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Three Essays in Microeconomic Theory

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Résumé

Cette thèse est un recueil de trois articles sur la théorie microéconomique. Les deux premiers traitent de la question de la course vers le bas lorsque les gouvernements se livrent à la concurrence pour certains facteurs mobiles. Le troisième article propose une extension du problème d'appariement plusieurs-à-un en y introduisant des agents de tailles différentes.

Dans le premier article, nous montrons comment le résultat standard de course vers le bas (race-to-the-bottom) peut être évité en introduisant du bien public dans un modèle de compétition fiscale. Notre économie comporte deux juridictions peuplées par de la main-d'œuvre parfaitement mobile répartie en deux catégories : qualifiée et non-qualifiée. Les gouvernements, en poursuivant un objectif Rawlsien (max-min), annoncent simultanément leur projet d'investissement en bien public avant d'adopter une politique de taxation non-linéaire du revenu. Les travailleurs, après avoir observé la politique de taxation des différents gouvernements et leurs promesses d'investissement en bien publique, choisissent chacun un lieu de résidence et une offre de travail. Ainsi, les gouvernements atteignent leurs objectifs de redistribution en cherchant à attirer de la main-d'œuvre productive à travers la fourniture de bien public en plus d'une politique de taxation favorable. Nous montrons qu'il existe des équilibres où les travailleurs qualifiés paient une taxe strictement positive. En outre, lorsque l'information sur le type des travailleurs est privée, il existe, pour certaines valeurs des paramètres, des équilibres où la main-d'œuvre non-qualifiée bénéficie d'un transfert net (ou subvention) de la part du gouvernement.

Dans le second article, nous étudions comment le modèle standard de compétition des prix à la Bertrand avec des produits différenciés pourrait fournir des informations

utiles pour les programmes de citoyenneté par investissement dans les Caraïbes. Nous montrons que lorsque les pays peuvent être classés en deux types en fonction de la taille de leur demande, l'imposition d'un prix minimum uniforme et d'un quota maximum appropriés amène les pays à un résultat efficace qui Pareto domine l'équilibre de Nash non coopératif.

Enfin, le troisième article explore une extension du problème standard d'appariement plusieurs-à-un en y incorporant des agents de tailles différentes (familles de réfugiés) d'un côté, à assigner à des foyers de capacités différentes de l'autre. La taille d'une famille de réfugiés représente le nombre de membres qui la compose. Une caractéristique spécifique à ce modèle est qu'il n'autorise pas de répartir les membres d'une même famille entre différents foyers. Il est bien connu que, dans ces conditions, bon nombre de propriétés désirables des règles d'appariement s'effondrent. Nous faisons donc l'hypothèse des priorités croissantes avec la taille pour chaque foyer, c'est-à-dire qu'une famille d'accueil préférerait toujours un plus grand nombre de réfugiés tant que la capacité de son foyer le permet. Nous montrons qu'un appariement stable par paire existe toujours sous cette hypothèse et nous proposons un mécanisme pour le trouver. Nous montrons que notre mécanisme est non-manipulable du point de vue des réfugiés : aucun groupe de réfugiés ne pourrait tirer profit d'une déclaration truquée de leurs préférences. Notre mécanisme est également optimal pour les réfugiés en ce sens qu'il n'existe aucun autre mécanisme stable par paire qui serait plus profitable à tous les réfugiés.

Mots-clés : Concurrence en matière d'impôt sur le revenu, Mobilité de la main-d'œuvre, Imposition optimale des revenus, Course vers le bas, Bien public, Concurrence des prix à la Bertrand, Prix minimum, Quota maximum, Efficacité au sens de Pareto, Agents de talles différentes, Correspondance plusieurs-à-un, Stabilité par paire, Monotonie par taille

Abstract

This thesis is a collection of three articles on microeconomic theory. The first two articles are concerned with the issue of race-to-the-bottom when governments engage in competition for some mobile factor. The third article proposes an extension for the many-to-one matching problem by introducing different-size agents.

In the first article, we show how the standard race-to-the-bottom result can be avoided by introducing public good into a tax competition model. Our economy has two jurisdictions populated by perfectly mobile workers divided into two categories: skilled and unskilled. Governments, in pursuit of a Rawlsian objective (max-min), simultaneously announce their plans for investing in public good before deploying a nonlinear income tax schedule. After observing the tax schedules of the governments and their promises to invest in public good, each worker chooses a place of residence and a supply of labour. Thus, governments achieve their redistribution objectives by seeking to attract productive labour through the provision of public goods in addition to favorable taxation policy. We show that there exist equilibria where skilled workers pay a strictly positive tax. In addition, when information on the type of workers is private, there are equilibria for certain parameter values in which unqualified workers receive a net transfer (or subsidy) from the government.

In the second article, we investigate how the Bertrand standard price competition with differentiated products could provide useful insight for Citizenship By Investment programs in the Caribbean. We show that when countries can be classified into two types according to the size of their demand, imposing appropriate uniform minimum price and maximum quota brings countries to an efficient outcome that Pareto dominates the Non-Cooperative Nash Equilibrium.

Finally. in the third article, we explore an extension of the standard many-to-one matching problem by incorporating different-size agents (refugee families) on the many side of the market, to be assigned to entities (homes) with different capacities on the other side. A specific feature of this model is that it does not allow refugee families to be split between several homes. It is well known that many of the desirable properties of matching rules are unachievable in this framework. We introduce size-monotonic priority ranking over refugee families for each home, that is, a host family (home) would always prefer a greater number of members of refugee families until its capacity constraint binds. We show that a pairwise stable matching always exists under this assumption and we propose a mechanism to find it. We show that our mechanism is strategy-proof for refugees: no refugee family could benefit from misrepresenting his preferences. Our mechanism is also refugees optimal pairwise stable in the sense that there is no other pairwise stable mechanism that would be more profitable to all refugees.

Keywords: Income tax competition, Labor mobility, Optimal income taxation, Race-to-the-bottom, Public good, Bertrand price competition, Minimum Price, Maximum quota, Pareto efficiency, Sized agents, Many-to-one matching, Pairwise stability, size-monotonicity

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Liste des sigles et des abréviations

C1	Caribbean 1
CBI	Citizenship By Investment
CDA	Consecutive Deferred Acceptance
DA	Deferred Acceptance
DSGC	Downward Sequential Greedy Correcting
ECCU	Eastern Caribbean Currency Union
FDI	Foreign Direct Investment
FMI	Fonds Monétaire International

MRS	Marginal Rate of Substitution
SCSE	Société Canadienne de Sciences Économiques
SGC	Sequential Greedy Correcting
USGC	Upward Sequential Greedy Correcting
WHD	Western Hemisphere Department

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First Article.

Should we subsidize the poor when labour is perfectly mobile? Tax Competition with Public Good

by

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This article is in preparation for submission to Games and Economic Behavior.

1. Introduction

Globalization has improved capital mobility and migration through the emergence of international trade, the formation of economic unions and the development of transport.

This global socioeconomic progress has made low-tax regions more attractive not only to investors but also to workers and, thereby, made redistributive policy harder to implement. In fact, migration reduces the possibility of redistribution in two different ways. First, high tax payers, which are typically wealthy and skilled people, are incentivized to migrate towards lower tax regions. Second, low wage workers, which are typically low skilled agents, are encouraged to move to areas where poor people are subsidized or where they could pay fewer taxes. Therefore, competition between jurisdictions for rich people not only entails a decrease in the tax liability for the top earners but also results in a decline in the subsequent subsidies to the poor. These effects are well described in the standard literature of tax competition for labour [1] and are getting more significant as migration costs decline throughout our modern societies. [2]

The literature on nonlinear tax competition for mobile labour has mainly been focused on the possibility of redistribution when public good provision is not considered. In this standard framework, we know from Bierbrauer et al. (2013) that, in the absence of mobility costs, there is no equilibrium in which the lowest skill workers are subsidized when governments pursue a utilitarian objective. This so-called race-to-the-bottom result is observed in perfect labour mobility tax competition models even if governments use a maximin criterium which is the most redistributive social welfare function. Nevertheless, redistribution become harder in the case of utilitarian objective as shown by Bierbrauer et al. They show that, there does not exist an equilibrium where the highest skill pay a positive tax, and more interestingly the highest skill may even be subsidized for some values of the parameters.

However, the provision of public goods is a determining factor in the choice of individuals location (see Bretschger and Hettich (2002)). This paper diverges from the standard framework by allowing governments to confront the trade-off between public good provision (financed by tax revenue) and tax incentives to achieve their redistributive goal. In our setting, workers decide on their place of residence by taking into account both the taxation policy adopted in each jurisdiction and the distribution of public goods throughout the whole economy. Governments then compete for high-income

1. See Wilson (1999), for a review of the literature on tax competition for labour.
2. For instance, in 2019, France experienced severe social protests against Macron's administration's fiscal policy which started in 2018 with the suppression of the Solidarity Tax on Wealth and ended up with an increase in fuel tax that affects both poor people and wealthy people.

earners by providing an attracting level of public good in addition to tax incentives. We find that the provision of public good reduces the effect of tax competition by creating an additional incentive for taxpayers to immigrate (or not to emigrate) and then allows for more redistribution.

Specifically, the model we are interested in is an extension of the discrete-skill setting of Stiglitz (1982) in which agents have either low productivity or high productivity. The economy consists of two regions, and regional governments simultaneously announce a level of public good provision before deploying simultaneously a nonlinear income tax schedule over its residents. Workers, then choose their locations and every non-empty region provides the announced level of public good financed by the tax revenue. In order to restaure the possibility of redistribution we assume that governments pursuit a Rawlsian criterium (maximin). When information is symmetric governments observe the productivity of agents, but when information is asymmetric, only the before-tax income and the place of residence are observed by the policymakers.

Poitevin and Gravel (2016) provide a first insight on this subject by proposing a model of tax competition with finitely many productivity levels for individuals who receive an exogenous income and are perfectly mobile. They show that if the information on individuals' income is public, then introducing public good in an income tax competition model increases the possibility of income redistribution by competing governments significantly. And if the information on individual income is private, then chances for redistributions are more limited. Nonetheless, the authors pointed out the empty community problem, as identified before by Wilson (1999), caused by the presence of a public good.

Indeed, an increase in public good provision attracts taxpayers, which in turn increases tax revenue and allows the government to provide more public good. This mechanism creates a snowball effect making an area increasingly more attractive and results in the gathering of all individuals in a single region. We avoid this problem by proposing a game where the decision to provide public good and the choice of a tax schedule are made sequentially. By allowing the governments to decide on a level of public good provision before deploying a nonlinear income tax schedules, we characterize pooling (all agents choose the same place of residence) subgame perfect equilibria. We also give sufficient conditions for the existence of separating (poor and rich agents choose different places of residence) subgame perfect equilibria.

In contrast with the models of tax competition without public good, we find that, when information is symmetric, all individuals pay a non-negative tax in equilibrium and the tax paid by the high skill could be higher than that paid by the low skill which is an indirect redistribution (through public good). In fact, in those equilibria, the tax paid by a high skill agent is greater than the average tax liabilities throughout the region. When information is asymmetric, however, the high skill earn a higher utility than the low skill but pay a non-negative tax. Under some specific conditions the low skill may even be subsidized in the case of asymmetric information. Furthermore, pooling equilibria are always possible, that is, all types of agents choose the same place of residence. Under some specific conditions, we find that, unlike the model of Poitevin and Gravel (2016), separating equilibrium may also occur, that is, agents of different types decide to live in different regions.

In practice, these results suggest that in the absence of a supranational redistributive entity, a better fiscal strategy would be to attract investors by announcing large public spending such as plans to build new cities. This strategy helps to avoid race to the bottom as funds are levied from the residents in exchange for a provision of public good. We have seen in the last decade projects of this nature emerge such as the construction of a new Egyptian capital. This project requiring an investment of 52 billion euros was announced by the Egyptian government in 2014 and is highly dependent on foreign investment. We can also cite the example of the hatching of many artificial islands in Dubai and that of the new Senegalese city Diamiadio. These projects are largely financed by the sale of land and therefore constitute an effective means of attracting resources to the national territory. Even though this example concerns an indirect taxation by the sales of expensive lands it ensures redistribution through investment in public good as our model predicts. The realization of these large-scale projects, in addition to their obvious socio-demographic objectives, have a direct (multiplication of public infrastructures) and indirect (creation of new jobs) economic interest for the most disadvantaged.

Section 2 review some closely related literature on the topic. Section 3 presents the model and describes the relevant equilibrium concept. We analyse the outcome of the tax competition subgame for all given distribution of public goods in Section 4. Section 5 analyses pooling and separating Subgame-Perfect equilibria with symmetric information and asymmetric information. We conclude in section 5.

2. Related Literature

The literature on tax competition can be subdivided into two categories depending on whether the mobile factor is labor or capital.[3] It should be noted that competition for labor is in itself an indirect competition for capital and vice versa because workers migrate not only with their labor power but often with their financial capacity too. However, our review of the literature will focus exclusively on the competition for labor and the optimal taxation of Mirrlees-type income which is directly related to this paper. These models, for the most part, do not take into account the potential power of redistribution generated by government spending on public goods.

One of the first attempts to study the optimal (non-linear) taxation of income in an open economy with free movement of labor is due to Piaser (2007). Considering Rawlsian governments, quasi-linear utility and two types of workers in a strategic tax competition model with asymmetric information, Piaser (2007) finds that the mobility of unskilled has no influence on the equilibrium income tax. He also shows that possibilities for redistribution are reduced by the tax competition. By supposing, in contrast, that the low type agent is also mobile in our model, we allow governments to compete for both types of agents. We show that the introduction of public good into the model of Piaser makes the equilibrium income tax dependent on the mobility of the unskilled. This is an immediate consequence for the possibility of separating equilibrium in our model.

Lipatov and Weichenrieder (2015) analyse a symmetric subgame perfect Nash equilibrium in a tax competition model for imperfectly mobile high skilled and immobile low skilled. They study Rawlsian, Utilitarian and Levianthan social welfare functions and find, in all cases, that tax competition lowers taxes on the high skilled whether information is symmetric or not. Considering Levianthan governments, they find that

3. There is a fairly large literature on tax competition for foreign direct investment in which the mobile factor is one or more firms that choose their location based on the tax incentives offered by governments (see, for instance, Krautheim and Schmidt-Eisenlohr (2011), Haaland and Wooton (1999)).

tax competition increases employment for the low skilled and reduces the distortion from informational asymmetry. Lipatov and Weichenrieder mitigate the tax competition by assuming that the low skill is immobile and by adding a constraint that makes it impossible for governments to set the utility of the workers to an arbitrarily low level. Unlike Lipatov and Weichenrieder (2015), we assume that both types of agents are perfectly mobile and value the public good financed by taxation. We also find that tax competition lower taxes on the high skilled in pooling equilibrium for a given level of public goods. However, there is still a room for subsidies to the poor when governments compete for mobile labor in our framework.

Bierbrauer et al. (2013) use a nonlinear income tax competition model in which governments maximize the average utility of the residents. They find that there is no equilibria in which the lowest skilled agents are subsidized or the highest productive agent pays a positive tax to one country whose utility is larger than the average utility in the other country. In some special cases, they show that it is even possible that the most highly skilled receive a net transfer funded by taxes on lower skilled individuals in equilibrium. By introducing a public good provision, unlike Bierbrauer et al. (2013) we find that redistribution occurs in some equilibrium and positive tax could be paid by highly skilled agents even though their utility is greater than the average utility in the other region.

Lehmann et al. (2014) introduce mobility cost in an optimal nonlinear income tax competition model for Rawlsian government with infinitely many productivity levels and quasilinear preferences. They find a dependance between the shape of the tax schedule and the slope of the semi-elasticity of migration (defined as the percentage change in the mass of taxpayers of a given skill level when their consumption is increased by one unit). This framework is very closed to our model but they do not take the provision of public goods into account. They find that «marginal tax rates may be negative if the semi-elasticity of migration is increasing along the skill distribution» even for the top earners. However, in our paper, most of the results are expressed in terms of total tax liability instead of marginal tax rates and more interestingly there exists no equilibrium in which the top earners pays a negative total tax. In our model, even if there is no direct migration cost, one could imagine that the presence of public goods in different quantities across regions induces an indirect cost of migration. This indirect cost is nothing other than the difference in total benefit derived from the public good between the region of destination and the region of departure. Unlike the model of Lehmann et al. (2014), this implicit cost is determined by the strategic behavior of governments through their policy of public good provision and applies uniformly to all agents residing in the same region.

Our model differs from the one proposed by Poitevin and Gravel (2016) in two respects. First, as opposed to exogenous income, we suppose that the income level of each type of agent is endogenously determined by a trade-of between labour and leisure. Second, in order to avoid the empty community problem, we consider a multi-stage game in which governments decide simultaneously on a level of public goods in the first stage and, after observing the levels of public goods in the economy, each government choose a nonlinear tax schedule.

3. General framework

The model

The economy consists of two regions (jurisdictions) A and B populated by agents with identical preferences over consumption, labor and public good. Here, we describe how interactions take place in the economy.

First, we present the behavior of agents. The utility function of an agent is given by

$$u(c, l, G) = c - v(l) + h(G)$$

where $l \geq 0$ denotes the quantity of labor supplied by the agent, $G \geq 0$ is the amount of public goods available in his region of residence and $c \in \mathbb{R}$ represents his consumption which can be negative or positive. [4]

We assume that h and v are strictly increasing functions and h is strictly concave while v is strictly convex. Both h and v are twice continuously differentiable ($h' > 0, h'' < 0, v' > 0, v'' > 0$), with $v(0) = h(0) = 0$. We also assume that the Inada conditions are

4. Restricting consumption to only positive value would unnecessarily make the model less trackable. One could equivalently assume that the income is big enough so that consumption remain positive for all relevant level of taxes.

satisfied :

$$\lim_{l\to 0} v'(l) = 0 \quad \text{and} \quad \lim_{l\to +\infty} v'(l) = +\infty$$
$$\lim_{G\to 0} h'(G) = +\infty \quad \text{and} \quad \lim_{G\to +\infty} h'(G) = 0.$$

These conditions ensure the existence of a unique interior solution to the consumer's problem. A quasilinear utility function is used to ensure the tractability of the model. Moreover, the separability between l and G ensures that the level of public goods affects only the extensive margin (the choice of the region of residence) as opposed to the intensive margin (the level of labour supply). Furthermore, agents are assumed to be perfectly mobile between the two regions, that is, they can choose their region of residence without cost. [5] The birthplace of the agents is, therefore, irrelevant.

We adopt an extension of the discrete-skill setting of Stiglitz (1982) in which there are n_L low skilled agents with a low productivity w_L and n_H high skilled agents with a high productivity w_H where $0 < w_L < w_H$. As it is usual in the nonlinear optimal tax setup initially proposed by Mirrlees (1971), the market of labor is assumed to be perfectly competitive so that the wage of each agent equates her (marginal) productivity of labour. The agent $i \in \{L, H\}$ before-tax income is, therefore, $y_i = w_i l_i$, and his utility if he lives in region j can be expressed in terms of his gross-income, y_i, his consumption level, c_i, and the public good provision of the region he lives in, G^j:

$$U_i(c_i, y_i, G^j) = c_i - v(y_i/w_i) + h(G^j). \tag{3.1}$$

Second, we present the behavior of governments. We assume that governments can use one unit of the consumption good (or income) to produce at most one unit of public good. So, in each region, a government levies taxes (which may be negative) on the income of agents who reside on its territory to maximize the social welfare through investment in public good and/or direct redistribution. We assume that Governments' objective is *Rawlsian*, that is, the social welfare function of each government is the minimum of the utilities of agents who live within its borders. This is the most redistributive social welfare function because it supposes an infinite aversion for wealth inequality. We adopt the *resident criterion* (Simula and Trannoy, 2012) meaning that

5. Nevertheless, there is an implicit cost from moving from one region to another. Everything being equal, moving from B to A would cost $h(G^A) - h(G^B)$ which can be positive or negative. This endogenous cost is conjointly determined by the level of public goods provided by both region.

governments care only about the well-being of the individuals living on their territory at the end of the game. Governments observe agents' location and their before-tax income $y = w_i l$. We distinguish *symmetric* and *asymmetric* information situations.

When information is symmetric, the government of each region $j \in \{A, B\}$ also observe agents' productivity and deploy a *type-specific tax schedule* $T^j = (t_L^j, t_H^j)$ which requires all i-type agents living in region j to pay t_i^j regardless of their income. Then, the budget constraint of an i-type agent who lives in region j is

$$c_i \leq y_i - t_i^j. \tag{3.2}$$

The consumer's problem consists of chosing the consumption-income bundle (c_i^*, y_i^*) that maximizes (3.1) under (3.2). Its First Order Conditions give the optimal levels of consumption c_i^* and income y_i^* independently of G^j [6]:

$$\begin{cases} c_i^* = y_i^* - t_i^j \\ v'\left(\dfrac{y_i^*}{w_i}\right) = w_i. \end{cases}$$

Therefore, the utility of an i-type agent who decides to reside in region $j \in \{A, B\}$ is:

$$U_i^j = v_i^* - t_i^j + h(G^j)$$

where $v_i^* \equiv y_i^* - v\left(y_i^*/w_i\right)$ is the before-tax utility of agent i due to consumption and labor. From the perspective of governments, when information is symmetric, a type-specific tax schedule $T^j = (t_L^j, t_H^j)$ can be achieved by two type-specific bundles of consumption-income (c_L^j, y_L^j) and (c_H^j, y_H^j) where $y_L^j = y_L^*$, $y_H^j = y_H^*$, $c_L^j = y_L^* - t_L^j$ and $c_H^j = y_H^* - t_H^j$.

On the other hand, when information is asymmetric, agents' productivities are private but governments still know the distribution of productivities in the economy. Then, governments deploy a nonlinear income tax schedule $T^j(y)$ only for agents who decide to live within their borders. The budget constraint of a type-i agent who lives in

6. y_i^* is a strictly dominant income level for every individual of type $i \in \{L, H\}$ regardless of his region of residence, public good level and taxes (here taxes imply no distorsions because they are type specific «head taxes»). And since v' is strictly increasing (because v is convex) we get $\frac{y_H^*}{w_H} > \frac{y_L^*}{w_L}$ and then $y_H^* > y_L^*$.

region $j \in \{A, B\}$ is therefore:

$$c_i \leq y_i - T^j(y_i). \tag{3.3}$$

According to the revelation principle, the government of region j may implement the tax schedule $T^j(.)$ through an indirect mechanism by proposing two bundles of consumption-income (c_L^j, y_L^j) and (c_H^j, y_H^j) such that type-L consumers prefer the first bundle and type-H consumers prefer the second one.[7] Whether information is asymmetric or not, the tax schedule deployed by government j is entirely defined by a 4-dimensional vector $\tau^j = (c_L^j, y_L^j, c_H^j, y_H^j)$. Most importantly, in the case of asymmetric information, τ^j must satisfy the incentive compatibility constraints within region j:

$$c_i^j - v\left(\frac{y_i^j}{w_i}\right) \geq \max_{i' \in \{L,H\}} \left\{c_{i'}^j - v\left(\frac{y_{i'}^j}{w_i}\right)\right\}, \qquad \forall i \in \{L, H\}. \tag{3.4}$$

This assumption garanties the existence of a function $T^j(y)$ such that the bundle of consumption-income (c_i^j, y_i^j) maximise the utility of the i-type at (3.1) under the budget constraint (3.3).

Now we describe how the game proceeds. Governments and agents engage in a three-stage game described as follow. First, each government $j \in \{A, B\}$ announces a level of public good provision $G^j \in [0, \overline{G}]$ where $\overline{G}$ is the highest level of public goods such that the total benefit of acquiring it is equal to its cost, i.e., $(n_L + n_H)h(\overline{G}) = \overline{G}$ with $\overline{G} > 0$.[8] After observing the distribution of public goods announced in the economy, both governments deploy a tax schedule $\tau^j = (c_L^j, y_L^j, c_H^j, y_H^j)$ where $(c_i^j, y_i^j) \in \mathbb{R} \times \mathbb{R}_+$. Finally, agents decide on their region of residence (extensive margin) and their bundle of consumption-income (intensive margin).

A strategy of a region j is a fiscal policy $S^j = (G^j, \tau^j(\cdot,\cdot))$ consisting of an announcement of public good provision G^j and a *contingent tax schedule* $\tau^j(\cdot,\cdot) = (c_L^j(\cdot,\cdot), y_L^j(\cdot,\cdot), c_H^j(\cdot,\cdot), y_H^j(\cdot,\cdot))$ which is a function of the public goods announced by the two governments, (G^A, G^B). Given a pair of fiscal policy S^A and S^B, denote by $U_i^j(S^A, S^B)$ the utility of an i-type agent who decides to live in region j and let $n_i^j(S^A, S^B) \in \mathbb{N}$ be the number of such an agent. If information is symmetric, each agent

7. The type-i agent must prefer (c_i^j, y_i^j) to every other bundle available throughout both regions. A more precise definition of the incentive constraints will be given later.

8. If $G^j > \overline{G}$, then government j would rather not invest in public good and would pursue a purely redistributive policy.

living in region j will choose the bundle of consumption-income intended to him in his region of residence. Therefore, his utility is given by

$$U_i^j(S^A, S^B) = c_i^j(G^A, G^B) - v\left(\frac{y_i^j(G^A, G^B)}{w_i}\right) + h(G^j). \tag{3.5}$$

In the remainder of the paper, the arguments (S^A, S^B) and (G^A, G^B) will be omitted from $U_i^j(S^A, S^B)$, $n_i^j(S^A, S^B)$ and $c_i^j(G^A, G^B)$, $y_i^j(G^A, G^B)$ if there is no risk of confusion.

However, if information is asymmetric, agents could *a priori* choose any bundle of consumption-income available in both regions. An i-type agent chooses the bundle (c_i^j, y_i^j) intended to him in region j only if his self-selection constraint is satisfied, that is, $\forall i \in \{L, H\}$ such that $n_i^j > 0$,

$$c_i^j - v\left(\frac{y_i^j}{w_i}\right) + h(G^j) \geq \max_{(i',j') \in \{L,H\} \times \{A,B\}} \left\{ c_{i'}^{j'} - v\left(\frac{y_{i'}^{j'}}{w_i}\right) + h(G^{j'}) \right\}, \tag{3.6}$$

and then the utility of an i-type living in region j is also given by (3.5). We assume that, when information is asymmetric, governments can only deploy a tax schedule $\tau^j = (c_L^j, y_L^j, c_H^j, y_H^j)$ that is incentive-compatible, given the announced level of public good in its jurisdiction, that is, τ^j must satisfy (3.4).

Let $SWF^j(S^A, S^B, n_L^j, n_H^j)$ be the resulting social welfare in region j. The social welfare of a region depends only on the number of residents and their utilities that depend on the fiscal policies in both regions We assume that regions have extreme aversion for emptiness and budget deficit, that is, $SWF^j = -\infty$ if region j is empty $(n_L^j = n_H^j = 0)$ or its *budget constraint* $(n_L^j(y_L^j - c_L^j) + n_H^j(y_H^j - c_H^j) \geq G^j)$ is violated. If the budget constraint is satisfied in an non-empty region, say j, then its payoff is given by

$$SWF^j(S^A, S^B, n_L^j, n_H^j) = \begin{cases} \min\{U_L^j, U_H^j\} & \text{if} \quad n_L^j > 0 \text{ and } n_H^j > 0 \\ U_L^j & \text{if} \quad n_L^j > 0 \text{ and } n_H^j = 0 \\ U_H^j & \text{if} \quad n_H^j > 0 \text{ and } n_L^j = 0. \end{cases}$$

Equilibrium concept

We are interested in finding the subgame perfect Nash equilibria of the game that are *feasible*, that is, such that the government budget constraint is satisfied in every non-empty region. Doing so, we will avoid trivial equilibria where governments propose an exorbitant level of consumption to agents while violating their budget constraints. To define an equilibrium, it will be important to define in any subset a distribution of agents in which the agents maximize their utility. So, we summarize the optimal reaction of agents to any pair of fiscal policies by the following definition.

Definition 1. *A distribution of agents* $(n_i^j)_{ij}$ *is* compatible *with a pair of fiscal policies* (S^A, S^B) *if agents behave rationally by deciding to live in the region where they find the bundle that maximizes their utility.*

In other terms, a distribution of agents is compatible with (S^A, S^B), if agents migrate optimally, that is,

1) $U_i^A > U_i^B \Rightarrow (n_i^A = n_i \text{ and } n_i^B = 0)$,

2) $U_i^A < U_i^B \Rightarrow (n_i^A = 0 \text{ and } n_i^B = n_i)$,

3) $U_i^A = U_i^B \Rightarrow n_i^A + n_i^B = n_i$.

It is also convenient to distinguish *separating distribution* where agents with different types decide to live in different regions, from *pooling distribution* where all types of agent decide to live in the same region. Note that only the amount of public good proposed by each government (G^A, G^B) and the corresponding taxation policy deployed by those governments $(\tau^A(G^A, G^B), \tau^B(G^A, G^B))$ are necessary for characterizing the agent's optimal migration decision. Now, we are ready to define an equilibrium concept for the game.

Definition 2. *Let* $\widetilde{P} = (\widetilde{S}^A, \widetilde{S}^B)$ *be a pair of fiscal policies where* $\widetilde{S}^j = (\widetilde{G}^j, \widetilde{\tau}^j(\cdot,\cdot))$ *with* $\widetilde{G}^j \geq 0$ *and* $\widetilde{\tau}^j(\cdot,\cdot) = (\widetilde{c}_L^j(\cdot,\cdot), \widetilde{y}_L^j(\cdot,\cdot), \widetilde{c}_H^j(\cdot,\cdot), \widetilde{y}_H^j(\cdot,\cdot))$ *a contingent tax schedule.* $\widetilde{P}$ *is an* equilibrium *if there is a distribution of agents* $(\widetilde{n}_i^j)_{ij}$ *compatible with* $\widetilde{P}$ *such that for all* $j = A, B$ *and* $-j \neq j$, *(1) if region j is non-empty then its government's budget constraint is satisfied, that is, $\widetilde{n}^j_L(\widetilde{y}^j_L - \widetilde{c}^j_L) + \widetilde{n}^j_H(\widetilde{y}^j_H - \widetilde{c}^j_H) \geq \widetilde{G}^j$;*

(2) $\widetilde{S}^j$ is government j's best response to $\widetilde{S}^{-j}$, that is, for all fiscal policy $\widehat{S}^j$, there exists a distribution of agents $(\widehat{n}^j_i)_{ij}$ compatible with $(\widehat{S}^j, \widetilde{S}^{-j})$ such that $SWF^j(\widehat{S}^j, \widetilde{S}^{-j}, \widehat{n}^j_L, \widehat{n}^j_H) \leq SWF^j(\widetilde{S}^A, \widetilde{S}^B, \widetilde{n}^j_L, \widetilde{n}^j_H)$.

(3) for all subgame defined by $(G^A, G^B) \in [0, \overline{G}]^2$, there exists a distribution of agents $(n^j_i)_{ij}$ compatible with (S^A, S^B) where $S^j = (G^j, \widetilde{\tau}^j(\cdot,\cdot))$, such that

(a) the budget constraint associated with $(n^j_i)_{ij}$ is satisfied in every non-empty region for (G^A, G^B),

(b) and, $\widetilde{\tau}^j(G^A, G^B)$ is government j's best response to $\widetilde{\tau}^{-j}(G^A, G^B)$, that is, for all contingent tax schedule $\widehat{\tau}^j(\cdot,\cdot)$, there exist a distribution of agents $(\widehat{n}^j_i)_{ij}$ compatible with $((G^j, \widehat{\tau}(\cdot)), S^{-j})$ such that $SWF^j((G^j, \widehat{\tau}(\cdot)), S^{-j}, \widehat{n}^j_L, \widehat{n}^j_H) \leq SWF^j(S^A, S^B, n^j_L, n^j_H)$.

And, then we say that $\widetilde{P}$ is an equilibrium supported by the distribution of agents $(\widetilde{n}^j_i)_{ij}$.

Definition 2 concerns both cases of symmetric and asymmetric information. The first condition requires that governments keep their promise of public good provision announced in the first stage. However if region j is empty, no promise needs to be kept. The second condition states that there is no *profitable deviation* for any region j, that is, there is no fiscal policy $\widehat{S}^j$ such that, $SWF^j(\widehat{S}^j, \widetilde{S}^{-j}, \widehat{n}^j_L, \widehat{n}^j_H) > SWF^j(\widetilde{S}^A, \widetilde{S}^B, \widetilde{n}^j_L, \widetilde{n}^j_H)$, for all distribution of agents $(\widehat{n}^j_i)_{ij}$ compatible with $(\widehat{S}^j, \widetilde{S}^{-j})$. Otherwise, no credible threat from agents could prevent government j from deviating from $\widetilde{S}^j$. Similarly, the third condition makes sure, that government j has no profitable deviation from the contingent tax schedule $\widetilde{\tau}^j(\cdot,\cdot)$. In other words $(\widetilde{S}^A, \widetilde{S}^B)$ induces a *(Feasible) Nash Equilibrium* in every subgame defined by some $(G^A, G^B) \in [0, \overline{G}]^2$. In the case of asymmetric information, any deviation $\widehat{\tau}^j(\cdot)$ is required to be incentive compatible in order to be implementable by a non linear tax schedule $\widehat{T}^j(y)$ as a function of the before-tax income y. Thus, Definition 2 defines pure strategy subgame perfect Nash equilibria in which public good provision promises are kept in non-empty regions.

All equilibria can be found using the standard backward induction algorithm and considering only the outcomes of the algorithm for which the budget contraint is satisfied in non-empty regions for all subgames. The first stage of the algorithm consists in finding, for all subgames defined by some $(G^A, G^B) \in \mathbb{R}^2_+$, a (feasible) *equilibrium* $(\tilde{\tau}^A(G^A, G^B), \tilde{\tau}^B(G^A, G^B))$ determined in such a way that no *profitable deviation* exists for either government, meaning that the third condition in Definition 2 is met for each region.

4. Equilibrium tax schedules

In this section we consider a subgame defined by a given distribution of public good $(G^A, G^B) \in [0, \overline{G}]$ and we propose some key properties of feasible equilibria within this subgame.

Remark 1. *If information is symmetric, then for all subgame defined by some* $(G^A, G^B) \in [0, \overline{G}]^2$*, taxes are positive in equilibrium.*

Proof: Suppose, by contradiction, that a negative tax is paid by i-type agents in region j at an equilibrium for a subgame defined by some $(G^A, G^B) \in [0, \overline{G}]^2$. By definition of the social welfare function, the utility of agents of type $-i$ with $-i \neq i$ is greater or equal to SWF^j. Since $-i$-type agents are subsidizing i-type agents, in addition to funding the public good, social welfare could be improved in region j by lowering the tax paid by the $-i$-type and getting rid of the i-type agents by imposing a high enough tax on them. Since information is symmetric it is possible to impose an arbitrarily high tax specifically on the i-type agent to get rid of them.[9] That would be a profitable deviation even if this deviation could attract more $-i$-type in region j. ■

Remark 1 states that subsidies are impossible when information is symmetric. This result contrasts with the findings of Poitevin and Gravel (2016) who assume that the taxe schedules and the amount of public good are chosen simultaneously by governments. Indeed, tax competition becomes fiercer when the level of public good is fixed because governments are, in this case, constrained to compete only with the fiscal instrument. As we it is shown later in the paper, Remark 1 does not hold if information is asymmetric.

9. If information is asymmetric, getting rid of the i-type with high enough taxes may not be possible because that move could violate the incentive constraints for the i-type.

Remark 2. *For all subgame defined by some* $(G^A, G^B) \in [0, \overline{G}]^2$*, the government's budget constraint is binding in equilibrium for all non-empty region.*

Proof: Suppose, by contradiction, that in a subgame defined by some $(G^A, G^B) \in [0, \overline{G}]^2$ government j's budget constraint does not bind in some equilibrium. If all agents pay non-negative taxes in j then government j could slightly reduce taxes for all agents in such a way that the budget and the incentive constraints still hold. When information is asymmetric, this deviation could be done by slightly increasing the consumption of each agent in region j without changing their labor supply which would not violate the incentive constraints. This move would increase the utility of every single agent in the region and, thus, increase the social welfare which is defined as the minimum utility in the region. Note that this deviation would be profitable to region j even though it may attract more agents into that region. In particular, if there is a type of agents who pays zero tax at the equilibrium, then, slightly reducing their tax would be tantamount to granting them a small subsidy. In this case, this subsidy should be low enough so that the government's budget constraint is not violated even if all agents of this type migrate toward region j in order to benefit from it.

If at this equilibrium some agents, say i-type, pay negative taxes in j, then two cases need to be distinguished. First, if no i-type agents live abroad, then, as before, a profitable deviation would be to reduce taxes for all agents in such a way that the budget and the incentive constraints still hold. However, if some i-type agents live abroad (which could happen only if they are indifferent between living in region j and living abroad), then slightly increasing the preexisting subsidy for the i-type could be detrimental to the government as more agents of that type could flee in the region. Indeed, such a deviation might attract so much i-type agents into the region that the government would run out of money to subsidize all of them. In that case, a profitable deviation for government j would instead consist of getting rid of the i-type by increasing their tax to the same non-negative tax that $-i$-type would need to pay if they were the only resident of the region, G^j/n_{-i}. That deviation is incentive compatible since it requires every agent to pay the same amount of tax. Furthermore, it is profitable to region j because it would increase the utility of the $-i$-type while chasing the i-type out of the region. That would increase the social welfare in region j since the utility of every remaining resident of region j would have increased. Therefore, in all cases, whenever the constraint budget does not bind in some non-empty region, a profitable deviation exists. ■

Remark 2 is not trivial. In fact, the budget constraint may not be binding in region A if we suppose that agents are not perfectly mobile in which case their birthplaces matter. Budget surplus could appear if we assume, for instance, that agents born in region A are perfectly immobile while those born in region B are perfectly mobile. In that case, If the L-type are subsidized in A the budget constraint may not need to be binding in region A in equilibrium, that is, the H-type agents living in A could be paying more than what is needed to finance the public good G^A and the subsidies, say $-t_L^A$, to the Ls. Further assume that all H-type agents are born in region A and some (perfectly mobile) L-type agents live in region B. It could then be the case that the government of region A cannot afford to pay the same subsidy $-t_L^A$ to all the Ls in the economy without reducing the region's social welfare. In this scenario, the government could excessively tax the H-type without reducing the social welfare as long as the budget surplus is not redistributed to the L-type.

It is important to consider the special case of zero public good, as this case coincide with the standard tax competition model. The following proposition gives some key insight about equilibrium tax schedules and the distribution of agents when public good provision is not allowed.

Proposition 1. *Equilibria in the subgame defined by $G^A = G^B = 0$ exhibit the following properties:*

(1) no tax (or subsidy) is paid, and

(2) all L-type agents live in the same region.

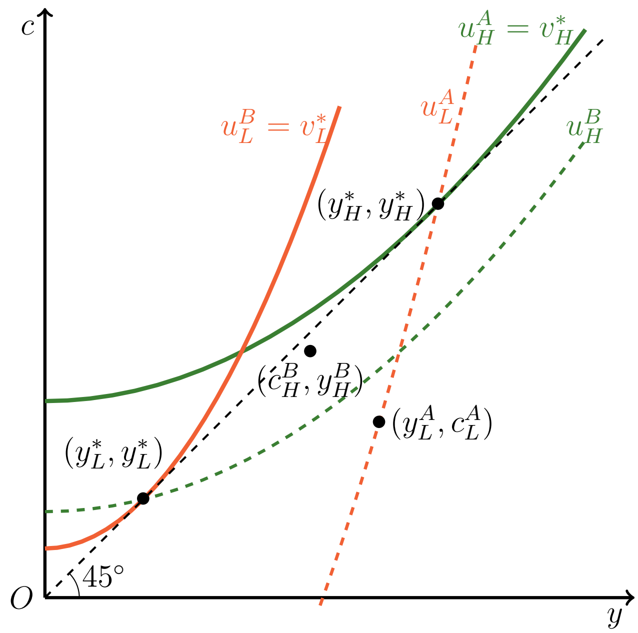
Proof: See Appendix. ■

Proposition 1 is, in part, in line with the findings of Bierbrauer et al. (2013): in the absence of a public good, there is no equilibrium in which the H-type pay a positive tax. However, unlike Bierbrauer et al. (2013), there does not exist an equilibrium in which the H-type receives subsidies either. This difference is due to the Rawlsian criterion that we have adopted which is infinitely more redistributive than the utilitarian criterion used by Bierbrauer et al. (2013). Indeed, if information is asymmetric, then the incentive constraints imply that the H-type agents earn higher utility than the L-type. Therefore, trying to subsidize the poor through a positive tax on the rich would push the latter the flee out of the country where there would pay no tax. Reciprocally, since the governments care only about the well-being of the least fortunate, there is no point in getting the rich financed by the poor in order to keep them in the region. This argument applies also when information is symmetric.In fact, a government that finances its public good and subsidizes the H-type would create high inequalities between the H-type and the L-type with the latter getting the least utility. It could get rid of the H-type and keep financing the same amount of public good with lower taxes on the L-type. That would be the best thing to do since only the well-being of the poor matters.

When zero tax is paid, there is no point in distorting the labour supply of the residents. The H-type agents would therefore be the most fortunate with a utility of v_H^*. So, when there is no public good provision, government would compete by trying to keep the H-type inside their region and the L-type outside. As a result of this competition, one country will end up with all the poor while the other country will get some (possibility all or none) of the rich.

Figure 1 illustrates the equilibrium when public good provision is zero in both region. The pair of tax schedules $(\boldsymbol{\tau}^A, \boldsymbol{\tau}^B)$, with $\boldsymbol{\tau}^A = (c_L^A, y_L^A, y_H^*, y_H^*)$ and $\boldsymbol{\tau}^B = (y_L^*, y_L^*, c_H^B, y_H^B)$, is depicted on the figure. u_i^j denotes the utility of i-type agents who live in region j if any. $(\boldsymbol{\tau}^A, \boldsymbol{\tau}^B)$ is an equilibrium (whether information is symmetric or not) that is compatible with a distribution of agents $(n_i^j)_{ij}$ such that $n_H^A = n_H$ and $n_L^B = n_L$. No taxes are paid and production/income is efficient for each type so none of them would benefit from fleeing abroad: $v_L^* > u_L^A$ and $v_H^* > u_H^B$. Moreover, $\boldsymbol{\tau}^A$ and $\boldsymbol{\tau}^B$ are incentive compatible. If B offers more to Hs than v_H^*, B needs to subsidize Hs via a positive tax on the Ls. B would then end up with a lower social welfare. In particular, if $(c_H^B, y_H^B) = (y_H^*, y_H^*)$, then $(\boldsymbol{\tau}^A, \boldsymbol{\tau}^B)$ is an equilibrium supported by any repartition of agents where all the Ls live in region B. Thus, an equilibrium with some rich people living in both region would then be possible. However, according to Proposition 2, this kind of agent distribution is impossible if the public good provision is non-zero in at least one region.

Figure 1. Equilibrium with zero public good $\left((c_L^A, y_L^A, y_H^*, y_H^*), (y_L^*, y_L^*, c_H^B, y_H^B)\right)$



Proposition 2. *For all subgame defined by some* $(G^A, G^B) \in [0, \overline{G}]^2 \setminus (0, 0)$, *there does not exist a type of agents living in both regions in equilibrium.*

Proof: See Appendix. ■

A crucial assumption underlying Proposition 2 is the lack of congestion in the use of public goods. Thus, the individual benefit of a public good does not diminish when the government accept more taxpayers. So, whenever possible, it is always profitable for a government to get more agents which belong to a type that pays a positive tax or to get rid of a type of agents that pay negative tax. If an agent type is present in both regions, then a slight modification of their tax liabilities in one region could make them all flee out or come in. If we further assume that public good provision is non-zero in at least one region, some agents would have to pay positive tax in that region. Also, note that this proposition would not hold if agents of the same type have different tastes for public good, that is, if h is specific to agents.

Proposition 2 allows us to focus only on a very specific range of Nash equilibria which are: *separating equilibrium* where agents with different types decide to live in different regions and *pooling equilibrium* where all types of agents decide to live in the same region. Note that the use of terms «separating equilibrium» and «pooling equilibrium» is non-standard. In this paper, these terms apply even in the case of symmetric information, and rather refer to the choice of agents on the extensive margin. So, a pooling equilibrium occurs if all types of agents choose the same region of residence in equilibrium regardless of whether they select the same consumption-income bundle or not. Similarly, an equilibrium is separating if agents of different types choose different regions of residence. Proposition 3 and 4 give necessary and sufficient conditions for the existence of both types of equilibrium.

Proposition 3. *For all subgame defined by some* $(G^A, G^B) \in [0, \overline{G}]^2$*, a pooling equilibrium always exists. Agents pool in region* $j = A, B$*, such that*

$$(n_L + n_H)h(G^j) - G^j \geq (n_L + n_H)h(G^{-j}) - G^{-j}, \tag{4.1}$$

with $-j \neq j$*.*

Proof: See Appendix. ■

The expression $(n_L + n_H)h(G^j) - G^j$ represents the total net benefit of the public good provided in region j if it is home to all types of agents. Proposition 3 states that a pooling equilibrium always exists, and it only exists in the region that has the highest potential total net benefit of public good. The existence of a Nash equilibrium in all subgames is a crucial result because it allows us to design subgame perfect Nash equilibria using backward induction. Indeed, no subgame perfect Nash equilibria would exist if there exists one subgame that does not admit an equilibrium.

If information is asymmetric, it is convenient to represent tax schedule in the space utility-income $(u_L^j, u_H^j, y_L^j, y_H^j)$ where $u_i^j = c_i^j - v(y_i^j/w_i)$. Consider a pooling equilibrium where agents pool in region A. That region plays the best incentive compatible allocation $(u_L^A, u_H^A, y_L^A, y_H^A)$ subject to the self-selection constraints. Characterizing a pooling equilibria requires, therefore, that we consider the problem of

Boadway and Keen (1993) that gives the maximum social welfare in such an equilibrium, $u_L^A = u_L(u_H^B - (h(G^A) - h(G^B)), G^A)$, where the function u_L is defined for all $(u_H, G) \in \mathbb{R} \times \mathbb{R}_+$ by:

$$u_L(u_H, G) = \max_{(c_L, y_L, c_H, y_H)} c_L - v(y_L/w_L)$$
$$s.t. \begin{cases} n_L(y_L - c_L) + n_H(y_H - c_H) \geq G & (BC) \\ c_L - v(y_L/w_L) \geq c_H - v(y_H/w_L) & (IC_L) \\ c_H - v(y_H/w_H) \geq c_L - v(y_L/w_H) & (IC_H) \\ c_H - v(y_H/w_H) \geq u_H & (PC) \end{cases} \quad (4.2)$$

In equation 4.2, (BC) represents the budget constraint. (IC_L) and (IC_H) are the incentives constraint respectively for the Ls and the Hs. (PC) is the participation constraint for H that require the utility of the Hs to be greater than the utility they would get if they flee abroad. It is important to note that, in a pooling equilibrium, the participation constraint for the L must also be satisfied: $u_L^A \geq u_L^B - (h(G^A) - h(G^B))$. We assume that $u_L(u_H, G) = \infty$, if u_H is so high that the constraints of the Boadway and Keen's problem define the empty set.

Let $u_H^M(G)$ be the lowest level of u_H for which the constraint (PC) is binding and define $u_L^M(G)$ by the corresponding utility level: $u_L^M(G) \equiv u_L(u_H^M(G), G)$.[10] In every feasible pooling equilibrium the utility of H-type in A must be at least equal to $u_H^M(G^A)$, so that, the government could grant the best incentive compatible utility level for type-L agent. As it will be clearer in the next section, a vast range of pooling equilibria may exist in a subgame. When information is asymmetric, the maximum utility $u_L^M(G^j)$ that the L could get in a pooling equilirium in region j is found by solving the Mirrlees'

10. $u_L^M(G)$ and $u_H^M(G)$ are the highest incentive-compatible utility levels for type L and H respectively that can be implemented in autarky if their numbers are n_L and n_H respectively. We have :

$u_L^M(G) = \frac{n_L v_L^M + n_H v_H^* + n_H [v(\frac{y_L^M}{w_H}) - v(\frac{y_L^M}{w_L})] - G}{n_L + n_H}$ and $u_H^M(G) = \frac{n_L [y_L - v(\frac{y_L^M}{w_H})] + n_H v_H^* - G}{n_L + n_H}$

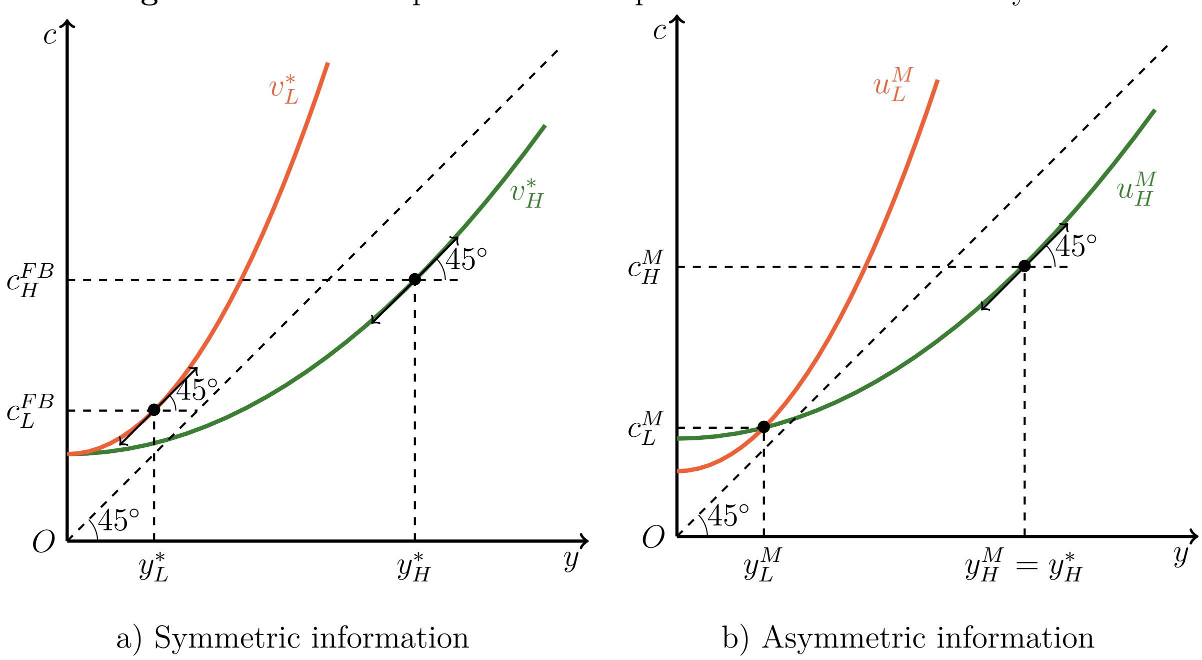
where $v_L^M = y_L^M - v(\frac{y_L^M}{w_L})$ and $y_L^M \in]0, y_L^*[$ is defined by $\frac{1}{w_L} v'(\frac{y_L^M}{w_L}) = \frac{n_L}{n_L + n_H} + \frac{n_H}{n_L + n_H} \times \frac{1}{w_H} v'(\frac{y_L^M}{w_H})$.

optimal taxation problem in a closed economy:

$$u_L^M(G^j) \;=\; \max_{(c_L, y_L, c_H, y_H)} c_L - v(y_L/w_L)$$
$$s.t. \begin{cases} n_L(y_L - c_L) + n_H(y_H - c_H) \geq G^j & (BC) \\ c_L - v(y_L/w_L) \geq c_H - v(y_H/w_L) & (IC_L) \\ c_H - v(y_H/w_H) \geq c_L - v(y_L/w_H) & (IC_H). \end{cases} \tag{4.3}$$

Figure 2 illustrates the solution $(c_L^M, y_L^M, c_H^M, y_H^M)$ for (4.3). Proposition 4 gives necessary and sufficient conditions for the existence of a separating equilibrium.

Figure 2. Mirrlees' optimal taxation problem in a closed economy.



Proposition 4. *If information is symmetric (resp. asymmetric), then for all subgames defined by some* $(G^A, G^B) \in [0, \overline{G}]^2$*, for* $j = A, B$*, a separating equilibrium supported by* $n_L^j = 0$ *and* $n_H^j = n_H$ *exists if and only if*

$$U_H^j \geq \max\{v_L^* + h(G^j), v_H^* + h(G^{-j})\} \tag{4.4}$$

$$(resp.\ U_H^j \geq \max\{u_L^M(G^j) + h(G^j), v_H^* - \delta + h(G^{-j})\}\ and \tag{4.5}$$

$$v(y_L^*/w_L) - v(y_L^*/w_H) \leq \Delta v + \Delta h + G^B/n_L - G^A/n_H \leq v(y_H^*/w_L) - v(y_H^*/w_H)), \tag{4.6}$$

$$where\ U_H^j = v_H^* - \frac{G^j}{n_H} + h(G^j)\ and\ \delta = v_H^* - u_L^{-1}(v_L^* - \tfrac{G^{-j}}{n_L}; G^{-j}) \geq 0.$$

Proof: See Appendix. ■

In a separating equilibrium supported by $n_L^j = 0$ and $n_H^j = n_H$, the utility of Ls and Hs are, respectively,

$$U_L^{-j} = v_L^* - \frac{G^{-j}}{n_L} + h(G^{-j})$$

$$U_H^j = v_H^* - \frac{G^j}{n_H} + h(G^j).$$

Proposition 4 states that in a separation equilibrium the utility of the H-type U_H^j must be not only greater than the maximum utility that they could have within a pooling distribution of agents in $-j$, but also, higher than the maximum utility which Ls could have within a pooling distribution of agents in j. Equations (4.4) and (4.5) are respectively equivalent to

$$\frac{G^j}{n_H} \leq \min\{h(G^j) - h(G^{-j}), v_H^* - v_L^*\} \tag{4.7}$$

$$\text{and, } \frac{G^j}{n_H} \leq \min\{h(G^j) - h(G^{-j}) + \delta, v_H^* - u_L^M(G^j)\}. \tag{4.8}$$

If information is symmetric, Equation (4.7) makes it clear that, in all separating equilibrium, the H-type agents live in the region with the highest provision of public good. Furthermore, the utility of type-L agents cannot be greater than that of agents of type H since (4.7) implies

$$\frac{G^j}{n_H} \leq h(G^j) - h(G^{-j}) + v_H^* - v_L^* + \frac{G^{-j}}{n_L}$$

which is equivalent to $U_L^{-j} \leq U_H^j$. In addition, if there exists a separating equilibrium where the H-type agents live in region j, then there also exists a pooling equilibrium in region j because

$$\frac{G^j}{n_H} \leq h(G^j) - h(G^{-j}) \Rightarrow (n_L + n_H)h(G^j) - G^j \geq (n_L + n_H)h(G^{-j}) - G^{-j}.$$

However, this implication does not hold in the case of asymmetric information.

Equation (4.6) guarantees that the self-selection constraints are satisfied in the case of asymmetric information. In addition, when information is asymmetric, due to the incentive and the self-selection constraints, H-type agents earn more utility that L-type in all equilibria. However, in this case, even if $U_L^{-j} \leq U_H^j$, the Hs could live in the region with the lowest public good provision in some separating equilibria. For instance, if $G^j = 0$, then equation (4.8) becomes

$$0 \leq \min\{-h(G^{-j}) + \delta, v_H^* - u_L^M(0)\}. \tag{4.9}$$

h could be defined is such a way that $\delta > h(G^{-j})$ and since $v_H^* > u_L^M(0)$, equation (4.9) would be satisfied. Figure 3 illustrate how δ is determined. δ is the difference between v_H^* and $u_L^{-1}(v_L^* - \frac{G^{-j}}{n_L}; G^{-j})$. The latter represents the maximal pre-public-good utility that a government spending G^j can grant to the Hs if it is required to ensure at least a pre-public-good utility of $v_L^* - \frac{G^{-j}}{n_L}$ to the Ls.

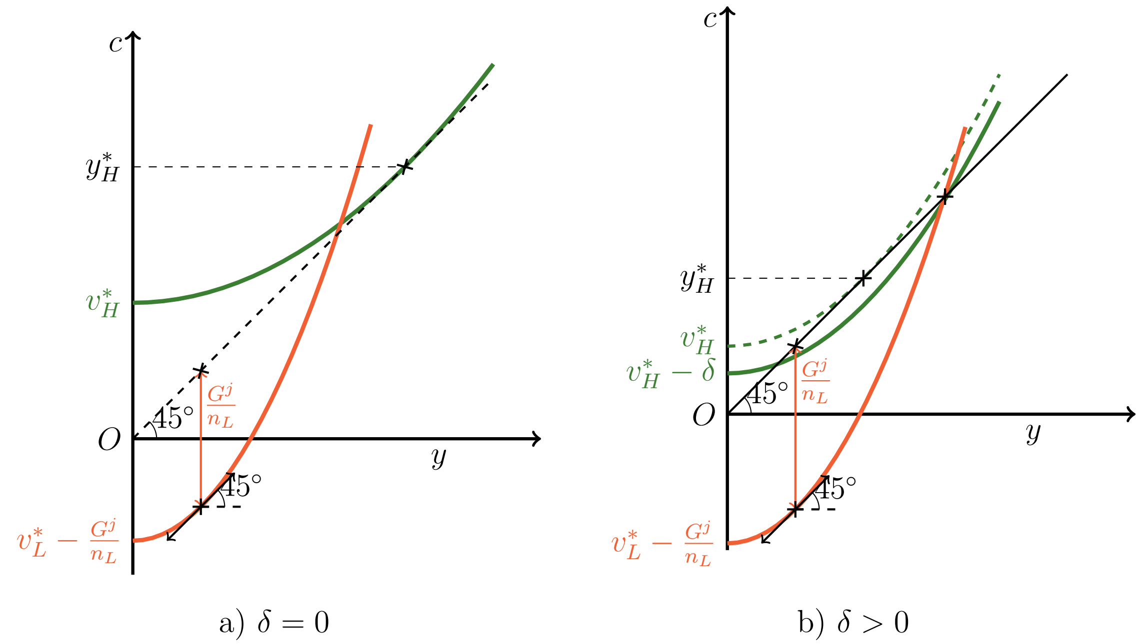
In what follows, we examine feasible subgame perfect Nash equilibria in both cases of symmetric and asymmetric information.

5. Subgame-Perfect Nash Equilibria

The previous section has presented necessary and sufficient conditions for the existence of an equilibrium in all subgames defined by a distribution of public good. Here, we study the existence of a subgame perfect equilibrium in the full game. The results are quite different depending on whether information is symmetric or asymmetric. We study both cases separately.

We may also need a benchmark to assess the impact of tax competition on the level of public good provision, the distribution of agents and the amount of redistribution (taxes). A natural benchmark consists of the solution of the problem of an inter-regional central benevolent planner who pursues a Rawlsian objective. The central planner would chose a pooling distribution of agents because for any allocation of public goods and agents

Figure 3. Illustation of $\delta = v_H^* - u_L^{-1}(v_L^* - \frac{G^{-j}}{n_L}; G^{-j})$.



between regions, merging the public goods provided by the two regions together and pooling agents in a single region without changing their tax liabilities is always a better feasible allocation. So, the optimal level of public good in the absence of tax competition according to the Samuelson condition is G_{LH}^*, defined by $h'(G_{LH}^*) = 1/(n_L + n_H)$.

Efficiency is achieved when the Mirrlees' solution is implemented (see figure 2 at page 39). In the case of asymmetric information the utility of the L-type and the H-type would be respectively $u_L^M(G_{LH}^*) + h(G_{LH})^*$ and $u_H^M(G_{LH}^*) + h(G_{LH}^*)$. When information is symmetric, the planner's problem boils down to

$$(c_L^{FB}, y_L^{FB}, c_H^{FB}, y_H^{FB}) = \underset{(c_L, y_L, c_H, y_H)}{\arg\max} \min\{c_L - v(y_L/w_L), c_H - v(y_H/w_H)\} \quad s.t. \left\{ n_L(y_L - c_L) + n_H(y_H - c_H) \geq G_{LH}^*. \right. \tag{5.1}$$

Therefore, $y_i^{FB} = y_i^*, \forall i \in \{L, H\}$ and (c_L^{FB}, c_H^{FB}) is determined by the system,

$$\begin{cases} n_L(y_L^* - c_L^{FB}) + n_H(y_H^* - c_H^{FB}) = G_{LH}^* \\ c_L^{FB} - v(y_L^*/w_L) = c_H^{FB} - v(y_H^*/w_H). \end{cases} \tag{5.2}$$

This benchmark is therefore identical to a closed economy consisting of all the Ls and the Hs governed by a social planner using a max-min criterion.

5.1. Symmetric information

Here, we suppose that governments can observe agents' productivities. The sole relevant constraint faced by each government is its budget constraint:

$$n_L^j t_L^j + n_H^j t_H^j \geq G^j, \quad \forall j \in \{A, B\}$$

where $t_i^j \equiv y_i^j - c_i^j$ is a type i agent's tax liability if he decides to be a resident of region j. The pair of type-specific taxes (t_L^j, t_H^j) completely defines the tax schedule of government j. Since information is symmetric, governments can impose a lump-sum type-specific tax. This implies that labour supply is efficient, and hence, $y_i^j = y_i^*$ for all $i \in \{L, H\}$ and $j \in \{A, B\}$. A strategy of each government j is a choice of a level of public good provision G^j in the first stage and a type-specific tax liability $(t_L^j(G^A, G^B), t_H^j(G^A, G^B))$ contingent on the distribution of public good in the second stage. We are interested in the subgame perfect equilibria of this game. Define $\Delta v \equiv v_H^* - v_L^*$ and $\Delta h \equiv h(G^A) - h(G^B)$.[11]

Lemma 1 (Maximal taxes in pooling equilibria)**.** *Let $(G^A, G^B) \in [0, \overline{G}]^2$ define a subgame and $j \in \{A, B\}$ such that $(n_L + n_H)h(G^j) - G^j \geq (n_L + n_H)h(G^{-j}) - G^{-j}$. If information is symmetric, for all tax schedule (t_L^j, t_H^j) such that $n_L t_L^j + n_H t_H^j = G^j$, there exists a tax schedule (t_L^{-j}, t_H^{-j}) such that $(t_L^j, t_H^j, t_L^{-j}, t_H^{-j})$ is a pooling equilibrium of the subgame (G^A, G^B), if and only if :*

$$t_i^j \leq \min\{\frac{G^j}{n_i}, \frac{G^{-j}}{n_i} + \Delta h\}, \quad \forall i \in \{L, H\}. \tag{5.3}$$

11. While Δh could be positive or negative, Δv is strictly positive because $\forall y \geq 0, v_H^* \geq y - v(\frac{y}{w_H}) > y - v(\frac{y}{w_L})$, and, specifically, $v_H^* > y_L^* - v(\frac{y_L^*}{w_L}) = v_L^*$.

For instance, $t_i^{-j} = t_i^j - \Delta h$ *for all* $i \in \{L, H\}$.

Proof: See Appendix. ∎

By Lemma 1, for all pairs $(G^A, G^B) \in [0, \overline{G}]^2$, if $G^A \leq G^B + (n_L + n_H)\Delta h$ then the maximal amount of tax that can be paid by an i-type agent and his minimal utility in a pooling equilibrium are respectively :

$$\bar{t}_i(G^A, G^B) \equiv \min\{\frac{G^A}{n_i}, \frac{G^B}{n_i} + \Delta h\}, \quad \forall i \in \{L, H\}$$

and

$$\underline{u}_i(G^A, G^B) = v_i^* + h(G^A) - \bar{t}_i(G^A, G^B), \quad \forall i \in \{L, H\}.$$

For all $i \in \{L, H\}$, define G_i^* by $h'(G_i^*) \equiv 1/n_i$, that is, the Samuelson optimal quantity of public good in a separating equilibrium. For each type $i \in \{L, H\}$, we have

$$\begin{aligned}
\underline{u}_i(G^A, G^B) &= \max\{v_i^* + h(G^A) - G^A/n_i,\ v_i^* + h(G^A) - G^B/n_i - h(G^A) + h(G^B)\} \\
&= \max\{v_i^* + h(G^A) - G^A/n_i,\ v_i^* + h(G^B) - G^B/n_i\} \\
&\leq v_i^* + h(G_i^*) - G_i^*/n_i
\end{aligned}$$

Then $\min\{\underline{u}_L(G^A, G^B), \underline{u}_H(G^A, G^B)\} \leq v_i^* + h(G_i^*) - G_i^*/n_i$, for all $i \in \{L, H\}$. Thus, the worst possible social welfare in a pooling equilibrium is not better than the social welfare in a separating equilibrium in each region when the Samuelson optimal quantities are implemented. Therefore a strategic profile can be constructed in such a way that region A and B end up respectively with social welfares $v_H^* + h(G_H^*) - G_H^*/n_H$ and $v_L^* + h(G_L^*) - G_L^*/n_L$ if they do not deviate from their respective strategy in a separating equilibrium. And for all deviation from (G_H^*, G_L^*) in the first stage, taxes are chosen in the second stage such that the social welfare reached by the regions is at most

$$\min\{\underline{u}_L(G^A, G^B), \underline{u}_H(G^A, G^B)\},$$

that is, the worst possible social welfare in a pooling equilibrium for a given pair of (G^A, G^B) which is less than $v_i^* + h(G_i^*) - G_i^*/n_i$ for all $i \in \{L, H\}$.

Proposition 5. *A separating subgame perfect Nash equilibrium with symmetric information exists if:*

$$\frac{G_H^*}{n_H} \leq \min\{\Delta v, h(G_H^*) - h(G_L^*)\}. \tag{5.4}$$

Proof: See Appendix. ■

Proposition 5 gives a sufficient condition for the existence of a separating subgame perfect equilibrium. This result contrasts with that of Poitevin and Gravel (2016) in which only pooling equilibria exist. Indeed, allowing governments to chose their public good provision before deploying a tax schedule gives room to less fierce tax competition. Note that condition (5.4) is not satisfied if $n_L > n_H$ which imply $h(G_H^*) - h(G_L^*) < 0$. According to Proposition 4, if information is symmetric, (5.4) is also a necessary condition for the existence of a separating subgame perfect equilibrium in which efficient level of public good is used in every region. Taxes paid by the Hs and the Ls in such an equilibrium are respectively

$$t_H = \frac{G_H^*}{n_H} \text{ and } t_L = \frac{G_L^*}{n_L}.$$

However, sub-optimal level of public good may also be played in a separating subgame perfect equilibrium under some conditions, even if $n_L > n_H$. The following proposition entirely characterizes taxes that could be paid in a subgame perfect pooling equilibrium.

Proposition 6. *If information is symmetric, a pair of taxes $\widetilde{t}_L$ are $\widetilde{t}_H$ are paid, respectively by the L-type and the H-type, in a subgame perfect pooling equilibrium if and only if*

— *$\widetilde{t}_L$ are $\widetilde{t}_H$ are both non-negative, and*

— $n_L\widetilde{t}_L + n_H\widetilde{t}_H = G_{LH}^*$.

Proof: See Appendix. ■

This proposition states that the empty region may exercise a credible threat that would prevent the other region from deviating from any non-negative pair of taxes that is big enough to finance G_{LH}^* when information is symmetric. Therefore, Proposition 6 allows for a wide range of pooling subgame perfect equilibria. If Δv is sufficiently low, taxes on the H-type agents could be so high that they would end up with lower utility than the L-type. This situation is not possible when information is asymmetric.

5.2. Asymmetric information

We now assume that governments cannot observe agents' productivity. Therefore, in addition to the budget constraint, governments face incentive constraints. Consider a given distribution of public good provision $(G^A, G^B) \in [0, \overline{G}]^2$. The strategy of the government of region j in the second stage can be represented by a 4-tuple $(u_L^j, u_H^j, y_L^j, y_H^j) \in \mathbb{R}^2 \times \mathbb{R}_+^2$ where

$$u_i^j = c_i^j - v(\frac{y_i^j}{w_i}).$$

The following lemma gives, for each type, the minimal utility that can be generated in a pooling equilibrium with asymmetric information.

Lemma 2 (Minimal utilities in pooling equilibria)**.** *Let (u_L^A, u_H^A) be a pair of utilities such that $u_L(u_H^A, G^A) = u_L^A$ and $u_H^A \geq u_H^M(G^A)$. There exists two levels of positive income $(y_L^A, y_H^A) \in \mathbb{R}_+^2$ such that $(u_L^A, u_H^A, y_L^A, y_H^A)$ is played by A in a feasible pooling nash Equilibrium in A with asymmetric Information in the subgame defined by $(G^A, G^B) \in [0, \overline{G}]^2$, if and only if*

$$u_H^A \geq v_H^* - G^B/n_H - \Delta h$$
$$u_L^A \geq v_L^* - \min\left\{G^A/n_L, G^B/n_L + \Delta h\right\}.$$

Proof: See Appendix. ■

Denote by τ_L^A the difference between the utility that a type-L agent would have if he does not pay any tax, all things being equal, and his actual utility: $\tau_L^A = v_L^* - u_L^A$. This difference can be considered as the *tax burden* borne by an agent of type L in a Nash equilibrium. It can be broken down into a real component t_L^A collected by the government and a fictitious component $v_L^* - (y_L^A - v(y_L^A/w_L))$ representing the distortion generated by the fiscal policy:

$$\tau_L^A \quad = \quad t_L^A \quad + \quad v_L^* - \left(y_L^A - v(y_L^A/w_L)\right).$$

Lemma 2 states that the tax burden for a L-type agent cannot exceed $\min\{G^A/n_L, G^B/n_L + \Delta h\}$ in a feasible pooling Nash equilibrium. This is analogous to the result presented in Lemma 1 for the case of symmetric information where the fictitious tax is zero. Since $u_L^A \geq v_L^* - G^A/n_L$ and $u_L(u_H^A, G^A) = u_L^A$ implies $y_H^A - c_H^A \geq 0$, a direct consequence of Lemma 2 is that no negative tax is paid by an H-type agent in a feasible pooling Nash equilibrium. However, in some cases, especially if $v_H^* - G^A/n_H > u_H^A \geq \max\left\{u_H^M(G^A), v_H^* - G^B/n_H - \Delta h\right\}$, L-type agents are subsidized. Lemma 2, states, in particular, that when $G^A = G^B = 0$, no distorsion occurs in a feasible pooling equilibrium, that is, $u_i^A = v_i^*$ for all i. This result is consistent with the no-tax policy for zero public good provision stated in Proposition 1 and also with the one found in the model of Piaser (2007) when transportation costs are low enough. However, as Bierbrauer et al. (2013) showed through an example, if governments use rather a utilitarian objective, that is, they seek to maximize the average utility of their residents, redistribution may occur even if there is no investment in public good. Notice that, here, migration costs are given by Δh and are zero when $G^A = G^B$. Lemma 2 will be useful for finding both separating and pooling subgame perfect equilibria with asymmetric information.

From Lemma 2, the minimal level of Social Welfare that can be reached in a feasible pooling equilibrium in A is:

$$\begin{aligned} \underline{SWF}^A &= v_L^* - \min\{\tfrac{G^A}{n_L}, \tfrac{G^B}{n_L} + \Delta h\} + h(G^A) \\ &= v_L^* + \max\{h(G^A) - \tfrac{G^A}{n_L}, h(G^B) - \tfrac{G^B}{n_L}\} \\ &= \max\{v_L^* + h(G^A) - \tfrac{G^A}{n_L}, v_L^* + h(G^B) - \tfrac{G^B}{n_L}.\} \end{aligned}$$

Then,

$$\underline{SWF}^A \leq v_L^* + h(G_L^*) - \frac{G_L^*}{n_L} \leq v_H^* + h(G_H^*) - \frac{G_H^*}{n_H}.$$

Therefore, it is possible to built a feasible subgame perfect separating Nash equilibrium where regions' payoffs are $v_L^* + h(G_L^*) - \frac{G_L^*}{n_L}$ and $v_H^* + h(G_H^*) - \frac{G_H^*}{n_H}$ and any deviation from the equilibrium path is punished by a payoff of $\underline{SWF}^A$. Therefore, following proposition immediately follows from Proposition 4.

Proposition 7. *A feasible subgame perfect separating equilibrium with asymmetric information exists if*

$$\begin{cases} \dfrac{G_H^*}{n_H} \leq \min\{h(G_H^*) - h(G_L^*) + \delta^*, v_H^* - u_L^M(G_H^*)\} \\ v(\dfrac{y_L^*}{w_L}) - v(\dfrac{y_L^*}{w_H}) \leq \Delta v + \Delta h^* + \dfrac{G_L^*}{n_L} - \dfrac{G_H^*}{n_H} \leq v(\dfrac{y_H^*}{w_L}) - v(\dfrac{y_H^*}{w_H}) \end{cases}$$

where $\Delta h^* = h(G_H^*) - h(G_L^*)$ *and* $\delta^* = v_H^* - u_L^{-1}(v_L^* - \frac{G_L^*}{n_L}; G_L^*) \geq 0.$

It is clear from Proposition 7 that a feasible subgame perfect separating equilibrium may not exist if, for instance, Δh^* is too high. According to Proposition 4, the equilibrium utility of the Hs, $\widetilde{U}_H$, satisfies

$$\widetilde{U}_H \geq \max\{u_L^M(G_H^*) + h(G_H^*), v_H^* - \delta^* + h(G_L^*)\}. \tag{5.5}$$

The next proposition presents our main result. It characterizes utilities for both types in a pooling subgame perfect equilibrium.

Proposition 8. *If information is asymmetric, there always exists a subgame perfect pooling equilibrium with a pair of utilities* $\widetilde{U}_L$ *and* $\widetilde{U}_H$ *are earned by the L-type and the H-type respectively, such that,*

$$\widetilde{U}_L = u_L\Big(\widetilde{U}_H - h(G_{LH}^*), G_{LH}^*\Big) + h(G_{LH}^*), \tag{5.6}$$

$$\widetilde{U}_L \geq v_L^* - \frac{G_{LH}^*}{n_L} + h(G_{LH}^*), \tag{5.7}$$

$$\text{and } \widetilde{U}_H \geq u_H^M(G_{LH}^*) + h(G_{LH}^*). \tag{5.8}$$

Proof: See Appendix. ■

A feasible pooling subgame perfect Equilibrium exists also in the case of asymmetric information. For instance, since $u_L\Big(u_H^M(G_{LH}^*), G_{LH}^*\Big) = u_L^M(G_{LH}^*) > v_L^* - \frac{G_{LH}^*}{n_L}$ the solution to the Mirrlees's problem is sustainable in Equilibrium. Unlike the case of symmetric information, for some values of the parameters, the L-type may be subsidized. This could happen especially when $v_H^* - \frac{G_{LH}^*}{n_H} > u_H^M(G_{LH}^*)$. Indeed, if information is asymmetric, the government may not be able to get rid of the L-type agents even though they pay negative tax. In fact the amount of tax that is necessary to get rid of them could be so high that it would violate their incentive constraint. So, this would prevent the government from any deviation from the negative tax upon the L-type agents.

Furtermore, (5.8) shows that the utility of the H-type agents in a pooling equilibrium is higher than the utility that they would earn in autarky for the same level of public good. Loosely speaking redistribution becomes harder under tax competition even though in some case the L-type may be subsidized. In other words, the L-type cannot receive more subsidies (or pay less taxes) under tax competition than they would in autarky. Therefore, tax competition lower taxes on the high skilled in pooling equilibrium compared to an autarky that have the same level of public spending. Loosely speaking asymmetric information makes tax competition less fierce.

Unsurprisingly, as compared to the benchmark, tax competition curbs redistribution efforts since in all equilibrium the scale of possible redistribution is always less or equal to what can be achieved in autarky. However, the introduction of public good increases the possibility of redistribution. Moreover, when public good provision can be announced before the tax scheduled the empty community problem may be avoided, under some conditions, by the existence of separating equilibrium. Table 1 summarizes the possible taxes in equilibrium depending on whether information is symmetric or asymmetric and whether public good provision is allowed or not.

Table 1. Possible taxes in equilibrium when Rawlsian governments compete

	Without public good	With public good
Symmetric information	no tax is paid	non-negative taxes are paid
Asymmetric information	no tax is paid	L-type may be subsidized

6. Conclusion

Perfect labor mobility fatally undermines efforts to redistribute from the rich to the poor as pointed out by Bierbrauer et al. (2013). In order to circumvent the perverse effects of the free movement of people, this paper introduces into a model of tax competition the possibility for each government to attract productive agents by using public goods in addition to tax incentives. This has the effect of mitigating the race-to-the-bottom thanks to the appearance of an implicit cost of migration due to the differential of public good between the regions. As a result, in order to maximize a social welfare function, governments' strategy consists in announcing a level of investment in public infrastructure before deploying a policy of redistribution of wealth which includes the financing of government spending. Thus, we have been able to show that no negative tax can be paid by the most productive agents. What is more, a negative tax can be levied by the least productive agents when information is asymmetric. Thus, redistribution becomes possible directly through subsidies and indirectly via the provision of public good.

Different extensions of the model are possible. In fact, in our framework, only two configurations are possible at equilibrium whenever the quantity of public good available in the regions is not uniformly zero: the separating equilibrium and the pooling equilibrium (see Proposition 2). In order to allow richer classes of equilibria, one could modify the model either by introducing congestion in the public good, or by introducing a heterogeneity of agents as regards their preferences for the public good. In the event of congestion, adding a user would lower the benefit to all agents who use the public good. Each type of agent would therefore continue to migrate to the region that offers the best fiscal policy until the fall in the benefit of the public good following the addition of an agent becomes so high that migration is no longer profitable between the reigions. A wide variety of mixed equilibria would therefore be possible and, depending on the nature of the congestion, the possibility for redistribution from skilled workers to unskilled workers could be preserved.

On the other hand, if agents of the same type value differently the public good, then for any pair of fiscal policies adopted in the different regions, people of the same type may not be willing to live in the same region. The studied model would therefore be analogous to the model of fiscal competition with heterogeneity of migration costs presented by Lehmann et al. (2014). Here, for each migrant, the difference between the benefit they would derive from the public good of the destination region and the benefit they would derive from the public good in the departure region would represent the implicit cost of migration. A difference, however, would exist between our model and that of Lehmann et al. (2014) since this implicit cost of migration is endogenous, and determined by the joint strategic actions of the two governments. The existence of mixed equilibrium in this context therefore seems "natural". This same result could be obtained by introducing heterogeneity in preferences for work or consumption, which would amount to considering more than two types of worker.

Second Article.

Potential Benefits from Coordination on the Market of Citizenship By Investment In the Caribbean

by

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This article is in preparation for submission to The RAND journal of Economics. It has been supervised by my directors of thesis Professors Michel Poitevin and Lars Ehlers and my internship supervisor at the IMF, Ding Ding, Ph.D. Results from this article have been included in an upcoming IMF Working Paper "Coordinating Revenue Policies in the Caribbean "co-authored with Ding Ding, Samira Kalla, Manuel Rosales Torres. Results from the paper were also included in a document called "Eastern Caribbean Currency Union" from the IMF's Selected Issues series published in March 2020.

1. Introduction

Citizenship By Investment (hereafter CBI) programs are a major economic policy tool for the Caribbean.[12] Through these programs, Caribbean countries attract wealthy people coming from all over the world (mainly from China and Russia) by offering to them a citizenship in exchange for a significant donation, substantial investment in real estate or relatively large purchases of government bonds. Applicants to these programs seek, among other things, to obtain a second passport allowing them to travel to a higher number of countries without a visa. 5 out of 6 of the Eastern Caribbean Currency Union (hereafter ECCU) member states have a CBI program and the price for new citizenship in these states has been steadily declining in recent years (see Figure 4) suggesting a race-to-the-bottom in the CBI market as Trevor and al. pointed out in 2017.

In fact, the ECCU member states seem to be undercutting each other revenue from CBI programs by lowering their prices as we can notice in Figure 4. St. Kitts and Nevis has the most extended history in running a CBI program and had its revenues from this program increasing until other states of the union entered the market. St. Kitts and Nevis' revenue fell drastically after Dominica introduced a \$200,000 Real Estate Investment option in 2015[13] and kept decreasing after Antigua and Barbuda reduced the required investment amount in National Development Fund (hereafter NDF) by 50%.[14] However, the CBI revenues of St. Kitts and Nevis took over, and those of Dominica dropped after the government of St. Kitts and Nevis also introduced a \$200,000 Real Estate Investment option in 2018.[15] Consequently, the overall revenue in the union from this market stopped rising since 2015 (see the total CBI Revenue in Figure 4).

Competition for Foreign Direct Investment (FDI) in the Caribbean through CBI programs can be thought of as a Bertrand price competition for differentiated products

12. For instance, in 2013, CBI revenues reached 26% of total GDP in St. Kitts and Nevis while it peaked at 32% of total GDP of Dominica in 2016 (National authorities and IMF staff calculations).

13. Until then, Dominica had only one option available in its CBI program, which is an economic contribution (donation) to the country under the government's investor visa program.

14. Now, applicants to Antigua and Barbuda's CBI programs can choose to contribute only \$100,000 to NDF.

15. In 2018, St. Kitts and Nevis introduced a \$200,000 Real Estate Investment resalable after 7 years option in addition to the existing \$400,000 Real Estate Investment resalable after 5 years option.

where countries use the amount of required investment as strategic variable,[16] which is perceived by applicants as the price for new citizenship. The conventional wisdom is that price competition leads competitors to a suboptimal outcome and a coordination policy could improve their joint revenues.[17] If countries are identical, then countries' revenue from CBI programs could improve either by a suitable minimum price or (equivalently) by an appropriate cap on the number of new citizens per country.[18] However, if countries face asymmetric demands for CBI, a minimum price and a maximum quota limits produce different effects. In this case, countries would benefit from using the right combination of both instruments for policy coordination.

In the CBI market, the demand in each country highly depends on the attractiveness of its passport, which is closely related to the number of other countries to which it grants visa-free access or visa-on-arrival access to its holder. The Score of Diversity of Travel Freedom has been converging in the ECCU member states during the current decade but, a notable difference persists between these states (see Figure 5), which can be grouped into two categories. The high Travel Freedom category consists of St. Kitts and Nevis and Antigua and Barbuda while Dominica, Grenada and St. Lucia steadily present a lower Score of Diversity of Travel Freedom.

To fix ideas, we consider n countries which can be classified into two groups according to their demand: the low-demand countries (L) and the high-demand countries (H). Countries are assumed to be identical within each category, and the former face less demand than the latter but they have symmetric elasticities with respect to prices and produce CBI at an identical constant marginal cost, which represents the administrative cost for applications' treatment.

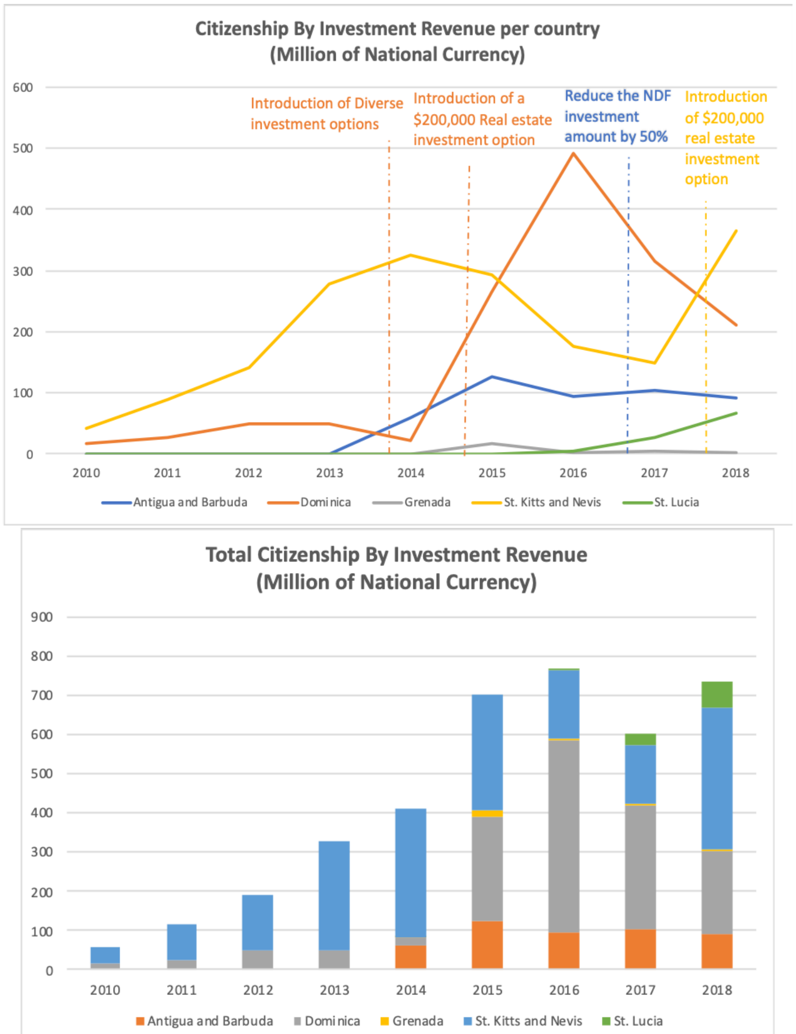
In an unconstrained equilibrium, H-type countries charge a higher price and grant a greater number of new citizenships than L-type countries do. So, while imposing

16. In practice, the criteria for admission to these programs are more complex than a simple amount of investment required. The conditions may vary according to the number of relatives accompanying the applicant, the age of those relatives, the area of investment (real estate, National Development Fund, etc.) and the required duration before an eventual resale of the investment.

17. Deneckere and Davidson (1985) shows that, when firms face symmetric demands, coalition increases the aggregate profit of firms while increasing the selling price. However, when countries do not face symmetric demands, a coalition that maximizes the overall revenue could reduce the individual profit of those facing a relatively weaker demand.

18. The Pacific island countries' agreement on tuna fishing daily price and annual quota is a real world example of price/quantity coordination.

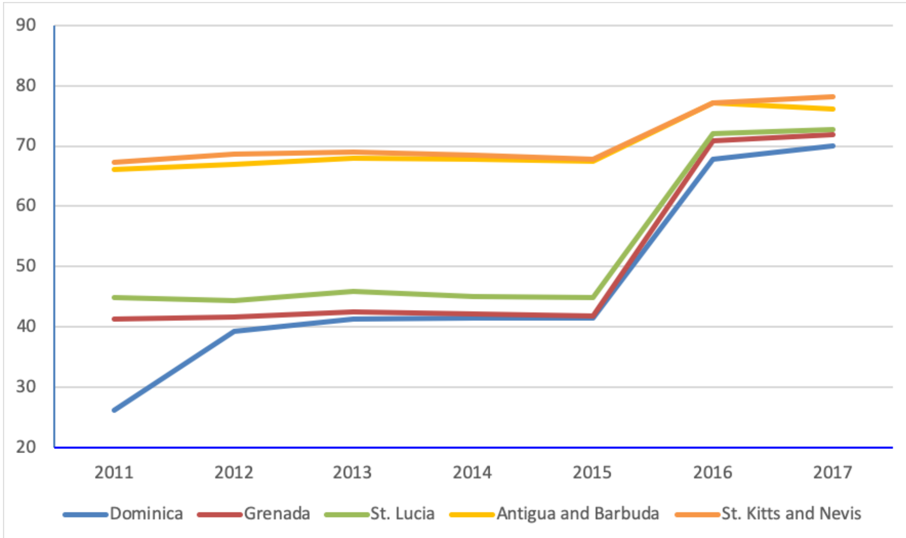
Figure 4. Citizenship By Investment Revenues in the ECCU member states



Sources: National Authorities and IMF staff calculations

a minimum price would be more biding to the Ls, a maximum quota limit on new citizenships would be more restrictive to the Hs. Therefore, using a combination of both instruments seems to be the right way to carry all countries to a «fairer» outcome. Nevertheless, the degree of freedom provided by these two instruments is quite limited. Indeed, most Pareto efficient price distributions could never be achieved by a coordination policy that uses only a uniform minimum price and a uniform maximum

Figure 5. Score of Diversity of Travel Freedom in ECCU member states*



Sources: Henley & Partners - Kochenov Quality of Nationality Index database.
∗ Data on St. Vincent and the Grenadines has been omitted because it has no CBI program.

quota. Moreover, even though such policies could achieve some Pareto efficient allocation, it could be the case that this allocation leaves some countries with less revenue than the unconstrained situation. We show that there exist a minimum price and a maximum quota such that if they were uniformly imposed to all countries would bring them to an efficient outcome that Pareto dominates the unconstrained Nash equilibrium.

The remainder of this document is organized as follows. The theoretical framework is described in Section 2, which provides the relevant definitions and show the existence of a unique constrained Nash equilibrium for any values of the parameters. Section 3 analyses efficiency in our framework and presents our main results. We study a generalized version of the model in Section 4 consisting of multiple coalitions of countries competing with one another. Finally, we give some concluding remarks in Section 5.

2. General framework

There exists $n \geq 2$ independent countries (or states) in the economy. Each country $j \in \{1, 2, ..., n\}$ sets a price for its CBI program and faces a demand [19] for CBI that depends negatively on its own price and positively on those of others. Countries can be classified into two types, L or H according to the size of the demand they are facing. L-type countries face a lower demand than H-type countries do. For each $i \in \{L, H\}$, let $n_i \geq 1$ be the number of i-type countries, so that, $n_L + n_H = n$. A price distribution is an n-dimensional vector $\widetilde{p} = (p_L^1, p_L^2, ...p_L^{n_L}, p_H^1, p_H^2, ...p_H^{n_H}) \in \mathbb{R}_+^n$, where p_i^j is the price charged by the j-th i-type country (assuming that i-type countries are numbered from 1 to n_i) with $i \in \{L, H\}$ and $j \in \{1, 2, \ldots, n_i\}$. For all $\widetilde{p} \in \mathbb{R}_+^n$, let $q_i^j(\widetilde{p})$ be the demand to the j-th i-type country, that is, the number of applicants for CBI in that country. As in Deneckere and Davidson (1985), we assume that demand functions can be expressed in the following linear [20] form

$$q_i^j(\widetilde{p}) = v_i - p_i^j - \delta(p_i^j - \overline{p})$$

where $\delta > 0$ is a substitutability parameter, $\overline{p}$ is the average price level in the economy ($\overline{p} = (p_L^1 + p_L^2 + \cdots + p_L^{n_L} + p_H^1 + p_H^2 + \cdots + p_H^{n_H})/n$) and $v_H > v_L > 0$. We assume v_L to be large enough, so that, $q_i^j(\widetilde{p})$ is always positive.

Note that the entire distribution of prices is not necessary for the determination of the demand in any given country. Only the country's own price p_i^j and the average price $\overline{p}$ matter. This feature of the demand function is not crucial for our main results but helps to find closed form solutions. If δ approaches 0 then CBI programs are strategically independent. The larger is δ the more homogenous are CBI programs.

We assume that the total cost for providing citizenships is zero. [21] Each type $i \in \{L, H\}$ country aims to set a price level as to maximize its revenue from CBI program defined by

19. The demand side of the market is not described explicitly as we will focus on the benefit from coalition from the point of view of suppliers.

20. This assumption is relaxed in the appendix and our main results are proved under a fairly general set of assumptions. Here, we use linear demand functions for their tractability. Martin (2009) and Fanti and Gori (2011) provide sound micro-foundations for linear Marshallian demand.

21. We can find same results with constant marginal cost (see appendix)

$$R_i^j(\widetilde{p}) = p_i^j q_i^j(\widetilde{p})$$

Definition 1. *A price distribution $\widetilde{p} \in \mathbb{R}_+^n$ is Pareto efficient if there exists no price distribution $\widetilde{p}' \in \mathbb{R}_+^n$ such that for all $i \in \{L, H\}$ and all j-th i-type country*

$$R_i^j(\widetilde{p}) \leq R_i^j(\widetilde{p}')$$

with at least one strict inequality.

It is worth noting that this definition of Pareto efficiency concerns only agents on the supply side of the market (countries) and not the well-being of agents on the demand side, which are not explicitly described in the model.

Countries are supposed to compete in prices, that is, they use prices as strategic variables and try to maximize their respective payoffs in a single-period game. Countries, therefore, engage in a Bertrand competition with differentiated products. It is also worth noticing that this is a supermodular game, meaning that a country's best response to an increase in prices in other countries is to increase its own price since the second cross derivatives $\frac{\partial^2 R_i^j}{\partial p_i^j \partial p_{i'}^{j'}} = \frac{\delta}{n}$ with $\{i, j\} \neq \{i', j'\}$, are positive.

In what follows, we define two types of equilibrium: the *(unconstrained) Nash equilibrium* and the *constrained Nash equilibrium*. For notational ease, assume that all countries (both types included) are numbered from 1 to n. For all price distribution $\widetilde{p} \in \mathbb{R}^n$, all country $k \in \{1, 2, ..., n\}$, and all price level $p_k \in \mathbb{R}$, define $(p_k, \widetilde{p}_{-k})$ by the n-dimensional vector obtained by replacing the k-th component of $\widetilde{p}$ by p_k without changing the order of the remaining components.

Definition 2. *An (unconstrained) Nash equilibrium is a price distribution $p^N = (p_1^N, p_2^N, ..., p_n^N)$ such that for all countries $k \in \{1, 2, ..., n\}$, if k is the j-th i-type country then*

$$p_k^N \in \arg\max_{p_k \in \mathbb{R}} R_i^j(p_k, p_{-k}^N). \tag{2.1}$$

unconstrained Nash equilibria are price distributions that correspond to situations where no country could increase its revenue through a unilateral deviation. If p^N is an unconstrained Nash equilibrium, then the first-order condition for (2.1) is

$$v_i - 2p_k^N - \delta\left(2p_k^N - \overline{p}^N - \frac{p_k^N}{n}\right) = 0,$$

which implies that

$$p_k^N = \frac{v_i + \delta\overline{p}^N}{2 + \delta(2 - \frac{1}{n})}.$$

So, in an unconstrained Nash equilibrium, countries of the same type charge the same price. Denote by p_L^N and p_H^N the equilibrium price charged by the L-types and the H-types respectively:

$$p_L^N \equiv \frac{v_L + \delta\overline{p}^N}{2 + \delta(2 - \frac{1}{n})}, \qquad p_H^N \equiv \frac{v_H + \delta\overline{p}^N}{2 + \delta(2 - \frac{1}{n})}. \tag{2.2}$$

Equation (2.2) implicitly defines p_L^N and p_H^N as functions of the average price level given by $\overline{p}^N = (n_L p_L^N + n_H p_H^N)/n$. An explicit expression of the equilibrium prices can be found by solving (2.2) for (p_L^N, p_H^N) after substituting the expression of the average price into the equation. Then we get

$$p_L^N = \frac{\delta n_H p_H^N + n v_L}{-\delta + 2n + 2\delta n - \delta n_L}, \qquad p_H^N = \frac{\delta n_L p_L^N + n v_H}{-\delta + 2n + 2\delta n - \delta n_H}. \tag{2.3}$$

Equation (2.3) uniquely defines the unconstrained Nash equilibrium price levels by

$$\begin{cases} p_L^N = \dfrac{n(\delta n_H v_H + 2n v_L - \delta(1 - 2n + n_H)v_L)}{(-\delta + 2(1+\delta)n)(2n + \delta(-1+n))} \\ p_H^N = \dfrac{n(\delta n_L v_L + 2n v_H - \delta(1 - 2n + n_L)v_H)}{(-\delta + 2(1+\delta)n)(2n + \delta(-1+n))} \end{cases}.$$

We can easily verify that

$$p_H^N > p_L^N \text{ and } q_H^N > q_L^N$$

where q_i^N denotes the unconstrained Nash equilibrium quantity for i-type countries. In other words, in an unconstrained equilibrium, high-demand countries charge a higher price and grant a greater number of new citizenships than low-demand countries do.

We know that the unique unconstrained Nash equilibrium is Pareto inefficient.[22] Now, suppose that countries can cooperate in order to increase their individual revenue. Assume that they can commit to a uniform lower bound for prices (minimum price) p_{min}

22. See for instance, Friedman (1983) and Friedman (1977).

and a uniform upper limit (maximum quota) q_{max} for the number of new applicants (quantities), that is, countries are no longer allowed to set their price below p_{min} or to provide citizenships to more than q_{max} applicants.

Definition 3. *A p_{min}-q_{max}-constrained Nash equilibrium is a price distribution $p^C = (p_1^C, p_2^C, ..., p_n^C)$ such that for all $k \in \{1, 2, ..., n\}$ if k is the j-th i-type country, then*

$$p_k^C = \arg\max_{p_k} R_i^j(p_k, p_{-k}^C) \quad s.t. \begin{cases} p_k \geq p_{min} \\ q_i^j(p_k, p_{-k}^C) \leq q_{max} \end{cases} \tag{2.4}$$

This equilibrium will simply be called a constrained Nash equilibrium if there is no risque of confusion about p_{min} and q_{max}. In a constrained Nash equilibrium each country plays its best response to the strategies of others under the minimum price and the maximum quota constraints. While the best response of an individual country to the strategy of all the others is easily defined, the definition of the best response of one type of countries, say i, to the strategies of the other type, say $-i$, deserves some special considerations. Denote by $\widetilde{P}(p_{-i})$ a price distribution where p_{-i} is the average price charged by the $-i$-type countries and for all countries k, if k is the j-th i-type country, then the price charged by k, say $P_k(p_{-i})$, satisfy

$$P_k(p_{-i}) = \arg\max_{p_k} R_i^j(p_k, P_{-j}(p_{-i})) \quad \text{s.t.} \begin{cases} p_k \geq p_{min} \\ q_i^j(p_k, P_{-k}(p_{-i})) \leq q_{max} \end{cases} \tag{2.5}$$

Equation (2.5) defines the price distribution $\widetilde{P}(p_{-i})$ such that each i-type country plays its best response to the strategies of all the other countries. As in Equation (2.2), the unconstrained solution of (2.5) would be

$$P_i^0(p_{-i}) \equiv \frac{nv_i + \delta p_{-i} n_{-i}}{2n + \delta[n + n_{-i} - 1]}$$

for every i-type country. However, since $q_i^j(p_k, P_{-k}(p_{-i}))$ is a decreasing function of p_k, the maximum quota constraint also requires p_k to be greater than some price level, say $P_i^{quota}(p_{-i})$ with

$$q_i^j(P_i^{quota}(p_{-i}), P_{-k}(p_{-i})) = q_{max}.$$

Therefore, since $R_i^j(p_k, P_{-k}(p_{-i}))$ is concave in p_k,

$$P_k(p_{-i}) = \max\left\{p_{min},\ P_i^0(p_{-i}),\ P_i^{quota}(p_{-i})\right\}. \tag{2.6}$$

Note that $P_k(p_{-i})$ is independent of k. Countries of the same type i independently charge the same price as a response to an average price level p_{-i} set by the other type. For convenience, we can replace the index k by i in the notation of the best response of i-type $P_k(p_{-i}) = P_i(p_{-i})$.

After some algebra, $P_i^{quota}(p_{-i})$ can be explicitly defined by:

$$P_i^{quota}(p_{-i}) = \frac{n(v_i - q_{max}) + \delta p_{-i} n_{-i}}{n + \delta n_{-i}}. \tag{2.7}$$

$P_i^{quota}(p_{-i})$ increases with v_i meaning that H-type countries are «more likely» to be affected by the quota limit and «less likely» to have a binding minimum price constraint while the L-types are more likely to have their best response distorted by the minimum price.

A constrained Nash equilibrium is entirely defined by a couple of price levels (p_L^C, p_H^C) such that [23]

$$P_L(p_H^C) = p_L^C \text{ and } P_H(p_L^C) = p_H^C.$$

Therefore, p_L^C is the equilibrium price of the L-type countries if and only if $P_L(P_H(p_L^C)) = p_L^C$. Thus, the search for constrained Nash equilibria boils down to finding the fixed points of the composed function $P_L(P_H(\cdot))$.

23. (p_L^C, p_H^C) depend on parameters p_{min} and q_{max}, which are omitted for notational ease whenever there is no risk of confusion.

Proposition 1. *For every minimum price $p_{min} > 0$ and maximum quota $q_{max} > 0$ there exists a unique p_{min}-q_{max}-constrained Nash equilibrium.*

Proof. Consider an auxiliary function F defined on $\mathbb{R}_+$ by $F(p) = P_L(P_H(p)) - p$. Now, we need to search for the zeros of F.

First, note that F is continuous on $\mathbb{R}_+$ and differentiable almost everywhere. F is a decreasing function because for every p where F is differentiable, we have

$$\begin{aligned} F'(p) &= P'_H(p) \times P'_L(P_H(p)) - 1 \\ &\leq \max\{P'_H(p)\} \times \max\{P'_L(p)\} - 1 \\ &\leq \frac{\delta n_L}{n + \delta n_L} \times \frac{\delta n_H}{n + \delta n_H} - 1 < 0. \end{aligned} \tag{2.8}$$

Second, since $P_i(p) > p_{min}, \forall p > 0, i \in \{L, H\}$, we get

$$F(0) \geq p_{min} > 0. \tag{2.9}$$

Furthermore, for high enough values of p, $P_i(p) = P_i^{quota}(p)$ because P_i^{quota} increases at a higher rate than P_i^0 and both of then are linear. So

$$\begin{aligned} \lim_{p\to+\infty} &= \lim_{p\to+\infty} F(p) \\ &= \lim_{p\to+\infty} P_L(P_H(p)) - p \\ &= \lim_{p\to+\infty} \left(\frac{P_L(P_H(p))}{p} - 1 \right) p \\ &= \lim_{p\to+\infty} \left(\frac{\delta n_L}{n + \delta n_L} \times \frac{\delta n_H}{n + \delta n_H} - 1 \right) p \\ &= -\infty \end{aligned} \tag{2.10}$$

From (2.8), (2.9) et (2.10) and according to the intermediate value theorem there exists a unique price level $p_L^C > 0$ such that, $F(p_L^C) = 0$. So, for any given p_{min} and q_{max}, there is a unique constrained Nash equilibrium (p_L^C, p_H^C) where $p_H^C \equiv P_H(p_L^C)$. ■

In particular, the unconstrained Nash equilibrium (p_L^N, p_H^N) can be reached by any constrained Nash equilibrium where the minimum price is smaller that p_L^N and that maximum quota is bigger than q_H^N. Figure 6 shows how a Pareto improvement of the unconstrained Nash equilibrium can be achieved through a cooperative one. Indeed, the constrained Nash equilibrium prices are determined by the intersection of the best response functions of L-types and H-types under cooperation. The dashed blue (resp. green) line represents the iso-revenue curve for L-types (resp. H-types) that corresponds to their unconstrained Nash equilibrium revenue level. Any point located above the dashed blue line (resp. to right of the dashed green line) represents a combination of prices, which will generate a higher CBI revenue for the L-types (resp. H-types) than the one produced by the unconstrained Nash equilibrium. Therefore, the purple area of the graph represents the sets of price distribution that generate more revenue to both countries as compared to the unconstrained scenario.

3. Efficiency and constrained Nash equilibrium

In this section, we study how Pareto efficient price distributions can be achieved by a constrained Nash equilibrium. First note that, Pareto efficiency does not require that countries of the same type charge the same price. However, since constrained Nash equilibrium requires prices to be identical for same type countries we will consider only Pareto efficient price distributions that have this feature. Price distributions for which same type countries charge same prices are entirely defined by a couple $(p_L, p_H) \in \mathbb{R}^2$ where p_i is charged by the i-type countries. For all $i \in \{L, H\}$, define by q_i and R_i the two-variable functions naturally induced, respectively by q_i^j and R_i^j when the price distribution is (p_L, p_H).

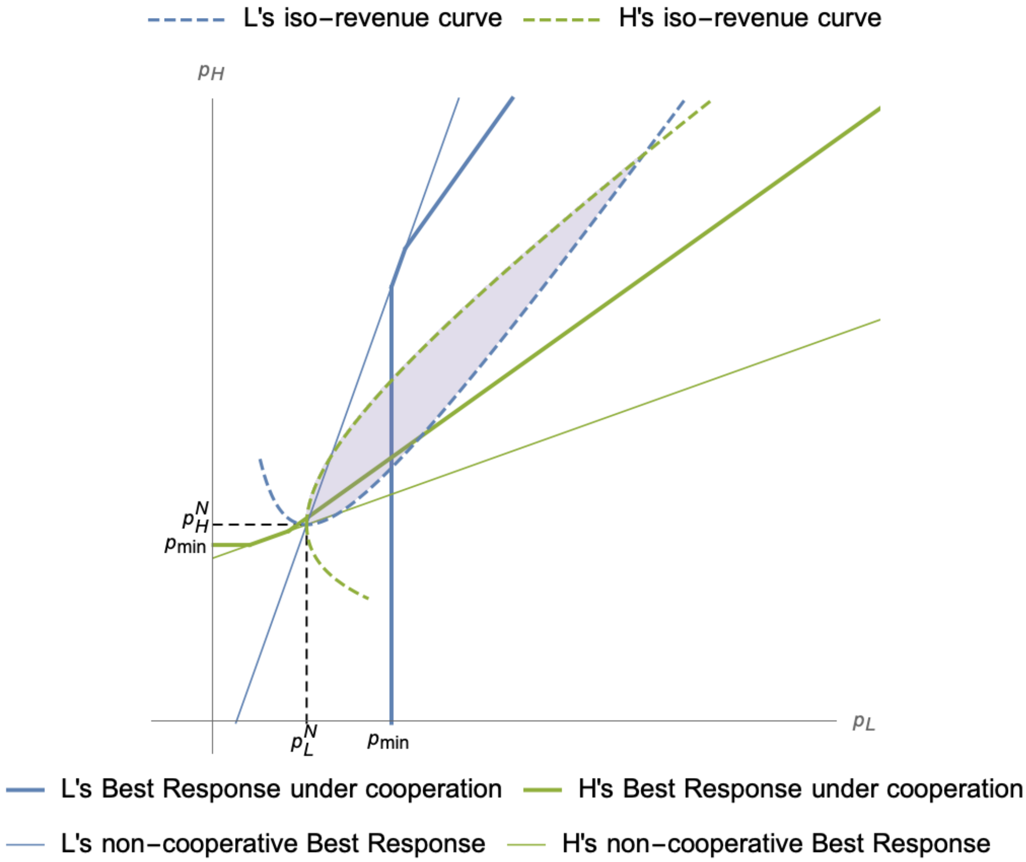
Pareto efficient price distributions under which the same prices are charged by same type countries are all the couples (p_L^E, p_H^E) characterized by:

$$\begin{aligned} (p_L^E, p_H^E) \quad &= \underset{p_L, p_H}{\arg\max}\, R_H(p_L, p_H) \\ &\text{s.t.} \quad R_L(p_L, p_H) \geq \overline{r}_L. \end{aligned} \tag{3.1}$$

where $\overline{r}_L = R_L(p_L^E, p_H^E)$. Therefore, Pareto efficiency can be characterized by the equality of the Marginal Rates of Substitution (MRS) between the two types:

$$MRS_L(p_L^E, p_H^E) = MRS_H(p_L^E, p_H^E)$$

Figure 6. A Pareto improvement of the unconstrained Nash equilibrium through a constrained one



where $MRS_i(p_L, p_H) = -\frac{\partial R_i}{\partial p_L} / \frac{\partial R_i}{\partial p_H}$ is the MRS of p_H for p_L and provided that the second order condition is satisfed.

Figure 7 represents the set of Pareto efficient allocations. There is an infinite mass continuum of efficient price distributions, and only a small fraction (i.e. a finite mass continuum) of which dominates the unconstrained Nash equilibrium. The efficient allocation curve never intersects the unconstrained best response functions meaning that, without a coordination policy, every country has an incentive to deviate from any efficient price distribution by lowering its own price. In fact, for all $i \in \{L, H\}$,

$$\begin{aligned} p_i^E = \quad & \arg\max_{p_i} R_i(p_i, p_{-i}^E) \\ & \text{s.t.} \quad R_{-i}(p_{-i}^E, p_i) \geq \overline{r}_{-i}. \end{aligned}$$

Since $R_{-i}(p_{-i}^E, p_i)$ increases with p_i, the constraint $R_{-i}(p_{-i}^E, p_i) \geq \overline{r}_{-i}$ requires p_i to be greater than a certain level. So, $p_i^E \geq \arg\max_{p_i} R_i(p_i, p_{-i}^E)$, and because the constraint binds in a Pareto's problem, we get the following with strict inequality

$$p_i^E > P_i^0(p_{-i}^E). \tag{3.2}$$

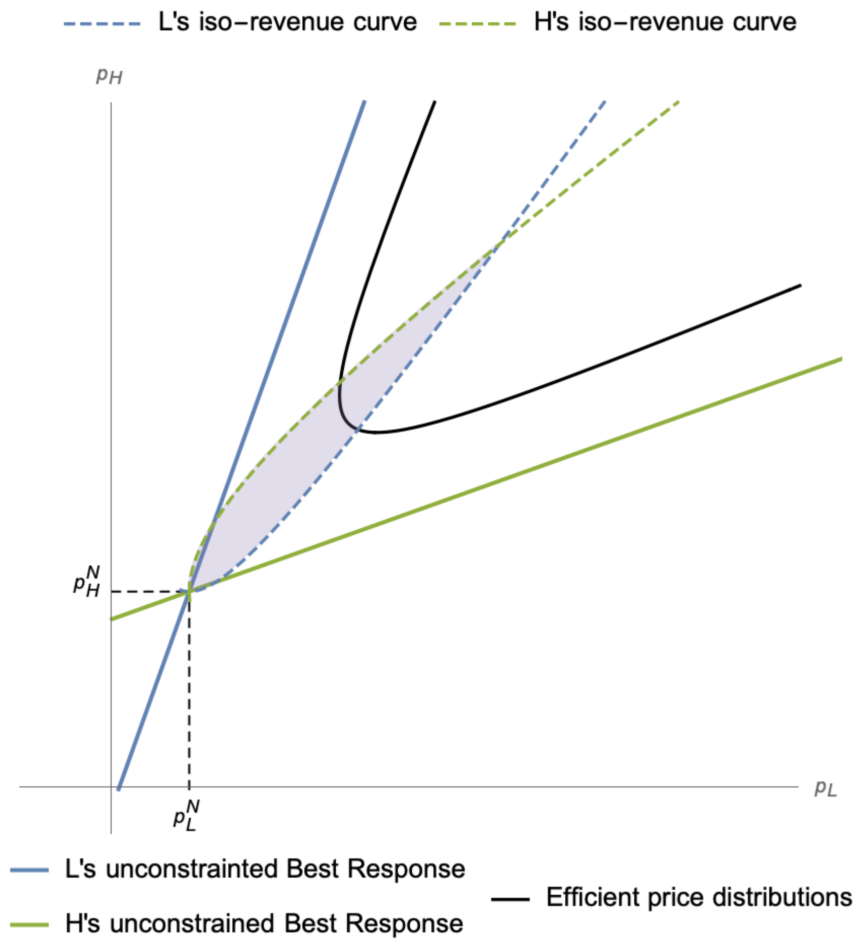
Lemma 1 (Implementability condition)**.** *A Pareto efficient price distribution* (p_L^E, p_H^E) *with corresponding quantities* (q_L^E, q_H^E) *is achievable by a constrained Nash equilibrium with an appropriate minimum price* p_{min} *and maximum quota* q_{max} *if and only if countries with larger quantities charge higher prices. And then, we have* $q_{max} = \max\{q_L^E, q_H^E\}$ *and* $p_{min} = \min\{p_L^E, p_H^E\}$.

Proof. First, if there is a country type, say $\hat{\imath}$, such that $\hat{\imath}$ has the lowest quantity and the highest price compared to the other type, say $-\hat{\imath}$, that is, $q_{\hat{\imath}}^E < q_{-\hat{\imath}}$ and $p_{\hat{\imath}}^E > p_{-\hat{\imath}}^E$, then for any constrained Nash equilibrium that reach (p_L^E, p_H^E) the minimum price constraint and the maximum quota constraint would not be binding for the $\hat{\imath}$-type. It could then benefit from deviating from $P_{\hat{\imath}}^E$ according to (3.2).

Now, suppose that countries with the largest quantity charge the highest price and set $q_{max} = \max\{q_L^E, q_H^E\}$ and $p_{min} = \min\{p_L^E, p_H^E\}$. Countries with the lowest price (resp. largest quantity) would like to lower their price but could not because of the minimum price (resp. maximum quota) constraint. And, then we get a constrained Nash equilibrium. ■

For any price distribution (p_L, p_H) and corresponding quantity (q_L, q_H), define $\Delta p \equiv p_H - p_L$ and $\Delta q \equiv q_H - q_L$. We have

Figure 7. Efficient price distributions and unconstrained best response functions



$$
\begin{aligned}
\Delta q \Delta p \geq 0 \quad &\Leftrightarrow \begin{cases} \Delta q \geq 0 \text{ and } \Delta p \geq 0 \\ \text{or} \\ \Delta q \leq 0 \text{ and } \Delta p \leq 0 \end{cases} \\
&\Leftrightarrow \begin{cases} v_H - v_L - (1+\delta)\Delta p \geq 0 \text{ and } \Delta p \geq 0 \\ \text{or} \\ v_H - v_L - (1+\delta)\Delta p \leq 0 \text{ and } \Delta p \leq 0 \end{cases} \\
&\Leftrightarrow \begin{cases} \Delta p \leq \dfrac{v_H - v_L}{1+\delta} \text{ and } \Delta p \geq 0 \\ \text{or} \\ \Delta p \geq \dfrac{v_H - v_L}{1+\delta} \text{ and } \Delta p \leq 0 \end{cases}
\end{aligned}
$$

Thus, since $v_H - v_L > 0$, $\Delta q \Delta p \geq 0$ is equivalent to

$$0 \leq p_H - p_L \leq \frac{v_H - v_L}{1+\delta}. \tag{3.3}$$

So, according to Lemma 1, a Pareto efficient price distribution (p_L^E, p_H^E) can be reached by a constrained Nash equilibrium if and only if (p_L^E, p_H^E) satisfies (3.3).

Countries will be willing to cooperate for a Pareto efficient outcome only if it provides them with a higher revenue than the unconstrained situation. So, we need to find out whether it is possible to achieve, through out a constrained Nash equilibrium, a Pareto efficient allocation that is welfare improving for both types as compared to the unconstrained scenario.

For this purpose, define by $(\widehat{p}_L, \widehat{p}_H)$ a price distribution that provides each country with the same revenue as the unconstrained Nash equilibrium with $(\widehat{p}_L, \widehat{p}_H) \neq (p_L^N, p_H^N)$. $(\widehat{p}_L, \widehat{p}_H)$ exists and is unique[24] and $MRS_L(\widehat{p}_L, \widehat{p}_H) > MRS_H(\widehat{p}_L, \widehat{p}_H)$ since $MRS_L(p_L^N, p_H^N) < MRS_H(p_L^N, p_H^N)$ (see Figure 7). Now consider, the Pareto efficient price distribution $(\widehat{p}_L^E, \widehat{p}_H^E)$ defined as a convex combination of $(\widehat{p}_L, \widehat{p}_H)$ and (p_L^N, p_H^N), that is,

$$(\widehat{p}_L^E, \widehat{p}_H^E) = \alpha(p_L^N, p_H^N) + (1-\alpha)(\widehat{p}_L, \widehat{p}_H)$$

where $\alpha \in (0,1)$ is uniquely defined by $MRS_L(\widehat{p}_L^E, \widehat{p}_H^E) = MRS_H(\widehat{p}_L^E, \widehat{p}_H^E)$. Clearly, $(\widehat{p}_L^E, \widehat{p}_H^E)$ is a Pareto improvement of the unconstrained equilibrium. Moreover, we know that the unconstrained equilibrium price (p_L^N, p_H^N) satisfies (3.3). So, the efficient price distribution $(\widehat{p}_L^E, \widehat{p}_H^E)$ can be achieved by a constrained Nash equilibrium if $(\widehat{p}_L, \widehat{p}_H)$ satisfies (3.3).

Lemma 2. *H-type countries benefit more (or lose less) from an increase in a common price, while they benefit less (or lose more) from a decrease in a common quantity, i.e,*

$$\frac{\partial R_L(p,p)}{\partial p} < \frac{\partial R_H(p,p)}{\partial p} \text{ and } \frac{\partial R_L(p_L(p),p)}{\partial p} > \frac{\partial R_H(p_L(p),p)}{\partial p}$$

24. The existence of $(\widehat{p}_L, \widehat{p}_H)$ is guaranteed by

$$\lim_{p_L\to\infty} MRS_L(p_L, p_H) = \lim_{p_L\to\infty} \frac{2\partial q_L/\partial p_L}{\partial q_L/\partial p_H} > \lim_{p_L\to\infty} \frac{\partial q_H/\partial p_L}{2\partial q_H/\partial p_H} = \lim_{p_L\to\infty} MRS_H(p_L, p_H).$$

$(\widehat{p}_L, \widehat{p}_H)$ is unique because L's iso-revenue curve is convex while the H's is concave.

where $p_L(p)$ is defined by $q_L(p_L(p), p) = q_H(p_L(p), p)$.

Proof. The first inequality is easily derivable from the definition of $R_L(p_L, p_H)$ and $R_H(p_L, p_H)$:

$$\frac{dR_H(p,p)}{dp} = v_H - 2p > v_L - 2p = \frac{dR_L(p,p)}{dp}.$$

To show the second inequality, we first derive an expression for $p_L(p)$ from $q_L(p_L(p), p) = q_H(p_L(p), p)$, which is $p_L(p) = p - \frac{v_H - v_L}{1+\delta}$. And after some algebra we get

$$\frac{dR_L(p - \frac{v_H - v_L}{1+\delta}), p}{dp} - \frac{dR_H(p - \frac{v_H - v_L}{1+\delta}, p)}{dp} = \frac{v_H - v_L}{1+\delta} > 0.$$

■

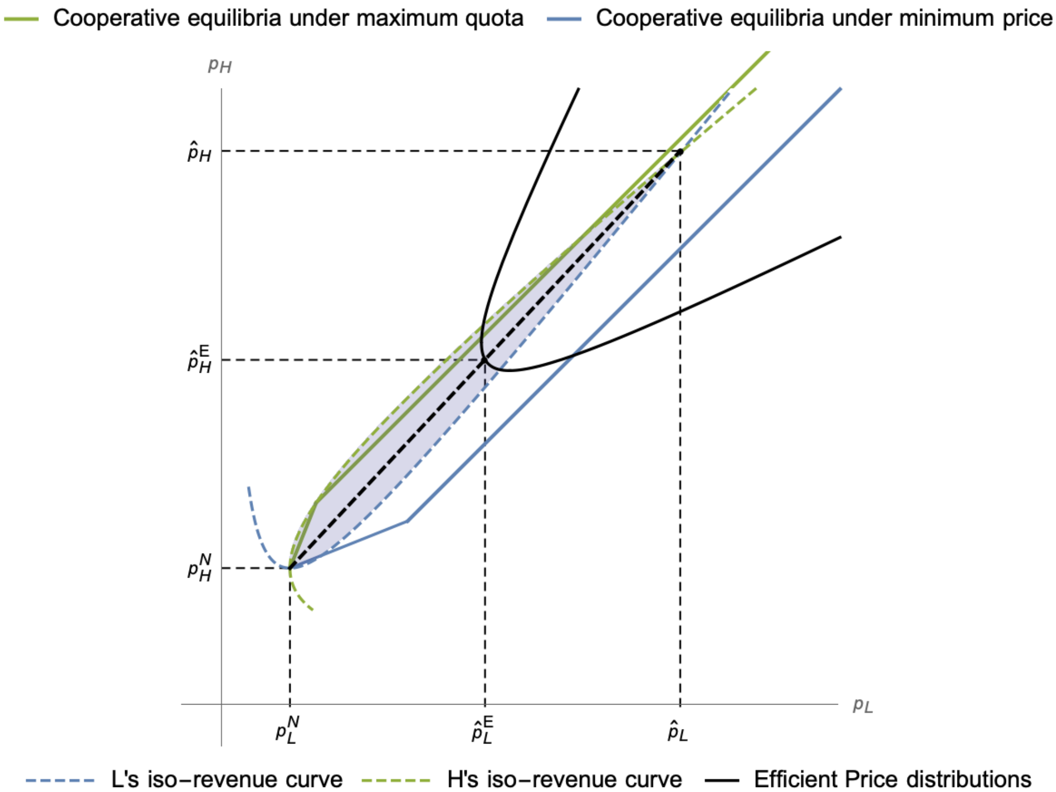
Now we are ready to present our main result.

Proposition 2. *There exist $p_{min} \geq 0$ and $q_{max} \geq 0$ such that the p_{min}-q_{max}-constrained Nash equilibrium is Pareto efficient and provides all countries with higher revenues than the unconstrained Nash.*

Proof. See Appendix ■

A simple graphical argument could help understand the rationale behind this result. Consider Figure 8 and notice that any point that lay between the blue line and the green line can be achieved by setting a combination of a minimum price and a maximum quota. Now, observe that the green line always passes above $(\widehat{p}_L, \widehat{p}_H)$ because otherwise, they would exist a maximum quota level for which high-demand countries would earn more revenue as compared to their revenue in an unconstrained equilibrium while low-demand countries would earn less, which is impossible. Analogously, the blue line always lay below $(\widehat{p}_L, \widehat{p}_H)$. Now, since the set of efficient price distributions (black line on the graph) always passes through the purple area (representing feasible Pareto-improvements of the unconstrained equilibrium), it always exists an efficient distribution of prices profitable to all types that lay between the green line and the blue line. Therefore, the right combination of a minimum price and a maximum quota would guarantee an

Figure 8. Efficient Pareto improvement of the unconstrained Nash equilibrium



efficient outcome that is more beneficial to every type of country than the unconstrained equilibrium.

The problem with a minimum price requirement alone is that it profits more to the H-type countries than it does to the L-type. More precisely, the marginal revenue from a minimum price requirement is always higher for the H-type, that is, each additional unit to the minimum price generates more revenue (or erodes less revenue) in the H-type countries than the L-type countries. That is because either L-type countries are the only ones that would face a binding minimum price constraint or both types would face a binding minimum price constraints.

When the minimum price is low, its marginal benefit is positive in all countries. However, when the minimum price is high enough, at some point, its marginal benefit becomes negative in the L-type countries while it remains positive in the H-type countries.

Pareto efficiency is achieved through a minimum price policy when the revenue generated in the H-type countries due to an additional unit to the minimum price is equal to the resulting loss of revenue in the L-type countries. So, if the discrepancies between the two types are high, it could take too much loss of revenue in L-type countries before we get to a Pareto efficient outcome. Therefore, Pareto efficiency with only a minimum price requirement would not benefit to the L-type countries (as compared to the unconstrained scenario) if the difference between the demands is too high.

The same reasoning goes for the maximum quota. The marginal benefit from a maximum quota requirement is higher for the L-type countries. In that case, Pareto efficiency is achieved when the revenue generated in the L-type countries due to a unitary decrease in the maximum quota is equal to the resulting loss of revenue in the H-type countries. Therefore, the Pareto efficient outcome achieved with such a policy may not dominate the unconstrained equilibrium. The reason is that Pareto efficiency through maximum quota requirement solely could require too much loss of revenue in the H-type countries when the two types are too different in their demand sizes.

Both instruments are needed to ensure a «fair» distribution of the additional revenue from cooperation and if rightly chosen they always guarantee an efficient outcome that is more profitable to both types as compared to the unconstrained equilibrium.

The set of all price distribution (p_L^E, p_H^E) is achievable by an efficient constrained Nash equilibrium that Pareto dominates the unconstrained Nash equilibrium is defined by

$$\begin{cases} MRS_L(p_L, p_H) = MRS_H(p_L, p_H) \\ R_L(p_L, p_H) \geq R_L^N \\ R_H(p_L, p_H) \geq R_H^N \\ 0 \leq p_H - p_L \leq \dfrac{v_H - v_L}{1+\delta} \end{cases}$$

where R_L^N and R_H^N are the Nash equilibrium level of revenue for the L-types and the H-types respectively.

4. Inter-regional compétition for CBI

The CBI market is large and extends beyond the ECCU member countries. European countries like Cyprus, Malta, Moldova, etc. have also engaged in the race for foreign investment through CBI programs.[25] Other countries like Canada, United States and France have indirect CBI programs with high standard including high net worth requirement. However, some countries like Vanuatu and Portugal, which offer investment options starting at \$130,000 for the former and \$350,000 for the latter, are now establishing themselves as real competitors for the Caribbean countries.

In this section, we study the influence of the rest of the world on a group of countries that decide to coordinate their CBI policy. Let N be the set of all (L-type and H-type) countries of the economy and let $\mathcal{P}$ be a partition of these countries. A coalition is an element of $\mathcal{P}$. In this section, we assume that there can be transfers between countries belonging to the same coalition. The objective of each coalition will therefore be to maximize the sum of the revenues of its member countries. Transfert can then be done *ex post* to ensure that each country get a higher revenu that it would in an unconstrained equilibrium.

However, coalitions have limited power over the individual strategy of the countries that make it up. They can only compel member countries to respect a minimum price and a maximum quantity not to exceed. The minimum price and the maximum quantity are imposed uniformly on the countries of the same coalition, but these thresholds can vary from one coalition to another. Thus, coalitions exert an externality on each other when they decide on these two instruments. As before, due to the substitutability of the goods sold, any increase in prices within a coalition positively influences the demand for other coalitions. And conversely, any drop in prices within a coalition negatively affects the demand of others. This situation of strategic interaction can be modeled in the form of a multi-stage game which we call the $\mathcal{P}$-*General CBI Competition Game*.

This game can be described as follows. In the first stage, coalitions simultaneously choose a minimum price and a maximum quantity. Then, in a second stage, countries observe these thresholds and simultaneously choose their individual prices while respecting the constraints imposed by the coalition to which they belong. We are interested in

25. However, it should be noted that these countries generally have far higher tariffs than those of the ECCU countries given their very large attractiveness.

the subgame-perfect equilibrium of this two-stage game. We use backward induction to analyse the strategies of each country and each coalition in such an equilibrium.

Let $\mathcal{P}$ be a set of K coalitions S^1, S^2,..., S^K, that is, $\mathcal{P} = \{S^1, S^2, ..., S^K\}$ where $S^k \cap S^l = \emptyset, \forall l, k \in \{1, ..., K\}$ with $l \neq k$ and $S^1 \cup S^2 \cup \cdots \cup S^K = N$. Denote by n_i^k the number of i-type countries that belong to S^k. Let p_{min}^k and q_{max}^k denote the respective price and quantity thresholds imposed by the coalition S^k on its member countries. The best response of a country of type i belonging to the coalition S^k, say p_i^k, depends not only on p_{min}^k and q_{max}^k but also on the price vector of the other countries within and without S^k:

$$p_i^k = \underset{p_i^k}{\arg\max} \quad p_i^k(v_i - p_i^k - \delta(p_i^k - \overline{p})) \qquad \text{s.t.} \quad \begin{cases} p_i^k \geq p_{min}^k \\ v_i - p_i^k - \delta(p_i^k - \overline{p}) \leq q_{max}^k \end{cases} \tag{4.1}$$

This best response function only depends on the minimum price and the maximum quota enacted by the other coalitions through the prices actually charged by their member countries. In addition, it should be noted that all countries of the same type belonging to the same coalition will have the same best response function, P_i^k, entirely defined by Equation 4.1. Indeed, as shown above, this equation uniquely defined P_i^k by:

$$P_i^k((\overline{p}_{-ik}; p_{min}^k, q_{max}^k)) = \max\left\{p_{min},\ P_{ik}^0(\overline{p}_{-ik}),\ P_{ik}^{quota}(\overline{p}_{-ik}; q_{max}^k)\right\} \tag{4.2}$$

where $\overline{p}_{-ik}$ is the average price charged by all countries except the i-type of coalition S^k, $P_{ik}^0(\overline{p}_{-ik})$ is the unconstrained solution of 4.1 and $P_{ik}^{quota}(\overline{p}_{-ik}; q_{max}^k)$ is the maximal price that can be charge by the i-type of coalition S^k under the maximum quota q_{max}. $P_{ik}^0(\overline{p}_{-ik})$ and $P_{ik}^{quota}(\overline{p}_{-ik}; q_{max}^k)$ are explicitly obtained by

$$P_{ik}^0(\overline{p}_{-ik}) = \frac{\delta n_{-ik}\overline{p}_{-ik} + nv_i}{2n + \delta(-1 + n + n_{-ik})} \tag{4.3}$$

$$P_{ik}^{quota}(\overline{p}_{-ik}; q_{max}^k) = \frac{\delta n_{-ik}\overline{p}_{-ik} - n(q_{max}^k - v_i)}{n + \delta n_{-ik}} \tag{4.4}$$

where $n_{-ik} = n - n_i^k$.

For all $k \in \{1, \ldots, K\}$ and all $i \in \{L, H\}$, the derivative of P_i^k with respect to any $p_{i'}^{k'}$ is less than $\frac{\delta}{n+\delta} < 1$. The function that maps each $\widetilde{p} = (p_L^1, p_H^1, \ldots, p_L^K, p_H^K) \in \mathbb{R}^n$ to

$(P_L^1(\overline{p}_{-1L}; p_{min}^1, q_{max}^1), P_H^1(\overline{p}_{-1H}; p_{min}^1, q_{max}^1), \ldots, P_L^K(\overline{p}_{-LK}; p_{min}^K, q_{max}^K), P_H^K(\overline{p}_{-HK}; p_{min}^K, q_{max}^K))$ is, therefore, a contraction. The contraction mapping theorem ensures the existence and the uniqueness of a Nash equilibrium price for every country $P_i^{k,N}(\widetilde{p}_{min}, \widetilde{q}_{max})$ in every node defined by $(\widetilde{p}_{min}, \widetilde{q}_{max})$ where $\widetilde{p}_{min} = (p_{min}^1, \ldots, p_{min}^K)$ and $\widetilde{q}_{max} = (q_{max}^1, \ldots, q_{max}^K)$.

The problem of the coalition S^k is to choose p_{min}^k and q_{max}^k as to maximize its total revenue

$$(P_{min}^k(\widetilde{p}_{min}, \widetilde{q}_{max}), Q_{max}^k(\widetilde{p}_{min}, \widetilde{q}_{max})) = \underset{p_{min}^k, q_{max}^k}{\arg\max} \sum_{i \in \{L,H\}} n_i^k p_i^k \times (v_i - p_i^k - \delta(p_i^k - \overline{p})) \quad \text{s.t.} \quad p_i^k = P_i^{k,N}(\widetilde{p}_{min}, \widetilde{q}_{max}),\ \forall i, k. \tag{4.5}$$

Actually, P_{min}^k and Q_{max}^k do not depend on p_{min}^k and q_{max}^k. However, for notational ease, we write $P_{min}^k(\widetilde{p}_{min}, \widetilde{q}_{max})$ and $Q_{max}^k(\widetilde{p}_{min}, \widetilde{q}_{max})$. Equation (4.5) gives the best response of coalition k to the strategy of the other coalitions.

Proposition 3. *For every partition $\mathcal{P}$ of the countries there exists a subgame-perfect Nash equilibrium in the $\mathcal{P}$-General CBI Competition Game.*

Proof. Denote by $\widetilde{p} = (p_L^1, p_H^1, \ldots, p_L^K, p_H^K)$ the $2K$-dimensional vector such that p_i^k is the price of the i-type countries belonging to the coalition k. Denote by $q_i^k(\widetilde{p})$ the demand of coalition k's i-type countries and by A the set of the $2K$-dimensional price vectors $\widetilde{p}$ for which each country receives a positive demand, i.e. $A = \{\widetilde{p} \in \mathbb{R}_+^{2K} \mid q_i^k(\widetilde{p}) \geq 0, \forall i \in \{L, H\}, k \in \{1, ..., K\}\}$. A is the set of all relevant prices that need to be considered. First, we show that A is bounded, that is there exists a price vector $\widetilde{p}^{max} = (p_L^{1,max}, p_H^{1,max}, \ldots, p_L^{K,max}, p_H^{K,max}) \in \mathbb{R}^n$ such that $\widetilde{p} \in A$ if and only if $0 \leq \widetilde{p} \leq \widetilde{p}^{max}$.

Note that $q_i^k(\widetilde{p}) \geq 0$ is equivalent to $p_i^k \leq l_i^k(\widetilde{p})$ where

$$l_i^k(\widetilde{p}) \equiv \frac{v_i + \delta n_{-ik} \overline{p}_{-ik}}{n + \delta(n - n_{ik})}.$$

The function that maps each $\widetilde{p} = (p_L^1, p_H^1, \ldots, p_L^K, p_H^K)$ to $(l_L^1(\widetilde{p}), l_H^1(\widetilde{p}), \ldots, l_L^K(\widetilde{p}), l_H^K(\widetilde{p}))$ is a contraction because the derivative of l_i^k with respect to $p_{i'}^{k'}$ is less than $\frac{\delta}{n+\delta}$. The contraction mapping theorem ensures the existence of a unique price vector $\widetilde{p}^{max} =$

$(p_L^{1,max}, p_H^{1,max}, \ldots, p_L^{K,max}, p_H^{K,max}) \in \mathbb{R}^{2K}$ such that $q_i^k(p^{max}) = 0$, for all i, k. Therefore, if for some $\widetilde{p} \in \mathbb{R}_+^{2K}$, there exists $(i, k) \in \{L, H\} \times \{1, 2, ..., K\}$ such that $p_i^k > p_i^{k,max}$, then $\widetilde{p} \notin A$. Furthermore, if for all (i, k), $p_i^k \leq p_i^{k,max}$, then $\widetilde{p} \in A$. So, $\widetilde{p} \in A$ if and only if $0 \leq \widetilde{p} \leq \widetilde{p}^{max}$.

Now, consider for all $k \in \{1, \ldots, K\}$, the function $F^k : \mathbb{R}_+^K \times \mathbb{R}_-^K \to \mathbb{R}_+^K \times \mathbb{R}_-^K$ that maps each $(\widetilde{p}_{min}, -\widetilde{q}_{max})$ to the $2K$-dimensional vector obtained from $(\widetilde{p}_{min}, -\widetilde{q}_{max})$ by replacing p_{min}^k and $-q_{max}^k$ by $P_{min}^k(\widetilde{p}_{min}, -\widetilde{q}_{max})$ and $-Q_{max}^k(\widetilde{p}_{min}, -\widetilde{q}_{max})$, respectively. F^k is a non-decreasing function because the best response for each country is to increase its price when other countries increase their own (strategic complementarity). This result immediately follows from (4.5) by the envelop theorem since $P_i^{k,N}(\widetilde{p}_{min}, \widetilde{q}_{max})$ increases in $\widetilde{p}_{min}$ and decreases in $\widetilde{q}_{max}$. Therefore, a coalition would not benefit from relaxing its constraint whenever prices increase in the rest of the word.

Let $F : \mathbb{R}_+^K \times \mathbb{R}_-^K \to \mathbb{R}_+^K \times \mathbb{R}_-^K$ be the function defined by $F \equiv F^1 \circ F^2 \circ \cdots \circ F^K$. By definition, the fix points of F are the subgame-perfect Nash equilibrium for the General CBI Competition Game. So, we need to show that F has a unique fixed point. Consider the sequence $(x_n)_{n\in\mathbb{N}}$ defined by $x_{n+1} = F(x_n)$ and $x_0 = (0_K, -\widetilde{q}_0)$ where $0_K = (0, 0, ..., 0) \in \mathbb{R}_+^K$ and $\widetilde{q}_0 \in \mathbb{R}_+^K$ is big enough so that the quota constraints do not bind for any country. $(x_n)_{n\in\mathbb{N}}$ is a non decreasing sequence bounded by $(\widetilde{p}^{max}, 0_K)$, therefore converges toward some $(\widetilde{p}_l, \widetilde{q}_l) \in \mathbb{R}^K \times \mathbb{R}^K$. Since F is a continuous function $F(\widetilde{p}_l, \widetilde{q}_l) = (\widetilde{p}_l, \widetilde{q}_l)$. ■

The intuition behind this proposition is that coalitions behave as if they were engaged in a supermodular game with two instruments: the minimum price and the maximum quota. For any coalition, the best response to an increase in a minimum price or a decrease in the maximum quota elsewhere is either to increase its own minimum price or to decrease the maximum quota (or keep them constant). As in standard supermodular games, the existence of a Nash equilibrium is guaranteed. From proposition 2, this Nash equilibrium is Pareto efficient, if the partition $\mathcal{P}$ is a singleton, that is, $\mathcal{P} = \{N\}$. Furthermore, because prices are strategic complements, if $\mathcal{P} \neq \{\{c\}, c \in N\}$, then prices are higher in the subgame-perfect equilibrium of the $\mathcal{P}$-General CBI Competition Game than those charged in an unconstrained equilibrium. Moreover, since every country benefits from an increase in prices in the others, such a subgame-perfect equilibrium will be better for everyone than the unconstrained Nash equilibrium.

5. Conclusion

We show in this paper, through theoretical analysis, that the ECCU member states would benefit from coordinating their CBI programs. This coordination policy would lead them to an efficient outcome that benefits all countries if it imposes to them an appropriate uniform minimum required investment and a maximum number of new citizenships by country. In practice, countries will need to harmonized the admission criteria in details and keep competing on the qualitative aspects of the program, like accelerating the application process, building sound resilience program against natural disasters, etc. This strategy will lead them to a race-to-the-top and make them more competitive as compared to the rest of the Caribbean.

It is well-known in Bertrand competition models that coalition formation within a subgroup of countries is beneficial both to the subgroup and to the outsiders. Moreover, outsiders usually earn more from the coalition than the insiders, which jeopardizes the stability of the group. Thus, the coordination policy proposed in this paper is not self-enforcing; that is, every country has an incentive to deviate from it unilaterally. Therefore, it could take the form of a contract enforced by a supranational entity like the ECCU and would be rather difficult to extend to all the Caribbean. Then, any deviation from the harmonized policy may be punished by a dissuasive monetary transfer. Nevertheless, an efficient allocation could also be self-enforced in a repeated game if the future discount factor, which is a combination of a measure of countries' patience and the probability that the game keeps going, is high enough.

The reputational costs that a country would incur for granting its citizenship based on low-selective criteria could be detrimental. For instance, in 2014, Canada has enacted a visa requirement for St. Kitts and Nevis citizens due to concerns related to their CBI program. Therefore, another potential benefit from coordination in the CBI market is that it prevents the damaging impact of these programs on the Caribbean citizenships' reputation. Further analysis that would take the reputational costs into account could demonstrate even higher benefits from coordination in the CBI market.

Our model can be extended in several ways. First, the case of more than two types of countries could also be considered. It would probably require some restrictive assumption for the existence of an efficient allocation that dominates the unconstrained

Nash equilibrium. Second, from an empirical standpoint, further analysis is needed to determine the optimal uniform admission criteria for CBI programs and the efficient corresponding maximum number of successful applicants per country by period. The resulting revenue surplus could then be estimated for each country.

Third Article.

Many-to-one Matching with Sized Agents and Size-monotonic Priorities

by

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1. Introduction

In the standard literature of many-to-one matching models, agents (usually students) on the many side of the market have to be assigned to entities (usually colleges) on the other side (Gale and Shapley (1962), Roth (1982), Abdulkadiroglu and Sönmez (2003)). The term many-to-one means that several agents may be assigned to the same entity but it is impossible to assign the same agent to more than one entity. For each entity, there is a priority order over groups of agents and an outside option which is being unmatched. Similarly, each agent have preferences over entities and being unmatched. Moreover, each entity has a capacity, say q, which means that it can be assigned to at most q different agents. Therefore, in this standard framework, every agents are implicitly supposed to have the same unit size.

However, many real-life matching problems involve different-size agents. The assignment of teaching assistants (entities) with different limited working hours [26] (capacities) to several classes (agents) which require different amounts of time (sizes); a centralized recruitment process where workers (agents) have different salary requirements (sizes) and firms (entities) announce their budgets (capacities) for job compensation; [27] the assignment of refugees (individuals or families) to landlords are some few practical examples. In the reminder of the paper our running example will concern the assignment of refugee families to homes (landlords).

According to the United Nations High Commissioner for Refugees (UNHCR), the world is «witnessing the highest level of displacement in record» with 68.5 million people forcibly displaced among which 25.4 million refugees and 3.1 million asylum seekers in 2017 [28]. Community hosting network such as Positive Action in Housing have registered up to 7,100 host families in 2018 and in US where the hosting programme is still in development, 2,000 hosts are still waiting to be matched [29]. The need for a matching market design for refugee resettlement has already been pointed out by, among others, Moraga and Rapoport (2014), Delacrétaz et al. (2016) and Anderson and Ehlers (2016). This paper proposes a simple framework to address this issue.

26. For instance, in Quebec, in particular, teaching assistants are generally PhD students, most of whom can not work beyond a certain amount of time (20 hours per week for international students during regular study sessions according to the law in force) . Other teaching assistants have family or professional constraints limiting the number of hours they would be willing to give to their job on the campus.

27. In this example, it is assumed that firms value their ressource only to the extend that it can be used to recruit workers and that no firm is allowed to propose to a worker a higher salary than the one announced. These assumptions make sense in the case of public services where the salary of each worker is fixed by the law in force in the country. See Abizada (2016) and Karakaya and Koray (2003) for cases where the salary of workers is endogenous.

28. UNHCR/ 19 June 2018

29. Positive Action in Housing, http://www.paih.org/host-a-refugee/, 14 February 2019.

In what follows we designate by *landlord* a person who possesses a home[30] and who is willing to host some refugee(s). The term «refugee» is used to designate either an individual or a family. So, the size of a refugee would be either one (if it is an individual) or the number of family members (if it concerns a family). Landlords are invited to submit to a centralized clearing house their preferences over groups of refugees and to declare the maximum number of people they are willing to host. For simplicity, we assume that landlords preferences define for each home a strict priority order over groups of refugees. As it is usual in matching theory, we assume that these priority orders over groups of refugees are responsive to the priority over refugees.[31] On the other hand, refugees have strict preferences over homes that are also submitted to the centralized clearing house. We call a *problem* the collection of *(i)* the set of refugee families with their respective preferences and sizes, *(ii)* the set of homes with their respective *capacities* (maximal numbers of people landlords desire to accommodate) and the priority order over groups of refugees for each home.

A *matching* is the complete specification, for a given problem, of which refugee (individual or family) is to be assigned to which home (or remained unmatched) without exceeding homes' capacities or splitting a refugee family between several homes. A *mechanism* is a systematic rule that precisely suggests a unique matching for any given problem. A mechanism is said to be *pairwise stable* when it suggests, for any problem, a *pairwise stable matching*, that is, *(i)* no refugee (resp. no landlords) prefers been unmatched to the home (resp. to the group of refugees) he is assigned to and *(ii)* no landlord could benefit from replacing a group of refugees that is currently assigned to him by a refugee who prefers that landlord to his current match.

It is well known that many of the desirable properties of matching rules are unachievable in this framework (Delacrétaz (2014)). No pairwise stable mechanism exists and a problem may admit several Pareto-undominated pairwise stable matchings. And,

30. Even if a landlord may possess more than one home in real life there is no loss of generality from associating each landlords to a single home.

31. Responsiveness of priorities (or preference) is a pretty common requirement in the matching literature. It means that *(i)* adding an acceptable refugee to any group of refugees, provided this does not result in exceeding its capacity, increases the priority of the group, and *(ii)* adding one refugee to any group of refugees is at least as desirable as adding another refugee to the same group of refugees if and only if the priority of the first student is higher than the one of the second.

inconveniently, the famous Gale and Shapley (1962) *Refugees-proposing Deferred Acceptance* algorithm fails sometimes to produce a pairwise stable matching even when it exists. The existence of a pairwise stable mechanism requires the priorities of refugees for homes to be size-monotonic, that is, the priority order are set in such a way that the bigger a group of refugees is the highest is their priority.

We show that a pairwise stable matching exists whenever the priority order of refugees for homes is size-monotonic and propose a mechanism to find it. We call this mechanism the *Downward Sequential Greedy Correcting* (DSGC) algorithm. This mechanism is strategy-proof for refugees: no refugee could benefit from misrepresenting his preferences. We know from Roth (1982) that there is no Pareto-efficient mechanism that is pairwise stable. Nevertheless, the DSGC is also a Pareto-undominated pairwise stable mechanism in the sense that there is no other pairwise stable mechanism that would be more profitable to every refugee.

Our methodology is based on an adapted version of the Sequential Greedy Correcting procedure initially introduce by Blum and Rothblum (2002) for the stable mariage problem.[32]

The rest of the paper is organized as follows. In Section 2, we present the related literature. Section 3 defines the model of the many-to-one matching with sized agents. Our main results are presented in Sections 4, 5 and 6. And finally Section 7 presents some concluding remarks.

2. Related Literature

The literature on many-to-one matching problems has been initiated by Gale and Shapley (1962) and many other contributions have been done since then.

The most closely related papers are probably Delacrétaz (2014) and Delacrétaz et al. (2016). The former proposes a many-to-one matching model where the size of agents can be one or two and find an algorithm that produces a pairwise stable matching whenever it exists. And the later proposes a generalization of this algorithm that extends to agents of any size. However, these algorithms do not converge in polynomial time in general which renders the problem computationally intractable when thousands of host

32. The stable mariage problem refers to the case where all refugees have a unit-size and all homes have a unit capacity.

families and millions of refugees are involved. Therefore, they also introduce the concept of size-stability which is a weaker version of stability that allows multiple size agents to envy agents of lower size and propose a mechanism that satisfy this requirement. In this paper, we, instead, assume that the priority for homes is size-monotonic and propose a mechanism that run in a polynomial number of iterations to achieve a pairwise stable mechanism.

Anderson and Ehlers (2016) and Anderson et al. (2018) propose a one-to-one matching problem with sized agents for refugees and landlords where preferences are correlated in some sense: landlords are indifferent between two refugee families of the same size who speak the same language that the concerned landlord finds acceptable. Furthermore, as we do, they assume that preferences of landlords are size-monotonic: a larger refugee family is preferred by landlords to a smaller one. They find that stable maximum matching exists for any problem and they propose an algorithm that produces it. Our paper diverges from that framework by allowing landlords to host several refugee families to the extend of their home capacity.

The school choice models with budget constraint is also closely related to our framework. In these models (see Abizada (2016), Abizada (2017), Karakaya and Koray (2003)), colleges use stipends in order to get more and better students enrolled and students have quasi-linear preferences over colleges and monetary transfer. A matching specifies not only the assignment of students to colleges but also the required monetary transfer that makes it possible. Whereas colleges have fixed budget constraints, the monetary transfer to students is endogenously determined by the matching. They show that a pairwise stable mechanism exists in this framework. In contrast, agents' sizes are given in our model and cannot be modified. This detail makes huge analytical difference between the two framework.

The mechanism proposed in this paper is a version of the Sequential Greedy Correcting (SCG) procedure [Blum et al., 1997; Blum and Rothblum, 2002]. Blum and Rothblum (2002) consider a two-sided matching market where agents arrive sequentially and the «natural greedy procedure» [33] is applied to retrieve stability. Using a one-to-one

33. Blum and Rothblum (2002) shows that by matching blocking pairs (that involved newcomers or not) together sequentially in a specific way we regain stability after a finite number of iterations. However, if the blocking pairs are chosen arbitrary the procedure may not converge (Knuth, 1976).

matching framework, they show that the last agent to enter in the market gets his best possible outcome in a stable matching. Our model differs from the one studied by Blum and Rothblum (2002) by allowing different sizes for refugees. We show that the SCG procedure, with some slight adaptation, is still applicable in our framework.

3. The model

We consider an economy consisting of a finite, non-empty set of *homes* $\mathcal{H}$ and a finite non-empty set of *refugees* (families) $\mathcal{R}$. Each refugee $r \in \mathcal{R}$ has an exogenous *size* $l(r) > 0$ which represents the number of members of the family and has a reflexive and anti-symmetric preferences $\succcurlyeq_r$ over $\mathcal{H} \cup \{r\}$. Denote by $\succ_r$ the strict component associated with $\succcurlyeq_r$. Similarly, each home $H \in \mathcal{H}$ has a *capacity* $q_H > 0$ and strict preferences $\succ_H$ over groups of refugees $\mathcal{P}(\mathcal{R})$, that is, the power set of $\mathcal{R}$. For each home H, we also define the reflexive extension $\succcurlyeq_H$ of $\succ_H$ by $a \succcurlyeq_H b$ if and only if $a = b$ or $a \succ_H b$. Denote by $\varnothing$ the empty set. For each group of refugees $R \subset \mathcal{R}$, $\varnothing \succ_H R$ means that R is *unacceptable* for the home H, that is, the group of refugees R cannot be hosted in the home H. For each home H the priority order over group of refugees is *responsive* to the priority over refugees, that is, for all $R \subset \mathcal{R}$ and $r, r' \in \mathcal{R} \setminus R$ such that $R \cup \{r\}$ and $R \cup \{r'\}$ are both acceptable for H, *(i)* $\{r\} \succcurlyeq_H \{r'\}$ if and only if $R \cup \{r\} \succcurlyeq_H R \cup \{r'\}$ and *(ii)* $\{r\} \succcurlyeq_H \emptyset$ if and only if $R \cup \{r\} \succcurlyeq_H R$.

For every home $H \in \mathcal{H}$, $r \succ_r H$ means that home H is *unacceptable* for refugee r, that is, H is not allowed to host r. For each group of refugees $R \subset \mathcal{R}$, denote by $l(R)$ the number of refugee families' members who belong to R, that is, $l(R) \equiv \sum_{r \in R} l(r)$ and $l(\varnothing) = 0$. This economy is entirely defined by a quintuple $(\mathcal{H}, \mathcal{R}, l, (q_H)_{H \in \mathcal{H}}, (\succ_i)_{i \in \mathcal{H} \cup \mathcal{R}})$ where $l = (l(r))_{r \in R}$. Denote by $\mathscr{P}$ the set of all such economies. A *problem* P is an instance of this economy, i.e, $P \in \mathscr{P}$.

For any given problem, a *matching* specifies which refugee is to be assigned to which home without exceeding the capacity of homes. Technically, a *matching* is a correspondence $\mu : \mathcal{H} \cup \mathcal{R} \to \mathcal{H} \cup \mathcal{R}$ such that *(i)* for all $r \in \mathcal{R}$, $\mu(r)$ is a singleton with $\mu(r) \subset \mathcal{H}$ or $\mu(r) = \{r\}$, *(ii)* for all $H \in \mathcal{H}$, $\mu(H) \subset \mathcal{R}$ such that $l(\mu(H)) \leq q_H$ and *(iii)* $\forall H \in \mathcal{H}, \forall r \in \mathcal{R}, r \in \mu(H)$ if and only if $\mu(r) = \{H\}$. In what follows, we write H or r instead of $\{H\}$ and $\{r\}$, when there is no risk of confusion. Denote by $\mathcal{M}(P)$ the set of matchings admitted by the problem P and $\mathcal{M} = \cup_{P \in \mathscr{P}} \mathcal{M}(P)$. A *mechanism* is a function M that specifies a matching for any problem, i.e., $M : \mathscr{P} \to \mathcal{M}$ such that $\forall P \in \mathscr{P}, M(P) \in \mathcal{M}(P)$.

Before we define *pairwise stability* which is a central concept in matching theory, it is convenient to define *individual rationality* and *blocking pair*. A matching μ is *individually rational* if: *(i)* for all $H \in \mathcal{H}$, $\mu(H) \succcurlyeq_H \phi$ and *(ii)* for all $r \in \mathcal{R}$, $\mu(r) \succcurlyeq_r r$. For each pair of home and refugee $(H, r) \in \mathcal{H} \times \mathcal{R}$, we say that a matching μ is *blocked* by (H, r) or r has a (μ-)*justified envy* for H if: *(i)* $H \succ_r \mu(r)$ and *(ii)* $(\{r\} \cup (\mu(H) \setminus R)) \succ_H \mu(H)$ for some $R \subset \mu(H)$ such that $q_H \geq l(r) + l(\mu(H)) - l(R)$.[34] A matching μ is *pairwise stable* if μ is not blocked by any pair of home and refugee and μ is individually rational. A mechanism is *pairwise stable* if it specifies a pairwise stable matching for any problem.

Group stability is a stronger notion of stability. A matching μ is *(group) stable* if there exists no coalition $(\mathscr{H}, R) \in \mathcal{P}(\mathcal{H}) \times \mathcal{P}(\mathcal{R})$ of homes and refugees for which there exists some matching μ' such that: *(i)* $\mathscr{H} = \mu'(R)$, *(ii)* $\forall r \in R, \mu'(r) \succcurlyeq_r \mu(r)$, with at least one strict ordering and *(iii)* $\forall H \in \mathscr{H}, \mu'(H) \succcurlyeq_H \mu(H)$. A mechanism is *(group) stable* if it specifies a group stable matching for any problem.

As we are interested in finding matchings that meet some desirable properties,[35] the priority order for any consumer $H \in \mathcal{H}$ may be restricted to groups of refugees which size is below the capacity of H, i.e, $\{R \in \mathcal{P}(\mathcal{R}) : q_H \geq l(R)\}$. For simplicity, we make the following assumption.

Assumption 1. *A group of refugees $R \subset \mathcal{R}$ is unacceptable for a home H if and only if $q_H < l(R)$ or there is $r \in R$ such that $\varnothing \succ_H \{r\}$.*

Remark 1. *Let $R, R' \subset \mathcal{X}$ be two groups of refugees such that $R \subsetneq R'$. If both R and R' are acceptable for a home H then $R' \succ_H R$.*

Proof. Consider $\{r_1, r_2, ..., r_k\} \equiv R' \setminus R$. Since R' is not unacceptable, Assumption 1 insures that r_1, r_2, ..., r_k are all acceptable, that is for $\kappa \in \{1, ..., k\}$, $\{r_\kappa\} \succ_H \varnothing$. Then, by using responsiveness, iteratively we can obtain $R' \succ_H R$ by induction. ■

34. Note that R may be the empty set, in which case home H does not achieve its full capacity and it is desirable to assign an additional refugee r with a small enough size to H.

35. We will define precisely some desirable properties of matchings later.

This model is an extension of School Choice (Abdulkadiroglu and Sönmez, 2003).[36] In fact, if we suppose that every refugees has the same size which can therefore be normalized to one ($\forall r \in \mathcal{R}, l(r) = 1$), then the model becomes identical to School Choice where refugees stand for *Students* and homes represent *Colleges*.

In the case where every refugee has a unit size, by using the Gale and Shapley (1962) refugees-proposing Deferred Acceptance (DA) algorithm, it is possible to find the refugee-optimal stable matching, that is, a stable matching that is at least as good as any other stable matching for each refugee. This algorithm adapted to our framework goes as follows.

At the first step, each refugee is proposed for his favorite home. For each home H the highest priority group of refugees R, among those who have been proposed for H, such that $l(R) \leq q_H$ is temporarily assigned to H. The rest of refugees are rejected.

In general, at the kth step, each refugee who has been rejected at step $k-1$ is proposed for his next favorite home. For each home H the highest priority group of refugees R, among those who have been proposed for H at this step and the ones who has been assigned to H at step $k-1$, such that $l(R) \leq q_H$ is temporarily assigned to H. The rest of refugees are rejected.

The algorithm terminates when no refugee is rejected and each refugee is matched to the home that he is temporarily assigned to if any.

But if we allow for differently-sized refugees, we know from Delacrétaz (2014) that no pairwise stable mechanism exists and a problem may admit several Pareto-undominated pairwise stable matchings. He also points out that *Refugees-proposing Deferred Acceptance* algorithm fails sometimes to produce a pairwise stable matching even when it exists. We make the following assumption in other to guarantee the existence of a pairwise stable mechanism as we will show it in our Theorem 1 later.

36. It can also be seen as an extension of the House Allocation (Abdulkadiroglu and Sönmez, 1999) and House Exchange (Shapley and Scarf (1973)) if we relax the assumption of strict priorities by allowing indifferences in priorities and assume that homes and agents have unit capacities and unit sizes. Indeed, if we suppose that agents have same priorities, that is, for all $r, r' \in \mathcal{R}, r \sim_H r', \forall H \in \mathcal{H}$, then we get the standard House Allocation model. Moreover, if there is the same number of homes as refugees and each refugee owns exactly one home, we get the House Exchange model by defining priorities by: $\forall H \in \mathcal{H}$, the refugee to whom belongs H has the highest priority for it and all other refugees have the same priorities for H.

Assumption 2 (size-monotonicity)**.** *For each home $H \in \mathscr{H}$, and for every groups of refugees R, R' that are acceptable for H, if $l(R) > l(R')$ then $R \succ_H R'$.*

This assumption is crucial for the remaining of our analysis.[37] It states that a larger group of refugees has higher priority than a smaller one, regardless of which refugees are in the groups, if the both groups are acceptable.[38] A version of Assumption 2 can be found in Anderson and Ehlers (2016).

4. Group stability and pairwise stability

Here, we show that some standard results about stability in many-to-one matching models does not hold if we allow for different-size agents. The following example will be useful to fix ideas on some non-standard properties of this environment.

Example 1. *Consider a problem consisting of 3 refugees r_1, r_2 and r_3 and two homes H_1 and H_2 whose capacities are respectively $q_{H_1} = 2$ and $q_{H_2} = 1$. Refugees have priorities overs homes, respectively $\succ_{H_1}$ and $\succ_{H_2}$, defined by:*

H_1	H_2
$\{r_1, r_2\}$	$\{r_1\}$
$\{r_3\}$	$\{r_2\}$
$\{r_1\}$	
$\{r_2\}$	

The sizes of the refugees are $l(r_1) = l(r_2) = 1$ and $l(r_3) = 2$ and their preferences over homes are defined by :

r_1	r_2	r_3
H_1	H_2	H_1
H_2	H_1	H_2.

37. Without Assumption 2 the search of pairwise stable matching becomes NP-hard (Delacrétaz et al., 2016), i.e. one of the hardest computational problem (like the multiple knapsack problem).
38. For more on size-monotonicity, see Pàpai (2000).

Remark 2. *Group stability implies pairwise stability but the converse is not true.*

Proof. Suppose μ is blocked by (H, r). Then, $\exists R' \subset \mu(H)$ such that $H \succ_r r$ and $\{r\} \cup (\mu(H) \setminus R') \succ_H \mu^{-1}(H)$ with $q_H \geq l(r) + l(\mu(H)) - l(R')$. Consider such a R' and define $R \equiv \{r\} \cup (\mu(H) \setminus R')$, $\mathscr{H} \equiv \{H\}$ and the matching μ' by:

$$\forall r \in R, \quad \mu'(r) = H$$

$$\forall r \notin R, \quad \mu'(r) = r$$

From this definition, it is clear that:

(1) $\mu'(R) = \mathscr{H}$.

(2) $\forall r \in R, \mu'(r) \succ_r \mu(r),$ or $\mu'(r) = \mu(r)$ (with at least one strict ordering)

(3) $\forall H \in \mathscr{H}, \mu'(H) \succ_H \mu(H),$ or $\mu'(H) = \mu(H)$ (with at least one strict ordering).

Therefore, μ is not group stable. We conclude that if μ is group stable then it is also pairwise stable.

Now, we show that the converse is not true. Consider Example 1 and consider the matching μ defined by:

$$\mu = \begin{pmatrix} R_1 & R_2 \\ r_3 & r_1 \end{pmatrix}$$

The pair (R_1, r_1) does not block μ because H_1 would not be better off either by replacing r_3 by r_1 or by adding r_1 to r_3 since $\{r_1, r_3\}$ is unacceptable. The same remark goes for the pair (H_1, r_2). On the other hand, H_2 is matched to r_1 by μ which is his best choice so there exist no $r \in \{r_1, r_2, r_3\}$ such that μ is blocked by (H_2, r). Therefore, it is clear that μ is pairwise stable.

However, μ is blocked by the coalition $(\{r_1, r_2\}, \{H_1\})$. So, μ is not group stable. ■

Remark 3. *Group stable matching may not exist.*

Proof. Consider, the problem described in Example 1. The only pairwise stable matching for this problem is the matching μ defined by:

$$\mu = \begin{pmatrix} H_1 & H_2 \\ \\ r_3 & r_1 \end{pmatrix}$$

since the three other feasible matching

$$\mu = \begin{pmatrix} H_1 & H_2 \\ \\ r_1 r_2 & \varnothing \end{pmatrix}, \mu = \begin{pmatrix} H_1 & H_2 \\ \\ r_1 & r_2 \end{pmatrix}, \mu = \begin{pmatrix} H_1 & H_2 \\ \\ r_2 & r_1 \end{pmatrix},$$

are blocked by (H_2, r_2) for the first one and (H_1, r_3) for the others. Therefore, this problem does not admit any group stable matching, since μ is blocked by the coalition $(H_1, \{r_1, r_2\})$. ■

Now, before we present Theorem 1, it is convenient to introduce some definitions that will be essential for Lemma 1.

Definition 1. *Let μ' and μ be two matchings. We say that μ* has higher priority than μ'*, and we write $\mu' \succ \mu$, if*

$$\begin{cases} \exists \widehat{H} \in \mathcal{H} \text{ such that } \mu'(\widehat{H}) \succ_{\widehat{H}} \mu(\widehat{H}) \\ \\ \forall H \in \mathcal{H}, \quad \mu'(H) \nsucc_H \mu(H) \end{cases}$$

The transitivity and the acyclicity of $\succ$ are straightforward.

Definition 2. *A matching μ is quasi-stable if μ is individually rational and no matched refugee has a justified envy.*

It is worth noting that Definition 2 diverges from the definition of woman-quasi-stable matching proposed in Blum and Rothblum (2002) and Blum et al. (1997). We adopt this definition because it helps to make the point more easily. Note that pairwise stability implies quasi-stability. The following example show that the converse is not true.

Example 2. *Consider the problem for which the set of homes is* $\mathcal{H} = \{H_1, H_2\}$ *and the capacities of homes are* $q_{H_1} = 1$, $q_{H_2} = 2$; *the set of refugees is* $\mathcal{R} = \{r_1, r_1', r_2\}$ *and their sizes are* $l(r_1) = l(r_1') = 1$ *and* $l(r_2) = 2$.

H_1	H_2
$\{r_1\}$	$\{r_1, r_1'\}$
$\{r_1'\}$	$\{r_2\}$
	$\{r_1\}$
	$\{r_1'\}$

r_1	r_1'	r_2
H_2	H_1	H_2
H_1	H_2	H_1.

It is easy to see that $\mu \equiv \begin{pmatrix} H_1 & H_2 \\ r_1' & r_1 \end{pmatrix}$ *is quasi-stable but* μ *is not pairwise stable since it is blocked by* (H_2, r_2) *and* $\mu(r_2) = r_2$.

The next definition is crucial to our argument.

Definition 3. *Let* μ *be a quasi-stable matching such that there exists an unmatched refugee* $\widehat{r}$ *who has a* μ*-justified envy. Let* $\widehat{H}$ *be refugee* $\widehat{r}$*'s most-preferred home for which he has a* μ*-justified envy. We say that* μ' is obtained from μ by satisfying the justified envy of $\widehat{r}$ *if,*

$$\mu'(H) = \mu(H), \forall H \neq \widehat{H} \text{ and } \mu'(\widehat{H}) = \max_{\{\widehat{r}\} \cup \mu(\widehat{H})} \succ_{\widehat{H}}$$

where $\max_{\{\widehat{r}\} \cup \mu(\widehat{H})} \succ_{\widehat{H}}$ *is* $\widehat{H}$*'s most preferred group of refugee in* $\{\widehat{r}\} \cup \mu(\widehat{H})$.

A variant of Definition 3 is found in Blum and Rothblum (2002) for the stable mariage problem. It is clear from Definition 3 that since μ is individually rational μ' is also individually rational. Moreover, since $\mu'(\widehat{H}) \succ_{\widehat{H}} \mu(\widehat{H})$ and $\mu'(H) = \mu(H), \forall H \neq \widehat{H}$ then μ' has higher priority than μ. The following Lemma is a generalization of Blum and Rothblum (2002) Lemma 2.1.

Lemma 1. *Let μ be a quasi-stable matching such that there exists an unmatched refugee $\widehat{r}$ who has a μ-justified envy. The matching μ' obtained from μ by satisfying the justified envy of $\widehat{r}$ is quasi-stable.*

Proof. See Appendix ■

In other words, the matching μ' inherits the quasi-stability of μ while improving the priority level of assigned refugees. This interesting property allows to introduce the Sequential Greedy Correcting (SGC) algorithm initially proposed by Blum and Rothblum (2002) for the stable mariage problem. Let $\sigma : \mathcal{R} \rightarrow \{1, 2, \cdots, |\mathcal{R}|\}$ be a one-to-one correspondence that specifies a unique index for each refugee. The σ-Sequential Greedy Correcting (σ-SGC) is defined as follows.

Round 0: Number all the refugees from 1 to $|\mathcal{R}|$ according to σ.

Round k: Identify (if any) the refugee, say r, with the smallest index among those who have a justified envy for some home but are not temporarily assigned to any. If no such refugee exists, end the algorithm and match each refugee to the home that he is temporarily assigned to (if any). Otherwise, temporarily assign r to his favorite home, say H, that he has a justified envy for and reject (unassign) from H the lowest priority group of refugees necessary to fulfill the capacity requirement.

Theorem 1. *The outcome of a σ-SGC algorithm is a pairwise stable matching.*

Proof. Consider the σ-SGC algorithm and denote by μ_k the temporarily matching obtained at the end of Round k (in particular, μ_0 is defined by $\mu_0(r) = r, \forall r \in \mathcal{R}$). Note that for all $k > 0$, μ_k is obtained from μ_{k-1} by satisfying the justified envy of some unmatched refugee. Since, μ_0 is clearly a quasi-stable matching, Lemma 1 guarantees that, by induction, every μ_k is quasi-stable. Furthermore, for all $k > 0$, $\mu_k \succ \mu_{k-1}$. So, the algorithm converges since $\succ$ is acyclical and the set of all matchings for a given problem $\mathcal{M}(P)$ is finite. Therefore, the algorithm ends up with a quasi-stable matching for which there is no unmatched refugee that has a justified envy, that is, a pairwise stable matching. ■

5. Pareto-efficiency and Strategyproofness

There is a well-documented trade-off between efficiency and stability in matching theory [Roth, 1982; Kesten, 2010; Che and Tercieux, 2015] . Here, we consider two weaker versions of efficiency. First, a *Pareto-undominated pairwise stable matching* is a pairwise stable matching μ that is not Pareto dominated by any other pairwise stable matching, that is, for all pairwise stable matching μ', if $\mu'(r) \succeq_r \mu(r), \forall r \in \mathcal{R}$ then $\mu' = \mu$. A *Pareto-undominated pairwise stable mechanism* specifies a Pareto-undominated pairwise stable matching of every problem. Second, a matching μ is *refugee optimal stable* if it provides for each refugee the best possible outcome in a stable matching, that is, for all stable matching μ' and for all refugee $r \in \mathcal{R}$, $\mu(r) \succeq_r \mu'(r)$. A *refugee optimal stable mechanism* specifies a refugee optimal stable matching of every problem.

Example 3. *The outcome of a σ-SGC is not Pareto-undominated in general. Consider a problem consisting of 4 refugees r_1, r_1', r_2 and r_2' and two homes H_2 and H_2' whose capacities are $q_{H_2} = q_{H_2'} = 2$. Refugees have priorities over homes defined by:*

H_2	H_2'
$\{r_1, r_1'\}$	$\{r_1, r_1'\}$
$\{r_2'\}$	$\{r_2'\}$
$\{r_2\}$	
$\{r_1\}$	
$\{r_1'\}$	

The sizes of the refugees are $l(r_1) = l(r_1') = 1$ and $l(r_2) = l(r_2') = 2$ and their preferences over homes are defined by :

r_1	r_1'	r_2	r_2'
H_2	H_2	H_2	H_2'
H_2'	H_2'		H_2.

If σ is define by $\sigma(r_2) = 1$, $\sigma(r_1) = 2$, $\sigma(r_1') = 3$ and $\sigma(r_2') = 4$ then $\mu^{\sigma}_{SGC} = \begin{pmatrix} H_2 & H_2' \\ r_2' & r_1 r_1' \end{pmatrix}$ *which is Pareto dominated by the pariwise stable matching* $\begin{pmatrix} H_2 & H_2' \\ r_1 r_1' & r_2' \end{pmatrix}$.

We know from Blum and Rothblum (2002) that the σ-SGC produces the refugee optimal stable matching if $l(r) = 1, \forall r \in \mathcal{R}$ and $q_H = 1, \forall H \in \mathcal{H}$. The following proposition extends this result.

Proposition 1. *If all refugees have the same size then the σ-SGC is a refugee optimal stable mechanism.*

Proof. Here, for the sake of convenience, we introduce a fictitious home that is the least preferred home for every refugee and which capacity is large enough to contain all refugees (the priorities for this fictitious home can be set arbitrarily). Clearly, the presence of this additional home does not change the outcome of the σ-SGC algorithm except for the fact that every refugee that remained unmatched at the end of the algorithm in the regular problem will now be matched to the fictitious home.

Now, let μ be a stable matching. Suppose that all refugees have the same size and suppose, by contradiction, that there is a refugee that has been matched by μ to a home that he prefers to the one to which he is assigned under σ-SGC. Therefore, there must be some round k, of the algorithm, in which a refugee, say r, has been temporarily assigned to a home, $\mu_k(r)$, such that $\mu(r) \succ_r \mu_k(r)$, for the first time, where μ_k is the matching representing the temporal assignment in round k. From Lemma 1 we know that μ_k is fair. For simplicity, denote by H the home to whom r is matched by μ, that is, $H \equiv \mu(r)$.

Therefore, every refugee that is temporarily assigned to H at round k has higher priority for H than r, that is, $\forall r' \in \mu_k(H), r' \succ_H r$, otherwise r would have not been assigned temporarily to a least preferred home, $\mu_k(r)$, at this round. Moreover, since r is the first refugee that is temporarily assigned to a home that is less desirable for him than the home to which it is assigned under μ, we must have $\forall r' \in \mu_k(H), H \succeq_{r'} \mu(r')$.

But, if $\forall r' \in \mu_k(H), \mu(r') = H$ then $\mu_k(H) \subset \mu(H) \setminus \{r\}$ and then, by responsiveness, $\mu_k(H) \cup \{r\} \succ_H \mu_k(H)$, which contradicts the fairness of μ_k because $H \succ_r \mu_k(r)$. Therefore, there exists at least one good $r' \in \mu_k(H)$ such that $H \succ_{r'} \mu(r')$ and then, since $r' \succ_H r$ and $l(r) = l(r')$, the pair (H, r') blocks μ which contradicts its stability.

Therefore, if all refugees have the same size then there is a no refugee that has been matched by μ to a home that he prefers to the one to which he is assigned under σ-SGC, that is, σ-SGC is a refugee optimal stable mechanism. ■

Several interesting remarks follows from Lemma 1. First, we know from Abdulkadiroglu and Sönmez (2003) that, when refugees have the same size, the refugees-proposing DA algorithm outcome is the unique (pairwise) stable matching that assigns to each refugees the home that is at least as good has any other home that he can possibly been matched to in a (pairwise) stable matching. Therefore, from Proposition 1, the σ-SGC algorithm is output equivalent to the refugees-proposing DA algorithm when refugees have the same size.

Therefore, by uniqueness of the refugee optimal stable matching, we deduce that the outcome of the σ-SGC is independent of the ranking order chosen in Round 0 when refugees have the same size. Generally, the outcome of the σ-SGC depends on the ranking order chosen at Round 0, σ.

Definition 4. *The* ***Downward Sequential Greedy Correcting*** *(DSGC) is the σ-SGC for which the refugees are ranked from the* ***biggest to the smallest*** *in Round 0. The* ***Upward SGC*** *(USGC) is the σ-SGC for which the refugees are ranked from the* ***smallest to the biggest*** *in Round 0.*

Interestingly, when the refugees' sizes may differ from one another DSGC is still a well-defined[39] mechanism which outcome can be obtained by consecutively running the refugees-proposing DA algorithm in the following way.

Round 1: Run the refugees-proposing DA algorithm only on the refugees that have the highest size. Then, match the highest-size refugees to homes as it is

39. The outcome of the DSGC does not depend on the specific ranking order chosen in Round 0 for refugees that have the same size.

suggested by the DA and redefine the capacity of each home by reducing it by the total size of the refugees that have been matched to it.

Round k: Run the refugees-proposing DA algorithm only on the refugees that have the kth highest size by considering the capacity of homes as it is defined in Round $k-1$. Then, match the kth-highest-size refugees to homes as it is suggested by the DA and redefine the capacity of each home by reducing it again by the total size of the refugees that have been matched to it at this round. Stop the algorithm if there is no kth-highest-size refugee.

This Consecutive Deferred Acceptance (CDA) algorithm stops in n rounds if n is the number of different refugees' size levels.

In fact, due to preference monotonicity no refugee of smaller size can "kick" a refugee of a bigger size out in a pairwise improvement. Therefore, a pairwise improvement that involve a refugee of a smaller size does affect the assignment of refugees of bigger size in a fair matching. And since the DSGC algorithm for identical-size refugees is output equivalent to the DA algorithm, running the DA consecutively on agents from to biggest to the smallest give the same result as the DSGC. This remark shows that the DSGC run in a polynomial time since the DA algorithm does.

Theorem 2. *DSGC is a Pareto-undominated pairwise stable mechanism.*

Proof. We prove Theorem 2 by strong induction. Let μ_{DSGC} be the outcome of the DSGC algorithm and μ a pairwise stable matching such that $\mu(r) \succeq_r \mu_{IOA}(r)$, for all $r \in \mathcal{R}$. We know that DSGC is output equivalent to CDA. And, at the first round of the CDA procedure, every highest-size refugee has its best possible match in a stable matching, that is, for all $r \in \arg\max_{r' \in \mathcal{R}} l(r')$, $\mu(r) = \mu_{DSGC}(r)$.

Let n be the number of different refugees' size levels and suppose that for a given $1 \leq k < n$ and for all $r \in \mathcal{R}$ such that $l(r)$ is greater or equal to the kth highest refugees'size, we have $\mu(r) = \mu_{DSGC}(r)$.

Now, we prove that for any good r such that $l(r)$ is the $(k+1)$-th highest refugees'size, one must have $\mu(r) = \mu_{DSGC}(r)$. Suppose, by contradiction, that for a $(k+1)$-th highest-size refugee $\mu(r) \succ_r \mu_{DSGC}(r)$, that will mean that the outcome of the DA algorithm run at Round $k+1$ is not refugee optimal stable which contradicts Proposition 1.

Therefore the outcome of the DSGC is not dominated by another pairwise stable matching. ■

In particular, the outcome of the USGC is not necessarily Pareto undominated.

Example 4. *They may exist some Pareto undominated pairwise stable matching that cannot be reached by any σ-SGC.*

Consider a problem consisting of 4 refugees r_1, r_3, r_3 and r_4 and two homes H_5 and H_6 whose capacities are $q_{H_5} = 5$ and $q_{H_6} = 6$. Refugees have priorities over homes defined by:

H_5	H_6
$\{r_1, r_4\}$	$\{r_3, r_3'\}$
$\{r_4\}$	$\{r_1, r_4\}$
$\{r_1, r_3\}$	$\{r_1, r_3\}$
$\{r_1, r_3'\}$	$\{r_1, r_3'\}$
$\{r_3\}$	$\{r_3\}$
$\{r_3'\}$	$\{r_3'\}$
$\{r_1\}$	$\{r_1\}$

The sizes of the refugees are $l(r_1) = 1$ and $l(r_3) = l(r_3') = 3$ and $l(r_4) = 4$ their preferences over homes are defined by:

r_1	r_3	r_3'	r_4
H_6	H_5	H_5	H_6
H_5	H_6	H_6	H_5.

$\begin{pmatrix} H_5 & H_6 \\ r_3 r_3' & r_1 r_4 \end{pmatrix}$ *is a Pareto-undominated pairwise stable matching that cannot be reached by any σ-SGC.*

Now, we study strategic behavior of agent in a direct revelation mechanism were refugees submit their preferences to a centralized clearing house. Informations about the mechanism used by the clearing house is common knowledge. Define by Π the set of strict preferences over homes $\mathcal{H}$ and by $\Pi^{|\mathcal{R}|}$ the set of preference profile of refugees. We assume that the sets of refugees and homes ($\mathcal{R}$ and $\mathcal{H}$), the capacity profile q, the sizes of refugees l and their priorities for homes $(\succ_H)_{H\in\mathcal{H}}$ are fixed. Then a problem P is entirely defined by the preference profile of refugees over homes, i.e, $P = (\succ_r)_{r\in\mathcal{R}} \in \Pi^{|\mathcal{R}|}$.

A mechanism $\mathcal{M}$ is *strategy-proof* if no refugee can get better off by misrepresenting his preferences, that is, $\forall P \in \Pi^{|\mathcal{R}|}, \forall r \in \mathcal{R}$ and $\forall P'_r \in \Pi$, $\mathcal{M}(P)(r) \succeq_r \mathcal{M}(P_{-r}, P'_r)(r)$ where (P_{-r}, P'_r) is the preference profile obtained by replacing from P the preferences of refugee r by P'_r, the rest remaining unchanged.

Now, consider the procedure of CDA which is output equivalent to DSGC. Suppose that the preferences of some kth-highest-size refugee, say r is misreported. The procedure of the CDA remain unchanged until Round k. At this round, since the refugees-proposing DA is strategy-proof for refugees, then the outcome for refugee r would not be better. Theorem 3 follows immediately from this remark.

Theorem 3. *The DSGC is group strategy-proof for refugees.*

Example 5. *A σ-SGC is not strategy proof in general*

Consider a problem consisting of 4 refugees r_1, r'_1, r''_1 and r_2 and two homes H_1 and H_2 whose capacities are $q_{H_1} = 1$ and $q_{H_2} = 2$. Refugees have priorities over homes defined by:

H_1	H_2
$\{r''_1\}$	$\{r_1, r'_1\}$
$\{r_1\}$	$\{r_2\}$
	$\{r_1\}$
	$\{r'_1\}$

The sizes of the refugees are $l(r_1) = l(r'_1) = l(r'') = 1$ and $l(r_2) = 2$ and their preferences over homes are defined by :

r_1	r_1'	r_1''	r_2
H_1	H_2	H_1	H_2
H_2	H_1	H_2	H_1.

Consider a ranking σ défini par $\sigma(r_1) = 1$, $\sigma(r_1') = 2$, $\sigma(r_2) = 3$ and $\sigma(r_1'') = 4$. We have $\mu^{\sigma}_{SGC} = \begin{pmatrix} H_1 & H_2 \\ r_1'' & r_2 \end{pmatrix}$. Notice that Refugee r_1 is unmatched by μ^{σ}_{SGC}. However, r_1 could misrepresent his preferences by submitting $H_2 \succ_{r_1} H_1$ and then get matched to H_2 by σ-SGC.

6. Maximality

A well-known result in matching theory is the Roth (1986) Rural Hospitals Theorem which states that when all refugees have unit sizes the set of unmatched refugees remain the same in any pairwise stable matching. This result no longer holds in our environment as it is shown in the following example.

Example 6. *The problem consists of three homes H_1, H_2 and H_3 with respective capacities $q_{H_1} = 4$, $q_{H_2} = 2$ and $q_{H_3} = 2$ and four refugees r_1, r_2, r_3 and r_4 with respective sizes $l(r_1) = 1$, $l(r_2) = 2$, $l(r_3) = 2$ and $l(r_4) = 3$. The priorities of refugees for the two homes are given by*

H_1	H_2	H_3
$\{r_1, r_4\}$	$\{r_2\}$	$\{r_3\}$
$\{r_2, r_3\}$	$\{r_3\}$	$\{r_2\}$
$\vdots$	$\{r_1\}$	$\{r_1\}$
$\{r_4\}$		
$\vdots$		

and the preferences of refugees over homes are given by

r_1	r_2	r_3	r_4
H_2	H_1	H_1	H_1
H_3	H_2	H_3	$\vdots$
H_1	$\vdots$	$\vdots$	

The two following matching are pairwise stable (they are also group stable).

$$\mu_{DSGC} = \begin{pmatrix} H_1 & H_2 & H_3 \\ \\ r_2 r_3 & r_1 & \varnothing \end{pmatrix}, \mu_{USCG} = \begin{pmatrix} H_1 & H_2 & H_3 \\ \\ r_1 r_4 & r_2 & r_3 \end{pmatrix}.$$

It is clear that the number of unmatched refugees differs from one pairwise matching to the other and also the total size of unmatched refugees is different. Note also that μ_{USGC} has higher priority than μ_{DSGC}.

For a given problem, a matching μ is said *maximal* if there is no matching μ' such that

$$l(\{r \in \mathcal{R}/\mu'(r) \in \mathcal{H}\}) > l(\{r \in \mathcal{R}/\mu(r) \in \mathcal{H}\}),$$

that is, a matching that guarantees that the maximal number of refugee families' members are matched. For instance, the matching μ_{USGC} in Example 6 is maximal whereas μ_{DSGC} is not. Anderson and Ehlers (2016) points out that maximality, stability and Pareto-efficiency are independent in the sens that none of these properties implies one of the others. And, ultimately, a maximal matching exists for any given problem P since the set of matchings $\mathcal{M}(P)$ is finite.

7. Conclusion

This paper investigates an extension of the standard many-to-one matching model that involve different-size agents (refugees) having size-monotonic preferences over objects (homes) with different quotas (capacities). We show that a pairwise stable matching exits in this framework and we propose an adaptation of the Sequential Greedy Correcting algorithm to spot it. Our mechanism is strategy-proof and Pareto-undominated from the point of view of refugees and converge in a polynomial time. Nevertheless, this algorithm does not necessarily yield a maximal matching. Further study could reveal how to find matches that are both pairwise stable and maximal.

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Chapitre A

Annexes

Proofs from Article 1

Proof for Proposition 1. Suppose that $G^A = G^B = 0$. First we show that no tax is paid in an equilibrium. Suppose, by contradiction, that non zero tax is paid in some region j. Since government j's budget constraint binds (see Remark 2), there exists in that region a type of agents, say i, that pay positive taxes which are used by the government to finance subsidies to the other type $-i$. Two cases need two be distinguished. First, If some $-i$-type agents live also in region $-j$, with $-j \neq j$, then they must receive the same subsidy from that government financed by some i-type living there. The government of region $-j$ could therefore increase the social welfare of its region by getting rid of the $-i$-type using a zero tax policy. Indeed, if no tax is paid in region $-j$ the $-i$-type agent will all migrate toward region j and the utility of the i-type will increase resulting in an increase in social welfare in $-j$. Second, if no $-i$-type agents live in region $-j$, its government could attract all the i-type agent with a zero tax policy which would increase the social welfare in $-j$ by increasing the utility of the i-type. In both cases, a zero tax policy, which trivially is incentive compatible, is a profitable deviation for $-j$. Therefore, zero tax is paid by both types in any equilibrium.

Now it becomes easy to show that all L-type agents live in the same region in the subgame defined by $G^A = G^B = 0$. If L-type agents live in both region at some equilibrium, from what precedes, they must pay zero tax in both region. Since the

H-type agents also pay zero tax, it is possible for the government of the region where reside some H-type agents to impose a small incentive compatible positive tax on the L-type. Doing so, all the L-type agents would flee out to the other region and, thus, leave alone the H-type agents with a higher social welfare. ■

Proof for Proposition 2. Suppose that the public good provision is non-zero in at least one region, that is, $G^A > 0$ and $G^A \geq G^B \geq 0$, without loss of generality. Further suppose, by contradiction, that there exists some type $i \in \{L, H\}$ such that agents of type i are present in both regions, which implies $U_i^A = U_i^B$. First, suppose that i-type agents pay negative tax in at least one region say j. In this case, a profitable deviation for government j would instead consist of getting rid of the i-type by increasing their tax to the same non-negative tax that $-i$-type would need to pay if they were the only resident of the region, $\frac{G^j}{n_{-i}}$. That deviation is incentive compatible since it requires every agent to pay the same amount of tax. Therefore it must be the case that i-type agents pay non-negative tax in both region.

It must also be the case that i-type agents pay zero tax in both regions, otherwise the region who charges strictly positive taxes, for i-type agents could have more agents of this type by reducing slightly their tax liabilities by some $\varepsilon > 0$. This reduction can be made in such a way that i-type agents still pay a positive tax that will generate a budget surplus. That surplus could eventually be used to increase the utility of the other type in such a way that the incentive constraint remains satisfied. More precisely, suppose, by contradiction, that region j's equilibrium tax schedule is given by $(c_i^j, y_i^j, c_{-i}^j, y_{-i}^j)$ with $t_i^j = y_i^j - c_i^j > 0$. From what precedes we also have $t_{-i}^j = y_{-i}^j - c_{-i}^j \geq 0$. According to Remark 2, the budget constraint is binding in region j, that is, $n_i^j(y_i^j - c_i^j) + n_{-i}^j(y_{-i}^j - c_{-i}^j) = G^j$. Now, consider a deviation $(\widehat{c}_i^j, y_i^j, \widehat{c}_{-i}^j, y_{-i}^j)$ where $\widehat{c}_i^j = c_i^j + \varepsilon$ and $\widehat{c}_{-i}^j = c_{-i}^j + \varepsilon$, with $\varepsilon > 0$. It is clear that this deviation is incentive compatible since $(c_i^j, y_i^j, c_{-i}^j, y_{-i}^j)$ is. So, if this deviation satisfies the budget constraint,

$$n_i(y_i^j - \widehat{c}_i^j) + \widehat{n}_{-i}^j(y_{-i}^j - \widehat{c}_{-i}^j) \geq G^j, \tag{A.1}$$

where $\widehat{n}_{-i}^j$ is the resulting number of $-i$-type agents in region j, then it would be profitable for j since it increases consumption for everyone. Note that $\widehat{n}_{-i}^j$ is equal to n_{-i} or n_{-i}^j depending on whether the deviation attracts the $-i$-type into region j or not.

Equation (A.1) is equivalent to $n_i(y_i^j - c_i^j) + \hat{n}_{-i}^j(y_{-i}^j - c_{-i}^j) \geq G^j + (n_i + \hat{n}_{-i}^j)\varepsilon$ which is true if and only if

$$(n_i - n_i^j)t_i^j + (\hat{n}_{-i}^j - n_{-i}^j)t_{-i}^j \geq (n_i + \hat{n}_{-i}^j)\varepsilon. \tag{A.2}$$

By hypothesis, $(n_i - n_i^j)t_i^j > 0$. Since $(\hat{n}_{-i}^j - n_{-i}^j)t_{-i}^j \geq 0$, therefore (A.2) is satisfied for some small enough value of ε which implies that $(\hat{c}_i^j, y_i^j, \hat{c}_{-i}^j, y_{-i}^j)$ is actually a profitable deviation for j. So, i-type agents pay zero tax in both regions.

Since $G^A > 0$, it must also be the case that $-i$-type agents, with $-i \neq i$, pay some positive tax in region A. Therefore, no such agent reside in region B otherwise government A could reduce slightly taxes for $-i$-type agents and, by doing so, attract those who live in B. A budget surplus will be generated thanks to the contribution of the news arrival and that surplus could finance a subsidy for the i-type who initially paid zero tax. So, for the budget constraint in region B to hold it must be the case that $G^B = 0$, since there is no $-i$-type to finance the public good and as the i-type agents pay zero tax.

Now, we have $h(G^A) > h(G^B) = 0$. It is possible for region A to deploy a tax schedule that would attract the i-type agents while charging them with a positive tax. The budget surplus generated by this tax could be used to increase the utility of the other type living in A in such a way that the tax schedule remains incentive-compatible. Consider, for instance, the following tax schedule defined $\widehat{T}^A(y)$ for a small $\varepsilon > 0$ as a function of the before-tax income :

$$\widehat{T}^A(y) = \begin{cases} \dfrac{G^A}{n_L + n_H} - \varepsilon & \text{if } y = y_i^* \\ \dfrac{G^A}{n_L + n_H} + \dfrac{n_i}{n_{-i}}\varepsilon & \text{if } y = y_{-i}^* \\ M & \text{otherwise.} \end{cases}$$

where $M > 0$ is a so big that it is optimal for each type to chose either y_i^* or y_{-i}^*. If ε is small enough i-type and $-i$-type agents would choose respectively y_i^* or y_{-i}^*. First, since $G^A \leq \overline{G}$ we have $(n_L + n_H)h(G^A) \geq G^A$, and then the utility of the i-type agents would be higher than their equilibrium utility U_i^B for all $\varepsilon > 0$:

$$v_i^* + h(G^A) - \frac{G^A}{n_L + n_H} + \varepsilon > v_i^* \geq U_i^B = U_i^A.$$

Second if ε is small enough, the utility of the i-type also would be higher than their equilibrium utility U^A_{-i}

$$v^*_{-i} + h(G^A) - \frac{G^A}{n_L + n_H} - \frac{n_i}{n_{-i}}\varepsilon > v^*_{-i} + h(G^A) - \frac{G^A}{n_{-i}} \geq U^A_{-i}.$$

Finally, the government budget contraint would still hold in region A because $n_i\widehat{T}^A(y^*_i) + n_{-i}\widehat{T}^A(y^*_{-i}) = G^A$. Therefore, the tax schedule $\widehat{T}^A$ would induce a higher utility for both type in region A and then would be a profitable deviation for government A whether information is symmetric or not. That contradicts the conditions for an equilibrium and, therefore, no type of agent could be present in both regions. ■

Proof for Proposition 3. First, we show that a pooling equilibrium always exists under condition (4.1). Without loss of generality, suppose that

$$(n_L + n_H)h(G^A) - G^A \geq (n_L + n_H)h(G^B) - G^B, \tag{A.3}$$

and consider the pair of tax schedules $T^A(y)$ and $T^B(y)$ defined as a function of the before-tax income y by

$$T^A(y) = \frac{G^A}{n_L + n_H}, \qquad T^B(y) = \frac{G^A}{n_L + n_H} - (h(G^A) - h(G^B)).$$

Now, we show that $(T^A(\cdot), T^B(\cdot))$ is a pooling equilibrium supported by $n^A_L = n_L$ and $n^A_H = n_H$. Note that, since $T^A(\cdot)$ and $T^B(\cdot)$ are uniform tax schedules, both agents will choose an efficient income level wherever they live. In other words, these tax schedules are not distortionary, that is, they also induce an efficient level of production/income, y^*_i, for each type wherever they live. Note also that a pooling distribution of agents is compatible with $(T^A(\cdot), T^B(\cdot))$ since it induces the same utility for each type in both regions. Denote by U^j_i the utility induce by $(T^A(\cdot), T^B(\cdot))$ for an i-type who lives in region j.

It follows that government A has no profitable deviation because it cannot increase the utility of one type without reducing that of the other type which will push them to flee out to region B. The former type would therefore be left in region A with a higher tax burden so that the budget constraint still holds. This will push them to leave region A in turn.

There is no profitable deviation for region B either. To see why, note that i-type agents receive $U_i^A = v_i^* + h(G^A) - \frac{G^A}{n_L+n_H}$ in region A where v_i^* represent the maximum before-tax utility that i-type agents could get from consomption and labor. For all profitable deviation from B designed to attract both types by granting them $\widehat{U}_i^B$, there exist two tax levels $\widehat{T}_L^B$ and $\widehat{T}_H^B$ that would satisfy the budget constraint in B $n_L\widehat{T}_L^B + n_H\widehat{T}_H^B \geq G^B$ such that

$$U_L^A < \widehat{U}_L^B \leq v_L^* - \widehat{T}_L^B + h(G^B) \tag{A.4}$$

$$U_H^A < \widehat{U}_H^B \leq v_H^* - \widehat{T}_H^B + h(G^B). \tag{A.5}$$

Adding Equations (A.4) and (A.5) together after multiplying them by n_L and n_H, respectively, gives

$$n_L U_L^A + n_H U_H^A < n_L(v_L^* - \widehat{T}_L^B + h(G^B)) + n_H(v_H^* - \widehat{T}_H^B + h(G^B)).$$

It follows that $(n_L + n_H)h(G^A) - G^A < (n_L + n_H)h(G^B) - G^B$ which contradicts (A.3). Similarly, any deviation from government B that aims to attract only one type of agents, say i, would have to grant them a utility level $\widehat{U}_i^B$ such that $\widehat{U}_i^B > U_i^A$ while imposing to them a tax requirement not smaller than G^B/n_i. However, we have $U_i^A \geq v_i^* + h(G^B) - \frac{G^B}{n_L+n_H} > v_i^* + h(G^B) - \frac{G^B}{n_i} \geq \widehat{U}_i^B$ which is a contradiction. Therefore, $(T^A(\cdot), T^B(\cdot))$ is a pooling equilibrium.

Now, we show that if (4.1) is false, then there does not exist a pooling equilibrium in j. Assume $(n_L+n_H)h(G^j)-G^j < (n_L+n_H)h(G^{-j})-G^{-j}$, and suppose, by contradiction, that the tax schedule $\tau^j = (c_L, y_L, c_H, y_H)$ is deployed by region j in a pooling equilibrium supported by $n_L^j = n_L$ and $n_H^j = n_H$. Consider the following possible deviation for region $-j$:

$$\widehat{\tau}^{-j} = (c_L^j + \frac{G^j - G^{-j}}{n_L + n_H}, y_L^j, c_H^j + \frac{G^j - G^{-j}}{n_L + n_H}, y_H^j).$$

First, $\widehat{\tau}^{-j}$ is more profitable for each type than τ^j, because it induces for an i-type agent a utility $\widehat{U}_i^{-j}$ such that

$$\begin{aligned}\widehat{U}_i^{-j} &= c_i^j + \frac{G^j - G^{-j}}{n_L + n_H} - v(y_i^j/w_i) + h(G^{-j}) \\ &= c_i^j - v(y_i^j/w_i) + h(G^j) + \frac{G^j - G^{-j}}{n_L + n_H} + h(G^{-j}) - h(G^j) \\ &> c_i^j - v(y_i^j/w_i) + h(G^j).\end{aligned} \tag{A.6}$$

Moreover, if information is asymmetric, $\widehat{\tau}^{-j}$ would be incentive compatible because τ^j would, and for all $i, -i \in \{L, H\}$, $c_i^j - v(y_i^j/w_i) \geq c_{-i}^j - v(y_{-i}^j/w_i)$ implies

$$c_i^j + \frac{G^j - G^{-j}}{n_L + n_H} - v(y_i^j/w_i)) \geq c_{-i}^j + \frac{G^j - G^{-j}}{n_L + n_H} - v(y_{-i}^j/w_i).$$

Finally, the budget constraint of government $-j$ would not be violated if all agents flee to region $-j$ because

$$\begin{aligned}n_L\left(y_L - c_L - \frac{G^j - G^{-j}}{n_L + n_H}\right) + n_H\left(y_H - c_H - \frac{G^j - G^{-j}}{n_L + n_H}\right) &= n_L t_L + n_H t_H - G^j + G^{-j} \\ &\geq G^{-j},\end{aligned} \tag{A.7}$$

where $t_i = y_i - c_i, \forall i \in \{L, H\}$. Therefore, $\widehat{\tau}^{-j}$ is a profitable deviation for $-j$, so there does not exist a pooling equilibrium in j if (4.1) is false and that ends the proof.

■

Proof for Proposition 4. We prove the result for $j = A$ and $-j = B$ without loss of generality. Let (G^A, G^B) define a subgame and consider a tax schedule profile (τ^A, τ^B) compatible with the following separating distribution of agents : $n_H^A = n_H$ and $n_L^B = n_L$, in that subgame. For each region there exists only three conceivable deviations that could potentially be profitable for that region ; (i) the government could change its fiscal policy so as to maximise its social welfare without changing the distribution of agents throughout the regions ; (ii) the government could also deploy a tax schedule that aims to substitute its current residents for the other type ; (iii) the government could design a tax schedule to bring all agents together in its region. (τ^A, τ^B) is an equilibrium in the subgame if none of these deviations is profitable for either region.

First, the kind of deviations described in (i) are not profitable if and only if in each region, every agent provides an efficient labour and the budget constraint is satisfied. So, the utility of the H-types U_H^A and the L-type U_L^B should be :

$$U_H^A = v_H^* - \frac{G^A}{n_H} + h(G^A)$$

$$U_L^B = v_L^* - \frac{G^B}{n_L} + h(G^B).$$

Second, if (iii) is not profitable then (ii) is not either. In fact, if a government, say j, could be better off by replacing its current residents, say i-type agents, by the other type of agents, say $-i$, then it must be the case that the $-i$-type agents would rather pay $\frac{G^j}{n_{-i}}$ instead of living in the region $-j$, with $-j \neq j$. Government j could then also keep both types via the uniform tax schedule $\widehat{T}^j(y) = \frac{G^j}{n_L+n_H}, \forall y \geq 0$ because $\widehat{T}^j(y)$ would be preferable not only for its current residents but also for the other type since $\frac{G^j}{n_L+n_H} < \frac{G^j}{n_i}, \forall i \in \{L, H\}$.

Therefore, if information is symmetric, then there is no profitable deviation for either region if and only if the government of one region cannot improve its social welfare by attracting the agents residing in the other region with any positive tax liability. That is,

$$v_L^* - \widehat{t}_L^A + h(G^A) \leq \max\{U_L^B, U_H^A\}, \quad \forall \widehat{t}_L^A \geq 0 \tag{A.8}$$

$$\text{and } v_H^* - \widehat{t}_H^B + h(G^B) \leq \max\{U_L^B, U_H^A\}, \quad \forall \widehat{t}_H^B \geq 0 \tag{A.9}$$

which is equivalent to

$$v_L^* + h(G^A) \leq \max\{U_L^B, U_H^A\}, \tag{A.10}$$

$$\text{and } v_H^* + h(G^B) \leq \max\{U_L^B, U_H^A\}. \tag{A.11}$$

Note that (A.11) is equivalent to $v_H^* + h(G^B) \leq U_H^A$ since $U_H^B \geq v_H^* + h(G^B)$. Therefore, equations (A.11) and (A.10) are equivalent to

$$\begin{cases} \min\left\{\dfrac{G^A}{n_H} - (v_H^* - v_L^*), \dfrac{G^B}{n_L} + h(G^A) - h(G^B)\right\} \leq 0 \\ 0 \leq \dfrac{G^A}{n_H} \leq h(G^A) - h(G^B) \end{cases} \tag{A.12}$$

Since $0 \leq h(G^A) - h(G^B)$, then $\dfrac{G^B}{n_L} + h(G^A) - h(G^B) \geq 0$. In addition, $\frac{G^B}{n_L} + h(G^A) - h(G^B) = 0$ if and only if $G^A = G^B = 0$, so

$$\min\left\{\frac{G^A}{n_H} - (v_H^* - v_L^*), \frac{G^B}{n_L} + h(G^A) - h(G^B)\right\} \leq 0 \Leftrightarrow \frac{G^A}{n_H} \leq v_H^* - v_L^*.$$

Therefore, Equation (A.12) is equivalent to $\dfrac{G^A}{n_H} \leq \min\{v_H^* - v_L^*, h(G^A) - h(G^B)\}$ which, in turn, is equivalent to $U_H^A \geq \max\{v_L^* + h(G^A), v_H^* + h(G^B)\}$. The tax schedules (τ^A, τ^B) can, therefore, be defined by $t_L^A = G^A/n_H$, $t_H^B = G^B/n_L$, t_L^A and t_H^B where t_L^A and t_H^B are so low that (τ^A, τ^B) is compatible with a separating distribution of agents. This proves the first equation of the proposition.

On the other hand, if information is asymmetric, gathering all agents in one region is not profitable to region A if and only if the resulting social welfare is not greater than U_H^A whenever it is possible for the government of A to propose an attractive contract to the type-L agents :

$$\max\{U_H^A, U_L^B\} \geq u_L^M(G^A) + h(G^A). \tag{A.13}$$

The incentive constraint in region B and the self-selection contraints for H-type imply $U_H^A \geq U_H^B > U_L^B$. Equation (A.13) is, therefore, equivalent to

$$U_H^A \geq u_L^M(G^A) + h(G^A).$$

Similarly, a profitable deviation for B consisting of gathering all agents in region B does not exist if and only if any fiscal policy that guarantees at least an utility level greater than $u_H^A + h(G^A) - h(G^B)$ for type-H agents would decrease the utility of type-L agents :

$$v_L^* - \frac{G^B}{n_L} \geq u_L(U_H^A - h(G^B), G^B). \tag{A.14}$$

Equation (A.14) is equivalent to

$$U_H^A \geq v_H^* - \delta + h(G^B) \tag{A.15}$$

where $\delta = v_H^* - u_L^{-1}(v_L^* - \frac{G^B}{n_L}; G^B)$. Note that $\delta \geq 0$ because otherwise it would be possible for government B to provide a utility level of $v_L^* - \frac{G^B}{n_L}$ to the Ls while granting to the Hs a utility higher than v_H^*.

Now we need to define the tax schedules deployed by the governments as to make them compatible with a pooling distribution of agents when information is asymmetric. This tax schedule can be defined by $(\tau^A, \tau^B) = \Big((u_L^A, u_H^A, y_L^A, y_H^A), (u_L^B, u_H^B, y_L^B, y_H^B)\Big)$, where $u_i^j = c_i^j - v(y_i^j/w_i)$. if (τ^A, τ^B) is a feasible separating Nash equilibrium with the H-type agents in region A and L-type agents in region B then it must be the case that the government of each region proposes the highest utility level to its residents given its budget constraint, that is,

$$(y_H^A, y_L^B) = (y_H^*, y_L^*) \text{ and } (u_H^A, u_L^B) = (v_H^* - G^A/n_H, v_L^* - \frac{G^B}{n_L}) \tag{A.16}$$

and then u_L^A and u_H^B are set low enough so that the incentive constraints are satisfied in each region. Moreover the self-selection constraints require that the utility of a type-L (*resp.* type-H) agent is greater when he lives in region B (*resp.* region A), that is,

$$u_H^A + h(G^A) \geq y_L^B - v(y_L^B/w_H) + h(G^B) - G^B/n_L \tag{A.17}$$

$$u_L^B + h(G^B) \geq y_H^A - v(y_H^A/w_L) + h(G^A) - G^A/n_H. \tag{A.18}$$

which yields the following equation, given (A.16).

$$v(y_L^*/w_L) - v(y_L^*/w_H) \leq \Delta v + \Delta h + G^B/n_L - G^A/n_H \leq v(y_H^*/w_L) - v(y_H^*/w_H). \tag{A.19}$$

Thus, (A.13), (A.14) and (A.19) are sufficient and necessary conditions for a separating equilibrium. ■

Lemma 0. *Let $(G^A, G^B) \in [0, \overline{G}]^2$ define a subgame. For all strategy profile compatible with a pooling distribution in j that satisfies the budget constraint in j and the self-selection constraints, there is no pooling profitable deviation for B if :*

$$(n_L + n_H)h(G^j) - G^j \geq (n_L + n_H)h(G^{-j}) - G^{-j}. \tag{A.20}$$

Proof for Lemma 0. Because profitable deviation in the case of asymmetric information requires additional constraints to be satisfied (namely the incentive constraints), it is obvious that if there is no profitable deviation in the case of symmetric information, then there cannot be one when information is asymmetric. Therefore, this proof addresses only the case of symmetric information. We prove the lemma for $j = A$. Consider a couple of tax schedules (τ^A, τ^B) compatible with a pooling distribution of agents in region A that satisfies the budget constraint and the self-selection constraints. Since information is symmetric, τ^j is entirely defined by a couple (t_L^j, t_H^j) where t_i^j is a lump-sum tax on i-type agents in region j. We have

$$n_L t_L^A + n_H t_H^A = G^A \tag{A.21}$$

$$t_i^B \geq t_i^A - \Delta h, \quad \forall i \in \{L, H\} \tag{A.22}$$

where $\Delta h = h(G^A) - h(G^B)$. A tax schedule $(\hat{t}_L^B, \hat{t}_H^B)$ designed to attract both types from A (i.e. $\hat{t}_i^B < t_i^A - \Delta h, \forall i$) is a profitable deviation for region B if and only if $n_L \hat{t}_L^B + n_H \hat{t}_H^B \geq G^B$. Such a profitable deviation exists for B if and only if

$$n_L(t_L^A - \Delta h) + n_H(t_L^A - \Delta h) > G^B,$$

that is, $G^A > G^B + (n_L + n_H)\Delta h$, since the budget constraint is binding in region A. Therefore, there is no profitable deviation that ends up with a pooling distribution of agents in B if :

$$G^A \leq G^B + (n_L + n_H)\Delta h$$

which proves the lemma.

■

Proof of Lemma 1 . Let $((t_L^A, t_H^A), (t_L^B, t_H^B))$ be a strategy profile compatible with a pooling distribution in region A such that $n_L t_L^A + n_H t_H^A = G^A$. Lemma 0 guarantees that there is no profitable deviation that ends up with a pooling distribution of agents in B. Moreover, there is no separating profitable deviation for region B if and only if

$$\forall i \in \{L, H\}, \quad v_i^* - t_i^A + h(G^A) \geq v_i^* - \frac{G^B}{n_i} + h(G^B). \tag{A.23}$$

Similarly, no separating profitable deviation exists for region A if and only if

$$\forall i \in \{L, H\}, v_i^* - t_i^A \geq v_i^* - \frac{G^A}{n_i} \tag{A.24}$$

Finally, if (A.23) and (A.24) are satisfied, by setting t_i^B to $t_i^A - \Delta h$, so that the self-selection constraints are binding, we prevent region A from every pooling profitable deviation since its budget constraint is binding. Therefore, (A.23) and (A.24) which are equivalent to

$$t_i^A \leq \min\{\frac{G^A}{n_i}, \frac{G^B}{n_i} + \Delta h\}, \quad \forall i \in \{L, H\}. \tag{A.25}$$

are necessary and sufficient conditions for (t_L^A, t_H^A) to be played in a pooling NE. ■

Proof of Proposition 5. Suppose $\frac{G_H^*}{n_H} \leq \min\{\Delta v, h(G_H^*) - h(G_L^*)\}$ and consider the strategy profile (S^A, S^B) defined by :

S^A : Region A's strategy

— First stage

Play $G^A = G_H^*$

— Second stage

— if $(G^A, G^B) = (G_H^*, G_L^*)$ then $t_H^A = \frac{G_H^*}{n_H}$ and $t_L^A \geq \frac{G_L^*}{n_L} + h(G_H^*) - h(G_L^*)$

— if $(G^A, G^B) \neq (G_H^*, G_L^*)$ and $G^A \leq G^B + (n_L + n_H)\Delta h$ then
$t_H^A = \bar{t}_H(G^A, G^B)$ and t_L^A is defined by $n_L t_L^A + n_H t_H^A = G^A$

— if $(G^A, G^B) \neq (G_H^*, G_L^*)$ and $G^A > G^B + (n_L + n_H)\Delta h$ then
$t_L^A = \bar{t}_L(G^A, G^B)$ and t_H^A is defined by $n_L t_L^A + n_H t_H^A = G^B - (n_L + n_H)\Delta h$

S^B : Region B's strategy

— First stage

Play $G^B = G_L^*$

— Second stage

— if $(G^A, G^B) = (G_H^*, G_L^*)$ then $t_L^B = \frac{G_L^*}{n_L}$ and $t_H^B \geq \frac{G_H^*}{n_L} - (h(G_H^*) - h(G_L^*))$

— if $(G^A, G^B) \neq (G_H^*, G_L^*)$ and $G^A \leq G^B + (n_L + n_H)\Delta h$ then
$t_H^B = \bar{t}_H(G^B, G^A)$ and t_L^B is defined by $n_L t_L^B + n_H t_H^B = G^A + (n_L + n_H)\Delta h$

— if $(G^A, G^B) \neq (G^*_H, G^*_L)$ and $G^A > G^B + (n_L + n_H)\Delta h$ then
$t^B_L = \bar{t}_L(G^B, G^A)$ and t^B_H is defined by $n_L t^B_L + n_H t^B_H = G^B$

First, we show that for all $(G^A, G^B) \in \mathbb{R}^2_+$ the restriction of (S^A, S^B) to the subgame defined by (G^A, G^B) is an equilibrium.

If $(G^A, G^B) = (G^*_H, G^*_L)$, Lemma 1 implies that $\left((t^A_L, t^A_H), (t^B_L, t^B_H)\right)$ is Nash equilibrium since $t^A_H = \frac{G^*_H}{n_H}$, $t^A_L \geq \frac{G^*_L}{n_L} + h(G^*_H) - h(G^*_L)$, $t^B_L = \frac{G^*_L}{n_L}$ and $t^B_H \geq \frac{G^*_H}{n_L} - (h(G^*_H) - h(G^*_L))$ with

$$SWF^A = v^*_H + h(G^*_H) - \frac{G^*_H}{n_H} \text{ and } SWF^B = v^*_L + h(G^*_L) - \frac{G^*_L}{n_L}$$

If $(G^A, G^B) \neq (G^*_H, G^*_L)$ and $G^A \leq G^B + (n_L + n_H)\Delta h$ then $n_L t^A_L + n_H t^A_H = G^A$, $t^B_i = t^A_i - \Delta h$ and $t^A_i \leq \Delta h$ for all $i \in \{L, H\}$. Lemma 1 implies that $\left((t^A_L, t^A_H), (t^B_L, t^B_H)\right)$ is a pooling Nash equilibrium with

$$SWF^A \leq v^*_H + h(G^*_H) - \frac{G^*_H}{n_H} \text{ and } SWF^B = -\infty$$

By analogy, if $(G^A, G^B) \neq (G^*_H, G^*_L)$ and $G^A > G^B + (n_L + n_H)\Delta h$ then $\left((t^A_L, t^A_H), (t^B_L, t^B_H)\right)$ is a pooling Nash equilibrium with

$$SWF^A = -\infty \text{ and } SWF^B \leq v^*_L + h(G^*_L) - \frac{G^*_L}{n_L}$$

Finally, we remark through the values of the Social Welfare Functions in the different subgames, that no deviation from (G^*_H, G^*_L) is profitable for either region. ■

Proof for Proposition 6. $\Rightarrow$

Suppose that the pair of taxes $\tilde{t}_L$ are $\tilde{t}_H$ are paid, respectively by the L-type and the H-type, in a perfect subgame pooling equilibrium in j. First, we know from Lemma 1 that negative taxes are not paid in pooling equilibria, so $\tilde{t}_L \geq 0$ and $\tilde{t}_H \geq 0$. Now, notice that G^*_{LH} is played by government j in every subgame perfect pooling equilibrium in j. Indeed, if $((G^A, \tau^A(\cdot,\cdot)), (G^B, \tau^B(\cdot,\cdot))$ is a perfect subgame pooling equilibrium in j, then the payoffs for the two regions satisfy $SWF^j > 0$ and $SWJ^{-j} = -\infty$. If $G^j \neq G^*_{LH}$, then government $-j$ could deviate G^{-j} to G^*_{LH} in the first stage. Since $(n_L + n_H)h(G^*_{LH}) - G^*_{LH} > (n_L + n_H)h(G^j) - G^j$ no pooling equilibrium could occur in j in the second stage. Therefore, the payoffs of region $-j$ would be greater than $-\infty$ whatever the equilibrium is in the second stage. That would then be a profitable deviation for B. Therefore, $n_L\widetilde{t}_L + n_H\widetilde{t}_H = G^*_{LH}$ because the budget constraint must be binding according to Remark 2.

$\Leftarrow$

Now, consider a set of positive tax $\widetilde{t}_L$ are $\widetilde{t}_H$ that satisfied : $n_L\widetilde{t}_L + n_H\widetilde{t}_H = G^*_{LH}$. We construct a a perfect subgame pooling equilibrium where $(\widetilde{t}_L, \widetilde{t}_H)$ are played by A on the equilibrium path. Consider the strategy profile $(\widetilde{S}^A, \widetilde{S}^B)$ that has the two following properties :

— G^*_{LH} is played by both governments in the first stage.

— In the second stage,

  — If $(n_L + n_H)h(G^A) \geq (n_L + n_H)h(G^B) - G^B$, then a pooling equilibrium in region A is played. In particular, if $G^A = G^B = G^*_{LH}$, then $(\widetilde{t}_L, \widetilde{t}_H)$ is played by both A and B which is compatible with a pooling equilibrium in A (see Lemma 1).

  — If $(n_L + n_H)h(G^A) < (n_L + n_H)h(G^B) - G^B$, then a pooling equilibrium in region B is played.

$(\widetilde{S}^A, \widetilde{S}^B)$ is a subgame perfect equilibrium because (i) it induces an equilibrium in every subgame and, (ii) in the first stage, any deviation by A is sanctioned by a pooling equilibrium in B and any deviation from B leads to a pooling equilibrium in A. ■

Proof of Lemma 2. Let $P = ((u^A_L, u^A_H, y^A_L, y^A_H), (u^B_L, u^B_H, y^B_L, y^B_H))$ be a strategy profile compatible with a pooling distribution of agents such that $u^A_L = u_L(u^A_H, G^A)$ and $u^A_H \geq u^M_H(G^A)$ and (y^A_L, y^A_H) is determined by solving problem (4.2) for $u_H = u^A_H$. Lemma 0 guarantees that there is no profitable deviation that ends up with a pooling distribution of agents in B. Moreover, no separating profitable deviation exists for B if and only if :

$$u^A_i + h(G^A) \geq v^*_i - \frac{G^B}{n_i} + h(G^B), \quad \forall i \in \{L, H\}. \tag{A.26}$$

In fact, $v^*_i - \frac{G^B}{n_i} + h(G^B)$ is the greatest utility level that region B could grant to the i-type agent in a separating profitable deviation. And, if equation (A.26) is violated for i, then $(v^*_L - \frac{G^B}{n_i}.v^*_H - \frac{G^B}{n_i}, y^*_L, y^*_H)$ would be a profitable deviation for B.

Similarly, no Separation Profitable Deviation exists for region A, for the L type if and only if

$$u_L^A \geq v_L^* - \frac{G^A}{n_L} \tag{A.27}$$

In fact, $v_L^* - \frac{G^A}{n_L}$ is the highest utility that can be achieved by the L-type in region A for any separating distribution, and if equation (A.27) is violated then $(v_L^* - \frac{G^A}{n_L}, v_H^* - \frac{G^A}{n_L}, y_L^*, y_H^*)$ would be a profitable deviation for A.

Finally, If u_L^B is small enough (i.e., $u_L^B - \Delta h \leq \min\{-\frac{G^A}{n_H}, y_H^* - \frac{G_H^A}{n_H} - v(\frac{y_H^*}{w_L})\}$) then no profitable separating deviation in A for the H type is possible. Therefore, if (A.26) and (A.27) are satisfied, given that $u_L(u_H^A, G^A) = u_L^A$ and $u_H^A \geq u_H^M(G^A)$, and by setting u_L^B small enough and u_H^B set to $u_H^A + \Delta h$, we could construct a feasible pooling NE in A in which $(u_L^A, u_H^A, y_L^A, y_H^A)$ is played. Therefore, (A.26) and (A.27) are necessary and sufficient condition for $(u_L^A, u_H^A, y_L^A, y_H^A)$ to be played in a feasible pooling NE. ■

Proof for Proposition 8. $\Rightarrow$ First, suppose the pair of utilities $\widetilde{U}_L$ and $\widetilde{U}_H$ are earned by the L-type and the H-type respectively, in a subgame perfect pooling equilibrium $P = ((G^A, \tau^A(\cdot,\cdot)), (G^B, \tau^B(\cdot,\cdot))$. Now suppose P induces a pooling distribution in j and then we show that G_{LH}^* is played by government j. Indeed, under P, the payoffs for the two regions satisfy $SWF^j = \widetilde{U}_L$ and $SWJ^{-j} = -\infty$. If $G^j \neq G_{LH}^*$, then government $-j$ could deviate from G^{-j} to G_{LH}^* in the first stage. Since $(n_L + n_H)h(G_{LH}^*) - G_{LH}^* > (n_L + n_H)h(G^j) - G^j$ no pooling equilibrium could occur in j in the second stage. Therefore, the payoffs of region $-j$ would be greater than $-\infty$ whatever the equilibrium is in the second stage. That would then be a profitable deviation for $-j$. So, $G^j = G_{LH}^*$.

Therefore, according, to Lemma 2, Equations (5.6) and (5.8) are satisfied and there exists $G^B \in [0, \overline{G}]$ such that

$$\widetilde{U}_L \geq v_L^* - \min\left\{\frac{G_{LH}^*}{n_L} - h(G_{LH}^*), \frac{G^B}{n_L} - h(G^B)\right\}, \tag{A.28}$$

which implies Equation (5.7).

$\Leftarrow$ Second, let $\widetilde{U}_L$ and $\widetilde{U}_H$ be a pair of utilities that satisfy (5.6), (5.7) and (5.8). We construct a subgame perfect pooling equilibrium where $\widetilde{U}_L$ and $\widetilde{U}_H$ are earned, respectively, by the L-type and the H-type in A on the equilibrium path. Define $\widetilde{G}^B$ by the smallest number that satisfies $\widetilde{U}_L = v_L^* - \dfrac{\widetilde{G}^B}{n_L} + h(\widetilde{G}^B)$. Note that $\widetilde{G}^B$. Now, consider the strategy profile $(\widetilde{S}^A, \widetilde{S}^B)$ that has the two following properties.

(1) G^*_{LH} and $\widetilde{G}^B$ are played by governments A and B, respectively, in the first stage.

(2) In the second stage,

— If $(n_L + n_H)h(G^A) \geq (n_L + n_H)h(G^B) - G^B$, then the worst pooling equilibrium in region A is played and the utility of the L-type would then be $U_L^A(G^A, G^B) = v_L^* - \min\left\{\frac{G^A}{n_L} - h(G^A), \frac{G^B}{n_L} - h(G^B)\right\}$. In particular, if $G^A = G^*_{LH}$ and $G^B = \widetilde{G}^B$, then the pooling equilibrium that provides $\widetilde{U}_L$ and $\widetilde{U}_H$, respectively, to the Ls and the Hs is played. This is possible because Equations (5.6), (5.7) and (5.8) are satisfied. (see Lemma 1).

— If $(n_L + n_H)h(G^A) < (n_L + n_H)h(G^B) - G^B$, then a pooling equilibrium in region B is played.

If $\widehat{G}^A$ is deviation from government A, then two cases can be distinguished. First, because $\widetilde{G}^B < G^*_{LH}$, if $\widehat{G}^A < \widetilde{G}^B$, then $(n_L + n_H)h(\widehat{G}^A) - \widehat{G}^A < (n_L + n_H)h(\widetilde{G}^B) - \widetilde{G}^B$ and a pooling equilibrium in B would be played which would not be profitable for A. Second, if $\widehat{G}^A \geq \widetilde{G}^B$ and $(n_L + n_H)h(\widehat{G}^A) - \widehat{G}^A \geq (n_L + n_H)h(\widetilde{G}^B) - \widetilde{G}^B$, then the worst pooling equilibrium in A is played and that would not be profitable either because

$$\begin{aligned} U_L^A(\widehat{G}^A, \widetilde{G}^B) &= v_L^* - \min\left\{\frac{\widehat{G}^A}{n_L} - h(\widehat{G}^A), \frac{\widetilde{G}^B}{n_L} - h(\widetilde{G}^B)\right\} \\ &= v_L^* - \frac{\widetilde{G}^B}{n_L} + h(\widetilde{G}^B) \\ &= \widetilde{U}_L. \end{aligned} \tag{A.29}$$

Finally, if $\widehat{G}^A \geq \widetilde{G}^B$ and $(n_L + n_H)h(\widehat{G}^A) - \widehat{G}^A < (n_L + n_H)h(\widetilde{G}^B) - \widetilde{G}^B$, then a pooling equilibrium in B would be played which would not be profitable for A. $(\widetilde{S}^A, \widetilde{S}^B)$ is subgame perfect equilibrium because (i) it induces an equilibrium in every subgames and, (ii) in the first stage, any deviation from B leads to a pooling equilibrium in A and any deviation from A is not profitable either. ■

Proofs from Article 2

We show how our results can be extended to a broader class of demand functions and to the case where CBI are produced with a uniform constant marginal c. Redefine the demand to the j-*th* i-type country by a mapping $q_i^j : \mathbb{R}_+^n \to \mathbb{R}_+$ and define by A the set of price distribution for which every country has a positive demand : $A = \{\tilde{p} \ / \ q_i^j(\tilde{p}) > 0, \text{for all country}\}$. Countries of the same type have symmetric demand, that is, for every couple of countries j, j', we have $q_i^j(\tilde{p}) = q_i^{j'}(\sigma_{j,j'}(\tilde{p}))$ where $\sigma_{j,j'}(\tilde{p})$ is the n-dimensional vector obtained from $\tilde{p}$ by permuting the price of countries j and j'. We make the following standard assumptions (see Friedman (1977), Friedman (1983), Deneckere and Davidson (1985)).

Assumption 3. *A is bounded and convex. q_i^j is twice continuously differentiable on A with $\partial q_i^j / \partial p_i^j < 0$ and $\partial q_i^j / \partial p_{i'}^{j'} > 0$ where $\{i, j\} \neq \{i', j'\}$*

Assumption 3 ensures that the demand functions are downward slopping and that CBI programs are gross substitutes. Our analysis can be restricted without loss of generality to the closure of A, $\bar{A}$ which is convex and compact.

Assumption 4. *For some $\varepsilon > 0$, for all i, j, and all $S \subset \Big(\{L\} \times \{1, 2, \ldots, n_L\}\Big) \cup \Big(\{H\} \times \{1, 2, \ldots, n_H\}\Big)$ subset of the countries such that $(i, j) \in S$.*

$$\sum_{(i',j') \in S} \frac{\partial q_i^j(\tilde{p})}{\partial p_{i'}^{j'}} < -\varepsilon, \quad \forall \tilde{p} \in A.$$

Assumption 4 states, in particular, that the total effect of an unitary increase in all prices on demand is negative for all country. Under this assumption, there exists a price vector $p^{max} = (p_L^{1,max}, p_L^{2,max}, ..., p_L^{n_L,max}, p_H^{1,max}, p_H^{2,max}, ..., p_H^{n_H,max}) \in \mathbb{R}_+^n$ such that for all $\tilde{p} = (p_L^1, p_L^2, ..., p_L^{n_L}, p_H^1, p_H^2, ..., p_H^{n_H}) \in \mathbb{R}_+^n$, if $p_i^j \geq p_i^{j,max}$, then the demand facing at least one country is non-positive. Moreover, if each country charge a price p_i^j such that $p_i^j \geq p_i^{j,max}$, except one country, this one country will face a negative demand (see Friedman (1977), page 55). The strategy set for the j-*th* i-type country can, therefore, be restricted to $[0, p_i^{j,max}]$. Now, define the revenue function of the j-*th* i-type country by $R_i^j(\tilde{p}) = (p_i^j - c) q_i^j(\tilde{p})$ and let A^* be the subset of A where every country gets a non-negative revenu, that is, $A^* = \{\tilde{p} \in A \ / \ p_i^j \geq c, \text{for all country}\}$. A^* is the only relevant set of strategies that need to be considered.

Assumption 5. *The revenue function is quasi-concave on* A^* *and* $\inf\{p_i^j \;/\; q_i^j(p_i^j, cI_{n-1}) > 0\} > c$ *where* I_{n-1} *is the* $(n-1)$*-dimensional vector with all components equal to 1.*

It is worth noting that the quasi-concavity of the revenue functions is required for the convexity of the the best response functions and, therefore, the convexity of the set of all Pareto-improvement of the Nash Equilibrium.

Assumption 6. *For all* $\widetilde{p}$ *in the interior of* A^*

$$\frac{\partial^2 R_i^j(\widetilde{p})}{\partial {p_i^j}^2} < \sum_{\{i,j\}\neq\{i',j'\}} \left| \frac{\partial^2 R_i^j(\widetilde{p})}{\partial p_i^j \partial p_{i'}^{j'}} \right|$$

Assumption 3, 5 and 6 guaranty that the best response functions are contractions (Friedman, 1977) and then the contraction mapping theorem ensures the existence of a unique Nash equilibrium for which quantities and profits are positive. We know that the Nash Equilibrium is inefficient in this framework and its uniqueness implies that same type countries necessarily charge the same price p_i^N and get the same quantity q_i^N. More generally, for any price level p_i uniformly charged by the i-type countries there is a unique price $P_{-i}^0(p_i)$ such that if charged by every $-i$-type country, then none of them would benefit from deviating from it.

We denote by $q_i(p_L, p_H)$ the demand to each individual i-type country, if L-type countries uniformly charge p_L and H-type countries uniformly charge p_H, and by $R_i(p_L, p_H)$ the corresponding revenue. Using the same argument as in Page 66, it is easy to see that if (P_L^E, P_H^E) is a Pareto efficient allocation under which same type countries charge the same price then, $P_i^E > P_i^0(P_{-i}^E)$. Lemma 1 follows immediately through the same proof. However, the following assumption is needed for Lemma 2.

Assumption 7. *For all* $(p_L, p_H) \in \mathbb{R}_+^2$, $q_H(p_L, p_H) - q_L(p_L, p_H) = f(p_H - p_L)$ *where* $f : \mathbb{R} \to \mathbb{R}$ *is a decreasing function with* $f(0) > 0$ *and* $f(\infty) < 0$.

In particular, Assumption 7 states that if all prices are equal, then H-type countries face a higher demand than L-type countries do and this difference in quantities decrease as the difference in prices becomes higher. Assumption 7 also requires that the difference in quantities, $q_H - q_L$, becomes negative if the difference in prices, $p_H - p_L$ is too high. Now, we can provide a general proof for Lemma 2.

Proof for Lemma 2. First, from Assumption 7 we have

$$q_H(p,p) - q_L(p,p) = f(0), \qquad \forall p. \tag{A.1}$$

Taking the derivative of (A.1) with respect to p after multiplying it by $(p-c)$ gives

$$\frac{\partial R_H(p,p)}{\partial dp} - \frac{\partial R_L(p,p)}{\partial dp} = f(0) > 0, \qquad \forall p \tag{A.2}$$

Now, note that following Assumption 7 we have $p_L(p) = p - f^{-1}(0)$, meaning that

$$q_H(p - f^{-1}(0), p) = q_L(p - f^{-1}(0), p) \qquad \forall p \tag{A.3}$$

Taking the derivative of (A.3) with respect to p after multiplying it by $(p - f^-1(0) - c)$ and using Assumption 4 gives for all p

$$\begin{aligned}\frac{\partial R_H(p - f^{-1}(0), p)}{\partial dp} - \frac{\partial R_L(p - f^{-1}(0), p)}{\partial dp} &= f^{-1}(0)[\frac{\partial q_L(p - f^{-1}(0), p)}{\partial p_L} + \frac{\partial q_L(p - f^{-1}(0), p)}{\partial p_H}] \\ &< -\varepsilon f^{-1}(0) \\ &< 0\end{aligned} \tag{A.4}$$

Equations (A.2) and (A.4) prove the Lemma. ■

Another consequence of Assumption 7 is that for all $(p_L, p_H) \in \mathbb{R}^2_+$, $q_H(p_L, p_H) - q_L(p_L, p_H) > 0$ and $p_H - p_L > 0$ if and only if $0 < p_H - p_L < f^{-1}(0)$. Considering Lemma 1, we immediately deduce that an efficient price distribution (p^E_H, p^E_L) is achievable by a Cooperative Nash Equilibrium if and only if

$$0 < p^E_H - p^E_L < f^{-1}(0).$$

Assumption 3 and 5 ensure that the set of the price distributions (p_L, p_H) that Pareto improve the Nash Equilibrium,$\{(p_L, p_H) \in \mathbb{R}^2_+ / R_L(p_L, p_H) \geq R_L(p^N_L, p^N_H)$ and $R_H(p_L, p_H) \geq R_H(p^N_L, p^N_H)\}$, is convex and compact which implies that iso-revenue curves must intersect twice. Therefore, there exists a unique price distribution $(\widehat{p}_L, \widehat{p}_H)$ such that $R_L(p^N_L, p^N_H) = R_L(\widehat{p}_L, \widehat{p}_H)$ and $R_H(p^N_L, p^N_H) = R_H(\widehat{p}_L, \widehat{p}_H)$

while $MRS_L(\widehat{p}_L, \widehat{p}_H) > MRS_H(\widehat{p}_L, \widehat{p}_H)$. So, an efficient convex combination of $(\widehat{p}_L^E, \widehat{p}_H^E)$ exists and is charaterized by $MRS_L(\widehat{p}_L^E, \widehat{p}_H^E) > MRS_H(\widehat{p}_L^E, \widehat{p}_H^E)$ and $(\widehat{p}_L^E, \widehat{p}_H^E) = \alpha(\widehat{p}_L, \widehat{p}_H) + (1-\alpha)(\widehat{p}_L^N, \widehat{p}_H^N)$ where $\alpha \in [0,1]$. We need a final special Assumption before we can get into a proof for Proposition 2.

Assumption 8. *At the Non-Cooperative Nash Equilibrium, H-type countries charge a higher price and get a higher demand than the L-type countries*[1] *:*

$$q_H^N > q_L^N \text{ and } p_H^N > p_L^N$$

Proof for Proposition 2. One only needs to show that $(\widehat{p}_L^E, \widehat{p}_H^E)$ is achievable by a Cooperative Nash Equilibrium. Using Assumption 8 and considering Lemma 1 it would be sufficient to show that

$$0 < \widehat{p}_H - \widehat{p}_L < f^{-1}(0).$$

First, note that $\widehat{p}_H > P_H^0(\widehat{p}_L)$ and $\widehat{p}_L > P_L^0(\widehat{p}_H)$ because indifference curves intersect with the corresponding best response line only once.

Suppose, by contradiction, that $\widehat{p}_H < \widehat{p}_L$. Then

$$R_L(\widehat{p}_L, \widehat{p}_L) > R_L(p_L^N, p_H^N) \text{ and } R_H(\widehat{p}_L, \widehat{p}_L) < R_H(p_L^N, p_H^N).$$

That is impossible because $R_L(p_H^N, p_H^N) < R_L(p_L^N, p_H^N)$ and $R_H(p_H^N, p_H^N) > R_H(p_L^N, p_H^N)$ and $R_H(p,p)$ grows faster than $R_L(p,p)$ according to Lemma 2. Therefore $0 \leq \widehat{p}_H - \widehat{p}_L$.

Now, suppose, by contradiction, that $\widehat{p}_H > \widehat{p}_L + f^{-1}(0)$. Therefore,

$$\begin{cases} R_H(\widehat{p}_H - f^{-1}(0), \widehat{p}_H > R_L(p_L^N, p_H^N) \\ R_H(\widehat{p}_H - f^{-1}(0), \widehat{p}_H) < R_H(p_L^N, p_H^N) \end{cases},$$

Morevover, it follows from Assumption 8 that $p_H^N < p_L^N + f^{-1}(0)$ since $q_H^N > q_L^N$. So,

$$R_L(p_L^N, p_L^N + f^{-1}(0)) > R_L(p_L^N, p_H^N) \text{ and } R_H(p_L^N, p_L^N + f^{-1}(0)) < R_H(p_L^N, p_L^H),$$

1. A sufficient condition for Assumption 8 is

$$\frac{\partial q_i^j(\widetilde{p}^N)}{\partial p_i^j} = \frac{\partial q_{i'}^{j'}(\widetilde{p}^N)}{\partial p_{i'}^{j'}}, \tag{A.5}$$

where $\widetilde{p}^N$ is the Non-Cooperative Nash Equilibrium price distribution.

which is impossible because $\dfrac{\partial R_L(p - f^{-1}(0), p)}{\partial p} > \dfrac{\partial R_H(p - f^{-1}(0), p)}{\partial p}$ (see Lemma 2). So, $\widehat{p}_H - \widehat{p}_L \leq f^{-1}(0)$. ■

Proofs from Article 3

Proof for Lemma 1. By definition of μ', there exists a group of refugees R and a home $\widehat{H}$ such that, for all $r \in \mathcal{R}$,

$$\mu'(r) = \begin{cases} r & \text{if } r \in R \\ \widehat{H} & \text{if } r = \widehat{r} \\ \mu(r) & \text{otherwise.} \end{cases}$$

where

— $\widehat{r} \in \mathcal{R}$ is a refugee such that $\widehat{r}$ forms a blocking pair with some home H and $\mu(\widehat{r}) = \widehat{r}$,

— $\widehat{H}$ is the favorite home of $\widehat{r}$ among all homes H such that μ is blocked by $(H, \widehat{r})$.

— $R \subset \mu(\widehat{H})$ such that $q_{\widehat{H}} \geq l(\widehat{r}) + l(\mu(\widehat{H})) - l(R)$ and $\Big(\{\widehat{r}\} \cup (\mu(\widehat{H}) \setminus R)\Big) \succ_{\widehat{H}} \mu(\widehat{H})$.

Clearly, μ' is individually rational. We shall show that μ' is «fair», that is, μ' is not blocked by a pair (r, H) such that $\mu'(r) \in \mathcal{H}$. Let $r \in \mathcal{R}$ be a refugee such that $\mu'(r) \in \mathcal{H}$. We show that there is no $H \in \mathcal{H}$ that forms a blocking pair with r for μ'. Suppose, by contradiction, that there is a home H such that

$$\Big(\{r\} \cup (\mu'(H) \setminus R_H)\Big) \succ_H \mu'(H) \text{ and } H \succ_r \mu'(r) \tag{A.1}$$

for some subset $R_H \subset \mu'(H)$.

First, suppose $r = \widehat{r}$. Therefore, $\mu'(r) = \widehat{H}$ which implies $H \neq \widehat{H}$ and then, by definition of μ', $\mu'(H) = \mu(H)$. It follows, from (A.1), that $\Big(\{r\} \cup (\mu(H) \setminus R_H)\Big) \succ_H \mu(H)$ and $H \succ_r \widehat{H}$ which contradicts the definition of $\widehat{H}$.

Now, suppose $r \neq \widehat{r}$. If $\mu'(r) = \widehat{H}$ then $\mu(r) = \widehat{H}$ and it follows, from (A.1), that $\Big(\{r\} \cup (\mu(H) \setminus R_H)\Big) \succ_H \mu(H)$ and $H \succ_r \mu(r)$ which contradicts the fairness of μ. Therefore, there must be some home $H' \neq \widehat{H}$ such that $H' = \mu'(r) = \mu(r)$.

If $H \neq \widehat{H}$ then $\mu'(H) = \mu(H)$ and it follows from (A.1) that $\Big(\{r\} \cup (\mu(H) \setminus R_H)\Big) \succ_H \mu(H)$ and $H \succ_r \mu(r)$ which, again, contradicts the fairness of μ. Therefore $H = \widehat{H}$, that is, μ' is blocked by $(\widehat{H}, r)$ then it follows from equation (1) that $\{r\} \cup \Big(\mu'(\widehat{H}) \setminus R_{\widehat{H}}\Big) \succ_{\widehat{H}} \mu'(\widehat{H})$ and $\widehat{H} \succ_r \mu(r)$.

Furthermore, since $\mu'(\widehat{H}) = \{\hat{r}\} \cup (\mu(\widehat{H}) \setminus H)$ by definition of μ', we obtain

$$\{x\} \cup \bigg(\Big(\{\hat{r}\} \cup (\mu(\widehat{H}) \setminus R)\Big) \setminus R_{\widehat{H}}\bigg) \succ_{\widehat{H}} \mu'(\widehat{H}) \succ_{\widehat{H}} \mu(\widehat{H}). \tag{A.2}$$

If, in addition, $\hat{r} \in R_{\widehat{H}}$ then $\bigg(\Big(\{\hat{r}\} \cup (\mu(\widehat{H}) \setminus R)\Big) \setminus R_{\widehat{H}}\bigg) = \mu(\widehat{H}) \setminus (R \cup R_{\widehat{H}})$ and then, it follows from (A.2) that $\{r\} \cup \Big(\mu(\widehat{H}) \setminus (R \cup R_{\widehat{H}})\Big) \succ_{\widehat{H}} \mu(\widehat{H})$ which contradicts the fairness of μ since $\widehat{H} \succ_r \mu(r)$. Therefore, $\hat{r} \notin R_{\widehat{H}}$ and then $R_{\widehat{H}} \in \mu(\widehat{H})$.

Moreover, by responsiveness of $\succ_{\widehat{H}}$, we know from (A.1) that $\{r\} \succ_{\widehat{H}} R_{\widehat{H}}$. Therefore, since $(\mu(\widehat{H}) \setminus R_{\widehat{H}}) \cup \{r\} \succ_{\widehat{H}} (\mu(\widehat{H}) \setminus R_{\widehat{H}}) \cup R_{\widehat{H}}$ would violate the fairness of μ, the set $(\mu(\widehat{H}) \setminus R_{\widehat{H}}) \cup \{r\}$ must be unacceptable for $\widehat{H}$, and by Assumption 1, that is :

$$q_{\widehat{H}} < l(\mu(\widehat{H})) - l(R_{\widehat{H}}) + l(r). \tag{A.3}$$

But we know from (A.1) that the set $\{r\} \cup \Big(\mu'(\widehat{H}) \setminus R_{\widehat{H}}\Big)$ is acceptable for $\widehat{H}$, and by Assumption 1, that is : $l(\mu'(\widehat{H})) - l(R_{\widehat{H}}) + l(r) \leq q_{\widehat{H}}$ and then

$$l(\mu(\widehat{H})) - l(R) + l(\hat{r}) - l(R_{\widehat{H}}) + l(r) \leq q_{\widehat{H}} \tag{A.4}$$

since $l(\mu'(\widehat{H})) = l(\mu(\widehat{H})) - l(R) + l(\hat{r})$. Therefore, from (A.3) and (A.4) we get $l(\hat{r}) < l(R)$. It follows by size-monotonicity of the priorities for $\widehat{H}$ that $R \succ_{\widehat{H}} r$ which, by responsiveness, contradicts, Equation (A.1).

Therefore, there is no blocking pair (H, r) for μ' with $\mu'(r) \in \mathcal{H}$, that is, μ' is fair.

■