ (as firms from country k have to pay the BTA in this case), while the corresponding expression is $(p(Q_l) - c)X_{kl}$ if $t_k > t_l$. Moreover, if $t_k > t_l$, country k also earns the revenue $\delta(t_k - t_l)X_{lk}$ on the BTA. This explains the formulas in (2.14), (2.15) and (2.16). Of course, for any $\delta > 0$, the marginal effects of a change in t_k on the various quantities of interest (Q_k, Q_l, X_{kk}, X_{kl} and X_{lk}) change discontinuously at $t_k = t_l$ (recall that the quantities have to be replaced by the expressions from (2.1) and (2.2)). In particular the function $W_k(t_k, t_l)$ has a kink at $t_k = t_l$ whenever $\delta > 0$. Moreover, while W_k is concave in t_k for $t_k < t_l$ and also for $t_k > t_l$, it is often not globally concave in t_k (see below).

Intuitively, the best response of country k to any given tax t_l should be higher in the game with BTA than in the game without BTA. Indeed, if t_k is already weakly below t_l, marginally decreasing t_k has the same effects in the domestic market (k) in both cases, but the advantage given to domestic firms in the foreign market is smaller for $\delta > 0$ than it is for $\delta = 0$, and even disappears completely in the case of full BTA. Moreover, if t_k is already weakly above t_l, country k has a stronger incentive to increases its tax further in the case with BTA compared to the case without BTA: the negative effect on the competitiveness of its firms is the same in the foreign market in both cases, but the BTA "protects" firms in the domestic market and also generates additional revenues. This intuition is of course somewhat incomplete, as it ignores the effects on pollution and on consumers.

As a full comparison of best response correspondences is involved and not needed for the subsequent equilibrium analysis, we only show here that if one country sets the tax t^{no}, the optimal tax for the other country is higher than t^{no}.The argument also allows us to make some observations that are needed for the characterization of equilibria (in sections 5.2.1 and 5.2.2).

Consider the following derivatives:

$$\frac{\partial W_k^1(t_k,t_l)}{\partial t_k} = \beta Q_k \frac{\partial Q_k}{\partial t_k} + p'(Q_k)X_{kk}\frac{\partial Q_k}{\partial t_k} + (p(Q_k)-c)\frac{\partial X_{kk}}{\partial t_k} + p'(Q_l)X_{kl}\frac{\partial Q_l}{\partial t_k} + (p(Q_l)-c)\frac{\partial X_{kl}}{\partial t_k} + \delta X_{lk} + \delta(t_k-t_l)\frac{\partial X_{lk}}{\partial t_k} - \gamma Z\frac{\partial Z}{\partial t_k}. \quad (2.17)$$

$$\frac{\partial W_k^2(t_k,t_l)}{\partial t_k} = \beta Q_k \frac{\partial Q_k}{\partial t_k} + p'(Q_k)X_{kk}\frac{\partial Q_k}{\partial t_k} + (p(Q_k)-c)\frac{\partial X_{kk}}{\partial t_k} + \left(p'(Q_l)\frac{\partial Q_l}{\partial t_k} + \delta\right)X_{kl} + (p(Q_l)-c-\delta(t_l-t_k))\frac{\partial X_{kl}}{\partial t_k} - \gamma Z\frac{\partial Z}{\partial t_k}. \quad (2.18)$$

Using the expressions of quantities in (2.1) and (2.2) to evaluate the derivatives in (2.17) and (2.18) at (t,t), we obtain the one-sided derivatives of country k's welfare function at symmetric tax profiles.

$$\frac{\partial W_k^1(t,t)}{\partial t_k} = \frac{n\left[2n^2t\beta(-2+\delta)-t\beta\delta+4An\gamma(2+\delta)+A\beta(-2+2n(-1+\delta)+\delta)-nt(4\gamma(2+\delta)+\beta(4+\delta))\right]}{(\beta+2n\beta)^2}. \quad (2.19)$$

$$\frac{\partial W_k^2(t,t)}{\partial t_k} = \frac{n\left[2A(2n\gamma(2-\delta)-\beta(1+n)(1-\delta))+t(n\beta(-4+\delta)-2n^2\beta(2-\delta)-4n\gamma(2-\delta)-\beta\delta)\right]}{(\beta+2n\beta)^2}. \quad (2.20)$$

A simple calculation shows that $\frac{\partial W_k^1(t,t)}{\partial t_k} - \frac{\partial W_k^2(t,t)}{\partial t_k} \leq 0$ if and only if $t \geq t^c(A,\beta,n,\gamma)$. Hence, W_k is a concave function of t_k if and only if $t \geq t^c(A,\beta,n,\gamma)$. Furthermore, equations (2.19) and (2.20) yield the following observations about the signs of $\frac{\partial W_k^1(t,t)}{\partial t_k}$ and $\frac{\partial W_k^2(t,t)}{\partial t_k}$:

$$\frac{\partial W_k^1(t,t)}{\partial t_k} \gtreqless 0 \quad \Leftrightarrow \quad t \lesseqgtr \frac{A\left[4n\gamma(2+\delta)+\beta(-2+2n(-1+\delta)+\delta)\right]}{2n^2\beta(2-\delta)+\beta\delta+n(4\gamma(2+\delta)+\beta(4+\delta))} =: t_1(A,\delta,\beta,n,\gamma). \quad (2.21)$$

$$\frac{\partial W_k^2(t,t)}{\partial t_k} \gtreqless 0 \quad \Leftrightarrow \quad t \lesseqgtr \frac{2A(2n\gamma(2-\delta)+\beta(1+n)(-1+\delta))}{n\beta(4-\delta)+2n^2\beta(2-\delta)+4n\gamma(2-\delta)+\beta\delta} =: t_2(A,\delta,\beta,n,\gamma). \quad (2.22)$$

From section 5.1 (or by plugging $\delta = 0$ into the formulas for $t_1(A, \delta, \beta, n, \gamma)$ and $t_2(A, \delta, \beta, n, \gamma)$ in (2.21) and in (2.22)), we have $t_1(A, 0, \beta, n, \gamma) = t_2(A, 0, \beta, n, \gamma) = t^{no}(A, \beta, n, \gamma)$. Moreover, it holds that $\frac{\partial t_1(A,\delta,\beta,n,\gamma)}{\partial \delta} > 0$ and $\frac{\partial t_2(A,\delta,\beta,n,\gamma)}{\partial \delta} > 0$. In particular $t_1(A, \delta, \beta, n, \gamma) > t^{no}(A, \beta, n, \gamma)$ and $t_2(A, \delta, \beta, n, \gamma) > t^{no}(A, \beta, n, \gamma)$ hold for all $\delta > 0$. Thus, we can conclude by looking at (2.21) and (2.22) that for $\delta > 0$, $\frac{\partial W_k^1(t^{no},t^{no})}{\partial t_k} > 0$ and $\frac{\partial W_k^2(t^{no},t^{no})}{\partial t_k} > 0$. Therefore, if one country sets the tax rate t^{no}, the welfare of the other country is strictly increasing beyond t^{no} (both the left hand and the right hand derivatives are strictly positive).

5.2.1 Symmetric equilibria

Given some tax level $t_l = t$ for the other country, the welfare of country k is maximized for $t_k = t_l$ if and only if we have $\frac{\partial W_k^2(t,t)}{\partial t_k} \geq 0$ and $\frac{\partial W_k^1(t,t)}{\partial t_k} \leq 0$.
According to (2.21) and (2.22), this condition is equivalent to

$$t_1(A, \delta, \beta, n, \gamma) \leq t_2(A, \delta, \beta, n, \gamma) \quad \text{and} \quad t \in [t_1(A, \delta, \beta, n, \gamma), t_2(A, \delta, \beta, n, \gamma)] \,.$$

Furthermore, a simple calculation shows that

$$t_1(A, \delta, \beta, n, \gamma) \leq t_2(A, \delta, \beta, n, \gamma) \quad \text{holds if and only if} \quad \gamma \leq \gamma_0,$$

where

$$\gamma_0 = \gamma_0(\delta, \beta, n) = \frac{\beta\delta(1 + n)}{4n(1 + 2n)(2 - \delta)}.$$

It follows that symmetric pure-strategy Nash equilibria exist if and only if $\gamma \leq \gamma_0$, and that every $t \in [t_1(A, \delta, \beta, n, \gamma), t_2(A, \delta, \beta, n, \gamma)]$ is an equilibrium tax rate in this case. Proposition 3 summarizes our findings.

Proposition 3.

1. *If $\gamma > \gamma_0(\delta, \beta, n)$, there is no symmetric pure-strategy Nash equilibrium.*

2. *If $\gamma \leq \gamma_0(\delta, \beta, n)$, for all $t \in [t_1(A, \delta, \beta, n, \gamma), t_2(A, \delta, \beta, n, \gamma)] \neq \emptyset$, (t, t) is a pure-strategy Nash equilibrium.*

Proposition 3 shows that equilibria where both countries set the same tax rate exist only if the cost of pollution is sufficiently small. The threshold γ_0 is very small if δ is small. The threshold is increasing in δ, but it is quite small even for more practically relevant values of δ, including the case of full BTA, and even for duopoly competition ($n = 1$). Furthermore, γ_0 decreases with n and converges to zero as n tends to infinity. From the formulas for $t^c(A, \beta, n, \gamma)$, $t_1(A, \delta, \beta, n, \gamma)$ and $t_2(A, \delta, \beta, n, \gamma)$ (see (2.7), (2.21) and (2.22)), it is easy to see that

$$\gamma \leq \gamma_0(\delta, \beta, n) \quad \text{holds if and only if} \quad t^c(A, \beta, n, \gamma) \leq t_1(A, \delta, \beta, n, \gamma).$$

and that

$$\begin{aligned} t^c(A, \beta, n, \gamma) &= t_1(A, \delta, \beta, n, \gamma) = t_2(A, \delta, \beta, n, \gamma) \quad \text{for} \quad \gamma = \gamma_0(\delta, \beta, n), \\ t^c(A, \beta, n, \gamma) &< t_1(A, \delta, \beta, n, \gamma) < t_2(A, \delta, \beta, n, \gamma) \quad \text{for} \quad \gamma < \gamma_0(\delta, \beta, n). \end{aligned}$$

We therefore find the following interesting result. The non-cooperative game with δ-BTA has a symmetric equilibrium precisely when the incentives to set higher taxes that are provided by the BTA are strong enough to push both taxes not only above t^{no}, the equilibrium levels for the non cooperative game without BTA, but even (weakly) above the efficient taxes. This happens for relatively small values of γ, for which t^c is close to t^{no} (recall that $t^{no} = t^c$ for $\gamma = 0$). As γ increases, t^{no} and t^c diverge, and the symmetric equilibria cease to exist after γ passes the level for which the equilibrium taxes of the δ-BTA game coincide with the efficient taxes.

Proposition 4. *(Attaining efficiency). Assume that* $\gamma \leq \gamma_0(1, \beta, n) = \frac{\beta(1+n)}{4n(1+2n)}$. *Then, there is a unique value* δ_0*, determined by the condition* $\gamma_0(\delta_0, \beta, n) = \gamma$*, such that the equilibrium taxes for the game with BTA are efficient: the unique pure-strategy Nash equilibrium of the game with* δ_0*-BTA is* (t^c, t^c)*. Furthermore, for any other level of BTA for which symmetric equilibria exist, i.e.,* $\delta > \delta_0$*, equilibrium tax rates are higher than the cooperative tax rate, yielding lower global pollution, but also lower welfare.*

Proof. See Appendix.

Proposition 4 shows that if the cost of the pollution is sufficiently low, a particular partial BTA maximizes welfare in equilibrium (if δ could be chosen by a planner prior to the game, the value δ_0 from Proposition 4 would be optimal).

We continue with a few basic welfare comparisons.

Proposition 5.

1. *Assume that* $\delta > 0$ *is given and that* $\gamma < \gamma_0(\delta, \beta, n)$*. Then* $\frac{\partial W_k(t,t)}{\partial t} < 0$*, for* $t \in [t_1(A, \delta, \beta, n, \gamma), t_2(A, \delta, \beta, n, \gamma)]$*, i.e., the symmetric equilibria are ranked in terms of welfare, and* $(t_1(A, \delta, \beta, n, \gamma), t_1(A, \delta, \beta, n, \gamma))$ *is the most efficient one.*

2. *There is a value* $\bar{\gamma} = \bar{\gamma}(\delta, \beta, n) \in (0, \gamma_0)$*, given explicitly in* (2.29) *in the Appendix, such that for* $k \in \{H, F\}$*:*

$$W_k^{no}(t^{no}, t^{no}) \gtreqless W_k(t_1(A, \delta, \beta, n, \gamma)), t_1(A, \delta, \beta, n, \gamma)) \Leftrightarrow \gamma \lesseqgtr \bar{\gamma}.$$

Proof. See Appendix.

For very small values of the cost of pollution (γ), t^{no} is almost equal to t^c, while the strategic effects of the BTA push equilibrium taxes significantly above the cooperative tax rate. As γ increases, the non cooperative tax rate without BTA moves away from the cooperative tax rate and $t_1(A, \delta, \beta, n, \gamma)$ gets close to t^c. This explains part 2 of Proposition 5: for low values of γ, each country has a higher welfare in the non cooperative equilibrium without BTA than in the best equilibrium with BTA. However, if γ is above the threshold $\bar{\gamma}$ welfare is higher in the case with BTA (at least in the best equilibrium $(t_1(A, \delta, \beta, n, \gamma), t_1(A, \delta, \beta, n, \gamma))$, and for γ sufficiently close to γ_0 also in the worst equilibrium).

5.2.2 Asymmetric equilibria

We now show that for $\gamma > \gamma_0$ (i.e., when the cost of pollution is so high that symmetric pure-strategy equilibria cannot exist), the game with δ-BTA has asymmetric pure-strategy equilibria. Furthermore, the two tax rates used in equilibrium are unique. There are

exactly two equilibria then, one in which country H sets the lower tax and one in which country F sets the lower tax. We also show that no asymmetric equilibria exist for $\gamma \leq \gamma_0$.
Without loss of generality, let us assume that country H is the one choosing the lower tax.
The two main necessary conditions for $(t, t') = (t_H, t_F)$ with $t < t'$ to be an equilibrium are that (a) t is the tax rate that maximizes $W_H^2(t_H, t_F)$ among all values $t_H < t_F = t'$ and (b) t' is the tax rate that maximizes $W_F^1(t_F, t_H)$ among all values $t_F > t_H = t$.
Given the strict concavity of W_H on the relevant domain, property (a) holds if and only if $\frac{\partial W_H^2}{\partial t_H}(t, t') = 0$. Thus, using (2.1), (2.2) and (2.18), property (a) is satisfied if and only if we have both

$$t = \frac{2A(2n\gamma(-2+\delta)+(1+n)\beta(1-\delta))-t'(2n^2\beta(1-\delta)\delta+\beta\delta(1-2\delta)-n(\beta+4\gamma-3\beta\delta+4\beta\delta^2-\gamma\delta^2))}{-n\gamma(-2+\delta)^2+2\beta(-1+\delta)\delta+2n^2\beta(-2+\delta^2)+n\beta(-3-2\delta+4\delta^2)} \tag{2.23}$$

and $t < t'$.
Similarly, property (b) holds if and only if $\frac{\partial W_F^1}{\partial t_F}(t', t) = 0$ and $t' > t$, i.e., if and only if we have both

$$t' = \frac{A(4n\gamma(2+\delta)-\beta(2+2n(1-\delta)-\delta))-t(2n^2\beta(1-\delta)\delta+\beta\delta(1-2\delta)+n\gamma(4-\delta^2)+n\beta(1+3\delta-5\delta^2))}{2\beta\delta^2+n\gamma(2+\delta)^2+2n^2\beta(2-2\delta+\delta^2)+n\beta(3-2\delta+5\delta^2)} \tag{2.24}$$

and $t' > t$.

Solving the system of the two equations (2.23) and (2.24) *without the restriction* $t < t'$, we obtain a unique solution, given by:

$$t' = \frac{A\left[\beta a_1(\delta)+2n^2(\gamma a_2(\delta)+\beta a_3(\delta))-4n^3(\beta a_4(\delta)+\gamma a_5(\delta))+n(\gamma a_6(\delta)+\beta a_7(\delta))\right]}{4n^4\beta(-2+\delta)^2+\beta\delta^2+n\delta(\gamma d_1(\delta)+\beta d_2(\delta))+n^2(\beta d_3(\delta)+\gamma d_4(\delta))-2n^3(\beta d_5(\delta)+\gamma d_6(\delta))} \tag{2.25}$$

$$t = \frac{A\left[\beta b_1(\delta)-2n^2(1-\delta)(\beta b_2(\delta)-\gamma b_3(\delta))-n(\gamma b_4(\delta)+\beta b_5(\delta))-4n^3(\beta b_6(\delta)+\gamma b_7(\delta))\right]}{4n^4\beta(-2+\delta)^2+\beta\delta^2+n\delta(\gamma d_1(\delta)+\beta d_2(\delta))+n^2(\beta d_3(\delta)+\gamma d_4(\delta))-2n^3(\beta d_5(\delta)+\gamma d_6(\delta))},$$

where the coefficients $a_i(\delta)$, $b_i(\delta)$ and $d_i(\delta)$ are given in the Appendix.
The tax rates in (2.25) are such that $t < t'$ if and only if we have $\gamma > \gamma_0$. Hence, no asymmetric equilibrium exists for $\gamma \leq \gamma_0$.
To check that the vector $(t_H, t_F) = (t, t')$ given by (2.25) really is an equilibrium if $\gamma > \gamma_0$,

we still have to verify that country F does not want to deviate to a tax below t and that country H does not want to deviate to a tax above t'. This is the case if and only if the following conditions are true:

(i) If $t^1(t') > t'$: $W_H^2(t,t') \geq W_H^1(t^1(t'),t')$, otherwise $W_H^2(t,t') \geq W_H^1(t',t')$ and

(ii) If $t^2(t) < t$: $W_F^1(t',t) \leq W_F^2(t^2(t),t)$, otherwise $W_F^1(t',t) \leq W_F^2(t,t)$,

where t^2 and t^1 are respectively how we denote the function (of t') on the right hand side of (2.23) and the function (of t) on the right hand side of (2.24).

Let $\gamma > \gamma_0$, considering the expressions of W_H^1 and W_H^2 given in (2.15) and (2.16) and by replacing in $t^1(t')$ and $t^2(t)$ (see their expressions in (2.24) and (2.23)), t and t' with their respective expressions (see (2.25)), we obtain by comparing the corresponding welfare that the conditions (i) and (ii) are all satisfied.

Finally, to ensure the validity of the preceding analysis, we have to ensure that the quantities given in (2.1) are indeed all positive. This is the case as long as the following condition is satisfied:

$$\gamma < \gamma^u(\beta, \delta, n),$$

where

$$\gamma^u(\beta,\delta,n) = \frac{\beta\left[\delta(2+\delta-2\delta^2)+n(4+\delta+5\delta^2-5\delta^3)+n^2(12-8\delta+6\delta^2-3\delta^3)+2n^3(-2+\delta)^2\right]}{\delta n(2-\delta)(1+2n)(2-\delta+4n)}. \tag{2.26}$$

Our result can be summarized in the following proposition.

Proposition 6. *For any $\gamma \in (\gamma_0, \gamma^u)$, the game with δ-BTA has exactly two pure-strategy Nash equilibria, $(t_H, t_F) = (t, t')$ and $(t_H, t_F) = (t', t)$, where the asymmetric tax rates $t < t'$ are given by* (2.25)*.*

Proposition 6 says that when the two countries suffer more from pollution, they set different taxes.

Note that γ^u is very large for small values of δ. Furthermore, it is bounded from below by a positive constant that is independent of n (and δ). Recalling the observations about γ_0 from section 5.2.1, we can thus conclude (informally) that asymmetric equilibria exist for a much larger range of parameters than symmetric equilibria. In particular, when there are many firms, equilibria are asymmetric for most interesting values of δ and γ.[3]

Proposition 7. *Assuming that $\gamma > \gamma_0$ and that (2.26) is satisfied, we have:*

1.1. In the asymmetric equilibrium of the game with δ-BTA, both countries set tax rates lower than the efficient tax rate: $t^c > t'$.

1.2. Depending on the parameters, t may be larger or lower than t^{no}.

2. Pollution in the equilibrium of the game with δ-BTA is lower than in the game without BTA, but higher than under the cooperative solution: $Z(t^c, t^c) < Z(t, t') < Z(t^{no}, t^{no})$.

3. Total welfare in the asymmetric equilibrium of the game with δ-BTA is higher than total welfare in the equilibrium of the game without BTA (and lower than total welfare for the cooperative solution): $W_H(t^{no}, t^{no}) + W_F(t^{no}, t^{no}) < W_H(t, t') + W_F(t', t) = W_H(t', t) + W_F(t, t') < W_H(t^c, t^c) + W_F(t^c, t^c)$.

Proof. See Appendix.

The most interesting insights from Proposition 7 are that whenever the equilibrium is asymmetric, *both* countries (not just the one setting the lower tax) set taxes below t^c, and that the global welfare is always higher in the non cooperative equilibrium with BTA than in the non cooperative equilibrium without BTA. However, we also note that (in contrast to the result of Proposition 4) when $\gamma > \gamma_0(1, \beta, n)$, there is no level of the BTA that allows us to attain the full efficiency.

6 Further findings

So far, we have not distinguished cases where taxes are positive or negative. We have found in section 5.2.1 that in the case where γ is sufficiently small ($\gamma < \bar{\gamma}(\delta, \beta, n)$), the total welfare can actually be higher in the equilibrium of the game without BTA than in the equilibrium of the game with a given level of BTA (e.g. full BTA). However, we

[3]For $\gamma > \gamma^u$, we would have to look for asymmetric equilibria where firms from one country do not sell anything in the other country. We do not pursue this, as it adds little insights.

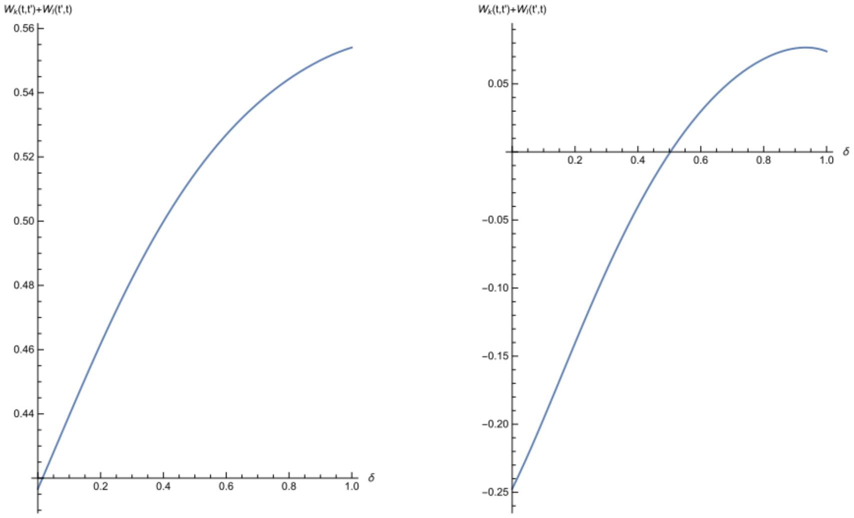
show now that this can never be the case, at least for the Pareto best equilibrium, if the cost of pollution is such that $t^c > 0$. Indeed, according to (2.7), $t^c > 0$ if and only if $\gamma > \gamma^c = \gamma^c(\beta, n)$,
where

$$\gamma^c(\beta, n) = \frac{\beta}{8n}.$$

Using the expression of $\bar{\gamma}$ (see (2.29) in the Appendix), it is not difficult to see that $\gamma^c > \bar{\gamma}(\delta, \beta, n)$ for all δ. Consequently, for each $\gamma > \gamma^c$, we have also $\gamma > \bar{\gamma}$. Thus, whenever t^c is positive and symmetric equilibria exist for a game with δ-BTA, the Pareto-best symmetric equilibrium yields higher total welfare than the equilibrium without BTA. Moreover, if γ is such that the equilibrium of the game with BTA is asymmetric, it always yields higher welfare than the equilibrium of the game without BTA (part 3 of Proposition 7). This shows the claim.

We also note that it is indeed possible that symmetric equilibria exist in cases where t^c is positive, for sufficiently large δ. This holds because $\gamma_0(1, \beta, n) > \gamma^c(\beta, n)$. In particular, there is a range of parameters of cost of pollution, $\gamma \in (\gamma^c(\beta, n), \gamma_0(1, \beta, n)]$, for which ($a$) the pollution problem is severe enough for t^c to be positive and (b) an appropriately chosen BTA (close to full BTA) allows achieving full efficiency in the non-cooperative game.

In the case where the two countries suffer a lot from pollution, if δ where chosen endogenously by a social planner that maximized the global welfare of countries at a prior stage before the Governments set their carton taxes, a partial or a full BTA can be optimal, it depends on the parameters (see Figures 2.1(a) and 2.1(b) below). Indeed, in Figure 2.1(b) we can see that the optimal level of BTA is strictly below 1. However, in Figure 2.1(a) the full BTA is the optimal one.



(a) $\gamma = 0.2$ (b) $\gamma = 0.9$

Figure 2.1 – Global welfare in the game with BTA in the case where $A = \beta = n = 1$ ($\gamma_0 = \frac{1}{6}$, and the minimum value of $\gamma^u = 1$).

7 Conclusion

The problem of carbon leakage and the loss of competitiveness of firms in international markets are the main problems that prevent countries that would like to take unilateral measures to reduce emissions, in a world where not all countries want to cooperate to fight against global warming. In the literature, border tax adjustments have been proposed to address both of these issues. However, almost all of the previous papers that have considered BTA have only worked with a unilateral BTA and with only one firm in each country. Considering at the same time a framework where any country can impose a border adjustment tax on imports and where countries have an arbitrary number of firms, this paper has studied a non-cooperative game on carbon policies between countries open to oligopolistic trade. We show that when countries are not sufficiently affected by pollution, this game admits a continuum of symmetric equilibria and no asymmetric equilibria. If pollution costs are significant, only asymmetric equilibria exist. In the case of symmetric equilibria, carbon taxes are higher than efficient taxes. In this case, if the value of BTA is even chosen optimally, the non-cooperative game could lead to full cooperation. However, in the case where asymmetric equilibria exist, the carbon taxes are lower than the efficient taxes. BTA on imports reduce the global level of pollution, and the global level of pollution is even lower than the efficient level of pollution when countries are not sufficiently affected by pollution. In all cases of interest (when the efficient taxes are positive), introducing a BTA increases total welfare. In a world where not all countries want to form a coalition to alleviate the global level of pollution, this paper advocates the use of BTA as a solution to reduce carbon emissions and increase the global welfare.

Appendix

Proof of Proposition 1:

Using (2.6) and (2.2), the first order conditions $\frac{\partial W^c(t_H,t_F)}{\partial t_H} = 0$ and $\frac{\partial W^c(t_H,t_F)}{\partial t_F} = 0$ are equivalent to:

$$\frac{-2n(n(t_H + t_F)(\beta + 4\gamma) + A(\beta - 8n\gamma))}{(\beta + 2n\beta)^2} = 0$$

This condition is sufficient for the optimality due to the fact that W^c is concave in (t_H, t_F). In particular, (t^c, t^c), where $t^c = \frac{A(8n\gamma-\beta)}{2n(4\gamma+\beta)}$ is optimal, and constitutes the unique optimal symmetric tax vector.

Proof of Proposition 2:

A straightforward calculation shows that the first order condition for country $k's$ welfare maximization is given by:

$$\frac{-n\left(2A(\beta + n\beta - 4n\gamma) + n(\beta(3 + 4n)t_k + 4\gamma t_k + t_l(\beta + 4\gamma))\right)}{(\beta + 2n\beta)^2} = 0$$

This condition is sufficient for the optimality due to the fact that W_k^{no} is concave in t_k. The resulting system of two linear equations has the unique solution (t^{no}, t^{no}) given in (2.13).

Proof of Proposition 4:

Using the expression for t^c from (2.7) and the expressions for $t_1(A, \delta, \beta, n, \gamma)$ and $t_2(A, \delta, \beta, n, \gamma)$ under the assumption that $\gamma \leq \gamma_0(1, \beta, n) = \frac{\beta(1+n)}{4n(1+2n)}$ we obtain easily that:

$t^c = t_1(A,\delta,\beta,n,\gamma) = t_2(A,\delta,\beta,n,\gamma)$ for $\delta = \frac{8n\gamma(1+2n)}{4n\gamma(1+2n)+\beta(1+n)} = \delta_0$, which is the level of the BTA for which $\gamma_0(\delta_0,\beta,n) = \gamma$. Hence, for this level of the BTA, (t^c, t^c) is the unique symmetric pure-strategy Nash equilibrium. Combined with the results of Section 5.2.2, which show that there are no asymmetric equilibria in this case, this proves the first claim. Furthermore, $t^c < t_1(A,\delta,\beta,n,\gamma)$ for every $\delta > \delta_0$, proving the second claim.

Proof of Proposition 5:

1 : If both countries set the same tax rate t, we obtain by using the expressions of equilibrium quantities (see (2.1)) $W_k(t,t) = (\frac{\beta(1+n)}{2n} - 2\gamma)Q^2 + tQ$,
where $Q = \frac{2n(A-t)}{\beta(1+2n)}$, is the global level of consumption in either country.
Differentiating with respect to t, we have:

$$\frac{\partial W_k(t,t)}{\partial t} = -\frac{4n^2(\beta+4\gamma)}{(\beta(1+2n))^2}\left(t - \frac{A(8n\gamma-\beta)}{2n(\beta+4\gamma)}\right) = -\frac{4n^2(\beta+4\gamma)}{(\beta(1+2n))^2}(t - t^c) \tag{2.27}$$

The derivative in (2.27) is negative for all $t \in [t_1(A,\delta,\beta,n,\gamma), t_2(A,\delta,\beta,n,\gamma)]$ because $t^c < t_1(A,\delta,\beta,n,\gamma) < t_2(A,\delta,\beta,n,\gamma)$.

2 : Evaluating the equilibrium welfare level of either country in the game without BTA and in the Pareto-best symmetric equilibrium of the game with BTA, we obtain:

$$W_k^{no}(t^{no},t^{no}) = \frac{A^2(1+n)(\beta+n\beta-4n\gamma)}{2(\beta+n\beta+2\gamma)^2}, \tag{2.28}$$
$$W_k(t_1(A,\delta,\beta,n,\gamma), t_1(A,\delta,\beta,n,\gamma)) = \frac{2A^2n(-2-n(2-\delta))(n^2(\beta-4\gamma)(-2+\delta)-\beta\delta-n(4\gamma\delta+\beta(2+\delta)))}{(-2n^2\beta(-2+\delta)+\beta\delta+4n\gamma(2+\delta)+n\beta(4+\delta))^2}.$$

Using the expressions in (2.28) and assuming $\gamma \in [0,\gamma_0]$ we obtain that:
$W_k^{no}(t^{no},t^{no}) \gtreqless W_k(t_1(A,\delta,\beta,n,\gamma)), t_1(A,\delta,\beta,n,\gamma)) \quad \Leftrightarrow \quad \gamma \lesseqgtr \bar{\gamma}$,

with

$$\begin{aligned} \bar{\gamma} &= \frac{\beta}{16+32n}\left[-4+3n(-4+\delta)+4n^2(-2+\delta)+\sqrt{D}\right], \\ \textit{and} \quad D &= 16n^4(-2+\delta)^2+24n^3(8-6\delta+\delta^2)+n^2(208-88\delta+9\delta^2)+16n(6+\delta)+16(1+2\delta)+8\delta n^{-1}. \end{aligned} \tag{2.29}$$

Proof of Proposition 7:

Let assume that $\gamma \in (\gamma_0, \gamma^u)$. Using the function Reduce of Mathematica, we have the following:

1.1 : Comparing the expression of t^c in (2.7) and the expression of t' given in (2.25), we have $t^c > t'$.

1.2 : is straightforward by comparing the expression of t^{no} in (2.13)and the expression of t given in (2.25).

1 and 2 : Using the expressions of the cooperative tax given by (2.7), the one of t^{no} given by (2.13) and the expressions of t and t' given by (2.25) in the formulas of the global level of pollution ($Z(t_H, t_F)$) and the welfare of each country ($W_k(t_H, t_F)$), we find that

$$\begin{aligned} Z(t^c, t^c) &< Z(t, t') < Z(t^{no}, t^{no}), \\ W_k(t^{no}, t^{no}) + W_l(t^{no}, t^{no}) &< W_k(t, t') + W_l(t', t) < W_k(t^c, t^c) + W_l(t^c, t^c) \quad k \neq l \in \{H, F\}. \end{aligned}$$

The coefficients for the formulas in (2.25) are given by:

$a_1(\delta) = 2\delta(-1+\delta^2)$, $a_2(\delta) = 8(1+\delta+\delta^2-2\delta^3)$, $a_3(\delta) = -6+7\delta-4\delta^2+2\delta^3$, $a_4(\delta) = 2-3\delta+\delta^2$,

$a_5(\delta) = 2(-4-\delta^2+2\delta^3)$, $a_6(\delta) = \delta(12+4\delta-13\delta^2)$, $a_7(\delta) = -4+\delta-4\delta^2+6\delta^3$,

$b_1(\delta) = \delta(-2+\delta+2\delta^2)$, $b_2(\delta) = 6-\delta+2\delta^2$, $b_3(\delta) = 8(1+2\delta^2)$, $b_4(\delta) = \delta(-4-8\delta+13\delta^2)$,

$b_5(\delta) = 4-\delta+\delta^2-6\delta^3$, $b_6(\delta) = 2-3\delta+\delta^2$, $b_7(\delta) = 2(-4+4\delta-3\delta^2+2\delta^3)$,

$d_1(\delta) = 2(4+4\delta-7\delta^2)$, $d_2(\delta) = 6+2\delta-3\delta^2$, $d_3(\delta) = 8+8\delta+5\delta^2-9\delta^3$,

$d_4(\delta) = 4(4+8\delta^2-9\delta^3)$, $d_5(\delta) = -12+6\delta-4\delta^2+3\delta^3$, $d_6(\delta) = 2(-8+4\delta-6\delta^2+5\delta^3)$.

Chapter 3

Transboundary Pollution and Border Tax Adjustment

1 Introduction

Border adjustments have been proposed as a measure to mitigate issues of carbon leakage and reduced competitiveness faced by a country that wishes to impose tighter environmental regulations to reduce carbon emissions in the absence of an effective international environmental agreement (see chapter 2 of this thesis for more details). In this chapter, we examine the robustness of some interesting results found by Eyland and Zaccour (2012, 2014). Eyland and Zaccour used a strategic trade model with two countries, called "Home" and "Foreign", that are heterogeneous with respect to the damages created by global carbon emissions, and with Cournot competition between two firms producing a polluting good (one from each country), to study how a border tax adjustment on imports, imposed by the Home country, influences equilibrium carbon taxes in a non-cooperative taxation game between the two countries, as well as the implications for welfare and for the global level of pollution. Assuming in addition that the polluting good is demanded only by consumers from the country that is more affected by pollution (Home), Eyland and Zaccour found that the level of import border tax adjustment (BTA) that maximizes global welfare is always *partial*. Moreover, even for the country that imposes the BTA, the optimal BTA is a partial BTA (albeit at a higher level than the partial BTA that maximizes global welfare). We examine the robustness of these intriguing findings as follows. First, we extend the model of Eyland and Zaccour (2012, 2014) to a model where the polluting good is also demanded by consumers from the Foreign country. We find that in this case, Home often (for a large set of parameters measuring the damage costs for country H) prefers a full BTA. Still, the BTA that maximizes total welfare is always partial. Secondly, for further examination of the robustness of the results found by Eyland and Zaccour (2012, 2014), we then extend our two country-model to a model with three countries and three firms (one per country). The new country, called "Middle" is also affected by pollution, albeit less than Home, the country imposing a BTA. In particular, in the three-country model, all firms can sell to three different markets, and carbon prices are adjusted, fully or partially (through the BTA), only for the market in the Home country. Our most interesting finding is that when pollution damages are significant for both the Home country and the Middle country, a full BTA is optimal among all possible BTAs, both for global welfare and for the welfare of the Home country. Furthermore, even if pollution damages are very low in the Middle country, a full BTA is optimal (for global welfare and for the Home country), if the damage costs for Home are sufficiently high. We conclude that in a broad range of cases, a full BTA is preferable to all partial BTAs.

2 Duopolistic framework

2.1 Model

We consider a world with two countries, referred to as Home (H) and Foreign (F). In each country, there is one firm that produces a homogeneous and polluting good at a constant marginal cost of $c \geq 0$. The firms compete à la Cournot for consumers in both countries. The quantity sold by the firm from country i to market j is denoted q_{ij}, where $i, j \in \{H, F\}$. The inverse demand function in each country is $P(Q_i) = \alpha - \beta Q_i$, where $\alpha > 0$, $\beta > 0$ and Q_i is the total quantity sold and consumed in country $i \in \{H, F\}$, i.e., $Q_i = \sum_{j \in \{H,F\}} q_{ji}$.

Each unit of production of the good generates one unit of carbon emissions. The global level of emissions is denoted by Z, $Z = Q_H + Q_F$.

The damages that are caused by global emissions for country i are $D_i(Z) = \frac{\gamma_i}{2} Z^2$, where $\gamma_i \geq 0$ is a constant.

We assume that the Foreign country does not suffer any damages (i.e., $\gamma_F = 0$) and that $\gamma_H = \gamma > 0$.

We denote by t_i the per-unit carbon tax in country $i \in \{H, F\}$. Moreover, we assume that the more affected country (Home) practices an *import border tax adjustment* if $t_H \neq t_F$. That is, the Foreign firm has to pay a per-unit border-tax adjustment equal to $\delta(t_H - t_F)$, where $\delta \in [0, 1]$ is a given constant[1]. The case without BTA corresponds to $\delta = 0$. For $\delta = 1$, the BTA fully adjust for the difference between taxes in the two countries (for country H's imports). For $\delta \in (0, 1)$, the adjustment is only *partial*. Note that like Eyland and Zaccour (2012, 2014), we assume for simplicity that Home practices the BTA even if $t_H < t_F$. Compared to Emel Pokam Kake (2020), who assumes that a country only practices a BTA if its tax is higher than the other country's tax (and that both countries can impose a BTA), this assumption simplifies the equilibrium analysis because it implies that countries' welfare functions are differentiable with respect to taxes even at the point where both taxes coincide. While the assumption implies that Home would actually subsidize the foreign firm if $t_H < t_F$, this does not actually happen in equilibrium if the two countries are sufficiently asymmetric with respect to pollution damages (γ is above a certain threshold), which we will assume throughout (see Section 2.2).

The two countries play a game with the following timing. In the first stage, taking δ as given, countries set their carbon tax rates simultaneously. In the second stage, firms compete à la Cournot. We study the subgame perfect Nash equilibrium.

Industry equilibrium

The profit functions of the firms in Home and Foreign are respectively:

$$\begin{aligned} \pi_H(q,t) &= (\alpha - \beta Q_H - c - t_H) q_{HH} + (\alpha - \beta Q_F - c - t_H) q_{HF}, \\ \pi_F(q,t) &= (\alpha - \beta Q_H - c - t_F - \delta(t_H - t_F)) q_{FH} + (\alpha - \beta Q_F - c - t_F) q_{FF}, \end{aligned} \tag{3.1}$$

[1] See Eyland and Zaccour (2012, 2014).

where $q = (q_{HH}, q_{HF}, q_{FH}, q_{FF})'$ and $t = (t_H, t_F)'$.

As marginal costs are constant, we easily find the equilibrium quantities for the second stage of the game from standard results for the linear Cournot model[2].

$$\begin{aligned}
q_{HH} &= \frac{1}{3\beta}(A - 2t_H + t_F + \delta(t_H - t_F)) \\
q_{HF} &= \frac{1}{3\beta}(A + t_F - 2t_H) \\
q_{FH} &= \frac{1}{3\beta}(A - 2t_F + t_H - 2\delta(t_H - t_F)) \\
q_{FF} &= \frac{1}{3\beta}(A + t_H - 2t_F)
\end{aligned}$$

where $A = \alpha - c > 0$.

2.2 The non cooperative carbon taxation game

The welfare functions of the two countries are

$$\begin{aligned}
W_H &= CS_H + \pi_H + TR_H + BTA_H - D_H(Z), \qquad (3.2) \\
W_F &= CS_F + \pi_F + TR_F,
\end{aligned}$$

where $CS_i = \frac{\beta}{2}Q_i^2$ is the consumers surplus in country i, TR_i is the tax revenue in country i and BTA_H is the revenue generated by the border tax adjustment. Note that, for brevity, we suppress the dependence of these expressions on the tax vector t and the resulting equilibrium quantities.

We now compute the Nash equilibrium taxes of the game where countries simultaneously set their taxes (to maximize their own welfare), anticipating the Cournot equilibrium quantities. Plugging the equilibrium quantities into (3.2), we obtain the following necessary and sufficient first order conditions for determining countries' best response functions:

$$\begin{aligned}
\frac{\partial W_H}{\partial t_H} &= -\frac{4\gamma(t_H+t_F)+7\beta t_H+\beta t_F+4\gamma\delta t_H+(\delta^2(\gamma+9\beta)-6\beta\delta)\,(t_H-t_F)-A\,(4\gamma(2+\delta)+(3\delta-4))}{9\beta^2} = 0, \\
\frac{\partial W_F}{\partial t_F} &= -\frac{t_H + 7t_F + 4A(1-\delta) - 6\delta t_H + 4\delta t_F + 8\delta^2(t_H - t_F)}{9\beta} = 0.
\end{aligned}$$

[2]See Belleflamme and Peitz (2015) for details of the equilibrium derivations.

Hence, the best response functions in the two countries are respectively:

$$
\begin{aligned}
t_H(t_F) &= \frac{\beta A\,(3\delta-4)+4A\gamma\,(2+\delta)+t_F\left[\gamma(-4+\delta^2)+\beta(-1-6\delta+9\delta^2)\right]}{\beta(7-6\delta+9\delta^2)+\gamma(2+\delta)^2},\\
t_F(t_H) &= \frac{-4A\,(\delta-1)+(1-6\delta+8\delta^2)t_H}{-(7+4\delta-8\delta^2)}.
\end{aligned}
$$

Solving the system of best responses, we obtain the following subgame perfect equilibrium taxes:

$$
\begin{aligned}
t_H^* &= \frac{A\left(\beta\left(24-25\delta+16\delta^2-12\delta^3\right)+4\gamma\left(-18-11\delta+13\delta^2+7\delta^3\right)\right)}{2\left(\beta\left(-24+7\delta-10\delta^2+9\delta^3\right)+\gamma\left(-12-34\delta+20\delta^2+17\delta^3\right)\right)},\\
t_F^* &= \frac{A\left(\beta\left(24-25\delta+10\delta^2-12\delta^3\right)+4\gamma\left(6-11\delta+7\delta^2+7\delta^3\right)\right)}{2\left(\beta\left(-24+7\delta-10\delta^2+9\delta^3\right)+\gamma\left(-12-34\delta+20\delta^2+17\delta^3\right)\right)}.
\end{aligned}
$$

We note that $t_H^* > t_F^*$ for all δ in $[0,1]$ if and only if $\gamma > \beta/12$, which we assume throughout for the numerical analysis below.

For completeness, we also compute the social optimum.

2.3 Social optimum

In this section, we assume that the two countries form a coalition and cooperatively set carbon tax rates that maximize their joint welfare in the first stage of the game. We assume here that, there is no BTA ($\delta = 0$). The joint welfare in this case is as follows:

$$
W^c(t_H, t_F) = W_H + W_F = 2AQ - \beta Q^2 - \frac{\gamma}{2}Z^2. \tag{3.3}
$$

where $Q = Q_H = Q_F = \frac{2A-t_H-t_F}{3\beta}$ is the equilibrium level of consumption in either country and $Z = 2Q$.

Maximizing (3.3) with respect respectively to t_H and t_F, we obtain that the first order condition is

$$
\frac{\partial W^c(t_H,t_F)}{\partial t_H} = \frac{\partial W^c(t_H,t_F)}{\partial t_F} = \frac{2\left((\beta+4\gamma)Q - A\right)}{3\beta} = 0. \tag{3.4}
$$

The first order condition (3.4) is sufficient for the optimality, because W^c is concave in (t_H, t_F). Solving this first order condition, we obtain a unique symmetric carbon tax rate that maximizes the joint welfare:

$$
t^c = \frac{A(4\gamma-\beta)}{2(\beta+2\gamma)}. \tag{3.5}
$$

2.4 Numerical findings

We now analyze numerically which value of δ is socially optimal (maximizes the joint equilibrium welfare of both countries), and also which value of δ maximizes the equilibrium welfare of the more affected country. We call these values δ_{glob} and δ_H, respectively. Like Eyland and Zaccour (2012, 2014), we fix $\alpha = \beta = 1$. We also fix $c = 0$ (results are similar for higher values of c). Given this, we consider all values of γ for which $t_H^* > t_F^*$ (i.e., $\gamma > 1/12$) and such that both firms actually compete in both markets (all equilibrium quantities are positive, and given by the formulas in Section 2.1). The latter holds for $\gamma \leq 0.473$.

Figure 3.1 shows δ_H and δ_{glob} depending on γ.

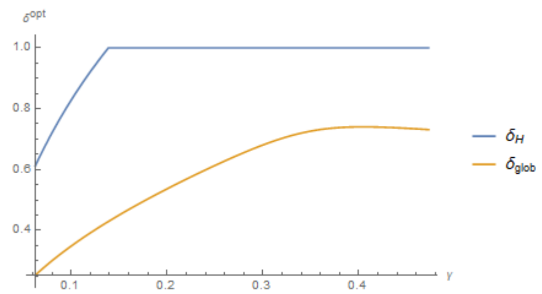


Figure 3.1 – Optimal level of δ

Eyland and Zaccour (2012) (see section 5.1) found that Home always prefers a partial BTA. By contrast, we find that for large values of γ ($\gamma \in [0.139, 0.473]$), a full BTA yields the highest welfare for Home.

Figure 3.1 also shows that, as in Eyland and Zaccour (2014), a partial BTA yields the highest total welfare, even though the optimal level is significantly higher here than in their model (for which consumption takes place only in country H).

We next verify some intuitive effects that the level of γ has on welfare and carbon tax rates. We only present results for two values of γ (see Tables 3.1 and 3.2).

We can see by looking at Tables 3.1 and 3.2 that increasing γ decreases the total welfare as well as the Home welfare. However, it increases the Foreign welfare in the non cooperative game without BTA. In the non cooperative game with BTA, as γ increases, the tax rate in the Home country increases quicker than the tax rate (subsidy) in the Foreign country.

Note also that the total welfare is larger in the cooperative scenario and in the BTA scenario than in the non cooperative game without BTA. The Home welfare is larger in the non cooperative game with BTA than in the case without, while the reverse is true for the Foreign welfare. The tax rate in the Home country is higher in the scenario with BTA than that in the non cooperative game without BTA.

All these results are compatible with the results of Eyland and Zaccour (2014) who focus on a two-country model where the good is only demanded by consumers in the Home country.

Table 3.1 – Results for $\gamma = 0.3$

	Non coop. equi. without BTA	Cooperative game	Non coop. equi. with BTA
Total welfare	0.529301	0.625	0.622612
Home welfare	−0.0302457	0.195313	0.168918
Foreign welfare	0.559546	0.429688	0.453694
Home tax	−0.0434783	0.0625	0.185699
Foreign tax	−0.565217	0.0625	−0.23152

Table 3.2 – Results for $\gamma = 0.43$

	Non coop. equi. without BTA	Cooperative game	Non coop. equi. with BTA
Total welfare	0.38612	0.537634	0.530733
Home welfare	−0.223475	0.144525	0.082659
Foreign welfare	0.609595	0.393109	0.448074
Home tax	0.119342	0.193548	0.318441
Foreign tax	−0.588477	0.193548	−0.240176

3 Three countries framework

To further examine the question whether a partial BTA is preferable to a full BTA, either from the perspective of a country that is strongly affected by pollution, or from the perspective of global welfare, we now extend the analysis to a three-country model.

3.1 Model

Here, we consider a world with three countries, referred to as Home (H), Middle (M) and Foreign (F). In each country, there is one firm producing a homogeneous and polluting good, at constant marginal cost $c \geq 0$. All firms compete la Cournot for consumers from each of the three countries. The quantity sold by the firm from country i to market j is denoted q_{ij}, where $i, j \in \{H, M, F\}$. The inverse demand function in each country is $P(Q_i) = \alpha - \beta Q_i$, where Q_i is the total quantity sold and consumed in country $i \in \{H, M, F\}$, i.e., $Q_i = \sum_{j \in \{H,M,F\}} q_{ji}$.
Each unit of production of the good generates one unit of pollution and the global level of pollution is denoted by Z, $Z = \sum_i Q_i$.
The cost of pollution in country i is $D_i(Z) = \frac{\gamma_i}{2} Z^2$, where $\gamma_i \geq 0$ is a constant.
We assume that the Foreign country does not suffer from pollution (i.e., $\gamma_F = 0$) and that the Home country suffers more from pollution than the Middle country (i.e., $\gamma_H > \gamma_M$).
As before, t_i denotes the per-unit carbon tax in country $i \in \{H, M, F\}$. The Home country imposes a per-unit BTA equal to $\delta(t_H - t_j)$ on imports from country $j \in \{M, F\}$ whenever $t_H \neq t_j$.
As in section 2, the three countries set their taxes non-cooperatively and simultaneously, anticipating the ensuing industry equilibrium.

Industry equilibrium

The profit functions of the firms in Home, Middle and Foreign countries are

$$\begin{aligned}
\pi_H(q,t) &= (\alpha - \beta Q_H - c - t_H) q_{HH} + (\alpha - \beta Q_M - c - t_H) q_{HM} + (\alpha - \beta Q_F - c - t_H) q_{HF}, \qquad (3.6)\\
\pi_M(q,t) &= (\alpha - \beta Q_H - c - t_M - \delta(t_H - t_M)) q_{MH} + (\alpha - \beta Q_M - c - t_M) q_{MM} + (\alpha - \beta Q_F - c - t_M) q_{MF},\\
\pi_F(q,t) &= (\alpha - \beta Q_H - c - t_F - \delta(t_H - t_F)) q_{FH} + (\alpha - \beta Q_M - c - t_F) q_{FM} + (\alpha - \beta Q_F - c - t_F) q_{FF},
\end{aligned}$$

where $q = (q_{HH}, q_{HM}, q_{HF}, q_{MH}, q_{MM}, q_{MF}, q_{FH}, q_{FM}, q_{FF})'$, and $t = (t_H, t_M, t_F)'$.
Again, we easily find the equilibrium quantities from standard results for the linear Cournot

model.

$$
\begin{aligned}
q_{HH} &= \frac{1}{4\beta}(A - 3t_H + t_F + t_M + \delta(t_H - t_F) + \delta(t_H - t_M)) \\
q_{HM} = q_{HF} &= \frac{1}{4\beta}(A + t_F + t_M - 3t_H) \\
q_{MH} &= \frac{1}{4\beta}(A - 3t_M + t_F + t_H - 2\delta(t_H - t_M) - \delta(t_F - t_M)) \\
q_{MM} = q_{MF} &= \frac{1}{4\beta}(A + t_F + t_H - 3t_M) \\
q_{FH} &= \frac{1}{4\beta}(A - 3t_F + t_M + t_H - 2\delta(t_H - t_F) + \delta(t_F - t_M)) \\
q_{FM} = q_{FF} &= \frac{1}{4\beta}(A + t_H + t_M - 3t_F)
\end{aligned} \tag{3.7}
$$

The quantities sold by the firm from the Home country in each market are decreasing functions of the Home carbon tax rate. With the exception of the quantities produced by the Middle firm and the Foreign firm for the Home market, that are positively related to the Home carbon tax rate, if $\delta < 0.5$ and negatively related to the Home tax rate for all others values of δ, the quantities produced in each country for a foreign market are decreasing functions of its domestic tax rate but are increasing functions of foreign taxes.

3.2 The non cooperative carbon taxation game

The expressions for countries' welfare are

$$
\begin{aligned}
W_H &= CS_H + \pi_H + TR_H + BTA_H - D_H(Z), \\
W_M &= CS_M + \pi_M + TR_M - D_M(Z), \\
W_F &= CS_F + \pi_F + TR_F,
\end{aligned} \tag{3.8}
$$

where $CS_i = \frac{\beta}{2} Q_i^2$ is the consumers surplus in country i, TR_i denotes the tax revenue in country i, BTA_H is the revenue from the BTA, and we again suppress the dependence on t and the resulting equilibrium quantities.

As in section 2, we study the game where countries simultaneously set taxes, anticipating the industry equilibrium. Plugging the quantities from (3.7) into (3.8), it is straightforward to check that each W_i is strictly concave with respect to t_i, and that the first order conditions $\frac{\partial W_i}{\partial t_i}(t) = 0$ yield the following best response functions (we omit the straightforward algebra).

$$
\begin{aligned}
t_H(t_M,t_F) &= \frac{3\beta A\,(2\delta-3)+9A\gamma_H\,(3+2\delta)-5\beta\left(1+\delta-2\delta^2\right)(t_F+t_M)-\gamma_H\,(3+2\delta)\,(3-\delta)\,(t_F+t_M)}{\beta(17-12\delta+20\delta^2)+(3+2\delta)^2\gamma_H},\\
t_M(t_H,t_F) &= \frac{3A\beta(2\delta-3)-\gamma_M(3-\delta)\left[(3-\delta)t_F+(3+2\delta)t_H-9A\right]+\beta\left[(8\delta-6\delta^2-5)t_F+(10\delta-12\delta^2-5)t_H\right]}{\beta(17+12\delta-18\delta^2)+(3-\delta)^2\gamma_M},\\
t_F(t_H,t_M) &= \frac{3A\,(2\delta-3)+(10\delta-12\delta^2-5)t_H+(8\delta-6\delta^2-5)t_M}{17+12\delta-18\delta^2}. \qquad (3.9)
\end{aligned}
$$

Both in Eyland and Zaccour (2012, 2014) and in our two-country model from Section 2, taxes are not necessarily strategic substitutes: while best-response functions of both countries are always decreasing for small values of δ, the more affected country's best-response function is actually increasing for high values of δ, and the less affected country's best-response function is increasing for intermediate values of δ. By contrast, we observe that in our three-country model (where firms can sell to more than one foreign market), taxes are always strategic substitutes, regardless of the BTA (imposed unilaterally by the most affected country).

Proposition 1. *Taxes are strategic substitutes regardless of the size of the BTA.*

$$\frac{\partial t_i}{\partial t_j} < 0 \quad for \quad i \neq j \in \{H, M, F\}$$

Proof. This result is straightforward from the best responses given in (3.9).

The following Lemma provides the expressions for equilibrium tax rates.

Lemma 1. *The subgame perfect equilibrium taxes are:*

$$
\begin{aligned}
t_H^* &= \frac{3A\left[a_{H1}(\delta)\beta+a_{H2}(\delta)\gamma_H+a_{H3}(\delta)\gamma_M\right]}{b_1(\delta)\beta+b_2(\delta)\gamma_H+b_3(\delta)\gamma_M},\\
t_M^* &= \frac{3A\left[e_1(\delta)\beta+e_2(\delta)\gamma_H+a_{M1}(\delta)\gamma_M\right]}{2(3+5\delta-6\delta^2)(b_1(\delta)\beta+b_2(\delta)\gamma_H+b_3(\delta)\gamma_M)}, \qquad (3.10)\\
t_F^* &= \frac{3A\left[e_1(\delta)\beta+e_2(\delta)\gamma_H+a_{F1}(\delta)\gamma_M\right]}{2(3+5\delta-6\delta^2)(b_1(\delta)\beta+b_2(\delta)\gamma_H+b_3(\delta)\gamma_M)},
\end{aligned}
$$

where $a_{H1}(\delta)$, $a_{H2}(\delta)$, $a_{H3}(\delta)$, $e_1(\delta)$, $e_2(\delta)$, $a_{M1}(\delta)$, $a_{F1}(\delta)$, $b_1(\delta)$, $b_2(\delta)$, $b_3(\delta)$ are given in the following table.

Table 3.3 – Formulas for the coefficients appearing in (3.10).

Coefficients	**Expressions**
$a_{H1}(\delta)$	$-18+21\delta-18\delta^2+8\delta^3$
$a_{H2}(\delta)$	$126+75\delta-54\delta^2-32\delta^3$
$a_{H3}(\delta)$	$-36+3\delta+45\delta^2-14\delta^3$
$e_1(\delta)$	$2(-54-45\delta+141\delta^2-136\delta^3+124\delta^4-48\delta^5)$
$e_2(\delta)$	$-216-234\delta+534\delta^2-624\delta^3-104\delta^4+384\delta^5)$
$a_{M1}(\delta)$	$756-243\delta-165\delta^2+642\delta^3-700\delta^4+168\delta^5)$
$a_{F1}(\delta)$	$3(-72+213\delta-381\delta^2+406\delta^3-268\delta^4+56\delta^5)$
$b_1(\delta)$	$162-73\delta+134\delta^2-48\delta^3$
$b_2(\delta)$	$3(18+75\delta-18\delta^2-40\delta^3)$
$b_3(\delta)$	$54-135\delta+177\delta^2-46\delta^3$

Recall that in the two-country model of Section 2, Home's equilibrium tax could be even lower than Foreign's equilibrium tax in cases where the pollution problem is very small: as we have defined the BTA (in accordance with Eyland and Zaccour (2012, 2014)) to take place even when Home's tax is lower than that of the other country, Home finds it optimal to use the BTA to subsidize the foreign firm in order to alleviate the underproduction problem. As noted by the next proposition, this effect disappears in the three-country model: countries that are more affected by pollution always set higher taxes in equilibrium.

Proposition 2. *For any $\gamma_H > \gamma_M > \gamma_F$ and any $\delta \in [0,1]$, we have that.*

$$t_H^* > t_M^* > t_F^*.$$

Proof. This is immediate from a careful examination of the expressions for equilibrium taxes provided in (3.10).

As an example, we note that if $\delta = 0$, the taxes in equilibrium are given by:

$$\begin{aligned} t_H^* &= \frac{(7\gamma_H - 2\gamma_M - \beta)A}{3\beta + \gamma_H + \gamma_M} \\ t_M^* &= \frac{(7\gamma_M - 2\gamma_H - \beta)A}{3\beta + \gamma_H + \gamma_M} \\ t_F^* &= -\frac{(2\gamma_H + 2\gamma_M + \beta)A}{3\beta + \gamma_H + \gamma_M} < 0 \end{aligned}$$

Thus, in this case the Middle country actually subsidizes its firm if $7\gamma_M < 2\gamma_H + \beta$ and taxes it if $7\gamma_M > 2\gamma_H + \beta$. Similarly, Home's tax is positive if $7\gamma_H > 2\gamma_M + \beta$.

3.3 Full cooperation

In this section, we assume that the three countries form a coalition to fight against climate change and set taxes to maximize joint welfare, anticipating the industry equilibrium. There is no border tax adjustment ($\delta = 0$) in this case. Accordingly, the total quantity consumed in each market is the same, and we denote it by Q. The countries jointly solve the following problem

$$Max_{t_H, t_M, t_F} \sum_{i \in \{H,M,F\}} W_i, \tag{3.11}$$

where

$$\begin{aligned} W_H &= \frac{\beta}{2}Q^2 + 3\beta q_{HH}^2 + 3t_H q_{HH} - \frac{\gamma_H}{2}Z^2, \\ W_M &= \frac{\beta}{2}Q^2 + 3\beta q_{MH}^2 + 3t_M q_{MH} - \frac{\gamma_M}{2}Z^2, \\ W_F &= \frac{\beta}{2}Q^2 + 3\beta q_{FH}^2 + 3t_F q_{FH}, \end{aligned} \tag{3.12}$$

and quantities are as in (3) (for $\delta = 0$).

The first order conditions are given by:

$$\frac{\partial \left(W_H + W_M + W_F\right)}{\partial t_i} = 0 \quad for \quad i \in \{H, M, F\}. \tag{3.13}$$

These conditions are sufficient due to the fact that the second order conditions[3] are satisfied. Solving the system of first order conditions for countries, we obtain that there is a unique symmetric tax rate that maximizes the joint welfare of the three countries, and this tax rate is equal to:

$$t^c = \frac{A(9\gamma_H + 9\gamma_M - \beta)}{3(3\gamma_H + 3\gamma_M + \beta)}. \tag{3.14}$$

The cooperative tax rate is an increasing function of γ_H and γ_M. We can see that the cooperative tax is lower than the pure Pigouvian tax, $\frac{3(\gamma_H+\gamma_M)A}{3\gamma_H+3\gamma_M+\beta}$. This is due to the fact that we have two market failures, the oligopoly distortion and the environmental damages.

[3] $\sum_{i\in\{H,M,F\}} W_i$ in concave in (t_H, t_M, t_F).

The welfare of countries in equilibrium for the full cooperation are respectively:

$$\begin{aligned} W_H^c &= \frac{A^2(\beta - 3\gamma_H + 6\gamma_M)}{2(\beta + 3(\gamma_H + \gamma_M))^2} \\ W_M^c &= \frac{A^2(\beta - 3\gamma_M + 6\gamma_H)}{2(\beta + 3(\gamma_H + \gamma_M))^2} > 0 \\ W_F^c &= \frac{A^2(\beta + 6(\gamma_H + \gamma_M))}{2(\beta + 3(\gamma_H + \gamma_M))^2} > 0 \end{aligned} \tag{3.15}$$

We have $W_H^c < W_M^c < W_F^c$. The global welfare is given by: $W_H^c + W_M^c + W_F^c = \frac{3A^2}{2(\beta+3(\gamma_H+\gamma_M))}$. The global welfare is a decreasing function of γ_H and γ_M.

3.4 Numerical findings

We now study numerically, for the non-cooperative game with BTA, which value of δ is socially optimal, and which value of δ maximizes the welfare of the country that is most affected by pollution. As in Section 2, we call these values δ_{glob} and δ_H, respectively.
We again fix $\alpha = \beta = 1$ and $c = 0$, and we restrict attention to those values of γ_M and γ_H for which all quantities q_{ij}, $i, j \in \{H, M, F\}$ are positive, regardless of δ. These values are shown in Figure 3.2.

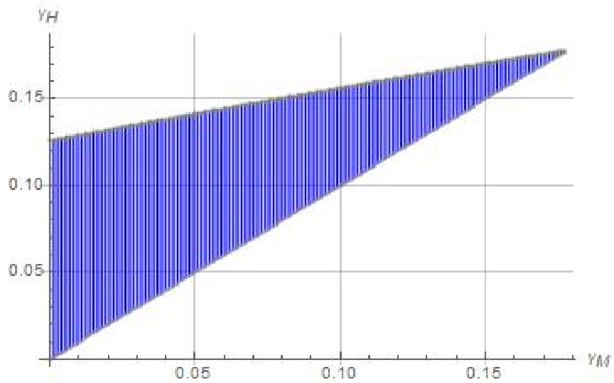


Figure 3.2 – The set of parameters (γ_M, γ_H) for which all q_{ij}, $i, j \in \{H, M, F\}$ are positive for any $\delta \in [0, 1]$.

Our main insight in this section is that δ_{glob} can only be smaller than 1 if γ_M is small ($\gamma_M \in [0, 0.036)$), and if in addition, γ_H is quite close to γ_M (and thus also small). In all other cases, a full BTA is optimal from the perspective of global welfare maximization. This result is in sharp contrast to the findings in our two-country model and in Eyland and Zaccour (2012, 2014) who found that δ_{glob} is always less than 1. Moreover, we find that for country H, a full BTA is optimal for an even larger range of parameters. We also study how the welfare of the two other countries, as well as quantities, taxes and pollution vary with δ, γ_M and γ_H.

3.4.1 Case 1: $\gamma_M \in [0, 0.0183)$

We assume here that γ_M is very low. The plots in this section are for $\gamma_M = 0.01$. But, our results remain qualitatively unchanged for other values of $\gamma_M \in [0, 0.0183)$. We represent the equilibrium values of global welfare, welfare for each country, tax rates, global level of pollution and quantities as functions of the border adjustment parameter, δ. We ran the simulations for different values of $\gamma_H \in [0.01, 0.127]$. However, we present only the figures for $\gamma_H = 0.01$, $\gamma_H = 0.07$ and $\gamma_H = 0.127$.[4]

Figures 3.3−3.6 illustrate how welfare depends on δ and γ_H. If γ_H is very close to γ_M, the global welfare and the welfare in the Home country are inverse U-shaped in δ. The global welfare and the Home welfare function are maximized for partial BTAs satisfying $0 < \delta_{glob} < \delta_H < 1$. The Middle country's welfare is inverse U-shaped in δ and the Foreign country's welfare is decreasing in δ.

However, there is a threshold $\gamma_H^1(\gamma_M)$ such that for $\gamma_H > \gamma_H^1(\gamma_M)$, Home's welfare is increasing in δ. Moreover, there is a second threshold, $\gamma_H^2(\gamma_M) > \gamma_H^1(\gamma_M)$, such that for $\gamma_H > \gamma_H^2(\gamma_M)$, global welfare is increasing in δ.

[4] All other simulation results are available upon request.

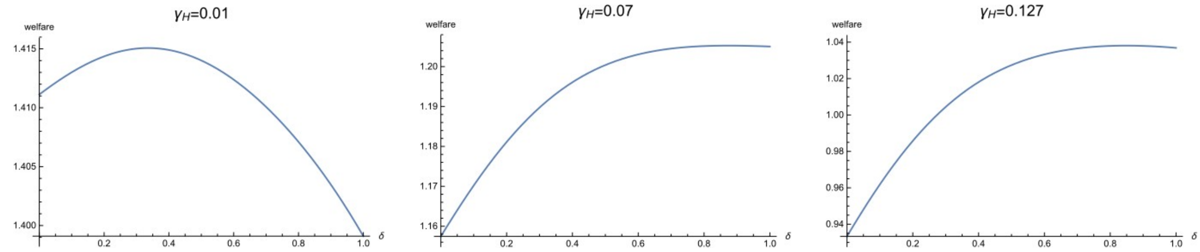


Figure 3.3 – Global welfare

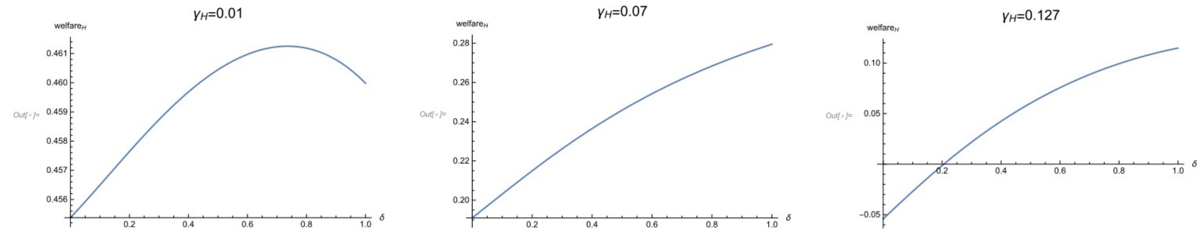


Figure 3.4 – Welfare of Home country

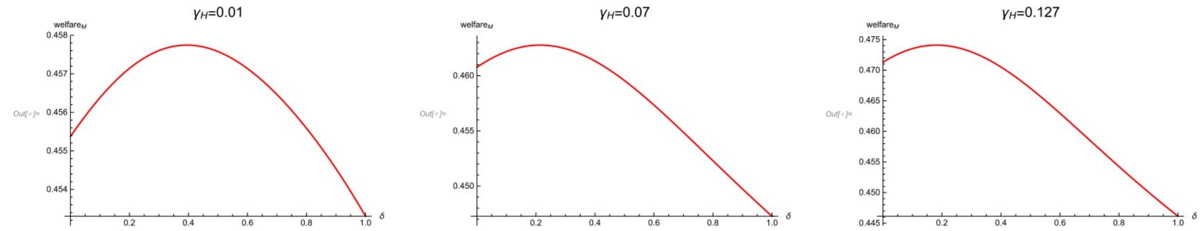


Figure 3.5 – Welfare of Middle country

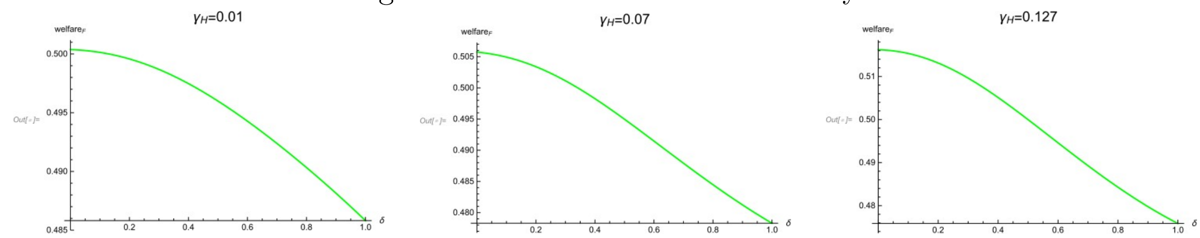


Figure 3.6 – Welfare of Foreign country

Numerical result 1. *Given $\gamma_M \in [0, 0.0183)$, there are two thresholds $\gamma_H^1(\gamma_M)$ and $\gamma_H^2(\gamma_M)$, with $\gamma_H^2(\gamma_M) > \gamma_H^1(\gamma_M)$, such that:*

1. *For any $\gamma_H < \gamma_H^1(\gamma_M)$, $\delta_{glob} < \delta_H < 1$.*

2. *For $\gamma_H > \gamma_H^1(\gamma_M)$, the Home country's welfare is increasing in δ. In particular, it holds that $\delta_H = 1$.*

3. *For any $\gamma_H^1(\gamma_M) < \gamma_H < \gamma_H^2(\gamma_M)$, $\delta_{glob} < 1$.*

4. *For $\gamma_H > \gamma_H^2(\gamma_M)$, the global welfare is increasing in δ. In particular, it holds that $\delta_{glob} = \delta_H = 1$.*

Thus, even if γ_M is very small, a full BTA can be optimal for maximizing global welfare (and Home's welfare).

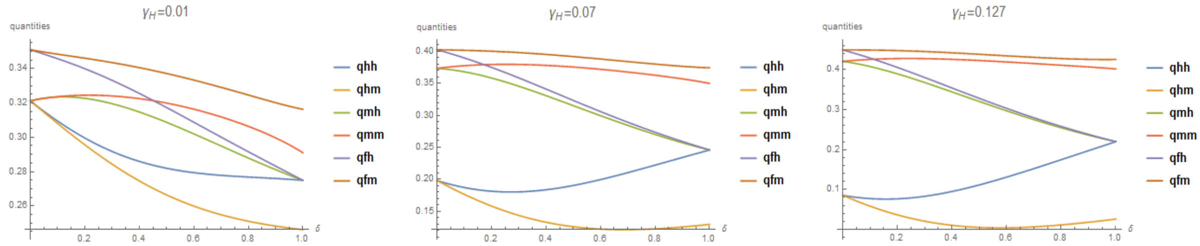


Figure 3.7 – Equilibrium quantities sold by the three firms in the different markets

Figure 3.7 presents the quantities sold by the three firms in each of the three markets (recall that the quantities for markets M and F coincide, i.e., $q_{iF} = q_{iM}$ for $i \in \{H, M, F\}$). Clearly, if $\delta = 0$, each firm i sells the same quantity in each market, namely, $\frac{A+t_j+t_k-3t_i}{4\beta}$, $j, k \in \{H, F, M\}$. Moreover, this quantity is lowest for country H (which sets the highest tax/lowest subsidy) and highest for country F. Figure 3.7 shows that the quantities sold by the firms from countries M and F in markets M and F change relatively little as δ increases, even though t_M and t_F increase substantially (see Figures 3.10 and 3.11) in response to a higher BTA (at least unless γ_H is very large and δ already close to 1). This is possible due to the loss of competitiveness of the firm from country H in markets M and F (t_H increases even more substantially than t_M and t_F). The first main effect of increasing δ is that the exports from countries F and M to country H decrease. Secondly, unless γ_H is very small (the case where the main problem is underproduction, not pollution), increasing δ increases firm H's competitiveness in market H substantially (q_{HH} is U-shaped, but increases a lot for larger values of δ), which explains intuitively why country H prefers $\delta_H = 1$: firm H's competitiveness in market H increases, its relative loss of competitiveness in markets F and M is not so important when γ_H is high (so that t_H would be high anyway even without BTA), and total pollution decreases (the total quantity produced decreases, see Numerical result 2 and Figure 3.8). Finally, if γ_H is very large, the consumption in countries M and F (the countries that are affected very little or not at all by pollution) is satisfied almost only by the firms from countries M and F, and varies very little with δ (see the graph for $\gamma_H = 0.127$ in Figure 3.7). This explains intuitively why the reduction in pollution damages due to reduced consumption in country H (as δ increases) can be the dominating effect, so that $\delta_{glob} = 1$.

Numerical result 2. *Given $\gamma_M \in [0, 0.0183)$, the equilibrium level of global pollution is decreasing in δ (and also in γ_H).*

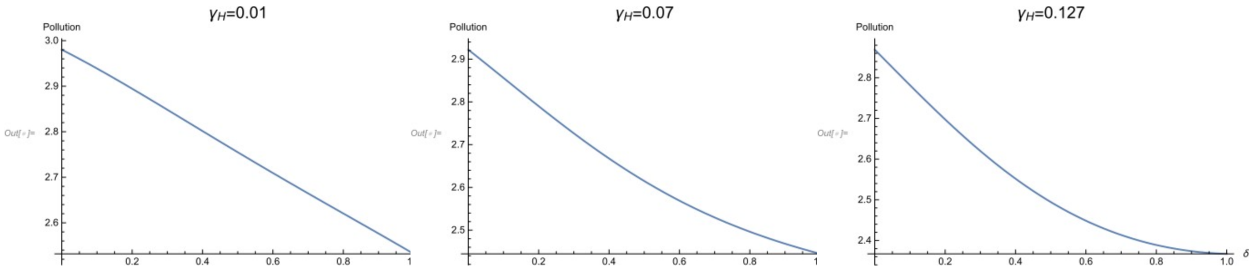


Figure 3.8 – Pollution

Figures 3.9 – 3.11 show the tax (subsidy) rates in Home, Middle and Foreign. For low values of γ_H, all three countries subsidize their firm, but the subsidies decrease as δ increases and the subsidy in Home is lower than the one in Middle and in Foreign.

For high value of γ_H, as δ increases, Home starts taxing its firm while Middle and Foreign still subsidize their firms.

For fixed value of δ, the subsidies in Middle and Foreign increase with γ_H. However, they are decreasing in Home as γ_H increases.

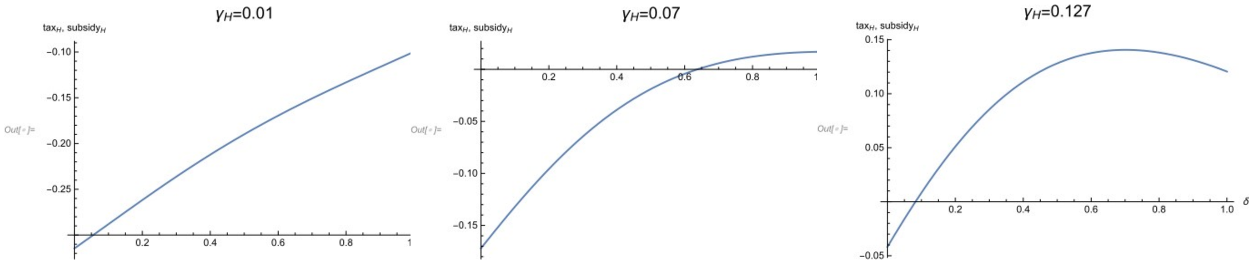


Figure 3.9 – Tax, subsidy of Home country

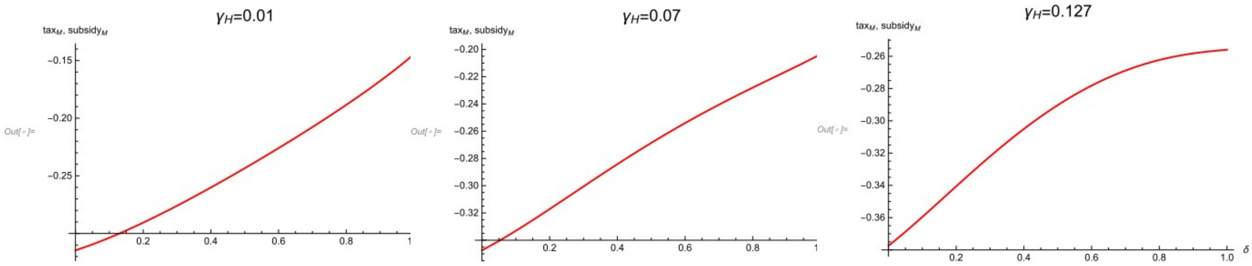


Figure 3.10 – Tax, subsidy of Middle country

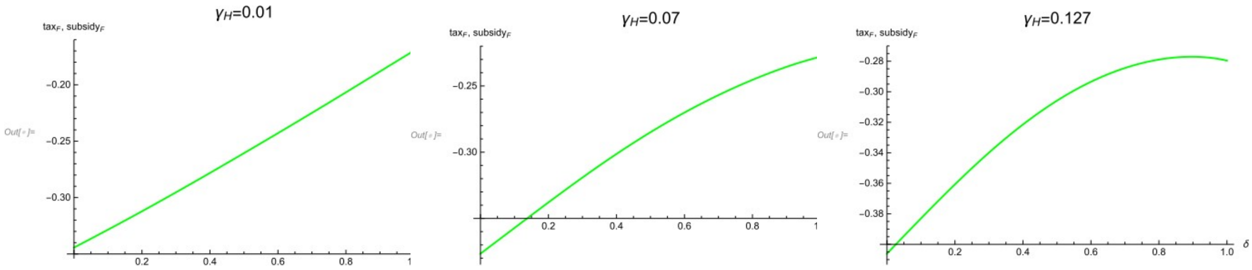


Figure 3.11 – Tax, subsidy of Foreign country

Finally, comparing the cooperative solution and the non cooperative equilibrium, we find that the global welfare in always higher in the cooperative solution than in the non cooperative equilibrium with BTA (see Figure 3.21 in the Appendix). However, as in Eyland and Zaccour (2014), the non cooperative equilibrium with BTA always yields a higher global welfare compared to the case with no BTA.

3.4.2 Case 2: $\gamma_M \in [0.0183, 0.177]$

For $\gamma_M \geq 0.0183$, we find that δ_H is always equal to one. For $\gamma_M \in [0.0183, 0.036)$ (damage costs in country M are small, but not very small) δ_{glob} can still be smaller than 1, but only if γ_H is very close to γ_M. For any other values of γ_H and γ_M, in particular whenever $\gamma_M \geq 0.036$, both δ_{glob} and δ_H are equal to one.
The plots in Figures 3.12 and 3.13, illustrate these findings for $\gamma_M = 0.1$, and for three different values of γ_H. Figures 3.14 and 3.15 show the welfare for the two other countries in these cases.

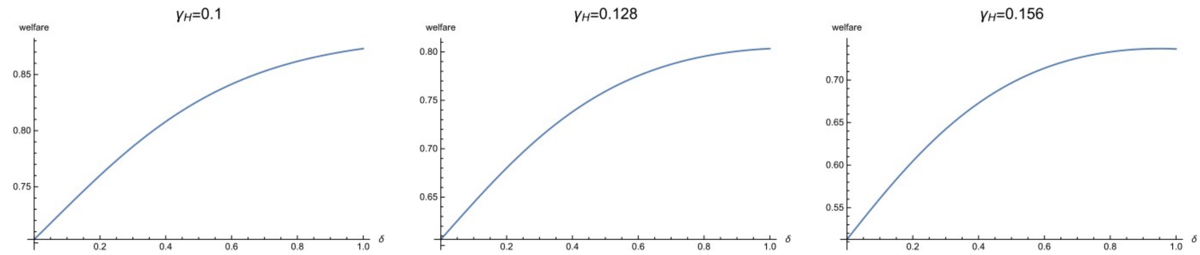


Figure 3.12 – Global welfare

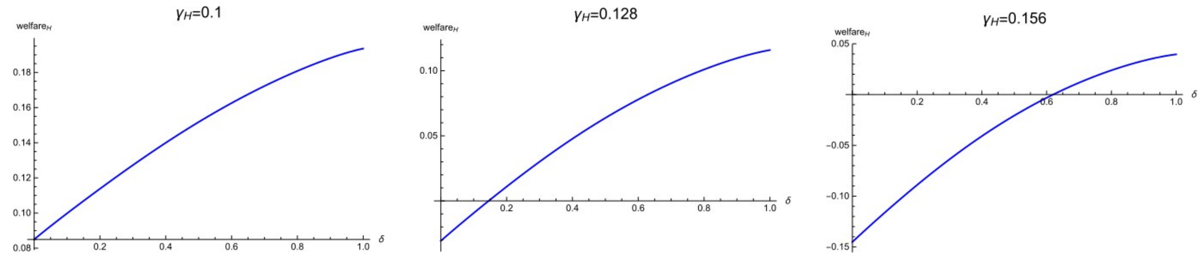


Figure 3.13 – Welfare of Home country

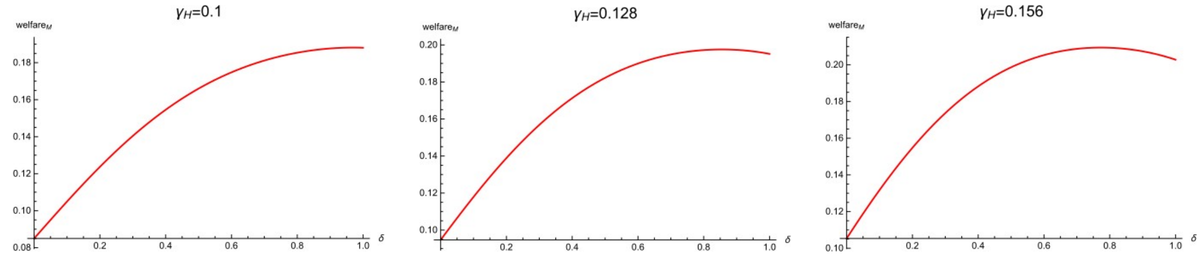


Figure 3.14 – Welfare of Middle country

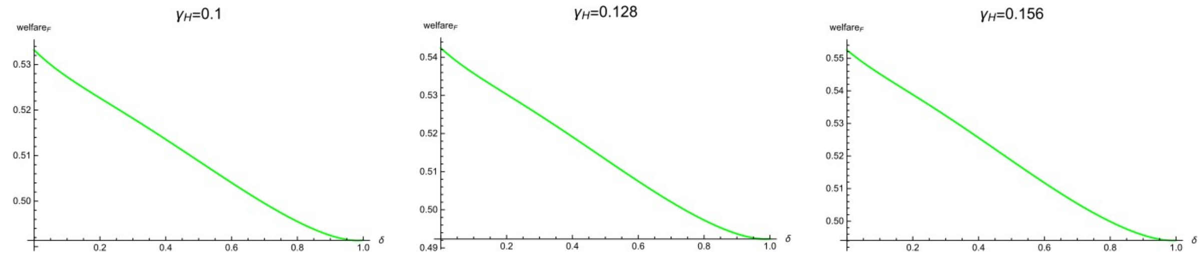


Figure 3.15 – Welfare of Foreign country

Numerical result 3.

1. *If* $\gamma_M \geq 0.0183$*, Home's equilibrium welfare is increasing in* δ*. In particular,* $\delta_H = 1$*.*

2. *If* $\gamma_M \in [0.0183, 0.036)$*, there is a threshold* $\gamma_H^3(\gamma_M)$ *such that* $\delta_{glob} < 1$ *for* $\gamma_H < \gamma_H^3(\gamma_M)$*, and* $\delta_{glob} = 1$ *for* $\gamma_H \geq \gamma_H^3(\gamma_M)$ *(and equilibrium global welfare is increasing in* δ*).*

3. *If* $\gamma_M \geq 0.036$*, equilibrium global welfare is increasing in* δ*. In particular,* $\delta_{glob} = 1$*.*

Figures 3.12 – 3.15 illustrate these results.

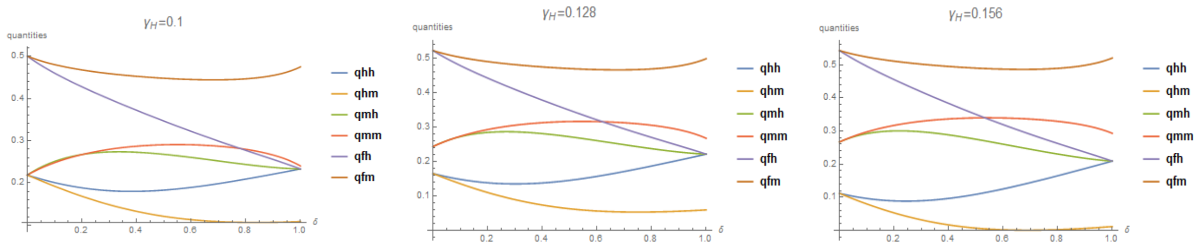


Figure 3.16 – Equilibrium quantities sold by the three firms in the different markets

Figure 3.16 presents the quantities sold by the three firms in each of the three markets for $\gamma_M = 0.1$ and the three values of γ_H also considered in Figures 3.12 – 3.15. The effects of increasing δ on the quantities sold by the Home firm are similar to case 1 above. Furthermore, total quantity/pollution again decreases significantly with δ, see Figure 3.17. Combined with the fact that γ_H is necessarily rather high now, this explains intuitively why δ_H is equal to 1. However, in contrast to case 1, the decrease in total quantity as δ increases is now driven mostly by the reduction of exports from Foreign to Home. The quantities that the firm from Middle sells in the different markets, including market H, change relatively little as δ increases (the most significant decrease occurs for δ very close to 1). As the Middle country now also benefits substantially from reduced global pollution (due mainly to the decrease of q_{FH}), this explains why country M now also prefers high values of δ, and receives a close to maximal welfare for $\delta = 1$. This, in turn, clarifies why $\delta_{glob} = 1$.

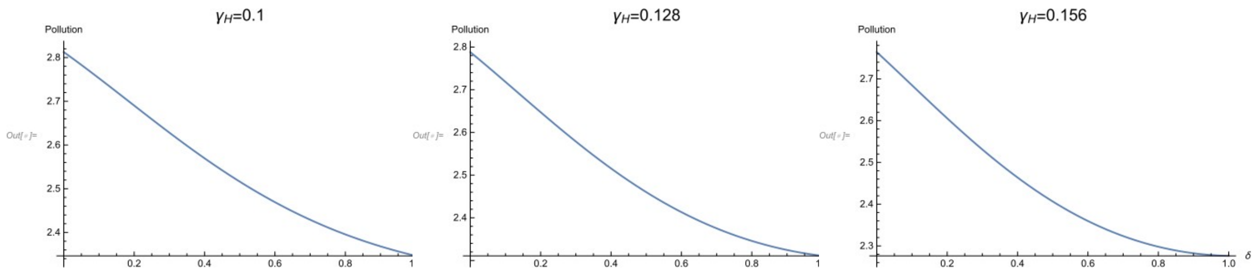


Figure 3.17 – Pollution

As in the first case, for a fixed value of δ, the carbon taxes (subsidies) increase (decrease) with γ_H in Home (see Figure 3.18). However in Middle and Foreign countries (see Figures 3.19 and 3.20), as we fix δ the subsidies increase with the cost of pollution of the Home country.

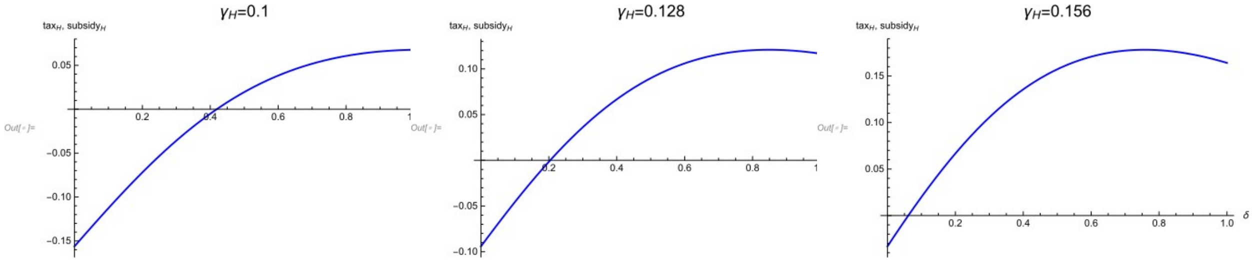


Figure 3.18 – Tax, subsidy of Home country

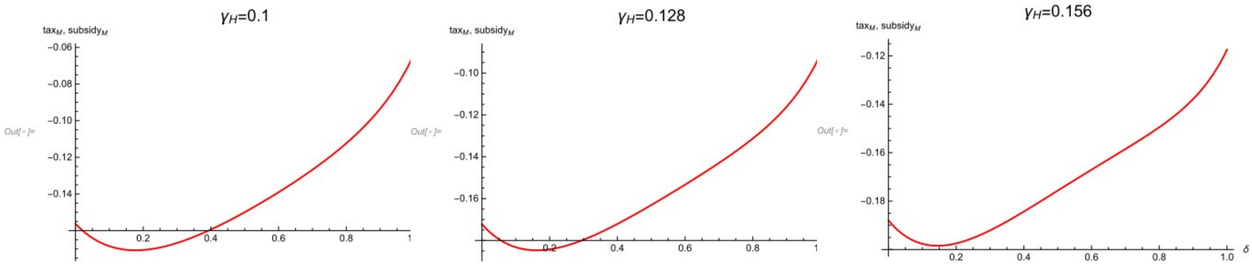


Figure 3.19 – Tax, subsidy of Middle country

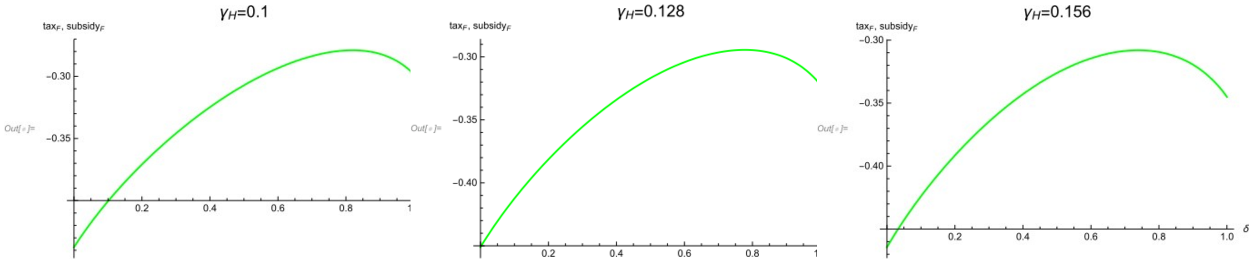


Figure 3.20 – Tax, subsidy of Foreign country

4 Conclusion

In this paper, we extend the duopoly model of Eyland and Zaccour (2012) by first introducing consumers also in the country that is unaffected by pollution. Then, we extend the analysis to a three-country model, with three firms and asymmetric pollution damages.

We show that for a broad range of cases, and in particular when another country, apart from the one unilaterally imposing the BTA, is also significantly affected by pollution, a full import BTA is optimal among all possible import BTAs, both from the perspective of the global welfare and the welfare of the most affected country.

Appendix

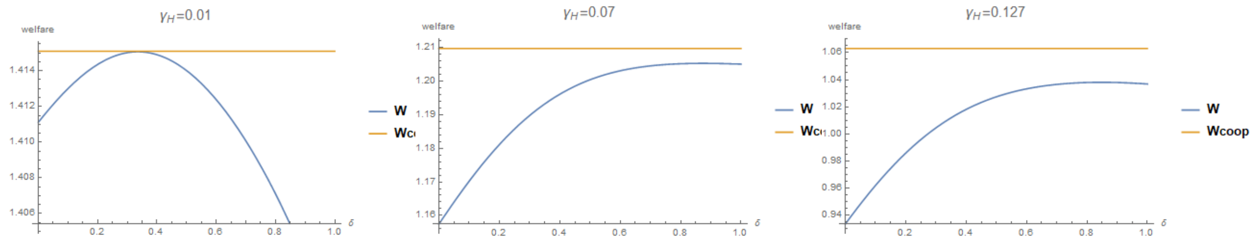


Figure 3.21 – Global welfare

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