# The Effect of Public Spending on Consumption: Reconciling Theory and Evidence

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## Abstract

Recent empirical evidence from vector autoregressions (VARs) suggests that public spending shocks increase (crowd in) private consumption. Standard general equilibrium models predict the opposite. We show that a standard real business cycle (RBC) model in which public spending is chosen optimally can rationalize the crowding-in effect documented in the VAR literature. When such a model is used as a data-generating process, a VAR estimated using the artificial data yields a positive consumption response to an increase in public spending, consistent with the empirical findings. This result holds regardless of whether private and public purchases are complements or substitutes.

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# 1. Introduction

Recent empirical evidence based on vector autoregressions (VARs) seems to support the traditional Keynesian idea that public spending crowds in private consumption. One of the first studies in this area is by Blanchard and Perotti (2002). They used post-war U.S. data and found that an increase in public spending leads to a large and significant increase in consumption, output and the real wage, and to a decrease in private investment spending. The responses of consumption, output and the real wage are strong, statistically significant, and hump shaped. While the increase in output is consistent with both neoclassical and standard Keynesian models, the increase in private consumption and the real wage contradicts standard neoclassical models. To identify fiscal spending shocks, Blanchard and Perotti exploit institutional information about the timing of tax collection to compute (and purge) the automatic responses of tax and fiscal spending. Fatás and Mihov (2001) and Galí, López-Salido and Vallés (2007) also use U.S. data but rely on a purely recursive identification scheme to isolate exogenous changes in public spending. They also find a positive effect of government spending shocks on consumption. Perotti (2005, 2007) extends the empirical study of Blanchard and Perotti (2002) to the United Kingdom, Germany, Canada and Australia and finds similar results. Mountford and Uhlig (2005) use U.S. data from 1955 to 2000 and the same definitions as in Blanchard and Perotti, but achieve identification by imposing sign restrictions on the impulse response functions. Many of their results are similar to those of Blanchard and Perotti, but the response of private consumption to an increase in fiscal spending is small, although positive and statistically significant on impact.

The question is whether these results undermine modern dynamic general equilibrium models, including both real business cycle (RBC) and New Keynesian models, in which changes in public spending crowd out private consumption due to a negative wealth effect (see Aiyagari, Christiano and Eichenbaum, 1992 and Baxter and King,

1993).[1] In this paper, we argue that this is not necessarily the case. In particular, we show that the crowding-in effect documented in the empirical literature can be rationalized within an RBC-neoclassical model in which public spending is set optimally by a benevolent government. The latter assumption is consistent with the underlying philosophy of the RBC approach, which emphasizes building models in which all agents optimize well-defined objective functions subject to technological and budget constraints. In our model, fiscal spending, which consists of public consumption and investment expenditures, reacts endogenously to stochastic shocks affecting preferences and technology. We show that when such a model is used as a data-generating process, a VAR estimated using the artificial data yields, upon imposing the commonly used identifying restrictions, a positive consumption response to an increase in public spending. As found in the empirical literature, the increase is persistent and hump shaped. Interestingly, this result holds whether private consumption and fiscal spending are substitutes or complements. These findings suggest that the identification schemes employed in the VAR literature may not be informative about the underlying model when fiscal spending is set optimally and is fundamentally endogenous.

A few alternative explanations have been proposed to account for the crowding-in effect of fiscal spending. Galí, López-Salido and Vallés (2007) set up a New Keynesian model in which a fraction of consumers are constrained to consume their current disposable income in each period, and households are willing to meet the firms' demand for labor at the wage rate set by a union. Provided the fraction of non-Ricardian consumers is large enough, the model can generate a positive response of consumption to a government spending shock. Bouakez and Rebei (2007) show that the RBC model can generate a crowding-in effect when preferences exhibit strong Edgeworth complementarity between public and private spending. Linnemann (2005) obtains the same result with a non-additively separable utility function and a small intertemporal elas-

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[1]Basu and Kimball (2005) showed that in a New Keynesian model with sticky prices, the negative wealth effect of an increase in public spending on consumption is so strong that output also declines.

ticity of substitution. In contrast to these contributions where public expenditures are assumed to be exogenous, our approach rationalizes the crowding-in effect on the ground that public spending is endogenously determined and covaries positively with private consumption in response to structural shocks.

The paper is structured as follows. In the following section, we describe the model and relate it to the existing literature. In the third section, we discuss the model's steady-state properties and its calibration. We present and discuss our results in the fourth section. The fifth section concludes.

## 2.   Model

We model endogenous public spending following Ambler and Cardia (1997). A benevolent government chooses public spending to maximize the welfare of the representative private agent. Kydland and Prescott (1977) showed that optimal government policies are subject to a time inconsistency problem. In our model, the government cannot precommit to its announced policies for public consumption and public investment spending.[2] We use dynamic programming methods to derive time-consistent policies. Private agents and the government have reaction functions that depend on the current state of the economy (so-called Markov strategies). Macroeconomic equilibrium in our model is Markov-perfect. Public spending is financed by proportional taxes on labor and capital income. Distortionary taxes balance the budget on average. Any discrepancy in the short run is made up by lump sum taxes and transfers.

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[2]Most of the existing models in which the behavior of government is endogenized allow the government to precommit to its policies. Chamley (1986), Chari, Christiano and Kehoe (1991, 1995), and Lansing (1998) used the framework first developed by Ramsey (1927) to consider optimal taxation with precommitment. The existing literature on optimal time-consistent fiscal policies is more sparse. Blanchard and Fischer (1989) contains a textbook discussion comparing optimal policies with and without precommitment on the part of the government. Fischer (1980) compared the levels of welfare that can be attained with and without precommitment in a simple model. Lucas and Stokey (1983) studied how the government can issue nominal debt contracts which make its optimal taxation plans time consistent. Chari and Kehoe (1992) analyzed how trigger strategies can be used as a credible means of enforcing precommitment. Ortigueira (2005) studied optimal Markov-perfect strategies for financing an exogenous stream of government spending.

Because of distortionary taxation, the first-best optimum is not attainable.[3]

## 2.1 Households

There is a representative private household that values consumption and leisure. Its utility function is given by:

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i \left\{ \ln\left(\tilde{c}_{t+i}\right) - \frac{\gamma_t}{1+\psi} n_{t+i}^{1+\psi} \right\}, \tag{1}$$

where $E_t$ is the mathematical expectations operator conditional on information available at time $t$, $\beta$ is a subjective discount factor, $\tilde{c}_t$ is the household's total consumption, $n_t$ is the number of hours worked by the household, $\gamma_t$ is a preference shock, and $\psi \geq 0$ is a preference parameter.

Total consumption is a CES aggregate of private and public consumption expenditures:

$$\tilde{c}_t = \left(\theta c_t^{-\sigma} + (1-\theta) C_{gt}^{-\sigma}\right)^{-1/\sigma},$$

where $c_t$ is the household's consumption spending, $C_{gt}$ is *per capita* government consumption spending, and the elasticity of substitution between private and public expenditures is $\nu \equiv 1/(1+\sigma)$. The CES specification implies that there are diminishing marginal returns to public spending for a given level of private spending in order to achieve a given level of total consumption. As shown by Bouakez and Rebei (2007), the magnitude of the elasticity of substitution $\nu$ has crucial implications for the comovement between private and public spending when the latter is determined exogenously. In particular, if the elasticity of substitution is sufficiently low, government spending can crowd in private consumption.

The household has the flow budget constraint given by:

$$c_t + i_t \leq (1 - \tau_n) w_t n_t + (1 - \tau_k) q_t k_t - T_t, \tag{2}$$

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[3]See Ambler and Desruelle (1991) for more details on this point.

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where $\tau_n$ and $\tau_k$ are, respectively, the labor and capital income tax rates, $w_t$ is the equilibrium real wage rate, $q_t$ is the equilibrium capital rental rate, and $T_t$ is the *per capita* level of lump-sum taxation.

The household's holdings of capital evolve according to:

$$k_{t+1} = (1 - \delta) k_t + i_t, \tag{3}$$

where $\delta$ is the constant rate of depreciation of private capital.

## 2.2    Firms

The representative firm uses capital and labor services purchased from households to produce goods subject to a production function that has constant returns to scale in private inputs:

$$Y_t = z_t N_t^\alpha K_t^{1-\alpha} K_{gt}^{\alpha_g}, \tag{4}$$

where $K_{gt}$ is the *per capita* stock of public capital at time $t$, $K_t$ is the *per capita* private capital stock, $N_t$ is the *per capita* number of hours worked,[4] and $z_t$ is an exogenous stochastic process for the state of technology at time $t$.

Under perfect competition, factors are be paid their marginal products, so that:

$$w_t = \alpha z_t \left( K_t/N_t \right)^{1-\alpha} K_{gt}^{\alpha_g}, \tag{5}$$

$$q_t = (1 - \alpha) z_t \left( N_t/K_t \right)^\alpha K_{gt}^{\alpha_g}. \tag{6}$$

With constant returns to scale in private inputs, factor payments exhaust output, there are no rents, and the $\alpha$ parameter can be calibrated in the standard way from data on labor's share of total income.[5]

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[4]We use the convention that when variables appear in both lower and upper case, the lower case variable is a choice or state variable for the individual household while the upper case variable is the equivalent aggregate *per capita* value.

[5]Note that we do not have endogenous growth in our model. The sum of the coefficients on reproducible factors in the production function, $\alpha + \alpha_g$, is less than one in our calibration.

## 2.3 Resource Constraints

The economy's aggregate resource constraint is given by:

$$Y_t \leq C_t + I_t + C_{gt} + I_{gt}, \tag{7}$$

and the government's flow budget constraint is given by:

$$C_{gt} + I_{gt} = \tau_n w_t N_t + \tau_k q_t K_t + T_t, \tag{8}$$

where $I_{gt}$ is public investment. The laws of motion for the aggregate private and public stocks of capital are:

$$K_{t+1} = (1 - \delta) K_t + I_t \tag{9}$$

and

$$K_{gt+1} = (1 - \delta_g) K_{gt} + I_{gt}. \tag{10}$$

## 2.4 Shock Processes

The technology and preference shocks evolve according to the following stationary AR(1) processes:

$$\ln(z_t) = (1 - \rho_z) \ln(z) + \rho_z \ln(z_{t-1}) + \epsilon_{zt}, \tag{11}$$

$$\ln(\gamma_t) = (1 - \rho_\gamma) \ln(\gamma) + \rho_\gamma \ln(\gamma_{t-1}) + \epsilon_{\gamma t}, \tag{12}$$

where $\rho_z$ and $\rho_\gamma$ are strictly bounded between $-1$ and $1$, variables without time subscript denote steady-state values, and $\epsilon_{zt}$ and $\epsilon_{\gamma t}$ are normal, uncorrelated white-noise disturbances with standard deviations $\sigma_z$ and $\sigma_\gamma$ respectively.

## 2.5 The Representative Household's Problem

The representative household chooses time paths for $\{n_{t+i}, k_{t+i+1}\}_{i=0}^{\infty}$ in order to max-imize the utility function (1). Given the household's choice of employment and its future holdings of capital, its investment expenditures are given by the law of motion

for capital, and its private consumption expenditures are given by its flow budget constraint. The household takes as given the wage rate, the rental rate on capital, the government's policy rule, and the feedback rule for the *per capita* equivalents of its choice variables. The household is aware of the government's flow budget constraint, and is able to calculate the level of lump sum taxes necessary to balance its budget.

This problem can be expressed as a stationary discounted dynamic programming problem. The one-period return function of the household can be written as follows:

$$r_t^h\left(Z_t, G_t, S_t, s_t, D_t, d_t\right) = \ln\left(c_t\right) - \frac{\gamma_t}{1+\psi} n_t^{1+\psi}, \tag{13}$$

where $c_t$ is given by equation (3), where

$$c_t = \left(1 - \tau_{nt}\right) w_t n_t + \left(1 - \tau_{kt}\right) q_t k_t - T_t - k_{t+1} + \left(1 - \delta\right) k_t,$$

where lump sum taxes are given by the government's flow budget constraint, and where

$$Z_t = \{z_t, \gamma_t\}$$

is a vector of state variables which are exogenous from the point of view of the representative household,

$$G_t = \{K_{gt+1}, C_{gt}\}$$

is a vector of government control variables whose laws of motion are also exogenous from the point of view of the household,

$$S_t = \{K_{gt}, K_t\}$$

is a vector of the *per capita* equivalents of the household's state variables,

$$s_t = \{K_{gt}, k_t\}$$

is a vector of the household's state variables themselves,[6]

$$D_t = \{N_t, K_{t+1}\}$$

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[6]Even though the representative household cannot control the evolution of $K_{gt}$, the numerical solution method we use makes it convenient to include $K_{gt}$ as an element of its state vector.

is the vector of *per capita* equivalents of the household's control variables, and

$$d_t = \{n_t, k_{t+1}\}$$

are the control variables themselves. The household's value function can be written as follows:

$$v^h(Z, G, S, s) = \max_d \left\{ r^h(Z, G, S, s, D, d) + \beta E\left[ v^h(Z', G', S', s') \mid Z, G \right] \right\}, \quad (14)$$

where we have dropped time subscripts, where primes denote next-period values, and where

$$Z' = A(Z) + \epsilon', \quad (15)$$

$$s' = B(Z, G, S, s, D, d), \quad (16)$$

$$S' = B(Z, G, S, S, D, D), \quad (17)$$

$$G = G(Z, S), \quad (18)$$

$$D = D(Z, G, S). \quad (19)$$

The household takes as exogenous the government's feedback rule given by (18): in equilibrium, this feedback rule must also satisfy the government's optimality conditions. The solution to the household's dynamic programming problem gives a feedback rule of the form:

$$d = d(Z, G, S, s). \quad (20)$$

## 2.6 Maximization by the Government

The government chooses time paths for $\{C_{gt+i}, K_{gt+i+1}\}_{i=0}^{\infty}$ in order to maximize the utility of the representative household. Public investment is then given by the law of motion for the public capital stock, and $T_t$ is determined in order to satisfy the government's flow budget constraint. Because taxes are distortionary, the government cannot attain a first-best optimum. The interaction between the government and the private sector is not a "team game" problem such as described by Chari, Kehoe and

Prescott (1989) or Ambler and Desruelle (1981), and the competitive equilibrium cannot be found as the solution to a social planning problem. The government takes as given the economy's resource constraint and the laws of motion for the aggregate capital stocks. It knows the private sector reaction function given by (20), and it takes into account the effects of its actions on the private sector. Because of this, it acts as a Stackelberg leader with respect to the private sector. Because we use dynamic programming techniques to derive its optimal strategy, the government's policies are time-consistent by construction. The government's one-period return function can be written as follows:

$$r^g\left(Z_t, S_t, D_t, G_t\right) = \ln(\tilde{C}_t) - \frac{\gamma_t}{1+\psi}N_t^{1+\psi}, \tag{21}$$

with

$$\tilde{C}_t = \left(\theta C_t^{-\sigma} + (1-\theta)\, C_{gt}^{-\sigma}\right)^{-1/\sigma},$$
$$C_t = Y_t - I_t - I_{gt} - C_{gt}.$$

Given this return function, the government's value function can be written as

$$v^g\left(Z, S\right) = \max_G \left\{r^g\left(Z, S, D, G\right) + \beta E\left[v^g\left(Z', S'\right) \mid Z\right]\right\}. \tag{22}$$

The solution to the government's problem gives a feedback rule of the same form as equation (18).

## 2.7  General Equilibrium

The following conditions must hold in general equilibrium:

- all agents maximize given their constraints;

- the optimal feedback rule for the representative household is compatible with the feedback rule for the *per capita* counterparts of its choice variables, so that

$$d\left(Z, G, S, S\right) = D\left(Z, G, S\right);$$

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- the law of motion for the government's control variables that is a constraint in the representative household's dynamic programming problem is compatible with the optimal feedback rule that is the solution to the government's problem;

- markets clear.

All agents solve dynamic programming problems. Their policy functions depend on the current state of the economy. General equilibrium in the model can therefore be characterized as Markov-perfect.

## 3. Steady State and Calibration

There are no analytical solutions to the optimization problems of the household and the government. In order to solve the model, we used numerical techniques that are an extension of those discussed in Hansen and Prescott (1995), and which are described in more detail in Ambler and Paquet (1994). These techniques involve using the household's and the government's exact first order conditions in order to calculate the deterministic steady state of the model (the long run equilibrium the economy would reach in the absence of stochastic shocks), and then calculating quadratic approximations of the one-period return functions of the household and government around this steady-state equilibrium. With quadratic return functions, it is well known that the optimal feedback rules for the household and government are linear, and that the value functions are themselves quadratic. We can then use simple iterative techniques to calculate the optimal feedback rules and the value functions. The steady-state properties of the model were used to calibrate some of its parameters. The model was calibrated to U.S. quarterly data.

The parameter values used in our base-case simulations are summarized in Table 1. The subjective discount rate, $\beta$, the depreciation rates $\delta$ and $\delta_g$, and the share parameter $\alpha$ were set to standard values from the real business cycle literature. The tax rates $\tau_n$ and $\tau_k$ were set to 0.197 and 0.303, respectively.

The first order conditions for the representative household were then used to calibrate the parameters of the utility function. The first order conditions for the representative household with respect to its control variables are

$$\frac{\partial v^h}{\partial d} = \frac{\partial r^h}{\partial d} + \beta \frac{\partial v^h}{\partial s'} \frac{\partial s'}{\partial d} = 0.$$

Differentiating the value function with respect to the current states $s$ and making use of the first order conditions gives

$$\frac{\partial v^h}{\partial s} = \frac{\partial r^h}{\partial s} + \beta \frac{\partial v^h}{\partial s'} \frac{\partial s'}{\partial s}.$$

In the steady state, this gives

$$\frac{\partial v^h}{\partial s} = \frac{\partial r^h}{\partial s} \left( I - \beta \frac{\partial s'}{\partial s} \right)^{-1},$$

so that the first order conditions for the household in the steady state become

$$\frac{\partial r}{\partial d} + \beta \frac{\partial r^h}{\partial s} \left( I - \beta \frac{\partial s'}{\partial s} \right)^{-1} \frac{\partial s'}{\partial d} = 0. \tag{23}$$

Applying this equation to our model and imposing the aggregate consistency conditions gives the following equations:

$$\frac{1}{C} (1 - \tau_n) w - \gamma N^\psi = 0, \tag{24}$$

$$\beta \{(1 - \tau_k) q + (1 - \delta)\} - 1 = 0. \tag{25}$$

The last equation gives a solution for the rental rate of capital in the steady state that depends only on the discount rate, the depreciation rate of capital, and the rate of taxation on capital income. Given this solution for $q$, it is possible to solve for the equilibrium steady-state private capital stock using equation (9), for given values of $N$ and $K_g$. Then, for a given level of hours, we can back out the value of $\gamma$ consistent with this equilibrium. We calibrated the model so that the average number of hours per employee $N$ matched its average *per capita* value in the U.S. data. We chose a low value for $\psi$, which increases the variability of total employment.

The first order conditions for the government can be written as follows:

$$\frac{\partial v^g}{\partial G} = \frac{\partial r^g}{\partial G} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial G} + \beta\frac{\partial v^g}{\partial S'}\left(\frac{\partial S'}{\partial G} + \frac{\partial S'}{\partial D}\frac{\partial D}{\partial G}\right) = 0,$$

where $\partial D/\partial G$ gives the effects of a change in the government's control variables on the behavior of the private sector. Differentiating the government's value function with respect to the current states $S$ and using the first order conditions gives

$$\frac{\partial v^g}{\partial S} = \frac{\partial r^g}{\partial S} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial S} + \beta\left(\frac{\partial v^g}{\partial S'}\frac{\partial S'}{\partial D}\frac{\partial D}{\partial S} + \frac{\partial v^g}{\partial S'}\frac{\partial S'}{\partial S}\right).$$

At the steady state, this gives

$$\frac{\partial v^g}{\partial S} = \left(\frac{\partial r^g}{\partial S} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial S}\right)\left(I - \beta\left(\frac{\partial S'}{\partial D}\frac{\partial D}{\partial S} + \frac{\partial S'}{\partial S}\right)\right)^{-1}.$$

Evaluating the first order conditions at the steady state and substituting this expression for the partial derivatives of the value function with respect to the states gives:

$$\frac{\partial r^g}{\partial G} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial G}$$
$$+\beta\left(\frac{\partial r^g}{\partial S} + \frac{\partial r^g}{\partial D}\frac{\partial D}{\partial S}\right)\left(I - \beta\left(\frac{\partial S'}{\partial D}\frac{\partial D}{\partial S} + \frac{\partial S'}{\partial S}\right)\right)^{-1}.$$
$$\left(\frac{\partial S'}{\partial G} + \frac{\partial S'}{\partial D}\frac{\partial D}{\partial G}\right) = 0. \tag{26}$$

This gives two equations to solve for the steady-state levels of the government's control variables $C_{gt}$ and $K_{gt+1}$, given the solutions for the steady-state levels of the household's control variables. Alternatively, the steady-state levels of $C_g$ and $K_g$ can be imposed, and the first-order conditions can be used to back out values of $\theta$ and $\alpha_g$ compatible with these levels.

In practice, these equations are quite complicated to solve. First of all, it is necessary to evaluate the partial derivatives of the private control variables with respect to the model's state variables and with respect to the government's controls. This involves either taking total differentials of the household's first order conditions

evaluated at the steady state, or solving for the household's optimal feedback rule, which necessitates having solved for the steady state of the model.

In order to circumvent these difficulties, we proceeded as follows. We chose values for $C_g$ and $I_g$ to match the average ratios of current government consumption to output and of public investment to output from our data set. Then, for given values of the $\theta$ and $\alpha_g$ parameters and given steady-state values of $C_g$ and $I_g$, as well as a given feedback rule for the government, we solved the model numerically for the private sector's optimal feedback rule. We then took this private feedback rule as given and derived the optimal feedback rule for the government, which gave implied steady-state values for $C_g$ and $I_g$. For any discrepancy between the initial and implied values of the government controls, the $\theta$ and $\alpha_g$ parameters were adjusted in value, and we iterated until we arrived at values for $\theta$ and $\alpha_g$ consistent with the initial postulated steady-state equilibrium, and until the household's and government's value functions converged.[7]

The parameters of the stochastic process for $z_t$ were calibrated to standard values from the RBC literature; the value of $z$ is an arbitrary normalization. The parameters for the preference shock are taken from Chang, Gomes and Schorfheide (2002); as noted above the constant $\gamma$ is chosen so that the steady-state value of hours as a fraction of the time endowment matches the average in the data.

Finally, the elasticity of substitution parameter $\sigma$ was set equal to either $-0.5$ for the case in which private consumption and public spending are substitutes ($\sigma_s$ in the table), or to 2.0 for the case in which they are complements ($\sigma_c$ in the table). The steady-state properties of the model are summarized in Table 2 below. The steady-state level of average hours and the ratios of the components of different aggregates to GNP reproduce their sample averages in U.S. data.

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[7]Klein, Krusell and Ríos-Rull (2004) solve for the steady state of a similar model by using only steady-state information. They approximate the decision rules by taking successively higher-order polynomial approximations and truncating the polynomials when the steady state changes by less than some convergence criterion.

# 4. Simulation Results

The model was used to simulate 1000 sequences of artificial series for output, public spending, private consumption, private investment, the real rental rate, and the real wage. Each series has a length of 300 periods. In each iteration, the first 100 observations were discarded to ensure that the results did not depend on initial conditions. The number of remaining observations roughly corresponds to the sample size used in empirical studies based on quarterly data.

Using the simulated series, we estimated a 4th-order VAR similar to those found in the empirical literature.[8] As Fatás and Mihov (2001), Galí, Lopez-Salido and Vallès (2007) and Bouakez and Rebei (2007), we identified government spending shocks by imposing a causal ordering on the contemporaneous shocks using a diagonalization of the variance-covariance matrix. More specifically, our identification scheme implies that government spending shocks affect all the remaining variables contemporaneously, whereas shocks to these variables affect government spending only with a lag.[9]

In each iteration, we used these restrictions to compute the impulse response functions to a 1 percent government spending shock. The responses, represented with solid lines in Figures 1 through 4, are averages across the 1000 replications. Their confidence intervals, delimited with dotted lines, were computed by excluding the 2.5 percent lowest and highest responses. Figures 1 and 2 show impulse response functions for the case in which public spending is exogenous and public and private consumption are, respectively, substitutes and complements. Figures 3 and 4 show impulse response functions for a the case in which public spending is chosen optimally, as described in section 2.6. Figure 3 depicts the case where private and public consumption are substitutes, and Figure 4 the case where they are complements.

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[8]We varied the lag length from 1 to 8 and found the results to be extremely robust.

[9]Blanchard and Perotti (2002) do not use a purely recursive identification strategy, but they assume that government expenditures are predetermined relative to taxes.

## 4.1  Exogenous public spending

We start by discussing the case where which public spending is purely exogenous. When public and private expenditures are Edgeworth substitutes, the simulated impulse response functions (shown in Figure 1) indicate what is expected from standard RBC models: government spending crowds out private consumption.[10]  Intuitively, the negative wealth effect induced by an increase in public spending is reinforced by the substitutability between private and public consumption, leading households to cut their consumption.  As predicted by the RBC model, the increase in public spending increases labor supply which in turn raises output and decreases the real wage. It is worth noting that while the increase in hours worked and output are consistent with the VAR-based evidence on the effects of government spending shocks, the negative response of the real wage is not: Several empirical studies find that the response of the real wage to an increase in public spending is positive albeit small.

In the case where private and public expenditures are strong complements (Figure 1), consumption increases following a government spending shock. As explained by Bouakez and Rebei (2007), this result arises because the complementarity effect more than offsets the negative wealth effect associated with an increase in public spending. The rise in consumption, however, requires a larger increase in labor supply and therefore a greater decline in the real wage compared with the case where private and public expenditures are substitutes (or are independent). Thus, while the complementarity between private and public consumption helps to resolve the crowding-in puzzle, it widens the discrepancy between the RBC model and the VAR-based evidence with respect to the response of the real wage.

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[10]For the case where public spending is truly exogenous, the theoretical impulse response functions can also be derived. These give exactly the same answer as the impulse response function reported and are therefore not reported separately.

## 4.2  Optimal public spending

We now turn to the analysis of the simulated impulse response functions for the model in which public spending is set optimally by the government. Starting with the case where private and public expenditures are substitutes, Figure 3 shows that an orthogonalized positive innovation to public spending generates a large and persistent increase in private consumption. Interestingly, the consumption response has a hump-shaped pattern, reaching its peak around 12 quarters after the shock, which accords with much of the evidence reported in the empirical literature. The response of the real wage is also positive (except on impact where it is negative but indistinguishable from zero), as documented in earlier VAR-based studies.

We obtained very similar results, both qualitatively and quantitatively, when private and public expenditures were assumed to be complements (Figure 4). In particular, there is a large, persistent and non-monotonic crowding-in effect on consumption. In addition, the response of the real wage is positive at all horizons (even on impact).

To summarize, when public spending is set optimally, a VAR estimated using the simulated series, and which identifies government spending shocks as is commonly done in the literature, leads to the conclusion that public spending shocks crowd in private consumption, regardless of whether private and public expenditures are substitutes or complements. This is despite the fact that the data generating process does not depart from the standard real business cycle model, except for the way the government makes its decisions. Therefore, the conclusion that RBC models are inconsistent with the data is unwarranted.

In order to gain some intuition about the mechanism that allows the model with optimal public spending to generate a crowding-in effect, it is instructive to examine the theoretical response of private and public spending to the different (true) structural shocks of the economy. Responses to technology and preference shocks are depicted in Figure 5 and 6 respectively. Figures 5 shows that private consumption and the two components of public spending (i.e., public consumption and invest-

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ment) increase in response to a technology shock. Private and public consumption are responding optimally to the positive wealth effect of the technology shock. Public investment responds optimally to the persistent increase in the marginal productivity of public capital. On the other hand, a preference shock leads to a fall in private and public spending. Private and public consumption optimally fall as private agents place more weight on leisure. Private and public investment optimally fall as the persistent decrease in hours lowers the marginal productivity of private and public capital.

Overall, these results indicate that private and public spending tend to move together conditional on each of the structural shocks. In turn, this positive comovement translates into a positive unconditional covariance between the two variables. When the simulated data are used to estimate a VAR, the impulse response function of consumption to an orthogonalized innovation in public spending is in fact picking up this positive covariance.

## 5.  Conclusion

We simulated a model in which public consumption and investment spending are determined optimally by a government that seeks to maximize the welfare of a representative private agent. We then used the simulated data to estimate a VAR using identifying assumptions that are common in the empirical literature. We showed that an orthogonalized innovation to public spending appears to cause a crowding in of private consumption expenditure. This result holds when public and private consumption are either substitutes or complements. Our results explain the empirical finding that increases in public spending lead to increases in private consumption expenditure. The explanation lies in the positive covariance of private and public spending in response to exogenous shocks. The conclusion that the empirical evidence undermines standard general equilibrium models of the business cycle follows from a modeling approach that runs counter to the principle of treating all agents

as optimizing well-defined objective functions subject to technological and budget constraints.

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Table 1: Parameter Calibration

| Parameter | Value |
|-----------|-------|
| $\alpha$ | 0.640 |
| $\alpha_g$ | 0.050 |
| $\delta$ | 0.021 |
| $\delta_g$ | 0.021 |
| $\beta$ | 0.990 |
| $\psi$ | 0.050 |
| $\theta$ | 0.693 |
| $\sigma_s$ | -0.400 |
| $\sigma_c$ | 2.000 |
| $z$ | 1.000 |
| $\rho_z$ | 0.950 |
| $\sigma_z$ | 0.007 |
| $\gamma$ | 0.622 |
| $\rho_\gamma$ | 0.940 |
| $\sigma_\gamma$ | 0.0089 |
| $\tau_n$ | 0.197 |
| $\tau_k$ | 0.313 |

Table 2: Steady State

| Variable | Value |
|----------|-------|
| $N$ | 0.352 |
| $Y$ | 1.312 |
| $C/Y$ | 0.664 |
| $C_g/Y$ | 0.169 |
| $I/Y$ | 0.167 |
| $I_g$ | 0.024 |
| $K/Y$ | 7.951 |
| $K_g/Y$ | 1.137 |
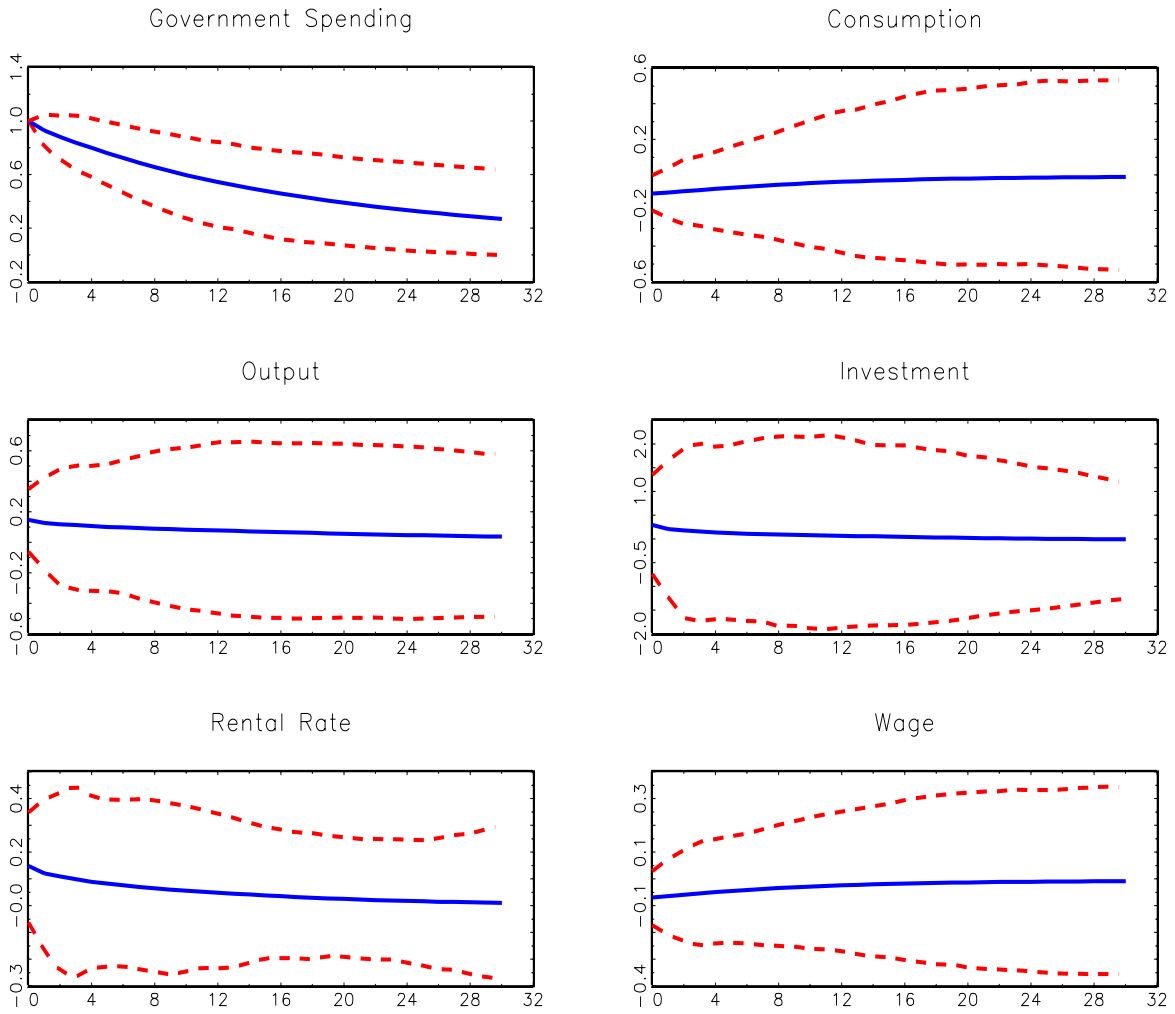
Figure 1: Simulated impulse responses to a 1 per cent increase in public spending: case with exogenous public spending and substitutability between private and public expenditures.

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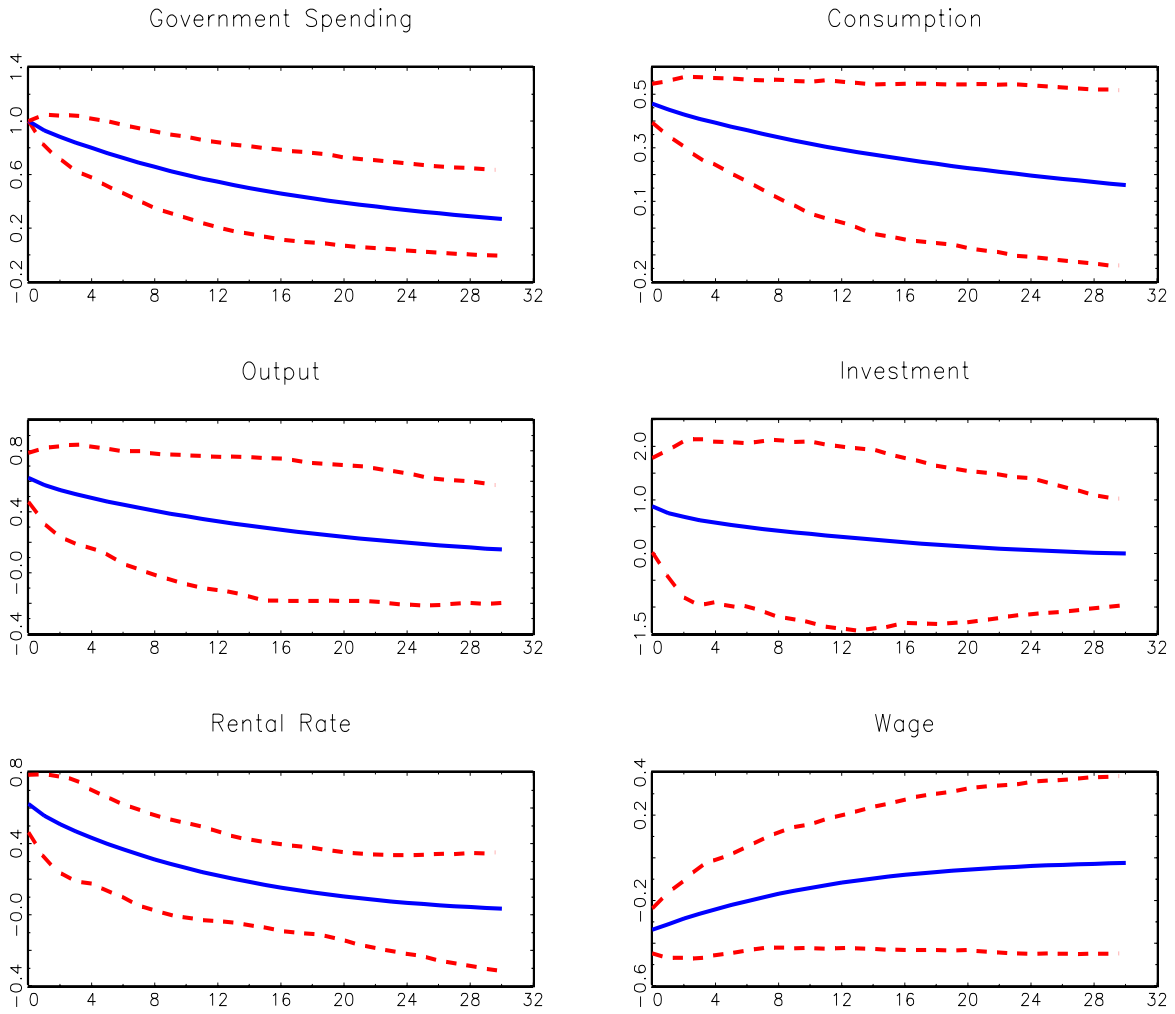
Figure 2: Simulated impulse responses to 1 per cent increase in public spending: case with exogenous public spending and complementarity between private and public expenditures.
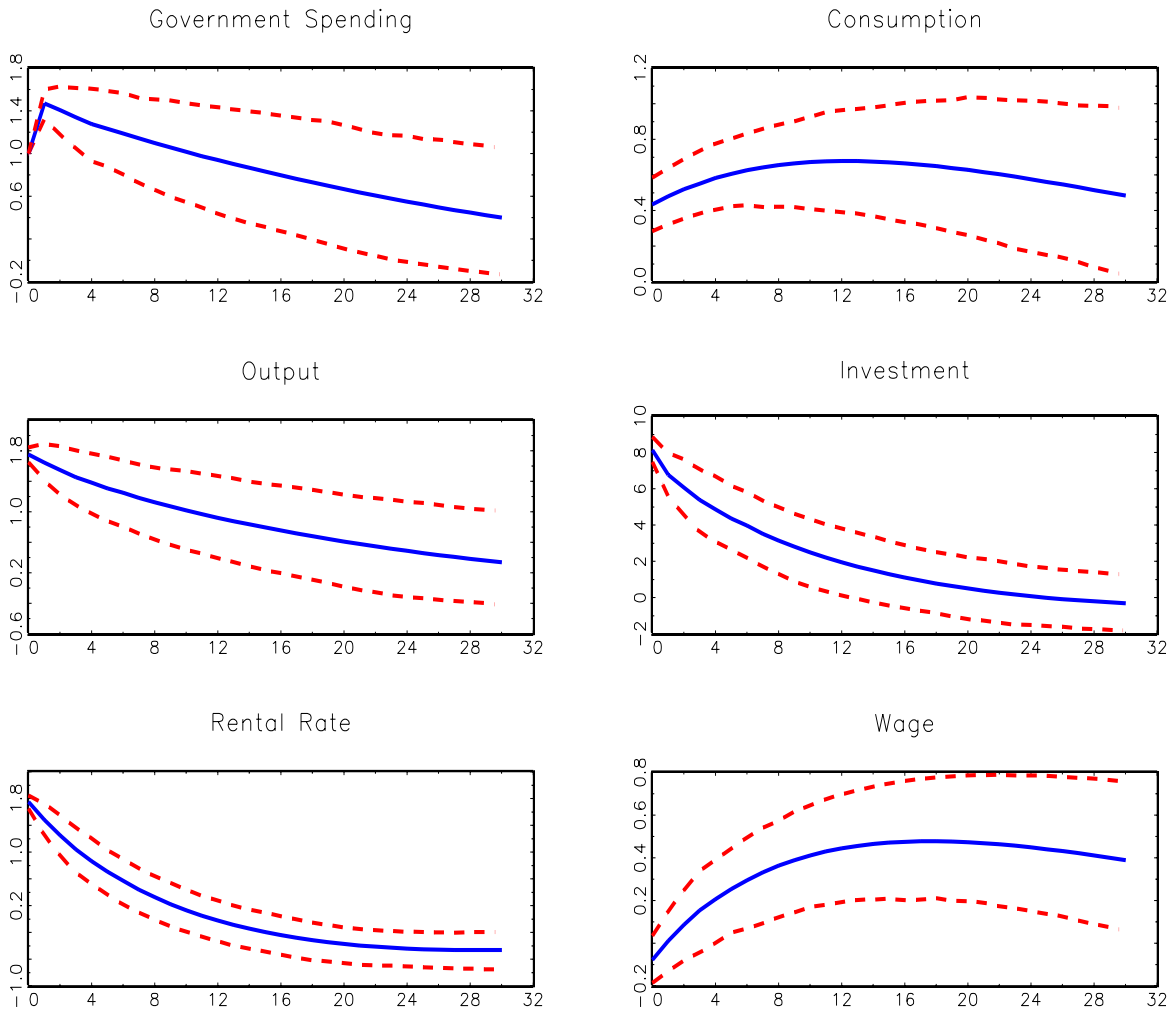
Figure 3: Simulated impulse responses to 1 per cent increase in public spending: case with optimal public spending and substitutability between private and public expenditures.
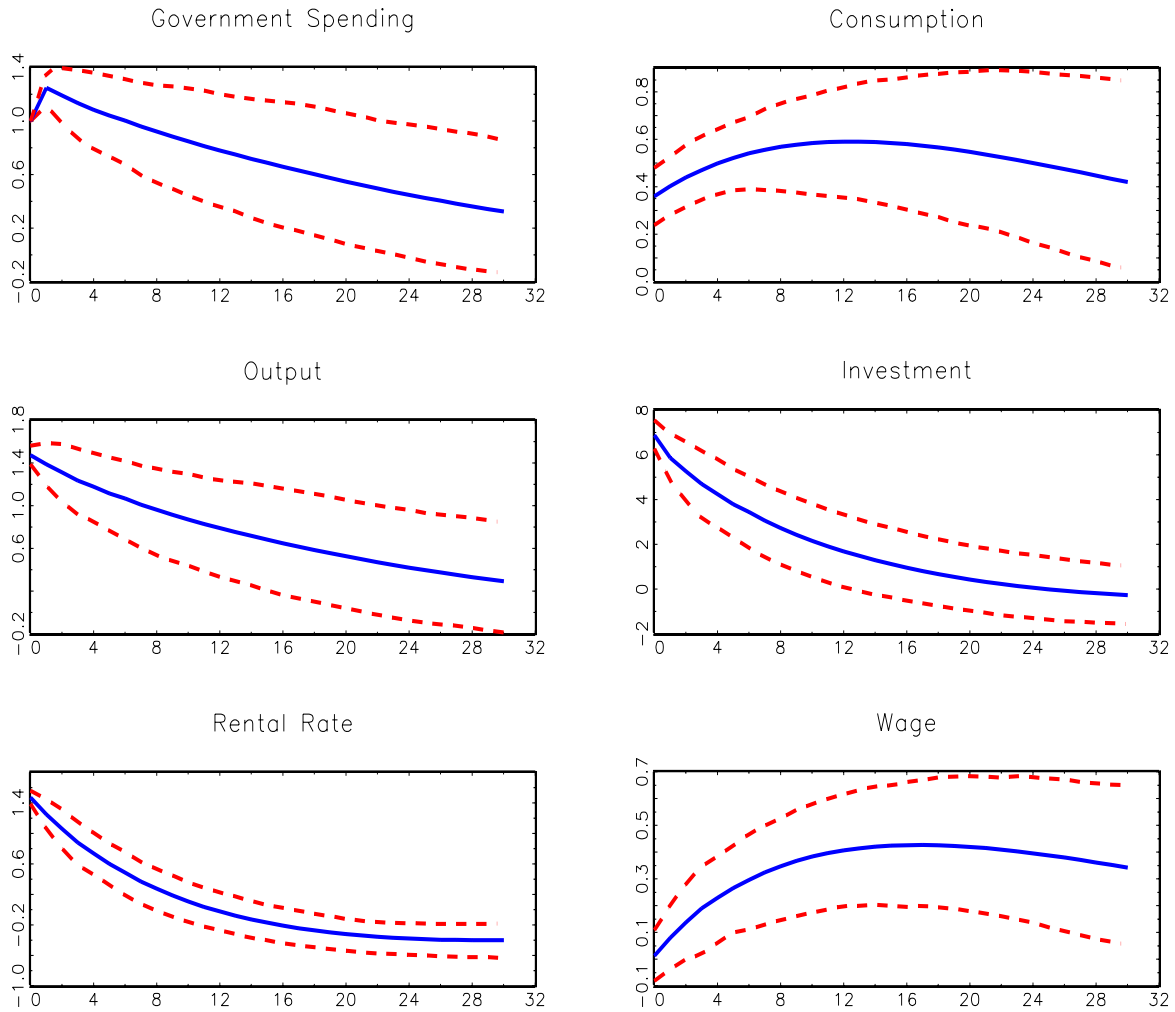
Figure 4: Simulated impulse responses to 1 per cent increase in public spending: case with optimal public spending and complementarity between private and public expenditures.
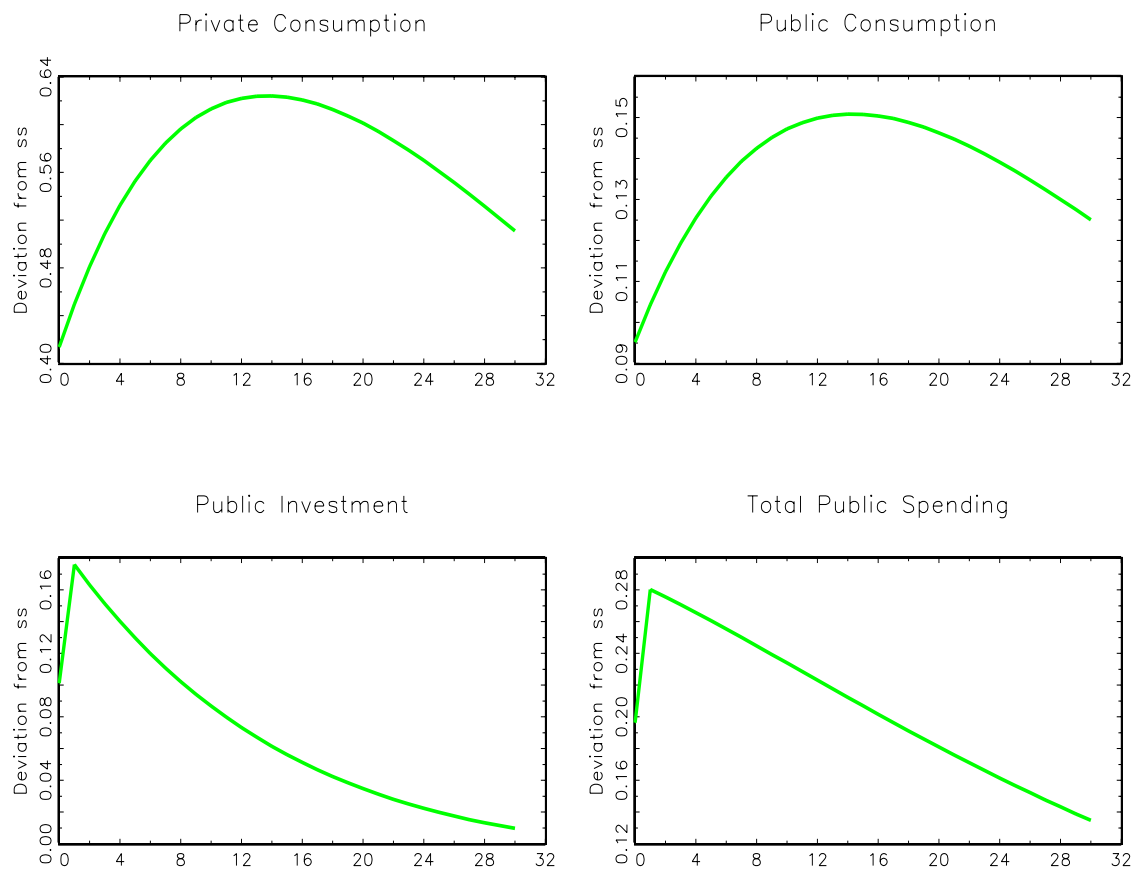
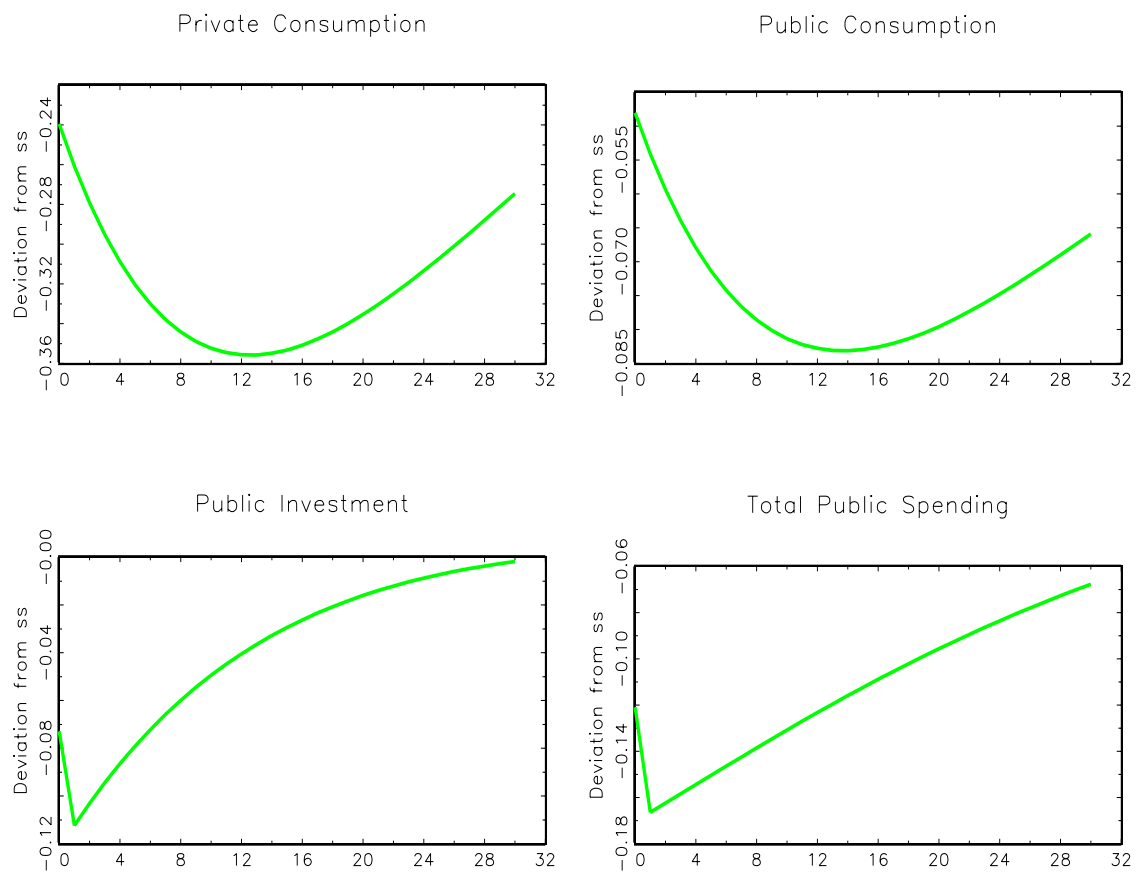Figure 5: Theoretical impulse responses to a 1 per cent technology shock.

Figure 6: Theoretical impulse responses to a 1 per cent preference shock.