Rationality, External Norms and the Epistemic Value of Menus*

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Abstract. Ever since Sen's (1993; 1997) criticism on the notion of *internal consistency* or *menu independence* of choice, there exists a widespread perception that the standard revealed preference approach to the theory of rational choice has difficulties in coping with the existence of external norms, or the information a menu of choice might convey to a decision-maker, viz., the *epistemic value of a menu.* This paper provides a brief survey of possible responses to these criticisms of traditional rational choice theory. It is shown that a novel concept of *norm-conditional rationalizability* can neatly accommodate external norms within the standard framework of rationalizability theory. Furthermore, we illustrate that there are several ways of incorporating considerations regarding the epistemic value of opportunity sets into a generalized model of rational choice theory.
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1 Introduction

In his characteristic parlance, Sen (1993) argued against the *a priori* imposition of requirements of internal consistency of choice such as the *weak* and the *strong axioms of revealed preference* (Samuelson, 1938, 1950; Houthakker, 1950), *Arrow's choice axiom* (Arrow, 1959) and Sen's (1971) *condition* α, and investigated the implications of eschewing these internal choice consistency requirements. The purpose of this paper is to summarize attempts designed to amend the traditional rational choice model so as to accommodate some of Sen's criticisms. In particular, we focus on possible responses to two of Sen's (1993) criticisms of the traditional model of rational choice by suggesting ways to modify revealed preference theory in order to address these points without giving up the notion of standard rationalizability altogether.

A first criticism of Sen's (1993) deals with what he refers to as *external norms* that may influence the choice behavior of an individual. Sen (1993, p.500) poses the following question: "[C]an a set of choices really be seen as consistent *or* inconsistent on purely internal grounds *without* bringing in something *external* to choice, such as the underlying objectives or values that are pursued or acknowledged by choice?" To bring his point into clear relief, Sen illustrates it with the following example. Suppose there are three alternatives x, y and z and a decision-maker chooses x from the feasible set $\{x, y\}$, whereas the agent chooses y when all three alternatives are available. As Sen rightly points out, this pair of choices violates most of the standard choice consistency conditions including the weak and the strong axioms of revealed preference, Arrow's choice axiom, and Sen's condition α. Sen (1993, p.501) argues that this seeming inconsistency can be easily resolved if only we know more about the person's choice situation. To give a concrete example where this choice appears to be plausible, he offers the following interpretation. "Suppose the person faces a choice at a dinner table between having the last remaining apple in the fruit basket (y) and having nothing instead (x), forgoing the nice-looking apple. She decides to behave decently and picks nothing (x), rather than the one apple (y). If, instead, the basket had contained two apples, and she had encountered the choice between having nothing (x), having one nice apple (y) and having another nice one (z), she could reasonably enough choose one (y), without violating any rule of good behavior. The presence of another apple (z) makes one of the two apples decently choosable, but this combination of choices would violate the standard consistency conditions ... even though there is nothing particularly 'inconsistent' in this pair of choices ... ."

On the face of it, Sen's argument to this effect may seem to go squarely against the theory of rationalizability *à la* Arrow (1959), Richter (1966; 1971), Hansson (1968), Sen (1971), Suzumura (1976a; 1977) and many others, where the weak axiom of revealed preference is a necessary condition for rationalizability. In the following section, we discuss ways that have been suggested in the literature to accommodate examples of this nature by modifying the traditional axioms of rationality so as to include considerations of external norms, thus arriving at various concepts of *norm-conditional rationalizability.* These alternatives have their origins in contributions such as Baigent and Gaertner (1996), Gaertner and Xu (1997; 1999a,b; 2004), Sen (1997), Baigent (2007), Bossert and Suzumura (2007) and Xu (2007). We attempt to build a bridge between rationalizability theory and Sen's criticism. In essence, what emerges is the possibility of a peaceful co-existence of a norm-conditional rationalizability theory and Sen's elaborate criticism against the internal consistency of choice.

Sen's (1993) second example used to call into question the imposition of internal choice consistency conditions is the following. Suppose a decision-maker is offered a cup of tea at a distant acquaintance's place, the feasible set thus consisting of the two alternatives 'tea' and 'staying home.' Suppose, further, that the person chooses 'tea.' Now suppose the acquaintance offers, in addition to tea, the option of having some cocaine at its place. It may very well be the case that, when faced with the new opportunity set consisting of the alternatives 'tea,' 'cocaine' and 'staying home,' the last option is selected. Again, the standard axioms of revealed preference are violated by this choice behavior. In the example, the opportunity set (the *menu*) itself conveys information about the consequences of these choices: if cocaine is offered in addition to tea, the decision-maker's perception of the acquaintance may change and, as a consequence, it chooses not even to enter its house. This is what Sen (1993, p.502) refers to as the *epistemic value* of a menu. The observation that opportunity sets may have epistemic value has been made before; for example, Luce and Raiffa (1957) argue that the existence or absence of certain menu items in a restaurant may influence a customer's perception of the nature of the place and thereby allow 'irrelevant' alternatives to affect its choices; see Luce and Raiffa (1957, p.288) for a detailed discussion.

In the specific example described above, the behavior of the decision-maker can be explained if one is prepared to acknowledge that the objects of choice may not be the objects of preference. The possible choices that can appear on menus are 'tea,' 'cocaine' and 'staying home.' The consequences the decision-maker may care about, however, are more adequately described as 'having tea at a place where cocaine is consumed' (outcome a), 'having tea at a place that is presumed to be cocaine-free' (outcome b), 'having cocaine' (outcome c) and 'staying home' (outcome d). If the menu consists of the options 'tea' and 'staying home' only, both 'having tea at a place where cocaine is consumed' and 'having tea at a place that is presumed to be cocaine-free' are possible consequences of choosing 'tea,' whereas if the menu item 'cocaine' is added, this uncertainty disappears—'having tea at a place that is presumed to be cocaine-free' ceases to be a possible consequence of accepting an invitation for tea.

Suppose the decision-maker's (transitive) preferences are such that b is better than d, d is better than a and a is better than c. The choice of 'tea' from the opportunity set consisting of 'tea' and 'staying home' induces the set of possible consequences $\{a, b\}$, whereas choosing 'tea' from the menu consisting of 'tea,' 'cocaine' and 'staying home' has but one possible consequence—ending up with a with certainty. If the set of possible outcomes $\{a, b\}$ is, according to the decision rule under uncertainty the agent may employ, better than the singleton set of possible outcomes $\{a\}$, the above-described choices can be explained in the context of preference optimization once the distinction between choice items and consequences is recognized and a preference relation on consequences is supplemented with a preference relation on *sets* of possible consequences under uncertainty.

A natural approach to choice under uncertainty where no probability information is available consists of establishing a ranking of sets of possible outcomes that is, in a sense to be made precise, consistent with a preference relation over these outcomes themselves. Discussions of the suitability of this approach are provided in Pattanaik and Peleg (1984), Bossert, Pattanaik and Xu (2000) and Barberà, Bossert and Pattanaik (2004), for instance.

The standard tool employed in this type of non-probabilistic decision problem is an *extension rule.* Suppose R is a preference relation defined on a non-empty set X of possible outcomes. An extension rule for R is a relation $\mathcal{R}$ on the set $\Pi(X)$ of non-empty subsets of X such that $\mathcal{R}$ ranks singletons (that is, certain outcomes) in the same way as R ranks the requisite outcomes themselves. That is, a relation $\mathcal{R}$ on $\Pi(X)$ is an extension rule for a relation R on X if, for all $x, y \in X$, $(x, y) \in R$ if and only if $(\{x\}, \{y\}) \in \mathcal{R}$. Clearly, an extension rule exists for any relation R but this extension rule may fail to have some suitable properties in order for it to be interpretable as a decision rule under uncertainty. Therefore, additional requirements are often imposed and impossibility results emerge frequently in this context. In our case, the additional requirement imposed is very mild and, thus, impossibilities are avoided. The problem of defining and axiomatizing decision rules of that nature has, by now, a long tradition; see, for instance, Arrow and Hurwicz (1972), Kreps (1979), Barberà, Barrett and Pattanaik (1984), Barberà and Pattanaik

(1984), Fishburn (1984), Heiner and Packard (1984), Holzman (1984a,b), Kannai and Peleg (1984), Nitzan and Pattanaik (1984), Pattanaik and Peleg (1984), Bandyopadhyay (1988) and Bossert (1989). A survey and further references can be found in Barberà, Bossert and Pattanaik (2004).

Again, a modified formulation of rational choice in this setting allows Sen's (1993) criticism to be accommodated without giving up completely on the traditional notion of rationalizability. Unlike in the standard framework, *two* relations are now sought in determining whether observed choice behavior is rational: a relation on the set of outcomes themselves and an extension rule for that relation. Following Bossert (2001), we discuss a characterization of rationalizability in this setting. The measurement of the amount of information contained in opportunity sets is analyzed in Bossert (2000) and Naeve and Naeve-Steinweg (2002).

Section 2 introduces the basic definitions and concepts used throughout the paper. In Sections 3 and 4, we review various contributions whose objective is to suggest resolutions of the external-norm issue and the epistemic-value issue, respectively. The final section concludes.

2 Preferences and Choices

Let X be a non-empty universal set of alternatives and let $R \subseteq X \times X$ be a (binary) relation on X. The asymmetric factor $P(R)$ of R is given by $(x, y) \in P(R)$ if and only if $(x, y) \in R$ and $(y, x) \notin R$ for all $x, y \in X$, and the symmetric factor $I(R)$ of R is defined by $(x, y) \in I(R)$ if and only if $(x, y) \in R$ and $(y, x) \in R$ for all $x, y \in X$. If R is interpreted as a *preference relation* (that is, $(x, y) \in R$ is interpreted to mean that x is at least as good as y), $P(R)$ and $I(R)$ are the *strict preference relation* and the *indifference relation* corresponding to R.

Let $S \subseteq X$ be a non-empty subset of X and let R be a relation on X. The set of *R-greatest elements* in S is defined by

$$G(S, R) = \{x \in S \mid (x, y) \in R \text{ for all } y \in S\}.$$

The *transitive closure* $tc(R)$ of a relation R is defined by letting, for all $x, y \in X$,

$$\begin{aligned}(x, y) \in tc(R) \quad \Leftrightarrow \quad & \exists K \in \mathbb{N} \text{ and } x^0, \ldots, x^K \in X \text{ such that} \\ & x = x^0 \text{ and } (x^{k-1}, x^k) \in R \text{ for all } k \in \{1, \ldots, K\} \text{ and } x^K = y.\end{aligned}$$

For any binary relation R, $tc(R)$ is the smallest transitive superset of R.

A relation $R \subseteq X \times X$ is *reflexive* if, for all $x \in X$,

$$(x, x) \in R$$

and R is *complete* if, for all $x, y \in X$ such that $x \neq y$,

$$(x, y) \in R \text{ or } (y, x) \in R.$$

R is *transitive* if, for all $x, y, z \in X$,

$$[(x, y) \in R \text{ and } (y, z) \in R] \;\Rightarrow\; (x, z) \in R.$$

It is clear that R is transitive if and only if $R = tc(R)$. A *quasi-ordering* is a reflexive and transitive relation and an *ordering* is a complete quasi-ordering.

Suppose $\Pi(X)$ is the power set of X excluding the empty set. A *choice function* is a mapping $C\colon \Sigma \rightarrow \Pi(X)$ such that $C(S) \subseteq S$ for all $S \in \Sigma$, where $\Sigma \subseteq \Pi(X)$ with $\Sigma \neq \emptyset$ is the *domain* of C. Note that we do not impose any restriction on Σ (other than its non-emptiness). Thus, we follow Richter (1966; 1971), Hansson (1968), Suzumura (1976a,b; 1977; 1983), Bossert, Sprumont and Suzumura (2005a,b; 2006) and Bossert and Suzumura (2008), among others, in that we want our model of choice to be applicable to any choice situation one may wish to analyze. Let $C(\Sigma)$ denote the image of Σ under C, that is, $C(\Sigma) = \cup_{S \in \Sigma} C(S)$. As is customary, we assume that $C(S)$ is non-empty for all sets S in the domain of C. Thus, using Richter's (1971) term, the choice function C is assumed to be *decisive.*

A choice function C is *rationalizable* if there exists a transitive relation $R \subseteq X \times X$ such that, for all $S \in \Sigma$, $C(S) = G(S, R)$. This is the definition of *greatest-element rationalizability,* as opposed to *maximal-element rationalizability* which is based on undominated rather than greatest elements. See Bossert and Suzumura (2008) for a detailed discussion of rationalizability with alternative coherence requirements on the rationalizing relation, such as *quasi-transitivity, acyclicity* and *consistency* (Suzumura, 1976b) in place of transitivity and with added richness properties such as reflexivity and completeness.

3 External Norms

An early suggestion to deal with external norms was proposed by Baigent and Gaertner (1996). In response to Sen's (1993) first criticism as outlined in the Introduction, they employ a non-standard notion of rationalizability that obeys the restriction imposed by the external norm not to choose the *uniquely* greatest element according to some relation but behaves as the traditional version of rationalizability when the set of greatest objects contains at least two elements. Baigent and Gaertner (1996) define, for a feasible set $S \in \Pi(X)$ and for an ordering R on X, the set $G^*(S, R)$ as

$$G^*(S,R) = \begin{cases} G(S,R) & \text{if } |G(S,R)| = 1 \\ \emptyset & \text{otherwise.} \end{cases}$$

According to Baigent and Gaertner (1996, p.244), a choice function C is *non-standard rationalizable* if there exists a transitive relation R on X such that, for all $S \in \Sigma$,

$$C(S) = G(S \setminus G^*(S,R), R). \tag{1}$$

The characterization of non-standard rationalizability due to Gaertner and Baigent (1996) applies to the full domain $\Sigma = \Pi(X)$ and, moreover, they assume X to be finite. The set of chosen elements is assumed to be non-empty but that means that, implicitly, they do not include singleton sets in their domain. A choice function that is rationalizable in the sense expressed by (1) selects all second-greatest elements according to a rationalizing relation if there is a unique greatest element; if, however, there are several greatest elements, C chooses *all* of these greatest elements. Baigent and Gaertner (1996, p.241) claim that they axiomatize the maxim "always choose the second largest except in those cases where there are at least two pieces which are largest, being of equal size. In that case, either may be chosen." Unfortunately, however, this informal maxim seems to be in conflict with the formal definition and characterization provided by Baigent and Gaertner (1996, p.243). Indeed, according to (1), if there is no unique greatest element, *all* greatest elements are chosen and not just one of them. Thus, there is a gap between their formal axiomatization and the informal maxim, the axiomatization of the latter being left unaccomplished so far.

Gaertner and Xu (1999a) discuss an alternative approach covering cases where external norms may lead to the choice of the *median* alternative(s) according to some antisymmetric relation on X. As is the case for Baigent and Gaertner (1996), they consider the full domain $\Sigma = \Pi(X)$. Moreover, X is assumed to be finite to ensure that the median alternatives are well-defined. Their results characterize the choice function C such that $C(S)$ is equal to the median alternative in S according to some antisymmetric ordering R on X. This approach is compared to the traditional rational choice setup and to the Baigent and Gaertner (1996) framework in the antisymmetric case in Gaertner and Xu (1997) and in a more general setting in Gaertner and Xu (1999b).

An alternative type of norm-constrained choice is characterized in Gaertner and Xu (2004). The choice functions analyzed in this contribution have a domain that contains the empty set in addition to all non-empty subsets of X and, moreover, choice sets may be empty even if feasible sets are non-empty. The behavior Gaertner and Xu (2004) attempt to capture is the refusal to make a choice in response to the suppression of alternatives: if there is but a single alternative available, the decision-maker may choose the empty set as a means of expressing his or her displeasure with the suppression of other feasible alternatives. An example put forward by Sen (1997, p.755) and used by Gaertner and Xu (2004) as a motivation of their approach is that of a government that outlaws all newspapers but one that is owned by the government itself. They argue that if several papers are available, the government paper may well be the choice of an agent, but the absence of any alternative sources leads the decision-maker to boycott the single available news outlet.

In general, external norms can be taken into consideration by specifying all pairs consisting of a feasible set and an element of this set with the interpretation that this element is prohibited from being chosen from this set by the relevant system of external norms. *Norm-conditional rationalizability* then requires the existence of a preference relation such that, for each feasible set in the domain of the choice function, the chosen elements are at least as good as all elements in the set *except* for those that are prohibited by the external norm. This approach, due to Bossert and Suzumura (2007), is very general because no restrictions are imposed on how the system of external norms comes about—any specification of a set of pairs as described above is possible. Of course, we do not claim that *all logically possible* specifications of a set of norms are intuitively plausible and attractive; what we propose in Bossert and Suzumura (2007) is a general method to accommodate *any* external norm one might want to specify in a choice-theoretic setting that is true in spirit to the traditional revealed preference approach. Indeed, the standard model of rational choice is included as a special case—the case that obtains if the set of prohibited pairs is empty. This framework does not rely on implicit assumptions such as, for example, everyone in a society having the same preferences and a decision-maker should refrain from choosing the unique greatest element according to such a *common* preference relation.

Xu (2007) discusses further special cases, namely, a variant of the Baigent and Gaertner (1996) 'never-choose-the-uniquely-largest' rule, the median-based rule (see Gaertner and Xu, 1999a) and two version of the 'protest-based' norm of Gaertner and Xu (2004). These special cases are obtained by ruling out the choice of unique best elements, elements better than the (bottom) median element, and non-empty choices in the case of single-valued feasible sets.

For example, suppose there is a feasible set $S = \{x, y\}$, where x stands for selecting nothing and y stands for selecting (a single) apple. Now consider the feasible set $T = \{x, y, z\}$ where there are two (identical) apples y and z available. The external norm not to take the last apple can easily and intuitively be expressed by requiring that the choice of y from S is excluded, whereas the choice of y (or z) from T is perfectly acceptable. In general, norms of that nature can be expressed by identifying all pairs (S, w), where $w \in S$, such that w is not supposed to be chosen from the feasible set S. To that end, we use a set $\mathcal{N}$, to be interpreted as the set of all pairs (S, w) of a feasible set S and an element w of S such that the choice of w from S is prevented by the external norm under consideration.

Given an external norm defined by $\mathcal{N}$, a *norm-conditional choice function* is a choice function C such that $C(S) \subseteq S \setminus \{z \in S \mid (S, z) \in \mathcal{N}\}$ for all $S \in \Sigma$. To ensure that the standard decisiveness requirement on C does not conflict with the restrictions imposed by the norm $\mathcal{N}$, we only consider norms $\mathcal{N}$ such that, for all $S \in \Sigma$, there exists $x \in S$ satisfying $(S, x) \notin \mathcal{N}$. The set of all possible norms satisfying this restriction is denoted by $\mathbf{N}$.

This model of norm-conditional choice may appear somewhat restrictive at first sight because it specifies pairs of a feasible set and *a single object* not to be chosen from that set. One might want to consider the following seeming generalization of this approach: instead of only including pairs of the form (S, x) with $x \in S$ when defining a system of norms, one could include pairs such as (S, T) with $T \subseteq S$, thus postulating that the subset T should not be chosen from S. Contrary to first appearance, this does not really provide a more general model of norm-conditional rationalizability because, in order to formulate our notion of norm-conditional rationality, we require that a chosen element $x \in C(S)$ has to be at least as good as all feasible elements except those that are already excluded by the external norm according to a norm-conditional rationalization—that is, x has to be at least as good as all $y \in S$ except for those $y \in S$ such that $(S, y) \in \mathcal{N}$. Allowing for pairs (S, T) does not provide a more general notion of norm-conditional rationalizability because the subset of S, the elements of which have to be dominated by a chosen object, can be obtained in any arbitrary way from the subsets T such that T cannot be selected from S according to the external norm. For simplicity of exposition, we work with the simpler version of our model introduced above but note that this formulation does not involve any loss of generality when it comes to the definition of norm-conditional rationality employed in this paper.

Returning to Sen's example involving the norm 'do not choose the last available apple,' we can, for instance, define the universal set $X = \{x, y, z\}$, the domain $\Sigma = \{S, T\} \subseteq \Pi(X)$ with $S = \{x, y\}$ and $T = \{x, y, z\}$, and the external norm described by the set $\mathcal{N} = \{(S, y)\}$. Thus, the external norm requires that $y \notin C(S)$ but no restrictions are imposed on the choice $C(T)$ from the set T—that is, this external norm represents the requirement that the last available apple should not be chosen.

In contrast with the classical model of rational choice, an element x that is chosen by a choice function C from a feasible set $S \in \Sigma$ need not be considered at least as good as *all* elements of S by a rationalizing relation, but merely at least as good as all elements $y \in S$ such that $(S, y) \notin \mathcal{N}$. That is, if the choice of y from S is already prohibited by the norm, there is no need that x dominates such an element y according to the rationalization. Needless to say, the chosen element x itself must be admissible in the presence of the prevailing system of external norms.

To make this concept of norm-conditional rationalizability precise, let a system of external norms $\mathcal{N} \in \mathbf{N}$ and a feasible set $S \in \Sigma$ be given. An $\mathcal{N}$-*admissible set* for $(\mathcal{N}, S)$, $A^{\mathcal{N}}(S) \subseteq S$, is defined by letting

$$A^{\mathcal{N}}(S) = \{x \in S \mid (S, x) \notin \mathcal{N}\}.$$

Note that, by assumption, $A^{\mathcal{N}}(S) \neq \emptyset$ for all $\mathcal{N} \in \mathbf{N}$ and for all $S \in \Sigma$.

We say that a choice function C on Σ is $\mathcal{N}$-*rationalizable* if and only if there exists a transitive relation $R^{\mathcal{N}} \subseteq X \times X$ such that, for all $S \in \Sigma$,

$$C(S) = G(A^{\mathcal{N}}(S), R^{\mathcal{N}}).$$

In this case, we say that $R^{\mathcal{N}}$ $\mathcal{N}$-*rationalizes* C, or $R^{\mathcal{N}}$ is an $\mathcal{N}$-*rationalization* of C. Norm-conditional rationalizability can be defined for rationalizing relations that are not necessarily transitive; see Bossert and Suzumura (2007) for details. We restrict attention to transitive rationalizations in this paper for expositional convenience.

To facilitate our analysis of $\mathcal{N}$-rationalizability, a generalization of the notion of the direct revealed preference relation of a choice function is of use. We define

$$R_C^{\mathcal{N}} = \{(x, y) \in X \times X \mid \exists S \in \Sigma \text{ such that } x \in C(S) \text{ and } y \in A^{\mathcal{N}}(S)\}.$$

We refer to $R_C^{\mathcal{N}}$ as the *norm-conditional direct revealed preference relation* corresponding to C and $\mathcal{N}$.

We are now ready to identify a necessary and sufficient condition for $\mathcal{N}$-rationalizability of a choice function for an arbitrary norm $\mathcal{N} \in \mathbf{N}$. We follow Richter (1966; 1971) by generalizing the relevant axiom in his approach in order to accommodate an externally imposed system of norms $\mathcal{N}$. This leads us to the following axiom.

$\mathcal{N}$-conditional transitive-closure coherence: For all $S \in \Sigma$ and for all $x \in A^{\mathcal{N}}(S)$,

$$(x, y) \in tc(R_C^{\mathcal{N}}) \text{ for all } y \in A^{\mathcal{N}}(S) \;\Rightarrow\; x \in C(S).$$

Intuitively, the transitive closure of $R_C^{\mathcal{N}}$ must be respected by any transitive $\mathcal{N}$-rationalization of C. In other words, an $\mathcal{N}$-admissible element x in S must be chosen from S if it is directly or indirectly $\mathcal{N}$-conditionally revealed preferred to every $\mathcal{N}$-admissible element in S. Moreover, the condition is sufficient for $\mathcal{N}$-rationalizability as can be seen by adapting Richter's (1971) argument to accommodate the external norm; see Bossert and Suzumura (2007) for details. Thus, for any system of external norms $\mathcal{N} \in \mathbf{N}$, a choice function C is $\mathcal{N}$-rationalizable if and only if C satisfies $\mathcal{N}$-conditional transitive-closure coherence.

We conclude this section with a remark on a related concept. Sen (1997) provided an important step towards a norm-conditional theory of rationalizability through the concept of *self-imposed choice constraints,* excluding some alternatives from permissible choices. According to Sen's (1997, p.769) scenario, "the person may first restrict the choice options ... by taking a 'permissible' subset $K(S)$, reflecting *self-imposed* constraints, and then seek the maximal elements ... in $K(S)$." Despite an apparent family resemblance between Sen's concept of self-imposed choice constraints and our concept of norm-conditionality, Sen did not go as far as to bridge the idea of norm-induced constraints and the theory of rationalizability as we do here.

4 Menus and Information

Let us now turn to the second issue of the epistemic value of menus. Suppose a set $S \in \Pi(X)$ is interpreted as a set of possible outcomes under uncertainty. For mnemonic convenience, we refer to such a set as a *situation.* Let an ordering $\mathcal{R}$ on $\Pi(X)$ be interpreted as a non-probabilistic decision rule that ranks these uncertain situations. A *choice function under uncertainty* is a mapping $D\colon \Delta \to \Pi(\Pi(X))$ such that $D(\mathcal{S}) \subseteq \mathcal{S}$ for all $\mathcal{S} \in \Delta$, where the non-empty set $\Delta \subseteq \Pi(\Pi(X))$ is the domain of D. In order to interpret $\mathcal{R}$ as a decision rule under uncertainty, it is minimally required that this relation is compatible with a ranking R on the set X of alternatives in the sense that $\mathcal{R}$ ranks singletons in the same way as R ranks these alternatives themselves. This joint requirement on the pair of relations R and $\mathcal{R}$ is expressed by means of the following *extension* axiom.

Extension: For all $x, y \in X$,

$$(x, y) \in R \;\Leftrightarrow\; (\{x\}, \{y\}) \in \mathcal{R}.$$

The above extension axiom may be considered necessary for the interpretation of $\mathcal{R}$ as a decision rule under uncertainty given the underlying relation R on the set of alternatives X themselves. On the other hand, the axiom does not appear to be sufficient for the intended interpretation. For example, suppose a decision-maker strictly prefers $x \in X$ to $y \in X$ according to the relation R. In this case, it would seem rather unnatural for the same person to strictly prefer the pair of possible outcomes $\{x, y\}$ (that is, obtaining either the better alternative or the worse alternative) to the singleton $\{x\}$ (that is, obtaining the better alternative with certainty). Such counter-intuitive features of a non-probabilistic decision rule can be avoided by imposing a *monotonicity* property.

Monotonicity: For all $x, y \in X$,

$$(x, y) \in R \;\Rightarrow\; [(\{x\}, \{x, y\}) \in \mathcal{R} \text{ and } (\{x, y\}, \{y\}) \in \mathcal{R}].$$

We say that $\mathcal{R}$ is an *extension rule for* R if the pair $(R, \mathcal{R})$ satisfies the extension axiom. If, in addition, the monotonicity axiom is satisfied, we refer to $\mathcal{R}$ as a *monotonic extension rule for* R.

The direct revealed preference relation $\mathcal{R}_D$ corresponding to a choice function under uncertainty $D\colon \Delta \to \Pi(\Pi(X))$ is defined in the usual manner, that is,

$$\mathcal{R}_D = \{(S, T) \in \Pi(X) \times \Pi(X) \mid \exists \mathcal{S} \in \Delta \text{ such that } S \in D(\mathcal{S}) \text{ and } T \in \mathcal{S}\}.$$

Rationalizability of a choice function under uncertainty is defined as usual. D is *rationalizable* if there exists a transitive relation $\mathcal{R}$ on $\Pi(X)$ such that

$$D(\mathcal{S}) = G(\mathcal{S}, \mathcal{R})$$

for all $\mathcal{S} \in \Delta$.

We now use a recursive construction to arrive at a relation that must be respected by any rationalization $\mathcal{R}$ on $\Pi(X)$ which is a monotonic extension rule for some transitive relation R on X. Let $\mathcal{R}_D^1 = \mathcal{R}_D$ and, for all $m \in \mathbb{N}$,

$$\begin{aligned}\mathcal{R}_D^{m+1} \;=\; & tc(\mathcal{R}_D^m) \cup \{(S, T) \mid \exists x, y \in X \text{ such that } x \neq y \text{ and } (\{x\}, \{y\}) \in tc(\mathcal{R}_D^m) \text{ and} \\ & [[S = \{x\} \text{ and } T = \{x, y\}] \text{ or } [S = \{x, y\} \text{ and } T = \{y\}]]\}.\end{aligned}$$

Now define $\mathcal{R}_D^* = \cup_{m \in \mathbb{N}} tc(\mathcal{R}_D^m)$. This relation can be constructed in a countable number of steps even if the sets X and Δ are not countable. Furthermore, the relation $\mathcal{R}_D^*$ is transitive (see Bossert, 2001, p.355).

Intuitively, the relation $\mathcal{R}_D^*$ is obtained by successively adding new pairs to the revealed preference relation that must appear in any rationalizing relation $\mathcal{R}$ on $\Pi(X)$ that is a monotonic extension rule for some transitive relation R on X.

That $\mathcal{R}_D^*$ must be respected is not only necessary, but also sufficient for rationalizability of a choice function under uncertainty, provided the extension and monotonicity axioms are imposed. To make this observation precise, we formulate the following property of *monotonic congruence,* an adaptation of Richter's (1966; 1971) congruence axiom in the traditional rational choice framework.

Monotonic congruence: For all $S, T \in \Pi(X)$ and for all $\mathcal{S} \in \Delta$,

$$[(S,T) \in \mathcal{R}_D^* \text{ and } T \in D(\mathcal{S}) \text{ and } S \in \mathcal{S}] \;\Rightarrow\; S \in D(\mathcal{S}).$$

As established in Bossert (2001, Theorem 3), there exist transitive relations R on X and $\mathcal{R}$ on $\Pi(X)$ such that a choice function under uncertainty D is rationalizable by $\mathcal{R}$ and $\mathcal{R}$ is a monotonic extension rule for R if and only if D satisfies monotonic congruence. Note that the completeness of R and of $\mathcal{R}$ does not follow as in the traditional rational choice results—rationalizability by a transitive relation $\mathcal{R}$ on $\Pi(X)$ that is a monotonic extension rule for a transitive relation R on X is, in general, weaker than an analogous rationalizability property involving orderings; see Bossert (2001, pp.357–358) for a discussion. This stronger form of rationalizability is characterized in Bossert (2001, Section 6) but it is considerably more complex because not every pair of ordering extensions of R and $\mathcal{R}$ preserves both the monotonicity and the rationalizability property. As a consequence, existential clauses have to be invoked.

5 Concluding Remarks

This paper provides a brief survey of some suggestions that appear in the literature to resolve what Sen (1993) refers to as instances of choice behavior where internal consistency conditions may be inappropriate. In particular, we focus on the inclusion of external norms and the epistemic value of menus. It turns out that, in both cases, it is possible to respond to Sen's criticisms by means of a revealed preference framework that is closely linked to the traditional theory of rational choice. Thus, we may suggest that these examples do not necessarily force us to abandon rational choice and revealed preference theory altogether.

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